Transportation
Library
TA
1
.A526

TRANSACTIONS

OF THE

AMERICAN SOCIETY

OF

CIVIL ENGINEERS

(INSTITUTED 1852)

VOLUME 105
1940

Edited by the Secretary, under the direction of the Committee on Publications.

Reprints from this publication, which is copyrighted, may be made on condition that the full title of Paper, name of Author, and page reference are given.

NEW YORK
PUBLISHED BY THE SOCIETY
1940

TRANSPORTATION LIBRARY

Copyright 1940, by the AMERICAN SOCIETY OF CIVIL ENGINEERS.

NOTE.—This Society is not responsible for any statement made or opinion expressed in its publications.

CONTENTS

PAPERS

No. PAGE

2056 **WIND FORCES ON A TALL BUILDING.**

By **J. Charles Rathbun** 1

Discussion:

By David Cushman Coyle 42
Clyde T. Morris 44
Robins Fleming 44
F. P. Shearwood 49
Lydik S. Jacobsen 50
Francis L. Castleman, Jr. 52
J. B. Wilbur 55
R. D. Spellman 57
David A. Molitor 58
Walter J. Gray 60
K. L. DeBlois 60
Albert Smith 61
Victor R. Bergman 64
Otto Gottschalk 65
Colin Skinner 65
J. Charles Rathbun 77

2057 **ANALYSIS OF RUN-OFF CHARACTERISTICS.**

By **Otto H. Meyer** 83

Discussion:

By Victor H. Cochrane 101
Bertram S. Barnes 104
LeRoy K. Sherman 106
Richmond T. Zoch 109
Merrill Bernard 117
Franklin F. Snyder 122
W. G. Hoyt 127
C. O. Clark 128
C. S. Jarvis 131
Howard M. Turner 134
Otto H. Meyer 138

2058 **SPECIFICATION AND DESIGN OF STEEL GUSSET-PLATES.**

By **T. H. Rust** 142

Discussion:

By R. H. Sherlock 159
L. E. Grinter 159
Russell C. Brinker 161
C. W. Wixom 162
T. H. Rust 163

2059 **GREAT LAKES TRANSPORTATION.**

By **M. C. Tyler** 167

Discussion:

By T. Kennard Thomson 186
Harland C. Woods 188
W. L. R. Haines 188
L. C. Sabin 193

No. PAGE

2060 **TESTS ON BUILT-UP COLUMNS OF STRUCTURAL ALUMINUM ALLOYS.**
By Marshall Holt 196
Discussion:
By Alfred S. Niles 218
Marshall Holt 219

2061 **TRANSPORTATION DEVELOPMENTS IN THE UNITED STATES.**
By Fred Lavis 220
Discussion:
By Milton Harris 237
Ralph Budd 239
L. Alfred Jenny 243
William J. Cunningham 256
S. R. Truesdell 257
David A. Molitor 259
C. A. Hoglund 263
Fred Lavis 266

2062 **EARTHQUAKES AND STRUCTURES.**
By Leander M. Hoskins and John D. Galloway 269
Discussion:
By Homer M. Hadley 298
R. McC. Beanfield 300
R. S. Chew 303
Jacob J. Creskoff 305
Arthur C. Ruge 307
Walter L. Huber 309
A. A. Eremin 314
Henry D. Dewell 315
John D. Galloway and Lydik S. Jacobsen 319

2063 **TRAFFIC PROBLEMS IN METROPOLITAN AREAS.**
By Earl J. Reeder 323
Discussion:
By George H. Herrold 331
Thomas Buckley 333
D. Grant Mickle 336
Robert A. Mitchell 339
Cleveland B. Coe 342
Earl J. Reeder 343

2064 **THE YELLOW RIVER PROBLEM.**
By O. J. Todd and S. Eliassen 346
Discussion:
By J. W. Beardsley 417
Elliott J. Dent 418
Herbert Chatley 425
H. van der Veen 427
C. S. Jarvis 433
E. W. Lane 434
George Higgins 441
O. J. Todd and S. Eliassen 443

2065 **THE RISK OF THE UNEXPECTED IN SUB-SURFACE CONSTRUCTION CONTRACTS.**
By Oren Clive Herwitz 454
Discussion:
By Alonzo J. Hammond 476
Frederick W. Newton 477

No. Page

David A. Molitor 488
F. B. Marsh 489
Evan S. Martin 490
C. Maxwell Stanley 490
Francis J. Morgan and Frederick C. Zeigler 492
Lazarus White 495
T. Kennard Thomson 497
Carroll A. Farwell 499
Oren Clive Herwitz 501

2066 **DESIGN OF CIRCULAR CONCRETE TANKS.**

By **George S. Salter** 504

Discussion:

By Carl A. Rosengarten 514
I. K. Silverman 515
Frank J. McCormick 517
H. B. Muckleston 518
Robert B. B. Moorman 519
Dana Young 521
L. J. Mensch 524
Jens Egede Nielsen 526
Lloyd S. Dysland 527
Basil Sourochnikoff 528
Regis F. Fey 529
George S. Salter 530

2067 **STATE-WIDE SURVEYING PRACTICE IN MASSACHUSETTS: A SYMPOSIUM** 533

ESTABLISHING A SYSTEM OF RECTANGULAR CO-ORDINATES.

By **Elmer C. Houdlette** 534

THE LAND COURT AND ITS ENGINEERING PROCEDURE.

By **Clarence B. Humphrey** 554

Discussion:

By H. J. Shea 565
Philip Kissam 568
Lloyd G. Frost 569
William Bowie 574
R. M. Wilson 578
Elliott B. Roberts 579

2068 **ENGINEERING GEOLOGY PROBLEMS AT CONCHAS DAM, NEW MEXICO.**

By **Irving B. Crosby** 581

Discussion:

By H. L. Johnson 600
Irving B. Crosby 604

2069 **LATERAL SPILLWAY CHANNELS.**

By **Thomas R. Camp** 606

Discussion:

By W. E. Howland 618
Lewis V. Carpenter, John K. Vennard, and Fenner H. Whitley 621
F. Knapp 625
Carl Rohwer 626
Harold Allen Thomas, Jr. 627
Thomas R. Camp 633

No. PAGE

2070 THEORY OF LIMIT DESIGN.

By **J. A. Van den Broek** 638

Discussion:

By JOHN H. MEURSINGE 662
I. K. SILVERMAN 665
EDWARD GODFREY 667
BASIL SOUROCHNIKOFF 668
E. MIRABELLI 670
C. M. GOODRICH 672
GEORGE WINTER 673
FRANCIS E. SIMPSON 679
JOSEPH A. WISE 680
ALFRED FREUDENTHAL 686
HANS H. BLEICH 690
ALFRED S. NILES 693
A. FLORIS 700
L. H. NISHKIAN 702
F. G. ERIC PETERSON 706
E. S. FABIAN 710
A. A. EREMIN 713
L. H. DONNELL 713
J. A. VAN DEN BROEK 717

2071 PROPOSED IMPROVEMENTS FOR LAND SURVEYS AND TITLE TRANSFERS.

By **Philip Kissam** 731

Discussion:

By GEORGE D. WHITMORE 741
PHILIP KISSAM 742

2072 DESIGN OF AN OPEN-CHANNEL CONTROL SECTION.

By **Karl R. Kennison** 744

Discussion:

By V. L. STREETER 752
EMERY H. WILLES 753
ROBERT O. THOMAS 755
FRANK S. BAILEY 757
GEORGE E. BARNES 758
KARL R. KENNISON 763

2073 GRAPHICAL ARCH ANALYSIS APPLICABLE TO ARCH DAMS.

By **Carl H. Heilbron, Jr., and William H. Saylor** 765

Discussion:

By I. M. NELIDOV 790
CARL H. HEILBRON, JR., AND WILLIAM H. SAYLOR 792

2074 HYDROLOGY OF THE GREAT LAKES: A SYMPOSIUM 794

TYPICAL QUANTITATIVE ANALYSIS AS APPLIED TO LAKE SUPERIOR.

By **C. R. Pettis** 795

EVAPORATION EXPERIMENTS.

By **Harold C. Hickman** 807

Discussion:

By A. A. YOUNG 818
ALFRED J. COOPER, JR. 821
ADOLPH F. MEYER 825
S. T. HARDING 829

No. Page

O. E. Meinzer 834
R. W. Davenport 835
G. H. Hickox 838
C. R. Pettis 842
Harold C. Hickman 845

2075 **LARGE CORE DRILLS AID CONSTRUCTION AT CHICKAMAUGA DAM.**
By **James S. Lewis, Jr.** 850

Discussion:
By J. B. Newsom 865
J. G. Tripp 865
James B. Hays 866
O. N. Floyd 867
James S. Lewis, Jr. 868

2076 **BEACH EROSION STUDIES.**
By **Earl I. Brown** 869

Discussion:
By Morris N. Lipp 891
George A. Soper 893
James J. O'Rourke 895
Elliott J. Dent 898
Ralph F. Rhodes 901
Charles T. Leeds 905
Earl I. Brown 913

2077 **SETTLEMENT STUDIES OF STRUCTURES IN EGYPT.**
By **Gregory P. Tschebotareff** 919

Discussion:
By L. C. Wilcoxen 946
Trent R. Dames 946
Edwin J. Beugler 950
W. S. Hanna 951
D. P. Krynine 954
Gregory P. Tschebotareff 958

2078 **POLLUTION OF BOSTON HARBOR.**
By **Arthur D. Weston and Gail P. Edwards** 973

Discussion:
By E. Sherman Chase 1009
F. E. DeMartini 1010
A. M. Rawn 1012
Harrison P. Eddy, Jr., and Samuel A. Greeley 1012
George A. Soper 1016
Arthur D. Weston and Gail P. Edwards 1017

2079 **SIMPLIFIED WIND-STRESS ANALYSIS OF TALL BUILDINGS.**
By **Otto Gottschalk** 1019

Discussion:
By Samuel T. Carpenter 1029
Charles B. Winick 1030
Otto Gottschalk 1035

2080 **DESIGN OF A HIGH-HEAD SIPHON SPILLWAY.**
By **Elmer Rock** 1050

Discussion:
By J. C. Stevens 1062
B. E. Torpen, G. E. Hyde, and R. B. Cochrane 1063
I. M. Nelidov 1066
Elmer Rock 1071

No. Page

2081 **DESIGN OF DOWELS IN TRANSVERSE JOINTS OF CONCRETE PAVEMENTS.**

By **Bengt F. Friberg** 1076

Discussion:

By L. E. Grinter 1096

W. O. Fremont 1102

Bengt F. Friberg 1111

2082 **STRESS DISTRIBUTION AROUND A TUNNEL.**

By **Raymond D. Mindlin** 1117

Discussion:

By W. O. Richmond 1141

Jacob Feld 1143

J. H. A. Brahtz 1143

Raymond D. Mindlin 1152

2083 **THE UNIT HYDROGRAPH PRINCIPLE APPLIED TO SMALL WATERSHEDS.**

By **E. F. Brater** 1154

Discussion:

By Franklin F. Snyder 1179

LeRoy K. Sherman 1181

Raphael G. Kazmann 1183

Waldo E. Smith 1184

E. F. Brater 1188

2084 **TENSION TESTS OF LARGE RIVETED JOINTS.**

By **Raymond E. Davis, Glenn B. Woodruff, and Harmer E. Davis** ... 1193

Discussion:

By Charles F. Goodrich 1246

Frederick P. Shearwood 1249

Jonathan Jones 1253

C. C. Winter 1261

W. M. Wilson 1264

J. M. Garrelts 1275

A. E. Richard de Jonge 1275

E. C. Hartmann and Marshall Holt 1291

Raymond E. Davis, Glenn B. Woodruff, and Harmer E. Davis . 1295

2085 **COMBINING GEODETIC SURVEY METHODS WITH CADASTRAL SURVEYS.**

By **Carl M. Berry** 1300

Discussion:

By William L. Sawyer 1331

Philip Kissam 1331

Gordon Macleish 1333

Carl M. Berry 1341

2086 **DEVELOPMENT OF THE COLORADO RIVER IN THE UPPER BASIN.**

By **Thomas C. Adams** 1345

Discussion:

By C. C. Elder 1366

Thomas C. Adams 1370

2087 **AN IMPROVED METHOD FOR ADJUSTING LEVEL AND TRAVERSE SURVEYS.**

By **Clarence Norris and Julius L. Speert** 1376

Discussion:

By George H. Dell 1395

Howard S. Rappleye 1398

No. PAGE
R. M. Wilson 1399
Cleveland B. Coe 1401
George D. Whitmore, C. C. Miner, and W. O. Byrd 1405
Robert C. Sheldon 1407
Clarence Norris and Julius L. Speert 1408

2088 **FLASH-BOARD PINS.**

By **Chilton A. Wright and Clifford A. Betts** 1412

Discussion :
By William P. Creager 1445
Lincoln W. Ryder 1445
E. T. Schuleen 1447
John E. Field 1451
Julian H. White 1452
Harry H. Hatch 1458
Chilton A. Wright and Clifford A. Betts 1458

2089 **EFFECTIVE MOMENT OF INERTIA OF A RIVETED PLATE GIRDER.**

By **Scott B. Lilly and Samuel T. Carpenter** 1462

Discussion :
By William R. Osgood 1477
Clyde T. Morris 1477
B. R. Leffler 1478
E. Neil W. Lane 1480
Lewis E. Moore 1481
W. E. Black 1482
L. E. Grinter 1484
Charles M. Spofford 1488
E. Mirabelli 1489
Edward Godfrey 1491
Walter H. Weiskopf 1493
C. D. Williams 1494
Alvin B. Auerbach 1496
Jonathan Jones 1498
R. L. Moore 1499
J. M. Garrelts and I. E. Madsen 1501
Francis P. Witmer 1504
A. W. Fischer 1505
J. R. Shank 1506
Harold D. Hussey 1508
Charles U. Kring 1511
Thomas C. Shedd 1512
Scott B. Lilly and Samuel T. Carpenter 1515

2090 **FIELD TESTS OF A SHALE FOUNDATION.**

By **August E. Niederhoff** 1519

Discussion :
By Harry H. Hatch 1535
Jacob Feld 1537
Jack R. Rountree 1538
August E. Niederhoff 1543

2091 **RELATION OF THE STATISTICAL THEORY OF TURBULENCE TO HYDRAULICS.**

By **A. A. Kalinske** 1547

Discussion :
By Hunter Rouse 1567
Martin A. Mason 1568
J. C. Stevens 1571

No. PAGE

BORIS A. BAKHMETEFF 1572
CLYDE W. HUBBARD 1574
JOHN S. MCNOWN 1576
SAMUEL SHULITS 1578
PAUL NEMENYI 1581
BENNIE N. NETZER 1591
A. A. KALINSKE 1593

2092 **SEWAGE DISPOSAL PROJECT OF BUFFALO, NEW YORK.**
By **Samuel A. Greeley** 1601

Discussion:
By C. A. HOLMQUIST 1626
ARTHUR J. BULGER 1628
C. R. VELZY 1629
THOMAS M. NILES 1634
SAMUEL A. GREELEY 1636

2093 **FUNCTIONAL DESIGN OF FLOOD CONTROL RESERVOIRS.**
By **C. J. Posey and Fu-Te I** 1638

Discussion:
By EDWARD SOUCEK 1648
J. C. STEVENS 1649
EDWARD J. BEDNARSKI 1652
ROLAND A. KAMPMEIER 1655
EDGAR E. FOSTER 1656
SHERMAN M. WOODWARD 1658
M. KINDINGER 1658
P. WILHELM WERNER 1662
WALDO E. SMITH 1665
J. O. JONES 1669
W. C. HAMMATT 1670
C. J. POSEY AND FU-TE I 1671

2094 **RECONSTRUCTION OF THE WALPOLE-BELLOWS FALLS ARCH BRIDGE.**
By **H. E. Langley and Edward J. Ducey** 1675

Discussion:
By J. B. FRENCH 1701
LEWIS D RIGHTS 1701
CONDE B. MCCULLOUGH 1702
H. M. NELSON 1707
H. E. LANGLEY AND EDWARD J. DUCEY 1708

2095 **FINAL REPORT OF SUB-COMMITTEE NO. 31, COMMITTEE ON STEEL OF THE STRUCTURAL DIVISION, ON WIND BRACING IN STEEL BUILDINGS** 1713

2096 **REPORT OF THE COMMITTEE ON WATER SUPPLY ENGINEERING OF THE SANITARY ENGINEERING DIVISION: IMPORTANT EVENTS, DEVELOPMENTS, AND TRENDS IN WATER SUPPLY ENGINEERING DURING THE DECADE ENDING WITH THE YEAR 1939** 1740

2097 **THE RELATION OF THE SOCIETY TO NATIONAL DEFENSE: ADDRESS AT THE ANNUAL CONVENTION, DENVER, COLORADO, JULY 24, 1940.**
By **John P. Hogan** 1781

XI

MEMOIRS OF DECEASED MEMBERS

	Page
Charles David Marx, Past-President and Hon. M. Am. Soc. C. E.	1785
Gustav Lindenthal, Hon. M. Am. Soc. C. E.	1790
Henry Bissell Alvord, M. Am. Soc. C. E.	1794
James MacFarlane Angle, M. Am. Soc. C. E.	1796
William John Ash, M. Am. Soc. C. E.	1798
Calvin Lewis Barton, M. Am. Soc. C. E.	1799
George Erle Beggs, M. Am. Soc. C. E.	1800
James C. Bell, M. Am. Soc. C. E.	1803
John George Bloom, M. Am. Soc. C. E.	1805
Norman Freed Brown, M. Am. Soc. C. E.	1806
Everett N. Bryan, M. Am. Soc. C. E.	1808
Walter Carl Buetow, M. Am. Soc. C. E.	1810
Ralph Howe Cady, M. Am. Soc. C. E.	1811
Thomas Jenks Carlile, M. Am. Soc. C. E.	1813
Allan Winter Cuddeback, M. Am. Soc. C. E.	1815
James Eugene Curtis, M. Am. Soc. C. E.	1818
Walter Whaley Curtis, M. Am. Soc. C. E.	1819
William Channing Cushing, M. Am. Soc. C. E.	1820
Cornelius Mark Daily, M. Am. Soc. C. E.	1824
John Vipond Davies, M. Am. Soc. C. E.	1826
Daniel Young Dimon, M. Am. Soc. C. E.	1828
Walter Alexander Doane, M. Am. Soc. C. E.	1830
Guy Carleton Emerson, M. Am. Soc. C. E.	1833
John Farris, M. Am. Soc. C. E.	1835
Howell Tracy Fisher, M. Am. Soc. C. E.	1837
Louis Prevost Gaston, M. Am. Soc. C. E.	1838
George Gibbs, M. Am. Soc. C. E.	1840
Ralph Edward Goodwin, M. Am. Soc. C. E.	1844
Roy Cotsworth Gowdy, M. Am. Soc. C. E.	1846
Edward Francis Haas, M. Am. Soc. C. E.	1851
Ephraim Harrington, M. Am. Soc. C. E.	1854
Philip Hart, M. Am. Soc. C. E.	1855
Ralph Stevenson Hawley, M. Am. Soc. C. E.	1861
William Chauncey Hawley, M. Am. Soc. C. E.	1863
James Clark Irwin, M. Am. Soc. C. E.	1864
Charles White Johns, M. Am. Soc. C. E.	1866
Arthur Newhall Johnson, M. Am. Soc. C. E.	1868
Richard Khuen, Jr., M. Am. Soc. C. E.	1869
Philip George Lang, Jr., M. Am. Soc. C. E.	1871
William Dominick Larrabee, M. Am. Soc. C. E.	1875
Arthur Hermann Markwart, M. Am. Soc. C. E.	1877
Frederick Mears, M. Am. Soc. C. E.	1881
Samuel Phillips Mitchell, M. Am. Soc. C. E.	1886
Charles Nicholas Monsarrat, M. Am. Soc. C. E.	1889
Asa Hall Morrill, M. Am. Soc. C. E.	1892
Ralph Hadlock Ober, M. Am. Soc. C. E.	1894
Charles Real Olberg, M. Am. Soc. C. E.	1896
Irwin Selden Osborn, M. Am. Soc. C. E.	1898
Guy Pinner, M. Am. Soc. C. E.	1900
T. Frank Quilty, M. Am. Soc. C. E.	1902
John Matthew Rayburn, M. Am. Soc. C. E.	1904
Ralph John Reed, M. Am. Soc. C. E.	1905
Edson Mason Scofield, M. Am. Soc. C. E.	1908
Harold Bemis Shattuck, M. Am. Soc. C. E.	1913
Benjamin Bruce Shaw, M. Am. Soc. C. E.	1914
Julian Cleveland Smith, M. Am. Soc. C. E.	1916
William Pliny Snow, M. Am. Soc. C. E.	1918
William Warburton Knox Sparrow, M. Am. Soc. C. E.	1920
Embert Hiram Sprague, M. Am. Soc. C. E.	1922
William Shackelford Sullivan, M. Am. Soc. C. E.	1923

Page
Frederick Horace Tibbetts, M. Am. Soc. C. E. 1924
Ross Kerr Tiffany, M. Am. Soc. C. E. 1928
Warren Ayres Tyrrell, M. Am. Soc. C. E. 1931
Aubrey Weymouth, M. Am. Soc. C. E. 1935
Frank Wilcox, M. Am. Soc. C. E. 1938
Sydney Bacon Williamson, M. Am. Soc. C. E. 1940
Frederick Newton Willson, M. Am. Soc. C. E. 1955
Frank Edward Winsor, M. Am. Soc. C. E. 1957
Charles Nicholson Wright, M. Am. Soc. C. E. 1962
John Morton Billing, Assoc. M. Am. Soc. C. E. 1963
Joseph Patrick Burns, Assoc. M. Am. Soc. C. E. 1964
Charles Page Comegys, Assoc. M. Am. Soc. C. E. 1966
Frederick Webster Deck, Assoc. M. Am. Soc. C. E. 1967
Clyde Henry Fenton, Assoc. M. Am. Soc. C. E. 1971
George Farnsworth Fisk, Assoc. M. Am. Soc. C. E. 1972
Francis James Fitzpatrick, Assoc. M. Am. Soc. C. E. 1973
Harry J. Hanmer, Assoc. M. Am. Soc. C. E. 1977
Harold Frederick Janda, Assoc. M. Am. Soc. C. E. 1978
William Joseph Judge, Assoc. M. Am. Soc. C. E. 1980
Arthur Sylvanus Kenman, Assoc. M. Am. Soc. C. E. 1983
John Kylstra, Assoc. M. Am. Soc. C. E. 1984
John Raymond Lapham, Assoc. M. Am. Soc. C. E 1986
Henry Ambrose Larsen, Assoc. M. Am. Soc. C. E. 1990
Daniel Collins Lipscomb, Assoc. M. Am. Soc. C. E. 1991
Harry Seymour McKibben, Assoc. M. Am. Soc. C. E. 1992
William Franklin Miller, Assoc. M. Am. Soc. C. E. 1993
Eri Olmstead, Assoc. M. Am. Soc. C. E. 1995
Victor Smith Persons, Assoc. M. Am. Soc. C. E. 1996
Charles Arner Ruggles, Assoc. M. Am. Soc. C. E. 1997
Herman Schneider, Assoc. M. Am. Soc. C. E. 1999
Reginald Crowley Scott, Assoc. M. Am. Soc. C. E. 2003
Alexander Jenifer Taylor, Assoc. M. Am. Soc. C. E. 2004
George Copeland Urquhart, Assoc. M. Am. Soc. C. E. 2006
Harold Emil von Bergen, Assoc. M. Am. Soc. C. E. 2008
Ralph Watts Wardwell, Assoc. M. Am. Soc. C. E. 2009
William Tibbitts Webb, Assoc. M. Am. Soc. C. E. 2011
Nathan Davis Whitman, Assoc. M. Am. Soc. C. E. 2013
John Thad Whitney, Assoc. M. Am. Soc. C. E. 2016
Harry McKean Conner, Jun. Am. Soc. C. E. 2018
Franklin Mechtley Felker, Jun. Am. Soc. C. E. 2021
Glenn William Holmes, Jun. Am. Soc. C. E. 2022
John Monks, Affiliate Am. Soc. C. E. 2024
Louis Aloysius Slattery, Affiliate Am. Soc. C. E. 2025
George Oliver Tenney, Affiliate Am. Soc. S. E. 2027

ERRATA

Transactions, Vol. 99 (1934)

Page 483, delete Footnote 53 and substitute the following:

[53] “Critical Review of the Literature on Riveted Joints,” by A. E. R. de Jonge, *Research Publications*, A.S.M.E., 1940 (publication pending).

PAPERS AND REPORTS OF COMMITTEES PUBLISHED IN PROCEEDINGS, AM. SOC. C. E., OCTOBER, 1939, TO SEPTEMBER, 1940, INCLUSIVE, NOT PUBLISHED IN TRANSACTIONS, VOL. 105

	Number of *Proceedings*.
GENERAL WEDGE THEORY OF EARTH PRESSURE. By Karl Terzaghi	October, 1939
THE RÔLE OF THE ENGINEER IN AIR SANITATION: A SYMPOSIUM	November, 1939
PROBLEMS AND TRENDS IN ACTIVATED SLUDGE PRACTICE. By Robert T. Regester	November, 1939
BRIDGE AND TUNNEL APPROACHES. By John F. Curtin	November, 1939
TREND IN HYDRAULIC TURBINE PRACTICE: A SYMPOSIUM	November, 1939
EFFECTS OF RIFLING ON FOUR-INCH PIPE TRANSPORTING SOLIDS. By G. W. Howard	November, 1939
TRANSIENT FLOOD PEAKS. By Henry B. Lynch	November, 1939
CHANNELIZATION OF MOTOR TRAFFIC. By Guy Kelcey	December, 1939
WATER SUPPLY ON UPPER SALT RIVER, ARIZONA. By John Girand	December, 1939
ANALYSIS OF LEGAL CONCEPTS OF SUBFLOW AND PERCOLATING WATERS. By C. F. Tolman and Amy C. Stipp	December, 1939
MINIATURE SYSTEM OF FIRST-ORDER ALINEMENT AND TRIANGULATION CONTROL. By Floyd W. Hough	December, 1939
PRESSURE-MOMENTUM THEORY APPLIED TO THE BROAD-CRESTED WEIR. By H. A. Doeringsfeld and C. L. Barker	December, 1939
NORRIS DAM CONSTRUCTION CABLEWAYS. By R. T. Colburn and L. A. Schmidt, Jr.	December, 1939
STANDARDS OF PROFESSIONAL RELATIONS AND CONDUCT. By Daniel W. Mead	January, 1940
THEORY OF ELASTIC STABILITY APPLIED TO STRUCTURAL DESIGN. By Leon S. Moisseiff and Frederick Lienhard	January, 1940

Number of *Proceedings.*

CHICAGO RIVER CONTROL WORKS.
By H. P. Ramey .... January, 1940

AXIOMS IN ROADWAY SOIL MECHANICS.
By Henry C. Porter .... February, 1940

MEASURING THE POTENTIAL TRAFFIC OF A PROPOSED VEHICULAR CROSSING.
By N. Cherniack .... February, 1940

SEALING THE LAGOON LINING AT TREASURE ISLAND WITH SALT.
By Charles H. Lee .... February, 1940

PROGRESS REPORT OF COMMITTEE OF THE HYDRAULICS DIVISION ON FLOOD CONTROL: FLOOD-CONTROL METHODS; THEIR PHYSICAL AND ECONOMIC LIMITATIONS . February, 1940

FOUNDATION EXPERIENCES, TENNESSEE VALLEY AUTHORITY: A SYMPOSIUM .... March, 1940

DESIGN OF HINGES AND ARTICULATIONS IN REINFORCED CONCRETE STRUCTURES.
By George C. Ernst .... April, 1940

PERMISSIBLE COMPOSITION AND CONCENTRATION OF IRRIGATION WATER.
By W. P. Kelley .... April, 1940

PROGRESS REPORT OF THE COMMITTEE ON FLOOD-PROTECTION DATA .... April, 1940

PROGRESS REPORT OF THE COMMITTEE OF THE STRUCTURAL DIVISION ON OXYGEN CUTTING (FLAME CUTTING) OF STRUCTURAL STEEL .... April, 1940

MASONRY DAMS: A SYMPOSIUM .... May, 1940

MODEL TESTS, BRIDGE PIER SUPPORTED ON LONG STEEL PILES.
By Thomas F. Comber, Jr., and John M. Coan, Jr. .... June, 1940

PROGRESS REPORT OF A SPECIAL SUBCOMMITTEE OF THE COMMITTEE OF THE STRUCTURAL DIVISION ON MASONRY AND REINFORCED CONCRETE: THE PRACTICE OF STATE HIGHWAY DEPARTMENTS IN THE DESIGN OF ABUTMENTS .... June, 1940

REPORT OF THE JOINT COMMITTEE ON STANDARD SPECIFICATIONS FOR CONCRETE AND REINFORCED CONCRETE: RECOMMENDED PRACTICE AND STANDARD SPECIFICATIONS FOR CONCRETE AND REINFORCED CONCRETE .... June, Pt. 2, 1940

MAXIMUM PROBABLE FLOODS ON PENNSYLVANIA STREAMS.
By Charles F. Ruff .... September, 1940

SUPERSTRUCTURE OF THEME BUILDING OF NEW YORK WORLD'S FAIR.
By Shortridge Hardesty and Alfred Hedefine .... September, 1940

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2056

WIND FORCES ON A TALL BUILDING

By J. Charles Rathbun,[1] M. Am. Soc. C. E.

With Discussion by Messrs. David Cushman Coyle, Clyde T. Morris, Robins Fleming, F. P. Shearwood, Lydik S. Jacobsen, Francis L. Castleman, Jr., J. B. Wilbur, R. D. Spellman, David A. Molitor, Walter J. Gray, K. L. DeBlois, Albert Smith, Victor R. Bergman, Otto Gottschalk, Colin Skinner, and J. Charles Rathbun.

Synopsis

Since the construction of the Empire State Building in New York, N. Y., considerable data have been collected pertaining to the intensity and distribution of the wind pressure during storms, as observed by instruments placed in this structure. At the same time the movement of the top of the building and the stresses in some of the members of the frame have been observed.

A model of the steel frame was constructed and measurements of its deformation under horizontal load, as well as its period of vibration, were noted. From these observations the actual lateral loads on the building were estimated under certain assumptions. The ratio between the stiffness of the building and its frame (acting alone) was then computed to give an indication of the percentage of the load that is carried by the frame. The plastic as well as the elastic action of the structure is indicated in the data.

The following studies were contemplated when this set of experiments was undertaken:

(*a*) The relation between the velocity of the wind and the pressure produced;
(*b*) The variation of stresses due to wind across a bent;
(*c*) A check, or refutation, of the basic principles of the portal and cantilever methods of analysis; and
(*d*) An estimate of the proper horizontal load that should be used in designing a tall building to insure proper rigidity and stability.

Because of the erratic nature of the data, definite conclusions have not been reached in the solution of these problems.

The instruments installed in the building for the special purposes of the observations were: 1 anemometer; 30 manometers; 28 cameras with operating mechanism; 22 extensometers; 1 collimator and target; and 1 plumb-bob.

Note.—Published in September, 1938, *Proceedings*.

[1] Associate Prof., Civ. Eng., Coll. of the City of New York, New York, N. Y.

The paper is divided into three parts. Part I contains a discussion of: (*a*) The nature of the wind and the pressure produced by it; and also (*b*) the deflection and stresses in the building due to wind action. Part II gives an estimate of the stiffening effect of the masonry on the steel and an approximate evaluation of the uniform load on the structure due to the wind. A model, with its equipment, was tested in the laboratory for the investigation described in Part II. Finally, Part III cites nine concise conclusions.

PART I.—DESCRIPTIVE

INTRODUCTION

The Technical Research Committee of the American Institute of Steel Construction by arrangement with the owners, architects, consulting engineer, and contractors for the Empire State Building equipped that building during its construction with instruments, by the use of which data were later obtained as to the effect of wind and the resulting stresses and strains.

With the exception of the work of Clyde T. Morris, M. Am. Soc. C. E., on the American Insurance Union Building[2] this is the first opportunity known to the writer in which engineers have been permitted to conduct an extensive study of the action of a large building over a period of several years, with a view to increasing their knowledge of the action of a structure subjected to wind loads. The data obtained, therefore, are of considerable value inasmuch as they tend to clarify the very intricate problem of determining the nature of wind pressure on large areas and permit the study of the plastic and elastic action of the building itself.

By the nature of the problem a high degree of accuracy was not necessary; nor was it obtainable. There were many factors beyond the control of the observers, and many data that were necessary for a complete solution of the problem could not be obtained. From the beginning of the project in April, 1931, until the removal of the instruments in the summer of 1936 a considerable number of observers gave their time to the task of making those records. Many of these data were fragmentary but as a whole they furnish a picture of the phenomena being observed.

DESCRIPTION OF BUILDING

The Empire State Building is the tallest in the world to date (1938); and it is not, like many tall buildings, simply a tower surmounting a comparatively low building. Its breadth is sufficient to present a very extensive surface to the wind (see Fig. 1). Its plan dimensions are 197.5 ft by 424.5 ft from the ground to the sixth floor. It has several setbacks, but from the thirtieth floor to the seventy-third, it covers approximately one-fourth the area of the ground floor. From the seventy-third floor it is gradually reduced in area to the roof on the eighty-sixth floor. A mast which supports the anemometer for measuring the wind velocity extends about 200 ft above the roof. All observations were made below the eighty-sixth floor. The design is almost ideally simple from a structural point of view. All the floors are symmetrical about two axes, with but few minor exceptions, which have little or no effect on the action of the building. The total height above the street floor, including the mast, is 1 248 ft.

---

[2] "Practical Design of Wind Bracing," by Clyde T. Morris, *Proceedings*, Am. Inst. of Steel Construction, Inc., October, 1927.

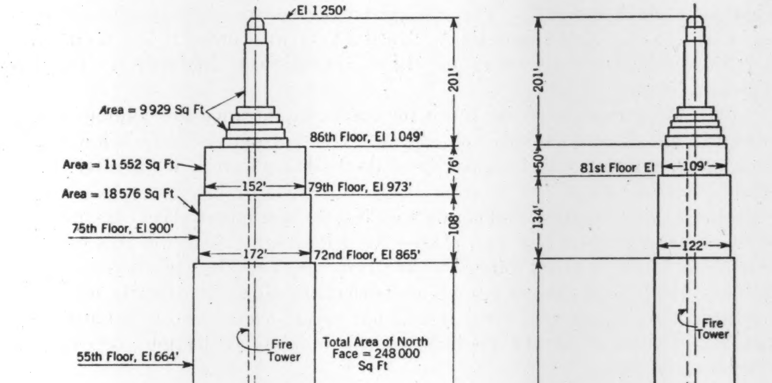

El 1 250'
Area = 9 929 Sq Ft
86th Floor, El 1 049'
Area = 11 552 Sq Ft
152'
79th Floor, El 973'
Area = 18 576 Sq Ft
75th Floor, El 900'
172'
72nd Floor, El 865'
Fire Tower
Total Area of North Face = 248 000 Sq Ft
55th Floor, El 664'
Area = 92 442 Sq Ft
Center Line
36th Floor, El 437'
186'
30th Floor, El 368'
Area = 26 555 Sq Ft
24th Floor, El 292'
235'
21st Floor, El 255'
Area = 57 040 Sq Ft
310'
6th Floor, El 75'
Area = 31 875 Sq Ft
425'
El 0'
201'
76'
108'
497'
113'
180'
75'
NORTH ELEVATION
201'
50'
81st Floor El
109'
134'
122'
Fire Tower
550'
Center Line
134'
25th Floor, El 315'
240'
164'
75'
197'
WEST ELEVATION

FIG. 1.—SIDE VIEWS OF EMPIRE STATE BUILDING

Structurally, the building consists of steel columns and girders, the latter being fastened to the columns by the shallow wind-brace connections of the type shown in Fig. 2. In addition, the central bays around the elevator shafts have a system of knee-braces which extend from approximately the third point on the girder to the point where the corresponding girder on the floor below joins the column. The system of knee-braces in any set of bays between two columns, with the accompanying columns and floor-beams, forms a vertical truss which materially stiffens and strengthens the frame of the building. These are shown with their form of bracing in Fig. 3 which gives an outline of the frame of the building in the narrow direction. As explained in Part II, there are three typical frames in the narrow direction of the building.

The floors are of cinder concrete slab construction supported by steel beams approximately 8 ft apart. The outside partitions are covered with chrome-nickel steel and Indiana limestone. Until a floor was rented it had no interior partitions except those surrounding the elevators and utility rooms. Fig. 4 is a typical layout.

With the exception of one offset for the main entrance the columns are in rows in both directions and each column extends from the foundation to the roof. The symmetry and regularity of the building have aided greatly in the study of its action.

Surrounding the steel, and acting with it, are other materials of construction, such as concrete floors, tile and plaster partitions, etc. Much of this material is not elastic and does not follow Hooke's law. As a result, when it is subjected to loads, the building shows a predominance of plastic action; that is, when it is deflected by a horizontal force it will not return exactly to a definite fixed position. A study of the readings of the building deflections reveals this phenomenon clearly.

THE VICINITY

The buildings in the vicinity of the structure are low compared to those a few blocks farther north, where one group of the tall buildings of the city is located. The buildings along Forty-Second Street present a solid wall of high structures which tend to break up the air currents at elevations above, as well as below, their tops. There are a number of tall buildings to the west as well as to the north; and, to the south and the southeast, there are few obstructions. Fig. 5, a view of the city near the building, affords some idea of the character of obstructions which, it is easily seen, are of a type that would disturb the air currents and interfere with any tendency to be streamlined.

Fig. 6 is a map giving the approximate elevations, in feet, of the roofs of those buildings that are high enough, and close enough, to affect the air currents seriously. Fig. 7 gives the heights, in number of stories, of the buildings in the immediate neighborhood.

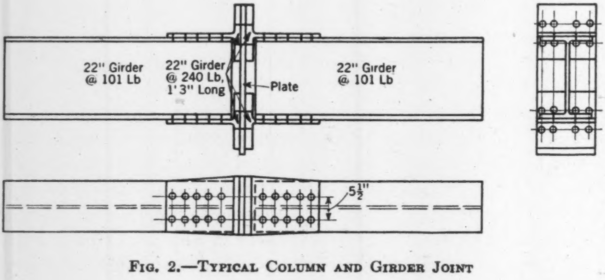


FIG. 2.—TYPICAL COLUMN AND GIRDER JOINT

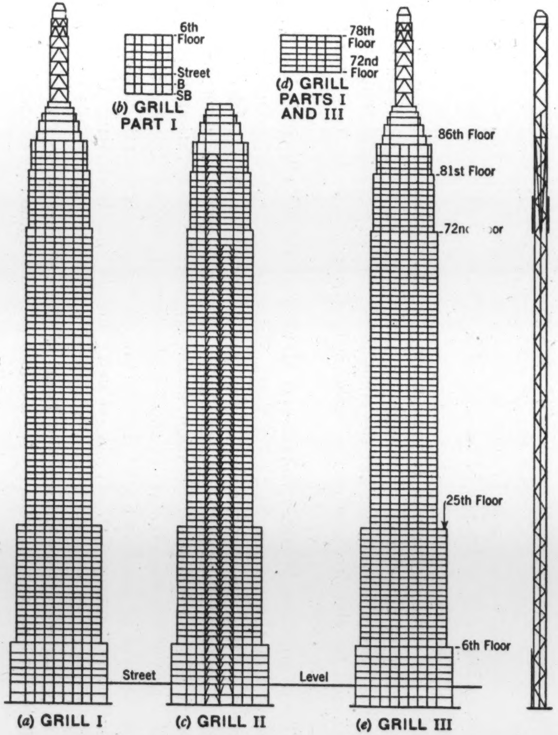

6th Floor
Street
B
SB
(b) GRILL PART I
78th Floor
72nd Floor
(d) GRILL PARTS I AND III
86th Floor
81st Floor
72n or
25th Floor
6th Floor
Street
Level
(a) GRILL I
(c) GRILL II
(e) GRILL III

FIG. 3.—OUTLINE OF MODEL AND OF STEEL FRAME OF PROTOTYPE, NARROW DIRECTION

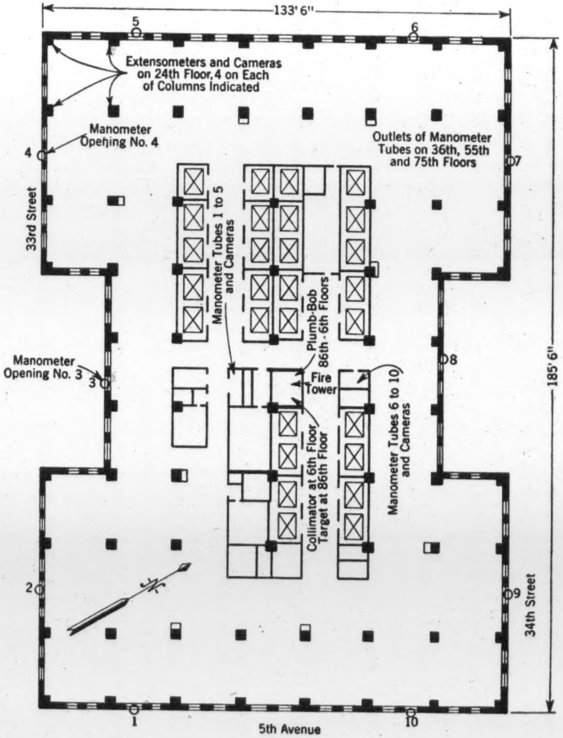

133' 6"
185' 6"
5
6
Extensometers and Cameras on 24th Floor, 4 on Each of Columns Indicated
Manometer Opening No. 4
4
Outlets of Manometer Tubes on 36th, 55th and 75th Floors
7
33rd Street
Manometer Tubes 1 to 5 and Cameras
Plumb-Bob 86th - 6th Floors
Fire Tower
Manometer Opening No. 3
3
8
Collimator at 6th Floor Target at 86th Floor
Manometer Tubes 6 to 10 and Cameras
2
9
34th Street
1
5th Avenue
10

FIG. 4.—TYPICAL FLOOR LAYOUT. NUMBERS DENOTE OBSERVATION STATIONS

Several methods were used in making the survey for the maps in Figs. 6 and 7, after which the public records for the heights of the more important buildings were looked up as a check. Accuracy sufficient for the purpose was obtained by these methods although they could not be relied upon where great care was necessary. The first method used was to observe the heights with a hand-level from the several floors of the Empire State Building. This method proved inaccurate for all except the closest structures.

An ingenious method was suggested and used by one of the student observers, which proved quite successful. Photographs were taken from the seventy-fifth, fifty-fifth, and thirty-sixth floors using windows in the same vertical line. With the known distance between floors as a base the heights of the buildings could be computed from the photographs by triangulation. The distances to the buildings were known from the published maps so that checks on this work were obtained.

From Figs. 5, 6, and 7, it is quite evident that the structure is many times higher than most of its neighbors and even those that compare with it in height (more than one-half as high) are isolated towers a considerable distance away. The air currents at the lower elevations (to a height that is extremely problematical) are very much disturbed.

VELOCITY OF THE WIND

The velocity and direction of the wind were obtained from an anemometer installed approximately 15 ft above the top of the mast, or 1 263 ft above the street. This instrument relies for its operation on a 32-blade rotor which does not spin (as does the cup anemometer). The dial (not self-recording), was read just before and just after the deflection readings. The hands often fluctuated over a range of about 15 miles per hr during a severe storm. The average velocity was taken and not the maximum due to this swing. In one instance, a fluctuation of 20 miles per hr on each side of a mean of about 50 miles per hr was noted.

There is some question as to the accuracy of the anemometer for recording the velocity of the storm. Although the instrument may register, correctly, the velocity at the mast head at the time noted, the air currents are doubtless influenced by the presence of the building itself, as well as by the other tall buildings in the vicinity.

There are three Weather Bureaus on Manhattan Island, all within less than five miles of the Empire State Building. In order to have as many data as possible on the wind movement during the observations the records of these several Weather Bureaus were consulted. Table 1 gives the direction and velocity of the wind at each station, at the time of the observation indicated. In the case of the anemometer at the Empire State Building, the readings were taken as near as possible to the time of taking the other data and at the time indicated in Column (2). If the velocity needle was fluctuating the mean reading was taken. In the case of the three neighboring stations, the average velocity and direction during the hour of the readings were recorded. In addition, the maximum velocity for a 5-min period attained by the wind during

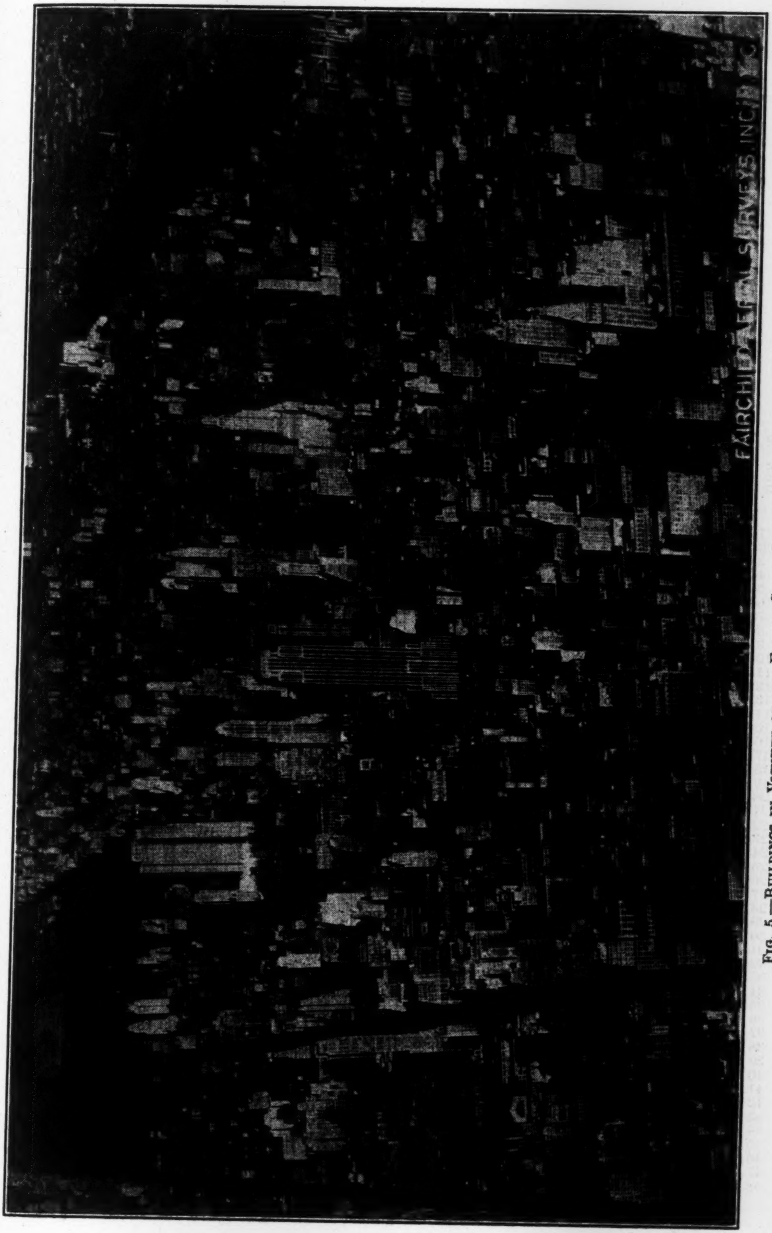

FAIRCHILD AERIAL SURVEYS INC. N.Y.C.

FIG. 5.—BUILDINGS IN VICINITY OF THE EMPIRE STATE BUILDING; VIEW FACING NORTHEAST

TABLE 1.—WIND RECORDS, NEW YORK, N. Y.

Item No.	Date*	Eastern Standard Time, in hours and minutes past midnight	EMPIRE STATE BUILDING		NEW YORK CITY METEOROLOGICAL OBSERVATORY		*Daily News* BUILDING		UNITED STATES WEATHER BUREAU OBSERVATORY			
									Hourly reading		Maximum	
			Direction of wind	Velocity, in miles per hour	Direction of wind	Velocity, in miles per hour	Direction of wind	Velocity, in miles per hour	Direction of wind	Velocity, in miles per hour	Velocity, in miles per hour	Time, in hours and minutes past midnight
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Height of anemometer, in feet		....		62†		504‡		454§			
	Maximum velocities (5-min period); 3/22/36		....	....	....	34	....	....	....	....	65	....
	Extreme velocities		....	102	....	54.4	....	....	....	....	71	....
1	2/10/32	15:00	SW	5	NNE	4	SW	2	NW	5	....	....
2	2/10/32	17:00	N	4	ENE	6	NE	5	NW	3	....	....
3	2/11/32	13:00	S	15	ENE	9	NE	9	NE	4	....	....
4	2/12/32	16:00	W	35	WSW	11	W	14	SW	14	....	....
5	3/ 9/32	14:30	W	20	WNW	12	NW	15	W	21	....	....
6	3/ 9/32	16:00	W	25	WSW	13	NW	18	W	21	....	....
7	3/10/32	15:00	WNW	45	NW	18	NW	17	NW	30	....	....
8	3/11/32	14:30	WNW	40	NW	16	NW	14	NW	27	30	14:50
9	3/25/32	14:30	SE	19–22	SSW	12	SE	15	S	17	....	....
10	4/25/32	14:30	SE	18–22	ESE	9	SE	11	SE	12	....	....
11	4/27/32	14:20	NW	29–38	WNW	18	NW	18	NW	32	46	14:36
12	4/29/32	14:00	SSW	12–18	S	12	SE	18	S	18	....	....
13	5/ 2/32	14:15	NW	28	NNW	12	NW	12	NW	21	....	....
14	5/ 3/32	14:04	WNW	8–11	NNW	10	NW	8	NW	14	....	....
15	5/ 4/32	13:55	SSW	12	WNW	7	S	8	SW	12	....	....
16	5/ 9/32	14:55	E	11–14	ESE	9	SE	6	SE	13	....	....
17	5/18/32	14:00	N	8	NNW	7	NE	3	NW	7	....	....
18	5/18/32	14:45	NNE	5	NNW	7	NE	3	NW	7	....	....
19	5/19/32	14:00	SSE	13–17	SSW	10	S	10	S	12	....	....
20	5/19/32	15:25	SSE	15	SSW	11	SE	12	S	14	....	....
21	5/23/32	11:15	W	15	WNW	9	NW	10	SW	14	....	....
22	5/23/32	12:00	W	12	WNW	9	NW	10	SW	15	....	....
23	5/24/32	11:00	SSW	8	SSW	6	S	8	SW	10	....	....
24	5/25/32	10:00	WSW	18	WSW	9	SW	11	SW	12	....	....
25	5/25/32	14:50	SSW	22	WSW	12	SW	11	SW	16	....	....
26	5/26/32	14:40	SSW	30	WSW	14	SW	19	SW	24	25	14:40
27	5/26/32	15:50	SSW	35	WSW	14	SW	20	SW	21	....	....
28	5/31/32	14:40	SSW	18	SSW	15	S	18	S	16	....	....
29	6/ 3/32	15:15	SSW	16	S	11	S	13	S	14	....	....
30	6/ 6/32	15:35	NW	22–24	NW	10	N	9	NW	17	....	....
31	6/ 7/32	16:45	N	45	NNE	18	N	19	N	26	31	16:40
32	6/ 8/32	13:25	WNW	25	NW	13	NW	11	NW	22	27	11:54
33	6/ 8/32	15:40	WNW	25	NW	14	NW	9	NW	21	26	15:29
34	6/ 9/32	14:40	WSW	30	WNW	13	SW	13	W	22	....	....
35	6/13/32	11:50	NE	15	ENE	12	NE	16	E	6	....	....
36	6/14/32	15:30	NE	2	E	6	SE	3	S	9	....	....
37	6/15/32	17:30	S	25	SSW	10	S	14	E	18	....	....
38	6/16/32	14:00	NNE	20	ESE	9	S	7	S	8	....	....
39	6/16/32	14:25	SSE	12	ESE	9	S	7	S	8	....	....
40	6/17/32	13:00	N	15	ENE	12	NE	13	E	8	....	....
41	6/17/32	13:20	N	20	ENE	12	NE	13	E	8	....	....
42	6/17/32	14:20	N	25	ENE	12	NE	19	NE	10	....	....
43	6/22/32	17:50	W	30	WNW	12	NW	17	NW	22	28	16:04
44	6/23/32	11:55	WNW	30	NW	16	NW	13	NW	25	29	10:00
45	6/23/32	15:25	W	32	WNW	15	NW	17	NW	29	36	14:25
46	6/24/32	11:15	WNW	25	NNW	14	NW	10	NW	24	28	11:09
47	6/24/32	13:50	WNW	40	NW	15	NW	15	NW	27	32	11:54
48	6/24/32	17:18	W	30	WNW	14	NW	16	W	24	27	17:04

* Month, day, year. † Height above the ground. ‡ Height above the street. § Height above the harbor.

TABLE 1—(*Continued*)

Item No.	Date*	Eastern Standard Time, in hours and minutes past midnight	EMPIRE STATE BUILDING		NEW YORK CITY METEOROLOGICAL OBSERVATORY		*Daily News* BUILDING		UNITED STATES WEATHER BUREAU OBSERVATORY			
									Hourly reading		Maximum	
			Direction of wind	Velocity, in miles per hour	Direction of wind	Velocity, in miles per hour	Direction of wind	Velocity, in miles per hour	Direction of wind	Velocity, in miles per hour	Velocity, in miles per hour	Time, in hours and minutes past midnight
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Height of anemometer, in feet		....		62†		504‡		454§			
	Maximum velocities (5-min period); 3/22/36...		....	....	....	34	....	....	....	....	65	....
	Extreme velocities		....	102	....	54.4	....	....	....	....	71	....
49	6/29/32	16:55	WNW	12	NW	9	NW	7	SW	11	....	....
50	7/ 1/32	15:00	S	30	SSW	15	S	27	S	29	32	15:19
51	7/ 5/32	17:15	WNW	20	NNW	7	NW	8	NW	15	....	....
52	7/ 6/32	13:00	SW	12	WSW	9	SW	12	SW	10	....	....
53	7/ 7/32	14:40	SSE	12	S	10	S	13	S	12	....	....
54	7/11/32	13:10	NW	40	WNW	18	NW	18	NW	29	35	10:55
55	7/14/32	12:55	NNW	10	NNE	11	N	10	N	13	....	....
56	7/19/32	12:45	WNW	20	WNW	10	NW	13	NW	17	....	....
57	7/25/32	14:30	W	8	NW	6	NW	6	N	7	....	....
58	7/27/32	10:30	SSW	15	WSW	7	SW	7	S	9	....	....
59	7/29/32	12:30	WNW	20	WNW	9	W	11	NW	15	....	....
60	8/ 2/32	10:15	SSW	13	SSW	8	S	9	S	11	....	....
61	8/ 4/32	10:27	S	32	NNW	12	N	14	NW	17	....	....
62	8/ 4/32	13:50	S	38	NNW	10	NW	9	NW	15	....	....
63	8/22/32	12:00	NNW	5	NW	5	N	6	N	9	....	....
64	8/23/32	11:45	NE	10	E	12	NE	15	NE	9	....	....
65	8/24/32	13:25	S	8	SSW	9	S	11	SW	13	....	....
66	8/31/32	16:25	E	6	E	9	SE	7	SE	9	....	....
67	9/ 7/32	11:25	NNW	12	NNW	10	NE	11	N	13	....	....
68	9/ 8/32	12:45	NNW	42	NNE	24	NE	27	N	29	....	....
69	9/ 8/32	14:50	NNW	55	NNE	21	NE	26	N	34	40	14:17
70	9/ 8/32	16:10	NNW	40	NNE	21	NE	24	N	36	44	16:18
71	9/12/32	14:07	W	10	WNW	7	NW	8	SW	11	....	....
72	9/27/32	10:00	SSW	22	WSW	13	SW	17	SW	15	....	....
73	9/27/32	16:15	SSW	22	SW	12	SW	14	SW	18	....	....
74	9/28/32	9:30	SSW	42	SW	15	SW	20	SW	26	28	9:30
75	9/28/32	10:15	....	40	SW	16	SW	21	SW	26	29	10:30
76	9/28/32	14:50	WNW	40	WNW	18	NW	18	NW	32	....	....
77	10/ 3/32	13:10	S	42	SSW	14	SE	21	SE	24	....	....
78	10/ 3/32	16:30	SE	26–35	SSW	14	S	24	S	24	....	....
79	10/ 4/32	16:40	SE	35	SSE	10	SE	17	SE	18	....	....
80	10/ 5/32	9:00	SSE	52	SSE	14	S	29	SE	28	....	....
81	10/ 5/32	14:40	SSE	45	S	14	S	27	SE	27	....	....
82	10/ 6/32	9:20	SSE	65–75	S	16	S	33	SE	33	36	9:44
83	10/ 7/32	10:15	WNW	32	WNW	16	NW	15	NW	27	34	10:33
84	10/11/32	9:20	WSW	34	WSW	10	W	15	W	16	....	....
85	10/12/32	9:00	SW	35	WSW	16	W	19	SW	24	....	....
86	10/12/32	13:10	W	47	WNW	21	W	28	NW	38	50	13:39
87	10/14/32	16:50	WSW	10	NW	6	W	6	W	10	....	....
88	10/17/32	9:00	SE	22	ENE	9	E	9	E	9	....	....
89	10/18/32	11:35	NE	35	ENE	19	NE	28	NE	15	....	....
90	10/18/32	11:50	NE	42	ENE	19	NE	28	NE	15	....	....
91	10/19/32	8:50	NNE	32	NE	18	NE	24	NE	13	....	....
92	10/21/32	10:00	W	22	WNW	12	NW	16	W	18	....	....
93	10/21/32	10:20	W	28	WNW	12	NW	16	W	17	....	....
94	10/21/32	16:20	WNW	50	NW	18	NW	18	NW	32	35	16:13
95	10/22/32	9:10	NE	22	WNW	8	W	11	NW	17	....	....
96	10/25/32	13:10	E	15	ESE	10	SE	12	E	12	....	....

TABLE 1—(*Continued*)

Item No.	Date*	Eastern Standard Time, in hours and minutes past midnight	Empire State Building		New York City Meteorological Observatory		*Daily News* Building		United States Weather Bureau Observatory			
									Hourly reading		Maximum	
			Direction of wind	Velocity, in miles per hour	Direction of wind	Velocity, in miles per hour	Direction of wind	Velocity, in miles per hour	Direction of wind	Velocity, in miles per hour	Velocity, in miles per hour	Time, in hours and minutes past midnight
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Height of anemometer, in feet		....		62†		504‡		454§			
	Maximum velocities (5-min period); 3/22/36...		....	....	....	34	....	....	....	....	65	....
	Extreme velocities		....	102	....	54.4	....	....	....	....	71	....
97	10/26/32	11;05	SSE	13	ESE	7	SE	6	SE	9	....	....
98	10/28/32	14:40	SSE	27–35	NW	18	NW	15	NW	29	34	14:58
99	10/29/32	8:55	SSW	4–10	W	5	W	5	WSW	7	....	....
100	11/ 1/32	9:00	SE	75	SSE	17	SE	40	SE	39	40	....
101	11/ 1/32	10:30	SE	75	SSE	18	SE	41	SE	40	....	....
102	11/ 1/32	12:00	SSE	75–95	SSE-NW	14	NW	16	NW	56	....	....
103	11/ 1/32	16:05	WNW	42	NW	17	NW	16	NW	29	....	....
104	11/ 1/32	16:50	WNW	45	NW	17	NW	16	NW	23	29	....
105	11/ 1/32	17:15	WNW	50	N	17	NW	16	NW	29	36	....
106	11/ 9/32	11:00	NE	45–57	ENE	24	NE	34	NE	12	....	....
107	11/ 9/32	12:00	ENE	45–60	ENE	27	E	36	NE	12	....	....
108	11/ 9/32	14:20	ENE	55–70	ENE	28	E	38	NE	14	....	....
109	11/ 9/32	14:45	ENE	60	ENE	28	E	38	NE	12	....	....
110	11/ 9/32	16:30	ENE	60	ENE	29	E	41	NE	13	....	....
111	11/11/32	11:00	S	25	SSW	10	S	16	S	13	....	....
112	11/15/32	9:50	SW	35	WSW	12	SW	12	SW	8	....	....
113	11/18/32	11:00	ENE	6–10	ENE	10	NE	9	ENE	7	....	....
114	11/18/32	16:00	ENE	15–16	ENE	9	E	12	ENE	6	....	....
115	11/19/32	13:10	SSE	65	SSW	22	S	37	S	44	49	13:24
116	12/ 2/32	14:00	WSW	35	WSW	13	SW	13	SW	14	....	....
117	12/ 2/32	16:00	SW	35	WSW	13	SW	16	SW	12	....	....
118	12/ 9/32	10:15	S	7	S	4	S	7	S	7	....	....
119	12/21/32	11:40	SSW	34	WSW	12	SW	14	SW	22	....	....
120	12/27/32	17:20	ENE	12	NE	11	NE	17	NE	10	....	....
121	12/28/32	9:40	SSW	12	ENE	6	SW	6	SW	9	....	....
122	12/30/32	11:45	SSW	8	E	1	SW	5	S	10	....	....
123	1/ 6/33	17:10	SW	29	WSW	12	W	15	W	11	....	....
124	1/ 7/33	12:15	WSW	50	WSW	17	W	24	W	30	35	12:06
125	1/11/33	9:10	SSW	42	SW	19	SW	24	SW	25	28	9:42
126	1/11/33	10:35	SSW	50	WSW	15	SW	20	SW	19	25	10:00
127	1/13/33	16:16	NE	15	ENE	15	E	15	NE	7	....	....
128	1/20/33	16:10	WNW	28	NW	11	NW	12	NW	18	26	16:07
129	1/25/33	9:30	ENE	58	ENE	15	NE	18	E	10	....	....
130	1/28/33	17:28	NW	47	NNW	18	NW	19	NW	32	34	17:05
131	2/ 2/33	13:45	WSW	40	WSW	14	W	17	W	25	30	12:59
132	2/10/33	10:10	W	35	W	14	W	12	W	15	....	....
133	2/15/33	10:00	NW	40	NNW	12	NW	15	NW	23	....	....
134	2/15/33	12:00	NW	46	NNW	12	NW	20	NW	31	35	11:55
135	2/17/33	16:30	WSW	18	WSW	8	SW	7	SW	9	....	....
136	2/18/33	16:30	WSW	18	NNW	8	SW	11	N	18	....	....
137	2/27/33	9:30	NW	55–65	NNW	24	NW	23	NW	41	44	9:15
138	2/27/33	10:15	NW	55–65	NNW	24	NW	25	NW	41	47	10:38
139	2/28/33	9:00	WSW	30	NNW	21	NW	19	NW	27	34	9:00
140	2/28/33	9:30	NW	50	NNW	21	NW	20	NW	27	....	....
141	2/28/33	11:00	NW	50–55	NNW	24	NW	22	NW	25	....	....
142	2/28/33	11:20	NW	45–50	NNW	24	NW	22	NW	25	33	11:47
143	3/10/33	12:30	WNW	52	NW	24	NW	18	NW	35	37	12:23

TABLE 1—(*Continued*)

Item No.	Date*	Eastern Standard Time, in hours and minutes past midnight	EMPIRE STATE BUILDING		NEW YORK CITY METEOROLOGICAL OBSERVATORY		*Daily News* BUILDING		UNITED STATES WEATHER BUREAU OBSERVATORY			
									Hourly reading		Maximum	
			Direction of wind	Velocity, in miles per hour	Direction of wind	Velocity, in miles per hour	Direction of wind	Velocity, in miles per hour	Direction of wind	Velocity, in miles per hour	Velocity, in miles per hour	Time, in hours and minutes past midnight
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Height of anemometer, in feet		....		62†		504‡		454§			
	Maximum velocities (5-min period); 3/22/36...		....	....	....	34	....	....	....	....	65	....
	Extreme velocities		....	102	....	54.4	....	....	....	....	71	....
144	3/15/33	11:00	WNW	50	ENE-NW	11	NW	12	NW	21	34	10:45
145	3/21/33	10:45	NE	40	NNE	13	NE	23	NE	14	....	....
146	3/25/33	13:25	SSE	15	S	11	S	13	S	14	....	....
147	3/28/33	17:00	WNW	41	NNW	18	NW	17	NW	27	35	17:20
148	4/ 4/33	17:00	WNW	45	NW	19	NW	19	NW	30	30	17:04
149	4/19/33	11:20	NE	44	ENE	21	E	28	NE	12	....	....
150	2/ 3/36	14:00	....	0	WNW	4	NW	5	N	5	....	....
151	3/ 2/36	18:40	SE	35	ESE	12	SE	15	E	18	....	....
152	3/ 9/36	17:00	SW	22	SSE	8	S	9	S	7	....	....
153	3/11/36	14:45	SE	55–60	ESE	18	SE	29	SE	30	32	14:55
154	3/18/36	14:40	E	55	NE	21	E	27	E	9	....	....
155	3/22/36	10:25	NNW	90	NW	28	NW	27	NW	56	58	10:10
156	3/22/36	15:55	NW	50–60	NW	25	NW	23	NW	47	54	15:50
157	3/22/36	16:10	NW	50–60	NW	27	NW	23	NW	45	54	16:35
158	4/ 1/36	14:15	S	25	SE	9	SE	8	SE	9	....	....
159	4/ 7/36	12:25	E	20	ENE	13	NE	11	E	6	....	....
160	4/ 8/36	9:36	NNW	40–45	NW	16	NW	17	NW	36	....	....
161	4/ 8/36	9:53	....	....	NW	16	NW	17	NW	36	....	....
162	4/13/36	15:50	NNW	20	NW	11	N	11	N	10	....	....
163	4/14/36	15:45	NNW	25	SW	10	SW	13	SW	15	....	....
164	4/15/36	10:00	NW	45	SE	7	SE	4	SE	5	....	....
165	4/17/36	11:00	WNW	30	WNW	15	W	23	W	31	32	11:37
166	4/29/36	14:10	....	....	SE	11	SE	11	SE	13	....	....
167	4/29/36	15:00	....	....	SE	13	S	15	SE	18	....	....
168	4/30/36	12:05	....	....	SSE	9	S	13	SE	13	....	....
169	5/16/36	11:30	....	....	NNE	10	N	7	N	10	....	....

that same hour, as recorded by the Federal Observatory, is given in Column (11). The exact time that this maximum was taken is given in Column (12).

The maximum velocity during the storm of March 22, 1936, for a 5-min period, was 34 miles per hr in Central Park, and 65 miles per hr at the Federal Station. The extreme velocity at the Federal Station was 71 miles per hr whereas a velocity of 102 miles per hr was noted at the Empire State Building. A pressure of 20.2 lb per sq ft has been reached once in Central Park since 1919.

The Federal Observatory is in the Whitehall Building overlooking Battery Park on the south end of Manhattan Island. In the vicinity there are a number of buildings as tall or taller than the one containing the U. S. Weather

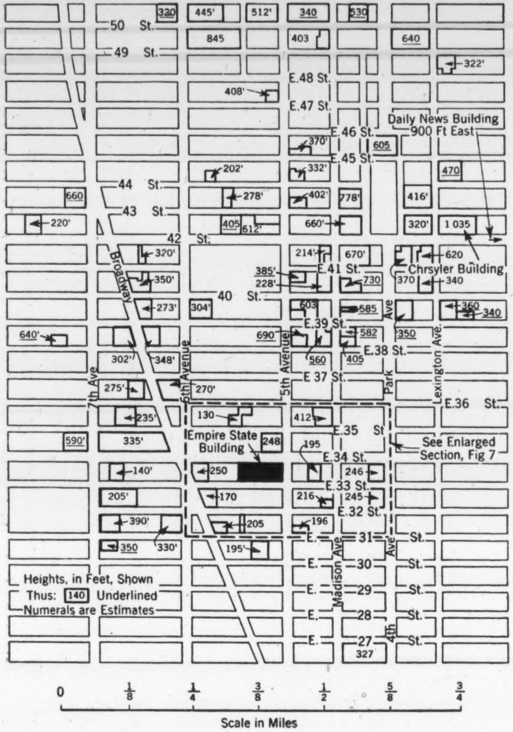

50 St.
49 St.
E.48 St.
E.47 St.
E.46 St.
E.45 St.
44 St.
43 St.
42 St.
E.41 St.
40 St.
E.39 St.
E.38 St.
E 37 St.
E.36 St.
E.35 St
E.34 St.
E.33 St.
E.32 St.
E. 31 St.
E. 30 St.
E. 29 St.
E. 28 St.
E. 27 St.
7th Ave
Broadway
6th Avenue
5th Avenue
Madison Ave
Park Ave
4th Ave
Lexington Ave.
Daily News Building 900 Ft East
Chrsyler Building
Empire State Building
See Enlarged Section, Fig 7
Heights, in Feet, Shown Thus: 140 Underlined Numerals are Estimates
320
445'
512'
340
530
845
403
640
322'
408'
605
370'
202'
332'
470
660
278'
402'
778'
416'
220'
405
612'
660'
320'
1 035
320'
214'
670'
620
350'
385'
228'
730
370
340
273'
304'
603
585
360
340
640'
690
582
350
302'
348'
560
405
275'
270'
235'
130
412
590'
335'
248
195
140'
250
246
205'
170
216
245
390'
205
196
350
330'
195'
327
0
1/8
1/4
3/8
1/2
5/8
3/4
Scale in Miles

FIG. 6.—LOCATION AND HEIGHTS, IN FEET, OF BUILDINGS IN THE VICINITY OF THE STRUCTURE

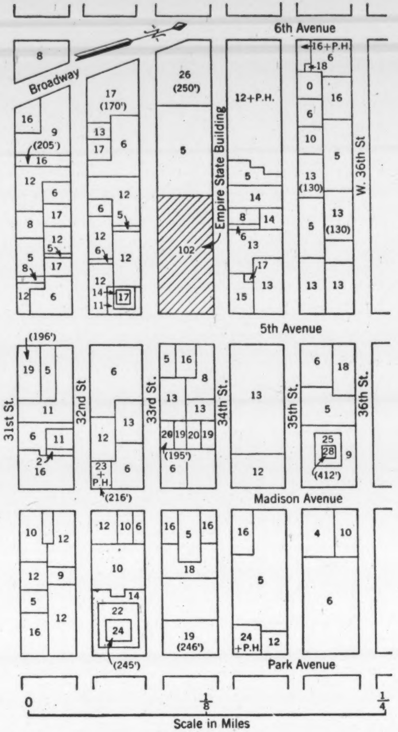

6th Avenue
Broadway
Empire State Building
W. 36th St
5th Avenue
31st St.
32nd St
33rd St.
34th St.
35th St.
36th St.
Madison Avenue
Park Avenue
8
16
9
(205')
12
6
17
5
8
17
(170')
13
26
(250')
102
12+P.H.
14
15
16+P.H.
18
0
10
13
(130)
(196')
19
11
2
23 + P.H.
(216')
20
(195')
25
28
(412')
4
22
24
(245')
(246')
24 +P.H.
0
1/8
1/4
Scale in Miles

FIG. 7.—HEIGHT OF BUILDINGS IN IMMEDIATE VICINITY OF STRUCTURE; SHOWN IN TERMS OF NUMBER OF STORIES; NUMBERS IN CIRCLES ARE HEIGHTS, IN FEET

Bureau Station, which to a considerable extent affect the readings of the anemometer. These tall buildings lie to the north and the northeast. To the south and west there is nothing between the instruments and the harbor. The anemometer is 454 ft above the harbor, and about 4.5 miles due south of the Empire State Building.

The News Building is on the south side of Forty-Second Street between Second and Third Avenues, and about 900 ft east of Lexington Avenue. This structure is much taller than any other building east of Lexington Avenue, except the Chrysler Tower, so that all buildings that are anywhere near the anemometer in height are shown in Fig. 6. The height of the anemometer is 504 ft above the street and 30 ft above the roof.

The New York City Meteorological Station is in Central Park, about 0.3 mile west of Fifth Avenue and 2.9 miles north of the Empire State Building. The observatory is in the approximate center of the park which has a width of about 0.5 mile and a length of a little more than 2.5 miles. The anemometer is 62 ft above the ground and is of the rotary cup type. This elevation places it well above the trees of the park but lower than the buildings on the adjoining streets.

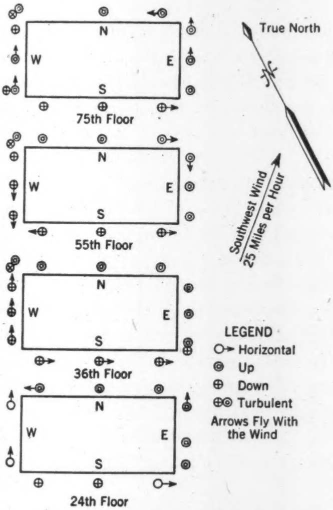

N
W
E
S
75th Floor
55th Floor
36th Floor
24th Floor
True North
Southwest Wind
25 Miles per Hour
LEGEND
Horizontal
Up
Down
Turbulent
Arrows Fly With the Wind

FIG. 8.—SNOW OBSERVATIONS, JANUARY 11, 1936, 2:00 TO 4:00 P.M.

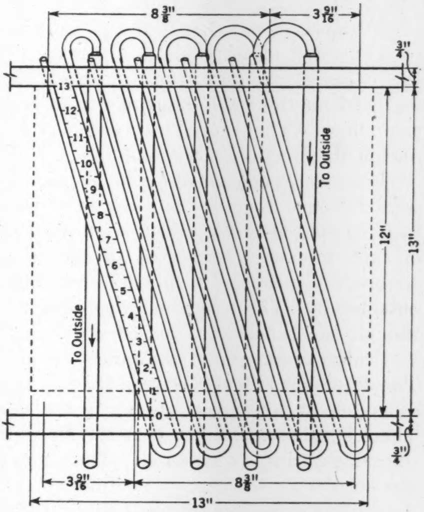

$8\frac{3}{8}''$
$3\frac{9}{16}''$
$\frac{3}{4}''$
13
12
11
10
9
8
7
6
5
4
3
2
1
0
To Outside
To Outside
12"
13"
$\frac{3}{4}''$
$3\frac{9}{16}''$
$8\frac{3}{8}''$
13"

FIG. 9.—U-TUBE MANOMETERS

The direction and approximate velocity of the air currents around the Empire State Building have been observed by watching the drift of fog and of snow flakes. The results of one typical observation are given in Fig. 8 which shows that the flow of the air is greatly disturbed by the building in a manner that is far from simple. Another observation on February 8, 1936, showed definite vertical currents, up as well as down, whereas the anemometer recorded a zero reading and the direction arrow rotated in an irregular manner over the entire range of the dial.

During a strong wind a rubber balloon was tied to a long thread and sent out from the highest point of the mast. It did not move horizontally away from the wind as was expected, but eddied about the lee side of the building and the mast. It made one vertical trip the full length of the thread (several hundred feet) and returned. It did not leave the lee side of the building until 15 min after the thread was severed. From these observations it is quite evident that the velocity of the wind acting on the surface of a building is entirely too complicated to be represented by a single reading.

Table 1 shows that recording instruments within a few miles of each other differ widely in their readings while recording a storm. This table (which justifies considerable detailed study) reveals the fact that each station has individual characteristics that must be understood, when interpreting data.

MANOMETERS

In order to study the pressure and the pressure distribution of the wind over the surface of the building ten gages were placed on each of three floors: The thirty-sixth, the fifty-fifth, and the seventy-fifth (see Fig. 4).

The opening in the outside wall consisted of a hole, ¼ in. in diameter, placed on a spandrel of aluminum. Great care was taken in boring the hole to have the edges sharp and true. A 0.5-in. galvanized iron pipe, with specially designed air-tight couplings, led from each hole to one of two closets on this same floor. These pipes were embedded in the floor and the holes were bored during the erection of the building.

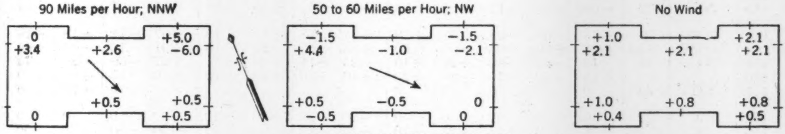
Each closet contained a rack consisting of ten glass tubes (see Fig. 9) which were fastened together in pairs at their lower ends by a short rubber tube, forming five U-tubes, which were partly filled with colored water. One end of each tube was connected to one of the galvanized pipes, previously described, by means of a rubber tube, thus subjecting it to the pressure of the outside air. The other end was open to the air pressure of the closet. Although the tubes were placed on a slant the scales were made to measure vertically; therefore, no allowance was made for this increase in length of tube. The difference in level between the two columns of water gave a measure of the difference in pressure of the air in the closet and that against the ¼-in. hole. There is some possibility that the air pressure in the two closets was not the the same during a high wind. The doors were left open during visual readings but not during the camera readings.

When the equipment was being installed it was felt that better information could be obtained by having all readings (on the manometers and on the steel) made at the same time by electrical control. In each closet a camera was focused on the five U-tubes; and each extensometer (described subsequently) had a camera attached to it. All were operated from a common button on the twenty-fourth floor. In its operation this set-up did not prove entirely satisfactory. During a strong wind storm the building not only deflects but also vibrates, and the camera may catch a reading on the steel at the maximum or minimum of the strain due to this vibration. With the manometers, on the other hand, the fact that the wind comes in gusts might cause a reading higher or lower than the mean pressure. The average was taken in all cases when visual readings were made. It took several minutes from the time the dial was read on the first floor until the observer had made all his readings on the several floors and returned to the first floor. In a considerable number of cases the character of the storm changed very materially during this time.

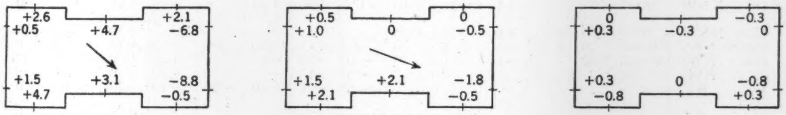
The cameras were a source of annoyance because they often jammed, and, in this way, much valuable information was lost.

In computing the pressures given in Table 2, 1 in. of difference in liquid elevation was taken as 5.2 lb per sq ft on the wall surface. The manometer readings given in Table 2 show the general character of the pressure distribution during a number of characteristic storms. Where the wind changed materially during a visual observation the results were not recorded. It is worthy of note that there is a great variation in the distribution of load among the floors. Some of the storms seem to affect only the upper floors and anemometers whereas others affect some lower level. One instance was noted in which the wind registered 70 miles per hr. Below the seventieth floor it was calm whereas above that floor the wind was strong. At another time a strong wind was noted at the thirty-sixth floor and it was comparatively calm at and above the fifty-fifth floor.

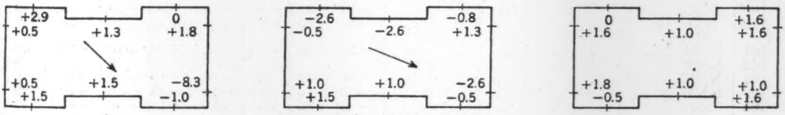
The manometer tubes (see Fig. 4) recorded both positive and negative pressures. Fig. 10 gives a graphic illustration of the distribution of the wind pressure over the surface of the building and impresses one with the lack of uniformity of this pressure. Three types of storms, as recorded by the anemometer, were selected.


90 Miles per Hour; NNW
0
+3.4
+2.6
+5.0
−6.0
+0.5
+0.5
+0.5
0
50 to 60 Miles per Hour; NW
−1.5
+4.4
−1.0
−1.5
−2.1
+0.5
−0.5
−0.5
0
0
No Wind
+1.0
+2.1
+2.1
+2.1
+2.1
+1.0
+0.4
+0.8
+0.8
+0.5

(*a*) THIRTY-SIXTH FLOOR


+2.6
+0.5
+4.7
+2.1
−6.8
+1.5
+4.7
+3.1
−8.8
−0.5
+0.5
+1.0
0
0
−0.5
+1.5
+2.1
+2.1
−1.8
−0.5
0
+0.3
−0.3
−0.3
0
+0.3
−0.8
0
−0.8
+0.3

(*b*) FIFTY-FIFTH FLOOR


+2.9
+0.5
+1.3
0
+1.8
+0.5
+1.5
+1.5
−8.3
−1.0
−2.6
−0.5
−2.6
−0.8
+1.3
+1.0
+1.5
+1.0
−2.6
−0.5
0
+1.6
+1.0
+1.6
+1.6
+1.8
−0.5
+1.0
+1.0
+1.6

(*c*) SEVENTY-FIFTH FLOOR

FIG. 10.—OBSERVATIONS DENOTING PRESSURE IN POUNDS PER SQUARE FOOT; ARROW INDICATES DIRECTION OF WIND

TABLE 2.—Record of Manometer Readings (Pressure

Item No.*	Wind Characteristics		Thirty-Sixth Floor (at Observation Stations; see Fig. 4)										Fifty- (at Obser-		
	Direction	Velocity, in miles per hour	1	2	3	4	5	6	7	8	9	10	1	2	3
	SSE	68	+26	+36	+30	0	−20	−36	−19	−28	−35	+21	+50	+53	+47
	WNW	31	−12	−05	−10	−04	+30	+40	+17	+05	+10	00	−04	−09	−08
103	WNW	42	+05	+20	−07	−01	+90	+37	0	+05	+10	+10	−20	−15	−10
106	NE	50	+04	−15	−17	−03	−23	−30	+18	+28	+29	+70	−07	−26	−30
107	ENE	52	+10	−30	−40	−04	−30	−18	+26	+60	+30	+50	+16	−45	−46
109	ENE	60	0	−50	−40	−10	−40	−10	+10	+30	+40	+60	0	−30	−50
111	S	24	+20	+22	+10	0	0	−05	0	0	0	+10	+07	+10	−05
113	ENE	12	+03	+02	+05	−01	+06	+04	−07	+15	+15	+18	−08	−10	−14
116	W	30	−02	+14	+10	+05	+06	+07	0	+05	+03	+03	−03	+19	+25
117	SW	35	−04	+16	+21	+05	−03	+03	+07	+08	+06	+03	−03	+17	+23
118	SSW	7	+10	+ 7	+ 7	0	+10	+07	+09	+06	+04	+12	−10	−11	−17
119	SW	43	−15	+29	+42	+05	−03	−05	+04	+11	+13	0	−19	+16	+08
122	SSW	8	+01	+04	+04	+ 0	+03	+04	+06	+05	+03	+06	−03	−04	−09
123	WSW	35	−02	+18	+29	+07	+22	+14	+04	+05	+03	+07	−12	+09	+18
125	SSW	53	−37	+54	+45	+05	−90	−25	−08	0	+02	−20	−25	+34	+49
..	NE	13	+19	+09	+10	0	+13	+12	+ 6	+31	+27	+22	−07	−14	−16
127	ENE	16	+09	+05	+06	0	−03	+08	+19	+21	+25	+21	−09	−16	−19
128	W	35	+02	+07	+16	+01	+29	+21	−06	−14	+06	+06	−06	−05	−25
..	NE	52	−07	−75	−60	−03	−40	−20	+11	+35	+80	+40	+01	−39	−52
130	NW	50	−06	−13	−13	−03	+51	+56	−20	+36	+11	−19	−29	−26	−27
135	W	10	..	..	..	..	..	+07	+08	+03	+08	+05	−08	−06	−09
137	NW	58	..	..	..	..	..	−13	+20	+07	0	−30	−38	−47	−54
138	NW	57	..	..	..	..	..	−33	−25	+15	+39	+18	−18	−28	−42
144	WNW	52	−08	0	0	−04	+14	+19	−12	+02	+13	+02	−24	−31	−44
145	NE	32	−08	+08	+12	+02	+13	+06	+32	+35	+31	+15	−05	−05	−04
147	NW	39	−66	−09	−28	−01	+18	+31	+13	+14	+15	−07	−12	−18	−19
148	WNW	45	−03	+10	+05	−06	+50	+25	−15	−29	−15	+01	−10	−05	−45
149	NE	44	−10	−25	−60	−28	−40	−25	+14	+40	+51	+27	−09	−29	−43
150	...	15	+15	+10	+15	+08	+20	+40	+20	+40	+40	+40	−15	+05	+ 0
151	SE	35	0	0	0	0	0	..	..	..	..	..	0	+20	+20
152	SW	22	0	0	0	0	0	−20	−25	−20	−20	−50	0	0	0
153	SE	55–60	+20	0	+25	+25	+10	−15	−25	−20	+40	−130	−80	+30	+50
154	E	55	..	0	+20	+20	+10	−10	−30	−40	−50	−20	0	+10	+10
155	NNW	99	−20	0	+20	+20	+ 0	−105	+60	+35	+20	+65	+30	+60	+60
155	NNW	90	..	0	+10	+10	+10	−115	+95	+50	0	+65	+30	+90	+60
155	NNW	90	−10	+10	+10	+10	+10	..	..	..	..	..	+10	+20	+10
156	NW	50–60	0	−10	−10	0	0	−60	−15	−20	−10	+95	+15	..	+50
156	NW	50–60	+10	−10	−10	0	0	−40	−30	−20	−30	+85	+30	+40	+40
157	NW	50–60	..	0	0	0	0	−20	−15	−20	−20	−20	0	−10	+10
158	S	35	..	..	..	..	..	−20	−20	−20	−20	−40	−10	0	0
159	S	20	..	..	..	..	..	..	..	..	..	..	0	0	0
159	S	20	..	..	..	..	..	−25	−25	−30	−30	−50	0	0	0
159	S	20	..	..	..	..	..	−20	−20	−30	−30	−50	0	0	0
160	NW	40.45	..	..	..	..	..	−50	−25	−30	−35	+90	+20	+25	+20
160	NW	40.25	..	..	..	..	..	−40	−15	−25	−45	+70	+10	+20	+20
161	NNW	..	..	..	..	..	..	−70	−10	−30	−40	+70	+10	+10	+10
162	NNW	..	..	..	..	..	..	..	..	..	..	..	0	0	0
162	NNW	..	..	..	..	..	..	−35	−30	−30	−30	−10	0	0	0
163	NNW	..	..	..	..	..	..	−15	−15	−15	−10	−25	0	−20	−20
164	NW	..	..	..	..	..	..	−85	+85	+65	+10	+15	−90	−10	+30
164	NW	..	..	..	..	..	..	..	..	..	..	..	−90	−10	+30
164	NW	..	..	..	..	..	..	−70	+60	+50	+10	0	−90	−10	+30
165	WNN	..	..	..	..	..	..	−35	−10	−20	−10	−20	0	−10	0
166	WNN	..	..	..	..	..	..	−35	−10	−10	−10	−20	..	..	..

* See corresponding Item No. in Table 1.

ON SIDE OF BUILDING, IN HUNDREDTHS OF AN INCH OF HEAD OF WATER)

FIFTH FLOOR VATION STATIONS; SEE FIG. 4)							SEVENTY-FIFTH FLOOR (AT OBSERVATION STATIONS; SEE FIG. 4)									
4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
+08	−23	−27	−17	−30	−22	+20	−29	−44	+20	+18	−40	−24	−02	−17	−17	+30
+08	−42	+55	−10	0	−02	−03	−15	−12	−17	+03	+12	+40	−03	−12	−10	−15
00	+50	+50	00	0	0	−10	−10	−20	−20	+05	+35	+55	0	−25	−15	−05
+07	−40	+53	+32	+29	+07	−38	−07	−27	−37	+04	−37	−31	+03	+15	+20	+14
+06	−50	−38	+15	+29	+36	+56	+07	−50	−43	00	−93	−60	+10	+60	+48	+70
0	−50	−60	0	+60	+50	+30	0	−50	−55	0	−50	−50	0	+80	+50	+50
+05	−10	−10	−05	−10	−08	0	0	+07	+02	+07	−27	−12	0	−10	−20	−07
+05	−12	−13	−06	−06	−08	−05	19	−28	−28	+03	−29	−24	+02	−17	−21	−23
+22	−09	−02	−03	−07	−12	−05	02	+09	+18	+12	−08	0	+04	−06	−15	−14
+22	−16	−08	−04	−06	−08	−03	11	+15	+24	+25	−20	−07	+01	−09	−11	−13
+05	−15	−20	−08	−18	−21	−09	−35	−36	−35	0	−39	−39	−04	−27	−33	−34
+11	−35	−29	−11	−18	−29	−19	−27	+09	0	+05	−35	−40	−03	−25	−30	−33
+03	−07	−12	−06	−12	−11	−09	−17	−21	−22	+03	−24	−18	−01	−15	−22	−17
+24	+10	−03	−07	−15	−17	−08	−25	−21	−08	+04	− 2	−13	−03	−24	−29	−23
+32	−10	−40	−13	−22	−23	−25	−55	+30	−51	+20	−48	−35	0	−75	−33	−43
+04	−13	−14	−07	−09	−08	−03	−32	−44	−45	+02	−43	−44	0	−21	−27	−29
+03	−14	−15	−06	−03	−03	+03	−36	−40	−46	+01	−41	−40	0	−24	−23	−40
+06	+21	+13	−09	−15	−22	−10	−19	−30	−25	+03	+05	0	0	−20	−25	−24
+06	+46	−47	+07	+32	+41	+14	−22	−59	+33	+02	−64	−62	+08	+60	+67	−51
+05	+19	+55	+04	+11	−12	−22	−41	−51	+57	+05	−03	+44	+03	−08	−27	−39
+06	−09	−10	−19	−16	−15	−09	−25	−26	−24	+03	−24	−27	−02	−21	−27	−29
0	+36	+71	−28	−37	−30	−37	−62	−64	−72	0	+04	+06	09	−25	−50	−59
+07	+12	+45	−37	−49	−24	−26	−42	−63	+47	0	+43	+33	04	−45	−70	−41
+05	0	+21	−32	−32	−26	−22	−37	−50	−53	−04	+32	+24	11	−42	−43	−39
+04	−18	−17	+14	+24	+11	−24	−13	−26	−29	0	−27	−33	0	+03	0	−40
+03	+22	+21	+05	−05	−08	−16	−35	−39	−37	0	0	−34	−02	−05	−20	−29
+05	−47	+46	−12	−17	−23	−13	−22	−33	−83	+02	+17	+50	−02	−31	−31	−32
+02	−34	−39	0	+15	+22	+14	−29	−44	−45	0	−60	−59	−01	−02	+09	+07
+15	+05	+05	0	−05	−15	0	+20	+30	+20	−10	+35	+30	0	+20	+30	+30
−10	+10	+10	+10	+15	+25	−25	+10	+30	+30	−10	+35	..	..	..	..	..
−10	0	0	0	0	+10	0	..	..	..	..	..	..	..	..	..	..
−10	+10	+10	−20	+40	+60	−90	−70	+30	+30	0	+30	..	..	..	..	..
−20	+20	+20	−10	−30	−40	−90	0	+20	+20	−10	+20	..	..	..	..	..
−10	−150	−110	+60	+80	+20	+30	+20	+40	+40	−20	−180	..	..	..	..	..
−10	−170	−130	+40	+90	+50	+10	+10	+30	+30	−20	−160	+30	0	+25	+55	+10
−10	−45	−30	+20	+10	+10	+10	+20	+30	+20	−10	−50	..	0	+10	+15	+10
−10	−40	−50	+20	0	+20	+10	+20	+30	+20	−10	−40	+40	0	+20	+70	−90
−10	−35	−50	0	0	+10	+20	+20	+30	+20	−10	−50	+25	−15	−50	−50	−10
−10	−10	−10	0	0	0	0	0	0	0	−10	0	+05	0	0	+10	+05
−20	0	0	0	0	0	0	0	+10	+10	0	+10	..	..	..	..	..
−20	0	0	0	0	0	0	+20	+20	+10	−10	+20	..	..	..	..	..
−20	0	0	0	0	0	0	+20	+20	+20	0	+20	..	..	..	..	..
−20	0	0	0	0	0	0	+20	+20	+20	0	+20	..	..	..	..	..
−10	−35	−50	..	0	0	+10	+20	+30	+20	−10	−40	..	..	..	..	..
−20	−60	−60	+10	0	0	+10	+20	+20	+20	−10	−40	..	..	..	..	..
−20	−50	−50	+10	0	0	+10	+20	+20	+20	−20	−30	..	..	..	..	..
−10	−20	−20	−10	0	0	0	0	+10	+10	0	0	..	..	..	..	..
−10	−20	−20	−10	0	0	0	+10	+10	+10	0	+10	..	..	..	..	..
−20	−10	−10	0	0	0	0	0	−10	−10	−20	0	0	−10	0	+10	0
+10	+10	−85	+10	+20	+30	+20	+20	+20	+40	−10	−70	−80	−10	+05	+30	+10
+10	+10	−80	+10	+20	+20	+20	+20	+20	+40	0	−60	−80	−10	+05	+30	+10
+10	+10	−85	+10	+20	+20	+10	+20	+30	+40	−10	−70	−10	−10	0	+15	+10
−10	−35	−30	0	+10	+10	+10	+20	+20	+20	−10	−10	−10	..	0	+15	+10
...	...	−30	0	0	+10	+10	+10	+10	+20	−10	−20	..	..	..	..	..

Messrs. Hugh L. Dryden and G. C. Hill, of the National Bureau of Standards, subjected a model of the Empire State Building, 5 ft high, to air currents of known velocity in a wind tunnel, and recorded the pressure at a great number of points,[3] including the stations shown in Fig. 4. A comparison of the pressures on the model and those on the building shows clearly that the natural wind movements are not at all like those in a wind tunnel.

DEFLECTION

For a study of the deflection of a tall building under wind loads a better planned structure than the Empire State Building could not be found. It is not only the tallest building in the world but it is symmetrical in plan about both planes parallel to its sides, and through its vertical axis. The stair well, or fire tower, is near this central axis (Fig. 4) and gives a clear sight from the sixth to the eighty-sixth floor.

The data reported herein are those of the movement of the eighty-sixth floor relative to the sixth. The lateral movement of the sixth floor is doubtless small and may be neglected.

In effect, the building is a huge cantilever and as such has two distinct movements when acted upon by a wind storm: It deflects from the vertical; and it vibrates with a definite period, similar to the tines of a tuning-fork when struck. A steady wind causes deflections only, whereas a gusty wind will set up vibrations with amplitudes that vary with the strength and character of the storm. Under the action of a strong wind the building vibrates about a mean position in the direction parallel to its narrow side and, with a different period, in a perpendicular direction. Apparently, these two vibratory motions are independent of each other, differing both in amplitude and period.

Two methods were used to study the motion of the upper part of the building during a storm. A specially designed vertical collimator with a right-angle eye-piece was set up on the sixth floor and sighted on a target on the ceiling of the eighty-sixth floor. From a point near this target a plumb-bob was suspended so that the bob hung near the sixth floor. The collimator proved very useful in studying the vibrations of the building both as to period and amplitude; the plumb-bob was more useful in obtaining the deflections. At the time of reading either instrument the velocity and direction of the wind at the mast head were recorded.

THE FIRE TOWER

One of the most valuable features of the building from the standpoint of this investigation is the fire tower. Although there are several offsets in this tower there is a clear opening from the sixth floor to the eighty-sixth floor (a vertical distance of 972.5 ft) of about 54 in. in either direction, leaving sufficient room for both the plumb-bob string, and the collimator sight. In order to defeat the tendency for vertical currents of air to occur in the shaft, a diaphragm was installed at the fifty-fourth floor, holes being left for the line of sight and the plumb-bob string. Even after the installation of this diaphragm there is a strong current of air into the tower when the doors leading from the building floors are opened, particularly above the fifty-fifth floor. When the doors are closed the leakage of air under the door is quite perceptible but the volume of air admitted is small. At each floor the stairway is separated from the tower by a partition containing a door. All doors to the tower were kept closed except when in use.

---

[3] "Wind Pressure on a Model of the Empire State Building," by Hugh L. Dryden and G. H. Hill, *Research Paper No. 525*, National Bureau of Standards.

As the tower is very close to the vertical central axis of the building any rotation of the building about this axis does not affect the readings. Deflections that persisted over a considerable period of time could be obtained to any desired degree of accuracy by means of the plumb-bob, whereas rapid movements could be studied by means of the vertical collimator.

The Plumb-Bob

The plumb-bob consisted of a 35-lb concrete cylinder (Fig. 11) containing considerable scrap iron. It was suspended by a three-strand sounding cable from the ceiling of the eighty-sixth floor to a convenient height above the sixth floor so that the distance from the point of suspension to the center of the bob was 969.25 ft. Due to this great length of suspending wire the bob did not follow rapid movements of the building, but maintained a position under the mean location of its point of support.

The first bob designed had a brass point on the lower end but trouble was experienced by the weight bending this point when it hit the lower target (marked in tenths of an inch) on which the readings were made. In order to prevent this effect, and to allow for vertical adjustment, a very small bob was suspended from the lower end of the larger one. The point of this second bob was adjusted so that it exactly cleared the target.

Oil baths were used to damp out the swing of the bob. With the dampers the readings were easily obtained to within 0.1 in., or even closer, during a light wind. During a heavy wind the bob had a tendency to oscillate, due to the movement of the point of support and also to slight upward currents of air. As the motion in any definite direction was that of a damped harmonic it was not difficult to estimate the position of rest to within 0.2 in. and in most cases to within 0.1 in. Once or twice the position of rest was difficult to determine with an accuracy of less than 0.5 in. This occurred during very strong winds and was probably due to the shifting of the mean position of the eighty-sixth floor during the time the observations were being taken.

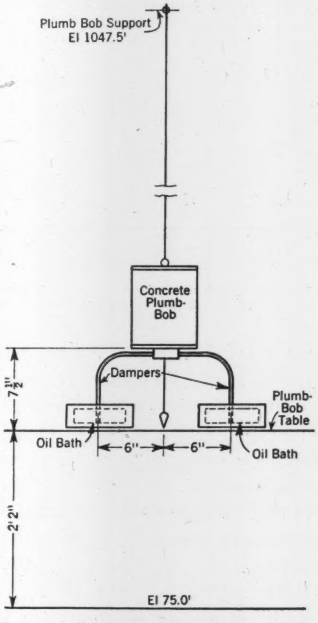


Fig. 11.—Diagram of Plumb-Bob Set-Up

Some question has been raised as to the accuracy of the plumb-bob as an instrument for noting the movement of the building because of the leakage of the air into the shaft and also because of the fact that the point of support is not steady. The size of the fire tower, except at the diaphragm across the fifty-fourth floor, is more than 125 sq ft, with the result that upward currents are slight and those that do occur do not deflect the plumb-bob string in one direction more than in another. When a door was opened a serious disturbance occurred, but it subsided at once when the cause was removed. As to the effect of vibration on the action of the bob, it is to be noted that the building vibrates from 6 to 8 times per min whereas the computed period of the bob is between 34 sec and 35 sec. As the bob did not record movements of its support that are not of greater duration than the period of its swing, the building vibrations are not recorded by the bob, but their mean position is determined by it.

The tidal effect on a pendulum of this length can easily be shown to be small enough to be neglected.

A large number of readings have been recorded to show the position of the plumb-bob, together with the recorded wind velocity and direction. A few representative data are given in Table 3. In plotting these readings the writer divided them into three groups for the sake of clearness: Table 3(*a*), for a wind velocity of less than 20 miles per hr; Table 3(*b*), for recorded wind velocities between 20 and 30 miles per hr; and Table 3(*c*), for a wind velocity greater than 30 miles per hr. Fig. 12 is an example of the form in which the data were plotted.

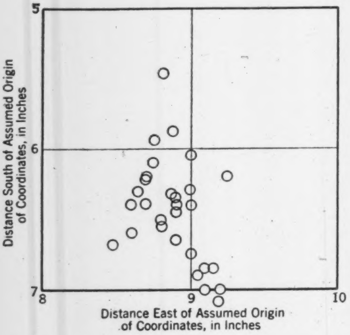


FIG. 12.—MOVEMENT OF PLUMB-BOB DURING A LIGHT WIND (0 TO 20 MILES PER HOUR)

The first group (see Table 3(*a*)) shows that the building fails to return to a definite position of rest. This is to be expected as there is considerable inelastic and semi-elastic material surrounding the steel frame. It is probable that this material resists recovery, and that there is a small amount of stress left in the steel after the building is relieved of a heavy wind pressure. As a result, if the wind were to shift to the opposite direction, the building would pass through the zero point of deflection much more readily than a simple stress-strain curve would indicate. This action would, in no way, endanger the building as the phenomenon will not occur except at low stresses. Fig. 12 gives an idea of the extent of this plastic set because it can be assumed that the deflection of the building due to the wind pressure is low and so may be neglected.

The forces due to the high wind (see Table 3(*c*)) are great enough to overcome the plastic deformations of the storm immediately preceding the one observed and so in each case, in the third group, the displacement would be due to the wind recorded and not to the effect of a previous storm.

TABLE 3.—PLUMB-BOB OBSERVATIONS, EIGHTY-SIXTH FLOOR TO SIXTH FLOOR

Item No.*	Date†	Eastern Standard Time, in hours and minutes past midnight	Wind Characteristics		Distance from Assumed Origin, in Inches	
			Compass direction	Velocity, in miles per hour	East	South
(a) Wind Velocity Less Than 20 Miles per Hour						
9	3/25	13:25	SSE	15	9.00	6.30
18	5/18	14:00	N	8	8.60	6.60
19	5/19	15:25	SSE	15	9.00	6.40
21	5/23	11:15	W	15	8.70	6.40
22	5/23	12:00	W	12	8.70	6.20
23	5/24	11:00	SSW	8	8.80	6.50
24	5/25	10:10	WSW	8	8.90	6.35
35	6/13	11:50	NE	15	8.60	7.20
36	6/14	15:00	NE	2	9.10	7.00
39	6/16	14:25	SSE	12	9.15	6.85
49	6/29	16:55	WNW	12	9.20	7.00
52	7/ 6	13:00	SW	12	8.90	6.65
53	7/ 7	14:40	SSE	12	9.05	6.90
55	7/14	12:55	NNW	10	9.18	7.08
57	7/25	14:30	WNW	12	9.10	6.85
58	7/27	10:30	SSW	15	8.90	6.45
60	8/ 2	10:15	SSW	13	8.45	6.65
63	8/22	12:00	NW	5	8.80	6.55
64	8/23	11:45	NE	10	8.65	6.40
65	8/24	13:25	S	8	8.75	6.10
66	8/31	16:25	E	6	8.90	6.40
67	9/ 7	11:25	NNW	12	8.65	6.30
71	9/12	14:07	W	10	8.87	5.87
87	10/14	16:51	WSW	10	8.81	5.47
96	10/25	13:10	E	15	9.00	6.05
97	10/26	11:05	SSE	13	9.25	6.20
120	12/27	17:20	ENE	12	8.75	6.95
127	1/13	16:16	NE	15	8.89	6.31
136	2/18	16:30	WSW	18	8.70	6.21
...	3/23	16:50	SW	15	8.60	6.40
165	4/17	11:00	WNW	3	8.50	7.00
(b) Wind Velocity from 20 to 30 Miles per Hour						
25	5/25	14:50	SSW	22	9.05	6.50
32	6/ 8	13:25	WNW	25	8.80	6.40
33	6/ 8	14:40	WNW	25	8.80	6.40
37	6/15	17:00	S	25	9.10	6.40
42	6/17	14:00	NNE	20	9.10	7.10
46	6/24	11:15	WNW	25	9.00	6.50
51	7/ 5	17:15	WNW	20	9.15	7.00
56	7/19	12:45	WNW	20	9.00	6.90
59	7/29	12:30	WNW	20	8.85	6.35
72	9/27	10:00	SSW	22	8.30	5.40
73	9/27	16:15	SSW	22	8.70	5.70
88	10/17	9:00	SE	22	9.20	6.10
92	10/21	10:00	W	22	8.90	5.20
93	10/21	10:20	W	28	9.20	5.65
95	10/22	9:10	NE	22	8.90	6.25
98	10/28	14:40	SSE	25	9.30	6.15
111	11/11	11:00	S	25	8.80	6.00
128	1/20	16:10	WNW	28	8.37	5.82
152	3/ 9	17:00	SW	22	8.90	6.40
158	4/ 1	14:15	S	25	8.10	6.30
159	4/ 7	12:25	E	20	8.40	7.30
162	4/13	15:50	NNW	20	8.70	7.25
163	4/14	15:45	NNW	25	8.20	6.80
(c) Wind Velocity More Than 30 Miles per Hour						
27	5/26	15:50	SSW	35	8.80	5.90
31	6/ 7	16:45	N	35	8.50	7.40
43	6/22	....	....	....	....	....
44	6/23	....	....	....	....	....
45	6/23	15:25	W	32	9.30	6.60
48	6/24	....	....	....	....	....
50	7/ 1	....	....	....	....	....
54	7/11	13:10	NW	40	9.50	7.20
61	8/ 4	10:27	S	32	8.45	5.75
62	8/ 4	13:50	S	38	8.55	5.60
68	9/ 8	12:45	NNW	42	8.30	7.52
69	9/ 8	14:50	NNW	55	8.80	8.64
70	9/ 8	16:10	NNW	40	8.60	7.48
..	9/28	9:30	SSW	42	8.80	4.78
77	10/ 3	13:10	S	42	8.72	4.45
79	10/ 4	16:40	SE	35	8.87	5.85
80	10/ 5	9:00	SSE	52	8.70	5.05
81	10/ 5	14:40	SSE	45	8.85	5.40
83	10/ 7	10:15	WNW	32	8.80	5.80
84	10/11	9:20	WNW	34	8.67	5.08
86	10/12	13:10	W	47	8.88	4.06
89	10/18	11:35	NE	35	9.07	7.51
90	10/18	11:50	NE	42	9.07	7.51
91	10/19	8:50	NNE	32	8.85	7.13
94	10/21	16:20	WNW	50	9.55	5.69
100	11/ 1	9:00	SE	75	8.20	4.75
101	11/ 1	10:30	SE	75	7.93	4.63
103	11/ 1	16:05	WNW	42	8.68	5.33
104	11/ 1	16:50	WNW	45	8.45	5.18
105	11/ 1	17:15	WNW	50	8.53	4.90
109	11/ 9	14:45	ENE	60	8.20	9.70
110	11/ 9	16:30	ENE	60	8.29	9.76
112	11/15	9:50	SW	35	8.60	5.20
115	11/19	13:10	SSE	65	8.36	3.26
124	1/ 7	12:15	WSW	50	9.39	4.23
125	1/11	9:10	SSW	42	8.36	4.23
126	1/11	10:35	SSW	50	8.57	3.82
129	1/25	9:30	ENE	58	8.72	9.21
130	1/28	17:28	NW	47	8.89	7.12
131	2/ 2	13:45	WSW	40	9.19	5.39
132	2/10	10:10	W	35	9.25	5.58
133	2/15	10:00	NW	40	9.10	7.04
134	2/15	12:15	NW	46	8.88	7.80
151	3/ 2	18:40	SE	35	8.40	6.25
153	3/11	14:45	SE	55–60	7.20	6.50
154	3/18	14:40	E	55	7.25	9.00
155	3/22	10:25	NNW	90	10.70	7.20
156	3/22	15:55	NW	50–60	9.8	7.5
157	3/22	16:10	NW	50–60	9.8	7.5
160	4/ 8	9:36	N to NW	40–45	9.25	7.25
161	4/15	10:00	NW	45	9.30	7.0
...	....	....	....	....	....	....
...	....	....	....	....	....	....
...	....	....	....	....	....	....
...	....	....	....	....	....	....
...	....	....	....	....	....	....
...	....	....	....	....	....	....
...	....	....	....	....	....	....
...	....	....	....	....	....	....

* See corresponding numbers in Table 1.

† Month/day.

It is to be noted that the values in Table 3 do not indicate the maximum deflection of the building. They are only the mean position about which the building vibrates; for a maximum deflection one-half the amplitude of vibration should be added.

In order to make a further study of this plastic set in the narrow direction of the building, three sets of measurements were made to the plumb-bob string from reference points, on the several floors. As it required more than two hours for each set of readings it was necessary to make them during calm weather in order to be assured that the building did not shift during the time of the observations. These variations are plotted in the curves of Fig. 13.

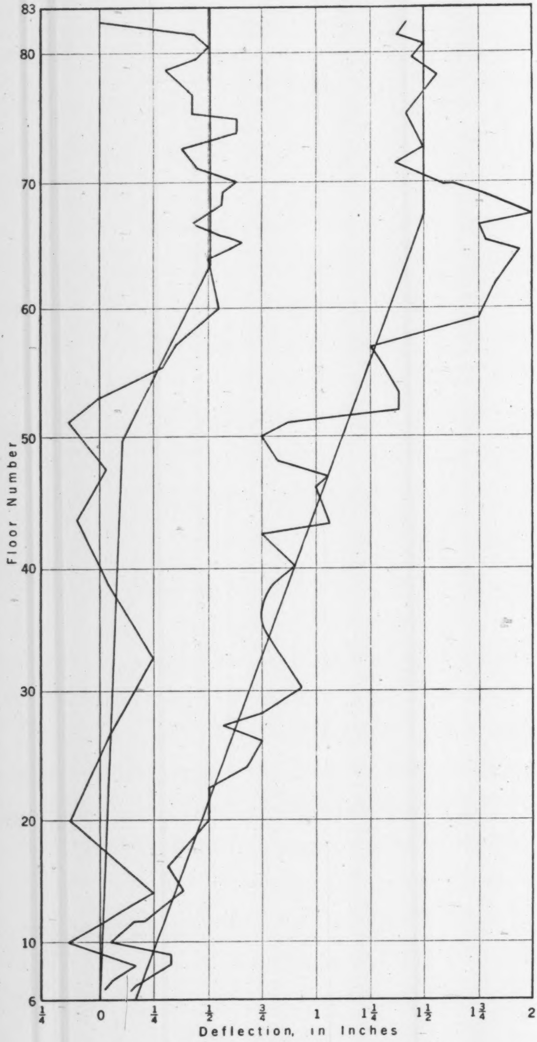


Fig. 13.—Check Measurements to Plumb-Bob String

Temperature Deflections

In order to ascertain the deflection due to the inequality of the temperature of the several sides of the building, observations were made throughout the day of August 24, 1936. The sky was clear and sunny, and the maximum temperature was 86° F at 3:00 P.M. There was little wind blowing, the anemometer reading from 25 to 30 miles per hr, with a steady wind from the west northwest. Table 4 gives the readings of the plumb-bob showing a north deflection of 0.25 in. at noon and an east deflection of 0.15 in. at 3:00 P.M. As the day was not cloudy, leaving the surface of the building exposed to the heat of the sun, it is inferred that the effect of temperature is only a minor factor in the results given in this paper.

TABLE 4.—PLUMB-BOB OBSERVATIONS ON A HOT, CLEAR DAY, AUGUST 24, 1936

Description	DISPLACEMENT OF PLUMB-BOB, IN INCHES, FROM THE ORIGIN, AS OBSERVED AT THE FOLLOWING TIMES (EASTERN STANDARD, IN HOURS AND MINUTES PAST MIDNIGHT)								
	9:15	10:00	11:00	12:00	12:30	13:00	14:00	14:30	15:00
Wind velocity, in miles per hour*	30†	....	25†	28‡	....	25†	30‡	....	30§
Position of Plumb-Bob, in Inches, from the Origin:									
Southward	7.00	7.00	6.80	6.80	6.75	6.80	7.00	6.90	7.00
Eastward	8.60	8.60	8.50	8.50	8.55	8.45	8.50	8.60	8.45

* Anemometer readings. † West, northwest. ‡ Northwest. § North, northwest.

The collimator was designed by the United States Coast and Geodetic Survey for the purpose of locating the reading points of triangulation survey marks above their permanent points. It consists essentially of a transit equipped with a right-angle eye-piece and a leveling bubble. The telescope has a focal length of about 8 in. One division of the leveling bubble represents 0.56 in. on the target; that is, the bubble has a sensitiveness of about 10″ of arc per division. The target can be read to 0.5 in.

As the principal value of the collimator lay in the study of the vibrations of the building it was used during high winds only. Hewitt Crosby, M. Am. Soc. C. E., reporting to the American Institute of Steel Construction,[4] stated that for some unknown reason the sixth floor tilted as much as four divisions of the dial, or approximately 40″ of arc. In light and steady winds the effect was eliminated by leveling the instrument carefully before taking a sight; but in strong or gusty winds, when readings must be made on a moving target, this was impossible. In extreme cases, the indicated error from this cause was found to be as much as 2 in. In strong winds, the motion of the target was irregular in direction, amplitude, and time.

Nevertheless, the instrument proved invaluable because it enabled one to study the movements of the top of the building qualitatively when acted upon by a high wind. Table 5 records the readings of the collimator, showing the amplitude of vibration of the building. A number of attempts were made to measure the deflection of other buildings by means of transits set up at a distance, but the writer knows of no case in which satisfactory results have been obtained.

---

[4] "Wind Stresses in the Structure of the Empire State Building," by Hewitt Crosby. Manuscript report to Am. Inst. of Steel Construction, New York, N. Y., June, 1933, pp. 30–33.

TABLE 5.—DEFLECTION READINGS BY COLLIMATOR

Item No.*	Date†	Eastern Standard Time, in hours and minutes past midnight	Wind Characteristics		Collimator Readings			
					East-West Direction		North-South Direction	
			Compass direction	Velocity, in miles per hour	Maximum	Minimum	Maximum	Minimum
79	10/4	16:30	SE	26	5.0	5.1	7.4	7.5
101	11/1	11:00	SE	75	4.0	4.0	7.0	10.0
101	11/1	11:15	SE	85	4.0	4.0	8.0	12.0
102	11/1	12:00	SSE	75	4.5	5.5	6.5	9.0
103	11/1	16:40	WNW	43	6.8	7.0	4.0	5.5
107	11/9	11:45	ENE	52	4.0	5.6	3.0	6.0
108	11/9	14:30	ENE	60	3.8	5.0	0.5	5.2
114	11/18	16:00	ENE	15	4.5	5.5	3.6	5.5
117	12/2	16:00	SW	35	4.8	5.1	7.9	8.5
119	12/21	11:40	SSW	34	5.3	6.5	6.7	7.5
123	1/6	16:25	SW	29	6.8	7.9	6.9	8.0
126	1/11	10:30	SSW	51	6.0	8.0	7.2	9.2
....	1/13	12:00	ENE	16	6.9	7.9	4.8	6.5
127	1/13	16:00	NE	16	6.5	7.8	5.5	6.8
128	1/20	16:00	WNW	28	7.4	8.2	5.3	6.9
129	1/25	9:40	ENE	58	5.0	7.2	0	4.3
130	1/28	17:00	NW	47	6.9	8.8	3.1	5.2
....	2/17	16:00	WSW	16	6.6	7.9	6.2	6.6
135	2/17	16:40	WSW	18	6.5	6.9	6.4	6.9
....	....	....	....	....	....	....	....	....

* See corresponding numbers in Table 1.

† Month/day.

The period of vibration was observed by means of the collimator on November 1, 1932 (see Fig. 14). The north-south movement is given in Fig. 15.

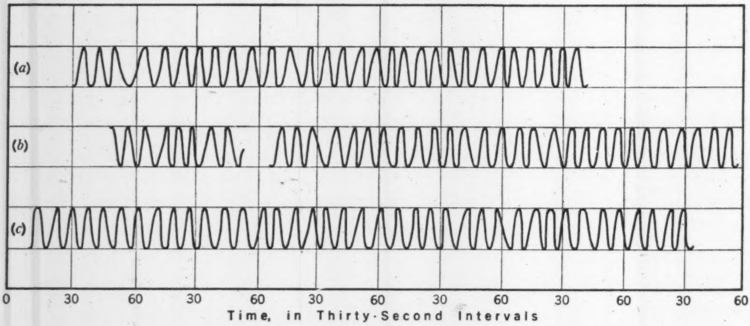


FIG. 14.—COLLIMATOR OBSERVATIONS ON THE SWING OF THE EMPIRE STATE BUILDING, NOVEMBER 1, 1932

The swing of one of the dials on the southwest corner column of the twenty-fourth floor (Fig. 4) was read on November 18, 1934. The curve thus obtained (see Fig. 15(*b*)) is the combination of two vibrations at right angles to each other, the north-south one dominating. The time of each high and low reading was taken and plotted. As the weight of the building was practically the same during these two observations the change of period from 8.28 sec to 8.14 sec is of considerable interest and is probably due to an adjustment of the several parts of the structure during severe storms.

EXTENSOMETERS

Four extensometers were attached to each of four columns on the twenty-fourth floor, by fastening an 0.001-in., Ames dial 8⅞ in. below the mid-height point. One end of a rod touches the stem of the dial, the other end being welded to the column 41⅛ in. above the midpoint of the column. In other words, the two points of attachment are 50 in. apart. They were all placed as near the outer end of the column flange (Carnegie H-beams) as possible.

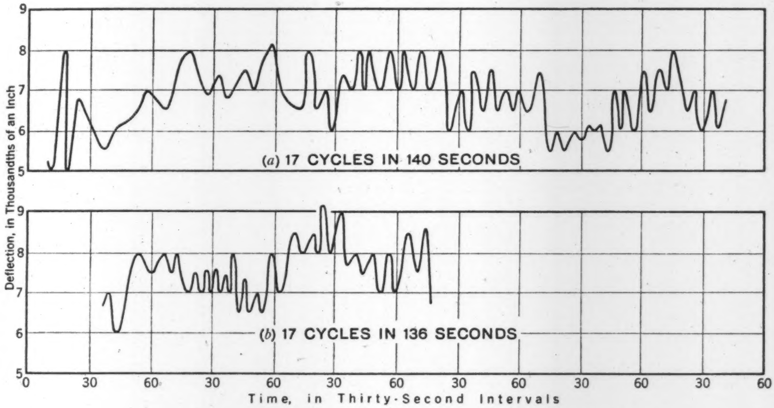


FIG. 15.—DEFLECTION OBSERVATIONS OF A CORNER COLUMN OF THE EMPIRE STATE BUILDING (TWENTY-FIFTH FLOOR)

With this arrangement the average of the differential readings of the dials gives the change of length of the column at its center line over a span of 50 in., thus furnishing a measure of the direct stress in the column. Although the angular change in the column can be computed it is not given herein because, unless the point of counterflexure is known, it is of little value.

The columns chosen for observation were the southwest corner column of the twenty-fifth floor and the three columns nearest to this corner. An offset in both directions places these columns in the interior on the twenty-fourth floor, thus facilitating the reading of the instruments.

Column (19) of Table 6 gives the sum of the readings of the several dials on the corner column in hundred-thousandths of an inch. Column (14) is the direct stress, in pounds per square inch, producing this strain. It is obtained by multiplying the strain by 29 000 000 and dividing by 50; or, the reading multiplied by 29 000 000 and divided by 50 × 100 000, or 5.8. In other words, the strain multiplied by the modulus of elasticity and divided by the length gives the stress, in pounds per square inch.

The stress in this same column due to the wind pressure has been computed from the manometer readings on the assumption that the building is a cantilever. The overturning moment of the wind about an elevation half-way between the twenty-fourth and the twenty-fifth floors has been computed for both a south

TABLE 6.—COMPARISON OF COMPUTED AND OBSERVED COLUMN STRESSES

Ob-ser-va-tion No.*	(a) MANOMETER READINGS, IN INCHES†						(b) STRESSES IN COLUMNS, IN POUNDS PER SQUARE INCH, FOR THE FOLLOWING DIRECTIONS OF THE WIND LOAD								(c) EXTENSOMETER READINGS, IN HUNDRED THOUSANDTHS OF AN INCH, ON THE FOLLOWING CORNERS OF THE POST¶				
	Elevation 36		Elevation 55		Elevation 75		Elevation 36		Elevation 55		Elevation 75		By computation, from manometer readings	By extensometer observations	NW	NE	SE	SW	Sum
	Manometers 2, 3, 4, 7, 8, and 9‡	Manometers 5, 6, 1, and 10‡	Manometers 2, 3, 4, 7, 8, and 9‡	Manometers 5, 6, 1, and 10‡	Manometers 2, 3, 4, 7, 8, and 9‡	Manometers 5, 6, 1, and 10‡	S	W	S	W	S	W							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
2	+1.48	−1.03	+1.82	−1.20	+1.49	+0.74	+28.7	−13.4	+100.6	−44.3	+189.5	−156.0	+103.8	−148.0	+7	+25	−76	−59	−103
7	−0.51	+0.82	+0.03	+1.04	−0.01	+0.82	−9.1	+10.7	+1.6	+38.4	−1.3	+73.4	+112.9	+52.0	−2	+42	+14	−18	+36
9	−0.03	+0.62	−0.25	+1.30	+0.05	+1.05	−0.6	+8.1	−13.8	+48.0	+6.3	+94.0	+142.0	+223.0	+4	+79	+73	−2	+154
10	−1.10	−1.27	−1.16	+0.58	−0.98	−0.75	−21.4	−16.0	−63.8	+21.4	−174.4	−67.1	−171.9	−343.0	−75	−27	−66	−69	−237
11	−1.90	−1.08	−1.65	−1.60	−2.11	−2.27	−36.9	−14.1	−90.7	−59.1	−268.0	−203.0	−671.8	−693.0	−138	−114	−117	−109	−478
12	−1.80	−1.10	−1.95	−1.40	−2.35	−1.50	−35.0	−14.4	−107.4	−51.6	−299.0	−134.2	−641.6	−712.0	−146	−110	−127	−109	−492
13	+0.32	−0.35	+0.33	−0.27	+0.46	−0.32	+6.2	−4.6	+18.2	−10.0	+58.5	−28.6	+39.7	−7.2	−4	+47	−33	−15	−5
14	−0.17	−0.23	+0.01	−0.12	−0.17	−0.11	−3.3	−3.0	+0.5	−4.4	−21.6	−9.9	−41.7	−110.0	−22	+26	−54	−26	−76
15	+0.21	+0.12	+0.88	−0.03	+0.56	+0.08	+4.1	+1.6	+48.4	−1.1	+71.2	+7.2	+131.4	−63.8	−27	+34	−6	−45	−44
16	+0.21	+0.01	+0.80	−0.18	+0.83	−0.03	+4.1	+0.1	+44.0	−6.6	+105.5	−2.7	+144.4	−98.5	−27	+35	−16	−60	−68
17	−0.05	−0.05	+0.24	−0.16	−0.03	−0.09	−1.0	−0.7	+13.2	−5.9	−3.8	−8.1	−6.3	+146.2	+9	+77	+6	+9	+101
18	+0.48	+0.07	+0.93	−0.26	+0.70	−0.15	+9.3	+0.9	+51.1	−9.6	+89.1	−13.4	+127.4	+355.0	+28	+97	+66	+54	+245
19	−0.10	0	+0.19	−0.07	−0.02	−0.08	−1.9	....	+10.5	−2.6	−2.5	−7.2	−3.7	−191.3	−54	....	−31	−47	−132
20	+0.42	+0.31	+0.90	+0.27	+0.31	+0.33	+8.2	+4.0	+49.5	+10.0	+39.4	+29.6	+140.7	+116.0	+11	+55	−4	+18	+80
21	+1.10	−0.58	+1.73	0	+0.57	+0.15	+21.4	−7.6	+95.1	....	+72.4	+13.4	+194.7	+341.0	+42	+78	+64	+51	+235
23	−0.65	−0.16	−0.02	−0.17	−0.39	−0.26	−12.6	−2.1	−1.1	−6.3	−49.5	−23.3	−94.9	+187.0	+15	+57	+24	+33	+129
24	−0.54	−0.25	−0.20	−0.24	−0.38	−0.05	−10.5	−3.3	−11.0	−8.9	−48.3	−4.5	−86.5	+165.0	+14	+57	+16	+27	+114
25	+0.38	−1.42	+0.22	+0.50	−0.07	+0.48	+7.4	−15.5	+12.1	+18.5	−8.9	−143.0	+77.6	−75.3	−18	+28	−19	−41	−52
26	−2.64	−1.33	−1.65	−1.08	−1.59	−0.53	−51.3	−17.4	−90.8	−39.8	−202.0	−47.5	−448.8	−488.0	−20	−108	−112	−99	−337
28	−0.56	+1.32	−0.51	+1.25	+0.43	+1.27	−10.9	+17.2	−28.1	+46.1	+54.6	+113.8	+192.7	+59.5	−5	+47	−7	+6	+41
40	−2.18	−0.82	−1.17	−0.78	−0.98	−0.97	−42.4	−10.7	−64.4	−28.8	−124.4	−86.8	−357.5	−623.0	−115	−64	−129	−122	−430
44	−0.57	+0.05	+0.40	+0.25	+0.90	−0.15	−11.1	+0.7	+22.0	+9.2	+104.2	−13.4	+111.6	....	....	....	....	....	....
50	−1.25	−1.90	−0.40	−3.40	−0.40	−1.45	−24.3	−24.8	−22.0	−125.6	−50.9	−130.0	−377.6	−533.0	−117	−137	−67	−47	−368
52	+0.25	−1.55	+0.10	−1.15	−0.70	+0.70	−14.9	−20.2	+5.5	−42.5	−89.0	+62.8	−78.5	−226.0	−89	−115	+11	+31	−156
53	+0.60	−1.35	+0.60	−1.35	+1.55	−0.35	+11.6	−17.6	+33.0	−49.8	+197.0	−31.4	+142.8	−249.0	−97	−115	+11	+29	+72
54	+0.55	0	−0.10	−0.20	−0.20	0	+10.7	....	−5.5	−7.4	−25.4	....	−27.6	−243.0	−75	−115	−09	+31	−168
	(20 #/D' = 384)						+74.6	+50.0	+211.0	+142 0	488.0		2 320.8§ 1 072.0‖		in hundred thousandths of an inch				

---

* See corresponding numbers in Table 1.

† 384 lb per sq in. is equivalent to a horizontal wind load of 20 lb per sq ft.

‡ See Fig. 4.

§ Computed stress for a horizontal load of 20 lb per sq ft on the south face.

‖ Computed stress for a horizontal load of 20 lb per sq ft on the west face.

¶ Compass directions.

pressure and a west pressure. It is assumed: (1) That the pressure is uniform between the twenty-fourth floor and an elevation half-way between the pressure openings of the thirty-sixth and the fifty-fifth floors; (2) that it is also uniform between this elevation and half-way between the fifty-fifth and the seventy-fifth floors; and (3) that between this latter elevation and the top it is uniform. These several pressures are obtained for the south face by taking the readings of Tubes 2, 3, and 4, Fig. 4, subtracting the readings of Tubes 7, 8, and 9, and dividing the resultant by 3. These summations (before dividing by 3) for the several floors are given in Table 6, Columns (1), (3), and (5); similarly, for the west pressure points. The readings of Tubes 1 and 10 are subtracted from those of Tubes 5 and 6, and the result is divided by 2 (see Columns (2), (4), and (6), Table 6).

The moment about the central elevation of the twenty-fourth floor of a 1-lb pressure, uniformly distributed on the south face, has been computed to be: (*a*) For the thirty-sixth floor, 5 411 000 ft-lb; (*b*) for the fifty-fifth floor, 15 300 000 ft-lb; and (*c*) for the seventy-fifth floor, 35 400 000 ft-lb. Similarly, for the west face these values would be: (*a*) For the thirty-sixth floor, 3 900 000 ft-lb; (*b*) for the fifty-fifth floor, 11 100 000 ft-lb; and (*c*) for the seventy-fifth floor, 26 700 000 ft-lb. To obtain the total overturning moment in either direction one would multiply the sum of the manometer readings by 5.2 to reduce them to pounds per square foot, divide by two or three, and then multiply by the proper value. The effect of the six sets of readings is additive.

Resisting this is the column steel on the twenty-fourth floor. The beam formula, $s = \frac{M\,c}{I}$, is used to obtain the stress in the steel. This assumes that the planes of the twenty-fourth and twenty-fifth floors remain planes.

The value of I about each axis has been obtained by multiplying the area of each column, in inches, by the distance to the axis, squared, in feet. The reduction to inches is indicated by the factor, 12, which cancels out of the computations. The moment of inertia, I, about the east-west axis is $31\,454\,000 \times 12 \times 12$ in.^{4}, whereas, for the north-south axis, I is $70\,970\,000 \times 12 \times 12$. The corner column is 65.25 ft and 91.25 ft from the two axes.

To ascertain the stress in the corner column corresponding to the manometer readings on the thirty-sixth floor, north and south faces, the data from Column (1), Table 6, should be multiplied by $\frac{5\,411\,000 \times 12 \times 65.25 \times 12 \times 5.2}{31\,454\,000 \times 12 \times 12 \times 3}$; or, 19.45. Similarly, for the east and west faces:

$$\frac{3\,900\,000 \times 12 \times 91.25 \times 12 \times 5.2}{70\,970\,000 \times 12 \times 12 \times 2}\text{; or, } 13.05$$

For the fifty-fifth floor the corresponding values are 0.55 and 0.3695, whereas, for the seventy-fifth floor they are 1.272 and 0.895. Columns (7) to (12) of Table 6 are obtained from Columns (1) to (6) by multiplying by these several values. Column (13) is the algebraic sum of Columns (7) to (12) and is the computed stress in the column.

A comparison of Columns (13) and (14) shows the difference between the computed and the observed stresses in this corner column. It furnishes a rough check on the "cantilever method" of computing the stresses in the columns of buildings.

An assumed horizontal load of 20 lb per sq ft over the south face will give a computed stress of 2 330 lb per sq in. in this column, whereas the same pressure over the west face would give 1 070 lb per sq in.

PART II.—AN ESTIMATE OF THE STIFFENING EFFECT OF THE MASONRY

INTRODUCTION

Two assumptions are usually made in computing the stresses in the frames of tall buildings due to lateral forces when any of the so-called exact methods are used. One assumption is that the girders are rigidly connected to the columns and the other is that the frame is not braced by the walls, partitions, floors, and other parts of the building.

The first assumption has been given considerable study both in America[5] and in England.[6] Data have been collected and investigations are in process in some of the colleges of the United States to ascertain the percentages of error in the final results due to this assumption. It is hoped that, before long, the properties of the connections will be incorporated into the solution of the rigid frame subjected to lateral loads as a routine part of the solution.

The second assumption (neglecting the stiffening effect of the masonry) is of far greater influence but the writer has been able to find only meager reference on the subject. It is of more than academic interest, because the stresses in the frame are proportional to the deflection of the structure if the distribution of stress is the same when the masonry is present as it is in the steel frame acting alone. Therefore, the actual stress in the steel is reduced by the presence of the masonry in proportion to its influence on the stiffness. David A. Molitor,[7] M. Am. Soc. C. E., has given an estimate that the frame would be stiffened 30% to 50% if the connections were rigid. He has also estimated that the building is stiffened 300% to 400% by the materials other than the frame.

Rigidity is of considerable importance inasmuch as it affects the popularity of a building with tenants. If the structure is too limber the fact may be shown by cracks in the plaster and the movement may be noted in other ways causing lack of confidence in the safety of the structure. On the other hand, if it is too stiff the period of vibration will be short enough to be noticeable by the tenants, thus producing the same lack of confidence. In either case an unsatisfactory result is obtained. David C. Coyle,[8] M. Am. Soc. C. E., has published information on the period of vibration of a few existing structures and has collected considerable data. The problem will doubtless assume major proportions if office buildings are built higher in the future.

---

[5] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 524.

[6] "Investigation on Beam and Column Connections," by C. Batho and H. C. Rowen, Second Rept., Steel Structures Research Committee, Dept. of Scientific and Industrial Research, pp. 61–137.

[7] "Structural Engineering Problems," by David A. Molitor, The Peters Co., Detroit, Mich., 1937, p. 64; also, reproduced by the Bureau of Yards and Docks, U. S. Navy Dept., 1937.

[8] "Testing the Strength of Skyscrapers," *Record and Guide*, February 7, 1931; "Sway of Tall Towers," *Architectural Forum*, July, 1931, p. 109; *Civil Engineering*, May, 1931; *American Architect*, June 20, 1929; "Stiffness of Skyscrapers," *Transactions*, First Cong., International Assoc. for Bridge and Structural Engrs., Paris, May, 1932; and, *Proceedings*, Am. Soc. C. E., October, 1928, p. 2393; and August, 1932, p. 1100.

The capacity of a building to resist loads is not dependent upon the steel frame alone but upon the combination of steel and masonry, acting as a unit. As a large percentage of the material in the structure does not obey Hooke's law the stresses incurred at low loads may be of an entirely different character from those that would occur if the building were subjected to a dangerous wind pressure. It is doubtless true that, due to the greater stiffness and high elastic limit of the steel composing the frame, the stiffening effect of the masonry becomes less valuable as the loads increase. It is customary to ignore this source of strength, and probably wisely so, when designing a building; but it may account for the fact that few, if any, steel buildings have been destroyed by wind or earthquake.

An estimate of this stiffening effect should be of value, and, therefore, a study of some of the data taken from the Empire State Building investigation has been used to find the stiffening of this one building in the given range of working loads. The accuracy of the result is open to some question as it is based on only one set of computations and on data from only one building. A second set of computations has not been made because of the time and labor that is required to make the necessary model. However, this value gives a general idea of the ratio between the rigidity of a building without this bracing and one with it. For a building of different construction the ratio can be estimated with greater accuracy as a result of this study. The entire problem is one that does not allow for accurate work and great refinement; nor does it require it.

In evaluating the rigidity of the building, as actually constructed, advantage is taken of the fact that the period of vibration is a simple function of the rigidity. The building vibrates in the same manner as does the tine of a tuning fork; it is an elastic cantilever secured at its base and vibrated by the gusts of wind (not a steady force). The structure is composed of an elastic steel frame and of parts that are more or less plastic. This plasticity has a dampening effect on the vibrations but does not alter the period.

In order to form an estimate of the period of vibration, similar in all respects to the one observed but supported by the steel frame alone, a model was constructed and observed. It was constructed so that the principles of similitude could be applied, and the period of vibration computed, for a steel frame of the size of the building, and weighted to represent the dead load of the building.

From the period of vibration of the model and that of the building one can compute the relative stiffness of the model and that of the finished building. The one has rigid (soldered) connections and no masonry stiffening, whereas the other has riveted connections and a great mass of masonry surrounding the steel. The model represents the steel skeleton, proportionally weighted. It is assumed that the effect of the difference between the model and the building (masonry and connections) will be the same on the several floors of the structure.

The period of vibration does not depend upon the severity and the character of the horizontal loads but rather upon the several properties of the building that can be obtained from the engineers' plans, such as the weight, weight distribution, linear dimensions, and stiffness of steel members. Figs. 16 and 17 are views of the model used.

If two structures of the general nature of the one under consideration are built similar to each other, except for the scales of weight, length, modulus of elasticity, and moment of inertia of the sections, it can easily be shown that their periods when subjected to flexural vibration will vary directly as the weight and the cube of the length, and inversely as the modulus of elasticity and the moment of inertia of the members.

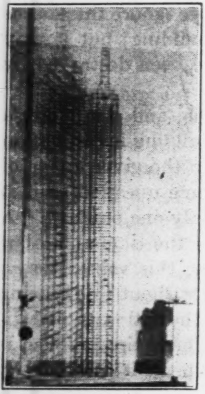


FIG. 16.—MODEL OF STRUCTURE

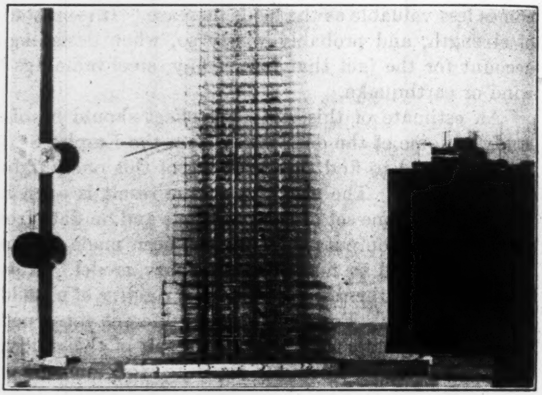


FIG. 17.—TESTING EQUIPMENT FOR MODEL

The period of vibration, expressed as a time dimension, T, or the ratio of homologous times, is expressed by the following equation,

$$T = \sqrt{\frac{W\,L^3}{E\,I}} \qquad \text{(1)}$$

in which W = the weight of the building or the model—or the ratio of homologous weights; L = the linear dimensions—or the ratio of homologous dimensions; E = modulus of elasticity—or the ratio of homologous elastic moduli; and I = the moment of inertia—or the ratio of homologous moments of inertia. Substituting known values in Equation (1),

$$T = \sqrt{\frac{50\,530\,000 \times 144^3}{2.07 \times 43\,000 \times 10^5}} = 130$$

The observed period of the model is forty-four vibrations in 6 sec; whence $\frac{6 \times 130}{44}$, or 17.8 sec, is the computed period of an unstiffened weighted frame. The building has an observed vibration period of about 8.25 sec (see Fig. 15). The ratio of these periods is then 8.25 to 17.8 or 1 to 2.16, which makes the stiffness ratio 2.16^2 to 1, or about 4.65 to 1. It can be concluded, therefore, that the structure is very close to four and one-half times as stiff as it would be if the steel frame alone held it in position.

Fig. 18(*a*) is the deformation curve of the model due to a pull at the eighty-sixth floor of sufficient magnitude to produce a unit deflection at that floor. The weight used was 5 lb, producing a deflection of 0.244 in. or 0.0488 in. per lb. Assuming similitude between the model and the building the force of the wind against the side of the structure can be estimated by the use of this curve and the deflection of the building. In computing the wind pressure its distribution must be assumed because a wind at an upper floor has more overturning effect than one lower down.

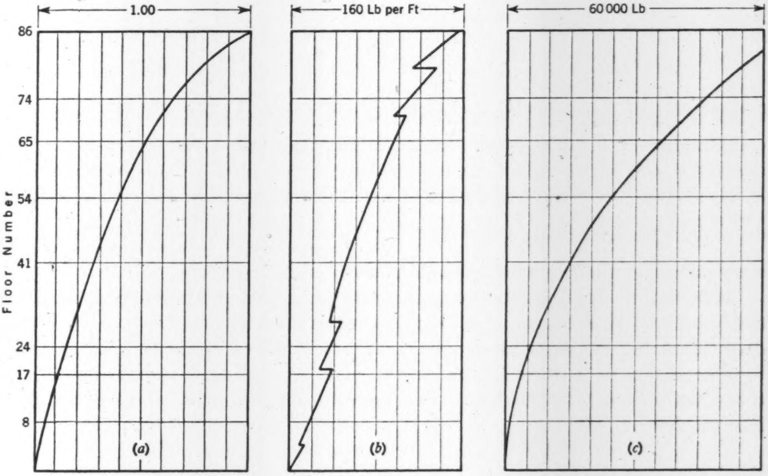


FIG. 18

The deflection due to a load at the end of a cantilever is given by the formula,

$$\Delta = K \frac{P L^3}{E I} \quad \text{.........................} (2)$$

in which K is an abstract constant depending upon the manner in which $E I$ varies along the length of the cantilever (the same in the model as in the prototype); that is, using ratios of similitude:

$$\Delta = \frac{P L^3}{E I} = P \frac{L^3}{E I} \quad \text{....................} (3)$$

in which Δ = the deflection; and P = a concentrated load. By Equation (1), $\frac{L^3}{E I} = \frac{T^2}{W}$; whence,

$$\Delta = \frac{P T^2}{W} \quad \text{........................} (4a)$$

and,

$$P = \frac{\Delta W}{T^2} \quad \text{........................} (4b)$$

In Fig. 18, if the ordinates of Fig. 18(*a*) are multiplied by the width of the building, the curve in Fig. 18(*b*) is obtained. In Fig. 18(*c*) is the integral of the curve of Fig. 18(*b*) obtained by plotting as ordinates the area below each point in Fig. 18(*b*). The total area of Fig. 18(*b*) is 64 600. Assuming a uniform pressure throughout the building of x lb per sq ft, and observing the deflection, the pressure is found to be as follows: From the plumb-bob readings in Table 3 the maximum south reading is found to be 9.76 in. and the minimum is 3.26 in. Assuming the difference is twice the deflection: $\Delta = 3.25$ in. The ratio of this deflection to that produced by a 1-lb load on the model is $\frac{3.25}{0.0488}$, or 66.6; the ratio of the times of vibration, T, is $\frac{8.25}{0.1385}$, or 59.6. The weight ratio has been computed as 50 530 000. Whence the horizontal pull at the eighty-sixth floor necessary to produce a deflection of 2.97 in. is $P = 66.6 \times 50\,530\,000 \times 59.6^2 = 11\,950\,000$ lb. From the curves of Fig. 18 the same deflection would be produced by a uniform load over the south face of: $\frac{11\,950\,000}{64\,600} = 18.5$ lb per sq ft, which includes the negative, as well as the positive, pressure.

It is to be noted that the foregoing data are based on the greatest recorded deflections. That storms producing greater deflections have occurred is doubtless true, so this value must be accepted with caution.

Conclusion 9, in Part III, is based partly on the values given herein and those of Column (14), Table 6. It is hoped discussion will bring out valuable information on this point.

Description of the Model

The steel frame of the Empire State Building lends itself readily to model analysis. It is unusually symmetrical and, with very few exceptions, the girders and columns can be reproduced as a rectangular grill with the columns extending from the foundation to the roof. The model was designed to have elastic properties similar to the frame when deflections in the narrow direction of the building (that is, parallel to Fifth Avenue) are being studied. It consisted of three grills fastened together in such a way that they were forced to vibrate as a unit. An examination of the plans shows that the frame at the foundation is composed of twenty-two rows of columns. The first eight rows consist of ten columns, the next six of nine columns and the last eight of ten columns. The twenty-two grills thus formed are forced to act as a unit by the concrete floors and other connections between them, and to vibrate with the same period. It was considered proper, therefore, to combine these grills into three, as indicated in Figs. 3, 16, and 17. In each grill the columns and girders were designed so they had a moment of inertia proportional to the sum of all eight (or six) corresponding columns or girders. At each floor the girders are secured to the columns by shallow wind-brace connections which, in this paper, have been considered as rigid. In addition, at certain joints, knee-braces extend from the intersection of a column and a girder to about the third-point on the girder in the floor above. These braces are represented in the model in their proper geometric relation by stiff brass wire (21-gage) which is a minimum allowable size for compression as a column.

In one or two cases, as at the Fifth Avenue entrance, there is a variation within the group of eight grills, which condition prevails only on the six lower floors. Another place where the building required more than three grills was between the seventy-second and the eighty-first floors, because the knee-braces did not lend themselves to a single pattern. In this case the model's grill is separated into two parts over this section of the structure; and so at these places the model contained five grills.

The distances from center to center of columns, and those between floors on the model, were laid out to a scale of 1 in. equals 12 ft, or a scale of 1 to 144. The grills were constructed of cold drawn brass 1/16 in. thick, which thickness was maintained for both girders and columns except in the case of some of the heavy columns in the lower part of the building where ⅛-in. material was used. The moments of inertia computed were divided by an arbitrary constant which was of such a value that the members of the model would be of a convenient size. These depths were calculated and the brass milled to the nearest thousandth of an inch. Each column was of one continuous piece, varying in depth to correspond with the computed value of its moment of inertia, whereas at each floor the girders were represented by a continuous member whose depth was changed at the points where it intersected the columns.

Each of the grills was formed by laying the girders on the columns and soldering them with soft solder. It was considered unwise to use hard solder for fear of drawing the temper in the brass.

The three grills were fastened together at approximately every eighth floor by means of horizontal trusses having no vertical stability but considerable lateral rigidity. Between the third, and also the eighth, sets of columns vertical trusses were placed between the two grills so that the model would have rigidity perpendicular to the direction of vibration. These trusses each consisted of a piece of cold drawn brass wire (21-gage) bent to form the webbing of a Warren truss whose chords were the columns. These trusses had no stiffening effect in the direction of vibration but acted as spacers between the grills, holding them about an inch apart. The posts were set in sockets in a steel frame, 4½ by 13/16 by 24 in., and doweled securely so that the rigidity of the base was as stable as the rock on which the building itself rests.

Representative specimens of the brass strips were tested, after milling to a depth of approximately that of an average beam, in order to find the value of E, the modulus of elasticity. This was done again after the strip was treated to considerable heat, which was applied in the same manner as was done in making the soldered joints. No change in modulus was noted.

Some trouble was experienced with the soldered joints flowing when subjected to a long continued stress. In order to determine whether these joints were strong enough to carry the stresses during the period of vibration (about 1/7 sec), the model shown in Fig. 19 was constructed. The joint at Point A was first made perfectly rigid by clamping the two pieces of brass together with a C-clamp which exerted enough pressure to prevent slippage. The lever was then vibrated and the period noted by the mark of a pen on a drum revolving

42.5 times per min. The clamp was removed and the joint soldered in the same manner as the joints in the model. The lever was again vibrated, and if the joint had not held satisfactorily, part of the motion would have gone through this joint into the section beyond and thus would have slowed down the period. The period of vibration used was approximately the same as that of the model, the weight being adjusted to produce it. Five readings over a fixed period of time showed, for the soldered joint: 27.5, 27.6, 27.6, 27.7, and 27.8 half vibrations, or a mean of 27.64. For the clamped joint, on the other hand, the readings were: 27.5, 27.8, 27.6, 27.8, and 27.6, with a mean of 27.66. The difference between the two means is low enough to warrant the conclusion that the soldered connection is satisfactory when the stress is low and of short duration.

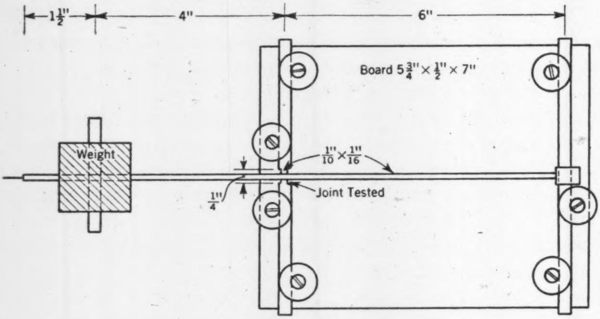

$1\frac{1}{2}''$
4"
6"
Board $5\frac{3}{4}'' \times \frac{1}{2}'' \times 7''$
Weight
$\frac{1''}{10} \times \frac{1''}{16}$
$\frac{1''}{4}$
Joint Tested

FIG. 19

In making the model it was impracticable to control both the area and the moment of inertia of the several members as the constant width of 1/16 in. had to be maintained, that being the thickness of the brass ribbon used. The areas, therefore, are not proportional to those of the building. In the case of the girders the direct stress is low and so the lack of proportionality is not objectionable. With the columns, however, the direct stress and the areas do enter into the deflections of both building and model and thus introduce a small error into the results. The fact that the areas of the braces in the model are too large for similitude has little influence on the results.

The weight of each floor was obtained from the column schedule of the steel plans. The total weight of the building above each floor was obtained by adding the stresses in all columns on that floor. The increment for each floor was obtained from these data. The partition loads and live loads were subtracted where the floors were not occupied. In the case of the occupied floors an estimate was made of the added weight and the model vibrated under conditions both allowing for this added weight and omitting it. The difference was not noticeable.

These computed floor loads were divided by an arbitrary constant and the weights of the floors in the model were increased to the computed value by the addition of pieces of steel.

In computing the weight of the model the weight of the brass girders and columns was first computed and distributed at each floor. The weights of the knee-braces and the several stiffening trusses were obtained by observing the length of the wire used in making them. The model was then weighed and the difference between the computed and actual weight was taken as the weight of the solder. This was distributed among the several floors in proportion to the number of joints on that floor. To each floor was then added enough weight in the form of steel rods to bring its weight up to the proper ratio. In a few cases this added weight was computed to be negative, in which case one-half the value was subtracted from the weight to be added to the floor below and the other half from that to be added to the floor above. The weights were secured in position by means of rubber bands whose weight was also considered.

The model, as thus designed and constructed, was similar to the building in the following particulars:

(*a*) Its geometrical proportions (both vertically and in a direction parallel to its narrow face) were to a scale of 1 to 144;

(*b*) Its weight distribution was the same as that of the building, and its weight, or mass ratio, was 1 to 50 530 000;

(*c*) The ratio of modulus of elasticity of the materials used in the model to that of the steel frame of the building was 14 to 29; and,

(*d*) The ratio of the moments of inertia (and, therefore, the ratios of the stiffness of the various members of the model, to the corresponding members of the building) was 1 to $43\,000 \times 10^5$. This ratio was chosen because it produced sizes of members in the model with which it was practical to work.

The model was not similar to the building in the following particulars: (1) In the dimensions perpendicular to the narrow face of the building; (2) in the method of fastening girders to columns; (3) in the fact that it is not braced by masonry surrounding the frame—floors, walls, partitions, and other building materials; and, (4) in the cross-sectional areas of the several members.

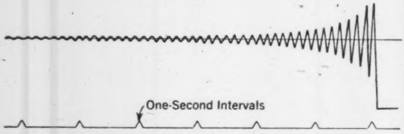
Fig. 17 shows the method of obtaining the vibration characteristics of the model. A 5-lb weight was used to produce the horizontal pull. The connection was broken suddenly by means of a trigger trip.

The horizontal movement was recorded by means of a thread attached to the model at one end and held taut by a rubber band at the other. A lever, holding a pen, was attached to the thread in such a manner that the pen traced the movements of the model on a revolving disk. A clock indicated the seconds on the disk. The lever exaggerated the movement of the model in the ratio of 478 to 240. Fig. 20 gives the curve produced with: (*a*) The load and recorder attached to the eighty-sixth floor; (*b*) the load on the eighty-sixth floor and the recorder on the sixty-fifth floor; and (*c*) the load on the sixty-fifth floor and the recorder on the eighty-sixth floor. The recorder was placed on the eighth, seventeenth, twenty-ninth, forty-fourth, fifty-fourth, sixty-fifth, seventy-fourth, and eighty-sixth floors. For each position the load was applied and suddenly released at all the floors named. Thus, a series of sixty-four curves similar to those of Fig. 20 were produced. The period of vibration was the same in all the curves.

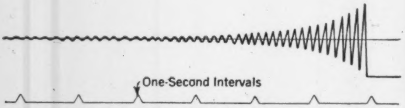
The curve of Fig. 18(*a*) was plotted from measurements taken from this series of curves. The position before and after releasing the 5-lb weight was used as the deflection due to the pull.

THEORY

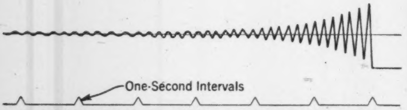
The derivation of the following formulas is given so that one may note the approximations involved in Equation (20) and thus estimate the errors if the result of neglecting them is open to question.



(*a*) LOAD AND RECORDER AT 86TH FLOOR



(*b*) LOAD ON THE 86TH FLOOR; RECORDER AT THE 65TH FLOOR



(*c*) LOAD ON THE 65TH FLOOR; RECORDER AT THE 86TH FLOOR

FIG. 20.—VIBRATION CURVES OF MODEL

Let A B in Fig. 21 represent a cantilever secured at Point A and vibrating about its neutral position, in a plane parallel to the paper, with a simple harmonic motion. This cantilever may be made up of a complicated set of springs, as girders and columns; or it may be a simple spring. A mass, $m\,dx$, is fixed at Point P, the symbol, m, being the mass per linear unit; it may be a function of x. When the particle, $m\,dx$, is in a position of maximum deflection the velocity is zero, and all energy is potential. This energy is stored up in the stressed fibers of the cantilever. If the cantilever is a building frame this energy is the internal energy of the strained columns and girders. When the particle, $m\,dx$, is in the neutral position all the energy is transferred to kinetic energy. If the period of vibration is T and the amplitude of vibration is y the velocity, v, at the central position is given by the equation,

$$v = \frac{2\pi y}{T} \quad \text{(5)}$$

The kinetic energy, W_k, of a moving mass is $\frac{m v^2}{2}$, and the energy in the mass is,

$$W_k = \left(\frac{m\,dx}{2}\right)\left(\frac{4\pi^2 y^2}{T^2}\right) = \frac{2\pi^2}{T^2} m y^2\,dx \quad \text{(6)}$$

in which both m and y are functions of x. In some cases it may be possible to express m as a mathematical function of x, but in any specific example the distribution of the mass over the length of the structure can be found. At the point of maximum deflection the acceleration of the mass, $m\,dx$, is equal to,

$$a = -\frac{4\pi^2 y}{T^2} \quad \text{(7)}$$

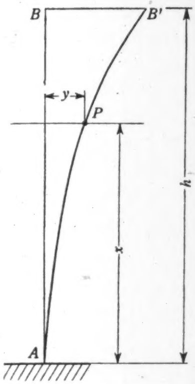


FIG. 21

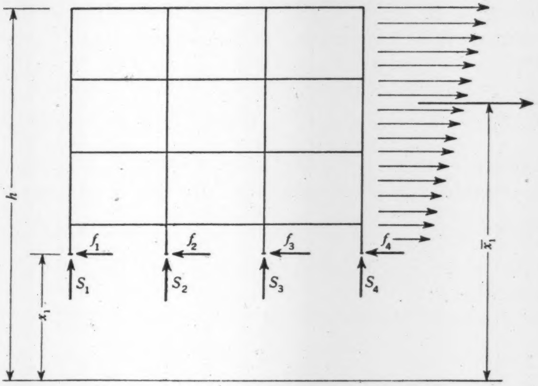


FIG. 22

and, since $f = m\,a$, the force exerted by, or on, Mass m, due to its acceleration, is,

$$df = -\frac{4\,\pi^2}{T^2}\,m\,y\,dx \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

This is the force that produces the stress throughout the cantilever and determines the value of y and, consequently, the shape of the curve of maximum amplitude. In addition to this energy of horizontal motion the point, P, will fall or rise a slight distance, b, producing a change in potential energy of $b\,g\,m\,dx$, which gives:

$$W_t = \frac{2\,\pi^2}{T^2}\int_0^h m\,y^2\,dx + g\int_0^h b\,m\,dx \ldots\ldots\ldots\ldots (9)$$

as the total energy which changes from kinetic to potential as the particle moves from the central position to the position of maximum amplitude. In Equation (9), $g =$ the acceleration due to gravity.

The second term in the problem considered in this paper is of minor significance and value, although in some cases, as a pendulum, it might be a dominating factor. Let Fig. 22 be that part of the frame forming the cantilever of Fig. 21, above which is a plane half-way between the nth and the $n + 1$ floor, or a distance, x, from the base. At maximum amplitude the structure is acted upon by the forces, F, shown in the diagram, in which,

$$F = \frac{4\,\pi^2}{T^2}\int_{x_1}^h m\,y\,dx \ldots\ldots\ldots\ldots\ldots\ldots (10a)$$

and the moment above Point C is,

$$F\,(\bar{x} - x_1) = \frac{4\,\pi^2}{T^2}\int_{x_1}^h m\,y\,x\,dx - F\,x_1 \ldots\ldots\ldots\ldots (10b)$$

Each column resists a fraction of the shear force, F, and a thrust, S, as well as a moment, M, which is not shown. The values of the forces (such as S) are assumed to be proportional to the moment about Point C as given by Equation (10*b*). The values of the column shears have a definite but unknown distribution among the columns and as their sum total is equal to F they are proportional to this value and may be represented by the fractional force, $\frac{p\,F}{q}$, in which p and q are abstract, dimensionless numbers. The values of S are probably approximately proportional to their distance from Point C. Let e be the distance from Point C to any given column and $\frac{c}{d}$ (a ratio of dimensionless abstract numbers) be the fraction that corrects for any lack of uniformity in the stress distribution. The value of Moment M is equal to the shear times the distance to the point of counterflexure or r, whence,

$$\Sigma\, S\, e = \frac{4\,\pi^2}{T^2} \int_{x_1}^{h} m\, x\, y\, dx - F\, x_1 \ldots\ldots\ldots\ldots\ldots (11a)$$

and,

$$S = \frac{1}{e}\frac{c}{d}\left[\frac{4\,\pi^2}{T^2} \int_{x_1}^{h} m\, x\, y\, dx - F\, x_1\right] \ldots\ldots\ldots\ldots (11b)$$

The energy in a column under an axial load, however, is equal to one-half the stress times the strain, or,

$$W_a = \frac{S}{2}\frac{S\,l}{A\,E} = \frac{S^2 l}{2\,A\,E} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12a)$$

or,

$$W_a = \frac{16\,\pi^4}{2\,T^4}\frac{1}{e^2}\left(\frac{c}{d}\right)^2 \frac{L}{A\,E}\left[\int_{x_1}^{h} m\, x\, y\, dx - x_1 \int_{x_1}^{h} m\, y\, dx\right] \ldots (12b)$$

The shear on the post is,

$$V = \frac{4\,\pi^2}{T^2}\frac{p}{q}\int_{x_1}^{h} m\, y\, dx \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13a)$$

and the moment in the column at any point is,

$$M_c = \frac{4\,\pi^2}{T^2}\frac{p}{q}\, r \int_{x_1}^{h} m\, y\, dx = \frac{j\, r}{T^2}\int_{x_1}^{h} m\, y\, dx \ldots\ldots\ldots\ldots (13b)$$

if $4\,\pi^2 \frac{p}{q} = j$, a substitution constant. The moment at the end of a girder is proportional to the moment in the column adjacent to that end, plus the difference in the thrusts, S, on this column above and below the joint, times the distance from the joint to the point of counterflexure in the girder:

$$M_b = M_c \frac{u}{v} + (S_n - S_{n+1})\, 0 \ldots\ldots\ldots\ldots\ldots\ldots (14a)$$

but, since $S_n - S_{n+1}$ is proportional to F:

$$M_B = \frac{u}{v} \times \frac{4\,\pi^2}{T^2} \times \frac{p}{q}\, r \int_{x_1}^{h} m\, y\, dx + \frac{i}{n} 0 \frac{4\,\pi^2}{T^2} \int_{x_1}^{h} m\, y\, dx$$

$$= \frac{K\, r}{T^2}\left[\int_{x_1}^{h} m\, y\, dx\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14b)$$

in which $K = \frac{u}{v}\frac{p}{q} + \frac{i}{n}\frac{o}{r}$ is an abstract number; but the energy in the beam or column due to bending is equal to, $W = \int_0^l \frac{M^2\,dx}{2\,E\,I} = \frac{1}{2\,E\,I}\int_0^l M^2\,dx$, in which l is the length of the member. Substituting the values:

$$M_c = j\,r \int_{x_1}^{h} m\,y\,dx \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15a)$$

and,

$$M_b = \frac{K\,r}{T^2}\int_{x_1}^{h} m\,y\,dx \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15b)$$

the energy in the column is expressed by:

$$W_c = \frac{j^2}{2\,E\,I\,T^4}\left[\int_{x_1}^{h} m\,y\,dx\right]\left[\int_0^l r^2\,dx\right] \ldots\ldots\ldots\ldots (16a)$$

and, the energy in the beam, by:

$$W_b = \frac{K^2}{2\,E\,I\,T^4}\left[\int_{x_1}^{h} m\,y\,dx\right]\left[\int_c^l r^2\,dx\right] \ldots\ldots\ldots\ldots (16b)$$

the second integral in each case being taken from the point of counterflexure to either end of the beam or column. Collecting these terms and equating the energy of motion given by Equation (9) against that of the stored energy of the deflected structure:

$$\begin{aligned} &\frac{2\,\pi^2}{T^2}\int_0^h m\,y\,dx + g\int_0^h b\,m\,dx \\ &= \frac{16\,\pi^4}{2\,T^4}\sum\frac{1}{e^2}\left(\frac{c}{d}\right)^2\frac{L}{A\,E}\left[\int_{x_1}^{h} m\,x\,y\,dx - x_1\int_{x_1}^{h} m\,y\,dx\right]^2 \\ &\quad + \frac{1}{2\,T^4}\sum\frac{j^2}{E\,I}\left[\int_{x_1}^{h} m\,y\,dx\right]^2\left[\int_c^l r^2\,dx\right] \\ &\quad + \frac{1}{2\,T^4}\sum\frac{K^2}{E\,I}\left[\int_{x_1}^{h} m\,y\,dx\right]^2\left[\int_0^l r^2\,dx\right]_x \ldots\ldots\ldots\ldots (17)\end{aligned}$$

Using capital letters for the dimensionless terms, which are, therefore, independent of the character of the model and its relationship to its prototype, Equation (17) becomes (after multiplying by T^4):

$$\begin{aligned} &T^2\,A\int_0^h m\,y^2\,dx + T^4\,B\,g\int_0^h m\,b\,dx = C\sum\frac{l}{e^2\,A\,E}\left[\int_{x_1}^{h} m\,x\,y\,dx\right. \\ &\quad \left. - x_1\int_{x_1}^{h} m\,y\,dx\right]^2 + D\sum\frac{r^3}{E\,I}\left[\int_{x_1}^{h} m\,y\,dx\right]^2 \\ &\quad + F\sum\frac{r^3}{E\,I}\left[\int_{x_1}^{h} m\,y\,dx\right]^2 \ldots\ldots\ldots\ldots\ldots (18)\end{aligned}$$

in which A, B, C, D, and F are abstract numbers in the terms representing the energy: (a) Of motion of the mass of the structure; (b) of position of the mass of the structure; (c) in the columns due to direct thrust; (d) in the columns due to bending; and (f) in the beams due to bending. Neglecting the second term as small, and the third term as being probably small:

$$T^2 = \frac{D \sum \frac{r^3}{E\,I} \left[\int_{x_1}^{h} m\, y\, dx \right]^2 + F \sum \frac{r^3}{E\,I} \left[\int_{x_1}^{h} m\, y\, dx \right]^2}{A \int_{0}^{h} m\, y^2\, dx} \quad \ldots\ldots (19)$$

Writing the dimensions of these terms for similitude:

$$T^2 = \frac{\frac{L^3}{E\,I} \left[\frac{M\,L^2}{L} \right]^2}{\frac{M\,L^3}{L}} = \frac{M\,L^3}{E\,I} \quad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

that is, the square of the period of vibration of the frame varies directly as the mass and the cube of the linear dimension and inversely as the modulus of the material and of the moments of inertia of the members. As the mass ratio is the same as the weight ratio the formula is the same as Equation (1).

PART III.—CONCLUSIONS

The following conclusions have been suggested by the data recorded in this paper:

(1) The distribution of the wind pressure on a tall building is very complicated and irregular, the air currents having been broken up by the surrounding structures and by the building itself.

(2) The action of a building under horizontal loads is plastic as well as elastic, with the result that strains and deflections are not proportional to the forces that produce them.

(3) The manometer readings do not give a true picture of the forces acting on the building, but do give the pressure at definite isolated points. This is because the wind comes in gusts that affect only a small surface. Frictional resistance has some influence.

(4) The deflection of the building due to the sun shining on one side of it (and expanding that side) is practically negligible.

(5) The horizontal deflection of this building during a wind storm is notable because of its popular interest and the small amount of data on this subject.

(6) Theoretically, there is danger of designing a building too stiff as well as not sufficiently rigid.

(7) The cantilever and portal methods of design have neither been corroborated or refuted by these data.

(8) The masonry increases the rigidity of the building about 350% more than that of the unsupported frame.

(9) The data herein given have not shown that 20 lb per sq ft is an improper value for the assumed lateral load on a building of the type of the one investigated when in a similar environment. This leaves the masonry as an added, but not perfectly reliable, factor of safety.

ACKNOWLEDGMENTS

The design and installation of the instruments herein described were made by the engineering staff of the American Institute of Steel Construction. The firm of Shreve, Lamb and Harmon, architects of the building, co-operated in the installation of the instruments and the gathering of the data. Many data and many suggestions were furnished and made by the late H. G. Balcom, M. Am. Soc. C. E., a member of the Institute's Committee on Research and Engineer of the design and construction of the building. F. H. Frankland, M. Am. Soc. C. E., member of the Committee and Chief Engineer of the Institute, supervised the design and installation of the manometers, extensometers, and cameras, and the collection of the first set of data (including that for the area shown in Fig. 7). Mr. Hewitt Crosby obtained some of the collimator data and assisted in obtaining some of the other readings. Alfred F. Samuel, Jr., Assoc. M. Am. Soc. C. E., installed the collimator target and made many of the readings. The second group of readings (the vicinity map of Fig. 6), the deflection curves, and Weather Bureau reports were obtained by the writer assisted by Walter J. Gray, Jun. Am. Soc. C. E., and other students from the College of the City of New York, in New York City. The model was made by the writer in the laboratories of the City College. The curves and readings from the model were made by Mr. Arthur Gatterdam. The balloon experiment was made, and many helpful suggestions were given during the first period of the research by Mr. Coyle.

DISCUSSION

David Cushman Coyle,[9] M. Am. Soc. C. E. (by letter).—In the design of slender towers to be occupied by paying tenants, the proper criterion is not so much strength against being blown down as a feeling of solidity noted by the tenants during high winds. Few cases will be found in which the requirements of sensation do not give a tower of a highly conservative design as to strength.

The theory advanced by H. V. Spurr,[10] M. Am. Soc. C. E., is adaptable to a sensation criterion, since it involves forcing the floors to remain plane, thus facilitating the mathematics of cantilever vibration analysis, and since it also involves as an arbitrary constant the allowable static deflection.

For the purpose of analysis for sensation, the writer would modify the (wholly nominal) assumed wind load used by Mr. Spurr. Instead of 0 at the street line and 30 lb at the top, the writer would use 0.03 h lb at the top, which is 30 lb at Elevation 1 000.

There are two assumed constants to be used in the proposed modification of the Spurr theory, both of which may be refined by research. One is the absolute allowable wind loading, a value to be used in checking the design to guard against overturning. Professor Rathbun points the way to a method of evaluating this constant. Obviously, it is not a real pressure per square foot, evenly distributed over the building; nor does it have a relation to anemometer readings that can be discovered by logical analysis. However, if a series of readings of static deflection can be obtained and correlated with anemometer readings, and if the stiffness of the towers thus treated is known, it is obviously possible to obtain a nominal wind load varying from 0 at the bottom to $f(a)\ h$ at the top, that would produce, theoretically, the observed deflection ($f(a)$ is a function of the anemometer reading).

The stiffness of existing towers must be measured in order to analyze the foregoing factors. Generally, the designer must deal with irregular spacings of columns, offsets, and other anomalies. The mathematical treatment of this problem was published by the writer in 1932.[11]

Using the observed period of vibration, and the estimated actual dead and live loads, the actual stiffness can be determined and expressed in assumed nominal wind load per inch of static deflection. This is a different kind of quantity from the wind load previously mentioned. That is, a maximum wind load of some kind will be imposed by Nature, and the only question in this regard is to find a nominal load which, when applied to the analytical formula, will result in a tower that will stand up. On the other hand, the stiffness is an intrinsic quality of the building definable as a ratio between load and deflection, and has no concern with whether a load of that form or size could ever exist in Nature.

Thus, by knowing the period of vibration—easily observed with a seismograph—and the weights, the true over-all stiffness of towers (including effects of steel and masonry) is easily found. Using this known stiffness, it will be possible, at considerable expense for research, to find, roughly, the proper absolute nominal wind load for design of stability.

---

[9] Cons. Engr., New York, N. Y.

[10] "Wind Bracing," by H. V. Spurr, McGraw-Hill, 1930.

[11] "Stiffness of Sky-Scrapers," International Assoc. for Bridge and Structural Eng., Paris, France, May, 1932.

Thereafter, a maximum hurricane of any specified velocity could be assumed as measured by the anemometer, and the wind diagram to be used in checking for over-turning could be derived therefrom.

There remains the problem of securing the proper stiffness for comfort. To obtain an analytical description of the criterion for comfort it is necessary to know, first, the form of the function of period and amplitude that corresponds to sensation. This function has been commonly assumed to be acceleration, which, since amplitude is proportional to static deflection, is described by the ratio, $\frac{y}{T^2}$, y being the deflection and T, the vibration period. The conclusion reached by Mallock that there has been some experimental work indicating vibration reaches the threshold of sensation at an acceleration of 1% of g. The vibrations involved, however, were tremors of the order of 5 to 25 per sec; the results, therefore, do not apply to wind sway. The writer has felt vibrations of an amplitude of about 0.5 in. at 13 per min, which gave an acceleration of only about 0.25% of g. It seems that the feeling of slow sway is different in kind from the feeling of tremor. It is possible that tremor may be a muscle or skin sensation, whereas sway may be a sensation of the semi-circular canals.

Another seeming fact throws light on this question. If a tower of given size and weight were to be made twice as stiff by the addition of extra bracing, the acceleration would not be changed because the amplitude would be cut in two and the period would be divided by $\sqrt{2}$, leaving $\frac{y}{T^2}$ unchanged. Adding stiffness to a tower without changing the weight, therefore, does not improve its acceleration; yet it seems fairly clear from observation that the stiffer tower is the more comfortable.

If the function is not $\frac{y}{T^2}$, it seems necessary to make tests on a shaking-table in a psychological laboratory to find out what the function is. The next step would be to measure a large number of towers for T and y, by means of the seismograph for T and analysis using the weights so as to find y. Then, using the necessary function of T and y, these towers may be arranged in statistical order, from the worst to the best. By observing this array one can then pick the desired flexibility or stiffness to correspond with the quality of building desired. Thus, a tower can be planned so that its amplitude and period are in a class with the Woolworth Building or the Metropolitan Life Building (both in New York City), or, on the other hand, one may design with just sufficient strength to pass inspection, according to the proposed use and the character of the job. Buildings to be used for sleeping should naturally be steadier than an office building needs to be. Note that amplitude can be reduced by additional bracing, but only with a corresponding increase in speed of vibration. Speed, however, can be reduced without changing the amplitude, by adding weight at the top. The architect's ideal of a "tin tower," made of light weight materials, is, therefore, not consistent with comfort, above a very limited ratio of slenderness.

The lines of research sketched herein would require considerable time and money, but if a new skyscraper era were to occur, with higher and more slender towers, such research, continuing the valuable pioneer work reported by Professor Rathbun, seems necessary to obtain a rational method of design for the major factor of commercial success—the occupants' feeling of security.

CLYDE T. MORRIS,[13] M. AM. SOC. C. E. (by letter).—The fact that so little has been published giving observed data on the stresses and deformations produced in actual buildings by the wind, makes this paper especially valuable, even though definite conclusions and recommendations cannot be drawn from these data.

The observations of the air currents surrounding the building and of the variation of the pressure recorded on the manometers show the futility of extreme refinements in the mathematical calculation of wind stresses. This does not mean, however, that the elastic theory of stress distribution may be discarded, but only that the basic values of the wind pressure used may be in error.

From the observed deflections of the building Professor Rathbun computes the equivalent uniform wind pressure to have been 13.4 lb per sq ft at a wind velocity of about 60 miles per hr (see following Equations (4)). If it is assumed that the pressure increases as the square of the velocity an equivalent pressure of 20 lb per sq ft would be caused by a wind velocity of about 73 miles per hr which would indicate that the usual assumption of 20 lb per sq ft being taken entirely by the steel frame is not too conservative. Conclusion (8), that the masonry increases the rigidity of the building about 350% more than that of the unsupported frame, must not be construed to mean that the masonry increases the load-carrying capacity by that amount.

ROBINS FLEMING,[14] ESQ. (by letter).—The forces treated in this paper are determined by actual measurement from observations extended over a period of five years. A wealth of data is given such as can be found nowhere else. It is a matter of congratulation to the structural engineering fraternity that the owners of the Empire State Building made it possible that such observations could be made. It is fortunate that so able a presentation of these observations as that given in the paper under discussion is available.

Professor Rathbun states that Table 1 justifies considerable detailed study. This is true not only of Table 1, but of the entire paper. Surprising varying wind velocities at near-by points are shown in Table 1. More striking still are the variations of pressure at different heights of the same building, as indicated in Table 2. This raises the question as to the allowable wind pressure that should be assumed in the design of a tall building. How shall it be distributed? Referring to the pressure obtained by Messrs. Dryden and Hill[3] from a model of the Empire State Building placed in a wind tunnel, the author writes: "A comparison of the pressures on the model and those on the building shows clearly that the natural wind movements are not at all like those in a wind tunnel." This is a striking sentence. How far-reaching is its application? Incidentally, a writer on this subject has stated[15] of a transmission tower in Belgium, erected for purposes of observation, that: "The installation will enable the relation between the aerodynamic coefficients of a model and a full-sized tower to be established, and will also enable work to be carried out as to the frequency of gusts."

---

[13] Prof., Civ. Eng., Ohio State Univ., Columbus, Ohio.

[14] Civ. Engr. (*Retired*), New York, N. Y.

Table 3, reporting the plumb-bob observations, is important. The writer notes that Professor Rathbun gives the maximum deflection of the eighty-sixth floor relative to the sixth floor (a distance of 974 ft), as 2.97 in. (see text following Equations (4)). The casual reader may have difficulty in reconciling this deflection with the sentences of ten pages previous (see heading "The Plumb-Bob") that:

> "It is to be noted that the values in Table 3 do not indicate the maximum deflection of the building. They are only the mean position about which the building vibrates; for a maximum deflection one-half the amplitude of vibration should be added."

A few lines, or a page, could profitably have been added to the paper on this subject. The terms, "sway," "vibration," and "deflection," should be clearly defined. Where is the origin of co-ordinates taken?

In Table 5, the greatest amplitude of vibration recorded is from 0.5 to 5.2 in. in a north and south direction. The deflection, therefore, is $(5.2 - 0.5) \div 2 = 2.35$ in.

At this point attention is called to an opinion expressed[16] by Henry V. Spurr, M. Am. Soc. C. E. For a tower building 100 ft, center to center, of wall columns and 1 000 ft high, Mr. Spurr believes that a reasonable "yardstick" for a total deflection at the top is $0.001\,h$ "which is about $\frac{1}{8}$ in. per story." The question may be raised: Will such a deflection give sufficient rigidity?

The literature of deflections is extremely meager. Few measurements have been made; or if they have been made they have not been published. Observations[17] made on two buildings in Chicago, Ill.—the 17-story Monadnock Block and the 14-story Pontiac Building—were reported in 1894:

> "At the Monadnock Block the deflection was first observed by transits set in sheltered positions at points north and south of the north and south ends of the building and these observations were checked by observing the oscillation of plumb bobs suspended in the stairway shaft from the 16th floor. These two observations agreed very closely and show the vibration east and west from $\frac{1}{2}$ in. to $\frac{1}{4}$ in."

It should be said that the dead weight of the Monadnock Block would now be called excessive. This was the last skyscraper built with structural masonry walls. "The observations made on the Pontiac Building agreed quite closely with those on the Monadnock Block." No details are given. "The wind registered a little over 80 miles per hour."

---

[15] *Engineering*, May 6, 1938, p. 498.

[16] "Wind Bracing," by Henry V. Spurr, 1930, p. 27.

[17] *Engineering News*, March 1, 1894.

In a monograph,[18] published in 1912, Cyrus A. Melick, M. Am. Soc. C. E., records measurements of the vibration of a bent in a 17-story building. The greatest indicated wind vibration was 1 in. in a period of about 4 sec. It will be remembered that the distance traveled during a vibration is twice the deflection from a normal position.

The invariable practice in the design of steel-frame tier buildings has been to proportion the steel frame to carry the entire wind load to the column bases. Professor Morris[19] has stated that: "The stiffening effects of walls, partitions and floors should be neglected in calculating wind stresses. The entire wind load should be regarded as carried by the steel frame." The deflection calculated from the steel frame alone carrying wind pressure is much larger than the actual deflection. Mr. Molitor[7] computes the deflection of a 40-story building bent for a wind pressure of 15 lb per sq ft, to be 6.5 in. He adds a sentence, "If such a building in its completed state was to vary as much as 1 in. in the wind it might border on financial failure from the standpoint of occupancy."

The writer has calculated the deflection of the 20-story steel frame analyzed[20] by Wilbur M. Wilson and George A. Maney, Members, Am. Soc. C. E., in 1915, and found the deflection of the tops of the outer columns to be 3.91 in. This deflection was determined from Mohr's work equation, evaluating the integrals by a substitution formula given by Mr. Molitor.[21]

The process of calculating deflections in a multi-storied frame is long and tedious. For the Empire State Building it would be impracticable. Still, by an ingenious and elaborate method, Professor Rathbun arrives at the conclusion "that the structure is very close to four and one-half times as stiff as it would be if the steel frame alone held it in position" (see following Equation (1)).

The reason for the small observed deflections is discussed in Part II of the paper (under the heading, "Introduction"): "The capacity of a building to resist loads is not dependent upon the steel frame alone but upon the combination of steel and masonry as a unit." Mr. Molitor does not question any conclusions based on the theory of analysis as applied to a strictly engineering structure. "However," he writes,[22] "a steel building may be classed as an engineering structure only when the frame is bare; but when clothed with architectural coverings of concrete, stone and brick, with steel and concrete floors, tile partitions, etc., it becomes a composite structure, the nature of which cannot be appraised in terms of mathematics."

Mr. Jacob J. Creskoff[23] thinks that the present practice of restricting wind stresses to the steel frame is contrary to common sense: "A building resembles its frame only remotely, for the reason that the deflections of the horizontal members of the frame are limited by the slabs, and those of the vertical members are limited by the walls." He presents a "dynamic approach to design."

---

[18] "Stresses in Tall Buildings," Cyrus A. Melick, *Bulletin No. 8,* Coll. of Eng., Ohio State Univ., June, 1912.

[19] "Practical Design of Wind Bracing," by Clyde T. Morris, M. Am. Soc. C. E., *Proceedings,* Am. Inst. of Steel Construction, Inc., October, 1927.

[20] "Wind Stresses in the Steel Frames of Office Buildings," by Wilbur M. Wilson and George A. Maney, *Bulletin No. 80,* Eng. Experiment Station, Univ. of Illinois, June 7, 1915.

[21] "Structural Engineering Problems," by David A. Molitor, M. Am. Soc. C. E., The Peters Co., Detroit, Mich., 1937, p. 31.

[22] *Loc. cit.,* p. 63.

[23] "Earthquakes and Wind Design: Suggested Rationalization," by Jacob J. Creskoff, *Engineering News-Record,* Vol. 113, November 1, 1934, p. 533.

The "vertical beams" are considered to be composed of reinforced concrete and structural steel.

Significant sentences in the paper by Professor Rathbun are: "Rigidity is of considerable importance inasmuch as it affects the popularity of a building with tenants. If the structure is too limber, the fact may be shown by cracks in the plaster and the movement may be noted in other ways causing lack of confidence in the safety of the structure. On the other hand, if it is too stiff the period of vibration will be short enough to be noticeable by the tenants, thus producing the same lack of confidence" (see heading "Introduction" in Part II). The period of vibration is a function of the rigidity. Mr. Coyle believes that the necessity for stiffness depends on the occupancy. In 1929 he wrote[24]:

> "The relation between type of occupancy and sensitiveness to motion is a very important one. In an office the occupants are awake, clothed, occupied and expected to be paid for their time. It is daylight and there are other people about to give a sense of security. In an apartment house one may find himself alone, in bed, and in the dark, with a storm howling outside. Moreover, if one is ever subject to disturbance of the organs of equilibrium, such a condition is most apt to recur after an evening. Such being the case it is inevitable that some of the tenants in an apartment tower will feel or imagine strange things, and tell them to their friends."

The writer recalls an instance where the scream of a girl in a dance hall led to the hall being closed. An investigation was ordered and the details of the steel frame were found to be reprehensible. The owners of the building were put to a heavy expense before the hall was allowed to be re-opened. Mr. Coyle thinks that safety and the approval of the Building Department are not all that need be considered in the design of a tall building; "there is also required a technique in dealing with the cause and cure of imaginary feelings on the part of the tenant."[25]

Of the nine "Conclusions" in the paper the writer is specially interested in Conclusion (7): "The cantilever and portal methods of design have neither been corroborated nor refuted by these data." Both these conventional methods have been severely criticized. Messrs. Wilson and Maney[20] pronounce the "portal" method so inaccurate that it should never be used; and declare that the "cantilever" method is "quite accurate in some cases but may give results that are seriously in error." A slope deflection is presented which they consider "very accurate." A shorter approximate method is then proposed. As previously mentioned the writer calculated the deflection of the steel frame from the data given in the text to be 3.91 in. In an apartment house such a deflection would certainly affect rental receipts.

The Third and Final Report of the British Steel Structures Research Committee—a volume of 599 pages—was published in 1936. The conventional methods of obtaining wind stresses, "such as the Portal and Cantilever methods," are condemned in this report, which also states that the steel frame is not alone in resisting wind forces. The "architectural clothing" has a share; and,

---

[24] "Mushroom Skyscrapers," by David C. Coyle, M. Am. Soc. C. E., *The American Architect*, Vol. CXXXV, p. 289, June 20, 1929.

[25] "Testing the Strength of Skyscrapers," by David C. Coyle, M. Am. Soc. C. E., *Record and Guide*, Vol. 127, February 7, 1931, p. 12.

> "Floors are not the only clothing likely to affect the stresses in the steel framework. The fire-resisting stanchions and the external walls will play their part. * * * These tests have shown that the effect of clothing cannot be ignored. * * * In a large class of steel-framed buildings the floors and walls in conjunction so brace the framework that appreciable sway due to wind is prevented and the steel work is thus relieved of the responsibility of resisting wind shear."

In view of the foregoing and similar statements it is pertinent to ask why the conventional methods of obtaining wind stresses do not give ample security. Are they not as nearly correct as the elaborate and sometimes impractical methods that have been proposed? Mr. R. Gray, a British engineer, in a paper, "Wind Stresses in Tall Buildings,"[26] gives three methods for obtaining moments, shears, and thrusts, of which the "cantilever" and the "portal" are the second and third. Professor Rathbun considers that proposed methods of obtaining so-called "exact" results are unworkable except in the simplest cases. "Moreover," he states, "it is doubtful whether they are much nearer the truth than the results given by the common methods, since the influence of the walls and floors, though quite considerable, is quite incalculable."

In connection with Conclusion (9), it is interesting to note that the horizontal pull at the eighty-sixth floor, necessary to produce a deflection of 2.97 in., is equivalent to that produced by a uniform load of 13.4 lb per sq ft over the south face of the building (see following Equations (4)).

It may be of historical interest that in order to deny the truth of current reported deflections a communication, signed "Empire State Incorporated," appeared in the *New York Sun* of July 10, 1931. The communication reads:

> "Please be informed that while no gale strong enough to permit measurements to be made has ever been furnished by the elements in New York City, it is a firm belief of the architects and the engineers who built Empire State that the possible swaying of the building in a 220 mile wind would be absolutely negligible. The highest measurement made so far indicates a sway of less than one-tenth of an inch."

Paul Starrett, of Starrett Brothers and Eken, the firm that built the Empire State Building, writes in 1938[27]: "The sway of the Empire State Building has been measured in strong wind-storms and the greatest variation of the mooring mast from the perpendicular was found to be two and a half inches on either side."

The highest velocity recorded in the tables of Professor Rathbun's paper is 90 miles per hr although a velocity of 102 miles per hr is mentioned in the text (see heading, "Velocity of the Wind").

The New York City Building Code for structures higher than 100 ft calls for an assumed wind pressure of 20 lb per sq ft of exposed surface from the top down to the 100-ft level. By an Amendment that became effective March 24, 1930, the requirement in the then existing Code was changed from 30 lb to the present 20 lb. The Empire State Building was formally opened to the public on May 1, 1931. The late H. G. Balcom, M. Am. Soc. C. E., reported[28] that:

---

[26] *The Structural Engineer,* Vol. XV, May, 1937, p. 186.

[27] "Changing the Skyline: An Autobiography," by Paul Starrett, 1938.

[28] "Wind Calculations," by H. G. Balcom, *Civil Engineering,* Vol. 1, May, 1931, p. 700.

"In wind calculation, the floor construction was assumed to act as a rigid horizontal plate, which distributed the wind pressure to the various bents in the ratio of their resistance. Overturning wind stresses were figured by the cantilever method. The resistance was based on the relative moments of inertia of the different bents; that is, the overturning unit pressure, positive or negative, taken by each column was proportioned to its distance from the center of gravity of the bents."

The paper by Professor Rathbun is an apt rejoinder to a hope expressed by Mr. Balcom in the last sentence of his article,[28] " * * * it is, therefore, greatly to be hoped that the building may be noted, not only for its height and majestic beauty, but also as a means of promoting engineering knowledge."

F. P. Shearwood,[29] M. Am. Soc. C. E. (by letter).—A fascinating problem is treated in this paper in that it deals with one of the major factors in the intricate problem of designing the frame of a high building, safely and economically. The records indicate that the wind forces acting on a high building are not nearly as severe as most building codes specify, and that the effect of gusts does not extend to a great depth. The paper indicates that the resulting forces on the sides of the building are very erratic and may create torsion in the structure.

A most useful and interesting part of the paper is the reference to the great initial resistance of the masonry. Some of this resistance was found to be plastic; that is, the deflection did not recover. This would indicate that movement had occurred between some parts of the material, or, that some permanent stretch or compression had taken place. Masonry has practically no ductility and, therefore, no resistance that is not elastic, except the friction between its particles. The steel frame is wholly elastic for loads far greater than those applied during the tests, with the exception of the possibility that the riveted joints can slip in their holes; but this is improbable at the small wind loads recorded during the tests.

If, therefore, the masonry has practically no ductility and the steel frame connections have not slipped, the plastic deflection of the building must be due to the fracturing of some of the elastic resistance of the masonry, and then to its overcoming the frictional resistance to its slipping on itself or on the steelwork.

A tremendous amount of labor, discussion, and thought has been expended on devising theories for calculating the exact distribution of the resistance to the wind forces on these steel frames, but each investigator disregards a great many actual facts which must modify the theories to such an extent that they probably do not agree, even approximately, with actual results.

To devise an exact solution of theory for such an indeterminate stress problem, and one which is based on very doubtful assumptions, seems like aiming at needless refinement. The resisting frame has many paths of resistance; it is composed of a material that has considerable ductility and elasticity of about twice the amount that is ordinarily utilized in design. Structural designing is restricted and influenced far too much by arbitrary application of unit stresses and other limitations, which are applied impartially to the computed stresses in members. Some of these members must resist the stress without assistance from adjacent members, whereas others, when overstrained, will automatically bring other paths into increasing assistance.

---

[29] Cons. Engr., Dominion Bridge Co., Ltd., Montreal, Que., Canada.

Roughly stated, in designing the frame of a high building the present custom is to consider it on the following bases:

(1) As composed of an articulated frame for calculating the vertical loads;

(2) As composed of independent portal bents with rigid joints for calculating the horizontal forces;

(3) That the wind force is a uniformly applied load; and,

(4) That the steelwork is assembled and fabricated without having initial strain or slackness, and that it is wholly and perfectly elastic.

In actual practice not one of these conditions is quite correct; but it is assumed arbitrarily that they are all correct, and on that basis the stresses are calculated and the material is strictly porportioned.

The important questions raised by the tests in this paper and which require further investigation are: (*a*) Whether the great proportion of resistance that the masonry exerts against the wind forces will be maintained when much higher winds occur; and (*b*) whether, or how much of, this resistance will be fractured and, therefore, destroyed before the wind forces reach their maximum intensities. There might be much insurance against partial damage to the masonry, etc., should severe winds occur, by utilizing more fully the potential stiffening effect of the masonry, anchoring it firmly to the steelwork, and even reinforcing the walls.

These tests show how very problematical are the assumed forces and resistances which are commonly used as the bases of the theories and calculations for designing steel building frames. They indicate that a building should be treated more as a monolithic whole rather than just as an independent frame composed of supports and bents. It is especially important to guard against details that have planes of weakness or relatively excessive or sudden changes in stiffness, where strains will be dangerously concentrated when meeting the general deflections. Finally, they suggest that the stresses given by the conventional theories (which endeavor to distribute, exactly, the indeterminate stresses induced by indeterminate forces) should be used only as a guide, and that engineering judgment and experience should be given greater latitude.

LYDIK S. JACOBSEN,[30] ESQ. (by letter).—A series of interesting tests is reported in this paper, which contains information of great value to the student of structural engineering. From the point of view of fundamental knowledge of the dynamic behavior of a building, the author's observations of the longer of the fundamental transverse periods of the Empire State Building is especially illuminating and is in good agreement with numerous observations made on buildings in California.

Equation (1), in Part II, appears to be somewhat misleading since it applies only to ratios. In other words, T is not the period of vibration; it is only the ratio of what Professor Rathbun calls homologous times. Similarly, W is not the weight of the building or of the model; it is only the ratio of homologous weights or masses, etc. Aside from this obvious over-sight, it is extremely interesting to note that the model period converted to the prototype dimensions becomes 17.8 sec whereas the observed building period is 8.25 sec. Since the influence of the foundation on the fundamental period of the actual building is not well known quantitatively, it was probably not considered in the model, so that the 17.8-sec period obtained by the model must have been based on the assumption of an infinitely rigid foundation. With this in mind, it is obvious that the stiffening effect of the masonry must be somewhat greater than the author's estimate of 350 per cent. Data obtained by the United States Coast and Geodetic Survey[31] on buildings at different stages of construction, support the view that the masonry walls may stiffen the structure as much as ten times the steel frame stiffness.

---

[30] Prof., Mech. Eng., Stanford Univ., Stanford University, Calif.

It is also of great interest to note that between 1932 and 1934 the observed period of the building decreased from 8.38 sec to 8.14 sec. One wonders if the present period of 8.25 sec indicates an actual lengthening since 1934, or if the methods of measuring the periods are really of three-place accuracy. The author's explanation that the shortening of the period is probably due to an adjustment of the several parts of the structure during severe storms is quite plausible, especially if the least known part of the structure—the ground on which it stands—is included. It is suggested that a time record of accurate determinations of periods be continued.

The fact that the building does not have a unique position of static equilibrium, as indicated by the collimator observations, is significant. Undoubtedly, it must be given the author's interpretation as being due to plastic action, within the structure itself, although even in that case the ground effect, if it is nonuniform over the foundations, may account for part of the wandering of the middle position of swing in still weather.

Considering the well-known fact that wind structure, especially during storms, is a very complicated phenomenon,[32] it is not strange that the simultaneous readings of wind velocities obtained at the four stations in the vicinity of the Empire State Building differ greatly. One wonders if the four lengthy pages of Table 1 really justify "considerable detailed study."

Conclusion (1) relating to the complicated and irregular distribution of wind pressure on a building is amply warranted by even a cursory glance at Table 2. It is further corroborated by common-sense observations as well as by the aforementioned technical studies.[32]

The author's derivation of formulas, beginning with Equation (5) and ending with Equation (20), is extremely involved considering that the result (Equation (20)) can be written down almost *a priori* and that Equation (19) simply shows, in integral form, the sum of some of the potential energies divided by the kinetic. Since the author agrees with other investigators about the great stiffening effect of the masonry walls, one is surprised to find that the theory involves only the usual "static" framework consisting of columns and girders, and that the more important members—the walls—are entirely neglected. The writer suggests that, if an explicit formula for the fundamental transverse period is to be developed, a method involving an equivalent cross-section of a homogeneous, although far from uniform, beam may lead to simpler and more accurate results. However, the step-by-step method of calculating the period is patently indicated for a building of as complicated an outline as the Empire State Building.

---

[31] Preliminary Repts. of U. S. Coast and Geodetic Survey's California Seismological Program.

[32] "Wind Structure in Winter Storms," by R. H. Sherlock, M. Am. Soc. C. E., and M. B. Stout, *Journal of the Aeronautical Sciences*, December, 1937.

In spite of this seemingly critical discussion of the paper, the writer wishes to record his great appreciation of this interesting and important contribution to the knowledge of the dynamic behavior of buildings.

Francis L. Castleman, Jr.,[33] **Assoc. M. Am. Soc. C. E.** (by letter).—The Technical Research Committee of the American Institute of Steel Construction, and the owners, architects, consulting engineers, and contractors for the Empire State Building are to be congratulated on the foresight shown in equipping this building in such a manner that pertinent data on wind and its effect could be recorded. Professor Rathbun has presented an interesting and worthy paper describing these data and it should stimulate interest in their proper evaluation. Quantitatively, many more large-scale experiments appear necessary before any definite interpretations or conclusions can be drawn; but, qualitatively, the data submitted tend to clarify and sustain some common conceptions on this intricate subject.

Constructive criticism would point to the relative brevity of the paper. As the work extended over a period of five years, considerably more explanatory data might have been inserted at the expense of brevity. On first reading, the paper appears somewhat voluminous, but on detail study the writer thought certain parts meager and fragmentary.

Although this building is monumental in that it is the tallest ever built, there is a particular property it has in common with many other buildings; namely, the ratio of its maximum height to its least width is less than 8. A maximum height of 1 049 ft and a least width of 134 ft are assumed. This feature is emphasized in the hope of encouraging like studies on buildings with a similar ratio, but of not such great height. The writer knows of several buildings exceeding this ratio.

A theoretically correct mathematical analysis, taking all factors and their combined effect into consideration, would be difficult for a structure of this nature. The bracing in the central bays around the elevator shafts, plus the shallow "knuckle" wind-bracing, constitute a combined system which, when considered as a unit, forms a complicated but effective arrangement. This should be borne in mind when interpreting much of the data submitted.

As stated in the paper, the plastic action of the surrounding material has a marked effect in reducing deflection and vibration at ordinary wind loads—even at high wind loads. In the present state of knowledge, however, it is doubtful whether these materials should be counted at extreme loadings, as their beneficial effects under such conditions are reduced. Extreme loadings occur perhaps only two or three times in the life of such a structure; but on such occasions it must be able to sustain them adequately. Mr. Charles B.

---

[33] Associate Prof., Structural Eng., Vanderbilt Univ., Nashville, Tenn.

Driscoll[34] has commented interestingly on his esoteric experiences during a recent storm as follows: "Yes, the tall towers swayed. Some of them rather sickeningly. There were plenty of towers in which office employees became seasick during that stormy afternoon. * * * A good many were excused from work." These remarks from a layman should be convincing in themselves.

Although the Empire State Building is much higher than its neighbors, the latter will have a considerable effect on the resulting action of the wind. Until wind-tunnel methods are modified to make allowance for this effect, as well as other factors, experimental data based on such methods can be of little help.

The wind records given in Table 1 are particularly enlightening and, as the author states, justify considerable detailed study. Twenty-three items in this table show wind velocities of 50 miles per hr, or more. Using the well-known relation, $p = 0.003\ V^2$ for a 50-mile wind, $p = 7.5$ lb per sq ft; for a 70-mile wind, $p = 14.7$ lb per sq ft; and, for a 102-mile wind, $p = 31.2$ lb per sq ft. It is assumed in the foregoing that true and not indicated velocities are dealt with. (Lack of this distinction is a source of continual annoyance and error in many published data.) In the storm that swept over a part of New England in September, 1938, an Associated Press Dispatch of September 23 stated that wind gusts of approximately 173 and 186 miles per hr were recorded at the Harvard Meteorological Observatory. In design, usually, the average speed over a few minutes is considered satisfactory, but there is a school of thought which maintains that extreme velocities should be used. From the data supplied, and considering its erratic nature, it would seem that 20 lb per sq ft taken by the frame alone is not over-conservative in the vicinity of New York City. Table 1 indicates that the pressure varies with the height (which has long been known), but that there appears to be no discoverable law of variation for such a densely built area. Many building codes allow for this condition by varying the pressure approximately directly as the height, with certain fixed limitations. Considering the present state of knowledge this may be a wasted effort complicating the work of the structural engineer. Until more data are forthcoming, the best that can be done is to use a sum total effect that approximates the actual conditions in the light of present-day knowledge, in an endeavor to produce an adequate structure from the standpoint of human safety and comfort, rather than from that of cost. This is not as easy as it sounds.

From a meteorological viewpoint three weather bureaus on Manhattan Island may be sufficient, but from the standpoint of the structural engineer, or rather the data that should be furnished to him by the meteorologist, this number would appear deficient.

Fig. 8 seems to indicate that on the windward side of the building the air currents tend down, whereas on the leeward side the tendency is upward. As Fig. 8 is indicative of a single observation it would be interesting to know if this is true in general, or even approximately so. The manometer readings exhibit qualitatively the complex action of the wind and the great variations in pressure both positive and negative. However, as Professor Rathbun states, they are not to be taken as giving a true picture of the actual forces acting on the building. Although the influence of temperature on deflection has been shown to be negligible on this building, the writer would be wary as to accepting this in principle. With the rapid developments made in light-weight covering, and considering similar future advances, this might not always be true, by any means.

---

[34] "New York Day by Day," McNaught Syndicate Inc., *Nashville Banner*, October 5. 1938, p. 9.

The author has made computations assuming the structure to be a cantilever, applying the well-known flexure formula, $s = \frac{M\,c}{I}$. As he states, this presupposes that the twenty-fourth and twenty-fifth floors remain planes after elastic action from wind has occurred. With the use of the "knuckle" wind connection, this is questionable. Although this type of connection has strength, it is relatively deficient in stiffness. Such computations should be accepted with reservations. Columns (13) and (14), Table 6, would seem to support this view. Furthermore, as one of the studies contemplated was, "a check, or refutation, of the basic principles of the portal and cantilever methods of analysis," the writer cannot help but wonder why the portal method was not at least touched upon for comparative data. However, it is only fair to state that for this building, with a double type of wind-bracing, any method, theoretical or otherwise, will be difficult to apply as the division of wind loading between the two systems leads to complicating ramifications.

It is stated that the plasticity of the surrounding materials has a dampening effect on the vibrations, but does not alter the period. This is true but does not tell the entire story as to ultimate possibilities which might occur. Although the possibility is remote, the disturbing wind forces might sometimes approximate a periodic nature for a short interval setting up a tendency toward a forced vibration. If the frequency of the disturbing forces is seriously different from the natural frequency the resulting vibrations will be small. If by chance the two frequencies are equal, or only approximately so, the amplitude is increased enormously.

In computing the equivalent uniform load by means of the model and graphical integration a minimum reading of 3.26 in., as given in Table 3, under Item No. 115, might be a more conservative value to use in the absence of detailed explanation. The deflection would then be $\Delta = 3.25$ in. The ratio of this deflection to that produced by a 1-lb load on the model would be $\frac{3.25}{0.0488} = 66.6$. The new horizontal pull at the eighty-sixth floor necessary to produce this deflection would be $P = \frac{66.6 \times 50\,530\,000}{(60.484)^2} = 919\,900$ lb. From the curves of Fig. 18 the same deflection would be produced by a uniform load over the north face of $\frac{919\,900}{64\,600} = 14.24$ lb per sq ft (including negative as well as positive pressure). As Professor Rathbun states, such a value should be considered with caution.

It would appear that Equation (18) lacks one term. The direct stress, s_1, in a beam at any floor level will be proportional to the difference of the shear below and above the level considered. Therefore:

$$s_1 = \frac{p_1}{q_1}\left(\frac{4\,\pi^2}{T^2}\right)\int_{x_1}^{h} m\,y\,dx \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

in which p_1 and q_1 are abstract dimensionless numbers. The energy in a beam under axial load is, $W_b = \dfrac{s_1^2\,L}{2\,A\,E}$, or,

$$W_b = \frac{L}{2\,A\,E}\,\frac{16\,\pi^4}{T^4}\,\frac{p_1^2}{q_1^2}\left[\int_{x_1}^{h} m\,y\,dx\right]^2 \ldots\ldots\ldots\ldots (22a)$$

and for all beams it will be,

$$\sum W_b = \frac{16\,\pi^4}{2\,T^4}\sum\frac{p_1^2}{q_1^2}\,\frac{L}{A\,E}\left[\int_{x_1}^{h} m\,y\,dx\right]^2 \ldots\ldots\ldots\ldots (22b)$$

Using capital letters for the dimensionless terms this will reduce (after multiplying through by T^4) to,

$$T^4\sum W_b = G\sum\left(\frac{L}{A\,E}\right)\left[\int_{x_1}^{h} m\,y\,dx\right]^2 \ldots\ldots\ldots\ldots (23)$$

The term on the right-hand side of Equation (23) should appear in the right-hand side of Equation (18). If it is admissible to neglect the third term in Equation (18), the term developed in Equation (23) can probably be neglected. For tower buildings the axial distortions of the columns are considerable and it is a question as to whether the third term in Equation (18) should be neglected. If it were necessary to consider it, much of the work of the author's paper might be invalidated as the area of the columns and beams of the model does not bear the proper similitude to the building itself. More information concerning the building and model would appear necessary to evaluate properly the weight that this term carries.

J. B. WILBUR,[35] ASSOC. M. AM. SOC. C. E. (by letter).—The subject of stresses in building frames due to lateral forces is as complicated as it is important. It is a problem that may be broken down into a large number of subdivisions, each one of which is so complicated as to discourage the hope of satisfactory solution without extended studies and research. It is only by the constant investigation of the various problems involved that satisfactory methods for the design of the frames of tall buildings may eventually be discovered. Professor Rathbun's data and discussions are especially helpful inasmuch as they are based on the action of an actual building.

The stiffening effect of the masonry of a building, as Professor Rathbun states, is a problem of considerable importance. Although it is undoubtedly important under most conditions, it is probably true that the stiffening effect of the masonry becomes of less importance as the loads increase. In the First Progress Report of the Committee on Steel of the Structural Division (Sub-Committee No. 31 on Wind Bracing in Steel Buildings),[36] the suggestion was made that judgment as to the stiffness of a building be based on a deflection index. This index was defined as the maximum deflection at the top of a frame (under maximum wind load and assuming the frame to carry the entire wind load) divided by the height of the frame. It was recognized that under normal conditions the deflection index corresponds to some multiple of the deflections that will actually occur, but the deflection index was suggested, nevertheless, as a convenient standard by which to judge the probable stiffness of a building. Although the use of the deflection index is open to criticism because of the fact that so much is left to judgment, in respect to the stiffening effect of masonry, the writer knows of no better practical approach to the problem. Some of Professor Rathbun's data should be of real value in applying the judgment factor in the use of the deflection index.

---

[35] Associate Prof. of Civ. Eng., Mass. Inst. Tech., Cambridge, Mass.

[36] *Civil Engineering*, March, 1931, p. 478.

When one wishes to make use of the deflection index, the question arises as to how to compute the deflection of the top of a building frame under the action of lateral loads. If a complete analysis of the frame has been made, this is easily accomplished by considering the action of one column stack by itself, and applying the moment-area method. It is often desirable, however, to estimate the deflection index without a complete stress analysis for the frame. Investigations by the writer have shown that deflection computations based on moments as analyzed by easily applied methods, such as the portal and cantilever methods, are likely to be seriously in error. In the belief that an approximate slope deflection solution might be used to advantage in this problem, an investigation was conducted, in 1933, under the supervision of the writer, by William Niessen, Jun. Am. Soc. C. E., and Mr. R. J. Stoddard.[37] The procedures followed were similar to those suggested by the writer in a previous publication dealing with an approximate slope deflection determination of moments in building frames due to lateral forces.[38] The method of distributing wind loads to the bents of a building which resulted from this investigation has been presented elsewhere,[39] but an interesting result of this research, which was not cited at that time, consisted of a method for determining the deflection of the top of a building bent, which is not only easily applied, but which is reasonably accurate. In the Appendix of that paper,[39] it was shown that, approximately,

$$\delta_o = \frac{H_o\,h_o}{24\,E}\left(\frac{2}{\sum K_{co}} + \frac{1}{\sum K_{ga}} + \frac{1}{\sum K_{gb}}\right)\ldots\ldots\ldots\ldots(24)$$

In Equation (24), all symbols apply to one bent only, as follows: δ_o = the increment to horizontal deflection occurring in a given story which is denoted by the subscript, o; H_o = total horizontal shear on the given story; h_o = height of the given story; E = modulus of elasticity of the material of the bent; $K = \frac{E\,I}{L}$ for a member, in which I and L are moment of inertia and length, respectively, for the member; $\sum K_{co}$ = the sum of the K-values for all the columns of the given story; $\sum K_{ga}$ = the sum of the K-values for all the girders of the floor directly above the story under consideration; and, $\sum K_{gb}$ = the sum of the K-values for all the girders of the floor directly below the story under consideration.

---

[37] "A Theoretical Investigation to Determine the Proportion of Wind Pressure Carried by the Various Bents of High Buildings," by W. Niessen and R. J. Stoddard. Submitted, in May, 1933, in partial fulfillment of the requirement for the degree of Master of Science.

[38] "A New Method for Analyzing Stresses due to Lateral Forces in Building Frames," by John B. Wilbur, Assoc. M. Am. Soc. C. E., *Journal*, Boston Soc. of Civ. Engrs., January, 1934.

[39] "Distribution of Wind Loads to the Bents of a Building," by John B. Wilbur, *loc. cit.*, October, 1935.

To obtain the deflection of the top of a bent, Equation (24) is summed up for all the stories of the bent:

$$\delta_{\text{top}} = \frac{1}{24\,E} \sum_{\text{basement}}^{\text{roof}} \left[H_o\, h_o^2 \left(\frac{2}{\sum K_{co}} + \frac{1}{\sum K_{ga}} + \frac{1}{\sum K_{gb}} \right) \right] \ldots\ldots (25)$$

In Equation (25), $\frac{1}{\sum K_{gb}}$ equals zero for the basement if the columns are considered as fixed at their base, since this condition is equivalent to girders of infinite stiffness.

Equation (25) may be evaluated directly for any given bent, if the loads acting on the bent and the make-up of the members are known. Comparing the deflections of the tops of nine different bents of varying arrangements and sizes as computed by Equation (25), with corresponding deflections based on so-called exact solutions by the slope deflection method, it was found that the average error resulting from the approximate nature of Equation (25) was 5%, whereas the maximum error was 13.5 per cent. In view of the fact that deflection indices based on these deflections would be used only as a guide to judgment in estimating probable building stiffness, the accuracy obtained is undoubtedly sufficient for practical purposes.

R. D. SPELLMAN,[40] ASSOC. M. AM. SOC. C. E. (by letter).—Regarding the effect of masonry upon the rigidity and strength of a building against wind pressure, as treated in the paper by Professor Rathbun, the writer would like to call attention to some observations made in Miami, Fla., after the 1926 hurricane. All the larger buildings in Miami, and especially those few which were structurally damaged by the hurricane, were inspected by the writer with the aid of the plans and specifications.

In most of the buildings damaged some kind of wind-bracing was included in the structural steel frame. Several were quite thoroughly designed, and, in others, the wind-bracing was covered by calling for top and bottom angles in the specifications. However, there was no apparent relation between the amount of the wind-bracing provided and the damage done to the different buildings. Some of the better designed buildings were damaged to a greater extent than others with practically no wind-bracing in the structural steel frame. The writer suspected that this inconsistency was due to the stiffening effect of the masonry. Calculations were made for each building to determine the unit shear on all masonry walls caused by the total wind load on the building above the floor where the greatest damage occurred. It was discovered that the damage was in close ratio to these unit shears.

---

[40] Chf. Engr., Rust Eng. Co., Pittsburgh, Pa.

When the calculations were made, a value of 60 lb per sq ft was assumed as the maximum pressure reached during the hurricane. The unit shears varied from about 40 lb per sq in. to 110 lb per sq in. in the Meyer Kiser Building, which failed completely near the sixth floor. Another building which had a unit shear of 75 lb per sq in. was damaged quite badly. As the shears were reduced to nearly 50 lb, the damage was quite small, and one building with practically no wind-bracing, but with a unit shear of about 40 lb, had only a few cracks in the plaster on the first floor. This building, however, was better protected by other buildings than most of those damaged severely. All the foregoing calculations were made hurriedly and no specific value should be attached to them, except to note that the damage was in proportion to the unit shears on the masonry walls of the buildings, and perhaps to show about where damage may be expected to begin. All the aforementioned buildings were between fifteen and thirty stories high.

Most buildings in Miami, and especially those referred to herein, are constructed with light masonry walls; buildings of the same class in New York City, or other parts of the United States, would have 50% to 100% more cross-sectional area of masonry walls.

It is not the writer's intent to leave the impression that damage was caused to some of these buildings in spite of the fact that they were provided with adequate wind-bracing in the structural frame. Had they been properly designed, it is certain that no damage would have occurred; for example, the Congress Building had been carefully designed for a wind pressure of 25 lb per sq in. and, at the time of the hurricane, had reached that stage in the construction at which all the exterior walls were completed but few of the interior walls and partitions had been installed, giving it the maximum wind load with the minimum masonry resistance.

Commenting on Conclusion (8) of the paper, it would seem that in some instances, depending on the proportions of the building and the quantity and distribution of masonry therein, a substantial saving could be attained safely by omitting the wind-bracing in the structural frame. This might apply to the entire building, or to the upper parts of most buildings. Perhaps the point at which this wind-bracing might be omitted could be determined at a certain unit shear on the masonry walls. When the unit shear exceeded this value, the entire load would then be taken by the structural steel frame, and the masonry disregarded both for strength and stiffness.

DAVID A. MOLITOR,[41] M. AM. SOC. C. E. (by letter).—The most complete answer hitherto published, relative to the actual functioning of the steel frame of a tall building, is contained in this paper. The author's findings, based on the observations and measurements made during high winds, at various times, over a period of years, constitute a valuable contribution to knowledge.

The assumptions which have served as criteria in formulating methods for the design of tall buildings, crude as they are, have generally given very satisfactory results in producing tenantable structures. However, these achievements have been attained by a process of gradual evolution, in which the progressive steps from smaller to taller buildings were verified by actual structures, each time after venturing a little higher than in previous cases; and now the profession learns from a large full-sized model (the Empire State Building) that the assumed design unit stresses for resisting wind loads are about three times those actually occurring in the steel frame when subjected to such wind loads; and, that the maximum deflection at the top of the cantilever is only about one-third as much as would be expected if the steel frame were fully loaded and stressed according to the design assumptions.

---

[41] Cons. Engr., Harlingen, Tex. Mr. Molitor died September 8, 1939.

Some years ago the writer attempted to appraise the relative stiffness of a building frame as affected by the contributing stiffness of the architectural clothing,[7] and, strangely enough, arrived at a conclusion which is now found to check remarkably well with the results of the actual measurements obtained on the Empire State Building. These findings prompted the writer to express the opinion that all refinements pertaining to wind stress analyses in building frames border on wasted energy, and that rational design methods only could lead to satisfactory and economic construction.

The deflection of a 40-story building frame was computed to be 6.5 in., on the assumption that the frame carried the entire wind load of 15 lb per sq ft on the basis of 24 000-lb unit stress. The probable deflection of this building would not exceed 1.5 in., leading to the conclusion that buildings thus designed would deflect three to four times as much as they actually do. In other words, the architectural clothing, and not the steel frame alone, is responsible for the rigidity found by experience to be manifested in present-day tall buildings.

Were it not for the inherent rigidity of the architectural clothing of the steel frame, current design assumptions would be seriously deficient. Fortunately, the combination of a flexible steel frame, designed for certain wind loads and unit stresses, with a comparatively rigid fill material such as concrete, does produce tall buildings with tenantable rigidity. There is one mitigating circumstance which makes vibratory disturbances more tolerable, and that is the period of the vibration which may vary from 5 to 10 sec.

However, rigidity constitutes an important asset to the owner of a building, and the degree which the steel frame alone contributes to this factor, despite the severe load assumptions, is, in fact, relatively insufficient to supply the necessity.

In view of the foregoing and the substantial agreement with the findings presented in the paper, it would be very risky to adopt any changes in the present methods of design, or to choose different kinds of steel such as silicon steel, with higher unit stresses which would result in more flexible frames, thus incurring the danger of introducing new difficulties which might easily result in untenantable buildings. If such an error of judgment were committed in a design, the result might produce failure of the financial undertaking.

It would be highly desirable to have similar measurements made on other new buildings so as to add more of the valuable data to present knowledge on this subject. However, the irrefutable facts presented in this paper should put a quietus on most of the theoretic speculations indulged in by writers on this subject, and designers should continue to use the simple approximate methods which have given such good results in practice.

Walter J. Gray,[42] **Jun. Am. Soc. C. E.** (by letter).—Adding to the interesting record of the variations of wind action on the Empire State Building, as reported by Professor Rathbun, is the observed fog characteristic shown in Fig. 23. As indicated, the fog formed a "wake" on the north face of the building, contracting to a neck at a distance of about 100 ft from the structure and then suddenly spreading again to dissipate itself in the direction of the wind. The phenomenon was visible from the street level since the concentration of fog in the "wake" caused it to appear darker than the surrounding fog body.

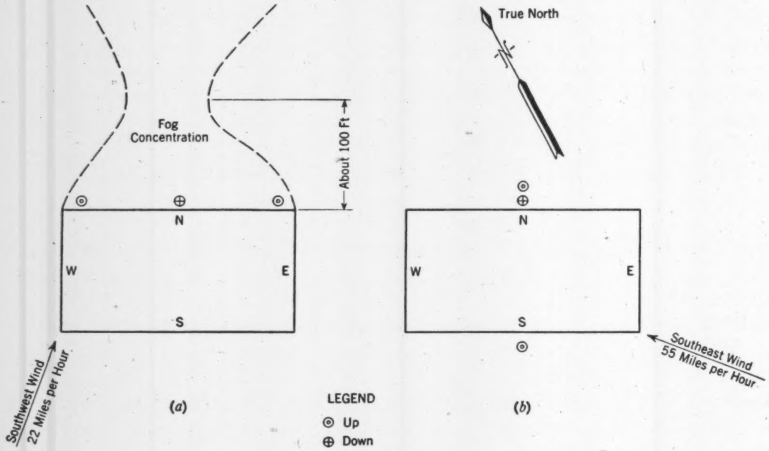


Fig. 23

This "wake," similar in character to the phenomena explained in texts on aerodynamics, substantiates the existence of a negative pressure on the far side of the structure, which characteristic is indicated by the manometer tube readings in Fig. 10 of the original paper. At the time of the observation, air currents close to the building indicated only vertical movements on the north face. The observation was made at a time of low wind velocity as shown by the anemometer at the top of the building. Subsequent observations in a heavy fog failed to indicate that the visible phenomenon occurred at higher wind velocities. The observations shown in Fig. 23(*b*) were taken under fog conditions similar to those of Fig. 23(*a*), but no "wake" was seen in the case of the higher velocity. It would appear that although a negative pressure exists at higher velocities, its visible evidence is destroyed by the gusty character of the wind.

K. L. DeBlois,[43] **Assoc. M. Am. Soc. C. E.** (by letter).—The vibration tests on the model as described under the heading, "Description of the Models," provide an opportunity to measure the damping. The shape of the curves shown in Fig. 20 indicates that damping is due to elastic or internal friction of the material, and, being proportional to velocity, the damping has the effect of reducing the oscillations in geometrical progression.[44]

---

[42] Eng. Asst., Dept. of Public Works, New York, N. Y.

[43] Engr.-Structural, U. S. Engr. Corps, Pittsburgh, Pa.

The equation of the curve connecting the extremities of the oscillations in Fig. 20 is given by,

$$\alpha_r = \alpha_0 \, e^{-2\pi\delta t} \qquad \text{(26)}$$

in which α_0 is the initial amplitude; α_r is any succeeding amplitude; t = elapsed time; and, δ = damping coefficient.

From inspection of Equation (26) it is evident that any convenient scale may be used to measure α_0 and α_r. Using Fig. 20(*b*): $\alpha_0 = 0.27$ in.; $\alpha_r = 0.01$ in. after 31 vibrations; $t = 4.0$ sec; and, T = period $= \frac{4}{31} = 0.129$. Substituting these values in Equation (26): $0.01 = 0.27\, e^{-2\pi\delta 4.0} = 0.27\, e^{-25.13\delta}$; from which, by logarithms, $\delta = 0.131$, and the damping factor, $e^{-2\pi\delta T} = 0.90$. This damping factor expresses the rate at which the vibrations tend to disappear; that is, the amplitude of any vibration is 0.90 of the amplitude of the preceding one on the same side of the axis.

Applying the same procedure to the building, which has an observed period of 8.25 sec, and assuming that the ratio of the amplitudes after 31 vibrations is the same as in the model: $t = 8.25\ (31) = 255.5$ sec; and, $\delta = 0.002$.

ALBERT SMITH,[45] M. AM. SOC. C. E. (by letter).—A large mass of original data is presented in this paper, which should be analyzed carefully to reveal all the information it will yield. From Table 1, the writer took all the items in which the velocity was as great as 30 miles per hr at the Empire State Building, and separated them into 16 compass directions. For each direction the average of the velocities was computed for the Empire State, the Daily News, and the Whitehall Buildings. (It is assumed that all readings have been corrected, and can be compared.) The numbers of readings are plotted in Fig. 24. The approximate effective depths of wind current are as follows:

Buildings	Depth, in feet
Empire State (1 263 − 100)	1 163, or less
Daily News (504 − 100)	404, or less
Whitehall (within a horizontal arc of 247°)	454

The dotted line for the Daily News Building and the full line for the Whitehall Building show ratios of the average velocity in each direction, to the average velocity at the Empire State Building in the same direction. Within an arc of 112° the high buildings near the Whitehall Building evidently reduce the velocity; and at the Daily News Building the influence of the Chrysler Building on the velocity of west northwest winds is clearly visible. Over an arc of 247° the Whitehall Building has a larger wind current depth than the News Building, and averages 0.617% as compared with 0.51% for the News Building. One would not expect so great an increase in velocity between 500 ft and 1 200 ft of height. These data seem to indicate that the idea of using a uniformly increasing pressure, advanced by H. V. Spurr, M. Am. Soc. C. E.,[10] is better than making no increase above 500 ft. Actually the readings indicate about four times as much pressure at the top of the Empire State Building as at the top of the News Building. It is likely, however, that the "20-yr wind," of much greater velocity than any measured in these tests, will have a velocity more nearly uniform above the 500-ft level. Returning to the consideration of shelter, winds from the south southwest appear to have a clear sweep for a long distance before reaching either the News or the Empire State Buildings, and one might expect the News velocity ratio be be high for this direction, instead of which it is about average. On the whole, there seems to be no evidence that anything except the very high Chrysler Building affects velocities at the News Building, and that therefore the velocities at the Empire State Building have not been reduced by the influence of other buildings.

---

[44] *Area*, Vol. 37, No. 380, October, 1935, p. 28.

[45] Pres., Smith & Brown, Engrs., Inc., Chicago, Ill.

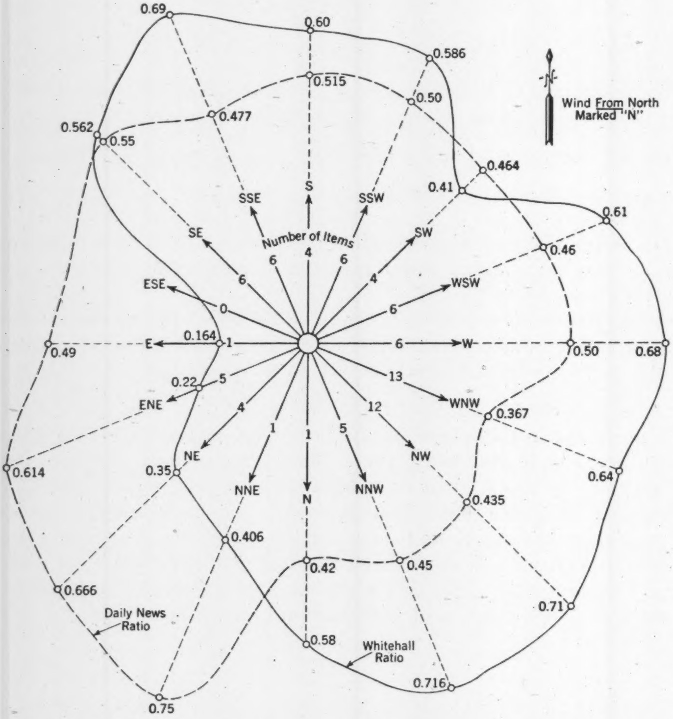


FIG. 24.—WIND VELOCITIES OF 30 MILES PER HOUR, OR MORE, AT EMPIRE STATE BUILDING

Manometer Readings.—The writer can see no possibility of error in the preparations for these tests. In a multi-story building there may be a difference in the interior air pressure at different stories; but the difference is small, and would not affect the total force. Since the pipes leading from the wall openings were very different in length, Tubes No. 3 and No. 8 (Fig. 4) would be much more responsive to gusts than the others; but this should not cause large errors. There are a large number of cases in which the approximately simultaneous readings at the stations on the same floor are of magnitude and sign that contradict all previous data from manometer readings. It may be that the 90-mile wind illustrated in Fig. 10 changed 90° between the readings of the two sets of five tubes each, and that this accounts for positive readings at both Tubes No. 7 and No. 4, but the writer cannot account, in Fig. 10(*c*) for readings in a 90-mile wind of simultaneous pressures at Tubes No. 6 and No. 10. He suggests that the sign for Tube No. 10 has been recorded in error. The same comment applies to the reading on Tube No. 10 at 50 to 60 miles per hr (Fig. 10(*c*)).

Of thirty-seven complete sets of observations for the seventy-fifth floor, eleven contain readings which contradict each other, and all sets exhibit slopes in the pressure diagram which differ very greatly from all those previously recorded. The air tunnel results of Messrs. Dryden and Hill[3] are consistent and explainable. Although the structure of the air that blew against the seventy-fifth floor was more variable than that in the wind tunnel at the National Bureau of Standards, it was probably less complicated than that in which the writer's observations were taken on the campus of Purdue University, Lafayette, Ind.[46] When plotted, his results were consistent internally, except for such variations as should be expected in light winds and near the ground, and agreed very well with wind-tunnel tests. For this reason he would like to know much more about the technique of the author's readings before he revises his ideas of how the wind forces are distributed.

Extensometer Readings.—Some of these readings correspond to a stress computed for the observed velocity, but many do not. Observations No. 9 and No. 10, Table 6, with the same velocity and direction of wind, show stresses of +223 and −343, respectively. Observation No. 26, during a south southwest wind of 30 miles per hr, showed a column stress of −488 lb per sq in., which would check quite well if it had been +488. Observation No. 28, during a south southwest wind of 18 miles per hr, showed a column stress of +59 lb per sq in., which, although correct in sign, is low in amount.

There is no mention in the description of the insulating precautions. Undoubtedly, some precautions were taken, but it is possible that they were not adequate. The largest fiber stress derived corresponds to about 4° differential in temperature between the rod and the column, and when the effect of a cold wind on this column above the twenty-fifth floor is considered, and the rapidity with which the twenty-fourth-story column would lose heat, it seems clear that the measuring rods should be insulated very carefully, or that exact temperature corrections should be made.

Plumb-Bob Readings.—The readings in Table 3 show something that one knows must occur, but which heretofore lacked proof. Fig. 12 indicates a "no man's land" under the bob about 1 in. in diameter. The tests prove that the eighty-sixth floor does not return to the same position each time when the wind becomes still, and that it is held away from the unstressed position a distance of the order of 0.5 in. The explanation of this phenomenon lies in the masonry envelop, which is of enormous stiffness against lateral forces, and has little strength. If it carried its own weight it would have great resistance to horizontal shear. Since it is supported at each story, the top layers of masonry in each story slide under horizontal shears, and the frictional resistance to sliding back prevents the building from returning to a zero position and keeps a small stress in the steel frame after the wind subsides. Except for this yielding resistance of the masonry the maximum deflections would be somewhat greater. In some of the top stories the masonry may carry nearly all of the wind shear, whereas in the lower stories it carries only a small part. In the lower stories the resistance of the masonry will not increase much as the wind force and the deflection increase. It is for this reason that structural engineers should neglect the masonry resistance in designing for wind loads. In the vibration of buildings, where the shears are small, the masonry plays an important part in determining the period and amplitude. The structural designer probably cannot do much to affect the vibration. The window area, type of masonry, and the character of the joint under the spandrels are more influential in respect to vibration than the stiffness of the structural frame.

---

[46] "Wind Pressure on Buildings," *Journal*, Western Society of Engineers, December, 1912; also, "Wind Loads on Buildings," *loc. cit.*, April, 1914.

A total deflection normal to the long axis of only 1.75 in. seems very small, but this was under a maximum velocity of only 50 miles per hr. Under 100 miles per hr the force would be four times as large, and many masonry joints, heretofore unstirred, would open and increase the deflection.

VICTOR R. BERGMAN,[47] ASSOC. M. AM. SOC. C. E. (by letter).—In Part II, under "Introduction," Professor Rathbun states that "Rigidity is of considerable importance inasmuch as it affects the popularity of a building with tenants." This aspect of the problem is naturally uncertain and debatable. However, a structure that is too flexible may show a definite economic loss to the owner because of increased maintenance costs directly ascribable to lack of rigidity.

The writer has in mind an office building with a tower extending upwards beyond the 500-ft level above the street. After every heavy wind storm considerable plaster cracking is reported. For example, on Sunday, January 22, 1939, the wind velocity rose to 56 miles per hr; the next day no less than sixty-one complaints concerning cracked plaster deluged the maintenance department of the building! As might be expected, most of the cracks occur in the lower stories of the tower. They appear in the plastered gypsum block partitions, usually horizontally at the ceilings, or vertically at the ends. Apparently the cracking does not occur in the plaster on the brick panel walls, the "skeleton walls" of the building.

That this building sways during a heavy windstorm is patent even to a casually observant tenant of any of the uppermost floors. Some of the electric-light fixtures are suspended from the ceilings by means of tubes which have a universal joint arrangement at the upper ends. At times, their movement is quite perceptible.

---

[47] Estimator and Engr., Godwin Constr. Co., New York, N. Y.

During a heavy gale in 1938, two young women were permitted to leave their work to go home, following their pleas that they were suffering from "seasickness." Strangely enough, the tenants, with the foregoing exception, have not complained about the swaying of the tower. Indeed they seem to expect it and to look for it. The maintenance department of the building has even received telephone calls from the tower tenants concerning the swaying of suspended objects. "Do come up," they sometimes say, "it's very interesting."

Evidently the "scientific" articles on the subject, as "ladled out" by the Sunday supplements, have had a reassuring effect.

OTTO GOTTSCHALK,[48] ESQ. (by letter).—In presenting field-tests data pertaining to the effect of masonry on a structural steel building frame, the author has certainly stirred up a desire to learn similar facts about as many existing tall buildings as possible. The opportunity to compare the behavior of a building with a true model of its skeleton will not occur frequently. This suggests that there should be more simple methods of determining the same information.

In 1938,[49] the writer presented a simple method of computing wind stresses (especially column pressures) from a general model analysis. If the column pressures are known, the distribution of the stress between various columns can be computed by this method. They can be computed, furthermore, from the shortening or lengthening of the columns in a building and, approximately, from the difference in elevation between corresponding points on adjacent columns.[49] It should be easy and inexpensive to weld special bolts or sockets to the field frame at several floors of existing buildings. Since the period of vibration of tall buildings is always equal to several seconds, it will be possible to obtain the change of level between column points originally at the same level by a simple device such as a water-level which can be adjusted to permit accurate readings to within 0.002 in. Such readings will supply extremely valuable information as applied to columns that are either isolated or embedded in walls or partitions, and also would offer a means of appraising the effect of the masonry covering.

COLIN SKINNER,[50] ASSOC. M. AM. SOC. C. E. (by letter).—In designing tall buildings numerous assumptions must be made in order to simplify the work to a practical degree. It is not rational, however, to go to great extremes in refinement of structural design unless the basic assumptions are substantially correct. Too much time and effort have been spent on devising methods of calculating wind stresses in tall buildings, based on the assumption that the steel frame carries all the stress, when it is obvious that the other materials increase both the strength and stiffness of a building. The data in Professor Rathbun's paper may not be complete, but it contains considerable useful information which should serve to stimulate a fresh approach, based on more logical assumptions. The American Institute of Steel Construction and the owners of the building should be congratulated for sponsoring this program and for permitting publication of the results.

---

[48] Buenos Aires, Argentine Republic.

[49] "Simplified Wind-Stress Analysis of Tall Buildings," by Otto Gottschalk (see this volume of *Transactions*).

[50] Coordinator, Region 2, USHA, Washington, D. C.

In 1932 the writer made a series of observations on the various instruments installed in the Empire State Building for measuring wind pressure, deflection of the building, and stresses in the steel frame. A considerable number of observations were made over a 2-yr period, and many of these are included in Professor Rathbun's paper. In addition to these data, however, there are several additional observations which may help in interpretation of the published data.

For approximately six months after the building was completed, the strain gages on several columns immediately below the twenty-fifth floor showed no change in stress, regardless of the intensity of the wind. The plastered partitions surrounding the elevator shafts and public halls, the concrete floors, and the exterior masonry walls stiffened the building to such an extent that practically no wind stress was transmitted to the steel frame.

First Big Storm.—Early in March, 1932, a severe storm hit the building with an indicated wind velocity averaging about 70 miles per hr, and gust velocities of more than 95 miles per hr. During this storm a series of muffled explosions were heard and some cracks developed in the masonry materials. At that time (1932) there were no tenants in the building between the twenty-fourth and the eightieth floors, with the exception of a few offices on the forty-first floor. The partitions surrounding the elevator shafts, public halls, and utility rooms (see Fig. 4) were all in place, however, and probably contributed to a large extent to the stiffness of the building.

A visual inspection made the day after this storm showed very few cracks in the partitions surrounding the elevator shafts. Diagonal cracks were observed in several office partitions perpendicular to the exterior wall on the forty-first floor, and in the partitions surrounding a motor room on the twenty-ninth floor. Cracks were also observed in the concrete floors at the first row of interior columns, parallel to the exterior walls.

These floor cracks seemed to indicate that the exterior columns were carrying a substantial portion of the direct stresses, in accordance with the portal theory. It should be noted, however, that the central core of the building contains deep knee-braces and many plastered partitions which undoubtedly restrained the floors from rotating in accordance with the cantilever theory. Furthermore, the floors were about a year old at the time of this storm, and the natural shrinkage of the concrete had probably produced internal tensile stresses which may have been close to the ultimate strength of the concrete. Many additional cracks have occurred in the floors since this storm as mentioned subsequently herein.

Groaning Period; Vibration.—On October 6, 1932, another severe storm occurred (Table 1, Item No. 82) with indicated wind velocities averaging about 70 miles per hr from the south southeast. During this storm the strain gages on the twenty-fourth-story columns moved back and forth very slightly, indicating that the steel frame was beginning to carry a small portion of the wind load. Groans could be heard as the vibration of the building apparently caused slipping between the steel frame and the masonry or grinding in the cracks, which had occurred in the first big storm. With each succeeding storm the groans caused by vibration decreased in intensity and the strain gages apparently became more sensitive, indicating that the building was becoming more flexible and that the steel frame was gradually assuming a larger percentage of the wind load.

After the storm of November 1, 1932 (Table 1, Item No. 102), it was noticed that winds of 30 to 35 miles per hr, against either of the broad faces of the building, would cause the strain gages to oscillate; but winds below this range caused no apparent change of stress in the steel. This indicated that the force of a 30- to 35-mile wind was required to overcome the internal friction and to cause the masonry to slip on the steel or along existing cracks. Thus, a severe storm would deflect the building several inches; then, as the wind subsided, the deflection would decrease until the internal friction was equal to the restoring force of the deflected building. The building would then remain in this partly deflected position until the next severe storm.

It was also noted that the building was considerably stiffer in the longer direction. Before November 1, 1932, there was little deflection about the minor axis; but during this storm (Table 1, Item No. 103) additional cracking occurred, and the building began to deflect and vibrate appreciably about both axes. After this storm it was noticed that winds of 55 to 60 miles per hr against either of the narrow faces would cause the strain gages to oscillate, but winds below this critical range caused no apparent stress in the steel frame.

Existing Cracks.—In December, 1938, the writer made a visual inspection of existing cracks on several unoccupied floors, including the twenty-ninth, forty-second, and fifty-fifth floors. It was found that the cracks in the floors and walls were more numerous then than in 1933 (five years previous). Inspection of the floors showed prominent cracks extending between column centers in both directions. It is probable that these cracks were caused chiefly by natural shrinkage of the concrete, as they generally averaged about 0.12 in. in width with columns spaced about 20 ft on centers. It was noted, however, that in several places the floor on one side of a crack was more than $\frac{1}{16}$ in. above the level on the other side, and in some cases the maximum vertical displacement was nearly $\frac{1}{8}$ in. There were relatively few cracks in the plastered ceilings; but the plastered partitions surrounding the elevator shafts were seriously cracked in the lower portion of the main tower and cracked to a lesser degree in the upper stories.

Velocity of the Wind.—In studying Table 1, it should be noted that the New York City Observatory in Central Park is nearly 7.5 miles from the U. S. Weather Bureau. The anemometer in Central Park is 62 ft above the ground, but only about 30 ft above the trees. In addition, the park is entirely surrounded by tall buildings averaging approximately 140 ft in height. The U. S. Weather Bureau, near the southern tip of Manhattan, is 454 ft above the harbor, but is near the down-town "skyscraper" zone, and winds from the north, northeast, and east are seriously modified in velocity and direction by these tall buildings. The anemometer on the Empire State Building has a 32-blade rotor which is restrained by helical springs, attached by gears to a self-synchronous motor. When the wind blows the rotor turns through less than 360° and the angle of rotation measures the wind velocity. The anemometer on the Daily News Building is similar to the one on the Empire State Building, but is 30 ft above the roof and may be less affected by the up-draft on the windward side of the building.

Before January 1, 1928, the U. S. Weather Bureau used a four-cup anemometer which gave indicated wind velocities greater than the true velocity. During the next few years, however, a new three-cup anemometer was developed to give more accuracy and, since January 1, 1932, wind speeds published by the U. S. Weather Bureau have been corrected before publication to give true wind speeds. In analyzing wind records it is important to know whether the velocity represents the average for a 1-hr period, a 5-min period, or a brief gust, as there may be considerable difference in these velocities. The velocities obtained at the Empire State Building are averages estimated after watching the anemometer dial for several minutes, but most of the other velocities are averages for a 1-hr period.

Wind-tunnel tests made at the National Bureau of Standards on a model of the Empire State Building indicate that the velocity 15 ft above the observation tower is about 23% greater than the velocity of the approaching wind. The wind velocities recorded in Column (4), Table 1, therefore, are about 23% too high and should be reduced when studying the actual wind pressures on the building. Another factor that should be considered is that during a severe storm the wind velocity 1 263 ft above the ground is undoubtedly greater than the velocity nearer the ground.

TABLE 7.—AVERAGE WIND VELOCITIES GROUPED BY INTENSITY

Observed velocity of wind, in miles per hour	Number of observations	AVERAGE VELOCITY OF WIND, IN MILES PER HOUR*				RATIO OF VELOCITIES COMPARED WITH EMPIRE STATE BUILDING			
		Empire State Building	Central Park Observatory	Daily News Building	U. S. Weather Bureau	Empire State Building	Central Park Observatory	Daily News Building	U. S. Weather Bureau
Less than 10	15	5.9	5.9	5.7	7.7	100	100	96	129
10 to 19	34	13.6	9.6	10.8	11.3	100	71	79	82
20 to 29	31	23.1	10.8	12.4	15.0	100	47	54	65
30 to 39	27	32.9	14.1	16.8	20.6	100	43	51	63
40 to 49	28	42.7	16.9	19.8	26.0	100	40	47	61
50 or more	28	59.2	21.0	26.6	29.8	100	36	46	51

* At Empire State Building wind velocities are averages for a brief period of observation, but at the other three observatories wind velocities are averages for a 1-hr period.

The winds in Table 1 were first arranged in six groups, according to velocity, and each group was averaged to obtain the data shown in Table 7. This study shows that the relative wind velocities at the four observatories change considerably as the storms increase in strength. With strong winds, low buildings receive considerable protection from surrounding buildings and the degree of protection is relatively greater during extremely severe storms.

The relation between height and velocity may be estimated from the storm of March 22, 1936, at the top of Table 1, which gives an indicated wind velocity of 102 miles per hr at the Empire State Building. The true velocity of the approaching wind, therefore, was about 84 miles per hr, compared with 71 miles per hr at the U. S. Weather Bureau and 54.4 miles per hr at the New York City Observatory in Central Park (see Table 8).

TABLE 8.—RELATION BETWEEN HEIGHT AND VELOCITY

Location of anemometer	Height above ground, in feet	True velocity of wind, in miles per hour	Ratio
Empire State Building	1 263	84	100
U. S. Weather Bureau	444	71	85
Central Park Observatory	62	54	65

On several occasions the writer has observed flags on various tall buildings flying in different directions with variations in direction as much as 90° at the same instant. In order to study the effect of surrounding buildings on velocity and direction, the winds of Table 1 were grouped by the direction indicated at the Empire State Building and the directions at the other observatories expressed as plus or minus degrees. This study showed fairly consistent differences for each particular direction and indicated that at the three lower stations both velocity and direction of the wind are seriously affected by the surrounding buildings.

On November 1, 1932, three sets of observations were made, as indicated by Items Nos. 100, 101, 102, Table 1. For several hours the wind was exceptionally steady with an indicated velocity of 75 miles per hr from the southeast.

TABLE 9.—AVERAGE WIND VELOCITIES, GROUPED BY DIRECTION

Number of observations	Direction of wind; Empire State Building	VELOCITIES OF WIND, IN MILES PER HOUR			
		Empire State Building	Central Park Observatory	Daily News Building	U. S. Weather Bureau
1	N	45	18	19	26
1	NNE	32	18	24	13*
5	NE	42	19	28	14*
5	ENE	58	25	34	12*
1	E	55	21	27	9*
6	ESE	51	15	28	28
6	SSE	58	16	26	36
4	S	35	13	18	21
6	SSW	39	15	19	23
3	SW	35	14	16	15*
6	WSW	36	15	17	22
6	W	35	14	17	24
14	WNW	43	17	16	28
13	NW	48	20	19	31
5	NNW	54	22	24	38
Total		666	262	332	277
Average		43.4	17.5	22.1	27.7
Ratio		100%	39%	50%	62%

* These points have been omitted from the average because tall buildings apparently shield the anemometer when winds are from these directions.

Just before noon, while the plumb-bob was being observed, the wind suddenly increased in intensity for a few minutes, swung around through south and then steadied at about 55 miles per hr from the west northwest. The exact time of this occurrence is not known, but the total change from an 85-mile southeast wind to a 55-mile west northwest wind probably occurred in less than 3 min. Similar rapid changes occurred on several other occasions and probably account for some of the marked variations in direction noted in Table 1.

Table 9 gives the average velocity of all winds of 30 miles per hr or more, grouped by direction. It shows fairly consistent ratios between the observed velocity at the Empire State Building and the average velocity for a 1-hr period at the other three observatories where recording instruments were used. The velocities shown in the right-hand column are generally about 60% to 65% as great as observed velocities at the Empire State Building, except in certain cases in which the skyscrapers of lower Manhattan shield the anemometer of the U. S. Weather Bureau. Thus, with winds from the northeast the ratio is 33%, and with winds from the east northeast, the ratio is only 21 per cent.

Gust Velocities.—The observations made during this program show clearly that the building not only deflects in accordance with the average wind velocity, but also vibrates due to variations in velocity. During a strong wind the anemometer in the Empire State Building moves back and forth continuously over a range of about 10 miles per hr, but occasionally a vigorous puff or a sudden lull may cause a momentary change in velocity as much as 20 miles per hr above or below the average velocity.

The velocity and duration of these momentary gusts are of considerable importance, as a 40% increase in velocity would cause an increase in pressure of nearly 100%, and if this sudden increase should occur in harmony with the vibration of the structure, the resulting stresses would be much greater than with a steady wind. On the other hand, however, it is quite probable that each gust of wind occurs over a relatively small area and therefore would not change the average pressure as much as indicated by the velocity-pressure equation. To illustrate this point the writer remembers one occasion when an observation was being made on the 5 manometers on the south side of the fifty-fifth floor. Readings had just been recorded for Stations 1, 2, 3, and 4 (Fig. 4) and all were less than 0.50 in. At Station 5 there was a difference of nearly −0.50 in., but while it was being observed, the difference suddenly increased to about −1.50 in. and remained steady at this point for about 10 sec. During this time several quick glances at the other manometers showed no great change at any of the other stations. This would seem to indicate that a gust velocity might be two or three times as great as the average velocity but that the gust might last only a brief time and extend over a relatively small area.

The data in Table 1 indicate that at the U. S. Weather Bureau in New York, N. Y., the maximum velocity for 5-min periods is generally about 17% greater than the 1-hr average. During the New England hurricane of September 21, 1938, the Harvard Meteorological Observatory records indicated an average velocity of 83 miles per hr for 1 hr; 94 miles per hr for $\frac{1}{2}$ hr; 111 miles per hr for a 5-min period; and gust velocities of 173 and 186 miles per hr. The velocity for the maximum 5-min period, therefore, was 34% greater than for the maximum 1-hr period, and the maximum gust velocities were more than twice the maximum hourly velocity.

Wind Pressure.—It is recommended that the data on actual wind pressures be studied in conjunction with the report of the wind-tunnel tests by Messrs. Dryden and Hill.[3] Unfortunately these tests did not include models of the surrounding buildings; but in spite of this the writer feels that their data are more complete and more reliable than the data actually obtained at the Empire State Building.

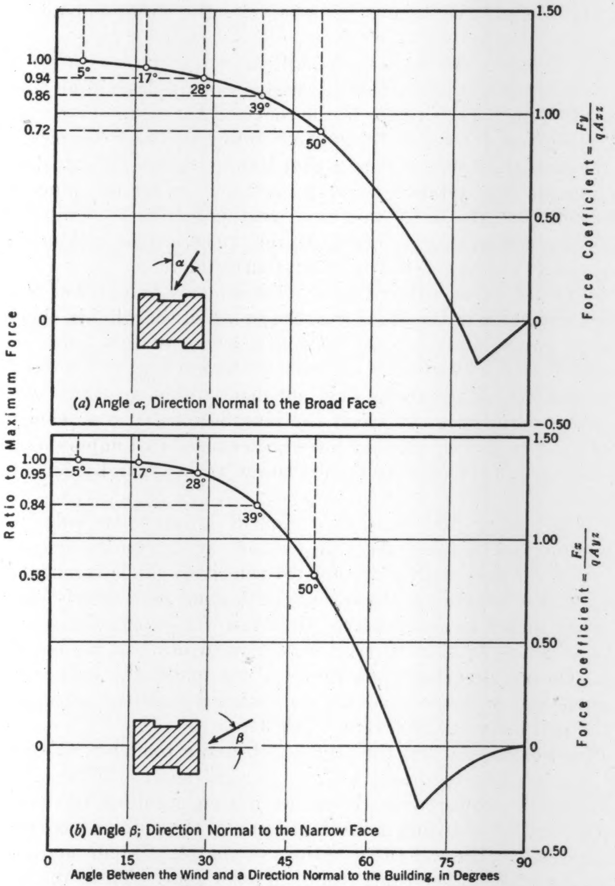


FIG. 25.—RELATION BETWEEN DIRECTION OF WIND AND OVERTURNING FORCE

The Dryden-Hill tests indicated: (*a*) That the greatest overturning moment occurs when the wind blows directly against one face; (*b*) that a suitable value for use in design of tall buildings is 0.0038 V^2, in lb per sq ft (V = wind speed, in miles per hour); and (*c*) that the velocity 15 ft above the top of the observation tower is approximately 23% greater than the velocity of the approaching wind. From the model tests, it was also possible to measure, accurately, the decrease in the overturning moment as the wind becomes more diagonal. The relations between the direction of the wind and the overturning wind force are shown in Fig. 25 and have been used in analyzing the plumb-bob and collimator readings.

The Fire Tower.—The vent shaft built in conjunction with the fire tower acts like an enormous chimney and causes a strong up-draft at all times. This shaft extends from the sixth floor to the roof and has an unobstructed opening of about 40 sq ft at the top. When the building was first completed, the up-draft in the vent shaft was so strong that leakage under the exit doors caused an objectionable high-pitched whining noise. In order to reduce this noise all doors leading to the fire tower were weather-stripped. This reduced the noise but did not eliminate it, and in March, 1933, a light bulkhead was built entirely across the vent shaft at the fifty-fourth floor.

This bulkhead reduced the noise to a satisfactory extent, but when a door was opened anywhere in the fire tower, the sound of the wind in the vent shaft was reduced immediately. After becoming aware of this phenomenon, the writer was able to tell by sound if all doors to the fire tower were tightly closed, and the plumb-bob observations were not made unless all doors were shut, as an open door would seriously affect the plumb-bob. The movements of the plumb-bob, shown in Fig. 12, may have been caused by temporary drafts, due to occasional opening of doors to the fire tower, rather than by deflection of the building.

There is also an up-draft on the exterior on the windward side near the top of the building, and on sunny days the heat of the sun warms one or two faces and the warm air rises with a perceptible velocity. On several occasions the writer has walked around on the seventy-fifth floor, successively opening each window a few inches to note whether the wind was blowing in or out. These observations generally showed a strong outward draft at about 75% of the windows. On one occasion when this test was made, the outward draft was noticed at all except seven windows, indicating a negative pressure on about 91% of the periphery at the seventy-fifth floor.

The Plumb-Bob.—At first the plumb-bob had a tendency to wander in an irregular manner due to drafts within the vent shaft. When all the doors leading to the fire tower were closed, the strong up-draft in the vent shaft produced a peculiar whistling noise; but if any of these doors were opened, this noise decreased greatly and the gyrations of the plumb-bob increased, due to the disturbing effect of the draft from the open door. These difficulties were solved by attaching a pair of oil dampers to the plumb-bob as shown in Fig. 11, and by recording only those observations that were made when all doors to the fire tower were closed.

In making plumb-bob readings the dampers were immersed in oil until the plumb-bob was moving in an ellipse with the major axis about 2 in. long. Small markers were placed on the target to mark the ends of the major and minor axes of this ellipse. As soon as the plumb-bob was observed to make three or

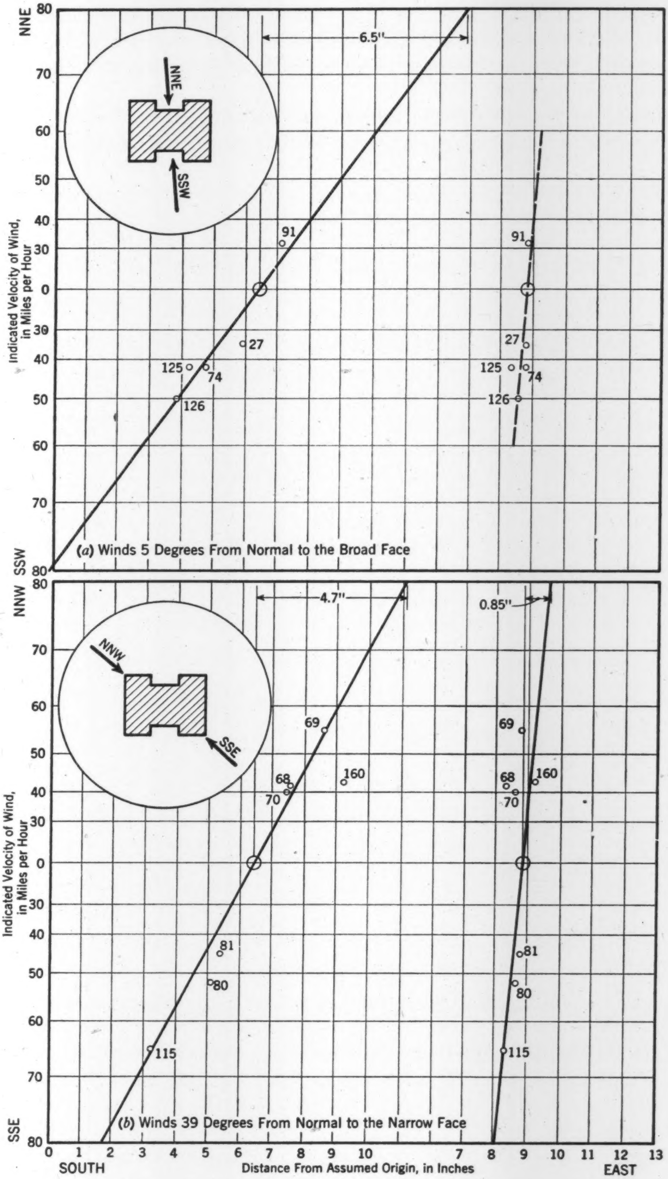


FIG. 26.—DEFLECTION OF THE BUILDING, INDICATED BY PLUMB-BOB

four complete circuits around the same ellipse, the center was observed and recorded. This process was repeated until at least ten readings had been made. These readings were then averaged and entered in the records as a single point for that particular wind.

Deflection of Building from Plumb-Bob Readings.—The deflection of the building was studied by plotting the plumb-bob observations on eight charts, such as the two in Fig. 26. Since the wind force is directly proportional to the square of the velocity, the indicated velocity of the wind was plotted on a parabolic scale so that the deflection curve on these charts will be a straight line. Winds of less than 30 miles per hr were omitted, as the force of these winds apparently was not sufficient to overcome the internal friction in the masonry materials.

TABLE 10.—DEFLECTION OF BUILDING, FROM PLUMB-BOB OBSERVATIONS

Item No. (see Table 1)	Velocity of wind, in miles per hour	Direction of wind at Empire State Building	Angle between wind and normal to building	Force ratio*	Observed deflection, in inches	"Normal" deflection, in inches	Item No. (see Table 1)	Velocity of wind, in miles per hour	Direction of wind at Empire State Building	Angle between wind and normal to building	Force ratio*	Observed deflection, in inches	"Normal" deflection, in inches
(*a*) WINDS AGAINST NORTH FACE; ORIGIN ESTIMATED AT 6.40							(*c*) WINDS AGAINST EAST FACE; ORIGIN ESTIMATED AT 8.85						
31	35	N	28	0.94	1.00	1.06	80	52	SSE	39	0.84	0.15	0.18
68	42	NNW	50	0.72	1.12	1.56	100	75	SE	17	0.99	0.65	0.66
69	55	NNW	50	0.72	2.24	3.12	101	75	SE	17	0.99	0.97	0.98
70	40	NNW	50	0.72	1.08	1.50	109	60	ENE	50	0.58	0.65	1.12
89	35	NE	17	0.98	1.11	1.13	110	60	ENE	50	0.58	0.56	0.96
90	42	NE	17	0.98	1.11	1.13	115	65	SSE	39	0.84	0.49	0.58
91	32	NNE	5	1.00	0.73	0.73	129	58	ENE	50	0.58	0.13	0.22
109	60	ENE	39	0.86	3.30	3.84	151	35	SE	17	0.99	0.45	0.45
110	60	ENE	39	0.86	3.36	3.91	153	57	SE	17	0.99	1.65	1.67
129	58	ENE	39	0.86	2.81	3.26	154	55	E	28	0.95	1.60	1.68
(*b*) WINDS AGAINST SOUTH FACE; ORIGIN ESTIMATED AT 6.40							(*d*) WINDS AGAINST WEST FACE; ORIGIN ESTIMATED AT 8.85						
27	35	SSW	5	1.00	0.50	0.50	69	55	NNW	39	0.84	0.05	0.06
61	32	S	28	0.94	0.65	0.69	94	50	WNW	5	1.00	0.70	0.70
62	38	S	28	0.94	0.80	0.85	103	42	WNW	5	1.00	−0.17	−0.17
74	42	SSW	5	1.00	1.62	1.62	104	45	WNW	5	1.00	−0.40	−0.40
77	42	S	28	0.94	1.95	2.07	105	50	WNW	5	1.00	−0.32	−0.32
80	52	SSE	50	0.72	1.35	1.87	124	50	WSW	50	0.58	0.54	0.93
81	45	SSE	50	0.72	1.00	1.39	130	47	NW	17	0.99	0.04	0.04
112	35	SW	17	0.98	1.20	0.22	131	40	WSW	50	0.58	0.34	0.59
115	65	SSE	50	0.72	3.14	4.36	132	35	W	28	0.95	0.40	0.42
124	50	WSW	39	0.86	2.17	2.53	133	40	NW	17	0.99	0.25	0.25
125	42	SSW	5	1.00	2.17	2.17	134	46	NW	17	0.99	0.03	0.03
126	50	SSW	5	1.00	2.58	2.58	155	90	NNW	39	0.84	1.85	2.20
131	40	WSW	39	0.86	1.01	1.17	156	55	NW	17	0.99	0.95	0.96
....	....	....	....	....	....	....	157	55	NW	17	0.99	0.95	0.96
....	....	....	....	....	....	....	160	42	NNW	39	0.84	0.40	0.48
....	....	....	....	....	....	....	161	45	NW	17	0.99	0.45	0.45

* From Fig. 25.

A study of these charts showed that the center of the building is at 6.40 on the east-west axis. The center on the north-south axis is not so well defined, however, due to the fact that the building is much stiffer in this direction, and the plastered partitions and other masonry probably were not seriously cracked in this direction until Storm 109 (see Table 1) on November 9, 1932. The few observations made after this date indicate that the east-west center is probably between 8.70 and 9.00 on the plumb-bob target and an average value of 8.85 has been assumed.

The wind-tunnel tests previously mentioned[3] showed that the greatest overturning moment occurs when the wind is blowing directly against one face of the building, and that this overturning moment gradually decreases as the wind becomes more diagonal (see Fig. 25). The deflections caused by diagonal winds, therefore, can be increased to show the deflection that would have occurred if the wind had been directly against one face of the building. Thus, when the wind is 28° from a normal to the broad face, the overturning force is about 0.94 times as great as for a normal wind and the observed deflection should be divided by 0.94 to obtain the maximum deflection for that particular velocity.

Table 10 was obtained in this manner. This study shows that an indicated velocity of 80 miles per hr directly against either of the broad faces causes a deflection of about 6.5 in. The same wind against either of the narrow faces would cause a deflection of about 2.0 in., which indicates that the moment of inertia about the north-south axis is more than three times as great as about the east-west axis. This ratio may not be a constant, however, as the plastered partitions and other masonry probably carry a large proportion of the wind load when the wind is against the narrow face, and each severe storm may cause further cracking and more flexibility in this direction.

If the indicated velocity at the top of the observation tower is assumed to be about 23% greater than the velocity of the approaching wind, the average deflection of the building at the eighty-sixth floor, as indicated by the plumb-bob, is as follows:

Description	Indicated velocity at the top of the building, in miles per hour:	
	80	100
True velocity of approaching wind, in miles per hour	65	81
Deflection of the building, in inches:		
North-south	6.5	10.2
East-west	2.0	3.1

Deflection of Building from Collimator Readings.—A study of the collimator readings given in Table 5 shows that the center is approximately at 5.6 along the north-south axis. Using this center, the collimator readings have been rearranged in Table 11 to show the average deflection, and the maximum amplitude of vibration for winds greater than 30 miles per hr against either of the broad faces. By using the "force ratios" obtained from Fig. 25(a), it is possible to estimate the probable deflection and vibration that would have occurred with a "normal" wind blowing directly against either of the broad faces.

Columns (7) and (8), Table 11, when plotted, indicate that the north-south deflection of the building is about 6.5 in. with an indicated wind velocity of 80 miles per hr. The average deflection indicated by the plumb-bob and the collimator is in close agreement for north-south deflections. In the east-west direction, however, there are insufficient collimator readings to determine the deflection with any degree of assurance.

In addition to the average deflection, the building vibrates, and the maximum range of vibration, as indicated by the collimator readings, is about 10% greater than the average deflection. This means that an 80-mile wind may cause the building to deflect about 6.5 in. and to vibrate about 7.2 in. about the deflected position. In this case the extreme deflection of the building would be 6.5 + 3.6 = 10.1 in.

Fig. 14 shows graphically the period of vibration of the building as observed with the collimator on November 1, 1932. It shows 104 cycles in about 860 sec, indicating a period of vibration of about 8.28 sec. It is also difficult to understand how a period of 8.14 sec was derived from the data in Fig. 15.

In December, 1938, when the writer last visited the building, the wind velocity was between 40 and 50 miles per hr, and it was possible to hear faint groans as the building vibrated; 28 groans were heard in slightly less than 2 min, timed with an ordinary watch, which seems to indicate that the period of vibration is still about 8.3 sec as measured on November 1, 1932.

TABLE 11.—DEFLECTION OF BUILDING FROM COLLIMATOR READINGS

Item No. (see Table 5)	Velocity of wind, in miles per hour (1)	Direction of wind at Empire State Building (2)	Angle between wind and normal to building (3)	ACTUAL OBSERVATIONS WITH COLLIMATOR		Force ratio* (6)	"Normal" deflection, in inches (7)	"Normal" vibration, in inches (8)
				Average deflection, in inches (4)	Maximum vibration, in inches (5)			
(a) WINDS AGAINST NORTH FACE; ORIGIN ESTIMATED AT 5.6								
107	52	ENE	39°	1.10	3.0	0.86	1.3	3.5
108	60	ENE	39°	2.75	4.7	0.86	3.2	5.5
129	58	ENE	39°	3.45	4.3	0.86	4.0	5.0
(b) WINDS AGAINST SOUTH FACE; ORIGIN ESTIMATED AT 5.6								
117	35	SW	17°	2.6	0.6	0.98	2.7	0.6
119	34	SSW	5°	1.5	0.8	1.00	1.5	0.8
126	51	SSW	5°	2.6	2.0	1.00	2.6	2.0

* From Fig. 25.

Conclusions.—From the foregoing, the writer advances the following nine conclusions:

(1) The plastered partitions, concrete floors, and masonry walls add both strength and stiffness to the building and carry a large percentage of the wind load. Any theory that ignores this fact is probably so far from the truth that great refinement is not warranted.

(2) The ratio between the stiffness of the building and the stiffness of the steel frame is not constant. It may have been 10 to 1 when the building was first completed and it may still be about 3 to 1. It is quite probable, however, that extremely severe storms will extend the existing cracks and allow the steel frame to carry a greater proportion of the wind load.

(3) After a severe storm, the building does not return exactly to normal. As the wind decreases, the deflection decreases until the frictional resistance of the building equals the restoring force of the deflected materials. The building then remains in this slightly deflected position until the next storm.

(4) The plumb-bob readings show that an indicated wind velocity of 80 miles per hr against either of the broad faces caused a deflection of about 6.5 in.; but the same velocity against either of the narrow faces caused a deflection only one-third as great.

(5) The collimator readings show that the building vibrates about the deflected position. The period of vibration is about 8.3 sec, and the range of vibration is slightly greater than the average deflection. An indicated velocity of 80 miles per hr, against either of the broad faces, therefore, would cause the building to vibrate back and forth over a range of about 7.2 in., and the extreme deflection would be $6.5 + 3.6 = 10.1$ in.

(6) During severe storms the vibration of the building causes groans as the masonry slips on the steel frame, or along cracks caused by previous storms.

(7) There is generally a strong up-draft on the windward side of the Empire State Building above the fifty-fifth floor. This up-draft, combined with the horizontal wind, probably causes a resultant velocity considerably greater than the velocity of the approaching wind. Indicated wind velocities at the Empire State Building, therefore, should be reduced to obtain the true wind speed.

(8) Both the velocity and direction of the wind are seriously affected by surrounding buildings. Low buildings receive considerable protection from surrounding buildings, and the sheltered area on the lee of a building is relatively greater during more severe storms.

(9) In New York City, during a very severe storm, there is considerable difference in velocity at points 100 ft, 500 ft, and 1 000 ft above the ground. A rough estimate indicates that simultaneous velocities at these three levels might be 55, 70, and 80 miles per hr. The corresponding wind pressures, therefore, would be about 20.9, 26.6, and 30.4 lb per sq ft, respectively, at elevations 100 ft, 500 ft, and 1 000 ft above the ground.

J. CHARLES RATHBUN,[51] M. AM. SOC. C. E. (by letter).—This paper has been fortunate in having so many able discussers. As many of them have been interested actively in this or a similar problem they have been able to suggest a number of related phases that should be given study. The most outstanding one is that of building vibration.

Strength and stiffness are two entirely different characteristics of a structure, as emphasized by Mr. Coyle and Professor Morris. This paper has been primarily concerned with the lateral loads on a building although it has taken into consideration both of these other questions to a considerable extent. To study the wind effects properly one must consider not only the physical characteristics of the building but also the effect of vibration on the tenants. It was considered unwise to expand the scope of this investigation into this latter field, although the value of it was recognized. Mr. Coyle has collected considerable data on the subject of vibration in various buildings and has given it careful study.

---

[51] Associate Prof., Civ. Eng., Coll. of the City of New York, New York, N. Y.

The writer agrees with Professor Morris that extreme refinement of mathematical calculations is unwarranted but that some theory of stress distribution for lateral stability is necessary in designing buildings. A continued study of the problem should reveal better theories that will enable the engineer to anticipate the action of his building under all conditions of loading, without increasing the labor of analysis unreasonably. The writer wishes to emphasize that he did not suggest that the fact—that the architectural clothing adds to the strength of the building—should be taken into consideration in its design. Both conclusions (8) and (9) demonstrate this point; and Professor Morris emphasizes it. The third sentence of Mr. Skinner's discussion is not at all in accord with the writer's opinion; nor does it seem to follow from the findings. In fact, his account of the gradual transfer of load from the masonry to the steel leads to the opposite conclusion.

Mr. Fleming's discussion is valuable particularly as he gives numerous references on this question and related problems. He quotes the report of the British Research Committee[52]: "In a large class of steel-framed buildings * * * the steel work is thus relieved of the responsibility of resisting wind shear." This idea should be studied carefully and the character of the building noted. For the tall steel building—the skyscraper—the writer feels that the steel should carry the responsibility of the wind load and lateral stability of the structure, even if the load is partly taken by the masonry.

It is true, as Mr. Shearwood suggests, that the data obtained from the Empire State Building show that the problem is far too complicated to be formulated by simple uniform horizontal loads and methods of analysis. In design the engineer endeavors "to distribute, exactly, indeterminate stresses induced by indeterminate forces." The writer hopes that nothing in his paper will lead to the idea that this is not good engineering practice. Of course, formulas must be followed with judgment and experience, not just as a guide. Engineers with sound judgment and broad experience may be able to design safely but most of them should not be encouraged to dispense with accepted methods which in the past have produced safe structures. If the experience and study of the specialists have decided that stresses obtained by conventional methods should not be exceeded is it wise for the Profession to violate these suggestions? In cases where they are sure the previous study does not apply, or where it has been improved upon, a special investigation should be made and the result applied in a conservative manner.

The writer believes (see fifth paragraph in Part II, under "Introduction") that it is probably wise to ignore the strength of the masonry until, as Mr. Shearwood suggests, further investigation of this problem is made.

Mr. Jacobsen has stated the fact (as has the writer preceding and following Equation (1)) that T and W are ratios of time and weight between the building and the model. The facilities for observing the period of vibration by means of the collimator are available for future use. It is planned to make checks from time to time on the building to note if a gradual change takes place in its period of vibration.

---

[52] Third and Final Report, British Steel Structures Research Committee, Dept. of Scientific and Industrial Research, 1936.

It is true that Equation (20) can be written without derivation as it is quite simple. The previous equations were given so that one might note the approximations involved and the effect of these approximations on the validity of this equation. The effect of the walls was omitted because the scope of the study was a comparison between the frame of the building and the model. The results of this study were compared with the actual building and the difference was attributed to the stiffness due to the walls.

The columns of the Empire State Building rest on solid rock whereas the columns of the model are anchored to steel bars. Therefore, both are quite rigid and have little effect in destroying the similitude between model and building frame.

The writer agrees with Professor Castleman that more data of this nature should be gathered. There are serious difficulties, however. It is seldom that the owner of a building is willing to allow an intensive study of his structure to be made and published. Furthermore, the gathering of these data, their interpretation, the equipment necessary, and its installation, are very expensive.

The writer feels, definitely, that the data obtained from wind tunnels by a skilled observer and under laboratory conditions are most valuable. The data are obtained under controlled conditions and so can be understood much better than those obtained from observations on a building. They are two entirely different sets of data but the reason for the differences between them is worthy of study. The local conditions that tend to break up the wind currents is only one factor, although an important one. The work of Messrs. Dryden and Hill is most valuable and the data therein should be compared with those obtained on the building itself.

The law of variation of pressure with distances from the ground has not been formulated as suggested by Professor Castleman. As can be seen from the data of this paper it is quite erratic. The observations given by Mr. Skinner in his Conclusion (9) are only one set of many and should have no more weight than the other items of Table 2. They certainly do not warrant the authority that is due a "conclusion."

The only objection the writer sees to adding the term suggested by Professor Castleman to Equation (18) is that it increases the length of the equation. When it was omitted it was thought the numerical value would be very small.

The fact that the temperature effect on the building is negligible should not lead one to generalize. The construction of the building, as well as the efficiency of the insulation around the frame, will have much to do with this effect. It was examined into, and recorded, largely to show that it was not the cause of the plumb-bob's failure to return to a fixed center.

Professor Wilbur discusses the deflection index and its importance. The data taken from the Empire State Building should be of considerable value in determining this index.

Mr. Spellman suggests that "in some instances a substantial saving could be attained safely by omitting wind-bracing." This is done in some cities where the ordinance allows one to design buildings less than a certain height without allowing for wind loads. To extend this practice to the top part of tall buildings is an idea of questionable value and one not encouraged by the data of this paper. If there are substantial wind loads the writer feels that they should be provided for in the steel.

It was particularly gratifying to note that the late Mr. Molitor[53] accepted the percentage added to the rigidity by the masonry as obtained by the writer. This value may vary between wide limits even in the same building while the materials are becoming adjusted to each other. He, like the writer, and several of the contributors to this paper seemed to approve the present method of stress analysis and assumed lateral loads. To date the results have proved satisfactory. The tests on the Empire State Building seem to indicate that 20 lb per sq ft is not too great and since there have been few if any failures due to wind it is not too low; and so, until better methods of loading, analysis, and design are discovered, the present method should not be considered outmoded.

Mr. Gray's observation of the fog phenomenon gives some idea of the action of the wind currents about the building.

Mr. DeBlois has expressed an interesting idea relative to the damping effect shown in the model data; and the curves viewed from this standpoint show this effect clearly. The writer was unable to devise any way of measuring this same phenomenon on the building and so could not use it in his paper.

Mr. Smith's plots of wind forces against their direction are very enlightening and, to some extent, show the general direction of the wind in New York, and also the effect of the neighboring buildings. He also calls attention to irregularities in the manometer and extensometer readings. These were noted before submitting the manuscript but it was decided that their omission would tend to lead to a false sense of accuracy.

The extensometer rods were close to the columns and were covered with the same insulating material. Thus the elongation due to temperature will be the same for both rod and column and the reading will not be affected. It has been shown by the test on building deflection that the temperature effect is very slight.

Mr. Bergman emphasizes the importance of rigidity in a building and that the designer is working between limits. As the writer has stated, this is a problem that could be studied to advantage by the Profession.

Mr. Gottschalk's suggestion that there should be more simple methods of obtaining the information about masonry stiffening than by models is one that it is hoped will be acted on and one which will lead to studies on buildings other than the Empire State. The difficulties already outlined by the writer make it probable that not many such studies will be made.

Mr. Skinner is to be commended that his interest in the problem induced him to make further observations after field work was considered finished and some of the instruments had been removed from the building. In many cases, however, his discussion gives an entirely erroneous impression due to an overemphasis on data that are considered by the writer to be of but little moment.

---

[53] David A. Molitor, M. Am. Soc. C. E., died on September 8, 1939.

This is particularly true of the wind currents in the fire tower and the creaking of the building during a high wind. The actual data on which Mr. Skinner bases his findings were available to the writer when the paper was written.

Mr. Skinner's comments relative to air currents should not lead one to discredit the work of the plumb-bob, and the findings shown in Fig. 12. When the writer first installed this bob it was found necessary to damp it, of course. After this all recorded readings were made with great care and accuracy. Sources of error such as temperature, wind currents, etc., were studied and taken into consideration. At the writer's suggestion, one of the observers made a study of the accuracy of his readings and found that they had a probable error of less than one-eighth inch. This is more accurate than the collimator readings. The eight charts similar to Fig. 26 were a part of a study made during the early part of the data-gathering period. The idea was abandoned because it was shown that the plasticity of the building is considerable and therefore the center of rest is of no importance.

The creaking mentioned by Mr. Skinner is a very usual phenomenon with structures when new and it would have been surprising if this building had proved an exception. In bridges it is not noticeable because of other noises when the load is being applied; in new occupied buildings this is also true to a limited extent but in new ships and vacant buildings it is noted often. That an observer who is watching for data should note it, is as it should be; that he should pick up the period of vibration from the sound is most interesting and Mr. Skinner is to be congratulated on using this information to advantage. That the noises should be classed as groans is incorrect and misleading. As he stated, when the building reached structural adjustment the sounds abated, just as one would anticipate.

The writer has investigated the question of the partitions that are "seriously cracked" and finds that no reports of any serious, dangerous, or even prominent cracks in partitions or floors have been made to those responsible for the building. As the building is composed of many materials, some weak in tension (concrete and plaster), it would be surprising if some cracks did not appear during the first winter. However, these are very minor and the writer did not consider them of any structural significance except to show adjustment of stress. The partitions are not a part of the working structure and there are no cracks in the floors due to wind.

To be complete, Mr. Skinner's attempt to correlate the observations of the four New York stations should indicate the spread of the data as well as their averages. There are so many factors that enter into this problem that a simple average does not give the true picture; nor can it be applied to one storm, or indicate the maximum velocities at the several stations and their ratios to each other.

The writer agrees with Mr. Skinner and the several other discussers that the work of Messrs. Dryden and Hill is of great value and should be studied by all those interested in the problem. Much of Mr. Skinner's discussion is mainly a repetition of the data, conclusions, and ideas expressed in the paper and with these, of course, the writer agrees. He has tried to give full credit for all of the ideas that were given by those associated with him in the work. However those employed in gathering data were too numerous to mention by name.

Mr. Skinner closes with nine "conclusions." Conclusion (1) is covered by the writer's Conclusion (8). His last sentence depends upon the meaning of "great refinement." In Conclusion (2), the figures given should have been substantiated by data. If they are only a guess the writer questions the wisdom of having given them. His first sentence is a contribution to the subject. Conclusion (3) is a repetition of the writer's Conclusion (2). Since the plumb-bob and Fig. 12 were the sources of the necessary data, and Mr. Skinner questions the accuracy of these, one wonders how he arrived at this conclusion. The writer questions the accuracy of Conclusion (4) but considers it of little importance. The data from which Conclusion (5) was drawn is probably Table 5. As the vibration and deflection are not simply related phenomena the writer questions the wisdom of generalizing from the data. As the data are available the reader can decide this matter. As for Conclusion (6), a normal amount of creaking and grating can be heard by those who are listening for it but this is not a structural defect. The idea of the masonry slipping on the steel frame is interesting but not probable unless data not published bear it out. The squeaking of joints in metallic partitions has been reported. The building, definitely, does not groan during a high wind. Conclusions (7) and (8) were fully covered by the writer's paper. The information in Conclusion (9) was based on one of the many observations, selections from which were taken to form Tables 1 and 2. The findings scarcely warrant a conclusion, however, as it was only one set of readings.

Mr. Skinner's discussion is full of "food for thought." His observations on the gradual increase of the sensitivity of the steel are of great interest. It is unfortunate that quantitative data were not secured. In many cases his overstatement of the data has led to an incorrect impression but it is probably for the best that a different point of view has been advanced for the reader to consider. The writer has tried not to draw conclusions from general averages of very erratic data but where Mr. Skinner has done so the reader can judge each case on its relative merits since he has all of the pertinent data.

In closing the writer wishes to thank all those who have contributed, either by supplying funds and physical facilities, collecting data, making suggestions, or discussing the paper. It is hoped that future studies will enable the designing engineer to work with greater exactness in predicting the stresses due to wind on his structure because of the ideas and data given in this paper and its discussions.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2057

ANALYSIS OF RUN-OFF CHARACTERISTICS

By Otto H. Meyer,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Victor H. Cochrane, Bertram S. Barnes, LeRoy K. Sherman, Richmond T. Zoch, Merrill Bernard, Franklin F. Snyder, W. G. Hoyt, C. O. Clark, C. S. Jarvis, Howard M. Turner, and Otto H. Meyer.

Synopsis

An analysis of the characteristics of run-off hydrographs is outlined in this paper, which proposes a revision of current methods for the determination of stream flow from rainfall. It is shown that a continuous rain of constant intensity produces a hydrograph of characteristic shape for any drainage area, and that this shape and the length of time until the run-off becomes constant (time of concentration) are functions of the shape and size of the drainage area. A method of constructing a hydrograph resulting from rain of duration less than the time of concentration is proposed. Based on this analysis, methods have been devised for determining hydrographs for areas lacking in run-off records, from the records of other areas similar in characteristics but different in size, and for the construction from rainfall data of hydrographs for storms or series of storms of varying intensity.

Unit Hydrographs and Distribution Graphs

Hydrographs resulting from isolated storms have long been known to have characteristic shapes. LeRoy K. Sherman, M. Am. Soc. C. E., found that for any drainage area all hydrographs resulting from isolated storms of a certain duration had approximately the same length of base, and that when reduced proportionally to correspond to the same volume of run-off they had very similar shapes.[2] He proposed the "unit hydrograph," or the hydrograph resulting from a 1-in. run-off from precipitation occurring in a unit of time (usually 24 hr). When its ordinates are multiplied by the inches of run-off from any storm of unit duration, this unit hydrograph gives the hydrograph resulting from that storm.

Note.—Published in November, 1938, *Proceedings*.

[1] Asst. Engr., U. S. Engr. Dept., U. S. Engr. Office, Sacramento, Calif.

[2] *Engineering News-Record*, April 7, 1932.

Merrill M. Bernard, M. Am. Soc. C. E., proposed a "distribution graph," in shape similar to the unit hydrograph, but representing, by its ordinates, percentages of the total run-off from a storm, or the proportional distribution of run-off with respect to time.[3] The distribution graph is derived by dividing the ordinates of the unit hydrograph by the total volume of run-off, as indicated by the area under the graph. A refinement of the method of using a single actual hydrograph to determine the distribution graph is to compute distribution graphs for a number of storms, plot them together, fitted as well as possible to each other, and draw a mean graph, thus producing the most probably representative distribution graph.[4]

A method of constructing the composite hydrograph resulting from a series of rains, or rains lasting through several units of time, has been developed and is in general use.[4] The procedure is to compute the hydrograph resulting from the rain in each unit of time, and to add the resulting hydrographs directly. The composite hydrograph thus constructed will fit the actual observed hydrograph fairly closely.

At best, stream-flow records are meager, and hydrographs are frequently required for points at which no gages have been established. Under such circumstances it is required to determine an approximate distribution graph by transposition from another drainage area or from another part of the same area. Mr. Sherman has proposed a method of determining a unit hydrograph for any size of drainage area from the known hydrograph of a drainage area of similar characteristics.[5] This method (based on the principle that all dimensions of similar areas vary as the square roots of the areas) is, in brief, to increase or reduce the ordinates and abscissas of the unit hydrograph in proportion to the square roots of the respective drainage areas. This similarity of drainage areas applies particularly to slope, nature of the topography, extent of vegetation, and distribution of tributaries. For the relation to hold requires that the mean velocity of the water remain approximately constant through the length of the water-shed. However, the velocity of flowing water, which determines the length of the unit hydrograph, depends not only on slope (which the transposition leaves unchanged although small drainage areas have steeper slopes than large ones), but also on hydraulic radius and stream-bed roughness, which vary with the size of the drainage area. The velocity, therefore, should not be assumed to be constant. It is thus apparent that reliance cannot be placed on the foregoing method. An attempt was made by the writer, using this method, to determine hydrographs at successive points on the Suncook River in New Hampshire, with indifferent success. It is true that each of the hydrographs constructed was, in itself, a reasonably good approximation, but comparisons between them failed to give the information required in this case, which involved the routing of natural and reservoir-modified flood hydrographs. Obviously, a real necessity for a more refined method exists.

Defects similar to the foregoing exist also in the following formulas proposed by the Committee on Floods,[6] of the Boston Society of Civil Engineers:

---

[3] "An Approach to Determinate Stream Flow," by Merrill M. Bernard, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 347.

[4] "Unit Hydrograph Analysis of Surface Runoff," U. S. Geological Survey, *Water Supply Paper 772.*

[5] *Transactions*, Am. Geophysical Union, 13th Annual Meeting, 1932, p. 332.

[6] *Journal*, Boston Soc. of Civ. Engrs., September, 1930, p. 388.

$T \propto \sqrt{M}$; and,

$$Q_p = C_f \sqrt{M} R \qquad (1)$$

in which M is the drainage area, in square miles; T is the total flood period, in days; Q is the peak discharge, in cubic feet per second; C_f is a flood coefficient, and R is the total run-off in inches above the base flow. These relations, involving, respectively, the abscissas and ordinates of the hydrograph, are in effect the same as those assumed by Mr. Sherman. According to the report of the Committee, these relationships, which are only approximately borne out by actual observations, are based on the assumption that the drainage area involved is rectangular and of uniform slope, like a shingle on a roof, and without any well-defined channels.

Analysis of the Hydrograph

It is well known that run-off is not a function of rainfall alone, but involves such factors as infiltration, evaporation, and surface pondage. The relationship between rainfall and the actual flow into streams is still the subject of much discussion, but probably it will be generally conceded that after a short time lag the run-off will be approximately proportional to the rainfall. Rainfall intensities vary considerably, and uniform, continuous rainfall is actually an unknown phenomenon. For these reasons, and because fluctuations in the intensity of a rain of considerable duration have only a minor influence on the shape of the hydrograph, it is assumed permissible to treat the run-off as a constant percentage of a uniform rain. Rainfall rates and percentages of run-off are assumed constant, therefore, unless otherwise specifically stated. It is also assumed that ground-water flow is separable from flood flow, and, consequently, it is assumed to be separated and is not mentioned in the analysis of flood flows. Uniform distribution of rainfall over the entire drainage area is also assumed, although it is understood that this condition is never realized, and is only approached on small water-sheds; in actual cases the hydrograph is not expected to vary greatly in shape from one produced by rain of uniform distribution.

Assuming a uniform, continuous rain, the run-off passing any point on the stream will increase as water from more distant parts of the drainage area comes to the point of measurement, reaching a maximum and becoming constant when water from the farthest extremity of the drainage area arrives. This time, until water from the most distant part of the drainage area reaches the gage, is known as the time of concentration.

If, after the run-off has become constant, the rainfall ceases, all water in surface detention soon runs off into the stream beds, soaks into the ground, or evaporates. As the streams then have no source of replenishment other than ground-water flow, which has been eliminated from consideration, all flow thereafter must come from depletion of the water in channel or valley storage. The run-off then follows a descending curve which W. W. Horner, M. Am. Soc. C. E., and F. L. Flynt, Assoc. M. Am. Soc. C. E.,[7] found to follow the general equation:

---

[7] "Relation Between Rainfall and Run-Off from Small Urban Areas," *Transactions*, Am. Soc. C. E., Vol. 101 (1936), Equation (2), p. 151.

$$Q_d = \frac{Q_m}{K^{t-t_l}} \qquad (2)$$

in which Q_d is the discharge, in cubic feet per second, on the descending limb of the hydrograph; Q_m is the maximum or peak discharge, in cubic feet per second; K is a constant; t is the time from the beginning of the storm, in minutes, corresponding to any Q_d; and t_l (time lag to peak) is the time corresponding to Q_m. A similar expression was derived by Mr. Richmond T. Zoch.[8] The writer found:

$$C_1 - t = C_2 \log Q \qquad (3)$$

in which C_1 and C_2 are constants applicable to a particular drainage area; t is the time from the beginning of the storm, in days; and Q is discharge, in cubic feet per second. It may be seen that these are different forms of the same function.

An investigation of the characteristics of this storage run-off function shows that the discharge at any time subsequent to the peak is a direct function of the peak discharge, and that the percentage of the storage yet to run off at any time, and the ratio of the discharge at that time to the peak discharge, is constant irrespective of the magnitude of the storm. As the hydrographs of greater storms yet lie entirely above those of lesser storms this should reconcile the two schools of thought, one of which claims that the length of base of the hydrograph is independent of the magnitude of the storm, and the other of which states, apparently conversely, that the larger the storm, the longer the hydrograph will be. The length of base is actually infinite, but the time until the ordinates have receded to some particular infinitesimal proportion of the peak ordinate is approximately constant for any drainage area.

Another corollary to the logarithmic expression for storage run-off is that discharge is directly proportional to the volume remaining in storage. For example, Equation (3) may also be expressed as,

$$Q = e^{\alpha} \qquad (4)$$

in which e is the base of Naperian logarithms, and

$$\alpha = \frac{C_1 - t}{C_2} \qquad (5)$$

The volume, V_s, remaining in valley storage at any time is equal to the area under the curve and to the right of the point corresponding to the time, or,

$$V_s = \int_t^{\infty} e^{\alpha}\, dt = -\, C_2 \left[e^{\alpha}\right]_t^{\infty} = C_2\, e^{\alpha} = C_2\, Q \qquad (6)$$

A convenient name for the part of the hydrograph occurring up to the time of concentration is "concentration curve." Although it governs the shape of the "storage curve" (or descending limb of the hydrograph), the shape of the concentration curve is itself dependent on several factors. One of these is the "histogram" of the drainage area, a function of the shape of the drainage area, and the arrangement of tributaries. A histogram is a graph on which stream-bed, or "thalweg," distances from a point down stream are expressed as abscissas; and drainage basin widths, or, rather, areas per unit of distance, are expressed as ordinates. If the histogram is altered by substituting for distances the time required for water to travel to the point of measurement, its integral will then become the concentration curve. In other words, the increase in ordinate of the hydrograph per unit of time is proportional to the additional area, water from which first reached the point of measurement during that unit of time. The time of travel itself is dependent on stream velocity. The velocity varies widely, from place to place on the same stream, but was found for the Mississippi River to fluctuate about a value approximately proportional to the one-fourth power of the distance from the head-waters. The time of travel from the head-waters to any point will then be proportional to the three-fourths power of the distance because, if $\frac{dx}{dt} = x^{0.25}$; $dt = x^{-0.25}\,dx$; and, $t = \frac{4}{3}\,x^{0.75}$. It follows, then, that for similar drainage areas the time of concentration may be expected to be proportional to the three-fourths power of the length, or approximately proportional to the three-eights power of the area.

---

[8] *Monthly Weather Review*, September, 1934.

It is now time to define a new concept, which will be referred to as the "basic hydrograph." This is the hydrograph resulting from a uniform rain with a duration equal to the time of concentration. This hydrograph has for its rising limb the concentration curve and for its falling limb the storage curve. There is a sharp break at the top. This break has actually been found on continuous gage records (see Fig. 1), although storms of sufficient duration to produce actual basic hydrographs on large drainage areas are quite infrequent, and the supply of data available from water-stage recorders at any place is small.

When a storm with a duration less than the time of concentration occurs, the hydrograph has a somewhat different shape, as shown by Figs. 1 and 2. A method of deriving the hydrographs of storms of shorter duration has been developed. It may also be used to synthesize the basic hydrograph from one produced by a storm of a known shorter duration. The storage curve has been described as conforming to Equation (3). This is only approximately true. Actually, the storage curve is the sum of a theoretically infinite number of curves, each of which conforms closely to the equation, and each of which expresses the run-off from an element of area at a certain time-distance from the point of measurement. A convenient, fairly close approximation is to group the run-off from the area, water from which first reaches the point of measurement within a certain unit of time, and determine the constants of the equation for that group. The unit should be about one-tenth the time of concentration, or less. The smaller the unit the better will be the definition of the curves. In practice, the equation itself is not determined, but only the ratio, λ, of two ordinates one unit of time apart; this is a constant for the particular equation. This ratio (see Appendix I) is found to be:

$$\log_e \lambda = -\frac{h}{V_s} \qquad \text{(7)}$$

As V_s is approximately equal to h times the mean time, t_m, it follows:

$$\log_e \lambda = \frac{-1}{t_m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

in which h is the difference between the ordinates on the concentration curve at the beginning and end of the unit of time; and V_s is the area between the corresponding abscissas and to the left of the concentration curve, or the volume of run-off from the corresponding area remaining in valley storage, as shown by Fig. 3. As this is the volume of run-off that has not appeared at the gage by the time of the cessation of the storm, it must be the volume under the storage curve also. Successive ordinates on the storage curve are found by multiplying each ordinate in turn by the ratio, λ. This ratio is determined for each unit of time until the time of concentration.

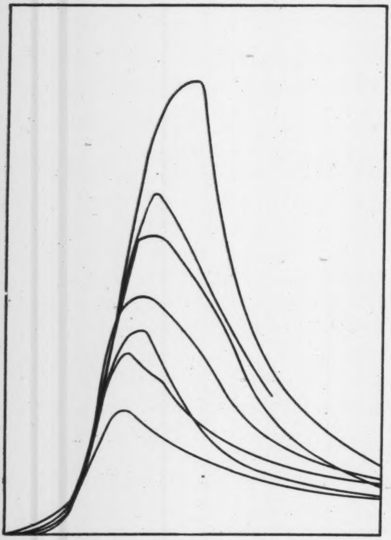


FIG. 1.—GROUP OF HYDROGRAPHS OF PEMIGEWASSET RIVER, AT PLYMOUTH, N. H., REDUCED TO COMPARATIVE SCALE

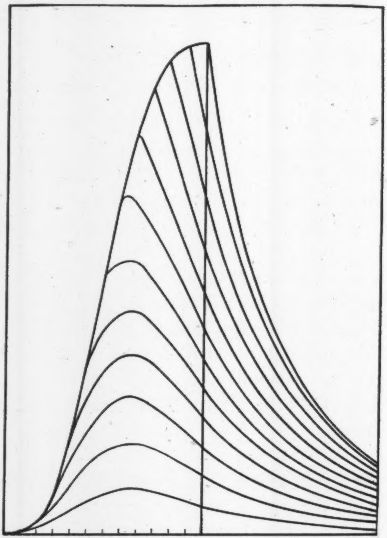


FIG. 2.—HYDROGRAPHS FROM STORMS WITH DURATIONS OF SUCCESSIVE TWELFTHS OF TIME OF CONCENTRATION COMPUTED FROM ASSUMED BASIC HYDROGRAPH

The hydrograph for a storm of any duration will follow the basic hydrograph until the cessation of the storm. At the next unit of time the ordinate will be the sum of the quantities, h, for the preceding units of time, each multiplied by its respective value of the ratio, λ, plus the h-value for the last unit of time multiplied by the ratio of actual run-off volume from its corresponding area to its V_s (which is the ratio of the duration of the storm to the mean abscissa for the unit of time). This last-mentioned multiplication is necessary so that the volume under the corresponding sub-storage curve will be equal to the actual run-off volume available. The following ordinates are the sums of all preceding subordinates multiplied by their respective ratios, plus the h-value for the last unit of time treated as in the foregoing text. After the time of concentration is reached this becomes the sum of all the preceding subordinates multiplied by their respective ratios. A sample computation is shown by Appendix II.

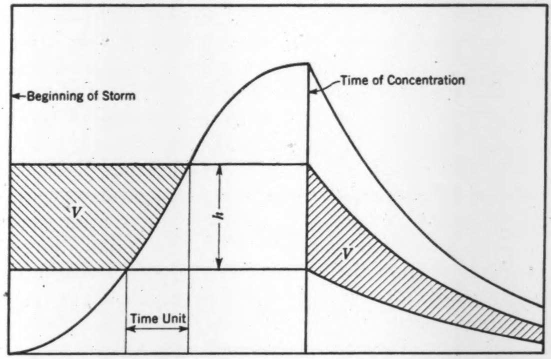


FIG. 3.—VALLEY STORAGE AND SUB-STORAGE CURVE FOR A TIME UNIT

The basic hydrograph may be synthesized from one resulting from a storm of known duration by a reversal of the foregoing process. The height of the peak will be the area under the known hydrograph divided by the duration of the storm, as the peak run-off rate is equal to the rainfall rate multiplied by the percentage run-off. The basic hydrograph will follow the known hydrograph until the cessation of the storm. The aforementioned ratios may be determined in advance, since they can be reduced (as shown) to functions of time. The subordinates for time units previous to the cessation of the storm are treated as described in the foregoing text, and their sum is subtracted from the corresponding ordinate of the hydrograph. The difference is divided by the ratio of the duration of the storm to the mean abscissa of the unit of time. This quotient is added to the last ordinate of the basic hydrograph to give the following one. This procedure is followed until the subtraction gives a negative result. At this point, the ordinate should have reached the maximum previously computed. The storage curve, taken as the sum of all sub-storage curves, then follows.

TRANSPOSITION OF HYDROGRAPHS

As many engineering problems require hydrographs at points where no stream-flow measurements have been taken, some method of transposing hydrographs is necessary. The method proposed by Mr. Sherman, according to which the ordinates and abscissas of the unit hydrograph vary with the square root of the area, has been discussed. Unit hydrographs derived by this method will be correct for total volume of run-off, of course, and will lie within reasonable limits of variation in peak run-off rate; however, they will be subject to considerable error in time of concentration and shape. For several positions on the same stream these hydrographs cannot be used comparatively.

In the course of flood-control studies of the rivers of Maine and New Hampshire, under the United States Engineer Department, it was found necessary to devise some more consistent method of transposing hydrographs. Under the pressure of time, a detailed study being impossible, Mr. Ralph Reinhart, of the Engineer Department, constructed a set of curves based on a large number of distribution graphs which had been prepared by the writer. These curves (shown in Fig. 4) each represent the percentage of the total run-off from a storm of one day duration for one of the first six days of run-off, plotted against drainage area. Each sheet, representing some particular day, contained four curves, representing four groups of drainage area types. It became possible, with experience, to pick the applicable curve after examining topographic maps of the drainage area in question. Each day's proportion of the run-off was then taken from the proper sheet. Only seven sheets were prepared, as the remainder of the hydrograph could be computed quickly by means of the storage curve formula. (The percentage for the seventh day is not shown in Fig. 4.) No attempt has been made to define, exactly, the topography falling in each group, as such an attempt would only be misleading; however, topography producing rapid run-off will fall in a group having a high rate the first few days, and *vice versa.* To determine the run-off distribution for any type of drainage area it will only be necessary to derive the distribution graph for a similar area having run-off records, plot the daily distribution percentages, and find which set of curves fits best. A set of curves of this type is very useful where a large number of hydrographs of a large number of drainage areas is required, and time or the budget does not allow a more refined method.

Certain situations arise where a high degree of refinement in hydrograph computation is required. An example of this would be the investigation preceding the construction of a large flood-control project. In such a case, particularly if hydrographs at various points on a stream are required, Mr. Sherman's transposition does not have a sufficient degree of precision. The aforementioned set of curves (Fig. 4) gives comparable hydrographs for successive points on a stream, but cannot be relied upon for peak run-off rates. A transposition of the basic hydrograph is then the best solution. The basic hydrograph is used in preference to the unit hydrograph because the relation between two basic hydrographs is a function of type and size of drainage area only, whereas the relation between two unit hydrographs involves the ratio of the time unit to the time of concentration, which is not the same for different sized drainage areas. The two unit hydrographs may vary greatly in shape, therefore, whereas the two basic hydrographs would be very similar in shape.

The transposition of the basic hydrograph of one drainage area to another similar area is very simple. The histograms of similar areas are similar, and their concentration curves are very nearly similar. The time of concentration, as shown previously, is approximately proportional to the three-fourths power of the length of the stream, or the three-eighths power of the area. It is pref-

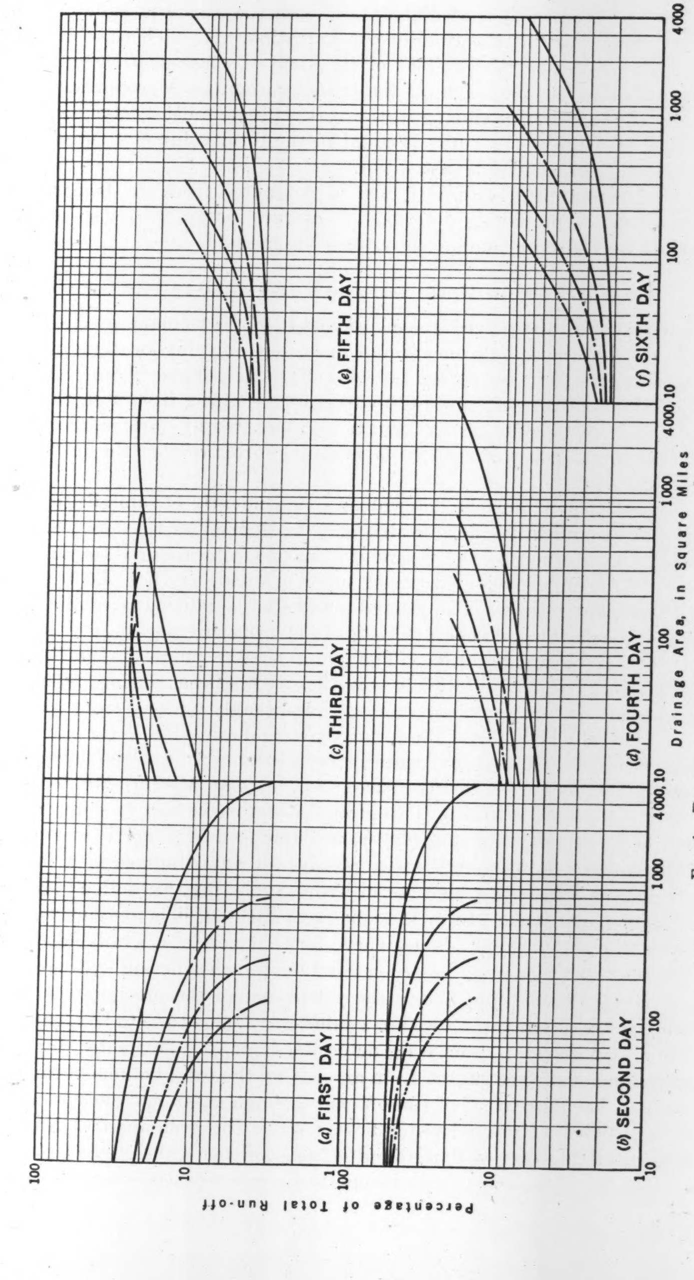


FIG. 4.—DISTRIBUTION PERCENTAGES ON SUCCESSIVE DAYS OF RUN-OFF

erable to use the length if it is available; otherwise the area will suffice. All abscissas will then vary as the time of concentration. As a check, the area under the curve must be the product of the time of concentration and the maximum ordinate. As the height of the basic hydrograph is proportional to the intensity of the storm producing it, the vertical scale must always be such that the total area under the curve is equal to the total volume of run-off; it needs no special treatment in transposition.

If a unit hydrograph is desired it can be computed from the transposed basic hydrograph, as previously described. Distribution graphs for storms of any durations may also be computed by the same method.

Composition or Reconstitution of Hydrographs

The purpose of all the procedures mentioned herein is to make possible the construction of hydrographs from rainfall data, not only for locations where gaging stations exist, but for others as well. For this purpose there must be available methods of constructing the hydrograph resulting from a succession of storms of various magnitudes and durations, separated by various intervals, these methods making use of such tools as unit hydrographs or the proposed basic hydrographs.

One element of flow must be determined independently of rainfall; that is, the "base" or ground-water flow, which may be defined as that part of the run-off which is absorbed by the ground during precipitation or thawing, and which, after traveling varying distances under ground, reaches the beds of streams at some later time. The ground acts as an equalizing reservoir, and the ground-water flow has no characteristics to identify any part of it with any particular period of precipitation. The ground-water curve is that curve which is approached asymptotically by the hydrograph during intervals between storms. It follows an annual cycle, remarkably uniform from year to year. It is generally high during the spring thaws and rains, low in the summer, a little higher in the fall, and low again in the winter. An example is offered by the ground-water curves for five successive years at Mercer, Me., on the Sandy River, shown in Fig. 5(*a*). This ground-water curve may be determined as a mean over several years, and plotted as cubic feet per second per square mile. It may then be used very satisfactorily, for any drainage area in the general region, as a base to which storm hydrographs may be added. If the hydrograph of a particular storm is to be reconstituted, it is obviously preferable to use the actual ground-water curve for that period, if available.

Another element that must be pre-determined in constructing hydrographs is the percentage of run-off. This is the percentage of the precipitation that appears in the streams as run-off within a sufficiently short time to be identified with a particular storm. It may be determined for any storm by dividing the total volume of surface run-off from an area by the total volume of rainfall on that area. The percentage of run-off goes through an annual cycle remarkably similar to that of ground-water flow. When the ground-water level is high the absorptive power of the ground is low, and the percentage of run-off is high. It is, in fact, almost proportional to the ground-water flow. An example of this cycle is given by Fig. 5(*b*), which is the percentage of run-off for the same location and period covered for ground-water by Fig. 5(*a*). Percentages greater than 100 are caused by melting snow.

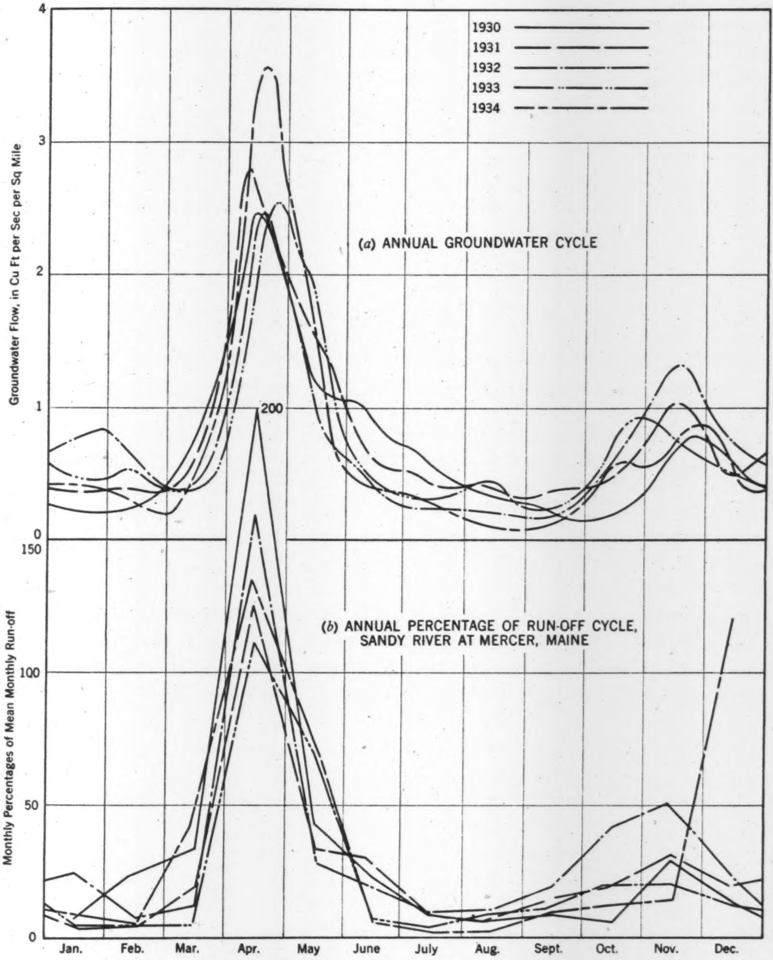


FIG. 5.—CURVES OF SANDY RIVER, AT MERCER, ME.

It is clear that in reconstituting a hydrograph from rainfall the time of year must be known in order to have the correct base flow and the correct run-off percentage for each storm. In composing the hydrograph resulting from an assumed storm, the time of year must be assumed also. Each day's rain must be multiplied by the percentage of run-off for that date, and the resulting hydrograph must be added to the ground-water hydrograph for the period covered.

The generally accepted method of reconstituting the hydrograph resulting from a group of storms is, as mentioned previously, to add, directly, the hydrographs resulting from the rainfall occurring in each unit of time, each one computed as if it were the run-off of an isolated, unit storm, using the distribution graph. The simplicity of this method is its strength. The hydrograph so developed, however, will follow an actual hydrograph only very imperfectly. One particular discrepancy that arises is that for a succession of storms of equal intensities, or a continuous rain, the method will frequently give a run-off rate exceeding the product of the rainfall rate and percentage of run-off. This is particularly true when the time to the peak of the unit hydrograph is very nearly the time of concentration. In such a case continuation of rainfall should not increase the discharge; but the method described adds to the peak the discharge of one day later on the falling limb, plus that of two days later, etc. Mr. Bernard recognized this effect, but attributed it to a flood wave.[8a] It is clear from the foregoing analysis that it is only the result of a flaw in the method.

The analysis of the characteristics of the basic hydrograph leads logically to a method of reconstituting hydrographs from rainfall. Underlying this method are two fundamental principles. The first is that the shape of the hydrograph resulting from a single storm is a function of the duration of the storm. The second is that the hydrograph resulting from a steady, continuous rain having superimposed on it a brief, heavy shower will be the sum of the hydrographs that would be produced by the two separately. This will be exactly true only on streams having a vertical stage-velocity relation; however, the similarity between hydrographs of storms of different magnitudes but of equal duration shows that little error is produced by accepting this assumption.

In records of the U. S Weather Bureau precipitation is recorded by days, and where unit hydrographs are the means of computing run-off, the records are used by days. In the proposed method, however, precipitation must be arranged "horizontally"; that is, where rainfall occurred on several successive days, it is divided into elements, each of which consists of a certain quantity of rain per day for the maximum number of days. This is more clearly illustrated in Fig. 6. Each of these elements, or "blocks," will produce a hydrograph of

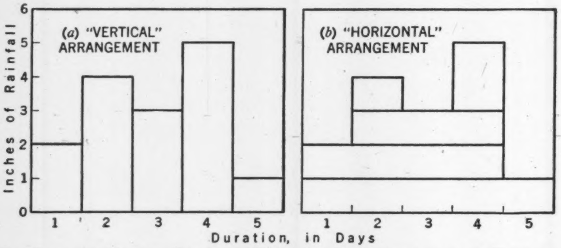


FIG. 6.—ARRANGEMENT OF RAINFALL DATA FOR RECONSTITUTION OF HYDROGRAPHS

different shape. The vertical scale of each hydrograph is proportional to the height of the corresponding block; analysis of the basic hydrograph provides the shape for each duration. These elements of the hydrograph may be added to each other directly until a time is reached when the rainfall rate, after having decreased, again increases, as on Day 4, Fig. 6. When this occurs the ordinate of the sum of the hydrographs resulting from preceding or lower blocks is carried as a constant through the duration of the additional block, and a rate of rainfall sufficient to maintain this flow is subtracted from the total rate during the period of the additional block. The hydrograph produced by the difference in rainfall is then computed and added to the other. At the end of this additional block of precipitation the underlying part of the hydrograph (all of the hydrograph other than that computed for the additional block of rain) reverts to the descending curve, which is the storage curve for the part of the rain that has ceased, plus the uniform flow from the part of the rain that continues. A sample computation is given in Appendix III.

[8a] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 189.

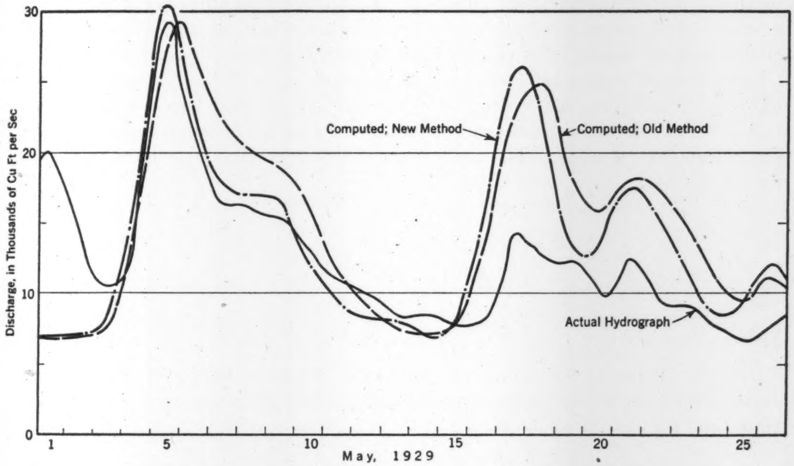


FIG. 7.—HYDROGRAPHS OF MERRIMAC RIVER, AT MANCHESTER, N. H.

The method described herein will produce a hydrograph which, in shape, will parallel an actual hydrograph, even reproducing minor fluctuations, whereas the hydrograph produced by the older method reproduces only the major undulations, and with a lesser degree of precision. Both methods rely very heavily on correct determination of percentage of run-off and groundwater flow. An example is offered by Fig. 7, which shows an actual hydrograph compared with hydrographs computed by the two methods described.

CONCLUSIONS

For studies in which time is short and great accuracy is not essential the use of unit hydrographs or distribution graphs, with their corresponding method of reconstituting a hydrograph, will be the most satisfactory method. In such case distribution curves similar to those shown in Fig. 5 will be found superior in accuracy to the unit hydrographs derived by the transposition method proposed by Mr. Sherman, and also much quicker, if hydrographs at a sufficient number of different locations are required to justify the labor of constructing the curves. Even for rather hasty studies the curves of annual percentage of run-off and ground-water flow will probably be found desirable. Where a high degree of accuracy is required, or for long-continued studies, the proposed methods will be found to be superior. Particularly in forecasting floods the greater degree of refinement should be of value. In such cases, the required curves and tables of daily run-off for various durations of precipitation, may be prepared in considerable detail. At any time, it will then be possible, very quickly, to compute the hydrograph that may be predicted to result from the rain to date. If future rain can be estimated for a day or two a more complete prediction can be made. The coefficients can be corrected from time to time as actual discharges become available.

APPENDIX I

Derivation of the Ratio of Successive Ordinates on a Storage Curve

As shown in the paper, the storage curve conforms to the general formula expressed by Equation (4); and the volume remaining in valley storage at any time is given by Equation (6). The volume flowing out during any time, t, is:

$$V_t = \int_0^t Q\,dt = \int_0^t e^{\alpha}\,dt = -\,C_2\left[e^{\alpha}\right]_0^t \ldots\ldots\ldots\ldots (9)$$

When $t = 0$, the exponent, α (see Equation (5)), becomes,

$$\frac{C_1 - t}{C_2} = \frac{C_1}{C_2} = \beta \ldots\ldots\ldots\ldots (10)$$

and, from Equation (10),

$$V_0 = C_2\,Q = C_2\,e^{\beta} \ldots\ldots\ldots\ldots (11)$$

When one-half the volume in storage has run-off, V_t is equal to V_s and equal to one-half the original volume; that is:

$$C_2\,e^{\beta} - C_2\,e^{\alpha} = 0.5\,C_2\,e^{\beta} \ldots\ldots\ldots\ldots (12)$$

or,

$$0.5\,C_2\,e^{\beta} = C_2\,e^{\alpha} = \frac{C_2\,e^{\beta}}{e^{\epsilon}} \ldots\ldots\ldots\ldots (13)$$

in which $\epsilon = \frac{t}{C_2}$. By elimination and transposition:

$$e^{\epsilon} = 2 \ldots\ldots\ldots\ldots (14a)$$

and

$$\epsilon = \frac{t}{C_2} = \log_e 2 \ldots\ldots\ldots\ldots (14b)$$

If the time interval required for one-half the volume to run off is designated as t_i, Equation (14*b*) becomes:

$$\frac{t_i}{C_2} = \log_e 2 \qquad (15a)$$

and,

$$t_i = C_2 \log_e 2 \qquad (15b)$$

As the ordinate, h, is equal to Q:

$$V_s = C_2 Q = C_2 h \qquad (16a)$$

$$C_2 = \frac{V_s}{h} \qquad (16b)$$

and,

$$t_i = \frac{V_s \log_e 2}{h} \qquad (16c)$$

Let $\frac{1}{t_i} = \frac{h}{V_s \log_2 2}$; then, the ratio of the two ordinates, one unit of time apart, is:

$$\lambda = 0.5^{\frac{1}{t_i}} \qquad (17)$$

that is,

$$\log_e \lambda = \frac{h \log_e 0.5}{V_s \log_e 2} = -\frac{h}{V_s} \qquad (18)$$

APPENDIX II

Sample Computation, Analysis of the Concentration Curve

Table 1 shows a sample computation form, in this case for the analysis of the concentration curve shown in Fig. 2. The time, y, and the dy-columns are self-explanatory. The values in Column (3) were computed from Equation (17). In the sub-storage curve for each time unit the first ordinate is dy; each ordinate is multiplied by the ratio shown to obtain the succeeding ordinate. In computing hydrographs for storms of short duration, the ordinates of each sub-curve are multiplied by the ratio of the duration of the storm to the mean time for the respective time unit (for Unit 3, Table 1, the mean time is 2.5; for Unit 7, it is 6.5, etc.), except where the time unit is prior to the cessation of the storm, in which case the full value is used. Thus, in a storm lasting four units, Curves (Units) 1 to 4 are used full value, Curve 5 is multiplied by $\frac{4}{4.5}$, Curve 6 by $\frac{4}{5.5}$, Curve 7 by $\frac{4}{6.5}$, etc. Run-off from each unit first appears at the time expressed by its number, and is at the maximum rate as long as the storm lasts, following the descending curve thereafter; or, if the storm has already ceased before its appearance, it follows immediately the curve, reduced proportionately as previously explained.

TABLE 1.—ANALYSIS OF A CONCENTRATION CURVE

Time unit	y	dy	Ratio, λ	SUCCESSIVE ORDINATES							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	1.1	1.1	0.195	1.1	0.2	....	....	....	....	....	....
2	4.2	3.1	0.520	....	3.1	1.6	0.8	0.4	0.2	0.1	0.1
3	11.4	7.2	0.676	....	....	7.2	4.9	3.3	2.2	1.5	1.0
4	29.5	18.1	0.756	....	....	....	18.1	13.7	10.3	7.8	5.9
5	53.3	23.8	0.803	....	....	....	....	23.8	19.1	15.4	12.3
6	80.0	26.7	0.837	....	....	....	....	....	26.7	22.3	18.7
7	103.8	23.8	0.859	....	....	....	....	....	....	23.8	20.4
8	123.7	19.9	0.878	....	....	....	....	....	....	....	19.9
9	137.4	13.7	0.892	....	....	....	....	....	....	....	....
10	145.7	8.3	0.911	....	....	....	....	....	....	....	....
11	150.3	4.6	0.9245	....	....	....	....	....	....	....	....
12	151.6	1.3	0.931	....	....	....	....	....	....	....	....

TABLE 1.—(*Continued*)

Time unit	SUCCESSIVE ORDINATES											
	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)
1	....	....	....	....	....	....	....	....	....	....	....	....
2	....	....	....	....	....	....	....	....	....	....	....	....
3	0.7	0.5	0.3	0.2	0.1	0.1	0.1	....	....	....	....	....
4	4.5	3.4	2.5	1.9	1.5	1.1	0.8	0.6	0.5	0.4	0.3	0.2
5	9.9	8.0	6.4	5.1	4.1	3.3	2.7	2.1	1.7	1.4	1.1	0.9
6	15.6	13.0	10.9	9.1	7.6	6.4	5.3	4.5	3.7	3.1	2.6	2.2
7	17.5	15.0	12.9	11.1	9.5	8.2	7.0	6.0	5.2	4.4	3.8	3.3
8	17.5	15.3	13.5	11.8	10.3	9.1	8.0	7.0	6.1	5.4	4.7	4.2
9	13.7	12.2	10.9	9.7	8.7	7.7	6.9	6.1	5.5	4.9	4.3	3.9
10	....	8.3	7.6	6.9	6.3	5.7	5.2	4.7	4.3	3.9	3.6	3.3
11	....	....	4.6	4.3	3.9	3.6	3.4	3.1	2.9	2.6	2.4	2.3
12	....	....	....	1.3	1.2	1.1	1.0	1.0	0.9	0.8	0.8	0.7

The same form shown in Table 1 is used in the synthesis of the concentration curve from hydrographs of storms of known duration, by the procedure described in the paper.

APPENDIX III

SAMPLE COMPUTATION OF RECONSTITUTED HYDROGRAPH

Appendix III offers an example of the computation of a hydrograph from rainfall data by means of the basic hydrograph. In this example, the rainfall shown in Fig. 6 will be used. It will be assumed that the drainage area for which the hydrograph is to be constructed has a space of 100 sq miles; a ground-water flow at this season of 3 cu ft per sec per sq mile; and a run-off factor at this season of 50 per cent. It will be further assumed that its basic hydrograph is known and has a time of concentration of three days, and that hydrographs for storms of one day and three days duration have been reduced to the distributions shown in Table 2.

TABLE 2.—DISTRIBUTION, IN PERCENTAGE OF DAILY RUN-OFF RATE

Description	(a) SUCCESSIVE DAYS; ONE-DAY STORM						(b) SUCCESSIVE DAYS; THREE-DAY STORM					
	1	2	3	4	5	6	1	2	3	4	5	6
Distribution	10	34	28	14	7	3.5	10	45	85	80	40	20

Stream flow on any later day will be one-half that on the preceding day. Flow on the fourth day of a longer storm will become 100% of the daily run-off rate, remaining at 100% for the duration of the storm, then going to 80%, 40%, etc., as in the case of the three-day storm (Table 2(*b*)).

A flow of 26.89 cu ft per sec for one day will be produced by 1 in. of run-off from 1 sq mile. The run-off, at 50% run-off factor, from 1 in. of rain per day on 100 sq miles is, therefore, 1 345 cu ft per sec. The rainfall shown on Fig. 6, being, on successive days, 2, 4, 3, 5 in., and 1 in., respectively, is the equivalent of: A five-day storm at 1 in. per day beginning on Day 1; a four-day storm at 1 in. per day beginning on Day 1; a three-day storm at 1 in. per day beginning on Day 2; a one-day storm of 1 in. beginning on Day 2; and a one-day storm of 2 in. beginning on Day 4.

TABLE 3.—COMPUTATION OF RECONSTRUCTED HYDROGRAPH: STANDARD PROCEDURE (UNITS ARE CUBIC FEET PER SECOND)

Day	DAILY RUN-OFF; STORMS AT 1 INCH PER DAY*					Total surface run-off	Ground-water flow	Total run-off
	Five-day storm (1)	Four-day storm (2)	Three-day storm (3)	One-day storm (4)	One-day storm* (5)	(6)	(7)	(8)
1	135	135	....	....	....	270	300	570
2	607	607	135	135	....	1 484	300	1 784
3	1 140	1 140	607	458	....	3 345	300	3 645
4	1 345	1 345	1 140	376	269	4 475	300	4 775
5	1 345	1 076	1 076	188	916	4 601	300	4 901
6	1 076	538	538	94	752	2 998	300	3 298
7	538	269	269	47	376	1 499	300	1 799
8	269	134	134	24	188	749	300	1 049
9	134	67	67	12	94	374	300	674
10	67	34	34	6	47	188	300	488

* Column (5), Table 3, is the equivalent rainfall in a one-day storm of 2 in. on Day 4.

TABLE 4.—COMPUTATION OF RECONSTRUCTED HYDROGRAPH: SPECIAL PROCEDURE (UNITS ARE CUBIC FEET PER SECOND)

Day	HYDROGRAPHS NOT MODIFIED		MODIFIED HYDROGRAPHS		Total run-off
	Preceding storm (1)	Succeeding storm (2)	Preceding storm (3)	Succeeding storm (4)	(5)
1	1 000	...	1 000	...	1 000
2	600	...	600	...	600
3	360	100	463	23	486
4	216	350	463	80	543
5	130	600	463	137	600
6	78	600	463	137	600
7	47	466	360	106	466
8	28	280	216	64	280
9	17	168	130	38	168
10	....	...	....	...	....

The computation is set up in Table 3. The run-off from the foregoing equivalent storms is shown in the columns headed "Daily Run-Off" (Columns (1) to (5)), each equivalent storm to one column, in the order given.

If an increase in rainfall should come when the hydrograph is falling, a special procedure is used. For example, assume a hydrograph to be falling after the cessation of a storm, with a ratio between successive ordinates equal to 0.60, as shown in Table 4 (Column (1)). A following storm, of intensity sufficient to produce a maximum run-off of 600 cu ft per sec, begins on Day 3, its hydrograph, if it had occurred alone, being as shown by Column (2). To combine the two, the ordinate on Day 2 is multiplied not by the ratio, 0.60, but by its square root, 0.77 (giving the ordinate at the beginning of Day 3 instead of at the middle of Day 3). This ordinate continues through the following storm, the ordinate that would have occurred on Day 3 now occurring on Day 7, the day following the end of the following storm, as shown in Column (3), Table (4). The increase in run-off under this hydrograph comes out of the run-off volume of the following storm, which must be reduced accordingly, so the modified hydrograph (Column (4), Table (4)) is in the ratio to that in Column (2) of $\frac{600 - 463}{600}$. The total run-off is the sum of Column (3) and Column (4), Table 4.

DISCUSSION

VICTOR H. COCHRANE,[9] M. AM. SOC. C. E. (by letter).—The graph which the author refers to as a "histogram" is very useful in the study of flood flows. That kind of histogram in which the abscissas express the time required for the water to flow to the point of measurement may be termed the time-area diagram. Its integral becomes the concentration curve, or ascending limb of the hydrograph, as stated by the author. It may also be employed in a simple manner to determine the ordinates of the descending limb of the hydrograph.

As an illustration, consider the simple case shown in Fig. 8. The watershed (Fig. 8(*a*)) is divided into six areas, or zones, such that the run-off therefrom, due to a rainfall of uniform intensity, arrives at the outlet in equal time intervals. The corresponding time-area diagram is shown in Fig. 8(*b*), and the resulting hydrograph in Fig. 8(*c*). The ordinates for the concentration curve are computed by summing the zone areas, as shown in Table 5. If the velocities during the falling stage were the same as during the concentration period, the ordinates of the descending limb of the hydrograph would be obtained directly by deducting from the peak flow the various zone contributions in succession, as in Table 5. The resulting curve is shown in the broken line, Fig. 8(*c*). This curve, plotted downward with reference to the horizontal line representing the peak flow, is the same as the concentration curve. It will be seen that if the ascending limb of the hydrograph rises slowly in the early stages, due to the narrowness of the water-shed or the flatness of the slopes near the outlet, the hydrograph tends to fall slowly after the peak, and *vice versa*.

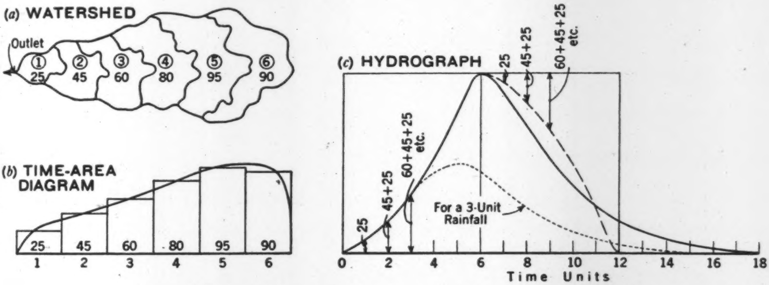


FIG. 8.—WATER-SHED, TIME-AREA DIAGRAM, AND HYDROGRAPH

The base of the falling limb of the hydrograph must exceed that of the rising limb. That this is true will be seen upon consideration of the flow from the zone farthest up stream. The initial run-off from this zone rides the crest of the flood, as it were, during the concentration period, thus traveling to the outlet at the maximum velocity. The final contribution flows at, or a little above, the base stage, with a relatively low velocity. The base of the hydrograph for falling stages is the time required for the water to travel from the most remote zone to the outlet at the ordinary or low-water stage of the stream. For practical purposes, a somewhat shorter time may be used, as the flow at final stages is relatively small. The base flow definitely limits the length of the flood period. Where the author states (following Equation (3)) that "the length of base is actually infinite," he is referring no doubt to the mathematical characteristics of his formula.

---

[9] Cons. Engr., Tulsa, Okla.

TABLE 5.—COMPUTATION FOR THE ORDINATES OF A HYDROGRAPH

Item No.	Zone No.	ORDINATE AT THE END OF EACH TIME-UNIT NUMBER:											
		1	2	3	4	5	6	7	8	9	10	11	12
1	1	25	25	25	25	25	25						
2	2		45	45	45	45	45	45					
3	3			60	60	60	60	60	60				
4	4				80	80	80	80	80	80			
5	5					95	95	95	95	95	95		
6	6						90	90	90	90	90	90	0
7	Totals	25	70	130	210	305	395	370	325	265	185	90	0

8	Average ordinate of descending limb	383	348	295	225	137	45
9	Relative times of flow	1.01	1.11	1.37	1.87	2.70	3.94
10	Discharge in unit time (Item 8 ÷ Item 9)	379	314	215	120	51	11.5
11	Corresponding abscissa; unit times*	6.50	7.56	8.80	10.42	12.71	16.03

* Base of hydrograph is 18 time units.

After the peak flow is reached, the velocity, which varies approximately as the square root of the hydraulic radius, decreases quite slowly at first; then much more rapidly during the last stages of the flood. The rate of change in velocity with respect to time can be determined approximately from the characteristics of the stream channel. As the time period corresponding to a given element of flow is lengthened, the discharge in unit time is decreased in inverse ratio. In Table 5 the increase in length-of-flow periods is assumed to vary as the cube of the number of units of flow (six in this case), and the base of the descending limb is twice that of the ascending limb. The resulting curve is shown in Fig. 8(*c*).

For a rainfall extending over a shorter period than that producing the maximum flow, the summations and adjustments are made in a similar manner. In Table 5 the summations would include as many terms in each horizontal line as there are periods of precipitation. The dotted line in Fig. 8(*c*) shows the hydrograph for a rainfall extending over three time units.

The author's unqualified assertion that there is a sharp break in the basic hydrograph is therefore open to question. Such a break might be observed in the case of a stream with a wide fan-shaped area at the down-stream end and a long narrow valley farther up stream. In a water-shed of this type the maximum flow might come from a rainfall covering only the lower part of the drainage area, the concentration period being shorter, and the intensity of precipitation greater, than for the full area. The author's logarithmic formula gives ordinates too small for the upper part of the curve and too large for the lower part. In Fig. 9, Curve A shows the falling limb constructed by the method just described, assuming a base length double that of the concentration curve, and Curve B is derived from Table 1.

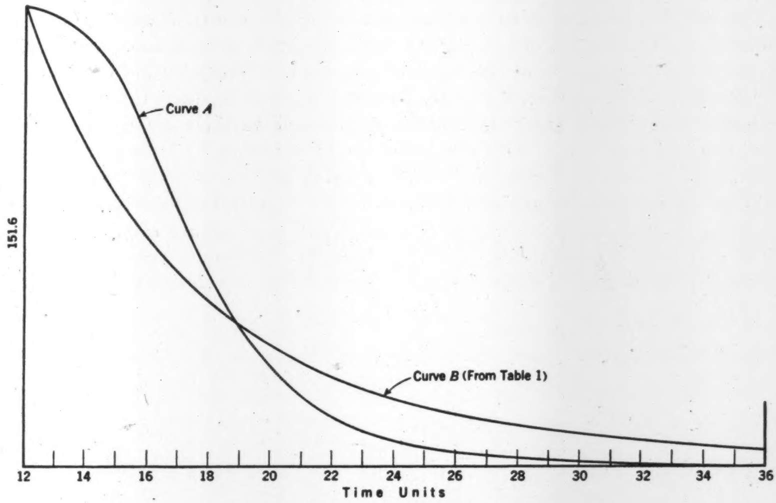


FIG. 9.—COMPARATIVE HYDROGRAPHS

For a given uniform intensity of precipitation and a constant run-off rate, the peak flow per unit of area is the same regardless of the size or shape of the water-shed. The pattern of the drainage area becomes a factor in case the rainfall is of variable intensity with respect to time or area. Inasmuch as the average intensity diminishes rapidly with increase of time, the time of concentration (the length of stream divided by the average velocity) is a much more reliable indicator of maximum discharge than either area or length of water-shed. The writer has heretofore proposed formulas[10] for maximum discharge from small water-sheds, as follows:

$$q = c\,I\,(A + B) \qquad (19)$$

in which, for $T = 1$ hr or less,

$$A = 645\,\frac{6 + I}{6\,T + I} \qquad (20a)$$

or, for T greater than 1 hr,

$$A = \frac{645}{T^{0.75}} \qquad (20b)$$

and

$$B = 100\,(z - 1)\,\frac{6\sqrt{T} - T}{0.4\,I + T} \qquad (21)$$

In Equations (19), (20), and (21): q = flood discharge in cu ft per sec per sq mile of drainage area; c = the ratio of run-off to rainfall; I = average intensity of maximum precipitation for the aggregate period of 1 hr during a given storm; T = time of concentration in hours; and z = ratio of maximum to average area of divisions of the time-area diagram (or, roughly, the ratio of maximum width of water-shed to the mean width).

---

[10] *Engineering News-Record*, November 25, 1937, p. 864.

It will be seen that these expressions are designed to improve the rational formula in two respects: The intensity of precipitation is made to vary with the time of concentration, and the effect of non-uniform precipitation is included. In Equation (19) the term A expresses the flow due to a uniform rate of rainfall, whereas the term B gives the additional flow due to the most extreme non-uniform precipitation. With the exception of the run-off coefficient, c, these equations contain no terms with widely varying and uncertain values, such as appear in many other run-off formulas. The time-intensity relationship in Equation (20*a*) has been shown[11] to agree well with actual rainfall records. The empirical expression for B in Equation (21) represents values computed from typical time-area diagrams, assuming that the maximum run-off occurs when the contribution from the maximum area, due to the most intense precipitation, arrives at the outlet.

TABLE 6.—VALUES OF q FOR: $I = 3.0$; $c = 0.5$; AND $z = 1.5$

Description	VALUES CORRESPONDING TO TIME OF CONCENTRATION, T, IN HOURS						
	0	0.25	0.50	1	6	12	24
Average intensity, I, in inches	9	6	4.5	3	0.78	0.465	0.276
Substitution Factors:							
A (Equations (20))	1 935	1 290	967	645	168	100	59.5
B (Equation (21))	0	95	110	113	61	33	10.5
q (Equation (19))	2 900	2 080	1 615	1 137	344	200	105

These formulas may be useful in estimating peak flows for water-sheds of different sizes. They cover the entire range of possible values. The four variables used, c, I, T, and z, may be evaluated for any particular case by the application of appropriate methods or pertinent data.

Values of q are shown in Table 6 for $I = 3.0$, $c = 0.5$, and $z = 1.5$. It will be noted that q diminishes very rapidly as the time of concentration increases.

In Equation (20*b*), the exponent of T $(= 0.75)$ is an average value, convenient for use with the slide-rule. Other values may be used to suit the rainfall characteristics. The average intensity per hour, i, for a 24-hr period, may be substituted for the term, I, according to the relation, $I = 10.9\, i$.

The foregoing formulas should be of use in transposing basic hydrographs, or in checking results obtained by other methods. They afford a clear picture of the effect of variations in the principal factors involved in flood discharge.

BERTRAM S. BARNES,[12] ASSOC. M. AM. SOC. C. E. (by letter).—The new and original attack upon the problem of constructing discharge hydrographs of run-off from observed rainfall, as presented by Mr. Meyer, will arouse considerable interest and will give a new impetus to research in this field. The author has admirably reconciled the apparently opposite viewpoints of the advocates of the unit hydrograph and of the depletion curve.

---

[11] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 369.

[12] Hydrologic Supervisor, Upper Mississippi Region, U. S. Weather Bureau, Iowa City, Iowa.

The assumption that run-off is proportional to rainfall will give much more satisfactory results in hilly country, where a relatively large part of the rainfall enters the stream channels at once, than it will in a flat country where most of the rain goes into the ground. Under the latter conditions, fluctuations in the momentary intensity of rainfall may have a pronounced effect upon the amount of run-off. The run-off factor is one of those devices used for lack of anything better. Its use is made necessary in most cases by the imperfect state of the present-day knowledge of the relation between rainfall and run-off, and because of the absence of sufficient basic data. Hydrologists should not lose sight of the fact that the idea of a run-off factor is a poor concept at best, and that its use is likely to produce very unsatisfactory results wherever the infiltration is large and rainfall varies greatly in momentary intensity. It is unquestionably true that the average monthly or weekly value of the run-off factor rises and falls with the ground-water level. In some cases it even appears for a time to maintain a fixed relation to the rate of ground-water flow, although this must be considered pure coincidence. In the Mississippi Valley the variation of the run-off factor from day to day is frequently so great that an average value, or one predicted from the calendar, has been found to be of little use when the unit hydrograph method was employed. There is urgent need for the development of better methods of predicting the quantity of run-off from a given storm.

Another fact frequently overlooked in present-day studies is that the time of concentration, as it is usually defined, is not necessarily identical with the elapsed time from the occurrence of rainfall to the peak of the run-off. In basins of irregular shape there will almost always be flow from certain tributaries which will still be increasing after the peak of discharge from the basin has passed.

The assumption that, after the storm water has ceased to flow from the surface of the ground, the streams have no source of replenishment other than ground-water flow is one that should be given more critical study, or at least a definite interpretation. Ordinarily, ground-water flow is thought of as the discharge from water held in storage at, or below, the ground-water table. Two or three weeks may sometimes elapse between a rainstorm and the increase in ground-water flow which results from it. Many streams in the Mississippi Valley receive storm water from two immediate sources: (1) The drainage from the ground surface into the stream channels during and immediately after the rain; and (2) what appears to be a lateral movement of percolating water in the upper soil layers, entering the stream channels in the form of seepage, and reaching its maximum flow about 2 days after the peak of storm flow. The latter phenomenon was noted by the writer when making some studies of Zumbro River, in Minnesota, in 1936, and was given the name "secondary base flow." The same, or something quite similar, was observed by Mr. C. R. Hursh[13] and called by him "subsurface storm flow."

---

[13] *Transactions*, American Geophysical Union, 1936, p. 302.

Separation of the storm run-off into surface flow and secondary base flow has been made for a number of streams in the Mississippi Valley. In order to obtain satisfactory results it was necessary to determine the discharge at intervals of 6 hr or less, rather than to use daily averages. It has been found that the recession of each of these components of flow can be plotted as a straight line on semi-logarithmic paper. The formula used in the office of the writer is

$$Q_t = Q_0 K^t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

in which Q_0 is the discharge at any instant; Q_t is the discharge t days later; and K is known as the "daily depletion factor," its value for secondary base flow being much higher than for surface flow.

Equation (22) can be derived from that proposed by W. W. Horner, M. Am. Soc. C. E., and F. L. Flynt, Assoc. M. Am. Soc. C. E.[14] It should be noted that flow from only one source (surface) was used in developing the Horner and Flynt formula.

In the case of the Iowa River at Marshalltown, Iowa, for a concrete example, it has been found that the secondary base flow becomes appreciable in about 12 hr after the peak of storm discharge has passed and reaches a maximum in from 60 to 65 hr after the peak of storm discharge. At that time it may become as much as four times the surface discharge, or 80% of the total storm discharge. The peak of the hydrograph of secondary base flow has a fairly flat top of about 40-hr duration, compared with about 10 hr for the peak of surface flow, and a depletion factor, K, equal to 0.69, as compared with 0.33 for surface flow. The ratio of the total secondary base run-off to the total storm run-off varies quite widely. For gentle rains the peak discharge of secondary base flow at Marshalltown is about 20% to 25% of the peak discharge of surface flow. For rains of high momentary intensity the ratio may be considerably smaller.

Studies, made in the writer's office, of the flow of the Iowa River at Marshalltown, and that of certain other streams, show that the recession of the ground-water flow can be expressed by Equation (22) during periods when no recharge is taking place (the value of K at Marshalltown for ground-water flow is 0.98). It follows, therefore, that for each of the components of flow the quantity of water remaining in live storage at any time during a true recession is directly proportional to the discharge of the component. This reasoning cannot be applied strictly to any flow that includes discharge from more than one of the three sources previously described.

LEROY K. SHERMAN,[15] M. AM. SOC. C. E. (by letter).—Since the introduction of the unit graph method,[2] several improvements have resulted, and a keener appreciation of the value of interpreting the hydrograph has developed. The author has presented a useful review of run-off characteristics and has outlined certain original procedures. This discussion will be confined first to brief comments on the fundamental principles and will be followed by a critical analysis of some of the author's conclusions and suggested procedure.

---

[14] *Transactions*, Am. Soc. C. E., 1936, Equation (1), p. 151.

[15] Cons. Engr., Chicago, Ill.

Mathematical derivation of the rising and falling stages of the hydrograph has been studied carefully by several hydrologists. In addition to those referred to by the author, the work of Franklin F. Snyder,[16] Jun. Am. Soc. C. E., published in 1938, should be noted.

Synthetic hydrographs serve a useful purpose. The test of such work is their correlation with observed graphs. When reliable observed hydrographs are available, they should be used in preference to any other basis. Nature integrates all of the factors with no oversights or assumptions.

Transposition of hydrographs may also be useful in the absence of a record. The criticism of the writer's paper on transposition (see heading "Transposition of Hydrographs") is well taken. Smaller areas generally have the steeper slopes. The use of diagrams of distribution percentages on successive days, as illustrated by Fig. 4, should prove useful in many cases.

The time of concentration has been used in many ways. The writer concurs with the definition (see "Synopsis") that concentration, under a continuous rain of uniform intensity, is "the length of time until the run-off becomes constant." The velocity of the stream generally increases with the stage, but it is fairly constant at higher stages. For this reason, best results with the unit graph method are obtained with unit distribution percentages derived from the higher storm intensities. The time of concentration for a given basin, therefore, will be relatively long for the lighter rains. In the case of overland flow, the time of concentration may be materially affected by the original moisture content of soil and vegetation. Variations of from 22 to 50 min have been observed on the same sod covered plots.

Fig. 5 is a constructive suggestion in estimating base flow. The author states (see paragraph before "Conclusions"): "Both methods rely very heavily on correct determination of percentage of run-off and ground-water flow." This truth is worthy of amplification. Net rainfall is not primarily a percentage of rainfall. It is rainfall minus infiltration, after surface run-off begins. Wherever automatic-recording data of rainfall and run-off are available, or wherever the soil infiltration capacity is known, run-off can be estimated accurately by the unit graph or by some other methods (see Table 7). When only daily quantities of precipitation are given, it is impossible to know whether this rain fell in 2 hr or 10 hr. In this case, the estimate will be more or less approximate and estimated percentages of run-off will suffice.

In Fig. 3, the uniform rate of net rainfall persisted for five units of time and ceased at the time of concentration. The tail of this graph ended six time units after the concentration period, or Time 11. If the rain ceased at the end of Time 2, the figure shows (at the bottom of the hatched area) the termination of run-off at the same point—Time 11. This violates two observed facts in nature: (1) For a uniform rate of rainfall, the time base of the hydrograph for a short rain is less than the time base for a longer rainfall; and (2) for any duration of rainfall the area (volume) of the tail of the hydrograph is equal to the area (volume) of net rainfall, up to said duration, above the hydrograph. This accords with the storage equation.

---

[16] "Synthetic Unit Graphs," by Franklin F. Snyder, *Transactions*, Am. Geophysical Union, 1938, Part I.

The author criticizes the unit graph method, particularly with reference to applications extending beyond the time of concentration. He states that the method will follow an actual hydrograph very imperfectly, sometimes giving results in excess of true run-off rates. He proposes another method.

TABLE 7.—ALTERNATE METHOD OF ANALYZING A CONCENTRATION CURVE

Time (ten-minute units)	Rainfall, in inches per hour	Infiltration, in inches per hour	Net rainfall, in inches per hour	Distribution (percentages)	SUCCESSIVE ORDINATES								
					10	20	30	40	50	60	Total to 60	65	Total to 65
10	1.73	0.18	1.55	35.8	0.55	....	....	....	....	....	0.55	....	0.55
20	1.73	0.18	1.55	43.6	0.68	0.55	....	....	....	....	1.23	....	1.23
30	1.73	0.15	1.58	12.0	0.19	0.68	0.57	....	....	....	1.44	....	1.44
40	1.73	0.15	1.58	5.0	0.08	0.19	0.69	0.57	....	....	1.53	....	1.53
50	1.73	0.13	1.60	3.6	0.05	0.08	0.19	0.69	0.58	....	1.59	....	1.59
60	1.73	0.13	1.60	....	....	0.05	0.08	0.19	0.70	0.58	1.60	0.29	1.60
65	Stop	0.13	1.60	....	....	....	0.05	0.08	0.19	0.70	1.02	0.35	1.38
....	....	....	....	....	....	....	....	0.05	0.08	0.19	0.32	0.10	0.58
....	....	....	....	....	....	....	....	....	0.05	0.08	0.13	0.04	0.23
....	....	....	....	....	....	....	....	....	....	0.05	0.05	0.02	0.13
Totals	....	....	10.26	100.0	1.55	1.55	1.58	1.58	1.60	1.60	9.46	0.80	10.26

The writer has not observed such discrepancies unless the unit graph method was misapplied or unless the data for a unit or distribution graph were inadequate. The writer presented a hypothetical concentration-time graph in his original paper.[2] He had not, however, tested the method against an actual hydrograph extended beyond the time of concentration. As a check, therefore, he has computed a hydrograph of run-off due to an actual rain of 1.73 in.

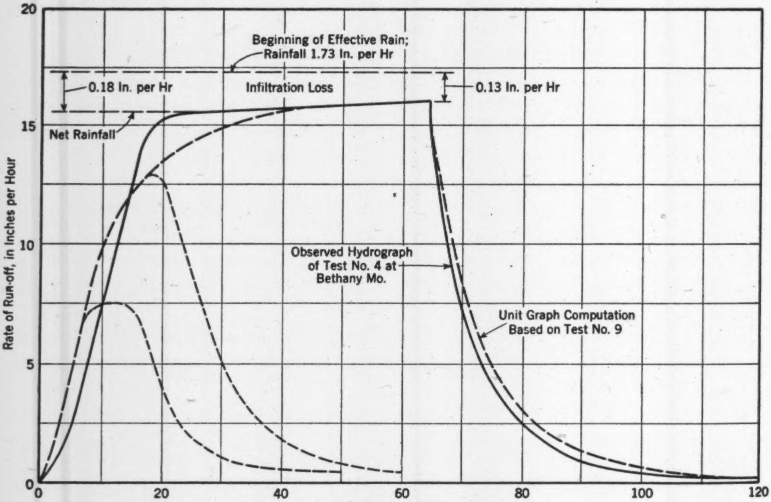


FIG. 10.—COMPARISON OF COMPUTED WITH OBSERVED HYDROGRAPH

per hr for 65 min, with a known run-off of 89 per cent. The computations are shown in Table 7. The computed hydrograph is shown in dotted line on Fig. 10. The observed hydrograph, shown solid, is one of a series of tests made in the Fall of 1937 at the Soil Conservation Experiment Station at Bethany, Mo., by Messrs. C. M. Woodruff, Dwight D. Smith and Darnell M. Whitt. A rainfall simulator was used on a plot 7 ft wide and 73.6 ft long. The slope was 7.19% covered with blue-grass sod.

The writer derived a 10-min distribution from an observed hydrograph known as Test No. 4. He applied this distribution to the rainfall data of an entirely different "storm" designated as Test No. 9.

The result speaks for itself and the writer offers no apologies for the unit graph method as a practical working tool.

The observed infiltration rates, as given for Test No. 9, were used. Almost identical results would have accrued had the writer used the average infiltration capacity of 0.158 in. per hr based upon the average of 15 tests made at Bethany.

Apparently Fig. 7 is inconclusive. One peak is checked very well by both methods. The other peak is checked very poorly by both methods. The writer suggests that if an infiltration capacity had been used, instead of a percentage of run-off, both results would have checked.

In Appendix III, Table 2, the percentage distributions for a hypothetical one-day storm and a three-day storm are given, together with the statement that three days constitute the time of concentration. The writer cannot reconcile these statements. Both conditions cannot be assumed. Each is a function of the one-day distribution. It is to be regretted that the comparison, such as Tables 3 and 4, was not checked against an observed hydrograph.

The author may have developed a desirable principle that would effect an improvement over the unit graph method, but, in the writer's opinion, this has not been demonstrated in the paper.

RICHMOND T. ZOCH,[17] ESQ. (by letter).—An interesting application of certain theories of rainfall, run-off, and stream flow is contained in this paper, and it is gratifying to find many of the assumptions, upon which Mr. Meyer's paper is based, explicitly stated. In the past, many of the writers dealing with this subject have not stated the assumptions involved nearly as explicitly. It is gratifying also to find the concept of the histogram of the drainage area mentioned.

Mr. Meyer's treatment of the depletion curve (which he calls the storage curve) is theoretically sound, practically convenient and commendable, with the exception that, strictly speaking, the depletion curve begins at a time equal to the concentration time plus the duration of the rain. His treatment of the concentration curve (a section of the hydrograph which he has also named most appropriately) is also theoretically sound and practically convenient; but only so in the section of the hydrograph which comes before the end of the rain, or the concentration time, depending on which of these two times comes first.

---

[17] Meteorologist in Chg. of Library, U. S. Dept. of Agriculture, Weather Bureau, Washington, D. C.

Mr. Meyer's presentation is usually clear, but he makes a number of statements which do not seem to be entirely correct. Most of these are in the paragraphs under the heading, "Analysis of the Hydrograph," and it is believed well to point out in detail the statements in which the writer's views differ from Mr. Meyer's. For example, he states, "* * * after a short time lag the run-off will be approximately proportional to the rainfall," and further, "It is assumed permissible to treat the run-off as a constant percentage of a uniform rain." These statements require much explanation in discussing them. First, it is necessary to distinguish between run-off and rainfall on the one hand, and the rate of run-off and rate of rainfall on the other. The first two are measured in inches and indicate depths of water which have run-off or fallen, respectively, up to time *t*, whereas the other two are measured in inches per hour and are, respectively, the depth of water running off, or falling, per unit time at a given instant of time.

Again, as regards run-off and rate of run-off, it is necessary to distinguish carefully between true surface run-off and underground (internal) drainage. When the soil is not saturated, there is always some infiltration and internal drainage, but the rate of infiltration may exceed the rate of internal drainage. As soon as the soil is completely saturated, there is still infiltration because there is also internal drainage; but the two are equal, and with a uniform continuous rate of rainfall the rate of internal drainage is a constant percentage of the rate of rainfall. Furthermore, after a steady state has been reached (which time may occur simultaneously with, or some time after, but never before, the time of complete saturation of the soil), the rate of the surface run-off is a constant percentage of the rate of rainfall. It is this condition which Mr. Meyer probably had in mind when he made the second statement quoted.

Consider next the capacity of the soil to retain water. The soil may retain water by internal absorption, or it may retain water as surface pondage. In the theoretical treatment of rainfall, run-off, and discharge, it is always possible to distinguish between the internal capacity of the soil and its external (surface) capacity. However, and naturally, the equations involved are more simple if this distinction is not made. Again, from the practical point of view the distinction is certainly not always necessary, because nearly all the internal drainage subsequently appears as discharge. In a stream large enough properly to be spoken of as a river, only the internal drainage in the immediate vicinity (broadly speaking) of the gage will not appear as discharge at the gage. Go to the soil a little distance away from the gage, as well as all the soil still farther up stream, and all the internal drainage, just the same as all the true surface run-off, will appear as discharge at the gage. For this practical reason, as well as the theoretical reason of simplicity, in the following remarks run-off and rate of run-off are considered in the broad sense of the combined surface run-off (or rate of run-off) and internal drainage (or rate of internal drainage).

Consider now the foregoing statements of Mr. Meyer from the point of view of rate of rainfall and rate of run-off. Whether the rate of rainfall is uniform or not, the rate of run-off is never even approximately proportional to the rate of rainfall. After a sufficiently long time (theoretically an infinitely long time—as Mr. Meyer himself states on a later page in another connection—but practically, after the lapse of a number of hours, depending on the nature and condition of the soil), with a uniform continuous rate of rainfall, the rate of run-off must equal the rate of rainfall. This sufficiently long time, of course, must be long enough for the soil to become completely saturated—or water-logged would probably be a better term—and the rate of infiltration is exactly equal to the rate of internal drainage. At no moment during this period of approach to the time when the rate of run-off will be exactly equal to the rate of rainfall, however, is the rate of run-off approximately proportional to the rate of rainfall. The rate of run-off is proportional, however, to the water remaining with the soil. In many cases, the rate of run-off will be a constant percentage of the water remaining with the soil, although in some cases undoubtedly it is a function of the amount remaining with the soil rather than a constant percentage. The percentage of the rate of rainfall running off at any instant of time steadily increases from the beginning of the rain until a time is reached when the rate of run-off is exactly equal to the rate of rainfall.

The steady state of run-off is that state at which the rate of run-off exactly equals the rate of rainfall. There is also a steady state of discharge, but it always comes some time after the steady state of run-off.

From the foregoing considerations, which are perfectly straightforward and simple, it is possible to set up a differential equation and from it to find the exact mathematical expression which represents the rate of run-off as a function of the rate of rainfall for any given time. This has actually been done by the writer in the paper to which Mr. Meyer refers,[8] and it forms the basis of later papers following it, to which Mr. Meyer does not refer.[18, 19] The differential equation[8] gives rise to an equation which contains the exponential function. This equation probably represents what actually takes place in Nature with remarkable exactness as the time of the steady state is approached; but in the initial range (that is, immediately after the beginning of the rain) it may be only a rough approximation to what occurs in Nature. For this reason the idea of treating the rate of run-off as a function rather than as a constant percentage of the water remaining with the soil has been recognized and, to a certain extent, treated.[19]

Consider next the statements by Mr. Meyer from the point of view of run-off and rainfall. The rainfall that has fallen up to time t is represented by $r\,t$, in which r is the rate of rainfall and t is the time, whereas the run-off which has disappeared up to time t is represented by $(r\,t - r\,c)$, in which c is the capacity of the soil. Clearly, one cannot say that, as the rain continues at a uniform rate (that is, as t increases), the run-off that has disappeared at time t is a constant percentage of the rainfall that has fallen up to time t.

The statement, "In actual cases the hydrograph is not expected to vary greatly in shape from one produced by rain of uniform distribution," is one on which the writer is not prepared to comment, but it certainly seems that

---

[18] *Monthly Weather Review*, Vol. 64, April, 1936; and, Vol. 65, April, 1937.

[19] *Transactions*, Am. Geophysical Union, 18th Annual Meeting, 1937, pp. 425–426.

this statement is worthy of further investigation before being accepted unconditionally.

There is confusion in the paper between the terms "volume of rate of run-off" and "discharge." The latter is the water flowing in a stream and takes account of the time between the moment the water falls to the ground as rain and the instant it reaches the point on the stream at which the discharge is being measured; and this time is a variable quantity increasing with increasing distance from the gage. The volume of rate of run-off is the water disappearing from the soil at any moment of time into the numerous steam channels of a drainage area. Both of these terms are expressed in cubic feet per second, and this may partly account for the confusion between the two. In the statement, "Assuming a uniform, continuous rain, the [discharge] passing any point on the stream will increase as water from more distant parts of the drainage area comes to the point of measurement, * * *," the writer has substituted the word "discharge" for Mr. Meyer's word, "run-off." The discharge does not reach a maximum at, nor does it become constant after, the time of concentration. For a drainage area, the width of which never decreases with increasing distance from the gage, this maximum cannot precede the concentration time; but even in drainage areas of this class the discharge does not become constant after this time. For drainage areas, the width of which decreases with increasing distance from the gage, the maximum discharge may precede the time of concentration. The duration of the rainfall itself has an important bearing on the time of the maximum discharge. The duration may be quite short, especially in thunderstorm types of rain, whereas during the winter season the duration may be very long, and the maximum discharge for any uniform continuous rate of rainfall can never precede the time when the rain stops. Hence, if the duration exceeds the concentration time there will be no distinction in the time of occurrence of the maximum discharge arising from different shapes of drainage areas.

Quoting Mr. Meyer: "As the streams then have no source of replenishment other than ground-water flow, which has been eliminated from consideration, all flow thereafter must come from depletion of the water in channel or valley storage." In considering this statement, in place of ground-water flow, which, as Mr. Meyer states, can be eliminated from consideration, eliminate from consideration the stream flow at the time the rain began. By doing this, only the flow due to the rain under discussion is considered. Just the same, much of the stream flow resulting from a given rain is due to water which at some time during the course of its journey from the spot where it fell as rain to the gage on the river was under the surface of the soil. This is true although the soil may have been completely saturated at the beginning of the rain, because even in this case of saturation there is infiltration into the soil, and the rate of infiltration in such a case is equal to the rate of internal drainage. Thus, after a time equal to the sum of the concentration time, plus the duration of the rain, flow results not only from depletion of the water in the channel storage, but also from depletion of the water within the soil which entered the soil during the current rain under consideration.

In the sentence beginning at Line 5 below Equation (3), the word "discharge" occurs four times. This statement is entirely correct if in each case the term "discharge" is replaced by "volume of rate of run-off," or also by "rate of run-off," but, applied to discharge it is never strictly correct from the theoretical point of view, and seldom approximately correct even from the practical point of view.

The statement, "the increase in ordinate of the hydrograph per unit of time is proportional to the additional area, water from which first reached the point of measurement during that unit of time," is only partly correct. Throughout the period of time when the discharge is represented by the curve which Mr. Meyer very fittingly calls the "concentration curve," the discharge is increasing for two different reasons: First, the soil in the drainage area is approaching the limit of its capacity for water and hence a greater and greater portion of the rain falling upon the drainage area is flowing off as free water; and secondly, the area that contributes to the discharge is increasing. After the concentration time, the entire drainage area above the gage is contributing to the discharge, and then the discharge continues to increase, but from the first cause only.

"This break has actually been found on continuous gage records (see Fig. 1), although storms of sufficient duration to produce actual basic hydrographs on large drainage areas are * * *" implies that Mr. Meyer believes that for a rain, the duration of which equals the concentration time, a sharp break occurs in the hydrograph. It is quite true that hydrographs are encountered with such sharp breaks, but these breaks are never due to the concentration time equaling the duration of the rain. They may be caused by moving showers of rain which must move down stream rather than up stream, but a uniform continuous rain will never produce a hydrograph with a sharp break in it, except in the very special case of a heavy rain with a very short duration (practically, the duration of the rain would have to be less than one hour).

Mr. Meyer regards the hydrograph as consisting of only two parts, the first of which he calls by the very descriptive title, the "concentration curve." Generally, for constant rates of rainfall, the hydrograph consists of four distinct parts, which are not in the same order in all cases. Suppose, first, that the duration of the rain is greater than the concentration time. Then the four parts of the hydrograph are as follows: (1) The concentration curve from the beginning of the rain until the concentration time; (2) the saturation curve when the area contributing to the discharge is constant, after the concentration time; (3) the period after the rain stops; and (4) the depletion curve.

Part (1).—During this interval the area contributing to the discharge at the gage is steadily increasing.

Part (2).—After the concentration time, the area contributing to the discharge does not increase, but remains constant; the discharge increases, however, approaching a steady state at which the discharge is equal to the volume of rate of rainfall falling upon the drainage area. During this interval (which, theoretically, may last an infinitely long time, but practically, lasts as long as the rain continues at a constant rate) the hydrograph is represented by the saturation curve, which is asymptotic to the straight line that would represent the discharge at a steady state.

Part (3).—After the rain stops, the discharge is no longer represented by the saturation curve. If a steady state has been reached (practically, for a drainage area large enough properly to be called a river basin, such a steady state of discharge is almost never reached) the discharge decreases immediately. If a steady state has not been reached, the discharge increases after the rain stops until the time of the crest. Here the hydrograph is represented by the transition curve which contains the crest of the flood. The transition curve extends from the end of the rain until a time equal to the duration of the rain, plus the concentration time.

Part (4).—Finally, there is a depletion curve, which extends from a time equal to the duration of the rain, plus the concentration time until, theoretically, an infinite time later, but practically until the beginning of the next rain.

If the duration of the rain is less than the concentration time, obviously, there is no saturation curve. There are still four sections to the hydrograph and a distinction is also caused from the shape of the drainage area. If the drainage area is fan-shaped—that is, if the width of the drainage area increases with increasing distance from the gage—the maximum discharge must follow the concentration time regardless of how short the duration of the rain may be. In that case there are: (1) A concentration curve from the beginning of the rain until the rain stops; (2) a curve of slow rise from the time the rain stops until the concentration time; (3) a transition curve from the concentration time until a time equal to the duration of the rain, plus the concentration time; and (4) a depletion curve.

If the drainage area is such that its width decreases with increasing distance from the gage, the crest may occur before the concentration time. Then there are: (1) A concentration curve from the beginning of the rain until the rain stops; (2) a transition curve from the time the rain stops until the concentration time; (3) a curve of slow fall from the concentration time until a time equal to the duration of the rain, plus the concentration time; and (4) a depletion curve.

Thus, generally, there are four parts to the hydrograph. If the rain is infinitely short (practically, less than one hour), the hydrograph consists of only two parts: Part (1), a curve of rise, and Part (2), a depletion curve. If the duration of the rain is exactly equal to the concentration time, the hydrograph consists of three parts: Part (1), a concentration curve; Part (2), a transition curve; and Part (3), a depletion curve—the saturation curve being completely absent.

It is for these reasons that the writer believes Mr. Meyer's treatment to be fully the equal of any that has been proposed heretofore for very short rains; but he does not believe it proper to consider Mr. Meyer's "basic hydrograph" as having only two parts. His "basic hydrograph" is naturally divided into three sections and cannot be closely represented by a graph of only two sections.

One point on which Mr. Meyer does not comment (except in the statement which has been quoted about the discharge reaching a maximum and becoming constant at the concentration time) is the determination of the time when the maximum discharge occurs. It has been shown[8] that for a rectangular drainage area, using essentially the same assumptions that Mr. Meyer uses, the time of the maximum discharge is expressed by the function

$$t_c = c \log \left(e^{\frac{L}{cv}} + e^{\frac{t_0}{c}} - 1\right) \dots\dots\dots\dots\dots\dots (23)$$

In this function t_c is the time of the crest or maximum discharge, measured from the beginning of the rain; t_0 is the duration of the rainfall; L is the distance from the gage to the source of the stream; and, v is the velocity of the water. Thus, $\frac{L}{v}$ equals the concentration time, and it should be observed that this function is symmetrical in $\frac{L}{v}$ and t_0; and, c is the capacity of the soil to absorb water either internally or as surface pondage. Naturally, with a differently shaped drainage area or with different assumptions of velocity, soil capacity, rates of rainfall and evaporation, this function requires modification. As stated previously, it is possible to separate c into that part which absorbs water internally and the part which is due to surface pondage; but for simplicity they are combined here. Naturally, when they are separated, the function which represents the time of the crest is a more complicated one, and a simple function is here taken merely to illustrate that, even when these capacities are combined, they influence the time of the crest appreciably, and when they are separated the influence upon the time of the crest is still more marked. This function shows that the time of the crest for this particular shape of a drainage area cannot occur before the concentration time, and neither can it occur before the ending of the rain. The capacity of the soil to retain water is measured in hours, and when the capacity c is said to equal a certain number of hours (say, 10 hr), one means that at the end of 10 hr from the beginning of the rain, the rate of run-off from the soil will be $\left(1 - \frac{1}{e}\right)$ times the rate of rainfall that is falling upon the ground. Thus, the capacity of the soil is a time constant, and is the time at which the rate of run-off reaches 0.632 of its final value. Clearly, this method of expressing the capacity of the soil is simple and convenient. For example, the capacity of the soil may be low. That is, the rate of infiltration may be low, or the soil may be fairly well saturated with water at the beginning of the rain; or, even if no water is on the soil at the beginning of the rain, it may be steep, and hence the surface pondage factor would be low. All of these conditions make the capacity of the soil low, and they also mean that the rate of run-off will approach the rate of rainfall more rapidly than it would if the capacity were high. By measuring the capacity of the soil in time, then, one measures the rapidity with which the rate of run-off approaches the rate of rainfall.

The time of the crest, as represented by Equation (23), is not greatly influenced by the capacity of the soil when the ratio of the duration of the rain to the concentration time is very large or very small (depending, of course, on which is the larger) but, when the duration of the rain and the concentration time are nearly equal, as Mr. Meyer chose them for his "basic hydrograph," the time of the crest is quite sensitive to the varying capacity of the soil. Table 8 (Columns (2) and (3)) illustrates this fact, where the concentration time and the duration of rain are given the same value of 50 hr, but the capacity has the different values shown. The times of the crest given in Table 8, Column (2), have been computed by means of Equation (23).

Naturally, the change in the time of the crest influences the shape of the hydrograph, and this illustrates one of the fundamental and outstanding defects of the concept of the unit hydrograph itself. The unit hydrograph is based upon the assumption that for any drainage area all hydrographs resulting from isolated storms of a given duration have the same shape when reduced proportionally to correspond to the same volume of run-off. Table 8 (Columns (1) and (2)) clearly indicates that the shape of the hydrograph may be decidedly influenced by the different capacities of the soil, although the duration of the rain, the shape of the drainage area, and the depth of the rain are exactly the same.

The mathematical theory shows that the concept of the unit hydrograph is practically applicable when: (1) The concentration time is appreciably greater than the duration of the rain and the drainage area is of such shape that the crest cannot precede the concentration times; and (2) the duration of the rain is appreciably greater than the concentration time, regardless of the shape of the drainage area. Under other conditions, the application of the unit hydrograph will not yield reliable results.

TABLE 8.—INFLUENCE OF SOIL CAPACITY ON TIME OF CREST

Capacity of soil, in hours	RECTANGULAR DRAINAGE AREA				TRIANGULAR DRAINAGE AREA			
	DURATION, FIFTY HOURS		DURATION, FIVE HOURS		DURATION, FIFTY HOURS		DURATION, FIVE HOURS	
	Time of crest, in hours	Maximum discharge in mile-inches per hour	Time of crest, in hours	Maximum discharge in mile-inches per hour	Time of crest, in hours	Maximum discharge in mile-inches per hour	Time of crest, in hours	Maximum discharge in mile-inches per hour
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
10	55.89	862.2	50.04	99.2	51.79	929.7	20.52	127.9
20	63.02	739.6	50.46	90.8	55.26	800.7	27.61	99.6
40	71.54	569.2	51.50	70.0	61.16	603.4	34.95	70.2
50	74.49	510.2	51.90	62.0	63.30	538.8	37.19	61.2

The influence of the time of the crest and the capacity of the soil upon the magnitude of the maximum discharge is also great. Column (3), Table 8, illustrates the corresponding values of the maximum discharge from a rectangular drainage area of 10 000 sq miles and 200 miles in length, and for a rate of rainfall of 0.10 in. per hr. These maximum discharges are computed from the formula

$$y_c = W\, r[L - v\,(t_c - t_0)] \dots\dots\dots\dots\dots\dots (24)$$

in which r is the rate of rainfall, and W is the width of the drainage area. In the foregoing cases, had the rain continued sufficiently long to bring the entire drainage area to a steady state (which, necessarily, would be longer than the time necessary to bring an individual unit to a steady state) the discharge clearly would be 1 000 mile-in. per hr.

The discharges for the corresponding capacities are expressed in mile-inches per hour, and 1 mile-in. per hr can be converted to cubic feet per second, thus:

$$1 \text{ mile-in. per hr} = \frac{5\,280 \times 5\,280}{12 \times 60 \times 60} \text{ or } 645.3 \text{ cu ft per sec.}$$

For short rains, the time of the crest does not vary appreciably with a varying capacity of the soil, provided the shape of the drainage area does not permit the time of the crest to precede the concentration time, but there still will be an appreciable variation in the maximum discharge. Thus for the same conditions that were outlined for Columns (2) and (3), but for a rain of only five hours' duration instead of fifty hours' duration, the corresponding values are shown in Columns (4) and (5), Table 8.

It should be emphasized that for Columns (2) to (5), Table 8, the rate of rainfall is the same, namely, 0.10 in. per hr. For Columns (2) and (3) the duration is 50 hr, so that the rainfall is 5.00 in.; and, for Columns (4) and (5), the duration is only 5 hr, so that the rainfall is only 0.50 in. Evaporation after the rain stops has been neglected, but formulas to correct the maximum discharge for this evaporation are readily obtainable.

By changing the shape of the drainage area, different values again result. Equations (23) and (24) are applicable only to a rectangular drainage area.[20] For a drainage area with a width $W(x)$, in which x is the distance above the gage and $W(x)$ a function that can approximate the histogram of the drainage area as closely as desired, the maximum discharge is given by

$$y_c = r \int_{v(t_c - t_o)}^{L} W(x)\, dx \dots\dots\dots\dots\dots\dots\dots (25)$$

for a duration of rain exceeding the concentration time regardless of the shape of the drainage area, and also for short rains if the drainage area is fan-shaped; or by a similar integral with upper limit $v\,t_c$, if the duration of the rain is less than the concentration time and the width of the drainage area decreases with increasing distance above the gage. Naturally, Equation (25) requires modification to apply it to varying conditions of soil capacity, rates of rainfall, and velocity of water.

To illustrate further how different soil capacities give rise to hydrographs of different shapes for the same drainage area, even if the rain is uniform throughout its duration, consider a hypothetical triangular drainage area, also of 10 000 sq miles.[20] For a long rain, duration of 50 hr, the appropriate formulas indicate the values given in Columns (6) and (7), Table 8. For a short rain, duration of 5 hr, the time of the crest precedes the concentration time, with the results shown in Columns (8) and (9).

MERRILL BERNARD,[21] M. AM. SOC. C. E. (by letter).—The unit hydrograph, as presented by Mr. Sherman[2] in 1932, has sustained itself well as the basis of a rational theory of relationship between rainfall and stream flow and as a practical tool for the solution of important hydrologic problems that previously had depended largely upon restricted, inflexible, empirical methods. Contributions following Mr. Sherman's introduction of the idea, including that of the author, have injected refinements into the technique and have added to the general knowledge of basic hydrologic relationships.

---

[20] "Run-Off—Rational Run-Off Formulas," by R. L. Gregory and C. E. Arnold, Assoc. Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), Fig. 5, p. 1068.

[21] Chf., River and Flood Div., U. S. Weather Bureau, Washington, D. C.

The stimulus given the field of applied hydrology by the unit hydrograph concept has created general dissatisfaction among hydrologists with the basic precipitation data, emphasizing, as it has, the inapplicability of average 24-hr rainfall to problems on the smaller water-sheds. The result can be expected, in the near future, to take the form of co-operative effort toward a network of recording rain gages over the United States, the beginning of which is evidenced in the 140 such stations established in 1937 by the Commonwealth of Pennsylvania.

The paper by Mr. Meyer has merit in a freshness of viewpoint, in the successful attempt to express the hydrograph of stream flow mathematically, and in the demonstration of the need for refinement in basic hydrologic data.

Refinement in method, as the writer conceives it, must evidence itself in two ways: (1) By producing consistently better agreement between computed and observed values; and (2) by adhering more rationally to the phases of the natural phenomenon of stream flow produced by surface run-off from rainfall.

The mathematical synthesis of the hydrograph has a definite place in this field, for the hydrograph is, in fact, a series of related curves capable of being expressed by three or more mathematical equations, the factors in which can be taken to represent the causes that combine to produce the hydrograph. A fuller understanding of the theory will depend upon the ability of hydrologists to inject greater detail into analysis, the confirmation of which must await the availability of correspondingly refined data.

Considering a practical relation between the adopted unit of time and the area of a water-shed, the degree of refinement will vary directly with the reduction of the time unit and will be proportional to the extent that the variation in rainfall over the water-shed and throughout time can be taken into account. Another source of appreciable error which, it is hoped, may be removed soon, lies in the inability of hydrologists to separate, accurately, the stream flow produced by surface run-off from that originating in ground-water sources.

It is difficult to associate the author's aims with his generalized assumptions. One can agree that total storm rainfall and total storm run-off may be expressed as a ratio. However, as far as the writer knows, the only approximate percentage through time is that existing between rainfall and the hydrograph of flow after the rainfall has been distributed as it will contribute to the flow at the outlet (the pluviagraph), and not in the order of its fall. It is more likely that refinement will take the course of dealing with the losses to run-off as a deduction from rainfall than as a ratio of run-off and rainfall.

Before continuing an examination into the author's work, it seems desirable to consider the least understood and most loosely expressed factor entering the synthesis of the hydrograph from rainfall—"concentration time." In its first conception this was considered to be the time in transit (based on the average velocity of flow) from the most remote point of the water-shed to the outlet. Later it was conceded that distance from the remote point to the outlet must be measured in units of time rather than in units of length; so that now "concentration time" is more generally understood to be the time interval at the end of which all parts of the water-shed are contributing to peak flow. In unit-hydrograph studies the concentration period is shown as the time interval between the respective centers of mass rainfall and resulting hydrograph, which is closely related to the peak ordinate of the hydrograph.

The author describes the time of concentration as that at the end of which the run-off becomes constant. Does he mean run-off (inflow) into storage, or discharge (outflow) out of storage? If he means the former, it would seem that wide fluctuations in the rate of rainfall must preclude the possibility of inflow prevailing at a constant rate for any appreciable time. If he means the latter, the assertion is not borne out in the shape of the normal hydrograph.

First, it must be understood that the concentration period cannot be a constant factor on any water-shed; it must vary according to the limits fixed by surface cover, the channel characteristics of the collecting system, and the distribution of rainfall over the area. Time of travel through the system of channels varies inversely with depth and effective hydraulic radii, fair stability being reached in both average velocity and wave velocity as bankful stages are approached.

If the time of concentration is considered to be that at the end of which all parts of the water-shed are contributing to peak flow, one must still be critical, because it is known that on a particular basin rainfall may be so favorably concentrated over the lower principal tributaries as to create the peak of the hydrograph before outlying areas have produced and delivered effective quantities of run-off to the outlet. Particularly on larger basins it is probable that surface retention and retardation will prevent contributory flow from appreciable areas during the peak stage, withholding their contribution to sustain the hydrograph in its recessional phase.

The writer has felt for some time that there is definite place for observational research in any plans there may be, in order to refine one's knowledge of rainfall-run-off relationships; and it seems to him that, with a little exercise of the imagination, the phenomenon of flow assembly can be explained as follows:

Select a unit of area on a natural water-shed of such size that a detailed view of the whole may be had from a central position, say, an acre. Take a position at the beginning of a downpour and note that the water at first soaks into the ground. Water begins to accumulate in the depressions and soon overflows and combines into a system of heretofore indiscernible small channels. The run-off from the unit area accumulates rapidly at the lowest corner where it enters a small, but well-defined, channel. The map of the water-shed indicates plainly the course the water will take to reach the outlet.

On another occasion one will observe an application of rainfall throughout a unit of time to the unit of area. Where, in the previous experience, the ground at the beginning of the rain was quite dry, with a resulting prolongation of the period necessary to satisfy the infiltration capacity of the soil, in this case antecedent rainfall has largely satisfied this initial condition, so that the run-off begins quickly, and assembles and discharges at a higher rate than before.

The observer visits his unit of area on numerous occasions and notes that, although there are obvious differences in the amount, and in the rapidity with which run-off flows to the outlet and joins the flow in the small outfall channel, there is a marked stability in the regimen of flow assembly. Run-off begins, combines, discharges from the area, and reaches the outlet in about the same manner from each rainfall of unit-time duration.

It is not difficult, then, to envision a water-shed comprised of unit areas which are related in the common characteristic of the time taken by their run-off waters to reach the outlet. In a natural water-shed these units would arrange themselves with reasonable continuity along lines conforming fairly well to surface contours. The isochrons, or "time contours," would differ from the surface contours in that, whereas the latter would be roughly parallel and would reach up into the principal tributaries of the system, the isochrons would be foreshortened because of the much higher rate of travel of the water after it has reached the principal channels.

To organize a mind picture of the process along these lines consider:

(1) A water-shed reflecting normal regional characteristics;

(2) A surface contour entering the water-shed at a line normal to the outlet;

(3) An isochron connecting the outlets of contiguous unit-areas entering the water-shed at the same point as the surface contour but bearing down-slope as it continues into the water-shed in order to maintain a constant delivery period (thus it provides for the obvious necessity of reducing the distance from the unit-area to the channel system in a direction normal to the latter in order to compensate for time of travel through the channels to the outlet); and,

(4) A map of water-shed, developed through acceptable analytical procedure (and based, if possible, on observational data) showing the unit-areas having a common time of travel from their outlets to that of the water-shed arranged along isochrons at fixed-time intervals, as the hour.

Concentration time may be expressed as the time required by the first water leaving a series of unit-areas along an isochron to reach the outlet; or it may be considered as the time taken by a series of unit-areas to discharge, completely, the run-off resulting from a unit-time rainfall. It is better understood as an enveloping line of equal time under which all of the units are contributing to the flow at the outlet within the period designated by the isochron. If the map shows lines of equal time of concentration at hourly intervals, and the areas progressively enveloped by each are determined, one can say that at the end of the first hour so many acres will be contributing to the flow at the outlet; at the end of the second hour so many more units, or a total of so many units will be contributing—and so on to the outer limits of the water-shed. Thus, in the isochronal map, one discerns the principal reason for the constancy in the time-base of a hydrograph resulting from rainfall confined to a unit of time.

The unit hydrograph has at least three phases—the rising, the peak, and the recessional. Thus far this discussion has had to do with the rising and peak stages. The rise is accounted for by the fact that the flow at the outlet is increasing because of (*a*) an increasing depth over the unit-areas already contributing to the outlet flow, and (*b*) an increasing contributing area. The latter influence on accelerated flow is removed when elapsed time has progressed upward and over the isochronal map to the outer limits of the water-shed. If the cessation of rain is coincident with the end of this time-period the result is a unit hydrograph as conceived by Mr. Meyer. There are two conditions, however, that must be satisfied before the analyst can be assured that the hydrograph will peak at, or shortly after, this time interval, and not before. First, the water-shed and the collection system must have fair uniformity in shape and in pattern. That is, the isochrons must be fairly regularly spaced and not concentrated around several important contributory branches discharging near the outlet and thus leaving appreciable areas at the outer limits of the water-shed to be represented by widely spaced and comparatively few isochrons. Second, the rainfall must be uniform throughout the time interval and over the water-shed; because, if normal variations are considered, it is obvious that heavy concentrations in the early part of the period and over the lower part of the water-shed will create the peak of the hydrograph before the more remote parts are contributing. Continuing rainfall will build up a gradual increase through the peak phase until a period of rainfall equal to the time-base of the unit hydrograph has passed, after which the peak will increase or decrease with the fluctuations in rainfall intensities to the end of the rainfall period. The hydrograph now starts downward through the recessional phase which constitutes withdrawal from the water in storage in the channel system.

Concentration time has been discussed as if it were a constant characteristic of a water-shed. This is true only within limits that are believed to be relatively unimportant when other errors inherent in hydrologic data themselves are taken into account. Variations in the concentration period are attributable to the following causes:

(1) Seasonal and developmental changes in vegetal cover of the water-shed affecting the quantity and rapidity of run-off;

(2) Seasonal changes in bank and channel growth affecting the movement of water in the channel system;

(3) The initial depth of water in the channels at the beginning of run-off; and,

(4) Variations in the distribution of rainfall over the water-shed as these variations affect the assembly of flow and propagation of the flood wave which delineates the hydrograph at the outlet of the water-shed.

If the actual condition of flow during a period of flood run-off is considered, one can appreciate the inappropriateness of applying average velocities to the moving water. The synchronization of flow assembly on a drainage area of fixed regimen varies only slightly from flood period to flood period—a fact which finds support in the similarity in shape of unit hydrographs of a particular water-shed.

Likewise, the conditions of the storage equation must be met (that is, within a unit of time, inflow plus or minus changes in storage must equal outflow). Inflow is a highly variable factor, being the sum of innumerable and widely varying contributions to flow discharging into channel storage along both banks of the entire channel system. Outflow is much less erratic in fluctuation, rising to and receding from a crest stage with the regularity of a compounded mathematical curve. Therefore, the balance demanded by the storage equation must be maintained by rapid changes in the configuration of the storage volume, creating irregularities in hydraulic slope and channel section which are recognized as characteristic of transitional waves.

FRANKLIN F. SNYDER,[22] JUN. AM. SOC. C. E. (by letter).—A "revision of current methods for the determination of stream flow from rainfall" is presented in this paper. The author uses time of concentration (defined as length of time until run-off becomes constant from a continuous rain of constant intensity) and a basic hydrograph to define the run-off characteristics of drainage basins.

The writer will show that if the user of the unit hydrograph and the user of the basic hydrograph could agree on the time of concentration for any area, the latter would be identical with the former as obtained by use of a unit hydrograph under the same assumptions. Of course, this would require that the agreed time of concentration be equal to the duration of surface run-off from a short duration or instantaneous storm, and would be a different quantity from that used in the rational method of sewer design.

Definitions not given by the author are required, with the introduction of the term, "surface run-off" or "flood run-off," as distinguished from "ground-water run-off." "Surface run-off" is usually defined as storm water that reaches the open drainage channels without penetrating the ground surface; but, the separation of surface run-off from ground-water run-off is usually made on hydrographs by drawing a horizontal or slightly inclined base flow line under the flood rise. The base flow rate is usually fixed by a low value of the stream flow previous to the flood rise. The time involved for the duration of surface run-off varies approximately from three to ten days for areas of from 10 to 10 000 sq miles. The flood run-off thus obtained may, or may not, fit the definition.

Table 9 gives a two-hour unit hydrograph derived by synthetic procedure[16] for the stream flow station of the U. S. Geological Survey at Big Piney Run near Salisbury, Pa. (drainage area equals 24.5 sq miles). The ordinates as tabulated are for the end of the designated two-hour periods. Fig. 11 shows the two-hour unit hydrograph and the cumulative plotting of the two-hour ordinates of the unit hydrograph, which is the run-off graph computed with the two-hour unit hydrograph for a continuous rain of 1 in. per 2 hr (100% run-off). This, then, is the basic hydrograph for the area according to the unit hydrograph procedure. To show that Fig. 11 is independent of the unit of time, Table 9 also gives the four-hour unit hydrograph for the same area. This hydrograph is the two-item progressive average of the two-hour unit hydrograph. The six-hour unit hydrograph would be the three-item progressive average of the two-hour unit hydrograph, etc.

---

[22] Associate Hydrologic Engr., U. S. Weather Bureau, Pittsburgh, Pa.

It should be emphasized that if an area is such that satisfactory results can be obtained with a unit hydrograph for four-hour rainfall quantities, the four-hour unit hydrograph ordinates would usually be tabulated at intervals of four hours rather than two hours as in Table 9. However, there is no reason why two-hour ordinates cannot be used with rainfall tabulated every four hours.

TABLE 9.—UNIT HYDROGRAPH ORDINATES, IN CUBIC FEET PER SECOND, AT TWO-HOUR INTERVALS FOR BIG PINEY RUN NEAR SALISBURY, PA.

	UNIT HYDROGRAPHS			UNIT HYDROGRAPHS			UNIT HYDROGRAPHS			UNIT HYDROGRAPHS	
Period (1)	Two-hour (2)	Four-hour (3)	Period (1)	Two-hour (2)	Four-hour (3)	Period (1)	Two-hour (2)	Four-hour (3)	Period (1)	Two-hour (2)	Four-hour (3)
1	120	60	13	150	156	25	54	57	37	12	13
2	480	300	14	138	144	26	49	51	38	10	11
3	1 380	930	15	128	133	27	44	46	39	8	9
4	1 480*	1 430	16	119	123	28	40	42	40	6	7
5	890	1 185	17	110	114	29	36	38	41	4	5
6	530	710	18	102	106	30	32	34	42	3	3
7	360	445	19	94	98	31	28	30	43	2	2
8	280	320	20	87	90	32	25	26	44	1	1
9	230	255	21	80	83	33	22	23	45	....	1
10	196†	213	22	72	76	34	19	20	46	....	....
11	176	186	23	66	69	35	16	17	47	....	....
12	162	169	24	60	63	36	14	15	....	....	....
Totals	....	....	....	....	....	....	....	....	....	7 915	7 909

* 1 650 at seventh hour. † Recession coefficient of 0.92 applicable after twentieth hour.

The continuous rain of 1 in. per 2 hr (in this case 2 in. per 4 hr) can be distributed with the four-hour unit hydrograph by the usual procedure of tabulating the product of the unit rainfall, times the unit hydrograph ordinates in vertical columns and adding horizontally for the totals. The hydrograph obtained is exactly the same as the basic hydrograph defined with the two-hour unit hydrograph. The same result would be obtained from a six-hour unit hydrograph and a continuous rain of 3 in. per 6 hr.

The time of concentration by the foregoing procedure is 88 hr, although the last two days do not increase the discharge very much. The basic hydrograph obtained from the two-hour unit hydrograph (run-off graph of a two-hour storm) and the author's method of synthesis is shown in Fig. 11. The author's values of λ, given in Table 1 of the paper, were used with ordinates of the two-hour unit hydrograph at 1-hr intervals. The procedure failed to define the concentration curve above a value of 6 245 cu ft per sec, so what was considered a reasonable extension was made and is shown by a broken line in Fig. 11.

This comparison shows the considerable discrepancy between the two procedures due to the fact that the author's time of concentration is evidently much less than the duration of unit hydrographs as ordinarily used.

If the run-off (which is usually called "flood run-off") were true surface run-off as previously defined, the time of concentration of 12 hr, and the basic hydrograph as obtained with the author's procedure, would probably be a true picture of what occurs. However, much of the run-off which at present is called surface run-off is not true surface run-off as previously defined; it is sub-surface storm flow which penetrated the surface of the ground on its way to the drainage channels, but did not reach the stable ground-water table. The recession sides of unit hydrographs are storage depletions, but not from channels alone. In other words, part of the run-off from every unit of rainfall is destined to pass through this temporary ground storage and reach the gaging station later than would satisfy a reasonable value of time of concentration as used in the rational method.

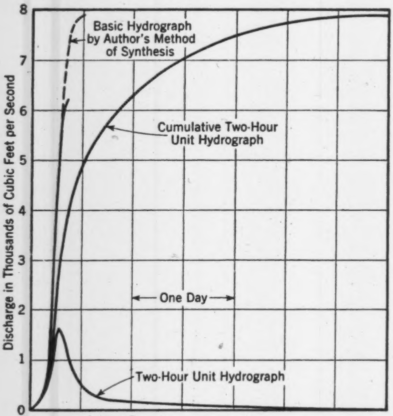


FIG. 11.—BASIC HYDROGRAPHS FOR BIG PINEY RUN NEAR SALISBURY, PA.

Eventually it may be necessary to separate the true surface run-off from sub-surface storm flow and ground-water discharge since it appears probable that there is a limit to the sub-surface storm-water storage and flow for any particular drainage basin. Thus, in very large floods, due to the maximum being reached or a limit set by existing infiltration capacity, a greater percentage of the total flood run-off occurs as true surface run-off as compared to sub-surface storm flow. This gives a unit hydrograph with an earlier and higher peak and less volume in the recession side than is the case in ordinary floods. This difficulty requires special adjustments in the unit hydrograph procedure and would probably require similar adjustments in the use of a basic hydrograph.

Fig. 12, showing the rainfall and run-off records for the storm of October 27 and 28, 1937, on Big Piney Run, illustrates the foregoing discussion.

A rational time of concentration for the small head-water area, whose longest channel is eight miles with a drop of 300 ft, would not be more than four hours at the maximum. The lag for the area, which is believed to be a more important hydrologic characteristic of a natural drainage basin than time of concentration, is about six hours. Lag is defined as time from center of mass of surface run-off producing rain to peak discharge and tends to be a constant for any drainage basin.

The U. S. Geological Survey topographical maps show some swamp area bordering on the two main channels, but in general the topography is quite rough, with a range in elevation from 2 800 to 2 200 ft. It is felt that the swamp areas might be the cause of as much as one hour out of the observed 6-hr lag for the area.

The Big Piney recording rain and snow gage, located on the Big Piney Run Basin, shows a continuous rain of 18 hr of fairly constant intensity, preceded by several short periods of precipitation and followed by 12 hr of precipitation at a lower but fairly constant rate; yet the discharge hydrograph shows no tendency toward a constant discharge at the crest. As a matter of fact, the unit hydrograph procedure for this storm gives a flatter crest than the observed record due to the rainfall of decreasing intensity at the end of the storm.

Fig. 12 shows the discharge hydrograph computed from the basin rainfall with the two-hour unit hydrograph previously given. Table 10 shows the two-hour quantities of precipitation and surface run-off and the distributions for a few of the items. The computations were shortened through the use of a surface run-off recession coefficient and the addition of a composite column which includes the recessions of discharge of all 2-hr items that have passed the twentieth hour. The increasing percentage of run-off was based on rainfall-run-off relations used in forecasting work in which ground-water discharge is used as the index of run-off conditions.

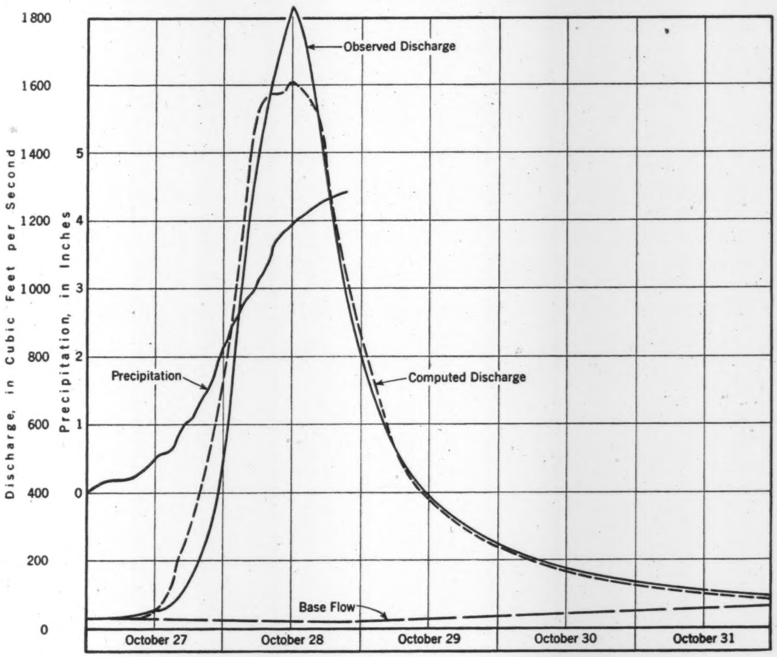


FIG. 12.—FLOOD OF OCTOBER 1937 ON BIG PINEY RUN

It would appear from a study of this storm that any basic hydrograph that would reproduce the discharge hydrograph would have to be quite similar to that obtainable from the unit hydrograph.

Assuming that a basic hydrograph can be agreed upon for any area, there are several weaknesses in its analysis and synthesis as given by the author. If a time of concentration of 12 hr is assumed for the drainage basin whose basic hydrograph is developed in Fig. 2, it is seen that the time of peak discharge is earlier for the 7-hr rain than for the 1-hr rain. The possibility of such an occurrence seems very slight.

TABLE 10.—UNIT HYDROGRAPH COMPUTATIONS FOR STORM OF OCTOBER 29, 1937, ON BIG PINEY RUN NEAR SALISBURY, PA.

Time ending, October:	Precipitation, in inches	Surface run-off, in inches	FLOW CHARACTERISTICS, IN CUBIC FEET PER SECOND							
			Distribution of surface run-off†				Composite term	Surface run-off	Base flow	Computed total discharge
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
27; 2:00 A.M.	0.10*	....	....	....	....	....	....	....	31	31
4:00 A.M.	0.08*	....	....	....	....	....	....	....	31	31
6:00 A.M.	0.02*	....	....	....	....	....	....	....	31	31
8:00 A.M.	0.02*	....	....	....	....	....	....	....	30	30
10:00 A.M.	0.11	0.02	2	....	....	....	....	2	30	32
12:00 Noon	0.18	0.07	10	8	....	....	....	18	29	47
2:00 P.M.	0.08	0.03	28	34	4	....	....	66	29	95
4:00 P.M.	0.28	0.15	30	96	14	22	....	162	28	190
6:00 P.M.	0.20	0.11	18	104	41	72	....	248	28	276
8:00 P.M.	0.27	0.16	11	62	44	207	....	396	27	423
10:00 P.M.	0.28	0.19	7	37	27	221	....	544	27	571
12:00 Mdnt.	0.50	0.37	6	25	16	133	....	699	26	725
28; 2:00 A.M	0.40	0.31	5	19	11	80	....	927	26	953
4:00 A.M.	0.33	0.26	4	16	8	54	....	1 254	25	1 279
6:00 A.M.	0.28	0.24	....	14	7	42	4	1 491	25	1 516
8:00 A.M.	0.35	0.30	36	....	6	35	17	1 546	24	1 570
10:00 A.M.	0.30	0.24	144	29	....	29	21	1 549	24	1 573
USE OF RECESSION COEFFICIENTS: (21 + 29) TIMES 0.92 EQUALS 46										
12:00 Noon	0.15	0.13	414	115	16	....	46	1 587	23	1 610
2:00 P.M.	0.16	0.14	443	331	62	17	62	1 550	22	1 572
4:00 P.M.	0.13	0.11	266	356	180	67	86	1 388	22	1 420
6:00 P.M.	0.08	0.07	159	214	192	193	113	1 234	21	1 255
8:00 P.M.	0.07	0.06	108	127	116	207	170	1 109	21	1 130
10:00 P.M.	0.05	0.04	84	87	69	125	212	976	20	996
12:00 Mdnt.	....	....	69	67	47	74	242	850	21	871
29; 2:00 A.M.	....	....	59	55	36	50	266	730	22	752
4:00 A.M.	....	....	....	47	30	39	299	604	24	628
6:00 A.M.	....	....	....	....	25	32	318	499	25	524
8:00 A.M.	....	....	....	....	....	27	315	429	27	456
10:00 A.M.	....	....	....	....	....	....	315	384	28	412
12:00 Noon	....	....	....	....	....	....	310	349	30	379
2:00 P.M.	....	....	....	....	....	....	298	319	31	350
4:00 P.M.	....	....	....	....	....	....	285	293	33	326
6:00 P.M.	....	....	....	....	....	....	270	270	34	304
Totals........	4.42	3.00	....	....	....	....	....	....	....	....

* Initial loss.

† The distributions are omitted for the items of run-off from 4 P.M., October 27 to 6 A.M., October 28, and for the items after 2 P.M., October 28.

Another point is the fact that if rain ceases after some unit area, say the sixth, has contributed its *h*-value at the gage, the author's procedure immediately begins to decrease the ordinate of discharge contributed at the gage by this area which is six units of time away from the gage. This means that the five units of flow from the area which have already started their trip through the drainage system are decreased simply because it stops raining.

The author's use of a recession coefficient is very timely. This coefficient eliminates about two-thirds of the work involved in unit hydrograph computations.

The author's illustration of the fact that ground-water discharge is a reliable index of run-off conditions is to be commended. However, the statement that ground-water flow has no characteristics to identify it with any particular period of precipitation is not so acceptable. If ground-water discharge is to be used as a successful index of run-off conditions, a well-defined procedure must be available for delineating ground-water discharge. It is difficult to accept time of the year as a satisfactory medium for this purpose. Ground-water depletion and storage curves provide a ready means of identifying, quantitatively, the ground-water discharge or recharge with particular periods of precipitation.

The statement that the unit hydrograph procedure and a continuous rain will frequently give a run-off rate exceeding the product of the rainfall rate and the percentage of run-off is difficult to understand. The continuous rain of 1 in. per 2 hr and the two-hour unit hydrograph for Big Piney Run give a peak discharge (100% run-off) of 7 915 cu ft per sec which is, of course, a run-off rate equal to the rainfall rate of 0.5 in. per hr.

In considering the probable accuracy of the author's synthesis of a hydrograph as compared with the unit hydrograph procedure, especially in flood forecasting, it is doubtful whether the proposed methods are far superior to the unit hydrograph procedure. It has been the writer's experience that the three main problems in computing an accurate discharge hydrograph from rainfall are the rainfall itself, the quantity of run-off that will occur, and the shaping up of the run-off.

Of the three, the possible rainfall eccentricities provide the most trouble, and the shaping up of the discharge hydrograph is the easiest to accomplish. By this, it is not meant to infer that the assumption of a uniformly distributed rain of constant intensity might not be the best for certain flood control studies. In comparing the flood reduction benefits available at some particular point from reservoirs at various locations, it may be more desirable to have a consistent procedure rather than to duplicate probable quantitative occurrences.

W. G. HOYT,[23] M. AM. SOC. C. E. (by letter).—Through detailed analyses, such as described by the author, a mass of valuable information is gradually being accumulated relating to the characteristics of surface run-off resulting from precipitation in the form of rain. It appears that at present the application of the unit graph and similar methods, such as that described by the author, must be largely confined to problems involving flood run-off resulting from rain, and to areas where the run-off reaches stream channels essentially as over-land flow. In the eastern part of the United States its application to flood problems during periods of no snow has been demonstrated amply. Unsatisfactory results may be obtained, however, if application is attempted in areas or periods when the flood run-off results largely from melting snow, or in areas where the greater part of the stream discharge reaches channels through the ground. Under such conditions failure to substantiate earlier findings may be attributed erroneously to flaws in basic principles, whereas the difficulties may relate to a lack of knowledge regarding the behavior of run-off that may reach a stream through the ground with a responsiveness approaching that of direct surface run-off.

---

[23] Hydr. Engr. (Prin.), Conservation Branch, U. S. Geological Survey, Washington, D. C.

Most of the basic data underlying the theory of the unit graph, the basic hydrograph described by the author and, in fact, much of the present knowledge relating to the flood run-off have been based on observations made in the north-central and northeastern parts of the United States. In these regions, relatively: (*a*) The greater part of the flood flow is essentially direct surface run-off; (*b*) infiltration rates and retention capacities over large areas, during major flood periods, are relatively small; and (*c*) periods involving large quantities of melting snow are generally found inapplicable for study. It is only natural that some modification in principles thus derived may be necessary before they are applicable in parts of the central and much of the western part of the United States. In this latter region, flood run-off—especially from the Sierra Nevada, Rocky, and northern coast range mountains—is largely associated with run-off from melting snow; and, in the southern coast range and semi-desert regions, infiltration rates and retention capacities, in general, are apparently very high, and only a small part of the storm precipitation reaches stream channels as direct over-land flow.

It is in the latter areas, especially, that much more hydrologic research with reference to flood run-off is urgently needed.

C. O. CLARK,[24] JUN. AM. SOC. C. E. (by letter).—In bringing to the attention of hydrologists the alteration of the hydrograph effected by channel storage, Mr. Meyer points to an agent ordinarily considered to be small, but an understanding of which is essential to one striving to avoid misuse of the unit hydrograph, and to reproduce or predict stream flow with greater accuracy.

Any one wishing to make a more extensive investigation into this problem will find the theory expressed in the Muskingum method of flood routing a valuable tool for correlating Mr. Meyer's analysis of the hydrograph with the familiar function of storage in the usual routing of floods.

The writer is familiar with no generally available publication which explains this method, but reference to it, and definition of terms, have been made by T. T. Knappen,[25] M. Am. Soc. C. E. The terminology used by Mr. Knappen is adopted herein, and the formula is not repeated.

Further development of the flood routing equation shows, in addition to the explanation in the aforementioned discussion, that

$$o_2 = C_3\, i_2 + C_4\, i_1 + C_5\, o_1 \qquad (26)$$

---

[24] Asst. Engr., U. S. Engr. Office, Norfolk, Va.

[25] "National Aspects of Flood Control: A Symposium," *Transactions*, Am. Soc. C. E., Vol. 103 (1938), Equation (3), p. 667.

in which

$$C_3 = -\frac{K\,x - 0.5\,T}{K - K\,x + 0.5\,T} \qquad \text{(27a)}$$

$$C_4 = +\frac{K\,x + 0.5\,T}{K - K\,x + 0.5\,T} \qquad \text{(27b)}$$

and,

$$C_5 = +\frac{K - K\,x - 0.5\,T}{K - K\,x + 0.5\,T} \qquad \text{(27c)}$$

It will be noted that C_5 is the same as the author's "λ," when $x = 0$—that is, when storage is solely a function of discharge. By analyzing the falling hydrograph, one can determine the value of K, which is the differential of the storage-discharge equation, and which, when storage varies directly with discharge, is the author's C_2. Knowing the value of K, and its variation with discharge, one can determine a storage-discharge curve, about which storage studies revolve, regardless of whether the ratio of total storage to instantaneous discharge is a constant or not, as long as the variation of the ratio is uniform. It will be seen that the assumption of direct variation is not necessary, except where the convenience of use outweighs the desirability of accuracy, or where the quality of fundamental data does not warrant the refinement of considering the variation. The writer is of the opinion that the assumption of uniform variation is sufficiently correct on many, and most, streams to warrant calculations on that basis except where reliable hourly records of rainfall are available. The graphical methods in use in routing floods through reservoirs, or reaches where storage is a function of discharge stage, will be found applicable and convenient.

The greatest advantage of studying the hydrograph with this tool may prove to be the indication that the storage in the ground-water table may be correlated with stream discharge stage and thereby enable one to use values of run-off that will be not less than the monthly average run-off, without resort to the somewhat confusing and highly artificial concept of variable base flow. For example, on a particular stream, with a drainage area of about 1 300 sq miles and 75 miles maximum length, the minimum value of "K" on the falling limb of several hydrographs of varying size, and from rainfall of various intensities and duration, appeared to be fairly constant at about 8.0 hr, but below a discharge of 6 cu ft per sec per sq mile the values were greater and had a common trend, but were not strictly uniform. The larger values were approximately 200 hr.

Detailed analysis showed that the variation could be explained and duplicated closely by a compound unit graph, the sum of two simple hydrographs, one of surface run-off and one of sub-surface run-off. The surface run-off, taken as 70% of the total run-off, was assumed to contribute according to the author's histogram, routed through channel storage of 8.0 hr by the Muskingum method. The sub-surface run-off (30% of the total run-off) was taken as contribution by the histogram routed through 200 hr of storage. These two hydrographs, added together, make up a compound unit hydrograph which has all of the advantages that the author claims for his basic hydrograph, and does not need to be used with a variable base flow to secure reasonably accurate and consistent results for the entire hydrograph, using percentages of run-off highly consistent with the monthly average run-off, base flow included. It will be apparent that the unit hydrograph is almost the same as the author's basic hydrograph for surface run-off plus another for sub-surface run-off. As Mr. Meyer states, the bases are infinite, but may be used as constant, of such length as will include 95% to 98% of the volume, without appreciable error.

Confining attention to the constant storage-discharge relation assumed by the author, the writer finds little difference between the unit hydrograph and the author's basic hydrograph, and therefore uses the terms interchangeably. A unit hydrograph that has been routed through such channel storage above the gage has the infinite base length to which the author refers, may be applied to variable quantities of run-off in successive unit periods of time, and will provide a more accurate indication of discharge than can be justified in the use of the usually available rainfall records. The well-derived unit hydrograph will show the effect of constant-ratio storage and is only less accurate than the author's method in that very small ordinates are conveniently neglected, for the sake of ease in computation, and a constant base length of hydrograph results. The lack of agreement between the new method and the old method in computing the hydrographs of the Merrimac River, shown in Fig. 7, could only result because the unit hydrograph used was not as well derived as the basic hydrograph. Although the writer has not proved that this statement is true algebraically, he has proved it to his satisfaction by routing hydrographs of various shapes through reaches of constant storage-discharge relation to prove that the sum of the separately routed hydrographs is equal to the routed hydrograph of the sum of the separate hydrographs.

The use of the concentration curve and the storage curve to derive the unit or basic hydrograph of an area has very real value. To be consistent, however, it would appear that the rising curve should show the effect of storage, because the same storage which holds the hydrograph up on the falling leg holds it down on the rising leg and accounts for much of the lag between initial rainfall and initial rapid rise of the stream.

The writer would like to raise one question with regard to the use of the histogram. If the contribution of the areas of varying distance from the gage combine to form the concentration curve, does it not necessarily follow that the contribution to the channel must fall off in the same manner? The writer has found hydrographs which exhibit the very rapidly falling leg and the angular break near the peak; but those found resulted from rainfall of several periods of duration which terminated in a short period of extremely intense rainfall, rather than, as the author suggests, from a prolonged rainfall of fairly uniform intensity.

The author's derivation of time of travel and, therefrom, the concentration time of a given area, will be useful in a system of rivers which has followed a fairly uniform geological formation (such as the Mississippi System), because there is a general symmetry within the basin; but this concept must be used with caution in river systems which have developed through much stream piracy, because in these may be found slopes in head-water streams which are less than the slopes in the main stream, near the mouth. This condition is quite common in streams in the South Atlantic Coast areas. Of course, the latter type may not meet the author's criteria for similar streams.

The analysis of stream run-off is susceptible of much more accurate computation than has yet been applied to it; by combining the concept of storage modification and the unit or basic hydrograph, degrees of accuracy in computation may be obtained, for many areas, which are limited at the present time by lack of hourly rainfall records. The increase in the number of such stations, and in recorder stream gages, will enable those who, like the author, are unwilling to accept the accepted methods, to delve into the problem and find the mechanism by which very high degrees of accuracy in discharge forecasting will be possible.

C. S. **Jarvis**,[26] M. Am. Soc. C. E. (by letter).—In the midst of the usual output of new plans hopefully aimed at refinements in run-off estimates, it is reassuring to find that the author, along with other proponents of new procedures, recognizes and admits the inherent limitations as to accuracy, and consequently the main dependence on logical reasoning, guided by mature experience, in any practical approach to evaluation of run-off factors and characteristics. Of all the divergent curves in Fig. 4, for instance, it is expected that appropriate choice shall be made after sufficient experience and with the aid of detailed topographic maps of the drainage area. However, the limitless variations in rainfall patterns, widely departing from what may have been adopted as the basis of reckoning, are capable of producing some very erratic results.

The writer has attempted to meet a requirement in the field of quantitative appraisal of run-off characteristics as they might be influenced by land-surface treatments and other soil and water conservation measures. The preliminary results are depicted in Fig. 13 and are held as only tentative, pending the completion of a more detailed analysis involving more rigorous procedure and a more nearly scientific approach. The agreement resulting from the two methods was so marked as to establish the preliminary solution nearly on a par with the final result.

Among the assumptions involved in both approaches to this problem were the following:

(1) Retention of rainfall by surface storage and induced infiltration on a representative portion of the drainage basin area, such as one-fourth or one-half, with no escape to drainage channels by over-land or other surface flow, must inevitably reduce the total volume of run-off within that flood-rise in about the same proportion;

(2) Reduction of run-off from similar areas by reason of detention works, increased vegetative cover, lengthened course of travel, reduced gradients as

---

[26] Hydr. Engr., SCS, Dept. of Agriculture, Washington, D. C.

well as velocities of flow, and prolonged contact of such water with the soil surface, account for a proportionate decrease of run-off volume; and,

(3) If the treated areas are widely distributed throughout the drainage basin, the base of the flood hydrograph could not conceivably be shortened as a result of such detention and retardation through conservation measures.

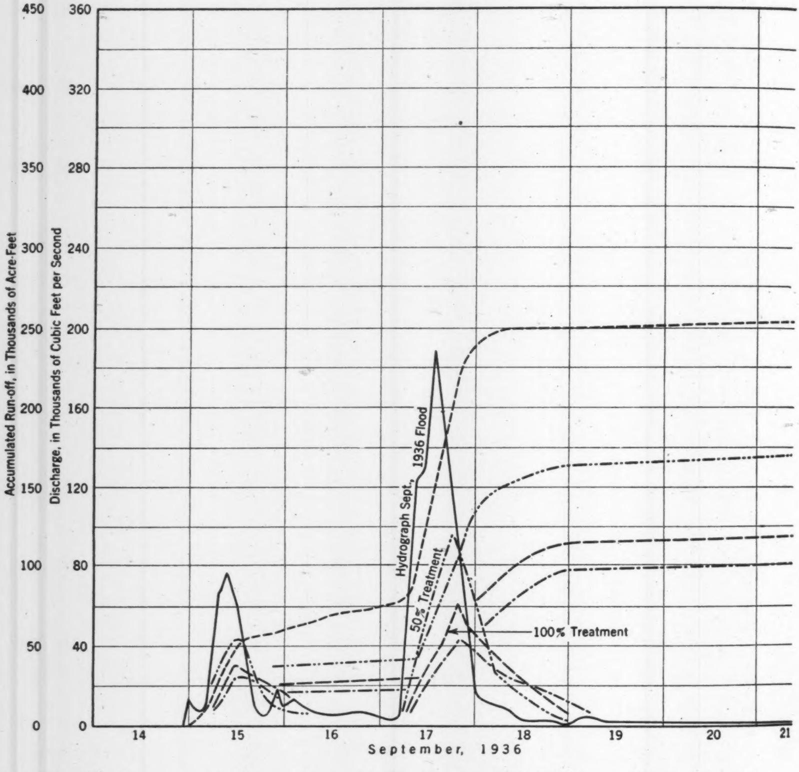
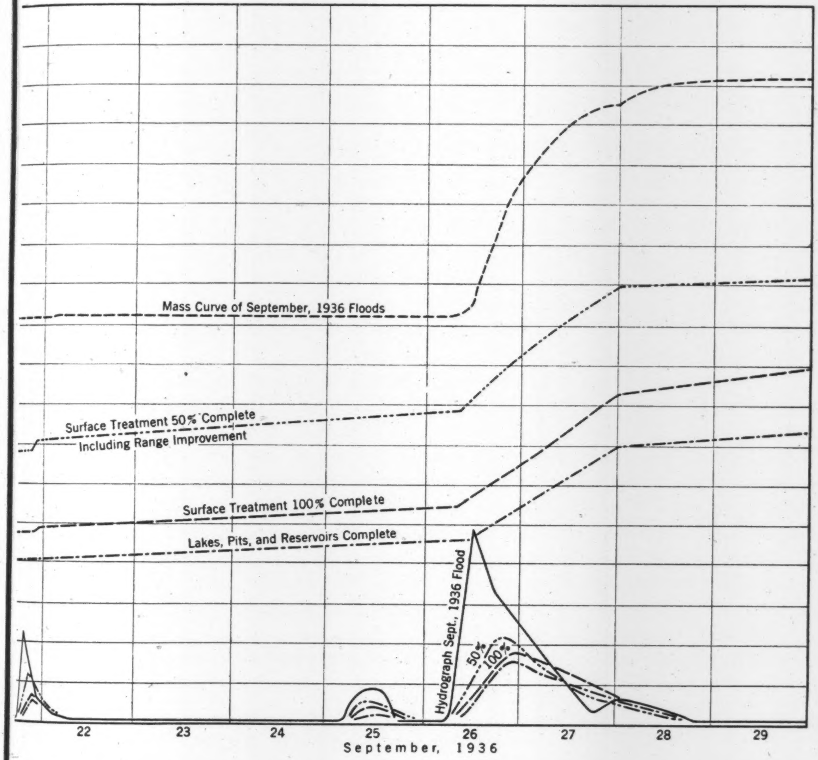


FIG. 13.—HYDROGRAPH AND MASS CURVE OF SEPTEMBER 1936 FLOOD ON NORTH CONCHO
SIVE EFFECTS OF CONSERVATION MEA-

Due to the reduction in velocities of the component parts of the gathering flood, not only must the occurrence of the flood peak be delayed, but the base of the flood hydrograph must be lengthened. Then the peak discharge (shown graphically in Fig. 13 as the altitude of the triangular form of flood-rise) must necessarily be reduced in measurable amounts both by reason of such lengthening of the base and by reason of reduced flood volume, represented by area under the hydrograph. Thus, if the area of a triangle is reduced by half and the base remains unchanged, only one-half of the original altitude remains; but if the base is lengthened, the apex is lowered still farther.

Although some percentage of the retained rainfall may appear in the stream channel at some later time, to increase the low-water discharge, the contributions will probably be in the nature of springs or seeps, and generally will be of value in a water-utilization program.



River at San Angelo, Tex.; also, Revised Curves to Illustrate Estimated Progressures Applied to the Drainage Basin

It happens frequently that the apparent lack of hydrologic data may be mended in part by recourse to such procedures and methods of approach as are illustrated in the foregoing example. Briefly, the justification for subtracting a treated area from the rôle of storm run-off contributor depended on the following provision: That the sum of infiltration and available surface-storage depths should equal or exceed the depth of rainfall on that area for any given period. In this case, soil infiltration records covered a period of several years, and actual performance of level terraces, contour ridges and furrows, diversion and spreading works, and other aids to increased infiltration, under the record-breaking storm conditions of September, 1936, proved their effectiveness in regulating and reducing flood run-off.

HOWARD M. TURNER,[27] M. AM. SOC. C. E. (by letter).—A "basic hydrograph" is developed, in this interesting paper, which the author defines as the hydrograph resulting from a uniform rain with a duration equal to the time of concentration. A hydrograph for a storm of this length should be a characteristic one for each point. The use of a time unit of subdivisions of the time of concentration instead of days or hours is an interesting suggestion, and the hydrograph itself should prove a useful tool in hydrograph analysis. The author develops this hydrograph as consisting of only two parts, its rising limb (the concentration curve) and its falling limb (the storage curve). He states that there is a sharp break at the top which he has actually found on continuous gage records. Groups of hydrographs of the Pemigewasset River at Plymouth, N. H., reduced to comparative scale, are assembled in Fig. 1. It would be interesting to have the complete information regarding this particular hydrograph and the rainfall producing it.

The writer does not believe that the basic hydrograph given is one which will be produced on a river by a uniform rain storm just equaling the time of concentration, particularly in respect to the sharp break at the top where, in the author's description, the concentration curve changes immediately to the storage curve. This assumes that the stopping of the rainfall all over the water-shed is felt to its full extent immediately at the point of measurement. The writer cannot conceive that the shutting off of the water supply from the rainfall at distant points can possibly be felt so abruptly at the point of measurement, due to the effect of time lag and channel pondage in the entire stream system. The latter is such a powerful factor that it modifies changes in river flow, definitely, through lengths of ordinary channel not involving large storage. The writer has in mind several cases in which continuous stream-flow gagings are made at points a few miles below water-power plants. The sharp, vertical changes in flow at the plant due to closing the station become modified into curved hydrographs with rounded corners and sloping sides a short distance down stream, due to effect of channel pondage.

That this sharp break cannot occur on the "basic hydrograph," except perhaps on very small drainage areas or in very special cases, can be shown simply by dividing any area into its tributary parts. The writer has assumed for this purpose a drainage area as shown in Fig. 14. Any division of the drainage area into tributary areas would serve as well, the selection being purely arbitrary. These tributaries, being smaller than the main river at the point of measurement, must have shorter periods of concentration so that a storm lasting for the period of concentration for the entire area must extend beyond the periods of concentration of the smaller component areas. A storm beyond the concentration period will consist of a rising concentration curve up to the time of concentration. If the rain continues uniformly beyond this point, the hydrograph will have a horizontal top equal to the maximum reached at the time of concentration, and if the author's assumption is correct, when the rain stops the storage curve will begin with the sharp break he has described. In other words, for a storm longer than the concentration period, the hydrograph, according to the author's concept, must be the same as his basic hydrograph except that it will have a flat place at the top equal in length to the time that the rainfall exceeds the time of concentration.

---

[27] Cons. Engr., Boston, Mass.

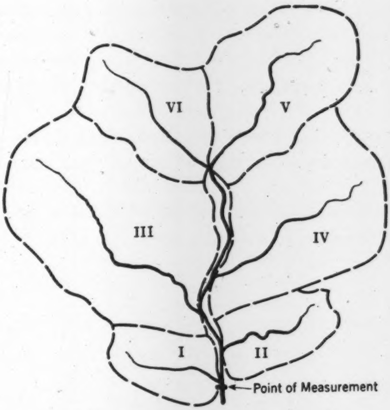


FIG. 14.—ASSUMED DRAINAGE AREA

On this basis the writer gives in Fig. 15 such flood hydrographs adapted from the author's basic hydrograph for the various subdivisions of the drainage area considered. These hydrographs of the tributaries are all drawn with similar "concentration curves" and corresponding "storage curves." They need not be uniform, of course, and in an actual case would not be so; but, on the author's assumption, they would present the general form given with the sharp break at the end of the rain.

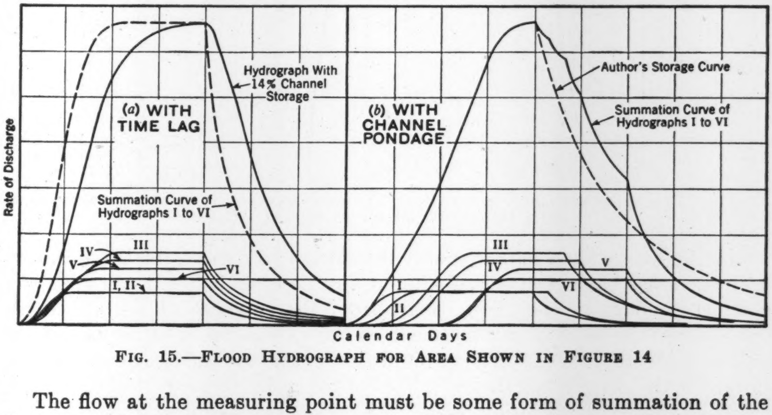


FIG. 15.—FLOOD HYDROGRAPH FOR AREA SHOWN IN FIGURE 14

The flow at the measuring point must be some form of summation of the flows from these separate drainage areas. The writer has combined them in two ways—one as shown in Fig. 15(*a*), allowing time lags for the water from each tributary to reach the point of measurement, and Fig. 15(*b*), allowing no time lag but assuming the main river, from the point of entrance of the uppermost tributaries to the point of measurement, is a pondage basin. In the latter case, water entering at any point of the basin would be felt immediately at the point of measurement—that is, the outlet of the basin. What actually happens is supposedly a combination of the two, with the storage-basin effect giving a decreasing time of transit along the main river as the river becomes more full. The two assumptions cover the extremes.

Considering Fig. 15(*a*), the descending limb of the curve is of a very different shape from the curve presented by the author. Because of the time lag required for the discharge from each tributary to reach the point of measurement, the storage curve does not begin sharply at the end of the storm. If the sharp curve does not exist at the point of measurement, neither does it exist on any of the tributaries, each of which may be similarly subdivided. If the tributary hydrographs do not have these sharp corners, the cusps in the combined hydrograph will disappear, and the curve will present a smooth descending limb, but with no sharp break.

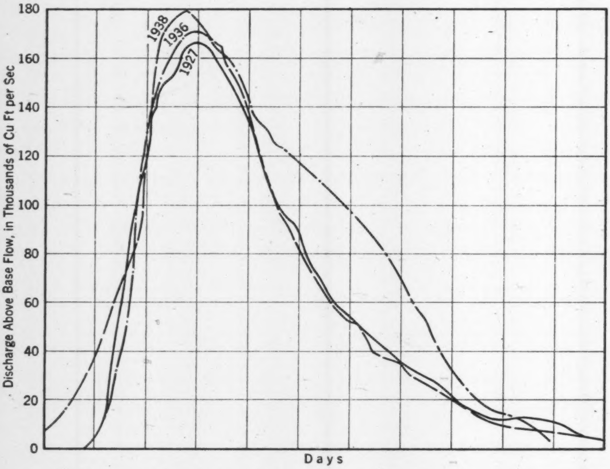


FIG. 16.—FLOOD HYDROGRAPHS ABOVE BASE FLOW—CONNECTICUT RIVER AT SUNDERLAND, 1927, AND MONTAGUE CITY, 1936 AND 1938

In the case shown in Fig. 15(*b*), the same hydrographs of the tributary drainage areas have been added without any time lag but with allowance for channel pondage in the main stream from the entrance of the uppermost tributary to the point of measurement, in this case assumed at the peak to be 14% of the total run-off. The effect of this storage is to give a convex curve, instead of the sharp break shown by the author, at the upper part of the hydrograph after the concentration period when the rain is assumed to stop. As in the case in the hydrograph shown in Fig. 15(*a*), if the tributary hydrographs do not have the sharp breaks shown by the author (which, by the same reasoning, they do not have) the descending limb in this case will depart still further from the author's.

It is not maintained that either the results shown in Fig. 15(*a*) or Fig. 15(*b*) represent the actual hydrograph for the assumed case, but neither of these hydrographs of the limiting cases shows the sharp break.

In the writer's opinion the flood hydrograph of a river is much less sensitive than many of the theories of hydrograph synthesis assume, due mostly to the effect of pondage in the entire system. As an example, Fig. 16 shows the flood hydrographs of the three large floods on the Connecticut River at Sunderland, Mass., in 1927 (8 000 sq miles of drainage area) and Montague City, Mass., in 1936 and 1938 (7 890 sq miles). The base flow has been deducted from these in each case by a straight line connecting the flow at the beginning and end of the flood period; but no other corrections have been made. The flood of 1927 was caused by a heavy two-day rainfall concentrated in Vermont, with the center of the storm 60 miles up stream from the point of measurement. The 1936 flood flow was due to a storm lasting four days, quite generally spread over the entire area, combined with melting snow. (The bulge on the recession side was due to additional rainfall occurring after the flood peak.) The 1938 flood was caused by five days of rainfall, of which the last two were extremely heavy in the lower part of the drainage area. The center was in Massachusetts immediately down stream from the measuring point. It may be that it is a matter of chance that the distribution and times of these rainfalls were such as to produce these similar hydrographs. However, the writer is inclined to the theory that the storage on a basin integrates storms of different durations and concentrations and tends to produce quite similar hydrographs for similar flow volumes.

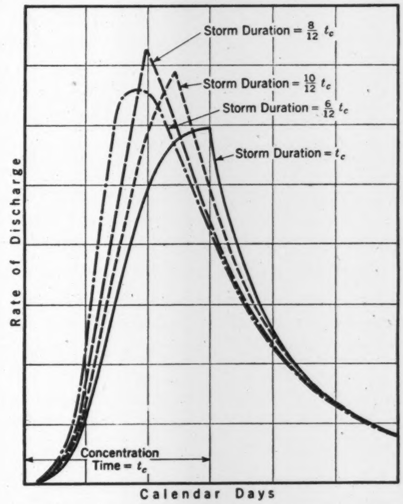


FIG. 17.—"BASIC HYDROGRAPH" AND CORRESPONDING HYDROGRAPHS FOR STORMS OF SHORTER DURATION BUT WITH SAME TOTAL RAINFALL

This is quite at variance with the author's basic hydrograph theory for a similar run-off taking place in 6, 8, 10, or 12 twelfths of the concentration time. Fig. 17 shows the author's hydrographs of floods of varying times taken from his Fig. 2 and multiplied by the proper factor so that each will represent the hydrograph of a flood from storms of the same magnitude but occurring at different times. On this basis the flood produced by a storm of the same size, but in 83% of the time of concentration, will produce a very different shape of hydrograph, with a peak 15% greater. The same storm occurring in 67% of the time will produce a still different hydrograph with a peak 20% higher. This appears to the writer to be a much finer distinction than is found in actual flood hydrographs. He believes that the storage for the entire length of the main river and of its tributaries through the entire system, and the time of travel of the flood into, through, and out from such storage, tend to cause hydrographs of similar shape for flood volumes of the same size, although there may be quite large differences in the length and distribution of the storm rainfall on the drainage area.

The author is in error regarding the history of modern flood hydrograph analysis. It was the report of the Committee on Floods of the Boston Society of Civil Engineers[28] at first revealed the fact that hydrographs from isolated storms had practically the same length of base. It contains the following statement:[28] "the Committee shows that a flood hydrograph once determined for a given river, even for an ordinary flood, will serve as a basis for the estimation of greater flood run-off, due to the fact that the base of the flood hydrograph (or time of flood period) appears to be approximately constant for different floods." On the next page exceptions are pointed out, among them, "the effect of storms which are of longer duration than the concentration period."

If individual recognition is to be given for this it should go mostly to Mr. S. S. Kent, Assistant Engineer of the Proprietors of the Locks and Canals, Lowell, Mass., who was Chairman of the Sub-Committee on "Flood Formulæ for New England" of the Committee on Floods of the Boston Society.

The author further states that the formulas of the Boston Society of Civil Engineers[28] "are based on the assumption that the drainage area involved is rectangular and of uniform slope, like a shingle on a roof, and without any well-defined channels" (see text following Equation (1)). Although it is true that such rectangular drainage areas were used in investigating the effects of different shapes of areas on the hydrograph, the final formula was a general one. The flood coefficients, C_f, were determined and flood hydrographs on a unit basis called "characteristic flood curves" were given for a number of New England rivers. These are quite similar to the distribution graphs of the "unit graph" system developed more than a year later.

OTTO H. MEYER,[29] ASSOC. M. AM. SOC. C. E. (by letter).—The discussions have indicated a gratifying interest in this subject, and have directed attention to the need for further investigation, particularly on infiltration and seepage.

Mr. Cochrane attempts to derive the shape of the falling limb of the hydrograph by an inversion of the rising limb. The writer believes that the discharge, at any time after the time of concentration and after the cessation of the rain, must be determined by a function of outflow from storage. This is sufficiently well established by numerous studies of depletion curves[7, 8], and by the study referred to by Mr. Barnes. The sharp break at the top of the basic hydrograph, questioned by Mr. Cochrane, and also by Messrs. Zoch and Turner, is definitely verified by the hydrograph of Fig. 10 presented by Mr. Sherman. The writer has no criticism to offer on formulas for maximum flood discharge; however, these are outside the scope of this paper.

---

[28] *Journal*, Boston Soc. of Civ. Engrs., September, 1930, p. 297.

[29] Asst. Engr., U. S. Engr. Dept., U. S. Engr. Office, Sacramento, Calif.

Mr. Barnes makes a timely comment as to the unreliability of the per cent run-off, or run-off factor. This factor is used where nothing better is available. As mentioned by Messrs. Sherman and Zoch, better results will follow from the deduction of infiltration from rainfall, and the consequent use of "net rainfall." However, infiltration and seepage are a separate subject, and consideration of them would complicate the subject unduly without affecting the characteristics being analyzed. In any case, a distribution of precipitation intensity giving a constant "net rainfall" is not impossible and would fit the assumptions on which the analysis was based.

The "subsurface storm flow" or "secondary base flow" mentioned by Mr. Barnes is an item of run-off of sufficient importance to merit considerable study. There can be no doubt that this type of run-off occurs and is often of considerable magnitude. The writer visualizes the source of this run-off as a sheet of water separated from the true "ground-water," possibly by a cushion of air filling the voids of the soil. Presumably, the same methods of analysis used for true surface run-off may be applied to a study of subsurface storm flow; however, it must be treated separately from surface run-off.

Mr. Sherman has apparently misinterpreted the hydrograph of Fig. 3, as no termination is indicated; the tail is intended to be of infinite length, only a portion being shown. As Mr. Sherman states, the use of unit hydrographs in combination, as shown by his example in Table 7 and Fig. 10, is a practical working tool, and the writer also uses this method, where applicable, on account of its simplicity. However, if the unit hydrograph used is that for a storm duration close to, or equal to, the time of concentration, inconsistencies may be expected to develop. For example, with a unit hydrograph for a one-day storm, if the tail is several days long, and the time of concentration is one day, a one-day rain produces a peak equal to the rainfall rate, but a two-day rain must necessarily cause the computed peak to be greater by the amount of the rate of discharge of the tail of the unit hydrograph one day after the peak of the unit hydrograph. This cannot occur in nature. The observed hydrograph of Fig. 10 obviously has the characteristics described in the writer's analysis, the time of concentration being about 25 to 30 min.

Mr. Sherman states that the time of concentration and the distribution for a three-day storm, as given in Appendix III, should be derived from the distribution for a one-day storm. This could be done, or, as in the example, the distribution for a one-day storm could be derived from that of the three-day storm, which in this case is the basic hydrograph, the time of concentration being three days. The process is reversible.

Mr. Zoch's distinction between run-off and rate of run-off, and between run-off and discharge, is commended. The writer does not agree with Mr. Zoch's statements on infiltration and internal drainage; however, that is considered outside the scope of this paper.

The writer cannot agree with Mr. Zoch as to the beginning of the depletion curve. It is true that with storms of short duration the part of the falling limb prior to the time of concentration contains both items of accretion and items of depletion; however, the succeeding portion must be pure depletion.

This has been established empirically, and may be reasoned logically. The contribution to flow from every part of the basin has reached its maximum when the time of concentration is reached, and thus can only diminish. For the same reason, the maximum discharge can never be later than the time of concentration. The transition curve mentioned by Mr. Zoch must lie entirely between the end of the rain and the time of concentration. The "saturation curve" is not a part of the basic hydrograph, as it occurs only if the duration of the rain exceeds the time of concentration. The writer, therefore, adheres to the statement that the basic hydrograph consists of the two parts: (*a*) The concentration curve and (*b*) the storage, or depletion, curve.

Mr. Zoch states that the writer has not mentioned the determination of the time of maximum discharge. This time is indicated by the peaks in Fig. 2 and adequately described in the accompanying text.

Mr. Bernard calls attention to the lack of sufficient and suitable basic data for hydrologic studies. It is this very lack which forces hydrologists to use unsatisfactory expedients such as the run-off factor.

The writer would like to call attention to the fact that the definition of time of concentration appears in the second paragraph of the section on "Analysis of the Hydrograph." The mention of time of concentration in the "Synopsis" is not intended for a definition.

In describing, graphically, the sequence of occurrences on a water-shed during a storm, Mr. Bernard makes use of a method which could well be adopted generally in investigations of physical phenomena. Too often there are only the "rationalists," who go into transports of pure mathematics, and the "empiricists," who generalize from meager data. It is only by visualizing actual physical occurrences that engineers can be sure that their derivations are "on the right track."

Mr. Snyder's conclusion that the time of concentration is not the duration of a unit hydrograph is correct. The synthetic unit graph of Fig. 11 includes subsurface storm flow (secondary base flow), and is not all true surface run-off; the basic hydrograph of true surface run-off would reach a maximum of 6 245 cu ft per sec. A separate hydrograph of subsurface storm flow must be constructed if the writer's methods are used. The relative time to peak of the various hydrographs of Fig. 2, commented upon by Mr. Snyder, are subject to errors in drafting. The 1-hr rain would certainly cause a slightly earlier peak than a 7-hr rain.

The writer concurs thoroughly with Mr. Hoyt in his remarks on the necessity for research on the behavior of melting snow.

Mr. Clark ties flood routing and hydrographic analysis together in an admirable fashion. A complete hydrologic study of a large basin must always involve the construction of hydrographs of head-waters and tributaries and routing the combination. An integration of methods used in these two steps is of real value.

The basic hydrograph used to construct one hydrograph in Fig. 7 was synthesized from the unit hydrograph used for the other; the differences may best be understood from a careful study of the method of construction.

Mr. Clark's question on the apparent lack of effect of storage on the rising limb is best answered thus: The steepening of the slope of the water surface during rising stages opposes and nullifies the storage effect, whereas the flattening of the slope during falling stages increases the storage effect and thus makes storage the dominant factor.

In cautioning against indiscriminate use of the expression for time of travel, Mr. Clark has emphasized an important point. The variation of velocity along a stream has not been given the attention it deserves. No assumptions on this subject should be accepted without a most critical analysis.

The effect of land treatment as described by Mr. Jarvis must be considered in any hydrologic study. The writer has encountered this problem in attempting to estimate run-off from irrigated lands, where the irrigation works cause almost complete retention.

Mr. Turner calls attention to the difference in shape of the writer's hydrographs for storms of the same volume but different intensity, and states that actual hydrographs are more similar. He shows, in Fig. 17, hydrographs resulting from storms of $\frac{6}{12}$, $\frac{8}{12}$, $\frac{10}{12}$, and the entire time of concentration. Actual storms lasting through the time of concentration are rare on large streams, and hydrographs resulting from storms of shorter duration than one-half the time of concentration are much more similar than those shown. The writer is of the opinion that the time of concentration of the Connecticut River at Sunderland, Mass., is about 5 to 6 days. Thus the hydrographs of Fig. 16 are not more similar than might be expected, especially as the shapes were probably determined largely by the precipitation occurring in the one or two maximum days of the storms.

No adverse criticism of the Committee on Floods of the Boston Society of Civil Engineers was intended, as the writer recognizes well the value of its work; it was to indicate the necessity for refinement that the shortcomings of the original formulas were mentioned.

In conclusion, the writer wishes to emphasize that it is the physical significance of the shape of the hydrograph, and not the empirical mathematical equations describing it, that is important.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2058

SPECIFICATION AND DESIGN OF STEEL GUSSET-PLATES

BY T. H. RUST,[1] ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. R. H. SHERLOCK, L. E. GRINTER, RUSSELL C. BRINKER, C. W. WIXOM, AND T. H. RUST.

SYNOPSIS

An attempt is made in this paper to supply information about the structural steel plates designed to transfer force from one member to another in a structural steel frame or bridge. The forces enter and emerge from these gusset-plates, often by means of rivets, but sometimes through welds. Since a gusset-plate is a point of discontinuity of stress transfer in a structural steel frame, it is here that failures most often occur.

As early as 1867, J. W. Schwedler noted[2] the unequal distribution of stress among the rivets in a riveted connection. He found that the end rivets took most of the stress and the interior ones least. Likewise, the ends of a welded connection are more highly stressed than the interior.

This inequality of stress transfer into a gusset-plate can be shown by stressing models of the type shown in Fig. 1, which are carved from single pieces of bakelite to simulate gusset-plates. When forces are applied through the legs to the plate, fringed photographs can be made by the aid of photo-elasticity. The photo-elastic fringes are the means of establishing the stress value $(p - q)$, at a particular point, p being one principal stress and q the other principal stress at right angles to it. Although a $(p - q)$-determination is a good indication of the condition of stress in a plate, a $(p + q)$-determination was made for some of the models. The lateral expansion or contraction of the stressed plate at any point is proportional to the $(p + q)$-stress at that point. This change in thickness was measured by a light interferometer. From the $(p - q)$ and the $(p + q)$-stress determinations, the value of the principal stresses, p and q, can be found.

Studies of several different shapes are reported herein and the stresses encountered are discussed. The models were designed with special reference to

---

NOTE.—Published in November, 1938, *Proceedings*.

[1] Structural Engr., Chicago, Ill.

[2] "Ueber Vernietungen," von J. W. Schwedler, *Deutsche Bauzeitung*, Architekten-Vereins zu Berlin I, November 29, 1867, No. 48, S 463–465.

gusset-plates. A part of the paper is devoted to recommendations for the design and specification of steel gusset-plates.

TESTS

Tensile forces were applied to the model shown in Fig. 1 by means of steel pins through the bushings. A force enters through the top pin, passes down the upper legs into the plate and is resisted by a reaction passing through the lower legs to the other pin, which is held in a stationary position. As shown in the side view of Fig. 1, the bushings are beveled to keep the force, *P*, on the center line of the model. At all junctions of the legs and the plate small fillets were provided to minimize local stress concentration. The plates were each about 0.2 in. thick, and the legs were attached to the plate for a distance of 0.75 in.

To avoid residual stresses in the edges of the bakelite plate, the models were annealed and polished as rapidly as possible. They were then subjected to forces, *P*, in the path of polarized light (see Fig. 2). A theory is that polarized light has its vibrations restricted to one plane, in contrast with ordinary light, in which light vibrations are said to be in many planes. When a beam of ordinary light is passed through a Nicol prism the light vibrations are held to a single plane, and it is then known as polarized light. If this light beam is passed through a second Nicol prism with its axis at right angles to the first prism, the beam disappears, and the field becomes dark. Since there are no vibrations perpendicular to one plane in a beam of interrupted polarized light, the second prism (termed the "analyzer") cannot transmit any light because its axis is at right angles to the axis of the first Nicol prism or polarizer.

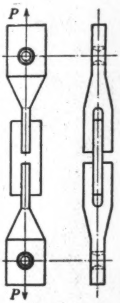


FIG. 1

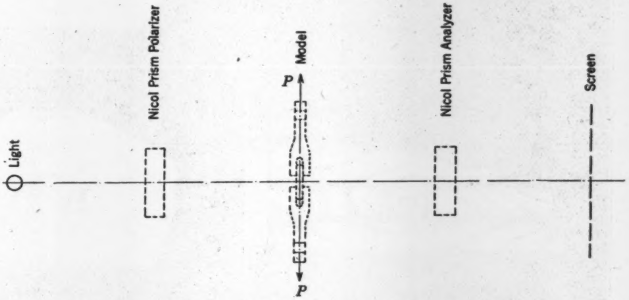


FIG. 2

If a polished bakelite model under stress is placed between the two prisms in the path of the polarized light, the second Nicol prism, or analyzer, transmits light between dark fringes. All the points on a particular dark fringe are of equal $(p - q)$-stress in the plate of a model. Monochromatic mercury light was used.

The change in $(p - q)$-stress from one fringe to another is constant for a given thickness of plate. This stress change between fringes is determined by subjecting a pilot specimen of the bakelite to known loads.

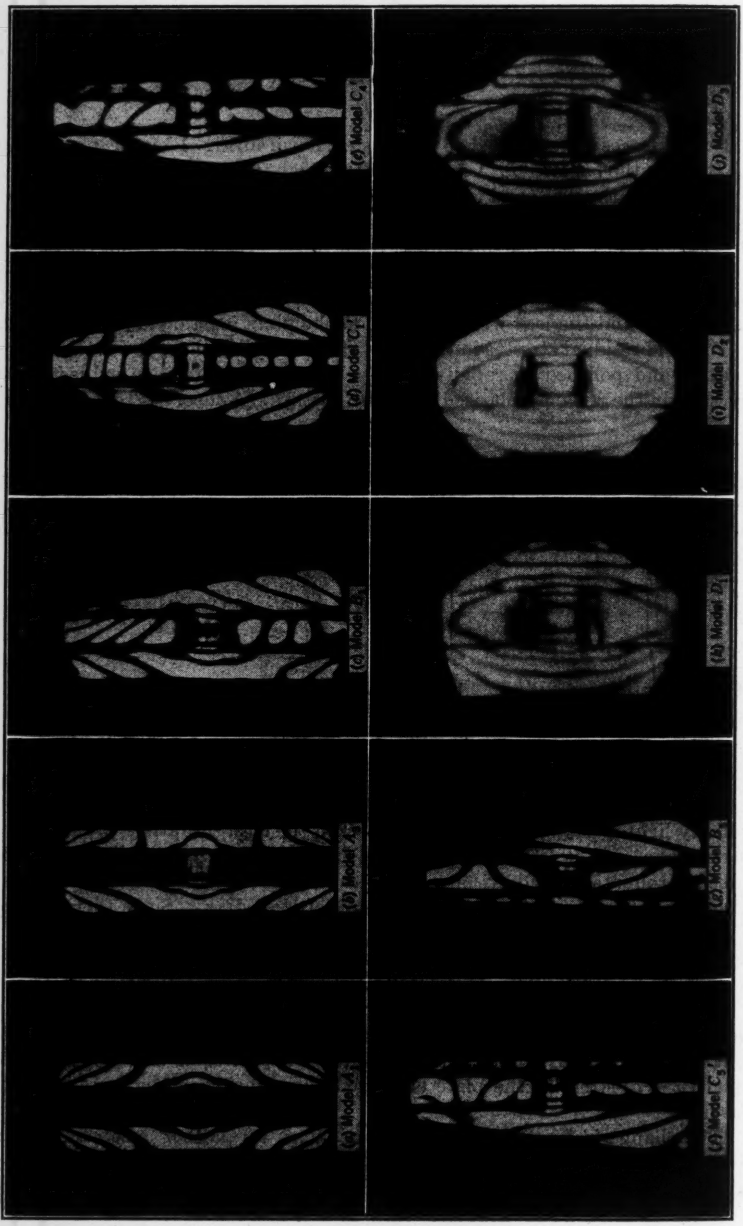

(a) Model A_1
(b) Model A_2
(c) Model B_1
(d) Model C_1'
(e) Model C_4'
(f) Model C_5'
(g) Model B_2
(h) Model D_1
(i) Model D_2
(j) Model D_3

FIG. 3.—FRINGED PHOTOGRAPHS OF PLATES OF DIFFERENT SHAPES

Fig. 3 shows fringed photographs for plates of different shapes. The plates of Models A_1, B_1, and C_1' were $\frac{5}{8}$ in. wide at the mid-length; for Models D_1, D_2, and D_3, the plates were 1.25 in. wide at the mid-length. For Models A_1, B_1, C_1', D_1, D_2, and D_3, the area of the plate at mid-length was nearly equal to the area of the two legs. The closeness of the fringes near the ends of the plates, at the legs, shows the rapid transfer of stress into the plates at those points. The unit stress in the legs is large at this point, whereas in the ends of the plate at the legs the unit stress is small. The difference in the unit stress of the legs and the plate causes rapid stress transfer. At the mid-length of the legs, the bakelite in the legs is stressed to about the same value as the plate. The comparatively great distance between the fringes in the plate at the mid-length of the legs, shows the less rapid transfer of stress at this point, Near the ends of the legs the unit stress is small in the legs; whereas the unit stress and unit deformation in the plates are large. This change in deformation accounts again for a rapid transfer of stress into the plate. The shear stress between the plate and the legs can be represented by a curve shown in Fig. 4. A smooth curve as indicated by the full line would be shown by a welded connection or a continuous connection such as exists in the models. A line of rivets would indicate a stepped curve as shown by the dotted lines in Fig. 4. A rivet or a row of rivets perpendicular to the length of the member would transfer an increment of shear into the plate equal to the area under the corresponding step.

If the cross-sectional area of the legs is equal to the area of the plate, the shear stress transfer at the end of the plate and at the ends of the legs would be of the same value. On the other hand if the area of the legs is less than the area of the plate there will be a greater transfer of stress into the plate at the end of the plate because of its greater stiffness, with correspondingly less transfer of stress at the ends of the legs, as shown in Fig. 5. If the legs were larger in area than the plate, the end ordinates of the curve would be reversed, as shown by the dotted curve.

Boundary Stresses.—At the outside boundary of a plate, symmetrical about the legs (as shown by Models A_1, B_1, and C_1', Fig. 3), the change of stress is less rapid than it is next to the legs. If this were not so, the $(p - q)$-fringes would extend from the sides of the legs, straight across the plate, to the boundary. The extension of the fringes, forward toward the mid-length of the plate, shows how stress lags at the boundaries of the plate. Corners of the plate are points of least accessibility of stress and the positions of zero stress. These corners, therefore, are good places to begin counting the fringes; each succeeding fringe represents a constant increase in $(p - q)$-stress over the preceding fringe.

The boundaries at the mid-length of the plate are the most highly stressed. The stress at that point also lags considerably behind the stress in the center of the plate at the corresponding positions. At the mid-length of the legs in Model A_1, Fig. 3 (which is approximately at the quarter-point of the plate), the fringes are nearly perpendicular to the boundary. There is comparatively little transfer of stress into the plate from the legs at this point, with less disposition of the stress to lag at the boundary. The same boundary stress condition is shown in Model C_1', Fig. 3, at the third fringe from the top, and again less definitely at the third fringe from the bottom. The wide fringe on the wide side of Model C_4' at the position of the upper leg shows that the increase in area of the plate practically compensates for the transfer of stress from the legs, causing little change in $(p - q)$-stress in the plate at this point on the boundary. The third fringe from the top on the narrow side of the plate turns back. This is caused by the bending moment due to the eccentricity of the plate. The bending stress more than compensates for the tendency of the stress to lag at the boundary. The fourth fringe of Model C_4' turns forward again because the more rapid transfer of stress from the legs near the ends causes the boundary stress to lag again. The same phenomena are shown in Models C_5', A_3, and B_5.

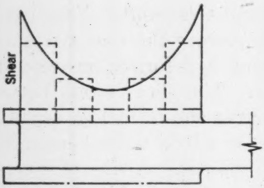


FIG. 4

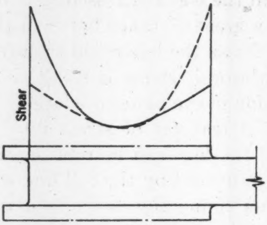


FIG. 5

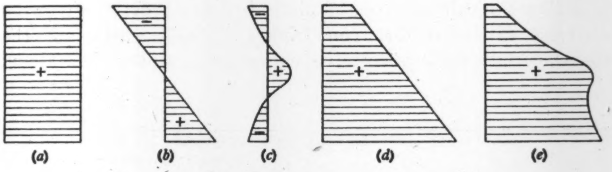
In order to show something about the plate corner effects, the corners on the left side of Model D_1, Fig. 3, were filed off for one-fourth the distance of the leg for Model D_2. There is a more rapid change of boundary $(p - q)$-stress near the left side of the leg in the plate of Model D_1 than in Model D_2. Due to greater resistance to stress transfer in Model D_1 than in Model D_2, the stress builds up more rapidly near the leg in Model D_1 than in Model D_2. The upper left-hand corner of Model D_1 has a larger area free of stress than Model D_2. Model D_1 has less continuity of stress transfer than Model D_2 in the upper left boundary. The $(p - q)$-stress in the right side of the gusset-plate shows almost no change from the removal of the corners on the left side. The upper outside corner of the right side of the plate is at the end of the leg, whereas the lower outside corner is only three-fourths the way along the leg toward its end. The $(p - q)$-fringes do not show any appreciable lack of symmetry due to this difference. The second fringe from the right on the right side of the plate bends out sharply near the boundary. This is true of the second fringe of all the D-models; for Model D_3, Fig. 3, the third fringe also bends out sharply at the boundary. The rapidly increasing area of the plate is more than sufficient to counteract the addition of stress from the legs at these points. It should be noted that the stress transfer from the legs is a minimum near the mid-length of the legs.

Model D_3, Fig. 3, was made by filing concave cuts in Model D_2. In these models there is less unit shear on the right side of the legs near the end of the plate than on the left side of the legs, because there is less material there and, therefore, less resistance. The smaller unit shear causes less shear stress in the end rivets or ends of welds, as well as less bearing stress on the plate from the end rivets. Since the rivets at the end of a gusset-plate as well as their bearing stresses on the gusset-plate are critical, it is well to make the plate comparatively elastic by not having its surface area much larger than the end rivets require.

The vertical boundaries of the plate of the *D*-models are almost free of stress. These boundaries are so far from the direct path of stress transfer that they are not highly stressed, in accordance with the theory of least energy.

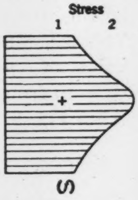
Interior Plate Stresses.—The greatest stresses in the interior of the plate are always between the ends of the legs of the models. All the stress has been transferred into the plate in this region, and it tends to concentrate in the most direct path of transfer. Some of the fringes curve out, which shows the tendency of the $(p - q)$-stress to spread over a larger part of the plate.

The stress condition in a plate may be shown graphically by the diagrams in Fig. 6, that is, Fig. 6(*a*) represents the direct stress on a transverse section assuming uniform distribution; Fig. 6(*b*) represents the bending stress; and, Fig. 6(*c*) represents the stress that minimizes the energy in the plate. The theory of least energy is not satisfied by the assumption of uniform and direct bending



(*a*) (*b*) (*c*) (*d*) (*e*)

FIG. 6



(*f*)

FIG. 7

stress distribution, but requires that the assumption shall be modified by stress similar to that shown in Fig. 6(*c*). Although there is ample evidence in these tests shown graphically by Fig. 7, plotted from readings on the section at the mid-length of the plate of Model A_1 (Fig. 3), there is no generalized theory for the least energy stress condition for Fig. 6(*c*). If the outside forces are applied somewhere near the neutral axis of the plate this added least-energy stress will always have the general shape of the curve in Fig. 6(*c*), the negative area being equal to the positive area (Fig. 6(*d*) = Fig. 6(*a*) + Fig. 6(*b*); and Fig. 6(*e*) = Fig. 6(*a*) + Fig. 6(*b*) + Fig. 6(*c*)); Figs. 6(*d*) and 6(*e*) will have the area of Fig. 6(*a*), because equilibrium of the free body is always maintained between external and internal forces.

In nearly all the models, the fringes that pass near the ends of the legs change curvature. Since the largest concentration of $(p - q)$-stress occurs in the plate between the legs, this stress tends to distribute itself over a greater part of the plate with the resulting change of curvature of these fringes.

With an eccentric plate such as that shown in Model C_5' the fringes denoting the highest $(p - q)$-stress shift over toward the narrower side of the plate. The direct stress, the bending stress due to eccentric loading, and the stress due to the least energy in the plate, create a maximum stress to the right of the center line of the legs. In Model D_3, Fig. 3, there is also a slight shifting of the fringes of highest value to the right of the center line of the legs. On the right side of this model there is less plate in the gusset corners than on the left side of the legs; therefore, the maximum stress shifts again slightly to the right.

In Model C_5' the almost horizontal fringes on the narrow side of the plate show that the bending stress due to eccentric loading is approximately equal to the decrease in least energy stress there. The principal stress, q, is not large except in the mid-length of the plate; even there it is considerably less than the p stress, as will be shown under the heading, "Discussion of Curves."

DISCUSSION OF CURVES

In addition to knowing something of the $(p - q)$-stress distribution from the photo-elastic photographs of the models, it was desirable to know something also about stress distribution in gusset-plates from the plotting of stress curves.

Graphs of $(p - q)$-stress and $(p + q)$-stress along the sides of the legs of Model A_1 are shown in Fig. 8. The $(p - q)$-stress difference between fringes was found by experiment to be $\frac{84.6}{0.183} = 470$ lb per sq in.; 84.6 lb per sq in. would be the stress difference between fringes in case the bakelite were 1 in. thick. This constant divided by the thickness of the plate used in Model A_1 gives the value of $(p - q)$-stress difference between fringes. The positions of the $(p - q)$-fringes together with their successive values were plotted for Model A_1 in Fig. 8.

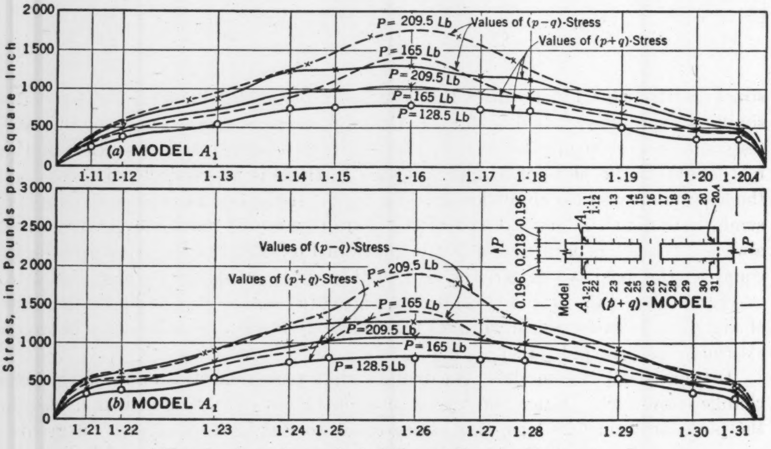


FIG. 8.—$(p - q)$ AND $(p + q)$ STRESS CURVES, MODEL A_1

The $(p + q)$-curves were plotted from values obtained by measuring the lateral contraction of the stressed plates at various points along the legs. The lateral contraction or expansion at a point of the stressed plates is proportional to the $(p + q)$-stress at that point. This lateral change in thickness of the bakelite plate was measured by a light interferometer,[3] and the resulting stresses were plotted as shown. The $(p + q)$-curves and the $(p - q)$-curves nearly agree except at the mid-length of the plate, which indicates that there is little q-stress except at the mid-length of the plate. Since there is a large concentration of stress directly between the ends of the legs, an appreciable q-stress arises in the effort to distribute the stress over a larger part of the plate. The difference in ordinates between the $(p + q)$-stress and the $(p - q)$-stress curves for a particular load, divided by 2, gives the q-stress for a particular point. The addition of the ordinates, divided by 2, gives the p-stress at a particular point.

---

[3] "An Application of the Interferometer Strain Gage to Photoelasticity," by R. W. Vose, Jun. Am. Soc. C. E., *Journal of Applied Mechanics*, September, 1935, pp. A 99–102.

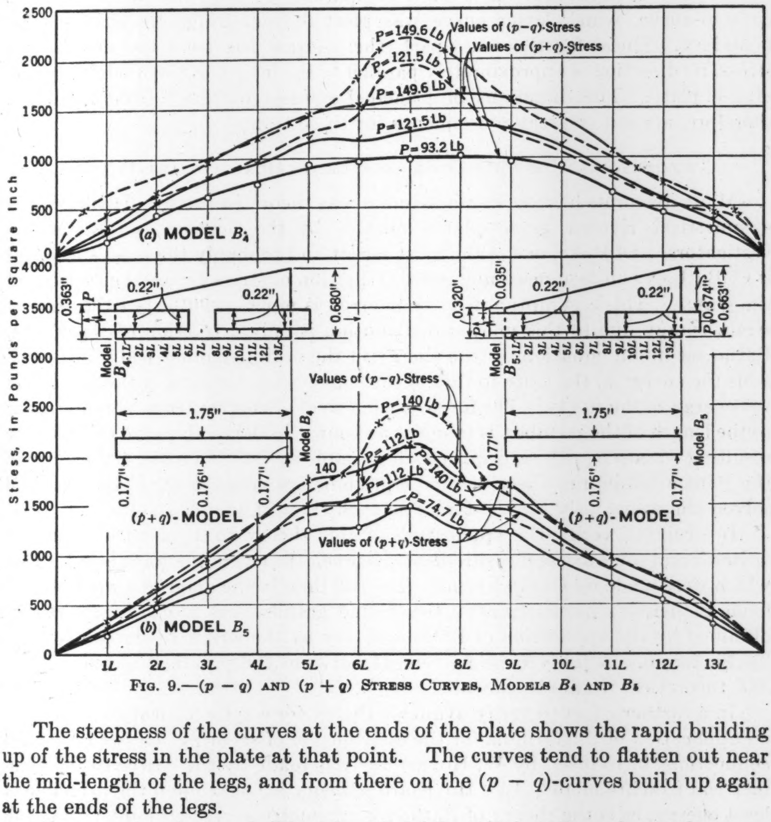


FIG. 9.—$(p - q)$ AND $(p + q)$ STRESS CURVES, MODELS B_4 AND B_5

The steepness of the curves at the ends of the plate shows the rapid building up of the stress in the plate at that point. The curves tend to flatten out near the mid-length of the legs, and from there on the $(p - q)$-curves build up again at the ends of the legs.

The secondary humps in Fig. 9 for Model B_5 at Point $9L$ are due to the stress (particularly the q-stress) tending to distribute itself over a greater part of the plate. There is considerable q principal stress between the ends of the legs in the plate; its tendency to spread over a greater part of the plate depresses the $(p + q)$-curves. No increment of $(p + q)$-stress is added in proceeding from Point $9L$ to Point $8L$ in Model B_5. The same phenomenon of $(p + q)$-stress distribution is shown also between Points $5L$ and $6L$ in the tendency of the $(p + q)$-curves to flatten out.

Sets of curves for $(p - q)$-stress and $(p + q)$-stress were plotted for the outer boundaries, with practical agreement. Since there is only one principal stress along these boundaries—the p-stress—the two corresponding curves for a particular model should agree. It was impossible to place the points of contact of the light interferometer exactly on the boundary. The contact points had to be moved in $\frac{1}{16}$ in. to prevent them from slipping off the model. This accounted for a slight difference in a comparison of a $(p + q)$-curve with a $(p - q)$-curve, which latter curve was plotted from fringe readings on the boundary. Since, for the most part, the p-stress has been the dominating stress, its direction is approximately parallel to the length of the model for most of the plate. The directions of principal stresses obtained from isoclinics, therefore, are not of great consequence for these tests.

THEORETICAL CONSIDERATION OF SYMMETRICAL GUSSET-PLATES

Many attempts have been made to find the theoretical stress distribution in symmetrical, riveted, gusset-plates, notably by the German and British investigators. Of these, one of the most recent and probably the most successful is C. Batho,[4] who assumes a uniform distribution of stress across the sections of the plates. This assumption may be in error as much as 20%, but the error of strain energy due to this uneven distribution is only a fraction of 1 per cent. Batho set up an equation for the energy in the entire symmetrical joint which adds the energy in the plate to that of the connecting members at the plate, and the energy in the rivets. The force on each rivet or row of rivets, perpendicular to the length of the member, is taken as unknown. Derivatives are taken of the resulting equation with respect to each variable force. To make the energy in the joint a minimum, these derived equations are equated equal to zero, and solved simultaneously for the forces at each rivet or row of rivets.

In order to verify the application of this theory to a practical case, the writer found some stress measurements between the rivets of a joint (see Fig. 10) which were made by C. Findeisen.[5] The full lines in the diagrams were plotted from Findeisen's measurements; the dotted graphs were plotted from results obtained by the application of Batho's theory by the writer to the joint tested by Findeisen. A fairly close agreement between the practical measurements and theoretical results is shown.

In a further effort to verify available theory for a symmetrical welded joint, an application of the formula for the distribution of shearing stresses in welded connections introduced by W. Hovgaard,[6] was made by the writer to the symmetrical bakelite model, A_1. Hovgaard's theory is based upon the principle of least energy, as is the theory of Batho for symmetrical riveted joints. Instead of deriving the work equation with respect to each force, X_n, on each row of

---

[4] "Riveted and Bolted Connections," by C. Batho, First Rept. of Steel Structures Research Committee, pp. 114–127.

[5] "Versuche über die Beanspruchung in den Laschen eines gestossenen Flacheisens bei Verwendung zylindrischen Bolzen," von C. Findeisen, Verein deutscher Ingenieure, *Forschungsarbeiten*, Heft 229, Berlin, Germany.

[6] "A New Theory of the Distribution of Shearing Stresses in Riveted and Welded Connections," by W. Hovgaard, *Transactions*, Inst. of Naval Archts., March 26, 1931, pp. 108–126.

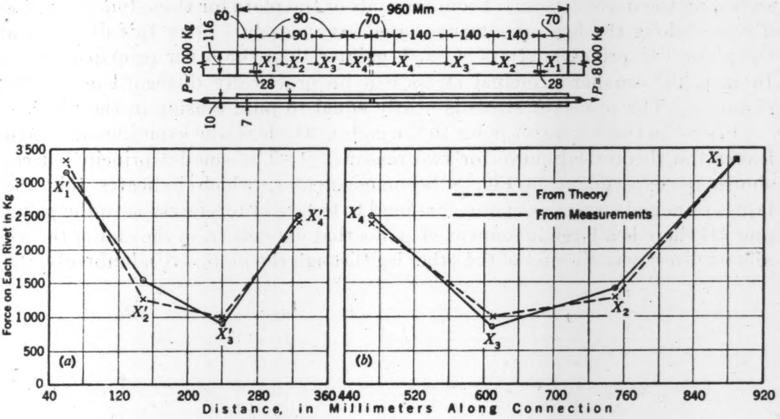


FIG. 10.—BATHO'S THEORY APPLIED TO FINDEISEN'S STRAIN MEASUREMENTS

rivets, and equating the derivatives equal to zero for a solution of least energy as Batho has done, Hovgaard writes an equation for the energy in the connection as a function of the shear stress at any point, X, of a welded connection of the member to the plate. The equation is minimized by variational calculus, and the solution of the differential equation obtained. The constants in the equation can be evaluated for a particular practical problem.

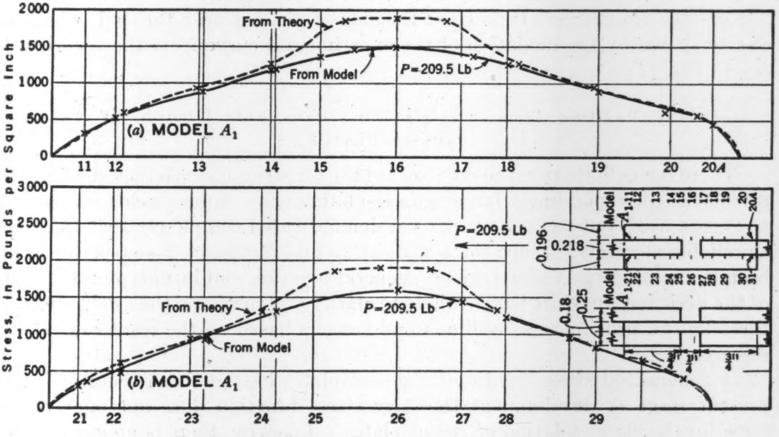


FIG. 11.—HOVGAARD'S THEORY APPLIED TO MODEL A_1

The results of Hovgaard's theory applied to Model A_1 are shown on the plotted graphs of Fig. 11. The full lines show the stresses along the legs in the plate of the model; the stresses from the equation applied to the model are shown by the dotted lines. From the ends of the plate for three-fourths of the distance along the legs, the curves agree very closely. Over this distance in the plate, one principal stress is much greater than the other principal stress. In fact, the smaller principal stress can be practically disregarded for this distance. The resultant stress is nearly equal to pure tension in the plate.

From the three-quarter point to the ends of the legs, the experimental curve leaves the theoretical curve for two reasons: (1) The smaller principal stress shown by experiment in Fig. 8 becomes greater, which indicates that the larger principal stress does not correspond to the exact tensile stress in the plate; and (2) there is a large increment of stress that extends from the end of the leg almost directly to the end of the other leg through the plate. Undoubtedly, the second is the most important reason for the deviation of the experimental curve from the theoretical. Path *b–b–b* shown in Fig. 12 is shorter and more direct than Path *a–a–a*; therefore, a definite increment of stress transfer takes place from the end of one leg directly to the end of the other leg. This increment relieves the transfer of stress from the side of the leg near the end, which is clearly shown by the deviation of the experimental curve from the theoretical one in Fig. 11.

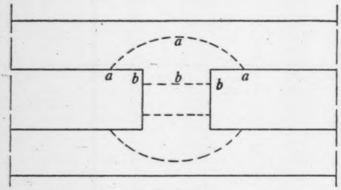


FIG. 12

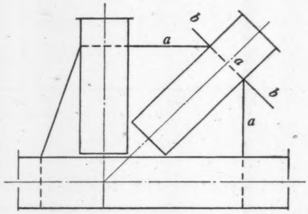


FIG. 13

RECOMMENDATIONS FOR THE SPECIFICATIONS AND DESIGN OF STEEL GUSSET-PLATES

There is a definite trend to-day toward the construction of larger steel structures with correspondingly larger gusset-plates. Sometimes, welds instead of rivets are used, but welds have not yet demonstrated completely that they are as reliable as rivets. Since the specification and design of gusset-plates have been the result largely of experience, general practise, and intuition on the part of the designer, an effort will be made to clarify and to extend the application of these known principles, as well as to add others based on the results of recent research.

A generalized stress solution for gusset-plates has not been found. In its present stage of development the Airy stress function does not offer much hope for the stress solution of gusset-plates. Likewise, there is no generalized stress solution about the phenomena of fatigue. Solutions for stress in symmetrical gusset-plates have been found both for riveted and for welded connections. The solution for symmetrical, riveted gusset-plates was found by Batho[4] whose solution is based on the principle of least energy, as is also that of Hovgaard[6] for symmetrical, riveted, and welded plates. The writer applied Batho's method to Findeisen's strain measurements on a symmetrical riveted plate connection; the writer also applied Hovgaard's method to the symmetrical bakelite model, A_1. The stress observations on bakelite models have been discussed in a previous part of this paper. The discussion will be broadened, however, to include considerations based upon general practice and present manufacturing procedure in order to give a more complete view of the art.

General Practice.—For small structures, gusset-plates can be designed and specified without great difficulty provided there is no reversal of stress. The rivet groups can be made comparatively short in the direction of the length of the member. Since there is less chance for distortion of the material, a shorter group allows a better distribution of stress between rivets. In small structures one or two gusset-plates are commonly used for each joint; but for larger ones, four or even more gusset-plates are necessary. The late J. A. L. Waddell,[7] Hon. M. Am. Soc. C. E., has recommended that there be as few cuts as possible in the plates, and that it is desirable to line up the gussets for appearance. For light gussets not subject to reversal of stress, they may be subordinated, without harm, to the appearance of the trusses and to fabrication economy; but for large and heavy structures the gusset-plates may well influence and even dominate some of the decisions as to the design and fabrication of the truss.

Long and narrow gusset-plates nearly always are computed, by elementary mechanics, more highly stressed than those with approximately equal axes. A long and narrow gusset-plate with diagonal members will have considerable eccentricity about the neutral axis of the gusset at some of its sections. Such connections are usually longer and less desirable than connections of diagonal web members with steeper slopes to the chord members. Long connections have a greater disproportion of stress among the rivets, with more low stressed rivets in the mid-length of the connections. An ideal gusset approaches a circle in shape, or a polygon of approximately equal axes, with all the forces intersecting at or near its center. A gusset-plate of such shape, due to its maximum width at the center, does not have great bending stresses. Furthermore, the longer section at the center allows the principle of least energy at the often critical center section of the gusset to function over a greater distance, with less stress concentration between the ends of the members (see Fig. 3). The shorter and wider rivet groups with respect to the length of the member tend to cover a larger rather than a smaller part of the gusset-plate, which implies that the gusset should be of no greater area than necessary. Gusset-plates with comparatively large unriveted areas increase the discontinuity which the joint represents.

The rivets should be of no greater spacing in a member connection than good design permits because the variation in stress from one rivet to the other will be less for the smaller spacing.

The cross-sectional area of the gusset where the member connects to it should tend to approach the area of the member at that point, and should not be (as is often the case) of much greater area. The plate shown in Fig. 13 should follow a line, *a–a–a*, rather than a line, *b–a–b*. The ideal of uniform stress transfer among the rivets can never be realized. The end rivets will always be the most highly stressed ones, as shown by the graphs of Fig. 10. The rapid building up of the stress in the plate, at the end of the plate, and at the ends of the legs of the models, is shown in Fig. 3. The stress difference between the end rivets and the intermediate ones can be reduced by making the rivet groups as short as practicable.

---

[7] "Bridge Engineering," by J. A. L. Waddell, 1925, pp. 518–525.

Manufacture.—Gusset-plates should be manufactured with great care. This is especially true of the larger and larger gusset-plates that are continually being demanded. They should be made as nearly plane as possible with subsequent care in shipping and handling to insure the plane condition. Plates as thick as 1 in. cannot be placed in close contact even with rivets as large as $1\frac{1}{8}$ in. in diameter. Furthermore, even 0.75-in. plates cannot be fabricated under present practical conditions to insure contact everywhere. Some of the 1-in. plates in experiments reported by J. Montgomerie[8] were subjected to special care in fabrication. Since they were pressed together with high hydraulic pressure, the value of friction which they were able to develop was raised materially. Plates fabricated in the ordinary manner had a resistance to slip of 9 tons per sq in. of rivet area for thicknesses from 0.40 in. to 0.60 in.; 6.6 tons per sq in. for plates 0.75 in. thick; and, 4.3 tons per sq in. for plates 1 in. thick. After subjecting the 1-in. plates to hydraulic pressure and re-riveting them, the resistance to slip was raised from 4.3 to 7 tons per sq in. of rivet area. These experiments show the importance of avoiding the use of plates with contact surfaces that are not planes.

Painted or oiled plates are less resistant to slip than those with no foreign substance added. Better resistance to slip is provided in these cases when the line of stress transfer is perpendicular to the direction of rolling, rather than parallel to it. A well-driven rivet in single shear should provide a resistance to slip of 35% to 40% of the value of the tension in the rivet shank. This tensile value should equal the product of the area of the rivet by the stress of the rivet material at the elastic limit. Rivets driven at the temperature of about cherry red color, with a minimum clearance at the beginning of driving, provide the best resistance to slip. Lower temperatures in driving prevent excessive heating of the plates, which causes brittleness in some carbon steels and more especially in alloy steels. Alloy steels are usually specified with higher unit stresses, and have less resistance to slip than ordinary steels. Large slip resistance takes much load from the rivet hole-wall where fracture often occurs.

Careful fabrication and handling of the parts will add definite strength to the connection. The edges and faces of the plate should be free of notches and scratches where failure can begin. Plates of ordinary thickness, or thinner than 0.75 in., can be sub-punched and reamed, while plates more than 0.75 in. thick should be drilled and reamed. The reamer[9] should be heavy enough and of sufficient velocity to hold it in its true course. All burrs and bits of steel should be removed with care, and the holes at the pre-set end of the rivet should be counter-sunk nearly $\frac{1}{16}$ in., in order to help the flow of the molten metal of the rivet around this shoulder of the hole-wall. In the fabrication of parts, improperly matched holes are not justified.

---

[8] *Transactions*, Inst. of Naval Archts., Vol. 65 (March 23, 1923), pp. 179–197, London, England.

[9] "Contribution à l'étude expérimental du Rivetage," par Paul Dugué, Bordeaux, France, 1927.

If an important gusset connection can be specified with plates of ordinary thickness and rivets of usual size, it is best to do so. The men who do the mechanical work are able to fabricate the materials of customary size with greater skill and achieve better results. Most of the tools for fabrication of the parts are designed for average steel sections and rivets. If exceptional steel sections and rivets are specified, increments of improvement from skilled craftsmanship may be more difficult if not impossible to obtain.

Stresses.—The stresses in a plate with a re-entrant corner such as Point *A*, Fig. 14, are always greater than the elastic limit even with small forces acting on the gusset.[10] Sufficient material at Point *A* changes into the plastic state to provide a means of stress transfer around the re-entrant corner. If there happens to be reversal of stress, failure occurs at relatively low forces. From the standpoint of excessive stress, gusset-plates with re-entrant cuts should never be specified. If the highly undesirable, and seldom specified, case of a gusset-plate with a re-entrant corner is encountered, a fillet in the plate at the re-entrant corner, with as large a radius as practicable, will facilitate the stress transfer. It is nearly always possible to avoid gusset-plates with re-entrant corners, but the discontinuities in stress transfer at the rivets in gusset-plates cannot be avoided.

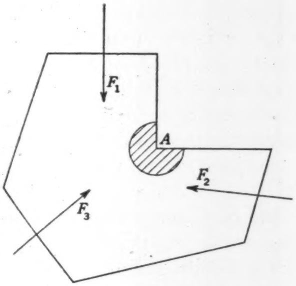


Fig. 14

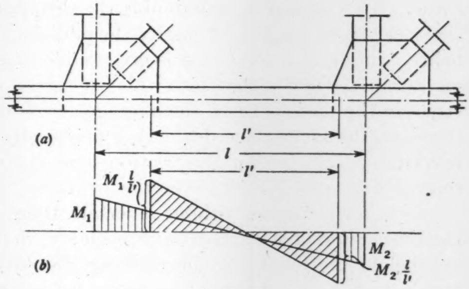


Fig. 15

Since the end rivets of a riveted connection are always the most highly stressed, it is of interest to consider their general method of stress transfer. If the section of the connecting member, carrying pure tension or compression, is equal to the section of the gusset, the percentage of total force taken by the end rivets varies in a manner similar to that shown in Fig. 10. If there are two rivets in a line the end rivets will take 50% of the forces. This percentage decreases to 36.6% for five rivets in a line.[11] From there on, for any addition to the number of rivets, the percentage of total force carried by the end rivet remains at about 36 per cent. Batho's data show that for the case in which the connecting member and the gusset-plate are equal in cross-section area, the end rivets cannot take less than 36% of the force on the member when the stress on them remains within the elastic limit. If there is no reversal, stressing the rivets beyond the elastic limit is of little consequence.

---

[10] "Stress Distribution in a Re-entrant Corner," by J. H. A. Brahtz, A. P. M. 55-6, *Transactions*, A. S. M. E., 1933, Vol. 55, pp. 31–37.

[11] "Riveted and Bolted Connections," by C. Batho, First Rept. of the Steel Structures Research Committee, pp. 100–136.

When reversal occurs, and each stress passes beyond the slip-resistance capacity of the end rivets, the ultimate strength of the connection is greatly reduced. In the design of a riveted connection for reversal of stress, one of the stresses should be kept well within the frictional capacity of the rivets with special reference to the end rivets, to prevent the backward and forward motion in the rivet holes with resultant deterioration of the connection. The cooling of the rivet and the hole-wall after driving causes a lack of tightness, which can never be avoided with heated rivets.

It may be possible to design an important connection with rivets in double shear. This procedure will double the slip capacity of the rivets as well as the shear per rivet, and might justify the additional expense in the shop and in the field. This recommendation is given added weight by the fact that at a point on the surface of the plate into which the stress is transferred in single shear, the stress in the plate is twice the average across the section of the plate, with the stress on the far surface decreasing practically to zero. In order to keep one reversible stress within the friction capacity of the joint, it may be well to re-proportion the joint or even the structure.

Even for members without reversals, there should be as many end rivets in the connection of a member as possible; or, in other words, the number of lines parallel to the length of the member should be a practical maximum. The number of these lines may be increased by welding comparatively long lug angles to the sides of the member, thus adding two or even four lines of rivets to the connection. Since a welded connection is much stiffer than a riveted one, the objection to lug angles is largely overcome. No alternate rivets in the end rows of the connection should be omitted. W. M. Wilson, M. Am. Soc. C. E., with Messrs. J. Mather and C. O. Harris, reports[12] that the section of a tensile connection is not weakened nearly as much by specifying solid end rows of rivets as is generally believed; or, conversely, the section of the member is strengthened only a few per cent by omitting alternate rivets of the end rows.

The decrease in the number of rows shortens the length of the gusset-plate, and thus limits the discontinuity that it represents. Smaller gussets also increase the clear length between joints as shown by Fig. 15(*a*). Moments M_1 and M_2 are from secondary effects. When l' becomes greater Moments M_1 and M_2 become less as a result of the greater slenderness ratio of the member. A. Ostenfeld[13] states that the moments in the members at the gussets as shown in Fig. 15(*b*) have been increased in the ratio, $\frac{l}{l'}$, over what they would have been if the member had been free to bend through the entire distance, l. This factor, $\frac{l}{l'}$, is sometimes termed the gusset factor. In so far as the secondary stresses in a member are concerned, it is important to keep this factor as near unity as possible. From research by K. Havemann[14] the length, l', was found to extend somewhat into the gusset-plates, and he gives the formulas (see Fig. 16):

---

[12] "Tests of Joints in Wide Plates," *Bulletin No. 239*, Univ. of Illinois, Urbana, Ill.

[13] "Teknisk Statik," A. Ostenfeld, II, 1925, p. 401.

$$y = a\,c^{x} \qquad (1)$$

and,

$$\log c = \frac{I_M}{I_G}\left[\frac{1}{K}\left(\frac{I_G}{I_M}\right)^{K} - K\right] \qquad (2)$$

in which $K = \frac{a}{l'}$; a = distance from the old origin to the new X-axis, or the distance from the intersection of members to the end of the gusset; y = y-ordinate from the new X-axis, or the distance into the gusset from its end for l';

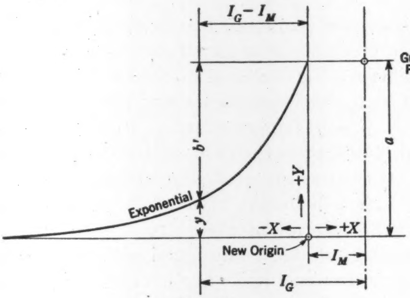


FIG. 16

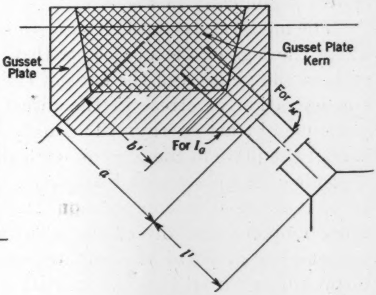


FIG. 17

I_G = effective moment of inertia of the gusset-plate; I_M = moment of inertia of the member; and, $b' = a - y$. The end of the gusset denotes the end of the welds or the end row of rivets. Fig. 17 shows a gusset connection with distances, a, b', and l' of Havemann's formula.

Smaller gussets and lighter sections in the members of the trusses resulted from the multiplication of trusses in the bascule span of the Link Bridge recently completed on the Outer Drive in Chicago, Ill. The greater number of trusses with smaller gussets brought about greater lengths, l', between the gussets for secondary bending in the members (see Fig. 15). The lighter sections also increased the $\frac{l}{r}$ ratios of the members, with a corresponding decrease in secondary bending of the members at the gussets. The smaller gussets in area and section with their smaller rivet groups are better in this bridge than the larger gussets of heavier, fewer trusses would have been.

The secondary moments in the members at the gusset-plates are usually not large enough to be of great importance in the gusset-plate stresses, but with the shortening of the rivet groups the stresses in the end rivets due to these moments should be investigated. The shorter lever arms of the end rivets about the center of gravity of a shorter rivet group increase the load on the end rivets for a given moment. These end rivets are already the most highly loaded ones from the direct stress in the member. On a section of the plate that does not cut any of the members, the bending stress can be found roughly by the usual theory of flexure. This case is seldom met in practice, and may not occur at the critical section. Many gussets have been designed by cutting sections and examining the direct stress, moment, and shear, considering the rivets as uniformly stressed. The bending moment by this method is found by multiplying the forces of the members by their distances from the neutral axis of the section. This moment, divided by the section modulus of the plate at the section, and multiplied by the secant[2] of the angle made by the edge of the plate with the normal to the section, gives an estimation of the stress from bending, at the edge of the plate.[15] This procedure does not consider the non-uniform stresses in the rivets of connections as shown in Fig. 10, or the minimizing stress function shown graphically in Fig. 6(*c*).

---

[14] "Beitrag zur Untersuchung ebener Fachwerke mit steifen Knotenpunkt Verbindungen," von K. Havemann, Darmstadt, 1928.

The model stress data all show the non-uniform stress transfer into the plates along the legs of the models, with the greatest shearing stress transfer at the ends of the plates and the ends of the legs. Fig. 3 also shows the tendency for concentration of stress in the direct path between the model legs from the principle of least energy. The highest $(p - q)$-fringes shift toward the narrow side of the plate in the unsymmetrical plates, or in the case of Model D_3 toward the side of the plate with least area. If the unsupported edge of a steel gusset-plate is stressed in compression, the edge will buckle before failure, throwing more moment and direct stress into the interior of the plate. Parts of the gusset-plate in direct line between members should be designed with the object of providing for all these additional stress increments. The connections of the members to the plate should be made wide, with as many lines of rivets for a riveted connection as practicable.

ACKNOWLEDGMENT

The work[16] reported herein was made possible by the varied facilities at the University of Michigan. The writer is especially indebted to Robert H. Sherlock, M. Am. Soc. C. E., for encouragement and assistance in this research. All the tests took place at the University of Michigan except those of Model *D*, which were made at the Armour Institute of Technology, Chicago, Ill.

---

[15] "Bridge Engineering," by J. A. L. Waddell, 1925, pp. 519–529.

[16] This paper is a part of a thesis submitted by the writer for a D.Sc. degree at the Univ. of Michigan. A complete copy of the thesis has been filed for reference in Engineering Societies Library, 29 West 39th Street, New York, N. Y.

DISCUSSION

R. H. Sherlock,[17] M. Am. Soc. C. E. (by letter).—The gusset-plate is one of the most common details in fabricated steel structures. The intensity and distribution of the stresses in the plates are seldom given any consideration since experience has shown that in ordinary cases a plate will not fail if it is big enough to contain the required number of rivets and has such a shape that it can be cut and fabricated economically. Unusual plates are sometimes analyzed according to the elementary principles of mechanics, one standard specification requiring that "The gusset plates shall be of ample thickness to resist shear, direct stress, flexure, etc."

There is relatively little published material, either theoretical or experimental, dealing with the stress distribution in gusset-plates, so that the engineer finds little to support his judgment in the design of joints lying well outside of previous experience. The stress trajectories in a plate have been found by the mathematical theory of elasticity for only two boundary conditions. In both cases, the load is applied through a single rivet—in the one case to a plate of infinite extent, and in the other case to one-half of an infinite plate bounded by a straight line passing through the rivet. The laboratory research worker knows in advance that there will be little if any opportunity to embellish his report with an elegant comparison between his test results and the results of rational analysis. Progress in this field then must consist in the accumulation of experimental results from a large variety of plate shapes and conditions of loading, thus extending the basis upon which the engineer may exercise his judgment in design.

The author has added to the knowledge in this field through the careful refinement and application of some of the newer laboratory techniques, and through a discussion of present practice in the light of his own results and those of other investigators. It is hoped that the same technique may be applied to more complicated shapes of plates and conditions of loading.

An interesting feature of the photographs in Fig. 3, not referred to by the author, appears in Models B_1, B_5, C_4' and C_5'. Here, the fringes, caused by the light passing through the members attached to the gusset-plates, are inclined to the axis of the members, thus indicating a non-uniform distribution of the stress on the cross-sections of the members. This is in sharp contrast to the fringes in the symmetrical Model C_1' where the fringes indicate a uniform distribution of stress across the members. This indicates again that, if the same welding or riveting is supplied on both sides of the member, the greater stress will occur in the welds attached to the side having the larger area of gusset-plate. All of this is merely another way of showing that the flow of stress is along the path of greatest rigidity.

L. E. Grinter,[18] M. Am. Soc. C. E. (by letter).—It seems strange at first thought that so much attention has been paid to the design of truss members for direct stresses and secondary stresses, whereas relatively little attention has been given to the analysis or design of gusset-plates. The explanation, of course, is that stress analysis of a gusset-plate is a complex process dependent upon the mathematical theory of elasticity, and, as the author has stated, only the simpler kinds of gussets have been subject to analysis at all. It is commendable, therefore, that Mr. Rust has found a device for representing, with reasonable assurance, the relative stress patterns in joints of various shapes and sizes. The meticulous care with which the author has prepared his models, which was observed by the writer, leads to more than usual confidence in the usefulness of the results. His use of the light interferometer for determining the sum of the principal stresses is highly commendable.

---

[17] Prof. of Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

[18] Vice-Pres. and Dean, Graduate Div., Armour Inst. of Technology, Chicago, Ill.

One feature of gusset design that seems to be neglected more often than it is considered is the simple one of flexure. There is nothing new about the requirement that the gussets of Fig. 18 must be designed for a combination of direct stress and flexure, but it is often misunderstood. Since the total stress, S_1, must be transferred across the broken lower chord member by the gusset-plate, it is evident that there is flexure to be resisted along Section a-a. The flexural moment is,

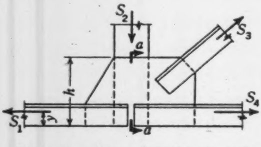


FIG. 18.—ECCENTRICITY AND GUSSET FLEXURE

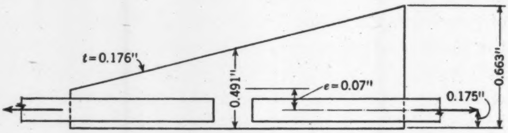


FIG. 19.—ECCENTRICITY OF MODEL B_4

$$M = S_1\left(\frac{h}{2} - y\right) \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Then the total fiber stress may be estimated as,

$$f = \frac{S_1}{h\,t} + S_1\left(\frac{h}{2} - y\right)\frac{6}{t\,h^2} \quad\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which t is the gusset thickness resisting the forces shown in Fig. 18. This type of design, in which the gusset is made to serve the function of a splice plate, is not recommended in any specification but it is still used.

It is possible to check this simple analysis against the results obtained by the author for two unsymmetrical models in Fig. 9. Consider Model B_5 for a load of 140 lb. The value of $p - q$ at Point 7L, which is the mid-point, is 2 450 lb per sq in. The corresponding value of $p + q$ is 2 150. The value of p, therefore, is the average of these, or 2 300 lb per sq in. This stress can be recomputed by Equations (3) and (4): $M = 140 \times 0.07 = 9.8$ in-lb; and, $f = \dfrac{140}{0.491 \times 0.176} + \dfrac{9.8 \times 6}{0.176 \times 0.491^2} = 3\,000$ lb per sq in. Thus, for this case, the approximate analysis given by Equations (3) and (4) proves to be safe since it over-estimates the maximum plate stress by 30 per cent. In Model B_4, at a load of 149.6 lb, the actual observed stress is 1 950 lb per sq in. and the fiber stress from Equation (4) is 2 360 lb per sq in., an excess of 20 per cent. The eccentricity is considerably less important for this model than for the one studied first.

RUSSELL C. BRINKER,[19] JUN. AM. SOC. C. E. (by letter).—Technical literature on the design of steel gusset-plates is much too scarce, and therefore Mr. Rust's paper is a welcome one. Rule-of-thumb design has doubtless been practised more on this "detail" than on any other unit in modern steel bridges. Nevertheless, safe designs (although perhaps often uneconomical) have resulted, and the following statement in the "Synopsis" might well be challenged: "Since a gusset-plate is a point of discontinuity of stress transfer in a structural steel frame, it is here that failures most often occur." Unquestionably gusset-plates are points of discontinuity and, therefore, are danger spots, but search of technical literature will disclose few if any instances of major (or minor) failures caused by faulty gusset-plate design. Perhaps that is merely an indication that rule-of-thumb design has been so overly cautious that even in otherwise faulty designs the gusset-plates have been too generously proportioned to permit failure, but it does prompt the statement that, practically, gusset-plates do not fail.

It might be noted that in general, gusset-plate design is based upon two conditions: (1) Selecting a size of plate such that the bearing value of a rivet upon the plate will approximately equal the double shearing value; and (2) providing sufficient area along any section of the gusset-plate to take the stress upon that section (brought by the rivets). As for condition (2), for specifications allowing 24 000 lb per sq in. in bearing and 12 000 lb per sq in. for shear, an $\frac{11}{16}$-in. plate will give balanced rivet values. For most modern spans, gusset-plates are usually made $\frac{5}{8}$ in. thick, or slightly less than the thickness required for balanced rivet values.

A point that has been neglected—apparently without serious results—is the possibility of buckling of the plate. In rare cases, stiffening angles or "flats" may be riveted along the edges of the plate to prevent the occurrence of such failures. The fact that a reduction in thickness requires more rivets, with a consequent enlargement of the plate, has tended to keep the plates heavy enough to prevent failure.

An important series of investigations performed to determine desirable sizes and shapes of gusset plates has been discussed by Theophil Wyss.[20] Two points worthy of note in his results were: (1) The maximum angle over which the stress spreads from the member through the gusset-plate is about 30°; and (2) an analysis can be made of the condition by which gusset-plates receive direct stress and also a twisting moment, in the same manner that a beam is designed for direct and bending stress by means of the formula,

$$f = \frac{P}{A} \pm \frac{Mc}{I} \quad \text{.......................} \quad (5)$$

---

[19] Asst. Prof. of Eng., Univ. of Hawaii, Honolulu, Hawaii.

[20] "Forschungsarbeiten Auf Dem Gebietes Des Ingenieurwesens," by Theophil Wyss.

C. W. Wixom,[21] Assoc. M. Am. Soc. C. E. (by letter).—A subject on which little information has been available is presented in this paper. Mr. Rust recommends that the non-uniformity of stress among rivets be minimized by making the groups short and compact without elaborate attempts to increase net sections. This recommendation has been strikingly supported in the paper entitled "Tension Tests of Large Riveted Joints."[22] These latter tests, however, led to the further conclusion that the present practice of assuming equal rivet stresses is satisfactory. Consistently, this assumption should be made not only in determining the number of rivets but when investigating gusset-plate stresses.

Tests and analyses of splices in which the stresses are principally direct and central, although valuable in themselves, are of limited application in analyzing gusset-plates in which stresses are transferred principally by shear and flexure. However, in the case where a gusset forms part of a chord splice, there is a large direct stress and also considerable bending stress in the gusset due to the location of the chord near the edge of the gusset. It is sometimes desirable to separate the splice from the gusset and eliminate this source of bending.

The statement that "An ideal gusset approaches a circle in shape, or a polygon of approximately equal axes, with all the forces intersecting at or near its center" will not be accepted by all (see heading "Recommendations for the Specifications and Design of Steel Gusset-Plates: General Practice"). The usual gusset connects one or two diagonals (Fig. 20 or 21), a vertical, and a chord. The "ideal" shape of such a gusset just encloses the rivet groups and is rectangular. The point at which the forces intersect is seldom near the center of the gusset. The gusset in Fig. 13 has been superimposed in dashed lines for comparison on Fig. 20. The entire stress from the vertical is transferred as a shear between the vertical and the diagonal. An extension of the gusset to the left of the vertical is of no assistance in this shear transfer. An experienced detailer can frequently lay out an efficient gusset with a rectangular corner at the diagonal. The stress in the corner rivets will compare favorably with the others because the plate is converging with the number

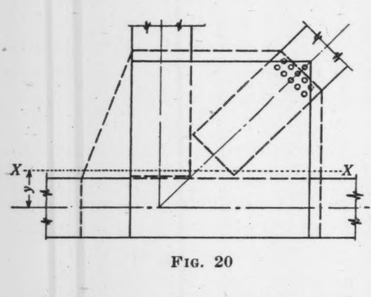


Fig. 20

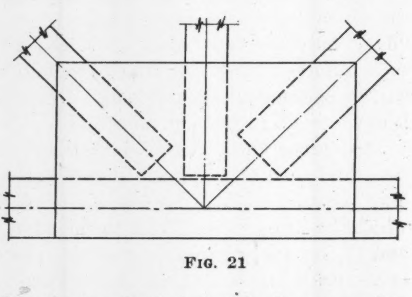


Fig. 21

---

[21] Asst. Engr., Am. Bridge Co., New York, N. Y.

[22] "Tension Tests of Large Riveted Joints," by Raymond E. Davis, and Glenn B. Woodruff, Members, Am. Soc. C. E., and Harmer E. Davis, Assoc. M. Am. Soc. C. E. (see this volume of *Transactions*).

of rivets.[23, 24] Lug angles, or clip angles, have been shown to be flexible and inefficient.[25, 26] They add to the complexity of the stress path.

Since the lines of force intersect at a point, difficulty is sometimes experienced in visualizing bending in gussets. Perhaps the simplest illustration is that of section "$X-X$," Fig. 20, on which there is a bending moment equal to the increment of chord stress times its lever arm "y." The author has quoted the method of analyzing more complex sections.[15]

Gusset-plates should be designed with adequate proportions without undue concern, except in extreme cases, about fabricating limitations or secondary stresses. Many large gussets have been carefully and successfully fabricated for modern structures. Secondary stresses resulting from truss rigidity may be eliminated to a large degree by proper cambering—that is, by fabricating with lengths of members, but not the angles between the members, corrected for their deformation under dead load plus a percentage of live load. In this manner, reverse secondary stresses are induced during erection and these are relieved when the loads are applied.

Of two structures designed for the same duty, the one having the fewer members will often be the more economical—not only in first cost but in maintenance. It will be easier to paint. Since its members and gussets are larger, it will vibrate less and develop fewer loose rivets. Painting of contact surfaces not only reduces the friction but results in loose rivets.[27] It is now prohibited in the leading specifications.[28, 29, 30]

T. H. Rust,[31] Assoc. M. Am. Soc. C. E. (by letter).—The discussions on this paper have further integrated the subject. Professor Sherlock and Dean Grinter note the difficulty of the mathematical approach to the accurate stress analysis in steel gusset-plates, and Professor Sherlock states the meager achievements in accurate stress determination of gusset-plates.

The difficulty arises when an attempt is made to state, mathematically, the principle of least energy for a plate with finite boundaries. The forces concerned are those parallel to the plate, applied at points within the plate rather than those applied perpendicular to it. The stress will flow from one force to another along many paths. The amount of stress along a particular path will be a function of its length and the rigidity of the material. Stored energy is a product of force by displacement, or a summation of such products; and the stress in the plate will distribute itself along the various paths in such a manner as to make the total energy in the plate a minimum. A prismatic plate with

---

[23] "Modern Framed Structures," by the late J. B. Johnson, the late C. W. Bryan, M. Am. Soc. C. E., and F. E. Turneaure, Hon. M. Am. Soc. C. E., Part III, p. 112.

[24] "Riveted and Bolted Connections," by C. Batho, First Rept. of Steel Structures Research Committee, pp. 115–120.

[25] "Modern Framed Structures," by the late J. B. Johnson, the late C. W. Bryan, M. Am. Soc. C. E., and F. E. Turneaure, Hon. M. Am. Soc. C. E., Part III, p. 127.

[26] "Riveted and Bolted Connections," by C. Batho, First Rept. of Steel Structures Research Committee, p. 102.

[27] "Tests on Effect of Paint between Riveted Plates," by the late Richard Khuen, Jr., M. Am. Soc. C. E., *Engineering News-Record*, Vol. 90, 1923, p. 460.

[28] Am. Ry. Engineering Assoc. for Steel Railway Bridges, 1928, par. 544.

[29] Am. Assoc. of State Highway Officials for Highway Bridges, 1935, par. 3, 11, and 9.

[30] Am. Inst. of Steel Construction for Structural Steel for Buildings, 1937, par. 26(c).

[31] Structural Engr., Chicago, Ill.

two opposite forces applied uniformly over its ends will be uniformly stressed similar to Fig. 6(*a*). However, if the forces are applied at points in the ends of the plate, and if the plate is other than narrow in an engineering sense, another component of stress will be introduced from the principle of least energy as shown by Fig. 6(*c*). If there is little bending stress, greater stress will lie in the path directly between the two forces than in any other longer path through the plate. It is this variation of path in stress transfer in gusset-plates from one attached member to another which makes the accurate determination of stress so difficult.

Dean Grinter's calculations of stress in the bakelite models reveal similar difficulties. In Models B_4 and B_5 the computed maximum stress at the edges is less than that given by the formula for direct and bending stress. This is because the path in the plate between the ends of the legs is the most direct one and hence takes more least energy stress than the longer paths at the edges. The wider plates of the *D* models show this phenomenon to an even greater degree, the stress at the edges being almost zero. It is well to note that the behavior of stress in bakelite models simulates that of steel in a qualitative, rather than in a quantitative, manner. The fringes fall off more rapidly in the bakelite than they would in a steel plate because the modulus of elasticity of bakelite is only about 550 000 lb per sq in. in comparison with 29 000 000 lb per sq in. for steel. As an extreme example of the localization of stress due to the elasticity of the material, a pin thrust through a thin sheet of rubber could be moved back and forth laterally in the plane of the sheet, producing only local distortions, and points at some distance would suffer little or no distortion. Steel models would not have this quantitative handicap, since all three types of stress (*a*), (*b*), and (*c*), shown in Fig. 6, would appear in their true values if the proper scale relations were maintained between model and prototype. After they are in the plate, the $\frac{P}{A}$-values and the $\frac{Mc}{I}$-values are independent of the modulus of elasticity of the plate, provided the material satisfies Hooke's law in the region considered. It is only the "minimizing stress" (Fig. 6(*c*)) that is thus affected. Photo-elastic methods could not be used with steel models; but perhaps other methods could be employed such as covering the plates with a thin layer of material which would show changes after the steel plates were subjected to stress.

In the absence of well validated methods for obtaining quantitative information, the qualitative knowledge is a definite help to the careful designer.

Although the mathematical methods are intricate and not far advanced for the study of stress in plates of finite boundaries, with forces applied at points within the boundaries and parallel to the planes of these plates, still progress might be made by methods analogous to those developed by the German physicist, Walter Ritz.[32] He made some noteworthy solutions in thin plates of finite boundaries, with forces applied perpendicular to the planes of the plates, by using variational calculus, polynomials, Green's theorem, and Fourier series. The continual refinement of results by Fourier series parallels the designer's successive approximations in his calculations and designs.

---

[32] "Deformation einer am Rande eingespannten elastischen Platte unter dem Einflusz eines gegebenes Normaldruckes," von Walter Ritz, 1878–1909, *Journal für die reine und angewante Mathematik*, Band 135, 1909, s. 6–57.

Professor Brinker states that search of the technical literature will disclose few if any instances of failures caused by faulty gusset-plate design. It is indeed unfortunate that more structural failures are not described in the literature, since the primary cause of failure is not always disclosed by a casual inspection afterward. The close study that accompanies the preparation of an engineering analysis and report might very well have placed the blame on these points of discontinuity. Engineers agree quite generally that failures are in the joints and details rather than in the bodies of the members.

Professor Brinker also notes that the gusset-plate should not have an angle of less than 30° with the connecting member. Because of the considerable research that has been completed in riveted joints, gusset-plates, and thin plates,[12, 22] the writer did not include this empirical specification in the paper or in the thesis[16] of which the paper is a part, because of the expectation that it would soon be superseded.

Such matters as thickness of gusset, and reinforcement of gusset to prevent buckling, which Professor Brinker raises should be the result of the specifications and requirements of a particular structure.

Equation (5) has been given in more generalized form, including examples of its application by the late J. A. L. Waddell, Hon. M. Am. Soc. C. E.[15] This formula is approximate and gives only the order of magnitude of plate stresses rather than accurate values.

Mr. Wixom's statement in the first paragraph of his discussion quite agrees with the statement of the writer (see "Recommendations for the Specifications and Design of Steel Gusset-Plates: Stresses"), "If there is no reversal, stressing the rivets beyond the elastic limit is of little consequence." It is this favorable quality in the plastic region of steel that has permitted the specification of gusset-plates to lag in progress behind the specification of members. However, when reversed stresses occur and each reversal goes beyond the slip-resisting capacity of the rivets, the connection is very likely to deteriorate.

It seems that it would be desirable in almost every instance to splice all members outside of the gusset connection and thereby avoid superimposing the discontinuity of the splice upon that of the gusset.

The writer agrees with Mr. Wixom that it would be folly to specify a gusset of approximately equal axes at the expense of long rivet connections or unriveted areas of plate, and the writer regrets that such an interpretation has been placed by Mr. Wixom upon the ideal. The ideal gusset-plate is similar to the ideal engineering structure; it can almost never be realized. However, the writer has found, after the examination of many gussets, with the experience and the elementary theory at his disposal before this research was undertaken, that gussets of approximately equal principal axes with forces intersecting near the center practically never showed high stresses due to bending and direct stress. The desirability of avoiding long and narrow gusset-plates has been reported by Clifford E. Paine,[33] M. Am. Soc. C. E.

Mr. Wixom's objection to lug angles is based upon three considerations: (1) Short lug angles induce moment in the transfer of stress; (2) the added rivet connection causes distortion with resultant stress in the lug angle before it can transfer any stress from the member into the plate; and (3) the added material in the lug angle must be distorted with resultant stress before it can transfer stress into the plate. The first consideration can be minimized by long lug angles. The second, which is undoubtedly the most important factor, can be reduced by welding the lug angles to the members. Welded connections are much stiffer than riveted ones[34]; so the less capacity for transmission of stress due to the added riveted connection can be largely counteracted by welding the lug angle to the connecting member. The entire connection from the member and auxiliaries to the gusset-plate, itself, should be either all riveted or all welded. The third factor can be minimized by making the lug angles as narrow as practicable and of comparatively thick material. This factor is not as important as the first and second factors. For these reasons it does not seem justifiable to condemn all use of lug angles.

In answer to Mr. Wixom's preference for heavy trusses, it seems that best results can be obtained by not attempting to exceed, or even equal, the maximum sizes of rivets, heavy plates, and sections in use at this time. At least, this should apply to important gusset-plates and their connections. Many specifications limit the thickness of plates, eye-bars, and other sections for steel structures, and C. F. Goodrich,[35] M. Am. Soc. C. E., reports that "the limit has about been reached in human strength to handle heavier, high-pressure, rivet guns."

In reply to Mr. Wixom's criticism that the models did not simulate closely enough the complex gusset-plates in actual trusses, it may be said that progress in any laboratory work results from simple beginnings. Also the development of a proper testing technique was of major importance in this research. It is true that the stresses are principally direct in most of the models, and in others the bending results entirely from the eccentricity of the direct load, whereas in actual trusses the presence of diagonals and verticals renders the conditions, more complex. Nevertheless, it is believed that the lessons from these simple models have supplied information for the estimation of stress in more complicated joints; and it is hoped that more involved joints will be investigated in the laboratory.

---

[33] "Designing Bridge Towers 700 Ft. High," by Clifford E. Paine, *Engineering News-Record*, October 8, 1936, p. 499.

[34] "The Relative Rigidity of Welded and Riveted Connections," by C. R. Young, M. Am. Soc. C. E., and K. B. Jackson, *Canadian Journal of Research*, 11, 1934, p. 131.

[35] "Tension Tests of Large Riveted Joints," discussion by C. F. Goodrich, *Proceedings*, Am. Soc. C. E., September, 1939, p. 1290.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2059

GREAT LAKES TRANSPORTATION

By M. C. Tyler,[1] M. Am. Soc. C. E.

With Discussion by Messrs. T. Kennard Thomson, Harland C. Woods, W. L. R. Haines, and L. C. Sabin.

Synopsis

The Great Lakes and their connecting channels are of great importance in the national economy and the national defense. The steel industry of the United States was largely built up around lake transportation, and the Great Lakes also afford low-cost transportation for grain and coal. This paper is an attempt to outline the history and effects of lake transportation, the works built during a century for the benefit of navigation, and to describe the existing commerce and shipping. The combined Lake and rail movement of bulk commodities is shown to be efficient and economical. The system has been developed gradually through the co-operative effort of the Government, the railroads, industry, States, municipalities, and the owners and operators of vessels. It demonstrates that good engineering practice and sound business judgment, applied to plans laid down only so far ahead as definite benefits can be foreseen, can produce remarkable results.

Historical

Water transportation on the Great Lakes and on the St. Lawrence River and its tributaries has had a tremendous influence on both the political and economic history of the United States and Canada. The first English colonies were established on the Atlantic seaboard in Massachusetts and Virginia at about the same time that Champlain set up his stockade at Quebec in 1608; so the English and the French got practically an even start at conquering the American wilderness. However, water transportation by way of the St. Lawrence River, the Great Lakes, and by the Ohio, Illinois, Fox, and Mississippi Rivers (see Fig. 1), gave the French explorers, missionaries, and traders a great advantage in freedom and rapidity of movement. By 1755, when the English felt strong enough to attempt to push their claims over the Alleghenies and establish themselves in the Ohio Valley, the French, due to water travel and trade, had

---

Note.—Published in November, 1938, *Proceedings*.

[1] Brig.-Gen., Corps of Engrs., U. S. Army; Pres., Mississippi River Comm., Vicksburg, Miss.

swung around in front and far ahead of them, and had built missions, forts, and trading posts at Sault Ste. Marie, Mackinac, Green Bay, Niagara, on the Ohio, Illinois, and Mississippi Rivers, and at Ticonderoga and Crown Point, on Lake Champlain. The British had only one post on the Great Lakes, a small one at Oswego on Lake Ontario. When General Edward Braddock made his futile attempt in 1755 to capture Fort Duquesne, a body of French and Wisconsin Indians was in the battle that resulted in his disastrous defeat. The British, moving overland through a wilderness, could make no headway against the French, who were able to concentrate quickly by water at any threatened point. Only when General James Wolfe cut the root of the French water-transportation system by the capture of Quebec in 1759, and Baron Jeffry Amherst, making use of water transportation, proceeded from Oswego with 10 000 men and 1 300 Indians in hundreds of bateaux and captured Montreal, in 1760, did the far-flung French Empire in America collapse, and then New France became British territory.

During the American Revolution, water transportation available to the British from Quebec and Montreal, nearly brought disaster to the Colonial Revolutionary cause. General John Burgoyne reached Saratoga by means of water transport and Sir John Johnson with his Tories and Indians, from Fort Niagara as a water base, raided the settlements in New York and Pennsylvania along the Mohawk and the Susquehanna Rivers. Even after the treaty of peace, water transportation enabled the British to hold the forts on the American side of the Lakes for a number of years in violation of its terms.

Again, in 1813, water transportation on the Lakes played a decisive rôle. The British, with their Indian allies, captured Detroit and Mackinac, and kept General William Henry Harrison's army ineffective on the Maumee River through the command of Lake Erie by a small British squadron which kept Commodore Oliver Hazard Perry's war vessels blockaded in Erie Harbor. It looked as if the United States might lose all of its Northwest Country. For some unknown reason, the British squadron left its station in front of Erie long enough for Commodore Perry, unmolested, to work his vessels across the bar, on which there was about 5 ft of water. The victory over the British squadron at Put-in-Bay followed; the United States regained the advantages of water transportation for movement and supply; and Commodore Perry, using his own and the captured vessels, ferried General Harrison's troops to the Canadian side. Detroit was re-occupied and the British and Indians were defeated in the Battle of the Thames. Had not the control of the Lakes been retaken, it is quite probable that the great iron ore deposits of this country might now be in Canada.

Steamboat navigation began on the Great Lakes in 1816, with a steamer running between Ogdensburg, N. Y., and Lewiston, N. Y., on the Niagara River. The first steamboat on Lake Erie was put in operation in 1818. The opening of the Erie Canal between the Hudson River and Buffalo, N. Y., in 1825 and the Oswego Branch to Lake Ontario in 1828, and, the completion of the first Welland Canal connecting Lake Erie and Lake Ontario around Niagara Falls, in 1829, expedited the settlement of Michigan, Wisconsin, and Northern Illinois. By 1830, a daily boat line was running between Buffalo and Detroit.

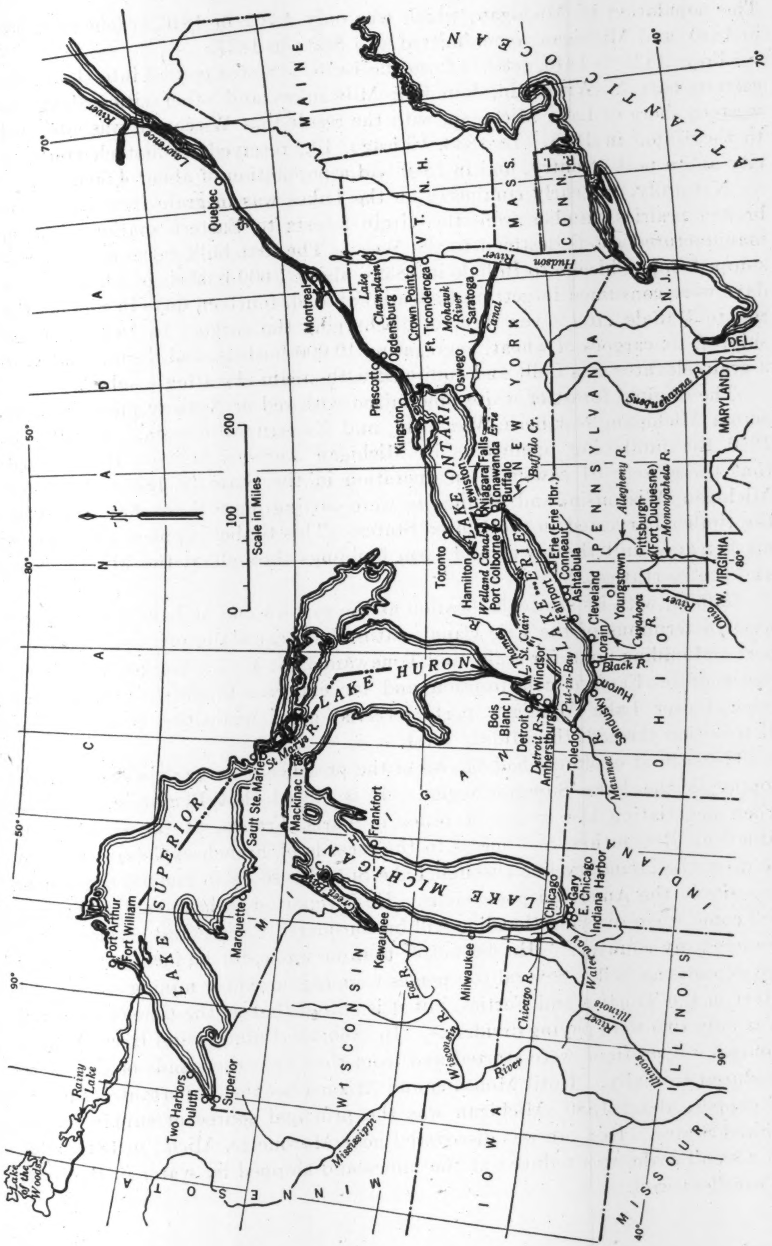

C A N A D A
Scale in Miles
0
100
200
N
LAKE SUPERIOR
LAKE MICHIGAN
LAKE HURON
LAKE ONTARIO
LAKE ERIE
Lake of the Woods
Rainy Lake
Two Harbors
Duluth
Superior
Port Arthur
Fort William
Marquette
Sault Ste. Marie
St. Mary's R.
Mackinac I.
Bois Blanc I.
Frankfort
Green Bay
Kewaunee
Fox R.
Milwaukee
Chicago R.
Chicago
Gary
E. Chicago
Indiana Harbor
Waterway
Illinois River
Wisconsin R.
Mississippi River
Detroit
Windsor
Detroit R.
Amherstburg
L. St. Clair
Thames R.
Toledo
Maumee
Put-in-Bay
Sandusky
Huron
Black R.
Lorain
Cleveland
Cuyahoga R.
Youngstown
Fairport
Ashtabula
Conneaut
Erie (Erie Hbr.)
Allegheny R.
Pittsburgh (Fort Duquesne)
Monongahela R.
Ohio River
Port Colborne
Welland Canal
Hamilton
Toronto
Niagara Falls
Lewiston
Tonawanda
Erie Canal
Buffalo
Buffalo Cr.
Kingston
Oswego
Prescott
Ogdensburg
Canal
Mohawk River
Saratoga
Ft. Ticonderoga
Crown Point
Lake Champlain
Montreal
Quebec
Lawrence River
Hudson River
M A I N E
V T.
N. H.
M A S S.
C O N N.
N. J.
DEL.
MARYLAND
W. VIRGINIA
N E W Y O R K
P E N N S Y L V A N I A
O H I O
I N D I A N A
I L L I N O I S
M I C H I G A N
W I S C O N S I N
M I N N E S O T A
I O W A
M I S S O U R I
A T L A N T I C O C E A N
90°
80°
70°
50°
40°

FIG. 1.

The population of Michigan, which was only 4 762 in 1810, reached 212 000 in 1840; and Michigan was admitted as a State in 1837.

From 1835 to 1845, settlers from the Eastern States poured into the southeastern part of Wisconsin, founding Milwaukee and other cities along the western shore of Lake Michigan, with the result that Wisconsin was admitted to the Union in 1848. In 1833, Chicago, Ill., received its first charter as a city and a port of entry, and in 1840 had a population of about 4 500.

Naturally, the early commerce on the Lakes was in grain from the newly broken prairies, lumber from the virgin forests to Eastern markets, and in manufactured goods destined to the West. The first bulk cargo of grain was shipped from Chicago to Buffalo in 1839—about 1 600 bushels of wheat. Five days were consumed in getting the cargo aboard, fourteen days in making the run to Buffalo, and seven days in discharging the cargo. In 1840, Chicago shipped six cargoes of wheat, aggregating 10 000 bushels, and during that year a grain elevator was built and equipped with steam elevating machinery.

The original forest of white pine mixed with red or Norway pine stretched across Michigan, Northern Wisconsin, and Eastern Minnesota. As early as 1850 the lumbering operations in Michigan were extensive. It is stated that there were 61 saw-mills in operation in the State in 1854. By 1890, Michigan, Wisconsin, and Minnesota were cutting more than one-third of all the timber harvested in the United States. This timber supplied the Eastern market and built the towns and farm buildings throughout the Middle West and the Northwest.

Buffalo, with its strategic location at the eastern end of Lake Erie and the western terminus of the Erie Canal, naturally became the principal receiving port and milling point for grain, and Tonawanda, N. Y., the market for lumber, destined for Eastern consumption and for export. Likewise, Chicago and other Lower Lake Michigan ports received great quantities of lumber for distribution through the Middle West.

The earliest explorers had known of the presence of large deposits of virgin copper in the Lake Superior region. It is stated that Benjamin Franklin, when negotiating the treaty of peace in Paris, France, at the close of the American Revolution, had access to the records of French explorers and that he drew the boundary line through Lake Superior so as to include the copper deposits in the American possessions. He is reported to have said, "The time will come when drawing that line will be considered the greatest service I ever rendered my country." He did better than he knew, for he included the iron ore deposits as well. Several companies were organized for mining copper and silver in the Thirties and Forties, but it is stated that at the time of the Civil War only two were paying dividends. In 1865, the Calumet and Hecla Mining Company operations were started and from that time thousands of tons were produced annually. Until Montana and Arizona became important producers of copper, about 1880, Michigan was the principal source of supply in the United States. Iron ore was discovered near Marquette, Mich., in 1844, and, on a small scale, was reduced at the mines and shipped by water in the form of excellent iron.

Until 1855 through commerce from Lake Superior was blocked by the rapids in the St. Marys River. Previously, a small quantity of ore was moved from the mines near Marquette by wagon to the Lake, loaded aboard a schooner and taken to Sault Ste. Marie where it was unloaded, carried around the rapids, and reloaded on another vessel.

The State of Michigan, with funds obtained from the sale of lands under a Federal land grant, constructed a canal with locks around the rapids and opened it to commerce in 1855. The first through shipment of iron ore to an Erie port was made in the same year in the brigantine, *Columbia*. The panic of 1857 arrested development temporarily, but the Civil War created a great demand for iron with which to arm, equip, and supply the largest Army and Navy which up to that time the nation had ever been called upon to raise and maintain. Thus, again, Lake transportation became an important factor in military operations and contributed materially to their successful conclusion.

The opening of the St. Marys River to through water transportation made the Lake Superior lumber available to Eastern and Chicago markets. Duluth, Minn., and Superior, Wis., at the head of the Lake, came into being as supply points for lumbermen and, in 1870, were dependent on the lumber business. However, in 1871, wheat began moving from Northern Minnesota by Lake and, rapidly thereafter, with the settlement of North Dakota and Montana, they became the outlet for grain from these States, as well as from Northwestern Minnesota destined for the Eastern markets and for export.

At the close of the Civil War the great era of railroad building throughout the United States required an enormous supply of iron. To meet it, furnaces were built and expanded in the area between Lake Erie and the Monongahela River, where ore by Lake and coal by rail and river could be brought together with a minimum of expenditure of time and money. Thus, the great steel industry in Western Pennsylvania, Ohio, and West Virginia was built by low-cost transportation, combining to the best advantage both rail and water movements. Later, the same principles caused the developments in the steel industry at East Chicago, Indiana Harbor, and Gary, Ind.

Prior to 1884, only the Michigan and Wisconsin ranges were worked, but in that year shipments began from the ranges in Northern Minnesota, which had been discovered about 1875. The Minnesota ores moved and continued to move to Two Harbors, Minn., Duluth, and Superior, for loading into vessels.

With the settlement of the Upper Lake States and the development of industries therein, a demand for coal was created which could be met most economically by combining rail and water movements. Thus, at low costs for transportation, the bituminous coals of Western Pennsylvania, West Virginia, Kentucky, and Ohio have been marketed in ever-increasing quantities throughout the Upper Lake region, farther west in the United States and, to a considerable extent, in Canada.

By 1914, all Europe was at war, and by 1917 the United States was also in the struggle. For two years, the demand of the Allied Powers for steel for arms and munitions was tremendous and then the immense requirements of the United States Government to construct ships and to arm and supply the

Army and Navy were added. The great system of Lake and rail transportation which had been developed in the century since Commodore Perry's victory in 1813, met every requirement of the emergency and had an incalculable weight in the favorable outcome of the war. As long as the iron mines on Lake Superior and the coal mines last, and as long as wars are still a possibility, the transportation system built around water carriage on the Great Lakes will continue to be one of the greatest elements of military and naval strength, better appreciated, perhaps, by the possible enemies of the United States.

The Transportation System

A water-transportation system, such as that of the Great Lakes, consists of many elements: The Lakes themselves, their connecting channels both natural and artificial, connecting canals, the harbors and harbor terminals, with their mechanical and other equipment, the railroad facilities serving the harbors and reaching inland, the maintenance of channels and harbors, the vessels in the carrying trade, harbor craft, shipyards, aids to navigation, lighthouses, radio beacons, fog signals, ranges and buoyage, life-saving and distress service, navigation charts and the organizations and men who operate and maintain the vessels, and all the other elements of the system.

It is impossible in a short paper to describe the development and present status of all these elements in detail. However, it may be stated that all of them are in existence and are operated with high efficiency.

Channels and Harbors

In their original condition, each of the Great Lakes in itself afforded easy navigation, but connecting channels were either non-existent or inadequate. The first Welland Canal completed by the Canadians in 1829 opened navigation between Lake Ontario and Lake Erie. It had a depth of 8 ft. The first canal around the Falls at the "Soo" was opened in 1855, with a depth of 11.5 ft, connecting Lake Huron and Lake Superior.

There were almost no natural harbors so located along the American shores as to be of commercial value, so the Congress promptly began to authorize the improvement of channels and the construction of harbors by the War Department. For the small sailing vessels and steamers, the rivers and creeks on which the growing Lake communities were located were the only harbor possibilities within the financial abilities of the National Government. In general, the mouths of such waterways were obstructed by bars on which there was as little as 3 or 4 ft of water. Some examples of early harbor improvements are as follows:

(1) A survey of Erie Harbor, where a deep natural bay existed, was made by the Corps of Engineers in 1819 and showed only 6 ft on the bar. By Act of Congress in 1823, a board of officers developed a plan for the improvement of the entrance by dikes and jetties, and work was begun in 1824. By 1828 a minimum depth of 7 ft at all times had been obtained, which was increased to 9 ft in 1830 and to 13 ft in 1868.

(2) Work was begun at the mouth of the Oswego River in 1827.

(3) At Chicago, work at the mouth of the Chicago River was begun in 1833 by the construction of jetties, and a depth of 9 ft was obtained before 1840.

(4) At Lorain, Ohio, parallel jetties were built at the mouth of Black River, work beginning in 1828, and a depth of 10 ft was secured, the original depth having been 3 ft.

(5) Work to improve the entrance to the Cuyahoga River at Cleveland, Ohio, by parallel jetties was begun under the Act of 1825. The original depth of 3 to 4 ft was increased to 6 ft in 1828 and to 10 ft in 1831.

(6) In 1826, work was begun on piers at the mouth of Buffalo Creek, at Buffalo. In practically all these jetties or piers (Items (1) to (6)), timber work filled with stone was utilized for reasons of cost. The Federal Government limited its work to providing entrance channels. The municipalities and vessel operators took care of the channels inside the jetties.

(7) A depth of 12 ft was obtained through the St. Clair Flats in Lake St. Clair in 1855, where the original depth had been from 2 to 6 ft, and St. Marys River was deepened.

(8) By 1869, vessels had so increased in size and number that the need of outer breakwaters to afford anchorages and safety of entrance into the rivers was apparent. Work was begun on the Buffalo breakwaters in 1869, and on those at Chicago in 1871. Timber cribs filled with stone were the standard construction for the earlier breakwaters and most of them, later cut down to low water and capped with concrete, are still in service.

From these modest beginnings, designed to meet the immediate needs of a rapidly growing commerce carried in small vessels, steady progress has been made in deepening and widening connecting channels, in deepening harbors and their entrances, and in giving them more extensive breakwater protection in order to keep pace with the increase in size and draft of the Lake carriers. This has been possible at reasonable costs through the use of the most modern dredging equipment and the substitution of rubble, concrete caissons, and cellular steel shell-pile construction for timber cribs in breakwaters, as the price of timber became excessive.

At the present time (1938), the Lake System consists, in addition to adequate harbors, of down-bound connecting channels 25 ft or more in depth from the head of the Lakes at Duluth-Superior and from Chicago, to Ogdensburg on the St. Lawrence River, with 14-ft channels in the Canadian canals to Montreal, and up-bound channels 21 ft deep. The New York State Barge Canal 12 ft deep from Buffalo and Oswego to the Hudson, being deepened to 14 ft between the Hudson and Oswego, and the Illinois Waterway with a 9-ft depth, are important connecting waterways to the east and south.

Wherever possible, the up-bound and down-bound channels have been separated for safety to vessels. The existing project for the Upper St. Marys River provides for a two-way channel having a minimum width of 1 000 ft, with a depth at low-water datum of 27 ft through Round Island Shoals and

Middle Ground, and 26 ft through Vidal Shoals, widening to 1 200 ft at Brush Point Turn and thence, to Point Louise with a width of 1 000 ft.

The American locks at the "Soo" have the characteristics shown in Table 1. The Canadian Canal at the "Soo" is 1⅛ miles long, 150 ft wide at the top, 142 ft at the bottom, and has a lock 900 ft long and 60 ft wide. Navigation in the canal is limited by the depth over the miter-sills at the locks to boats with drafts of about 17 ft. It was opened to navigation in 1895.

In the Lower St. Marys River, the project provides for a down-bound channel with a depth of 25 ft and a minimum width of 300 ft, and an up-bound channel with a depth of 21 ft and a minimum width of 500 ft. In St. Clair River, the minimum width of the two-way channel is 600 ft, and the depth is 25 ft. In St. Clair Lake the two-way channel is 800 ft wide and 25 ft deep, narrowing to 700 ft at the mouth of the South Channel of St. Clair River.

TABLE 1.—CHARACTERISTICS OF THE AMERICAN LOCKS AT SAULT STE. MARIE, MICHIGAN (DIMENSIONS ARE IN FEET)

Locks	Clear width	Length between miter sills	Lift	Depth on Breast-Wall		Type of construction	Year completed
				Upper	Lower		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Weitzel.......	80*	515	21.7	15.8	11.2	Stone masonry	1881
Poe..........	100	800	21.7	21.8	16.6	Stone masonry	1896
Davis........	80	1 350	21.7	24.3	23.1	Concrete	1914
Fourth.......	80	1 350	21.7	24.3	23.1	Concrete	1919

* 60 ft at the gates.

The two-way channel at the head of Detroit River is 800 ft wide and 25 ft deep. Fighting Island Channel, below Detroit, is 26 ft deep and 800 ft wide; Ballards Reef Channel north of the junction with Livingston Channel is 26 ft deep and 600 ft wide. The Livingston Channel west of Bois Blanc Island for down-bound vessels, is 450 ft wide and 26 ft deep to opposite Bar Point, thence 26 ft deep and 800 ft wide to its junction with Amherstburg Channel, and thence 26 ft deep and 1 200 ft wide to the 26-ft depth contour in Lake Erie, the lower 1 200-ft section being used for both up-bound and down-bound traffic. Amherstburg Channel, east of Bois Blanc Island, for up-bound vessels, is 21 ft deep with a minimum width of 600 ft.

The new Welland Canal, opened in 1932, is 25 miles long, with a bottom width of 200 ft, 310 ft at the water line, and a depth of 25 ft. The canal has seven locks, each having a usable length of 820 ft, a width of 79 ft, and a depth of 30 ft over the sills.

SHIPPING

The great length of the connecting channels in which natural depths were small and the artificial character of the Lake harbors built in comparatively shallow water have made it impracticable to provide depths for Lake navigation equal to that at ocean ports. Storm waves on the Lakes probably never much exceed a height of 20 ft from trough to crest. Most of the freight is handled in bulk. Docks and their unloading equipment, once installed, can be modified only at great expense.

Channel and harbor depths have limited the draft of vessels, dock equipment has limited their beam, and safe handling in restricted waters has limited their length, although the moderate height of storm waves has permitted ratios of length to beam and to depth greater than ocean practice. As a result of all these factors a bulk freighter has been developed with large rudder power, which can carry about as much on 22-ft draft as large ocean vessels can carry on 30 ft, or more. This development, of course, has been gradual with improvements in ship construction and with the increase in depths of channels and harbors.

The first vessels on the Lakes were sailing schooners and brigs; then came wooden-paddle wheel steamers. The *Ontario*, the first American steamer on the Lakes,[2] was built in 1816. It was 110 ft long, with 24 ft beam, 8½ ft depth, a displacement of 237 tons, and was driven by one low-pressure cylinder with a bore of 34 in. and a stroke of 4 ft. It is stated that it took her ten days to make the trip from Lewiston to Ogdensburg. The *Vandalia*, the first propeller boat in the United States, was built in Oswego in 1841. For years, commerce was carried in sailing vessels, towed through the rivers by tugs, in steamers, and in steamers with one or more barges in tow. Hulls went through the evolution in design from wood, to composite wood and iron and steel, and then to steel. The steel whaleback freighter and whaleback barge had their day, but since about 1904 the type of bulk carrier has been well established, the changes being only in their size.

The carrying capacity of Lake bulk freighters on a 19-ft draft ranges from 3 000 to 12 000 short tons (see Items Nos. 1 to 3, Table 2). The extra load per foot of draft, up to 22-ft draft, varies from about 1 000 to 430 net tons. These vessels are usually constructed with the deck-house and high bridges forward, a cargo hold, without bulkheads, and a coal-burning power plant aft. The hatches over the cargo hold are spaced on about 12 ft centers. Trim of the vessels and safety are provided by inside tanks.

Four bulk freighters built during 1937 and 1938 have a carrying capacity on 22-ft draft of 15 440 short tons (Items Nos. 4 to 7, Table 2). They are equipped with geared turbines of a capacity sufficient to obtain speeds of 12 miles per hr loaded and 13.5 miles per hr light. The Canadian steamer, *Lemoyne* (Item No. 8, Table 2), the largest of the Lake fleet, has carried record cargoes in net tons, as follows: Ore, 16 400; bituminous coal, 16 500; anthracite coal, 14 600; stone, 18 100; and, wheat, 17 100.

Self-Unloaders.—In 1928 there were 43 self-unloading vessels on the Great Lakes, with a gross tonnage of 144 730, which were increased to 65 vessels having a gross tonnage of 243 343 in 1937. Many of these vessels are of the usual ore and coal-handling types with cargo hoppers installed in their holds, discharging on to belt conveyors and through a bucket elevator on to a belt conveyor carried on a long boom, which is swung outboard when unloading cargo. Loads from the large vessels can be placed 115 ft from shipside at an elevation of 65 ft above deck level.

---

[2] "Freighters of Fortune," Beasley, p. 50.

Motor Ships.—There are 48 motor-operated vessels in bulk and package-freight services on the Lakes, with gross registered tonnages ranging from 1 000 to 8 877, of which 36 are of American registry and 12 Canadian. The two largest, the *Henry Ford II* and *Benson Ford*, are both bulk freighters, of the standard Lake type, propelled by Diesel engines (see Items Nos. 9 and 10, Table 2).

TABLE 2.—TYPICAL VESSELS IN THE GREAT LAKES TRADE

Item No.	Name of vessel	Year built	DIMENSIONS, IN FEET				Gross registered tonnage	Type
			Length	Breadth	Draft loaded	Depth of hold		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	*George B. Leonard*	1903	380	50	21	23.6	4 037	Lake bulk freighter
2	*Joliet*	1916	504	54	21.5	30.0	6 262	Lake bulk freighter
3	*Thomas W. Lamont*	1930	588	60	22	27.9	7 964	Lake bulk freighter
4	*William A. Irvin*	1937	586	60	22	32.5	8 255	New bulk freighter
5	*Ralph H. Watson*	1937	586	60	22	32.5	8 302	New bulk freighter
6	*Governor Miller*	1937	586	60	22	32.5	8 255	New bulk freighter
7	*John Hulst*	1938	586	60	22	32.5	8 302	New bulk freighter
8	*Lemoyne*	1926	621	70	20	....	10 480	Largest bulk freighter
9	*Henry Ford II*	1924	597	62	17.5	27.8	8 877	Motor vessel
10	*Benson Ford*	1924	596	62	17.5	27.7	8 626	Motor vessel
11	*W. J. Conners*	1901	382	50	19.3	28.0	4 625	Passenger and package vessel
12	*Crescent City*	1897	406	48	20	24.0	4 109	Automobile carrier
13	*James Watt*	1896	405	48	....	23.5	3 807	Automobile carrier
14	*Edgewater*	1931	291	43	12.5	15.7	1 819	Lake and canal freighter
15	*Chester*	1931	291	43	12.5	15.7	1 819	Lake and canal freighter
16	*Badger State*	1912	250	43	10	16.7	1 539	Lake and canal freighter
17	*Panoil*	1920	242	37	18	19.7	1 370	Typical Lakes tanker

Passenger and Package Vessels.—On December 30, 1934, there were 119 vessels of 1 000 gross tons and upward, aggregating 357 231 tons, in the passenger and package freight trade on the Great Lakes. The *W. J. Conners* (Item No. 11, Table 2) is probably typical of this class of vessel. It is powered with an 1 800-hp, quadruple expansion, steam engine, giving it a speed of 13 miles per hr.

Automobile Carriers.—This type of vessel was converted from the bulk freighter. They were converted by the addition of "'tween decks" and elevators for the lowering of cars to the lower decks. A representative vessel of this type is the *Crescent City* (Item No. 12, Table 2). In 1936, a similar conversion was made on the *James Watt* (Item No. 13). This vessel is a pioneer in a rather unique transportation plan—the carrying of loaded truck trailers between Detroit and Cleveland.

Tankers.—Tanker tonnage on the Great Lakes now constitutes one of the important items of traffic. No individual type has been developed in this trade with the exception of the motor tanker that trades on the New York State Barge Canal and the Great Lakes. Several of the tankers in use on the

Great Lakes have seen prior service on the ocean. A typical tanker on the Great Lakes is the *Panoil* (Item No. 17, Table 2), with a speed of 10 miles per hr loaded and 12 miles light.

A large group of Diesel-powered tankers are operated on the Great Lakes by one of the more important oil companies. All these boats are of the New York Barge Canal type, whose general characteristics are a rather excessive breadth, a length of 250 ft, a draft limited to approximately 12 ft, with a speed of approximately 9 miles per hr, and a carrying capacity of approximately 11 000 bbl.

New York State Barge Canal-Boats.—Because of the peculiar physical conditions to be met, a distinct type of boat has been developed for this traffic. All these self-propelled vessels are Diesel-powered with the exception of two boats, the *Edgewater* and the *Chester* (Items Nos. 14 and 15, Table 2). A typical vessel of this type is the *Badger State* (Item No. 16), which draws 10 ft of water, loaded. The power is a Diesel engine of 930 ihp. The cabins are set through the deck with about one-half their height projecting above the main deck. The hold is divided into compartments and closed with the conventional type of hatches.

Items Nos. 14 and 15, Table 2, are more elaborate in construction than other boats in this trade. Power is supplied with two oil-fired water-tube boilers to steam turbines, developing 1 600 shaft hp. The pilot-house is on an elevating mechanism that raises and lowers it. When operating in the canal, the pilot-house is lowered to pass under the bridges, and, in the Lakes, it is raised to give better visibility.

The records show that in 1868, the total number of documented American vessels, exclusive of canal-boats, amounted to 2 543, with a gross tonnage of 454 051, averaging 178 tons per vessel. In 1935, there were 550 American vessels of 1 000 gross tons and more, with a total gross tonnage of 2 575 455, of which 441 were classed as bulk freight carriers, exclusive of tankers, having a gross tonnage of 2 231 883, or 87% of the total tonnage. Canadian vessels of 1 000 gross tons and more numbered 285, having a total gross tonnage of 733 971, of which 221 were classed as bulk freight carriers, aggregating 574 903 gross tons, or 78% of the total gross tonnage.

AIDS TO NAVIGATION

The United States Lake Survey (War Department[3]) publishes for sale and keeps up-to-date general navigation charts of each of the Great Lakes, of Rainy Lake, Lake of the Woods, the New York State Barge Canals, and Lake Champlain, and detailed charts of all harbors and connecting channels and of the St. Lawrence River from Lake Ontario to the International Boundary. It also publishes annually a *Bulletin*, supplementing information given on charts, containing detailed descriptions of shores, outlying shoals, harbors, storm warning, and Coast Guard stations, together with changes in channel conditions and buoyage. The Survey is constantly engaged in sweeping for shoals and wrecks, revising charts, and making observations of Lake levels and river discharges.

---

[3] See, Manual of Engineering Practice No. 7, Am. Soc. C. E., Fig. 1, p. 5.

The United States Bureau of Lighthouses (Department of Commerce) has installed and maintains a most modern system of lights, radio beacons, fog signals, ranges and channel buoyage throughout the American waters, harbors, and connecting channels of the Lakes and of the St. Lawrence River. Any changes in lights or buoyage or any other aids are immediately made known to navigators by that Bureau. The Canadian Government performs a similar service in Canadian waters.

The Coast Guard maintains a system of life-saving stations at all important points throughout the Lakes and a fleet of two 165-ft cutters and seventeen patrol boats for the enforcement of Federal laws, for assisting vessels in distress, and for protecting life and property upon the navigable waters of the United States. It also operates, through arrangements with commercial telephone and radio companies and its own radio, a system of communication for vessels in distress.

The United States Weather Bureau operates a system of storm warning stations and forecasts for the Lakes region especially applicable to navigation.

MAINTENANCE OF CHANNELS, CANALS, AND HARBOR WORKS

For the maintenance of channels, canals, and harbor works, and the construction of new works, the War Department maintains five Districts with District Offices in Duluth, Milwaukee, Chicago, Detroit, and Buffalo. The Lake Survey, operated as a District covering the entire Lake region, is also situated at Detroit. All District Engineers are supervised by a Division Engineer, with office in Cleveland, who reports to the Chief of Engineers, U. S. Army, in Washington, D. C. New work of constructing breakwaters or channel dredging is done by contract, and the marine contractors on the Lakes have the most modern equipment for such work.

The War Department maintains four sea-going hopper dredges for the maintenance of harbors, and sufficient other plant to do repair work on breakwaters. Each District Engineer is responsible for the maintenance of project depths in the Government channels in his District and for performing other duties relative to navigation charged by law to the War Department.

HARBOR TERMINALS

There is nothing remarkable about the package freight terminals on the Lakes. The bulk freight terminals, however, are unusual in their excellence of mechanical equipment, capacity, and efficiency. The ore-loading docks are designed with their ore pockets spaced so as to cover the hatches of vessels. The unloading docks for ore are laid out and equipped so as to take ore from vessels and load it directly to railroad car, or into stock-piles. Car dumpers for coal from rail to vessel are of the best design. The bulk terminals in general have been installed by the railroad companies and are supported by adequate and well laid out storage and switching trackage. The efficiency of the Lake system of transportation is due to a combination of economical water haul with highly efficient terminal and railroad operation, making possible the quick turn around of vessels during a seven-month navigation season.

The combined efficiency of terminals and vessels is shown by the records in Table 3. The records of the Lake Carriers' Association show that the average stay of iron-ore carriers at Lower Lake ports has decreased from 36 hr in 1906 to less than 11 hr in 1937. Corresponding figures for ore carriers at loading ports are 22.5 and 4.2 hr.

TABLE 3.—COMBINED EFFICIENCY OF TERMINALS AND VESSELS

Commodity (1)	Unit (2)	(a) Loading Vessels		(b) Unloading Vessels	
		Quantity (3)	Time, in hours (4)	Quantity (5)	Time, in hours (6)
Ore	Gross tons	12 508	0.28	12 009	2.42
Coal	Net tons	11 293	3.5	9 366	6.08
Wheat	Bushels	157 565	1	451 382	15

VESSEL OPERATORS AND CREWS

The Lake Carriers' Association was organized at Buffalo in 1892 and was incorporated in 1903. The purpose of the Corporation as set forth in the Charter is:

> "To establish and maintain assembly rooms for the use and general welfare of seamen on the Great Lakes, their connecting and tributary waters; to establish and maintain and procure the establishment and maintenance of aids to navigation; and improve and secure the improvement of channels, docks, wharves, loading and unloading and terminal facilities; to establish and maintain by contract or otherwise such amicable relations between employers and employed as will avoid the public injury that would result from lockouts or strikes in the lake carrying service; to provide for the prompt and amicable adjustment of matters affecting shipping and the interests of vessel owners of the Great Lakes and their connecting and tributary waters, and to lease, rent, purchase or sell such real or personal property as may be necessary or convenient in carrying out the foregoing purposes."

The liberality of view in this statement of purpose and the success of the Association in accomplishing it are remarkable. It has demonstrated how such an Association can be of great service to an industry by studying its problems, improving the conditions of its employees, and presenting facts to the Government agencies with which it has to deal.

The recommendations made by the Association for improvements for the benefit of shipping on the Lakes have consistently been constructive and conservative. At the time of incorporation the membership included passenger vessels, tugs, and sailing ships. On December 31, 1937, the membership included mainly the bulk freight vessels of American registry in the iron ore, coal, grain, and stone trades, as follows:

Bulk freight steamers and motor ships	311
Self-unloading bulk freighters	33
Package freighters	2
Tankers	15
Barges	13
Towing tug	1
Total	375

The total gross registered tonnage of the vessels enrolled in the Association is 2 080 850, or 81% of the total United States gross registry of commercial vessels on the Lakes. The carrying capacity for a single trip at prevailing drafts is approximately 3 000 000 gross tons.

Growth in Tonnage of Lake Vessels

The number and gross tonnage of vessels enrolled in the Lake Carriers' Association at 10-yr periods, with the change in average tonnage, are as shown in Table 4. In certain cases the Association has expended its own funds for dredging in channels, and the establishment and maintenance of lights, buoys, and lightships when the Government was not in a position to do so.

TABLE 4.—Vessels Enrolled in the Lake Carriers Association

Year (1)	Number of ships (2)	Gross registered tonnage (3)	Average per ship (4)	Year (1)	Number of ships (2)	Gross registered tonnage (3)	Average per ship (4)
1893...	612	579 919	946	1925	423	2 223 143	5 255
1905...	542	1 411 749	2 605	1935	397	2 185 822	5 500
1915...	428	2 012 593	4 700	....	..	......	....

In 1908, it established a welfare plan for the benefit of sailors, providing for assembly rooms in the principal ports, equipped with current newspapers, magazines, writing materials, and other conveniences, and for the payment of death benefits from accidents incident to employment. The Association maintains an excellent library at the "Soo" and exchanges books with its vessels as they pass through the locks. Through its Committee of Shore Captains, composed of a group of masters with long experience in lake navigation now filling executive posts, its Committee on Aids to Navigation, composed of active masters, and its Fleet Engineers' Committee, made up of former chief engineers now supervising the Engine Departments of the vessels in their respective fleets, the Association is constantly working for safer and better operation and better maintenance and construction.

It has established for its member vessels up-bound and down-bound courses in Lakes Huron, Michigan, and Superior, which also are generally followed by non-Association vessels, thus reducing the hazards of collision in fog.

For twenty years the Association has conducted schools, during the closed season, for the wheelsmen and oilers in its fleet desirous of advancement to officer rating, and for officers desirous of raise in grade. During that period 730 oilers and 544 wheelsmen obtained their original licenses, and more than

1 800 Lake seamen who attended the schools obtained the license for which they were instructed.

The scale of wages paid on Association vessels in 1937 was as follows:

	Per 30-day month		Per 30-day month
Wheelsman	$118.50	Chief cook (on vessels of more than 4 000 tons)	$183.00
Watchman	115.50	Chief cook (on vessels of less than 4 000 tons)	162.00
Deck watch	102.00	Second cook	105.00
Deckhand	87.00	Porter	87.00
Fireman and oiler	115.50		
Coal passer	81.00		

During the World War, under arrangements made with the U. S. Navy Department, 3 503 men were trained aboard the Association's vessels for positions in the Navy and the Shipping Board. In addition, the Association responded to every call from the Emergency Fleet Corporation to provide crews for the ships built or purchased on the Lakes by the Government for ocean service in the war. The offices of the Association furnished 1 934 men for the 112 vessels.

FREIGHT AND PASSENGER TRAFFIC

Now, "the proof of the pudding is in the eating," and the efficiency of any system must be judged by the results. In this sense, the record of the volume of commerce on the Great Lakes and the cost of water transportation speak for themselves.

The freight passing through the canals at the "Soo" (which is the best index of Lake transportation) was only 14 500 tons in 1855; the 500 000-ton mark was passed in 1870; in 1876 more than 1 000 000 tons of traffic utilized this route; and the peak was reached in 1929, when more than 92 000 000 tons of freight passed through the locks.

During the decade, 1926–1935, the average annual commerce of the United States Great Lakes ports was 114 837 300 short tons, which included the highest as well as the lowest tonnages in recent years, with 161 344 361 tons in 1929 and 54 913 140 tons in 1932. After eliminating all known duplications, the net traffic on the Great Lakes in 1936 amounted to 138 373 898 short tons, valued at $1 907 952 846.

Passenger traffic at the United States ports on the Great Lakes consisted largely of ferry passengers during the decade, 1926–1935, and averaged more than 19 000 000 persons, of which 58.5% arrived at, or departed from, Detroit; 20.2% were handled at Lake Erie ports; 6.6%, at Lake Michigan ports; 5.1%, at Port Huron; 5%, at localities on the St. Lawrence River; and the remainder at other Lake ports.

Some idea of the traffic on the connecting waterways of the Lakes can be had from the following data: During the 81 yr from 1855 to 1935, the St. Marys Falls Canal has transited a total of 2 394 472 474 tons of freight, or an average of 29 561 389 tons per year during its existence; the St. Clair Flats Canal, in Michigan, in 1891, transited 22 160 000 tons, which increased to a peak of 99 677 966 tons, in 1929, and, in 1935, amounted to 59 287 907 tons;

the Detroit River with 23 209 619 tons in 1891, reached its high level of 110 719 845 tons in 1929 and, in 1935, shows a total of 75 779 280 tons; the Welland Ship Canal showed a movement of 719 360 tons in 1900, and a peak of 9 280 452 tons in 1929, whereas the 1935 traffic amounted to 8 537 460 tons; the New York State Barge Canal traffic since 1837 (or during 99 yr) amounted to 363 641 280 tons, or an average of 3 673 144 tons per yr. The largest recorded traffic was moved in 1871, when 6 467 888 tons were handled, and the smallest, or 1 159 270 tons, in 1918. Since 1918 the movement has increased to its 1936 volume of 5 014 206 tons; traffic on the St. Lawrence River Canals saw its peak in 1928, when 8 411 538 tons were handled, whereas the traffic since that time has ranged from 5 718 651 to 6 951 064 tons, with a recorded total of 6 873 655 tons handled in 1933, of which 4 879 714 tons originated in Canada and 1 993 941 in the United States.

The Grain Movement.—The territory tributary to the Great Lakes is the most important grain-producing district in the world. It includes most of the surplus grain-producing States of the United States and nearly all the grain territory of Canada. This territory in 1928 produced 35.1% of the world's grain, excluding that of Russia and China, but, in 1935, production had dropped to 28.7 per cent. The sixteen States included within this territory have produced 71% of the wheat grown in the United States since 1919. The enormous quantity of grain flowing eastward is practically all shipped through the four ports of Fort William-Port Arthur, Duluth-Superior, Milwaukee, and Chicago. Shipments from the United States ports are of record from 1868, when a total of 48 585 bushels were shipped, and shipments from Fort William-Port Arthur began in 1884 with 951 bushels. The peak year in grain shipments from Upper Lake ports was in 1928 when more than 572 000 000 bushels were shipped. However, in 1935, shipments from these ports had dropped to 235 000 000 bushels. During the 16-yr period from 1920 to 1935 Fort William-Port Arthur shipped 49.6% of the grain from the four Upper Lake ports; Chicago, 25.1%; Duluth-Superior, 19.4%; and Milwaukee, 5.9 per cent. Wheat constituted 58.3% of the total grain shipments from the four ports; oats, 15.4%; corn, 12.8%; barley, 7.0%; rye, 4.8%; and flaxseed, 1.7 per cent.

Buffalo is the principal receiving port for grain at the Lower Lake ports. It received 200 000 000 bushels in 1928, and more than 104 000 000 bushels in 1935.

Port Colborne, Ont., Canada, is an important transfer point on the Lower Lakes for Canadian grain and received 20 600 000 bushels during 1935. However, since the construction of the new Welland Ship Canal and the construction of transfer elevators at Kingston and Prescott, Ont., Canada, grain en route to Montreal now goes directly to these points for transfer. Kingston received 12 000 000 bushels in 1935 direct from Upper Lake ports in deep-draft vessels, Prescott received 8 000 000 bushels, and Montreal received 51 000 000 bushels in vessels drawing not more than 14 ft of water.

The grain movement on the Great Lakes has changed considerably since 1928. The natural flow of grain has been from west to east, but reduced grain crops during the droughts of 1934, 1935, and 1936 resulted in shipments of grain from the seaboard to the grain-growing sections of the West. During 1928 there were shipments from the seaboard to the Great Lakes *via* the New York State Barge Canal of 230 688 bushels. In 1934, 1935, and 1936, the shipments by this route were 7 452 622, 9 620 572, and 3 671 361 bushels, respectively. In the same years the westbound shipments through the Welland Canal amounted to 11 662 490, 9 214 453, and 9 268 284 bushels, respectively.

In 1937, the westbound movement through the New York State Barge Canal amounted to 3 220 192 bushels and through the Welland Ship Canal to 32 101 679 bushels, a total of 35 321 871 bushels. The receipts at the Upper Lake ports totaled about 30 000 000 bushels and included 21 500 000 bushels of corn from Argentine Republic delivered at Chicago and Duluth-Superior and 1 239 477 bushels of flaxseed from Argentine Republic delivered at Duluth-Superior.

Iron Ore.—More than 1 600 000 000 tons of iron ore have passed down through the St. Marys Falls Canals since 1855. The peak year in the ore trade was 1929 when more than 72 000 000 short tons were handled. In 1936, the ore movement by Lakes was 50 176 000 short tons, of which 47 000 000 short tons passed through the United States locks at the "Soo" from the Superior ranges. The receipts of ore at Lower Lake ports in 1936 were as follows:

Port	Short tons	Port	Short tons
Detroit	1 720 000	Ashtabula	5 017 000
Toledo	1 693 000	Conneaut	7 586 000
Huron	789 000	Erie	1 721 000
Lorain	3 078 000	Buffalo	4 170 000
Cleveland	10 149 000	Port Colborne	77 000
Fairport	1 006 000	Hamilton	521 000

and at Lake Michigan ports there was received:

Port	Short tons
Indiana Harbor	2 527 000
Gary	3 630 000
South Chicago	6 016 000

The movement of ore from Erie ports to interior consuming furnaces is an all-rail movement. The greater bulk of the ore is loaded direct from ship to car. Cleveland, Conneaut, and Ashtabula, Ohio, are the leading Lake ports for shipment to the interior.

Coal.—In 1936, 44 000 000 net tons of bituminous cargo coal were carried, which was 6 000 000 tons more than had ever before been moved on the Lakes in a single season. In addition, the car dumpers put 1 430 000 tons of fuel into the bunkers of vessels which were loading coal. For the first time a single port —Toledo, Ohio—has loaded more than 20 000 000 tons of cargo coal. Sandusky, Ohio, loaded 9 355 000 tons. Shipments from these two ports were almost entirely coal from Southern mines. Shipments of bituminous coal from Lake Erie ports to, or through, the Welland Ship Canal, to a great extent to Hamilton and Toronto, Ont., Canada, reached 3 000 000 tons for the first time.

Distribution of Lake soft coal by districts was as follows:

District	Tons
Lake Superior	11 331 000
Lake Michigan	14 738 000
Lake Huron and Georgian Bay	1 918 000
Lower rivers	6 730 000
Lake Erie	5 057 000
Welland Canal	3 035 000

During 1937, 674 000 tons of anthracite were handled on the Lakes. The value for 1917 was 4 689 000 tons.

Limestone.—In 1936 the Lake movement of limestone amounted to 12 080 000 net tons.

Car-Ferry Traffic.—The first cross-lake service was inaugurated in 1892 between Frankfort, Mich., and Kewaunee, Wis. Since that time this method of transportation has increased until there are three lines operating on Lake Michigan, one on the Straits of Mackinac, three across Detroit River, between Detroit and Windsor, Ont., Canada, one each on Lakes Erie and Ontario, and one across the St. Lawrence River at Ogdensburg. These ten lines operate a total of thirty-five car-ferries, thirty-one of which are in active service. All the car-ferries, except those on Lake Erie, operate during the entire year, trips on Lake Erie being made as traffic demands. During the calendar year 1935, car-ferry traffic on Lake Michigan amounted to more than 3 000 000 tons.

A comparison of the traffic through the Suez, Panama, and St. Marys Falls Canals and of the commerce of three Lake ports is given in Table 5.

TABLE 5.—TRAFFIC OF MAJOR CANALS AND LAKE PORTS

Canal or port	1928		1929		1932		1935	
	Short tons*	Rank among United States ports	Short tons*	Rank among United States ports	Short tons*	Rank among United States ports	Short tons*	Rank among United States ports
(*a*) TONNAGE THROUGH THE SUEZ, PANAMA, AND ST. MARYS ("SOO"), CANALS								
Suez	31 905 902	..	33 466 014	..	26 049 554	..	29 021 354	..
Panama	33 022 935	..	35 320 308	..	20 334 758	..	27 746 588	..
St. Marys	86 992 997	..	92 622 017	..	20 480 873	..	48 293 308	..
(*b*) COMMERCE OF LAKE PORTS								
Duluth-Superior	53 204 000	2	60 385 000	2	10 520 000	12	29 185 000	3
Buffalo	21 006 000	5	19 110 000	7	11 146 000	9	13 396 000	10
Toledo	18 539 000	6	19 717 000	6	10 790 000	10	19 631 000	5

* 2 000 lb

Ore.—During 1937 rates on iron ore from the mines at the head of Lake Superior to Lake Erie ports were as follows per gross ton:

Rail to Duluth-Superior and Two Harbors	$0.81
Dumping at dock	0.10
Carriage by vessel to Lake Erie ports and unloading charge to vessel rail	0.80
Vessel rail to car	0.08
Rail to Pittsburgh, Pa.	1.15
Rail to Youngstown, Ohio	0.82

Adding the terminal charges to the water haul gives $0.98 from port to port as against $1.96 for the two rail hauls for ore delivered at Pittsburgh, and $1.63 for ore delivered at Youngstown.

The 1936 vessel rates on coal from Lake Erie ports were as follows:

	Per net ton
To principal ports on Lake Superior	$0.40
To principal docks at Milwaukee	0.50
To principal docks at South Chicago and Indiana Harbor	0.50
To ports on Georgian Bay	0.35

Grain rates vary throughout the season, depending upon the volume of movement and competition for cargoes. The 1936 rates from Duluth-Superior to Buffalo fluctuated between 1.75 cents and 3 cents per bushel. Canadian grain rates from Fort William fluctuated between 3.5 cents and 5 cents to Montreal and between 1.5 cents and 1.75 cents to Buffalo, except for lake loadings when the rate jumped as high as 5 cents.

Cost and Savings

The total expenditures for new work by the War Department on the channels, canals, and harbors of the Great Lakes to June 30, 1937, is $193 573 686.83. It has been estimated by engineers outside the Government service that the savings in freight charges due to water transportation on the Great Lakes are considerably in excess of $200 000 000 per yr.

Conclusion

The existing system of industry and transportation which has been built up around water carriage on the Great Lakes in the last century is the result of the co-operative efforts of the Federal Government, the railroad companies, industries, municipalities, and vessel owners and operators during that entire period. In each decade the facilities provided for navigation and at the terminals have been reasonably adequate for the number and class of vessels in the Lake trade and for the tonnage moved. They have never been overbuilt; nor has money been spent extravagantly far ahead of requirements. As one examines the record, one must be impressed with the fact that the present results have been obtained by the application of good engineering practice and sound business judgment, and by planning for the future only as far as definite benefits could be foreseen.

DISCUSSION

T. KENNARD THOMSON,[4] M. AM. SOC. C. E. (by letter).—The French and then the English managed to reach Lewiston and Queenston, on the Niagara River, by water transportation with a few land portages, between Montreal and Lake Ontario (St. Lawrence River).

They encountered the first formidable portage, a climb of more than 300 ft and a land haul of about 30 miles, from Lewiston to near Buffalo. This afforded the main route for trade (chiefly with Indians) until the Canadians constructed the Welland Canal. The construction of the Erie Canal was due to the untiring and unselfish efforts of De Witt Clinton (1769–1828) who contributed his time and money without stint. No one else did nearly as much for the project. He started the construction of the Canal in 1817, and completed it in 1825.

The Erie Canal, as then built, was 4 ft deep, 40 ft wide, and was used by boats of from 30 to 70 tons. By 1825 it had earned its entire cost of $7 143 789, and, at the same time, greatly reduced the freight rates, thereby enormously increasing the population and wealth of the State. By 1862 the Canal had been deepened to 7 ft for boats of 240 tons. The enlargements from 1837 to 1862 cost $31 834 041. Work was started to increase the depth to 9 ft, but it was not completed.

The cost to 1899 was $48 000 000. In 1881 all tolls on the Canal were abolished. Prior to that time the net revenue had exceeded all costs by $42 000 000. The Barge Canal was started in 1904 and completed to a depth of 12 ft in 1918, at a cost of $170 000 000. Prior to 1908 the revenue from the Canal had been about $360 000 000, or $20 000 000 more than the total cost to that date.

Under the heading, "Shipping: New York State Barge Canal-Boats," General Tyler states that "because of the peculiar physical conditions to be met, a distinct type of boat has been developed." The writer presumes that he refers to the limited height under the bridges, which make it a "Barge Canal."

The reason for changing the Erie Canal to a "Barge Canal" was that the politicians were afraid that New York, N. Y., and Buffalo, N. Y., would vote against the proposed enlargement, if ships could pass through the Canal without being unloaded and loaded at Buffalo and New York. Hence a "Barge Canal" was constructed, with fixed bridges that make it impossible for any boat standing 15.5 ft above the surface of the water to pass under the bridge.[5]

Another mistake made on the Barge Canal was in not relocating the line near Rome, N. Y., so that it would be unnecessary for all boats from Buffalo to "lock up" about 50 ft, before "locking down" again. As it is, water from Lake Erie cannot reach the Hudson River, and it was necessary to create great artificial lakes, by dams at Delta, N. Y., Hinckley, N. Y., etc., to supply water for the Canal east of the hump.

---

[4] Cons. Engr., New York, N. Y.

[5] *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 867.

For many years there has been talk about constructing a ship canal from the Hudson River to Lake Ontario or Lake Erie, so that ocean-going vessels could reach the Great Lakes. Before the depression (1930) it was estimated that such a ship canal would cost $650 000 000. As the Americans can use the new Canadian Welland Canal—the best route for the ship canal would be from the Hudson River to near Oswego, N. Y., on Lake Ontario.

General Tyler states that the entrance to Buffalo Creek was improved in 1826, and that the Government breakwater in front of Buffalo Creek was constructed in 1869 (see heading, "Channels and Harbors," Improvements (6) and (7)). As Buffalo still has a very inadequate harbor, the writer has, for many years, been advocating the reclamation of from 6 to 10 sq miles from Lake Erie, just outside the Government breakwater, for a great railroad and shipping terminal.

As Buffalo will eventually extend from Lake Erie to Lake Ontario, it will then have two ports for ocean-going vessels, one at Elevation 242, and the other at Elevation 576, above sea level.

As the author states (see heading, "Channels and Harbors"), the enlargement of the Welland Canal, with a depth of 25 ft of water was completed in 1932. Many wonder why this well-constructed canal was built to such a depth before the St. Lawrence River was developed, when boats drawing less than one-half that depth of water could not reach Lake Ontario from Montreal, and could not enter many of the Lake ports. Some say that it was due to "keeping sub-election promises."

As stated in the paper, the important commerce over the Barge Canal now consists of coal, ores, wheat, etc. In the earlier days it was mostly for Indian trade which was for many years controlled by the Great Hudson Bay Company.

It is now claimed that ores can be delivered in New York Harbor, *via* the Lakes and the Barge Canal, at a smaller freight charge than they can be delivered in Pittsburgh.

The indispensable test of all construction should be the question: "Will it pay?" The only way in which this condition can be met in developing the navigation facilities of the St. Lawrence River, to permit ocean-going vessels to reach Lake Superior, will be by developing the water power of the Niagara and St. Lawrence Rivers.

For twenty-seven years, the writer has been advocating, a dam ("Niagara Falls Junior") to be located about 4.5 miles down the river from the Old Falls, or below the Whirlpool.[6] For about the same time, he has also been advocating the reclamation of about 10 sq miles from the St. Lawrence River, between Lachine and Montreal, Que., Canada, and the development of 1 500 000 hp, hydro-electric, at that site.

If these projects are constructed economically, as they should be, industries will be attracted sufficient to create a dozen Pittsburghs along the banks of the Niagara and St. Lawrence Rivers, bringing great financial prosperity to the entire continent of North America and benefitting many other parts of the world.

---

[6] "Niagara Falls, Jr.," an interview with T. Kennard Thomson by G. P. Roney, *The Banker and Financier*, September 15, 1921.

HARLAND C. WOODS,[7] M. AM. SOC. C. E. (by letter).—In connection with this paper, it is of interest to note how the developments in bulk freighters have influenced the design of locks at Sault Ste. Marie, Mich. In the early days, freight carriers were general cargo ships suitable for navigating the relatively shallow connecting channels of the central Great Lakes. With the development of the ore, grain, coal, and limestone traffic, the bulk freight carriers were gradually evolved. At first, the dimensions of vessels were gradually increased in conventional ocean-going craft proportions. As the bulk freighter type of vessel emerged, widths were limited by the distance from the face of the dock within which it has been found economically practicable for the unloading machines to operate. This has been found to be less than 70 ft. Structural strength of the vessel limits the depth of hull proportionally to the width. For large vessels the maximum has been found to be about 32 ft. With width and depth defined, the length of vessels has been limited by structural requirements and practical thickness of deck plates to about 600 ft to enable a fully loaded vessel to "ride out" Great Lakes storms safely.

The grain and ore movement involves passage through the St. Marys River and the locks at Sault Ste. Marie. As vessels increased in size, larger locks were built. Fig. 2 indicates, chronologically, the parallel development of vessels and locks. Experience has shown that the lockage of more than one vessel at a time is most expeditious and safe if vessels are tied stem to stem in the lock rather than side by side. Hence, the latest locks built are slightly wider than the maximum width of vessel and long enough to accommodate two of them. Until new developments in trade, vessels, and cargo handling equipment appear, there is little prospect of changes in dimensions of vessels and locks.

W. L. R. HAINES,[8] M. AM. SOC. C. E. (by letter).—An able and concise presentation of an historical and statistical review of transportation on the Great Lakes has been presented by General Tyler.

One factor contributing very largely to the efficient and economical development of freight transportation—particularly of ore and coal—on the Lakes is deserving of special emphasis; that is, the great flexibility of the transportation system, combining water haul with rail haul. This is made possible by the development of numerous transhipping docks along the southern shore of Lake Erie, by the railroads. Such facilities offer shippers a wide range of choice as to rail routes between Lake Erie and the consuming or producing districts. They make it possible to divert vessels or cars from one transhipping dock to another in order to avoid delays to vessels which might result from congestion or break-down of facilities at a given dock.

Those interested in the location and ownership of these docks are referred to a report prepared by the Board of Engineers for Rivers and Harbors in 1937.[9]

---

[7] Senior Engr., U. S. Engr. Office, Buffalo, N. Y.

[8] Special Engr., Office of Vice-Pres., P. R. R., Pittsburgh, Pa.

[9] "Transportation on the Great Lakes," prepared by the Board of Engineers for Rivers and Harbors, *Transportation Series No. 1* (revised 1937); published by the U. S. Government Printing Office, Washington, D. C.

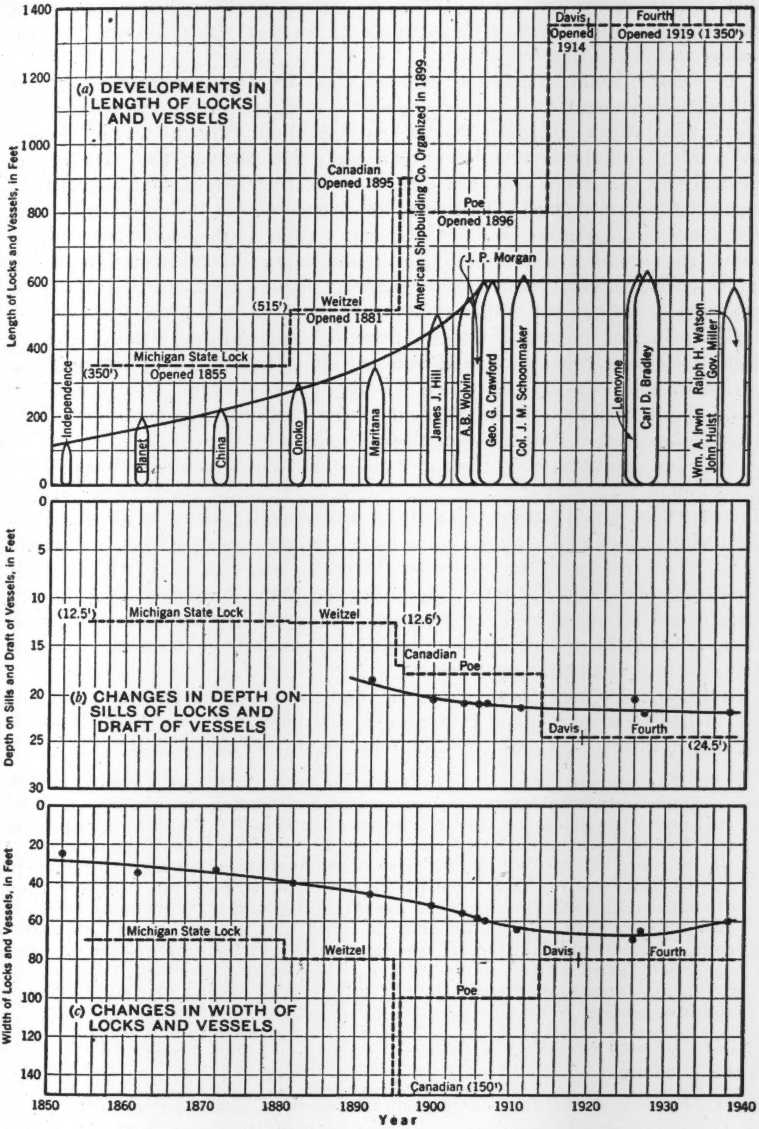


FIG. 2.—CHRONOLOGICAL DEVELOPMENT IN DIMENSIONS OF LAKE BULK FREIGHTERS AND LOCKS AT SAULT STE. MARIE, MICH.

This and an earlier edition of the same report (dated 1930) contain maps devised to illustrate the flow of ore and coal between Lake Erie and the consuming and producing districts.

For many years there has been agitation for the construction, by the Federal Government, of a canal between Lake Erie and the Ohio River, the most favored route being from a point near Ashtabula, Ohio, by way of Youngstown, Ohio, and the Beaver River to a point near Beaver, Pa. The cost of constructing this canal has been estimated by the Government engineers at about 203 million dollars. This sum, however, does not include facilities and equipment for operating and maintaining traffic on the canal; nor does it include the entire cost of rearranging railroad bridges and tracks made necessary by its construction. Experienced engineers have estimated that the total cost would be about 281 million dollars.

The principal purpose of the canal would be the transportation, in barges, of ore from Lake Erie and of coal from the Ohio River. This tonnage is now carried by the railroads and would be diverted from them. One effect that would result would be an impairment of the flexibility of operations on the Great Lakes, and of the service of the railroads.

It is inconceivable that the railroads could afford to maintain all of the transhipping docks now in existence. Those at Ashtabula Harbor and Fairport Harbor, Ohio, would almost certainly be abandoned, because of the diversion of tonnage to the canal. In the opinion of experienced railroad men in positions to judge accurately, 17 ore unloaders, 3 ore docks with supporting storage, 4 coal unloaders, and possibly 1 grain elevator and 1 warehouse at Lake Erie ports would be abandoned for the same reason. With the abandonment of these facilities, the opportunity for diversion of vessels from one dock to another would be destroyed to a great extent.

Another important factor aiding in the expeditious movement of ore and coal in Lake transport is the "Ore and Coal Exchange." Mr. E. A. Burslem, Examiner, Interstate Commerce Commission, states[10] that this is:

> "* * * a central agency established and maintained by the railroads with all their individual rights as to operation, dealing with shippers, and customary practices of the individual railroads * * *. The expenses of the Exchange are borne by the railroads * * *." The Exchange is for the purpose "of providing the best possible services to lake shippers; to furnish information as to coal at the ports, in transit, and the operation of coal machines at the docks; to supervise the supply, movement, routing, and delivery of ore to the furnaces; to establish contacts between railroads, coal shippers and lake vessel owners; and to furnish such other services as would facilitate the movement of coal and ore, thereby reducing the detention of cars, increasing the car supply, and adding to the facilities of the lake traffic."

Mr. Burslem cites the following as some of the more important functions of the Exchange:

(1) To issue permits regulating the movement of the lake coal to the ports; (2) to receive reports from all docks west of Buffalo of coal at the ports and the number of cars of coal in transit and due in the ports in the following

---

[10] Proposed report in Interstate Commerce Commission Docket No. 27266.

12-hr, 24-hr, 36-hr, and 48-hr periods for each transhipper or consignee, with the number of cars in each consignment; (3) to issue daily reports showing the quantity of coal at each dock, the quantity in transit, previous day's dumping and estimated dumping for the current day; (4) to keep account of the various vessels unloading at the Lake Erie ports which are named for coal loading, and to ascertain the time the boats will be unloaded and ready to sail, so that the dock at which the vessel will receive its lake-coal cargo will be advised of the time of the vessel's probable arrival in order to get coal to the docks and not delay the loading of the vessels; and (5) to receive orders from transhippers for loading coal into vessels named by the transhippers, and to transmit such information to the docks. The Exchange keeps all interested lake-coal shippers, transhippers or consignees, and carriers informed as to the situation, daily, so that they may co-ordinate their efforts and thus effectively permit the unloading of cars and the dispatching of vessels.

Many of the vessels transporting coal from the lower Lake Erie docks come to those docks loaded with ore. Thus the speed of "turn-around" is affected by both the unloading and loading time. The provision, by the railroads, of ground storage for enormous quantities of ore makes it possible to discharge the ore cargoes even if there is not immediate demand for shipment to the furnaces. Furthermore, this ground-storage makes possible the shipment of ore to the furnaces as it is needed after the close of the Lake season.

The reduction of the average stay of iron-ore carriers at Lower Lake ports from 36 hr (in 1906) to less than 11 hr (in 1937), mentioned by the author, is largely due to the capacity and efficiency of the docks owned by the railroads, the efficient service of the railroads themselves. and the service of the Ore and Coal Exchange, maintained by and at the expense of the railroads.

The writer objects to the implication which might arise from the comparison of Lake rates and rail rates on ore, given by the author. The statement is made that the "water haul" is $0.98 per gross ton from port to port as against $1.96 for the two rail hauls (that is, from the mines to the Lake and from the Lake to destination) for ore delivered at Pittsburgh and $1.63 for ore delivered at Youngstown (see heading, "Freight and Passenger Traffic: Ore"). The inference might be drawn that rail rates were excessive. Such a comparison is scarcely fair.

The author states (see "Cost and Savings") that the total expenditures for new work by the War Department on the channels, canals, and harbors of the Great Lakes to June 30, 1937, were $193 573 686.83. In addition to this, the report of the Chief of Engineers, U. S. Army, for 1937, lists an expenditure of $66 940 948 for maintenance and of $174 061 "not distributed"—a total of $67 115 009 for the five Lake engineer districts (that is, Chicago, Milwaukee, Duluth, Detroit, and Buffalo) to the same date. Thus the total expenditure amounts to $260 688 696, making an annual interest charge, at 5%, of $13 034 435. During the decade 1926 to 1935 the average annual commerce of the United States Great Lake ports is stated by the author to have been 114 837 300 short tons (see "Freight and Passenger Traffic"). The resulting cost amounts to $0.114 per short ton.

Reducing the charges per gross ton given by the author to terms of short tons, and adding to the water haul the cost (borne by the taxpayers) per short ton resulting from Government expenditures, gives the following:

Water haul charges	$0.875	
Interest on Government expenditures	0.114	
Total		$0.989
Rail Haul Charges:		
Mines to Lake ports		0.741
Lake ports to Pittsburgh		1.027
Lake ports to Youngstown		0.732

It may be assumed that the sums reported as spent for new work and for maintenance are actual costs, without any addition for interest. Had interest been added to actual expenditures from year to year and the resulting totals carried forward to show the total cost to any given date, the charges would be vastly increased.

There should also be added the costs to the Government incurred by the U. S. Lake Survey, the U. S. Bureau of Lighthouses, the U. S. Coast Guard, and by the U. S. Weather Bureau in its operation of storm-warning stations and forecasts for the Lakes region, especially applicable to navigation. These services are all mentioned by the author, but without including any costs.

It is the opinion of the writer that it is scarcely proper to combine the two rail hauls; that is, from the mines to the Lakes and from the Lakes to destination in the manner done by the author. These two steps are performed under very dissimilar conditions.

The Board of Engineers, cited previously,[8] called particular attention to the contribution of railroads in the rapid development of the various ore ranges (or mines). A special type of carrier was developed in order to handle ore from mine to vessel economically and with the greatest dispatch. The fact that there is a wide divergence in the chemical constituents of ores makes sampling and mixing necessary in order to create a certain grade. Furthermore, the ore vessels must be loaded promptly with a fixed tonnage, the chemical content to be always within a small fraction of 1% of the analysis on which it was sold. All of these factors, according to the Board, "do not allow much flexibility in the rail transportation of ore."

The special equipment provided, and the complexity of movement from the mines to vessel, mean additional expense, which must be taken into consideration in any comparison of rail rates. The movement of ore from the lower Lake Erie ports to the mills is less complicated. Special equipment is not required, but the movement is through a more congested district, and requires close supervision by the railroads to insure prompt delivery to the mills.

The operations of blast furnaces require specific mixes of ore which must be met by shipment of the grades of ore as needed. Many of the mills have little or no storage space and must depend upon incoming shipments to meet day-to-day requirements. Frequently it happens that emergency movements of ore must be made by the railroads to meet the demands of the mills.

The maintenance and operation of the facilities required for the movement of a cargo from Lake Erie to its destination are all items of expense in addition to the "line haul cost" itself, but they are included in the rate charged. On the other hand, the vessel movement from port to port is a comparatively simple one. Once the cargo is loaded, the vessel makes a continuous voyage to its destination, carrying, in some instances, nearly 15 000 gross tons of ore. For the years 1928 to 1936, inclusive, the average cargo was 9 331 tons, and the average maximum cargo 13 489 tons.

Data are not available to the writer to show the total expenditures by the railroads in providing transhipping facilities. However, the report of Mr. Burslem[9] shows an Interstate Commerce Commission valuation of the coal-loading facilities alone, as of December 31, 1935, with depreciation allowed, amounting to more than $13 000 000. To this sum should be added the value of the ore-unloading facilities, of the ore-storage areas, and of the supporting yards for both ore and coal.

The writer does not take any issue with the "Conclusion" stated by the author. He merely wishes to emphasize the part played by the railroads in the development of Lake transportation, to indicate certain "hidden costs" and to anticipate any erroneous conclusion that might be drawn as to relative costs to the public of Lake or rail transportation.

L. C. SABIN,[11] M. AM. SOC. C. E. (by letter).—The co-ordination of effort by the United States, the shipping interests, and the railroads to build up a most efficient system is emphasized in this paper. Not since the Annual Presidential Address[12] of the late Alfred Noble, Past-President, Am. Soc. C. E., in 1903 has the subject been covered with such thoroughness and understanding. Little comment remains to be made, and that only to emphasize some details regarding the extent and value of the system.

The author defines the value of water transportation in times of emergency. Even in the French and Indian Wars and in the struggle of 1812, before any improvements had been made, the waterway served a military purpose. In the Civil War the supplies for armament were made available by water service, whereas in the World War a stoppage of the flow of commerce on the Great Lakes, involving food as well as raw materials for munitions, would have been most serious. The comparative isolation now enjoyed by the system is not without its value as a feature of defense in future possible emergency.

Bearing in mind the contribution of this system of transport, attention may be called to the fact that the expenditure of the United States in improving channels and harbors—less than 200 million dollars—is equal to the cost of only three battleships; and, while it is held in readiness during peace time, for service in emergency, the system is returning annually in savings the full cost of the development.

The author mentions the practice, followed in channel improvement, of keeping the up-bound and down-bound channels separate wherever possible.

---

[11] Vice-Pres., Lake Carriers' Assoc., Cleveland, Ohio.

[12] *Transactions*, Am. Soc. C. E., Vol. L, June (1903), p. 327.

A similar system is in effect in the open lakes where on Lakes Superior, Michigan, and Huron the Lake Carriers' Association has designated separate lanes for up-bound and down-bound traffic. Adherence to these lanes is general on the part of Lake Carriers' ships, and many others follow them habitually. Their use has resulted in greatly reducing the chances of collision.

The lengths of the sailing courses from Duluth and Chicago to Buffalo are 986 miles and 893 miles, respectively. Deep-water navigation extends to Ogdensburg, N. Y., and Prescott, Ont., Canada, on the St. Lawrence River, adding 230 miles to these distances. The improved reaches in the connecting channels total about 139 miles in length.

Readiness of the ship operators to co-operate with the United States is shown by the fact that, as the improvements progressed, vessels were built to take full advantage of the better facilities. Prior to the opening of the Poe Lock in 1896 there were no cargoes exceeding 5 000 tons. Nine years later there were 191 vessels that carried more than 5 000 tons, and cargoes of 12 000 tons were being carried. Prior to the opening of the Davis Lock about 14 000 tons was the maximum cargo, whereas subsequently, cargoes of 17 000 tons were carried.

The U. S. Lighthouse Service has contributed generously to the safety of navigation. More than 900 lighted aids and more than 200 fog signals are maintained. There are also in operation 41 radio-beacon stations at important points, and all vessels enrolled in the Lake Carriers' Association are equipped with radio direction finders, the most important recent contribution to safe navigation.

Radio telephones are in use on 80 United States vessels and 50 of Canadian register. These have proved so satisfactory as a means of communication from ship to shore and ship to ship that their general use seems only a matter of time. The U. S. Lighthouse Service in co-operation with the U. S. Weather Bureau, U. S. Coast Guard, and U. S. Hydrographic Office broadcasts information regarding weather and other matters affecting navigation; and the U. S. Coast Guard Service contemplates full co-operation through supplying radio equipment to a large number of life-saving stations on shore.

In the further interest of safe navigation load lines are marked on all ships limiting the depth to which they may be submerged. In channels of limited depth current drafts are recommended by the Lake Carriers' Association according to the progress of dredging and the prevailing water level.

Among the ports on the Great Lakes there are 18 having an annual traffic in excess of 3 million tons. On the Atlantic Coast there are 13 such harbors, and 7 on the Pacific. The net total United States Commerce of the Great Lakes, excluding that pertaining only to Canada, has averaged 117 million tons annually for the ten years 1928–1937. The foreign commerce of all ocean and gulf ports for the same period averaged 81 million tons. When computed in ton-miles of movement the traffic is still more impressive. In 1936 the ton-mileage on the Lakes was 77 billion, whereas that of all other inland waterways was 15 billion. Another view of the volume of traffic may be gained by the statement that during the seasons of navigation covered by the six years from 1924 to 1929 ships passed Detroit on the average of once every 12 min with an average cargo of 3 300 tons.

Freight rates are given for ore and coal. Reduced to net tons the cost of ore transport, exclusive of terminal charges, is 71.4 cents per ton. The length of haul varies from 280 miles (a small tonnage from Escanaba to the south end of Lake Michigan) to 986 miles (Duluth to Buffalo) with an average of about 800 miles. The ton-mile rate is thus about 0.9 mill.

On coal from Lake Erie to Duluth the rate is 45 cents per net ton for 800 miles, which gives a ton-mile rate of 0.56 mill. The rate to Milwaukee on Lake Michigan is 55 cents (a trifle higher than Lake Erie to Duluth because of lack of return loads), giving a ton-mile rate of about 0.65 mill. Shorter hauls from Lake Erie to near-by ports carry a higher ton-mile rate. The combined rates on 87 million tons passing St. Marys Falls Canal in 1937, including package freight, is given as 1.07 mills per ton-mile. This low cost of transportation is not found in any other large transportation system.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2060

TESTS ON BUILT-UP COLUMNS OF STRUCTURAL ALUMINUM ALLOYS

BY MARSHALL HOLT,[1] ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. ALFRED S. NILES, AND MARSHALL HOLT.

SYNOPSIS

Results from tests on fifteen built-up column specimens of structural aluminum alloys and on two of structural steel are presented in this paper. The investigation was a study of the effect of the shape of section on the strength of built-up columns. All specimens had maximum slenderness ratios of about 40 and cross-section areas of 7 or 8 sq in. Strains and deflections were measured under various loading conditions. In the range of working loads the shape of the cross-section has no significant effect on the ratio of the measured to computed deflections of the axis of the specimen nor on the magnitude of the transverse shear indicated by the lacing-bar stresses. The shape of the cross-section has a significant effect on the ratio of the measured to computed stresses, the local deflections under higher loads, and on the type of failure, whether by buckling of a component part or by bending as a unit. Short built-up columns of high-strength aluminum alloys are likely to fail by local buckling at average stresses less than the yield strength of the material when the width-to-thickness ratio of outstanding plates and legs of angles exceeds about 12 and when the width-to-thickness ratio of the cover-plates exceeds about 40.

INTRODUCTION

Structural designers are always confronted with the problem of determining the strength of compression members. These members may be single units (structural shapes), or they may be built-up of a number of such shapes. Considerable work, both analytical and experimental, has been done in the study of the strength of columns. It has been established that the strength of the member as a column is affected by its stiffness and the compactness of the cross-section, as well as by the strength of the material. Columns acted upon by axially applied loads might fail at stresses considerably less than the

---

NOTE.—Published in November, 1938, *Proceedings*.

[1] Research Engr., Aluminum Research Laboratories, New Kensington, Pa.

yield strength of the material by sidewise bending because of lateral instability of the member as a whole, or by twisting or buckling of a component part because the parts themselves are not sufficiently rigid. The solution of the problem then lies in knowing not only the fundamental behavior of columns, but also the effects of the shape and proportions of the cross-section of the column upon its compressive strength.

The results of many tests on steel columns can be found in the technical literature. These test results form the basis for specification requirements. The tests discussed in this paper were made as part of a research program to acquire information for the establishment of a set of specifications for the structural use of the high-strength aluminum alloys.

Notation.—The letter symbols used in this paper are defined where they first appear and are assembled for reference in the Appendix.

DESCRIPTION OF SPECIMENS

Fifteen of the seventeen specimens used in this investigation were fabricated of high-strength aluminum alloys having mechanical properties comparable with those of mild steel. The other two specimens were of structural steel and duplicated two of the aluminum alloy specimens in size and shape of the cross-section. Each of the specimens had a greatest slenderness ratio of about 40 and an area of 7 or 8 sq in.

Table 1 contains a description of the specimens. Each piece of material was given a number (Column (8), Table 1) by means of which specimens for determining the mechanical properties of the materials were identified. The weight of the details, such as rivets, lacing-bars, and batten-plates, is the difference in the weight of the complete specimen and that of the main members before the rivet holes were punched. The length was determined after the ends had been carefully machined by turning the specimen on centers in a lathe. The gross area was calculated from the weight and length of short pieces and the nominal specific gravity of the material, 2.79 for Items Nos. 17 S and 25 S, and 7.85 for steel. The aluminum alloys were in the heat-treated condition.[2] The radii of gyration were computed on the assumption of unified action of the main members. The slenderness ratios of the specimens as a whole are given for the two principal axes, as is the slenderness ratio of the segments, which is the distance between two adjacent rivets divided by the radius of gyration of the part (see Columns (17), (18), and (19), Table 1).

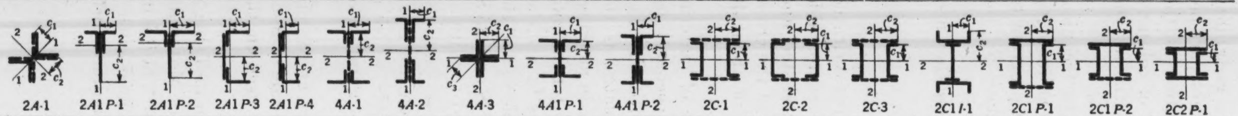
In order to emphasize the effect of the shape of the cross-section and to obtain a suitable area of cross-section, the component parts used in some of these specimens are thinner than allowed by specifications commonly used in the design of structural steel.

The mechanical properties of the materials in tension are given in Table 2. The tests were made on A. S. T. M. (American Society for Testing Materials) standard tensile specimens having a rectangular section and a width of 0.5 in. The stress-strain relations of the aluminum alloys were determined with a mirror-type extensometer, using a 2-in. gage length, whereas a modified form of the bell-crank type of extensometer was used with the steel.

---

[2] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), Table 7, p. 1279.

TABLE 1.—DESCRIPTION OF TEST SPECIMENS; BUILT-UP COLUMNS



SPECIMENS: No.* (1)	SPECIMENS: Type† (2)	DISTANCE TO EXTREME FIBER, IN INCHES: c_1 (3)	DISTANCE TO EXTREME FIBER, IN INCHES: c_2 (4)	MAKE-UP: Piece (5)	MAKE-UP: No. (6)	MAKE-UP: Size, in inches (7)	Piece No. (8)	WEIGHT, IN POUNDS: Main member (9)	WEIGHT, IN POUNDS: Complete specimen (10)	WEIGHT, IN POUNDS: Details (11)	Percentage of weight in details (12)	Length, in inches (13)	Gross area from weight, in square inches (14)	RADII OF GYRATION: k_1 (15)	RADII OF GYRATION: k_2 (16)	SLENDERNESS RATIO: Column: Axis 1 (17)	SLENDERNESS RATIO: Column: Axis 2 (18)	SLENDERNESS RATIO: Segment (19)	RATIO OF WIDTH TO THICKNESS: Outstanding legs (20)	RATIO OF WIDTH TO THICKNESS: Plates (21)	Rivet spacing parallel to stress (22)	Length of lacing-bar (23)
2A-1	17S	3.54	3.67	Angles‡	2	5×5×⅜	156	56.8	63.5	6.7	10.6	79.3	7.00	1.92	2.40	41.3	33.0	28.3	13.3	..	....	..
2A1P-1	25S	3.63	7.32	Angles Plates	2 1	5×3½×$\frac{5}{16}$ 10×¼	132, 133 144	37.4	38.1	0.7	1.8	49.4	7.67	1.14	2.66	43.3	18.6	..	11.2	25.2	9.6 t_a = 12 t_p	..
2A1P-2	25S	5.13	7.82	Angles Plates	2 1	5×3½×$\frac{5}{16}$ 10×¼	132, 133 168	57.1	58.5	1.4	2.4	75.0	7.67	1.89	2.70	39.7	27.8	..	16.0	31.2	9.6 t_a = 12 t_p	..
2A1P-3	25S	4.00	5.00	Angles Plates	2 1	5×3½×$\frac{5}{16}$ 10×¼	130, 131 143	46.9	48.5	1.6	3.3	61.4	7.68	1.52	3.91	40.4	15.7	..	16.0	24.0	9.6 t_a = 12 t_p	..
2A1P-4	25S	3.00	5.00	Angles Plates	2 1	5×3½×$\frac{5}{16}$ 10×¼	130, 131 143	28.1	29.0	0.9	3.1	37.5	7.68	0.94	3.52	39.9	10.7	..	11.2	12.0	9.6 t_a = 12 t_p	..
4A-1	25S	4.25	6.00	Angles	4	4×3×¼	134, 135 162,‡ 163‡	50.1	68.5	18.4	26.9	72.8	6.82	1.93	5.36	37.7	13.6	16.4	16.0	..	....	40 t
4A-2	25S	3.25	6.00	Angles	4	4×3×¼	134, 135 162, 163	36.9	54.0	17.1	31.7	53.6	6.82	1.31	4.95	40.9	10.8	12.5	12.0	..	....	32 t
4A-3	17S	4.00	4.00	Angles	4	4×4×½	160, 161‡	53.8	55.0	1.2	2.2	67.6	7.87	1.63	1.63	41.5	41.5	..	8.0	..	12 t	..
4A1P-1	17S 25S	3.13	4.00	Angles‡ Plates	4 1	3×2½×¼ 8×¼	157, 167 148	33.1	33.8	0.7	2.1	46.5	7.19	1.17	3.17	39.7	14.7	..	12.0	21.0	12 t	..
4A1P-2	17S 25S	2.63	4.00	Angles‡ Plates	4 1	3×2½×¼ 8×¼	157, 167 148	26.8	27.4	0.6	2.2	37.5	7.19	0.90	3.02	41.7	12.4	..	10.0	18.0	12 t	..
2C-1	25S	3.50	4.57	Channels	2	7×0.314§	136, 137	78.4	98.0	19.6	20.0	108.4	7.17	2.60	2.95	41.7	36.7	15.8	..	..	....	34 t
2C-2	25S	3.50	4.88	Channels	2	7×0.314§	138, 139	78.0	98.0	20.0	20.4	108.4	7.17	2.60	4.39	41.7	24.7	15.8	..	..	....	34 t
2C-3	Steel	3.50	4.57	Channels	2	7×0.314§	169	220.0	274.0	54.0	19.7	108.3	7.12	2.60	2.95	41.7	36.7	15.8	..	..	....	34 t
2C1I-1	17S	2.50	4.25	Channels‡ I-beam	2 1	5×0.190§ 5×0.210§	158, 159 155	44.4	45.5	1.1	2.4	61.4	7.18	1.54	2.65	39.9	23.2	..	..	..	15.8 t	..
2C1P-1	25S	4.33	5.00	Channels Plate	2 1	7×0.230§ 10×$\frac{3}{16}$	140, 141 145	87.5	98.5	11.0	11.2	108.7	7.98	2.81	3.36	38.7	32.4	..	..	45.3	16.0 t	40 t
2C1P-2	Steel	4.34	5.00	Channels Plate	2 1	7×0.210§ 10×$\frac{3}{16}$	170 171	233.0	258.0	25.0	10.3	108.7	7.57	2.79	3.35	39.0	32.4	..	..	45.3	16.0 t	40 t
2C2P-1	17S 25S	2.69	5.00	Channels Plate	2 2	5×0.190§ 10×$\frac{3}{16}$	158, 159 146A, 147	68.3	69.5	1.2	1.7	85.5	7.87	2.29	3.37	37.3	25.4	..	..	46.7	16.0 t	..

* A = angles; P = plates; and C = Channels. † All heat treated. See *Transactions*, Am. Soc. C. E., Vol. 102 (1937), Table 7, p. 1279. ‡ Extruded; all other sections rolled. § Web.

The mechanical properties of the materials in compression are also given in Table 2. The tests were made on short sections of the pieces having slenderness ratios of 10. The stress-strain relations were determined by measuring the relative movement of the heads of the testing machine with two dial-gages reading directly to 0.001 in.

TABLE 2.—MECHANICAL PROPERTIES OF MATERIALS IN BUILT-UP COLUMNS

Piece No.*	TENSILE PROPERTIES, IN POUNDS PER SQUARE INCH			Percentage elongation in 2 inches	Piece No.*	COMPRESSIVE PROPERTIES, IN POUNDS PER SQUARE INCH		
(1)	Ultimate strength (2)	Yield strength§ (3)	Proportional limit (4)	(5)	(6)	Ultimate strength¶ (7)	Yield strength¶ (8)	Proportional limit (9)
143†	59 670	43 300	34 000	21.0	130	34 280	34 000	24 000
143‡	57 040	38 600	24 000	19.5	131	34 130	33 800	22 000
144†	55 080	38 200	24 000	17.0	132	40 470	35 000	22 000
144‡	58 000	41 000	24 000	18.5	133	40 800	35 400	22 000
145†	58 750	41 100	36 000	20.0	134	37 310	37 100	28 000
145‡	55 640	37 000	26 000	17.5	135	38 710	38 500	28 000
146A†	57 530	38 900	28 000	19.0	136	40 490	32 000	20 000
146A‡	57 020	38 500	26 000	16.5	137	41 610	33 500	20 000
147†	63 040	46 500	39 000	17.0	138	38 480	32 200	20 000
147‡	58 850	40 900	28 000	18.0	139	40 380	31 950	19 000
148†	59 240	43 700	32 000	18.5	140	39 870	34.000	24 000
148‡	58 680	42 500	32 000	16.5‖	141	40 660	34 700	24 000
161	56 360	33 800	24 000	22.0	155	55 910	42 600	28 000
162	55 900	33 400	22 000	21.0	156	38 450	38 300	26 000
163	54 960	34 000	24 000	20.5	157	36 250	32 000	20 000
167	58 130	41 800	36 000	25.0	158	47 470	37 600	20 000
168†	57 630	39 200	32 000	20.0	159	46 900	38 300	22 000
168‡	57 400	36 300	22 000	19.0	159A	48 040	42 600	26 000
169	62 250	....	31 100	37.0	160	32 450	32 200	24 000
170	60 170	42 000	38 300	33.0	161	32 520	32 200	24 000
171†	62 780	52 500	53 350	25.0	162	33 650	33 500	24 000
....	....	....	....	....	163	34 010	33 500	24 000
....	....	....	....	....	167	39 030	36 900	26 000
....	....	....	....	....	169	52 040	40 800	31 500
....	....	....	....	....	170	40 520	37 200	32 000

* See Table 1. † Specimen taken parallel to direction of rolling. ‡ Specimen taken across direction of rolling. § For aluminum alloys, at 0.2% set; and, for steel, at 0.45% set. ‖ Broke through the gage point. ¶ Slenderness ratio = 10, tested with flat ends.

It should be borne in mind that the aluminum alloys, like other non-ferrous metals and some high-strength steels, do not have a definite point of yielding as does mild steel.

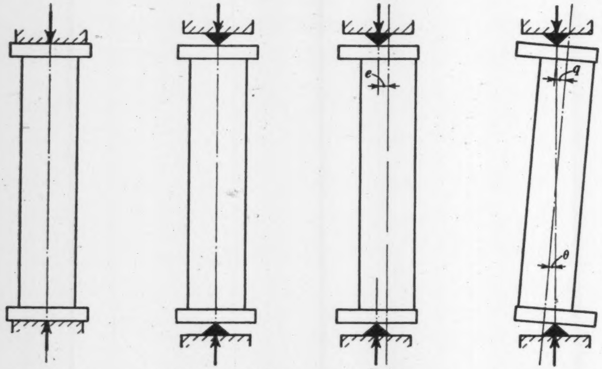
METHODS OF LOADING

The column tests were made in a testing machine of the hydraulic type, having a maximum capacity of 300 000 lb, and intermediate load-ranges. The load was applied to the ends of the specimens using the various loading conditions shown in Fig. 1.

Flat end conditions (Fig. 1(*a*)) were obtained by centering the specimen on the fixed bearing heads of the testing machine.

Round end conditions (Fig. 1(*b*)) were produced with knife-edge bearing blocks consisting of a bearing plate supported by a single knife-edge. The specimens were thus allowed to deflect freely normal to the plane of the knife-edges, but were restrained with the condition of flat ends in the plane of the knife-edges. The specimens were carefully centered in an attempt to make the axis of the specimen coincide with the plane of the knife-edges.

Eccentric loadings (Fig. 1(*c*)) were produced with the knife-edge bearings by placing the specimen off center the same distance at each end and in the same direction. The eccentricity, *e*, is defined as the distance between the axis of the column and the plane of the knife-edges. The values of the eccentricity ratio, $\frac{e\,c}{k^2}$, were between about 0.4 and 0.6. In computing these values of eccentricity ratio, the value of *c*, distance from the neutral axis to the point where the stress is desired, was taken as the distance to the extreme fiber and *k* is the corresponding radius of gyration.



(*a*) FLAT ENDS (*b*) ROUND ENDS (Knife Edges) (*c*) ECCENTRIC LOAD (*d*) OBLIQUE LOAD

Fig. 1.—End Conditions for Loading Columns

Oblique loadings (Fig. 1(*d*)) were produced with the knife-edge bearings by placing the specimen off center the same distance at each end, but in opposite directions at the two ends. The obliquity, *q*, is defined as the distance between the axis of the column at the ends and the plane of the knife-edges. The values of the obliquity ratio, $\frac{q\,c}{k^2}$, were between about 0.4 and 0.6

Some tests were made with the specimen off center each way from the plane of the knife-edges. In some cases, the knife-edges were parallel, in turn, to the two principal axes of the cross-section of the specimen. All specimens were loaded to failure with the condition of flat ends, as shown in Fig. 1(*a*).

MEASUREMENTS FOR STRAIN AND DEFLECTION

Strains were measured on several gage lines on the main members using a 2-in. strain-gage. The lines were placed to show the distribution of stress (strain) in the members not only around the section but also along the length of the specimen. Tensometers (1-in. gage length) were used in measuring strains on both sides of the lacing-bars and on rosettes (gage lines intersecting at a point) on batten and cover-plates.

Deflections at the center of the specimens were measured by the use of fine wires, stretched between pins at the ends of the specimens, and scales (graduated to $\frac{1}{50}$ in.) mounted on mirrors which, in turn, were mounted on the specimen. In addition to the measurements at the center, the deflections in the direction of bending were measured at the quarter-points in some of the tests using eccentric and oblique loadings.

Discussion of Test Data

The measured strains were interpreted into stresses using values of modulus of elasticity of 10 300 000 lb per sq in. for the aluminum alloys and 29 000 000 lb per sq in. for steel. Only a very small part of all the data accumulated in these tests are presented herein. Fig. 2 gives the load-deflection data from the flat end tests; Fig. 3 gives typical eccentricity-deflection data from the eccentric load tests; and Fig. 4 gives typical transverse shear-obliquity data from the oblique load tests. The curves indicated by the stress in Bars 20 and 32 are shown as broken lines, and the curve indicated by the stress in all bars is shown solid.

Measured Stresses

In general, the measured stresses are slightly lower than the computed values obtained by the following equations:

For axial loads,

$$s = \frac{P}{A} \qquad \text{(1a)}$$

for eccentric loads,

$$s = \frac{P}{A}\left(1 + \frac{e\,c}{k^2} \sec \frac{L}{2\,k} \sqrt{\frac{P}{A\,E}}\right) \qquad \text{(1b)}$$

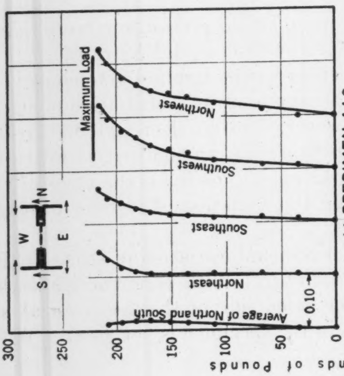
and, for oblique loads,

$$s = \frac{P}{A}\left(1 + \frac{q\,c}{k^2}\right) \qquad \text{(1c)}$$

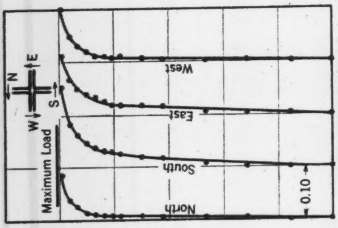
in which s = computed unit stress, in pounds per square inch; P = total load, in pounds; A = cross-section area of main members, in square inches; E = the modulus of elasticity of the material, in pounds per square inch; and L = length, in inches. The agreement was improved when added precautions were taken to avoid the effects of temperature variations on the measured strains.

For the case of low axial loads, there seems to be no significant tendency for any one general type of specimen to have a more uniform distribution of stress than any other type. For higher loads, however, there is a decided tendency for the specimens with thin outstanding plates to have a pronounced variation in the stress distribution. These same remarks can be made for the cases of eccentric and oblique loads when considering the variation of stresses on gage lines the same distance from the neutral axis.

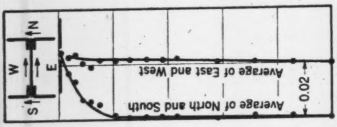
For the case of axial loads, there is no significant variation of stress at the various sections along the length of the member. With eccentric loads the larger stresses are found, of course, on the same side of the specimen at all sections and the variation along the length of the specimen is not great. With



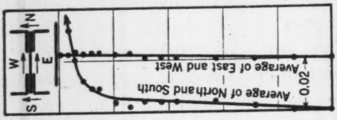
(a) SPECIMEN 4A-2



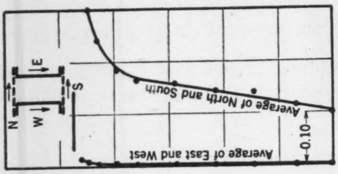
(b) SPECIMEN 4A-3



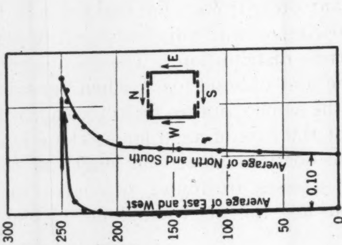
(c) SPECIMEN 4A1 P-1



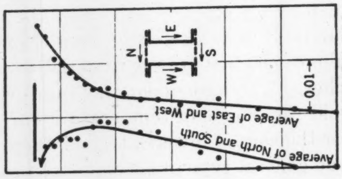
(d) SPECIMEN 4A1 P-2



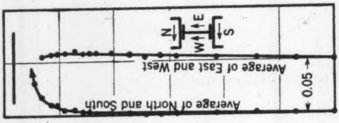
(e) SPECIMEN 2C-1



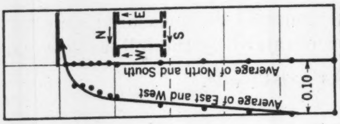
(f) SPECIMEN 2C-2



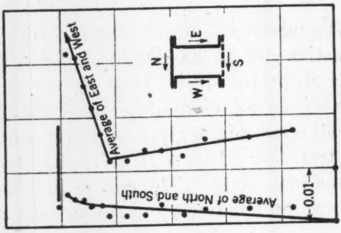
(g) SPECIMEN 2C3



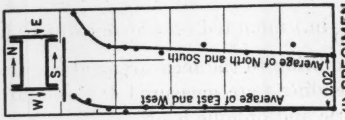
(h) SPECIMEN 2C1 I-1



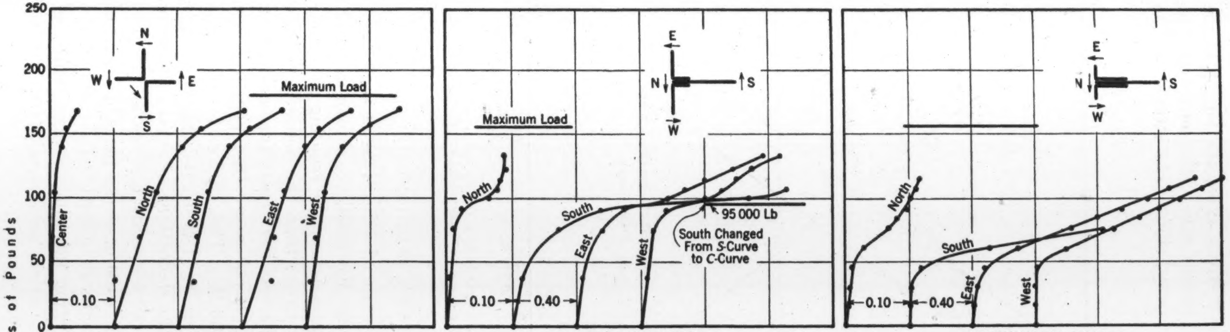
(i) SPECIMEN 2C1 P-1



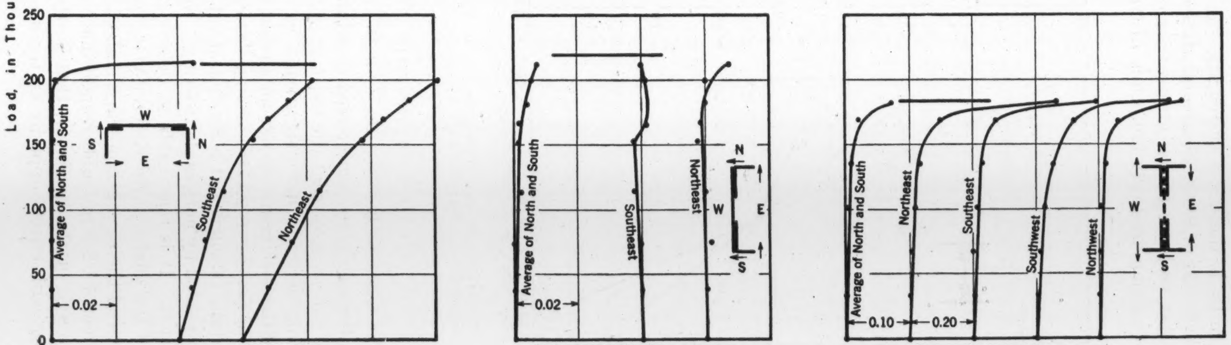
(j) SPECIMEN 2C1 P-2



(k) SPECIMEN 2C2 P-1



(*l*) SPECIMEN 2*A*-1 (*m*) SPECIMEN 2*A*1 *P*-1 (*n*) SPECIMEN 2*A*1 *P*-2



(*o*) SPECIMEN 2*A*1 *P*-3 (*p*) SPECIMEN 2*A*1 *P*-4 (*q*) SPECIMEN 4*A*-1

FIG. 2.—RELATION BETWEEN LOAD AND CENTER DEFLECTION (DEFLECTION, IN INCHES)

oblique loads there is a systematic variation of stress along the length of the member. At one end the greater stresses are on one side of the section and at the other end the greater stresses are on the other side. The stresses at the center section are fairly uniformly distributed.

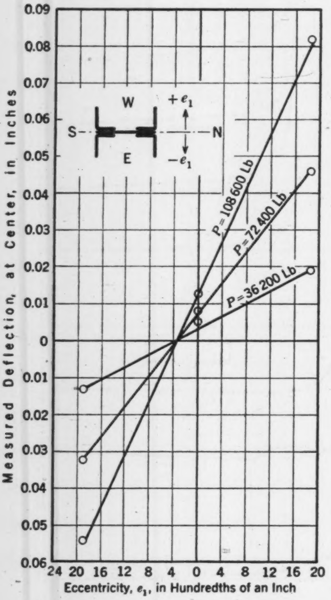


Fig. 3.—Relation Between Deflection and Eccentricity, e_1, Specimen 4A1P-1

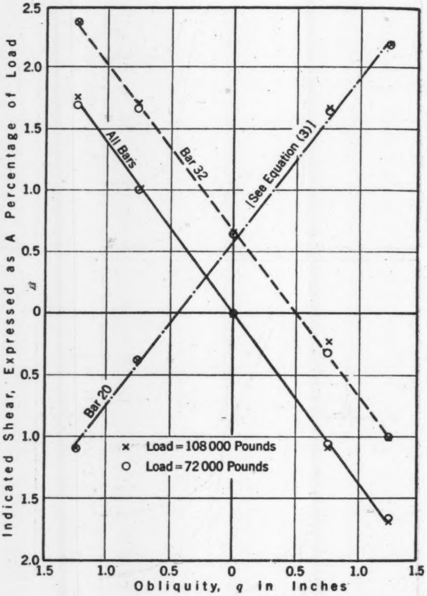


Fig. 4.—Transverse Shear Indicated by the Measured Stresses in Lacing-Bar (2C-2)

Deflections with Flat-End Conditions

The relations between the load and the measured deflections at the center, when the specimens were tested as columns with flat ends, are shown in Fig. 2. These curves show that the twisting of the specimen or the bending of an outstanding part was more significant, in many cases, than the deflection of the axis of the specimen. It was noted that the shape of the edge of the outstanding plate of Specimen 2A1P-1 changed from double curvature (**S**-curve) to single curvature (**C**-curve) at a load of about 95 000 lb. This change in shape was accompanied by noticeable twisting. Specimen 2A1P-2 showed this same type of action. It is reasonable to believe that the deflections of the outstanding legs of the angles of Specimen 2A1P-3 produced a condition of instability that influenced the strength of the member. The twisting of the pairs of angles of Specimen 4A-2 is probably responsible for the change in the direction of bending of the axis of the specimen; that is, the twisting introduced eccentricities of loading with effects opposite to those of the initial "accidental" eccentricity.

("Accidental eccentricity" is defined as the unintentional eccentricity that must have occurred, including the effects of crookedness of the member, fabrication defects, etc., as well as the error in the location of the point of application of the load.) Specimen 2C-3 also gave a queer curve for the deflections of the north and south sides, which was probably produced by erratic action of the lacing-bar joints. The sudden breaks in the curves of Specimen 2C1P-2 are probably a result of the buckling of the plate.

Analyses of load-deflection data have been developed by Professors W. Ayrton and John Perry,[3] and by Professor R. V. Southwell,[4] by which the eccentricity of loading and the critical column strength of the specimen can be determined. It is then possible to determine the fixation of the ends of the specimen from the critical column strength and the Euler formula for column strengths. There are only a few of these specimens in which the action of the component parts was sufficiently unified and free from twisting that such analyses might have been applied to the data.

Values of accidental eccentricity can be obtained from these data as the eccentricity required to give a good agreement between the measured and computed values of deflection. Such values of accidental eccentricity are not reliable because the measured deflections were greatly influenced by the lack of integral action of the specimen. They would be the upper limiting value of the accidental eccentricity, however. Such values have been used subsequently herein in computing the strength of the members (see heading, "Computed Column Action and Tests to Failure").

The specimens with lacing and relatively light tie-plates show a tendency toward large values of the eccentricity ratio, $\frac{e\,c}{k^2}$, resulting from accidental eccentricities. For the other specimens, the shape of the section seems to have no significant effect on the values of eccentricity ratio resulting from the accidental eccentricity.

DEFLECTION UNDER ECCENTRIC LOADS

The averages of the measured deflections for positive and negative eccentricities agree quite satisfactorily with computed values obtained by the secant formula for deflection:

$$y = e\left(\sec \frac{L}{2\,k}\sqrt{\frac{P}{A\,E}} - 1\right) \dots\dots\dots\dots\dots\dots (2)$$

In some cases the measured deflection for positive eccentricity is quite different from the value for negative eccentricity. These differences are produced by accidental eccentricities of loading, the magnitude of which can be obtained from curves showing the relation between the deflection and the intentional eccentricity. In Fig. 3, which shows a typical eccentricity-deflection relation, the accidental eccentricity is that which corresponds to zero measured deflection.

---

[3] "On Struts," by W. Ayrton and John Perry, *The Engineer* (London), December 10 and 24, 1886, Vol. 62, pp. 464–465, and 514–516.

[4] "On the Analysis of Experimental Observations in Problems of Elastic Stability," by R. V. Southwell, *Proceedings*, Royal Soc. (A) 135 (1932), pp. 601–616; see, also, "Theory of Elasticity," by R. V. Southwell, Oxford Press, 1936, p. 428.

The relations of the measured and computed deflections of the axes of the specimens do not seem to be influenced by the shape of the section. There were a few cases in which the local bending of the component parts was noticeable. These local deflections were influenced by the shape of the section; for example, in the two T-shaped specimens (2A1P-1 and 2A1P-2), the measured values show that the deflection was accompanied by twisting about a longitudinal axis. This action was observed even for small loads.

Deflections Under Oblique Loads

In general, the deflections measured at the quarter-points were small. Percentage variations from the computed values are quite large in some cases but the differences between the measured and computed values for the aluminum alloy specimens are, in only one case, greater than 0.008 in. for average stresses of 10 000 lb per sq in. This case is the two-angle, star-shaped specimen (2A-1) for the condition in which the axis of bending is across the tie-plates. Otherwise, the shape of the section seemed to have no significant effect on the relation of the measured and computed deflections of the axes of the specimens. In this case (Specimen 2A-1), the average of the measured values is about 2.5 times as great as the computed value.

The computed values used in these comparisons were obtained as one-eighth the value computed by Equation (2) for the specimen loaded with an eccentricity equal in magnitude to the obliquity. Although this formula does not strictly apply to this type of loading, the error introduced is quite small.

The twisting of the T-shaped specimens was again noticeable even for the low loads and did not change direction with a reversal in the direction of the obliquity.

Lacing-Bar Stresses and Shears

It was found early in the investigation that because of the bending it was necessary to measure the stresses on both faces of the lacing-bars in order to arrive at a satisfactory value of the load on the bar. In some cases the maximum stress in a lacing-bar was more than three times the average stress. With the specimens tested with the condition of flat ends or with eccentric loads, the stresses in the lacing-bars were rather small (less than 4 800 lb per sq in. for average stresses, $\frac{P}{A}$, of 15 000 lb per sq in.). The transverse shears indicated by the average of the lacing-bar stresses were equal to not more than two-thirds of 1% of the total longitudinal load. The transverse shear indicated by the most highly stressed lacing-bar (Specimen 2C-1) was equal to about 1% of the load when the specimen was tested as a column with flat ends. These indicated transverse shears were arrived at as the transverse load required to produce the measured average stress in the lacing-bars, assuming that the bars support all the shear.

With the specimens tested with oblique loads, the stresses in the lacing-bars are somewhat larger and indicate values of transverse shear (based on the average stresses for all the bars of a specimen) as high as 3.9% of the longitudinal load. The typical data in Fig. 4 show the relation between the indicated shear and the ratio of obliquity to the length of the specimen. It should be noticed that the indicated shear, expressed as a percentage of the total load, is independent of the load and can be represented by an empirical equation of the form,

$$V_i = F + G\left(\frac{2q}{100L}\right) \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The data indicate the maximum values of the constants, F and G, to be 0.7 and 1.7, respectively. It is shown in Fig. 4 that the relation between the indicated shear and the obliquity is a straight line, and that for individual lacing-bars there is some indicated shear even for zero intentional obliquity.

The average stresses in the lacing of the specimens with heavy batten-plates at the ends (4A-1 and 4A-2) indicate transverse shears between one-third and two-thirds of the transverse component of the load. The specimens with lighter batten-plates at the ends have an indicated shear between 0.8 and 1.7 times the transverse component of the load. The average stresses in the lacing of the specimens with one cover-plate and one plane of lacing indicate a transverse shear on the lacing equal to about one-fourth the transverse component of the load.

Some of the equations commonly used for estimating the shear in built-up columns follow from analyses of the shape of the deflected structure. In generalizing and reducing the formulas, the value of the modulus of elasticity is taken as the initial modulus and the value of the average stress is taken as the ultimate strength of the member.[5] For use with structural steel (which has a very definite yield strength) this combination of values might be acceptable, but with the high-strength steels and non-ferrous metals which do not have such a definite point of yielding this is subject to criticism. An effective modulus of elasticity for strengths above the elastic stress range might be used in the generalization but the resulting formulas would be complicated considerably.

Other equations in common use are based on analyses that consider the column as a beam and the shear load producing a bending stress equal to the reduction in strength given by the slenderness-ratio term of the straight line or Rankine formulas.[6] When used to estimate the shear at failure, these analyses are subject to the foregoing criticism.

The data obtained on these specimens indicate that, for average stresses within the elastic range, the transverse shear on axially loaded columns is a constant percentage of the load on the specimen. In the case of Specimen 2C-1 (with the highest indicated shear), the transverse shear at failure would be about 2 350 lb if the relation between shear and axial load were the same (1%) at failure as at working stresses. The average stress in the most highly stressed lacing-bar would be 4 930 lb per sq in., whereas the Euler column strength of the bars is 7 400 lb per sq in. Since the failure of the specimen (described subsequently) was accompanied by the buckling of the lacing-bars, either the shear was greater than 1% of the load or the bar was in effect an eccentrically loaded column. Either explanation is tenable. This shear of 2 350 lb represents a transverse load of 330 lb per sq in. of cross-section of the main members. Although the results of this investigation do not definitely establish a value for the transverse load, they indicate that the value of 600 lb per sq in. of cross-section used in the old A. R. E. A. (American Railway Engineering Association) specifications for pin-ended columns is quite reasonable. These test data also indicate that the shear on eccentrically loaded columns should be taken greater than 1%, the amount depending on the value of the eccentricity ratio, $\frac{e\,c}{k^2}$.

---

[5] Rept. of Committee XV on Iron and Steel Structures, *Bulletin No. 374*, Am. Ry. Eng. Assoc., February, 1935; see, also, "Rational Design of Steel Columns," by D. H. Young, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 422.

[6] "Structural Engineers' Handbook," by the late Milo S. Ketchum, Hon. M. Am. Soc. C. E., 1924, p. 733.

FORMULAS FOR PREDICTING COLUMN STRENGTHS

Before proceeding with the discussion of the tests, a few remarks will be interjected concerning formulas for predicting the column strength and the type of failure that might be expected.

The straight-line type formula is commonly used for predicting the strength of columns that fail by the sidewise bending of the member as a unit. This equation is of the form,

$$\frac{P}{A} = s_0 - C\frac{K\,L}{k} \qquad \text{(4)}$$

in which s_0 = column strength, in pounds per square inch, corresponding to zero slenderness ratio. The range of slenderness ratios is limited for which this equation is applicable; but the value of C can be chosen so that the combination of this equation with the Euler column formula covers the full range of slenderness ratios. This combination of formulas has been found applicable to the various aluminum alloys.[7]

Equations for stresses in columns, such as Equation (1*b*) and that of Professors Ayrton and Perry,[8]

$$s = \frac{P}{A}\left(1 + \frac{P'}{P' - P}\,\frac{e\,c}{k^2}\right) \qquad \text{(5)}$$

can be used also for predicting the ultimate strength of columns that fail by sidewise bending as a unit. The value of s in these equations is set equal to the compressive strength of the material and a value of Load P is determined that satisfies the equation. The solution is usually most easily obtained by trial. It will be noticed that the value of the modulus of elasticity enters each of these equations, appearing indirectly in Equation (5). This presents no particular problem in the case of structural steel, which has a definite yield strength; but for some of the high-strength steels and non-ferrous metals the evaluation of an effective modulus of elasticity [9] must be made.

Although Equations (1*b*), (4), and (5) can be used to predict the ultimate strength of a column that fails by sidewise bending of the member as a unit, if the member is subject to failure by local buckling or twisting, it must be treated as a local instability problem. Naturally, the strength of the member will be limited by the lowest of the strengths predicted by a consideration of all the types of failure that might occur. Equations have been derived for some special cases of local buckling.

---

[7] "Column Strength of Various Aluminum Alloys," *Technical Paper No. 1*, Aluminum Research Laboratories, Aluminum Co. of America, 1938.

[8] "On Struts," by W. Ayrton and John Perry, *The Engineer* (London), December 10 and 24, 1886, Vol. 62, Equation (22); also "Rational Design of Steel Columns," by D. H. Young, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 422.

[9] "Rational Design of Steel Columns," by D. H. Young, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 467.

For the case of wide thin plates, the critical stress, s_{cr}, at which buckling occurs, can be computed by the equation:[10]

$$s_{cr} = \frac{a\,E}{1 - \mu^2}\left(\frac{t}{b}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which a = a coefficient that depends on the proportions of the plate and the type of restraint along the edges of the plate; μ = Poisson's ratio; t = thickness; and b = width of plate. Values of Coefficient a for various loading conditions and various conditions of the edges of the plate can be obtained from the reference giving the equation. It should be borne in mind that this value of s_{cr} is the maximum average stress that can be developed in the plate and at which buckling occurs. The agreement between the critical stress and the average stress at which collapse occurs will depend on the shape and proportions of the entire cross-section of the column and the loading conditions of the member; that is, in the specimens of which the plate is the principal part of the member, the agreement between the critical stress and the average stress at failure should be rather close, whereas for a specimen in which the plate is merely an appendage, the average stress at final collapse of the entire member may be considerably greater than the critical stress at which the plate buckles. The average stress at the collapse of a specimen with round or pinned ends will probably agree very well with the computed critical stress and, of course, will be less than the test result for a similar specimen with restrained or fixed ends.

Some cases of local buckling can be treated satisfactorily by using a proper value of s in Equations (1b) and (5). These formulas would include those cases in which the maximum stress, due to combined axial load and bending, is developed in some part that is subject to local buckling. The value of s then is the compressive strength of that part and can probably be determined most satisfactorily by a compression test on a short length of the part.

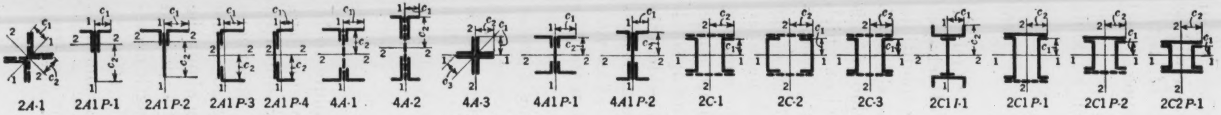
COMPUTED COLUMN ACTION AND TESTS TO FAILURE

The computed strengths of the specimens and the types of failure to be expected are given in Table 3. In Column (2), the larger of the two numbers corresponds to the fixed-edge condition and the smaller to the simply supported edge condition. In Column (4), Table 3, two values are listed for Specimens 2C-1 and 2C-2. The first value corresponds to the case of bending about the axis of least computed stiffness and the second to the case of bending about the axis of greatest computed stiffness. The difference in the values of accidental eccentricity for the two directions makes this investigation of strength necessary. In calculations involving stresses above the elastic range, an effective modulus of elasticity has been used. The effective modulus was assumed to

---

[10] "Strength of Materials, Part II," by S. Timoshenko, Van Nostrand, 1930, p. 605; see, also, *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 1009; and "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill, 1936, Chapter VII.

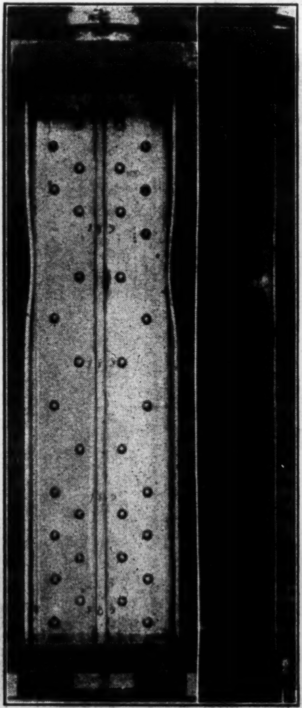
TABLE 3.—COMPUTED STRENGTHS AND TEST RESULTS; BUILT-UP MEMBERS TESTED AS COLUMNS WITH FLAT ENDS



Specimen No. (see Table 1) (1)	Strengths, in Pounds per Square Inch: Critical strength by Equation (6) (2)	Column Strengths by: Equation (4) (3)	Column Strengths by: Equation (5) (4)	Type of failure predicted (5)	Maximum load, P, in pounds (6)	Column strength, $\frac{P}{A}$, in pounds per square inch (7)	Remarks (8)
2A-1	25 600	39 400	34 500	Twisting of full length*	179 500	25 460	....
2A1P-1	13 850 5 200	34 300	35 000	Twisting of full length*	156 400	20 390	Stress at the "break" in the load-deflection curve = 12 500
2A1P-2	12 400 4 700	34 900	35 200	Twisting of full length*	156 500	20 400	Stress at the "break" in the load-deflection curve = 7 000
2A1P-3	27 800 17 800	33 500	33 900	Buckling of outstanding legs*	211 800	27 580	....
2A1P-4	30 900 26 250	33 600	32 700	Buckling of outstanding legs	222 800	29 000	Failure accompanied by buckling of the long legs of the angles and of the plate
4A-1	29 000 17 800	33 500	29 400	Buckling of outstanding legs*	185 000	27 380	....
4A-2	30 900 27 100	33 000	31 800	Buckling of angles	219 800	32 230	Critical strengths of long legs of angles between 29 300 and 29 900
4A-3	31 400 29 300	31 900	29 700	Twisting of full length*	252 500	32 080	If the angles should act individually so $\frac{b}{t} = 16$, the critical strength is between 17 800 and 28 100
4A1P-1	31 500 25 600	34 200	34 600	Buckling of outstanding legs	249 200	35 000	Failure accompanied by sidewise bending
4A1P-2	32 900 28 000	33 800	33 400	Buckling of outstanding legs	252 500	35 460	Failure accompanied by sidewise bending
2C-1	....	32 100	32 700 29 000	Bending about axis parallel to the web of the channels*	235 000	32 600	....
2C-2	....	31 400	33 300 33 800	Bending about axis parallel to the planes of lacing*	249 500	34 800	....
2C-3	....	40 800	39 400	Bending about axis parallel to the planes of lacing*	273 000	38 340	....
2C1I-1	....	39 300	41 000	Bending about axis parallel to the web of the I-beam*	290 500	40 600	....
2C1P-1	26 100 18 000	34 200	35 400	Buckling of the plate*	251 400	31 820	The plate was noticeably wrinkled at a stress of 29 000
2C1P-2	37 200	37 200	36 500	Bending about axis parallel to the plate*	251 400	33 200	....
2C2P-1	29 800 17 000	38 200	39 900	Buckling of the plates*	247 100	31 400	....

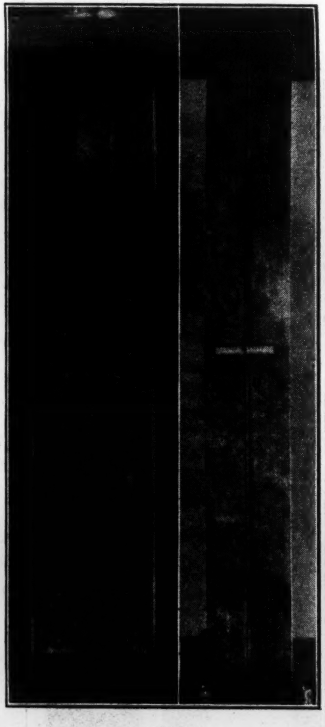
* Prediction fulfilled.

2A1P-2

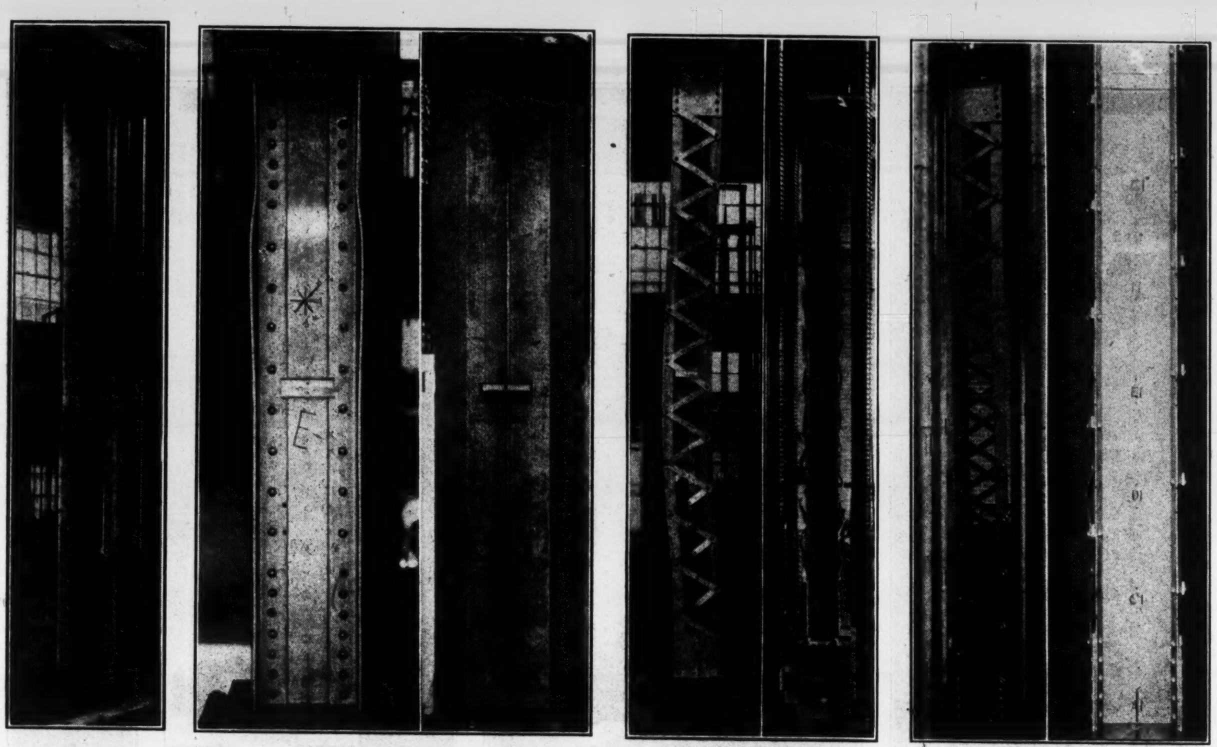


2A1P-4

4A-2



4A1P-1



4A-3 4A1P-2 2C-1 2C-3

FIG. 5.—PHOTOGRAPHS OF SELECTED SPECIMENS AFTER TESTING

decrease along a straight line from the initial value at the upper limit of the elastic range to the value of the tangent modulus at the yield strength (0.2% set) and from this decreased value along another straight line to zero at the maximum stress.

The column strength of the member acting as a unit was obtained by the straight-line formula, Equation (4), based on the properties of the material in the specimen and also by Equation (5), using a value of accidental eccentricity consistent with the data in Fig. 2. In these computations, the value of K was taken equal to 0.5 which assumes that the specimens acted as columns with fixed ends. For each specimen the critical strength based on the local buckling of the plates was obtained by Equation (6) using values of the coefficient corresponding to two widely different conditions of edge support, namely, simply supported edges and fixed edges. The actual edge conditions usually provide some restraint, so the critical strength that can reasonably be expected can be obtained by interpolation between these extreme values. The predicted type of failure is that corresponding to the lowest computed strength, whether by local buckling or by bending as a unit. The maximum loads and column strengths are for these specimens tested as columns with flat ends. In nearly all cases the type of failure was predicted correctly. In a few cases the failure appears to be a combination of buckling and bending or of buckling of more than one part.

The strength of Specimen 2A-1 was computed on the basis of simple support along the back of the plate. This seems reasonable because the four outstanding legs are all equally subject to buckling with the resultant twists centering about the shear centers of the individual angles. This reasoning does not apply to Specimen 4A-3 (Fig. 5), because, although the four legs are all equally subject to buckling, the resultant twist must center about the heels of the individual angles. Thus, the angles are stronger than similar angles, free to twist about their own shear centers. The outstanding plates of the other specimens are all restrained to a certain extent. The two values of computed critical stress given in Table 3 are based on the two conditions of the back edge of the plate mentioned previously. For those specimens with computed strengths in the plastic range the effect of the restraint at the edge of the plate is not so pronounced as in the cases in which the strength is in the elastic range. This results from the fact that the effective modulus of elasticity varies with the stress.

A study of the load-deflection curve and the stresses measured at the free edge of the plate of Specimen 2A1P-1 indicates that the value of computed critical stress corresponding to fixed edges is close to the average stress at which buckling occurred, whereas a similar study for Specimen 2A1P-2 (Fig. 5), indicates that the value corresponding to fixed edges is much too great. The differences between the ultimate strengths and these computed critical strengths should not be surprising in view of the definitions of terms and the previous discussion of Equation (6). The ultimate strength of Specimen 2A1P-4 (Fig. 5) was probably reduced somewhat by the buckling of the plate.

The critical strength of Specimen 4A-2 is computed for the outstanding (short) legs of the angles with a ratio of width to thickness of 12 and a large ratio of length to width. It is also computed for the inside (long) legs with a ratio of width to thickness of 16 and a length equal to the width. The range of values based on both simple and built-in support at the back edge of the plate is from 27 000 lb per sq in. to 30 900 lb per sq in. The reversal of the load-deflection curve in Fig. 2(*a*) at a stress of about 26 000 lb per sq in., is probably associated with the buckling of the legs of the angles. The ultimate strength is 32 230 lb per sq in.

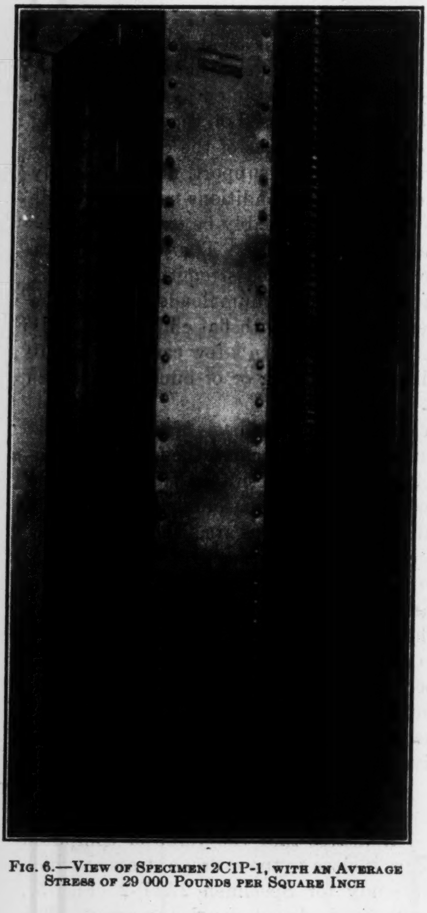


Fig. 6.—View of Specimen 2C1P-1, with an Average Stress of 29 000 Pounds per Square Inch

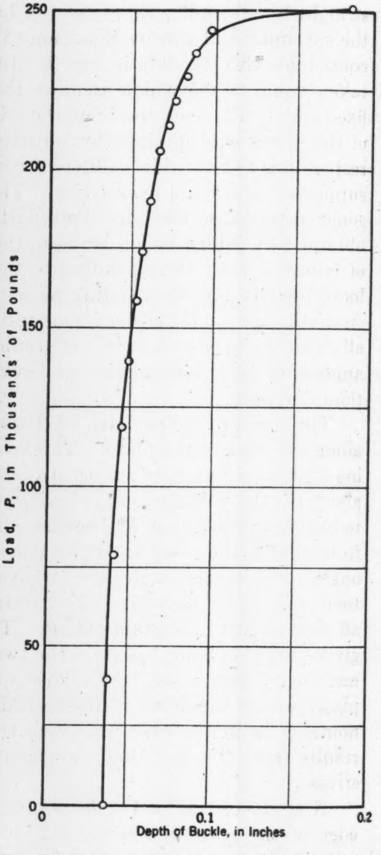


Fig. 7.—Relation Between the Load and the Depth of Buckle, in Plate of Specimen 2C1P-2

Specimens 4A-3, 4A1P-1, and 4A1P-2 (Fig. 5), with outstanding legs of angles, having width-to-thickness ratios of 8, 12, and 10, respectively, developed strengths equal to the values computed by the straight-line formula and by

Equation (5). Specimen 4A-2 (Fig. 5) developed 98% of the strength predicted by the straight-line formula, and Specimens 2A1P-1 and 2A1P-2 developed only about 60% of the strength similarly computed. These comparisons should emphasize the fact that short columns with outstanding plates having ratios of width to thickness greater than about 12 are subject to local failure at loads less than those predicted by the usual column formulas.

Specimens 2C-1 and 2C-3 (Fig. 5) can scarcely be said to act as units in view of the fact that failure occurred by bending about the axis of greatest computed stiffness. The ratio of moments of inertia is 1 : 1.3. The failure of Specimen 2C-3 (Fig. 5) was accompanied by the formation of Lueders' lines. The agreement between the test results and the computed values was very good.

Fig. 6 shows Specimen 2C1P-1 under an average stress of 29 000 lb per sq in. The buckling of the plate is quite apparent although at an average stress of 28 000 lb per sq in., the plate appeared smooth. These values agree fairly well with the critical strength computed on the basis that the edges of the plate are built in and the width is the clear distance between the rows of rivets.

Fig. 7 shows the relation between the load and the depth of a buckle in the plate of Specimen 2C1P-2. The plate buckled rather suddenly at a load of 205 000 lb. This load coincides with that at which the "break" occurred in the load-deflection curve of Fig. 2(*j*). Besides the formation of this buckle, failure was accompanied by the formation of Lueders' lines. The computed strengths for this specimen given in Table 3 are about equal to the yield strength of the material. The test result is about 10% less than this, probably as a result of the initial unevenness of the plate.

The computed critical strength of Specimen 2C2P-1 agrees fairly well with the break in the load-deflection curve. The ultimate strength exceeds this value by 19 per cent. None of the three specimens, with a ratio of width of plate (between the lines of rivets) to thickness of about 46, developed the yield strength of the materials nor the strength predicted by the straight-line formulas.

CONCLUSIONS

The following conclusions regarding built-up columns of high-strength aluminum alloys are justified by the foregoing data and discussion:

(1) For low loads the shape of the cross-section has no significant effect on the distribution of stress; but for the higher loads the specimens with thin outstanding plates have a pronounced variation in the stress;

(2) The shape and proportions of the cross-section have a significant effect on the ultimate strength and the type of failure, whether by bending as a unit or by local buckling;

(3) The type of failure, whether by local buckling of a component part or by bending as a unit, can be predicted as the type of failure corresponding to the least of the strengths computed by the various equations for critical stress and column strength;

(4) In order to avoid failure of short columns $\left(\frac{L}{k} = \text{about } 40\right)$ by local buckling at a load less than that predicted for the column acting as a unit, the

width-to-thickness ratio of outstanding plates and legs of angles should not exceed about 12 and the ratio of width to thickness of cover-plates with a row of rivets near each edge should not exceed about 40;

(5) Local buckling will not always result in the collapse of the entire member, but may be only a wrinkling of the buckled part;

(6) In the range of working loads the shape and proportions of the cross-section have no significant effect on either the ratio of measured deflection of the axis of the member to the computed deflection or on the magnitude of the indicated shear, expressed as a percentage of the load; and,

(7) For two-channel members with two planes of lacing to develop equal column strengths in the two principal directions, the moment of inertia of the section parallel to the lacing, computed in the usual manner, should be greater than that parallel to the webs of the channels in order to counteract the minute unintentional slack in the lacing-bar joints.

ACKNOWLEDGMENTS

The writer wishes to acknowledge the assistance of the following members of the staff of the Physical Testing Division of the Aluminum Research Laboratories: R. L. Templin, M. Am. Soc. C. E., Chief Engineer of Tests; R. G. Sturm, Assoc. M. Am. Soc. C. E., Research Engineer Physicist; and E. C. Hartmann, Assoc. M. Am. Soc. C. E., Research Engineer.

APPENDIX

NOTATION

The following notation conforms essentially with American Standard Symbols for Mechanics, Structural Engineering, and Testing Materials[11] compiled by a committee of the American Standards Association with Society representation, and approved by the Association in 1932:

A = area of cross-section, main members, in square inches;

a = a coefficient in Equation (6), that depends on the proportions of the plate and the type of restraint along the edges of the plate;

b = breadth; width of plate, in inches;

C = a coefficient chosen so that the combination of Equation (4) with the Euler formula covers the full range of slenderness ratios;

c = distance, in inches, from the neutral axis to the point where the stress is desired;

E = modulus of elasticity of the material, in pounds per square inch;

e = eccentricity of loading, in inches;

F = term in Equation (3);

G = term in Equation (3);

K = a coefficient describing the end conditions; taken as unity for a column with round ends, and as 0.5 for a column with fixed ends;

---

[11] A.S.A.—*Z*10*A*—1932.

k = radius of gyration of the cross-section, in inches, assuming unified action of the component parts of the member;

L = length, in inches;

P = total load, in pounds; P' = Euler column load, in pounds;

q = obliquity of loading, in inches; distance between the axis of the columns at the ends and the plane of the knife-edges;

s = average unit stress, in pounds per square inch = $\frac{P}{A}$; s_{cr} = critical buckling stress; s' = Euler's column strength = $\frac{\pi^2 E}{\left(\frac{K L}{k}\right)^2}$; s_0 = column strength corresponding to zero slenderness ratio;

t = thickness, in inches;

V = total shear, in pounds; V_i = indicated shear, expressed as a percentage of the total load, P;

y = deflection, in inches; and,

μ = Poisson's ratio.

DISCUSSION

Alfred S. Niles,[12] Assoc. M. Am. Soc. C. E. (by letter).—From two distinct viewpoints, Mr. Holt's paper is a valuable contribution to the literature of column design. Its most obvious merit is that it extends the knowledge of column behavior to large members of aluminum alloy. It also throws additional light on some problems of column design that are of interest to the engineer regardless of whether he intends to use aluminum alloy or steel as his structural material. The discussion of the stresses in lacing-bars, for example, should be of as great value to the designer of steel columns as to the worker with aluminum. In the past this problem has been slighted and lacing-bars have been designed by methods based on so many assumptions and on so few experimental data that they scarcely rise to the dignity of being called empirical, but can only be described as rule of thumb. The more information that can be obtained on this and several of the other topics treated in the paper, the more efficiently will the engineer be able to design his columns, and the more safely can he use unconventional sections for special purposes.

One phase of the tests that is rather meagerly covered in the paper is the tendency of the "cross" and the "tee" sections to fail by twisting. In Table 3 four columns are listed for which failure by twisting was predicted, and in each case the prediction was fulfilled. It may be noticed, however, that there is no mention of predicted loads at which failure in this manner would take place. Furthermore, it should be noted that three of these columns failed under much lower average stresses than those predicted by Equations (3) and (5). It might be thought that Equation (6) would be helpful in predicting the probable stress at failure by twisting, since buckling of an outstanding leg due to local instability would tend to cause the columns in question to twist. That this equation does not apply is evident from the fact that two of these columns failed by twisting under much higher unit stresses than the larger values obtained from that formula. The real situation is that, for any column there is a critical load which cannot be exceeded without producing general instability with respect to torsional failure. For most of the standard column sections this load is so much larger than the critical loads corresponding to other types of failure that it need not be considered in design. For the "cross" and "tee" sections, however, it is likely to be less than the other critical loads and becomes a limiting factor. This type of failure is not discussed in the paper although the aforementioned predictions show that the author was properly aware of its likelihood. Its importance to the designer is indicated by the relatively low average stresses at which the first three columns listed in Table 3 failed.

One probable reason for the lack of predictions of the stresses at which torsional failure was to be expected is the present very primitive state of the theory applicable to this phenomenon. Although it had been noticed in a number of earlier column tests it was not until quite recently that any notable progress was made in explaining it rationally or in predicting the loads at which it might be expected to occur. At present the best discussion of the phenomenon easily available to American engineers is a report by E. E. Lundquist, Jun. Am. Soc. C. E., and Mr. C. M. Fligg[13] of the engineering staff of the National Advisory Committee for Aeronautics. The theory of this report, however, is incomplete since it is limited to the case of a column with a doubly symmetrical cross-section, and is not supported by test data.

---

[12] Prof., Aeronautic Eng., Leland Stanford Junior Univ., Aero. Laboratory, Stanford Univ., Stanford University, Calif.

MARSHALL HOLT,[14] ASSOC. M. AM. SOC. C. E. (by letter).—The writer wishes to thank Professor Niles for his discussion of this paper. As stated, a theory for columns that fail by twisting, and reliable equations for computing the strength of such columns, are not readily available. When satisfactory methods become available for predicting the strength of members that fail in this manner, the use of such members in special locations may become more common.

The prediction of twisting failure for four of the specimens in Table 3 was based on the following reasoning: The outstanding plates would fail by local buckling because the strength for this mode of failure is less than the strength if failure were to occur by lateral deflection of the specimen as a unit (compare the computed strength obtained by Equation (6) with the strengths obtained by Equations (4) and (5)). If the supported edge of the plate were free to rotate (but not deflect), the free edge of the plate would buckle into a single wave; but if the supported edge were fixed against rotation, the free edge would deflect into a number of waves, the number depending on the relative proportions of the plate. Since the torsional rigidity of these specimens is so small, the degree of fixation at the supported edge of the outstanding plate would not cause the development of more than one wave in the free edge. Thus, the center of the specimen would merely twist relative to the ends.

Failure under this type of action may or may not result in the collapse of the specimen but occurs at that load at which the lateral deflection of the free edge of the outstanding parts increases at a rapidly increasing rate (see Figs. 2(*m*) and 2(*n*)). The maximum load sustained by the specimen may be greatly in excess of this buckling load and, of course, will be accompanied by collapse of the specimen. There seems to be no method available for predicting the spread between the buckling load and the maximum load, but obviously it depends on the relative proportions of the cross-section. Considering failure in this light, the strengths of the specimens that failed by twisting were predicted within reasonable limits by Equation (6).

---

[13] "A Theory for Primary Failure of Straight Centrally Loaded Columns," National Advisory Committee for Aeronautics, *Technical Report No. 582*, Washington, D. C., March, 1937.

[14] Research Engr., Aluminum Research Laboratories, New Kensington, Pa.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2061

TRANSPORTATION DEVELOPMENTS IN THE UNITED STATES

By Fred Lavis,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Milton Harris, Ralph Budd, L. Alfred Jenny, William J. Cunningham, S. R. Truesdell, David A. Molitor, C. A. Hoglund, and Fred Lavis.

Synopsis

A broad review of the problems facing transportation in the United States is offered in this paper, with sharpest emphasis placed upon the problems facing the railroads. Of the five important transportation agencies—railways, highways, waterways, pipe lines, and airways—railways are shown to be the most important.

Special stress is laid on the need for adequate research on a large scale and broad basis into the problems confronting the railroads, both for their proper rehabilitation and necessary progress. The writer considers that the prosperity of the railroads is essential to the prosperity of the country as a whole. The railways are in grave financial difficulties, maintenance is greatly deferred, and progress is not being made. This is due to fundamental defects in governmental regulation rather than loss of traffic due to the general depression. It is believed that there should be better co-ordination of all transport facilities and direct governmental aid or extension of credit should be provided for the railways as it is now being extended to all other forms of transportation.

Importance of the Railways

Twenty-five years ago, when people spoke of transportation in the United States they thought only of railways. At the present time, the railways comprise, as a group, only one of five important transportation agencies—railways, highways, waterways, pipe lines, and airways—all important factors in transportation. Twenty-five years ago the automobile was scarcely out of the crawling stage; the truck, the tractor, and the motor bus were unknown. Aviation was an experiment, pipe lines were not a factor to be reckoned with, and the waterways were little used. Inland waterways had declined as a factor

---

Note.—This paper was presented at the Annual Convention, Salt Lake City, Utah, on July 20, 1938, as part of the Symposium on Transportation, and published in November, 1938, *Proceedings*.

[1] Cons. Engr., New York, N. Y.

in transportation as the railways advanced, and they had not yet come to the stage of heavy development in physical structure through the use of Government funds and Federal capital invested in operation.

In spite of the development of these competing forms, however, the railways are still the dominating factor in the field of transportation in the United States; that is, they are dominating in the sense that they move by far the largest volume of freight over the longest distances and, consequently, they are dominating in their relation to general business and industry. With highways so easily visible, and with the dramatization of air travel, the public tends to forget this position of the railways and the fact that they move considerably more than twice as much freight as all the other agencies combined. Exact data as to this distribution are not available, but the following is a reasonably correct estimate of the distribution, between the different kinds of carriers, of the ton-miles of freight handled in 1936:

Description	Ton-miles, in millions	Percentage of total
Steam railways	341 181	67.5
Waterways	92 651	18.3
Motor carriers (highways)	40 227	8.0
Pipe lines (petroleum)	30 710	6.1
Electric railways	704	0.1
Total	505 473	100.0

This list does not include intra-urban transportation. In referring to this classification Mr. Joseph B. Eastman, Member of the Interstate Commerce Commission and formerly Federal Co-Ordinator of Transport, has demonstrated [2] that the ton-mile data do not show the relative effect of highway competition inasmuch as truckage hauls, on the average, are comparatively short. Comparison on a tonnage basis shows a higher percentage of actual tons hauled by trucks, at considerably higher and more profitable rates.

In general, the average cost (to the shipper) of freight hauled on the railroads is slightly less than 1 cent per ton-mile whereas the cost of haulage by truck [3] is from 5 to 7 cents per ton-mile. It is to be realized, of course, that the truck haul is from door to door, whereas the railway haul is the cost of movement on the rails alone.

Another estimate of the relative importance of the various carrier agencies is given by John S. Worley,[4] M. Am. Soc. C. E., as follows:

Description	Gross revenues for 1932	Percentage of total
Steam railways	$3 168 537 000	83.03
Great Lakes	11 733 099	0.31
Pipe lines (oil)	211 788 647	5.55
Inter-city trucks for hire	164 728 000	4.32
Inland waterways	8 572 965	0.22
Electric railways	70 865 505	1.86
Inter-city buses	171 200 000	4.49
Airways	8 600 000	0.22
Total	$3 816 025 216	100.00

---

[2] Second Rept. of Co-Ordinator, March, 1934.

[3] "Motor Truck Freight Transportation," U. S. Dept. of Commerce, 1932.

[4] *Engineering News-Record*, June 11, 1936.

These values afford some interesting comparisons although they are for 1932 and the prior list is for 1936. Comparison of the two tabulations serves at least to indicate the very low ton-mile costs to shippers on the waterways as compared to those on either railways or highways. In making these comparisons, however, it must be borne in mind that total costs of transportation are from door-to-door and although, undoubtedly, costs of movement in barges or vessels on waterways are low, they do not necessarily show total costs from origin to destination; neither do they include costs of building and maintaining the waterway itself.

HIGHWAY CAPACITY

It is not possible in the United States, on the basis of present knowledge or expectations, to substitute highway transport for railways for the movement of freight on a large scale. For example, consider the freight movement on the Pittsburgh Division of the Pennsylvania Railroad, which extends between Pittsburgh and Altoona, Pa., a distance of 113.9 miles. On a representative day in 1938, a total of 162 756 net tons of freight was moved in 108 freight trains, containing 4 089 loaded cars, and manned by approximately 575 employees. To move a similar volume of freight by highway, in 5-ton trucks, would require 32 551 vehicles, each manned by at least one employee. If these vehicles were spaced the legal distance of 500 ft apart required by the Pennsylvania State law, they would cover a total of 3 084 miles of highway, or about the distance from Los Angeles, Calif., to Philadelphia, Pa.; and, moving at the rate of 25 miles per hr, would require 123 hr, or more than 5 days, to pass a given point.

Assuming 6 000 motor vehicles per day (that is, vehicles moving in both directions which is a reasonably dense traffic as the normal capacity of a first-class, two-lane highway), it is evident that this freight alone would require about five or six of such highways to handle it. If only half the total traffic were trucks—the remainder being buses, private automobiles, and other vehicles—it would require ten or twelve such highways through this one section of the country to equal the capacity of the one railroad.

Of course, there are several ways of estimating this highway capacity as, for instance, assuming 10-ton, instead of 5-ton, trucks; the fact, however, that the rail traffic cited is fairly representative of an ordinary day must also be considered, but in both cases provision must be made for peak loads. The railroad could handle twice as much traffic on the same trackage; but to handle twice as much in the same period of 24 hr on the highways would require twice as many highways.

The railways are not only important because they now handle by far the largest proportion of the freight but because there now seems no other way in which this traffic can be handled. Furthermore, the World War demonstrated the absolute necessity of railways in time of war.

PRESENT STATUS OF THE ART OF TRANSPORTATION

Highway Developments.—The very important subject of highways and motor-vehicle development has been ably treated by Charles F. Kettering[6] and L. I. Hewes,[6] Members, Am. Soc. C. E.; but attention may be called to one phase of the subject, namely, the effect of the speed of motor vehicles on highway design. There are those who claim that all present-day highways are obsolete; that they are out of date as soon as they are built because they are not designed for speeds of 100 miles per hr. They claim that the modern automobile (and possibly, the modern truck) can be, or soon will be, driven safely at this speed; that engineers are lacking in foresight and are hopelessly conservative and behind the times if they do not recognize this need in the design of highways. These writers ignore the fact that only a small proportion of drivers of motor vehicles are capable of driving at speeds in excess of 50 or 60 miles per hr under ordinary every-day conditions.[7] Engineers must take the human element into consideration and not be carried away by too much enthusiasm for modern scientific and mechanical triumphs which, in some cases, may be beyond the ability of the public to use.

---

[6] "*Proceedings*, Am. Soc. C. E., June, 1938, p. 1116.

Aviation.—It has been predicted [8] by J. Monroe Johnson, M. Am. Soc. C. E., Assistant Secretary of Commerce, that within a short time 10-hr transcontinental schedules will be established, with great air liners flying in the substratosphere, as well as non-stop sea schedules between New York, London, and Paris; and 40 000 000 persons instead of 400 000 will be using the air regularly for travel. Spencer R. Newman, District Traffic Manager of the United Air Lines, has described [9] the status of aviation in considerable detail.

Realizing that great strides have been made in aviation in recent years, and quite believing that the prediction of Colonel Johnson is not altogether an iridescent dream, the writer doubts that aviation will, for a long time to come, take care of much more than the "cream" of normal increase and the increase in travel which aviation itself creates.

Aviation, highways, and waterways, are handicapped by bad weather conditions to a much greater extent than the railways, and all-year-round, daily, and hourly reliability is still an important factor in the business of transportation.

Waterways.—The subject of navigable inland waterways has been discussed quite fully by R. W. Putnam and S. L. Wonson,[10] Members, Am. Soc. C. E., and C. L. Hale,[11] M. Am. Soc. C. E., who present, quite fully, the arguments for and against these Government subsidized lines of transportation.

The investments of Government capital and the use of Government subsidies under whatever guise they may appear (and in the case of the waterways it is not difficult to recognize them) is a very important consideration in dealing with the transportation problem, but the writer doubts very much if the general public has any idea of the extent of this phase of Government activity, the facts in regard to it, and its effect.

---

[6] *Civil Engineering*, September, 1938, p. 585.

[7] "Safety and Speeds as Affecting Highway Design," by F. Lavis, *Proceedings*, Highway Research Board, 1937.

[8] *New York Times*, June 23, 1938.

[9] *Civil Engineering*, October, 1938, p. 679.

[10] "Water Transportation Versus Rail Transportation": A Symposium, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1279.

[11] "Economics of the Ohio River Improvement," *Transactions*, Am. Soc. C. E., Vol. 103 (1938) p. 1527.

Pipe Lines.—Pipe-line operation is a purely commercial development, and is really an extension of the oil-refining and distributing plants of the oil companies. It is part of the system of production, refining, and distribution of their products. The business—the freight—is supplied by the owners of the transportation agency and is part of their business.

Railways.—The development of light-weight Diesel engine trains has been traced by Ralph Budd,[12] M. Am. Soc. C. E., and C. P. Kahler,[13] Assoc. M. Am. Soc. C. E., has described turbo-electric-driven trains.

In the railroad field, at the present time, there are potentialities for improved service, and for the reduction of operating costs, which would almost revolutionize the railway industry and with it create improved business conditions in all industry. Under the conditions obtaining on the railroads to-day, however, these potentialities can bear full fruit only if the human element, financial considerations, and political thought keep step with them. Prosperity and sound business conditions are indispensable for such advances.

Prospect of Improvement

Need of Research.—Not long ago advances and developments in engineering and mechanics were products of the thought and research or genius of individuals. Such progress to-day, however, is mostly the result of intensive study by large groups and combinations in laboratories. No longer is this progress the result of the work of the lone man in a garret; it is the co-ordinated effort of large groups of well-paid scientists. They work on definite plans, toward definite objectives, in well-equipped laboratories with, on the one hand, huge testing machines and, on the other, the most delicately made and finely adjusted scientific instruments. Such progress means the expenditure of money. It is supported by groups of large businesses that are prosperous and by this means help maintain their prosperity.

Although a certain amount of research is being conducted by the railways, it is scattered, not well co-ordinated, and is not on a scale that the industry warrants. Some of it is scattered among the colleges and thus is useful, but there is need of a large centrally located and well-equipped laboratory where all these efforts can be supervised.

Advances Depend on Human Element as Well as Science.—Steps are being taken and methods are being put into practice to try to solve at least one of the very difficult problems which the railways have had to face, namely, the loss of passenger business. Attempts to encourage passengers to return to the railways do not begin, however, to touch the fundamental difficulties in which the railways of the United States now find themselves involved. Meritorious though these improvements in passenger service may be, and valuable though they may be in indicating what may be accomplished in the entire field of railway transportation, they should not be allowed to obscure the effects of the much more important factors in advances—the human element, the political element, and the underlying financial difficulties of the railroads to-day.

---

[12] *Civil Engineering*, September, 1938, p. 592.

[13] *Loc. cit.*, August, 1938, p. 509.

Recently, the writer experienced the entire gamut of transportation development in one short period of less than two months. He traveled by primitively equipped train, by automobile, and by truck over badly worn dirt roads and no roads, by ox-cart, by mule wagon, motor-boat, horse-back, mule-back, and on foot, over an area about 300 miles in length which required actually five weeks of strenuous endeavor. Then he returned, in the air, over or above the same route, by a regularly scheduled airplane, in 1½ hr. In studying the ways and means of improving the land transport over this route the writer was impressed by the part that had to be played by Man, the human being, by politics, by finance, by commerce, and by economic considerations. All these factors came into play in finding a solution for the problem. The engineering aspects were simple compared to the human, political, and financial aspects.

Advances in transportation, or advances in other fields, mean more than the improvement of existing machines, or the substitution of a new machine for an old one, a Diesel engine or a turbo-generator for the steam locomotive, or the improvement of the steam locomotive itself.

In evaluating the worth of such improvements as increase in speed, stream-lined trains, Diesel engines, electrification, light-weight cars, modernistic design of passenger coaches, and improved freight services, one must have some picture of the railroad situation as a whole and its relation to other transportation developments. If the railroads are "sick," as they are if they are all going into bankruptcy, and they seem to be tending that way, it is of little moment whether or not trains are stream-lined, or whether the decorations in the lounge cars are modernistic, futuristic, or sur-realistic.

Surplus of Facilities.—The outstanding feature of the transportation situation in the United States is the surplus capacity of the combined transportation facilities in relation to the business there is to be done. During the World War, the railways, then virtually the only transportation agency in the country, broke down under the volume of business offered. In the years immediately following, large investments of earnings and capital were made to improve and increase their facilities so that, with the boom period of 1927–1929, they handled the very large volume of business then offered without difficulty.

Government Capital in Highways, Waterways, and Airways

In the meantime, highways were being extended, rebuilt, and repaved, waterways were being improved for navigation, and air lines were beginning to be a factor. The first two were built entirely with Government capital, and the airways were developed under substantial Government subsidies. (Government will be understood to mean not only the Federal Government, but also that of the States, counties, and municipalities.) It is fair to state that, although the highways were, and are being, built with capital furnished by the Government, a considerable part of it came from direct taxes on the vehicles and their operators, and that the subsidies to aviation have been warranted both for mail service and on the score of developing the national defense. It has been announced, however, that the State of Pennsylvania is to provide the sum of $58 000 000 to build a new highway from Pittsburgh to Harrisburg, Pa. Of this, $32 000 000 is to be a public loan and $26 000 000 a grant. The loan is expected to be repaid from tolls.

The subsidies to the waterways—that is, the Government investments in waterway improvements—have been justified or excused, in part, on the plea of flood control, or power development, but in the Mississippi Valley they have been made frankly and almost wholly for the improvement of navigation.

The point to be made, is, that the capital for this expansion of highways, waterways, and part of that for airways, was furnished by the Government.

INCREASES OF TRANSPORTATION FACILITIES

In his report of March, 1934,[2] as Federal Co-Ordinator of Transport, Mr. Joseph B. Eastman called attention to the fact that in 1920, when the railways had a virtual monopoly of transport, their total investment was approximately 19 billion dollars. Between 1920 and 1932 investments in additional transportation facilities amounted to an additional 20 billion dollars, divided as follows:

Transportation agency	Millions of dollars
Railways, additional	6 300
Pipe lines	424
Streets and highways	12 500
Waterways (U. S. Government)	604
Total	19 828

There were some decreases due to abandonment of electric railways, but the investment in transportation facilities was doubled in that 12-yr period, 1920 to 1932. One-third of it was private capital invested in the railways, and almost two-thirds was Government capital invested in highways.

One of the fundamental aspects of the present-day transportation problem, therefore, is a very greatly increased capacity, a surplus capacity over and above the needs of the country, and the continued and continuing investment of Government capital in additional transportation facilities. It is probable that this surplus capacity would not be so evident or nearly as large if the country had gone ahead at a normal rate of growth during the ten years, 1928–1938, but with depression succeeding depression this increase in capacity becomes one of the major factors in the present situation.

With the almost inevitable further continuance of growth of highways on a large scale, this surplus capacity is not likely to be reduced by population increase or growth of business. This problem, therefore, is likely to continue for some time, aggravated by certain factors that reduce freight traffic permanently.

Passenger Traffic.—Whether or not the passenger traffic can be restored to railroads, or further loss prevented, is difficult to judge. Speaking before the Western Railway Club, at Chicago, Ill., on January 20, 1936, W. W. Colpitts, M. Am. Soc. C. E., expressed great hopes that it might be, by reason of better and faster service. The writer believes that much of the passenger business is lost permanently to the private automobile; but there is a substantial volume that can probably be retained and possibly increased by improvements in service. It is a question, however, if some of this business is not being carried at a loss.

Among some of the adjustments that will be brought about by re-organization and also by voluntary action on the part of the railroads (with the permission of the Government) will be the abandonment of services that do not pay. On July 9, 1938, Federal Judge Carroll C. Hincks over-ruled the Massachusetts Department of Public Utilities and ordered the elimination of eighty-eight passenger stations on the Old Colony Railroad. In his decision he stated that re-organization is impossible, legally and economically, for a railroad that cannot earn its operating expenses. To continue the passenger service of the Old Colony Railroad on the old basis would have jeopardized its capacity to maintain the indispensable freight service in that territory.

Although the changes in service will be inconvenient to some, they were judged to be in the public interest, permitting more satisfactory service for others, and it was reasonable to assume that transportation by highway would be available for those who found themselves without service by rail. Furthermore, according to the decision, the public interest would be served better by a policy protecting indispensable freight and passenger service than by a policy which daily was bringing nearer the destruction of the entire enterprise.

Freight Traffic.—Certain factors that adversely affect the freight business of railways, however, will undoubtedly be permanent. The transportation of coal has not only failed to increase with the increase of population but has actually diminished due to the use of fuel oil and electricity. Both oil and gasoline are now transported by pipe lines and these commodities have the double effect of replacing coal and taking the business from the railways. The decrease in foreign trade, which is not likely to be revived, has decreased the traffic in imports and exports. Much l.c.l. (less car-load lots) freight, especially short-haul freight, is being carried by trucks on the highways, some of these, however, being operated by the railways themselves. The transportation of raw materials has been diminished due to the moving of factories to new locations near sources of supply of the materials they use. Many economies have been introduced by manufacturers to reduce transportation as well as other costs of production.

There is no new territory to develop, the rate of increase in population has decreased, and there is no new immigration. The net remainder, therefore, is a greatly increased transportation capacity and with it certain special factors that reduce the volume of freight to be transported. On "the other side of the ledger" is a certain amount of new business from the automobile industry and the fairly large volume of business that comes to the railways in hauling supplies for the construction of highways and other public works financed by the Government. Some of these benefits will continue unless the nation reaches its limit of borrowing or devaluation, or decides to balance its budget.

One particular phase of freight movement on the railways, which has been developed within recent years, is the need of speed in the movement of certain kinds of goods.[14] Until a comparatively short time ago, large inventories were the rule and orders were placed for quantities to cover seasonal rather than daily or weekly requirements. Then came the present era of small inventories, quick turnover, and so-called "hand-to-mouth" buying.

---

[14] *Railway Age*, December 23, 1933, p. 872.

To-day, orders are placed by telephone and deliveries from distances as remote as 500 miles may be expected the next day. The result is the development of freight trains traveling at night at speeds of 40 to 50 miles per hr which presents a very different situation from that which obtained 30 to 35 yr ago when grades of 0.2%, 0.3%, and 0.4% were built for heavy trains traveling at 14 to 15 miles per hr, and when shipments were often a week or more on the road.

IS THERE A RAILROAD PROBLEM?

The extent of the decline in the business of the railroads is indicated by the data in Table 1. In 1936, Mr. Worley [4] ridiculed the idea that there was then any national transportation problem. He claimed that the railroads were no worse off than other industries and that all were suffering alike from the economic debacle of 1929. Nevertheless, about 75 000 miles of railroads are in the hands of Receivers or Trustees in Bankruptcy to-day (July, 1938), and others seem to be seriously threatened. On July 1, 1938, the Superintendent of Banks of the State of New York announced that more than $3 000 000 000 face value (principal sum) of railroad bonds had been stricken from the list of legal investments for the savings banks of the State. This evidence lends the general appearance of a crisis or, at least, of a gravely serious situation.

TABLE 1.—DECLINE IN RAILROAD BUSINESS, CLASS 1 RAILROADS

Item No.	Description	1920	1926	1929	1932	1934	1935
1	Freight originated, in millions of tons	....	....	115.3	....	....	76.3
2	Freight carried, in billions of ton-miles	....	....	447.3	....	....	282.0
3	Gross revenues, in billions of dollars	....	6.3	....	....	....	3.4
4	Net operating income, in millions of dollars	....	....	1 375.0	447.0	....	....
5	Passenger traffic, in billions of passenger miles*	47.0	....	....	....	18.0	....

* *Railway Age*, February 1, 1936.

It must be borne in mind that the railways cannot be compared with ordinary industry, such as a steel mill, a copper mine, a textile plant, or other similar industries, any of which can be shut down in times of low demand. The railroad is compelled to run trains, its need for maintenance goes on more or less regardless of traffic; the service to be rendered, and the rates to be charged are subject to regulation by the Government; and the wages it must pay its employees are (or may be) determined by another governmental agency.

In times of stress the railroads must stop maintenance, stop buying, stop improvements, stop advancing, and this cessation has a very serious reaction on the country at large. During the seven years, 1931 to 1938, the railways are reported[15] to have spent only 49% as much money annually for maintenance as was spent annually in the five years, 1925 to 1929, inclusive. It is estimated that the deferred maintenance at the end of 1937 was approximately a billion dollars.

---

[15] *Railway Age*, January 1, 1938, p. 22.

This is partly a matter of judgment but the detailed data cited make it seem reasonable. Keeping maintenance up to date, however, is not advancing, but there can be no advancement at all with any substantial amount of deferred maintenance. Advances cannot be expected in an industry which is in the condition in which the railroads find themselves to-day. If it is true that nothing stands still, then the railroads are going backward, and if they are (and the writer believes they are) it is a serious situation.

Perhaps it is true that the difficulties in which the railroads find themselves are not symptoms exclusively of the railroads but are symptoms of a badly adjusted national economy, the attack on the profit motive, and the drift toward socialistic government ownership of everything. The railroads, however, seem to be bearing the brunt of the attack and, therefore, the problem now is a railroad problem.

Needs of the Railroads

Need of Lower Costs of Operation.—So far as the railways are concerned, economy in the cost of operation means producing ton-miles and passenger miles at the lowest cost consistent with the full movement of traffic, and with the value of the facilities used and service rendered. The last is fundamental. This is particularly so because the service of the future must be much better than that of the past, and must continue to improve.

Labor.—One of the most important factors in this phase of the problem is the cost of labor. According to Thomas F. Woodlock [16] wages to all workers on the railroads now constitute more than 60% of all operating costs and, therefore, they are a most important factor in the situation. Service depends very largely on the human element.

Every one is entirely in favor of adequate compensation for labor—that is, compensation for all those employed from executives to track walkers. However, when special groups of employees banded together for mutual protection (as they have a perfect right to be), use their combined voting strength or group pressure to obtain unfair advantages for this group at the expense of others, or obtain privileges not granted to labor generally, it is not conducive to sound business for the country or the industry.

The very powerful unions of the railway workers have not always worked in the interest of the railroad industry retarding many otherwise possible advancements and, incidentally, injuring their own cause.

There seems to be no good reason why railway employees should be specially singled out and favored over other classes of labor by general legislation as, for instance, in the special Social Security and Pension Acts, applying only to railway employees, passed at the last session of the 75th Congress when this same Congress was too busy to heed the President's plea for substantial relief to the railroads.

There are many angles to the labor question and having been a laborer all his life the writer is entirely in favor of having labor get all the pay the traffic will bear, with reasonable hours of labor and security for old age, but if any or all of these demands are applied unreasonably simply to provide jobs and not to promote efficiency, adjustments will inevitably have to be made, or the jobs will disappear.

---

[16] *New York Sun*, January 8, 1936.

That railroad labor has not lagged behind in increases of wages is indicated by the following data cited by Mr. Robert V. Fletcher[17]:

> "The average hourly compensation of railroad employees has increased from 28.3¢ in 1916 to 68.6¢ in 1935, an increase of 142.4%, while the cost of living has increased only 26.8 per cent. The average annual pay increased in this same period from $892 to $1 653, nearly double; but note this, the average number of persons employed dropped from 1 647 097 to 994 371, not much more than half."

Part of this decrease in employment has been due to economies in operation by increasing the number of cars and tons per train, longer runs, elimination of certain terminal and intermediate yards, etc., but these operating economies by the railroads have been more than offset by decreases in the rate level, both freight and passenger, and the general decline in business. The working of the seniority rule has tended to keep an unduly large number of older employees in service and to stifle the ambitions of the younger men, a situation which tends to devitalize the industry.

It is not only that wages to railway employees increased considerably during the twenty years, 1916 to 1935, but these increases have continued through the depression.

At the end of 1937 the bill for wages on the railways was increased by more than $100 000 000 per yr at a time when operating revenues were seriously declining.[18] The result was further curtailment of buying by the railways, an accentuation of the decline in general business activity, and, of course, further decline in business (that is, freight, handled by the railroads) and, a further laying off of large numbers of men. The National Mediation Board which obtained "peace" with the labor unions by granting this increase, did not improve either the general business of the country, the position of the railroads, or, in reality, the economic status of the railroad employees. It is not a healthy condition to have earnings of trainmen increased 16% while average net earnings of the railroads decreased 70 per cent.[19]

The labor question on railroads is a tremendous problem which cannot be ignored. If the railroads of this country are to maintain and improve their position (and there can be no general prosperity without prosperous railroads), labor must do its part in producing ton-miles at the lowest possible cost, and must provide improved service. If this means sacrifices either in number of employees due to more efficient machines, or lower rates of pay, these sacrifices will have to be made; but intelligent co-operation on the part of labor can help produce lower costs without such sacrifices through greater efficiency in operation which, in turn, means better service at lower cost.

The automobile industry is probably the most highly mechanized of any in the country, the value of the product of each man's labor is higher than that of any other industry, and the rates of wages are at the top. This has not been achieved by "made" work but by securing the maximum production from the proper combination of men and machines. Railways will have to achieve something similar to regain and hold prosperity.

---

[17] Address to the Academy of Political Science, November 12, 1936, p. 62.

[18] *Railway Age*, January 8, 1938 (quoting from *Washington, D. C. Post*).

[19] Associated Press Dispatch, in New York paper, June 30, 1938.

It must be evident from the foregoing that reduced revenues and increased costs of operation (labor being an important influence in the latter) are two outstanding factors affecting the railroad situation in the United States to-day.

Railroad Capitalization.—There is little use in discussing the present financial structures of the railroads. It has been shown time and time again that the railroads are not over-capitalized. Considerable of the existing capital will probably be lost in the various re-organizations and bankruptcies through which so many of the lines are now passing. The diminishing rate of return on the capital invested in the railroads is shown in Table 2.[20]

TABLE 2.—RAILROAD CAPITALIZATION

Year (1)	Net operating income, in dollars (2)	Annual rate of return (percentage) (3)	Year (1)	Net operating income, in dollars (2)	Annual rate of return (percentage) (3)
1928	373 871 000	5.03	1933	132 595 000	1.93
1929	311 735 000	4.04	1934	118 946 000	1.79
1930	223 692 000	3.10	1935	175 696 000	2.57
1931	121 863 000	1.74	1936	232 782 000	3.53
1932	130 875 000	1.80	1937	119 161 000	1.66

It is very evident that the railways need considerably more capital in order to bring their maintenance up to date, to make advances, and to take advantage of such improvements in the art as are now available in order to produce ton-miles and passenger miles at the lowest possible costs consistent with the improved service which is now being, and will be more and more, demanded in the future.

Present Needs of the Railways.—High-speed, well-equipped, smooth-running, air-conditioned, and probably lighter-weight passenger trains, are needed. Perhaps it is not going too far to state that it would pay to replace most of the present passenger equipment before 1948. Certainly that is true of main-line equipment, except for the comparatively few trains recently put in service. Genuine research is needed, and a mass of facts as to complete costs per unit of transportation of some of the newer forms of equipment and motive power.

The state of the art of building lighter equipment has progressed to the point of planning for an even greater program of replacement of freight equipment than that indicated for passenger equipment. There are possibilities in mechanical refrigeration and other improvements in the mechanical features of freight cars, trucks designed for high speeds, improved draft rigging, air-brakes, etc.; and yet, it is doubtful whether the industry is even making the beginning of a showing into research as to the possibilities of these improvements.

To carry the new equipment and to operate trains at the high speeds to be demanded in the future, considerable attention needs to be paid to the track and signals. No one seems to have been able to improve on the general form of track structure from that of the earliest days; but heavier rails are needed, as are probably welded rails, improved fastenings, still further lengthening of the useful life of ties, more ballast, better drainage, and, of course, elimination of many grade crossings and some revisions of alignment.

---

[20] *Railroad Age*, February 12, 1938.

The art of signaling is well advanced and undoubtedly the general installation of some of the newer forms of automatic signals will not only promote safety, permitting increases of speed, but by reason of their efficiency, pay for their original cost.

Railroads have only begun to explore the possibilities of power production. The few examples of electrification, of course, mark important advances, but, in the present state of the art, installations of this kind must be confined to areas of very dense traffic, not only because of cost but from their very nature. The overhead wire system of power transmission seems to be tremendously costly and complicated and perhaps one of the advances in store for railroads is a better means of transmission from the power station to the electric locomotive.

Railroads are experimenting with Diesel engines but these have not yet progressed much beyond the experimental stage for general railroad traction inasmuch as all is not known about their total expense in producing and using power.

The steam locomotive is continually advancing in efficiency and steam as a prime mover is yet the "king" as far as most railroads are concerned. However, there are many more old steam locomotives in service than is desirable, and these should be replaced with modern power.

All these and a myriad other factors can, and should, be developed and put into practice to produce a really efficient railroad transportation machine. Brains, knowledge, and the engineering and scientific ability are available, and nothing is needed but money.

THE MONEY QUESTION

Needed Highway Expenditures.—At the meeting of the Greater New York Safety Council, in New York, N. Y., in 1937, Frank T. Sheets, M. Am. Soc. C. E., read a paper on "The Highway of Tomorrow." He discussed, ably and in great detail, not only the probable future traffic of the highways and their needs but also the economic factors—the costs of delays and the congestion due to lack of sufficient highways of the highest types. According to Mr. Sheets:

> "The blindest can see that tomorrow's highway system must include many classes of facilities. Neglect of one class will cripple the efficiency of all others. The metropolitan citizen must grasp the viewpoint and needs of the small town resident and the viewpoint of the farmer. The latter must see metropolitan highway needs. All must grasp the whole highway picture. Narrowness of vision, selfishness of concept, suspicion, intolerance and misunderstanding are the most formidable barriers to the realization of adequate highway transportation. Planning, engineering and financing are relatively small problems when compared with the public educational work and legislative action which must precede accomplishment."

These sentiments are important and are equally applicable to all transportation problems. They should by no means be confined to those of the highways; and, they certainly apply to railways. After this summing up of the situation the speaker then proceeded to forecast a reasonable estimate of the additional investment which he considered "was required to realize the highway system of tomorrow," and this he fixed at the very modest sum of $57 000 000 000.

Government Funds and Credit.—All this money, of course, is to be provided by Government credit and, again, without discussing the question as to whether the motor vehicle operator or owner will eventually pay his share of it consider what would happen to the railways of the United States if one-tenth this sum (that is, the mere bagatelle of, say, $6 000 000 000) were placed at the disposal of the railways for rehabilitation and progress. Like the owner of the motor vehicle the railroad should then be allowed a fairly wide freedom of operation within the limits of fair dealing with shippers, passengers, and employees.

The Government is extending its credit to the highways and, therefore, incidentally to highway motor transportation and to airways. Assume that this investment will be repaid by taxes or other contributions of the users of these facilities. The fact remains that the Government is financing this progress and looks to the development of business, and to the future users of the facilities, for a return on its investment. In these enterprises the Government takes the place of the bondholders and the stockholders of the railways.

There can be no question but that the Government investment in waterways is a special gift by the entire nation to a small section of the population that can take advantage of these facilities. There is little return on the investment, and there is a further annual expenditure for maintenance. Only recently it was proposed also to invest further millions of public funds in building a system of waterways, permitting the passage of ocean-going vessels from the Great Lakes to the sea, an addition to transportation capacity not needed at all and, from the point of view of many, entirely unsound economically.

Political Aspects.—The Government is a most important factor in this railway situation. If the most important section of the national transportation system is to advance, the Government must do its share. It has undertaken for many years now, to regulate and formulate the policies of the railways in minute detail. It fixes the maximum revenues, it allows or disallows capital issues, it regulates rates and hours of labor, etc. In view of the financial aid it gives to other forms of transportation (which it does not regulate so closely) it should extend some financial aid to the railroads, perhaps only to the extent of the $6 000 000 000, one-tenth of that which highway experts say must be spent on the highways to meet the needs of the motor vehicles. Even quite a little less than this would be very useful.

As early as October, 1932, the President of the United States, speaking at Salt Lake City, Utah, said:

"There is no reason to disguise the fact that the railways as a whole are in serious difficulty, and when so large a part of the American people have a direct cash stake in the situation—our job is neither to howl about a calamity nor to gloss over the trouble but patiently and carefully to get to the bottom of the situation—and try to plan for the removal of the basic causes of that trouble."

Many things have been accomplished by the Government since the President made this statement but the basic causes of the "serious difficulty" of the railways have not been removed.

Between 1932 and 1938 the railroad situation, except for a slight flurry in the early part of 1937, got continuously worse and on April 11, 1938, the President in a special message to Congress[21] declared: "Immediate legislation is, I believe, necessary at this session in order to prevent serious financial and operating difficulties between now and the convening of the next Congress."

However, in a recent "Fireside Chat" to the people over the radio, the President complained to his audience that: "The Congress also failed to meet my suggestion that it take the far-reaching steps necessary to put the railroads of the country back on their feet."

In spite of the request of the President for immediate relief and in spite of the appropriation by this same Congress of between $4 000 000 000 and $5 000 000 000 for various public works, which undoubtedly will include further construction of highways, waterways, and airways, nothing was done about the relief to the railroads which the President said was so greatly needed. However, in June, 1938, this same Congress did find time to pass special legislation for the railway employees.

SUGGESTED REMEDIES

Consolidation and Co-Ordination.—The questions of consolidation, co-ordination, and competition are always foremost in any discussion of the railroad situation. Personally, the writer is opposed to consolidation when carried to the extent of building up such large organizations that they lose the sense of personal management. There are undoubtedly cases where further consolidation may be desirable, but these should be studied as individual cases rather than on the basis of some arbitrary plan for the grouping of all the railroads of the country, or of some section of the country.

Co-ordination is being developed between the railways and the highways, waterways, and airways. The experience of the Federal Co-Ordinator of Transport, however, has shown that this is no one-man job; probably it must be developed along the lines of natural growth—in other words, by the American method rather than that of governmental paternalism.

Competition.—Competition has two aspects. In one sense competition is waste, inasmuch as it involves the use of duplicated facilities, but, in the case of the railroads, it is not as injurious or wasteful as it may seem on the surface. The second aspect, which compensates for any disadvantages of the first, is that competition is a healthy state for an industry such as railroad transportation, which depends so much on the human element which, in turn, competition tends to keep alert. There is certainly competition in commercial highway transportation and in commercial aviation. It would be wise to continue reasonable competition in the railway field.

---

[21] H. R. Doc. No. 583, 75th Cong., 3d Session, April 11, 1938.

In a review of the railway situation presented to the Alumni Association of the Harvard Business School on June 17, 1938, Mr. Eastman referred to the various "schools of thought" in suggested remedies for the present situation of the railways which he said were sick, financially speaking, and are the subject of general and grave concern.

He referred to the school of thought that believes that the solution of the problem is to be found in putting the capitalization of the companies "through the wringer"; another school believes in raising rates and fares; and another believes that the entire trouble is that of wages and working conditions.

He emphasizes the fallacies of the first two and says of the third that although drastic changes in the wage scale are not feasible, he does believe that adjustments are necessary. He finally expresses the belief that, in view of the position it has taken and built up in the past, the Government must be an important factor in the necessary rehabilitation and building up of the railways.

Mr. Eastman states that, inasmuch as the Supreme Court has ruled that transportation is a "function of the State," essential to the welfare and well-being of the entire community, the Government must play a more active part in the field of planning, promotion, and prevention.

THE SOLUTION

Now, what is the answer to all the foregoing questions? They all revolve about one question—advances in transportation. The railways are clearly the largest factor, and it is evident that they must be maintained in that position if the country is to continue to prosper and if general industry is to be served. Nevertheless, the railroad industry, on the whole, is going backward instead of advancing and this is not due, except in degree, to the present depression. It will not be really cured by such advance in prosperity as can now be visualized.

The labor situation needs adjustment. If the railways must compete with other industries for business they should be allowed to compete and "sink or swim" by their own efforts as other businesses must, and this need not involve indiscriminate "throat cutting" on rates.

The writer has avoided the mention of taxes because it is too big a subject for the scope of the paper; but taxes are growing ever larger. It seems as if the tax structure might well be adjusted; but there are many opinions about that and, in any event, it will add little to the present thesis to do more than mention it as a factor in the situation.

Government to Furnish Capital and Credit to Railways.—In the matter of capital needs and credit, the railways should be placed on an equality with highways, airways, and waterways, and they should have the full advantage of Government credit and the supply of Government funds for their necessary rehabilitation and progress. If this is done the Government should be in the same position as to railways properties that it is to the highways; that is, a stockholder looking for a return on his investment to the earnings of the property, after existing funded indebtedness has been taken care of.

Investments that are not legal for savings banks are usually not legal for insurance companies and insurance companies are large holders of railway securities. This, then, is not a question of a few powerful and affluent bankers, or wealthy families, but of almost every holder of a savings account or an insurance policy, and that means nearly all the ordinary, every-day, plain careful citizens of the country.

The loaning of Government credit and capital to these people is at least as much justified as is the loaning of Government money to the manufacturers of automobiles, gasoline, and oil, through the construction of highways on which the automobiles may run and the gasoline and oil may be used.

There seems to be ample precedent for the extension of Government credit and the making of Government loans to the railways, and if the railroads are to avoid the Government ownership and operation to which the President says he is opposed, there is little else that can be done.

There seems to be no valid argument against maintaining the present financial structures intact and subject to the orderly processes of present re-organization proceedings. Railroads have been "shaken down," their weaknesses have been exposed, some non-useful and non-profitable services are being abandoned. The President of the United States has spoken very strongly against Government ownership and operation. Viewing the transportation system of the United States as a whole, therefore, it seems reasonable that, in the matter of credit and capital, the Government should put the railways on a par with the highways, waterways, and airways, and then let them run their own business with a minimum of regulation and interference.

DISCUSSION

Milton Harris,[22] Assoc. M. Am. Soc. C. E. (by letter).—The rise and fall of the American railroad has been treated very thoroughly by Mr. Lavis. After justifying the railroad as the major national transportation facility for long-haul freight and showing the relationship of highway transport to that phase of the problem he lightly touches on aviation, waterways, and pipe lines, as also belonging in the transportation picture.

Improvement in transportation facilities, he notes, has been largely financed by Government aid, except that railroads have not been a beneficiary (of late), but have been beset by governmental regulations as to taxes, rates, and wages. The solution, according to Mr. Lavis, is Federal aid to put the railroads on a par with Government-financed highways, waterways, and airways.

In drawing the deplorable picture of the condition of the railroads of the United States, Mr. Lavis has omitted one vital figure that stalks the background from the beginning of railroad history. To-day, this figure stands out as the foremost agent responsible for "sick" railroads. That figure is "management." This word may be subdivided into financial and operations management; the first is under indictment, and the latter is responsible for the finest system of railroads in the world.

Management that bred the expression "the public be damned" soon led that self-same public to devise regulations to protect their own interests from the depredations of voracious financiers who were not aware that they were in the transportation business. Therein lies the main difference between highway and rail transportation. The highways are not listed on the stock exchange. True, they are in many cases under the control of political pressure groups, but those groups, by their own greedy tactics, are soon ousted, and it is a fair sign of progress that highway engineers are now becoming aware of their heritage in that they are becoming "traffic-minded"—that highways are not merely pavements, structures, and grading quantities, but are vital elements of the transportation problem and should be so designed and operated with that in mind.

About 20 or 25 yr ago railroad managements were presented with many, many miles of feeder road-bed. It was laid in their laps practically free of cost and all they were asked to do was to provide the rolling stock. Because these short-sighted gentlemen could see no ballast, ties, or rail winding into the hinterland, they lay back and allowed a younger, keener breed of executives to populate the highways with trucks and buses that stole the short-haul business. Too late did railroad management "let out a howl to high heaven" that they were robbed.

Lack of money is not entirely the pressing problem of the moment for the railroads. At one time in their history, it was the very least of their troubles; but unredeemed bond issues, structures, plant, and equipment that have paid themselves out many times over, but still appear on the books as capital investment (combined with resulting inflexible rate structures forced on the companies as the result of their own shortsighted policies) have now tied the railroads into a "balled up mess of red tape." Money will not cut the way out except in the hands of modern executives who realize the place of the railroad in the modern transportation scheme and can so co-ordinate that element to the whole that the entire structure is basically in tune with modern needs and economics.

---

[22] Associate Highway Engr., State Div. of Highways, Sacramento, Calif.

"Railroaders" to-day are antiquated. They "railroad" like their grandfathers did. New blood is needed—new thoughts and the application of the newer sciences. How many college graduates do the railroads absorb each year? What possible incentive under the present regime is held out for such men that other major industries seek so avidly? Where are the railroad research laboratories? What contributions have the railroads recently made to civilization where the crying need of the hour is more economical distribution of the products of labor?

These are the questions to be answered by railroad management. With a transfusion of modern executive blood into its system, and a lightening of governmental regulation, the railroad problem will fit itself quietly into the complete transportation picture and its financial worries will not affect civic life to the extent that they do at present. That the railroad has a definite place in the distributive system, no one can deny, but until railroad executives realize that they are in the transportation business instead of in the railroad business, Federal monetary aid to help them in their dilemma will merely be "pouring sand down a rat hole." Reference to Table 3 will clarify this viewpoint.

TABLE 3.—STATEMENT NO. 53 OF THE FIFTEENTH ANNUAL REPORT OF THE STATISTICS OF RAILWAYS IN THE UNITED STATES FOR 1936 BUREAU OF STATISTICS, IN DOLLARS

Year ended December 31 (1)	Investment in road and equipment—book value (Classes I, II, III, lessor, and proprietary companies) (2)	Common stock outstanding (3)	Preferred stock outstanding (4)	Funded debt outstanding (5)	Net capitalization equals total capital less sum of railroad stocks and bonds owned by railroads (6)
1923	21 372 858 000	7 398 000 000	1 852 000 000	13 589 000 000	17 810 000 000
1924	22 182 267 000	7 539 000 000	1 935 000 000	14 162 000 000	18 202 000 000
1925	23 217 209 000	7 602 000 000	1 937 000 000	14 105 000 000	18 191 000 000
1926	23 880 740 000	7 560 000 000	1 925 000 000	14 192 000 000	18 234 000 000
1927	24 453 871 000	7 683 000 000	1 980 000 000	13 951 000 000	18 137 000 000
1928	24 875 984 000	7 809 000 000	2 034 000 000	13 904 000 000	18 511 000 000
1929	25 465 036 000	7 853 000 000	2 065 000 000	14 065 000 000	18 680 000 000
1930	26 051 000 000	8 009 000 000	2 074 000 000	14 248 000 000	19 066 000 000
1931	26 094 899 000	8 031 000 000	2 049 000 000	14 264 000 000	18 941 000 000
1932	26 086 991 000	8 067 000 000	2 047 000 000	14 723 000 000	18 894 000 000
1933	25 901 962 000	8 057 000 000	2 042 000 000	14 624 000 000	18 831 000 000
1934	25 681 608 000	7 994 000 000	2 044 000 000	14 532 000 000	18 653 000 000
1935	25 500 465 000	7 987 000 000	2 036 000 000	14 224 000 000	18 342 000 000
1936	25 432 388 000	7 993 000 000	2 036 000 000	13 974 000 000	18 336 000 000

The sum of Columns (3), (4), and (5) (Table 3) equals the total railroad capital outstanding. Careful year-by-year study of the relationship of the elements of capitalization against book value should indicate the mortgage that railroad management has allowed to accrue against their plant without change in their operating methods. The heritage of past financial evils rides the railroads of to-day.

The railroads pioneered the opening of vast domains of natural resources of virgin country which were soon settled and producing traffic which increased with the years. Then came stabilization of population and industry, followed by the seeking of transportation at a lower price engendered by competition. Highway transport filled this need to a certain extent and decentralization of industry followed to decrease further the rail revenues. As a result there may be a surplus of rail facilities, as Mr. Lavis states, but is that a reason why the consumer or taxpayer should be forced to keep these dying tentacles alive? Would it not be more economical to abandon this surplus, write off the investment and replace it with a cheaper form of transportation plant? Possibly this would lead to a profitable organization, selling transportation at the lowest possible value by using the railroads as a back-bone and the more profitable self-sustaining highways as feeders, aided by pipe lines, airways, and water transport, all co-ordinated and consolidated into a national transportation system.

RALPH BUDD,[23] M. AM. SOC. C. E. (by letter).—The pity is that the information in Mr. Lavis' paper cannot be more widely disseminated and brought home to the general public. Lack of general understanding of the facts, so ably marshalled by him, is one of the primary factors in the so-called railroad problem.

Many thoughtful persons believe that substantial portions of the investments in railways already have been confiscated through low rates and high wages. Almost one-third of the railroad mileage is being operated under trusteeship, nearly one-half did not earn operating expenses and taxes in 1938, about nine-tenths failed to earn taxes, operating expenses, and fixed charges, and only about one-tenth earned anything for improvements or dividends. Education of the public must come from the disinterested studies of transportation experts and not from the clamor of minority pressure groups which have selfish interests.

Mr. Lavis' suggestion that there should be intensive and centralized research will meet with general approval, but more research is now (1939) being conducted than is generally recognized. The Association of American Railroads has done some work, although the extent of its research has been disappointing. Manufacturers of steam and Diesel locomotives, manufacturers of cars, machinery, and supplies, and the individual railroads have contributed to far-reaching improvements made in recent years.

With great deference to Mr. Lavis, the writer is not quite content with his statement that Diesel engines have not yet progressed much beyond the experimental stage for general railroad traction. The Diesels have demonstrated their efficiency and economy in switching service and in passenger train service, expecially on long-distance high-speed schedules. Coincidentally, the newer steam locomotives are greatly improved in design and capacity. Recently some very remarkable results have been obtained by the use of motion pictures in the study of "pound" on track by locomotive driving wheels at high speeds—either actual or slippage speeds. Space forbids enumeration of the many improvements that have been made possible in the last few years by intensive study and research. The daily operation of about 240 000 miles of railroad provides a laboratory in which a vast total of improved methods and devices are being developed all the time. Much of this development is not spectacular or widely publicized, but it is going on day in and day out as the result of the thought and ingenuity which hundreds of railway engineers are devoting to their work.

---

[23] Pres., C. B. & Q. R. R., Chicago, Ill.

As Mr. Lavis states, there is no lack of brains, knowledge, or engineering and scientific ability; what is needed is money. It would be a splendid thing if the railroads could afford to establish a nation-wide railway laboratory, in charge of a staff of skilled engineers. However, there are several financial aspects of that problem. It is not alone the initial cost of establishing such a research laboratory, or the expense of maintaining it, which makes such an investment difficult under the present financial condition of the railroads; the utilization of improvements frequently is disappointing from the standpoint of direct financial return on the investment. For one thing, there are many archaic rules respecting pay and working conditions which are so little adaptable to improved devices that no actual saving can be accomplished thereby. The development of new devices and of light-weight metal alloys of great strength has opened new vistas of building, and operating, light-weight locomotives and cars, which would reduce the ratio of dead weight to total train weight, reduce wear and tear on roadway and rolling stock, and permit faster and safer operation. However, existing equipment which is in serviceable condition cannot be scrapped immediately and replacements can be made only gradually. In the public interest it is desirable to make such replacements largely through depreciation accounts rather than by increasing capital accounts. To do this within a reasonable time requires a higher standard of earnings than has been permitted to the railroads in the past. Frequently there are opportunities for the railroads to make expenditures to take advantage of new ideas, with prospect of large return upon the investment; but as investments in railroads already are so large, and the return so meager, conservative management cannot consistently take any unnecessary risk with the capital available. If private enterprise is to provide necessary funds with which to take advantage of improvements and developments in railroading, there must be greater prospect than in the past that those investing the money will have some chance to share in the benefit of the improvements. It is not enough that the shippers and employees shall be benefited.

Some part of earnings must be paid for the use of money which makes earnings possible. Equipment trust certificates, at relatively low rates of interest, are well regarded by investors because they carry reasonable assurance of security of principal and interest.

Mr. Lavis suggests that the Government should provide funds for the rehabilitation of the railroads, either by loans (see heading, "The Solution: Government to Furnish Capital and Credit to Railways") or by gifts (heading, "The Money Question: Government Funds and Credit"). For a number of years the railroads have been borrowing money from the Government, chiefly through the Reconstruction Finance Corporation, and such loans, designed to tide over railroads that could not obtain money elsewhere, have been of great assistance; but such loans are merely temporary assistance—not a cure. In so far as the aggregate of railroad debt is increased in that manner, the day of reckoning is simply postponed; and, ordinarily, increasing the debt load reduces the credit.

Mr. Lavis makes a plausible statement of reasons why the Government well might grant the railroads a subsidy of several billions of dollars, in view of the much greater financial aid it is giving to other agencies of transportation. The writer does not think that Government subsidy should be considered as anything but a last resort. The capitalization of the railroads as a whole is below the valuations made by the Interstate Commerce Commission and, on the whole, the fixed indebtedness is not excessive. The railroad problem would not be solved by reducing fixed charges through writing off losses of investors, nor by Government loans without interest, nor by subsidies, unless it is to be solved through resort to Government ownership. If investors are to put new money into the railroad industry, what is needed is an increase in net railway operating income and elimination of unnecessary expenditures, not devices for expediting the so-called "wringing process."

To increase net railway operating income, the rate-making rule of the Transportation Act of 1920, which was practically eliminated by the Act of 1933, should be restored. In substance, the 1920 rate-making rule directed the Interstate Commerce Commission, in regulating rates, to permit the carriers as a whole, or in groups selected by the Commission, to earn an aggregate annual net railway operating income equal to a fair return on the aggregate value of the railway property used in transportation. In 1933, the statute was changed so as to omit this definite requirement and to instruct the Commission to give due consideration, among other factors, to the effect of rates on the movement of traffic. This new provision has been interpreted by the Commission as a direction to consider many matters aside from transportation conditions in determining the reasonableness of rates, and to substitute its own judgment for the judgment of the railroads on the question of whether or not increased rates will yield increased revenue. This construction in effect puts the Commission in the position of business manager of the railroads, passing judgment upon the question of what rates will or will not promote the movement of traffic, which is essentially a matter for determination by those who are responsible for the management of the railroads. The Commission has no direct contact with the subject and can reach its conclusions only on the opinions expressed by witnesses before it; in many instances these witnesses represent the people who would be required to pay the increased rates. The railroad managements would not propose rates which they thought would appreciably reduce the volume of traffic on the railroads and, even if they should make such a mistake, they are the ones who must bear the responsibility for the results of railroad operation. The law should be amended so as to restore the 1920 direction to the Commission to regulate rates in such a manner as to yield a fair return upon railroad property devoted to the public service, and to eliminate the existing admonition that the Commission consider the effect of rates on the volume of traffic.

A number of other proposals have been advanced for increasing the earnings of the railroads legitimately, such as repeal of the so-called "land grants" which now give the Federal Government practically 50% reduction on a large volume of traffic, and repeal of the "long-and-short haul clause" of the Interstate Commerce Act. These, and other proposals, would be of substantial assistance to the railroads in their present struggle with unregulated and subsidized competition; but the most important single legislative step which should be taken at once is the restoration of the 1920 rate-making rule, which would give the railroads a chance to exercise their best judgment to increase their net railway operating income by such rates as the Commission might approve as reasonable.

Increased earnings have a direct bearing on increased operating efficiency. To take advantage of the economies offered by new devices and improvements it is necessary to attract capital, and that depends upon ability to pay interest. The rate of retirement of old equipment is to some extent dependent on the depreciation charges which earnings will stand. The railroads must earn more money before they can eliminate many unnecessary expenditures of this type.

Another class of unnecessary expenditures, of an unproductive character, is for separation of highway grade crossings, for the principal benefit of traffic on the highways; increasing the size of railroad bridge openings, for the benefit of traffic on waterways, and special assessments for municipal street paving, drainage, and other local improvements. Mention of such enforced contributions for structures and improvements of a local nature leads to the thought of the vast sums of money being exacted each year from the railroads for Federal, State, county, city, town, village, school, park, library, and other purposes. Railroad taxation is too large a subject for this kind of discussion. Of course, the railroads should contribute their fair proportion of the cost of Government, but it is fundamental that the burdens of taxation should be distributed equally. One striking inequality is the heavy taxes on railroad terminal facilities in contrast to tax free municipal barge terminals and airports.

Another cause of unnecessary expenditure is found in the rules and regulations respecting rates of pay and working conditions in railway service. The agreements with the railway labor organizations have not been adapted to changed conditions, and their application results in some instances in exorbitant payments and, in others, payments far greater than the industry can afford. "Make-work" laws, limiting length of trains, size of crews, etc., are of like character.

One of the few railroad questions upon which the writer seems to differ with Mr. Lavis is that of consolidation; perhaps the two views are not so divergent. Mr. Lavis is opposed to consolidation when carried to the extent of building up such large organizations that they lose the sense of personal management, and he expresses approval of reasonable competition in the railway field. With these general expressions the writer is in full agreement, although there might be a difference of opinion as to the precise number of consolidated systems. In England a little more than 20 000 miles of railroads were consolidated into four systems. In the United States approximately 240 000 miles of road were disposed of in eighteen systems in the final consolidation plan of the Commission. Reducing the number of railroads to about eighteen or twenty would eliminate a tremendous amount of over-head organization, would enable traffic to be concentrated on the most favorable routes, using the best parts of the several lines as they now exist, and would automatically bring about the most desirable type of co-ordination of terminals—that is, co-ordination in a few strong ownerships. The beneficial features of competition would be preserved and enhanced, because a relatively few strong roads, able and willing to give good service, would insure a higher quality of competition than could be obtained from too many lines, competing with each other and weakening each other by excessive competition, and unable to use their facilities to capacity. Capital costs would be reduced. All of the indispensable advantages of private operation would be preserved.

Of course, large-scale consolidations would reduce the number of railroad officers and employees. In England, provisions were made for their protection, and the railroads in this country now have the so-called Washington Agreement with the railroad Brotherhoods providing for payment of a "dismissal wage" in case of consolidations. Undoubtedly, some reasonable provision could be made to avoid turning any man out on the street. Although the cost of such provision might postpone for a few years the full economies to be derived from consolidation, the problem is not so large as might be imagined. Mr. Leslie Craven states[24] that if the railroads were consolidated into seven or eight major systems, it is probable that about 75 000 men would lose their jobs; whereas, from July, 1937, to June, 1938, railway employment decreased from 1 174 000 to 914 000, or by 260 000 men.

L. ALFRED JENNY,[25] M. AM. SOC. C. E. (by letter).—The railroad problem is a very live issue at present. The writer is in general agreement with Mr. Lavis' analysis of the situation and his conclusion that the railroads are, and will continue to be, the major transportation medium. Some national research bureau would be very valuable. Much could be accomplished by the use of lighter and more modern equipment. Certainly something must be done to save the railroad industry. The writer does not agree with some of Mr. Lavis' conclusions, however, or the solution he offers. He agrees with the general diagnosis, but disagrees on the cure.

Are Many of Our Railroads Over-capitalized?—Mr. Lavis maintains that the railroads of the United States are not over-capitalized, and he thinks it is not necessary to put many of them "through the wringer." There are many schools of thought on this subject, and it all depends upon the standard by which the capital structure is judged. If it is judged on the basis of valuation for certain periods, then many capital structures compare favorably; but it is not fair to stop there without also taking into consideration the past, present, and future potential earning power of the property.

Abundant examples are available of industrial plants built to meet increasing demands for their product, which could not even be duplicated to-day for the price paid originally, but which now have very little value, not from a physical point of view, but because they are being operated only at a fraction of their possible capacity, and because there appears little chance of operating fully again. These industrialists have long since taken their losses and have written off a very substantial portion of the original investment.

---

[24] "Railroads Under Pressure," by Leslie Craven, *Atlantic Monthly*, November, 1938.

[25] Cons. Engr. (L. Alfred Jenny & Co.), New York, N. Y.

The argument may be raised that one cannot compare an industrial plant with a railroad; perhaps not. The railroad picture is even worse, because the industrialist can close a part or all of his plant if need be, whereas the railroad cannot, and, therefore, should have a financial structure that will enable it to remain solvent in depression periods.

Railroads were built to meet what looked like an ever-increasing flow of traffic and in a period when competition from other forms of transportation was negligible. Now, with increasing competition, many of them cannot possibly hope to operate to a capacity sufficient to compensate properly all classes of investors. Many of these railroads have not even been able to pay interest charges on income bonds during good years, when there was very little other competition, to say nothing of payments on the preferred or common stock. In fact, there have been many bankruptcies even during prosperous years. The common stock has received improper consideration in many instances—a very unfair procedure.

If a railroad is properly capitalized it should be able to pay a fair return on all classes of securities (even on the common stock) during prosperous times. If it cannot do that, it is difficult to understand how one may claim that it was not over-capitalized, or that it was not necessary to reduce its capital structure to such an extent as to permit a reasonable return for all classes of security holders in good times. The Interstate Commerce Commission has stated that, in general, the railroads are not over-capitalized. The writer believes that this statement was based on the valuations; yet the Commission wants to reduce fixed obligations drastically; in many instances it has held that the common stock had no value and has recommended that these securities be eliminated from consideration in reorganization proceedings. If that is not evidence of over-capitalization, one may ask, "What is?" Many railroads are, in fact, over-capitalized and many of them must be put "through the wringer" in order to effect a cure and not merely a palliative which perpetuates a diseased body and even adds new complications. One of the fundamental difficulties of the carriers is that so many of them have been continually adding new security issues to meet new requirements and, instead of following a balanced program of cancellation when they became due, have resorted to the policy of refunding. Although many of them had no alternative, it has made their financial position increasingly precarious.

Added Federal Loans.—Mr. Lavis is in favor of Government credit and the making of Government loans to the carriers; and he states (see heading, "The Solution: Government to Furnish Capital and Credit to Railways"), "if the railroads are to avoid Government ownership and operation to which the President says he is opposed, there is little else that can be done." Here too there are many schools of thought; but one may ask seriously how it is expected to repay these loans, or even to pay interest on them, if no drastic measures are taken at the same time to revamp the entire industry and "put it on its feet." Under the circumstances, such loans will only add to the financial woes of the carriers and help to force early Government ownership, the very effect Mr. Lavis hopes to avoid. It is probably true that the Government will have to extend credit to the railroads and make substantial loans to permit rehabilitation, but that should not be done without, at the same time, setting up proper machinery for bringing about a reconstitution of these properties and a sound national plan of co-ordination and correlation of all forms of transportation.

Deferred Maintenance.—Mr. Lavis states that the deferred maintenance at the end of 1937 was estimated to be approximately one billion dollars. The writer believes that it was more than that, based upon a study in which he compared the maintenance costs of the five years, 1932–1936, inclusive, with those of the seven years, 1925–1931, inclusive.

As the cost per equated ton-mile was deemed to offer the best relative comparison, that method was used. It is fully appreciated, however, that some labor-saving devices have been installed in recent years which did not exist during the late twenties, and that some processes have been developed for extending the lives of rails, ties, etc., and that these have somewhat reduced the cost of comparable maintenance. By making a direct comparison between say, 1925–1929 and 1932–1936, the resultant value, therefore, would tend to show a slightly greater sum for deferred maintenance than actually existed. In order to be conservative, and in some way to compensate for this condition, the first two depression years, 1930 and 1931, were added to the base period. This introduced an equalization factor of about 10%, or, in other words, the resultant value, covering all maintenance expenditures including materials, etc., is about 10% less than it would be if the years 1925–1929 were used as the base period.

The result of that study, and on the basis outlined, is shown in Table 4. The total under-maintenance on Class 1 railroads, alone, for the five depression years, 1932–1936, has reached the staggering sum of nearly 900 million dollars and if Class 2 and 3 railroads, and switching and terminal companies are added, the total will be much greater. If 1937 and 1938 are added, it is believed that the total for deferred maintenance will be nearly 1½ billion dollars.

TABLE 4.—STUDY OF DEFERRED MAINTENANCE, CLASS 1 RAILROADS

MAINTENANCE OF:	(a) UNDER-MAINTENANCE, IN MILLS PER EQUATED TON-MILE				(b) TOTAL FOR THE FIVE YEARS, 1932 TO 1936, INCLUSIVE, IN DOLLARS			
(Districts)	Eastern	Southern	Western	Total	Eastern	Southern	Western	Total
Way and structure	0.42	0.33	0.36	0.380	283 400 000	114 300 000	191 800 000	589 500 000
Equipment	0.27	0.14	0.11	0.185	182 100 000	48 500 000	58 600 000	289 200 000
Total	....	....	....	.....	465 500 000	162 800 000	250 400 000	878 700 000

If the deferred maintenance was estimated to be about 1 billion dollars at the end of 1937, it would probably be about 1⅛ billion dollars to-day (1939) on that basis. Whether it was that value, or 1½ billion dollars, or some intermediate value, is no doubt a question of judgment. The important question is, "How is it to be met?" No one can give the answer at this time. One thing is certain: It will represent a staggering sum, a "millstone around the necks" of these carriers which, for an indefinite period, will prevent them from enjoying even a relative economic freedom, so essential to the up-building and stabilization of that industry, and which will greatly retard the rehabilitation of these properties.

When it is considered that some of these railroads have maintained their properties along a higher standard than others, it must be obvious that some carriers will be found to be below the average standard of the district. Such a continued under-maintenance is certain to become dangerous if prompt measures are not taken to obtain an early remedy.

It must be obvious from this that maintenance costs in the future, at least for many years to come, will be more, instead of less, than in the past, and that if any operating savings are to be made they must be made in other ways than at the expense of proper and adequate maintenance.

Labor.—The United States has not been immune from the surges of varying philosophies going in one direction or another, and there has been a considerable change in attitude, particularly with regard to labor problems. Some call it the emancipation of labor; others call it by a less euphemistic name. Whatever it may be called, it is something to conjure with. It cannot be disregarded. The proper thing to do would seem to be to recognize its existence and assist in directing it along sound, equitable, and democratic lines.

In reviewing the railroad situation the labor problem must therefore be considered seriously, particularly in its effect upon possible future net earnings. During low traffic periods in the past, the railroads were able to reduce wages and thus correspondingly reduce operating costs, resulting in a more uniform ratio between income and outgo. All statistics, including maintenance, therefore, bear some relation to the prosperity of the carrier. It seems doubtful if such a procedure can be followed in the future.

The 1938 decision in regard to the reduction of 15% in railway wages is a "shining example" of this changed philosophy. No one can deny that this decision will have a tremendous and far-reaching effect upon the future of the railroad industry. Only a few years ago, and under similar economic duress, railway labor would have been almost certain to lose, or at best effect a compromise. Failure to recognize this condition can only result in disappointment later.

The railroads had estimated a net saving of about 250 million dollars annually with the 15% wage reduction they had asked for, and they stated frankly that, if such a reduction were not made, serious consequences would result. No reduction has been made, and there appears to be little likelihood that any will be made, of sufficient importance to have any material effect upon the net operating returns of the carriers.

Railroads must be prepared, therefore, to effect important savings in other ways because the carriers cannot continue to function properly under the existing heavy burden of expenses. Some people have suggested a profit-sharing plan for labor as a possible solution. It would have the effect of providing an equalizing factor, fluctuating with the prosperity of the carrier. It may well be that a solution along these lines could be effected which would establish a minimum fixed return for both capital and labor, and permit labor to share with the stockholder the fruits of a more abundant day.

The Problem.—The problem that confronts the nation is much broader and much more deeply rooted than what is commonly referred to as the "railroad problem." In order to grasp this fact fully it is necessary to review some of the underlying factors briefly. With few exceptions the railroads were built with private capital. They enjoyed a monopoly on transportation and attracted great sums of money for new construction to meet an expanding trade. Although the inland waterways, developed at public expense, had long predated the railroads, they offered no formidable competition, particularly with reference to the growing transcontinental traffic. Then the Government built the Panama Canal, at public expense, and this project detracted great tonnages from the transcontinental railroads of the United States. Not only is that tonnage lost, but the diversion is increasing.

New ports were established, at public expense, along the Gulf of Mexico, and much of the rail tonnage, particularly between Texas and Louisiana and the Eastern seaboard, is now (1939) handled by coastwise shipping, in which field competition is increasing.

Pipe lines were then built, taking the oil away from the railroads. In addition to losing the oil business this development has had the effect of substituting oil for coal in many fields. As coal is one of the most profitable carrier commodities, this trend has reduced railroad revenues seriously.

Large-scale highway developments were made necessary by the introduction of the automobile. They were built at public expense, and passenger and freight-carrying vehicles, both public and private, were permitted to operate over them in competition with the railroads. This agency has proved to be by far the most serious competitor of the carriers. In addition, it has made necessary very large railroad expenditures for grade-crossing eliminations, thus not only reducing carrier income, but increasing outgo.

Finally, there has been a great development in the air industry. Most of the airports were built by municipalities, again at public expense, and since no traffic arteries had to be built, with perhaps the exception of beacons, the airplane too began to compete over public facilities.

The truck and the bus are, in fact, making such inroads upon the carriers, in so far as traffic within a radius of a few hundred miles is concerned, as to remove, completely, any necessity for demanding, in addition to that, competition between the carriers. The airplane, on the other hand, is making heavy inroads on the long-distance passenger and preferred merchandise traffic, creating a natural competition for that class of business. The coastwise shipping is handling increasing tonnages of a very desirable class of bulk transcontinental traffic and in recent years has cut heavily into the profitable refrigerator business, thus creating a natural competition for that class of business.

All of the aforementioned forms of competition are here to stay. The competition sought in early legislation has been largely, if not completely, supplanted by these other forms of competition, reducing the railroads to the handling of such mass transportation as can still be performed best by them, and which, no doubt, will continue to be handled by them.

Here again, however, other factors are working against the railroads. The nation has witnessed a formidable movement, decentralizing important industries and establishing manufacturing plants close to the sources of supply, thus greatly and permanently reducing the profitable long-distance railroad shipments of raw materials and finished products, previously enjoyed by the railroads. Not only has this seriously reduced railroad traffic, but a considerable percentage of the traffic that would remain has been taken away by the trucks.

All of these elements must be considered seriously, entirely apart from the present reduction in net railroad revenues due to a reduced volume of business because of the depression, lower rates, and higher wages. The depression is only one of many elements contributing to the present plight of the railroads which has been accentuated by top-heavy financial structures. It can be understood, therefore, that, even with a return to normal times, the railroads will not get even a relative percentage of their former tonnages.

If the railroads were to be placed on a parity with these other forms of transportation in so far as facility ownership is concerned, the public would have to provide either railroad lines or terminals, or both; and it would have to permit private operation over these facilities. Under effective consolidations it would be possible, at least, to have new consolidated terminal facilities owned by the public and leased to the carriers for private operation. This improvement would save the railroads large sums annually and would still permit the continuation of the principle of private ownership and operation.

It has been suggested that transportation companies be organized to supply all forms of transportation. If competition is desired it would be most logical to maintain it under this type of organization, as the present and effective types of competition would be removed. However, there is considerable opposition to this type of organization, particularly by the present competitors of the railroads who would lose their identity.

It must be apparent that the problem which confronts the transportation industry, and which must be solved effectively and courageously, is far greater than the commonly accepted and circumscribed railroad problem. The scope and character of the solution required is so far beyond the control of the railroads that it is impossible for them to attempt a solution alone. Co-ordination and correlation of all forms of transportation, wherein each can labor with a certain degree of freedom within its own sphere, are essential to a sound solution. That requires centralized and authoritative direction and impartial administration.

The proper procedure is to recognize all the foregoing basic elements and prepare a sound, comprehensive plan in the interest of the transportation industry as a whole. The centralized authority could be either: (1) A new Federal department under a Secretary of Transportation; or (2) a Federal Transportation Authority, whose duty it would be to effect a national policy of transportation looking toward a truly co-ordinated and correlated system of transportation for both civil and military purposes, and to promote a plan of sound and all-inclusive railroad consolidation.

The term "military purposes" is included because, although much progress has been made in industrial mobilization for war, nothing has been done to plan and organize properly the transportation system, which is the back-bone of all military operations. The railroads should be prepared to superimpose upon civilian traffic, on a given day, the transportation of vast quantities of raw materials and the finished products of these industrial plants and of the military forces and equipment, without causing a serious disarrangement of civil transportation requirements, or the possible collapse of the entire system. No other major country is as unprepared as the United States in this respect.

Under the aforementioned plan all Federal regulatory functions, covering all fields of transportation, could then be entrusted to the Interstate Commerce Commission, with such added facilities and authority as may be needed.

The writer's views on this general subject, and particularly on the creation of a central Authority, are shared by many outstanding men in different walks of life. W. J. Wilgus, Hon. M. Am. Soc. C. E., has frequently recommended[26] such a central Authority as the only way to cope with this situation adequately. In the report of the Coolidge-Smith Committee (1933), former Governor Alfred E. Smith recommended the creation of a Department of Transportation in Washington, D. C. Similar opinions have been advanced forcefully by Mr. W. M. W. Splawn, Chairman of the Interstate Commerce Commission[27] and (from a legal view) Mr. Leslie Craven, who was for many years a railroad attorney representing various carriers in Washington.[28] This problem was well on the road to a solution when the railroads frustrated their own salvation by having the office of Railroad Co-ordinator abolished.

Mr. Lavis seems to consider centralized planning for the improvement of this industry an un-American procedure—one of governmental paternalism. If the railroads could solve this problem without some centralized direction many would no doubt agree with Mr. Lavis in principle, but since it must be obvious that the carriers cannot do so, it becomes anything but an un-American procedure to help put this industry "on its feet" in the interest of the industry itself and the country at large. Transportation no longer can be viewed as an individual problem, or a private monopoly, because it has long since ceased to be that. This problem must be viewed in its broadest sense, with the realization that one of the fundamental factors, essential to sound and intelligent planning, is unbiased and objective thinking, freed from selfish consideration. It must be solved in the interest of the transportation industry as a whole, not merely the railroads, and the economic well-being of the country in general.

Under the aforementioned circumstances, and restricting comments to the railroad field, with about one-third of the railroad mileage in receivership and with huge sums already advanced to them by the Government, it is difficult to understand how any one can expect to ward off Government ownership of the railroads unless some sound plan of general consolidation is advanced. In fact, some of the foremost engineers, and even railroad and fiduciary officials, seem to see no other escape than to resort to Government ownership. Competition will probably be restricted or completely removed in certain areas, under Government ownership, unless some effective plan can be devised for private operation of publicly-owned systems, outside of political influence.

---

[26] *Military Engineer*, September–October, 1938, p. 349.

[27] Report of the Fiftieth Annual Convention of the National Association of Railroad and Utility Commissioners, New Orleans, La., November 15–18, 1938, *Railway Age*, November 26, 1938, p. 776.

[28] *Atlantic Monthly*, December, 1938, p. 767.

Railroad Consolidation.—In order to establish a co-ordinated and correlated system of transportation in this country it is essential that each type of transportation, or each system, be given a certain sphere of activity which will enable it to "stand on its own feet." What must be done to accomplish this with the railroads? There is general agreement that one of the fundamental requirements is railroad consolidation.

Apparently, Mr. Lavis is in favor of permitting the carriers to make such voluntary consolidations as they may care to make without regard to some national plan. He believes in maintaining strong competition between the carriers and considers that as creating "a healthy state for an industry such as railroad transportation" (heading "Suggested Remedies: Competition"). Again it must be said that there are many different schools of thought on this subject and it would seem most important to give this matter serious consideration.

As stated previously herein, maintenance expenses will be relatively higher in the future, and it is most unlikely that railroad wages will be reduced, or at least sufficiently to have any appreciable effect upon the net income of the carriers. Therefore, railroads must look elsewhere for important savings.

Such consolidations as have been made in the past have not helped to relieve those carriers that are most in need of assistance, because the strong roads are unwilling to merge with the weak ones. The ultimate result of such a policy will be that the nation will find itself with many poor, broken-down railroads on its hands, which nobody wants, yet which are necessary to serve economic needs. What will then happen to these roads? The result is certain to be more chaos and more demands for Federal help, all at the expense of the taxpayer. It is not likely that the Government, irrespective of who may be in power, will continue to support these roads without demanding a sound reconstitution of the industry as a whole, and the absorption of the weak by the strong, along some kind of national plan.

There are several reasons why little progress has been made in the past by the railroads, or can be made by them in the future, if the problem of consolidation is left in their hands without some form of compulsion following a prescribed pattern.

There are too many divergent interests to be considered, which, if not directed by an impartial outside Authority, will result in disagreement and unsound consolidations. Many high railroad officials would find their positions abolished, and it is only natural for them not to expedite, or even belittle, effective consolidations, although they well know that the industry as a whole would be greatly benefited. Finally, present laws are inadequate to permit really effective consolidations, because they are based upon the predicate that strong competition be maintained, as between the carriers.

With the many forms of natural competition established, as outlined previously herein, there would seem to be no reason for demanding, in addition, competition between the railroads themselves, except possibly between Mid-Western points and the seaboard. The regions served by the railroads, and the country at large, would be served to much better advantage by removing restrictions as to competition between the carriers and permitting regional consolidations of privately-owned and privately-operated systems. The entire country benefits from a stable transportation industry.

In view of the many competing forms of transportation it must be obvious that the railroad plant, designed and built as the exclusive transport agency, is now greatly in excess of the needs of the people. Some of the facilities can, and should be, abandoned. That can be done under a proper and all-inclusive plan of consolidation. If the railroads do not so consolidate, it will be impossible to abandon a large number of the weak lines which, unfortunately, are essential to the economic existence of many regions.

If the present policy of voluntary consolidations is continued, it must be obvious that a fight for the "survival of the fittest" will result, which neither the railroads, nor the country as a whole, can afford to do. It would result in profits by the few and a "falling by the wayside" of the many, unless, of course, they are saved by the Government; but that is not sound planning, not sound engineering. It would seem far better to plan a sound solution than to sit back and await the inevitable, with resultant chaos.

With increased costs, and increasing competition, railroads of the United States, particularly the weak ones, would have to ask for rate increases at a time when a reduction in rates would have a most wholesome and far-reaching effect upon the entire economic structure of the country.

Where it becomes necessary to abandon certain properties because of the obsolescence of the plant, administrators must also take care of the consequent obsolescence of man-power. If it is economically desirable to abandon certain lines, the carrier can give such labor protection as may be necessary and fair to all. That should be mandatory. It would not be a permanent burden.

In discussing possible losses of labor in connection with such an all-inclusive program it should be stated that it will be necessary to create many new and important consolidated railroad facilities, totaling many hundred millions of dollars, particularly in terminal districts, giving work to scores of thousands of workmen all over the country and greatly benefiting all basic industries. This will largely, if not completely, offset losses of labor due to abandonments which, after all, will represent only a nominal percentage of the total railroad mileage.

It has been recommended recently that the railroads be relieved of certain obligations so that they can spend these savings, amounting to many hundred millions of dollars, for much needed improvements. As some of that money would no doubt be wasted with a proper plan of consolidation, which is certain to be adopted ultimately, it would seem prudent to proceed first with effective consolidations and then make such adequate improvements as will serve such a permanent and co-ordinated railroad plant.

Under the foregoing circumstances it would seem futile to expect any worth-while solution, unless some central and neutral authority is set up with power to compel proper consolidations. Such Federal Authority must establish the general pattern, but the railroads themselves should do much of the detail work of effectuating such a plan.

A Possible Solution.—Convinced that the greatest savings could be made under the regional type of consolidation, the writer made a study of consolidating the railroads into six regional systems. That study is presented herein, not as an unalterable plan, but merely to show what could be accomplished under the type of plan recommended in opposition to Mr. Lavis' position with reference to consolidations.

Since many regions have problems peculiarly their own, it is not difficult to group these railroads in such a way as to have each region covered by such a system. Thus the different carriers could be grouped effectively into North Eastern, Central Eastern, South Eastern, North Western, Central Western, and South Western systems, herein referred to as Systems I, II, III, IV, V, and VI, respectively. All traffic within these systems would be handled by that one system, but, as stated before, there should be competition in a larger sense, such as providing, for instance, two competing lines between Mid-Western centers and the seaboard or between important, interior, industrial centers. To establish such competition it will be necessary to provide certain "bridge lines" reaching from one system into such important centers of another system. This principle has been followed herein in the allocation of railroads to the various systems. The ten principal Class 1 railroads tentatively assigned to each of the six systems are as follows:

I.—North Eastern System: New York Central; Illinois Central; Wabash; Pere Marquette; Erie; Boston and Maine; New York, Chicago and St. Louis; Maine Central; Delaware, Lackawanna and Western; and Alton Railroads.

II.—Central Eastern System: Pennsylvania; Baltimore and Ohio; Louisville and Nashville; Chesapeake and Ohio; Reading; New York, New Haven and Hartford; Lehigh Valley; Western Maryland; Virginian; and Long Island Railroads.

III.—South Eastern System: Southern; Norfolk and Western; Atlantic Coast Line; Seaboard Air Line; Central of Georgia; Florida East Coast; Yazoo and Mississippi Valley; Mobile and Ohio; Nashville, Chattanooga and St. Louis; and Gulf, Mobile and Northern Railroads.

IV.—North Western System: Chicago, Milwaukee, St. Paul and Pacific; Chicago and North Western; Great Northern; Northern Pacific; Chicago, St. Paul, Minneapolis and Omaha; Minneapolis and St. Louis; Chicago Great Western; Spokane, Portland and Seattle; Duluth, Missabe and Northern; and, Green Bay and Western Railroads.

V.—Central Western System: Atchison, Topeka and Santa Fe; Union Pacific; Chicago, Burlington and Quincy; Chicago, Rock Island and Pacific; Denver and Rio Grande Western; Western Pacific; Colorado and Southern; Fort Worth and Denver City; Chicago, Rock Island and Gulf; and Northwestern Pacific Railroads.

VI.—South Western System: Southern Pacific; Missouri Pacific; St. Louis and San Francisco; Texas and New Orleans; Missouri-Kansas-Texas; St. Louis Southwestern; Kansas City Southern; Louisiana and Arkansas; Missouri and Arkansas; and Midland Valley Railroads.

No doubt, a further study would reveal the desirability of certain reallocations between the foregoing systems. For instance, System I has no

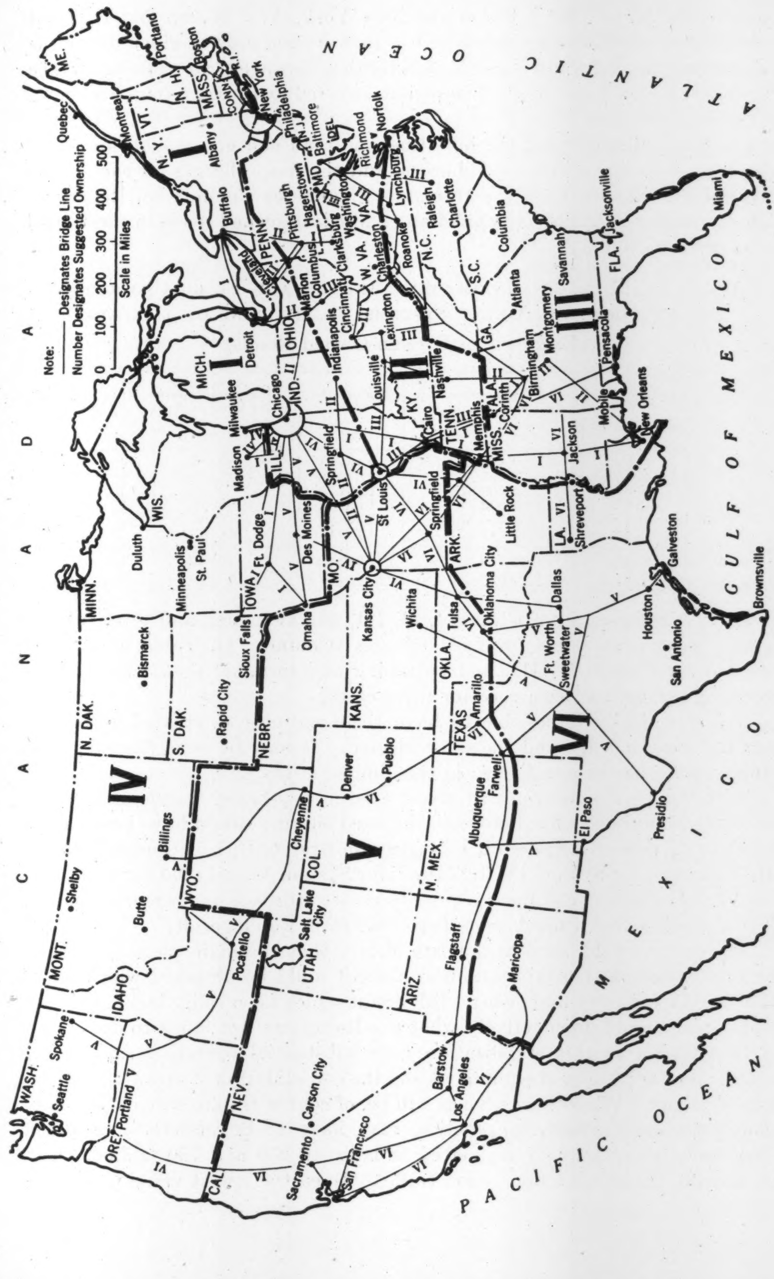

Note:
Designates Bridge Line
Number Designates Suggested Ownership
0 100 200 300 400 500
Scale in Miles
CANADA
ATLANTIC OCEAN
GULF OF MEXICO
MEXICO
PACIFIC OCEAN
I
II
III
IV
V
VI
ME.
N. H.
VT.
MASS.
CONN.
R.I.
N. Y.
N.J.
DEL.
MD.
PENN.
OHIO
MICH.
IND.
ILL.
WIS.
MINN.
IOWA
MO.
KY.
W. VA.
VA.
N.C.
S.C.
GA.
FLA.
ALA.
MISS.
TENN.
ARK.
LA.
TEXAS
OKLA.
KANS.
NEBR.
S. DAK.
N. DAK.
MONT.
WYO.
COL.
N. MEX.
ARIZ.
UTAH
NEV.
IDAHO
WASH.
ORE.
CAL.
Quebec
Montreal
Portland
Boston
New York
Philadelphia
Baltimore
Norfolk
Richmond
Albany
Buffalo
Pittsburgh
Hagerstown
Washington
Clarksburg
Columbus
Cleveland
Marion
Cincinnati
Charleston
Lynchburg
Roanoke
Raleigh
Charlotte
Columbia
Savannah
Jacksonville
Miami
Atlanta
Lexington
Detroit
Indianapolis
Louisville
Nashville
Montgomery
Birmingham
Pensacola
Mobile
New Orleans
Corinth
Memphis
Cairo
Chicago
Milwaukee
Madison
Springfield
St Louis
Jackson
Shreveport
Little Rock
Galveston
Houston
Brownsville
San Antonio
Dallas
Ft. Worth
Sweetwater
Oklahoma City
Tulsa
Wichita
Kansas City
Des Moines
Ft. Dodge
Omaha
Sioux Falls
Minneapolis
St. Paul
Duluth
Bismarck
Rapid City
Amarillo
Farwell
Pueblo
Denver
Cheyenne
Albuquerque
El Paso
Presidio
Billings
Shelby
Butte
Salt Lake City
Pocatello
Flagstaff
Maricopa
Spokane
Seattle
Portland
Carson City
Sacramento
San Francisco
Barstow
Los Angeles

FIG. 1

access to Baltimore, Md. Either the New York, New Haven and Hartford Railroad may be allocated to System I or that system may be given one of the coal carriers to Baltimore. Such possibilities, however, do not change the principle of regional systems. The regions covered by these systems are shown in Fig. 1.

The total miles of road, the investment in road and equipment, long-term debt, income available for fixed charges, and the fixed charges for each of the proposed six systems, as of the Interstate Commerce Commission Report for 1936, are shown in Table 5. Canadian or Mexican owned roads in the United States are not included.

TABLE 5.—MILEAGES, INVESTMENT, INCOME AND FIXED CHARGES OF THE PROPOSED SIX GREAT SYSTEMS, IN MILLIONS OF DOLLARS (Exclusive of Foreign-Owned Roads)

System	Miles (1)	Investment in road and equipment (2)	Long-term debt (3)	Income available for fixed charges (4)	Fixed charges (5)
I	41 997	4 109.72	2 474.61	200.35	158.05
II	36 400	4 695.78	2 822.88	305.17	192.65
III	36 098	2 269.87	1 064.76	96.50	62.46
IV	43 633	2 922.74	1 658.71	80.61	78.32
V	51 467	3 294.82	1 268.43	91.47	74.76
VI	42 587	2 617.27	1 780.07	104.27	109.92
Total	252 182	19 910.20	11 069.46	878.37	676.16

Table 5 includes all of Classes I, II, III, and switching and terminal railroads. Income available for fixed charges (Column (4), Table 5) was not reported for Classes II, III, and switching and terminal roads; net railway operating income has been used for these cases.

It is practically impossible to have these systems so created as to give each the same mileage and the same income. It may be seen, however, that a fair balance has been achieved in this limited study.

With this plan, and even with many existing top-heavy financial structures and consequent heavy fixed charges, the fixed charges would have been earned in 1936, 1.27 times for System I, 1.58 times for System II, 1.54 times for System III, 1.03 times for System IV, 1.22 times for System V, and 0.95 times for System VI. In every one of these regions there are railroads under reorganization. With these in effect a much more favorable ratio would result.

The study would seem to indicate clearly that the main purpose (namely the establishment of a fair earnings balance) could be obtained which would stabilize the industry and re-establish credit which is so sadly lacking to-day. Such a plan would do effectively what the Recapture Act failed to do.

In addition to this stabilization very substantial operating savings will result from the pooling of equipment and the consolidation of operations within these systems. What these savings will be, of course, is pure speculation based upon hypotheses which, in practice, may have to be materially modified. They have been variously estimated at between 250 and 750 million dollars per annum, based upon depression traffic. Even the lowest value would be a worth-while inducement for making a change, but the net result is certain to be more nearly 500 million dollars. In addition to this it should be borne in mind that the securities of such a system, with a modified capital structure, would be nearly as safe as Government securities and could be marketed at much lower interest rates than are paid at present. This arrangement would naturally further reduce expenses and still further strengthen the financial position of these systems.

Mr. Lavis seems to be of the opinion that in such large organizations it would not be possible to maintain personal contact. In so far as straight distances are concerned these new systems would be no different from the present large systems reaching from the Mississippi River to either the Eastern or Western seaboard. Instead of one railroad ribbon across this distance there would be several. This will necessitate the creation of regional managers who certainly would be able to maintain a closer contact than is now possible with some of the existing carriers.

Effectuation of Such a Plan.—Within a relatively short period of time, the Federal Authority could prepare a tentative plan and then allow the railroads, say, 6 months or 1 yr, within which to offer modifications and to submit a plan of organization, abandonments, etc. In the meantime, the carriers could appoint some Board of Managers, composed of one member from each road to be consolidated, whose duty it would be to co-operate with the Authority.

After the aforementioned modifications have been considered by the Authority, it could then issue a general plan and order the Board of Managers to manage these properties as consolidated systems for a trial period of, say, 5 yr. This will allow further modifications to be made in the plan and will mark the beginning of 10-yr program of abandonments. These should be spread over a considerable period of years.

After the 5-yr period has elapsed the Authority could then issue its final plan and order the election of system officers and the establishment of these great systems. The Authority would also prepare a financial plan indicating the amount of new system securities it deems proper to allocate to the then existing individual securities. In this allocation weight might be given to: (1) The valuation; (2) the last 5 yr of earnings as an independent carrier; (3) the first 5 yr of system earnings contribution; and (4) other special merits of the respective carriers.

Under such a procedure an immediate beginning could be made and ample time would be allowed for the consideration of varied interests. These six great systems should be organized under a Federal charter as national systems.

Other Considerations.—In addition to the consolidation and rate-equalization problems, consideration should also be given to legislation establishing a uniform system of railroad taxation based upon the dictates of national economy and the welfare of the industry, and a uniform system or policy with reference to amounts to be paid by the railroads for grade-crossing eliminations. Numerous other measures, intended to give immediate relief and which have already been recommended by various bodies, should naturally be adopted in addition to the long-term program outlined herein.

Conclusion.—The railroads must have new capital to build up these properties. The poor roads, which are most in need of new capital, cannot get it except at exorbitant rates, because the investor, whether it be an individual or a fiduciary institution, is afraid to risk his money.

It is necessary to restore confidence in the industry as a whole. That can best be done by a scientific grouping of poor and prosperous roads within certain regions, with resultant important savings, and the consequent balancing and equalization of resources. It cannot be done by making only a few minor consolidations of desirable properties, as would be the case if left to the railroads for solution. An impartial Federal Authority must be established with power to compel a sound solution in the interest of the transportation industry as a whole and the country in general.

WILLIAM J. CUNNINGHAM,[29] ESQ. (by letter).—A well-rounded and concise summary of the present rail situation is presented in this interesting paper. The writer feels, however, that Mr. Lavis has placeed too much emphasis upon what he calls "fundamental defects in governmental regulation" (heading, "Synopsis"). He is inclined to attribute the difficulties more to regulation than to the loss of traffic because of the general depression. The writer believes that the depression accounts for much the greater part of the difficulties.

The statement that "It has been shown time and time again that the railroads are not over-capitalized" (heading, "Needs of the Railroads: Railroad Capitalization") needs greater amplification than is given in the paper. It is true that they are not over-capitalized from the viewpoint of dollars invested in transportation facilities, but they are over-capitalized from the viewpoint of dollars invested in facilities that are now used and useful. A substantial part of railroad capital is in useless branch lines, needless multiple running tracks, unused sidings, facilities designed for earlier days with shorter train runs and different characteristics of traffic, and duplicate terminals and other facilities. There must be a realistic recognition of the high degree of obsolescence in investment and capitalization written down to conform to the investment in productive property. Holding these views, it is obvious that the writer does not agree with the assertion that "There seems to be no valid argument against maintaining the present financial structures intact" (heading, "The Solution").

Greater emphasis should have been placed upon the need of organized and centralized research. The single paragraph on that subject is not consistent with the statement in the Synopsis that "special stress" is laid on that need.

In the discussion of the causes which have adversely affected the volume of railroad freight business, specific mention should have been made of technological advancements which have made possible the production of more units of finished product for a given volume of raw material. The substantial improvement in the efficiency of fuel consumption on locomotives, for example, has had the result of reducing, correspondingly, the volume of railroad-borne coal for railroad use.

---

[29] James J. Hill Professor of Transportation, Harvard Univ., Boston, Mass.

S. R. TRUESDELL,[30] ASSOC. M. AM. SOC. C. E. (by letter).—Statistics presented by Mr. Lavis referring to the relative amount of transportation in the United States might have been somewhat more comprehensive as to motor transport if he had used data secured by the U. S. Department of Commerce in its Motor Transport Survey of 1935. This survey showed that common-carrier motor trucks earned \$531 000 000, and motor buses \$168 000 000, a total of \$699 000 000, which with the private truck operations would make a very much more formidable showing than Professor Worley's estimate for 1932. In fact, the Bureau of Economic Research of the Department of Commerce has made estimates from year to year of the earnings of all types of transportation, which show the following distribution for 1936, in millions of dollars:

Type of transportation	Millions of dollars
Steam railroads	2 787
Motor transport	1 274
Electric railway	436
Water transportation	535
Air and pipe line	97
Total	5 129

On this basis motor transport earned 25% of the total transportation revenue in the United States for 1936. Similar data in 1929 showed motor transport with \$1 194 000 000, or 17.5% of the total transportation bill of \$6 847 000 000 in that year. As the actual total in 1936 should have been about \$6 000 000 000 on the basis of the index of business activity, the motor trucks have made heavy inroads into railroad traffic during the depression. However, if total traffic had increased in this period in continuation of the trends existing from 1910 to 1929, the increase in truck traffic would have been comparatively unnoticed.

In using the very heavy traffic on the Pennsylvania Railroad between Pittsburgh, Pa., and Altoona, Pa., to illustrate the impossibility of handling rail freight by trucks, Mr. Lavis forgets the tremendous mileage of railroad in the territory west of the Mississippi River which has less than 500 tons per day. It is on these lines that the cost of rail transportation and over-head has risen to 3 cents and 4 cents per ton mile, and thus into the economic realm of the motor truck, which is not 5 cents to 7 cents per ton mile, as cited by Mr. Lavis, but more nearly 3 cents. (Statistics were submitted to the Interstate Commerce Commission by thirteen typical truck operators, in the matter of "Increase of rates in Central Motor Freight Territory, 1937" showing approximately 625 000 000 ton miles costing \$19 000 000 or 3 cents per ton mile.) The railroad rate on such traffic is probably only 1.5 cents to 2 cents per ton mile so that truck substitution, which seems inevitable, will be more expensive. The uneconomic and unrecognized result of such transitions is the scrapping of valuable tracks and rights of way for the intangible values represented by

---

[30] Special Asst., President's Office, C. & N. Ry., Chicago, Ill.

"foot-loose" trucking operators; and this is a result of frozen rates due to overzealous regulation combined with unreasonable burdens of cost for labor and taxes. Some more modest type of rail motive power could easily accomplish the service needed at a lower cost, provided encouragement could be given to the rail management by relief from taxes, full crew laws, labor agreements, and out-of-date tariffs. Otherwise, the people in the communities served by the rail branch lines will pay increased sums for the transportation of coal, lumber, feed, gasoline, cement, and other necessary bulk products, or they will move away. In most territories affected by rail abandonments both of these contingencies are now happening.

If, through this process of erosion of the feeder rail lines, the railroad main lines are gradually deprived of traffic and, in turn, are compelled to raise rates from the present unprofitable 1 cent per ton mile, where will the process end? Furthermore, are even such apparently essential railroads as the Pennsylvania between Pittsburgh and Altoona safe? In each case in the past, the same conditions were present, the same slow attrition, the same support by regulation to prevent monopoly, and the same results eliminating the stages and the wagon transport of the 1800's, the canals of the 1830's, and now the railroads. Unless this process is to be arrested by release from anti-monopoly regulation, taxes, unreasonable labor provisions, and restrictive rate legislation, no railroad is safe from this uneconomical and artifically fostered process of change.

In regard to Mr. Lavis' paper, some reference is made, or implied, as to the extent of unremunerative capital expenditures since 1920. Mr. Eastman also has referred to this at times as an element for criticism. However, this is only a natural effort of a threatened industry to hold or improve its product in the face of dangerous competition. It would be the same in the case of two competing automobile manufacturers—new plants, new presses, new materials, without one dollar added to profits. If the effort were tardy or insufficient, as in the case of the railroads, there would be an absolute loss or decrease in profits with nothing to show for the capital invested. It is true that some of the expenditures for heavier locomotives, rail, and bridges seem foolish in the light of present facts; but they should be unquestioned as necessary in view of the strife for lower costs to keep the railroad systems solvent. Present-day speeds, quoted by Mr. Lavis as 40 to 50 miles per hr to satisfy the present "hand-to-mouth" merchandising, have probably doubled the costs of the tonnage trains of yesterday at 15 miles per hr; yet eminent economists have repeatedly justified the unit of gross ton miles per train hour (dependent largely on speed) as a measure of railroad efficiency. In the West the longest and most round-about rail lines have the highest "efficiency" in gross ton miles per train hour, and the least profitable results.

This paper has delved deeply and most intelligently into the ills of the railroads. The writer does not want to detract a bit from its value, which is great. He is almost ready to agree that the railroads are going backward; but it is not of their own volition. Railroad morale is very low. Under the stress of straitened circumstances research in both methods and materials is almost nil; and, by and large, the cause for this condition (which is destroying the largest and at one time the most efficient American industry) is as much from without as from within. It speaks well that there is still continued fighting effort from managements that have been so harassed.

David A. Molitor,[31] M. Am. Soc. C. E. (by letter).—This paper contains a most valuable presentation of the transportation facilities of the United States and deals comprehensively with the problems confronting each class of service. It emphasizes the fact that defects in governmental regulation are responsible for the grave financial difficulties of the railways, and that there should be better co-ordination of all transportation facilities. Finally, it stresses the fact that direct Government aid, like that now being extended to highways, waterways, and airways, should also be provided for the railways.

Informed persons will agree with most of these statements, but when it is proposed to extend Government aid to the railways, much will depend on the particular form in which it is given, whether by an out-right grant, by subsidy, or by an extension of credit; and also, what returns or guarantee should be given to the Government for such aid as may be granted by Congress.

In the case of the highways, the State and National Governments all collect special taxes on gasoline and for owning and operating motor vehicles so that highway construction and maintenance are being paid for by those using the highways. However, the highways belong to all the people, which means the Government, and those contributing the money therefore, have no vested rights in the property.

The natural waterways, in general, were made navigable by improvements and are kept usable by work done under Congressional appropriations paid from any funds in the Treasury. Any individual or corporation owning a boat can enjoy the free use of the national waterways; but again, the users have no vested rights in the property provided by the Government with money from the people. The improvement of the channels through the Great Lakes is the most extensive system in the United States and is not now, nor ever was, undertaken to serve any other purpose than that of navigation for both freight and passengers. In only one or two instances, such as the Tennessee Valley Authority, has flood control and power development been an important factor.

The airways have enjoyed the most generous treatment by the Government since they have been given subsidies toward operating expenses and generally have no monetary investments except in flying equipment.

The case of the railways differs in many respects from that of the other three facilities. It is now (1939) being proposed to place them in a position to partake in some measure of Government aid and thereby correct the inequalities which supposedly exist regarding their ownership, maintenance, and operation when compared with the others.

The railways enjoy no direct Government aid at present, although they carry the mails and considerable Government freight. In most States they pay a property tax similar to private land owners. According to the author, their total property investment in 1920 was about 19 billion dollars to which more than 6 billion dollars was added during the next twelve years, making a total of more than 25 billion dollars in 1932. In Table 2, the net operating income for 1932 is given with the annual rate of return on the capital invested, from which it appears that this capital sum was only slightly more than 7 billion dollars. How much of the remaining 18 billion represents property value? How was it acquired and who owns it at present?

---

[31] Cons. Engr., Harlingen, Texas. Mr. Molitor died September 8, 1939.

Many of the railways received land grants representing enough value to finance their construction with the aid of borrowed money from private investors.

Until about 1920, the railways were in practical control of all transportation business except between such terminals as were also served by the waterways. Their practices were such that they were declared common carriers under control and regulation by the Interstate Commerce Commission as a protection to the public against unfair practices.

The quarrel between the waterways and railways is an old one in which the latter were generally the aggressors. They fought the Panama Canal project for years until there came into the presidency a Theodore Roosevelt, who was big enough to subdue this propaganda and launch the undertaking. The Mississippi River improvement is another case in which a once prosperous water transportation was driven out of business but not with any great benefit to the railways, which were not properly equipped to carry cheap bulk freight.

A case in point to the contrary is the present spectacle of the St. Lawrence-Great Lakes Waterway, for which no one of sufficient stature has yet appeared who could settle the quarrel and subdue the opposition in Congress. It is regrettable that the power development was brought into the "squabble," but the navigation improvement must come sometime, and when the finances of the railways become sufficiently depleted on propaganda, perhaps they will spend what funds may remain on "putting their own house in order."

As long as business was good and money plentiful, due largely to low wages paid to operating employees generally, the railways could afford to fight everybody, including City, State, and National Governments, to maintain supremacy in the transportation field. They could do this and yet pay large dividends to stockholders and high salaries to officials. Then came the war period 1914–20, with the railways enjoying a virtual monopoly, but underequipped for the huge increase in business, so that with Government control and funds the wheels were kept in motion with sufficient new equipment to handle the heavy traffic, which continued to overtax the railways until about 1926.

The diversion of business to the highways, both passenger and freight, rapidly increased after about 1920, and, with the depression years 1932–38, caused the present surplus of transportation facilities on the part of the railways, while the highways were becoming busier each year.

During recent years the railways have been unable to compete with motor transportation both in kind and cost of service. It is doubtful whether any appreciable amount of the diverted business can ever be recaptured by a highly modernized railway equipment because of the greater convenience and house-to-house service afforded by motor for both freight and passengers, even at comparable rates.

Since 1928 the writer has traveled 75 000 miles by automobile and had his household goods moved 2 900 miles (including a single move of 1 900 miles) by motor van, all because of the superior efficiency and cheapness of this service. In short, he was prevented from using the railways because of the greater cost involved, their lack of house-to-house service, and loss of time due to poor schedules or infrequent service. The great majority of the traveling public is probably doing the same things for the same reasons.

The existence or non-existence of the railways of the future will hinge on "the survival of the fittest" as has already been the case with respect to interurban railways and some city street-car systems.

Big and little business has sought and found economies in freight costs, as the author mentions, which means a permanent loss to the railways. In their ambition to retrieve some of their lost business and to combat public dissatisfaction, the railways have gone intensively into politics, resulting in much discriminating legislation aimed particularly against highway transportation.

For instance, highway trucks in Texas cannot carry in excess of 7 000 lb and violations are vigorously prosecuted at the public expense to benefit the railways. In other States as much as 20 000 lb is permitted. Recently the Interstate Commerce Commission ruled an increase in billing weights on vegetables effective January 1, 1939, in Texas, New Mexico, Louisiana, and Arkansas, whereas Florida and California, the chief competitors, will not be affected. This ruling will add about $140 per car lot to an already excessive freight rate on shipments from the Rio Grande Valley, which is ruinous to the Valley growers, packers, and shippers alike. It is also proposed to repeal the long-haul and short-haul clause of the Interstate Commerce Commission Act.

Considerable statistical data have been presented by the author, and by others,[32] so that the writer has refrained from making any additions.

The enormous decline in railway business, from whatever cause, has greatly reduced income and thereby depreciated the investment securities. It has destroyed much capital value by mergers, reorganizations, and bankruptcies. What remains of a once lucrative business is now asking for Government aid, preferably in the form of an outright gift, on the plea that the other transportation agencies have enjoyed such aid not hitherto accorded the railways. This is termed the railway problem. Is this the real problem, or is it rather a case of inability to continue to attract investors in a rapidly declining business, which is over-equipped for future needs?

Originally all money invested in railway property, fixed and movable, came from the people. It was not first collected by taxation and then disbursed by the Government, after deducting a large overhead expense. Instead, the business was handled by the railway promoters themselves, and they now wish to sell the property to the Government because it is no longer profitable to operate and hold; but if the Government should buy that which was already created by the people through loss of their invested money and rates paid for service, it would mean that, ultimately, the money has come twice from the people, first to create the property and then to purchase what they have created.

---

[32] *Proceedings*, Am. Soc. C. E., November, 1938, p. 1731.

In the case of the highways and waterways the people paid only once, with the Government acting merely as a collecting and disbursing agency.

From the foregoing one must conclude that the railway problem is somewhat akin to that of any large corporate business during a period of obsolescence where the remedy, of course, would be receivership or bankruptcy. Then in the final wind-up, if the Government desired to acquire and operate the property for military or other reasons, Congress could provide the necessary legislation to create a Department of Transportation to handle all such matters, replacing the Interstate Commerce Commission, and, ultimately, to solve the railway labor question which, with other labor legislation, has become a political "foot-ball."

However, if the railways hope to rescue their property they must work out their own salvation by introducing new economies, lowering rates, catering to, and winning, the public by rendering efficient and reasonable service, and finally by recognizing the social implications of their business. The trend toward heavier locomotives and rolling stock, and adding more elaborate passenger trains, is not considered a move in the right direction because of the vastly increased maintenance cost of the road-bed. Lines that cannot exist on the available business income should be abandoned as was done with the interurban lines. Railway employees should realize that they are "killing the goose that laid the golden egg" and should accept reasonable pay reductions.

The foremost competitor of the railway is no longer the waterway but the automotive transportation on the highways. The latter, despite strenuous opposition and propaganda, burdensome traffic restrictions, license fees, and taxes, has become the choice of the people.

Were one to concede that the relations between the various transportation facilities and the City, State, and National Governments do require readjustment to rectify any inequalities supposed to exist, then the following legislative changes by Congress should receive consideration on the assumption that all fixed property be Government-owned and all transportation equipment be privately owned and operated without fees being charged:

(1) The Highways.—Repeal unfair traffic restrictions and remove license fees and gasoline taxes, the Government to build and maintain the highways, which are to be used free.

(2) The Waterways.—National improvements to be provided for navigation purposes and made free to the users.

(3) The Airways.—Remove subsidies.

(4) The Railways.—Acquire road-beds, fixed structures, such as bridges, tunnels, and stations, by purchase or, preferably, by donation. The Government would then build extensions when needed and maintain the property, the operation to be conducted by private capital as at present.

That these changes, or any of them, will ever find favor, or materialize, is highly problematical and much will depend on the funds available to each interest for propaganda purposes. The proposal to vote 6 billion dollars of credit to the railways would be a dangerous undertaking, to say the least. Although it might relieve the present financial crisis temporarily, it would not solve the railway problem. This sum of money, loaned to the railways, would only add to their present predicament; and it would further reduce the interest on their stocks, since there is no prospect of any increased earning power, and the loan would probably never be repaid.

Years ago the Canadian Government guaranteed certain outstanding obligations of its railways with the result that the properties are now practically owned by that Government. If, and when, any monetary aid is given to the railways, a safe plan would be for the Government to take a first mortgage on the property and thus be in a position to protect its own interests in case of default.

Writers on transportation problems have generally defended or favored only one system as against the others. The writer has attempted to express his unbiased opinion respecting their relations to the public.

C. A. HOGLUND,[33] ASSOC. M. AM. SOC. C. E. (by letter).—The suggestion by Mr. Lavis that a study be made of the entire transportation field in the United States is a proper one. This study should be made by men who have: (1) Knowledge of the subject by study or experience; and (2) no personal interest in the result. The entire transportation question should be approached with the thought expressed by Mr. Sheets, as quoted in the paper (see heading "The Money Question: Needed Highway Expenditures"), but with the thought that it should be applied to the entire transportation question instead of to highways. Mr. Lavis' approach to the general subject is somewhat confusing inasmuch as he discusses a special picked problem applying to each method of transportation, to the exclusion of the general problem. He also does not state the problem in such a manner that issues can be drawn as a basis of constructive discussion.

To the writer it seems that the problem should be first considered in a general manner, as for instance: At the present time what are the materials to be moved for the country as a whole? What are the methods and facilities available to move them? What consideration is being given to the relative economy and fitness of the various methods of transportation with a view to encouraging and promoting the use of each for purposes which it can serve best and most economically, while avoiding such use as is merely harmful to agencies better suited to the work?

The definite allocation of materials to be moved by a given type of transportation should be approached with caution, if at all, as the nation has not yet reached a point at which regimentation to this extent is possible. A certain theoretical recommended allocation is probably desirable only as a basis of determining theoretically whether or not there is an over-abundance of classes of transportation in a particular field, and to what extent.

---

[33] Prin. Engr., Inventory and Pricing Branch, Bureau of Valuation, Interstate Commerce Comm., Washington, D. C.

Fundamentals.—Certain fundamentals should be considered before suggesting remedies or solutions for transportation generally. As a background it might be well to review the past and the present quantities to be moved and the classes of transportation.

TABLE 6.—COMMERCIAL FREIGHT TRAFFIC IN THE UNITED STATES
(Revenue Ton-Miles)

Item No.	Agency of transport	1926	1936	1937
1	Steam railways*	447 443 627 000	341 181 596 000	362 815 382 000
2	Inter-city trucks	23 530 000 000	40 230 000 000	43 380 000 000
3	Great Lakes	90 037 507 000	77 263 645 000	93 243 674 000
4	Other inland waterways	9 542 877 000	15 387 091 000	16 882 943 000
5	Pipe lines	21 700 000 000	40 037 134 000	44 793 161 000
6	Electric railways	1 312 744 000	704 500 000	697 482 000
7	Airways	Nil	1 861 000	2 156 000
8	Total	593 566 755 000	514 805 827 000	561 814 798 000

* Reports of the Interstate Commerce Commission.

Volume of Transportation.—The volume of transportation in 1910 is unknown. The volume in 1926, 1936, and 1937 is given in Table 6.[34] The volume of traffic is less in 1936 and 1937 than in 1926, and in all probability, although the data do not show the volume in 1932, 1933, 1934, and 1935, it will probably be much less than that shown in 1926.

Changes in Ways of Transportation.—In 1910 there were approximately the following types of transportation: 240 000 miles of railroad, 200 000 miles of indifferently constructed highways, and about 30 000 miles of pipe lines—a total of approximately 470 000 miles of transportation lines. In 1930 there were 250 000 miles of railroad, 700 000 miles of fairly well co-ordinated highways (perhaps 200 000 of which were concrete or some other hard surface), 110 000 miles of oil and gasoline lines, 30 000 miles of airways, and about 10 000 miles of inland waterways—a total of not less than 1 110 000 miles of transportation lines, some of which were of almost limitless capacity or, rather, such a capacity as it now seems impossible to visualize. The foregoing shows a total increase of 136% in the total length of transportation lines in 1930 over 1910.

Over-capacity of Transportation System.—When one refers to the capacity of the transportation system, including ways, methods, and facilities of transport, it will be agreed that as a whole the system is not less than 50% more than that necessary to move the nation's production, using a prosperous year such as 1937 as the measure of an average production year. If this is true, it is axiomatic that the methods of transport should be adjusted and either reduced by law or be allowed to be reduced by competitive action without the restriction of any control.

Background of the Railroad Problem.—One of the basic questions affecting the railroads is the following: In 1910 the railroads as a whole were making money and were in practical control of the entire transportation field. There were approximately 470 000 miles of transportation lines. The railroads had no competition in the bulk transportation field of which they were not in control through relative efficiency; nor had there been any competition for about thirty years prior to this time. By better and more economical ways and methods the railroads had driven all other modes of transport from the bulk transportation field. To all intents and purposes they were a monopoly. This naturally caused the introduction of Government regulation.

---

[34] Report to the President of the United States of a Committee appointed to submit recommendations on the general transportation situation, December 23, 1938, Table 2, p. 66.

In 1930 a different situation existed, with more than 1 110 000 miles of transportation lines available. The modern highways, airways, waterways, pipe lines, and railroads now give an opportunity for one to choose other ways and methods of transport than the railroads. Highways and trucks of all sizes have opened the transport fields to every farmer and manufacturer in the United States. If a farmer does not like the freight rates, by truck or railroad, on hogs or cattle to the packing houses, he can haul by his own trucks, as illustrated by the fact that, in 1926, 90% of livestock received at 17 large markets were hauled by the railroads and 10% by motor trucks whereas, in 1937, 48% were hauled by railroads and 52% by motor trucks. If the packing houses do not believe that the rates are economical on meats by trucks or railroad, they can transport their own meats. If the steel mills or coal mines do not think the rates to the consumer are economical by the present water facilities, trucks, or railroads, they can transport by their own facilities. The ways of transportation are in existence and available for use, and their capacity has not been reached.

The methods of transport are probably as good to-day (1939) or even better than they were in 1910. Whether or not the methods of one particular class of transportation are more economical than those of another is naturally open to question. The methods and facilities on all of them are now in being and open to development regardless of existence in former times. Therefore, the railroads are in a competitive field, and Government regulation is probably not of as much service as it was in 1910 unless it is extended to cover, not only the ways, but all methods and facilities of transport. It is axiomatic, therefore, that the solution of the problem of one of the types cannot be made fairly without consideration of all classes, methods, and facilities, since they are now more or less competitive.

Increase in Railroad Capacity of Transporting Materials.—The speed of freight trains has probably doubled since 1910. As to capacity of locomotives, in 1910 the carriers had a total of 60 019 locomotives with an aggregate tractive effort of 1 588 894 000 lb and an average unit tractive effort of 27 282 lb. In 1935 they had 49 541 locomotives of which 48 477 were steam locomotives, with an aggregate tractive effort of 2 206 201 000 lb and an average tractive effort per unit of 48 367 lb.

In 1910 there were 2 148 478 freight cars with an aggregate capacity of 76 579 000 tons, or an average per unit of 35.9 tons. In 1935 there was a total of 1 867 381 units with an aggregate capacity of 88 677 000 tons, or an average per unit of 48.3 tons.

Needs of a Defense System in Times of War.—The problem of a transportation system should never be settled without giving consideration to the fact

that in time of war certain railroads, highways, waterways, pipe lines, and airways will be called upon to bear their part of the transport problem and to this extent these classes of transport are necessary to the national defense system. They should be maintained to a standard necessary to serve their objective. If the business naturally handled and the earnings obtained from this business are not sufficient to maintain them for this objective, they should be maintained with Government assistance.

***Conclusions.*—The foregoing shows that since the time when the railroads were a monopoly in the transportation field there has been an increase of 136% in the length of transportation lines generally. In addition there has been an increase in the efficiency of the railroad transportation to the extent that it would be able to handle perhaps 50% more transportation than it did in 1910, but there is a decrease in the possibilities of obtaining this traffic. The entire railroad field is burdened with over-capacity. What is shown herein relative to railroads is probably equally applicable to highways, waterways, and pipe lines. The crux of the situation—the question of over-capacity of ways, methods, and facilities of transportation—is the real problem.**

The writer has purposely omitted reference to the questions of: (1) Unjust railroad taxes; (2) inconsistencies in valuation procedure; and (3) rate of return on carriers' investment.

Item (1).—In 1933 railroads were taxed $150 000 000 less than in 1926; and in 1938, $40 000 000 less than in 1926.[35] A further review of their proportion of taxes on their investment will develop the fact that, generally, their tax burden is much less than that now being paid by the ordinary individual.

Item (2).—The value of the railroads as established by the Interstate Commerce Commission, based upon stated elements, is probably correct and proper as applied to a particular railroad. However, there is a condition of over-capacity of transportation facilities, when applied to the railroads in a freight region as a whole. The introduction of this element in the present discussion would probably reduce the values of the railroads in a freight region by not less than 30 per cent.

Item (3).—The propaganda of showing the rate of return on the carriers' investment by including the deficits of poorly paying railroads in with better paying railroads is no more proper than adding the statistics of failures among automobiles, steel, and other corporations to the good ones in obtaining their earnings as an average; and yet, nothing has been mentioned of the fact that the "dividend per cent on railroad stocks paying dividends" has never been less than 4.57% since 1891 (this low value occurred in 1932) and that the average over that period is probably not less than 6 per cent.

FRED LAVIS,[36] M. AM. SOC. C. E. (by letter).—The discussions of the paper have been very interesting and amply confirm the contention of the writer that the railway problem in the United States is one of national and very vital importance. Some of the commentators seem to have overlooked the

---

[35] Report to the President of the United States of a Committee appointed to submit recommendations on the general transportation situation, December 23, 1938, p. 78.

[36] Cons. Engr., New York, N. Y.

fact that this paper was originally presented as the opening address in the "Symposium on Transportation" offered at the Annual Convention of the Society in July of 1938, and seem to have assumed that it was a voluntary defense of the railroads rather than, as it really was, an attempt to evaluate the position of the railroads as part of the general scheme or plan of transportation in the United States.

Professor Cunningham notes that although a statement is made in the "Synopsis" that "Special stress is laid on the need for adequate research," little is said about it in the paper. This matter is referred to in more detail by Mr. Budd.

While most of the commentators agree as to the nature of the problem and its seriousness they disagree as to its solution or the remedies to be applied. Mr. Harris' indictment of present-day management, although he admits that "it is responsible for the finest system of railroads in the world," ignores a great many factors which are of vital importance to the nation in considering the problem. The greatest of these is the tyranny of railway labor and its influence on legislation. Reference was made to this in the paper but further reference may be made to a statement by Garet Garrett[37] which, while crediting the railway labor unions with a most creditable record as workmen, shows the abuses which have crept in and become established in the "constructive" rulings and in the rulings of the Railway Mediation Board.

Referring to railway and highway financing, Mr. Harris states that the main difference between them is that "The highways are not listed on the Stock Exchange." However, he ignores the fact that the Government bonds which paid for them are listed; that the capital of the banks and of the insurance companies is tied up in these bonds; that the stocks of the motor vehicle corporations are listed; and that they, with the labor organizations which are occupied in the motor vehicle industry, exercise a great deal of influence with the various State Legislatures as well as with the Congress. These factors cannot be ignored; nor can one, in facing the problems of to-day and trying to find a solution for them, get very far by blaming the financiers of yesterday.

No one realizes better than the writer the sins of railway financing in the past; but the securities issued are now in the hands of a large number of people in the United States, either directly or through the insurance policies so many of them hold. The problem is not one for the relief of Wall Street but for the relief of a very large number of the people of the United States who are not banded together and are not vocal. Above all, however, it is a problem of how to preserve a great asset of the country—one on which so many industries must depend.

Professor Cunningham points out that while it may be, or probably is, true that the railways are not over-capitalized, from the viewpoint of dollars invested, they probably are from the viewpoint of "dollars invested in facilities that are now not used or useful." Perhaps there is some little merit in this contention but it ignores the many improvements out of earnings and the more intensive use (under normal conditions) of the facilities which remain.

---

[37] "Peace on the Rails," by Garet Garrett, *The Saturday Evening Post*, September 9, 1939, p. 8.

Mr. Jenny also argues that the fact that the railroads do not earn enough to pay a fair return on their outstanding securities is evidence that they are over-capitalized. This fact, however, does not seem to be valid in view of what is said in the paper and the discussions as to the difficulties of modernizing their plant, of conducting reseach, and of being ground between the upper and lower millstones of Government regulation and labor union demands. Whether railroads are over-capitalized or not, the problem facing the nation is the rehabilitation of the transportation machine to produce ton-miles and passenger-miles efficiently and reliably at the lowest possible costs.

Perhaps the statement in the paper that Diesel engines (locomotives) have not yet progressed "much beyond the experimental stage for general railroad traction" may be subject to misinterpretation. That they are actually performing important services, with reliability, day in and day out, cannot be questioned; but in the original statement it was intended to call attention to the fact that, as yet, there is not much information at hand about final costs of service, and that Diesel engines are being used now for only a very small part of the total railway traction requirements.

Mr. Budd's remarks on the rate question, or some parts of the rate questions, are very much to the point and answer a good part of Mr. Harris' criticism of management—that is, of financial management. In times of stress, the railways are not only faced by demands for higher wages, supported by, and to all intents and purposes practically enforced by, the National Government, but are told by another Government agency what they may charge for service.

The writer is not for a moment advocating higher rates generally on railways, although some may be justified. Rather he is emphasizing that the greatest transportation agency in the United States, an agency vital to the existence of the country, is falling between the demands for higher wages, lower costs of transportation, and competition by Government financed agencies.

In the opinion of the writer, after studying the very interesting and able discussions on this subject which have appeared, it is more than ever evident that the only solution of the problem is the entire modernization of the railroad plant so that the lower costs, and therefore rates, can be produced and reasonably good wages paid. There can be little quarrel with reasonable, even something more than ordinary, compensation for railroad workers. They must have a high mental and moral standing as well as requisite skill; but many of the anomalies of "constructive" rulings should be abolished and reasonable disciplinary measures should be upheld.

No matter what form the assistance may take, the writer can see no alternative to such a subsidy to the railways as will permit this necessary modernization and the establishment of adequate research facilities on a very broad scale.

There certainly can be no more objection to this, and perhaps much more reason for it, than for the subsidies being generously donated to waterways, aviation, and highways. As for burdening the railways with debt for loans to be made, why should this be any more necessary than it has been for the construction of the other and competing facilities without placing on them any burden?

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2062

EARTHQUAKES AND STRUCTURES

BY LEANDER M. HOSKINS,[1] ESQ., AND JOHN D. GALLOWAY,[2]
M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. HOMER M. HADLEY, R. McC. BEANFIELD, R. S. CHEW, JACOB J. CRESKOFF, ARTHUR C. RUGE, WALTER L. HUBER, A. A. EREMIN, HENRY D. DEWELL, AND JOHN D. GALLOWAY AND LYDIK S. JACOBSEN.

SYNOPSIS

This paper is made up of two sections. Section I, written by Mr. Galloway, refers briefly to the work of the Earthquake Committee of the Society formed in 1923, and to its report. The nature of the earthquake waves is described and some reference is made to their cause, complexity, and characteristics. Some notes are added of the relation between earthquake waves and the structures that may be subjected to the forces arising therefrom.

Section II, by Professor Hoskins, considers the nature of the ground motion, gives certain fundamental rules and conclusions, and investigates the theory of flexure of vertical beams subjected to transverse oscillations. An extension of the theory to an elastic column sustaining a load is given in the Appendix.

I.—HISTORICAL AND DESCRIPTIVE

HISTORICAL

In 1923, after the destructive earthquake in Japan, a committee of the Society was formed with American and Japanese members. About five years were spent in assembling data, and a report was submitted to the Society. This report,[3] made up of various chapters, contained, in addition to historical and descriptive matter, a theoretical treatment of the problem of stresses in structures subject to earth movements. One paper, contributed by Professor Joseph N. LeConte, and John E. Younger, of the Uni-

---

NOTE.—Published in December, 1938, *Proceedings*.

[1] Prof. of Applied Mechanics, Emeritus, Stanford Univ., Palo Alto, Calif. Prof. Hoskins died on September 8, 1937.

[2] Cons. Engr., San Francisco, Calif.

[3] Unpublished Report of the Special Committee of the American Society of Civil Engineers on Effects of Earthquakes on Engineering Structures with Special Reference to the Japanese Earthquake of September 1, 1923. Available for reference at Engineering Societies Library, 29 West 39th Street, New York, N. Y.

versity of California, at Berkeley, Calif., was published in 1932.[4] The other paper was contributed by Professor Hoskins at the same time and revised in 1926. Section II of this paper by Professor Hoskins, contains a fundamental treatment of the problem of earthquake forces and the resulting effects upon structures subject to earthquake waves. It is a recent abridgment of the paper prepared in 1926.

THE PROBLEM

Section I describes the complex nature of the ground movements and the resulting elastic waves. The very complexity of the waves and the form and nature of various structures, such as buildings, water towers, bridge piers and trusses, etc., would seem to preclude any theoretical treatment. It would appear to be beyond the ability of any mathematician to develop formulas that would take accurate account of the extremely variable nature of the factors involved. This is true. On the other hand, the very complexity of the subject makes it necessary to approach the problem by making some general assumptions and to attempt simple solutions; for instance, it is necessary to assume a simple harmonic wave in place of the complex variable waves of the earth when in motion. It is advisable to assume a structure of uniform dimensions and material. Using these assumptions, it is possible to develop equations for moments and shear in accordance with dynamic theory. It then becomes a matter of judgment to establish a relation between any actual structure and the results of the theoretical deductions from simple assumptions that lie within the range of mathematical treatment.

It must not be inferred that more extended theoretical treatment of the problem is impossible. On the contrary, this has been done by others and with good results, especially in buildings. However, it seems desirable to present some of the fundamental conditions and treatment as a foundation for further work.

EARTHQUAKES AND THEIR EFFECTS

Any one charged with the design of engineering structures in any part of the world should take account of possible earthquake forces and the resulting stresses. Although earthquakes are generally confined to the two great belts of the earth, they may occur anywhere.

The earthquake belt that borders the Pacific Ocean passes from New Zealand, through Japan, The Islands of Alaska, California, Mexico, and the Andes, to Cape Horn. It is represented by a ring of high mountains bordering the continental shelf on the edge of the ocean depths. Japan is very close to the greatest ocean depth. There seems to be a relation between the intensity of the shocks and the nearness to great mountain chains and the ocean depths. Some minor earthquakes are volcanic, but most of them are tectonic, the result of adjustments in the crust of the earth. They are evidences of mountain building. It is thought by some that most earthquakes in California are connected with the uplift of the Sierra Nevada and the Coast mountains. The evidences are found in the numerous fault lines that traverse sections of the State. The earthquake of 1878 was caused by a slip on the Haywards fault. The earthquake of 1906 was caused by a slip on the San Andreas fault where the surface trace was evident for nearly 300 miles. The Japanese earthquake of 1923 was caused by movements in the sea bottom of Sagami Bay about 70 miles south of Yokohama, where an area of about 270 sq miles sank from 300 to 1 300 ft and an area of more than 90 sq miles was elevated from 600 to 800 ft. The earthquake of March, 1933, at Long Beach and other towns in Southern California was caused by a slip on the Inglewood fault.

---

[4] "Stresses in a Vertical Rod When Subjected to a Harmonic Motion of One End," by Joseph N. LeConte and John E. Younger, *Bulletin*, Seismological Soc. of America, Vol. 22, No. 1, March, 1932.

The rupture of the earth crust, as on the San Andreas fault (the depth of which is estimated to have been 30 miles), resulted in a maximum vertical movement of about 3 ft and a maximum differential horizontal movement of about 22 ft. It is obvious that no structure placed directly over the fault can withstand such a tearing apart of the ground. However, as a rule, the major damage is caused by the waves that travel from the disturbance.

The earth crust is not a perfect solid; it responds to the shock of rupture on the fault by a series of elastic waves that have all the characteristics of waves in a perfect medium. There are three generally recognized types of waves which move outward from the origin with different velocities, suffer reflection and interference, and probably have harmonics. The three groups have the following characteristics:

(1) Longitudinal waves that follow a curved path through the earth crust and move with a velocity of about 3.5 miles per sec;

(2) Transverse or shear waves that traverse the surface layers of the earth and move with a speed of about 2 miles per sec; and,

(3) Waves that carry the maximum amount of energy, travel through the surface, and move with a velocity of about 3 miles per sec.

In addition to these characteristics there is evidence given in almost every shock, of waves that move slowly over the surface of the ground. These waves are seen usually in alluvial soils.

The waves that damage structures are impressed waves, changing rapidly in period, amplitude, and acceleration. Seismograms rarely show traces of harmonic waves, or those with regular characteristics. This is probably a very fortunate circumstance because it prevents, to a large extent, the resonance of vibrating structures. As a result of the impressed forces generating the wave trains, the periods may vary from 0.1 sec to 2 sec, and the acceleration may vary from practically nothing to more than that of gravitation. The amplitude may vary from a small fraction of an inch to 9 in. and, possibly, to 12 in. The foundation material is the important element of the movement. Solid rock moves the least; deep alluvial soils saturated with water, such as the lands bordering the southern part of San Francisco Bay, the plain of the Los Angeles River, or the region from Tokyo to Yokohama, will have the greatest movements.

The duration of the major wave movements is an important element in the problem of the design of structures. Within limits it may be said that this time varies from a few seconds to several minutes. It will be appreciated that the duration of the destructive movements has an important bearing upon the capability of structures to withstand the earthquake. Reports indicate a duration of the sensible waves of the Long Beach earth movements as 11 sec. At Charleston, S. C., in 1886, the duration was 1 min and 10 sec; in California, in 1906, the duration was said to have been 3.5 min.

The energy released by the breaking of the earth crust is gradually dissipated during the onward progress of the waves. The destructive effects become less as the wave advances and the period of wave tends to increase to as much as 10 sec. The destructive effects are limited to a relatively small area, but the sensible shock may be felt at distances of 1 000 to 2 000 miles. Major earthquakes shake the entire globe and are detected by seismographs now installed in every civilized country.

The earth movements are very complex. All the phenomena of waves in an elastic solid are present, modified by the varying nature of the materials of the crust of the earth. One very important factor is the extreme complexity of the wave action. A movement such as that of the San Andreas fault in 1906, where the earth was ruptured for a length of 300 miles, does not generate a single wave. On the contrary, waves from innumerable foci radiate in all directions. Any given structure may be subjected to movements in all directions. It is this fact, together with the variable amplitude, period, and acceleration of the principal waves, that makes any exact mathematical treatment of structures impossible.

Relation to Structures.—It will be recognized from the foregoing brief summary of the characteristics of earthquake waves, that any structure subjected to such forces must have elements of design differing in many respects from structures subjected only to the vertical force of gravity.

One important element is the vibration period of the structure. All structures have one or more natural periods of vibration. If the period of the structure approximates closely to the periods of some of the maximum earthquake waves of the ground upon which it rests, even if such are variable, there is danger of resonance in the structure. If long continued, the cumulative effects of resonance may result in the destruction of the structure. In general, low, broad structures have vibration periods differing considerably from the periods of the maximum earthquake waves. In the case of high, narrow structures the vibration period is often close to that of some of the maximum waves and, unless the structure is well designed, serious damage will result if the shock is of long duration. The Earthquake Committee filed with its report[8] the vibration periods of a number of buildings in San Francisco, Calif., and the results would be of interest to those having such subjects under consideration. Other data of a similar kind have been published.

It is obvious that the forces generated in a structure subjected to earth movements from an earthquake are measured by the mass or weight of the building. The forces of the moving ground are unlimited. It follows from this that the use of light materials in a structure is the best and that heavy, inelastic materials should be avoided as far as possible.

The form of the structure is of importance when the design is considered. This remark applies with greatest force to buildings. A central section, with wings at right angles, suffers extensive damage, especially in the central section. The wings vibrate in different directions and with different periods, and batter the central section to its destruction.

Since the forces resulting from an earthquake are, in general, horizontal and dynamic, it follows that the design of a structure must provide for the transfer of stresses from one member to another at the connections. A reversal of stress always takes place. The structure, as a whole, is subjected to torsion and to cross-bending, from which arise tension, compression, and shearing stresses. Provision must be made in the design to resist these stresses.

Section I is presented in an attempt to explain the complex nature of the earth movements and the relation to the form of structures, the design, and the materials used. As stated, an exact treatment of the problem by mathematics is impossible, owing to the complexity of the elements of the problem. On the other hand, it is quite possible to arrive at an approximation to the stresses in a structure such as is given in Section II. If such methods are used in the light of data on possible earthquakes that are now being accumulated in all civilized countries, it will be possible to design a structure that will be consistent and thus will have every chance of passing through an earthquake without excessive damage.

II.—STRESSES AND DYNAMICAL THEORY

NATURE OF GROUND MOTION

Rules for the design of structures to resist earth shocks must be based upon definite assumptions regarding the motion of the ground. Probably the assumption most commonly made is that the motion is translational; that is, that all parts of the ground supporting the structure are at every instant moving in the same direction with a common velocity. At every instant all parts of the supporting ground will then have the same acceleration, although its value may be varying more or less rapidly in magnitude and direction.

Although it is probably true that, in many cases, the predominating motion is translational, it is not improbable that cases occur in which there is a rotational component of sufficient amplitude to be a factor in causing dangerous stresses. There is considerable testimony, in fact, to the occurrence of visible surface waves; and although such testimony often alleges wave amplitudes of incredible magnitude, it would seem that a traveling wave crest large enough to be visible must cause an appreciable rotational motion of the ground at any given spot. It will be assumed herein that the ground motion is translational unless otherwise specifically stated. In all cases, however, the solution of the dynamical problem depends upon the same fundamental principle, which it is convenient to state in the form of a general rule of procedure.

FUNDAMENTAL GENERAL RULE

The stress condition throughout the structure at any instant depends upon the accelerations of all particles of the structure at that instant. Assuming these accelerations to be known, the general rule of procedure may be stated as follows:

Assume to act upon every particle of the structure and its contents, a force equal to the product of its mass by its acceleration and opposite in direction to the acceleration. Treat the body as if in equilibrium under the action of these supposititious forces in addition to the actual forces.

The actual forces include: (*a*) Forces, such as dead weight and wind pressure, which are independent of earthquake motion; and (*b*) the supporting forces, with values that may be changed materially by the earth motion. The former are a part of the known or assumed data of the problem, whereas the latter are unknown until computed by the aid of the above rule. Just as in the ordinary problem of structural design, the supporting forces must be determined as a preliminary to the computation of the stresses. The fundamental rule is simply the application to this problem of the general principle of dynamics known as d'Alembert's principle.

APPLICATION OF GENERAL RULE ON ASSUMPTION OF RIGIDITY

If the structure is assumed to be perfectly rigid, and if it does not leave its supports, every part of it must have, at every instant, the same acceleration as the ground. The application of the general rule, therefore, is simple. Thus, if the ground has an acceleration α in a certain direction, the supposititious force assumed to act upon a particle of mass m would be a force $m\,\alpha$ opposite that direction. In other words, the magnitude of the supposititious force would be the fraction, $\frac{\alpha}{g}$, of the weight of the particle, g being the acceleration due to gravity.

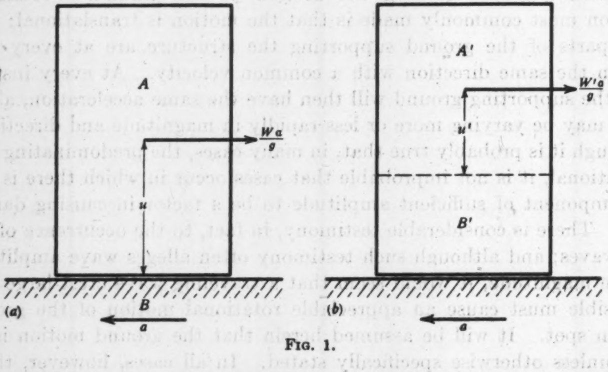


FIG. 1.

To illustrate, suppose *A* (Fig. 1(*a*)) is a rigid body supported upon the ground *B*. Let the weight of the body be *W* lb, and let *u* be the height of its mass center above the base. At a certain instant let the ground have the ac-

celeration, α, directed horizontally to the left; then the supposititious forces to be assumed to act upon the body would have a resultant of magnitude, $\frac{W\alpha}{g}$ lb, directed horizontally to the right and acting at the mass center. The supporting forces at the base must be such as to counterbalance this resultant; hence, they must be equivalent to a force, $\frac{W\alpha}{g}$ lb, directed horizontally to the left, and a couple of moment, $\frac{uW\alpha}{g}$ ft-lb, counter-clockwise. These are in addition to the supporting forces that counterbalance the weight of the body.

In like manner, if A' and B' (Fig. 1(*b*)) are the parts of the body above and below any horizontal plane, the forces exerted upon A' by B' must be such as to counterbalance the supposititious forces acting on the part, A'. If the weight of this part is W' lb, the supposititious forces have a resultant of magnitude, $\frac{W'\alpha}{g}$, directed horizontally to the right and acting at the mass center of A'; hence, the forces exerted by B' upon A' are equivalent to a force, $\frac{W'\alpha}{g}$ lb, acting horizontally to the left (shear) and a couple of moment, $\frac{u'W'\alpha}{g}$ ft-lb, counter-clockwise (resisting moment). These are additional to the forces which counterbalance the weight of A'.

If the ground has an acceleration, α, directed vertically upward, the supposititious forces are directed downward; the effect upon the supporting forces and internal stresses is the same as if the weight of every part of the structure and its contents were increased by the fraction, $\frac{\alpha}{g}$, of the actual weight. In like manner a downward acceleration of the ground would have the same effect as a decrease in the weight of the structure and its contents. Whatever the actual direction of the acceleration, its horizontal and vertical components may be treated separately. The following discussion will refer mostly to horizontal accelerations, since it is probable that, ordinarily at least, the horizontal acceleration of the ground is the principal cause of dangerous stresses. (The foregoing statements are true, of course, only when the ground motion is translational. For a rigid body, however, the computation of accelerations throughout the body due to a known rotational motion of the ground can be made without difficulty, thus determining the supposititious forces for applying the general rule.)

EFFECT OF ELASTIC YIELDING

If the acceleration of the ground varied very slowly, the departure of actual materials from perfect rigidity would have no appreciable effect upon the values of the supposititious forces referred to in the general. It would still be practically correct to assume that all parts of the structure have always the same acceleration as the ground. This is no longer true, however, if the ground is in rapid oscillation. It appears in fact that, because of elastic yielding, all parts of the structure above the foundation may have a greater range of motion than the ground, and therefore the accelerations of the upper parts may be materially greater than that of the ground. The effect of elastic yielding will depend upon the rapidity of the oscillation and upon the elastic properties, dimensions, and mass of the structure. The problem of estimating the yielding is too complex for exact mathematical treatment unless simplified by assumptions which are far from true in the case of many structures. It is possible, however, to infer something regarding the nature of the effects to be expected, from well-known considerations relating to oscillating systems in general. It seems possible, also, to throw some light on the possible order of magnitude of these effects by solving the dynamical problem for idealized simple cases.

CONCLUSIONS FROM GENERAL THEORY OF FORCED AND FREE OSCILLATIONS

The problem under consideration falls under the general case of a system which is constrained to oscillate with a certain definite frequency. (By frequency is meant the number of complete oscillations per unit time. Frequency is the reciprocal of period.) General theory indicates that the phase and amplitude of the forced oscillation will depend upon the relation between its frequency and that of the "natural" or "free" oscillation which the system might have. For the simple case of a system having only one degree of freedom and, therefore, one possible mode of vibration, the following statements hold:

(*a*) *Phase of Forced Oscillation.*—If the disturbing oscillation has a less frequency (that is, a longer period) than the free oscillation, the system will oscillate in the same phase as the disturbance that causes it. If the disturbance has a frequency greater than that of the free oscillation, the phase of the actual oscillation will be opposite to that of the disturbance.

(*b*) *Amplitude of Forced Oscillation.*—The amplitude of the forced oscillation is greater the more nearly its frequency approaches that of the free oscillation; that is, if f_0 is the frequency of the free oscillation and f that of the disturbance, the amplitude of the forced oscillation will be greater the nearer $\frac{f}{f_0}$ is to 1. Moreover, as $\frac{f}{f_0}$ approaches 1 the amplitude increases without limit.

These statements refer to undamped oscillations. In the case of an oscillating building the damping (due to internal friction and to air resistance) is presumably too small to affect the validity of the propositions stated. It should further be noted that a system having more than one degree of freedom may have more than one possible mode of free oscillation; for elastic structures the number of possible modes may, in fact, be indefinitely great. In applying Statements (*a*) and (*b*) to such a case it is to be understood that the mode referred to is generally that of longest period, which may be regarded as the fundamental mode.

These statements indicate, at least in a general way, the nature of effects that might be caused in an elastic structure by a simple harmonic oscillation of the supporting ground. Proposition (*b*) is of especial importance, since it indicates that if a structure has a natural oscillation period that is nearly the same as that of the ground motion, continued repetition of the earth tremors will inevitably result in dangerous stresses. Whether this condition is actually of frequent occurrence, and whether it is possible to guard against it by proper design, would seem to be questions of fundamental importance. To answer the former question with certainty it would be necessary to have full knowledge regarding the frequencies of the harmonic components of ground oscillations occurring in violent earthquakes, and also regarding free-oscillation frequencies of actual buildings. It can scarcely be doubted, however, that observed destructive effects have in some cases been due to the cause stated.

As regards the second question, it is obvious that the designer is powerless to avoid the dangerous condition unless he is able to estimate in advance the natural oscillation frequency of any projected building. Of great value for this purpose would be data concerning actual oscillation periods of many existing structures so selected as to serve as types. The problem of the structural designer in the matter of avoiding dangerous oscillations thus depends very largely upon data which can be obtained only by observation of actual structures and actual earthquakes. It remains to consider whether further light can be thrown upon the problem by dynamical theory.

As already stated, the possibility of exact mathematical treatment depends upon the introduction of assumptions which are often far from representing the facts for actual structures. The solution of such idealized problems, however, may be of use as indicating the general character and order of magnitude of the effects likely to be caused by elastic oscillation. One such problem will now be considered.

OSCILLATION OF A UNIFORM VERTICAL BEAM

The ideal problem treated in the following analysis is that of a uniform beam placed in a vertical position with the upper end free and the lower end rigidly attached to the ground, the supporting ground being assumed to have a horizontal motion resolvable into components which are simple harmonic functions of the time. The method of solution consists in forming the differential equation of the elastic curve by a simple extension of the ordinary theory of flexure, and solving this equation subject to the conditions assumed to hold at the ends. From the equation of the elastic curve, it is easy to deduce formulas for the shear and resisting moment at any cross-section of the oscillating beam. The solution given herein was formulated by the writer in 1926 when the original draft of this paper was prepared for the Special Committee on Effects of Earthquakes on Engineering Structures (not published). The same fundamental theory has been employed by many writers both before and since that date. A partial list of publications is given at the end of this paper.

THEORY OF FLEXURE APPLIED TO OSCILLATING BEAM

The following deduction of the differential equation of the elastic curve is applicable to the general case of a uniform elastic beam performing small transverse oscillations. The notation introduced, however, is especially adapted to the foregoing problem. The quantities involved are the following: h = length of beam; A = its cross-sectional area; $I = A\,k^2$ = moment of inertia of cross-section about axis of bending; W = total weight of beam, in pounds; E = modulus of elasticity of material (Young's modulus); g = acceleration due to gravity; and, M and V = resisting moment and resisting shear acting at any transverse section.

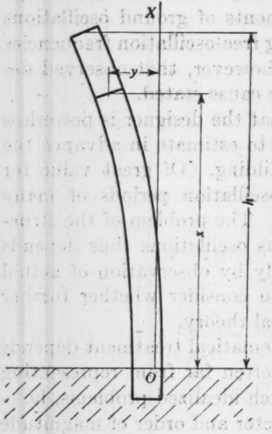


FIG. 2.

Let x, y denote the rectangular co-ordinates of any point in the elastic curve with respect to axes taken as in Fig. 2. The origin, O, is taken at the base of the beam in its mean position, the positive X-direction is vertically upward, and the positive Y-direction is horizontally to the left. The convention of signs for M and V is as follows: The part of the beam above the cross-section exerts upon the part below, forces of which V is the sum of the components in the positive Y-direction and M is the sum of the moments in the counter-clockwise sense.

The first step is to write the dynamical equations for an element of the beam bounded by two transverse sections at x and $x + dx$. The acceleration of this element in the X-direction is inappreciable while its Y-acceleration is $\frac{\partial^2 y}{\partial t^2}\cdot$ Its angular acceleration is $\frac{\partial^2 \phi}{\partial t^2}$ in which ϕ differs inappreciably from $\frac{\partial y}{\partial x}$. The mass of the element is $\frac{W\,dx}{g\,h}$ and its moment of inertia about the axis of bending is $\left(\frac{W\,dx}{g\,h}\right)k^2$. The total force acting on the element in the Y-direction is $\left(\frac{\partial V}{\partial x}\right)dx$; the total turning moment acting on it is $\left(\frac{\partial M}{\partial X}\right)dx + V\,dx$. Hence, the equations of linear and angular motion for the element may be written as follows (after cancellation of dx):

$$\frac{W}{g\,h}\frac{\partial^2 y}{\partial t^2} = \frac{\partial V}{\partial x} \qquad \text{(1)}$$

and

$$\frac{W\,k^2}{g\,h}\frac{\partial^2}{\partial t^2}\left(\frac{\partial y}{\partial x}\right) = \frac{\partial M}{\partial x} + V \qquad \text{(2)}$$

Eliminating V between Equations (1) and (2), there results,

$$\frac{W\,k^2}{g\,h}\frac{\partial^4 y}{\partial t^2\,\partial x^2} - \frac{W}{g\,h}\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 M}{\partial x^2} \qquad \text{(3)}$$

As in the ordinary theory of flexure,

$$M = E\,I\frac{\partial^2 y}{\partial x^2} \qquad \text{(4)}$$

so that Equation (3) reduces to the form,

$$\frac{\partial^2 y}{\partial t^2} + \frac{E\,I\,h\,g}{W}\frac{\partial^4 y}{\partial x^4} - k^2 \frac{\partial^4 y}{\partial t^2\,\partial x^2} = 0 \qquad (5)$$

It is found that the third term in Equation (5) is practically negligible unless the beam is short in comparison with its transverse dimensions, and in the following solution this term is omitted. (If the third term is retained the solution is entirely practicable, but involves considerably more labor, especially in numerical applications.) It is to be noted that the omission of this term reduces Equation (2) to the form,

$$V = -\frac{\partial M}{\partial x} \qquad (6)$$

as in the theory of beams under static conditions.

SOLUTION OF THE DIFFERENTIAL EQUATION WHEN THE GROUND MOTION IS A SIMPLE HARMONIC OSCILLATION

If the ground motion is resolved into components, each of which is a simple harmonic oscillation, each component may be treated separately. It suffices, therefore, to consider the solution of the differential equation,

$$\frac{\partial^2 y}{\partial t^2} + \frac{E\,I\,h\,g}{W}\frac{\partial^4 y}{\partial x^4} = 0 \qquad (7)$$

on the assumption that the lower end of the beam is constrained to have a simple harmonic oscillation in a horizontal plane. This gives one of four boundary conditions; the other three being that $\frac{\partial y}{\partial x}$ is always zero at the lower end, and that M and V are always zero at the upper end. Noting the general values of M and V as given by Equations (4) and (6), the boundary conditions may be stated as follows:

When $x = 0$,

$$y = a \sin\left(\frac{2\,\pi\,t}{T} + q\right) \qquad (8a)$$

when $x = 0$,

$$\frac{\partial y}{\partial x} = 0 \qquad (8b)$$

when $x = h$,

$$\frac{\partial^2 y}{\partial x^2} = 0 \qquad (8c)$$

and when $x = h$,

$$\frac{\partial^3 y}{\partial x^3} = 0 \qquad (8d)$$

The amplitude and period of the forced oscillation are here represented by a and T, and q is a constant the value of which depends upon the choice of the origin of time.

It is easily verified that the differential equation and the four boundary conditions may be satisfied by assuming:

$$y = (A_1 \cos m\, x + B_1 \sin m\, x + A_2 \cosh m\, x + B_2 \sinh m\, x)$$
$$\times \sin\left(\frac{2\,\pi\, t}{T} + q\right) \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

and assigning proper values to the constants m, A_1, B_1, A_2 and B_2. Thus, the differential equation is satisfied if,

$$m^4 = \frac{4\,\pi^2\,W}{E\,I\,h\,g\,T^2} \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

and the boundary conditions give the four equations,

$$A_1 + A_2 = a \ldots\ldots\ldots\ldots\ldots\ldots (11a)$$

$$B_1 + B_2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots (11b)$$

$$-A_1 \cos m\, h - B_1 \sin m\, h + A_2 \cosh m\, h + B_2 \sinh m\, h = 0 \ldots (11c)$$

$$A_1 \sin m\, h - B_1 \cos m\, h + A_2 \sinh m\, h + B_2 \cosh m\, h = 0 \ldots (11d)$$

which determine A_1, B_1, A_2, B_2, when m is known, their values being as follows:

$$A_1 = \frac{a}{2}\left(1 - \frac{\sin m\, h \sinh m\, h}{1 + \cos m\, h \cosh m\, h}\right) \ldots\ldots\ldots\ldots (12a)$$

$$B_1 = \frac{a}{2}\left(\frac{\cos m\, h \sinh m\, h + \sin m\, h \cosh m\, h}{1 + \cos m\, h \cosh m\, h}\right) \ldots (12b)$$

$$A_2 = a - A_1 \ldots\ldots\ldots\ldots\ldots\ldots (12c)$$

and,

$$B_2 = -B_1 \ldots\ldots\ldots\ldots\ldots\ldots (12d)$$

Formulas for M and V are easily written from Equations (4) and (6).

CASE OF FREE OSCILLATION

If the ground is at rest the foregoing solution is still valid with $a = 0$. In this case Equations (11) determine only the ratios of the four constants A_1, B_1, A_2, and B_2, their actual values containing a factor with an arbitrary value. Moreover, in this case the equations can be satisfied only by restricting T to certain values. For by eliminating A_1, B_1, A_2, and B_2 from Equations (11) there results the equation,

$$1 + \cos m\, h \cosh m\, h = 0 \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

which must be satisfied by any permissible value of m, while T depends upon m according to Equation (10).

Equation (13) is satisfied by each of an infinite series of values of mh, which may be determined by trial to any desired degree of approximation. In order of magnitude the first three values are 1.8751, 4.6944, and 7.8548; the succeeding values differ very slightly from $\frac{7\pi}{2}$, $\frac{9\pi}{2}$, etc. Each of these values corresponds to a possible free oscillation, the period of which may be computed by Equation (10). The greatest of the possible free periods may be called the fundamental period, a formula for which may be obtained by taking $m = m_0 = \frac{1.8751}{h}$. Calling this period, and the corresponding frequency, T_0 and f_0, and using Equation (10),

$$T_0 = \frac{1}{f_0} = \frac{2\pi}{m_0^2}\sqrt{\frac{W}{E\,I\,h\,g}} = \frac{2\pi}{(1.8751)^2}\sqrt{\frac{W\,h^3}{E\,I\,g}} = 1.787\sqrt{\frac{W\,h^3}{E\,I\,g}} \;\ldots (14)$$

It may easily be verified that the successive free oscillations have periods the ratios of which to T_0 are: 1, 0.1595, 0.05699, 0.02908, 0.01759, etc. These numbers, beginning with the fourth, are to a close approximation proportional inversely to the squares of the odd numbers 7, 9, 11, etc.

FORCED OSCILLATION OF ASSIGNED PERIOD

It is seen that, for a given beam, the assignment of values to a and T determines all the constants, m, A_1, B_1, A_2, and B_2, so that (except in the case of free oscillation) the values of y, M, and V, at all sections, may be computed by Equations (9), (4), and (16). In examining the nature of the results for various particular cases it is convenient to take as a basis of comparison the ideal case of no elastic yielding.

Case of Perfect Rigidity.—For this case let the values of y, M, and V, for any value of x, be y', M', and V', and let values for $x = 0$ be y_0', M_0', and V_0'. Denoting the ground acceleration by α and reasoning as before (see heading "Application of General Rule on Assumption of Rigidity") there result the formulas:

$$M_0' = -\frac{W\,h}{2\,g}\alpha \;\ldots\ldots (15a)$$

$$V_0' = -\frac{W}{g}\alpha \;\ldots\ldots (15b)$$

$$M' = M_0'\left(1 - \frac{x}{h}\right)^2 \;\ldots\ldots (15c)$$

$$V' = V_0'\left(1 - \frac{x}{h}\right) \;\ldots\ldots (15d)$$

Since the motion of the ground is assumed to be represented by the formula,

$$y_0' = a \sin\left(\frac{2\pi t}{T} + q\right) \;\ldots\ldots (16)$$

the acceleration is,

$$\alpha = \frac{\partial^2 y_0'}{\partial t^2} = -\frac{4\pi^2}{T^2} a \sin\left(\frac{2\pi t}{T} + q\right) \;\ldots\ldots (17)$$

Effect of Elastic Yielding.—Equations (4), (6), (9), (10), and (15) lead easily to the following formulas:

$$\frac{y}{y_0'} = \frac{A_1}{a} \cos m\,x + \frac{B_1}{a} \sin m\,x + \frac{A_2}{a} \cosh m\,x + \frac{B_2}{a} \sinh m\,x \ldots (18a)$$

$$\frac{M}{M_0'} = \frac{2}{m^2\,h^2}\left(- \frac{A_1}{a} \cos m\,x - \frac{B_1}{a} \sin m\,x + \frac{A_2}{a} \cosh m\,x + \frac{B_2}{a} \sinh m\,x \right) \ldots (18b)$$

$$\frac{V}{V_0'} = \frac{1}{m\,h}\left(- \frac{A_1}{a} \sin m\,x + \frac{B_1}{a} \cos m\,x - \frac{A_2}{a} \sinh m\,x - \frac{B_2}{a} \cosh m\,x \right) \ldots (18c)$$

Equations (18*b*) and (18*c*), compared with Equations (15*c*) and (15*d*), show the effect of elastic yielding upon the values of the bending moment and shear at any section of the beam. It is seen that, for a given value of $\frac{x}{h}$, the values of $\frac{M}{M_0'}$ and $\frac{V}{V_0'}$ depend solely upon the value of $m\,h$. Moreover, $m\,h$ depends only upon the value of $\frac{T_0}{T}$, since by Equation (10),

$$\frac{m^2\,h^2}{m_0^2\,h^2} = \frac{T_0}{T} \ldots (19)$$

while $m_0\,h$ has the fixed value 1.8751, so that,

$$m\,h = 1.8751 \sqrt{\frac{T_0}{T}} \ldots (20a)$$

$$m\,x = 1.8751 \sqrt{\frac{T_0}{T}}\,\frac{x}{h} \ldots (20b)$$

Thus, if K is written for $1.8751 \sqrt{\frac{T_0}{T}}$, x' for $\frac{x}{h}$, and A_1', B_1', A_2', and B_2' for $\frac{A_1}{a}$, $\frac{B_1}{a}$, $\frac{A_2}{a}$, and $\frac{B_2}{a}$, Equations (18) become:

$$\frac{y}{y_0'} = A_1' \cos K\,x' + B_1' \sin K\,x' + A_2' \cosh K\,x' + B_2' \sinh K\,x' \ldots (21a)$$

$$\frac{M}{M_0'} = \frac{2}{K^2}\left(- A_1' \cos K\,x' - B_1' \sin K\,x' + A_2' \cosh K\,x' + B_2' \sinh K\,x'\right) \ldots (21b)$$

and,

$$\frac{V}{V_0'} = \frac{1}{K}\left(- A_1' \sin K\,x' + B_1' \cos K\,x' - A_2' \sinh K\,x' - B_2' \cosh K\,x'\right) \ldots (21c)$$

in which (from Equations (12)):

$$A_1' = 1 - A_2' = \frac{1}{2}\left(1 - \frac{\sin K \sinh K}{1 + \cos K \cosh K} \right) \ldots (22a)$$

$$B_1' = - B_2' = \frac{1}{2}\left(\frac{\cos K \sinh K + \sin K \cosh K}{1 + \cos K \cosh K} \right) \ldots (22b)$$

All the constants in the formulas thus depend upon the single number K, which itself depends upon $\frac{T_0}{T}$. In other words, for a given value of $\frac{x}{h}$, the ratio of change in M and in V, due to elastic yielding, depends solely upon the ratio of the actual oscillation period to the fundamental free period, being wholly independent of the dimensions and material of the beam, except as these factors determine the value of T_0.

Characteristic Curves.—The graphs of Equations (21) may be called characteristic curves for the problem under consideration. Such curves are shown in Fig. 3. Each curve is marked with the appropriate value of $\frac{T_0}{T}$, the range of such values being from 0 (perfect rigidity) to 0.9 and from 1.1 to 1.5. The curves illustrate in a striking manner the rôle played by the free period T_0. The foregoing statements (see heading "Conclusions from General Theory of Forced and Free Oscillations"), regarding phase and amplitude of forced oscillations in general are clearly illustrated in Fig. 3(a), and Figs. 3(b) and 3(c) show the rapid increase of M and V as the case $T = T_0$ is approached.

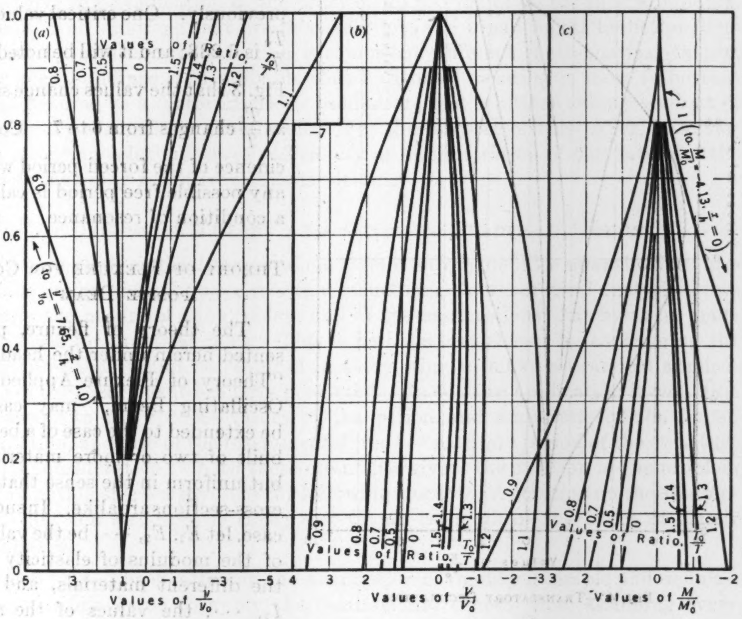


FIG. 3.

Curves for $\frac{y}{y_0}$, covering a greater range of values of $\frac{T_0}{T}$ are shown in Fig. 4. These curves are of especial interest as showing the transition from the first to the second of the natural periods. In the case of perfect rigidity, $\frac{T_0}{T} = 0$, so that the ratio of change in M and V, due to elastic yielding, is found by comparing each value computed by Equations (21) with the corresponding value computed from Equations (15). Infinite values of y, M, and V result if T is equal to T_0, or to any one of the possible free periods specified previously. One critical value of $\frac{T_0}{T}$ is 6.238; and it will be noted in Fig. 3 that the values change sign as $\frac{T_0}{T}$ changes from 6 to 7. Coincidence of the forced period with any possible free period is called a condition of resonance.

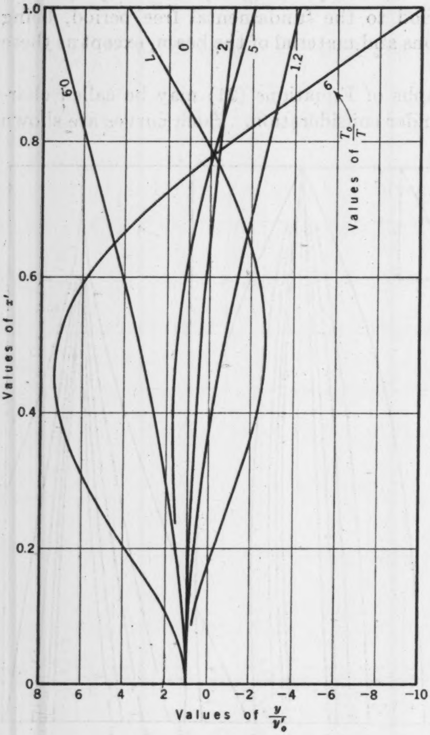


FIG. 4.—TRANSLATORY OSCILLATION.

THEORY OF FLEXURE FOR COMPOSITE BEAM

The theory of flexure, presented herein under the heading "Theory of Flexure Applied to Oscillating Beam," may easily be extended to the case of a beam built of two or more materials, but uniform in the sense that all cross-sections are alike. In such a case, let $E_1, E_2, \cdots$, be the values of the modulus of elasticity for the different materials, and $I_1, I_2, \cdots$, the values of the moment of inertia for the corresponding parts of the cross-section; then, the formula for the resisting moment is:

$$M = (E_1 I_1 + E_2 I_2 + \cdots) \frac{\partial^2 y}{\partial x^2} \quad \ldots\ldots\ldots\ldots\ldots (23)$$

The use of Equation (23) instead of Equation (4) introduces no change in the differential Equation (7), except that EI must be replaced by $E_1 I_1 + E_2 I_2 + \cdots$. With this change the case of a composite beam is covered by the solution given above. It is of especial interest to note the application of this statement to the formula for T_0 (Equation (14)). (In the interest of rigor it should be noted that the radius of gyration, k, occurring in Equations (2) and (5) applies not simply to the area of the cross-section, but to the mass of an elementary layer. In the foregoing solution, however, terms involving k have been treated as negligible.)

ACTUAL AND IDEAL STRUCTURES

It cannot be supposed that numerical results computed for the foregoing ideal problem apply closely to actual buildings, but the general character of the results probably represents the kind of effects to be expected in a steel frame building which is uniform in horizontal area and tall in comparison with its horizontal dimensions. It would seem, for example, that Equation (14) may give some indication of the order of magnitude of T_0 for such a building. The case of buildings that are relatively low and broad might be better illustrated by solutions based upon a somewhat different mathematical theory. For such buildings it is probable that horizontal shearing is more important than bending as a factor in elastic oscillation, and solutions taking account of this factor are entirely practicable. For the purpose of this paper, however, the foregoing solution serves sufficiently as an illustration of the nature of the effects that may result from elastic yielding.

DYNAMICAL THEORY AS A GUIDE TO PRACTICAL DESIGN

Although dynamical theory alone cannot supply definite quantitative rules for the design of earthquake-resistant structures, the theoretical analysis shows clearly the nature of the problem and of the assumptions that must be made as a basis for practical rules. This is perhaps best shown by considering the procedure that would be required in attempting to make a practical application of the theory as it has been presented. In such an application it would be necessary to use specific values of the period and amplitude of the forced oscillation, and also of the fundamental free or "natural" period of the structure under consideration. With these quantities given, the solution of the problem of computing stresses involves the following steps: (*a*) A solution on the assumption that the structure is rigid; and (*b*) an estimate of the effect of elastic oscillation:

Step (*a*).—The solution on the assumption of rigidity is simple and definite, the procedure being to apply the fundamental general rule, assuming every part of the building and its contents to have the same acceleration as the ground. The computation of the supporting forces and of the stresses in the framework is then an ordinary problem in structural mechanics.

Step (*b*).—To determine, quantitatively, the ratio of change of the stresses due to elastic oscillation would require a solution of the elastic problem for the actual structure similar to that which has been given for the ideal structure treated as a uniform beam. In the ideal problem it was found that elastic yielding increased the values of V and M by ratios depending upon the value of $\frac{T_0}{T}$, the effects being shown quantitatively in Figs. 3(*b*) and 3(*c*). Although these quantitative results cannot be assumed to hold for the actual problem, it is reasonable to assume that certain important characteristics of the results do hold; namely (1) that the values of V and M are increased by elastic yielding and (2) that the ratio of change in V and in M depends upon the ratio $\frac{T_0}{T}$ and increases without limit as T approaches equality with T_0, or with any other of the natural periods of the structure.

Dynamical theory thus makes clear the nature of earthquake hazards, and at the same time shows the difficulty of formulating practical rules for meeting the hazards because of uncertainty regarding the actual data that should be assumed to apply to any particular case. There is one fundamental practical rule, however, which may be stated with confidence:

Fundamental Practical Rule.—The structure should be designed to resist forces, $m\,\alpha$, acting on all its particles in any horizontal direction, α being the probable maximum ground acceleration and m the mass of any particle.

The value of α to be used in applying this rule is a matter for the judgment of the engineer, guided by records of actual earthquakes. Different authorities have recommended values from 2% to 10% of gravity.

Danger of Resonance.—It is clear that very great stresses may result if there is a near approach to a condition of resonance; that is, equality of T to one of the natural periods of the structure. Since any given structure has a multiplicity of natural periods, and since the harmonic analysis of seismograph records has shown a considerable range of values of T, it seems impracticable to formulate any simple rule for minimizing the probability of dangerous stresses due to resonance.

PUBLICATIONS DEALING WITH THE VIBRATION OF PRISMATIC BARS

The theory of the elastic vibration of prismatic bars has been discussed by many writers during the past 200 yr, among the earliest having been Daniel Bernoulli (about 1743) and his contemporary, Bernhard Euler. The deduction of the differential equation of the elastic curve, in the form given in this paper when gravity is neglected, and its solution subject to stated boundary conditions, are found in numerous publications, of which a few are listed subsequently. Of these, Nos. (5) and (6) treat the same case of the oscillating vertical beam as that treated in this paper, with substantially identical results.

(1) **Strutt, John William (Baron Rayleigh).** Theory of Sound. Second Edition, 1894, MacMillan.

(2) **Love, A. E. H.** A Treatise on the Mathematical Theory of Elasticity. Second Edition, 1906, Cambridge Univ. Press.

(3) **Hort, Dr. Wilhelm.** Technische Schwingungslehre, Second Edition, 1922, Berlin.

(4) **Timoshenko, S.** Vibration Problems in Engineering. 1928, Van Nostrand.

(5) **LeConte, Joseph N., and John E. Younger.** Stresses in a Vertical Elastic Rod When Subjected to a Harmonic Motion of One End. *Bulletin*, Seismological Soc. of America, Vol. 22, March, 1932.

(6) **Creskoff, Jacob J.** Dynamics of Earthquake Resistant Structures. 1934, McGraw-Hill.

(7) **Morse, Philip M.** Vibration and Sound. 1936, McGraw-Hill.

APPENDIX

EXTENSION OF THEORY DESIRABLE

The theory of the uniform vertical beam or column has served to illustrate the general character of effects which may be caused in structures of a certain type by a forced harmonic oscillation. It seems desirable to extend the theory so as to cover a more general class of cases than has been treated previously. In this more general treatment special attention will be given to the important case of free oscillation.

ELASTIC COLUMN SUSTAINING A LOAD

Let the vertical beam or column treated in Fig. 2 be assumed to carry at its upper end a rigid body weighing P lb. Let the mass center of this body lie in the extension of the axial line of the column at a distance h' from the top; let k' be the radius of gyration of its mass with respect to an axis through its mass center perpendicular to the plane of flexure of the column; and let other data and notation be as in the problem already solved. This case is represented diagrammatically in Fig. 5(a).

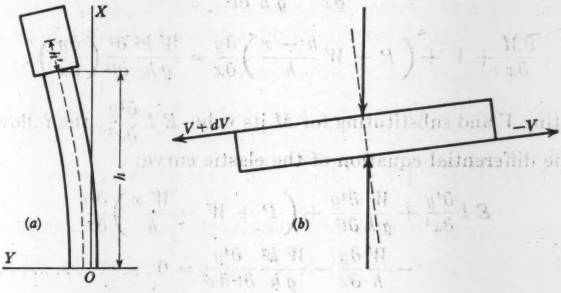


FIG. 5.

DIFFERENTIAL EQUATION OF ELASTIC CURVE

In the theory already given the action of gravity on the column was neglected. The following is a more rigorous analysis, taking account of the weight of the column itself as well as that of the supported load. In order to form the differential equation of the elastic curve, write the equations of linear and angular acceleration of an element of the column of thickness dx, bounded by transverse planes normal to the curve. (It will be assumed as before that horizontal shearing is negligible in comparison with bending in determining the form of the elastic curve.) Such an element is represented in Fig. 5(b).

The mass of the element is $\frac{W\,dx}{g\,h}$, and its moment of inertia about its central axis is $\left(\frac{W\,dx}{g\,h}\right)k^2$. Its acceleration in the X-direction is inappreciable, and its

Y-acceleration is $\frac{\partial^2 y}{\partial t^2}$. Its angular acceleration is $\frac{\partial^2 \phi}{\partial t^2}$, in which ϕ differs inappreciably from $\frac{\partial y}{\partial x}$.

The forces acting on the element are as follows: Forces $-V$ and $+(V + dV)$ acting in practically horizontal directions on the lower and upper faces; couples of moments $-M$ and $+(M + dM)$, acting on the same faces; a vertical force, $P + W\frac{h - x}{h}$, acting upward on the lower face, and a vertically downward force, less by $\left(\frac{W\,dx}{h}\right)$, acting on the upper face. Each of these vertical forces, being uniformly distributed over the cross-section, may be regarded as applied at the centroid, the distance between their lines of action being dy. (The reasoning here is that of the common theory of flexure, which assumes that the normal stresses on a cross-section are equivalent to a normal force applied at the centroid, together with a couple of which the moment is the resisting moment M.) The equations of linear and angular motion for the element, therefore, may be written as follows (after dividing by dx):

$$\frac{\partial V}{\partial x} = \frac{W}{g\,h}\frac{\partial^2 y}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (24a)$$

and,

$$\frac{\partial M}{\partial x} + V + \left(P + W\frac{h - x}{h}\right)\frac{\partial y}{\partial x} = \frac{W\,k^2}{g\,h}\frac{\partial^2}{\partial t^2}\left(\frac{\partial y}{\partial x}\right) \quad \ldots\ldots (24b)$$

Eliminating V and substituting for M its value $E\,I\,\frac{\partial^2 y}{\partial x^2}$, the following is obtained as the differential equation of the elastic curve:

$$E\,I\,\frac{\partial^4 y}{\partial x^4} + \frac{W}{g\,h}\frac{\partial^2 y}{\partial t^2} + \left(P + W - \frac{W\,x}{h}\right)\frac{\partial^2 y}{\partial x^2}$$
$$- \frac{W}{h}\frac{\partial y}{\partial x} - \frac{W\,k^2}{g\,h}\frac{\partial^4 y}{\partial t^2 \partial x^2} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

Comparing Equation (25) with Equation (5), it is seen that the latter consists of the first, second, and last of the terms in the former, while in Equation (7), which was actually solved, only the first two terms are retained. Next, consider whether a more rigorous treatment which retains some or all of the omitted terms is practicable or desirable. For this purpose, note the meanings of the different terms as well as their relative magnitudes.

Any given element of mass influences the oscillation of the system in two ways—by its inertia and by its weight. The latter effect was wholly disregarded in the theory leading to Equation (5). In the problem there treated (in which the only mass present was that of the column itself) the effect of gravity is represented in Equation (25) by the terms: $\left(W - \frac{W\,x}{h}\right)\frac{\partial^2 y}{\partial x^2} - \frac{W}{h}\frac{\partial y}{\partial x}$. When these terms are retained, and when $P = 0$, Equation (25) can be solved by development in series, and it is not difficult to prove that the effect of gravity is of slight importance in that problem. In the more general case now under consideration there is the additional gravity effect due to the load P, and it seems probable that there may be cases in which this effect will be of much greater importance than that of the weight of the column itself. In the analysis which follows, therefore, the term containing P is retained but those terms representing the effect of gravity on the column itself are omitted.

The inertia effect is represented in Equation (25) by the second and last terms, of which the former represents the linear inertia of the mass element and the latter its angular inertia. Of these terms the latter is ordinarily of negligible importance and will not be retained. The inertia effect of the load P does not affect the differential Equation (25), but is included in the boundary conditions to be given subsequently.

With the approximations stated the differential equation to be solved reduces to the following:

$$E\,I\frac{\partial^4 y}{\partial x^4} + \frac{W}{g\,h}\frac{\partial^2 y}{\partial t^2} + P\frac{\partial^2 y}{\partial x^2} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

It remains to formulate the boundary conditions.

The ground motion and the method of supporting the column being the same as in the problem already solved, the boundary conditions at $x = 0$ are expressed by Equations (8a) and (8b). In the present case, however, it cannot be assumed that V and M are zero at $x = h$; instead it is necessary to write the equations of linear and angular motion for the body P. Let the values of V, M, y, and $\frac{\partial y}{\partial x}$ at the top of the column be V_1, M_1, y_1, and ϕ_1. If the body is rigid and rigidly attached to the top of the column, the value of y for its mass center is $y_1 + h'\,\phi_1$. Therefore, for the body P:

The acceleration of the mass center is $\frac{\partial^2 y_1}{\partial t^2} + h'\left(\frac{\partial^2 \phi_1}{\partial t^2}\right)$, the force causing this acceleration is $-V_1$; the angular acceleration is $\frac{\partial^2 \phi_1}{\partial t^2}$, and the torque about the mass center is $-M_1 + V_1\,h' + P\,h'\,\phi_1$, from which follow the two equations:

$$\frac{P}{g}\left(\frac{\partial^2 y_1}{\partial t^2} + h'\frac{\partial^2 \phi_1}{\partial t^2}\right) = -V_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (27a)$$

$$\frac{P\,(k')^2}{g}\frac{\partial^2 \phi_1}{\partial t^2} = -M_1 + V_1\,h' + P\,h'\,\phi_1 \quad \ldots\ldots\ldots\ldots (27b)$$

The values of V_1 and M_1 to be substituted in Equations (27) are obtained from the general formulas:

$$M = E\,I\frac{\partial^2 y}{\partial x^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (28a)$$

$$V = -\frac{\partial M}{\partial x} - P\frac{\partial y}{\partial x} \quad \ldots\ldots\ldots\ldots\ldots\ldots (28b)$$

the latter being equivalent to Equation (24b) when the terms containing W are treated as negligible. The four boundary conditions may now be written

as follows:

When $x = 0$,

$$y = a \sin\left(\frac{2\pi t}{T} + q\right) \quad \text{(29a)}$$

when $x = 0$,

$$\frac{\partial y}{\partial x} = 0 \quad \text{(29b)}$$

when $x = h$,

$$\frac{P}{g}\left(\frac{\partial^2 y}{\partial t^2} + h'\frac{\partial^2}{\partial t^2}\left(\frac{\partial y}{\partial x}\right)\right) = E\,I\,\frac{\partial^3 y}{\partial x^3} + P\frac{\partial y}{\partial x} \quad \text{(29c)}$$

and when $x = h$,

$$P\,\frac{(k')^2}{g}\,\frac{\partial^2}{\partial t^2}\left(\frac{\partial y}{\partial x}\right) = -\,E\,I\left(\frac{\partial^2 y}{\partial x^2} + h'\,\frac{\partial^3 y}{\partial x^3}\right) \quad \text{(29d)}$$

The general problem thus reduces to the solution of the differential Equation (26) subject to the boundary conditions, Equations (29). It must be remembered that, in the problem as thus formulated, the column is regarded as having inertia, but not weight; for the supported body, however, the equations take account of both inertia and weight. It is proposed first to outline the solution of this problem in its generality, and then to consider solutions based upon certain simplifying assumptions which, in some cases, may be permissible.

GENERAL SOLUTION

For conciseness the following notation will be used:

$$\theta = h\sqrt{\frac{P}{E\,I}} \quad \text{(30a)}$$

$$\theta' = h\sqrt{\frac{W}{E\,I}} \quad \text{(30b)}$$

and,

$$u^2 = \frac{g\,T^2}{(4\,\pi^2\,h)} \quad \text{(30c)}$$

Equation (26) may be written:

$$\frac{\partial^4 y}{\partial x^4} + \frac{\theta^2}{h^2}\frac{\partial^2 y}{\partial x^2} + \frac{\theta'^2}{g\,h^3}\frac{\partial^2 y}{\partial t^2} = 0 \quad \text{(31)}$$

It is found that a solution of Equation (31) satisfying Equations (29) may be obtained by assuming,

$$y = (A_1 \cos m_1 x + B_1 \sin m_1 x + A_2 \cosh m_2 x + B_2 \sinh m_2 x) \sin\left(\frac{2\pi t}{T} + q\right) \quad \text{(32)}$$

and properly determining the constants, m_1, m_2, A_1, B_1, A_2, and B_2. It is easily seen that Equation (32) is a solution of Equation (31) if m_1 and m_2 satisfy the

following equations:

$$m_1^4\, h^4 - \theta^2\, m_1^2\, h^2 - \frac{(\theta')^2}{u^2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (33a)$$

$$m_2^4\, h^4 + \theta^2\, m_2^2\, h^2 - \frac{(\theta')^2}{u^2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (33b)$$

Solving these two quadratics and noting that negative roots must be rejected because they give imaginary values of m_1 and m_2:

$$m_1^2\, h^2 = \frac{1}{2}\left(\sqrt{\theta^4 + \frac{4\,(\theta')^2}{u^2}} + \theta^2\right) \ldots\ldots\ldots\ldots\ldots (34a)$$

and,

$$m_2^2\, h^2 = \frac{1}{2}\left(\sqrt{\theta^4 + \frac{4\,(\theta')^2}{u^2}} - \theta^2\right) \ldots\ldots\ldots\ldots\ldots (34b)$$

By differentiation of Equation (32), expressions may be obtained for all the derivatives occurring in Equations (29). For conciseness, substitute $\frac{h'}{h} = c$, $\frac{(k')^2}{h^2} = e$, $m_1\, h = \alpha_1$, $m_2\, h = \alpha_2$, and replace $\frac{P}{E\,I}$ by $\frac{\theta^2}{h^2}$; then the resulting four equations for determining A_1, B_1, A_2, and B_2 may be written as follows:

$$A_1 + A_2 = a \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35a)$$

$$\alpha_1\, B_1 + \alpha_2\, B_2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35b)$$

$$\begin{aligned} &[\theta^2 \cos\alpha_1 + (u^2\,\alpha_2^2 - c\,\theta^2)\,\alpha_1 \sin\alpha_1]\, A_1 \\ &+ [\theta^2 \sin\alpha_1 - (u^2\,\alpha_2^2 - c\,\theta^2)\,\alpha_1 \cos\alpha_1]\, B_1 \\ &+ [\theta^2 \cosh\alpha_2 + (u^2\,\alpha_1^2 + c\,\theta^2)\,\alpha_2 \sinh\alpha_2]\, A_2 \\ &+ [\theta^2 \sinh\alpha_2 + (u^2\,\alpha_1^2 + c\,\theta^2)\,\alpha_2 \cosh\alpha_2]\, B_2 = 0 \ldots\ldots\ldots (35c) \end{aligned}$$

$$\begin{aligned} &[u^2\,\alpha_1^2 \cos\alpha_1 - (c\,u^2\,\alpha_1^2 + e\,\theta^2)\,\alpha_1 \sin\alpha_1]\, A_1 \\ &+ [u^2\,\alpha_1^2 \sin\alpha_1 + (c\,u^2\,\alpha_1^2 + e\,\theta^2)\,\alpha_1 \cos\alpha_1]\, B_1 \\ &- [u^2\,\alpha_2^2 \cosh\alpha_2 + (c\,u^2\,\alpha_2^2 - e\,\theta^2)\,\alpha_2 \sinh\alpha_2]\, A_2 \\ &- [u^2\,\alpha_2^2 \sinh\alpha_2 + (c\,u^2\,\alpha_2^2 - e\,\theta^2)\,\alpha_2 \cosh\alpha_2]\, B_2 = 0 \ldots\ldots (35d) \end{aligned}$$

Forced Oscillation of Known Period.—The coefficients of A_1, B_1, A_2, and B_2 in Equations (35) depend upon the constants of the oscillating system and also upon the period. For an oscillation of known period and amplitude the four unknowns may therefore be determined by direct solution of the four linear equations.

Free Oscillation.—The case of free oscillation also is covered by the foregoing solution as the limiting case, $a = 0$. In this case, however, u is one of the unknown quantities to be determined by Equations (35); moreover, these equations determine only the ratios of the four constants, A_1, B_1, A_2, and B_2, their actual values being indeterminate even after u becomes known. The elimination of these four quantities results in an equation involving the single unknown, u. (One method of expressing this equation is to equate to zero the determinant of the coefficients of A_1, B_1, A_2, and B_2.) Since both α_1 and α_2 depend upon u, the only practicable method of solving such an equation is to substitute trial values of u. After u is known, any one of the four constants, A_1, B_1, A_2, and B_2, may have an arbitrary value, the other three being expressed in terms of it; this means merely that the amplitude of the free oscillation remains arbitrary.

Because of algebraic complexity the general formulas resulting from the solution of Equations (35) when u is given, are here omitted; but the procedure above outlined will be illustrated by means of an assumed set of numerical data. First, however, will be shown the form assumed by the general solution when certain simplifying assumptions are made.

UNLOADED COLUMN

If $P = 0$, the problem reduces to that solved in the main part of this paper. This case is mentioned merely to emphasize the fact that the solution of that problem is covered as a particular case by the foregoing more general solution. Thus, if $\theta = 0$, and if $m_1 = m_2 = m$, Equations (33) reduce to a form which is identical with Equation (10). If, now, in Equations (35), $\theta = 0$, $c = 0$, $e = 0$, and $\alpha_1 = \alpha_2 = m\,h$; are substituted, these equations reduce to Equations (11).

SOLUTION WHEN INERTIA OF COLUMN IS NEGLECTED

For a given mass element, the effect of inertia upon the oscillation of the system is greater the greater the distance of the element from the ground. There may well be cases, therefore, in which the effect of the load, P, so far predominates that the inertia of the column itself may be treated as negligible. The simplified form assumed by the solution in such a case will now be given.

The differential Equation (31) reduces to the form,

$$\frac{\partial^4 y}{\partial x^4} + \frac{\theta^2}{h^2}\frac{\partial^2 y}{\partial x^2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

and a solution satisfying the boundary conditions, Equations (29), may be written in the form,

$$y = (A_1 \cos m\,x + B_1 \sin m\,x + A_2 + B_2\,x) \sin\left(\frac{2\,\pi\,t}{T} + q\right) \ldots (37)$$

It is easily seen that this satisfies Equation (36) if $m\,h = \theta$; and that the substitution of the proper derivatives in Equations (29) gives four equations for determining A_1, B_1, A_2, and B_2. Omitting details of algebraic transformation, the four equations may be written as follows:

$$A_1 + A_2 = a \ldots\ldots\ldots\ldots\ldots\ldots (38a)$$

$$B_1\,\theta + B_2\,h = 0 \ldots\ldots\ldots\ldots\ldots\ldots (38b)$$

$$(1 - \cos\theta + c\,\theta \sin\theta)\,A_1 + (\theta\,u^2 + \theta - \sin\theta + c\,\theta - c\,\theta\cos\theta)\,B_1 - a = 0 \ldots\ldots (38c)$$

and,

$$((c\,\theta \sin\theta - \cos\theta)\,u^2 + e\,\theta \sin\theta)\,A_1 - ((c\,\theta\cos\theta + \sin\theta)\,u^2 - e\,\theta + e\,\theta\cos\theta)\,B_1 = 0 \ldots\ldots (38d)$$

Equations (38) correspond to Equations (35) of the general solution; but in Equations (38) the solution for the unknown quantities has been partly carried out by the elimination of A_2 and B_2 from Equations (38*c*) and (38*d*).

Forced Oscillation.—If a is not 0, Equations (38) may be solved directly for A_1, B_1, A_2, and B_2, so that all the constants in Equation (37) become known in terms of a and u. This completes the solution for a forced oscillation of given amplitude and period.

Free Oscillation.—If $a = 0$, Equations (38) can be satisfied only for restricted values of u. In this case u must satisfy the following equation, obtained from Equations (38*c*) and (38*d*) by eliminating A_1 and B_1:

$$\theta\,(c\,\theta \sin\theta - \cos\theta)\,u^4 + (\sin\theta - \theta\cos\theta + (c + c^2 + e)\,\theta^2 \sin\theta)\,u^2 + e\,\theta\,(\theta\sin\theta - 2 + 2\cos\theta) = 0 \tag{39}$$

An example with numerical data, illustrating this case, will be given subsequently.

SOLUTION WHEN GRAVITY EFFECT IS WHOLLY NEGLECTED

In the foregoing theory both the inertia effect and the gravity effect of the load P have been taken into account, although the gravity effect of the column itself has been treated as negligible. In order to estimate the relative importance of the two effects for the load P, it will be useful to give a solution in which the gravity effect is neglected. This effect for the load P is represented by the term containing θ in Equation (31) and by the last term in Equation (36). Consider the result of omitting these terms, still treating the inertia of the column as negligible.

The differential equation becomes,

$$\frac{\partial^4 y}{\partial x^4} = 0 \tag{40}$$

The boundary conditions at $x = 0$ are unchanged, and the conditions at $x = h$ are still expressed by Equations (27), omitting the last term of Equation (27*b*); the value of V_1 to be substituted in these equations must be determined from the relation $V = -\dfrac{\partial M}{\partial x}$. This affects only Equation (29*c*), from which the term containing P in the second member must be omitted.

The appropriate solution is expressed by the following equations:

$$y = (A_0 + A_1 x + A_2 x^2 + A_3 x^3) \sin\left(\frac{2\pi t}{T} + q\right) \tag{41}$$

$$A_0 = a \tag{42a}$$

$$A_1 = 0 \tag{42b}$$

$$(1 + 2c)\,\theta^2 A_2 + (6u^2 + (1 + 3c)\,\theta^2)\,A_3 h + \frac{\theta^2 a}{h^2} = 0 \tag{42c}$$

and,

$$2\,(u^2 - e\,\theta^2)\,A_2 + 3\,(2\,(1 + c)\,u^2 - e\,\theta^2)\,A_3 h = 0 \tag{42d}$$

Forced Oscillation.—When a is not zero, Equations (42) determine A_0, A_1, A_2, and A_3 in terms of a, u, and the constants of the system, thus completing the solution for a forced oscillation of any known period and amplitude.

Free Oscillation.—If $a = 0$, u is restricted to values satisfying the following equation, which results from the elimination of A_2 and A_3 between Equations (42*c*) and (42*d*):

$$12\,\frac{u^4}{\theta^4} - 4\,(1 + 3\,c + 3\,c^2 + 3\,e)\,\frac{u^2}{\theta^2} + e = 0 \quad \text{.........} \quad (43)$$

It is noteworthy that Equation (43) makes u directly proportional to θ, their ratio depending solely upon the constants, c and e. Remembering the meanings of u and θ, this result shows that the free period is proportional to $\sqrt{\frac{P\,h^3}{E\,I}}$. It is of interest to compare this result with Equation (14).

SOLUTION WHEN SUPPORTED LOAD IS TREATED AS A PARTICLE

If the dimensions of the load are small in comparison with the length of the column, a sufficiently close approximation may be obtained by treating the load as a heavy particle. Algebraically, this amounts to assuming c and e to be zero, and results in material simplification of the formulas obtained in the foregoing solutions. In the general case defined by Equations (30) to (35), inclusive, in which the inertia of the column is taken into account, the case of free oscillation must still be solved by trial, and the value of u will depend upon both P and W. When the inertia effect of the column is neglected, however, the formulas become very simple, Equations (39) and (43), respectively, taking the following forms:

$$(\theta \cos \theta)\, u^4 - (\sin \theta - \theta \cos \theta)\, u^2 = 0 \quad \text{............} \quad (44a)$$

and,

$$12\, u^4 - 4\, \theta^2\, u^2 = 0 \quad \text{....................} \quad (44b)$$

Disregarding the zero roots, Equation (44*a*) gives $u^2 = \frac{\tan \theta}{\theta} - 1$; and Equation (44*b*) gives $u^2 = \frac{\theta^2}{3}$. A comparison of these results shows the effect of gravity upon the period of free oscillation. That this effect increases with θ, but is relatively small for small values of θ, may be seen by developing $\tan \theta$ in a power series giving,

$$\frac{\tan \theta}{\theta} - 1 = \frac{\theta^2}{3} + \frac{2\,\theta^4}{15} + \cdots \quad \text{..........................} \quad (45)$$

Since the definition of u (Equation (30*c*)) gives,

$$T = 2\,\pi\, u \sqrt{\frac{h}{g}} \quad \text{..........................} \quad (46)$$

Equations (44) yield the following formulas for T_0 for the oscillating particle:

Taking account of gravity effect,

$$T_0 = 2\,\pi \sqrt{\frac{\tan \theta}{\theta} - 1}\, \sqrt{\frac{h}{g}} \quad \text{................} \quad (47a)$$

neglecting gravity effect,

$$T_0 = \frac{2\,\pi\,\theta}{\sqrt{3}}\sqrt{\frac{h}{g}} \quad \text{(47b)}$$

FLEXURE OF COLUMN UNDER STATIC LOAD

It is of interest to note that the foregoing theory includes, as a special case, the theory of flexure of a column under a static load, commonly known as Euler's theory. For a column fixed at the bottom and free at the top, sustaining a centrally applied load, P, the Euler formula for incipient flexure may be written:

$$h\sqrt{\frac{P}{E\,I}} = \frac{\pi}{2} \quad \text{(48)}$$

In the foregoing oscillation theory the static case is the limiting case, $T_0 = \infty$, which, in Equation (47a), gives $\theta = \frac{\pi}{2}$, agreeing with the Euler formula by the definition of θ (Equation (30a)).

NUMERICAL APPLICATION

Assume the following numerical data for the purpose of illustrating the application of the formulas obtained in the foregoing theory, giving special attention to the case of free oscillation.

Example 1.—Assume that $\frac{h'}{h} = c = 0.3$; $\frac{k'}{h} = 0.2$; $e = \left(\frac{k'}{h}\right)^2 = 0.04$; $h\sqrt{\frac{P}{E\,I}} = \theta = 1$; and $W = P$, so that $\theta' = \theta = 1$. These values must be substituted in Equations (34) and (35); the former gives:

$$\alpha_1^2 = m_1^2\,h^2 = \frac{1}{2}\left(\sqrt{1 + \frac{4}{u^2}} + 1\right) \quad \text{(49a)}$$

and,

$$\alpha_2^2 = m_2^2\,h^2 = \frac{1}{2}\left(\sqrt{1 + \frac{4}{u^2}} - 1\right) \quad \text{(49b)}$$

Thus, for a given value of T (which fixes u), α_1 and α_2 become known and the coefficients of A_1, B_1, A_2, and B_2 in Equations (35) may be computed; the solution is thus easily completed for a forced oscillation of any given period and amplitude. For the case of free oscillation, however, u is not given, but must have a value such that Equations (35) can be satisfied when $a = 0$; such values can be found only by trial as indicated under the heading "General Solution." The trial solution involves considerable labor and the details will be omitted. It is found that the required condition is satisfied to a close approximation by $u^2 = 2.507$. By Equation (46), this value of u gives, as a possible value of the free period, $9.95\sqrt{\frac{h}{g}}$.

Example 2.—Assuming the same values of c and e as in Example 1, let $\theta' = 0$ (that is, neglect the inertia of the column). The period of free oscillation is computed: (*a*) for $\theta = 1$; (*b*) for $\theta = 0.25$; and (*c*), for $\theta = 0.1$. Substitution in Equation (39) gives the following:

$$(a)\quad u^4 - 2.3032\,u^2 + 0.010828 = 0$$

$$u^2 = 2.298 \quad \text{or} \quad 0.00471$$

$$T_0 = 9.53\sqrt{\frac{h}{g}} \quad \text{or} \quad 0.431\sqrt{\frac{h}{g}}$$

$$(b)\quad u^4 - 0.04977\,u^2 + 0.00001364 = 0$$

$$u^2 = 0.04949 \quad \text{or} \quad 0.0002757$$

$$T_0 = 1.398\sqrt{\frac{h}{g}} \quad \text{or} \quad 0.1043\sqrt{\frac{h}{g}}$$

$$(c)\quad u^4 - 0.007684\,u^2 + 0.000000335773 = 0$$

$$u^2 = 0.007640 \quad \text{or} \quad 0.00004395$$

$$T_0 = 0.549\sqrt{\frac{h}{g}} \quad \text{or} \quad 0.0417\sqrt{\frac{h}{g}}$$

Example 3.—With the same data as in Example 2, let the gravity effect of P be neglected. The value of u for free oscillation is now determined from Equation (43) which becomes, $12\frac{u^4}{\theta^4} - 9.16\frac{u^2}{\theta^2} + 0.04 = 0$, giving $\left(\frac{u}{\theta}\right)^2 = 0.7589$ or 0.004392; and for the three specified values of θ,

$$(a)\quad T_0 = 5.473\sqrt{\frac{h}{g}}, \quad \text{or} \quad 0.416\sqrt{\frac{h}{g}}$$

$$(b)\quad T_0 = 1.368\sqrt{\frac{h}{g}}, \quad \text{or} \quad 0.1041\sqrt{\frac{h}{g}}$$

$$(c)\quad T_0 = 0.547\sqrt{\frac{h}{g}}, \quad \text{or} \quad 0.0416\sqrt{\frac{h}{g}}$$

Example 4.—With each of the three values of θ used in Examples 2 and 3, determine the free period on the assumption that P is a particle. The simplified formulas, Equations (44) to (47), give the following results:

If gravity effect is taken into account,

$$(a)\quad \text{For } \theta = 1, \qquad u^2 = 0.5574, \qquad T_0 = 4.691\sqrt{\frac{h}{g}};$$

$$(b)\quad \text{For } \theta = 0.25, \qquad u^2 = 0.02137, \qquad T_0 = 0.9186\sqrt{\frac{h}{g}};$$

$$(c)\quad \text{For } \theta = 0.1, \qquad u^2 = 0.003347, \qquad T_0 = 0.3635\sqrt{\frac{h}{g}}.$$

If gravity effect is neglected,

$$(a)\ \text{For } \theta = 1, \qquad u^2 = 0.3333, \qquad T_0 = 3.628\sqrt{\frac{h}{g}};$$

$$(b)\ \text{For } \theta = 0.25, \qquad u^2 = 0.02083, \qquad T_0 = 0.9067\sqrt{\frac{h}{g}};$$

$$(c)\ \text{For } \theta = 0.1 \qquad u^2 = 0.003333, \quad T_0 = 0.3628\sqrt{\frac{h}{g}}.$$

Summary of Computed Results.—The results computed in Examples 2, 3, and 4 are summarized in Table 1, in which the tabulated numbers are proportional to the values of the free period for the several cases. The tabulation brings out clearly the fact that the importance of the gravity effect decreases as θ decreases.

TABLE 1.—VALUES OF $2\pi u$ IN EQUATION (46)

θ	$c = 0.3, \quad e = 0.04$		$c = 0, \quad e = 0$	
	With gravity	Without gravity	With gravity	Without gravity
1	9.53 0.431	5.47 0.416	4.69	3.63
0.25	1.398 0.1043	1.368 0.1041	0.919	0.907
0.1	0.549 0.0417	0.547 0.0416	0.364	0.363

DISCUSSION

Homer M. Hadley,[5] Assoc. M. Am. Soc. C. E. (by letter).—Any designer undertaking to design an entire structure or an individual part thereof is confronted, at the very outset of his labors, with the question of loading. To what loads will his structure be subjected; what is their magnitude; their direction; their point of application; their duration; their periodicity, etc.?

It is refreshing, therefore, to read the authors' admirable statement of the nature of the movements that cause and create the earthquake loadings (see paragraph preceding "Earthquakes and Their Effects: Relation to Structures" in Part I).

This is a plain, true description of the earthquake movements of the ground which every seismographic record ever taken clearly attests. A veritable chaos of confused, conflicting movements follows upon the primary initial disturbance, which may occur—who knows when, or how? The authors state (see "The Problem"): "It would appear to be beyond the ability of any mathematician to develop formulas that would take accurate account of the extremely variable nature of the factors involved." Assuredly this is so.

To the writer it has always seemed a waste of effort to use methods of design more refined than basic conditions warrant, and the clear presentation, in this paper, of the utter ignorance there is of the earthquake loading, prompts the suggestion that structural engineers may well recognize the limitations of their knowledge. When the magnitude and direction of the loads is unknown, what choice is there between a dynamical or statical approach to a solution as arbitrary as, and no more valuable than, the good guesses called assumptions made at the outset?

In the short insert appendix to his book "Earthquake Damage and Earthquake Insurance" the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E., gives "Some After-Thoughts by J. R. F. and a Few Notes from the Suyehiro Lectures."[6] The views summarized therein are most interesting and valuable, particularly the concluding paragraphs containing the suggestion—originating with Professor Suyehiro, a former naval architect—that quite as sea-going ships are designed successfully by assuming them to be riding on "standard" waves without knowing much about the actual waves and quartering seas which they encounter, so similarly one may, arbitrarily and successfully, design buildings against earthquakes by assuming a conventionalized loading. The fact that buildings in Tokyo designed under "the $0.1g$ basis" resisted the 1923 earthquake fairly well is called "a datum more valuable than any other arguments."

It would seem desirable, therefore, to adopt as design procedure nothing more complicated or involved than the assumption of horizontal forces acting in any direction upon the structure and applied at its base. Logically and properly, the magnitude of these forces is a function of the weight of the structure and its loads: $0.10g$, $0.08g$, or whatever judgment dictates. The damping effect which the height of very tall buildings has upon stresses may be recognized by using a smaller coefficient of g; or, as Mr. Freeman suggests, by model experimentation on a shaking table and special treatment. The important consequence of a design developed on this basis is not that computed stresses will be those actually developed in an earthquake, but that reversals of stress will be provided for, connections will be connections, anchorages will be adequate. All the sorry structural spectacles that have characterized past earthquakes have not been consequent upon the failure to use this or that cunning formula or method of design. They have been due to failure to provide proper resistance to horizontal forces even roughly approximate to those occurring.

---

[5] Regional Structural Engr., Portland Cement Assoc., Seattle, Wash.

[6] *Proceedings*, Am. Soc. C. E., May, 1932, Part 2.

Some additional comment may be offered on certain minor points. The authors state ("Earthquakes and Their Effects: Relation to Structures," in Part I) that since the forces generated in a structure subjected to earth movements are measured by the mass or weight of the building it follows that "the use of light materials in a structure is the best and that heavy, inelastic materials should be avoided as far as possible." Actually what follows is that with light materials the forces generated in a structure are of less magnitude than those developed with heavy materials. This is not, however, a criterion by which to assess the relative worth of the two constructions. Certainly on the basis of cost to the owner it may well prove more economical to specify the heavier material; and since the earthquake behavior of a structure does not result from the inert, lifeless materials of which it is built, but from the manner in which these materials are shaped and assembled and are combined together, it is not apparent that the weight of the construction has any appreciable bearing on the matter at issue.

Inasmuch as buildings "with wings at right angles" may be so built and connected as to sustain no earthquake damage, it is suggested that the paragraph dealing with the form of the structure be modified so that it indicates the hazards and dangers of this shape rather than pronounces inevitable doom. There is an almost equal hazard and danger in a square or rectangular building wherein the resistance to horizontal forces is most unequally and unsymmetrically distributed with respect to the plan of the building.

Regarding "visible surface waves" in the ground, to the writer (who has never chanced to see them), it has always seemed quite strange that they should occur without causing a general and extensive shattering of concrete sidewalks, curbs, etc. He has seen the rupturing of concrete pavements at fissures; but the waves, if any, apparently do no damage. In the fall of 1923 in Tokyo, a friend of the writer was in the Imperial Hotel when one of the countless aftershocks occurred. This was a very severe one, however, and it caused considerable alarm. Telling about it in the calm of the evening, he said that he had rushed to the window of his room, and looking out, it seemed as if the long side-wall of the hotel was "waving like a flag in the wind." He did not believe that the wall did wave, but he asserted that it seemed to. Bricks and concrete being intact, the phenomenon undoubtedly was a personal one—"in my mind's eye, Horatio."

R. McC. Beanfield,[7] M. Am. Soc. C. E. (by letter).—This paper is helpful and thought-provoking and will tend no doubt to stimulate interest in this important subject, which has been somewhat neglected by the profession in general. The paper contains much reiteration of known data; Part II, covering stresses and dynamic theory, treats the subject more from an academic point of view, and fails to offer any practical solutions. It also fails to include much advanced development in the art of aseismic design subsequent to 1928. The fact is that many engineers are mindful of the inadequacy or incompleteness of the theory and practice relative to aseismic design. The present state of the art is rather empirical and is based to a large extent on opinions and conjectures which are not substantiated by fact. Engineering is essentially a constructive art, and builds from known facts.

The following fundamental facts have been developed by scientific research, structural dynamics, and experimental investigation:

(1) When a vibratory motion is applied to the support of a system, the system itself will generally begin to vibrate. In the early stages of its vibration the motion of the system does not bear a simple relation to that given to the support. During this interval, the resulting motion is made up of two parts: "Free," or transient, vibrations; and "forced," or steady-state, vibrations. If the system is initially at rest when the disturbing motion begins, free vibration will always exist during the early part of the motion. These transient vibrations may cause the displacements of the structural system to be more than twice as great as those that will eventually exist in the steady state.

(2) Earthquake motion is not simple motion even if one dimension is considered. It consists of elastic impacts followed by free vibration and forced vibration with all their transients superimposed. The period, or apparent period, varies throughout the duration of the earthquake although in a number of actual, strong-motion seismographs certain periods dominate.

(3) Seismic ground vibrations (in so far as records to date reveal them) are not harmonic; on the other hand, the vibration of elastic yielding structures, caused by the disturbing force of seismic impulses, is harmonic, which fortunately tends to simplify the dynamic problem.

(4) The presence of transients may cause very high stresses within the duration of the first cycle or the first two cycles of the disturbing motion, which indicates that earthquakes do not require sustained vibrations in order to cause dangerous stresses in structures. Displacements of an oscillating structure are directly proportional to the ground amplitude. Therefore, the structure may be out of resonance with the ground and yet be over-stressed because of the large amplitude of the ground motion. Earthquake periods need not be in direct resonance with the natural periods of a structure in order to produce dangerous dynamical stress effects.

(5) The application of a fictitious or supposititious static force, derived from the conventional formula $H = \frac{W\alpha}{g}$ (wherein a maximum acceleration of the ground is assumed to be the same through all parts of the structure above the ground), may be used to obtain a practical solution only for relatively stiff

---

[7] Cons. Engr., Los Angeles, Calif.

structures such as one-story or two-story block-type buildings of average height. Its use as a general formula for aseismic design in elastic yielding structures such as water-towers, chimneys, multi-story frames, etc., is unscientific and may result in a false sense of security and structural stability.

(6) The conventional measure of the intensity of the earthquake known as the seismic factor is the ratio of the greatest horizontal acceleration of the ground to the acceleration due to gravity. It is a well-established fact that the only proper method of judging earthquake intensity is by the kinetic energy method. It can be demonstrated by structural dynamics that the kinetic energy decreases as the frequency increases, or as the period shortens, and that the amplitude of motion decreases very much faster than the frequency increases. These results prove that the acceleration used in the conventional statical formulas is not a constant and loses its usefulness as a measure of the intensity of the earthquake. It is irrational to assume a single constant value of the intensity ratio or seismic factor independent of the periods.

(7) The natural period of a structure is a criterion of its rigidity.

(8) Dampening has little effect in reducing the maximum amplitude of the transient vibration and possible dangerous stresses during the first few cycles of vibration. The stiffening effects due to the friction of joints and of interior secondary partitions, such as plaster, wood, or tile, should not be given any value in computing displacements of the structural frame.

(9) Elastic yielding structures, particularly the more flexible type, do not always vibrate only in their fundamental modes. As a matter of fact the motion of the ground causes harmonics of frequencies higher than the fundamental, and there is always the probability that one of these harmonics, usually within the first five modes, is close enough to the ground oscillation to cause dangerous dynamic forces. In the more elastic types of structures, such as towers, the maximum natural period is limited by conditions of buckling.[8]

The statement in the paper (see heading "Dynamic Theory as a Guide to Practical Design"), that the solution on the assumption of rigidity is simple and definite, may be questioned. The structural designs, in which a fictitious horizontal static force is used, do not comprise a simple problem. As a matter of fact, in order to obtain similar rigidities throughout the structure, considerable ingenuity and a good practical knowledge of the application of the elastic theory are required.[9]

The "Fundamental Practical Rule," which the authors infer can be used with confidence (see "Dynamical Theory as a Guide to Practical Design"), does not apply to all types of structures. It should be understood definitely, and emphasized, that this rule applies only to inherently rigid structures, such as one-story or two-story buildings that have natural periods of exceedingly high frequencies.

Attention is called to a common error in the use of the fixed-force method wherein the horizontal force or supposititious shear (used, for example, at each floor level in a structural frame) is based on a constant value of the maximum ground acceleration, usually 0.1 g, whereas the actual acceleration at each floor level varies and increases from the ground up. It is obvious that, as the deflection increases upward, so does the acceleration.

---

[8] "Die Erbebensicherheit von Bauwerken," by Dr. R. Briske.

[9] "Earthquake-Resisting Construction," by Dr. T. Naitō.

All structures that are elastic yielding should be designed by some method of dynamic analysis. In this connection the writer suggests the following method.[10]

(a) Compute the natural period of the structure by determining the maximum deflection or displacement due to horizontal disturbing forces based on the seismic factor of 0.1 g. If the maximum deflection of an elastic structure within its elastic limit is known, then, by structural dynamics, the maximum period, acceleration, and dynamical shears can be obtained. The maximum statical deflection of an elastic structure is also approximately the maximum amplitude of its transient vibrations.[11]

(b) By the use of the magnification factor B, the maximum stresses, which should not exceed the elastic limit, can be determined by the formula

$$B = \frac{1}{1 - \frac{T_0^2}{T^2}} \dots\dots\dots\dots\dots\dots (50)$$

in which T_0 = natural period; T = period of disturbing forces; and, B = index of the increase in deflections, bending moments, and stresses due to forced vibrations.

(c) The natural period of a structure ascertained by the foregoing method should not exceed 0.5 sec.

It is an established fact that the dynamic responses of structures to earthquakes of high frequencies or short periods do not have large displacements of amplitude. Therefore, structures with high frequencies will not be dangerously affected, and it is also true that the duration of seismic frequencies is usually short.

Attention should be called to the fact that application of the various formulas based on the dynamic analysis of a thin rod of constant cross-section, even for comparative purposes, in the design of structures, may lead to erroneous and dangerous results. In a number of the flexible structures such as chimneys and towers, for example, the mass of these structures is not usually uniform, the moments of inertia vary, and the deformation is caused more by shear than by bending moments. For instance, the deformation of structural frames is due more to shear which completely changes the formula of the thin rod concept. Therefore, it is not applicable to such structures. "Formulas are the anesthetics of thought, not its stimulants."

What Materials Should Be Used?—Invariably, buildings are described as brick, concrete, or steel-frame structures. It must be emphasized that it is not the materials but the design that is of foremost importance. There are certain requisites for materials, and certain materials fit these particular requirements better than others. As to this suitability, there has been much ignorant propaganda and some deliberate misrepresentation. If the engineer will prepare a design and compute the stresses, he will see for himself which materials are required for a given place in the structure.

---

[10] "Vibrations and Their Effects on Structures," by R. McC. Beanfield, Applied Science Div., Local Section, A. S. M. E., Los Angeles, Calif., April 25, 1934.

[11] "Vibrational Problems in Engineering," by S. Timoshenko, Rayleigh's Energy Method, p. 63.

R. S. Chew,[12] M. Am. Soc. C. E. (by letter).—It is thirty years since the writer offered a paper which dealt with this subject.[13] The analysis is crude and has several apparent errors; and yet the treatment of the stiff structure is similar to that suggested in Section II of the present paper, under the heading "Fundamental General Rule."

The analysis of the flexible structure was not attempted by the writer except to indicate reversal curves for both a uniformly flexible structure and the flexible first story. In later years the writer has realized that the foregoing treatment was insufficient and has made an analysis of the problem assuming the structure to have its base oscillating horizontally, with simple harmonic motion, and has published a monograph on this subject.[14]

From the foregoing it will be appreciated that the writer has read the rigid analysis presented in this paper with keen interest and believes it will be of great value to the profession in its study of the problem. The writer hopes that the treatment will be extended so that it will reveal the condition in which $\frac{T_0}{T}$ varies from 1.6 to 4.0 because he believes in a zone of $\frac{T_0}{T}$ between 1.6 and 4.0. The characteristic curves will appear as reverse curves with nodal points. This is necessary in the consideration of higher buildings in which periods of 1.35 to 1.8 are usual. In the writer's treatment[15] he has attempted to indicate the deformations in this zone.

Turning from theory to a practical consideration, it is to be noted that the value of T_0 is ambiguous.

The authors state that Equation (14) may give some indication of the order of magnitude of period for a tall building. On the other hand, it is impractical to evaluate $E\,I$ in a building when walls (fire-proofing), partitions, and frame must all be considered as acting.

In the thirty-three years since 1906 the writer has observed the rebuilding of San Francisco, Calif., with structures having a marked difference in the amount and type of bracing, and has made a relative study of measured periods of the tall buildings ranging from seventeen to twenty-nine stories in this city. On the basis of this study he has reached the opinion that at least 50% of the stiffness in a wind-braced, wall-enveloped, tall building, with usual office windows, is due to wall and fire-proofing, etc.

From records of the Japanese it was found that the period of a structure lengthens during a base movement due to the loosening and cracking of the non-elastic materials—and from measurements before and after an earthquake, it has been stated[16] that the increase varies from 30 to 50 per cent. This indicates that the structure passes through several period states until the various resistances produce a period that maintains. Consequently, the period T_0 must be assumed from comparison with existing structures of known period; or it must be based on some empiric formula.

---

[12] Cons. Engr., San Francisco, Calif.

[13] "Effect of Earthquake Shock on High Buildings," by R. S. Chew, *Transactions*, Am. Soc. C. E., Vol. LXI, December, 1908, p. 238.

[14] "An Approximate Measure of Earthquake Effect on Framed Structures," by R. S. Chew; published privately by the author at San Francisco, Calif., 1938; a copy has been filed for reference in Engineering Societies Library, 33 W. 39th Street, New York, N. Y.

[15] *Loc. cit.*, p. 82.

[16] "Tests on Maronouchi Building in Tokio," by Tadasi Taniguti, *Paper no. 645*, *Transactions*, World Power Conference, Vol. 8, 1929.

The foregoing indicates why the writer is unable to subscribe to the authors' practical rule of

$$F = m\,\alpha = \frac{W}{g}\,\alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (51)$$

when applied to all buildings, stiff or flexible. In fact the writer does not believe that any constant seismic factor is logical.

When applied to the frame the factor produces increased strength but only a small margin of stiffness, whereas stiffness is the required feature. It is understood that strength is necessary; but for the present problem it is sufficient to have a resilient body under the action.

For the purpose of illustration consider the practical problems involved in a six-story design and a twenty-five-story design, assuming the following factors as known and as actually resulting from earthquake acceleration equal to 3.2 ft per sec^2: Amplitude = 1 in.; and, period T = 1 sec. Both structures are to be designed under a specified load of $\frac{1}{10}W$ as a lateral force. The designer of the six-story structure will apply the factor giving the structure the necessary strength, but is likely to have a period T_0 of between 0.6 and 0.8 sec, in which case the structure actually has stresses 40% higher than called for by the factor.

Assuming that it is practical to apply the lateral force to the twenty-five-story building, the designer would have to add approximately 40% to his steel tonnage in order to meet the strength requirement; but it will be found that unless diagonal bracing is incorporated, the increased stiffness is not in proportion, and from the writer's study, the period would be decreased 10% to 15% below that of the 20-lb, wind-braced, structure.

It will be seen that the structure has changed its flexible characteristics very little, so that its deformation is approximately the same, in which case, although the building is stronger, it will probably suffer as much wall and partition damage as it would under a 20-lb wind design. These results are logical because the factor (0.1W) was based on a rigidity, which neither design possessed, under the action described. The stiffer building received the accelerated movement, increased by additional acceleration due to yield. The flexible structure absorbed the accelerated movement in its deformation.

From the writer's viewpoint the requirement by ordinance, for the foregoing, should have been: Acceleration = 3.22 ft per sec^2; and amplitude = 0.25 in., 0.50 in., or 1.0 in., depending on the soil, with corresponding periods, T, equal to 0.5 sec, 0.7 sec, or 1.0 sec, in which case the designs would have been more consistent.

Resonance.—In high flexible buildings the danger of resonance is remote due to the fact that an earthquake period is rarely, if ever, as great as the period of the loaded frame with damping removed.

In low stiff buildings $\left(\frac{T_0}{T} < 1\right)$ there is a danger of the period lengthening until it is approximately equal to T.

Damage.—There is no doubt that wall damage is unavoidable due to deformation except in such low structures (as to height and design) that their maintained period is less than 0.5 sec, in which case deformation is small.

Jacob J. Creskoff,[17] Esq. (by letter).—Under the headings "Dynamical Theory as a Guide to Practical Design: Fundamental Practical Rule" and "Danger of Resonance," the authors express some interesting conclusions. Since m equals $\frac{W}{g}$, the fundamental practical rule of the authors may be restated as:

$$F = W\frac{a}{g} = W\,K \qquad (52)$$

in which, W = weight, in pounds; and, K = a constant, varying from 0.02 to 0.10. Hence, the fundamental practical rule of the authors will be recognized as the basis of the statical method of aseismic design. Naturally, statical design is to be preferred to a system that disregards aseismic features. However, it should be kept in mind that, although some statically designed structures have reacted well during earthquakes, it has become increasingly apparent that survival in these cases has been largely a matter of chance.

The following factors are involved in most of the earthquakes that occur in the United States: Invisible elastic earth waves are generated by the slipping of contiguous fault-blocks. Upon reaching a certain location, these waves jar the site and its superimposed structures into vibration. The response of a particular building is determined by the complex relationship between the earth waves, the site, and building characteristics, but principally by the resonance or synchronism ratio between the dominant periods of the site and building. If the ratio approaches unity, the building will vibrate dangerously within a few seconds—only several vibrations. That synchronism is of profound importance in aseismic design has been demonstrated not only theoretically, as in the paper by the authors, but experimentally by Arthur C. Ruge, Assoc. M. Am. Soc. C. E.,[18] and others, and by analyses of actual earthquake damage. A. Blake has shown[19] that the periods of invisible earth waves may be of any values included in the entire range of building periods. One or more dominant periods may appear in a site vibrating by reason of the passage of these waves through it. Finally, the superimposed structure vibrates in a number of natural periods.

At first glance therefore it seems improbable that a practical rule for estimating the probable effects of synchronism can be formulated. However, the writer is of the opinion that (except for the improbable case of synchronism between the earth waves and the site) the degree of synchronism existing between the dominant site and building periods is a reliable index to the probable extent of stresses due to synchronism.

---

[17] Cons. Engr., New York, N. Y.

[18] "Earthquake Resistance of Elevated Water-Tanks," by Arthur C. Ruge, *Proceedings*, Am. Soc. C. E., May, 1937, p. 801.

[19] "The Recording of Strong Seismic Motion," by A. Blake, *Bulletin* of the Seismological Society of America, July, 1933, p. 111.

The dominant periods of a site can be determined by jarring it with dynamite blasts, or with a shaking machine, recording the vibrations by seismographs, and interpreting the records. The dominant periods of an existing building can be found by recording and interpreting its vibrations as caused by wind, or with a shaking machine. In the case of a projected building, the writer agrees with the authors that a designer is powerless to estimate the possible danger from synchronism unless he can compute the periods of vibration of the building. Fortunately, there is now (1936) available a wealth of observational data on building vibrations.[20] With the plans and loading of the building, the empirical data, and the available vibration equations, the designer can compute the periods of the projected building with reasonable accuracy.

For example, suppose that, as in the statical method, the horizontal force to be resisted by a building is assumed to be acting at right angles to the elevations of the building, at the foundation, floor, and roof levels, and calculated by the formula

$$F = W K \dots\dots\dots\dots\dots\dots\dots\dots (53)$$

in which F is the horizontal force acting at the section under consideration, in pounds; K is a numerical constant depending on dynamical and experimental considerations (see Table 2); W is the total dead load plus actual live load at, and above, the section under consideration, in pounds (minimum live load to be one-fourth the design live load); T_b is the period of vibration of the building, in seconds; and T_g is the period of vibration of the site, in seconds. The synchronism ratio $\frac{T_b}{T_g}$ that gives the largest value of K in the plane of vibration under consideration is to be used. The values of K given in Table 2 must be divided, however, by the order of the period of vibration (that is, the fundamental period will be divided by (1), the first harmonic by (2), etc.). The minimum value of K used for any building will be $K = 0.05$, except for public buildings, for which K is not less than 0.10.

TABLE 2.—VALUES OF K

Site Material	VALUES FOR THE FOLLOWING SYNCHRONISM RATIOS, $\frac{T_b}{T_g}$:									
	2.0	1.8	1.6	1.4	1.2	1.0	0.9	0.8	0.7	0.6
Bed-rock	0.05	0.06	0.07	0.12	0.21	0.30	0.25	0.12	0.07	0.05
Consolidated ground	0.05	0.07	0.08	0.13	0.23	0.33	0.28	0.13	0.08	0.05
Soft ground	0.05	0.05	0.06	0.09	0.15	0.22	0.19	0.09	0.06	0.05

The seismic force, $F = W K$, acts against the mass of a building. Assuming that the mass is concentrated at the floor levels, and that floors are horizontal girders that deflect as units, then the columns and beams between two consecutive floors may be regarded as vertical beams—deflecting equally because of a concentrated horizontal force acting against the upper floor—and with end fixities determined by the degree of fixity between the vertical beams and the floors. Under these assumptions, the seismic force is distributed to all of the vertical beams, and to each in proportion to its relative rigidity.

---

[20] *Bulletin No. 201*, U. S. Coast and Geodetic Survey.

It will be noted that the values of K given in Table 2 are frequently much larger than the maximum value described by the authors under their fundamental practical rule. This is to be expected, of course, since the values of K in the table reflect the influence of synchronism between site and building periods during an earthquake.

One might judge that the use of such high values of K would add materially to the cost of a structure. This might be the case if the frame of the building is to be designed for these large values of K. However, if, in addition to the frame, the walls and floor and roof slabs are designed to resist the seismic force, it has been the experience of the writer that these higher values of K will not affect, appreciably, the cost of the building as a whole.

Arthur C. Ruge,[21] **Assoc. M. Am. Soc. C. E. (by letter).**—This interesting paper draws attention to the great difficulties in the way of rational design against earthquakes. Certainly it is clear that a purely analytical approach to the problem is out of the question, except for such general and valuable qualitative information as that developed by the authors and several other writers on the subject. The natural tendency of the average designing engineer is to throw up his hands at the thought of making any dynamical analysis at all, and it is only through the persistence of these investigators that the subject is kept alive and with it the hope of an ultimate solution.

The writer would like to comment upon a few statements contained in the paper with a view to further clarification, rather than criticism, and to add some comments on the interpretation of forced vibration analyses like those presented by the authors.

It is clear that the authors recognize the importance of the transient response of structures, and that they regard the forced or steady-state response as only a guide to judgment in design. There is one important point, however, which seems to be quite generally overlooked in connection with the behavior of the higher modes of vibration. It will be found invariably that a considerably closer "tuning in" of earthquake frequency with natural frequency is required in the higher modes for the same degree of dynamic magnification; the higher the mode, the sharper the tuning must be.

This fact is well known to those who have had experience in generating the higher modes of vibration of mechanical systems or models. Many casual experimenters have had the mistaken notion that the effect is due to a greater degree of damping in the higher modes. Although in some cases there may be greater damping and an attendant reduction of magnification in the higher modes, the presence of damping will always tend to make the tuning less critical at resonance which, therefore, makes it impossible to explain the tuning phenomena observed on the basis of damping effects.

Physically, this means that the probabilities of damage in the lower modes are far greater than in the higher, simply because the degree of tuning is less critical in the lower modes. As an example, consider the ratio $\frac{V}{V_0'}$ (Equation

---

[21] Asst. Prof. of Eng. Seismology, Mass. Inst. Tech., Cambridge, Mass.

(18c)) at the base of the beam for the first and second modes; that is, the ratio of the actual shear at the base to the shear that would occur in a perfectly rigid structure of the same weight. The following data were obtained from Equation (18c):

Mode	$\frac{T_0}{T}$	$\frac{V}{V_0'}$
First.................	0.95	7.0
Second..............	0.98	4.6

That is, the earthquake period is 5% above the first mode resonance and 2% above the second mode. It is apparent that the tuning in at the second mode must be more than 2½ times as sharp as at the first or lowest mode in order to produce the same dynamic magnification. The effect becomes even more pronounced above the second mode. Now, it must be remembered that the foregoing data are not the actual shears but magnification factors—the shears are $V = 7.0\ V_0'$ and $4.6\ V_0'$; and it is here that a second and equally important consideration enters; namely, the actual values of V_0' decrease very definitely at the higher earthquake frequencies and hence at the higher modes of buildings. Exactly what law governs the variation of earthquake amplitude with frequency is not quantitatively known as yet, but the fact that the amplitudes do decrease with increase in frequency is too well known to be debated. Over very limited ranges the trend may be reversed; but when widely separated ranges are compared (as the first and second modes of a structure which may bear a 5 : 1 or a 6 : 1 period relation) one may confidently expect to find a large drop in amplitude in the range of the higher modes.

The natural conclusion, therefore, seems to be that the modes above the first (or, in rare structures, possibly the second) are not nearly so important as would be indicated by a superficial study of the steady state analysis. It is on the basis of such reasoning that the writer questions the statement (see Part II, under "Dynamical Theory as a Guide to Practical Design: Danger of Resonance") that: "Since any given structure has a multiplicity of natural periods, and since the harmonic analysis of seismograph records has shown a considerable range of values of T, it seems impracticable to formulate any simple rule for minimizing the probability of dangerous stresses due to resonance." It seems to the writer that by limiting the modes to be studied to one or, perhaps, two, the problem is not so hopeless as it might at first seem. The mere fact that difficulties still beset the investigator should not lessen the determination to achieve a full solution ultimately.

It is further true that the frequency selectivity of the higher modes may be augmented by proper design. Indeed, the introduction of a flexible first story has just such an effect, as may easily be visualized.[22] The only objectionable feature of this construction is that it increases, considerably, the liability of the first story to damage, due to the lowered period of the fundamental. The writer has yet to see a suitable and economically feasible design proposed for the first story of such structures.

---

[22] See "Experimentally Determined Dynamic Shears * * *," by L. F. Jacobsen, *Bulletin*, Seismological Society of America, Vol. 28, No. 4, October, 1938.

The statement (see Part I under "Earthquakes and Their Effects: Relation to Structures"), to the effect that the forces of the moving ground are unlimited, may require some modification. Although it is doubtless true for structures on rock and for light structures on compact soil, there is reason to believe that heavy structures resting on soil do not behave as if the ground motion were inexorable, but instead they influence the ground movement locally to a marked extent. Some evidence of this influence has been noted in the effects of the Japanese earthquake of 1923, and analytical studies by K. Sezawa and K. Kanai[23] lead to the same conclusion. Professor Jacobsen has also presented a valuable study of this phenomenon.[24]

In Part I (under "Earthquakes and Their Effects"), the authors express the limits of earthquake periods as 0.1 sec and 2 sec. This is a much disputed point, however, and there is reliable evidence that wave periods much longer than 2 sec occur in destructive earthquake motion.[25] In general, it appears that the more severe and extensive the quake, the more prominent become the long-period components of motion. This fact is of great importance and should receive careful consideration in connection with ground-period studies made by shaking machines and by observation of small earthquakes and blasts. A study of the Long Beach motion (integrated by the U. S. Coast and Geodetic Survey) shows a surprising amount of long-period motion. Of course, waves shorter than 0.1 sec occur, but the authors probably considered them of no structural importance. Perhaps this question might best be left open at present.

In Part I, the authors omit consideration of the possibilities of the experimental or semi-experimental approach to the problem of anti-seismic design. The writer believes that sufficient facilities exist for extending the knowledge of quake-resistant design to the point of practical significance by combining the experimental and analytical techniques effectively. The principal reason for the lag of accomplishment behind facilities has been, and is, the lack of adequate funds for establishing the requisite long-time basic research with adequate personnel. To say that the surface has only been scratched is to risk exaggeration, but the profession owes a debt of gratitude to those men who have pushed forward the theoretical analysis until the possibility of a practical solution may be seen.

Walter L. Huber,[26] **M. Am. Soc. C. E. (by letter).**—A basic assumption not mentioned in the authors' development of the equations for the vibration of an elastic cantilever is that the formulas hold only for the "steady state" condition—that is, the condition obtained only after a considerable interval of time has elapsed since the initial harmonic impulse. This one assumption alone, not including those pointed out by the authors, is difficult to substantiate in such an erratic ground movement as experienced during an earthquake. At the same time, since this "steady state" condition is not likely to be realized in an earthquake, it suggests the application of the impact theory as developed by Prof. Joseph N. Le Conte in 1926.[27] During earthquakes there is considerable evidence of failure due to impact. In reviewing the damage to the concrete stack of the Seal Beach Station of the Los Angeles Gas and Electric Corporation, due to the Long Beach earthquake of March 10, 1933, the application of Professor Le Conte's paper was studied as one line of analysis.

---

[23] *Bulletin*, The Earthquake Research Institute, Tokyo Imperial University, Vol. XIV, Part 2, June, 1936.

[24] "Natural Periods of Uniform Cantilever Beams," by Lydik S. Jacobsen, *Proceedings*, Am. Soc. C. E., Vol. 64, No. 3, March, 1938, p. 432.

[25] "Earthquake Investigations in California," *Special Publication No. 201*, U. S. Coast and Geodetic Survey; see Chapter 9 by B. Gutenberg.

[26] Cons. Engr., San Francisco, Calif.

This stack, weighing about 2 800 tons, was 317 ft high, rising from a concrete platform which was, in turn, supported just beneath the roof on a steel framework. The base of the stack was about 65 ft above the ground. It was 24 ft in diameter at the top and 35 ft 6 in. in diameter at the base. The principal reinforcement consisted of vertical steel bars. Circumferentially only two layers of steel fabric were supplied for reinforcement. The stack suffered considerable damage in the earthquake, particularly in the middle third of its height, where many horizontal cracks opened. In certain places it appeared that such cracks had opened and had then come together. This was particularly noticeable where reinforcing steel appeared to have been first elongated and then compressed.

Determination of forces due to earthquakes to which complicated structures are subjected is not an easy task, and opinions advanced from such determinations are not always beyond question. On the roof of Seal Beach Station three 20 000-gal water-tanks would seem to afford opportunities for rather direct determinations. Even in this case, however, certain very essential information was not available. For instance, it was not known whether, at the time of the earthquake, the tanks were filled. It seems probable that two of them were not filled, but that the third one may have been, and probably was, approximately full. Assuming that it was filled and assuming the ultimate strength of certain rods in the tank frame which were broken by the earthquake to be 65 000 lb per sq in., the force necessary to have caused the destruction of these rods would have amounted to 27% of the force of gravity; or, if the ultimate strength of the rods were assumed equal to 70 000 lb per sq in., this force would have been 30% of the force of gravity. Of course, if the tank were not filled, the force exerted was a larger percentage of the force of gravity since in any case the rods were broken, and if the tank were not filled they would have been acting upon a smaller mass. These percentages must be considered the minimum force. It is not known how much this force exceeded that necessary to rupture the rods.

The tank in question was supported on the roof of the station about 65 ft above the ground. On this roof it is probable that both accelerations and displacements were considerably more than those of the ground beneath the station. Since the earthquake of June 10, 1933, instruments have been installed by the U. S. Coast and Geodetic Survey to record accelerations in the top stories and in the basements of several buildings in California. These instruments, although of the strong-motion type, are so connected that a movement sufficient to start one instrument will start both, thus giving simultaneous records. These instruments have shown (as in the case of a

---

[27] "The Stresses in a Free Prismatic Rod Under a Single Force Normal to Its Axis," by J. N. Le Conte, *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 968.

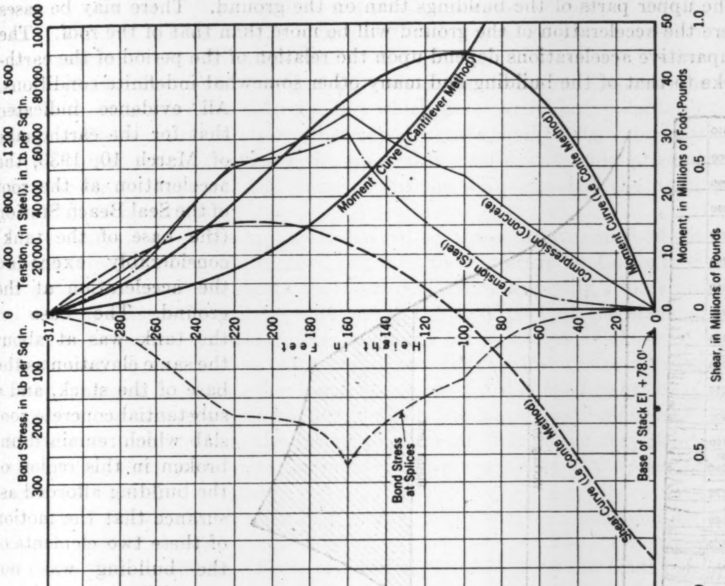


FIG. 7.—SEAL BEACH STACK; STRESSES BY LE CONTE METHOD

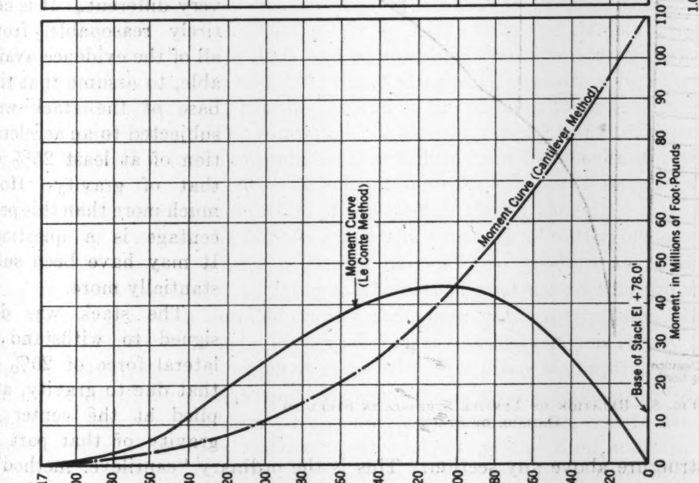


FIG. 6.—SEAL BEACH STACK; COMPARISON OF MOMENTS BY CANTILEVER AND LE CONTE METHODS

minor earthquake of June 25, 1933, for example) considerably more acceleration in the upper parts of the buildings than on the ground. There may be cases where the acceleration of the ground will be more than that of the roof. The comparative accelerations depend upon the relation of the period of the earthquake to that of the building and many other somewhat indefinite conditions.

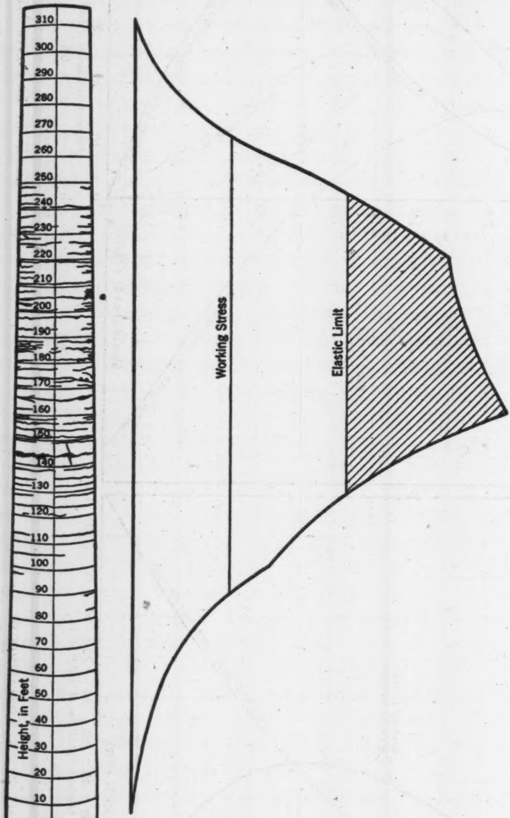


FIG. 8.—RELATION OF TENSILE STRESSES IN STEEL TO DAMAGE OF STACK

All evidence indicated that for the earthquake of March 10, 1933, the acceleration at the roof of the Seal Beach Station (the base of the tank) considerably exceeded the acceleration at the ground. The base of the tank was at about the same elevation as the base of the stack, and a substantial concrete roof slab which remained unbroken in this region of the building afforded assurance that the motion of these two elements of the building was not very different. It is entirely reasonable, from all of the evidence available, to assume that the base of the stack was subjected to an acceleration of at least 25% of that of gravity. How much more than this percentage is a question. It may have been substantially more.

The stack was designed to withstand a lateral force of 20% of that due to gravity, applied at the center of gravity of that part of the structure above any section. This is the ordinary "cantilever method"; and, although it is applicable if static loads or fundamental vibration forces are to be resisted, it is not in accordance with methods now commonly accepted for earthquake analysis of tall slender structures. Nor does it accord with observed destruction of such tall slender structures. Analysis in accordance with the "cantilever method" indicated that, for a force of 20% of gravity, the reinforcing steel would not be stressed beyond 16 000 lb per sq in. in tension, and the concrete, beyond 650 lb per sq in. in compression, which are conservative working stresses.

Professor Le Conte's method of analysis requires some assumptions which are not realized in their entirety, but which were particularly adaptable to this stack. These assumptions are that the structure shall be rigid, prismatic, and free of all support. Actually the stack was elastic, not truly prismatic, and had an elastic support, but these conditions approximated the assumptions required to be made.

A comparison of the moments derived by the cantilever method with those derived by following the method outlined by Professor Le Conte is shown in Fig. 6. The moments shown there are derived by applying a force of 25% of that of gravity. It will be noted that for that part of the stack near the base, the cantilever method gives moments far in excess of those of the Le Conte method, and it is also of interest to note that the lower part of the stack was little damaged. On the other hand, above one-third of the height the moments are found to be more than those for which the stack was designed—in places more than double. It is also interesting to note that at the very points where this analysis indicates largest moments and over-stressing of the steel, the stack suffered considerable damage.

Fig. 7 shows the stresses in the steel and concrete corresponding to the moments and shears obtained by the Le Conte method of analysis. For heights of from 150 to 225 ft, or from 47% to 71% of the total height, the tensile stress in the steel reinforcement was found to be more than 50 000 lb per sq in. and more than 60 000 lb per sq in. at a height of 160 ft, these stresses being beyond the elastic limit and approaching the ultimate strength of the material. It should be noted that unit stresses of steel are a function of the amount of steel actually supplied as well as the moment to be resisted—hence, the irregular curve. The computed bond stress ranged to as much as 266 lb per sq in., and it is not improbable that failure from this weakness resulted.

In Fig. 8 the very close agreement between computed stresses and the experienced cracking and failure of the stack is clearly shown.

Here is a practical example where the necessity of satisfying more than merely the laws of statics is clearly necessary to produce an earthquake-resistant structure. In applying mathematical analyses to this problem the writer was guided by detailed studies of damage by the earthquake, which is a test to be highly regarded. Every such mathematical analysis is only as valid as the assumptions on which it is based. Too often assumptions are made without a firm foundation, and thereafter the analyst is inclined to forget the doubts which troubled him when making the assumptions and to accept the results of the intricate mathematical analysis as the final solution of the problem. Although the stresses found in this investigation were, in all cases, considered to be approximate, the application of the theoretical analysis is substantiated by damage observed in careful and comprehensive inspections of the structure.

The theoretical analysis discussed herein did not enter into the problem of reconstruction because it was found necessary to remove a considerable part of the stack without replacing it. The problem was thus removed from the field of this analysis.

The experience noted was in accord with that of other stacks built on the tops of buildings and which have been subjected to earthquake shocks. Homer M. Hadley, Assoc. M. Am. Soc. C. E., in commenting upon damage resulting from the Japanese earthquake of September 1, 1923, states:[3]

> "Of all structures of reinforced concrete, chimneys gave the least satisfactory performance. When built up from the tops of buildings, they failed in all cases."

An attempt was made to apply the so-called "hammer theory" in an analysis of the stack, utilizing some of the work of Walter Ruppel, Assoc. M. Am. Soc. C. E.[8] By this theory moments somewhat larger than those heretofore enumerated in this discussion were found in the lower 90 ft of the stack, but these were all less than the moments for which this part of the stack was designed. Above this lower section, although following the same general shape, the moment curve showed somewhat smaller moments. This condition does not accord with observations of damage, although even in this case the vertical reinforcement was stressed beyond its elastic limit. It is the writer's opinion that the base of the particular stack under consideration did not have enough effect to make the "hammer theory" applicable.

It would be interesting to investigate the stack by means of the vibration theory developed by the authors, but this is not possible with available data. There is some difficulty in determining the period of ground motion from available seismograph records. These records verify the authors' statement (see Part I "Earthquakes and Their Effects") that:

> "The waves that damage structures are impressed waves, changing rapidly in period, amplitude and acceleration. Seismograms rarely show traces of harmonic waves, or those with regular characteristics. This is probably a very fortunate circumstance because it prevents, to a large extent, the resonance of vibrating structures."

In the present case, there is even less information to indicate the period of motion at the roof of the power-house, or the base of the stack.

A. A. EREMIN,[28] ASSOC. M. AM. SOC. C. E. (by letter).—The moment of inertia of mass has a negligible effect on the distribution of earthquake stresses in the vertical long (compared with transverse dimension) beam, as correctly stated in the paper. Likewise, one may easily find that the error from neglecting the influence of moment of inertia of mass in a vertical short beam is less than that from neglecting the influence of shear stresses. If the moment of inertia of mass is not considered a more direct development of the equations may be made.

Equation (7) may be derived from Equation (1), and from the well known elastic equation: which is,

$$E\,I\,\frac{\partial^4 y}{\partial x^4} = -\frac{\partial V}{\partial x} \qquad (54)$$

---

[28] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

Likewise, Equation (26) may be developed from Equation (1) and from the elastic equation:

$$E I \frac{\partial^4 y}{\partial x^4} = - \frac{\partial V}{\partial x} + P \frac{\partial^2 y}{\partial x^2} \quad \text{. . . . .} \quad (55)$$

Henry D. Dewell,[29] M. Am. Soc. C. E. (by letter).—This paper is an excellent and terse presentation of those fundamentals of the behavior of earthquakes, and of structures during earthquakes that are of particular importance to the structural engineer. In Section I, Mr. Galloway has quite properly emphasized the complexity of the earthquake and of the problem of giving a structure earthquake resistance.

The writer believes that the statement (see heading: "Earthquakes and Their Effects: Relation to Structures"), "In the case of high, narrow structures the vibration period is often close to that of some of the maximum waves and, unless the structure is well designed, serious damage will result if the shock is of long duration," should be amplified. The danger of resonance is not confined to high and narrow structures; neither is it at all certain that only the maximum earthquake waves are dangerous to structures. Herein it is understood that by the term "maximum earthquake waves" is meant the transverse waves that have the greater amplitudes. If the periods of the earthquake and the building are substantially in agreement, serious damage will undoubtedly occur, even if the shock is of short duration; consequently even a "well designed" structure will probably suffer seriously. After all, the term "well designed," when used in connection with ability to resist an earthquake, is rather relative. The writer understands that, as used, it means a construction with the structural units well arranged, designed for ample dead loads and live loads, with conservative unit stresses, with walls built integral with the structural frame or well tied and anchored thereto, and with all connections of sufficient capacity to develop the full strength of the members connected; and, in addition, it means that the structure has been designed to resist a definite lateral force from any direction.

Mr. Galloway has correctly stated (see heading "Earthquakes and Their Effects") that, "The foundation material is the important element of the movement [of the ground in an earthquake]. Solid rock moves the least; deep alluvial soils saturated with water * * * will have the greatest movement." Lest inference be drawn from this statement that damage is always greater when the structures are founded in alluvial soils than in harder or diluvial soils, it may be mentioned that in the 1923 Tokyo earthquake the damage to brick and concrete buildings was greater to those buildings situated on the up-town diluvial soil than to similar buildings situated in the down-town soft alluvial soil.[30] R. R. Martel, M. Am. Soc. C. E., states[31] that:

> "* * * somewhat similar results were obtained from a study of effects of the Long Beach earthquake of 1933. There the damage to brick build-

---

[29] Cons. Civ. Engr. (Dewell and Earl), San Francisco, Calif.

[30] "The Vibration of Structures and a Method of Measuring It," by Kyoji Suyehiro, *Inokuty Technical Papers*, 1928.

[31] "Effect on Foundation of Earthquake Motion," by R. R. Martel; presented at the meeting of the Soil Mechanics and Foundations Division, San Francisco, Calif., July 26, 1939 (not published).

> ings located on the softer, more recently deposited alluvium with ground water from 2 to 10 ft. from the surface, was less than to similar buildings on the older, more firmly consolidated marine terrace deposits with ground water not so close to the surface."

Referring to the relative accelerations in soft and hard grounds, he aptly states, "for one earthquake the flip of a coin will furnish as satisfactory a guide as any in forecasting the relative accelerations on different materials." The foregoing statements are illustrations of the often observed phenomenon, that almost every rule or law of earthquake action on structures has some exceptions which are apparently inexplicable.

Referring to the possibility of resonance, Professor Hoskins states (heading "Conclusions from General Theory of Forced and Free Oscillations") that to prevent this

> "* * * it would be necessary to have full knowledge * * * regarding free-oscillation frequencies of actual buildings * * *.
>
> "As regards the second question, it is obvious that the designer is powerless to avoid the dangerous condition unless he is able to estimate in advance the natural oscillation frequency of any projected building. Of great value for this purpose would be data concerning actual oscillation periods of many existing structures so selected as to serve as types."

In this connection Mr. Galloway has noted (heading: "Earthquakes and Their Effects") that "The Earthquake Committee filed with its report[3] the vibration periods of a number of buildings in San Francisco * * *. Other data of similar kind have been published." It may be well to amplify Mr. Galloway's statement by noting that from this rather small beginning of the Earthquake Committee resulted the extensive program of measurements of periods of buildings, bridge piers, water tanks, and dams, by the U. S. Coast and Geodetic Survey, the results of which have been published in the *Bulletins* of the Seismological Society of America and are summarized in the Survey's *Special Publication No. 201*, "Earthquake Investigations in California, 1934–1935." In all, the Survey made about 1 200 observations on 400 buildings, 150 observations on 41 tanks, 200 observations on special structures, and 500 ground observations. A portable ground and building shaker was developed at Stanford University, Stanford University, Calif., and tests were made with this machine on buildings, dams, and bridges, and on the foundation ground itself.

Because of the complexity, intricacy, and the length of mathematical work necessary for the solution of the stresses in any but the most elementary forms of structures when subjected to even simplified types of vibratory motion, the writer believes that the greatest gain in current knowledge of earthquake-resistant design will come from experimental studies of models of buildings and other engineering structures subjected to artificial vibration of predetermined pattern, amplified by theoretical study. The most promising procedure is the determination, if possible, of those static loads which may be used as the equivalent of the dynamic earthquake shears for buildings of different shapes, heights, and mass distribution. Valuable work in this field has already been done by Prof. Lydik S. Jacobsen in the Vibration Research Laboratory at Stanford University, Professor Martel at the California Institute of Technology, at Pasadena, Calif., and Prof. Arthur C. Ruge at the Massachusetts Institute of Technology, at Cambridge, Mass. Some of the results of their work have been published from time to time in the technical press. Of particular interest are the experiments[32] of Professor Jacobsen on a model of a 16-story and a 29-story building.

A special type of structure studied by the writer, acting as a member of Mr. Galloway's Earthquake Committee,[33] was the tall, free-standing chimney. Sufficient data were available, through the courtesy of the Japanese members of the Committee, on the behavior of a large number of these chimneys during the Tokyo earthquake of September 1923, to warrant a study. The writer endeavored to correlate the location of the fractures in these chimneys with the theoretical locations when computed according to the impact theory presented by Professor Le Conte[27] in 1926 using assumed ground accelerations thought to be reasonable. Conversely, he computed the ground accelerations theoretically necessary to cause the fractures that had occurred. In these computations the formulas developed by Professor Le Conte were modified with the assistance of Walter Ruppel, Assoc. M. Am. Soc. C. E., to adapt them to the case of a tall, free-standing chimney, of truncated form, considered to act integrally with its foundation, the latter usually being a massive block of concrete. These modified formulas represent the so-called "hammer theory."[3]

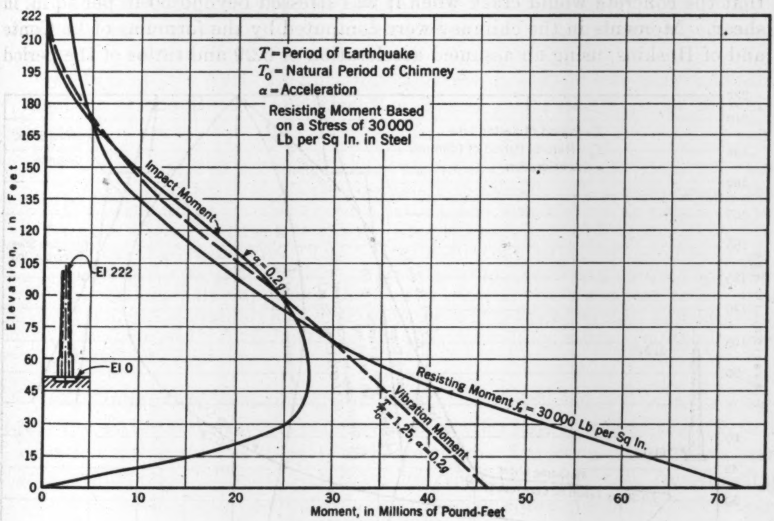


FIG. 9.—MOMENT CURVES

---

[32] "Experimentally Determined Dynamic Shears in a Sixteen-Story Model," by Lydik S. Jacobsen, and Robert S. Ayre, Jun. Am. Soc. C. E., *Bulletin*, Seismological Soc. of America, October, 1938.

[33] See *Civil Engineering*, March, 1932, p. 204.

The results of the investigation of the Japanese chimneys were neither consistent nor reasonable and the writer came to the conclusion that the action of these chimneys had not been similar to that of a free, prismatic rod subjected to a single force.

It may be questioned whether there is any earthquake phenomenon corresponding to a sudden blow or an impulsive force. It has been ascribed by some to the first longitudinal waves of the earthquake as distinguished from the large transverse waves which result in the violent swinging of structures. Seismographic records, however, have not generally indicated any such violent longitudinal waves. Possibly the sudden transition from the preliminary vibration of the earthquake wave to the first violent swing of its major portion might produce, in effect, a sudden blow.

A few years ago the writer's office had occasion to investigate the earthquake resistance of a free-standing, reinforced concrete chimney, 222 ft high. The theoretical resisting moment of the chimney was computed assuming that it would fail when the stress in the reinforcing steel reached 30 000 lb per sq in. In like manner, the shearing resistance of the chimney was computed assuming that the concrete would crack when it was stressed beyond 50 lb per sq in. in shear. Moments in the chimney were computed by the formulas of Le Conte and of Hoskins, using an assumed acceleration of 0.29 and ratios of the period of the earthquake to the natural period of the chimney of 1.25 and 1.50. The curves representing the results of these computations are shown in Figs. 9 and 10. Referring to Fig. 9, the moment curves, it will be seen that the zone in which the active moments exceed the resisting moment is essentially the same

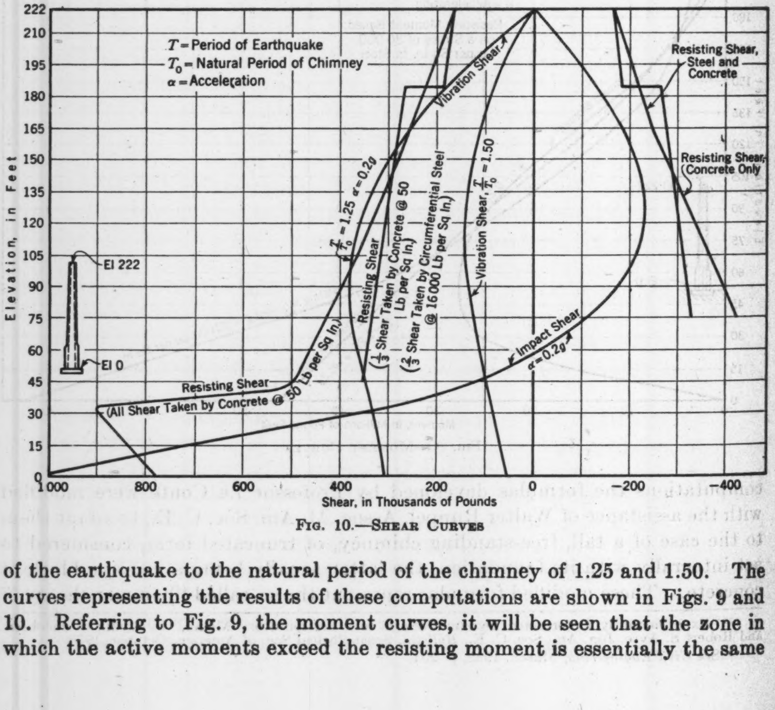


FIG. 10.—SHEAR CURVES

for both the impact moment and the vibration moment. It is true that the excess amount of earthquake moment over the resisting moment is greater for the impact moment curve than for the vibration moment curve, although a not greatly different assumption of earthquake period and acceleration would have produced a larger earthquake moment. For example, a ratio of periods $\frac{T}{T_0}$ of 1.00 instead of 1.25 would throw the vibration moment curve farther beyond the curve of resisting moment. It is also to be observed from Fig. 10, showing the shear curves, that the theoretical zone of over stress is essentially the same as that determined by the moment curves, although the over stress of shear is very much less than that of moment.

For many years engineers generally accepted the stated conclusion of the Japanese seismologist, the late F. Omori, that free-standing chimneys usually broke at two-thirds their height, and that their action during an earthquake somehow brought into play their centers of percussion, although the center of percussion of a chimney of the usual truncated form is nearer one-half its height than two-thirds. Indeed, the term "center of percussion" came to be associated with tall, free-standing chimneys and it was even thought by some that the value of acceleration should be doubled when used in the design of chimneys to provide for the effect of impulse.

In 1929 at the World Engineering Congress, I. Hiroi,[34] late Superintending Engineer of the Home Department of Tokyo, presented a paper in which he stated:

> "Of over 240 chimneys taller than 50 ft * * * more than 110 were totally wrecked, and over 40 seriously damaged. It is remarkable that the point at which the rupture took place was almost anywhere in the whole height, contrary to the old idea that it should be at two-thirds the height in tall chimneys. The determination of the dimensions of a chimney sufficient to resist the actions of great earthquakes is not simple; but when proportioned by supposing the acceleration at the lower end to be equal to that of the ground, (say 3 000 m.m. per second per second), and assuming it to increase by 50% nearer to the top, it would be strong enough in most cases."

Concluding this discussion of chimneys, the writer believes it more probable that the earthquake fracture of free-standing chimneys is due to the effects of vibration rather than of impact, and results from failure in moment or shear, or a combination of both. It must be recognized that tall, free-standing chimneys, both of brick and of reinforced concrete, are special types of structures that are inherently vulnerable to earthquakes.

JOHN D. GALLOWAY,[35] M. AM. SOC. C. E., AND LYDIK S. JACOBSEN,[36] ESQ. (by letter).—Some points made by the several writers who have discussed this paper seem to require further explanation in order to make the subjects more clear.

---

[34] "Prevention of Damage to Engineering Structures Caused by Great Earthquakes," by I. Hiroi, Paper No. 117, *Proceedings*, World Eng. Cong., Tokyo, 1920, Vol. 9, p. 25.

[35] Cons. Engr., San Francisco, Calif.

[36] Prof. of Mechanical Eng., Stanford Univ., Stanford University, Calif. [Prior to his death on September 8, 1937, Professor Hoskins had persuaded Professor Jacobsen to act for him in preparing the closing comments of this paper. Throughout, the word "writers" is interpreted to mean Messrs. Hoskins and Galloway, with the understanding that, in the closing discussion, Professor Jacobsen is interpreting his knowledge of Professor Hoskins' views.]

Mr. Hadley seems to doubt the value of the theoretical treatment of design of structures to resist earthquake waves, owing to the great complexity of the forces. The value of the theoretical treatment lies in the fact that, if the treatment is correct, there will result a consistent design of the structure. The assumptions are necessarily uncertain and should be simple. However, when once made, there is a possibility that the design will be so consistent that no part of the structure will be found wanting when subjected to forces of the shock. It is to be doubted whether Mr. Hadley would advocate designing a structure where "good guesses" formed the basis of design. Under such a procedure, some brace or other essential member might be left out, and the whole structure brought to failure thereby. Experience, such as that indicated by the writers, is a guide in making the assumptions upon which theory is based, such as the acceleration expressed as a percentage of gravity, which is of the greatest value. Once this is determined, theory may be brought in to produce a fairly consistent design. The writers must differ from Mr. Hadley in his statement that "* * * it is not apparent that the weight of the construction has any appreciable bearing on the matter at issue." The mass of the structure measures the forces set up therein by the earthquake. Reduction of mass, with the corresponding decrease in stresses, permits a design in which such stresses can be taken care of in a better manner. This remark applies particularly to the joints of an articulated structure. In regard to visible surface waves in earthquakes, it is suggested that Mr. Hadley examine reports of various earthquakes where such waves have been reported. Possibly the experience of one excited observer in Tokyo and a quotation from Shakespeare would not be considered sufficient to offset the uniform testimony of the existence of such waves.

The comments by Mr. Beanfield constitute an interesting summary of the problem from his point of view. The writers are in agreement with most of his remarks, but they cannot concur with his positive statement in Item (6) that "It is a well-established fact that the only proper method of judging earthquake intensity is by the kinetic energy method." It is true that much is in favor of this method, and that rationally it has a good foundation, but it has not yet become "a well established fact." It remains an hypothetical, although a plausible, method.

Suggestion (*a*) offered by Mr. Beanfield is a rule for computing the "maximum period, acceleration, and dynamical shears," presumably on the criterion of maximum fundamental mode deflection within the elastic limits of the materials. However, the statement that "the maximum statical deflection * * * [due to 0.1 g] is also approximately the maximum amplitude of its transient vibrations" seems to be without theoretical backing; at least the reference given to Timoshenko's book[11] does not warrant such sweeping conclusions. Again, under Suggestion (*c*) the statement that "The natural period of a structure ascertained by the foregoing method should not exceed 0.5 sec" is blunt and arbitrary.

The writers agree with Mr. Chew in advocating that empirical formulas for fundamental periods of buildings be used whenever sufficient observational data are available for existing buildings in the vicinity.

The influence of the non-rigid ground on the periods of a structure is largely a matter of guess work even if, as Mr. Creskoff suggests, dominant periods of the site had been obtained before the building was erected. For this reason, it is questionable what is meant by "reasonable accuracy" in the determination of the period of a projected structure. The usefulness of Table 2, given by Mr. Creskoff, would depend on the designer's ability to estimate the period ratios $\frac{T_b}{T_g}$, and even then the use of Mr. Creskoff's table is extremely arbitrary.

The discussion by Professor Ruge mentions the fact that a closer "tuning in" of earthquake frequency with structural frequency is necessary for the higher modes of vibration if equal dynamic magnification is to be expected. The writers agree with this fact and admit that the absolutely "steady state" of vibration, considered in the paper, is so idealized that application to actual conditions must be tempered with considerable insight and experience on the part of the designer. Professor Ruge also calls attention to the fact that a large building will interact with, and influence, the ground on which it stands to such an extent that the forces of the moving ground are not unlimited. This is also in agreement with the view of the writers; moreover, this interactive play of a large building with its foundation makes the period ratio $\frac{T_b}{T_g}$, used by Mr. Creskoff as well as by the writers, rather indeterminate.

Mr. Huber differs from the analysis of the writers which holds only for the "steady state" and in his investigations of the Seal Beach Power Station chimney he used the "impact theory" as developed by Professor Le Conte. It will be recalled that the "impact theory" is based upon the supposition of infinite rigidity of a perfectly free rod—that is, a rod without elastic distortions and consequently without vibrational properties. One must admit that actual structures fall short of approximating these assumptions. In the case of the Seal Beach chimney there was an agreement between the observed damage and the results of the theory. However, the location of the damage is equally well explained by the elasto-dynamic theory if it is supposed that the second mode vibration was excited instead of the fundamental mode. Calculations and observations make it seem highly probable that the earthquake of March 10, 1933, excited the second mode of the chimney much more violently than the fundamental.

The analysis of this particular chimney indicates the difficulties that confront an engineer when called upon to determine what caused the damage to the structure during the earthquake. Here was the case of a very high and heavy chimney held aloft upon a steel frame—a very unusual design, but one not entirely unknown. Mr. Huber adopted an analysis based upon a certain theory and the observed effects of the earthquake seemed to correspond thereto. Another theory probably would have reached substantially the same result and also might have justified the recommendation made by the engineer that the chimney be reduced in height. No mistake was made in so doing, but the writers are of the opinion that the weight of evidence favors the elastodynamic theory.

In this connection attention may be called to the fact that the speed of the earthquake waves is such that when any structure is encountered, the effect would be similar to a blow or sudden impact. The waves move at a speed of several miles per second and it is a well-known fact, as in bridges, the suddenness of the blow such as from a moving train is of material effect in increasing the stresses in the structure. In modern bridge design, impact stresses play a material part in the design. The entire discussion indicates the complexity of the subject and demonstrates how differences of opinion may arise.

The writers agree with Mr. Dewell's suggestion, namely, to amplify the statement under the heading "Earthquakes and Their Effects: Relation to Structures." The analysis given by Mr. Dewell, of a 220-ft chimney, is also very interesting. Again, the moments and shears due to the existence of higher mode vibrations will explain Mr. Dewell's quoted statement, due to I. Hiroi of Tokyo, that the damage to a chimney might be located "almost anywhere in the whole height."[34] The writers concur with Mr. Dewell in believing it more probable that earthquake fractures of free-standing chimneys are due to the effects of vibration rather than of impact.

The substance of the writers' paper lies in the statements of the complexity of the problem and the necessity for some simple assumptions that would make the problem one in which a consistent mathematical treatment was possible. There does not seem to be any material differences among those who have discussed the paper, as to the treatment adopted. It may be assumed that there is a general approval of the assumptions made by the writers. It is not contended that the treatment represents the last word on the subject, but rather the reverse is true. As more data are accumulated upon the subjects of earthquakes—especially as to the duration of the movement, the acceleration, the characteristics of the waves and (which is important) the similarity to assumptions that are within the range of mathematical treatment—a revised treatment may be possible that will accord with the additional knowledge acquired.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2063

TRAFFIC PROBLEMS IN METROPOLITAN AREAS

By Earl J. Reeder,[1] M. Am. Soc. C. E.

With Discussion by Messrs. George H. Herrold, Thomas Buckley, D. Grant Mickle, Robert A. Mitchell, Cleveland B. Coe, and Earl J. Reeder.

Synopsis

The purpose of street traffic is to convey persons from where they are to where they want to be, and goods from where they are to where they are wanted. Time is an important factor because delays defeat this purpose in part. Safety is an even more important factor because accidents defeat the purpose entirely.

In general, the traffic problems before all municipalities to-day relate to safeguarding the purpose of traffic by reducing the probability of congestion and accidents and by helping traffic to move expeditiously and safely. In metropolitan areas these problems are generally more serious than in smaller communities because the concentration of traffic is greater. Although these problems may be variously classified, they will be treated herein under six headings, some of which overlap to some degree. These headings are: (1) Helping traffic through danger spots; (2) protecting the person on foot; (3) stopping the increase in night accidents; (4) reducing traffic concentration; (5) keeping speeds safe for conditions; and (6) providing for loading and parking.

Helping Traffic Through Danger Spots

Contrary to the belief of many, a city street is not a continuous scene of human and mechanical disaster. In fact, traffic runs along quite well at most places, thanks to the judgment and skill of the majority of drivers and pedestrians. At certain points, however, accidents and congestion problems tend to recur because the difficulties are more serious than individuals can cope with successfully.

In general, a higher concentration of accidents occurs in the more congested districts, although the severity is greater in areas with less congestion. Little

---

Note.—This paper was presented at the meeting of the Highway Division, New York, N. Y., January 20, 1938, and published in December, 1938, *Proceedings*.

[1] Chf. Traffic Engr., National Safety Council, Inc., Chicago, Ill.

trouble is encountered where the movements are simple and clearly evident to drivers and pedestrians. Where the movements are complex and confusing, drivers and pedestrians do not fully understand the situations before them and do not have time to analyze these problems for themselves.

Proper accident records are imperative for determining where these "sore spots" are, and for deciding upon measures which will solve the problems as they exist. There is no better single indication of what is wrong with traffic than the accidents that occur in it. In fact, there is no indication that equals the accident records in importance. Traffic volumes are important in the solution of traffic problems, but they do not show where the conflicts are. Speed observations are helpful but, likewise, they do not show where traffic gets into difficulties. The accident records show where, when, and how these conflicts occur. Any community which does not have good accident records should provide for their proper accumulation as the first step in attacking the traffic problem.

One important problem of metropolitan areas (important there because traffic is denser than in smaller municipalities) is to simplify traffic at the "sore spots" and to provide assistance for vehicles and pedestrians to move safely. The purpose of the signs, signals, and markings which are installed for traffic control is to give the individual driver or pedestrian a little greater assurance that others are going to move in a certain way at a certain time so that he may act accordingly. For example, a "stop" sign at an intersection is intended to bring traffic from one direction to a full stop so that the drivers in that direction will work their way into the traffic on the cross street at moderate speeds after finding out that the way is clear.

One of the important principles in traffic control is to provide enough restriction or assistance, but no more than is necessary for safe and expeditious movement of traffic, because, after all, human beings do desire to be safe and will act in accordance when they know what to do. Acting on this principle, the time is passing when a "stop" sign, a "stop-and-go" signal, or some other favorite measure is expected to prevent all kinds of accidents and eliminate congestion from any cause. Studies of accidents before and after the application of these various measures have shown that unreasonable regulations—restrictions which are not adapted to the accident experience pattern, for example—do not accomplish results. There is a direct relation between stop-sign obedience and the reduction in speed that is necessary for safety, determined in terms of locations, of view obstructions on corners, street widths, and other definable factors. As a result of these studies, the stated speed "slow" sign which reads, "Slow to 11 Miles," "Slow to 14 Miles," or slow to any other speed which is computed as the critical speed, has been developed for use where full stops are not necessary for safety, but some restriction of speed is needed. This sign slows down the driver to the speed at which he can determine what the other drivers are doing and can act accordingly.

The development in "stop-and-go" signals and in their use is another good example of the simplification of traffic movement to avoid conflicts. Studies made in a number of cities in 1930–31 showed that such signals are not panaceas for all traffic ills but are invaluable for the types of problems that they can solve. Gradually, their adaptation to complicated places has been expanded. More phases of the cycle of colors have been added for complicated situations. Pedestrian "walk" lights have been developed. Special "turn" arrows have been added for unusual movements. Traffic and pedestrian actuation has been developed for fitting the operation more nearly to the traffic demand.

In metropolitan areas the demands upon traffic control reach their highest degree of concentration. It is there that greatest complications of traffic occur; and it is there also that the efficient use of the streets at the same time that the greatest of safety is provided, is imperative. For that reason extensive inter-connected signal systems have become necessary to enable vehicles to move with uniformity and continuity and, at the same time, to provide protection for pedestrians. Although such systems must often be complicated as to design, they must be simple so far as their appearance to the public is concerned because their purpose is to simplify traffic movement and make it easier for each driver and pedestrian to understand what he is supposed to do at each point.

No signal installation meets this condition more fully than the type in use on Michigan Avenue, in Chicago, Ill. The timing is extremely complicated because, at points, traffic is controlled independently for opposite sides of the street. However, every effort has been made to make clear, to drivers and pedestrians, how they are supposed to use the signals. Turning signals show when certain turns may be made to avoid conflict with other movements. Pedestrian signals show when pedestrians may cross. Traffic is kept in step by flashing the part of the green light that is not normally required for handling the volume of traffic at the less important intersections.

To simplify traffic by signs, signals, and markings, the engineer must do some more or less complicated planning in order to determine what should be done at certain locations and he must give the public the advantage of this solution by unmistakable means. Careful engineering planning is thus substituted for snap driver judgment, to a large extent. Traffic control systems of the future will probably be considerably more complicated in design than those of the present, although traffic movement will be greatly simplified thereby. As public understanding increases through traffic safety propaganda, public acceptance of such aids will be greater, and the results will be even more successful than those that are now being experienced from such measures.

PROTECTING THE PERSON ON FOOT

The pedestrian problem is one of increasing seriousness. The greater the speed and mobility of the vehicle, the more difficult it is for the pedestrian to foresee the hazards that surround him. The freedom of pedestrian action of former days must be restricted as the differential between vehicle speeds and pedestrian speeds increases. This is particularly true in congested areas where there are more pedestrians as well as more vehicles.

The pedestrian problem is particularly serious among persons who have never driven automobiles and who do not appreciate the limitations upon vehicle performance. Perhaps this fact is best shown by data from Connecticut for a recent period of five years during which 1 231 pedestrians were killed. Of these, 83% were of sufficient age to be eligible for drivers' licenses, but only 5% were licensed operators. This emphasizes more effectively than almost any other available data the importance of public education, as well as actual control measures, for preventing accidents.

In the past, too little attention has been devoted to the pedestrian, in devising control measures. Too little emphasis has been placed upon the need for educating the pedestrian concerning the hazards of traffic. Propaganda has been directed at the driver and control measures have accrued to his benefit; but the pedestrian has been considered a free agent who can protect himself if he is sufficiently concerned.

There are, and will continue to be, many "stop-and-go" signals without special lights for the pedestrian; and he must be brought to closer conformance with the usual signal indications. To encourage this co-operation the cycle of operation of signals should not be longer than is necessary to handle the traffic efficiently because pedestrians will not wait when there is no traffic moving.

The pedestrian education program must often be selective. In a recent traffic survey of Vancouver,[2] B. C., Canada, it was found that the residences of many of the pedestrians, as well as of many of the drivers, who were involved in accidents, were in areas of special racial, language, or social characteristics which would not commonly be reached by the ordinary means of public propaganda. Many of them lived in the Oriental sections, for example. The usual clubs, English language newspapers, and similar agencies did not reach those people, and their lack of appreciation of the traffic problem made many of them unsafe drivers and unsafe pedestrians. The public must be educated concerning traffic problems through agencies that they understand and in which they have confidence.

STOPPING THE INCREASE IN NIGHT ACCIDENTS

Closely related to this problem of pedestrian protection is that of the increasing seriousness of night accidents. Pedestrians, in particular, are susceptible to accidents at night because they carry no lights, and dark clothing often gives little or no reflection of vehicle lights. For their protection proper street lighting is a necessity. Experiences in Detroit, Mich., Evanston, Ill., and other localities have shown that good lighting does reduce certain kinds of night accidents, particularly those involving pedestrians.

How far the engineer can hope to go in providing lights for rural highways may be a problem involving some controversy, but concerning the proper lighting of city streets where vehicles and pedestrians must intermingle—and street intersections are frequent—there can be no question. It is there that the problems of night accidents are more highly concentrated. It is there that headlights often are of least value in providing adequate visibility.

Adequate lighting is more than a question merely of increasing the candle power of the luminaries. It involves proper spacing, mounting height, position with respect to the street, and types of luminaries. It is more than mounting a light on top of a post to show approximately where the street is. It is a matter of providing illumination to reveal objects that would otherwise be hidden from the view of drivers at night.

---

[2] "Improving Street Traffic, Vancouver, B. C.," by the National Safety Council, 1936–38.

Here, again, the best indication of what is wrong is the accident records. In the traffic survey of Honolulu, Hawaii, in 1936, a spot map of night accidents was prepared and it was found that the spots were particularly dense at certain points and along certain streets, and that they were very few at other places. In general, the density of the spots seemed to bear an inverse relation to the quantity of the lighting, much of which was very defective. Obviously, the proper approach was to improve the lighting first where the ratio of night accidents to day accidents was the greatest. That is a fair approach for any city to make to its street-lighting problem.

The night-accident problem is not to be attacked entirely from the standpoint of general lighting although that is an important part. Special hazards must be specially lighted. Many persons have been killed or injured in vehicle collisions within improperly lighted safety zones, for example. Center-posts in railway underpasses have taken their toll. Signs that were not visible at night have failed to prevent those accidents which they prevented in the day time. The traffic problems at night are entirely different from daylight problems because the very source of illumination is different. Special illumination is required at hazardous places to make sure that the hazards can be seen, and that the way to avoid them is apparent.

REDUCING TRAFFIC CONCENTRATION

One of the fallacies that must be exposed and overcome is the feeling that is often held by merchants and business people that traffic density is a good sign of business prosperity. It may, rather, be a sign of impending business decay and decentralization. An important problem is to differentiate between potential business traffic and through traffic that has no commercial significance. In some cities as much as 30% of the traffic in central business districts passes through without making any business stops. Means should be provided for traffic that is not potentially commercial to by-pass congested districts so that these areas may be reserved for easier access by drivers and pedestrians who go there expressly for business purposes.

This is to be done by providing convenient routes properly marked and safeguarded. It is not to be done by compulsion. Traffic that is not potential business can be directed easily over any convenient route that is properly marked and is a more comfortable route to travel than the streets through the central business district.

Too many cities are trying simultaneously to maintain convenient access to the business houses for business purposes and to handle considerable through traffic which can do nothing but make it more difficult for potential customers and clients to patronize the business places. Persons who are not potential buyers prefer to use routes that do not have the confusion, congestion, and delays of business streets. However, traffic tends to go toward business districts if alternate routes are not properly marked because it is in such districts that drivers can get their direction to their destination most conveniently; and the business districts are usually plainly seen from almost any part of the city. It is only through proper routing that the necessary segregation of business and non-business traffic can be made.

KEEPING SPEEDS SAFE FOR CONDITIONS

One of the big problems of municipalities is to adapt traffic speeds to conditions. The day of installing speed-limit signs at the boundaries of municipalities and leaving the individual driver to determine whether he is driving safely as long as he keeps within these speeds, is passing rapidly. The obligation of speed regulation for safety is not discharged that easily. Particularly on densely-traveled arterial routes, the problem of directing the traffic means moving it as rapidly as the physical conditions and safeguards will permit in safety. It is not always possible for the driver to judge accurately the maximum safe speed from place to place because hazards are sometimes hidden. Speed zoning, in which the maximum permissible speeds are determined by engineering investigation and are posted conspicuously, is an important new development. It is almost as necessary as route-marking for directing traffic conveniently and expeditiously, with safety.

One of the most important traffic problems is to create an adequate public understanding of the meaning of speed zoning, so that drivers will observe the restrictions that are imposed. They are likely to look upon any speed limit as a remnant of former days when the many who had no automobiles did not hesitate to impose extreme restrictions upon the few who did have them. Traffic administrators must show these drivers that the purpose of a speed-zoning sign is to give every driver the advantage of the detailed engineering investigation, which may have taken days or weeks in order to determine the critical speed so that he may not have to depend upon his momentary judgment in an emergency to determine how fast he can go.

In "stop-and-go" signal systems, proper inter-connection and timing are very effective in speed control. Progressive signal systems are planned to allow vehicles to pass through in platoons at pre-determined speeds. If they go at appreciably higher or lower speeds, they come to red lights and are stopped. Such designs are controlled by the volume of traffic that must be handled, the block lengths, and the cross-traffic at the different intersections. There probably is no more intriguing problem in traffic engineering than the design of a flexible progressive control system for a street with varying block lengths and dense traffic. Regardless of their actual complications, their indications to the public must be simple, direct, and unmistakable, as well as reasonable.

Speeds must be appropriate for street conditions. Where drivers are unable to recognize, quickly, the practical limitations on their speed, restrictions must be applied. As in traffic control for all other purposes, these restrictions must be reasonable and they must be adequately "sold" to the public. Most drivers will comply with such restrictions when they are understood, but some must be required to do so by enforcement.

PROVIDING FOR LOADING AND PARKING

Parking is a problem involving much controversy in congested business districts. It is probably one of the most misunderstood problems in the entire field of traffic administration. There are those who want unlimited parking and those who want to eliminate it altogether, depending upon their respective interests in the problem.

The primary purpose of parking, of course, is to provide access to business houses in business districts. It is a problem which usually develops simultaneously with other problems. The denser the traffic, the more acute is the parking demand likely to be and the more serious is the conflict.

The primary purpose of providing for access to the curb is to receive or discharge passengers or merchandise. When this has been provided to the full extent that is required, the remaining free curb space may be used for parking. However, the process has often been reversed—the curb has been parked full of cars in actual storage for various lengths of time, and so the driver, in order merely to load or unload, has been forced to take any place he could find to stand his vehicle for a few minutes. It may have been opposite a fire hydrant or a safety zone, or double-parked in the street, if no place at the curb was available.

It is inconceivable that cars will continue to be permitted to park almost bumper to bumper in busy city streets, making it difficult for vehicles to load or unload at the curb. More off-street parking and loading spaces must be provided. The time will soon come when restrictions will require all business buildings to have off-street provisions for parking and places within them, or on adjacent property, for the loading or unloading of merchandise, leaving the near-by streets free for stops to load or unload passengers.

Until that day cities must resort to time restrictions upon parking which will insure a sufficiently rapid turnover in the use of curb space to give every one a chance and to provide for convenient loading and unloading. A new development in the supervision of parking regulations is the parking meter which shows, mechanically, the length of time that has elapsed since a coin was inserted by the driver who parked there and reveals at once the expiration of that time and the act of over-parking. Meters do not solve the parking problem, but they do help the police materially in supervising and enforcing the restrictions that exist.

MASS TRANSPORTATION

In dealing with these problems of traffic one must not overlook mass transportation. In the street space used per person carried, the private passenger vehicle is much less efficient than the bus or street car. In the placing of street car and bus stops, in the establishment of routes, and even in the restriction of traffic, buses and street cars should be impeded the least that is consistent with safe and expeditious movement of all traffic.

Many of the measures that have been mentioned apply as well to mass transportation as to private passenger vehicles. Others must be adapted. The timing of "stop-and-go" signals applies to both. The establishment of bus stops requires the special clearing of parked vehicles so that the stops may be made at the curb in the most convenient place for all concerned. Like all other problems in traffic regulation and control, those pertaining to mass transportation must be solved on the basis of the conditions that exist. Accident records must be consulted, traffic volumes must be considered, and the demand for service must be taken into account and properly evaluated.

CONCLUSION

Traffic planning is distinctly an engineering function. The day is long past when trial-and-error methods are appropriate for solving the traffic problem. In former days traffic officials had to learn from experience by trying something and, if it did not work, trying something else. From that experience they have learned many things about the effectiveness of different measures for solving many kinds of problems.

Engineers still face the necessity for testing their planning efforts against actual results in accidents prevented, congestion reduced, and traffic expedited. Every measure that is applied should be preceded by the best analyses that can be made of the important facts about the problem to determine what is the most reasonable measure to be applied. It should be followed by equally thorough analyses of the results, to decide whether accidents have been prevented and, if so, what kinds. The engineer must determine whether traffic moves more freely and whether over-all time between destinations has been reduced. He must know whether he is handling more traffic or less, because it has been driven to some other streets. All these facts help him to guide his next steps in traffic planning, to apply appropriate corrective measures, and to anticipate the needs of traffic when he builds new streets or makes improvements.

Traffic planning provides the facilities for traffic movement. It provides the aids and safeguards that are needed. It attempts to determine what normal drivers and pedestrians do in traffic and to fit the facilities and regulations accordingly. Traffic planning must be reinforced by adequate education to inform the public about the use of the facilities that are provided. It must also be supported by adequate enforcement to require obedience from those who are wilful or negligent in their disobedience or misuse of the facilities. However, the responsibility of the engineer in traffic planning is to adapt his measures as nearly as he can to normal driver and pedestrian practices.

DISCUSSION

George H. Herrold,[3] M. Am. Soc. C. E. (by letter).—Observations and analysis of accident reports lead the writer to believe that the accident is most likely to occur where movements are simple and clearly evident to drivers and pedestrians, and that where the movements are complex and confusing, and drivers and pedestrians do not fully understand the situation, they proceed cautiously and safely. This would indicate that the cause of many accidents is in the driver. The careless driver does his damage where movements are simple and clearly defined. Careful drivers are becoming better drivers. They are constantly improving their technique, but the accident-prone driver presents a problem with which the careful driver cannot cope.

The writer has had an analysis made of several hundred accident records which were kept on forms presumably approved by the National Safety Council as filled out by police officers. They show where, when, and how the accident occurred; but it takes a keen analysis to determine the cause of an accident from these reports, because the "how" is not the "cause." In fact, it requires a trained investigator, of which there are very few. The report shows that "Vehicle No. 1, going north, collided with Vehicle No. 2, going west, at the intersection of 'X' Street and 'G' Avenue; (date), 10 A.M.; daylight; pavement dry; no defects; clear weather; drivers both males; not under influence of liquor; brakes O. K.; no car defects apparent: Driver No. 1 wore a brace on right foot; no signal or warning signs at the intersection; both drivers going straight through; and both had a license." The report shows where, when, and "how" the accident occurred (Driver No. 1 collided with Driver No. 2) but the cause of the accident can only be surmised. Probably both drivers were stubborn and would not "give an inch."

Accident reports are valuable; but the "vital point" (that is, human reactions at time of accident) is difficult to establish. An analysis of these reports by trained investigators is the real solution in helping traffic through danger spots. The railroads are correct in requiring a long course of training for a locomotive driver.

Records of "Stop Sign" violations and repeat violations by the same cars indicate that people refuse to stop where there is no vehicle in sight (see Table 1). The suggestion of another sign to take the place of the stop sign in many places, to read "slow to 10 miles" or slow to the computed critical speed is an excellent suggestion.

There is an unfortunate lag between increasing traffic congestion and the merchant's acknowledgment that he is losing business. Blight affects mercantile streets in many cities because of this lag. If the problem had been analyzed in the beginning a suitable by-pass could have been arranged and thus saved or delayed another blighted area. Proper routing through cities, and education as to the purpose of these routings, are among the essential necessities.

---

[3] Planning Engr., City Planning Board, St. Paul, Minn.

Every one knows that too high speeds are used by drivers in business districts. When a business man walks from his office to the bank he could save considerable time if he ran all the way; but he does not run. In fact, it has never occurred to him that he could save time by running. He walks in a dignified manner from one place to the other. The automobilist must also learn to move in a dignified manner when on congested business streets.

TABLE 1.—Observance of Stop Signs by 58 165 Drivers at Eleven Intersections in St. Paul, Minn.
(7:00 A.M. to 7:00 P.M.)

Description	(a) Came to Full Stop			(b) Shifted Gears but Did Not Stop			(c) Slowed but Did Not Stop			(d) Went Straight Through		
	1934	1935	1936	1934	1935	1936	1934	1935	1936	1934	1935	1936
Number of vehicles.......	24 827			25 764			5 249			2 325		
Percentage of vehicles.....	44.2	44.5	42.7	36.8	42.2	44.3	17.1	10.1	9	1.9	3.2	4

Providing for Loading and Parking.—The purpose of parking a car is to enable the driver to leave it, while attending to one errand or another. In most cities where an off-street parking space has been provided it is only about half in use; that is, twice the number of cars could probably be cared for in existing parking lots and garages when the people have been educated to make use of them. In St. Paul, Minn., the off-street storage capacity in the central business district is for 9 560 cars. The used capacity is 5 655 cars. In addition, parking space provided by utility companies and other private concerns, on their own grounds, for employees' cars, totals 1 630. Much the same situation occurs elsewhere. City officials have not yet decided whether it is a part of their business to provide parking spaces off the street, although a number are doing so on city-owned or tax-delinquent lands.

In St. Paul, with 90-min parking in the central business district, surveys developed 82% legal parking and 18% illegal parking, with 62% of all legal parking remaining at the curb less than 30 min. Based on these facts the parking ordinance was changed to provide 30-min and 60-min parking as more nearly in line with actual needs. In the 1937 survey it was found that 56% of the cars are legally parked and 44% illegally parked. The percentage of illegal parking increased, of course, with shortening of parking time and with the increased use of cars. The report of the National Association of Building Owners and Managers on parking meters, dated October, 1938, leaves no doubt that parking meters will cure this situation.

It is possible that the eventual solution of the parking and loading problem lies in condemning older buildings and land in the retail district, clearing the ground, and using it for parking of cars and "off-street loading," assessing the cost of such operation to the surrounding property owners. This would provide a free parking space with a limit of 1 or 2 hr to insure a logical turnover and a cash charge for any parking over-time. Such an area for off-street parking and loading should be planned to fit into the mosaic of the retail and office building district.

Mass Transportation.—An indication of the traffic picture of a city is the morning flow taking people to work. In 1924, 541 street cars brought into the St. Paul business district, in the morning, 31 000 passengers, whereas 9 250 automobiles brought in 14 677 passengers. In 1938, 419 street cars brought in 17 000 passengers; 71 buses, 763 passengers; and 18 100 autos and other vehicles, 29 100 passengers. In addition there were 7 200 pedestrians walking to work in 1938, a situation which was not considered important enough to check in 1924. Street space in St. Paul has been increased quite materially since 1924 but not in any ratio comparable to this increase in volume of traffic or use of street space—and this is true of many cities. Only by providing mass transportation that is more comfortable and attractive than a private automobile (such as the electric trolley bus) and limiting the use of private cars by those who do not need them downtown during the day (by enforcing drastic "no parking" regulations or parking meters, as is now proving successful in fifty cities) can this situation be altered for the better.

THOMAS BUCKLEY,[4] M. AM. SOC. C. E. (by letter).—A concise and interesting arrangement of traffic problems is presented in this paper. Sub-headings are neatly phrased and carefully chosen to include essential phases. They are in a form worthy of attention because they are adaptable to use outside the boundaries of purely technical review; and they lose none of their effectiveness if restricted solely to observation and discussion by engineers. Descriptive captions such as those in the paper prompt a reminder that the most urgent problem of the traffic administrator is to catch and hold public attention and to engender a respect for the profound importance of the personal factors in all traffic problems. After all, traffic control is outstanding among all technical fields as a procedure wherein the primary fundamental problem is to find effective ways and means of educating the public at large in human (and humane) behavior; or, in other words, to a full realization, and to the point of permanent, co-operative acceptance of their individual personal responsibilities as joint users and sharers of streets and highways. The extent of this problem is discernible in the following quotation:[5]

> "By what method can we make people realize that it isn't real fun to take chances with a motor car? What persuasions will be strong enough to convince the motorist that he should lose a minute and save a life? What means can be used to convince everybody that they should not drive after drinking?
>
> "How can taxpayers be persuaded that a drivers' training program in the high school is at least as important as a luxurious swimming pool? What kind of propaganda will it take to make drivers as courteous at the wheel as they are in the drawing room?"

Through the centuries the "human" characteristics of carelessness, thoughtlessness, heedlessness, selfishness, stubbornness, rudeness, and ruthlessness have become deeply imbedded in Man's nature. Unfortunately for his fellowmen, and particularly for traffic engineers, he elects to use streets and highways as places in which to display the mental perversities mentioned. Solution of the problems of self-control will never keep apace with the progress made in mechanical and design controls.

---

[4] Asst. Chf. Engr. and Surv., Bureau of Eng., Surveys and Zoning, Dept. of Public Works, Philadelphia, Pa.

[5] See editorial: "Stop and Go," *Public Safety*, December, 1938.

The stressing of the time (speed) and safety factors by the author in his Synopsis suggests a second comment. This is that three utilitarian tests of a good highway should be emphasized more frequently as such. It is also a reminder that the third one of these tests seldom receives consideration commensurate with its importance in traffic problems. The three tests in mind are those of speed (time), safety, and comfort. The first two of these tests are under discussion publicly more or less continuously but the third (comfort) is seldom mentioned. This circumstance is probably due to the general acceptance of "comfort" as being covered by the "safety."

A well-designed highway is one that permits reasonable speed with minimum hazards and, in addition, assures maximum comfort to all its pedestrians and vehicular users—comfort not only in the sense of that "bodily ease," which is afforded by efficient design and construction, but including, as well, that comfort resulting from "mental ease," which can only be realized when travel upon public highways is guided and protected by effective controls. Mental dread or fear of highway hazards is particularly strong among young and aged pedestrians. It is not entirely absent in the minds of many automobile operators. Its presence under test may induce the fatal mental panic which so often contributes to the production of highway accidents. Tests for highway comfort must also take into consideration those lesser forms of highway friction, such as the crowdings, jostling, blocking, bumping, etc., constantly occurring during business hours on the sidewalks and in the roadways of the principal streets in main, wholesale, and retail centers of urban areas. Although the great majority of such happenings do not result in serious property or personal harm, they add materially to the mental stress and strain of daily life in large cities. These conditions are plainly evident at the crossings of countless highway intersections. The problem of comfort to pedestrian and rider at the junctions of main streets will never be completely solved; but, with such lessons in mind, new streets and highways should be designed and controlled, as far as it is economically possible, so that pedestrian and vehicular users may derive and share maximum enjoyment of them. To accomplish this end, highway discomfort must be reduced as much as controlling circumstances permit. A gospel of "friendly roads" with a mutual sharing of their privileges must be taught.

The author's consideration of the traffic situation in metropolitan areas is naturally concentrated on problems common to the urban or improved sections of such areas. Difficulties encountered in urban traffic problems are generally increased and their solutions made impossible of completion because of original planning defects or deficiencies. Owing to the nature of urban areas, controls such as signals, signs, markings, etc., with occasional islands, zones, channelizations, etc., constitute the common ultimate methods of solving the major group of traffic problems. These measures cannot entirely overcome the handicaps and hazards to traffic resulting from structural faults such as poor alignment, bad grades, lack of visibility, insufficient lighting, inadequate cross-section space, weak intersection layouts, etc. Few towns and cities are free of problems arising from such conditions. The lessons derived from traffic problems arising in urban sections of metropolitan areas should be applied advantageously not only in the planning and improving of new streets, but in replanning, wherever possible, all existing planned streets that are not developed as yet to a point which precludes corrections.

Metropolitan areas generally include, in addition to their built-up and partly built-upon sections, areas that are wholly unimproved. In some jurisdictions, these unimproved districts may actually include lands that are in two important stages of evolution: One class comprises planned sections (that is, areas laid out into streets and highways upon an official map or city plan, but not physically improved) and the second class embraces areas that are at present unplanned but are "ripening" rapidly for planning and development. What future traffic problems are hidden in such areas? Can proper precautions be taken so that these new sections will respond to a higher degree of traffic control with fewer problems than old urban areas? The municipal engineer and planner are deeply interested in these questions and from their viewpoint a discussion of traffic problems in metropolitan areas must also include the potential problems that may exist within already planned but unimproved sections, or which may happen by reason of incomplete future planning.

In his "Conclusion" the author rightly declares that "traffic planning is distinctly an engineering function." As such, it must acquire a vision which reaches out beyond the problems and necessities of traffic control installations on existing highways. It must begin with, and have a definite place in, the original planning. It must undertake to assure that areas already planned for unimproved districts are redesigned, if necessary, to meet reasonable traffic requirements and that they are adequately adapted for rational controls. The planner's objective should be to so plan or replan, and to so guide trends in development, that future traffic problems in the areas affected will approach an irreducible minimum in so far as damaging highway friction is concerned.

It is the responsibility of the engineer to develop and apply sound practice in the design and construction of streets and highways. To accomplish this objective, he has been meeting the requirements of speed, safety, and comfort for years by endeavoring to provide highways with dry foundations, solid and durable paving structures, safe grades, visible alignment, and even surface. To these qualities, he must now add that of traffic control, and co-operate with the planner to accomplish this end.

Modern highway use has produced new standards in the factors of volumes, intensities, loads, speeds, etc., which in turn have established new requirements with respect to the structural elements of highways—alignment, grades, cross-section design (including channelization), paving structures, curbing, drainage, fencing, railings, etc. All of these elements are directly connected with, and are an intimate part of, the problems of traffic control. Not only is sound engineering technique in design essential in minimizing traffic problems, but first-class construction and efficient maintenance are equally important. Worn-out or otherwise defective paving on certain streets and highways can disarrange the best of traffic controls completely and thus create a number of new local traffic problems. The design, construction, and maintenance of highways can be carried to any degree of efficiency required. Safeguards for traffic can be provided that will function to a high state of effectiveness. The difficulty encountered in building and equipping such highways is a very material one. Generally there are economic limitations to the extent to which such improvements are justified; and, in concluding this discussion, the writer raises another basic question in traffic control: "How can finances be obtained for furnishing, satisfactorily, the essentials—the means—necessary to complete the solutions of the many traffic problems after the ways of solving them have been fully determined? The problem of financing the solution of traffic problems deserves considerably more attention.

D. Grant Mickle,[6] Assoc. M. Am. Soc. C. E. (by letter).—In his excellent paper Mr. Reeder shows that city streets are not filled with human and mechanical disaster as is often thought. With the opportunities for conflict within the various traffic streams and eddies existing in metropolitan areas, it is indeed surprising that there are not more deaths and injuries than there are.

It is the problem of traffic engineers to schedule the movements of vehicles, persons, and merchandise, within the traffic stream, so that they are coordinated and made orderly and safe. Among the factors to be considered in such a program are the arrangement of the circulatory system, land usage, location and density of population, location of employment in offices, retail establishments and industries, and the location and usage of recreational and educational centers. The relation of these factors to one another, and their influence upon congestion and accidents, make up the traffic problem.

Because accidents constitute the spectacular phase of the traffic engineer's work, there has been, and still is, an insistent public demand that direct and positive action be taken to stop them. Mr. Reeder has presented many of the methods used in this direct approach. Automatic traffic-control signals, stop signs, speed control zones, and stated speed "slow" signs are among the control methods described. In connection with the latter device, it should be emphasized that it is difficult to enforce a sign reading "Slow to 11 Miles," inasmuch as the officer must pace the offender for a reasonable distance in order to obtain a conviction for an offense. Furthermore, care must be taken that a local ordinance provides authority for such a control device.

It is indeed true that the pedestrian problem is one of increasing seriousness. Many city officials, recognizing this problem, are providing legislation which places responsibility upon the pedestrian, as well as upon the motorist, in his activities in the traffic stream. No longer does the pedestrian have uncontested right to cross the street whenever and wherever he pleases. His actions and behavior are prescribed by law, and the designers of traffic-control signals are beginning to include indications in the signal to allot certain parts of the cycle to the pedestrian, restricting his movement during the remainder of the cycle.

---

[6] Traffic and Safety Engr., State Highway Dept., East Lansing, Mich.

One of the most prevalent locations for pedestrian fatalities is at the very outskirts of cities along the major highways. Sidewalks are often lacking at such places, highway lighting may be poor or non-existent, and the speed of vehicles is generally high. These factors combine to make the zone at the fringe of cities produce fatalities at a high rate.

In Michigan it was found in 1938 that, of all fatal pedestrian accidents on rural trunk-line highways, 34% occurred within the zone extending 1 mile out from the city limits. The rate per mile was found to be more than twice that of the second mile zone, decreasing gradually from that point as the distance from the city increased. This study focuses attention upon a pedestrian problem in which both the rural and urban officials must co-operate.

The genuine causes of accidents and congestion in metropolitan areas are usually of such a fundamental nature that their correction is costly. For example, in Detroit, Mich.,[7] it was found that the city's pattern of streets radiating from the central business district emphasized the conditions of congestion and hazard caused by the present-day use of the automobile in cities planned for other kinds of transportation. Studies of traffic volumes, and of the origin and destination of traffic, disclosed that these down-town streets carried not only vehicles bound for destinations within the district, but also thousands of other cars which passed through them to distant destinations because adequate cross-town arteries were lacking. Investigation of parking conditions, requirements, and habits indicated that desirably located off-street facilities were inadequate and that all of the legal, and much of the illegal, curb space was fully occupied. Not only were terminal facilities insufficient but the radial arterial streets were over-crowded for several miles away from the central business district.

The results of these unfavorable conditions upon business were revealed when comparisons of traffic counts over a period of twelve years showed that the number of people entering the district had actually declined although passenger car volumes had increased. Recommendations for dealing with these conditions included the construction of new cross-town arteries and the substitution of ample storage space in the areas occupied by deteriorated property surrounding the business district. In making these recommendations, the engineers went well beyond the act of merely counting and analyzing traffic movements. They took into consideration both the general physical plan and organization of the city, and the social and economic implications of real estate conditions in and about the central business district.

Too often officials and the public fail to realize the magnitude of the traffic problem or the losses attendant on congestion and accidents. The work of the traffic engineer, therefore, is often handicapped through lack of sufficient funds or proper interest in his plans. For example, a traffic report in Toledo, Ohio,[8] revealed that traffic accident losses amounted to $1 022 325 annually, as compared with annual fire losses of less than one-third of that sum, or $311 802; yet more than $2 was allotted to the fire department for each dollar of actual fire loss, whereas the traffic control and betterment services received only 34 cents for each dollar of traffic accident loss. Similarly in Louisville, Ky., whereas an average of 89 cents was spent to prevent one dollar's worth of fire loss, only 30 cents was spent to prevent an equal amount of traffic loss.

---

[7] "Street Traffic—Detroit," Michigan State Highway Dept., 1936–37.

[8] "Traffic Planning Report—Toledo, Ohio," Jensen, Bowen & Farrell, Engrs., 1937.

Parking or storage facilities in the business districts of cities are far from adequate; and providing these facilities is a problem of major importance. It is thought by some authorities that the provision of terminal facilities in the business district is a public duty, and if this is true, means for public ownership of parking facilities must be devised. Another view is that the primary use of the public streets is for moving traffic, and the parking of vehicles is a secondary consideration which should not interfere, to any appreciable degree, with the free movement of vehicular or pedestrian traffic.

Confusion is apparent in the attitude of the business men, who tend to demand strict regulation of all curb space, except that adjoining their own property. This conflict cannot be reconciled entirely; but it can be equalized by giving consideration to all the related factors, among which are traffic volume, vehicular speed, available space, the demand for spaces, and parking-time requirements.

The main function of time-limit parking regulations is to provide reasonable turnover and equable distribution of available curb space. Some cities are turning to parking meters as a means of enforcing time-limit regulations.

Business men realize that parking facilities are essential to business and that costly decentralization will result unless the problem is solved. Officials are attempting various means of improving off-street parking facilities or means of improving mass transportation facilities in an effort to curb decentralization tendencies.

A traffic survey in Youngstown, Ohio,[9] compared the number of persons accumulated in the business district by the use of mass transportation facilities with the number that used passenger automobiles. The average number of persons transported per mass carrier was 35.72, whereas the average per passenger automobile was 1.48. There can be no question of the relative efficiency of these two types of carriers in regard to economy in the use of street space. If the number of persons using mass transportation facilities had to resort to passenger automobiles as a means of transportation, and if the average of 1.48 persons per passenger automobile remained the same, 29 730 additional passenger vehicles would be required. The streets could not possibly accommodate this additional load. Thus, it will be seen that mass transportation is a vital element in the traffic problem of Youngstown, as it is in any large city; and that, in any plan of traffic control and regulation, its requirements must receive due consideration, as must the effect of its operation on the free movement of traffic.

Mr. Reeder has shown the need for a three-fold attack upon the traffic problem involving the use of engineering, enforcement, and education. A balanced traffic improvement program must include all of these methods of approach.

---

[9] "Traffic Planning Report—Youngstown, Ohio," Jensen, Bowen & Farrell, Engrs., 1937–38.

In conclusion, the first consideration that any city should give to its traffic problem is the efficient use of its present facilities. Minor physical improvements often may be made to the methods of control or to the street system, which result in making the present facilities more useful and more efficient. In planning for future facilities, the basic facts concerning traffic movement should be studied carefully and taken into consideration in developing a traffic-planning program. If traffic control and enforcement are to be successful, all users of street space must be made aware of their responsibilities, one to another, and educated to the safe and efficient use of these facilities.

ROBERT A. MITCHELL,[10] ESQ. (by letter).—In any discussion of this paper one must keep in mind the enormous scope of such a subject and realize that it is impossible to cover even the most important problems fully in such a small space. This is understandable when one realizes that entire books have been written on the subject.

Considering the limitations, Mr. Reeder has presented a sufficiently clear picture so that those who peruse it can obtain an adequate idea of some metropolitan traffic problems, their bearing on social and economic life, and how such conditions may be alleviated. On the other hand, the writer questions the accuracy of certain statements; for instance (see "Helping Traffic Through Danger Spots"), Mr. Reeder infers there is a widespread belief that a city street is "a continuous scene of human and mechanical disaster." On the contrary, the safety problem is so difficult to correct because very few people have seen, or been involved in, a traffic accident, and thus do not take sufficient interest in the subject. It is also unfortunate that more factual data and examples were not used to substantiate certain assertions, because civil engineers have analytical minds and are impressed by facts and figures.

Under the heading "Helping Traffic Through Danger Spots" the author presents, admirably, the advantages of careful engineering planning in making streets safer by designing, locating, and operating traffic signals, signs, and markings so as to take the "guess-work" out of driving. A person could probably learn to run a locomotive as easily as he could a motor vehicle, but no one would think of letting him run that locomotive even a mile along the track unless he knew all the rules of the road and had carefully planned signals to guide him.

The importance that Mr. Reeder attaches to accident records as basic functional data for correcting "bad spots" cannot be stressed too greatly. However, some who have had practical experience in handling traffic problems in a large city find that there are many "mix-ups" that must be solved by past experience and personal judgment. These are mostly problems of congestion and delay rather than those of hazard.

Careful engineering planning also plays an important part in decreasing delays which, Mr. Reeder states (see "Synopsis"), defeat, in part, the purpose of street traffic. From a practical standpoint, citizens seem just as much interested in having traffic officials "clean up" points of serious congestion as accident "sore spots." The author's discussion of by-passing through traffic, parking, and loading and unloading conditions yields an insight into some of the factors causing congestion and indicates practical methods of correcting the situation; but, there are other phases worthy of consideration.

---

[10] Traffic Engr., Dept. of Public Safety, Div. of Traffic Eng., Philadelphia, Pa.

A well-designed traffic signal system, for instance, accelerates traffic, reduces delays, and eliminates congestion, in addition to simplifying the traffic movement, and reducing accidents. When Broad Street in Philadelphia, Pa., was controlled by police at each intersection, the usual sight was a solid mass of vehicles for ten or fifteen continuous blocks—a condition breeding congestion and accidents. Under a proper signal system, over-all speed was increased, the volume of traffic increased, and accidents decreased. Vehicles now travel in small successive groups, often called "waves" or "platoons." In addition, the timing can be altered each day so as to favor the heavy inbound traffic in the morning and the peak outbound traffic in the afternoon. On one of the other down-town streets, the traction company's daily records show that the delay time of their trolley cars has been reduced 18% by the operation of a progressive signal system.

A little white or yellow paint applied in the right places can also help in reducing delays. Unless three lanes of traffic are definitely marked off on each half of Broad Street, motorists will only drive in two lanes, thus reducing the capacity one-third. There are also many examples of increasing the capacity of intersections to handle traffic by the installation of proper laning. Sometimes certain types of cement, rubber, or metal markers are used instead of paint.

Congestion on streets carrying a large volume of traffic can often be alleviated by creating one-way streets or establishing other convenient adjacent routes. Two parallel and adjacent streets in Philadelphia, both with 44-ft cartways, were two-way for many years and carried considerable arterial traffic. In spite of some public criticism, these streets were made one-way in opposite directions and to-day (1939) each of them is carrying about 80% more traffic with less congestion and delay. Traffic on another important artery was relieved by smooth-paving an adjacent granite block street with asphalt, and building, at small cost, a few connecting streets.

Safety zones at street-car stops allow motorists to move by the trolley while it is stopped for passengers; channelization islands at certain intersections reduce conflicts and speed up traffic movement; at highly congested points where land values are not too high, traffic circles and grade separators are sometimes used to improve conditions and there are several examples of parkways and elevated highways in New York, N. Y., Chicago, Ill., Newark, N. J., Philadelphia, and other cities, where millions of dollars have been spent to relieve traffic congestion.

Mr. Reeder made no truer statement than that: "The pedestrian problem is one of increasing seriousness" (see "Protecting the Person on Foot"). While the number of car occupants killed in traffic accidents in large cities has been steadily decreasing, the proportion of pedestrian fatalities has increased until to-day (1939) such deaths represent approximately 75% of the total. In the space allotted, the author has covered the subject admirably well particularly the need for selective education.

Paper after paper has been written on the value of adequate street lighting in improving metropolitan traffic problems. Fact after fact has been presented by experts to show the proportionate seriousness of night accidents over day accidents and to prove that the improvement of illumination in certain cities and on certain highways greatly reduced night accidents. Nevertheless, most public officials are doing almost nothing to improve street lighting in metropolitan areas and the writer does not believe they will until money becomes more plentiful, and there is a strong public demand.

Mr. Reeder "puts his finger" on what is probably the greatest difficulty facing public officials in their attempt to correct traffic problems in the business districts of municipalities; that is, "One of the fallacies that must be exposed and overcome is the feeling that is often held by merchants and business people that traffic density is a good sign of business prosperity" (see "Reducing Traffic Concentration"). Counts made in one large city in 1938 show that on the average, a street car carries 30 people, a bus 15, and a private car less than 2 into the central business district; and yet, merchants continually bring pressure to bear on the law-making body to defeat regulations designed to make it easier for mass transportation riders to reach the shopping district.

Motor vehicles of to-day are smooth running machines which move along the road at reasonably high speeds without any apparent effort. This has made speed control a real problem since most motorists fail to realize how fast they are traveling. Mr. Reeder has explained, adequately, some of the methods for coping with the situation, such as speed zoning and signal systems; but neither of the solutions will be effective unless, as he says, they are reasonable, adequately "sold" to the public, and based on engineering investigation. One of the main reasons that speed zoning has not been effective is that unreasonable speeds were established and then the motorists were given to understand they could travel about 10 miles faster before they would be arrested. During the last year of the 40-mile speed limit in Pennsylvania, state authorities even published the fact that motorists would not be apprehended unless they drove faster than 50 miles per hr. In another case, police officials informed the writer that they never apprehended a driver unless he drove faster than 30 miles an hr through a speed zone posted for speeds not greater than 20 miles per hr. How can one expect motorists to obey regulations that the officials themselves do not believe proper? Establish a reasonable speed, enforce it to the letter, and then motorists will respect it, and other laws. Most of the credit for Pennsylvania's reduction of traffic fatalities in 1938 is given to the absolute enforcement of the speed limit of 50 miles per hr adopted in the latter part of 1937.

Herein, the writer has introduced data to show the importance of mass transportation carriers, which substantiate Mr. Reeder's assertions as to "the street space used per person carried" (see "Mass Transportation") by various types of vehicles. With the increased use of private vehicles, the problem of mass transportation is increasing, both financially and from a standpoint of congestion. There has been considerable controversy as to whether or not buses will supplant street cars entirely because they are more mobile and seem to interfere less with the general traffic movement. One thing is certain: Quick pick-up, more speed, and faster stopping ability must be the qualifications of modern transportation vehicles.

Although Mr. Reeder hasn't covered all the problems faced by municipal traffic authorities, the toughest of which is "public relations," he has given enough information to impress the reader with the immensity of the traffic problem and to show that it will not be solved by trial-and-error methods but by adequate traffic planning based on engineering principles.

Cleveland B. Coe,[11] Assoc. M. Am. Soc. C. E. (by letter).—The point most forcibly "brought home" by this paper is that traffic control is now a specialized branch of engineering, calling for the usual engineering procedure of assembling all available facts, analyzing them, and creating a rational design according to tested rules. The rules will continue to be more or less empirical, as the human element plays a greater part than in the design of structures. Efforts in the past to force traffic, both wheeled and foot, into certain channels have failed to a greater or less degree, and the solution lies in the education of the individual, on the one hand, and in making traffic rules and channels easy and natural to follow, on the other.

It required centuries to educate the public to the employment of engineers in structural design; and in the fields of highway location and construction, and land surveying, as well as others, the process of education is still going on, with results showing in proportion to the density of population of any section of the country. It is a clear function of technical societies to educate the public, as no individual can do so. Neither can headquarters of a national group reach the civic consciousness of the voters of the many cities, with the possible exception of a few of the larger ones, where in all probability experts are already in charge of the problem, driven thereto by the exceeding seriousness of the situation.

This is a matter for local sections and local engineering clubs. Each one should have a committee working through local publicity channels, with subject matter furnished by their own investigations and by central headquarters. They should not confuse their objective with an attempted solution of local traffic problems, but should educate the local authorities to the point of employing a traffic engineer, first laying the groundwork by educating the voting public and the press to approval of this course.

The importance of this groundwork cannot be over-estimated. Time and again in all fields, notably in City Management, the employment of experts has been tried and abandoned for just such lack of support, the voters objecting most strenuously to bringing in an outside expert, no matter whether one was available in the community or not. It is just such lack of understanding and intolerance of the democratic process that has caused most engineers to be exceedingly ineffective politicians.

An interesting fact revealed by the data in the paper is the apparent numerical relation between licensed drivers and pedestrian fatalities. The decrease in fatal accidents in 1938 is encouraging. It may be attributed to the widespread campaign of education. No doubt part of it is due to the education of the younger generation in the proper handling of a high-powered dangerous weapon and their general knowledge, when on foot, of the limitations in driver judgment and mechanical equipment. Many writers have commented on the psychological change that occurs in an otherwise courteous and well-behaved individual when he or she slides under the wheel of an automobile, and it is to be hoped that the coming generation will be so gradually accustomed, from childhood, to the control of so much power that no sudden change of habit will occur when the legal driving age is reached.

---

[11] Associate Highway Engr., TVA, Chattanooga, Tenn.

Education of the public as to the by-passing of through traffic and limiting storage of cars on pavement provided primarily for moving traffic is a more difficult problem and one requiring much time to effect. Cities in the more densely populated Eastern States have apparently learned their lesson through bitter experience, and highways are now being routed around centers of population. Cities in the Southern States are still in the stage where the merchants complain loudly against any proposal to take through tourist traffic off of Main Street. This attitude seems to be not so uncompromising of late, but it is still thought necessary by many that parking be unrestricted in front of their doors. Facts on parking are comparatively easy to ascertain, and only facts will convince the merchant that little or no trade comes to his store from a car occupying valuable space in the street.

It should be noted by those who argue that expensive pavement is laid for moving traffic and not for storage that, regardless of restrictions, the public will "pull over" to the curb and stop, if only to discharge a passenger, and that only one or two, doing this in a block, occupy a lane against moving traffic almost as effectively as a line of parked cars. Since this lane is not available anyway, its use should be divided among the drivers as far as practicable and this means limited parking. The use of parking meters to assist the regulating officers is still on trial, some cities and merchants approving them after trial and others having discontinued their use. Here again personal bias and prejudice play a part far too great, whereas the facts could, and should be obtained by an engineer and a report rendered which should quash, effectually, all opinions based on preconceived notions, always excepting those held by people who cannot be reached by fact and reason. All of this emphasizes the importance of employing a trained engineer who cannot only gather the facts but can analyze them and present a summary and conclusions in a form that any citizen can understand.

EARL J. REEDER,[12] M. Am. Soc. C. E. (by letter).—The discussions by Messrs. Herrold, Buckley, Mickle, Mitchell, and Coe amplified the original paper in a very effective manner. Although little difference of opinion has been expressed, some questions have been raised which will be treated herein.

Mr. Herrold is quite right in his statement that "the accident-prone driver presents a problem with which the careful driver cannot cope." There can be little doubt that the person who, through improper driving habits or unwillingness to conform with the requirements of traffic safety, becomes repeatedly involved in accidents is a menace to others who want to drive or walk safely. However, accident-proneness, as indicated by the repetition of accidents, does not appear to be a characteristic of a very large percentage of drivers. In fact, T. W. Forbes, of the Bureau for Street Traffic Research at Yale University, states[13] concerning a study of 29 500 drivers from the State of Connecticut for the years 1931–1936 that "We are thus left with the conclusion that not more than 1.3 per cent of the drivers were accident-prone and that only 3.7 per cent of the accidents in the states studied could be attributed to the accident-prone driver."

---

[12] Chf. Traffic Engr., National Safety Council, Inc., Chicago, Ill.

On the other hand, many street locations are definitely accident-prone as indicated by the repetition of accidents of almost the same kind. Analyses of many hundreds of collision diagrams of street intersections, and other problem locations, have shown definite patterns of accident experience at individual locations. In many cases these patterns are not at all similar to the traffic-flow pattern, for they show vehicles from certain directions predominating in the accident experience and those from other directions having little or no part in it. Surely, one cannot conclude that the careless drivers come from only one or two directions and that the drivers approaching from other directions are careful. Rather, the only conclusion that can be drawn in such cases is that there is something unusual and unanticipated about the condition to which the drivers and pedestrians are subjected from certain approaches. Further evidence of this is found in the fact that corrective measures often result in the prevention of these accidents by simplifying the traffic and making it easier to drive or walk safely.

Mr. Mitchell's interpretation of the statement "Contrary to the belief of many, a city street is not a continuous scene of human and mechanical disaster," is somewhat different from what was intended, and some explanation should be made. The purpose was not to convey the impression that a great many people over-estimate the seriousness of the traffic-accident problem. That certainly is not the case. It was intended, rather, to mean that many believe that accidents are rather indiscriminately distributed along the street and are likely to occur almost anywhere. In fact, however, they are concentrated at certain points and along certain sections of streets, whereas other sections are relatively free of accidents.

The difficulty with the belief of many, as the statement was intended to convey, is that it would place the responsibility for accidents too largely upon abnormal or unsocial drivers and pedestrians who are not interested in their own safety. However, the patterns of accidents at individual "high-accident" locations, and the freedom from accidents of long sections of street bearing the same traffic that becomes involved in accidents at other places, show quite conclusively that a substantial part of the responsibility for accidents lies with the conditions peculiar to individual locations.

The very basis of the engineering approach to the solution of traffic problems is the belief that the vast majority of drivers and pedestrians drive or walk safely to the extent that they can foresee and avoid hazards. The function of traffic engineering is to make it easier for drivers and pedestrians to use the streets and highways safely and expeditiously either by eliminating the hazards or by providing warnings, safeguards, and aids for avoiding them where they must remain. The more effectively this function is performed, the easier will be the functions of traffic-safety education and enforcement.

---

[13] *Journal of General Psychology*, April, 1939.

The field of traffic engineering has the very fascinating job of determining and evaluating driver and pedestrian practices as important factors in the design of traffic facilities, control measures, and safeguards. The more quantitative data that are gathered concerning traffic the more obvious it becomes that driver and pedestrian practices, under similar conditions, are approaching similarity. Vehicle speeds along a given section of street will fall into quite definite patterns, the dissenters from which are likely to be relatively few. Nearly all drivers will obey a "stop" sign where the computed "critical speed," as determined by view obstructions and other conditions, is very low. However, where the critical speed is relatively high, a much smaller percentage will obey because the others resent restrictions which are obviously unnecessary. These and other facts which traffic engineers observe enable this branch of the profession to design measures for the prevention of accidents and congestion and to obtain anticipated results.

Another important function of the traffic engineer is to make the results of his efforts to reduce accidents or congestion on existing streets or highways serve as the basis for foreseeing and preventing similar conditions in the design and construction of new ones. It is the assembly of that body of information concerning the value of different remedies for different problems and the conditions which differentiate one problem from another which will give to his proposals for avoiding accidents or congestion in new street or highway plans the prestige which comes from successful experience.

Fortunately, in setting up a new traffic-engineering department in a city, the engineer can start with treatment of the many accident or congestion problems that already confront him and work from them to the treatment of the problems which he foresees in the new streets or highways that are being planned. As time goes on, his job will change somewhat from the treatment of existing problems to the anticipation of those that will develop if the proper measures are not taken in the design and construction of improvements or new streets or highways.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2064

THE YELLOW RIVER PROBLEM

By O. J. Todd,[1] M. Am. Soc. C. E., and S. Eliassen,[2] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. J. W. Beardsley, Elliott J. Dent, Herbert Chatley, H. van der Veen, C. S. Jarvis, E. W. Lane, George Higgins, and O. J. Todd and S. Eliassen.

Synopsis

The river problems of North China, aggravated by serious floods and heavy silt loads, have perplexed the Chinese for more than 4 000 yr. Especially has the Yellow River taxed the ingenuity of their hydraulic engineers, as shown by the numerous changes of course this river has taken across the Great Plain of North China through recorded history, changes that have meant appalling disasters to the population of the plain. What Western engineers have learned in recent years about its many-sided problems is outlined in this paper. Such evidence as is available is offered to indicate that certain phases of the Yellow River problem are much clearer to-day than they were in 1919 when the first attempts were made by Western engineers to probe into the hydraulics of this exceptional river and suggest methods for its control and regulation. Proposals for a general regulation program are also outlined, based on the most recent investigations. These proposals give promise of more adequate flood insurance for the population of the Great Plain than has existed previously. Since 1919, both writers have been connected with organizations, such as the Yellow River Commission, the Chihli or North China River Commission, and the China International Famine Relief Commission, all of which have been concerned with investigations of this river in a modern way. Data presented in this paper have been gathered by these Commissions, or under their direction unless otherwise mentioned.

Introduction

The Yellow River is unique among the larger rivers of the world in that, during freshets, it can carry a silt load that may reach 40% by weight where

---

Note.—Published in December, 1938, *Proceedings*.

[1] Cons. Engr., Palo Alto, Calif.; formerly, Cons. Engr. to the Yellow River Comm. and Shantung Govt., Peiping, China.

[2] Engr., Messrs. Höyer Ellefsen, Oslo, Norway; formerly, Engr. in Chg., Survey Dept., The Yellow River Comm., Kaifeng, Honan, China.

it enters the alluvial plain of North China. The effect of this high silt load is to raise the river bed gradually, a phenomenon which can be divided into two distinct actions: (1) That in the upper reaches of the diked course, caused by the silt load itself; and (2) that in its lower reaches, due mainly to a flattening of the slope produced by the extension of the delta into the shallow Gulf of Chihli. The two actions, as far as they affect the rise of the river bed, overlap each other, and there is no distinct border line between them. Changes of the river's course through the delta may have far-reaching influences up stream in some years, much beyond the back-water influence of the tides; but the effect of such delta changes may be completely obliterated a year or two later, by the arrival of highly silt-laden freshets, or by dike breaches. The seaward growth of the delta is thus only partly responsible for the steadily increasing super-elevation of the river's course above the plain.

Locally, the gradual rise of the river bed (which averages about 3 ft per century) can often be as much, or more, in a few years time; but such deposition may again be partly scoured out during following years. This creates an exasperating river-control problem in that the dikes must be raised continually, sometimes as much as 5 ft or 6 ft, over lengths as great as 20 to 30 miles, and it may happen that dike sections recently raised must again be built up after a couple of years due to local deposition. It is an ever-continuing struggle to keep the height of the dikes ahead of the river. Often the river wins, overtops and breaks the dikes and, for a time, wanders unfettered across the densely populated plain causing immense damage and loss of life. Under such circumstances, it is impossible for the vast region on both sides of the Yellow River to develop.

The Yellow River flows on a ridge and is elevated above the plain through which it runs. Once it has broken its dikes, therefore, it will not, as a rule, go back to its low-water channel of its own accord when the flood is over. It remains outside and must be brought back to its course forcibly. This is a classical procedure in China and the person supervising it derives much credit if he is successful. If he fails, this is his misfortune. It may even cost him his life; but no glory accrues to the person who watches the dikes skillfully and keeps them intact. He is merely a "work horse."

That the Chinese have tried various methods of controlling this troublesome river, during the course of time, is evident, and the books and pamphlets dealing with its history and control are myriad; but, unfortunately, they have mostly been written by scholars who knew more about poetry than rivers. It is said of the great Emperor Yü (2200 B.C.) that he solved the problem of getting the river to flow in a bed below the surface of the plain. This skill seems to have died with him, however, and ever since attempts to do so have failed. Since Yü's time discussions regarding control methods have generally fallen into two groups—the diked single-channel method, and a system of multiple channels combined with irrigation and broad flooding when the freshet season is on. In practice, it has usually been a combination of the two methods. During the last ten centuries the single-diked channel method has been adhered to as far as the river has allowed.

The modern conception of channel contraction to induce a more concentrated flow with greater velocities to scour the channel is not new to the Chinese. History tells of much heated discussion among river engineers regarding the good and bad points of a single channel and a system of several channels. In the Ming Dynasty (A.D. 1368–1644), Pan Chi Hsun was the outstanding proponent of the single-channel theory. Already in his time the river was mainly held to a single, diked channel; but he had more advanced ideas and wanted a channel as narrow as possible in order to carry the silt and the floods to sea. His more conservative opponents advocated a wide distance between the dikes for storage of silt and for dissipating the floods. They said:

> "If the dikes are far apart there will be fewer places where we must defend the dikes by costly works and the dangers will be few. If the dikes are too close together the foreshore is easily washed away, the dangerous places will be many and the cost of protecting the dikes will be very great."

This argument holds to-day and has been taken up by Western engineers, as witnessed by the proposals of the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E.,[3] by German engineers who have later visited the Yellow River, such as Professor L. Franzius, and by the laboratory experiments made in Germany for the Chinese Government in 1932 and 1934, experiments which do not entirely uphold the narrow-dike, distance theory.

One of the real obstacles to progress, of course, lies in the lack of hydraulic knowledge of the river itself. Until a relatively few years ago very little hydraulic data existed. This is now being overcome and gradually "light is being shed" on many of the perplexities that have shrouded the river in mystery and have made it subject to more superstition, perhaps, than that with which rivers in China generally are endowed.

There is probably no river in the world which is of so little use to mankind as the Yellow River considering the populous districts through which it flows. Thus far, people have been only partly successful in protecting themselves from its ravages. Even as a communication artery it is unimportant. The river is an enemy instead of a helpful agent. The task of making it obey the commands of Man is a most fascinating challenge to the Engineering Profession.

SCOPE OF INVESTIGATIONS

Former Studies.—For more than half a century foreign engineers and others of scientific leanings have interested themselves in the problems of the Yellow River. The great breach that occurred in Eastern Honan in 1851 (see Fig. 1), causing this river to leave its southeasterly course to the Yellow Sea, and changing it permanently to a northeasterly direction emptying into the Gulf of Chihli, 400 miles to the north of its old mouth, was of pronounced significance. It was the year 1868, however, before a foreign investigator visited the site and prepared notes for publication telling of the effects of this channel change and discussing the various problems involved.

---

[3] "Flood Problems in China," by the late John R. Freeman, *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1405.

The studies of the break of 1851 and that of 1887 by Ney Elias and, later, by G. J. Morrison (British), Gen. J. H. Wilson (American), Messrs. Shermbeek and Visser (Dutch), and others at various times over a period of 25 yr from 1868 to 1893, need no special remarks. The findings of these engineers and scientists, as well as those of Capt. W. F. Tyler in 1902, have been commented upon by the late Mr. Freeman.[3]

The mapping work made in the upper reaches of the river by European travelers and geologists such as Richthofen, Tafel, and Filchner (Germans), Obrutschew (Russian), D'Ollene (French), and others, are of much geographical importance, being the main sources for defining the western drainage boundaries of the river. The geological findings of Pumpelly, Richthofen, Baily Willis, and lately of the Chinese and foreign geologists of the China Geological Society, are of the greatest value for understanding and furthering the engineering problems of the Yellow River.

Among the engineers of modern times, Mr. Freeman must be cited as the first who has gone into the flood problems of the Yellow River with such zeal and intelligence as to make his paper on "Flood Problems in China"[3] a contribution of outstanding value. He had devoted much time to a careful study of all the more important records extant. In connection with the Grand Canal improvement plans which he came to China to supervise and complete, he had an unusual opportunity to have certain important data gathered, tabulated, and mapped. It may fairly be said, therefore, that the systematic investigation of the flood problem of the Yellow River, in a modern sense, began with Mr. Freeman.

Studies in Recent Years; General Surveys.—Prior to the autumn of 1933 the control of the Yellow River, where it ran through the plain, had been in the hands of the Provincial Governments of Shantung, Chihli (later Hopei), and Honan (see Fig. 1). These Governments had their own survey systems, differing in co-ordinates, scales, and elevation datums. Independent of each other they managed as well as possible, and the political and financial questions to be solved generally outweighed the technical ones. However, one Chinese Government organization in Tientsin (the Chihli River Commission, which was investigating the river problem of the Province of Chihli on account of the silt and flood situation affecting the City of Tientsin and its navigable river, the Hai Ho) took an active interest also in the Yellow River since this river had a direct bearing on the port problem of Tientsin. Centuries ago, the Hai Ho (river) from Tientsin to the sea was the outlet of the Yellow River into the Gulf of Chihli, and it is not unlikely that it again may take this course. The Chihli River Commission, therefore, began its own independent studies in 1919, had its engineers run level lines down to the Yellow River at various points, and established two hydrometric stations on the river, one at Lokou, 150 miles from the sea, and one at Shanchow, in Western Honan, 600 miles from the sea. Although involving the observation of the river only at two places the data collected have been very valuable in that, at least as far as water-level observations are concerned, they have been uninterrupted since 1922 in spite of all civil wars, and they have been continued to date (1938) by the Yellow River Commission, a Chinese National Government organization

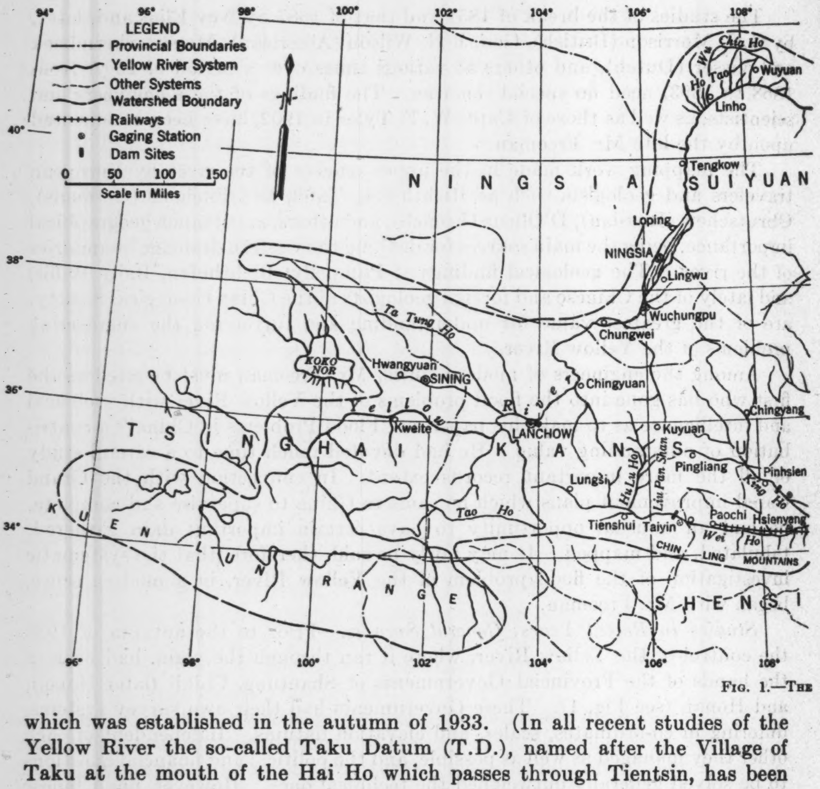
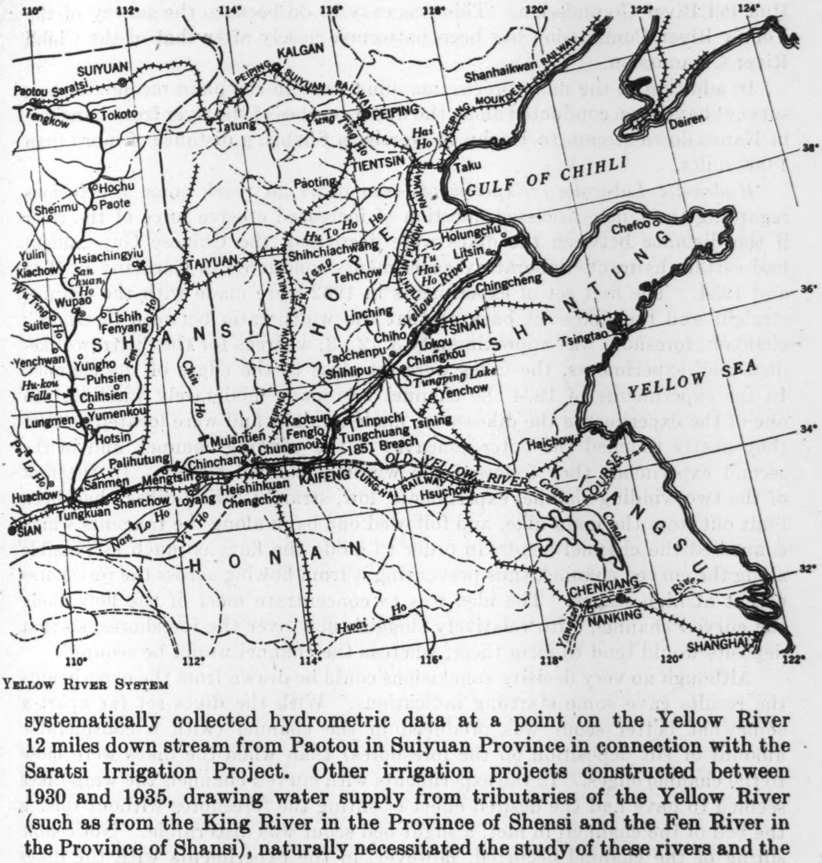

94°
96°
98°
100°
102°
104°
106°
108°
LEGEND
Provincial Boundaries
Yellow River System
Other Systems
Watershed Boundary
Railways
Gaging Station
Dam Sites
0
50
100
150
Scale in Miles
N
40°
38°
36°
34°
NINGSIA
SUIYUAN
TSINGHAI
KANSU
SHENSI
KUEN LUN RANGE
Chia Ho
Wu
Ho Tao
Wuyuan
Linho
Tengkow
Loping
NINGSIA
Lingwu
Wuchungpu
Chungwei
Ta Tung Ho
KOKO NOR
Hwangyuan
SINING
Yellow River
Kweite
LANCHOW
Chingyuan
Chingyang
Kuyuan
Hu Lu Ho
Liu Pan Shan
Pingliang
Pinhsien
King Ho
Lungsi
Tao Ho
Tienshui
Taiyin
Paochi
Hsienyang
Wei Ho
CHIN LING MOUNTAINS

FIG. 1.—THE

which was established in the autumn of 1933. (In all recent studies of the Yellow River the so-called Taku Datum (T.D.), named after the Village of Taku at the mouth of the Hai Ho which passes through Tientsin, has been used as elevation datum. It is a zero reference plane corresponding to the low water of spring tides at the mouth of the Hai Ho. The Grand Canal Improvement Board, in its studies of the Grand Canal and the Yellow River, used the so-called Tsingtao Datum which corresponds approximately to zero elevation of extreme low tides of the ocean at Tsingtao Harbor. The relation between Taku Datum and Tsingtao Datum near Lokou is approximately: Taku Datum minus 5.25 ft equals Tsingtao Datum. Taku Datum is 4.50 ft below mean sea level.)

The hydrometric investigations by the Grand Canal Improvement Board during the spring, summer, and autumn of 1919 covered water level, discharge and silt observations, and studies of scour and refill of the river bed at two sections near the Grand Canal crossing. Furthermore, its engineers surveyed the dike system of the Yellow River between the Peiping-Hankow and Tientsin-Pukow Railways, all being engineering data (including cross-sections) pertinent to the Grand Canal improvement problem.

Between 1929 and 1932, the China International Famine Relief Commission


110°
112°
114°
116°
118°
120°
122°
124°
38°
36°
34°
32°
SUIYUAN
KALGAN
Paotou
Saratsi
Tengkow
Tokoto
PEIPING SUIYUAN RAILWAY
Tatung
PEIPING
Yung Ting Ho
Hai Ho
TIENTSIN
Taku
Shanhaikwan RAILWAY
PEIPING MOUKDEN
Dairen
GULF OF CHIHLI
Paoting
Hochu
Paote
Shenmu
Hu To Ho
Shihchiachwang
Yulin
Kiachow
Hsiachingyiu
San Chuan Ho
TAIYUAN
Wu Ting Ho
Wupao
Suite
Lishih
Fenyang
Fen Ho
S H A N S I
H O P E I
Chefoo
Holungchu
Litsin
Tu Hai Ho
Tehchow
Fu Yang Ho
Chingcheng
S H A N T U N G
Linching
Chiho
TSINAN
Lokou
Tsingtao
Yellow River
PEIPING HANKOW RAILWAY
TIENTSIN PUKOW RAILWAY
Yenchwan
Yungho
Puhsien
Hu-kou Falls
Chihsien
Taochenpu
Shihlipu
Chiangkou
Tungping Lake
Yenchow
YELLOW SEA
Lungmen
Yumenkou
Hotsin
Chin Ho
Kaotsun
Fenglo
Linpuchi
Tungchuang
1851 Breach
Tsining
Mulantien
Chungmou
Haichow
Pei Lo Ho
Palihutung
Chinchang
OLD YELLOW RIVER COURSE
Grand Canal
Sanmen
Mengtsin
Heishihkuan
KAIFENG
LUNG-HAI RAILWAY
Hsuchow
Huachow
Tungkuan
Shanchow
Loyang
Chengchow
SIAN
Nan Lo Ho
Yi Ho
H O N A N
K I A N G S U
CHENKIANG
Yangtze River
NANKING
Hwai Ho
SHANGHAI

Yellow River System

systematically collected hydrometric data at a point on the Yellow River 12 miles down stream from Paotou in Suiyuan Province in connection with the Saratsi Irrigation Project. Other irrigation projects constructed between 1930 and 1935, drawing water supply from tributaries of the Yellow River (such as from the King River in the Province of Shensi and the Fen River in the Province of Shansi), naturally necessitated the study of these rivers and the knowledge thus gained contributed to the general understanding of the Yellow River problem.

A more ambitious program of investigations has been conducted by the Yellow River Commission since October, 1933. This Commission has conducted hydrometric studies at fifteen stations along the main river and at sixteen stations along the more important tributaries. These stations are shown in Fig. 1. In addition to the hydrometric work a detailed topographic and hydrographic survey has been made of the entire course of the Yellow River through the plain, including the sea coast outside the delta. This survey, mapped to a scale of 1 : 10 000, with contour intervals of 1.64 ft (0.5 m), covers the area between the dikes and approximately a 5-mile strip outside. The total area surveyed is about 9 500 sq miles. It has been linked to the topographic survey of the former Chihli River Commission, later the

Hua Pei River Commission. This was easy to do because the survey of the Yellow River Commission has been patterned closely after that of the Chihli River Commission.

In addition to the more precise mapping work in the plain reconnaissance, surveys have been conducted along the upper reaches of the river from Lanchow in Kansu down stream to Hochu in Northern Shansi, a distance of more than 1 000 miles.

Hydraulic Laboratory Experiments.—In order to reach some conclusions regarding the much-discussed question of increased erosive force of the river if the distance between the dikes were decreased, the Chinese Government had certain hydraulic laboratory experiments conducted in Germany in 1932 and 1934. The first set of experiments in 1932 were made with the channel straight and the dikes set back so that the width ratio between foreshore : channel : foreshore was approximately 3 : 2 : 3; whereas for the "narrow-dike-distance" experiments, the dikes were set close to the edges of the channel. In the experiments of 1934 the channel was made moderately winding. In one of the experiments the dikes were made straight and were located so that they nearly touched the outer concaves of the winding channel, and in the second experiment they followed the winding channel closely. In the first of the two winding-channel experiments, low, straight "wing-dikes" had been built out from the main dike, and followed one bank along the tangents which connected the channel bends, in order to guide the flow as much as possible along the curved channel, thus preventing it from flowing across the foreshores except at high stages. The idea was to concentrate most of the flow along the curved channel, with relatively sluggish flow over the foreshores, so that deposits would tend to form there, whereas the channel would be scoured.

Although no very decisive conclusions could be drawn from the experiments the results gave some startling indications. With the dikes set far apart a somewhat better scour was produced in the channel (with a considerable amount of silt deposition on the foreshores) than when the dikes were close to the channel edges. In the experiments with curved channels the wing-dikes seemed to have had the desired effect of silting the foreshores without raising the bed of the channel; in fact, a slight bed scour was discernible. Noticeable silting of the channel occurred, however, in the experiments with the dikes following the edges of the channel closely. It should be remarked that the kind and quantity of silt used, the length of the flood and low-water season, and the intensity and duration of the flood flow, were assumed quantities and bore little relation to those of the Yellow River. Therefore, the experiments indicate only, in a general way, what the results are likely to be if dikes are built close to, or far from, the banks of a silty river.

When the results became known they had the effect of postponing any moving of the present dikes closer to the banks of the river. A highly silty river does not seem to act quite according to customary hydraulic assumptions or calculations, and in this respect the experiments were illuminating. Since they were made the hydraulics of the Yellow River had become better known, and further laboratory experiments may give more specific results regarding the reaction of the river to contemplated control measures.

Soil Erosion Experiments.—Base surveys have been made of gullies and sloping lands where the erosion is severe, with a view to conducting experiments with soil retention measures, both structural and agricultural, on sloping lands as well as in the gullies. Since the silt is the "stumbling block" to nearly all conservancy measures on the Yellow River, it is evident that, in the future, soil-erosion control will assume greater and greater importance.

FORMER ATTEMPTS TO COPE WITH THE FLOOD PROBLEM

Such records as are trustworthy show that for many generations (at least during the past 1 000 yr) the Chinese have depended chiefly on their dike system as a means of controlling floods on the densely populated Great Plain. Inspection of old banks of former river courses, such as those near Techow in Northern Shantung, where the river ran during the Tenth and Eleventh Centuries A.D., indicates clearly that earth dikes were the principal means of keeping this troublesome stream within bounds. Likewise, was its control within one channel for the 500 yr preceding the break of 1851 effected by a single line of strong dikes. Practically no evidences remain of stone work being used to protect this dike system, of which plain remnants exist below the point of the break 30 miles east of Kaifeng. Records reveal that when dikes did give way in places the breaches were repaired by a prodigious use of kaoliang stalks, hemp rope, willow stakes, bags of clay, etc. This method of closing a breach has gradually been improved and is used with certain modifications to this day.

Ancient history, perhaps mythical, reveals that the great Emperor Yü divided the flow across the plain into nine channels and by this method succeeded, as mentioned in the "Introduction," in getting the bed of the channels to lie below the surface of the plain. There is evidence to support the belief that the serious erosion period which Northwest China now experiences began about the time of the great Yü and, if this is correct, it is most likely that he would not have had the silt masses to contend with which now are such obstacles to an effective control of the Yellow River. Undoubtedly, it was the increasing silt masses which finally destroyed Yü's work. Since then pandemonium has reigned. Many control methods have been tried to prevent the river from raising its bed, but to no avail. In 1073 A.D., during the Sung Dynasty, a proposal was made by an official and scholar, Wang An-Shih, in which he urged that 1 000 iron scarifiers be used to stir up the mud in the river bed at low-water stages so that it might be carried down stream and the channel thus lowered. His idea was not adopted because it was thought to be too costly and not very promising in result; but it is perhaps the first recorded proposal for dredging in the history of the Yellow River. Pan Chi Hsün and his dike contraction idea, which created such a controversy, has already been mentioned in the "Introduction." On a stone tablet which to-day can be seen on top of Yün Lung Shan (outside the City of Hsüchow, at the crossing of the Tientsin-Pukow and the Lunghai Railways, and on the bank of the old Yellow River course previous to 1851) Pan Chi Hsün has left an inscription which bemoans the fact that his ideas were not understood.

By the middle of the Nineteenth Century the bed had become so high that when the breach of 1851 occurred, it was found impossible to close it, especially as the Taiping Rebellion was raging at the time. The breach is known to the Chinese as the Tung Hwa Hsiang Breach. Due to the great difference in elevation between the river bed and the land outside the dikes the flow rushing out through the breach caused a back-cutting toward the up stream which lowered the river bed for a considerable distance. For more than 25 miles up stream from the breach the foreshore lands now (1938) have elevations which are still higher than the highest flood levels since 1851; and from the Peiping-Hankow Railway Bridge to the Shantung border the river is still actively adjusting its grade as a result of this breach which occurred 87 yr ago.

After ten years of indecision the new course toward the northeast was finally accepted as the new permanent channel of the river; but nearly thirty years elapsed before an adequate dike system had been built which could be said to control the river during years of low flood flow; and even to-day there is great difficulty in keeping a medium-sized flood from overtopping or breaking through the dikes in the section between the 1851 break and the Shantung border.

The most notorious breach that has occurred since the Tung Hwa Hsiang catastrophe is perhaps the one in 1887. It occurred more up stream, between Kaifeng and the Peiping-Hankow Railway Bridge, and, as a result, for two years, the river flowed southeastward to the Huai River and the Hungtze Lake before the breach was finally closed during mid-winter when the flow was exceptionally low. Another serious break occurred in 1921 in Eastern Shantung near the coast when the entire flow of the river left the old bed. The successful closure of this breach, by means of a rock-fill dam, was reported in 1933[4] (see Fig. 2).

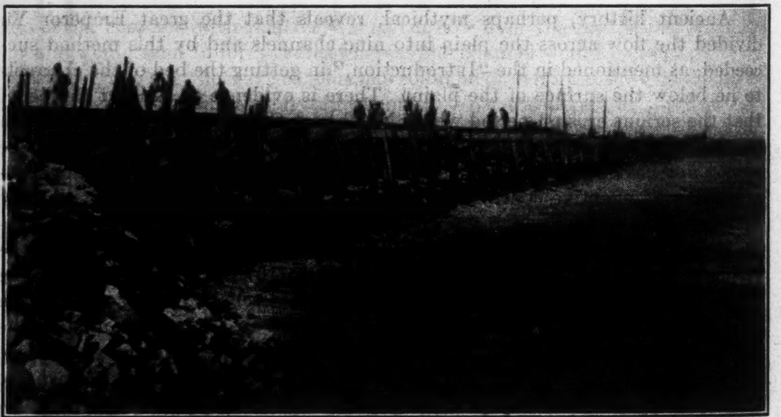


FIG. 2.—COMPLETING ROCK-FILL DAM FROM TRESTLE ACROSS THE YELLOW RIVER TO DIVERT FLOW BACK INTO ITS OLD BED, APRIL, 1923

---

[4] "Wrestling with Some of China's Rivers," by O. J. Todd, *Civil Engineering*, June, 1933, p. 304.

The next major break in the main dikes occurred in August, 1925, in Western Shantung One of the writers made surveys and estimates during the following months and drew up a plan for closure work. The Shantung Provincial River Bureau effected a closure within six months. Fortunately, on this occasion, only one-half the flow of the river had left its bed, running into a low area that emptied into Tungping Lake and from there back to the Yellow River, 75 miles east of the break. It was in this region that the break occurred in 1919 as described by Mr. Freeman.[5]

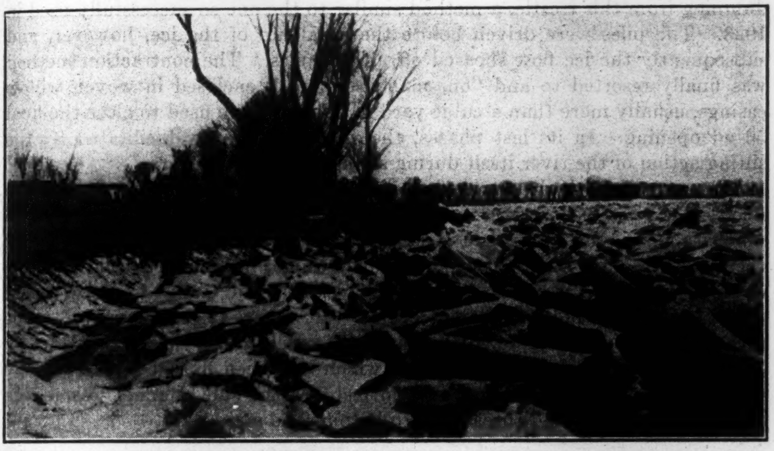


FIG. 3.—ICE JAM IN YELLOW RIVER JUST ABOVE LITSIN, SHANTUNG, FEBRUARY, 1928

In late February, 1928, a heavy ice jam caused ponding and overtopping of the south main dike in two places east of Litsin, Shantung, near the coast (see Fig. 3). This breach was closed within a few months using old Chinese methods; but considerable aid was given by the river itself in that it partly silted up the breach openings. It was in this vicinity, but 10 miles up stream on the same bank, that the dikes failed in August, 1937, during heavy, prolonged summer floods. The region then inundated was estimated at 1 000 sq miles, approximately equal to that of 1928. No attempt has been made to close this breach. The country affected is not densely populated and the land has much alkali. It lies in the lower delta region close to the sea, and sluicing silt on it will convert it into good, arable land, after a few years.

Dike breaks occurred in 1933, 1934, 1935, and 1937. In flood discharge magnitude the year 1933 was the most serious in that the highest flood, perhaps since 1842, then occurred. Between the place of the Tung Hwa Hsiang breach and the Shantung border the dikes were breached in thirty-five places. Several long sections of the north dike were buried in silt so that nothing could be seen except the two lines of trees that grew along the top of the dike. Due to the extremely heavy silt flow of the river the breaches gradually closed themselves except the one farthest down stream on the north bank. This breach, called the Fenglo breach, was closed the following spring by gradually contracting the opening, working from both sides. Originally, it had been planned to build a wooden pile trestle across the opening and close it by stone-dumping from this trestle, a method similar to the one so successfully used in 1923. The piles were driven before the break up of the ice, however, and subsequently the ice flow sheared off all the piles. The contraction method was finally resorted to and "one-man-sized" rock enclosed in woven willow casings, usually more than a cubic yard in volume, were used to close the final 50-yd opening. In its last phases, the work was greatly facilitated by the silting action of the river itself during a small spring freshet.

---

[5] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), Fig. 5, p. 1414.

The 1933 flood had destroyed so much of the dike system that when the flood season arrived in 1934 a 10-mile section of the dike, abutting the northern end of the Tung Hwa Hsiang breach, remained to be rebuilt. Since there was an outer dike (the Golden Dike) which, in case of overflow, would deflect the escaped water back into the river near the Grand Canal crossing, the situation was not considered serious. However, 1934 also proved to be a year of high floods and, in August, a side channel toward the north gradually developed across the former dike line which had been buried in silt; and by the time the flood season was over in October the side channel was taking nearly half the flow and threatened to become the main river.

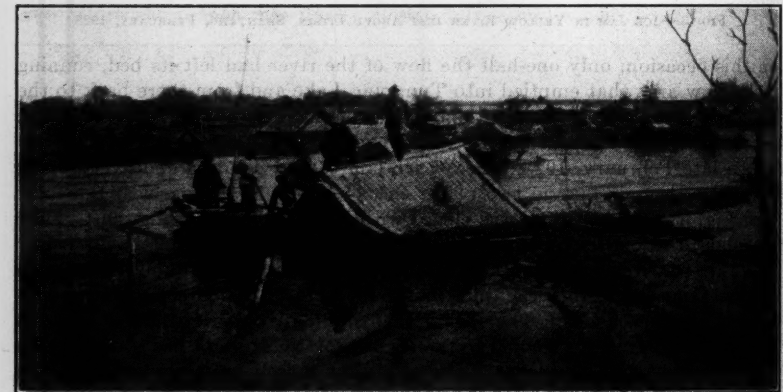


FIG. 4.—A VILLAGE IN SOUTH HOPEI BURIED IN SILT BY A YELLOW RIVER DIKE BREAK, MARCH 1, 1935

The Golden Dike was seriously endangered and if it should break the flow would most likely go to Tientsin. A major closing operation became necessary. An attempt was first made to contract the opening sufficiently by layers of kaoliang stalks and earth so that a plug of the same material could be lowered into the opening. This could not be wider than 80 ft if one were to be sure of success. Unfortunately, the river bed proved to be very sandy to great depths and became deeply scoured when the opening had been contracted to

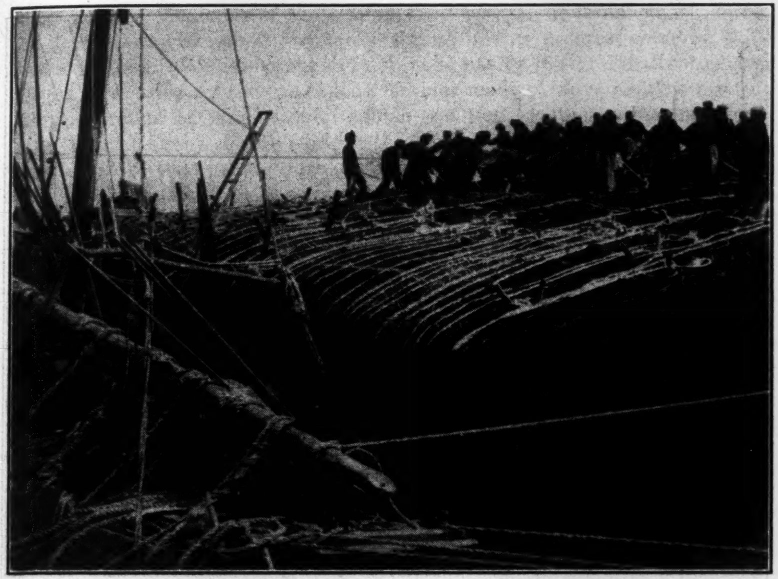


FIG. 5.—BUILDING KAOLIANG CORE OF CLOSURE DIKE AT THE GREAT BREACH NEAR TUNGCHUANG, SHANTUNG, CAUSED BY THE 1935 FLOODS

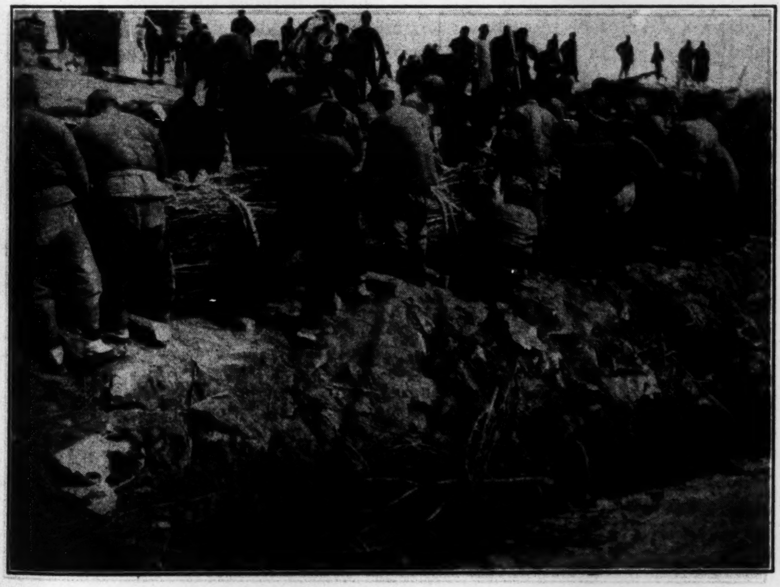


FIG. 6.—TYING UP WILLOW CASED STONE "SAUSAGE" TO ROLL INTO LAST GAP IN CLOSURE DIKE AT TUNGCHUANG BREACH, MARCH, 1936

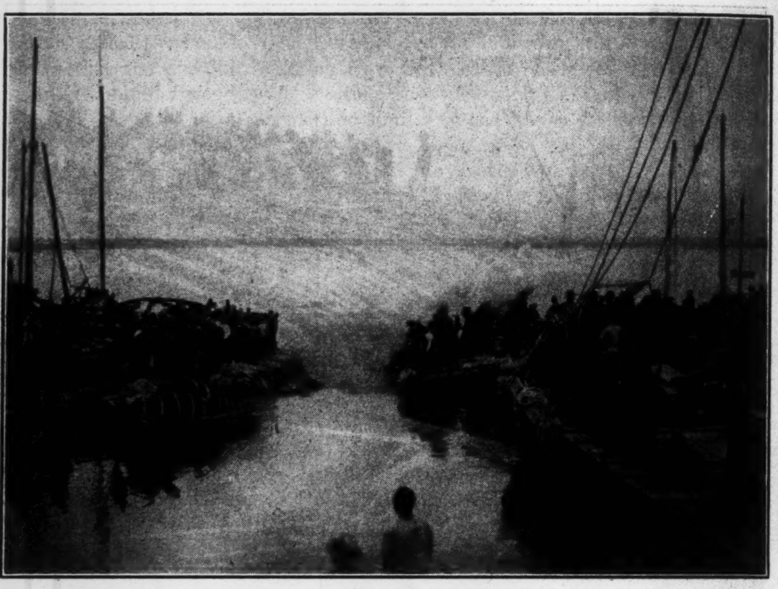


FIG. 7.—PLACING LAST STONE "SAUSAGES" TO CLOSE BREACH OF 1935, AT TUNGCHUANG, SHANTUNG, ALONG SOUTH BANK OF YELLOW RIVER

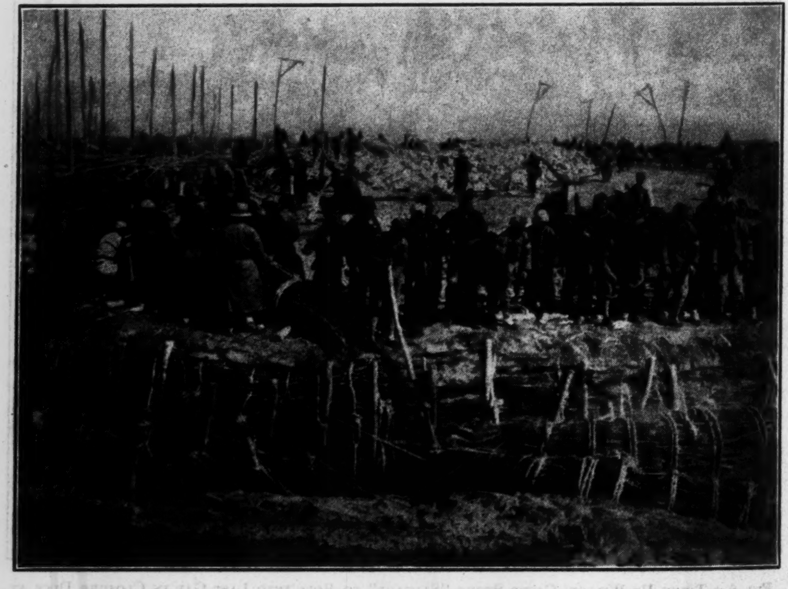


FIG. 8.—WILLOW STONE "SAUSAGES" AT TUNGCHUANG CLOSURE HELD IN POSITION AGAINST HEAD OF 5 FEET OF WATER BY THREE OR FOUR TWISTED WIRE CABLES AFTER HEMP ROPES FAILED

about 120 ft. The ends of the closing work became undermined, broke off, and floated away like great haystacks, and no further progress could be made by this method. Soundings showed depths of nearly 70 ft. Finally, the river bottom was sealed by the use of long fascines made of woven willow matting wrapped around an inner core of rubble and tied together with hemp rope. As in the case of the closure work at the Fenglo breach, the river threw a silt bar in front of the opening during a silty spring freshet and the final closure work could practically be done in quiet water.

The break at Tungchuang, in Western Shantung, the following year (1935) proved by far the most disastrous since the break of 1887, because it inundated approximately 6 000 sq miles of farming country in Western Shantung and Northern Kiangsu, the flow going to sea at a point near the former mouth of the river during the five centuries prior to the change of course in 1851. Fig. 4 is a view, taken March 1, 1935, of a village in South Hopei. Before the end of 1935 the entire flow was pouring to the southeast through the breach. The Shantung Provincial Yellow River Bureau began the work of closure; but after two months of preliminary arrangements the National Government set up an organization that assumed charge of the task, and completed the closure successfully by the end of March, 1936. In connection with this work the writers were engaged as Assistant Chief Engineer and Consultant, respectively. Benefiting by the experience gained during the Fenglo and Kuant'ai breach closure works (the Chinese refer to the 1934 breach or side-channel closure as the Kuant'ai work), stone was used in much larger quantities, and after the breach opening had been contracted to about 130 ft, with a maximum depth of about 40 ft, work was begun immediately to seal the river bed and prevent it from being scoured further (see Figs. 5, 6, 7 and 8). The fascine "sausages" were made about 3 ft in diameter and from 40 to 60 ft long, and they were considerably larger than those used at Kuant'ai. The discharge during the time of closure averaged 30 000 to 35 000 cu ft per sec, but in spite of this rather heavy low-water flow the work proceeded without any serious setback. In the last three days, during the final closure, the river had to be "headed up" more than 6 ft in order to force the flow back into the old channel. This was facilitated by a cut-off channel 3 miles long which short-circuited a 10-mile sharp bend immediately below the breach and acted as a "primer" for accelerating the scour action in the old channel.[6] Fig. 9 shows a stone rip-rap apron with a woven willow cover, tied back by twisted wire ropes to hold small stones in position until silt fills the voids. A kaoliang wall tops the dike with 2 ft of earth cover. Fig. 10 shows a bent protected by a loose brick toe held by wire mesh.

PHYSIOGRAPHY

The Yellow River has its origin in a series of lakes and swamps on a plateau in Eastern Tibet. Before it reaches China proper it has cut its way through the mighty mountain ranges of the Kuenlun in a series of magnificent gorges. Reaching the Province of Kansu it receives its first important tributary, the Tao Ho, from the southeast. Then passing Lanchow, the capital of Kansu,

---

[6] *Engineering News-Record*, May, 1936.

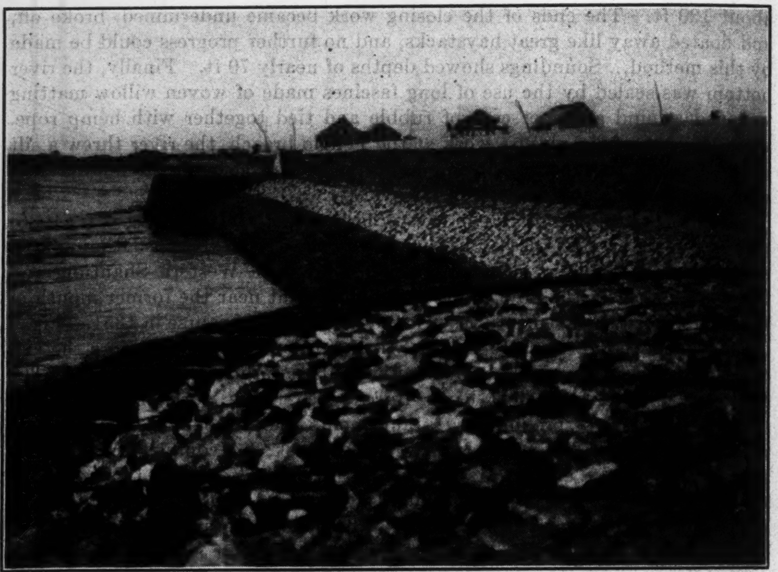


FIG. 9.—FINAL PROTECTION WORK TO CLOSURE DIKE AT TUNGCHUANG, STONE RIP-RAP APRON WITH WOVEN WILLOW COVER TIED BACK BY TWISTED WIRE ROPES TO HOLD SMALL STONES IN POSITION UNTIL SILT FILLS VOIDS. A KAOLIANG WALK TOPS DIKE WITH 2 FEET OF EARTH COVER.

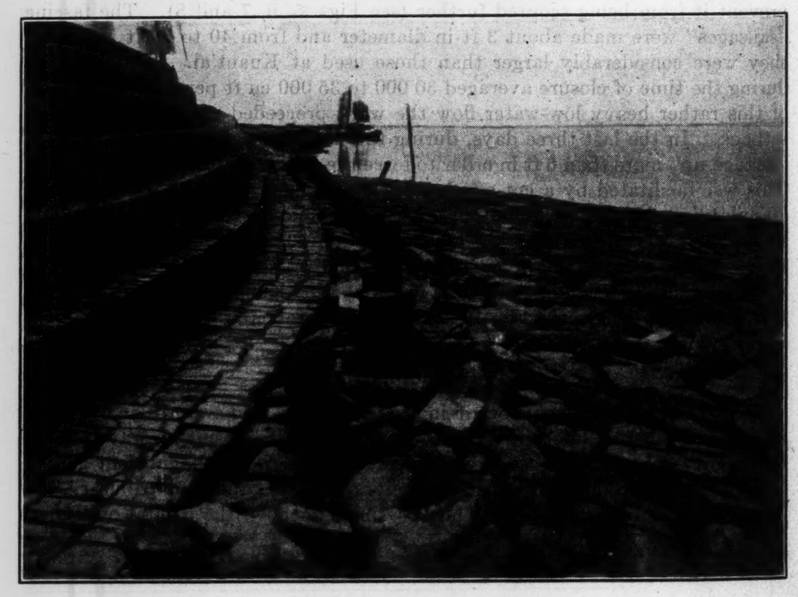


FIG. 10.—BRICK BANK PROTECTION WITH LOOSE BRICK TOE HELD BY WIRE MESH JUST ABOVE TUNGCHUANG ON SOUTH BANK OF YELLOW RIVER

it cuts through the most easterly spurs of the Nanshan Mountain Range and flows out on the plains of Ninghsia. Here, the course widens and islands and shoals appear everywhere in the channel. Forming the border between Ninghsia and Suiyuan it is again hemmed in by low mountains before emerging on the Suiyuan Plain.

It is typical of all the rivers in North China that they flow through a series of gorges and basins, the slope being flat through the basins and steep through the gorges. The basins are overflow and depositing areas for the loessic silt which the rivers carry in freshets and the gorges are cut through denuded mountain ranges where the rocky surfaces are exposed. The main basins of the Yellow River are in Ninghsia and Suiyuan, and between Lungmen and Tungkuan. However, there are many smaller, intermediate basins, such as at Chungwei, in Kansu. The alluvial plain of North China may be regarded as the last basin. The gorges first extend from the plateau near the source to Kansu, through which Province the river runs in a series of shorter gorges. Where it follows the Ninghsia-Suiyuan border it has a shallow, but remarkably steep, sloping gorge. From the Suiyuan-Shansi border to Lungmen the entire course is gorge like, and, finally, the last gorge extends from Shanchow to where the river enters the alluvial plain of North China. The slope through the gorges is often steeper than 1 : 1 000 with occasional rapids and waterfalls; through the basins (such as the Suiyuan Plain) the slope may be flatter, locally, than 1 : 10 000, but usually it ranges from 1 : 2 500 to 1 : 3 000.

Through Ninghsia and Western Suiyuan, for a distance of more than 400 miles, the river has practically no tributaries. This region is arid, with an average rainfall of scarcely more than 5 in. per yr; but, as it leaves Suiyuan and enters the north-south course between Shansi and Shensi the river receives numerous tributaries from both sides. These carry little flow during the dry season from November to June, but during the rainy season in summer they may be raging silt torrents. Then, at Tungkuan, where the course bends abruptly toward the east, its most important tributary—the Wei Ho—enters. The Wei Ho has many small tributaries draining the lofty Chinling Mountain Range which flanks it closely on the south. From the north, draining the loess plateau, it has only a few, but highly important tributaries. From Tungkuan to where the Yellow River enters the plain the tributaries are also numerous, but, as is the case between Tungkuan and Suiyuan, they carry almost no flow during the dry season whereas in the rainy season they are of much importance. Especially notorious flood carriers are the two streams, the Lo Ho from the south and the Chin Ho from the north, which enter the main river just up stream from the point where the dike system begins.

Besides the Chinling Mountain Range which forms the divide between the Yellow River and the Yangtzekiang drainage areas, the Liu Pan Shan Mountain Range in Southeast Kansu should be mentioned. This range has a north-south direction. It stands directly in the way of easterly rain-bearing winds and thus causes copious, orographic rainfall on the head-waters of the King Ho, the most important tributary to Wei Ho from the north. Table 1 gives the distances and elevations of a number of points along the river, which have been definitely connected by surveys and leveling.

The drainage area of the Yellow River above the Peiping-Hankow Railway Bridge is 294 000 sq miles (756 000 sq km). Some geographers like to include also the tributaries that belonged to it when it had a different course to the sea than it has now, for example, when it was flowing north to Tientsin or south to the Yellow Sea. In such case, its drainage area would be increased about 65%; but at least for the time being one must assume that the river will most likely be kept to its present course for another century. In that case its area can only be regarded as lying up stream from a point where it enters the fully diked course at the Peiping-Hankow Railway Bridge. To this should be added the Shantung area of about 4 500 sq miles. The exact value to be given to the last area is difficult to determine because during floods the flow becomes divided and much of it does not enter the Yellow River, but flows south into the Grand Canal and the West Shantung Lakes.

TABLE 1.—PROFILE ELEVATIONS OF LOW-WATER FLOW IN THE YELLOW RIVER

Name of place (see Fig. 1) (1)	Elevation of low-water flow, in feet T. D.† (2)	Distance from the Gulf of Chihli, in miles (3)	Slope (4)	Name of place (see Fig. 1) (1)	Elevation of low-water flow, in feet T. D.† (2)	Distance from the Gulf of Chihli, in miles (3)	Slope (4)
Mouth of Yellow River	4	0		1851 Breach	228	365	
			1 : 6 000				1 : 5 300
Holungchu	28	27		Railway Bridge‡	305	441	
			1 : 10 000				1 : 3 800
Litsin	36	42		Mengtsin	390	503	
			1 : 12 600				1 : 1 100
Lokou	82	152		Shanchow	952	624	
			1 : 9 000				1 : 2 800
Shihlipu*	131	236		Tungkuan	1 053	677	
			1 : 7 800				1 : 3 000
Tungchuang	179	307		Lungmen	1 198§	760	
			1 : 6 000				1 : 1 600
1851 Breach	228	365		Paotou	2 920§	1 275	

* Grand Canal Crossing. † T. D. = Taku Datum. ‡ Peiping-Hankow Railway Bridge. § Approximately.

The chief characteristic of the Yellow River drainage area is its loess and tertiary clay deposits, which are especially extensive in Eastern Kansu, Northern Shensi, and Shansi. They are found in the river valleys, between the mountain ranges, and exist as a continuous plateau extending from Eastern Kansu across Shensi into the Province of Shansi. Frequently, the deposits have depths of more than 300 ft. It should be noted that the aeolian loess (in Chinese, *huang tu*, or "yellow earth") is a relatively recent deposit. Hence, it is generally the top-soil cover. It is underlaid by the similar, but older, tertiary clay, which, as a rule, is of a more reddish color. Over large areas the loess cover has been entirely stripped off and the red tertiary clay is fully exposed (see Figs. 11, 12, and 13). The tertiary clay is a soil of finer texture than the loess and is more resistant to erosion. Therefore, rivers passing mainly through tertiary clay areas, such as the Honan Lo Ho, as a rule, are less silt-laden after heavy rains than those which pass through areas where the loess cover is still deep.

The origin of the loess, or even the tertiary clay, is a much disputed point. The tertiary clay seems to have filled the valleys as a result of the leveling of the mountain areas throughout the Tertiary epoch, and the loess deposited intermittently toward the end of the Tertiary and the beginning of the Quaternary ages, either blown in from the north or due to weathering of all the mountain ranges which traverse the Kansu, Shensi, and Shansi areas. This was a period of very wet, very dry, and windy, climate when erosion by stream flow and soil deposition by wind alternated. Hence, one may often find the strangest mixture of layers of gravel, loess, and tertiary clay in exposed, high, vertical river banks (see Figs. 14 and 15). Most likely, the present is a period of erosion, as witnessed by the heavy silt load which all the North China rivers carry in the rainy season; but the dust storms that occur in the spring are reminders of the earlier dry periods when the loess cover was laid down. Even now, in less than a day's time, a cover of nearly 0.01 in. of dust may settle on floors and tables in a closed room during a heavy dust storm. Two such storms per year would deposit a 20-in. layer in 1 000 yr and in 100 000 yr (which approximate the duration of the geological periods when these dust deposits were formed), the depth of the deposits would be 167 ft (modified by a suitable settlement factor), which is an average depth of the actual deposits on much of the Yellow River water-shed. As these dust storms always occur when there is a north gale, it seems that there is a gradual shifting of the soil toward the south by the wind; and, as all the rivers flow toward the east, there is also a gradual shifting of the soil toward the east by water action.

To-day the terrain in the loess and tertiary clay areas has become most severely cut by erosion. The farmers are fighting a losing battle against this destruction, and every year they see edges of their land worn away, or the top soil stripped off, by the heavy rain storms, which are so frequent during the rainy season. The entire area has been most extensively, but rather carelessly, terraced, to hold the soil in place, often with good result; but frequently, perhaps due to internecine wars, the terraces have been abandoned. For all practical purposes, the entire area may be said to be denuded of forests if there ever have been forests on it. Trees exist around villages and temples and on some of the more favorable mountain slopes facing northeast; but their effect is negligible in preventing the erosion which is now progressing at an alarming speed.

The Yellow River Basin and the surrounding area, especially its western part, are known as a great earthquake region. To the east its alluvial plain has been built up on a sinking geosynclinic trough, and earthquakes at times also shake this plain severely, as in the beginning of August, 1937. It seems reasonable to believe that the earthquakes are caused in part by the constant shifting of earth loads both by the Yellow River and by the wind from the mountain areas in the northwest to the plain in the east and southeast, amounting to an average probably exceeding 2 000 000 000 tons per yr. That isostatic adjustments due to the accumulating effect of the constant transfer of such huge loads must occur from time to time seems quite evident.

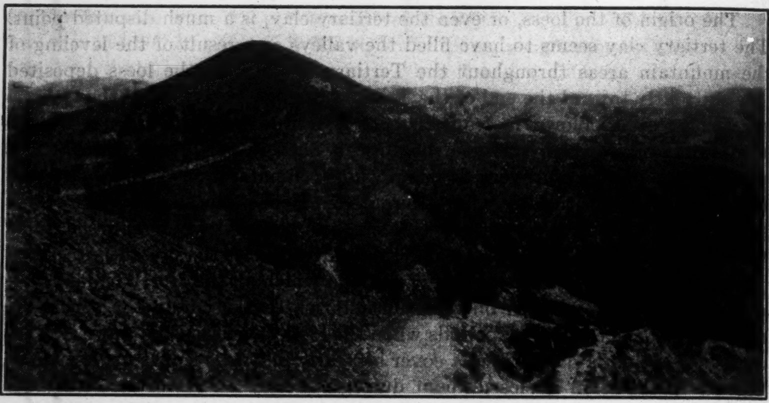


FIG. 11.—LOESS-COVERED HILLS OF CENTRAL SHENSI

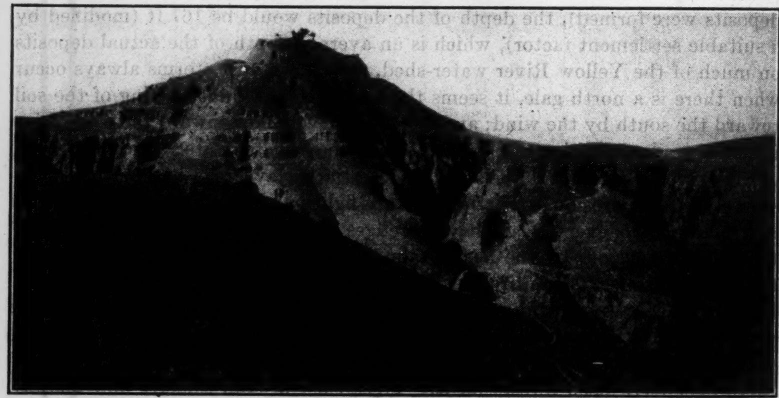


FIG. 12.—LOESS-COVERED HILLS OF CENTRAL SHENSI

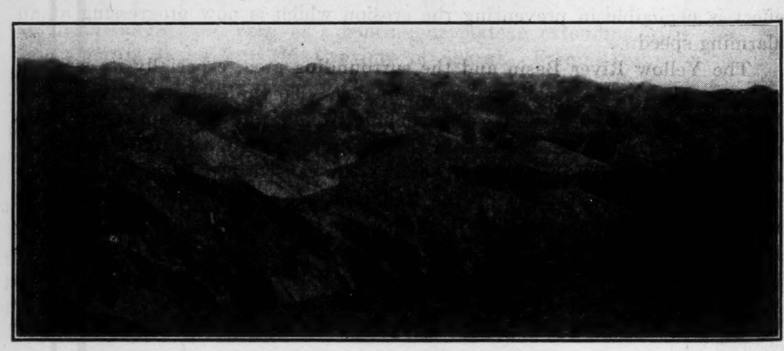


FIG. 13.—LOESS-COVERED HILLS OF CENTRAL SHENSI. NOTE EFFECTS OF EROSION

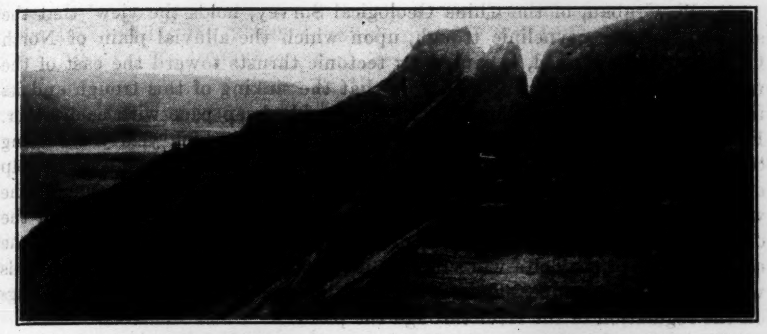


FIG. 14.—A CUT IN LOESS IN WEST SHENSI ALONG MOTOR ROAD FROM SIAN TO LANCHOW, 1932

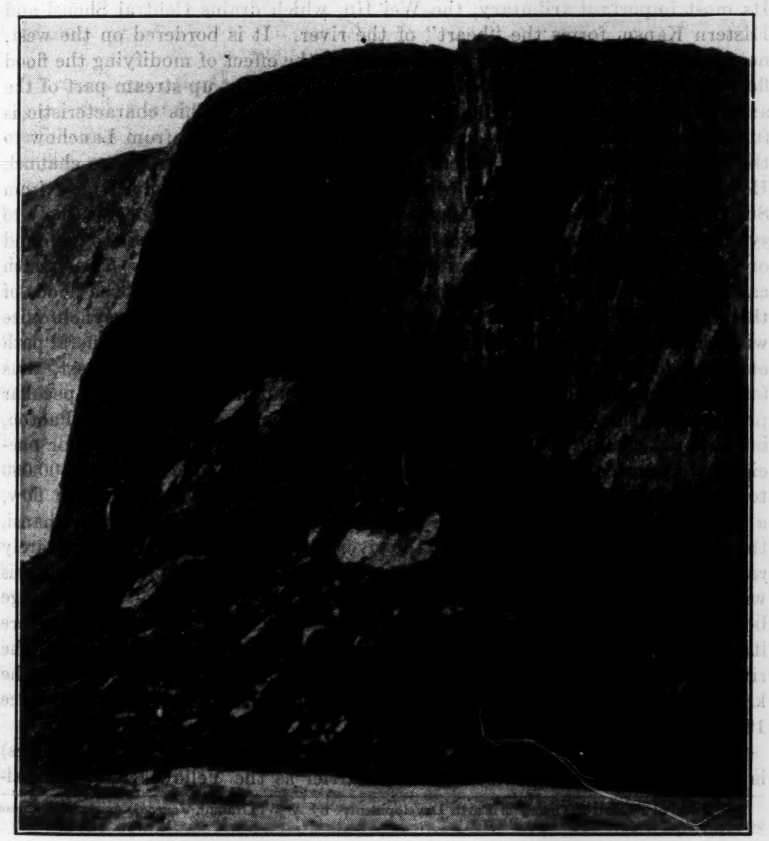


FIG. 15.—A CUT IN LOESS SOIL ALONG NEW MOTOR ROAD FROM SIAN TO LANCHOW, 1932

A. W. Grabau, of the China Geological Survey, holds the view[7] that the sinking of the geosynclinic trough, upon which the alluvial plain of North China has been formed, is caused by tectonic thrusts toward the east of the mountain masses of Central Asia and that the sinking of this trough and its filling, mainly by the Yellow River silt, roughly keep pace with each other. However, it is possible that the enormous silt loads which yearly are being deposited on the North China alluvial plain and its coastal fringe tend to help depress the geosynclinic trough and thus further promote the thrust of the western mountain masses. Furthermore, if the sinking of the plain and the deposition of silt on it keep pace rhythmically, there may be periods when the coastal fringe of the delta will suffer an invasion by the sea, and other periods when it will be built up. These are interesting speculations and have bearings on the regulation of the river through the plain.

The Yellow River is peculiar in the way it encircles a part of its own area. Its most important tributary, the Wei Ho, which drains Central Shensi and Eastern Kansu, forms the "heart" of the river. It is bordered on the west, north, and east by its parent river. This has the effect of modifying the flood flow of the entire Yellow River as floods from the most up-stream part of the area seldom coincide with floods from the Wei Ho. This characteristic is further modified by the flow loss along the 900-mile course from Lanchow to the Suiyuan-Shansi border, where the river is merely a conveyance channel. However, the heavy flood flow which the river can receive on its way from Suiyuan through the 400-mile course down to Tungkuan, at times, is likely to synchronize with the flood flow coming from the Wei Ho. Here is to be found one of the causes for the magnitude of the flood of the Yellow River which can be so destructive in the plain. The other cause is found in the floods of the tributaries—Lo Ho and Chin Ho, in Honan—which also can synchronize with the floods from up stream. This is due to the fact that the general path of many of the flood-producing rainstorms is from west toward east, thus following the flood crest down stream. The result, therefore, is the peculiar phenomenon that one-third of the entire drainage area up stream from Paotou, in Suiyuan, cannot contribute any more than 100 000 cu ft per sec, or one-eighth to one-tenth of the river's estimated maximum flood flow of 800 000 to 1 000 000 cu ft per sec where it enters the plain. As regards the silt flow, a similar situation exists. Where the river leaves Suiyuan and enters Shansi, the maximum silt percentage that the river has been observed to carry rarely reaches 5% by weight; generally, it runs less than 1 per cent. Contrast this with more than 40% at Lungmen, which lies at the lower end of the gorge between Shansi and Shensi, and nearly the same value for the Wei Ho where it enters the Yellow River. These findings regarding the distribution of the river's flood and silt flow are some of the most important contributions to the knowledge of the hydraulics of the Yellow River during the seven years since 1930.

It should be mentioned that the Wei Ho (drainage area, 56 000 sq miles) is almost as important a flood and silt carrier as the Yellow River itself, although it is less important as a low-water feeder. Its most important tributaries are the King Ho (23 000 sq miles), draining mainly East and Northeast Kansu, the Lo Ho (10 000 sq miles), draining North Shensi, and the Hu Lu Ho (5 000 sq miles), draining Southeast Kansu. These three streams all enter the Wei Ho from the north. As the loess and tertiary clay areas lie mostly to the north of the Wei Ho, it follows that the northern tributaries are the silty ones whereas those coming from the Chin Ling Mountain Range to the south may almost be classed as clear rivers, in comparison. The silt coming from the Wei Ho area is, in general, much finer than that coming from the areas in Shansi and Shensi between Hochu and Tungkuan. Partly for this reason freshets from the Wei Ho have a tendency to scour the diked channel of the Yellow River, whereas those from the Hochu-Tungkuan area, containing much coarser silt, tend to raise the river bed. In general, however, freshets from the Wei Ho area have more variation in the intensity of their silt loads than those from the Hochu-Tungkuan area which are always heavily silt-laden.

---

[7] "The Polar Control Theory of Earth Development," by A. W. Grabau, *Journal*, Assoc. of Chinese and American Engrs., May–June, 1937.

HYDROLOGICAL CONSIDERATIONS

Meteorology.—North China has two distinct yearly climatic seasons—the dry late fall, winter, and spring season with its northerly monsoons caused by the Siberian high-pressure area; and the rainy summer and early fall season with its southerly monsoons caused by the low-pressure area over interior China. In the northern regions of North China the rainy season tends to begin later and end earlier than in the regions closer to Central China. As the course of the Yellow River traverses most of the territory which may be termed North China, its low-water season is shorter and its high-water season longer than the corresponding seasons of the smaller rivers in the north, such as those belonging to the Hai Ho System which converge on Tientsin. The flood season of the Yellow River, therefore, is reckoned from the middle of June to the end of October; but usually it falls between the end of June and the end of September. There is also a brief high-water period in the beginning of April due to release of the ice cover in Suiyuan and North Shansi. From Tungkuan to the sea the ice has already left the river by the end of February, but, where it flows through Suiyuan which lies 5° to 6° farther north, the ice cover does not break up until late March or the beginning of April. At this time ice jams usually form along the flat, sloping course of the river through Suiyuan. These jams may have considerable storage of water behind them. Frequently small freshets from the region above Lanchow occur at the same time. Due to its more southerly location, this section thaws out a little earlier than the river in Suiyuan. Together they form the early April freshets. It should be noted that along the diked part of the river through the alluvial plain a somewhat similar situation exists. The course near the mouth lies near Latitude 38°, whereas the up-stream diked course lies near Latitude 35°; hence, this part thaws out first and causes ice jams in the river near the mouth. These ice jams may be serious enough to break the dikes, as in late February, 1928.

Central China and North China get their rainfall from two systems of cyclonic disturbances, the continental system of low barometric pressure areas which originates in the interior of Asia and which drifts eastward into the Pacific Ocean, and the Pacific Ocean system of depressions or typhoons which generally originates east of the Philippine Islands and which drifts either westward toward South China or curves north and northeastward following the coast of Asia and then back into the Pacific Ocean over the Japanese Islands. These two systems serve to bring into violent contact the dry, cooler air masses of the North Asia Continent and the moist equatorial and South China ocean air masses causing condensation and rainfall. The constant presence of dust in the continental air masses may accentuate the condensation. The south monsoons serve to drive the moist equatorial and Indo-China air masses northward and thus make moisture available for condensation when the depressions arrive on the scene. During the winter the "high" Siberian pressure area, with its strong anti-cyclonic northerly winds, drives moisture southward. This factor, combined with low temperatures, makes the available moisture for condensation and precipitation much less when the depressions arrive from the continent than during the summer, which explains the dry winters and springs although the depressions are numerous enough. Often these depressions merely cause dust storms. When there are no depressions hovering about, there is a gradual tapering off of one air mass into the other. The weather is then generally fine and settled all over China and the typical monsoonal winds blow steadily.

It should be noted that the continental depressions drift against the sources of available moisture in the Pacific Ocean, and, therefore, the rainstorms set up by them are usually short-lived; in the winter and early spring they often cause little or no precipitation. During the summer, however, their drift may be slow, and they may remain stationary for some days causing prolonged, copious rainfall. In order to cause rain on the Yellow River area, the Pacific cyclones must pass close to, or inside, the China coast. Such paths seem possible only during July, August, and early September, and hence typhoons cause flood-producing rain during this period only. The paths of all the depressions are determined by the indistinct contact line of the equatorial—Indo-China and the continental air masses, between which the depressions tend to pass. The primary cause for the rainfall is thus the two opposing air masses; and the secondary cause is the depressions that serve to bring them in contact in such a way that condensation occurs.

To forecast floods in the Yellow River from meteorological observations only seems feasible and may become quite reliable after more study has been made of the various combinations that produce severe rainfall. Considerable study has already been made by the various observatories in China to unravel the rain-storm mysteries; but meteorological stations are lacking in West China and in the regions still more to the west. This has prevented students from forming reliable conclusions as to what climatic combinations will produce the most serious floods in the Yellow River. For the present, flood warnings are sent from "key" hydrometric stations equipped with wireless sending apparatus. This is fairly satisfactory and will generally give 24 to 48 hr advance notice of floods before they arrive at the upper section of the diked course. However, the floods from the Lo Ho and the Chin Ho (which enter the river near the head of the diked channel) constitute a serious difficulty. In July, 1935, the flow at Shanchow was reported telegraphically as 250 000 cu ft per sec, whereas at the hydrometric station just below the Peiping-Hankow Railway Bridge it was measured and reported as 586 000 cu ft per sec. The warnings from the Lo Ho and the Chin Ho came too late and, mainly because the river guards were taken unaware, a destructive breach occurred in Western Shantung. Such flow synchronism, only not so severe, also occurred in the summer of 1937. The flood of July, 1935, was due to a continental depression. A still larger flood of 635 000 cu ft per sec occurred in August, 1935, at Shanchow, traced to typhonic causes. It had increased to only 653 000 cu ft per sec at the head of the diked section.

Little information is available regarding rainfall measurements on the Yellow River drainage area prior to the organization of the Yellow River Commission in 1933. One of the first steps this Commission took was to establish a rainfall-measuring service and, as far as possible, it enlisted the co-operation of the Provincial Governments. Before the flood season of 1934, rainfall was measured at more than 250 stations scattered mostly over the area down stream from Suiyuan, but including the Wei Ho area. Rain-gages, 8 in. in diameter, patterned after the United States Weather Bureau standard, were installed, and it has been possible, from the data collected during 1934 and 1935, to compare rainfall and run-off. The rainfall values have been computed from 1-in. isohyetal maps by the use of planimeters. As the rainfall stations were too few up stream from Paotou in Suiyuan to draw the equal rainfall lines accurately, only the area down stream from this hydrometric station was used and the flow coming from up stream deducted.

The results show a remarkably small run-off percentage. The rainfall from August 1 to 10, 1937, resulting in the high-water period from August 1 to 18, gave a ratio, run-off-to-rainfall, of 13.8 per cent. The rainfall period from August 1 to 10, 1935, as compared with the run-off period from August 1 to 17, had a ratio of 9.2%, and the rainfall period of July 1 to 10, 1935, with a run-off period from July 7 to 13, gave a ratio of 16.1 per cent. The latter storm covered only the Wei Ho area and the Yellow River area down stream from Tungkuan. The flow observed at the Lungmen station and that of the Fen Ho coming from Shansi entering below the Lungmen station were deducted. These data show, conclusively, the effective absorbing power of the loess areas and contrast greatly with similar observations for many of the tributaries to the Hai Ho which drain the more loess-denuded eastern slopes of the Mountain districts immediately west of the plain. Run-off percentages as high as 75% have been observed on these loess areas. Unquestionably, many of the smaller mountain tributaries of the Yellow River can show high run-off rates; but, taken as a whole, the Yellow River has a low run-off rate, which is one of the main reasons why this river, having such a large drainage area, does not yield a higher flood flow than 900 000 cu ft per sec. The Yangtzekiang, up stream from Ichang, with a somewhat similar drainage area, has been observed to have a discharge of nearly 2 500 000 cu ft per sec in a heavy flood.

Frequency and Magnitude of Floods.—Since it is the control of the floods in the plain that is most imperative, one naturally is concerned with the flood flow that enters the diked section of the Yellow River. Hence, the most useful records of flow would be those obtained close to the head of that section. Unfortunately, this flow has been observed only for a few years. The nearest are those for the Shanchow station where practically continuous records are available dating from late 1918. That station is 185 miles up stream from the head of the diked section, and the drainage area down stream is 12 000 sq miles. The latter has several highly important tributaries, such as the Lo Ho (5 000 sq miles) from the south and the Chin Ho (4 000 sq miles) from the north, both of which enter the main river practically where the dikes begin. Unquestionably, the Lo Ho can have flood flows as high as 350 000 cu ft per sec and the diked Chin Ho perhaps as much as 70 000 cu ft per sec. (An automatic control by overflowing the plain before its course has been diked precludes any higher flood-crest flow from entering the Yellow River.) Twice since 1934 the Lo Ho has been observed to have had freshets of 250 000 cu ft per sec—once in 1935 and, again, in 1937. These were exceptionally large floods, however.

It has previously been mentioned that the flow of these two rivers, especially that of the Lo Ho, may synchronize with the flood crest of the main river. Except the flow of these two rivers and other smaller ones belonging to the drainage area between Shanchow and the head of the diked section, however, the Shanchow data record the flow of the entire Yellow River area of 278 000 sq miles up stream from it.

The Lokou records from 1918 to 1937 contain the flow of the river after all losses (such as those from percolation, evaporation, and dike breaches) have occurred. Obviously, they are not very useful for developing a proper flood-magnitude frequency relation. In the past these records have been much referred to as representing the flow of the Yellow River.

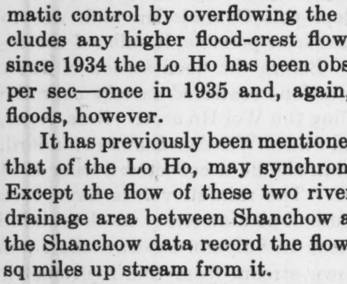


Fig. 16.—Frequency Relation; Yearly Maximum Floods

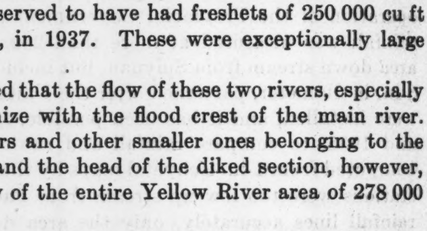


Fig. 17.—Flow Loss Diagram, Yellow River

The duration curves for maximum annual floods have been drawn in Fig. 16, using the 19-yr flow records at Shanchow (1918–1937). It is obvious that, at this point, the Yellow River can have a flood flow exceeding that of 1933, which has been interpolated to have been approximately 812 000 cu ft per sec. Even in 1933 this value could have been exceeded if a closer coincidence had occurred between the freshet of the Wei Ho and the freshet coming from the main river up stream from Tungkuan. The floods are very flashy both from the Wei Ho and from the main river; therefore, a few hours' difference in the arrival of their flood crests at the confluence point at Tungkuan will make a decided difference in the magnitude of the resulting floods passing down the main channel below Tungkuan.

Usually, when the main river has a freshet, there is none, or a very small one, from the Wei Ho—or *vice versa.* As far as has been observed it is the rainfall produced by typhonic causes that tends to bring about simultaneous freshets in both systems. Furthermore, serious floods from typhonic causes are less frequent than those from continental depressions, which generally produce freshets in only one of the systems; but such freshets are again likely to synchronize with the Lo Ho and Chin Ho floods. When floods are produced by typhonic causes, however, the Lo Ho and Chin Ho floods run off first. An extreme flood at Shanchow caused by typhonic influences, therefore, is not likely to be seriously increased by run-off from the area down stream, such as those produced by continental depressions.

As far as can be judged the river is not very likely to have a flow exceeding 900 000 cu ft per sec at the Peiping-Hankow Railway Bridge. To provide for floods greater than this seems economically unsound. This expected maximum flood is about 100% greater than that estimated by earlier investigators. In a long-range forecast of possible maximum floods, perhaps, one should also take into consideration the probability of undertaking drainage-area improvement, such as erosion-control measures. If it proves feasible to reduce erosion to an extent that will decrease the silt content of the river in heavy floods, from 40% to approximately 10%, this alone will reduce a flood of 900 000 cu ft per sec to about 700 000 cu ft per sec by silt removal, not to mention the ameliorating effect which the improvement works themselves may have in retarding the surface run-off and in assisting the soil absorption. Considering the question carefully it seems that 900 000 cu ft per sec is a conservative value to adopt in future design for flood-control measures.

TABLE 2.—MAXIMUM ANNUAL FLOOD DISCHARGES AT SHANCHOW, IN CUBIC FEET PER SECOND

Year (1)	Discharge (2)	Year (1)	Discharge (2)	Year (1)	Discharge (2)	Year (1)	DISCHARGE, AT: Gage (2)	DISCHARGE, AT: Head of diked section (3)
1919	459 000*	1924	152 000	1929	399 000	1933	812 000	825 000†
1920	247 000	1925	459 000*	1930	247 000	1934	399 000	512 000*
1921	353 000*	1926	254 000	1931	265 000	1935	635 000	685 000†
1922	247 000	1927	212 000	1932	371 000	1936	477 000	547 000
1923	371 000	1928	159 000	....	....	1937	424 000	459 000*

* Dike breach. † Serious dike breach.

The maximum yearly flood discharge of the Yellow River, at Shanchow, for the years 1919 to 1937, inclusive, is shown in Table 2.

Flow Losses.—A matter of much importance for the regulation of the Yellow River is the loss by soil absorption, percolation, and channel storage, along the diked course. These losses are so important that they will figure definitely in the design for the capacity of a regulated channel. The river flows on a ridge with the water level higher, generally, than the plain on both sides; and, the plain itself slopes away from the river toward both sides, which further promotes percolation. Due to the constant dike breaches it is somewhat difficult to reach definite conclusions; but, from the available records some opinion may be formed. Such losses should be observed for various conditions of flow because evidently a high but flashy freshet, lasting scarcely more than 24 hr, is certain to suffer greater losses than another freshet with longer duration. Furthermore, if the river's foreshores are saturated by a recent flood, they will absorb less water than if the foreshores were dry. A silty freshet may or may not have heavier losses than a freshet that is less silt-laden. Heavy rainfall on the wide area between the dikes when a freshet passes down may partly offset losses. These are some of the many factors having a bearing on this question. The loss due to channel storage is a temporary one and its effect is to flatten the flood crest as it proceeds down stream. It is a detention effect only, without loss in volume.

Considering only the diked course from Lokou to Litsin near the mouth, a distance of 110 miles, the average monthly low-water loss in April, May, and June, 1935 (months when the flow was not affected by dike breaches) was 62, 82, and 54 cu ft per sec per mile of river, respectively. As the flow during these months averaged, at Lokou, 39 148, 39 783, and 43 525 cu ft per sec, and, at Litsin, 32 405, 30 886, and 37 736 cu ft per sec, the losses ranged from 13% to 22% of the flow at Lokou, which correspond to average losses in ordinary irrigation channels. Considering the elevated position of the Yellow River with reference to the plain the loss may be regarded as small. Due to complications of dike breaches in the upper course of the diked river and also due to seepage flow from the Shantung Mountains it has not been possible to arrive at any valuable results from that part of the river. The bed is much wider at that part and it is probable that the low-water losses are heavier than for the lower river.

During the flood season it is only to be expected that the flow losses will be relatively greater than during the low-water season. The wide foreshores become inundated and since, as a rule, they slope from the river banks toward the dikes, and usually are well cultivated, this feature will facilitate percolation losses. Much silt is deposited on the foreshores and for every cubic foot of settled silt about 0.7 cu ft of water will be held with the silt. For example, if there is a loss in silt discharge of 10 000 cu ft per sec (dry state) between two points on the diked river it means that, at the same time, 7 000 cu ft per sec of clean water is tied up with the deposited silt. Although small, this loss is a permanent one and may be regarded as part of the percolation losses.

On account of the wide distance between the dikes, the flattening out of flood crests by channel storage is always most marked from near the head of the diked course at the Peiping-Hankow Railway Bridge to the border of Shantung, a distance of 135 miles. As the dike distance narrows it becomes less marked along the next 70 miles to the Grand Canal crossing. (When there are dike breaches up stream, as in 1934, there will be a return flow in the vicinity of the Grand Canal crossing and this obscures the actual amount of loss.) Along the down-stream 185 miles, from the point where the return flow enters to the last hydrometric station at Litsin near the coast, the channel is much narrower and the average distance between the dikes is only slightly more than 1.25 miles against an average width of 4.5 to 5 miles up stream from the Grand Canal crossing. Along this relatively narrow channel, judging by the observed flow data and allowing for observation inaccuracies, the decreases in cresting flow due to channel storage will usually not be great. The lengthening and flattening of the flood crest by up-stream channel storage has already become so marked that any possibility for further modification is practically eliminated. In fact, due to the flattening of the slope, which is quite pronounced up stream from Litsin, there may be a slight increase of the cresting flow at times, due to the overtaking effect. Observations tend to show that this may take place. From these considerations it is evident that the point for determining what the down-stream channel capacity should be under regulated conditions lies somewhere near the Grand Canal crossing because, from this point down stream, there will be a tendency for the flow to remain relatively unaltered. The loss or gain that may occur along the course from the Grand Canal crossing to the sea does not seem to be large enough to warrant any alteration in channel section. The observed losses during flood periods in 1934, 1935, and 1936 have been plotted in Fig. 17.

Assuming that the losses are proportional to channel area some conclusions may be reached as to what the losses will be in a regulated channel of much smaller dimensions than the present unregulated channel. To convey a flow of 900 000 cu ft per sec the average regulated width between the new dike lines will be about 2.5 miles for the upper reaches of the river from the Peiping-Hankow Railway Bridge to the Shantung border and 2 miles from this place to the Grand Canal crossing. The present dike distances for these two stretches are about 6 miles and 4 miles, respectively. The diagram for the upper stretch, using a flood of 900 000 cu ft per sec at the Peiping-Hankow Railway Bridge, gives a flow loss of about 450 000 cu ft per sec and for the second stretch about 200 000 cu ft per sec. The proportional losses for the new channel would then be about 187 000 cu ft per sec and 100 000 cu ft per sec for the two stretches. For rainfall effect it would be conservative to allow only 150 000 and 80 000 cu ft per sec, or a total loss of 230 000 cu ft per sec between the Peiping-Hankow Railway Bridge and the Grand Canal crossing.

It was noted during the flood season of 1937 that prolonged, medium freshets of 300 000 to 350 000 cu ft per sec did not register as large losses as the more flashy freshets. This is to be expected, of course; it has a bearing on the regulated channel capacity for the condition when there are flood-control detention basins in the mountain area west of the plain. The maximum flow that is likely to enter the diked section in this case will probably not exceed 550 000 cu ft per sec, but it may last for more than 24 hr. Under this condition the flow loss between the Peiping-Hankow Railway Bridge and the Grand Canal crossing will probably not be more than 150 000 cu ft per sec and may even be as small as 100 000 cu ft per sec. This will be sufficient control, however, to prevent a flow of more than 450 000 cu ft per sec from passing into the reach below the Grand Canal crossing.

The writers realize that further investigations and study are necessary to solve more conclusively the dependable magnitude of flow losses along the diked channel; but they are of the opinion, nevertheless, that the present observations show that these losses are so consistently present, and so large, that they should be taken into consideration in a control and regulation plan for the diked course. In fact, if they are not taken into account it will result in an over-sized down-stream channel which will be difficult to maintain. Further investigations may show that the values adopted tentatively by the writers are too small.

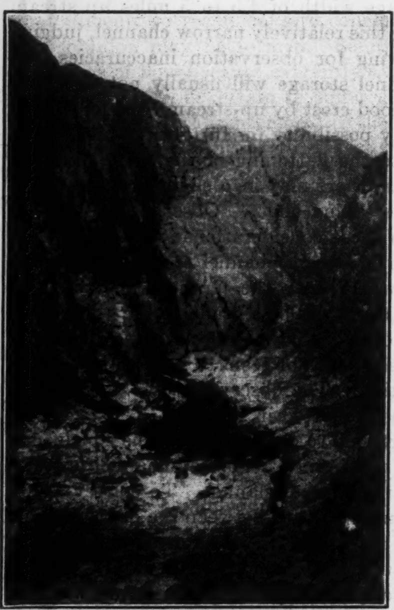


FIG. 18.—GORGE OF THE KING HO, SHENSI, AT INTAKE OF WEI PEI IRRIGATION PROJECT, WHERE MEASUREMENTS OF SAMPLES HAVE SHOWN AS HIGH AS 50% OF SILT BY WEIGHT

Silt Flow.—There is perhaps no important river in the world in which silt is so intimately related to every phase of conservancy work as the Yellow River. Whether it is flood control, irrigation, hydro-electric development, or navigation, the question as to how to overcome the evil effects of the silt overshadows all other technical considerations. This is due to the staggering proportions of the load carried. On other rivers a silt content of 1% or 2% by weight is considered a heavy and obnoxious charge which causes much trouble; but where the Yellow River enters the plain the silt load reaches 40% by weight at times. On several of its tributaries charges as high as 50% by weight have been observed (see Fig. 18). The silt percentage is expressed as: $\frac{\text{Weight of dried silt}}{\text{Weight of water and silt}}$; whereas, the customary method is to express it as: $\frac{\text{Weight of dried silt}}{\text{Weight of filtered water}}$. The first ratio is used in order to avoid such absurd silt percentages as 80%, 90%, or, at times, even 100 per cent.

For computing volumetric silt quantities it has been assumed that the silt density is that of the dried silt ordinarily found on the plain, the weight of which averages 101.5 lb per cu ft (1 600 kg per cu m); that is, its specific gravity is 1.6. Furthermore, from observations based on numerous experimental samples, the average specific gravity of the individual silt grains has been taken as 2.7. Using these values the relation between silt percentages and silt discharges can be developed. Fig. 19, which shows this relation, is used in preference to curves based on experimental data, due to the average condition which the computed curve represents.

Since a knowledge of quantities of silt is more useful than a knowledge of weights, the curve has been made to represent the volume of silt contained in a unit volume of river discharge corresponding to any observed percentage by weight, of silt, in the river discharge, assuming that the silt has been naturally deposited on the plain and has dried. The silt discharge is found by multiplying the number obtained in Fig. 19 by the observed discharge. This curve is very convenient for use when one has to convert silt percentage and discharge into silt discharge a considerable number of times. Most investigators use a conversion factor for computing silt discharge from silt percentages. This may be correct enough for very low silt charges; but, obviously, it gives incorrect results for higher silt percentages since the variation is not a straight-line relation.

Fig. 19 has been based on the assumption that the silt has been allowed to settle, drain, and dry naturally. Therefore, it should not be used for calculations involving silt deposits temporarily settled in flood-control detention basins. Silt settled in water has a volume from 5% to 20% greater than that which the curve gives and a density of 1.8 to 2.1, depending on the character of the silt, time of settling, and depth of the deposits. Loess silt and water become plastic when the percentage by weight reaches 56 to 58. Coarse silt, settled under water, attains a definite volume rather quickly, whereas fine silt continues shrinking for a long period. Hence, each silty stream on which flood detention basin control is contemplated must be especially investigated with regard to its silt-settling action.

In the Yellow River, as well as in other heavily silt-laden rivers of North China, it does not necessarily follow that a maximum flood will have the maximum silt concentration. It may have the maximum silt discharge, however. Several factors determine when the Yellow River will have its maximum silt concentration, some of which are: (*a*) Intensity of rainstorms and area covered by them; (*b*) place where intense rainstorms hit the drainage area; (*c*) dry or wet climatic cycles; (*d*) dilution effect from other parts of drainage area; and (*e*) heavy bed scour during serious freshets.

The most important factor in modifying the silt charge of the river's flow is perhaps the climatic cycle. The silt concentration in the flow has gradually diminished since 1934 and this is unquestionably due to the wetter cycle which began in 1931 and may continue for some years more. It has resulted in a better growth of bushes and grasses over the entire drainage area to such an extent that, during the dry winter and spring, the farmers have not been able to cut the bushes down or to scrape the grass and leaves away completely. This increased vegetation is now holding the soil more securely in place over areas large enough to have a marked influence upon both surface and gully erosion.

The bank erosion along the river channels has probably not been affected; but, in any case, bank erosion is a secondary factor among those which ordinarily contribute to the silt charge as it usually alternates with bank deposition all along the course. On the other hand, channel bed scour may be a very important factor in throwing large masses of bed silt into the stream. For example, in the reach up stream and down stream from Lungmen, in August, 1933, the river bed was deepened an average of more than 15 ft along a stretch estimated at 30 miles. It is unfortunate that no definite record of this extraordinary phenomenon exists other than that obtained from information after the flood. At Lungmen the bed was deepened approximately 30 ft over a width of 3 000 ft, tapering off in depth up stream and down stream. It has been estimated that a silt quantity approximating 7 000 000 000 cu ft was eroded from the river bed along this stretch in a time which cannot have greatly exceeded 12 hr. It demonstrates the capacity of the river to scour in spite of the high silt load it was already carrying. The average discharge during this period seems to have been about 300 000 cu ft per sec. Since this occurrence in 1933 the river bed has again been built up more than 12 ft. A part of this enormous silt quantity was redeposited in the wide river channel up stream from Tungkuan; but, at Tungkuan, and probably down stream toward Shanchow, the bed was again lowered 3 or 4 ft. Similarly, in the plain up stream and down stream from the Peiping-Hankow Railway Bridge, the bed was scoured nearly 6 ft by the same flood that deepened it at Lungmen. At these places the bed remained depressed after the flood, but was partly refilled during the spring of 1935. It seems quite possible that this extra silt load was at least partly responsible for the deposition in the plain which buried the north dike and caused overtopping and numerous dike breaks in August, 1933, below the 1851 break.

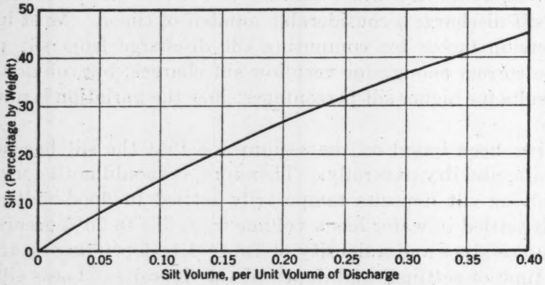


FIG. 19.—CONVERSION OF SILT PERCENTAGES TO SILT VOLUMES IN TERMS OF UNIT VOLUME OF DISCHARGE

However, such occurrences are infrequent. In fact, the inhabitants at Lungmen cannot remember that the river bed has ever been lowered so seriously before. They say: "The river lost its bed"; but old boat fastenings, and iron rings bolted to the rock and uncovered when the bed silt was removed, show that the normal river bed has once been deeper. This points to cycles of silty and less silty periods, which again means dry and wet periods. There is no other stretch along the river where such abnormal action can occur. Regulation of the Yellow River, from the gorge outlet at Lungmen to Tungkuan, with a flood-control detention basin up stream from Lungmen (as is advocated herein under the heading, "Flood Control and Regulation"), will remove, or at least minimize the effect of, such a flood as occurred in August, 1933.

Normally, however, the river receives its silt charge from surface and gully erosion of the loess lands in Kansu, Shensi, and Shansi, and to a lesser extent from Honan. The parts of the drainage area from which the river receives its silt are shown in Table 3.

TABLE 3.—PERCENTAGE OF SILT OBSERVED IN THE YELLOW RIVER

Month	Maximum	Minimum	Average	Maximum	Minimum	Average	Maximum	Minimum	Average	Maximum	Minimum	Average
	(a) PAOTOU			(b) LUNGMEN			(c) SHANCHOW			(d) RAILWAY BRIDGE*		
1934:												
January	....	....	....	....	....	....	0.35	0.30	0.32	0.44	0.24	0.32
February	....	....	....	....	....	....	0.56	0.29	0.40	0.81	0.36	0.49
March	....	....	....	....	....	....	1.22	0.40	0.82	1.50	0.49	0.74
April	....	....	....	....	....	....	3.15	0.62	1.13	1.90	0.52	0.91
May	....	....	....	....	....	....	2.30	0.69	1.22	1.60	0.74	0.98
June	....	....	....	....	....	....	3.05	1.02	1.75	2.81	0.58	1.38
July	2.55	0.17	0.89	38.00	1.98	8.20	20.16	2.37	6.86	9.42	1.14	5.46
August	2.35	0.88	1.48	36.22	1.70	6.36	38.14	3.03	9.01	18.75	2.78	7.88
September	2.13	1.08	1.47	6.38	0.97	2.35	8.14	2.23	3.77	6.84	2.02	3.58
October	1.96	0.99	1.38	2.57	0.89	1.52	7.62	2.77	4.04	8.15	1.73	4.03
November	....	....	....	1.52	0.43	0.82	2.71	1.39	2.13	3.46	1.13	1.99
December	....	....	....	0.70	0.07	0.27	1.85	1.20	1.62	1.77	0.67	1.19
1935:												
January	....	....	....	0.31	0.03	0.17	1.95	0.91	1.23	1.38	0.40	0.92
February	....	....	....	0.78	0.04	0.22	1.95	1.23	1.50	1.47	0.73	0.93
March	....	....	....	2.78	0.26	0.87	3.05	1.64	1.93	2.61	0.79	1.62
April	....	....	....	1.11	0.34	0.60	2.94	1.59	1.91	1.49	0.90	1.21
May	....	....	....	1.35	0.44	0.82	2.68	1.61	2.17	1.70	0.97	1.25
June	....	....	....	1.38	0.54	0.97	2.55	1.00	1.68	1.45	0.87	1.14
July	1.11	0.72	0.87	13.37	0.75	3.81	14.71	1.17	4.40	10.75	1.07	4.26
August	1.14	0.81	1.01	33.46	0.53	3.28	19.34	1.88	5.31	15.26	2.91	5.37
September	1.00	0.78	0.91	3.73	0.75	1.54	4.10	0.99	2.17	3.86	1.46	2.53
October	1.22	0.68	0.98	1.87	0.64	1.10	3.22	1.17	1.99	3.18	1.42	1.92
November	0.94	0.42	0.61	1.25	0.32	0.69	1.98	0.85	1.40	3.02	1.29	1.95
December	0.46	0.20	0.32	0.55	0.03	0.17	1.45	0.32	0.98	2.71	0.57	1.42
	(e) KAOTSUN			(f) TAOCHENGPU			(g) LOKOU			(h) LITSIN		
1934:												
January	....	....	....	....	....	....	0.25	0.07	0.14	....	....	....
February	....	....	....	....	....	....	2.05	0.06	0.40	....	....	....
March	....	....	....	....	....	....	0.79	0.19	0.44	....	....	....
April	....	....	....	....	....	....	2.93	0.55	1.08	....	....	....
May	1.12	0.51	0.70	2.06	0.82	1.72	2.21	0.61	1.30	....	....	....
June	2.33	0.79	1.40	3.03	0.63	1.30	4.07	0.50	1.83	....	....	....
July	6.25	0.87	3.93	7.52	0.90	4.23	6.35	0.58	3.66	5.53	0.52	3.41
August	11.42	2.54	5.58	12.26	3.22	6.24	10.97	2.35	5.69	10.92	2.47	6.43
September	5.43	1.40	3.28	5.65	1.89	3.10	5.86	2.34	3.51	5.43	1.73	2.29
October	5.12	1.74	3.21	3.47	1.69	2.53	4.15	1.68	2.66	5.36	2.10	2.92
November	2.87	0.63	1.75	3.41	1.14	1.95	2.12	0.68	1.26	2.29	0.52	1.27
December	1.74	0.41	0.65	2.13	0.09	0.46	1.00	0.20	0.40	0.63	0.17	0.35
1935:												
January	0.72	0.19	0.39	0.44	0.08	0.22	0.38	0.11	0.22	0.23	0.10	0.17
February	0.49	0.15	0.30	0.31	0.08	0.16	0.31	0.09	0.18	0.16	0.11	0.14
March	1.34	0.32	0.66	1.73	0.13	0.51	1.62	0.27	0.50	1.14	0.15	0.39
April	1.53	0.48	1.07	2.68	0.22	1.13	1.77	0.32	0.89	1.33	0.33	0.75
May	1.68	0.84	1.18	1.94	0.77	1.25	1.86	0.67	1.31	1.60	0.50	1.02
June	2.59	0.85	1.17	3.69	1.10	1.75	2.17	0.96	1.38	2.03	0.95	1.33
July	8.22	0.40	2.48	5.29	0.81	2.27	5.48	0.67	1.77	6.01	0.89	2.28
August	10.19	0.62	3.60	9.56	1.34	3.21	5.32	0.45	1.41	4.48	0.61	1.68
September	3.57	0.91	1.93	6.44	2.05	3.60	2.38	0.85	1.35	2.81	0.93	1.64
October	2.55	1.08	1.60	4.07	1.23	2.82	1.55	0.72	1.02	2.04	0.43	1.17
November	2.57	0.85	1.68	1.96	0.21	0.74	0.81	0.06	0.29	0.62	0.06	0.24
December	2.00	0.05	0.71	0.39	0.02	0.18	0.13	0.03	0.06	0.10	0.02	0.04

* Peiping-Hankow Railway Bridge.

TABLE 4.—CHANGES IN SILT CONCENTRATION ALONG THE COURSE OF THE YELLOW RIVER
(Percentage of Silt by Weight)

(a) OBSERVATIONS IN 1934

Date (1)	Shan-chow (2)	Chin-chang (3)	Kao-tsun (4)	Tao-cheng-pu (5)	Lo-kou (6)	Lit-sin (7)
July 6	5.25	3.83	2.35	2.45	2.98	1.74
	4.30					
7	3.77	3.59	2.68	2.23	2.55	1.93
	5.35	3.89				
8	5.47	3.91	2.98	2.12	2.20	2.41
	9.47					
	13.14					
9	14.38	7.93	2.74	2.24	2.01	1.08
	14.93					
10	14.63	8.37	5.27	3.66	2.57	2.14
	13.28					
11	10.42	8.62	4.26	7.52	3.36	2.15
	9.02					
12	8.56	8.23	2.77	5.18	6.35	3.70
	8.39					
13	6.92	7.49	4.80	4.59	5.12	4.98
	5.09					
14	3.93	7.44	5.32	4.70	4.20	4.69
	5.01					
15	2.84	8.17	6.25	5.28	5.04	3.66
	3.40					
16	3.88	5.68	5.53	5.28	4.66	3.45
	4.73					
17	5.87	3.91	6.08	6.13	4.66	3.62
	5.98					
18'	3.89	4.16	4.91	5.50	4.52	4.24
	10.99					
19	9.38	4.62	3.86	5.06	5.13	4.55
	11.60	5.44				
20	14.66	5.22	3.42	4.46	5.05	4.51
	20.16					
21	9.87	7.79	3.97	4.95	4.81	5.04
	8.78					
22	8.15	9.42	4.62	4.74	4.76	4.92
	9.54					
23	8.32	8.16	5.56	5.10	4.64	4.62
	7.32					
24	6.81	6.86	6.23	4.56	4.44	4.28
	6.45					
25	3.36	5.28	5.59	5.15	4.74	4.30
	5.11					
26	4.36	6.05	4.99	4.78	5.59	4.35
	7.71					
27	5.90	5.11	4.33	4.27	3.95	5.52
	5.79					
28	5.44	4.58	4.94	5.14	4.54	3.95
	4.21					
29	3.75	5.25	4.33	5.67	4.39	4.32
	4.21					
30	3.25	3.93	4.00	6.42	3.53	4.34
	3.03					
31	3.89	4.57	3.29	6.00	3.78	4.33
	2.45					
Aug. 1	3.29	3.39	2.88	3.22	3.02	3.72
	3.03					
2	4.50	2.78	3.33	3.39	3.04	3.57
	4.95					
	4.21					
3	6.96	3.98	2.54	4.03	2.43	3.05
	6.09					
4	6.36	4.78	3.08	....	2.35	2.47
	8.45					
5	7.80	7.10	3.30	4.35	3.10	2.75
6	6.23	9.55	3.72	4.54	3.91	3.04
	8.28		3.93			
7	7.71	8.89	4.96	4.13	3.71	4.01
			5.26			
8	6.96	6.22	5.30	5.40	3.61	4.33
	4.23		5.10			

(b) OBSERVATIONS IN 1935

Date (1)	Shan-chow (2)	Chin-chang (3)	Kao-tsun (4)	Tao-cheng-pu (5)	Lo-kou (6)	Lit-sin (7)
July 6	2.44	1.47	1.48	....	....	....
	2.65	1.48	1.44			
7	2.48	....	1.39	....	0.95	1.07
	2.81		1.55		1.10	0.95
8	5.10	....	1.78	....	1.02	0.98
	4.91		2.46		1.18	1.02
9	4.31	5.62	4.12	....	1.20	....
	3.93	4.93	3.77		1.21	
10	3.41	4.22	1.61	....	2.06	1.60
	3.30	3.68	0.95		2.12	2.07
11	2.86	2.54	0.40	....	4.26	2.07
	1.98		0.57			5.01
12	3.10	2.36	0.80	....	5.48	4.79
	2.50	2.02	1.31		3.24	6.01
13	3.22	1.72	1.57	....	1.82	3.53
	4.01	2.38	2.74		1.53	4.69
14	5.40	2.53	3.32	....	1.29	2.39
	14.00	2.90	2.43		0.96	1.95
15	14.71	4.85	2.66	....	0.83	1.33
	11.27	5.25	2.44		0.74	1.43
16	11.84	....	4.24	....	0.76	1.41
	10.45	10.71	8.22		0.67	1.31
17	4.97	9.91	6.57	....	0.91	1.48
	5.35	10.41	6.64		1.56	1.57
18	5.06	8.47	6.41	....	1.79	1.78
	5.89		6.20			1.78
19	7.13	7.16	4.23	....	2.56	1.89
	10.44	7.58	5.22		2.33	2.47
20	7.72	6.93	4.17	....	2.60	3.57
	4.84	7.01	3.60		2.90	3.63
21	4.87	8.89	4.20	....	2.44	3.40
	3.38		4.46		2.85	4.01
22	4.00	7.16	3.60	....	3.29	3.50
	3.86	5.32	6.29		2.87	3.47
23	4.18	4.66	3.90	....	2.55	3.48
	3.83	3.93	4.51		3.04	3.90
24	3.02	3.79	3.41	....	2.36	3.15
	3.48	3.84	3.47		2.11	3.60
Aug. 3	2.43	4.39	5.47	2.25	1.50	1.64
	1.94	4.85	4.41	2.79	1.33	1.62
4	3.63	4.65	3.53	2.60	1.45	1.64
	4.43	3.59	3.78	2.84	1.36	1.44
5	5.87	3.22	1.80	3.80	1.52	1.62
	5.85	3.35		3.60	1.71	1.62
6	6.27	3.76	2.63	3.07	1.68	1.70
	3.71	3.58	3.59	2.45	1.63	2.13
	7.63					
7	11.65	6.21	2.87	2.92	1.71	2.07
	17.55	5.39	3.11	2.43	1.20	
8	19.20	6.02	1.35	2.32	1.16	1.89
	19.54	7.20		2.74	1.24	1.67
9	14.08	13.13	8.75	4.51	1.17	1.63
	10.10	12.90	3.77	5.57	1.30	1.77
10	9.98	13.41	10.19	6.10	1.94	1.50
	7.21	15.26	4.57	9.56	2.40	2.05
11	6.41	10.86	6.00	3.49	3.65	2.67
	5.15	11.53	5.76	3.49	4.32	3.26
12	3.97	7.64	6.36	3.08	2.66	3.73
	3.50	7.98	6.19	3.61	2.57	4.48
13	3.66	5.73	6.36	2.59	2.19	3.27
	4.12	5.86	8.09	2.32	2.16	3.13
14	3.81	4.07	4.11	2.28	1.43	2.14
	3.97		3.22	2.08	1.35	1.60
15	2.52	4.11	4.48	2.16	0.91	1.86
	1.98	4.25	2.79	2.05	1.01	2.42
16	2.41	4.54	3.26	1.81	0.98	2.11
	1.88	3.10	3.72	1.96	0.91	1.39
17	2.13	3.45	3.59	1.78	0.74	1.24
	2.08		2.84	1.90	0.87	1.66

TABLE 4.—*Continued*

Date (1)	Shan-chow (2)	Chin-chang (3)	Kao-tsun (4)	Tao-cheng-pu (5)	Lo-kou (6)	Lit-sin (7)
(*a*) OBSERVATIONS IN 1934 (*Continued*)						
Aug. 9	20.90 16.41	7.55	4.80 4.82	5.15	4.69	4.46
10	18.64	10.41	6.07 5.86	6.25	6.14	5.30
11	31.82	16.65	7.13 5.48	5.54	4.96	5.10
12	21.21 20.88	15.22	8.95 8.44	10.22	5.14	5.14
13	24.21 38.14	15.77	6.56	12.26	7.88	5.31
14	16.11	18.75	4.19 4.88	9.61	8.66	8.35
15	10.66 6.83	15.89	7.87 4.36	8.83	8.42	10.70
16	5.52 4.55	10.55	3.89 10.60	10.67	7.74	9.40
17	5.88 4.73	6.97	11.42 8.89	9.65	10.97	9.58
18	3.17 5.02	6.75	9.19 7.63	8.83	10.11	10.92
19	5.56 5.25	6.49	5.56 6.04	6.41	7.67	9.46
20	3.57 3.86	4.59	5.18 5.62	6.42	7.15	9.07
21	11.52 9.61	2.82	5.20 4.62	5.96	6.94	9.00
22	8.16 9.96	4.84 5.04	4.30 4.71	4.51	5.03	9.00
23	7.44 5.81	7.18	5.89 5.08	6.29	5.62	9.03
24	4.63 4.52	8.09 8.46	4.66 5.35	6.56	6.17	9.00
25	6.82 7.16	6.05 6.05	6.59 6.32	4.79	4.90	8.84
26	11.48 11.09	5.86	5.89 5.06	6.03	6.23	8.73
27	8.60 7.09	7.32 6.40	5.28 6.06	4.51	6.33	8.55
28	6.15 6.32	9.34 7.60	5.77 5.32	5.77	4.90	6.07
29	6.66 7.42	5.76	6.36 6.38	5.62	5.05	5.14
30	5.83 6.90	5.10 5.03	5.46 4.30	5.36	5.59	5.46
31	5.13 4.00	5.82	4.04 4.61	4.08	5.86	5.06
Sept. 24	2.35 2.31	2.81 3.98	2.83 2.67	3.10 2.54	2.34 2.86	2.34
25	2.34 2.56	2.49 2.04	2.36 2.01	2.70	3.24 2.85	2.59
26	2.48 2.23	3.20	1.40 1.90	2.56	2.82	2.55
27	2.23 2.23	3.52 3.52	1.64 2.03	2.79	2.84 2.77	2.35
28	3.17 3.13	2.73 2.57	2.14 1.98	2.40	2.71 2.95	1.73
29	3.05 3.14	3.97	2.08 1.66	2.77	2.53 2.47	2.13
30	2.81 2.37	2.86	2.19 2.31	1.89	2.79 2.70	2.44
Oct. 1	2.80 3.23	3.42 3.16	2.25 2.83	2.92	2.74 2.93	2.40
2	4.81 5.98	3.12 2.69	4.85 3.55	3.07	2.71 2.70	2.30
3	7.87 7.07	6.85 7.47	2.85 2.82	2.91	2.94 2.80	3.11
4	7.52 6.61	4.98 5.48	4.19 4.63	3.21	2.83 3.32	2.80
5	5.93 5.27	8.15	4.62 4.37	3.47	3.58 4.15	2.69
(*b*) OBSERVATIONS IN 1935 (*Continued*)						
Aug. 18	1.95 3.26	3.48 3.36	2.27	1.73 1.90	0.63 0.45	1.13 0.96
19	3.05 2.46	3.18 3.47	2.26 1.94	1.64 1.34	0.72 0.71	0.84 0.78
20	2.31 2.66	3.14 3.15	2.16	1.44 1.36	0.59 0.61	0.79 0.86
21	3.55 3.15	3.53 3.39	2.78 2.28	1.57 1.59	0.63 0.54	0.96 0.66
22	2.90 4.19	3.36 3.39	2.51 2.04	2.42 1.72	0.65 0.54	0.61 0.79
23	3.79 3.77	5.29 5.66	1.77 1.86	2.14 1.94	0.54 0.76	0.69 0.67
24	4.65 4.73	2.99 3.52	2.81	2.50 2.67	0.64 0.64	0.67 0.63
25	4.75 5.83	....	4.82 3.28	2.69 2.16	0.68 0.77	0.73 0.89
26	6.01 5.68	2.91 3.65	2.22 2.88	5.56 3.82	1.09 1.51	0.87 1.09
27	5.21 5.08	6.30 4.48	3.82	3.60 3.60	1.31 1.71	1.23 1.26
28	3.63 4.05	6.94 4.91	3.52 2.96	6.89 5.42	2.01 1.83	1.54 1.57
29	5.74 6.84	3.85	3.23 2.31	6.65 5.75	2.36 2.26	1.95 2.29
30	7.88 7.88	3.74 4.55	2.99 1.69	5.13 4.57	2.11 2.26	2.52 2.90
31	4.74 3.43	4.13 3.70	0.62 2.37	4.75 4.18	1.79 1.98	2.47 2.83
(*a*) OBSERVATIONS IN 1934 (*Continued*)						
Oct. 6	5.03 3.89	4.78 5.88	5.12 4.51	2.40	3.46 3.11	4.05
7	3.31 3.22	5.27 4.37	4.42 4.18	2.36	3.46 3.47	3.80
8	3.23 3.86	5.59 3.72	3.53 3.77	2.30	3.11 2.91	3.79
9	3.46 3.31	2.99 2.48	3.61 3.96	2.58	2.71	2.99
10	3.62 3.25	3.37	3.15 3.03	2.50	2.98 3.12	2.80
11	3.05 2.95	3.18 3.01	3.07 3.22	2.44	3.14 3.24	2.26
12	3.35 3.31	2.85 3.97	2.71 2.83	2.67	2.27 2.39	2.93
13	5.10 3.78	3.02 3.53	2.91 1.74	2.60	2.33 2.56	2.64
14	2.79 2.77	4.59 5.58	2.25 2.38	2.39	2.93 2.87	2.83
15	3.23 3.28	4.35 2.86	2.34	2.49	2.32 2.46	2.91
16	4.50 5.84	5.33 5.22	3.44 3.83	3.08	2.48 2.61	2.22
17	4.45 3.48	4.73 5.24	3.52 3.93	2.84	2.42 2.65	2.99
18	3.22 3.30	3.11 4.22	3.29 2.65	3.47	2.65 2.56	2.95
19	4.28 4.34	4.37 4.69	2.61 3.34	2.40	3.18 3.11	5.36
20	4.25 5.27	3.53	2.90 2.70	2.35	2.81 2.67	3.19
21	4.13 3.79	4.26 4.20	2.90 2.49	2.35	2.64 2.38	2.99
22	3.39 3.79	2.03 1.79	2.83 2.59	2.42	2.60 2.56	2.84
23	3.65 3.67	2.64 3.42	2.73 2.51	2.51	2.67 2.91	2.50

When comparing the data in Table 3 it is necessary to consider the effect of the dike breach at Kuantai in 1934 and at Tungchuang in 1935. Although the Kuantai breach had no appreciable effect in reducing the silt concentration during the flood season at the stations down stream, it had much effect in reducing them during the low-water season that followed, on account of the clarified outflow from the flooded areas north of the river between the inner and outer dikes in Shantung. The Tungchuang breaches, on the other hand, caused considerable reduction in the silt concentration down stream immediately after the breaches had taken place, due to much deposition between Tungchuang and the Grand Canal crossing and to the clarified flow that re-entered the river at Chiangkow below the Grand Canal crossing.

The daily silt percentages during the freshet season in 1934 and 1935, as they were observed at the hydrometric stations from Shanchow down stream, are listed in Table 4. The sudden increases in silt concentration during freshets, and its gradual diminution and tapering off as the flow passes down stream, is very striking. Table 5 gives interesting silt records gathered in 1934 and 1935 on the Wei Ho.

It has been observed that the river is quite capable of conveying a 12% silt-laden flow of 200 000 cu ft per sec from the Grand Canal crossing to the sea, without any marked deterioration of the channel in that stretch, in spite of the fact that the slope is materially flatter than up stream. It did so during the August, 1934, freshet. The records of this flood are shown in Table 6.

As noted heretofore, the maximum silt concentration does not always occur at the same time as the maximum discharge. Usually, it comes some time after the crest has passed. Quite frequently, however, the maximum silt discharge occurs at the same time as the maximum stream flow. The silt percentages that coincided with the cresting flow are shown as Item No. 3, in Table 6. Actually, the discharge that conveyed the heaviest silt concentration down the diked channel was smaller than the cresting flow. The channel that had been scoured was refilled during the passage of the crest, but not so much as to leave it in any worse condition than during the freshet, except in the reach immediately down stream from the Kuantai breach. It is unfortunate that dike breaks have obscured, somewhat, the happenings along the middle reaches of the diked section. Otherwise, it would have been possible to interpret more accurately the normal action of the river, for instance, with regard to the scour and refill action between the hydrometric stations. All one can conclude for the present is that, at times, it seems deposition can occur in the channel between the hydrometric stations during freshets. Table 4 (*b*) indicates that this perhaps took place between Lokou and Litsin during the August, 1934, freshets, but not to any great extent. The recorded maximum discharge at Litsin at that time is probably too small, and this again would affect the silt discharge. The silt concentration in the flow shows no appreciable change.

The average monthly silt discharges for 1934 and 1935, as they have been computed for all the hydrometric stations along the river below Shanchow, are given in Table 7; the total yearly silt volumes passing the principal hydrometric stations along the Yellow River and the Wei Ho for the same two years, are given in Table 8.

TABLE 5.—PERCENTAGES OF SILT RECORDED BY STATIONS ALONG THE WEI HO

Year	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
(*a*) TAIYIN												
1934:												
Maximum	....	....	....	....	....	41.32	41.76	30.21	5.99	4.37	0.29	0.13
Minimum	....	....	....	....	....	0.01	0.34	0.42	0.17	0.38	0.08	0.06
Average	....	....	....	....	....	6.52	8.83	7.13	1.60	2.07	0.15	0.09
1935:												
Maximum	0.10	0.34	0.89	2.26	22.35	30.48	36.76	33.46	5.68	0.42	0.15	0.09
Minimum	0.03	0.03	0.30	0.92	0.04	0.08	0.09	1.00	0.10	0.03	0.03	0.02
Average	0.07	0.16	0.59	0.73	1.95	5.53	9.97	10.26	0.98	0.14	0.07	0.04
(*b*) HSIENYANG												
1934:												
Maximum	0.07	0.07	0.21	0.59	12.83	30.20	32.10	26.80	2.36	6.33	0.20	0.11
Minimum	0.05	0.05	0.09	0.01	0.12	0.19	0.68	0.44	0.28	0.19	0.07	0.06
Average	0.52	0.76	1.63	0.58	2.39	4.10	8.11	7.46	0.99	1.78	0.14	0.08
1935:												
Maximum	0.09	0.10	0.44	0.91	3.08	9.87	26.77	32.30	3.83	0.25	0.24	0.10
Minimum	0.02	0.03	0.13	0.25	0.24	0.51	0.40	0.53	0.12	0.08	0.05	0.01
Average	0.05	0.05	0.20	0.42	0.67	1.95	4.89	7.06	0.84	0.16	1.04	0.05
(*c*) HUACHOW*												
1935:												
Maximum	....	....	0.38*	0.98	1.82	9.36	29.31	26.58	6.27	0.38	0.26	0.19
Minimum	....	....	0.17*	0.29	0.17	0.32	0.24	0.15	0.22	0.08	0.13	0.07
Average	....	....	0.30*	0.50	0.46	1.97	5.27	6.97	1.03	0.18	0.17	0.13

* This station was established in the middle of March, 1935; the March records include only the last half of the month.

The average flow volumes and silt volumes for the periods of the more important freshets occurring in 1934 and 1935 for the three stations Shanchow, Chinchang, and Kaotsun, have been listed in Table 9, in order to give some idea of quantities to be handled in the flood-control detention-basin projects. It will be seen that more than 10% of the flow volume can be silt. The Wei Ho (Huachow Station) measurements in Table 8 do not include the flow of the Pei Lo Ho, which enters the Wei Ho immediately up stream from the confluence

TABLE 6.—REDUCTION IN SILT FLOW, AUGUST, 1934, FLOOD, ALONG THE DIKED CHANNEL

Item No.	Description	Shanchow	Chinchang	Kaotsun	Taochengpu	Lokou	Litsin
1	Maximum stream flow, in cubic feet per second	399 000	494 000	300 000	184 000	191 000	138 000*
	Percentage of Silt:						
2	Maximum value	38.14	18.75	11.42	12.26	10.97	10.92
3	Value of time of maximum discharge	21.00	15.00	6.00	8.50	10.50	10.00
	Maximum Silt Discharge:						
4	In cubic feet per second	60 000	52 000	11 800	10 400	13 400	9 000*
5	Expressed as a percentage of maximum stream flow	15.0	10.5	3.9	5.6	7.0	6.5

* Discharge value is questionable due to poor hydrometric equipment.

TABLE 7.—Average Monthly Silt Flow of the Yellow River, in Cubic Feet per Second

Hydrometric station	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
(a) 1934 Observations												
Shanchow	30	51	127	230	201	391	1 831	9 362	2 227	3 662	880	259
Chinchang	30	56	111	218	208	322	1 564	9 143	2 983	3 726	866	231
Kaotsun*	10	40	60	200	159	365	1 098	4 464	2 481	2 261	627	93
Taochengpu*	14	125	65	280	196	267	1 208	5 420	2 395	2 151	747	105
Lokou	15	112	67	273	185	447	1 114	5 454	3 524	2 711	751	136
Litsin†	20	140	80	270	180	440	885	4 935	2 687	2 462	937	174
(b) 1935 Observations												
Shanchow	178	208	467	446	615	478	3 028	7 731	1 680	1 334	576	151
Chinchang	171	131	296	244	324	283	4 625	6 015	2 517	1 723	909	323
Kaotsun*	25	11	78	235	332	315	2 142	3 897	1 736	929	707	228
Taochengpu*	26	19	131	246	336	494	1 256	1 103	937	638	44	11
Lokou	30	25	128	239	376	415	1 101	950	570	307	61	4
Litsin†	47	33	109	156	207	298	1 529	1 093	690	352	36	5

* From January to April, inclusive, the flow is estimated.
† Estimated from January to June, inclusive.

point with the Yellow River. Furthermore, the silt flow of the Fen Ho, which enters the Yellow River from the east below Lungmen, is not shown in Table 8. The silt flow of these two rivers will practically account for the rather large discrepancy of 7 500 000 000 cu ft by which the Shanchow Station measurements exceed the sum of those of the Huachow and Lungmen Stations.

Scour and Refill of the Yellow River.—It has been observed that some rivers, flowing in alluvial beds of their own making, tend to scour their beds when the discharge is increasing and again refill them upon a decreasing discharge. In America the Lower Colorado River is an outstanding example of such a self-scouring river. On the other hand, the tendency of the Mississippi River and the Yangtzekiang seems to be to raise their beds when the flow is increasing (at least, at the crossings) and to scour the deposited material when the river is falling. The causes of this difference in behavior are not readily determinable, but the character, quantity, and flow characteristics of the bed material, and of silt in suspension, may be some of the factors.

TABLE 8.—Annual Silt Volumes of the Yellow River and Wei Ho, in Cubic Feet

Hydrometric station: (1)	1934 (2)	1935 (3)	Hydrometric station: (1)	1934 (2)	1935 (3)
Paotou	7 413 000 000	7 095 300 000	Chinchang	51 722 000 000	46 656 890 000
Lungmen	29 387 000 000	26 409 100 000	Kaotsun	31 552 720 000*	28 220 590 000
Huachow			Taochengpu	34 372 580 000	13 879 890 000‡
(Wei Ho)	....	10 991 940 000	Lokou	39 191 910 000	11 147 350 000
Shanchow	51 263 550 000	44 911 350 000	Litsin	34 987 040 000†	12 083 430 000

* Breach at Kuantai, between Chinchang and Kaotsun.
† The measurements for Litsin are probably somewhat low; January to June, inclusive, in 1934, are estimated quantities (see Table 7).
‡ Breach at Tungchuan on July 10, 1935, between Kaotsun and Taochengpu.

TABLE 9.—COMPARISON OF SILT DISCHARGE DURING FLOODS

DATE From: (1)	 To: (2)	Time, in days (3)	Silt flow, in cubic feet (4)	Total stream flow, in cubic feet (5)	Silt flood, expressed as a percentage of total stream flow (6)
		(a) SHANCHOW HYDROMETRIC STATION			
July 7, 1934	July 12, 1934	5.00	1 623 800 000	23 177 980 000	7.0
Aug. 9, 1934	Aug. 17, 1934	8.00	16 687 793 000	153 865 774 000	10.8
Oct. 1, 1934	Oct. 8, 1934	7.00	4 066 560 000	110 164 593 000	3.7
July 6, 1935	July 11, 1935	5.50	1 255 586 000	51 771 651 000	2.4
Aug. 6, 1935	Aug. 11, 1935	4.75	11 311 638 000	118 195 855 000	9.6
		(b) CHINCHANG HYDROMETRIC STATION			
July 8, 1934	July 13, 1934	5.50	1 141 443 000	20 067 344 000	5.7
July 8, 1934	July 17, 1934	9.00	15 694 715 000	152 496 000 000	10.3
Oct. 1, 1934	Oct. 9, 1934	8.00	4 565 737 000	120 624 336 000	3.8
		(c) CHUNGMOU HYDROMETRIC STATION			
July 7, 1935	July 13, 1935	5.50	2 970 195 000	100 342 368 000	3.0
Aug. 7, 1935	Aug. 15, 1935	8.00	7 622 170 000	140 448 816 000	5.4
		(d) KAOTSUN HYDROMETRIC STATION			
July 9, 1934	July 13, 1934	4.25	657 921 000	21 380 504 000	3.1
Aug. 9, 1934	Aug. 19, 1934	9.75	5 746 840 000	132 671 520 000	4.3
Oct. 2, 1934	Oct. 10, 1934	8.00	2 654 560 000	97 597 440 000	2.7
July 8, 1935	July 15, 1935	6.50	1 355 520 000	127 840 997 000	1.1
Aug. 8, 1935	Aug. 18, 1935	9.50	5 841 444 000	170 490 528 000	3.4

The Yellow River and most of the other silt-bearing rivers of North China exhibit the same characteristics as the Colorado River, even to a greater extent. In some places the Yellow River has been observed to enlarge its net cross-section, during the freshet season, to more than three times the area it has during the low-water season. However, the question (among many others) arises as to whether this river will stop its scouring, or even begin to deposit its load, when an extremely severe, silty freshet occurs. It is a non-regulated stream without any fixed bed width, being extremely wide and braided in many sections and very narrow in others, and its slope through the diked section varies considerably. It is not easy to predict how it would behave if it were to be regulated and made to flow in a bed of fixed width. The phenomenon of scour and refill, its magnitude, characteristics, dependability under all conditions of silty freshets, and the relation this phenomenon has to the silt transportation, therefore, seem to be of sufficient importance to warrant an investigation as to how far it will influence any regulation plans.

In order to examine the scour and refill action of the river, use has been made of the collected hydrometric data in order to decide whether some conclusions could be drawn. In general, the data gathered to date (1938) show a rapid channel enlargement and deepening upon the rising stage of a freshet and a partial refill immediately after the cresting flow has passed and the flow is receding. After the freshet season has passed there will be a further gradual refill and by the end of the year the bed will have more or less the same elevation position it had before the freshet season began in the middle or end of June or the beginning of July, as the case may be. However, the reach up stream and down stream from the 1851 break is an exception. Here the river bed has been rising definitely during the five years, 1932–1937. In other respects, the scour and refill action is less pronounced in the up-stream reaches than in the down-stream reaches, but local conditions seem to have a considerable bearing on the magnitude of the phenomenon.

When the writers were trying to interpret the data highly dubious cases arose due to lack of cross-section measurements taken just at the time when important changes occurred in the river bed. Due to the long distances between the stations it has been difficult to correlate the happenings at one station with the happenings at the next station; and questions regarding the influence of the character of the bed silt and suspended silt also arose. At times, erosion and refill of the river bed occurred in a manner that could not be explained from the magnitude of the freshets, changes in velocity, the accompanying silt charge, or the rapidity with which the river rose or fell. Furthermore, the condition in the delta seemed to have a strong influence on the elevation of the river bed in the lower reaches. When the delta channel was favorable, the adjacent up-stream channel was deep, and the data tended to show that this increased depth propelled itself up stream by back-cutting until the influence several months later was noted as far up stream as the Grand Canal crossing. This conclusion needs verification by a better correlation of observations, however. The necessary "machinery" for all the investigation work had been set up, but had to be canceled on account of the war.

Some studies have been made of the variation in the effect of silty discharges on scour and refill by plotting silt discharge against total discharge on logarithmic paper for the conditions: (*a*) When the river is scouring its beds; (*b*) when it remains unaltered; and (*c*) when it is silting up. The data, although falling somewhat scattered on the graph, nevertheless show distinct straight line logarithmic relations in all three cases. The lines tend to show that the up-stream reaches of the river at low-water stages are more efficient for conveying silt without channel deterioration than the lower reaches; whereas, for a discharge of more than 90 000 to 100 000 cu ft per sec, the lower reaches become more efficient. These are relatively deep and narrow whereas the upper reaches are shallow, wide, and often much braided.

Variation in slope along the diked course does not seem to have any very decided influence on the capacity of this river to transport its silt or to scour its bed. Perhaps these phenomena may be determined more by the character of the silt (in the bed and in suspension), by the shape of the cross-section, and by the rapidity with which the flood flow increases—that is, by the accelerating force of the flow. Furthermore, the rate of refill seems to depend to a great extent on the rapidity with which the discharge decreases: Rapid decrease means a rapid refill, and *vice versa.* The steady refill back to normal low-water condition in the autumn is probably due to a slow "geschiebe" or bed-load movement when material deposited up stream is gradually moved down stream.

There has not been a chance to observe the scour and refill action during an intense, highly silt-laden freshet; but as far as has been observed a medium-sized freshet, silt-laden to the extent of 18% in the up-stream reaches and 12% in the down-stream reaches, does not seem to have its scouring capacity decreased materially. Higher silt concentrations may show detrimental effects, however.

These are the general conditions. The entire aspect is highly complex, and any silt-transportation and bed-scour formula to cover such multitude of unknowns, does not seem of much practical use. It is realized that more accurate and more specialized data are required to clarify the scour and refill problem. Data should be secured simultaneously at many places along the river with the sole purpose of obtaining the needed information. The river scours and refills so rapidly during freshets that special observation equipment must be organized for such study, including that of rapid silt analysis. Furthermore, the places where the observations are taken should be selected judiciously and should be numerous enough to eliminate possible local effects.

For the time being all that the writers can say with regard to the scour and refill which occur along the diked course of the Yellow River is that such actions do take place at all the five hydrometric stations along this stretch. However, the hydrometric stations have been placed where the river is relatively narrow and stable; no data have been obtained from places where the river is wide and braided, or at typical "cross-overs" between bends. It is possible that at such places there is little or no scour. There may even be a slight fill during the rising stages of a freshet. The possibility of such happenings should not be ignored in the design for a regulated channel. The writers, therefore, advocate cautiousness when it comes to deciding upon widths of a regulated channel for the diked course of the Yellow River.

The laboratory experiments in Germany (reported herein under the heading, "Scope of Investigations: Studies in Recent Years"), although illuminating, are not sufficiently conclusive to warrant any other description than "further experiments."

The gradual refill, especially in the lower reaches, which has been observed to take place in the autumn after the freshet season and which causes the river bed to be brought back to where it was before the beginning of the freshet season, may or may not occur if the channel were to be regulated. The bed-load may then be moved along the bottom more evenly through the diked channel and not, as at present, in fitful stages, first by deposition in the wide up-stream bed and then in another stage down stream. In the reaches up stream and down stream from the 1851 break the river is still adjusting its bed and this also obscures what the actual action would be in the lower reaches if the entire river profile were based on the delta at the sea as the main depositing area for the river through the plain.

FLOOD CONTROL AND REGULATION

Methods of Decreasing Flood Hazards.—As the problem of decreasing flood hazards along the Yellow River through the Great Plain is now analyzed it appears that the remedy should be far more extended than merely to construct greatly strengthened dikes. The remedies that should be considered carefully are the following:

(1) Detention basins west of the Great Plain to arrest temporarily, and to "iron out," the cresting flow of maximum floods;

(2) Spillway flow into basins on both sides of, and near, the river in West Shantung;

(3) A by-pass channel following the general course of the Tu Hai Ho from the Grand Canal crossing to the sea north of the river;

(4) Earth dikes and their protective works;

(5) Channel regulation and fixing a selected course;

(6) Control of soil erosion in Shansi, Shensi, Kansu, and parts of Honan; and,

(7) Control of run-off by increasing the brush and forest cover in Shansi, Shensi, Kansu, and parts of Honan.

Detention Basins West of the Great Plain.—The writers have come to certain tentative conclusions as to the value of detention basins along the main Yellow River above the delta, on several of the feeders from Shensi and Shansi, as well as on the Lo Ho and Chin Ho that enter in Honan at the beginning of the plain. Actual preliminary studies of the gorges below Shanchow were made under the direction of one of the writers in 1935 and 1936. In a paper[8] dealing with flood-control detention basins for the Yellow River, it is suggested that basins in comparatively narrow, steep reaches of the main river are more likely to be successful than any that may be created on the tributaries, due to a more sustained flow in the main river to wash out silt accumulation immediately after the passage of a flood. The aforementioned tributaries, however, have sufficiently sustained flows after flood periods to cause rapid silt removal from the basins behind the dams. Such basins will function both to smooth the flood crests (which are so extreme that they endanger the diked channel through the plain) and to detain a part of the extreme flood silt load, distributing it over a long period.

Beginning 8 miles down stream from Shanchow at the Sanmen Rapids and continuing down stream for 75 miles, the river runs through a series of gorges mainly in limestone formations. Either at Sanmen, or at other points in the gorge (such as at Palihutung), dam sites can be found that would be suitable for use in establishing flood and silt-detention control works. Above the Sanmen Rapids, 200 miles by river, are the Hu-kou Falls where another suitable site for a detention-dam reservoir has been found. This region was inspected by one of the writers and surveyed under his direction in connection with hydro-electric power investigations in 1934. The preliminary studies made of the Yellow River at this point and also those made of the Fen Ho in Shansi, 85 miles north of Taiyuanfu, at Hsiachingyiu,[9] as well as those made by the writers on the Chin Ho, in Shansi, and the King Ho and the Wei Ho, in Shensi, indicate that reasonably satisfactory sites are to be had for an extensive system of detention basins to retard flood discharge. Topographical maps indicate that equally good reservoir and dam sites are to be found in the upper waters of the Lo Ho, in Honan Province; but, thus far, no reliable reports have been made which would definitely show their worth for this purpose of river regulation. The general field has been investigated sufficiently, however, to encourage the writers to believe that the detention-basin feature of flood control of the Yellow River is feasible. Nevertheless, the silt difficulties are fully appreciated.

---

[8] "Possibility of Yellow River Flood Control by Means of Detention Basins," by S. Eliassen, *Journal*, Assoc. of Chinese and American Engrs., July-August, 1936.

[9] "A Study of Shansi Rivers," by O. J. Todd, *Journal*, Assoc. of Chinese and American Engrs., January, 1934; also, "Shansi Water and Power Problems," by O. J. Todd, *Journal*, Assoc. of Chinese and American Engrs., July-August, 1935.

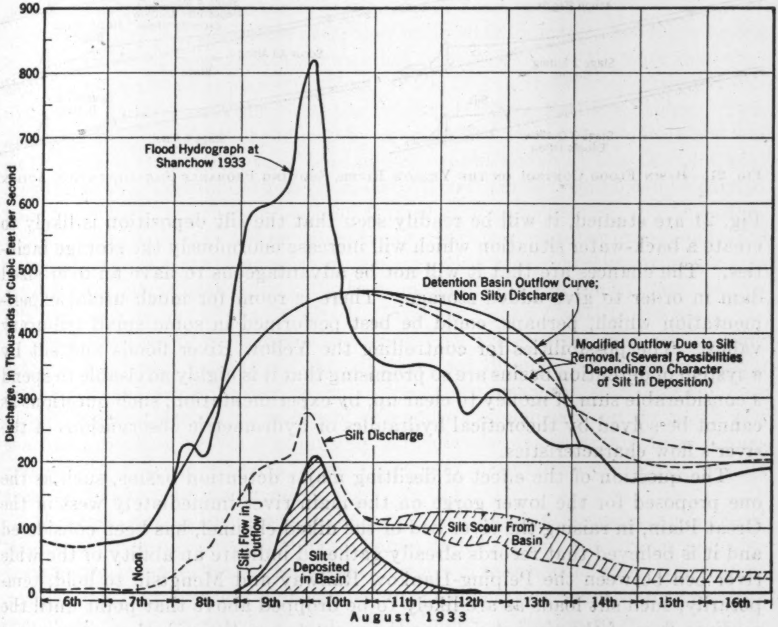


FIG. 20.—SILT DEPOSITION AND OUTFLOW FROM A DETENTION BASIN

Students of this problem generally assume that the openings in dams and canyon walls used for flow control must be ample so that the deposition of silt will not be too heavy in the reservoirs and that the deposits will be removed by scour. This limits the degree of control possible. Due to the complicated nature of the detention action, when there is much silt in the flow, it is not possible to compute, accurately, the data for establishing an outflow curve after the maximum stage has been reached in a basin. That the variations in the outflow and silt-clearing action may be many, can be seen from the diagrammatic sketches, Figs. 20 and 21.

If the basins are selected in relatively narrow valleys, with a slope steeper than 1 : 2 000, so that the normal velocity always will be a scouring one, there need be no apprehension that the basins will not clean themselves; nor is it possible that the large outflow openings may become choked. The important questions are the time element involved in the silt clearing and the height of dam necessary. The time question has a bearing on the possibility of a second large flood following closely on the heels of the first; and the dam question is both one of general security and of economics. If the diagrammatic sketches in Fig. 21 are studied, it will be readily seen that the silt deposition is likely to create a back-water situation which will increase enormously the storage facilities. The chances are that it will not be advantageous to have an over-sized dam in order to give added storage. There is room for much useful experimentation which, perhaps, could be best performed in some small tributary valley. The possibilities for controlling the Yellow River floods and silt by a system of detention basins are so promising that it is highly advisable to spend a considerable sum of money to clear up, by experimentation, such questions as cannot be solved by theoretical hydraulics or hydrometric observations of the river's flow characteristics.

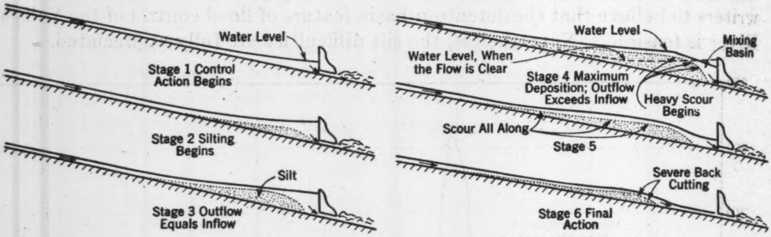

Water Level
Stage 1 Control Action Begins
Stage 2 Silting Begins
Silt
Stage 3 Outflow Equals Inflow
Water Level
Water Level, When the Flow is Clear
Mixing Basin
Stage 4 Maximum Deposition; Outflow Exceeds Inflow
Heavy Scour Begins
Scour All Along
Stage 5
Severe Back Cutting
Stage 6 Final Action

Fig. 21.—Basin Flood Control on the Yellow River, Showing Probable Silt-Detention Action

The question of the effect of desilting major detention basins, such as the one proposed for the lower gorge on the main river immediately west of the Great Plain, in raising the river bed of the diked channel, has been considered and it is believed that records already gathered indicate an ability of the wide river bed between the Peiping-Hankow Railway and Mengtsin to hold, temporarily, such silt loads as are likely to be dropped above that point until the medium flow of the river can carry them out to sea through the autumn and winter months. This section of the river is nearly 75 miles long, is bounded on the south by high loess hills, and only for the last 8 to 10 miles on the north bank are there dikes, which may be raised when necessary. These can easily be protected against any temporary tendency to overtopping due to rapid bed-fill.

The ability of the Yellow River to pick up additional load and degrade its bed even in low and medium stages has been noted when certain local fills, as long as 50 miles, have existed due to new depositions that interfered with the flow unduly. The rapid cutting out and carrying to the sea of the silt filling the channel below the 1935 breach at Tungchuang, Shantung, was observed in late March and early April, 1936, as the closure of the breaches was completed and the river's flow of 30 000 to 35 000 cu ft per sec was forced through the old channel which had been choked with silt left there in the latter part of 1935. As it came into the plain in April (nearly 2% of silt by weight), the spring freshet was not so heavily charged as to prevent its picking up, in the course of a week, the greater part of the fill in this section of the river channel. Furthermore, in the autumn of 1937, a sustained, medium-high flow of 150 000 to 200 000 cu ft per sec, silt-laden to 3% or 4% by weight, had a most potent influence in clearing out bed deposits that had been accumulating for one or two years in long stretches of the river below the Peiping-Hankow Railway. The main river channel is now as free from silt in its upper diked reaches as it was immediately after the series of high-water flows in 1933.

An examination of feasible detention-basin sites indicates that these basins could be relied upon to reduce a maximum crest flow by a little more than one-third (that is, detain a maximum flood of 900 000 cu ft per sec) so that it will not exceed 550 000 cu ft per sec at the Peiping-Hankow Railway Bridge. This discharge would be further reduced by channel losses to approximately 500 000 cu ft per sec at the Shantung border, and would arrive at the Grand Canal crossing with a flow of not more than 450 000 cu ft per sec, which can be carried by the down-stream channel without further control measures. By the adoption of this plan, it may not be necessary to resort to such other expediencies for flow reduction as are suggested in the following text, namely, overflow and desilting basins on both sides of the river in Shantung, and the by-pass channel following the course of the Tu Hai Ho from the Grand Canal to the sea.

Spillway Basins in Western Shantung.—In the control plan for guiding safely a maximum flood of 900 000 cu ft per sec through the regulated, diked section, it has been proposed, as one of the safety measures, to relieve the flow of 100 000 cu ft per sec by means of two spillways, one each on the north and south inner dikes in West Shantung immediately down stream from Tungchuang. If detention basins in the up-stream mountain areas prove feasible these spillways may not be necessary.

However, due to the fact that the river's section near Tungchuang is one of the most dangerous along the diked course under present conditions, and also because one must reçkon with the fact that it will take considerable time before a large-scale improvement plan can be put into operation, since so many points remain to be investigated, it would seem highly advisable to construct such spillways at an early date in order to give added insurance against flood disasters during the period of preparation for the greater flood-control program.

The overflow from the spillways would pass into the areas between the inner and outer dikes, where it would be detained briefly and partly desilted, and then allowed to re-enter the main river just up stream from the Grand Canal crossing after the flood crest has passed the outflow points. Control works to regulate both the inflow and outflow would be necessary. As protection against the attacks of a prolonged flood, it may also be found advisable to have a set of escape sluices in the outer south dike just up stream of the outflow point in order to pass some of the flow into Tungping Lake from which it again would return to the main river at Chiangkou. Additional diking to limit the Tungping Lake area would be necessary. Likewise, there must be a strengthening of the south outer dike. The north outer dike (or the Golden Dike) has already been strengthened in view of the frequent dike breaks that have occurred on the north bank in late years, such as those at Fenglo and Kuantai.

The regions to be used for this purpose are low lying, are amply large, and lend themselves well to a part control of an excessive flow. The only obstacle is the number of villages involved. However, with an improved inner dike system it is not likely that these overflow basins would be used more than once in a decade; nor would the volume of flow into them be very large. The danger of their becoming rapidly filled with silt, therefore, would not be great. Each flooding would be during the summer only and the farmers would thus always be assured of a good spring crop. Often the flooding would not be severe enough to injure the summer kaoliang crop, except in the immediate vicinity of the spillways.

The villages within the basins would either have to be removed or raised. Such raised villages are very common in areas in the North China plain which are subject to frequent flooding and could be promoted with State support. The opposition to be overcome, however, will remain a very difficult part of this control measure. The farmers will submit themselves patiently to frequent chance disasters, but do not wish to listen to any controlled flooding although this may occur less frequently and with less destruction to themselves.

By-Pass Channel from Grand Canal to the Sea.—The Yellow River has no such natural by-pass channel as the Mississippi possesses in the Atchafalaya River. The nearest to it is a small stream called the Tu Hai Ho beginning near the Grand Canal north of the Yellow River and flowing to the sea to the northeast in a course nearly parallel to the main river. It has been suggested that this small stream may be excavated so that it can handle a flow of 120 000 cu ft per sec in times of high flood. Surveys of this project have been made by the Yellow River Commission and advance cost estimates have been developed. The writers have studied the data and feel that the cost of the proposed excavations is somewhat excessive compared to the benefits gained. However, two lines of substantial dikes may be built so that this stream-bed may be used as an overflow channel with a maximum width of about one mile. A suitable modern gate structure would be required near the Grand Canal crossing of the Yellow River to pass flood water from the main river into this new channel. To permit it to function properly it may even be necessary to put a regulator across the main river in order to throw the clearer low-water flow into the by-pass channel when necessary to scour away these silt deposits left during floods.

In case further studies tend to discourage the construction of large detention basins in the mountain districts west of the plain so that floods, after being reduced by channel storage and seepage losses, cannot be held to a maximum of less than 450 000 cu ft per sec at the Grand Canal crossing, it may be found advisable that this by-pass channel should be provided to take a flow of as much as 120 000 cu ft per sec. If it is assumed that a maximum flood will be nearly 900 000 cu ft per sec as it enters the diked section of the river without any modification by detention basins above that point, it may be estimated that not more than 150 000 cu ft per sec will be "ironed out" and lost by channel storage and seepage in the 135-mile stretch to the Shantung border. From a crest flow at that point of 750 000 cu ft per sec, approximately 100 000 cu ft per sec may be diverted to the two adjoining overflow basins in Western Shantung, and a further 80 000 cu ft per sec lost by channel storage and seepage in the next

70 miles. This means that a flow not exceeding 570 000 cu ft per sec will reach the Grand Canal crossing. Here, the proposed by-pass channel will start functioning in the case of an extreme flow. Such a by-pass channel will help in distributing the silt deposits at the coast over a wider area and thus retard the rapid delta building at the mouth of the river.

However, if detention basins in the main river near Shanchow, above Yumenkou, or at Hu-kou, and on some of the larger feeders in Honan, Shansi, or Shensi, prove feasible as aids in reducing crest flows, this by-pass may not be necessary. Assuming that detention basins plus flow losses reduce a maximum flood to 450 000 cu ft per sec by the time it reaches the Grand Canal, the present channel of the main river below that point can accommodate that quantity without any great improvement to that part of the system as it is.

Earth Dikes and Their Protection.—Although dike building has been practiced by the Chinese for more than 1 000 yr, they are to-day behind Western countries in construction methods and safety requirements. On the Yellow River the present earth dikes are not of uniform quality, slope, or width over the 430 miles that they extend from the beginning of the delta plain in Honan west of the Peiping-Hankow Railway to the tidal flats 20 miles east of Litsin; nor are they as scrupulously maintained as they should be, considering the highly important function they perform. Formidable dikes exist for the entire stretch along the north bank; but along the south bank from the Grand Canal crossing to the Tientsin-Pukow Railway, a distance of 90 miles, and from the Peiping-Hankow Railway westward, hills take the place of dikes. On the north bank the dike extends west from the Peiping-Hankow Railway for a distance of about 15 miles.

These dikes are made of loess soil, selected (if the supervision is good) from non-sandy areas and hammered down while wet in 1-ft layers by the use of a 90-lb stone, thrown 8 ft into the air by ropes managed by eight men, the flat surface of such stone being approximately 1 sq ft. By this means the dike is made reasonably tight, and voids that would permit serious leaks are generally prevented. Often the workmen are not carefully supervised, the layers become as thick as 2 ft, and the tamping is indifferently done. Furthermore, sandy soil may often be used with a thin surface layer of clay both for water-proofing and against wind erosion. Leaks through such dikes are difficult to stop and the result is often disastrous. When dikes are reconstructed in winter after dike breaks, frozen chunks appear which cannot be properly pulverized by the light tamping methods, and breaks have occurred from this cause. For these reasons the supervision should be made far stricter than it is at present. No dike construction should be allowed during freezing weather, and such construction can easily be avoided by a better use of the time when the freezing weather has passed. As long as the present light tamping methods are used for compacting the fills, the layers should never be more than 9 in. thick. It would add greatly to the efficiency of the dikes, however, if mechanical compacting devices, such as the American sheep-foot roller, were introduced. When the time comes for regulating the diked channel, the fill must be compacted mechanically or strict rules for construction methods must be enforced if disasters are to be avoided. The new dikes will have to withstand water pressures and swift currents fully as severe as those to which the present dikes are subjected.

As a rule, the main dikes of the Yellow River are 25 to 30 ft wide on top, although some stretches built during the two decades (1918–1938) are as wide as 50 ft. Such excessive widths are partly due to poor, sandy soil, and partly to the fact that in times of high water it may be necessary to utilize the earth from the back of the dikes for making quick repairs. At such times the land outside the dikes is often flooded, due either to rainfall or to seepage, and it is costly to get earth from any other place than the dike itself. Spoil-banks of earth are also frequently placed on top of the dikes for such emergency conditions. These piles of earth which the Chinese call *tu niu* ("earth oxen") are familiar sights to travelers along the dikes. In all new dike work it would be well to follow this practice since, in the flood season, the elevation of the channel above the plain tends to create a swampy or flooded condition along the foot of the dike on the land side.

The present dike slopes are usually from 1 on 2.5 to 1 on 3.5, no uniform rule being followed as to which side should be the flatter. Generally, the dike slopes are covered with an indifferent growth of grass which seldom develops a first-class sod because, during the winter, the farmers rake the surface to gather the wilted grass for fuel and permit their sheep to graze on the dike slopes. Old regulations forbidding such practices are no longer enforced. During times of dike breaches people from the flooded areas crowd on top of the dikes where they build temporary shelters. After the breaches have been closed it is frequently impossible to persuade all of them to leave. Even when the dikes are raised these huts remain, gradually becoming caves in the back slopes with the roofs in some cases even lower than the top of the dike. The desire to live on the dike exemplifies the insecurity of the region along the river.

In recent years an effort has been made to plant willows on the foreshores close to the dikes. This should be encouraged as much as possible because such trees protect the dikes both from wave action and from the high-water currents. Five or more staggered rows of willows with the inner row 20 ft from the foot of the dikes and successive rows 10 to 15 ft apart afford an excellent protection. In the past, trees have been planted on the dikes and this practice still continues. As the dikes frequently have to be raised the trees become a nuisance and have to be removed. The roots remain in the body of the dike and are thus a source of leaks. However, the silty water tends to seal the finer interstices and generally tightens the dikes in the course of time so that this form of trouble is not so serious as it would be if the water were clear. Nevertheless, the potentialities for leaks are there and trees on dikes should not be encouraged.

Slipping of dikes due to excessive saturation is seldom experienced along this river although it does occur on the Yangtzekiang dikes where the clay content is often large and the difference between high and low water is as much as 50 ft at times; on the Lower Yellow River this difference seldom exceeds 15 ft. Dike failure may sometimes be caused by burrowing animals; but less apprehension need be felt for this possibility on the Yellow River than on the Han River near the Yangtze, for instance, where badger-like animals dig holes a foot or more in diameter. Great care should be exercised in watching for these holes.

As the main dike system is frequently a mile or more from the low-water channel the foreshores are inhabited and cultivated, and the farmers living there have gradually thrown up small dikes along the edges of the channel to protect their villages and growing crops against flooding from medium high waters. In times of greater floods these dikes are overtopped and breached, and, as the main dikes are not protected, except where the current constantly impinges on them, the sudden onrush of water has been known, as at Tungchuang in 1935, to set up highly eroding eddy currents which destroy the main dikes. Unauthorized dike work inside the main diking system may thus result in appalling disasters, and, therefore, in future regulation plans, human habitation must be prohibited inside the main diking system and any cultivation strictly supervised. With no villages on the foreshore lands, every one will be interested in keeping the main diking system intact, whereas, at present, there is a considerable number of people who often hope that the main diking system may be breached. The population living on the foreshore lands constitutes one of the difficulties in maintaining the river in its diked course.

Dike Protection.—Since the river channel is not stabilized it meanders at will over the wide strip of land between the dikes. Where the swift eroding currents impinge on the earth dikes it is necessary to protect them. This phase of maintenance work often becomes costly due to the long haul of stone. Stone quarries are few and far between in the Great Plain of North China.

Until a few decades ago when foreign steel became available for rapid quarrying, the Chinese used very little stone for dike protection works. The kaoliang stalk was the chief material used before that time and the kaoliang groin and "fish scale" pack work of the same material, so well described by Mr. Freeman,[3] are still used to a certain extent for dike protection in regions between the Shantung border and the Grand Canal crossing, and also on the lower river near Litsin; but in other regions they are being replaced rapidly by stonework, although for dike-break closure and in emergency cases the kaoliang stalk (Fig. 5) is still extensively used everywhere. Huge stacks of it may be seen to-day piled up near all the maintenance offices, which are spaced at intervals of about 20 miles along the river. Due to the great quantities required kaoliang is not a cheap material to use, and it needs constant maintenance because it rots out in two or three years. New work is then placed on top of the old, forcing the decayed mass into the mud below water where it is readily scoured into by the current. Often it happens that the entire construction is undermined and torn out. As an emergency material kaoliang has its decided place in maintaining the dikes intact, but not for permanent protection. Stone has now become the logical material for this purpose and will probably always remain so since sand cannot be found within reasonable distances for concrete revetment construction on a large scale.

Everywhere along the river, groin construction as defense work against swift currents is preferred to the continuous revetment method practiced on the Mississippi River. Many types of groins are used (see Figs. 19 to 27). One may see the triangular, pointed groin, the narrow, down-stream-pointing groin, and groins rounded in form. They consist of barricades of earth paved with stone and are exactly long enough to keep the current off the dike. In Shantung

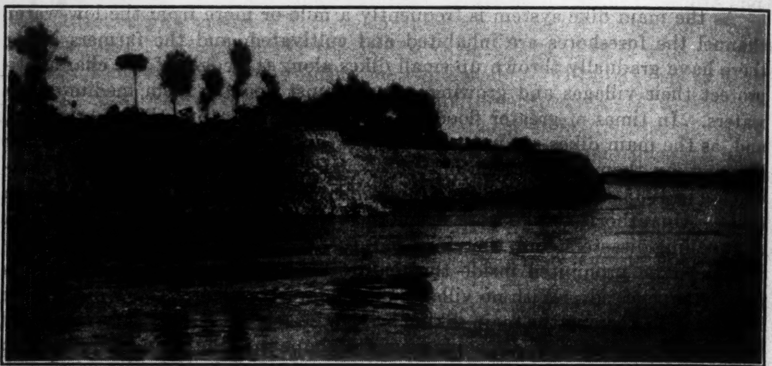


FIG. 22.—STONE GROIN BANK PROTECTION ALONG YELLOW RIVER IN SHANTUNG

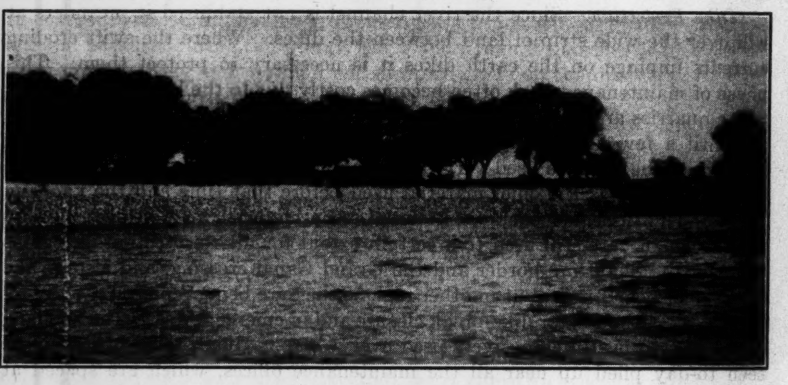


FIG. 23.—GOOD STONE WALL BANK PROTECTION WITH RIP-RAP ALONG YELLOW RIVER BANKS, JUST WEST OF TIENTSIN-PUKOW RAILWAY

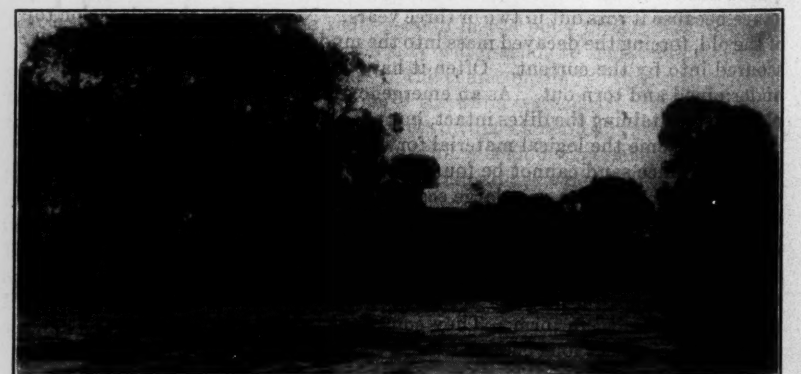


FIG. 24.—WELL BUILT STONE-PROTECTED GROIN POINTING DOWN STREAM ALONG SOUTH BANK OF YELLOW RIVER, 20 MILES BELOW LOKOU

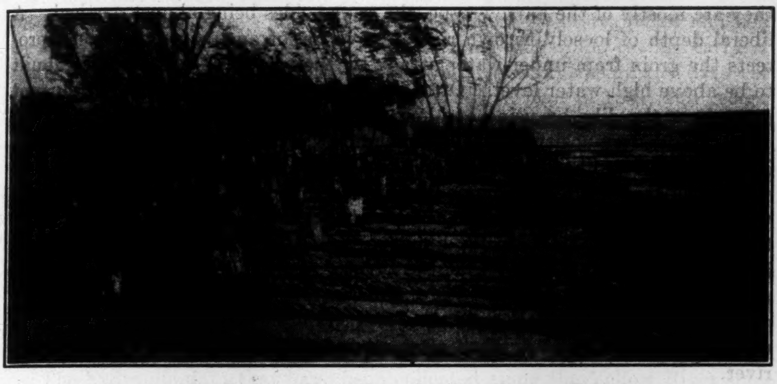


FIG. 25.—NEW KAOLIANG BANK PROTECTION TIED BACK BY HEMP ROPES, ALONG YELLOW RIVER, IN SHANTUNG

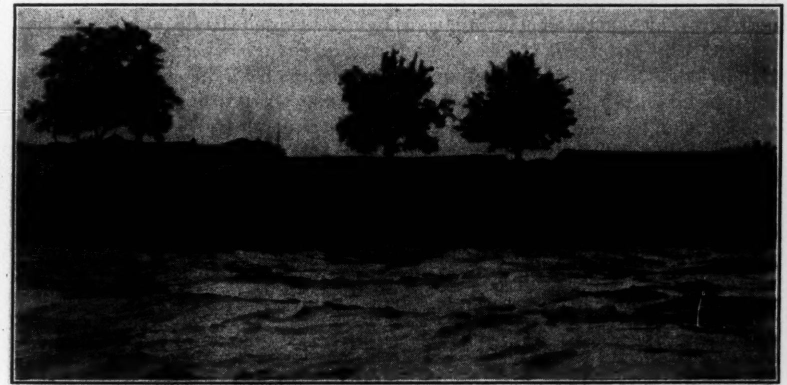


FIG. 26.—NEW KAOLIANG LAYER WITH EARTH COVER ON TOP OF OLD FISH-SCALE BANK PROTECTION ON YELLOW RIVER DIKE OPPOSITE LITSIN, SHANTUNG

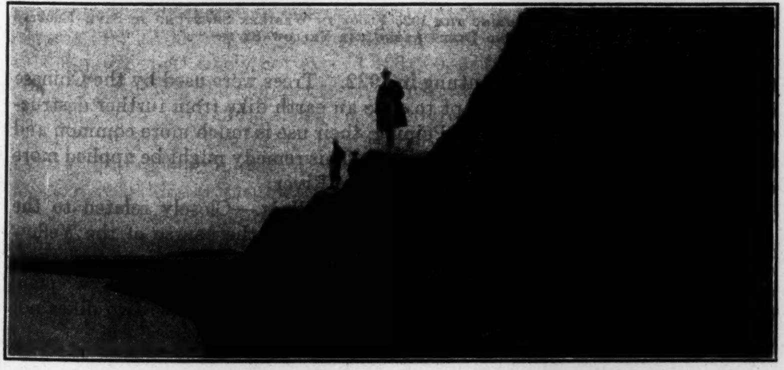


FIG. 27.—SAWTOOTH KAOLIANG BANK PROTECTION ALONG YUNG TING HO, 10 MILES BELOW MARCO POLO BRIDGE, NEAR PEKING

they are mostly of the narrow type, the paved sides being almost vertical. A liberal depth of loosely thrown rip-rap covering of one-man-sized stone, protects the groin from under-water attack by the current. All groins are built to be above high-water level. Often an attempt is made to guide the current away from the dike by using one long groin; but this usually has resulted in causing added sinuosity of the river channel with more protection required for the opposite dike which becomes attacked. The "cure-all" effect of the single long groin is firmly defended by the old school of Chinese river engineers which discusses confidently what the river's action will be if a long groin were to be built in order to control a difficult reach. The reason for this optimism is probably due to the fact that the "trick" sometimes has worked without undue harm to down-stream parts. A decentralized control and lack of funds other than for immediate, pressing protection have prevented them from attacking, systematically, reach after reach and thus gradually subduing the river.

Tree retards as an emergency protection (see Fig. 28) are not greatly used along the Yellow River, although one of the writers experimented with this type of bank protection in Shantung in 1922. Trees were used by the Chinese in 1925 at Linpuchi in an attempt to save an earth dike from further destruction. On the Yungting Ho, near Peiping, their use is much more common and has there proved valuable in emergencies. This remedy might be applied more commonly than it has been along the Yellow River.

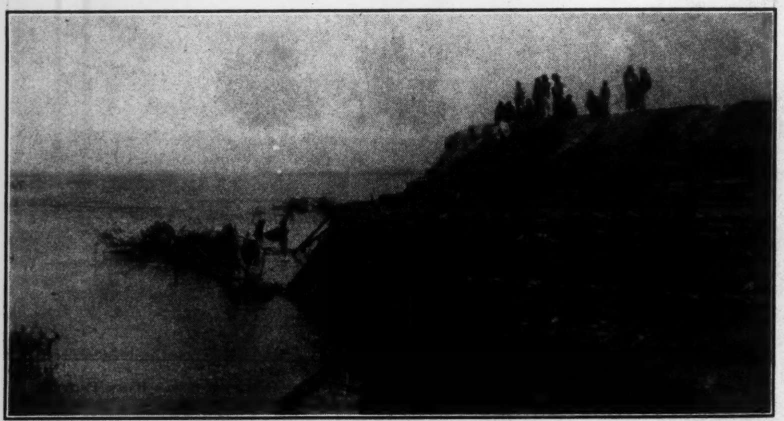


FIG. 28.—USING TREE RETARDS DURING THE 1925 FLOOD IN WESTERN SHANTUNG TO SAVE ERODING BANKS AND DIKES ALONG THE YELLOW RIVER

Regulation of Diked Channel Through the Plain.—Closely related to the problem of maintaining dikes is that of regulating the course of the Yellow River through its diked section. In 1922, Mr. Freeman[3] proposed that there should be a straighter, narrower, and better protected course between two main dike lines than had existed theretofore. His plan was to have two dikes not more than one-half mile apart and protected by regularly spaced groins. It was also his intention to have an inner channel, 1 800 ft wide, regulated and held in a fixed course so that it would be self-scouring and would readily carry all flood flows. Professor Franzius, of Hanover, Germany, using the data collected by Mr. Freeman, formulated an alternative plan in 1931, and recommended a slightly wider central channel, having inner unprotected dikes built along a theoretical mean-water channel expecting to retain the flow long enough in such a course as to make it scour its channel deeper. Each year these dikes would be repaired, but would not be protected in any way. This was a device to obtain a deep, narrow channel at low cost along which permanent dikes with suitable protection might eventually be built, thus stabilizing the channel. These eminent engineers both believed that a narrow channel with dikes built closely along the banks would make the river perform the work of maintaining a deep channel and that it would also scour sufficiently to accommodate the floods.

The hydraulic data that have been collected along the Yellow River since these two engineers discussed its channel regulation have thrown a much better light on the perplexities and difficulties which are inherent in the problem of making a stable, diked channel through the plain. In discussing this regulation problem, two alternative plans present themselves: (1) When the diked channel is to convey the modified, detained outflow from a system of flood-control detention basins within the mountain area to the west of the plain; and (2) when no detention-basin system is feasible and the diked channel must accommodate the full flood flow of the river.

The diked course is not without importance from the viewpoint of navigation and this should be given a certain weight in fixing the dimensions of the regulated channel. Let it be stated at once, however, that protection against flood disasters outweighs so greatly all other considerations that the question of navigation improvement must be considered after the flood-control measures have been fully met.

Under the heading, "Hydrological Considerations: Frequency and Magnitude of Floods," it has been stated herein that the writers consider a flood of 900 000 cu ft per sec to be the probable maximum discharge that the Yellow River can have at the head of its diked course. With a flood-control detention basin system west of the plain it may not be possible, due to the silt load, to reduce this discharge to less than 550 000 cu ft per sec where it enters the diked river. Allowing for channel storage and percolation, and other losses, this flow will most likely not exceed 450 000 cu ft per sec at the Grand Canal crossing, assuming the channel to be regulated. Below this point the present channel, with a moderate degree of improvement, should have sufficient capacity to convey this flow. For a prolonged period during the summer of 1937 it did convey almost this quantity, but finally breached the south dike near the coast due to insufficient bank protection.

Without a detention-basin system, however, the full flow of 900 000 cu ft per sec would have to be accommodated. This flow, it is estimated, would be reduced to 810 000 cu ft per sec at the 1851 break, to 750 000 cu ft per sec at the Shantung border, to 650 000 cu ft per sec by spillways into the proposed overflow basins between the inner and outer dikes on both sides of the river between the Shantung border and the Grand Canal crossing, and, finally, by channel losses to 570 000 cu ft per sec at the Grand Canal crossing. At the latter station the Tu Hai Ho by-pass channel would take away a further flow of 120 000 cu ft per sec, allowing a flow of 450 000 cu ft per sec to pass down the present main channel of the river to the sea. Fig. 29 shows this arrangement. Even if the channel has been narrowed by regulation, these losses do not seem excessive to apply but sufficient foreshore widths must be left to permit them to take place. Even if experience should prove that the final regulated channel does not give such great losses there still remains the expedient of installing light-capacity flood spillways at a few points, such as at the 1851 break, where the old channel still exists, and also below Tungchuang, where the proposed spillway over-flow basins would function for a long period before they became filled with silt.

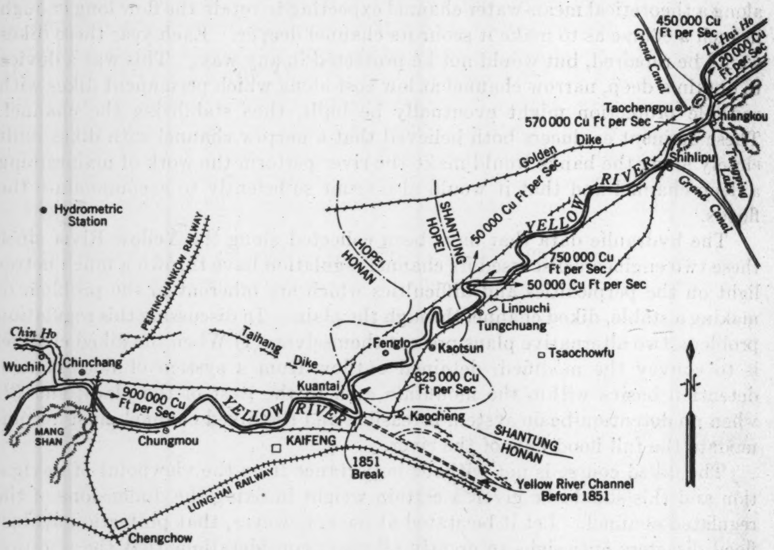

450 000 Cu
Ft per Sec
Tu Hai Ho
120 000 Cu
Ft per Sec
Grand Canal
Taochengpu
570 000 Cu Ft per Sec
Chiangkou
Dike
Golden
Shihlipu
Tungping Lake
Grand Canal
RIVER
YELLOW
50 000 Cu Ft per Sec
Hydrometric Station
PEIPING-HANKOW RAILWAY
HOPEI
HONAN
SHANTUNG
HOPEI
750 000 Cu
Ft per Sec
50 000 Cu Ft per Sec
Linpuchi
Tungchuang
Chin Ho
Taihang
Dike
Fenglo
Kaotsun
Tsaochowfu
Wuchih
Chinchang
825 000 Cu
Ft per Sec
Kuantai
900 000 Cu
Ft per Sec
YELLOW RIVER
Kaocheng
MANG SHAN
Chungmou
KAIFENG
SHANTUNG
HONAN
1851
Break
LUNG-HAI RAILWAY
Yellow River Channel
Before 1851
Chengchow
N

Fig. 29.—Flow Losses During the Passage of a Maximum Flood Under the Regulated Condition

As regards the distance between the dikes and the width and shape of the actual channel itself conveying the full flow of 900 000 cu ft per sec, there is room for much theoretical discussion. From the considerations of scour and refill, flow and silt losses, and also from the standpoint of the hydraulic laboratory experiments conducted in Germany, it is not at all certain that a narrow channel with dikes built closely along the banks will be the best type to adopt. The German experiments are especially illuminating; they tend to show, in a qualitative or indicative manner, that a channel with dikes spaced far apart scours a little better and silts up its foreshores somewhat faster than the same channel with dikes built closely along the banks. This seems the logical result to be expected. By the transverse circulation of the flow a part of the silt load is transferred from the channel to the wide foreshores giving the less silty flow a better chance to pick up silt from the bed. If there are no foreshores the river has no other chance to relieve itself of its excess silt than by desilting it in the bed of the channel. However, even having a channel with wide foreshores there is certain to come a time when these have been silted up so much that the silt transfer has been reduced to a point where it no longer can influence the flow in picking up silt from the bed of the river. From this time, both the river bed and the foreshores will rise together. Taking a long view of the situation this is perhaps the condition which favors the slowest rise of the river bed. In selecting the width of the foreshores, therefore, it is of little use to make them wider than that required for a normal deposition width. The cross-sections of the river often show a break in the ground profile indicating the line where the heaviest deposition ends. This varies somewhat along the river, but a careful inspection of maps and cross-sections will point out, in a general way, the line to be followed for the best position of the dikes. At times, bad soil condition will make it impossible to follow this line closely, and the best practical route will have to be followed considering every aspect. Judging from cross-sections the indications are that foreshore widths need not exceed 1.25 miles, and, perferably, should be less than this on account of the cost involved in connecting a channel protective system to the dike lines.

There are also other reasons why excessive widths between dikes should be avoided. An outstanding example of detrimental, excessive width is the section between the 1851 break and the Shantung border. At the 1851 break the channel is more than 12 miles wide, whereas at the Shantung border it has been narrowed to about 5 miles. The consequence has been that the banks along the channel, due to the rapid silt deposition, have become as high as the top of the far-away dikes, as indicated by cross-section lines, and differences of more than 10 ft in elevation between the banks and the foot of the dikes have been noted. Generally, the transverse slope of the deposition is from 1 : 3 000 to 1 : 4 000 and hence the flow during freshets, when the foreshores overflow, runs diagonally across the foreshores cutting numerous channels which unite along the foot of the dikes, thus creating a very dangerous situation. It is almost impossible to stop the flow through these side channels once they have developed, and, in time, the river takes up a new channel along the foot of the dike. This has occurred time and again and when it happens the higher flood levels previously existing along the central channel are transferred to the dikes, and overtopping and breaching inevitably occur; hence, the necessity for both a narrower channel and a fixed channel as a link in the flood-control program. With foreshores not exceeding 6 000 ft in width the difference in elevation between the banks along the central channel and the foot of the dikes will be about 2 ft, which is not too much; but practical considerations may reduce this width to about 4 500 ft. The writers do not consider a width less than this to be advisable in the up-stream stretches of the diked river where most of the losses and silt deposition on the foreshores will occur. It may gradually be reduced to 3 500 ft at the Grand Canal crossing. Below the Grand Canal crossing the dikes are not too far apart. They are even too close together in places. The main considerations here would be the straightening out of the existing dike system so that the entire length of the channel would have a more regular dike distance better co-ordinated to the channel itself in order to create a uniform flood profile with easier run-off conditions.

As regards the width to be given the actual channel, Messrs. Freeman and Franzius proposed 1 800 and 2 400 ft, respectively, as most practicable. However, they did not have available data regarding the actual flood flow or silt content of the river, which have been found to be more than twice the values they proposed to adopt.

When the river has a truly silty spell of 40% by weight as was measured during the flood of August, 1933 (812 000 cu ft per sec), there may be some doubt whether it will scour its bed even if this is regulated. The heavy silting which occurred below the 1851 break should be a warning. When designing the regulated channel width, therefore, no absolute reliance can be placed on a hope that the channel will enlarge itself. It seems better to be conservative and give it a width, at least in the up-stream reaches, which will carry the flow without making allowance for scour. The existing channel in the upper reaches of the diked section has widths varying from less than one mile to nearly two miles. At low water this channel is full of sand-bars with many breaches. As the narrower parts are fairly long, and since they carried the maximum flow in 1933 quite effectively, it would seem that under no circumstances should the regulated channel be wider than the narrowest existing sections. From an inspection of the actual conditions it would seem that it would be about right to give the proposed regulated channel a width not exceeding 4 000 ft in the upper reaches, gradually reducing this width to 3 750 ft at the 1851 break, to 3 500 ft at the Shantung border, and to 3 000 ft at the Grand Canal crossing. After the reduction of flow through the Tu Hai Ho by-pass channel a width of 2 000 to 2 200 ft may be sufficient along the entire distance to the sea. This is about the average width of the present channel. It is quite true that the width now at many sections is much narrower than this (as, for example, along the 3-mile stretch at Chiho Hsien, 15 miles up stream from Lokou, where there is scarcely more than 1 500 ft between the protected dikes); but this stretch is too narrow and should be widened. The observed water levels indicate that this section causes a considerable heading up during heavy floods such as occurred in 1937.

When it comes to dike distance width and channel width for the case in which a detention basin system has been built up stream, quite different considerations must be met. In this case the outflow from the detention basin at its maximum, for one or two days, will have a relatively light silt load. As the water level becomes lowered in the basins, the deposited silt begins to be scoured, causing the outflow to be heavily silt-laden. One may be reasonably certain that there will be heavy scour along the regulated channel during the period of less silt-laden outflow, and, therefore, it will be feasible to have a much narrower channel than that required under an assumption of no scour of the channel. A channel width in the up-stream reaches not exceeding 1 800 ft seems amply sufficient; but the same foreshore widths should be kept as in the case of unregulated flood flow. When the silty flow arrives the discharge has already become so low that it will be contained in the channel without flooding the foreshores. Gradually, however, the scoured channel will be refilled and this may again raise the water level and flood the foreshores. Silt will then be transferred to them, and this again will retard further deposition in the channel bed. The swifter velocities in the narrow channel will also help to transport the silt load without undue deposition. Having several detention basins, there will also be silt deposition below each dam as well as along the temporary silt storage area between Mengtsin and the Peiping-Hankow Railway Bridge. This will reduce the possibility of any extremely high silt concentration entering the diked regulated channel as a result of the silt depositions in the basins being scoured out. The rise of the river bed which has been observed in the upper reaches of the diked channel during small, medium silt-laden freshets, is unquestionably due to the present bed, which is far too wide.

The considerations that have been dwelt upon in the foregoing discussion are some of those which unavoidably arise when an investigator attempts to unravel the perplexities encountered in solving the regulation problem of the Yellow River. It will be noted that no attempt has been made to correlate velocities with silt transportation and bed scour. Velocity is only one of the many factors that enter into this problem and cannot be singled out as the safe guiding criterion. Recent studies in mechanics of turbulence seem much more likely to clarify the situation than mere studies of velocities. The writers have tried to use Reynolds numbers to express efficiencies in silt transportation; but the results are meaningless, chiefly on account of the rapidly changing hydraulic radius. For rivers, a different criterion than the Reynolds number must be found to correlate turbulence and its effect on silt transportation and bed scour.

The question of using dredges as an aid in improving the capacity in the river to carry its silt load has been considered. Since the silt which from year to year is left in the channel is small compared with the total silt transportation it may seem worth while to experiment with specially designed, powerful dredges, to see what can be done, before each freshet season, to remove deposits in the channel which would seem to interfere with channel efficiency and hence silt transportation. Dredged cuts, rapidly done, through such deposits may help in their removal by the flood flow when it arrives. Such cuts may be entirely obliterated during the freshet season, but, nevertheless, may have performed their duty in slowing up the rise of the bed. It will be a new departure in the aims of dredging operations and may incidentally assist navigation.

Channel Stabilization Methods.—Until the main dikes have been moved to their designed position in the new regulation plan it will be necessary (since the Yellow River changes its course so rapidly) to construct some cheap, temporary stabilization works to keep the present channel in place until the new dikes have been built and the permanent channel works begun. The actual regulation work should begin near the Grand Canal crossing, each year continuing systematically up stream according to a well-formed plan. Below the Grand Canal crossing the river is in fairly good condition over long stretches, and the main improvements will be to ease a number of sharp bends and bring the dike system to a more uniform width. The existing protection should be utilized to the greatest extent and, as much as possible, be made to fit into the general stabilization plan.

When it comes to stabilizing the much wider channel up stream of the Grand Canal crossing the question naturally arises as to which of the following methods will be adopted: (1) A continuous revetment of the channel banks in the concave sides of the bends and where it is necessary to guide the flow in the cross-overs from one bend to the next; (2) a system of groins; or, (3) a combination of Methods (1) and (2).

Method (3) will probably be the most logical since there naturally will be many places where the banks are fairly stable and will merely require paving and under-water rip-rap for complete stabilization. In such places it will be necessary to construct long groins tied to the dikes. Furthermore, in bends where the flow necessarily comes close to the dike, and where no previous groin work exists, it will be preferable to use the continuous revetment method. Groins in bends create a highly pulsating flow condition during floods and many groins have collapsed from this cause. A certain degree of turbulence is necessary in order to carry the high silt content in suspension, but this should be in moderation. Excessive resistance merely creates higher water stages. The bends themselves unquestionably produce sufficient eddy currents to propel the silt in suspension past the bends and, therefore, a smooth flow around the bends should be sought.

A question of a more difficult nature arises where the channel must be stabilized through reaches where the soil is easily scoured and unstable and the channel far removed from the dikes. Long low groins will be necessary in such cases to protect the foreshore lands and the stabilization works must be tied to the dikes. Several forms of such groins suggest themselves, such as the pile groin, the fascine-stone groin, and the protected earth groin. The first two forms are indispensable where the existing channel must be narrowed, and the earth groin may be used to tie across foreshores, which are dry at medium stages. However, as an earth groin must be above high water to escape destruction, its use will probably be very limited. The guiding principle, in all cases, should be such that silt flow on the foreshore land will be fairly unobstructed in order to give facilities for the channel to scour; but the velocities on the foreshore land must not be so swift that they will tend to scour side channels.

Availability and cost of materials will determine whether the fascine-stone groin should be used to bridge the space between the banks and the dikes, or the pile-hurdle groin. The latter will require a mattress construction along its full length to prevent its being undermined and destroyed. The outer ends of these groins should define the new channel alignment, and should be in the form of stone heads. Since economical considerations make it inadvisable to space these long groins close enough to keep the channel from becoming sinuous, it will be necessary to construct short, intermediate groins to prevent this effect. These intermediate groins may be exactly long enough to be tied to the nearest normal bank or ground, which is above the ordinary summer stages, between freshet periods. As regards the question of spacing the groins, and whether they should be short or long, much depends on the soil conditions. Obviously, a sandy, easily scoured soil requires closer groin spacing than if the soil is of a more erosion-resisting quality.

Experience on the Yellow River indicates that nothing but solid construction work will endure the onslaughts of a flood. Miserable failures have resulted from attempts at constructing light pile groins for flood control. They have been laid flat or rooted up by the silty, swift water, and the great bulk of weed root and kaoliang stalk which the flow carries during freshet periods and which pile on to the works. Anything that remains will be entirely removed by the ice during the late winter. In much exposed places the fascine-stone groins will have the best chances for surviving. They can also be added to readily as the silt accumulates on the foreshores.

A question has been raised as to the best direction to give long groins—up-stream pointing, down-stream pointing, or a direction perpendicular to the current. In their report on the Yellow River, the engineers[10] employed by the League of Nations have advised groins pointing up stream. On the silty Missouri River groins pointing slightly down stream or perpendicular are preferred. Two questions seem to enter into this problem: (*a*) The rate of silting on the foreshores; and (*b*) the turbulence designed to produce the best scour of the river bed and the best silt transportation.

It appears to the writers that hydraulic laboratory experiments can do much to solve the question of groin direction. It is fairly obvious that, for various conditions, one angle will be better than another. The correct or incorrect direction may even be sufficient to determine whether or not the Yellow River, in its upper reaches (where the condition of the bed is not influenced by the extension of the delta at the coast), will raise, maintain, or lower its bed. Channel boundary conditions are important functions in determining the relation between frictional resistance and turbulence, and, as the aim must be to have the greatest turbulence with the smallest flow friction, the direction and shape of the groins may play a decided rôle in the bed scour, refill, and silt transportation questions. Although the river is wide compared to the short groins, and the general bed conditions are seemingly more influential in creating turbulence than the groins, it may be quite possible that the groins can influence the bed conditions as well, and hence, in a double sense, the turbulence. It has also been contended by writers on fluid mechanics[11] that the boundary conditions during increasing and decreasing stages of flow are extremely important, and that certain frictional boundary conditions may even cause still water or eddy flow in the boundary layers during a rapidly decreasing flow. In a silty river this explains partly, if not wholly, the rapid refill which occurs as soon as a flood period has passed its crest. That groin shape and direction can modify the refill somewhat is possible and is certainly worth investigating. Extensive hydraulic laboratory experiments to find the best solution to this problem are likely to repay themselves a hundred-fold.

---

[10] "Hydraulic and Road Questions in China," Series of League of Nations Publications, 1936.

[11] "Modern Conceptions of the Mechanics of Fluid Turbulence," by Hunter Rouse, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 463.

Influence of the Delta.—It has been stated in the "Introduction" that the steady advance of the delta into the shallow Gulf of Chihli causes a flattening of the slope of the river bed in the down-stream section of the diked course, making this higher than the plain. As far as can be estimated from the existing hydrometric records the silt volume that is being deposited on the undiked delta land, and in the neighboring sea, amounts on the average to about 15 000 000 000 cu ft per yr. Previous estimates of 8 000 000 000 to 10 000 000 000 cu ft per yr are unquestionably too low, whereas the observed silt quantity of 35 000 000 000 cu ft passing the Litsin Hydrometric Station in 1934 seems much greater than the average. In 1935 only 12 000 000 000 cu ft passed the Litsin Station; but if the Tungchuang breaches had not occurred, it would probably have been approximately 25 000 000 000 cu ft, a volume still greater than the average under the present condition of frequent dike breaches.

After the diked course has been regulated and well protected the average yearly silt volume reaching the delta and the sea may be as much as 22 000 000 000 cu ft, which means that the delta will advance faster eastward than it does to-day. It does not seem likely, however, that in the near future the river course in the lower diked section will become much more elevated above the plain than it already is. Table 1 shows that the slope from Lokou to Litsin is 1 : 12 600, from Litsin to Holungchu, 1 : 10 000, and from Holungchu to the sea, 1 : 6 000. Toward the end of the Nineteenth Century the dikes were extended from Litsin 10 miles down stream, but it was only during the present decade (1928–1938) that they were continued the next 5 miles to Holungchu. The low-water surface between Lokou and Litsin is almost uniformly 5 to 6 ft higher than the plain. At a point 10 miles down stream from Litsin it is practically level with the plain, while at Holungchu, where the dikes end, the low-water elevation is about 2 ft lower than the delta plain. This merely shows that the dikes east of Litsin have been built on the advancing steep, frontal delta débris fan. Beyond the ends of the dikes the silt spreads and deposits freely in all directions; and, as the débris fan is extended, the slope inside the diked course will be flattened to correspond with that up stream from Litsin. As long as there is distance enough for a steep, frontal débris-fan slope to exist between the ends of the diked course and the sea coast, however, it does not seem probable that the super-elevation of the diked course from Lokou to Litsin will become much greater than it is at present. Hence, the slope of 1 : 12 600 should not become markedly flatter, at least not for some time to come (see Fig. 30).

Surveys conducted by the British Admiralty about 1870, by the German Land Survey Bureau some time after 1900, and by the Chinese Hydrographic Survey Bureau and the Yellow River Commission in 1936 and 1937, show that the delta has advanced 12 miles, corresponding to a rate of one-sixth of a mile per year since the British survey was made. Since the time of the German survey the advance has been about one-fifth of a mile per year. The delta sector has a radius of approximately 40 miles, and a sector arc-length of 60 miles. The average sea depth has been approximately 40 ft, the under-water slope of the deposited material, judging from recent surveys, is about 1 : 1 500, and the frontal slope of the delta fan as it is to-day—namely 1 : 6 000. This gives a total deposition volume of approximately 1 150 000 000 000 cu ft.

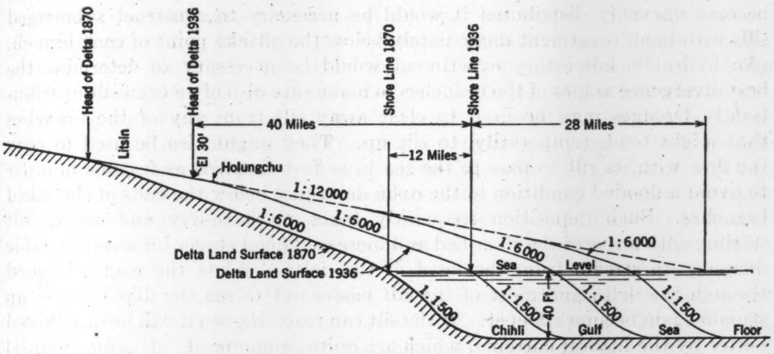

Head of Delta 1870
Head of Delta 1936
Shore Line 1870
Shore Line 1936
40 Miles
28 Miles
12 Miles
Litsin
El 30'
Holungchu
1:12000
1:6000
1:6000
1:6000
1:6000
Delta Land Surface 1870
Delta Land Surface 1936
Sea Level
1:1500
1:1500
40 ft
1:1500
1:1500
Chihli Gulf Sea Floor

FIG. 30.—ADVANCE OF THE DELTA FAN AND THE GRADUAL DISAPPEARANCE OF THE STEEP FRONTAL SLOPE INTO THE SEA

Since the annual silt volume passing to the delta and the sea has been estimated at 15 000 000 000 cu ft, it has taken almost 76 yr (since about 1862) to deposit the foregoing volume. This is in fair agreement with the advance rate of the delta as given by the surveys, if allowance is made for silt carried beyond the 60-mile arc-length front. The latest Japanese maps, printed in 1935, show the average depth in the Gulf of Chihli (that part lying west of Longitude 121° E) to be about 22 to 23 m or, say, 70 ft. If there is no geosynclinal action interfering with the elevation of the sea floor, the Gulf of Chihli should become dry land (except the part north of Latitude 39° N) within 2 000 yr, assuming that the Yellow River continues to flow into it and has the same silt-carrying capacity as at present. This is an interesting speculation; but what is of more importance is to decide if there is any immediate cause for alarm due to the rapid delta advance. The length of the steep frontal slope of the delta fan from Holungchu to the coast line is 37 miles; and, assuming a rate of advance of the delta under a regulated condition of the Yellow River to be one-fourth of a mile per year, it will take 148 yr for the delta front to have passed completely into the sea. (Actual computation on the basis of 22 000 000 000 cu ft per yr gives 138 yr.) After all, this is not a very long period, and when it has passed, a highly critical situation will rapidly develop which is likely to spell the doom of the present course from the Grand Canal to the sea. Hence, it is important that the advance of the delta should be retarded as far as this is possible. One way would be to try to distribute the silt over a longer front than at present.

The length of the coast line along which the river has spread its silt during the last 70 yr has been stated to be 60 miles. It may be possible to lengthen this coast line gradually to about 100 miles by having a three-forked diversion point 10 miles below Litsin, one arm going in a northerly direction, one to the east, and the third toward the southeast. Each branch would need to be diked from the diversion point for a distance of 5 or 6 miles, the diking being continued from time to time as the delta fan advances. To prevent any of the branches from developing their sections so that the flow shall become unevenly distributed it would be necessary to construct submerged sills with bank revetment immediately below the offtake point of each branch. (An hydraulic laboratory experiment would be necessary to determine the best divergence angles of the branches to make sure also of an even silt distribution.) Dredges may be used to clear away silt from any of the branches that might tend, temporarily, to silt up. They might also be used to coax the flow with its silt to pass to the sea in as few channels as feasible in order to avoid a flooded condition in the open delta just below the ends of the diked branches. Such deposition frequently takes place to-day and causes silt settling with a rise of the river bed and increased flood stages for a considerable distance up stream from the head of the delta. When the course is good through the delta and most of the silt passes out to sea the diked course up stream again becomes deeper. If the silt can reach the sea it will be distributed better by the littoral currents, which are quite pronounced. It seems natural that the up-stream course should tend to deepen when the delta course is good since, then, there will be a tendency for the flow to scour a deeper channel in the transition zone between the steep and the flat slopes, and by back-cutting, this propels itself up stream. There are indications that this cycle of bed rise and scour, due to good and bad delta channels, influences the diked course as far up stream as the Grand Canal crossing. It is of special importance to have the delta channel in good condition before the flood season arrives as an early serious flood may cause overtopping of the dikes in the lower reaches if it should find the bed in a deteriorated condition and the scour not rapid enough to accommodate the flow. This nearly happened in 1936.

If as much of the silt as possible can be made to reach the sea, two objects are attained: (1) To prolong the life of the steep frontal delta fan slope, which prevents flatter slopes from developing up stream; and (2) to keep the up-stream channel in a deeper condition.

It does not seem technically feasible to construct training works at the mouth to guide the flow and the silt into deeper water where the littoral current could distribute it. Even with this agent acting the silt deposition would be too rapid locally. The training works would also have to be tied to a dike system extended to the coast, which, again, would mean a deposition of the entire silt volume over a limited area. This should absolutely be avoided since it would soon re-act unfavorably on the course up stream. Spreading the silt over a wide front so as to maintain the present course as long as possible seems the best plan. The one advocated is one of several that may be adopted. The Tu Hai Ho by-pass channel which has been proposed should be guided to the north of any branches that may be conducted across the delta.

If nothing is done in the loess areas of the Yellow River water-shed to diminish the soil erosion, there is certain to be serious difficulties in maintaining the Yellow River on its present course from the Grand Canal to the sea after another two centuries have passed. During the next few decades every effort should be made, therefore, to enforce an energetic soil-erosion control campaign. This promises so much for the Yellow River that no statements recommending it can be too strong.

Control of Soil Erosion.—Were it not for the heavy load of silt in the Yellow River at high stages, the flood-control problem would be far less difficult. The vast quantities of fine loess soil that are washed off the hillsides in Kansu, Shensi, and Shansi, in the rainy season, and are brought into the Yellow River by such feeders as the Wei Ho, the Lo Ho, and the Fen Ho, create a most formidable problem. Therefore, erosion control should be a most important factor in flood control for this river; and, yet, there is ample evidence that it has not been practiced in a sufficiently effective manner. It is true that many farmers have sought to protect their side-hill lands by terracing them and have done so to a degree. However, the thousands of gullies and ravines that have been cut into these loess hills by the agency of running water bear testimony to the wearing away of these lands at an alarming rate.

In his soil studies in China Mr. James Thorp has called attention to this problem,[12] and states that the methods of control adopted by Chinese farmers have been only partly effective, although he finds the terracing in Northwest China to have been conducted over a wide area. Lack of rock supply and timber reasonably close to the regions being treated has made this work less effective in the great loess-covered water-shed of Kansu and Shensi, for example, than in the hilly parts of Shantung where stone walls are used in making lasting terraces. More rigid application of rules as to level terraces, contour plowing, and pit and strip planting, as well as suitable supervision of a drainage system, will all aid in preventing further inroads of eroding processes on the lands that are now used for cultivation. However, in the great problem of controlling the gullies and ravines that are already deeply cut into the landscape, timber and stone must be used in broad areas where these are very difficult to obtain. Here, the power of the State must enter and a large subsidy must be created to aid local farming communities to put these gullies and ravines under control if this erosion is to be seriously and effectively retarded. The native population does not possess the wealth or the leadership to carry through this phase of the program, which will be costly and will require at least two or three decades with Government support of a most substantial character. If the outside support is only moderate, a century or more may pass before this important part of the Yellow River flood control is well in hand.[13]

The field laboratory, established in Western Honan early in 1937 to make certain experiments in connection with gully-erosion control, has yielded few results. Those in charge made certain brief field studies in Japan and, later, were driven from their work by war conditions. The only laboratory work of significance to date in this connection is the great natural open labora-

---

[12] "Soil Erosion in China," by James Thorp, *Journal*, Assoc. of Chinese and American Engrs., July-August, 1936.

[13] "Soil Erosion and River Regulation with Special Reference to the Yellow River," by S. Eliassen, *Journal*, Assoc. of Chinese and American Engrs., January-February, 1936.

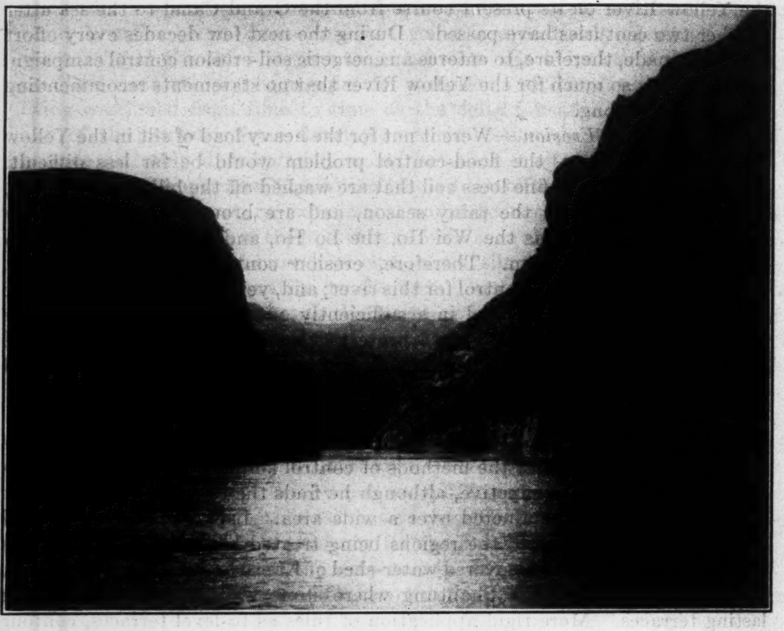


FIG. 31.—YELLOW RIVER GORGES 3 MILES ABOVE YUMENKOU

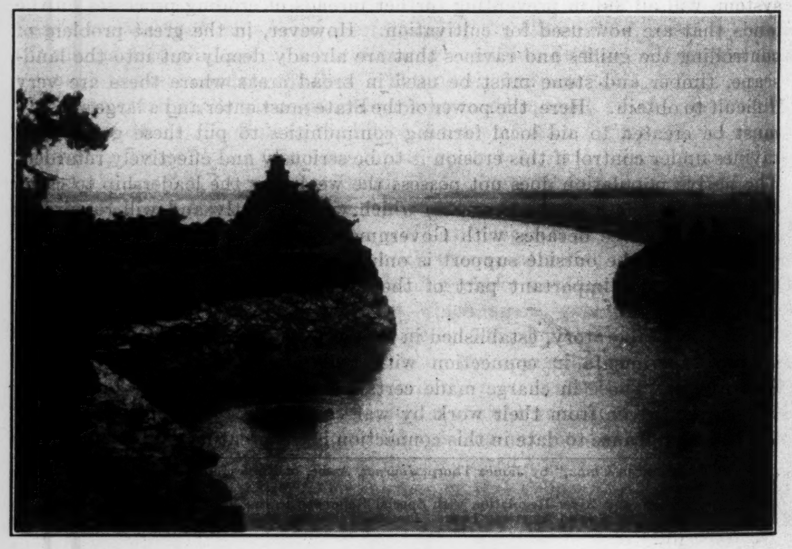


FIG. 32.—YUMENKOU AT FOOT OF GORGES BELOW HU-KOU AND 83 MILES NORTH OF TUNGKUAN

tory of the Chinese farming community extending throughout the loess-covered areas. The observations of the writers over many years lead them to believe that the condition of these gullies, many of which will lend themselves readily to treatment by a series of low earth dams supported by cribwork or piling, can be improved within one generation at reasonable cost under intelligent leadership where a way is found to obtain the general co-operation of the various land-owners and strong State support.

Reforestation and Run-Off.—It has long been the hope of students of North China's flood problems that some method might be found whereby the rainfall that is concentrated almost entirely in the months of July, August, and September, with occasional heavy rains in late June, might be spread out over a longer period, or that the run-off might be better distributed through the year. Since storage of flood-waters in reservoirs for weeks or months is not practicable for the Yellow River system, due to the heavy silt content in the water during the rainy season, the remedy of reforestation has been frequently recommended as a panacea.

The speculation that has existed in China as to the effect of extended reforestation on run-off in the Northwest has not produced a unanimous opinion among climatologists as to the possibility of prolonging the rainy season. The group that has insisted for the past two decades that China's flood problems will be remedied, to a great extent, by heavy reforestation in the arid regions of the Northwest, has gradually become less vocal as the difficulty of the problem has been shown, and as more has been published on the matter of storm occurrences due to cyclonic action. To the writers it appears that, although the duration of the rainy season may not be appreciably increased and the climate modified by forests in a way to produce more frequent rains, there may be a distinct benefit in the line of arrested run-off if North and Northwest China ever developed a suitable forest cover. Such a cover over one-half the area of Kansu, Shansi, and Shensi, cannot be obtained at any reasonable cost and will require several decades of persistent effort due to the dryness of the climate and poverty of the country.

For many centuries seasonal typhoons have been coming in from the regions in the Pacific Ocean east of the Philippines and drifting westward to the China coast and then north through China. Other storms belong to the continental system originating in West-Central Asia and drifting eastward, as discussed heretofore in this paper. These typhoons are all of such force as to be only little affected by such forest cover as may develop in the Northwest Provinces.

RIVER UTILIZATION

Power Development at Hu-kou Falls.—Only since about 1933 have serious studies been made of power development on the main course of the Yellow River. Difficulties with storage on account of the rapid silting of reservoirs have made impracticable any large power developments on the main tributaries, such as the Wei Ho in Shensi, or even the Fen Ho in Shansi, although in either case winter flow, with its low silt load, may be stored a few months for spring irrigation. The Yellow River, however, has a sustained low-water flow at Hu-kou Falls warranting hydro-electric development at that point, whereas along the gorges of the Yangtze, no great industrial city is near-by to use the power thus developed; but the possibility of utilizing power for pumping in connection with irrigation in Southwest Shansi, as well as for industrial uses in surrounding *hsien* cities (a "*hsien* city" is a "county-seat"), seems to warrant the development of the power afforded by this fall in the main river.

The Hu-kou Falls are 125 miles north of Tungkuan, Shensi, or nearly 45 miles north of Yumenkou. After having passed a series of rapids and narrow gorges between Hu-kou and Yumenkou (see Figs. 31 and 32) the river debouches on a plain which lies chiefly in Eastern Shensi, but also includes the valley at the mouth of the Fen Ho, in Shansi. The region to the east of the river, from the Fen Ho south, is a loess plateau of which more than 150 000 acres lie on an average of 250 ft above the river at Yumenkou. This land might be irrigated by pumping from the Yellow River, using low-priced power from Hu-kou Falls. The climate and soil are both well adapted for cotton growing.

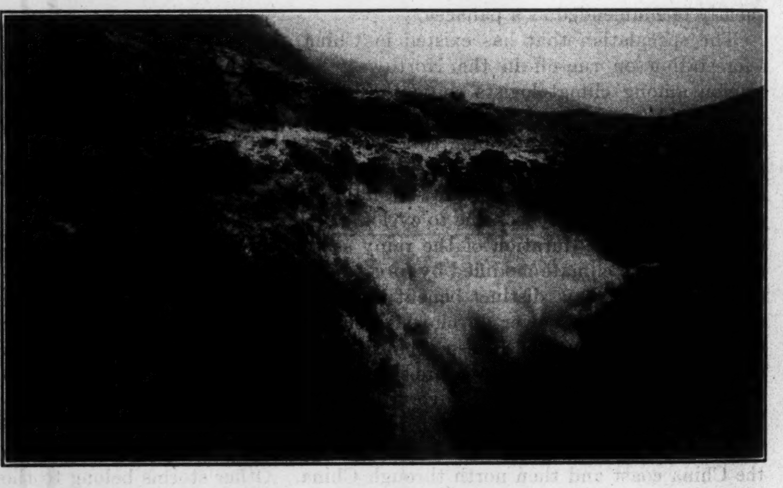


Fig. 33.—Hu-kou Falls on the Yellow River

In 1934, surveys, studies, and geological investigations were made of the Hu-kou site under the direction of one of the writers. Estimates based on these and other investigations indicate that a development of 100 000 hp dependable every day of the year can be made very economically, costing approximately $3 000 000 in United States currency, including transmission to Yumenkou and vicinity. With a diversion dam 40 ft above the river bed and 600 ft up stream from the Falls (Fig. 33) the available head at the power site, chosen 5 000 ft down stream, would be a little more than 110 ft. If a flood-control detention basin is to be established above these Falls, the problem of combining such flood-control project with the hydro-electric power development remains to be considered. Probably it can be solved by letting the lower 40 ft of the dam act permanently as head for power while the higher dam section will serve for flood-control purposes.

Irrigation.—A thorough program of control for the Yellow River will include improvements to existing gravity irrigation projects, such as those near Ninghsia, the Ho Tao region, and Saratsi. In addition to them, however, two projects already mentioned might be undertaken that would require pumping on to table-lands in Southwest Shansi, immediately south and east of Yumenkou. One of these projects, near Hotsin, was surveyed in 1934 under the direction of one of the writers, and its construction has been strongly urged by the local officials. In embraces 50 000 acres of cotton land on terraces and sloping plains from 200 to 300 ft above the Yellow River at Yumenkou. A larger tract lies across the Fen Ho to the south of Hotsin on a plateau, where approximately 200 000 acres, lying at an average elevation of 250 ft above the low-water stage of the Yellow River (where the Fen Ho meets it), offers irrigation possibilities. As yet (1938) no reliable surveys of this area have been made. This land is particularly adapted to cotton growing, but it frequently suffers from deficient rainfall. Development of cheap hydro-electric power at Hu-kou would make the building of an irrigation system practicable.

Ten miles above the Peiping-Hankow Railway the Yellow River might be tapped on the north bank for a gravity irrigation project that would parallel the river to the east. Although surveys have not been made for laying out canals for such a project, studies of available maps indicate that an area of at least 200 000 acres may be irrigated at a reasonable cost. This irrigation would insure crops of cotton, beans, wheat, etc., in drought years. Most of the land that would be served thus is of good quality for mixed agriculture and can maintain a farming population of 1 000 per sq mile.

An indispensable accessory to all irrigation plans for the Yellow River in the plain is that the river channel must first be stabilized where it runs past the intake works. If not, the river is certain, after a few years, to form a high foreshore in front of the intake due to silt deposition and change of course.

Fertilization by Flooding.—The dike break of 1935 in Western Shantung carried large quantities of fine sand out of the river bed across country to the southeast. Inspection trips over this flooded region were made by one of the writers both in June and in October, 1936, to estimate the damages and the benefits due to sand and good silt deposits. The heaviest sand deposit followed the course of the principal channel eroded by the flood waters for a distance of nearly 50 miles making almost worthless for agriculture approximately 350 sq miles of farming country that, in previous floods, had sustained similar injury, but had again been improved by fertilization and farming. The October inspection trip showed that the late spring planting on these lands yielded scarcely enough to pay for the seed. Farther away from the break where there had been ponding with but little current, deposits of very fine silt or loess had greatly improved the land. Over large areas 50 to 60 miles from the break such deposits of clay-like silt were from 1 to 2 ft in depth, but not, as a rule, dry enough for planting in 1936.

It seems probable that systematic flooding of these low, sandy lands by the use of adequate gate structures, that will retain most of the sand within the river bed while accumulating a deposit of fertile silt over the area, will prove feasible and beneficial. A zoning plan can be developed so that each year a district on each side of the river may benefit by such controlled flooding. Five or six years later those same areas would have another turn to receive fertile mud from the river through controlled flooding at times when the suspended silt load is high.

Attempts have already been made in Eastern Shantung to siphon Yellow River water on to alkali lands near the sea; but even with the 24-in. cast-iron pipes thus used the process is slow. Flooding through an extensive system of gates seems necessary if this land improvement is to be undertaken seriously.

Navigation Improvements.—At present, only light-draft junks can ply the Yellow River in the low-water season. A power launch drawing more than 2 ft would have difficulty navigating the diked section of the river in low-water stages. With the proposed training of the channel, perhaps, depths slightly exceeding 3 ft may be maintained at all stages, although bars will continue to be built in new places as the bed-load moves and new silt deposits are laid down. This applies to the river from the Peiping-Hankow Railway to the sea. In order to have depths greater than 3 ft it would be necessary to construct a system of contraction works inside the proposed regulated channel. This would interfere with the river's scour action, however, and would prevent the enlargement of the channel so necessary for carrying the flood flow safely. Perhaps it would also encourage increased refill which would cause the bed to rise more quickly than otherwise. The only navigation improvement worth while would seem to be a light system of construction works at places which show tendencies to much shoaling. These works would be washed out by the first freshets and would have to be replaced early each spring. Of course, the deepest channel would have to be marked constantly.

From the Peiping-Hankow Railway to Tungkuan, the country on both sides is mostly very mountainous and sparsely populated. The currents are swift on account of the river's steep slope, making up-stream navigation slow and difficult. The treacherous Sanmen Rapids are also much feared by the boatmen. Before the construction of the Lunghai Railway this part of the river carried a large part of the exports from Southeast Kansu, West Shansi, and Central Shensi; but the railway has now practically absorbed all the traffic and there is left only a small tonnage of coal transportation from mines along the banks below the Sanmen Rapids. Navigation through this section is now unimportant and not worth considering.

From Tungkuan to Yumenkou no very swift-water stretches are encountered, but immediately to the north of Yumenkou there are gorges where the river is very narrow, and the rock walls are nearly vertical in many places (see Fig. 31). Above Yumenkou there are a number of important coal mines from which the Lunghai Railway and large areas in Central Shensi draw their coal supplies. At high-flood stages no craft can go up these gorges and, even in low water, the coal barges have difficulty in getting through the 15-mile stretch from Yumenkou to the coal mines above the gorges. Good power boats may be feasible here as the traffic seems to warrant their use.

For the next 30 miles to Hu-kou Falls the river has a number of rapids. Boats rarely go up this stretch, therefore, although they may shoot these rapids on their way down stream after being hauled around the Falls. In consequence, only cheaply constructed shells are made for traffic through this section and such boats are sold in Honan or Shantung as it is not economical to make the return trip

The question of getting boats around the Hu-kou Falls has been a problem for years. The passage is highly dangerous except for a short season of the year when unloaded cotton boats may be taken down a small channel prepared for that purpose. All cargo is carried past the Falls by donkeys, a distance of two miles, and then reloaded. As yet the construction of modern locks to handle boat traffic past these Falls has not been seriously discussed by the authorities.

Other rapids 200 miles farther north make navigation of this part of the river dangerous in low water. Sharp rocks often tear holes in boats that attempt to pass down this section. Pilots must use the greatest caution and pick a medium stage of river for travel.

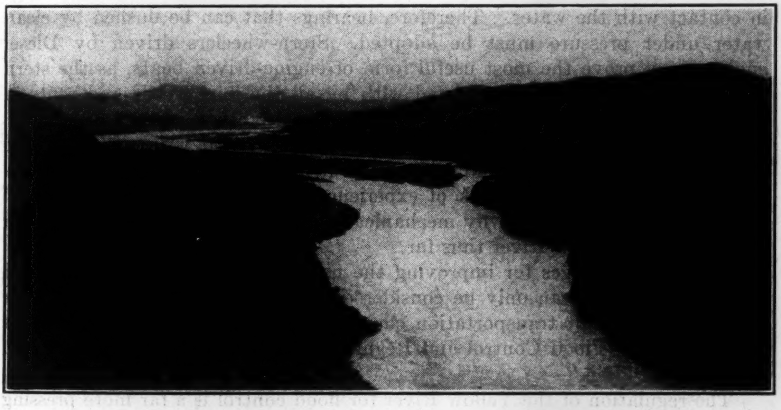


FIG. 34.—THE YELLOW RIVER 20 MILES BELOW LANCHOW, KANSU, 1931

Although Shansi officials have discussed improvements to navigation, so that grain boats may go from Ninghsia and Suiyuan to points in their own Province, the regulation of the Yellow River along the west border of Shansi has appeared too formidable and has not been undertaken. The problem is worthy of further study, since water transport may prove cheaper than rail haulage if it can be handled safely and without costly portages. It would not be an all-year service due to the long ice period and the suddenly appearing freshets of the high-water season, but it might be relied upon for six months of the year.

An interesting form of navigation exists between Lanchow and Paotou. In the spring, the natives at Lanchow construct rafts of ox hides stuffed tightly with wool or sheep skins, inflated with air, and lashed to a top framework of light wooden poles. As many as 120 ox hides, or 500 sheep skins, are used for each raft. It is down-stream navigation only, but as much as 20 tons of goods have been carried on a single raft. The passage through the narrow, jagged gorges between Lanchow and the Ninghsia border is rather perilous (see Fig. 34) and many rafts have been wrecked in this stretch; but the overland journey to the coast is so long and expensive that the dangers of the river route are risked. On arrival at Paotou the wooden frame is sold and the skins deflated and carried overland back to Lanchow. Two trips per year are all that can be made by one crew.

Boats are also used on the stretch from Paotou to the Kansu-Ninghsia border for down-stream as well as up-stream navigation, but they cannot ascend the gorge section to Lanchow. It is possible, but very costly, to improve the gorges so that boats may reach Lanchow. Under present trade conditions of Northwest China, however, such heavy expenditures are not warranted.

Steam or motor-driven launches which have been tested on the Yellow River have always encountered trouble because the silt wore out all bearings in contact with the water. Therefore, bearings that can be flushed by clean water under pressure must be adopted. Stern-wheelers driven by Diesel engines may prove the most useful form of engine-driven boats, as the stern wheel is less likely to be interfered with by all the long weeds and roots in the water during floods. The axle-bearings are also above water.

A boat with an air propeller has recently been tried with some measure of success between Paotou and Ninghsia; but maintenance trouble has been a drawback. It seems that lack of experience with engines and their upkeep has been the main reason why mechanically propelled crafts have not been a success on the Yellow River thus far.

The use of dredges for improving the navigable channel along the diked course of the river can only be considered in connection with maintaining a better flood and silt transportation channel as mentioned toward the end of the section on "Flood Control and Regulation: Regulation of Diked Channel Through the Plain."

The regulation of the Yellow River for flood control is a far more pressing problem than putting this river into condition for safe navigation. In fact, the former must precede the latter.

Magnitude of a Comprehensive Program

The area lying within the Great Plain where floods have done so much damage during recorded history, and where, at any time in the future, the population may suffer if adequate control and regulation work is not undertaken, is estimated to have an area greater than 35 000 000 acres, or 55 000 sq miles—approximately the area of the entire Shantung Province (see Fig. 35). The Provinces of Hopei, Honan, Kiangsu, and Anhwei, as well as Shantung, all have plain lands subject to damage by these floods. Parts of all these

Provinces might well bear a moderate annual flood protection tax of 20 cents (U. S.) per acre for a period of two decades or more. If this is done the Central Government will be better able to lend aid to the poorer Provinces of Shansi, Shensi, and Kansu in coping with the soil-erosion problem, which will require three decades or more of persistent work to solve in a satisfactory manner.

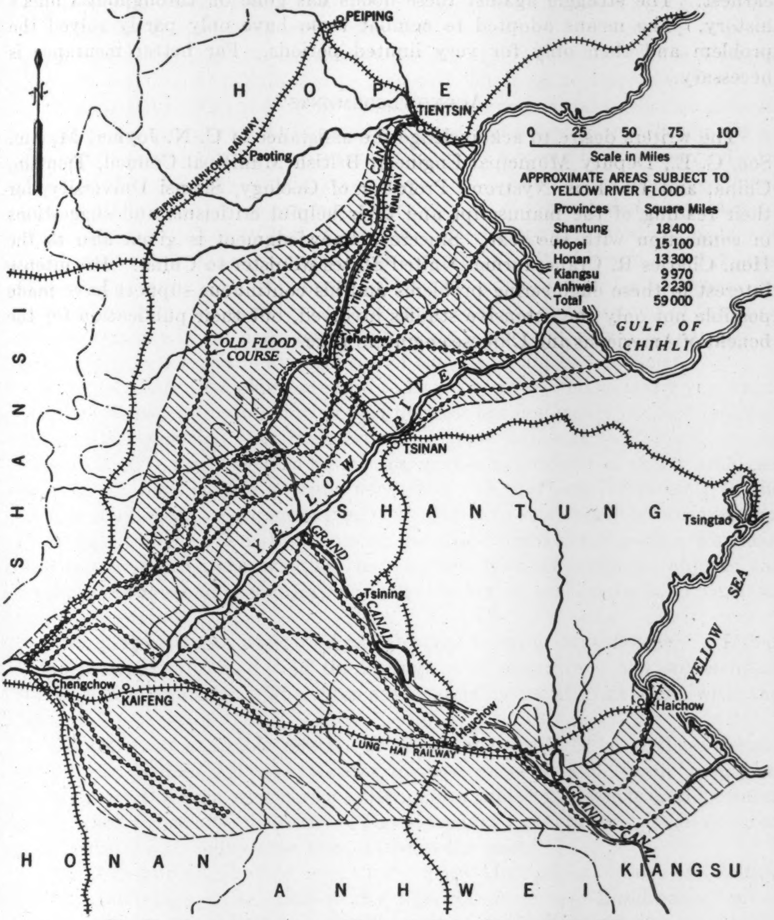

PEIPING
H O P E I
TIENTSIN
Paoting
PEIPING-HANKOW RAILWAY
GRAND CANAL
TIENTSIN-PUKOW RAILWAY
0 25 50 75 100
Scale in Miles
APPROXIMATE AREAS SUBJECT TO YELLOW RIVER FLOOD
Provinces Square Miles
Shantung 18400
Hopei 15100
Honan 13300
Kiangsu 9970
Anhwei 2230
Total 59000
GULF OF CHIHLI
OLD FLOOD COURSE
Tehchow
YELLOW RIVER
TSINAN
S H A N S I
S H A N T U N G
Tsingtao
GRAND CANAL
Tsining
YELLOW SEA
Chengchow
KAIFENG
Hsuchow
LUNG-HAI RAILWAY
Haichow
GRAND CANAL
H O N A N
A N H W E I
KIANGSU

Fig. 35.—Approximate Areas Threatened by Yellow River Floods

Since construction costs in China are far less than those in America the necessary work can be done very reasonably, and Yellow River floods, such as have cursed North China in the past, can be made so highly improbable in the future as to be no longer dreaded by the people who live within their reach. The Great Plain of China can be made a perfectly safe place in which to dwell. Furthermore, it can be improved in a number of ways aside from mere flood protection. The follow-up work of soil-erosion control, in the Shensi and Kansu water-sheds particularly, will be possible with continued National Government encouragement; but it is time to begin this problem in earnest. The struggle against these floods has gone on throughout China's history. The means adopted to combat them have only partly solved the problem and then only for very limited periods. Far better insurance is necessary.

ACKNOWLEDGMENTS

The writers desire to acknowledge the assistance of C. N. Joyner, M. Am. Soc. C. E., Deputy Municipal Engineer, British Municipal Council, Tientsin, China, and Dr. Erik Nystrom, Professor of Geology, Shansi University, for their reading of the manuscript and their helpful criticisms and suggestions in connection with the text. Sincere acknowledgment is given also to the Hon. Charles R. Crane, former United States Minister to China. His intense interest in these engineering problems and his continuous support have made possible not only the extensive studies involved, but their publication for the benefit of Americans and Chinese engineers alike.

DISCUSSION

J. W. **Beardsley**,[14] M. Am. Soc. C. E. (by letter).—Exceptionally detailed and comprehensive data on a difficult problem are presented in this paper. To appreciate the work involved one must consider the Chinese language which is not adapted to technical terms, the lack of co-operation between adjacent provincial authorities during past centuries, and the failure of the National Government to collect data and to plan and construct protective works.

During 1918–1920, the Grand Canal Improvement Board collected samples of silt in the vicinity of Shihlipu and the railroad crossing near Tsinan Fu by means of a hand pump and a pipe which could be lowered to any desired depth. A silt load of 15% by weight was rather high. Samples were also collected from old silt beds and measured carefully. It was concluded that 15% by weight averaged close to 10% by volume. Recently deposited silt beds, sun dried, were cracked into irregular blocks about 2 or 3 ft wide and 3 or 4 ft long. The cracks were 1 ft to 2 ft deep. Along the edge of such banks, where the current had been relatively swift, were generally found small areas of fine sand suitable for use in an hour-glass. Probably such sand would be harmful to soil fertility, whereas the average run of the silt would be beneficial.

The deltaic central plains are said to have been very fertile many years ago; but the intense cultivation during past decades has materially reduced fertility. Missionaries reported that their census of areas adjacent to Tsining, on the Grand Canal in southwestern Shantung, gave a population of about 2 000 per sq mile. A food crop is a prime necessity. The farmers live mainly in the protected villages. A common type of irrigation in this area is by water drawn from wells which have a stone post on each side suitable for using a windlass. The farmer carries the windlass and bucket from the village, adjusts the windlass in the stone posts, draws a bucket of water, and carries it to his small cultivated plat.

This part of China has no "April showers to bring May flowers." It has slight snowfall or rain during the spring. Soil moisture is not sufficient to sprout early plantings. The later seasonal rains usually interfere with the harvest.

Data are not available relative to the decreasing fertility of these deltaic areas. Sir William Willcoks referring to the displacement of "basin" irrigation in Egypt by the then new perennial irrigation said:[15] "The rich muddy water of the Nile has been the mainstay of Egypt for many generations, it can no more be dispensed with today than it could be in the past."

The Nile and the Yellow rivers have many similarities. Both are deltaic rivers. Both lack tributaries of any importance several hundreds of miles above their mouths. Both have built up silt banks higher than adjacent lands. A notable dissimilarity is that the Nile has been orderly. It is the life of Egypt. The Yellow River is an unruly bandit to such an extent that it is called "China's sorrow."

---

[14] Cons. Engr., Syracuse, N. Y.

[15] "The White Nile and The Cotton Crop (No. 2)," by Sir William Willcoks; a lecture delivered at a meeting of the Khedivial Geographical Society, January 25, 1908.

The authors' discussion of "Irrigation" and "Fertilization by Flooding" are exceptionally interesting. Similar comments on reclamation would have been of equal interest. With considerable hesitation, the writer ventures to make two suggestions for making a useful servant, at least a part of the time, out of the unruly Yellow River:

(1) Adjacent to the river's old bed prior to 1851 (and in several other localities near its present bed) are a number of shallow lakes and marshy areas too wet for cultivation. Is it practicable to fill such areas with the July floods? The silt loads in the vicinity of the Peiping-Hankow Railway Bridge frequently carry 15% or more of silt by weight, or 10% by volume when compacted. An impounded basin having a maximum depth of 5 ft would deposit a half foot of soil within a few days. Within a few years such areas could be converted into "culturable" land.

Non-fertile sandy areas could be made productive by impounding a depth of a few feet of flood waters, heavy with silt, for two or three days before draining. If such an operation could be repeated annually for a few years, high fertility would be maintained for many following years.

(2) Is it practicable to flood those areas near the river, where irrigation is difficult, with the low-water flow of late winter or early spring? Some soils take up moisture slowly and the retention of the water might be necessary for a week or more before sufficient moisture is absorbed to sprout early plantings. The basin must then be drained and the retained moisture should be sufficient to carry the crop until the arrival of the summer rains.

The silt load is then generally about 1 per cent. If a depth of 2 ft were impounded, the silt deposited would be of considerable value as a fertilizer, especially if the operation can be repeated each year. This plan conforms to the "basin" irrigation of ancient Egypt.

As long as a stable government control is disturbed and co-operative maintenance of the system of dikes remains impracticable, the danger that the Yellow River will usurp an unpredictable channel to the sea is increased manyfold. Sooner or later these river problems must be solved.

ELLIOTT J. DENT,[16] M. AM. SOC. C. E. (by letter).—This paper is an extremely interesting one as it places before the profession many facts that have not, heretofore, been available with respect to this extraordinary river. "China's Sorrow" is such an unusual stream, and the local conditions are so at variance with those that river engineers are accustomed to find elsewhere, that the description has many of the intriguing qualities of a Jules Verne novel. The quantities of sediment shown as passing the hydrometric station at Chinchang are so great that there should be outstanding evidences of buried structures throughout the alluvial plain, and Mr. Thorp has mentioned the case of a pavement buried to a depth of about 12 ft at Kaifeng, a few miles west of the 1851 break. The authors state that there is reason to believe that the serious erosion now taking place in Northwest China began at about the time of the Emperor Yü (2200 B. C.) but, even so, the depths of deposit in a period of 4 000 yr should have attracted more attention than is evidenced by the mention of a single case by Mr. Thorp.

---

[16] Col., U. S. Army (*Retired*), Washington, D. C.

Although the writer confesses a certain degree of skepticism with respect to the capacity of a river to carry sedimentary loads of the magnitude reported, the data have been accepted, for the sake of discussion, and an effort has been made to indicate some of the inevitable consequences that must follow from such a condition.

The slope of a river is one of its most important characteristics, and a profile is very useful in studying its habits. In Table 1 the authors give the necessary data and Fig. 36 has been prepared to show the slope from Shanchow (Mile 624) to the mouth. All elevations are referred to Taku Datum, the zero of which is 4.5 ft below mean sea level. As should be expected, when dealing with a stream as unstable as this one, the distances shown are somewhat different from those used by Mr. Freeman.[3] A number of stations not included in Table 1 have been approximately located by reference to the text or by scaling the distance to the nearest known point.

Shanchow, Mile 624, to Chinchang, Mile 441.—Between Shanchow and Mengtsin, Mile 503, the river flows through the last of its mountain gorges with numerous rock ledges and minor rapids; in this section it is an eroding stream carrying an enormous load of loess in suspension. A short distance above Mengtsin it enters its area of deposition and begins to meander from side to side of an alluvial valley in a bed composed of sandy material dropped by the river itself. A short distance above Chinchang (site of the Peiping-Hankow Railway Bridge) two tributaries, the Lo Ho and Chin Ho, enter the main stream. The importance of these branches as flood carriers was illustrated in 1935, when they added about 300 000 cu ft per sec to the July flood of that year. These tributaries drain an area covered largely by Tertiary deposits of red clay, and they do not carry as much sediment as the main stream at Shanchow. Moreover, the clay is finer than the loess and more easily transported. There are some strata of sand and gravel outcropping in the clay cliffs and these should add to the bed-load of the river. Certain maps, not published in the paper, indicate that the lower portion of the flood plain of the Chin Ho has been modified by a rise in the bed of the Yellow River.

In the rocky gorge above Mengtsin the average slope is about 4.6 ft per mile; and, in the alluvial section from Mengtsin to Chinchang, it is about 1.37 ft per mile. Throughout this alluvial section the bed is sandy and the river shifts from side to side of the flood plain; for the full length along the right bank the plain is bordered by the clay or loess hills; but on the left bank the dikes extend about 15 miles above Chinchang.

There is abundant evidence that the river is depositing huge quantities of sediment in the overflowed areas. The outflow from this section is measured at Chinchang and the portion of the inflow that passes Shanchow is also measured. No direct information is given with regard to the inflow from the Lo Ho and Chin Ho, and minor tributaries, but there is some indirect evidence that this amounts to several hundred million cubic yards per year.

Table 7 shows the monthly flow of sediment at Shanchow and at Chinchang for the years 1934–1935. During July, 1935, there was a flood in the tributaries amounting to more than 300 000 cu ft per sec, and during that month the outflow of sediment amounted to the inflow from above Shanchow, plus 158 000 000 cu yd. (For convenience in visualizing the extent of the flood-plain deposits it is well to recall that 1 000 000 cu yd may be taken, with an error of only 3%, as the volume of sediment required to bury 1 sq mile to a depth of 1 ft.) If, as seems probable, there were deposits during that month, the inflow from the tributaries must have been 158 000 000 cu yd plus the quantities deposited. During the 2-yr period there were nine other months in which the outflow was greater than the inflow from above Shanchow, the aggregate excess for the entire ten months having been 406 000 000 cu yd. During these ten months the inflow of sediment from the tributaries was apparently 406 000 000 cu yd plus an indeterminate volume of deposits.

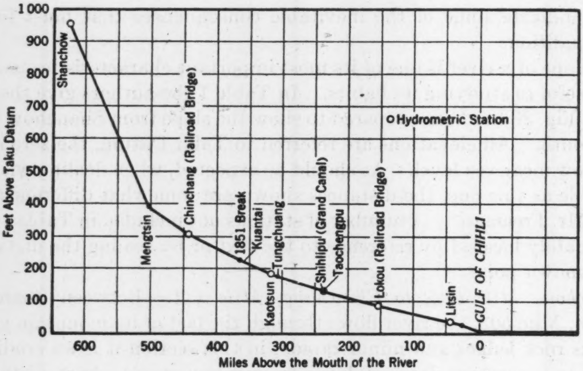


FIG. 36.—LOW-WATER PROFILE OF THE YELLOW RIVER

During the remaining fourteen months of the record the inflow of sediment from above Shanchow, alone, exceeded the outflow at Chinchang by 323 000 000 cu yd, and the deposits included this volume plus the entire inflow from the tributaries. The limited information available indicates that, for the years 1934–1935, the deposits on the overflowed land above Chinchang amounted to the huge total of more than 500 000 000 cu yd. For the flood plain below Chinchang, better information is, fortunately, available.

Chinchang, Mile 441, to Tungchuang, Mile 307.—The flow at Tungchuang will be assumed to be the same as at the hydrometric station at Kaotsun, a few miles up stream (see Fig. 29).

Throughout this 134-mile reach, the river flows along the back-bone of a ridge formed by the deposit of part of the sedimentary load. There are no tributaries; on the contrary, there is a continuous threat that a crevasse will form, permitting the wastage of part of the flood waters over the adjacent plain. In this section of the Yellow River the slope averages 0.94 ft per mile; the distance between dikes averages about 6 miles; the channel is constantly shifting its position between the dikes and there are many sandy islands and bars; there is a heavy bed-load; there are extensive sand deposits on the foreshores and on the adjacent plain; and, as shown by Mr. Freeman[5] all of the major changes in the course of the river have originated in this section.

At Mile 365 the great break of 1851 occurred and changed the course of the river from its former southeasterly direction to the Yellow Sea, to its present northeasterly course to the Gulf of Chihli. Mr. Freeman has presented a cross-section[17] about five miles above this break which shows that the dikes were about 6.25 miles apart; that the foreshores between the dikes and the channel had been built to a level about 30 ft above the adjacent plain; and that following the break the river had eroded deep into the deposits. In 1937, as explained by the authors, the foreshore lands, for a distance of about 25 miles above the site of the break, were still above the level of the highest floods. The river bed at this point is said to be rising at a rate of about 3 ft per century, but exact figures are not given. Down stream from the site of the break, toward Tungchuang, the natural and artificial levees have not yet developed to dimensions that may be considered reasonably secure. In the great flood of 1933 there were 35 breaks in a distance of 48 miles; in 1934 there was a break at Kuantai, only a few miles below the site of the 1851 break; and, in 1935 there was a break near Tungchuang, the lower end of the reach.

Table 8 shows that the quantity of sediment passing Chinchang during 1934 was 1 915 000 000 cu yd, and that only 1 168 000 000 cu yd passed Kaotsun; the difference, 747 000 000 cu yd, was deposited between the dikes or spread over the plain as a result of the Kuantai break. In 1935 the silt passing Chinchang amounted to 1 727 000 000 cu yd, of which 1 046 000 000 cu yd passed Kaotsun; the difference, 681 000 000 cu yd, was deposited between the dikes. In 1935 a considerable part of the sediment that had passed Kaotsun was subsequently spread over the plain as a result of the crevasse at Tungchuang.

The profile of the river, the location of the various outlets since the beginning of the historical record, the descriptions of the shifting channels and numerous sand bars and islands in the upper reaches, the references to the relatively better defined channels in the lower reaches, the descriptions of the sand and gravel strata in the eroding Tertiary cliffs, the references to the bed-load and to the sand deposits on the foreshores and adjacent plain above the 1851 break, the data with respect to the silt loads at the various hydrometric stations, and the reports that the bed is rising—all of this evidence is in accord as pointing to the conclusion that the alluvial plain from Mengtsin to Tungchuang is now serving as a settling basin in which the coarsest part of the sedimentary load is deposited. The record of silt observations at the hydrometric stations and the statement that the river bed at the site of the 1851 break is rising at the rate of about 3 ft per century are in agreement as indicating that sediment is being deposited, but in notable disagreement as to the rate of such deposit.

The foregoing deductions should be checked by additional field surveys, and these should include mechanical analyses of the sedimentary deposits throughout the flood plain. Similar analyses should be made of the suspended load at each of the hydrometric stations; the sedimentary load should also be measured at one of the minor rapids above Mengtsin where the entire load will presumably be in suspension and therefore susceptible to being measured. The gage records at the Lokou (Mile 152) and Chinchang railroad bridges should be examined to see whether the low-water levels at those points are changing; and all available information with respect to changes in elevation of the river bed, the foreshores, or the adjacent plain should be assembled and studied.

---

[17] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), Fig. 12, p. 1421.

Tungchuang, Mile 307, to Litsin, Mile 42.—The hydrometric station nearest to the mouth is at Litsin and the 265-mile reach from Tungchuang to that point will be considered as a unit. The Grand Canal crosses the river at Shihlipu (Mile 236) and, under certain conditions, water that has escaped through crevasses above that crossing is returned to the river at that point, after having been nearly completely clarified by sedimentation. From Tungchuang to the crossing the average distance between dikes is about 4 miles; below the canal the width is about 1.25 miles. Below the crossing the channel is more clearly defined than above that point. The average slope for the entire 265-mile reach is 0.54 ft per mile.

Table 8 shows that in 1934 the volume of silt entering the reach was 1 168 000 000 cu yd and the volume leaving at Litsin was 1 295 000 000 cu yd. In 1935 the crevasse near Tungchuang allowed the escape of an unknown, but very large, volume of water and sediment. The hydrometric station at Taochengpu, about Mile 236, showed an annual load of 514 000 000 cu yd as compared with 447 000 000 cu yd at Litsin. These values are not conclusive proof of either fill or scour between the dikes of this reach during the 2-yr period.

Delta.—In the discussion of plans for the control of the Yellow River, the authors state that the sediment to be discharged at the mouth may be as much as 815 000 000 cu yd per yr, a value far less than the quantity shown as having entered the diked channel at Chinchang during 1934 and 1935. Their discussion of the rate of advance of the delta into the Gulf of Chihli indicates that no insuperable problem will be encountered at that point during the next century, but that very serious difficulties may arise within the next two centuries.

General.—When a project is under consideration for the control of a river flowing in an alluvial bed, between natural or artificial levees, the thought is often expressed that the stream should be so guided and controlled that the entire sedimentary load will be carried from the head of the valley to the sea without the deposition of any part en route. Assuming, without demonstration, that this can be accomplished in the flat sections near the mouth, the proponents then look at the steeper slopes in the upper reaches of the area of deposition and propose, further, that the same methods of control be applied to the reaches of relatively steep slope. They argue that this should cause erosion and that the river should cut its channel deep into the friable deposits, and thus lower the high-water surface to a level below that of the surrounding plain.

The Emperor Yü (2200 B.C.) is alleged to have controlled the Yellow River by causing the flood waters to flow to the sea at levels below those of the surrounding plain. Unfortunately for humanity, this tradition must be classed as a myth or the method used 4 000 yr ago must be cataloged as one of the lost arts. In recent years, when it is proposed to make the Yellow River dig deep, the method advocated calls for a reduction in width and a debate starts as to whether this contraction will result in a lowering of the bed or whether it will cause it to rise at a faster rate than at present. Those claiming that the bed will be lowered base their arguments on the higher velocities that will be found in the narrowed channels, and the observed scour that has been noted in certain short reaches where the channel now has the width proposed. Those believing that the bed will be raised hold to the opinion that the highest velocities obtainable in practice will be lower than those necessary to carry the entire sedimentary load to the sea, and that the reduction in the area over which the deposits will be laid down will increase the depth of the annual accretions.

In nature, an alluvial river carrying a heavy bottom load builds for itself a wide bed with a slope sufficient to create an adequate bottom velocity. In discussing the effect of structures designed to reduce the width of such a stream, the increase in depth and consequent increase in mean velocity is habitually stressed. There are other factors of equal importance, however, although frequently they are left unmentioned. Exact data are not always available and, for a mental picture of the case, it may at times be advisable to depart from strictly scientific accuracy and resort to the use of rough estimates that can, at least approximately, be checked by field observation.

From the surface to within about 1 ft of the bottom, the suspended load and velocity of the river current can be measured with reasonable precision; by extrapolation the figures for a point 0.5 ft above the bottom can be closely approximated. For purposes of discussion the velocity 0.5 ft above the bottom will be used as one of the indices. At a point about 0.2 ft above the bottom, or possibly less, a movement of sand by the saltation process comes into action, the character of the load is greatly altered, the load is much increased, and the velocity of the water is reduced. No practicable method of measuring the velocity of the water and the extent of the sand movement within 0.2 ft of the bottom has been devised.

The sand, traveling by saltation, together with the much smaller quantity that moves by rolling or sliding, will be referred to as the bed-load. In the absence of information as to the magnitude of this load in the Yellow River, a very rough estimate will be made in the hope that it will be better than nothing. Based on experiments by the late Grove Karl Gilbert,[18] the bed-load in the Yellow River when the mean velocity is 5 ft per sec, and the velocity 0.5 ft above the bottom is 3.0 ft per sec, will be assumed to be 300 grams per sec per ft width of channel; this is equivalent to 20 cu yd per day per ft of width.

If the depth of a stream is 10 ft, its mean velocity 5 ft per sec, the suspended load 6%, and if 1 cu yd of deposit weighs 2 800 lb, the suspended load per 1 ft of width will amount to 5 800 cu yd per day. These rough estimates indicate that if one-third of 1% of the suspended load in the gorge above Mengtsin consists of sand too coarse to be carried in suspension through the alluvial channel, the capacity of the river to carry bed-load may be fully taxed. The profile of the river and the descriptions of its characteristics indicate that its bed-load capacity is over-taxed. (On the Missouri and Colorado rivers in the United States, a rough rule is that sand too coarse to pass through a 200-mesh sieve will be actual or potential bed-load material. The slope of the Colorado River is about 1.5 ft per mile, and that of the Missouri River about 0.8 ft per mile. Mr. Freeman stated[3] that more than 99% of the Yellow River sediment would pass through a 200-mesh sieve.)

---

[18] "The Transportation of Débris by Running Water," *Professional Paper No. 86*, U. S. Geological Survey, 1914.

It is not easy for a river carrying a heavy bed-load to dig vertically downward. As an alluvial plain is built up the natural process tends to pave the bed with the coarsest material available; any subsequent attempt to dig vertically downward results in the formation of a more and more resistant surface. On the other hand, the natural levees are composed of material carried in suspension until the velocities along the foreshores have sufficiently slackened to permit its deposit, the coarsest part of the suspended load being normally deposited nearest to the channel. These bank deposits are conspicuously lacking in cohesive qualities and are very vulnerable to attack by undermining or sapping. In expanding in a lateral direction the current need only loosen part of the material to be excavated; the remainder is loosened by the caving process. Material lifted from the bottom by the current is so coarse that much of it must be carried away by the slow process of bed traction, whereas material loosened from the banks, for the greater part, can be borne away readily in suspension. When a river must enlarge its section by excavation it is often easier for it to accomplish its purpose by digging laterally than by digging vertically downward.

If q is the bed-load capacity of a river per 1-ft width and B is the width in feet, $q\,B$ is the bed-load capacity of the entire stream. The unit capacity, q, can be increased by an increase in velocity, and this in turn can be secured by a reduction in width. However, if the reduction in width is at a faster rate than the increase in unit capacity, the bed-load capacity of the entire river will be reduced instead of increased.

The foregoing considerations create a doubt as to whether it would be possible to increase the bed-load capacity of the Yellow River by reducing its width, or that it would be possible to induce the stream to dig vertically downward to a measurable extent; but if these objectives could be attained, the present flattening of the river as it flows downward through its alluvial section would introduce further difficulties. From Mengtsin to Chinchang, a distance of 62 miles, the present slope averages 1.37 ft per mile, and the Yellow River is unable, nevertheless, to carry its full sedimentary burden through that reach. If the use of contraction works and revetments were to make it possible for the stream to carry its full load through the aforementioned section, the problem would arise as to how the stream could be trained to carry this load, larger than the present one, through the next reach to Tungchuang, a distance of 134 miles, with a present slope of 0.94 ft per mile. If the problems of both of the aforementioned sections were solved there would remain the task of training the river to carry this full load through the remaining 307 miles to the sea, the present slope being only 0.54 ft per mile.

In their discussion of a possible project, the authors state that the regulated channel should be competent to carry to the sea an annual load of 815 000 000 cu yd. The volume entering the diked channel at Chinchang is stated as 1 727 000 000 cu yd in 1935 and 1 915 000 000 cu yd in 1934; and, the quantity carried by the much greater flood of 1933 is not known.

It would appear that the project for the control of the Yellow River for the next century should include provision for the deposit of a huge volume of the coarsest sediment, in an area above Tungchuang, diked off and reserved for that primary purpose. Below Tungchuang the project might provide for carrying the partly desilted waters, as at present, with only moderate deposits on such foreshores as may be left open between the dikes and the edges of the channel.

In selecting the proper area to carry the flood waters as distinguished from the sediment, the authors are on firm ground when they propose to give the regulated river a width that will carry the floods without making allowance for beneficial scour. The writer is not convinced that, if detention basins of the type suggested be built, it can be safely assumed that beneficial scour will result and that the width of the regulated channel can be reduced correspondingly.

HERBERT CHATLEY,[19] ESQ. (by letter).—This paper has the merit of being the first fairly complete technical description of the river in question, and the authors are to be congratulated on having been able to present so full an account.

The references to the Emperor Yü are a little unfortunate. Modern archeological research tends to show that this person is very largely mythical and an imaginative creation of the early Chou dynasts who wished to show that in overthrowing the Shangs they were recreating the ideals of their remote ancestors. There is no reliable history of China prior to about 1500 B.C.

In the references to modern studies some mention should be made of the work of the engineers of the League of Nations, Messrs. A. T. Coode and W. Nijhoff. It is true that a note as to their recommendations occurs in the paper, but their report[10] as a whole is also a valuable contribution to the rather scanty literature on the subject.

On the matter of dike positions and the Dresden experiments, the writer has published [20] a note in 1938 as to the theory of flow between set-back dikes, showing that, if the slope is constant and the flow formulas in current practice are accepted, the bank-edge position does give a larger flow for certain limited rises than a set-back position for the dikes unless the set-back is very large indeed. This conclusion is strongly and very naturally questioned; but, if it is not true, the flow formulas are somewhat incorrect and the hydraulic radius is no longer the sole criterion of flow in a bed of uniform material. The changes in the velocity-depth relation are possibly such that unless river sections are homologous, or nearly so, the mean velocity is not only an exponential function of the hydraulic radius but depends also on the shape of the section. It is worth remembering that, for constant area and constant wetted perimeter, there are two forms—one very narrow and deep and the other very broad and shallow—and it would be of great interest to know if model experiments really showed that the flow in each (with the same slope and bed material) were the same.

---

[19] Former Engr. in Chf., Whangpoo Conservancy Board, and Consultant to the Chinese Government; Cons. Engr., London, England.

[20] *Engineering*, December 9, 1938.

In this connection the writer strongly supports the desire of the authors to have exhaustive model investigations, including a careful study of the distortion effect due to vertical scales differing from horizontal ones. The writer is convinced that one great objection to the bank-edge dike position is the "plugging" of the section after a spate. In a flashy river a large deposit of silt must occur when the river suddenly falls, and this will leave a diminished section when the next rush comes. The old bed of the Yellow River south of Shantung is very interesting; in many places it is quite high above the plain. The writer is satisfied that this is largely explicable by the conditions which existed after the break of 1853. Some water was in it when the break occurred farther up; and, in addition, some water continued to enter it after the break, and the silt content of both forthwith settled and plugged the channel. It is improbable that the bed, say, at the crossing of the Grand Canal by the old channel, was as high when the river flowed in it as it is now. The writer urged this argument to Mr. Freeman when the latter advanced his straight deep channel proposals, the writer stating that after a spate the channel would tend to choke and that a new spate would back up and overflow the dikes before the plug was washed away. In this connection it should be noted from the data in Tables 7, 8, and 9 that the silt flow in a few days may represent a large fraction of the entire annual silt flow.

The gradients given in Table 1 show that the Yellow River is much steeper than the Mississippi River, even as the latter is steeper than the Yangtze. The most comparable rivers appear to be those of Northern India, several of which act in much the same way as the Yellow River. Owing to the far greater rainfall, however, and the shorter distance from the mountains, the bed material there is generally coarser. The hypotheses advanced by Mr. Gerald Lacey[21] as to regime conditions are well worth studying in this connection, although they are modified in cohesive materials.

The statement that the discharge of the Yangtzekiang above Ichang approaches 2 500 000 cu ft per sec is a little questionable (sentence preceding "Hydrological Considerations: Frequency and Magnitude of Floods"). This value almost represents the maximum flow in the lower part of that river, and as many large tributaries enter below Ichang such discharge can rarely if ever occur in the gorges.

It is noticeable that the packing of the consolidated loess is quite close, the Terzaghi pore-factor being less than unity. In the suggested measures for regulation no mention is made of the plan suggested by Capt. W. F. Tyler[22] for silt deposition basins. In view of the fact that there are areas not far from the river which are below its summer level it would be feasible to draw off considerable volumes of silt and water and thereby build up, between dikes, a dike of enormous width. The technical and administrative difficulties would be quite serious but not quite insuperable.

---

[21] "Uniform Flow in Alluvial Rivers," *Minutes of Proceedings*, Inst. C. E., Vol. 237 (1934), p. 421; also, "Stable Channels in Alluvium," *loc. cit.*, Vol. 229 (1929), p. 259.

[22] "The Conservancy of the Yellow River"—Chinese Maritime Customs Special Reports (about 1905).

As to the delta, the writer finds some difficulty in crediting the statement that the earlier surveys have much value for comparative purposes, and the backwater function seems to be over-rated; but this is a matter for investigation.

The breach of 1938 presumably occurred after the authors had submitted their paper. The flow has apparently been transferred almost entirely into the Huai River and unless the dikes are repaired there is a serious prospect that the Hungtze Lake may become silted up and that the valuable rice lands on the east side of the Middle Grand Canal may be injured during 1939. In this connection the plan of threatened areas, given in Fig. 35, should be extended down to the Yangtze River, near the junction with the Grand Canal.

The writer sincerely hopes that American and other engineers will devote some thought and investigation with models and otherwise to the problems of this great river, which is the example par excellence of a silt-bearing stream. Messrs. Todd and Eliassen have provided sufficient data for excellent work to be done in this direction.

H. VAN DER VEEN,[23] ESQ. (by letter).—The authors' contribution to the study of the problem in question is undoubtedly the most valuable and most extensive treatise on this subject which thus far has been published and contains a mass of data never collected before. When the writer severed his connections with the Chinese Government in 1929, practically no hydrometric research work had been done as it was impossible to induce the Chinese officials to undertake to sponsor it. Lack of funds in those days, and an insufficient understanding of the importance of such investigations, were no doubt responsible for this attitude. It was extremely gratifying, therefore, when, in 1919, Mr. Freeman began his independent hydrometric study of the Yellow River in connection with the crossing of that river by the Grand Canal.[3]

It may truly be said, therefore, that Mr. Freeman was the one who gave the impetus to a further study of the Huang Ho, which Mr. Eliassen in his capacity of Engineer with the Yellow River Commission in Kaifeng, so ably continued and enlarged upon. At the same time it is to be regretted that the authors (as it seems to appear) paid so little attention to the work of J. G. W. Fijnje van Salverda[24] which was based on the reports of Captain P. G. van Schermbeek and Mr. A. Visser. This memorandum was submitted in 1887 to the Chinese Government; and, apart from describing completely the conditions of the Yellow River at that time, and explaining its peculiar characteristics, it offers valuable suggestions about possible methods for ameliorating existing conditions. More particularly, attention is drawn to the fact that much might be attained by constructing adequate protective devices at the foot of each loess cliff so as to prevent the enormous influx of silt, with which the river must cope. Mr. Freeman prized this memorandum so highly that he photographed it. He told the writer that it inspired him to learn more about that interesting subject. Mr. Freeman's passionate interest was roused by this work. Eminent engineer that he was, he saw his opportunity and grasped it to collect the valuable data which he published later;[3] but apart from influencing Mr. Freeman to shed some more light on the pecularities of the Yellow River, the memorandum contains some valuable advice which, if followed, would have improved conditions to such an extent that much of the harm done since would have been prevented. In fact, the same may be said of every sensible suggestion pertaining to improvements of the Yellow River conservancy which have been made since that time; but, as it is, much has been written about the subject and many theories have been advanced, but no work of any practical constructive value has been done.

---

[23] Former Advisor in River Conservancy to Chinese Government; Govt. Engr., Bureau Rijks Water Staat, Nijmegen, Netherlands.

[24] "Memorandum Relative to the Improvement of the Huang Ho or Yellow River in North China," by J. G. W. Fijnje van Salverda; submitted to the Chinese Government in 1887.

Problems have been permitted to "drag along" as in the past thousands of years. Formerly, this was due, perhaps, to the mental attitude of the river authorities in China which is best explained by the following anecdote: Once when a high Chinese official was asked why it was that no one had taken radical steps to end all those dike breaks instead of resorting only to temporary measures, he is said to have given this reply: "When one is keen on tigerhunting and there is only one tiger, would it then be sensible to kill that tiger outright during the first hunt?" There may have been some justice in this viewpoint in the old days, but it certainly does not apply to the present generation with its many brilliant young engineers who desire nothing better than to end the untold miseries which the Yellow River continues to inflict. With laudable zeal and energy these younger men have started by practising what they learned in foreign schools and universities about scientific river engineering, following the example set by foreign experts. They began to collect data after approved methods—valuable data, as is proved by the excellent paper under discussion. They even went so far as to have a model made of the river, at Obernach in Germany, where Prof. H. Engels in 1932 (assisted by Dr. Ing. Futu Li and in 1934 by Dr. Ing. Yi Shen), conducted the necessary investigations to determine whether it would be feasible to cope with the silt evil (1) by narrowing the river between dikes, (2) by building the dikes parallel, or not parallel, to the channel, and (3) by creating a straight or a curved channel either with or without wing dikes. These studies are all quite interesting, but it is questionable whether such scientific studies[25] may not be considered rather premature at this stage because, judging by the facts, the practical sides of the question seem to be receiving too scant attention.

It is quite in order to collect data and to conduct intensive investigations since this is necessary in order to arrive at the best solution. It is even an indispensable condition for ultimate success; but it is quite a different matter (and a serious one) when, in pursuing such a course, the immediate needs seem to be forgotten. By paying too much attention to certain sides of a question appreciation of the main issue is likely to suffer. It is somewhat like seeing a few particular trees but not the forest of which they form a part. Why not focus all available attention on one point only instead of spending years in a laboratory in order to ascertain, for example, which shape of river is best suitable to silt transportation? In this particular case, this is the immediate prevention of dike breaks.

---

[25] "Modelversuche über das Verhalten eines Schemmstofführenden Flusses, etc," by Dr. Ing. Chr. Keutner, *Die Bautechniek,* February 11, 1936.

It is true, as the authors state (see heading, "Scope of Investigations: Hydraulic Laboratory Experiments"), "The results gave some startling indications." Wherein lies this startling quality? The authors (and the writer, for that matter) have made a serious study of the Yellow River problem, over a long period of years, and are consequently in a position to arrive at conclusions after mature consideration. Surely, to such students there can be nothing strange about the result of investigations. In the first place the investigations proceeded from premises that are not comparable with those of the Yellow River, and secondly (as the authors remark, see heading "Scope of Investigations: Hydraulic Laboratory Experiments") the conditions bore little relation to this river. Furthermore, it is only logical to assume that, in a heavily silt-laden river, less scour can be effected in the main channel when all the silt must be transported than when a great part of this silt is allowed to settle on wide foreshores. There can be no question as to the fact that the more the river is allowed to spread over a large area like that offered by the foreshores the weaker the current will become and the more silt will be deposited. The latter circumstances, undoubtedly, will tend to place the main channel in a better position to maintain its full capacity, always provided that the banks are stabilized so that it cannot widen itself at the cost of depth, and, consequently, energy. At the same time it should not be forgotten that the process of silting the foreshore is accelerated. The banks will become higher and higher until ultimately they will only become inundated during extraordinarily high stages. Then there will be only one channel in any case. Would it not be better, therefore, to try to find a solution whereby this inundation can be postponed, at least, and, in the meantime, try to evolve methods that would minimize the influx of silt? Researches in this direction are useful indeed, and it is gratifying to note that the authors did considerable in that respect.

Under "Introduction" the authors state: "One of the real obstacles to progress, of course, lies in the lack of hydraulic knowledge of the river itself." Is this entirely true? Would it not have been possible to construct certain works which, at least, would have prevented the annual breaks, notwithstanding that deplorable lack of data, a shortcoming that could always have been remedied during the execution of protective measures? By beginning to pay too much attention to "many of the perplexities that have shrouded the river in mystery" the vision on the problem as a whole may become more or less blurred.

What is the problem in a broad sense? In a report[26] on the Yellow River problem submitted in 1921 to the Ministry of the Interior, the writer dealt with this subject in detail and also gave advice as to what should be done to mitigate the danger of, if not to prevent, the disastrous dike failures of the Yellow River.

In principle, the Yellow River is a watercourse, pure and simple, draining its catchment-basin and, like other rivers, transporting detritus. Every river fulfills this duty in its own particular way. Thus, depending on the quantity of loose soil which happens to be available on the drainage area, the steepness of its slopes and that of the tributaries, different rivers will bring more or less detritus down to the low lands and eventually, if conditions are favorable, as far as the sea. If the slope is too gentle in the low countries to maintain the energy required to keep the silt load in suspension, this load is dropped. Depending on the quantity being carried, it is either merely deposited along the banks of the river when, during floods, the water overflows (with the result that natural levees are formed with gentle slopes away from the river), or it also settles to a greater or lesser extent in the river channel itself. In the latter case, during an unusually severe freshet, the river will probably break through one of its levees after a certain lapse of time and from then on will follow a new course in which the same cycle is repeated. This is called plainbuilding, a process which continues on and on until eventually a certain equilibrium is reached, whereby the slope of the plain has become sufficiently steep to enable the river to develop a gradient that is capable of imparting to the current the energy that is required to carry all the silt as far as the sea. In the case of most rivers plainbuilding is completed, or practically so, because erosion in the hills has discontinued (or at least has diminished in intensity), or because the topographic features of the country lend themselves to a speedy solution of the problem. Unfortunately, the Yellow River basin has neither exhausted its supply of silt not does its geographical location render a rapid solution possible. Between the hills and the sea there is an enormous plain which the river must traverse. At the same time the supply of silt seems to be unlimited. Consequently it is reasonable to conclude that because the river is passing through the middle phase of its stabilization (a gigantic process) it would be foolish not to abide by it; or, what is still more foolish, to try to fight against it. Only if river engineers are prepared to accept this fact as indisputable, will it be possible to succeed in ameliorating the conditions affecting the Yellow River problem. In this connection it will be well to recognize the fact that such steps will only tend to mitigate an unpleasant state of affairs; they will never be able to solve it except in a degree.

---

[26] *De Ingenieur*, Royal Dutch Inst. of Engrs., 1934, No. 33 (abstract).

A few thousand years ago, when the Chinese began to build dikes along the Yellow River they interfered with Nature's process of plainbuilding, inasmuch as they restricted the process to a narrow strip of land. The effect, of course, was to hasten the forming of a steeper gradient which, in turn, created a stronger current so that more silt could be transported to the sea. Furthermore, the river was made to run on a ridge, with the result that if Nature now asserts its right to continue plainbuilding the gravest consequences ensue for those standing in her way.

To make a sweeping statement, it would have been infinitely better for the present generation if past generations had permitted the Yellow River to do its work unhindered, or if those in the past who were entrusted with its care had been far-sighted enough at least to permit the river to follow another course. This was not done, however, and therefore the present generation is faced with the fact that the river runs on a ridge where it is kept in bounds between dikes which are more or less far apart. Within this cramped space Nature goes on with its process of plainbuilding in an endeavor to establish a slope sufficiently steep to create an equilibrium between the volume of silt that must be carried, and the carrying capacity of the current. It is evident, therefore, that, as long as such a slope has not been established, this narrow strip of a plain (which, by Nature's right, belonged entirely to the River) will continue to rise higher until the final stage is reached. The question is, "When will that event occur?" Perhaps this moment might have been accelerated if: (1) The space allotted to this restricted plainbuilding had been restricted still further—in other words, if the dikes had not been so far apart; and (2) if the main channel had been given a fixed alignment. However, for the moment, it is probably best to leave this question alone.

The river channel is left free to shift from one side to the other between the dikes. Time after time, therefore, the dikes are exposed to a direct attack of the current, at one place or another. Inevitably, they collapse and permit the disastrous inundations in the low-lying, adjacent, great plain. The higher the ridge on which the river runs becomes, the more devastating will be the consequences of a dike failure. Why then, one may well ask, are not better precautions taken to protect those dikes? In the course of many centuries the Chinese have developed clever devices for dike protection; but these are only applied whenever a dike is actually threatened; never before. This seems illogical, but it is often difficult, if not impossible, to predict the particular spot at which the dike will be attacked during a freshet season. It would be necessary in order to afford entire safety from flood to protect every foot of dike. It is clear that such a procedure would be enormously costly; and the Chinese are not a race that throws money away at random. Why, then, protect a dike when one is not sure that it will be necessary? This system is mainly responsible for the bad reputation of the Yellow River, whatever its other bad points may be.

The crux of the question is in dike protection; and it is quite immaterial for the moment, whether dikes should be far apart or not, which dimensions should be given to the channel so as to enable it to carry a maximum quantity of silt, whether highly silt-laden water follows the same hydraulic laws as clear water, etc. All these are extremely interesting problems in themselves and, therefore, should receive due attention in time; but at this stage they are of no consequence.

Since it is beyond question that the main cause of dike failures is the direct attack of the river (that is, the deep-water channel) on the dike, it is obvious that the only method of preventing those attacks is to prevent the main channel from running too close to the dikes.

In the writer's report[26] to the Chinese Government he explained in detail how this could be done. The method he advocated was to construct an adequate system of groins and parallel works with a view to keeping the channel in a fixed position. As a matter of fact the authors advise, in principle, a similar procedure. The width that should be given to the channel is a matter of conjecture, at least for the present. It would be advisable, however, to make it ample at the outset because it is far less costly to lengthen the groins, in case they should prove to be too short, than to make them shorter. Over a certain length these regulation works should not be higher than the foreshore, although they should slope upward slightly from the new channel alignment (normal lines) in the direction of the dikes. Perhaps this will render it necessary, at some places, to give them a slight elevation with respect to the general ground level. The remaining length of the groins—that is, the distance outside of the future high-water channel—can be raised above high-water level.

Groins should be built close together in those stretches where the new river alignment is designed close to the present side of the channel. Of course, it is not necessary to extend each groin as far as the main dike; but a series of groins may be grouped together by means of a dam parallel with the river, connecting the latter with only one dam across the foreshore with the dike. It is the writer's firm belief that in this way the danger of breaks can be reduced to a minimum. At the same time the channel, being kept within bounds, cannot widen itself during freshets as it does now; consequently no dissipation of energy can occur (at least from that cause) so that, incidentally, more energy will be available for carrying silt. Whatever the relation may be between the velocity of the current and its capacity for keeping silt in suspension, there can be no doubt that a river which is free to erode its channel (or if capable of maintaining one course, widens it at the expense of serious scour) cannot possibly have the same capacity for carrying a load as a channel whose width is fixed.

After these structures have been built, or rather during their construction, the hydrology of the river should be studied rigorously. The writer is a fervent adherent of hydraulic studies and has a firm belief in the usefulness of laboratory experiments; but, at the same time, he is of the opinion that great care should be taken to insure that the practical aspects of the problem are not pushed into the background. Consideration of practical aspects should always predominate.

Under the heading "Flood Control and Regulation: Channel Stabilization Methods," the authors describe a procedure that coincides practically with that of the writer.[26] The latter differs only in comparatively minor points such as the elevation of the groins, and their direction. In Holland where groins have been used extensively there are adherents to each mode of location, different directions being advocated, with different supporting arguments in each case. There is something attractive in deflecting groins, however, as they tend, somehow, to concentrate the current; but, after all, their radius of action is small compared with the width of the river, so that it is doubtful whether they are worth worrying about. That laboratory experiments in this connection "are likely to repay themselves a hundred-fold," therefore, seems to be a rather strong statement.

There is one other item on which the writer would like to comment; namely, the question of spacing the groins. More than a century of experience in Holland has taught engineers that the spacing should preferably be not more than one and a half times their length, and even less if that length should be disproportionate to the width of the channel. Hydraulic laboratory experiments conducted in 1938 supported this view. The spacing has little to do with "a more or less erosion-resisting quality of the soil," however. It is governed principally by the fact that it determines "boundary conditions": The better the spacing, the better these conditions will be, inasmuch as friction is reduced to a minimum and no energy is wasted at the cost of loss of head and other losses. If that result has been attained, the river banks are adequately protected, whatever the quality of the soil may be. It is a well-known fact that flow can be made smoother by careful planning of training works and, although turbulence plays an important rôle in keeping the heavier particles moving, in this particular case, where the silt is extremely fine, more can be gained by saving energy than by wasting it in order to create turbulence. However, laboratory tests should certainly be made in an effort to shed more light on this question.

Under the heading "Flood Control and Regulation: Influence of the Delta," the authors have verified the writer's views on the same question on the basis of the interesting data which they have collected. In fairness to Mr. van Salverda,[24] however, it should be noted that as early as 1887 he called attention to this particular item. In this connection, it may be of interest to recall the fact that, acting on the advice given in Mr. van Salverda's report, the Chinese Government purchased a bucket dredger in order to clear away the silt in the channels through the delta, just as the authors now propose exactly fifty years later. This dredge arrived in the early nineties; but it foundered soon after it was placed in service and now lies deeply buried in the silt which it was supposed to remove. It would not be surprising if the Chinese in those days ascribed this mishap to a revenge of the river gods who were angered that mortals dared to interfere with their work. The writer does not belong to the class of higher beings who, according to the belief of the masses in China, rule the waters; neither is he angered about what was done; and yet he too does not approve of the method advocated by the authors for the simple reason that it seems to him like "fighting windmills." It seems strange that the authors, who so admirably indicate the means of mitigating the silt evil under the general headings "Hydrological Considerations," and "Flood Control and Regulations," should include dredging through the delta as a remedy. The effect of dredging would be insignificant compared with the excellent measures which they propose in the aforementioned chapters, with which the writer is entirely in accord.

The writer does not believe in detention basins in the hills, however. These must be considered as temporary measures only, without any lasting merit. The writer is keenly in favor of spillway basins in the plain, however, not so much because they will reduce the discharges and flatten the peaks of the freshets, but because they will tend to desilt the water. In doing this the natural plainbuilding process with which Man has interfered is at least restored to some extent. For the same reason the writer considers the by-pass channels mentioned by the authors to be an excellent means of ameliorating conditions.

Before ending his comments on the authors' valuable paper the writer wishes to state that he is fully convinced that if the Chinese Government could be induced to accept the theories so ably expounded by the authors, and could be made to put into practice the remedies that have been advocated, with the least possible delay, the untold miseries, which heretofore have made the Yellow River the scourge of China, would cease to exist.

It seems regrettable that the authors have left China and thus, perhaps, lack the opportunity to continue their work in connection with the fascinating Yellow River problem.

C. S. JARVIS,[27] M. AM. SOC. C. E. (by letter).—Many essential data hitherto unavailable to the engineering profession have been presented by the authors, and they have suggested such remedial measures as seem to have most practical value, according to their intimate knowledge of local conditions prevailing throughout the Yellow River Basin.

---

[27] Hydr. Engr., SCS, Dept. of Agriculture, Washington, D. C.

From the basic information and also from the recommendations thus presented, it appears that the most dependable and positive relief from floods would entail a combination of head-water storage and detention works, extensive land surface treatment for the express purpose of soil and water conservation, and such bank protection, flood spillways, and by-pass facilities as would best afford the needed channel capacity and stability, together with the desired security for inhabitants and their properties. Moreover, there is assurance that the most critical sections of protective works may be located and strengthened well in advance of the flood stages, instead of waiting until the danger is imminent or actually upon them, according to the ancient fatalistic practice.

It is interesting to note that the total Yellow River drainage area is nearly twice that of the Ohio, yet its maximum estimated discharge (900 000 cu ft per sec) was only about one-half of that recorded in the Ohio River (1 600 000 cu ft per sec at Paducah, Ky., in April, 1913, and 1 850 000 cu ft per sec at Metropolis, Ill., in February, 1937). Furthermore, the Yangtzekiang drainage area and maximum discharges both compare fairly closely with those of the Mississippi River.

The authors state (in the text preceding "Hydrological Considerations") that the Wei Ho (drainage area, 56 000 sq miles) is almost as important a flood and silt carrier as the Yellow River itself. Although it is less important as a low-water feeder, it appears that the critical areas awaiting corrective and regulatory measures must be within this tributary basin; and these main flood-source areas may constitute only a minor portion. Detention and storage facilities would be capable not only of reducing damaging flood crests, but also of augmenting the low-water flow as a natural consequence.

The use of auxiliary flood channels and lateral waterways running nearly parallel to the river course, and supplied by controlled sluices through the embankments, with gateway sills set low enough to divert a portion of the bed-load as well as some of the suspended material along with the released flood water, might go far toward maintaining channel capacities through the removal of bars, or by preventing their formation. At the same time, such sluicing might contribute toward filling in the depressions and undrained areas adjacent to both the irrigable tracts and also the alluvial ridge along which the river now wends its way, continually chafing under the restraints imposed by barrier embankments, and searching for some means of escape to the neighboring low-lands.

E. W. LANE,[28] M. AM. SOC. C. E. (by letter).—Probably no engineering project under serious consideration would do more to relieve human suffering than the control of the Yellow River in China. In their work on this problem, in the field, the authors have already made major contributions, and this paper, by preserving through times of disorder the most important facts and engineering data previously collected, may well be another major contribution to the success of this project. Small amounts of information on the Yellow River are widely scattered through engineering literature, but in no other place can one find a wealth of material at all comparable with that presented by the authors.

---

[28] Prof., Hydr. Eng., Hydr. Laboratory, Univ. of Iowa, Iowa City, Iowa.

The Yellow River is not only a stream of water, but it is also a stream of sediment, and any engineering plan which will be more than a temporary measure must take this into account. The Yellow River is not unique in this respect—the same thing is true of most rivers. The writer believes that a great deal of trouble will occur in the future because sufficient provision has not been made in a number of engineering projects for the flow of sediment as well as the flow of water. However, the Yellow River is unusual in that the quantity of sediment carried is so great that the necessity of a consideration of it is more readily apparent. In some other streams the action will be the same but it will go on at so slow a rate that the results will be apparent only after many years.

A comprehensive solution for the flood problem of the Yellow River, therefore, must be complete both from the standpoint of the water stream and also from that of the sediment stream. From the practical standpoint, the best plan may be to undertake first only the control of the water stream, since such a procedure may give the greatest return on the money which can be raised in the early years for carrying out the project; but in order to select the best plan it is necessary to view the whole problem and weigh all the disadvantages of temporarily neglecting the control of the sediment. Some of these disadvantages may not be readily apparent without a careful study of the control of the sediment stream, but may prove to be so great that an apparently satisfactory solution from the limited viewpoint may turn out to be much less desirable than one which, without considering the sediment aspects, appears relatively unpromising.

In order to appraise correctly the problems due to the sediment stream, it is necessary that the laws governing the transportation of sediment be known. Unfortunately knowledge of these laws is very imperfect. However, rapid progress is being made in this field, and it is confidently believed that before long it will be possible to handle problems of sediment quantitatively as engineers now handle those of water. At least approximate solutions of bed-load problems have been available for some time, and quantitative solutions of suspended load relations are being developed. One of the greatest needs for a solution of the Yellow River problem is a further working out of these laws. It is probable that the difficulties of controlling the sediment will be as great or even greater than those of controlling the water, and the need for a knowledge of the laws governing sediment will be as essential to a proper solution as a knowledge of the laws controlling the flow of water.

If the Yellow River were only a river of water, the data presented in this paper indicate that its flood problem probably could be solved with comparative ease by retarding basins. Because of the extremely short duration of the flood peak, the conditions are unusually favorable to retarding basin control. The presence of the heavy sediment load, however, seriously complicates the situation, and in any flood-control plan it must be given most serious consideration.

As shown by the profile given by Colonel Dent, there is a distinct flattening of the gradient of the river where it leaves the gorge section at Mengtsin and starts across the plain. This point is just below the region in which the most promising storage site appears to be located. The slope in the bottom of the reservoirs would be over six times that just below Mengtsin. As the writer has explained previously,[29] the water flowing across the sediment deposited in the bed of the flood-control basin would pick up a much greater load of sediment than it could carry across the plain, and therefore extensive deposit would take place down stream from the reservoirs, where the slope is flatter. This deposit would fill up the bed of the stream and might raise the elevation sufficiently to cause the overtopping of the levees. The possibility of such action should justify a thorough study of this point before a final decision on retarding basins is made. The existing bed is the product of the filling and scouring of the river with its present sediment load, and any conditions, such as the construction of the retarding basin, which would add materially at frequent intervals to the sediment load, would probably cause an increase in the average level of the river-bed. The raising and lowering of the bed levels mentioned in the paper are believed to be a demonstration of the sensitiveness of the river to changes in the character or amount of the sediment load.

The authors have presented a very complete discussion of the Yellow River problem from the standpoint of the water stream, but further consideration might be given to the problem from the standpoint of the sediment stream. Viewing the situation broadly, it seems that the sediment load of the Yellow River must be handled in one or more of the following three ways: (1) It may be reduced by keeping the soil in place on the land by erosion-control methods; (2) it can be carried out to sea and deposited on the sea bottom; and (3) it may be stored on the land surface by taking it out of the river. Probably the best solution will be a combination of the three methods.

In a paper entitled "Soil Erosion and River Regulation with Special Reference to the Yellow River,"[30] Mr. Eliassen has discussed at some length the possibilities of reducing the sediment load of the Yellow River by soil-conservation methods. Based on the cheap labor conditions existing in China and the knowledge gained in soil-control engineering in the United States, it seems probable that at least part of the sediment load should be taken care of in this manner. The benefits would be not only that of flood control but also of increased food supply which could probably be produced by the vegetal cover developed in the erosion-control process. While erosion control has no doubt been practiced in the past as a means of conserving agricultural land, its value as a flood-control measure has not been considered, and if financial support were given to the work because of the flood-control benefit it is possible that an extensive program along this line might be feasible. If practically all the sediment could be retained on the land, the clear water in the river would pick up a large volume of sediment from its bed and carry it to the sea, thus enlarging the river channel, and eventually solving the flood problem. It is improbable that anything approaching complete erosion control will be practicable, but something between complete control and the present conditions will probably be a part of the best plan for flood control.

---

[29] *Journal*, Assoc. of Chinese and American Engrs., Vol. XVIII, March–April, 1937, p. 130.

[30] *Journal*, Assoc. of Chinese and American Engrs., January–February, 1936.

The second method, by carrying the sediment to the sea, will no doubt be a part of any flood-control plan. As long as the Yellow River flows over a bed of fine material it will not be feasible to prevent it from carrying a large load out to sea. Although this disposition does bring in some serious problems, as the authors have pointed out, these difficulties are probably less than those of the third method, and, therefore, deposition in the sea should be encouraged as far as possible. Since the fall of the river in crossing the plain cannot be materially increased without raising the level at the western side of the plain much higher that it is now, it is probable that, aside from maintaining the levees intact and possibly regulating the channel across the plain to keep it deep and straight, little can be done to increase the ability of the river to transport sediment.

If this is the case, the amount that can be carried out to sea under the most favorable conditions will be considerably less than that now brought into the plains section by the river, and unless a major reduction in volume can be made by soil-conservation methods, any long-time plan for Yellow River flood control must include provision for controlled sediment storage. In other words, for complete control of the floods, if more sediment enters the plains section of the river than can be carried out to sea, the excess must be stored by some controlled method. The excess which now exists may be reduced by soil conservation and possibly by producing some increase in the carrying capacity of the river, but unless the excess is entirely eliminated by these methods, disasters cannot be permanently prevented except by means of sediment storage under controlled conditions.

There can be little doubt that in the past the flow of sediment in the river at the western side of the plain has materially exceeded that which passed out of the mouth at the east side, and therefore that there were large quantities of material deposited between these two points. When the river changed its course in 1851, the meager accounts available indicate that the new course to the sea was along the line of a stream which had a much flatter gradient than the Yellow River and therefore flowed at a lower level. The flow through the breach in the dikes entered this low-level stream, and the bed of the Yellow River above the break was cut back, lowering the bed materially. Since this change of the river's course took place, the bed of the new portion of the channel filled considerably, and, as stated by the authors, an adjustment is still taking place in the vicinity of the break. In this process tremendous quantities of sediment have been deposited, which accounts for part of the excess of sediment inflow over outflow in the plains section, as previously mentioned.

Another part of this excess has been deposited on the land by the flow through the breaks in the dikes. When such breaks occur, the heavily silt-laden water flows over the land and deposits tremendous quantities of sediment. In some cases the flow returns again to the Yellow River in a comparatively clear condition and goes on out to sea. In other cases it goes overland to some other stream leading to the sea, but in either event a large part of its sediment is deposited on the plain.

Unless the inflow of sediment at the western side of the plain can be made to equal that flowing into the sea at the eastern side, a levee system alone will not be a permanent solution for the Yellow River floods, since the sediment storage which takes place will raise the bed of the stream and cause progressively higher flood levels until so great a height is reached that levee construction becomes very expensive and insecure. In this condition, if a break occurs, the elevation of the river-bed will be so much higher than the land that the water cannot be forced back into the channel. The raising of the bed will increase slightly the amount of sediment which can be carried out to sea, but since a 5% increase in slope would raise the river level about 15 ft at the western side of the plain, no great increase in average slope can be attained without raising the river at that point to unsafe levels.

Since it seems unlikely that in the plains section of the river the combination of reduction in sediment inflow by soil conservation and the increase in sediment outflow can reduce the excess of inflow over outflow to zero, some form of controlled sediment storage must form a part of any permanent plan for the control of the Yellow River floods. If this is the case, such storage will have to be carried on continuously and sooner or later the basins for the deposit of the sediment will be filled. It appears, therefore, that no set of works can in themselves be a complete solution of the flood-control problem, but sooner or later other works will have to be constructed and the control of the Yellow River will therefore be a continuing process of construction work.

Another reason why continuous work will be necessary is that as the sediment continues to be carried out to sea, the mouth of the river will move outward, as explained by the authors. The river levees, therefore, will have to be extended seaward and those now near the sea will have to be raised. Although this process will no doubt be slow, it seems to be inevitable. In considering the best plan for controlling Yellow River floods, therefore, it should be realized that no solution will entirely eliminate the necessity of future construction, and therefore works which are obviously not a permanent solution should be considered along with those which may appear to be, but in reality are not, permanent solutions. When one considers the necessity of controlled sediment storage in any plan for permanent flood control, with the consequent expense and difficulties due to opposition of land owners, the advantages of soil conservation from the flood-control standpoint become apparent. Of course, such measures also would be a continual expense but it seems certain that a large program of work to keep much of the soil in its place will be cheaper and more satisfactory than artificially storing the soil after it has reached the river.

The foregoing statements must not be construed as showing that better levees should not be built along the Yellow River or that those now in existence should not be maintained. The damage caused by a levee break is so great that better levees might pay for themselves long before they became inadequate. It may not be possible for some years in the future to finance the works necessary for complete control. What the writer wishes to emphasize is that eventually sediment storage will probably be necessary, and that the best possible plan of action is more likely to be found both for the immediate future and for more distant times if all the factors are considered. Therefore, every effort should be made to collect the information necessary for a complete picture of the problem. In the writer's opinion this would include the collection of more data on the composition and concentration of the sediment carried by the river and its tributaries and a better working out of the laws governing sediment transportation.

Sediment storage for the improvement of streams is no new thing in China. On the Yung Ting Ho above Tientsin there is a large area in which silt has been depositing for many years and which has permitted the Hai Ho from Tientsin to the sea to maintain a navigable channel. The writer has not been able to find any statements as to when this sediment-storage basin was constructed and whether or not it was built with that function in view, but in any event it has served very well for this purpose until recent years when it has become so full that it is no longer effective and the channel of the Hai River has been badly deteriorated. To replace this sediment-storage basin a new one has been constructed and, it is understood, is now functioning satisfactorily.

Three principal plans for controlled storage of sediment might be developed: (*a*) Storage in reservoirs west of the plain, (*b*) storage in basins along the river in the plain, and (*c*) storage at the west side of the plain. The authors have mentioned good sites for reservoirs in the section between the Peiping-Hankow Railway and the mouth of Wei Ho, and also in the vicinity of the Hu Kou Falls. Sediment storage could probably be obtained at reasonable cost at these sites. As stated before, the reservoirs would undoubtedly fill rapidly and eventually new storage space would be required. In the early stages of filling they would serve admirably for retarding basins and would have a dual purpose—the reservoirs could be used for the generation of water power and in the early stages of filling would have available considerable storage and pondage space. As they became filled, however, the power generating plants would have to be operated on a run-of-river basis. Thus the reservoirs might be used either for combined retarding basin and silt storage, with the retarding ability eventually being lost, or as combined water power and sediment storage, with part of the water power benefits eventually being lost. Since the vicinity of these basins is well supplied with coal, it would seem that the combination of sediment storage with retarding basin action would probably be the best. The use of these basins as detention basins would also permit their use for flushing out the river down stream by storing clear water during low-flow times and releasing it with higher flows during short periods.

The necessary sediment storage could be accomplished by deposit in predetermined areas in the plain section during floods, when the water level was high enough to permit overflow of the land along the stream. By building a pair of dikes parallel to and outside of the present ones (where secondary dikes are not already in existence), with cross-dikes connecting the outside and inside dikes at intervals, a series of basins could be formed in which the water could be let in at the upper end through sluice-ways and the desilted water let out at the lower end. Such a scheme would build up the land along the river banks and thus add to the security from disastrous levee breaks. The desilted water released at the lower end of these basins, upon flowing back to the river, would pick up a load of sediment from the river banks and bed, and thus enlarge the capacity of the river channel. The amount of material picked up, however, would probably be materially less than that deposited, since the former material would average much coarser than the latter. The construction of the second dike system would be a considerable expense, although it would not have to be as heavy as the primary system, since in case of a break only the water in the desilting basin would be released. The principal disadvantage of this form of sediment storage is the damage resulting from taking so large an area of land out of cultivation or the periodical destruction of the crops which would occur if this land was cultivated, since the Yellow River floods come during the growing season. This plan could be put into operation only under a strong central government and would no doubt meet with a great deal of opposition from the present owners of the land included in these storage basins.

A third plan for storage of the surplus sediment would be to construct basins where the river emerges from the hills, causing the river to form a sort of debris cone by spreading it out over the plain in this vicinity under controlled conditions. This plan would have the same disadvantages as that previously described but might be cheaper. This form of sediment storage might be combined with retarding basins, the sediment carried out from the retarding basins after the floods being desposited under controlled conditions on the plain. All three plans have the disadvantage that they would require continuous work, but no form of flood protection for the Yellow River seems feasible which can be constructed immediately in its final form.

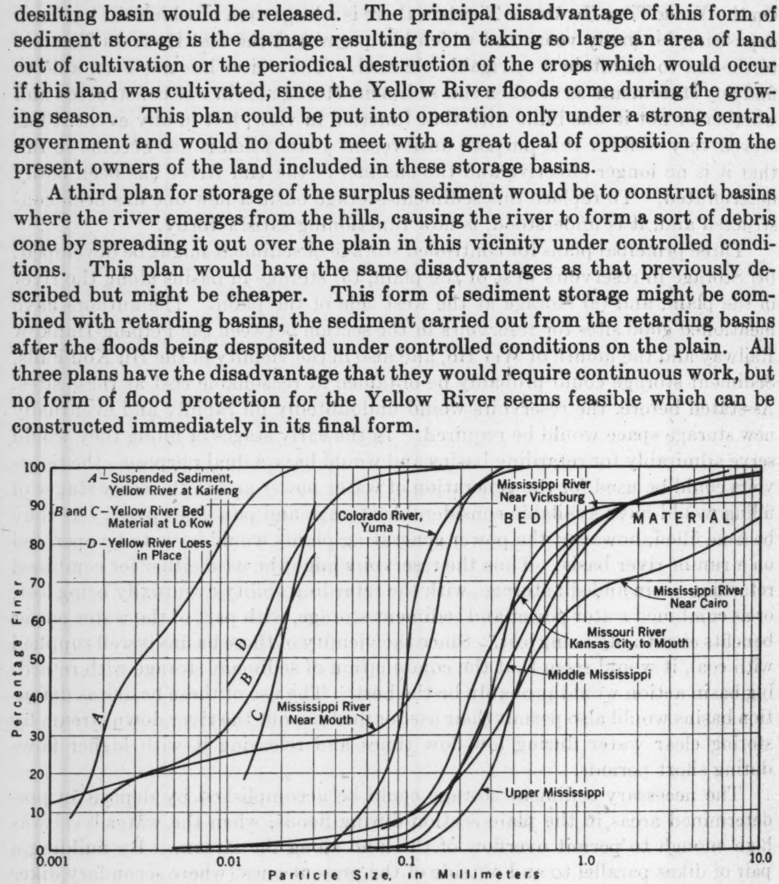


FIG. 37.—COMPARISON OF SEDIMENTS, YELLOW AND AMERICAN RIVERS

The authors have presented a large amount of data on the silt load of the Yellow River, but no data have been given on the size of the particles of which this silt was composed. The writer understands that analyses of numerous silt samples have been made by one of the authors, but that the data are inaccessible because of the war. It would be a great misfortune if this information should be permanently lost, since sedimentation investigations are showing conclusively that a knowledge of the particle size or settling rate is essential in studying sedimentation problems.

Very little information on the size of sediment in the Yellow River is available, but three curves showing the compositions of Yellow River sediment are shown in Fig. 37, the data being taken from "Die Rigelung des Hwangho," by Li Fu Tu. The left-hand curve, for material at Kaifeng, is probably typical of suspended material in the river. The remaining curves give analyses of bed material of a number of the rivers in the United States. The finest of these is the bed material of the Colorado River near Yuma, Ariz.

This diagram strikingly illustrates the difference in the character of the Yellow River sediment and that found in the rivers of this country. The median diameter of these various samples is as follows:

Sample	Median diameter, in millimeters
Yellow River suspended sediment at Kaifeng	0.0033
Yellow River loess in place	0.012
Yellow River bed material at Lo Kou Chen (average)	0.018
Colorado River bed material	0.10
Missouri River bed material	0.28
Lower Mississippi River bed material at mouth	0.13
Lower Mississippi River bed material at Vicksburg, Miss.	0.33
Lower Mississippi River bed material near Cairo, Ill.	0.53
Middle Mississippi River bed material	0.38
Upper Mississippi River bed material	0.48

A comparison of bed material shows that the average of a sample from the Yellow River was about one-fifth the size of the Colorado River bed material, one-seventh that of the Mississippi at its mouth, and from one-sixteenth to one-twenty-seventh that of the various other rivers for which data were given. It seems obvious that the reason why the Yellow River can carry such heavy loads of sediment is that the material carried is so extremely fine. These samples were taken in the plain section of the river, but the gorge section above Mengtsin carries a certain amount of quite coarse material, as shown by the fact that this was sufficiently large to break the glass bottles used in the sampling device, and metal bottles had to be substituted. This coarse material apparently settles out in the vicinity of Mengtsin in a sort of debris cone and does not reach the plains section of the river.

GEORGE HIGGINS,[31] M. AM. SOC. C. E. (by letter).—Valuable material and suggestions have been furnished by the authors. The writer suggests that the entire length of that part of the river in which the flow is above the level of the adjacent country shall have its banks constructed to act as spillways. This means a spillway on the left bank about 465 miles long, and one on the right bank about 450 miles long. These are approximately the lengths of the dikes or levees already constructed. It is suggested that, in general, a shallow flow of flood waters over the adjacent country on both sides of the river would benefit the land, which sometimes suffers from drought, and would enrich it with the silt derived from the uplands. Not only that, but the relief from high flood conditions which the river would experience in consequence of the diversion of so great a proportion of the flood waters would greatly lessen the danger of concentrated flood waters which now sometimes break through the levees.

---

[31] Cons. Engr., Melbourne, Victoria, Australia.

In effect, the proposal is to follow Nature's procedure. Heavy rains wash material from the hill country into the valleys where much of the mixture of solids and liquid is carried along. Sooner or later, heavier particles will settle on the stream beds, raising them; and, when these beds are high enough and the banks low enough, overflow will occur. Generally such overflow is accompanied by deposition of silt on the land adjacent to the stream, heavier and coarser particles lodging close to the stream, the finer being carried on. This accounts for the form which cross-sections present—gradual surface slopes falling away from the relatively higher river banks. Rivers in all parts of the world exhibit this natural action.

On the other hand, if it is attempted to confine flood flows to a definite channel by erecting artificial banks, the bed will continue to rise in consequence of the subsidence of the heavier particles which the water is carrying forward—this is the ultimate result even if, at times, the action of the current may be a scouring one. One of the most notable examples of this effect is furnished by the Po River in Italy, whose banks have had to be raised several times to keep pace with the higher water level due to the raising of the river-bed. About 5 000 000 persons live on areas protected by these banks, and, when the banks break through (as they have done, unfortunately), the results are disastrous.

"China's Sorrow," the Yellow River, carries an enormously greater proportion of sediment than the Po, and the fact that much of this sediment is deposited on the river-bed as a result of floods is well known. Were it not that the artificial banks have frequently broken through, the rate at which the bed rises would be still more rapid. What will happen if the levees are made unbreakable? The channel will fill up, and, if the banks are raised higher, the channel filling will continue. Terrible as the results of bank fracture have been in the past, they will be worse if still higher banks break through. In any case, there must be a limit to this bank building.

The simpler plan seems to be to strengthen the river's natural banks so as to make them suitable to act as spillways, and to let the river deposit silt on them and outside them at its natural slope—probably not steeper than 6 in. to the mile. To a great extent, the river itself will raise its own banks as well as its bed. Man may help, here, to direct the natural action: By strengthening the natural banks; by raising them in places so as to keep the flow over the crests approximately uniform in depth; by checking any tendency for the flow over the country's surface to concentrate itself in depressions and cause scour there; by building groins or other means for deflecting the river's current away from the bank which it may tend to undermine; by encouraging the growth of vegetation on the loess and other surfaces from which the silt is derived; by interposing positive checks to the flow of silt from eroding surfaces; and, generally, by working with Nature.

Such details as the raising of dwellings and providing for the safety of live stock during the month or two that the shallow flooded conditions may last may well be left to the resourceful people who have displayed remarkable skill and endurance in the past.

In the writer's opinion, the existing levees should be removed. If they remained, they would only obstruct the even flow of silt-laden water over the surface of the country; and this flow is desired. The material in the levees would be useful, in places, for the strengthening of the river banks, thus enabling them to act as spillways, and also for forming groins or training-walls in the channel.

As for the height of the banks, which would become spillway crests, the writer suggests taking present bank levels, where they have not been altered artificially, as a guide at the outset. Absolute uniformity of depth flowing over the crests will be unattainable. Local falls of rain would prevent such uniformity, producing "flood waves," as they do. Moreover, every large flood has its "peak," and the peak passes down stream while gradually flattening itself out.

Earthquakes are known to affect the country through which the Yellow River flows. Any shattering due to this cause will be much less serious with river banks of indefinite width than with narrow banks formed by levees. Ice jams may be expected to occur in the future, as in the past, and the consequent "heaping up" of water locally will have to be accepted.

In common with all irrigation systems, there is a possibility of the irrigated land becoming alkaline, necessitating drainage, which is not a simple matter in country so nearly level as that here considered; but such difficulties, and probably many others, will have to be faced. The aim must be to eliminate difficulties as far as possible.

The growth of the delta has occasioned anxiety to many engineers who have studied the Yellow River problem. This growth would be greatly lessened if much of the silt were deposited on the country adjacent to the river. The "plains" through which the Yellow River flows for the last 500 miles of its course have evidently been deposited as deltas. Allowing Nature to continue its processes should not lead to conditions inferior to those existing before Man interfered. In any case, even if it were possible to form and maintain a channel between levees which will not overflow, it will not be possible to discharge all the silt into the ocean. A certain quantity will be deposited on the river-bed, making it necessary to raise the levees frequently. Where would that end? It would create a channel on top of a steep embankment, rising higher and higher as time goes on! There is no satisfactory end to this process.

O. J. TODD,[32] M. AM. SOC. C. E., AND S. ELIASSEN,[33] ASSOC. M. AM. SOC. C. E. (by letter).—Since this paper was written, in 1938, there has been a marked change in the lower course of the Yellow River which is no longer flowing in the diked channel north of the Shantung Mountains, but across the plains to the southeastward from a point nearly 40 miles west of Kaifeng. It has joined the Huai River and, together with that river, empties into Hungtze Lake east of the Tientsin-Pukow Railway in Kiangsu Province and from there, via several channels, into the Yellow Sea. Some of the flow follows the course of the Grand Canal to the Yangtzekiang 700 miles south of the 1937 outlet of the Yellow River on the Gulf of Chihli.

---

[32] Cons. Engr., Palo Alto, Calif.; formerly, Cons. Engr. to the Yellow River Comm. and Shantung Govt., Peiping, China.

[33] Engr., Messrs. Höyer Ellefsen, Oslo, Norway; formerly, Engr. in Chg., Survey Dept., The Yellow River Comm., Kaifeng, Honan, China.

This latest great change of course reminds observers of the consequences of a "back to Nature" program. If human beings are going to live and carry on their daily activities in this fertile region, the Yellow River must be held to a channel that is as stable as possible. The inhabitants must have a sense of security if they are to live in peace and build permanent homes. Railroads, motor roads, and canals must all be made secure against serious flood attacks. A section of the Lunghai Railway has already been buried in silt since the dike break of June, 1938. It will not be long until sections of the Tientsin-Pukow Railway in Kiangsu will be similarly threatened. A river which, annually, can fill an area of 300 sq miles with silt to a depth of 5 ft must not be permitted to go uncontrolled across a fertile plain where the average population density is greater than 600 per sq mile.

The question may be raised as to which of the two general routes should be chosen as the regulated course of the river—the one north of the Shantung Mountains to the Gulf of Chihli, or the one south of these mountains to the Yellow Sea. Both from technical and social-economic considerations the northern course is decidedly preferable. The distance to the sea is shorter and will remain shorter for the next two centuries even if nothing is done by way of soil-erosion control of the inland areas. Also, the northern route has no complications with other river systems such as would be found along any southern route that might be selected. The northern route is flanked on the south for a distance of 90 miles by the foothills of the Shantung Mountains, making one dike unnecessary in that region; and, since a dike system exists which can be utilized, in part, in a permanent plan of control, it will obviously be less costly to regulate the northern course (of 1851 to 1938) than any southern course, including that being followed since the summer of 1938. The latter would require a new diking system with costly regulating works to control the floods of the rivers draining the southern slopes of the Shantung Mountains, as well as the floods of the Huai River. Since these rivers have serious flood problems of their own, flood control would be complicated by routing the Yellow River that way. The Huai River is notoriously difficult to handle in time of high flood.

The fact that the Yellow River has been so altered in its course since the preparation of this paper does not mean that the problem of control is essentially different from that outlined by the writers. The present (1939) course of the river is likely a temporary one, although it might be maintained to advantage while the former (1851 to 1938) course is being regulated as suggested by the writers. Much of this work could be done "in the dry," at a great saving in costs, and the river then diverted back into the regulated channel from the new (since 1938) course.

Mr. Beardsley asks if it will be practical to fill, with silt, a number of shallow lakes and marshes along the old and former diked courses, and thus convert these areas into good, arable land. This is quite feasible and desirable. The areas are small, however, and confined to narrow strips of land just outside the dikes. During the decade 1929 to 1939, a number of siphons were constructed for the purpose of drawing silty water from the Yellow River in order to irrigate, fertilize, and reclaim just such areas. Experience has shown that these siphons need careful watching in order to avoid drawing in fine sand which may suddenly appear at the surface of the river, due either to small freshets or to the throwing into suspension of bed silt which contains coarser material. Lands have been ruined by careless operation of such siphons. The best success thus far has been obtained along the lower course in Shantung. Several siphons installed in 1936 near Kaifeng had not been in operation long enough to warrant any remarks as to their merits when the war broke out, preventing further experiments.

However, it is the larger aspect of broad-flooding wide areas during floods in order to control the silt that is of major interest. It involves the moving of a population estimated at 200 000 from an area of 400 sq miles, every 8 or 10 yr, into other areas prepared for them, and then fitting the abandoned area for silt deposition and drainage of the desilted water. The feasibility of such a plan depends very much on the ability of the Government to relocate and raise many towns and villages, giving each suitable protection. Since this process would be continuous and accompanied by constant difficulties with the population, the writers have hesitated to urge the adoption of such methods.

Colonel Dent's analysis of the silt inflow, deposition, and outflow for the Shanchow-Chinchang reach is interesting. However, if there is an excess of silt leaving a reach during a certain period, it is not certain that this excess is due chiefly to tributary inflow. A considerable portion may have been eroded from the river-bed.

The writers agree that for the reach from Shanchow to Mengtsin there will not be much difference in the silt inflow and outflow and that whatever difference there is will be due to tributary inflow; but for the reach from Mengtsin to Chinchang the case is different. Here both deposition and erosion are going on at a rapid rate, especially during the flood season. The net result is a slow but steady rise of the bed and foreshores. Referring to Tables 7 and 8, it will be seen that for the two years 1934 and 1935 the following volumes entered and left at Shanchow and Chinchang (in cubic feet of silt):

Entering at Shanchow	96 174 900 000
Leaving at Chinchang	98 378 890 000
Excess leaving at Chinchang	2 203 990 000

—or, 82 000 000 cu yd. From silt records of the Lo Ho and Chin Ho for 1934 and 1935 (incomplete during the low-water season), it has been computed that these two rivers carried approximately the following silt volumes (in cubic feet):

Year	Lo Ho	Chin Ho
1934	325 000 000	280 000 000
1935	1 750 000 000	770 000 000
Total	2 075 000 000	1 050 000 000

The total of the two is 3 125 000 000 cu ft, or about 115 000 000 cu yd. Taking into account other small streams which brought silt into the reach, one may say that as much as 150 000 000 cu yd of silt entered the Shanchow-Chinchang reach from tributaries during the 2-yr period, 1934–1935. Hence, only 68 000 000 cu yd of silt seem to have been deposited between Shanchow and Mengtsin due to the steep slope. This differs considerably from the value of 500 000 000 cu yd computed by Colonel Dent. Unfortunately, he did not have the Lo Ho and Chin Ho data to guide his judgment.

Colonel Dent finds it difficult to reconcile the 52 000 000 000 cu ft of silt entering the diked section at Chinchang in 1934 (and a similar amount in 1935) with the estimated 22 000 000 000 cu ft passing Litsin at the head of the delta under a regulated condition. Both 1934 and 1935 must be regarded as more than average flow and silt years, however. The yearly average silt volume passing Chinchang has been estimated by the Yellow River Commission at 35 000 000 000 cu ft. Furthermore, the average value of 22 000 000 000 cu ft passing Litsin was based on the Shantung settling basins being in operation; but it is quite possible that the silt volume reaching the delta every year may be greater than 22 000 000 000 cu ft. If this proves to be the case, the lower course will likely deteriorate more rapidly than estimated. The aim of the regulation must be to attempt as nearly as possible to bring a silt quantity to sea which will bear such a relation to the silt deposition up stream that neither the one nor the other deposition will be out of step and thus unduly accelerate the superelevation of the river's course along that reach.

With regard to the former rise of the river-bed across the plain, the writers have mentioned 3 ft per century as an average value, having checked a similar figure by Mr. Freeman referred to by Colonel Dent. However, this value has been based on the plain outside the dikes as reference; but, obviously, since the plain also has had silt deposited on it, the actual rise, with reference to a fixed plane, must be greater. If, as has been stated, the average yearly silt inflow at Chinchang has been 35 000 000 000 cu ft and the outflow into the delta 15 000 000 000 cu ft, 20 000 000 000 cu ft per yr have been left behind along the diked channel, and on the plain, as a result of dike breaks. The entire river channel has thus risen with the plain, but also slowly above the rising plain.

Assuming that 70% of the silt that has been left behind has been deposited in the up-stream 200 miles of the river's lower course (in the channel and on the plain), there will have been deposited in 100 yr about 1 400 000 000 000 cu ft of silt in this locality. From personal investigation the writers know that practically all of the silt that flows out on to this plain through dike breaks becomes deposited within 30 miles of the dike lines. A section through the deposition normal to the river's course will have approximately the form of a trapezoid, the top being the distance between dikes and taken as 5 miles and the base as 65 miles. Deducting 85 000 000 000 cu ft for the 3-ft rise of the channel over this 200-mile stretch leaves 1 315 000 000 000 cu ft of silt deposited within this section whose height is computed at approximately 6.7 ft. Adding to this the former estimate of 3 ft gives a total rise in a century of nearly 10 ft with reference to a fixed plane. As the deposition is not regularly distributed along the river's course, there will be some variation from this average depth.

It is generally during super-floods, such as in 1933, that vast quantities of silt become deposited on the plain. During that flood alone from 60 000 000 000 to 80 000 000 000 cu ft of silt must have found its way to the plain. Following the break near Tungchuang in 1935, the silt volume deposited on the plain was estimated at 15 000 000 000 cu ft. Added to another large quantity that escaped through breaks at Kuantai the same year, probably as much as 20 000 000 000 cu ft of silt was deposited on the plain outside the dikes in 1935. In 1934, through the Kuantai breach, another 15 000 000 000 cu ft of silt reached the plain. During these three years alone, from 90 000 000 000 to 100 000 000 000 cu ft of silt must have been added to the deposits on the plain. None of this went farther than 30 miles from the dikes.

If the calculation for the rise of the plain and the river channel is further examined, it will be found that only 20% of the volume left behind appears to be deposited between the dikes, and the remaining 80% is deposited on the plain outside. In view of the foregoing estimates of silt volumes which are known to have flowed out on to the plain, this is not a surprising deduction. It is an approximate calculation, but the result cannot be materially altered even if the dike distance and the deposition limit away from the dike lines are changed 25% either way. It means that approximately 3 000 000 000 cu ft of silt are deposited along the river channel between the dikes every year, and this quantity is a measure for the efficiency of the river channel, as it has existed in the past, to transport silt.

Colonel Dent asks what will happen if the dike distance is narrowed and the dikes strengthened so as to make breaks improbable. Until river-laboratory research work has been conducted to clarify the river's silt action for wide and narrow dike distances, and for different widths of channel under various conditions of discharge, the writers are not prepared to make any definite statement of opinion. It does not seem reasonable, however, to expect a deterioration in the river's capacity to transport silt if the channel is regulated and the dike distance is adjusted so as to give the greatest transporting efficiency for silt. With regard to the 80% silt quantity (which, under existing conditions, is deposited on the plain outside the dikes), the writers have recommended settling basins in Western Shantung, and they have also strongly advocated soil-erosion control measures in the up-stream loess areas in order to reduce, gradually, the silt masses in the river's flow. They realize, however, as suggested by Colonel Dent and Professor Lane, that it may be necessary to have another silt-depositing basin along the up-stream reaches of the river's diked course to be used until the up-stream soil-erosion control program becomes effective. They do not believe, however, that it would be wise, at the outset, to advocate a system of controlled silt-depositing reservoirs on the plain, as this would be likely to stir up strong opposition that would be very difficult to overcome. It could only be done in case the soil-erosion control should prove an absolute failure, and after the delta has advanced so far as to threaten to make the entire lower course untenable. There are social aspects to the problem that add to its complexity.

Colonel Dent wishes a silt-depositing reservoir west of Tungchuang mainly for the purpose of getting rid of the coarser silt particles so that the river channel down stream, where the slope is flatter, will be able to convey the silt. Theoretically, of course, this is correct. The writers deal with this point in their comments on the discussion by Professor Lane and merely wish to state here that west of Tungchuang the natural conditions for silt deposition are not so favorable as east of that place. The silt-settling basins proposed by the writers are so located as to require little construction work to create them and to bring the clarified flow back to the river. A deposition basin, however, is possible immediately east of the Peiping-Hankow Railway, on the north side of the river. The land is low and unproductive at this point, and the population not dense; but costly construction works are required. So very important is the problem of taking care of excess silt that further studies, both in the field and the laboratory, seem necessary for a satisfactory solution.

With regard to the danger of the channel being plugged by silt after a freshet, the writers' opinion is that there may be a danger of plugging during a freshet if the flow is exceptionally silt laden. The high elevation of the old channel prior to 1851, east of Hsüchow, where Mr. Chatley has had occasion to observe it, is probably not due to any plugging after the occurrence of the 1851 breach. Most likely it is due to the same prolonged silt action that has been building up the channels in which the river has been flowing. When a breach occurs, the abandoned channel quickly tends to become choked at that point, and it is only the nearest 15 to 20 miles that are seriously silted up by the rapidly diminishing flow. This was especially noticeable after the Tungchuang breach in 1935. Three months after the break occurred at this point there was almost no flow passing down the abandoned river channel. Cross-sections showed that 15 miles below the break the bed of the channel had retained practically its former elevation and shape.

Mr. Chatley questions the value of 2 500 000 cu ft per sec as a probable maximum for the Yangtzekiang at Ichang, 400 miles up stream from Hankow. One of the writers has had an opportunity to investigate the probable maximum discharge of the Yangtzekiang just below Ichang and has found that it is possible for the aforementioned discharge to occur. The reason the lower stretches of that river do not show a higher discharge is the existence of the two enormous lakes, Tungting and Poyang, which act as regulators for the Yangtzekiang. Usually, when the flood is heavy from the up-stream regions, there are also dike breaks which further aid in regulating the flow. Another moderating factor is found in the fact that the tributaries to these lakes are in flood late in June, whereas, the most serious floods from the main upper river come in late July or August.

In regard to the Yellow River delta surveys, both the British Admiralty and the German land surveys have astronomical positions close to the coast in that region. These surveys have been checked by the more precise surveys made by the Yellow River Commission. Positions of the more important villages have been compared and found to agree reasonably well, especially in latitude. The writers believe, therefore, that the estimate they have made of delta advance is fairly reliable.

Mr. van der Veen complains that the writers have not given due credit to certain previous writers on the Yellow River, notably his countryman, Mr. Fijnje van Salverda, who issued a memorandum in 1887. Lack of space made it impossible for them to devote any considerable portion of the paper to a discussion of former writings, however interesting this might have been. They had so much new material to discuss that they referred the interested readers to Mr. Freeman's paper on "Flood Problems in China"[3] where references to, and discussions of, works on the Yellow River by former writers could be found. The writers wrote entirely from their own knowledge and experience with the river itself and only referred to such works as, to them, seemed of sufficient importance to be of distinct value in adding to the hydraulic knowledge of the river. They regard the hydraulic laboratory experiments which were made in Germany for the Chinese Government (by a collaboration of Chinese and German engineers under the leadership of Prof. H. Engels), as a distinct contribution toward improving the knowledge of the Yellow River in spite of the fact that they were made without simulating the conditions of the Yellow River too well. When the results of these experiments are compared with those which eminent engineers such as Freeman and Franzius expected from their narrowing in of the dike distance and river channel, then it must be agreed that the experimental results gave "some startling indications."

Mr. van der Veen is correct in urging better dikes and better dike protection for the immediate future, pending a complete solution of the Yellow River problem. This is one of the practical aspects of the situation wherein all western engineers conversant with it agree. In fact, the inadequacy of the dike system, its low factor of safety in many places, and its haphazard location has brought adverse criticism from nearly every investigator who has had the opportunity of making a personal inspection.

The flood problem of the Yellow River does not end merely with strengthening the present dikes, however. To declare that it is merely a diking job is to close one's eyes to the real issue. Mr. van der Veen feels that the writers have explored too many angles of the problem and mentions "seeing a few particular trees but not the forest of which they form a part." He would limit the problem of Yellow River control mostly to a program of strengthening the dikes and banks where the current impinges on them. The writers believe that the problem must be attacked from several directions and that an adequate dike system is only a part of the remedy needed. The Yellow River problem has its roots in the most stupendous and rapid geological transformation process which is taking place on this planet—a process which has been created, for the most part, by human ignorance and carelessness. Soil has been stripped of the cover of vegetation that held it in place, and food-producing plants with less soil binding properties have been put on all these highlands that required careful treatment. The cultivation of these crops induced erosion which has continued without abatement for centuries. To repair the enormous erosion damage that has already been done is impossible; but it may be possible to retard it appreciably, if not entirely to arrest it. This will be the work of decades; and, in the meantime, ways and means must be found to control the floods and the silt masses that make the otherwise attractive and fertile Great Plain of North China such an insecure place in which to dwell. Most likely the Yellow River will always have a silt problem which will be reduced gradually in importance with the development of erosion control. The laws of silt transportation, as they apply to this river, will eventually be determined in suitable field laboratories and will give the river engineer this added power not possessed by the mere dike builder.

The writers have not recommended that dredges be introduced without mature consideration. When the channel has been regulated, the use of dredges may mean much toward a prolongation of its life. How far out into the delta it will be practical to use dredges only experience can prove. This equipment will have to be of a different type than the antiquated bucket dredge, which, due to bad seamanship, ran on a silt bank, capsized, and was buried in silt at the mouth of the river in the 1890's. That feat should not be cited as a warning to keep dredges from the Yellow River. Unlike the Mississippi, the Yellow River scours on rising stages. If dredged cuts are kept open through crossings immediately before the flood season begins, it may result in the river scouring a channel deeper than otherwise. It is likely that the low-water season's bed-load will be induced to progress down stream more efficiently by dredging cuts through bars which act as obstacles, tending to silt up the channel as a whole. A judicious use of dredges may prove to be the turning point in preventing the river from raising its bed due to other causes than prolongation of the delta.

Mr. van der Veen believes that the soil condition of the river banks has little to do with groin spacing. In the case of very long groins this is true; but for short groins the condition of the soil is an important consideration in spacing. This has been the experience of engineers on the Yellow River.

Mr. Jarvis calls attention to the fact that the Wei Ho is the greatest flood producer and silt carrier of the tributaries and hence requires special attention. In Fig. 1, the writers have shown possible dam sites for detention basins on the upper Wei Ho in Kansu, and also on the King Ho and Pei Lo Ho in Shensi, these latter being feeders of the Wei Ho. Such detention basins may be of great assistance in smoothing out crest flows. It is also in this region that, eventually, extensive soil-erosion control work must be constructed, as it is one of the critical parts of the Yellow River system, especially in the matter of silt flow. The writers have not drawn more attention to the Wei Ho because they believe the main river to be in more immediate need of control. The Wei Ho silt is of such extreme fineness, being mostly of clay composition, that it will not cause the same difficulty along the diked channel as the silt from the main river. Therefore, corrective measures on the drainage area between Tungkuan and Tokoto in Suiyuan should take precedence over similar measures on the Wei Ho drainage area.

The writers had suggested only one auxiliary flood channel—the Tu Hai Ho—or one in its general vicinity. Hills on the southeast bank prevent the location of a similar channel on that side of the river from the old Grand Canal crossing to a point a few miles up stream from Lokou. It is possible, however, to have an auxiliary channel to the south of the river from a point near Lanfeng, just east of Kaifeng, to Tungping Lake and then through the Po Ho to the main river. As Mr. Jarvis suggests, such channels might aid in maintaining capacities in the main river at critical times. In this vicinity there are areas that could take large silt storage in filling depressions.

The writers are highly appreciative of the careful study given this subject by Professor Lane and share with him some of the anxieties he has in the matter of silt transport and disposition. It is agreed that, aside from handling the flood waters and routing them to the sea, there is the closely allied problem of controlling the silt or sediment stream. He is entirely in accord with the writers in emphasizing the need for the further development of laws governing the transportation of sediment. Professor Lane's discussion is especially valuable since he is able to write not only with a personal knowledge of Chinese river conditions, but also with years of experience in the study of silty rivers in both field and laboratory. Some of his field work has been in the Huai River valley south of the Yellow River.

Professor Lane intimates that a possible danger in a flood-control and silt-control detention basin is that the silt concentration is likely to be a maximum when the outflow from the basin has become nearly that of the natural stream. Hence the Yellow River course through the plain may be in danger of choking. Opinions may differ both in regard to the time of maximum silt concentration in the outflow (see Fig. 20) and the possibility of choking. Since it is not possible to solve these problems by analytical deductions, the writers have recommended that they be solved by laboratory experiments, or, preferably, on small, silty streams such as may be found in Southern California or Arizona.

In this connection actual field data from the Yellow River for August, 1934, show that the diked river is capable of conveying a flood of about 185 000 cu ft per sec, with an 11% silt load, through the entire length of its lower course from Taochengpu to the sea without injuring the channel to any marked degree (see Item 1 in Table 6, Table 7, and Table 4, August 9–20, 1934). Attention is called to the controlling influence of the section between Shanchow and Chinchang on the silt concentration. It is the Mengtsin-Chinchang section of this reach which acts as a temporary storage reservoir when the silt masses become excessive. The next depositing area under present non-regulated conditions is between the 1851 break and Kaotsun near the Shantung border. These two natural storage regions for silt seem, so far, to have prevented the silt concentration in the diked river below Kaotsun from exceeding 12½%. Mr. Freeman records about 10% in 1919 near the Grand Canal crossing. The values for August, 1934, in Table 4, are the highest recorded by the Yellow River Commission for the lower diked river. It seems quite possible, after the channel has become regulated, that discharge of 200 000 cu ft per sec may convey a silt load of nearly 20% without serious deterioration of the channel.

If the Palihutung or Sanmen detention basin project were to be completed and the diked channel regulated, one might expect (see Figs. 19 and 20) that the maximum silt concentration in the outflow would not exceed 40%. (It may even be possible to keep the maximum silt percentage lower than this by gates whereby the stages in the basin could be controlled and excessive silt scour from the basin thus reduced.) Reasoning from what occurs at present along the down-stream channel, one may assume further that the silt concentration in the outflow would be reduced down stream to about 20% by temporary storage between Mengtsin and Chinchang and to 17 or 18% at Kaotsun, from which place the flow would continue with the same silt load right through the diked channel. There might be some deterioration of the diked channel during exceptional floods having very high silt loads; but it seems scarcely reasonable to expect that the diked channel at any place would become choked and the flow sent over the dikes, since in a regulated channel the silt would tend to become uniformly deposited all along the channel. It is also very probable that the clear outflow from the basin, preceding the scour period, would remove considerable silt from the channel between Chinchang and Kaotsun, the same reach where refill would later take place, thus further reducing chances of choking. Since not all large floods are as heavily silt laden as that of August, 1933 (of which Fig. 20 is an actual record), the channel will again improve in time.

To interpose another silt-depositing reservoir near the up-stream end of the diked course is scarcely advisable although it may be done in case of need. To arrange for such a reservoir would be a formidable technical problem. If, in its simplest form, it is made as a lake, through which the river flowed all the time, depositing most of its silt there, it would become silted up rapidly, even if it had an area of 1 000 sq miles. If it were meant to receive the silty flow only during the periods of heaviest silt concentration, control works would be required both across the main river channel and on the side into the reservoir, as well as for the outlet back to the river channel.

It has been contended several times, in the discussion, that the change in slope along the diked channel will affect, materially, the river's capacity to transport a heavy silt load. This may not necessarily be the case. Field studies made of the Yellow River indicate that slope changes within the limit of those which exist along the diked course are not sufficient to detract seriously from the river's capacity to carry silt. The slope which will begin to affect silt transportation seriously will, most likely, be found to be considerably flatter than those which exist, especially after the channel has been regulated. Of course, there are limits for discharge and silt load beyond which the existing slopes will begin to take effect; but the writers believe that these limits, due to the fineness of the silt, lie much higher than on other rivers—hence, their optimism in regard to the non-choking probabilities in case flood-control and silt-control detention basins should prove feasible.

Under a system of flood-control and silt-detention basins west of the plain, there is the objection that nearly all of the silt will be deposited in the sea and rapidly make the lower diked course untenable. If soil-erosion control in the loess areas is vigorously pursued, however, this objection may not be serious, since the erosion control may be expected to bring imposing results within three or four decades. Nevertheless, the possibility exists that such control may not be sufficiently effective, and the problem arises of how else to handle the silt. Professor Lane pointedly draws attention to this and suggests three methods:

1. Keeping it in place on the water-shed (may not be very effective);
2. Taking part out of the river channel and storing it on the plain; and
3. Carrying a part out to sea and depositing it on the sea floor.

The writers concur fully with Professor Lane in these suggestions, but at the same time they maintain that it is of paramount importance to regulate the channel, as without regulation it will be impossible to enforce any other control. Nothing can be done with a river which is likely to break loose anywhere and which wanders at will over a broad strip of land within irregular lines of dikes spaced excessively far apart. The silt problem must be handled quantitatively, and this can be done first after the channel has been regulated. It must be determined how much silt goes to the land, how much is left between the dikes, and how much goes to the sea. These quantities should be so proportioned that they will give the longest life to the entire diked channel. To obtain the desired uniformity and quantitative control of deposition in channel, outside areas, and sea, it is essential that the entire channel be so regulated that there is no chance for any outflanking of the control works. The outflanking of the Yung Ting Ho diversion works in 1933 and 1934 was due to the fact that the river up stream was unregulated. This and other illustrations of costly gambling with silty rivers in North China remind engineers of the prudence of stabilizing and regulating these channels.

It has been suggested that the writers might have considered the matter of silt disposal more fully in their recommendation for control measures. To do so would have meant, evidently, a quantitative appraisal of silt disposal. Data for doing so, unfortunately, are still insufficient. Professor Lane suggests certain principles along which such silt disposal should be guided, but at the same time recommends laboratory research for quantitative determinations. To this the writers agree, and here the matter must rest for the time. With China at war there seems little opportunity of this research work being undertaken in that country. Possibly a way will be found to solve some of these problems in the United States so that a comprehensive control of the Yellow River will be planned with greater assurance that the element of chance has been removed to a very large degree.

The proposals of Mr. Higgins have been answered in replies to previous discussions. The writers believe that they have remarked fully enough on the possibility of letting the river "handle its own problem." They are convinced that more control, rather than less, is essential, especially where there is such a dense population to be considered. Resourceful though they be, the Chinese farmers need direction in matters such as control of Yellow River floods, because these come as great avalanches of water surcharged with mud. Too long, the people of China have been victims of these river "dragons." It is time for the entire engineering profession to give the very highest service in its power to bring this river under control. It is a task for hydraulic engineers with years of intimate river experience.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2065

THE RISK OF THE UNEXPECTED IN SUB-SURFACE CONSTRUCTION CONTRACTS

BY OREN CLIVE HERWITZ,[1] ESQ.

WITH DISCUSSION BY MESSRS. ALONZO J. HAMMOND, FREDERICK W. NEWTON, DAVID A. MOLITOR, F. B. MARSH, EVAN S. MARTIN, C. MAXWELL STANLEY, FRANCIS J. MORGAN AND FREDERICK C. ZEIGLER, LAZARUS WHITE, T. KENNARD THOMSON, CARROLL A. FARWELL, AND OREN CLIVE HERWITZ.

SYNOPSIS

In the early days before the development of a technique for preliminary underground exploration, it was customary for the contract to provide that the risk of the unexpected should fall on the contractor. Broadly worded contract clauses took the place of scientific and careful investigation. This practice often resulted in a higher total cost to the owner, since contractors, perforce, added an arbitrary sum to their bids to cover possible difficulties (1(*a*));[2] that is, the owner passed on the risk to the contractor and paid a substantial premium for this insurance against loss.

Modern standards of sub-surface investigation, as detailed in the Society's Manual of Engineering Practice No. 8 (1), have eliminated much of the costly uncertainty involved in underground construction, with corresponding benefit to owner and contractor. Indeed, the cost of supplying the contractor with complete data on subterranean conditions will generally be less than the sum contractors would charge for blindly assuming all risks.

Nature, however, often shows a whimsical unconcern for Man's data; a careful and complete preliminary investigation may not always forecast actual subsoil conditions accurately. In that event, the parties to the contract often disagree as to which must bear the loss. Even an agreement carefully drawn along the lines suggested in Manual No. 8 (1) may yet leave room for reasonable men to differ as to its interpretation; and, when amicable adjustment of the dispute is not effected, it is more than likely that recourse will be had to the Courts.

Experience in this type of litigation shows that emphasis is placed on carefully worded specifications, often with results expensively significant to one

---

[1] Asst. Corporation Counsel, Div. of Contract Litigation, Law Dept., City of New York, New York, N. Y.

[2] For reference to numerals in parentheses, see "Court Citations and References," in the Appendix.

party or the other. A judicial ruling may result in an unexpected assessment against the owner or in wrecking the contractor.

This paper shows how reports of borings and similar data may return to plague both owner and contractor, even when given in the utmost good faith and with careful attention to the best modern practice. There should be a clear "meeting of the minds" in the perfect contract. It is necessary, therefore, that both parties to the contract understand the extent of the risks they are assuming, so that the hazards of litigation may not be added to the still substantial uncertainties of the subsoil.

BORING DATA

In Manual No. 8 it is suggested that variations in subsoil conditions, revealed during the progress of the work, shall not entitle the contractor to additional compensation, unless the variance is considerable and the error in the preliminary boring data is firmly established (1(*b*)). However, extra compensation has been allowed by the Courts even without specific evidence that error was made in gathering the preliminary information.

In one case (2), the contractor had agreed to excavate a by-pass for Oklahoma City, Okla. Boring and sounding records appended to the contract led the contractor to believe that the material to be excavated consisted of a quantity of earth and dirt with a small quantity of rock. There was no language in the contract purporting to absolve the City from liability should the indications of the borings and soundings prove inaccurate. The contractor bid a unit price per cubic yard for "all excavation," which was to "cover the excavation of all earth of whatsoever nature down to the grades and lines as shown on the plans." Instead of the small quantity of rock indicated by the preliminary tests, about 60 000 cu yd were found, and suit was brought to recover the additional cost of excavating it.

The State Supreme Court of Oklahoma held that the contractor was entitled to recover, stressing the combined significance of two facts: (*a*) That the contractor was led to believe from the records of borings and soundings "that there was no rock to be encountered except in negligible quantities"; and (*b*) that the price bid, from the restrictive language of the specifications, merely covered the excavation of earth. The word, "earth," it was declared, did not include rock (3); therefore, the excavation of rock had not been "contemplated nor contracted for," and should be treated as extra work.

If, unlike the foregoing case, the language of the contract is broad enough to embrace removal of materials not shown by the borings (4), contract provisions designed to shift the risk to the contractor should the borings prove misleading are frequently effective, particularly in the absence of evidence that serious error was made in making the tests. Thus, it has been held that if the owner "does not warrant the indications of the borings to be correct," and if the contractor agrees that he shall have no claim "should the character and extent of the various materials be found to differ from what is indicated," owner responsibility for the unexpected is averted (5).

A similar ruling was made in a case in which the contract stated that there was no express or implied guaranty as to the accuracy of the borings and soundings, and directed each bidder to place his own interpretation upon the soundings and borings made by the City in forming his opinion of the character of the materials to be excavated (6).

The State of New York includes in its contracts a disclaimer clause which has proved an almost impenetrable barrier against claims based on unexpected sub-surface conditions. A leading case in point is Foundation Company *vs.* State of New York (7), where a recovery was denied in spite of the fact that the borings incorrectly indicated the elevation at which bed-rock would be encountered. The contractor there had agreed that he would make no claim against the State "because any of the estimates, tests, or representations of any kind affecting the work made by any officer or agent of the State, may prove to be in any respect erroneous." This disclaimer clause is ineffective, however, where fraud is proven (as discussed subsequently), and also where the extra expense incurred is due to a subsequent act of neglect or default on the part of the State (9).

Disclaimer clauses have even been held to cover a case in which the borings were interpreted incorrectly by the owner's engineer. The evidence in that case (10) was that the engineer's error was due to his ignorance of the wash-boring technique; the indications of hardpan were not recognized. The decision holding against the contractor's claim rested largely on a contract clause stating that the borings were exhibited to the contractor so that he could form his own judgment without implying any guaranty on the part of the owner as to their completeness or correctness. The Court held that, under this clause, an honest mistake would not justify a recovery unless it rose to "that degree sufficient to brand it as the equivalent of fraud or bad faith." A similar ruling was made in another case (11) where the contract stated that the borings were not guaranteed to be absolutely correct.

No disclaimer clause can force the contractor to assume the risk of deceptive borings, however, if the information given is known to be incomplete or misleading. The well established doctrine that no man may contract against his own "fraud" (12), strikes down any attempt on the part of the owner to escape liability for "unexpected" subsoil conditions. Although the principle is undeniably sound, difficulty lurks in determining just what facts will justify its application.

A well known case arose from a contract with the United States Government for the construction of three locks and dams on the Warrior River, in Alabama (13). Borings met "obstructions which, from the particles broken off and floating to the surface, would indicate that they might be logs." When this happened the drill was moved elsewhere until a place was found into which it would penetrate. The evidence of buried logs was not noted on the boring plans because the Government's engineer did not consider them of sufficient importance to be so recorded.

Although the Court found that this was an honest opinion and that there was no conscious intention to conceal material information from bidders, the contractor was permitted to recover the additional cost of uprooting stumps, buried logs, cemented sand and gravel, and sand-stone conglomerate, none of which was indicated on the borings. The extent to which the Court's decision was influenced by the statement in the specifications that "the material to be excavated, as far as known, is shown by borings" is not apparent, but it is significant that, in its opinion, the Court italicized the words, "as far as known" (14). In any event, it would seem that the engineer's failure to note a significant detail on the boring plan may have an expensive sequel.

Even the broad disclaimer clause included in New York State contracts has been held impotent to prevent recovery by the contractor when material information has been knowingly concealed. In one case (15) the State had conducted earlier boring and sounding tests which suggested the presence of hardpan, but the results of these preliminary explorations were not disclosed to bidders. Instead, they were given a later series of wash-borings which did not show hardpan. Although the State's engineer testified that the earlier borings were not disclosed because, in his honest opinion, they were not as reliable an indication as those that were shown, the contractor was allowed to recover the additional cost of excavating hardpan. Recovery was also allowed in another case where the statute authorizing the construction of the New York State Barge Canal required the State to give bidders accurate information (16).

In still another case (17), it appeared that the Passaic Valley Sewerage Commissioners had contracted for the construction of a subterranean and submarine sewerage tunnel. A series of borings had been made and the results tabulated on a set of plans appended to the contract. These findings signified that under the waters of New York Bay the material to be excavated consisted of a layer of mud; below that a layer of sand and some boulders; and, beneath that, a "cemented Triassic formation."

On these data the Commissioners entered into a contract with a party which, after completing only part of the work and failing to find the "cemented Triassic formation," abandoned the job. The Commissioners then entered into a contract with a second party on exactly the same data previously used. Again, the elusive rock formation was not found; again, the contractor abandoned the work.

In re-advertising the contract for the third time the Commissioners, now apparently doubtful of the accuracy of the borings, added a note to the contract stating that the borings were "supposed to be approximately correct, but should they be found to be otherwise the contractor shall have no claim on that account, it being expressly understood that the Commissioners do not warrant the plot to be approximately correct." The contract was awarded, and for the third time the "cemented Triassic formation" was not found; instead, gravel, quicksand, bull-liver, boulders, and other pervious materials were encountered. This time the contractor stopped work and brought an action for damages. A verdict for the plaintiff was affirmed by the Circuit Court of Appeals. It was held that the self-exonerating note appended to the boring plans was ineffective to relieve the defendant from liability, inasmuch as the defendant knew from its two previous experiences that the borings were misleading (18).

In another case, it appeared that a plan of preliminary borings was included in a contract to construct a section of the Lexington Avenue Subway in the City of New York, N. Y. (19). The borings, although "not guaranteed," indicated that the tunnel was to be constructed entirely through solid rock, and the contract drawings were obviously based upon that supposition; but the drawings also contained a notation that "if rock is not encountered, the design of the side walls and floor will be modified." For a considerable part of the distance traversed the contractor met soft earth and mud that required a pneumatic method of tunneling, and to recover the higher cost of this type of construction an action was commenced. In allowing a recovery the Court emphasized the fact that on file in the office of the Public Service Commission there was a copy of the "Viele" map which showed that in earlier times a creek had crossed the line of work at the point where soft earth and mud were found. From this, the Court stated, the engineers for the Public Service Commission should have expected that soft earth or mud would there be encountered. It was concluded, therefore, that the extra work was caused by conditions which should have been anticipated by the Commission, and could not have been discovered by bidders (20). Testimony that the map was also filed in the New York Public Library and was readily available for inspection by prospective bidders was apparently not considered of any significance in this instance. The ready availability of the allegedly concealed information, however, has been emphasized in cases denying recovery (21). Greatly stressed was the bidder's inability to make his own borings, since the sub-surface lay under a principal thoroughfare of the city (22). By the same token, the bidder's opportunity to make borings has been given equal importance in cases denying recovery (23).

Although Manual No. 8 (1*c*) suggests that boring data be included with the contract and guaranteed as to correctness, it also advises that "soil conditions between borings may be assumed by the contractor at his own risk." The distinction thus drawn between guaranteeing merely the nature of the subsoil within the limited area encompassed by the boring pipe, and not the subsoil throughout the entire site, is one that is not readily accepted by a Court of law. Judicial interpretation of a boring plan as being merely a statement of what was found where the borings were made is rarely encountered (24).

Fig. 1(*a*) represents a situation where bidders are merely advised as to the results of the boring tests at the place where the borings were taken. Fig. 1(*b*), on the contrary, constitutes a statement not only of the results of the borings, but also of what is to be expected elsewhere.

A notable exception to the judicial reluctance to distinguish between these two situations is the opinion of the Circuit Court of Appeals in litigation over the construction of a bridge across the Arkansas River, at Fort Smith, Ark. (25). The contractor sought to cancel his contract because he found quicksand unexpectedly and because he did not encounter the soft shale and soapstone shown by the borings. The only statement in the contract pertaining to the borings was that "the findings are as indicated on sheet number 2," which, the Court said, amounted to nothing more than a statement that the findings of the borings were truly indicated. In its opinion the Court considered the interpretation of borings in relation to construction contracts (26). It concluded that the borings were submitted merely as a bit of useful information, and not as a statement of sub-surface conditions generally to be expected throughout the site (27). There being no evidence that the results of the borings were not fully and fairly recorded on the boring plan, the Court held the defendant innocent of misrepresentation, and, accordingly, denied recovery.

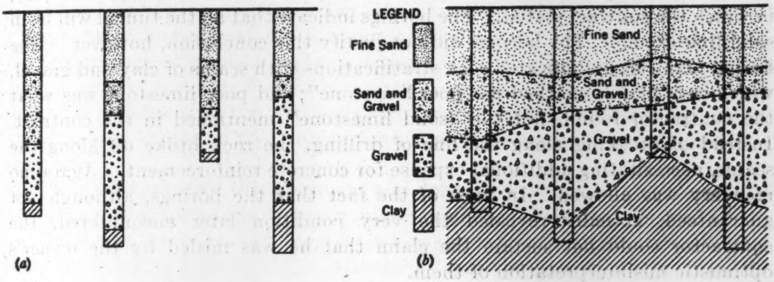


FIG. 1

This case eloquently illustrates to the two conflicting interpretations of a boring plan which Courts have been urged to adopt. Since the Court's understanding of borings is largely limited to what is testified before it by experts, irreconcilable decisions may be expected to continue as long as witnesses disagree.

OWNER INTERPRETATION OF BORINGS

In addition to a boring plan, the owner sometimes supplies his own conclusions as to what the borings show with reference to subsoil conditions throughout the entire site. Whether the owner thereby assumes any greater risk if his conclusion proves erroneous or misleading turns again on the Court's interpretation of the language used in the agreement. Thus, in a contract for the construction of a railroad bridge the contractor was directed to use a certain borrow-pit. The specifications stated that the borrow-pit was to be excavated to Elevation 655, except that no blue clay (which the borings indicated was approximately 15 ft below the surface) was to be taken (28). The contractor found the blue clay only 6.5 ft below the surface, and brought suit to recover the cost of obtaining additional fill elsewhere.

In the lower Court, the contractor won judgment on the theory that the railroad company had "falsely stated in the specifications that there was no blue clay in the tract nearer the surface than 15 ft"; but this judgment was reversed on appeal. The Appellate Court held that the lower Court had misconstrued the language used in the contract; the only statement made in the specifications was what the borings indicated, rather than what was the actual condition of the entire subsoil (29).

After constructing a lock and dam on the Ohio River up stream from Ashland, Ky., a contractor sued to recover the additional expense of removing unexpected logs, snags, and stumps, found embedded in the river silt. These obstructions were not indicated on the boring plan, and the contract contained a disclaimer clause (30). The evidence disclosed that an earlier series of borings were not even shown to bidders and that one of these borings had encountered a "log next to rock." Nevertheless, a recovery was denied in view of the broadly worded disclaimer clause (31).

An interesting variation on this theme was presented in a Missouri case (32), in which the contract stated: "The borings indicate that all the tunnel will be in solid limestone." The borings did not justify this conclusion, however. The boring plan clearly indicated rock stratifications with seams of clay and gravel, which were plainly marked as "poor limestone"; and poor limestone was what the contractor found, not the "solid limestone" mentioned in the contract. Instead of breaking along the line of drilling, the rock broke off along the seams, necessitating additional expense for concrete reinforcement. Again, no recovery was allowed. In view of the fact that the borings, although not guaranteed, actually revealed the very condition later encountered, the contractor could not sustain the claim that he was misled by the owner's optimistic misinterpretation of them.

Profiles and Cross-Sections

Instead of a boring plan, there may sometimes be annexed to the contract a profile or cross-section showing the elevations of the surface and underlying rock, or the general composition of the subsoil throughout the site. In this case, the contractor is presented with the owner's conclusions as to sub-surface conditions, sometimes without access to the preliminary information upon which they were based. Must the contractor bear the loss if the true situation is found to differ from that indicated on the plan?

Here, again, the language of the contract is generally the decisive factor. In one case a profile plan showed the location of a proposed sewer in relation to the existing surface of the street (33). Superimposed on this plan was a convoluted, dotted, line marked: "Probable surface of rock as shown by borings." The contractor had agreed, however, that if "the location of the rock be found to differ from what is indicated," he "shall have no claim on that account." During excavation, quicksand was encountered where the profile showed rock, and the contractor sued for the added expense thereby occasioned. Recovery was denied. The Court held that the contract clause was a disclaimer of warranty relating "expressly to the convoluted line which the plaintiffs claim constitutes a warranty of the true location of the rock."

It has also been held that if the specifications state that "the profiles are reasonably correct, but are not guaranteed to be absolutely so," honest mistakes made in observing and recording the results of the borings upon which the profile plan had been based, were not actionable (11). Similarly, owner liability was not imposed when the actual physical conditions, although different from the profile, were "obvious to any careful observer." For example, where it was a matter of common knowledge that a saw-mill had formerly occupied the site of the work, the contractor in one case (34) could not recover the expense due to logs that were not indicated on the profile. Recovery was also denied in a case (35) in which the contract stated that "levels of the rock at various points are shown approximately on the plans, but the bidder must satisfy himself of the accuracy of these and of the rock surfaces in general."

On the other hand, the use of the word, "approximate," in connection with the information given, is not always effective to prevent recovery by the contractor. This was demonstrated in a Virginia case (36). Bidders on a proposed contract to construct a bridge over the James River, near Richmond, were furnished "a profile of existing conditions," which showed a line marked "rock" extending across the bottom of the river. The successful bidder discovered rock at a much lower elevation than that shown on the profile and sued to recover the cost of additional earth excavation. The City of Richmond argued in defense that the words, "approximate" and "plus or minus," before the elevation figures on the profile were sufficient to excuse it from liability for the error.

The Court rejected this argument and permitted the contractor to recover. It was said that the words, "plus or minus" and "approximate," as used on the profile, served to cover only "negligible deviations from entire accuracy"; they were intended to express the idea merely "that the line as shown upon the profile was nearly, but not exactly, correct." Since the actual conditions departed so substantially from those pictured on the profile, the qualifying words were held inadequate as a defense.

A contract for the construction of a new coagulation concrete basin in Alliance, Ohio, incorporated a plan showing, in cross-section, the location of the basin in relation to the existing grade, signified by a wavy, horizontal line marked "approximate original grade" (37). From the scale of the drawing, the contractor calculated that a general average excavation of about 9 ft and a fill of about 9 ft would be required. He found, however, that the actual average excavation was only about 3 ft, and brought suit for the cost of obtaining the necessary additional fill elsewhere.

A verdict for the defendant City was reversed on appeal and a new trial was ordered. As in the foregoing case (36), the Court held that the word, "approximate," would not protect the City in the face of so substantial a discrepancy. The attempt to charge the contractor with knowledge of the error, on the theory that the contour lines on another plan revealed the inaccuracy, was also unsuccessful. Proof that bidders do not customarily compute excavation from contour lines was accepted by the Court as sufficient to "excuse the contractor for not discovering the mistake."

Recovery was allowed in a similar case (38) where the piers were to be built "to the line and elevations given on the plans" and excavation was to be "done to the lines shown." Recovery was denied, however, where the specifications stated that the plans showed "approximate grades for excavation," but where the contractor was also directed to carry the excavation "to the foundation lines and grades as shown on the plan, or to such increased depth as may be necessary to obtain a firm foundation" (39).

In still another case the contractor complained because rock was found nearer the surface than suggested on the profile (40). His attempt to recover the cost of excavating additional rock was unsuccessful. Here, again, the line on the profile was marked "approximate rock line" but the contractor had also agreed, under another clause, to bear "all losses resulting to him * * * because the nature of the land in or on which the work is done is different from what is assumed or was expected." The fact that the contractor had failed to protest upon discovery of the error was likewise deemed fatal to the claim (41).

The foregoing case (40) was later cited by the same Court when it refused to grant a contractor the cost of extra excavation required to reach bed-rock (42). This claim was based upon the fact that a cross-section showed the foundations of the proposed dam resting on bed-rock, which, from the scale of the plan, appeared to be all above Elevation 148. The plans were to be "considered as only approximate," and although it was necessary to go a considerable distance below Elevation 148 to reach bed-rock, no recovery was permitted.

In holding that this was a contract to rest the foundations on bed-rock wherever found, and not a contract to build "these piers with foundations at a certain depth" shown on the plans, the Court emphasized the fact that the contract included a contingent item of "200 cubic yards of caisson work below elevation 148." The inclusion of such an item was considered a clear indication that bed-rock might be found below Elevation 148. The following part of the Court's opinion has been quoted frequently (43):

> "A contract and specification may contain representations as to existing physical conditions. If so, a bidder may rely upon them, even though it be provided that he shall satisfy himself by personal inspection and investigation as to their truth, where because of time or situation such investigation would be unavailing. (Faber *v.* City of New York, 222 N. Y. 255); or statements may be made on which the bidder, because of the language of the contract, cannot rely. He may have agreed that he will not. Then if they are made in good faith he takes the risk of their accuracy."

In litigation over the construction of an East River bridge, the same Court had earlier held that a plan indicating the existing rock surface and showing caissons sunk into it, was a warranty as to the true position of bed-rock (44). However, the specifications contained the added limitation that "bids are based upon depth of excavation indicated on the plans," and that "if excavation deeper than that shown on the plans is required it is to be paid for" at so much per cubic yard. Bed-rock was found from 8 ft to 9 ft higher than was shown on the plan. In order that the top of the caissons should not protrude over the water level and create an obstacle to navigation, the contractor was directed to excavate deeper into the rock.

The defense was based on a contract provision which required that "the contractor must assume the responsibilities for the difficulties encountered in sinking the foundations to bedrock or into it to whatever depth shall be determined upon." In view of the other contract provision requiring payment for additional excavation into rock, the Court decided that the argued clause contemplated the assumption of responsibility merely for any "difficulties" encountered in excavating into rock, but not for the additional "expense" thereby occasioned!

SUB-SURFACE DESCRIBED IN CONTRACT

A statement describing the materials to be encountered may be incorporated in the contract. The form of the alleged representation in this case is the language of the contract, and not mere lines or symbols on a plan whose meaning may be doubtful. Typical is the well known case (45) based on a United States Government contract for the repair of a dam, in which it was stated definitely that: "The dam is now backed for about fifty feet with broken stone, sawdust and sediment * * *." Instead, the contractor found a log cribwork filled with stones, and sued to recover the consequent additional expense. The contract provision requiring bidders to examine the site and to satisfy themselves as to the nature of the work to be done was urged in defense. With this argument the Court was unimpressed. A recovery was permitted because "the specifications spoke with certainty as to a part of the conditions to be encountered by the claimants," and of them, it was said, "the government might be presumed to speak with knowledge and authority."

In another case (46), recovery was permitted because the specifications stated falsely that "the vitrified pipe line is mostly, and the tunnels are entirely, to be in soft shale rock."

A similar conclusion was reached in a case arising out of a contract to excavate a ship channel at the mouth of the Detroit River (47). The material to be removed was stated to consist of: "Sand, gravel and boulders, all in unknown quantities"; but a large part of the material proved to be limestone rock. For the consequent additional cost of performance, the contractor was allowed reimbursement.

A contrary decision was given in an instance in which the statement as to the nature of the sub-surface was qualified (48). The contract stated that: "Good gravel will be encountered in the excavation," but the contractor was also notified that if sufficient gravel was not found, equally good gravel would have to be furnished for making concrete. The contractor, not finding any gravel at all, sued for the cost of obtaining it elsewhere. The Court held that no representation had been made to the contractor that he would find sufficient gravel in the excavation for making concrete; on the contrary: "All that the specifications say is that some gravel will be found in the excavation. The language of the specifications put him upon notice and required him to make investigation, and having failed to do so he cannot be heard to complain."

Recovery was likewise denied where the contract stated that a supply of necessary stone "could be obtained at Union Spring [N. Y.], at 90 cents the cubic yard" (49). Before bidding, the contractor made no attempt to verify this statement and, later, when he discovered that the stone could not be obtained at Union Spring, he tried to rescind the contract and recover his deposit. The broad disclaimer clause found in contracts with the State of New York, to which reference has been made heretofore, was cited by the Court in support of its conclusion that the contractor was not entitled to rely upon this statement as to source of supply.

In a very similar case, however, the clause was held impotent as a defense to the contractor's claim for the additional cost of obtaining stone at a place other than that mentioned in the specifications (50). Before bidding, the contractor had inquired at the place named in the specifications and was told that the stone was available. Later, when he sought to purchase the material, he discovered that the person in possession had no legal right to sell it, and thus he was forced at added expense to go elsewhere.

Having done everything possible to verify the statement and having found that, apparently, it was true, the contractor was deemed entitled to rely upon it; whereas, in the preceding case (49), recovery was denied because investigation by the bidder would have disclosed the error. Thus, in the face of contract disclaimer clauses, investigation seems to be the wisest course.

As has already been shown in other types of cases, however, exonerating language in a contract is unavailing if the representations made are knowingly false. This was the situation presented in a contract which stated that "the material to be removed is believed to be mainly mud, or mud with an admixture of fine sand, except from Station 54 to Station 55 plus 144 * * * where the material is firm sand and gravel or cobbles" (51). The "accuracy of this description" was expressly not guaranteed and bidders were enjoined to decide for themselves the character of the materials to be handled. The description presumably was based on preliminary borings made by the Federal Government, which also were included in the contract with the statement that:

> "No guarantee is given as to the correctness of these borings in representing the character of the bottom over the entire vicinity in which they were taken, although the general information given thereby is believed to be trustworthy."

Upon finding a heavy compacted material instead of mud and fine sand, the contractor discontinued operations and sued to recover the value of the work already done. It was shown that the Government had encountered an impenetrable material during preliminary borings and had not noted this fact on the plans. Recovery was allowed by the United States Supreme Court. The decision apparently turned upon the fact that the Government's assertion of "belief" as to the nature of the material was knowingly false, in that it was contrary to other information in its possession. (Compare this statement with that in Pearson *vs.* State of New York (8) in which the use of the word, "expected," was considered to be merely a statement of "hope or expectation" and not a "positive agreement or warranty.") Emphasized, too, was the fact that before work was begun, Government engineers had inspected the contractor's plant and equipment and approved it, knowing that it was suitable for dredging only the materials stated in the specifications.

Frequently, the statement or representation is not made part of the written contract. When such information proved incorrect, recovery was denied in an early case (52). The Court declared that a statement of the expected nature of the sub-surface was a mere expression of opinion "with reference to matters equally within the power of both of the parties to the contract to ascertain and determine for themselves," and that the failure to include this statement in the written contract indicated that the parties did not regard it as material.

IMPLIED SUBSOIL REPRESENTATIONS

It has occasionally been claimed that a plan, or the quantities stated in an engineer's estimate, or the methods prescribed for doing the work, led the contractor to believe that a certain type of sub-surface would be encountered. Such attempts to "spell out" an implied warranty have rarely been successful.

Plans.—In litigation over a contract to construct a sewer, it appeared that a plan showing the work to be done bore certain dotted lines which the testimony showed "was the usual manner of indicating an existing sewer" (53). The contractor claimed this authorized him to assume that there was such a sewer and that it would drain off all water flowing into or accumulating in the excavation during performance. In fact, no such sewer existed. An action was brought to recover the cost of keeping the trenches dry with pumps. Recovery was denied because of contract provisions requiring the contractor to keep all the trenches free from water while the excavation or the construction of the foundations and sewer was in progress. This provision negatived the idea that an outlet for water was to be supplied to the contractor by any existing sewer (54).

This ruling was repeated in an almost identical case (55), in which the contract also contained a similar clause requiring the contractor to keep the excavation free from water. In its opinion, the Court observed that, even if there had been a warranty that a sewer existed, there was certainly no warranty that it would be "fit and suitable for drainage."

In 1899 the City of New York entered into a contract to demolish the old reservoir at Forty-Second Street and Fifth Avenue to clear the way for the construction of the New York Public Library (56). Annexed to the contract was a plan which indicated the dimensions of the old reservoir walls, the angle of the slope, and the extent of the excavation and fill to be made. It did not, however, "show the quantity of material to be removed in removing the walls and foundation of the reservoir, or the quantity of this material that might be utilized upon the site for the filling required by the contract." The contractor asserted that these quantities could be calculated from the plans on the assumption they were correct and drawn to scale. The plans were inaccurate in that they indicated an angle of slope of 1 on 1 when actually it was 1 on 1.51. Consequently, about 20 000 more yards of material had to be removed than the contractor had calculated. Recovery for the extra work was denied. Emphasis was placed upon the fact that the contractor had agreed in the contract to perform "the entire removal and disposal of the masonry and rubbish of the reservoir" (57).

In a contract for the construction of a part of the Catskill Aqueduct a plan showed the types of tunnels to be constructed (58). Beneath the invert and resting on the bottom of the excavation, square and round drains were shown. These drains were to be installed so that "the bottom of the tunnel may be free from water and sufficiently dry at all times." Dimensions were given for all details on the plans except the drains, but their size could be determined by scaling. The contractor contended that a drain of this size could not carry off more than 70 to 80 gal per min. It was argued, therefore, that the drains shown on the plans implied a warranty that the water to be handled would not exceed that quantity. Finding it necessary to dispose of as much as 560 gal per min, the contractor sued to recover the extra cost of drainage and of excavating rock under these unusually wet conditions.

Recovery was denied. The Court stressed contract provisions which called attention to the uncertainty of the sub-surface to be encountered and which directed the contract or to provide all necessary pumps, pipes, and drains at his own expense (59).

An ingenious argument was advanced for the implication of a warranty in a case in New York City (60). The contractor was directed to remove "earth, rock or other material overlying the sub-grade" of a roadway. When rock was found, he objected that its removal was not required by the contract. He pointed out that, many years before, the City had let a contract for the removal of all rock "overlying the sub-grade," and that the work done under that contract had been accepted by the City. He claimed, therefore, that he was justified in assuming that the other contractor had removed all the rock, and that there would be none left for him. His suit to recover the cost of removing the rock was unsuccessful. The Court held "that the acceptance by the City of the prior work was not a representation of anything, upon which this contractor was entitled to rely" (61). Of interest in this connection is a case decided by the New York Court of Appeals in 1938, involving the construction of the New York State Building in the City of New York, wherein the superstructure contractor was held entitled to recover damages for the delay occasioned by the fact that the foundation plans had to be altered to meet unexpected subsoil conditions (62). The foundation plans, it was said, were a warranty to the superstructure contractor as to the anticipated time for the commencement of his work.

Engineers' Estimates.—Attempts to base claims for damages upon an engineer's inaccurate estimate of the quantity of work to be done have been generally unsuccessful. The usual contract clause stating that such estimates are "approximate only, * * * are given to form a basis of comparison for testing bids, * * * are not guaranteed to be accurate, and are not considered as a binding feature of the contract," is generally sufficient to negative the existence of the warranty.

In several cases (63) where recovery for work in excess of the quantities included in the engineers' estimate was denied, the rulings apparently were centered around the proposition that the estimates formed no part of the contract and were ineffective to limit the amount of work to be done thereunder. As a corollary to this proposition, it has frequently been held that no action can be maintained for loss of profits if the engineer has over-estimated the quantity of work to be done (64).

Implications in Specified Methods.—Closely related to the principle that an owner impliedly warrants the sufficiency of his plans and specifications to accomplish the intended result (65), was the situation presented in litigation over the construction of a sewer in Yonkers, N. Y. (66). The contractor argued that the contract contained an implied warranty that the sewer tunnel was to be constructed in "free air." The claim was based on the fact that the contract "set forth various features of the construction work required of the contractor which could be used only in a free air tunnel." Those "features" were: (1) The installation of under-drains; (2) the drilling of alignment holes; and (3) ventilation, concrete work, and care of water to be done by methods suitable only in a "free-air" job.

For 6 142 ft of 8 553 ft of tunnel, the contractor encountered a soft, water-bearing soil which required construction by compressed air. Suit was brought to recover the higher cost of doing the work in this manner.

The Court held that "the stipulation that these free air methods be used was a representation that the job was free air tunnel construction work," and that the contractor was entitled to recover. Of the contract provisions urged in defense (67), the Court stated that they could not be assumed to nullify the evidence in the plans that the contract was primarily for a free air tunnel (68); but the Court also observed that the contractor would not have been entitled to rely upon the representations if the borings, annexed to the contract, had indicated the presence of the water-soaked material. The attempt to advance this argument in defense failed because the testimony did not establish clearly what the borings really indicated.

In another case, the contractor argued that a warranty as to the character of the rock to be removed was to be inferred from the fact that the specifications provided for the use of light charges of explosives and for the completion of the entire job within six months (69); but the contention was unsuccessful. Recovery was denied. The Court held that no warranty could be implied, "particularly in view of the fact that there is no evidence indicating that by the use of more men and the employment of more equipment the task could not have been completed on time."

In litigation resulting from a contract for the construction of three giant piers in New York Harbor, the contractor argued, among other things, that a warranty as to the nature of the rock to be removed was to be inferred from the fact that vertical faces of the rock walls were to be produced by "channeling, or by drilling and broaching" (70). It was argued that a vertical face could not be produced by the methods specified unless the rock was firm and solid, and that, therefore, the contractor was justified in assuming that the rock would be of that type. The attempt to charge the City of New York for the extra expense of removing a large quantity of loose, rotten, shattered, and disintegrated rock failed. The evidence indicated that the specified methods would have been prescribed even for the kind of rock actually encountered. Therefore, the provision for "channeling or drilling and broaching" meant nothing at all with reference to character of the rock to be removed.

In another case, a contract provision that "tunneling will not be allowed without the approval of the engineer, and the method of tunneling shall be subject to his approval," was successfully used in defense of a claim for extra payment for the cost of removing an unexpectedly large quantity of rock (71). It was held that the contract clause should have warned the contractor that rock might be encountered and, accordingly, cast upon him the burden "of ascertaining for himself just the conditions to be encountered in the building of the drain or sewer."

Of course, if the contractor is left free to develop his own methods of accomplishing the result intended and is not circumscribed by specific contract requirements (72), the argument for the implication of this type of warranty cannot be made.

CONCLUSION

In the last analysis, each case, both past and future, must rest upon its own distinctive facts (73); but, in this matter, as in everything, a knowledge of what the Courts have done in similar circumstances lessens the element of uncertainty for the future. Even if it is too much to hope that risk can be eliminated entirely from sub-surface construction contracts, its existence and extent can be recognized in advance with the aid of earlier judicial decisions, to the end that costly mistakes for owner and contractor may be minimized.

APPENDIX

COURT CITATIONS AND REFERENCES

(1) "Engineering and Contracting Procedure for Foundations": Manual of Engineering Practice No. 8, Am. Soc. C. E.: (*a*) p. 15, "Compensation for Unforeseen Conditions"; (*b*) p. 4, "Contractor's Responsibility"; and (*c*) p. 12, "Borings, or Test Pits."

(2) Maney *vs.* Oklahoma City, 150 Okla. 77, 300 Pac. 642 (1931).

(3) To the same effect, see Sweeney *vs.* Jackson County, 93 Oregon 96, 178 Pac. 365, 376 (1919); Nesbett *vs.* Louisville, C. & C. R. R. Co., 2 Speers (S. C.) 697, 705 (1844); and Dickinson *vs.* City of Poughkeepsie, 75 N. Y. 65, 76 (1878).

(4) Where a contract provision is inconsistent with the plans, the former usually prevails. See Dean *vs.* Mayor, 167 N. Y. 13, 60 N. E. 236 (1901); Lentilhon *vs.* City of New York, 102 App. Div. 557, 92 N. Y. S. 897 (1905), affirmed 185 N. Y. 549, 77 N. E. 1190; Cunningham *vs.* City of New York, 39 Misc. 200, 79 N. Y. S. 401 (1902), affirmed 90 App. Div. 606, 85 N. Y. S. 1129; Barash *vs.* Board of Education, 226 App. Div. 249, 235, N. Y. S. 30 (1929), affirmed 255 N. Y. 587, 175 N. E. 324.

(5) O'Brien *vs.* City of New York, 15 N. Y. S. 520, affirmed 65 Hun 112, 19 N. Y. S. 793 (1892), affirmed 139 N. Y. 543, 35 N. E. 323, reargument denied 142 N. Y. 671, 37 N. E. 465.

(6) Odell *vs.* City of New York, 206 App. Div. 68, 200 N. Y. S. 705 (1923), affirmed 238 N. Y. 623, 144 N. E. 917. **The contract clause provided as follows: "Most of the borings were made by means of wash drills, but many were made with core drills of various types. The test pits have been left open and may be inspected. There is, however, no expressed or implied guaranty as to the accuracy of the borings and soundings, nor of any interpretation of them. Each bidder must form his own opinion of the character of the materials to be excavated, from an inspection of the ground, put his own interpretation upon the soundings and borings made by the city, and make such other investigations as he may see fit."**

(7) 233 N. Y. 177, 135 N. E. 236 (1922).

(8) Pearson & Son *vs.* State of New York, 112 Misc. 29, 182 N. Y. S. 481 (1920); Murray *vs.* State of New York, 131 Misc. 262, 226 N. Y. S. 625 (1928).

(9) Cunningham, Woodward Company *vs.* State of New York, 119 Misc. 761, 197 N. Y. S. 343 (1922); Sundstrom *vs.* State of New York, 213 N. Y. 68, 106 N. E. 924 (1914); Feeney & Sheehan Building Co. *vs.* State of New York, 120 Misc. 240, 198 N. Y. S. 83 (1923).

(10) The Arthur A. Johnson Corporation *vs.* City of New York, 95 N. Y. L. J. 3049 (1936); affirmed 251 App. Div. 811, permission to appeal denied, 276 N. Y. 688.

(11) City of Lima, Ohio *vs.* Farley, 7 Fed. (2d) 40 (C. C. A. Ohio, 1925).

(12) Bridger *vs.* Goldsmith, 143 N. Y. 424, 38 N. E. 458 (1894); McGovern *vs.* City of New York, 202 App. Div. 317, 195 N. Y. S. 925; affirmed in 235 N. Y. 275, 139 N. E. 266 (1925); Brassil *vs.* Maryland Casualty Co., 210 N. Y. 235, 104 N. E. 622 (1914) L. R. A. 1915A, 629.

(13) Christie *vs.* United States, 237 U. S. 234, 35 Sup. Ct. 565 (1914).

(14) 237 U. S., at p. 239. See also, comment on the case in Pearson & Son *vs.* State of New York, Note 8, and in Elkan *vs.* Sebastian Bridge District, Note 25.

(15) Jackson *vs.* State of New York, 210 App. Div. 115, 205 N. Y. S. 658 (1924); affirmed in 241 N. Y. 563, 150 N. E. 556 (1925).

(16) Stewart & Co., Inc. *vs.* State of New York, 121 Misc. 827, 201 N. Y. S. 334 (1923); affirmed in 218 App. Div. 810, 245 N. Y. S. 638.

(17) Passaic Valley Sewerage Comm'rs. *vs.* Holbrook, Cabot & Rollins Corporation, 6 Fed. (2d) 721 (C. C. A., 1925).

(18) The Court said: "The representations were false in the sense of being different from the conditions which, from the borings and the disclosures made in the O'Rourke and Haskins operations, the Commissioners knew to exist." (6 Fed. (2d) at p. 723.)

(19) McGovern *vs.* City of New York, 202 App. Div. 317, 195 N. Y. S. 925; affirmed in 235 N. Y. 275, 139 N. E. 266 (1925).

(20) 202 App. Div., at pp. 332–333; 195 N. Y. S., at p. 937.

(21) See Rodgers & Hagarty, Inc., *vs.* City of New York (1936), 247 App. Div. 874 (Record on Appeal, Folio 3951–3955); Irving Trust Company, as Trustee *vs.* City of New York (1937), 253 App. Div. 714 (Record on Appeal, Folio 1176–1178); Allen N. Spooner & Son Inc. *vs.* City of New York, 97 N. Y. L. J. 1439 (1937).

(22) See also Christie *vs.* U. S., Note 13; Jackson *vs.* State of New York, Note 15; Faber *vs.* City of New York, 222 N. Y. 255, 118 N. E. 609 (1908).

(23) Dunn *vs.* City of New York, 205 N. Y. 342, 98 N. E. 495 (1912); Leary *vs.* City of Watervliet, 222 N. Y. 337, 118 N. E. 849 (1918); Semper *vs.* Duffy, 227 N. Y. 151, 124 N. E. 743 (1919).

(24) Groton Bridge & Mfg. Co. *vs.* Alabama & V. Ry. Co., 80 Miss. 162, 31 So. 739 (1902); Pope *vs.* United States, 76 Ct. Cl. (U. S.) 64 (1932).

(25) Elkan *vs.* Sebastian Bridge District, 291 Fed. 532 (1923). The Court said: "Of course any one would realize that the actual subsoil conditions

might, except where and to the depth shown by the borings, be different than so shown. The actual conditions were hidden. The borings were merely indications, at certain places and to certain depths, from which deductions might be drawn as to actual conditions along the line and to the depths of such borings. Both parties knew that deductions so drawn might prove untrue when the necessary excavations were made."

(26) The Court said: "The question here is who took the risk of these deductions? Either the district or the contractor must take this risk. Either could legally do so. Construction of the contract in the light of applicable legal principles must determine which did. That the district made known to Elkan the result of the borings is certain! This was done with one of two purposes: (1) To be the statement of a fact (subsoil conditions to be encountered) upon which he might rely and act; or (2) as useful information which he might accept as sufficient or not as he thought wise, leaving him to make such further and other investigations as he might wish." (291 Fed., at p. 538.)

(27) The Court said: "The bare statement that the boring sheet may be relied upon as accurate is entirely different from saying that the sub-soil along the bridge line is as shown by the boring sheet. One is a statement that certain evidence of an ultimate fact exists; the other is a statement that the ultimate fact exists. The statement here made goes, in effect, little further than if the boring sheet had, without comment, been furnished the bidder." (291 Fed., at p. 540.)

(28) The exact wording was as follows: "The excavation of this borrow-pit shall be carried to Elevation 655 D. and I. Datum, or to blue clay, which test borings show to be approximately fifteen (15′) feet below the surface. If the blue clay is above Elevation 655, none of the blue clay shall be excavated or placed in the embankment." (Detroit & I. Ry. Co. *vs.* Guthrie & Co., Inc., 72 Fed. (2d) 126 (C. C. A. 1934).)

(29) The Court wrote: "The only representation of fact made therein was that test borings had been made, and that they showed blue clay approximately 15 feet below the surface. The issue to be submitted to the jury was not whether there was blue clay nearer the surface than fifteen feet, but whether the representation that borings had been made and they showed no blue clay less than fifteen feet below the surface was true or untrue." (72 Fed. (2d), at p. 129.)

(30) The contract provided: "From borings and test pits, made at the site, it is assumed that rock will be found approximately as indicated on the drawings, but the United States does not guarantee the nature of materials to be encountered in the river bed, the correctness of the depth of rock as assumed or shown on the drawings, the depth to which it may be necessary to excavate, nor that the bottom of the river will not be changed before or after commencement of work." (Bates & Rogers Construction Co. *vs.* United States, 56 Ct. Cl. (U. S.) 49 (1921).)

(31) The Court said: "It would be extending the rule further than the adjudicated cases have gone, and further than we are prepared to go, to say that the Government can be held, as upon a warranty, of a condition as to which it makes no representation * * * when all it does is to furnish drawings which accurately state all they purport to show, notwithstanding it fails to state to the bidder that a hole was bored somewhere in or about the large area involved at some other time which disclosed that there was a log at the bottom of the Ohio River." (Bates & Rogers Construction Co. *vs.* United States, 56 Ct. Cl. (U. S.), at p. 60.)

(32) United Construction Co. *vs.* St. Louis, 334 Mo. 1006, 69 S. W. (2d) 639 (1934).

(33) Kelly *vs.* City of New York, 87 App. Div. 299, 84 N. Y. S. 349 (1903); affirmed 180 N. Y. 507, 72 N. E. 1144.

(34) Hill *vs.* City of Beaumont, Texas, 5 S. W. (2d) 590 (1928); cited in dissenting opinion in City of Dallas, Texas *vs.* Shortall, 87 S. W. (2d) 844 (1935). See, also, Jahn Contracting Co. *vs.* City of Seattle, 100 Wash. 166, 170 Pac. 549 (1918).

(35) Conners *vs.* U. S. 130 Fed. 609 (1904); affirmed in 141 Fed. 16 (1905); see, also, Pawling *vs.* United States, 62 Ct. Cl. (U. S.) 123 (1926) and Cauldwell-Wingate Company *vs.* State of New York, 276 N. Y. 365 (1938).

(36) City of Richmond *vs.* I. J. Smith & Co., 119 Va. 198, 89 S. E. 123 (1916).

(37) Pitt. Const. Co. *vs.* City of Alliance, Ohio, 12 Fed. (2d) 28 (C. C. A. 1926).

(38) United Construction Co. *vs.* Haverhill, 9 Fed. (2d) 538 (C. C. A. 1925); see, also, same case in 22 Fed. (2d) 256 (C. C. A. 1927).

(39) Maryland Casualty Co. *vs.* Board of Water Commissioners, 43 Fed. (2d) 418 (Dist. Ct. 1930).

(40) Leary *vs.* City of Watervliet, 222 N. Y. 337, 118 N. E. 849 (1918).

(41) To the same effect see Borough Construction Co. *vs.* City of New York, 200 N. Y. 149, 93 N. E. 480 (1910); Simpson *vs.* United States, 172 U. S. 372, 19 Sup. Ct. 222 (1899); Marshall Construction Co., Inc. *vs.* State of New York, 133 Misc. 131, 231 N. Y. S. 345 (1928).

(42) Foundation Company *vs.* State of New York, 233 N. Y. 177, 135 N. E. 236 (1922).

(43) 233 N. Y., at pp. 184, 185; 135 N. E., at p. 238.

(44) Faber *vs.* City of New York, 222 N. Y. 255, 118 N. E. 609 (1918).

(45) Hollerbach *vs.* United States, 233 U. S. 165, 34 Sup. Ct. 553 (1913).

(46) Delafield *vs.* Village of Westfield, 77 Hun (N. Y.) 124, 28 N. Y. S. 440 (1894).

(47) United States *vs.* Smith, 256 U. S. 11, 41 Sup. Ct. 413 (1921).

(48) Kuhs *vs.* Flower City Tissue Mills Co., 104 Misc. 243, 171 N. Y. S. 688 (1918); modified in 189 App. Div. 539, 179 N. Y. S. 450.

(49) Matter of Semper *vs.* Duffy, 227 N. Y. 151, 124 N. E. 743 (1919); see also, Gross & Son *vs.* State of New York, 214 App. Div. 386, 212 N. Y. S. 222 (1925); Brennan Construction Company *vs.* State of New York, 117 Misc. 816, 191 N. Y. S. 253 (1921).

(50) Atlanta Construction Co. *vs.* State of New York, 103 Misc. 233, 175 N. Y. S. 453 (1918).

(51) United States *vs.* Atlantic Dredging Co., 253 U. S. 1, 40 Sup. Ct. 423 (1930).

(52) Nounnan *vs.* Sutter County Land Co., 81 Cal. 1, 22 Pac. 515 (1889); see also, Foundation Company *vs.* State of New York, 233 N. Y., at p. 185; and, more recently, T. J. W. Corporation *vs.* Board of Higher Education of the City of New York, 251 App. Div. 405; affirmed 276 N. Y. 644 (1937).

(53) Cunningham *vs.* City of New York, 39 Misc. 197, 79 N. Y. S. 401 (1902); affirmed 90 App. Div. 606, 85 N. Y. S. 1129.

(54) 39 Misc., at p. 201; 79 N. Y. S., at pp. 403–404.

(55) Thileman *vs.* City of New York, 82 App. Div. 136, 81 N. Y. S. 773 (1903); see, also, Des Moines Plumbing & Heating Co. *vs.* Margarian, 201 Iowa 647, 207 N. W. 750 (1926).

(56) Lentilhon *vs.* City of New York, 102 App. Div. 548, 92 N. Y. S. 897 (1905); affirmed 185 N. Y. 549, 77 N. E. 1190.

(57) The Court said: "The plan was designed to indicate the location of the walls that were to be removed and the levels and extent of excavation and the levels to which hollows were to be filled; but we think it was not intended as a basis upon which bidders were to figure the quantities of material to be removed, and that the express provisions of the contract and specifications requiring the removal of the entire reservoir structure were controlling." (102 App. Div., at p. 557; 92 N. Y. S., at pp. 901–902.)

(58) Odell *vs.* City of New York, 206 App. Div. 68, 200 N. Y. S. 705 (1923); affirmed 238 N. Y. 623, 144 N. E. 917.

(59) The Court declared: "Thus it will be seen that it was expected that water would be encountered but no one attempted to approximate the quantity. * * * Apparently the amount of water encountered was an element unforeseen by all parties, and, while one may regret the additional burden placed on the contractor, that burden arises through no misrepresentation on the part of the city, and he must bear it alone as best he may. Everything that could be done to put bidders on their guard as to the difficulties and dangers of this situation was done." (206 App. Div., at pp. 92, 93; 200 N. Y. S., at pp. 725–726.)

(60) Dunn *vs.* City of New York, 205 N. Y. 342, 98 N. E. 495 (1912).

(61) To the same effect see MacArthur Bros. Co. *vs.* United States, 258 U. S. 6, 42 Sup. Ct. 225 (1922).

(62) Cauldwell-Wingate Company *vs.* State of New York, 276 N. Y. 365 (1938).

(63) Sullivan *vs.* Sing Sing, 122 N. Y. 389, 25 N. E. 366 (1890); Molloy *vs.* Briarcliff Manor, 145 App. Div. 483, 129 N. Y. Supp. 929 (1911); Weston *vs.* State of New York, 262 N. Y. 46, 186 N. E. 197 (1933). Cf. Long *vs.* Inhabitants of Athol, 196 Mass. 497, 82 N. E. 665 (1907).

(64) Litchfield Construction Company *vs.* City of New York, 244 N. Y. 251, 155 N. E. 116 (1926); Kinser Construction Co. *vs.* State of New York, 204 N. Y. 381, 97 N. E. 871 (1912); Matter of Merriam, 84 N. Y. 596 (1881). Cf. Del Balso Construction Corporation *vs.* City of New York, 278 N. Y. 154 (1938).

(65) United States *vs.* Spearin, 248 U. S. 132, 39 Sup. Ct. 59 (1918); MacKnight Flintic Stone Co. *vs.* Mayor, 160 N. Y. 72, 54 N. E. 661 (1899); Filbert *vs.* Philadelphia, 181 Pa. 530, 37 Atl. 545 (1897); Sundstrom *vs.* State of New York, 213 N. Y. 68, 106 N. E. 924 (1914).

(66) Montrose Contracting Co., Inc. *vs.* County of Westchester, 80 Fed. (2d) 841 (C. C. A. 1936).

(67) The principal clauses relied on read as follows: "That where material other than solid rock is encountered, containing loose sand or mud, which shifts with the water contained in it, and the Contractor chooses to use compressed air to pass through it, in order to partly relieve him of the additional cost of preventing and caring for the incoming water

and material or restraining it, an allowance will be made throughout the stretch where the compressed air is actually used under the above conditions of $30.00 per foot and for not more than six hundred (600) feet of length, where actually used, regardless of whether more be used or not."

"Maximum partial compensation for labor, plant and materials under compressed air when and if actually used as specified, but not in excess of 600 ft in length and Thirty and no/100 dollars ($30.00) per foot."

(68) The Court said that the contract provisions: "* * * cannot be given the effect of completely negating the representations in the plans that the tunnel was primarily a free air tunnel. They might have been effective to limit appellee's liability if the work had required somewhat more than 600 ft. of compressed air, but in point of fact, there was 6,142 ft. to be so built. These clauses cover a variation, not a transformation." (80 Fed. (2d), at p. 843.)

(69) Golden & Son, Inc. *vs.* Marblehead, 68 Fed. (2d) 875 (C. C. A. 1934).

(70) Allen N. Spooner & Son, Inc. *vs.* City of New York, 97 N. Y. L. J. 1439 (1937).

(71) Matter of Keep, 138 Misc. 194, 245 N. Y. Supp. 321 (1930); reversed on other grounds, 237 App. Div. 377, 260 N. Y. Supp. 237; affirmed in 262 N. Y. 596, 188 N. E. 80.

(72) Meads & Co. *vs.* City of New York, 191 App. Div. 365, 181 N. Y. Supp. 704 (1920).

(73) Cauldwell-Wingate Company *vs.* State of New York, 277 N. Y. 365, at p. 377.

(74) The Circuit Judge said, in his decision favorable to the contractor: "I am led to believe that these prior investigations could have been obtained by the defendants and, in every sense of the word, should rightly have been in the possession of the complainants so they might be correctly guided in their proposal and estimates on the material to be handled. The suppression of all such prior information unquestionably worked a hardship, if not a fraud, on the complainant." (Citation Vol. 89, Federal Reporter 251.)

(75) The Court of Appeals reversed the lower Court, stating: "Trustees are not bound to exercise diligence to obtain information concerning the nature or cost of work for the benefit of bidders with whom they deal at arms length * * * and a contractor, for the excavation of a section of the drainage canal, is not entitled to a rescission of his contract because he encountered a substance more difficult and expensive to excavate than anything he was led to expect from an examination of profile and data in the office of the Chief Engineer."

Furthermore, there was "No provision in statute * * * which required that information concerning the nature of the materials to be excavated should be collected for the benefit of the bidders, and that such duty on the general principles of law or equity there is no foundation in authority or reason." (Citation 91, Federal Reporter No. 511, February 7, 1899.)

(76) The Court declared as follows: "On this appeal, defendants strenuously contend that the evidence shows that plaintiff drilled the greater portion of the time through shale and that the terms 'rock' and 'rock formations' do not include shale. To support this contention, defendants pro-

duced a petroleum engineer who testified that technically rock includes 'shale' or 'stone,' but in the oil field parlance the term refers to hard formations, usually in large bodies as, for instance, 'granite,' and further that 'shale' is not considered rock by oil people. The trial court interpreted the term used in the contract in its general sense and refused to interpret it in the limited fashion suggested by the expert. We agree with the interpretation of the trial court * * * the term 'shale' is defined as rock formed by the consolidation of clay, mud or silt, having a finely stratified or laminated structure * *, *. Under the circumstances, the general and popular meaning of the term is reasonably intended."

(77) White *vs.* United States, 241 U. S. 149.

(78) L. P. & J. A. Smith *vs.* United States, 256 U. S.

(79) St. George Contracting Co. *vs.* City of New York, 205 N. Y. 121.

(80) 80 Fed. (2d) 841.

(81) 94 Fed. (2d) 580.

(82) The contract contained the following provisions concerning the use of compressed air:

"The Contractor hereby agrees to receive and accept as and for his full compensation for furnishing all the labor, plant and materials for, and building, constructing and fully completing and maintaining for 12 months all the work called for under this contract and for all the labor, plant and materials incidental thereto as required by the specifications and as shown on the plans of the work referred to therein, the following prices:

78-inch sewer in tunnel complete as indicated on the plans and specifications of this Contract, including shafts, portals, boat hole, manholes, alinement holes, sewer connections and all their appurtenances; except only any partial compensation for compressed air if used as listed in item 2 below; @ Nine hundred sixty-seven thousand and 00/100 dollars.

($967,000.00) lump sum Amount $967,000.00

Maximum partial compensation for labor, plant and materials under compressed air when and if actually used as specified but not in excess of 600 feet in length of tunnel @ Thirty and no/100 dollars.

($30.00) per foot Amount $18,000.00

Total sum bid in words Nine hundred Eighty-five thousand and 00/100 dollars.

Total Amount $985,000.00

"The above total includes all excavation, backfilling, waterproofing, masonry, concrete, reinforcement, forms, pipe, underdrain, fittings, sheeting, shoring, forepoling, timbering, piling, foundations, testing, maintenance, labor, plant, materials, insurance and all other things shown, specified or necessary for the full, watertight and enduring completion of the entire work.

* * * *

"That where material other than solid rock is encountered, containing loose sand or mud, which shifts with water contained in it, and the Contractor chooses to use compressed air to pass through it; in order to partly relieve him of the additional cost of preventing and caring for the incoming water and material or restraining it, an allowance will be made throughout the stretch where compressed air is actually used under the above conditions of $30.00 per foot and for not more than 600 feet of length, where actually used, regardless of whether more be used or not."

(83) 191 App. Div. 205, 181 N. Y. Supp. 137, aff'd 231 N. Y. 564, 132 N.E. 890.

DISCUSSION

Alonzo J. Hammond,[3] **Past-President, Am. Soc. C. E.** (by letter).—In 1934 a Committee of the Construction Division of the Society, on Substructure Engineering, presented a report which, subsequently, was published as Manual of Engineering Practice No. 8. The object of this work was:[4]

> "* * * to establish fair and equitable practice, and to correct undesirable procedures and abuses in the relations between owners, engineers, and contractors concerned with the design and construction of bridge foundations."

It is gratifying to note that the author credits the manual of practice with eliminating "much of the costly uncertainty involved in underground construction, with corresponding benefit to owner and contractor" (see "Synopsis").

He emphasizes the need of carefully worded specifications, but this does not carry with it the placing of all the burden on the contractor, as has been clearly stated. It is now considered best practice to give the engineer means and authority to obtain reasonably full and complete information of subsoil conditions, and to be responsible for such data.

In 1912, while conducting studies preliminary to the design of the Lake Street double-deck bascule bridge for the City of Chicago, Ill., the writer found information of borings made on each side of the river, about 250 ft apart, showing rock at Elevation −75.0 ft. That rock should be at such an exact uniformity of elevation raised a doubt in his mind as to whether it was bed-rock. New borings were ordered and confirmed the occurrence of rock at Elevation −75.0, but only 4 in. thick; so the driller continued down and found the bed-rock at Elevation −110.0.

The author has cited a number of border-line cases and the effect, on the Court, of the evidence presented by the engineer, as illustrated in the Fort Smith case (25). It seems reasonable to reach the conclusion that the engineer must set himself up primarily as an impartial judge, fair to both owner and contractor; otherwise a contestant does not reach conclusive knowledge of the law in the case until he reaches the Court of last resort, and it may not be on equity then.

To cite, briefly, a case in the Chicago district: The contractor was given certain information, but there were some additional borings referring to a material as "very hard," which borings were not shown. About 12 ft below the surface, the contractor found conglomerate rock, formed of boulders and clay cemented together, which had to be drilled and blasted and broken up before it could be shoveled. A favorable decision by the Circuit Court (74) in this case was later reversed by the Court of Appeals (75).

Engineers, generally, would consider the decision of the Circuit Judge in this case as representative of good law and common sense as against the decision of the Appellate Court.

---

[3] Cons. Engr., Chicago, Ill.

[4] "Engineering and Contracting Procedure for Foundations," *Manual No. 8*, Am. Soc. C. E., 1934, p. 3.

A contrary case of a decision of a lower Court being confirmed by a higher Court on a rule of reason is that of Schiltz *vs.* Akers, *et al.*[5] Action was brought by the plaintiff to recover the sum of $7 550, alleged to be due from defendants under the terms of a written contract for drilling of oil wells. The contract provided, among other things, "that in the event [and this is the important feature of the contract] rock is encountered in drilling any of said wells 'plaintiff agrees to drill one foot through the same without extra compensation, but beyond that defendants agree to pay $75.00 per day extra compensation for each day of nine hours required in drilling through all rock formations.'" The lower Court ruled in favor of the plaintiff, and the decision was confirmed by the Supreme Court (76).

As a young man the writer had a feeling of awe for the Supreme Court, until a very able lawyer tempered it by the remark that, although law in itself was an exact science, the interpretation by the Supreme Court was by some very human individuals. Contrary to the legal technicalities involved in some of the cases mentioned in the paper, the writer can cite the following as a common sense solution of a very difficult problem: On the main drainage canal from Chicago to Lockport, Ill., one firm of contractors had Sections 2, 3, and 4 at Lemont, Ill., the original contract calling for 27 cents and 28 cents per cu yd for glacial drift. The indurated clay was found to be almost as difficult to excavate as rock; and, therefore, by mutual consent, the contract was readvertised and the bids of four contractors ranged from 59 cents to 75 cents per cu yd for the same sections. The Chicago Sanitary District, however, finally made a new contract with the original contractor on the basis of 50 cents and 49 cents for Sections 2 and 4, respectively. There were, perhaps, two valid reasons for a renewal of the contract on the terms mentioned. The Trustees, no doubt, were willing for the contractors to recoup some of their losses, whereas the contractors, having their plant on the ground, were willing to make what appears as a low price compared to the bids of the other contractors.

The author has presented a most valuable list of case material for study by every engineer and contractor engaged in sub-surface work.

Frederick W. Newton,[6] **Assoc. M. Am. Soc. C. E.** (by letter).—An ideal toward which the engineer should strive in the preparation of a construction contract. The engineer is required to work with the tools at his disposal, however, and too frequently these do not include the complete investigation of sub-surface conditions which he would like to have. The complete investigation that should be made by an engineer before advertising a contract means more than simply making such borings and soundings as are necessary to determine the stability of the structure. It is recognized that in some cases such complete investigation is impossible.

The value in Mr. Herwitz's paper is that it helps to show how, in preparing a contract, the engineer may make the best use of such information as is available to him. The paper places too much emphasis on the wording of specifications, however (particularly the so-called "exonerating" clauses), and not enough on the facts. The statement in the "Synopsis" that "Experience in this type of litigation shows that emphasis is placed on carefully worded specifications, often with results expensively significant to one party or the other" is, to say the least, subject to question. Mr. Herwitz presents many instances where, in two or more cases involving contracts with the same or similar specifications, the result has been that in one the contractor was awarded increased compensation and in others the increase was denied.

---

[5] Supreme Court of California, Vol. 292, Pacific Reporter 463, October 20, 1930.

[6] Attorney and Counsellor at Law (Wait, Wilson & Newton), New York, N. Y.

The fact is that the Courts have endeavored to decide each case on its merits. In practice, a case is first decided and then an opinion is written explaining the decision. In most of the cases the exonerating clauses have been urged as a defense. Hence, these clauses are discussed in the opinions of the Court but, with the exception of a few cases in which there have been provisions of the contract or specifications relating to the particular matter in suit, it will be found that the facts, not the contract clauses, form the real basis of the decision.

The author recognizes this fact in his "Conclusion." He states that, in the last analysis, each case must rest upon its own distinctive facts. That being so, a discussion of this subject should direct attention to the facts that have led to litigation in the past, in order that a repetition of those facts may be avoided in the future. No exonerating clause can be drawn which will avoid the result of carelessness, negligence, or deception in the preparation of a contract.

Contract Should Describe Work Correctly.—The first and most important rule for the engineer who is preparing a contract is that the work to be done by the contractor must be described, clearly and accurately, in the contract. A bidder looks first at the "description of the work" and next at the bidding sheet. He makes a proposal for doing the work which the contract describes. If he is required to do more expensive work, he is usually entitled to increased compensation.

Where the contract described the work as "earth excavation," the contractor was entitled to additional compensation for excavating rock, Maney *vs.* Oklahoma City (2); a contract that called for the excavation of earth, hardpan, rock, etc., and defined each of these materials, did not include the excavation of adobe or gumbo where it was shown that these materials were not included in the definitions, Sweeney *vs.* Jackson County (3); a contractor who had undertaken to construct a tunnel in rock was entitled to additional compensation for a part of the tunnel that was in loose earth and water and required the use of compressed air, McGovern *vs.* City of New York (19); a contractor who had agreed to construct a tunnel in free air received increased compensation because he had to construct it in compressed air, Montrose Contracting Company, Inc. *vs.* County of Westchester (66); and, a contractor who had undertaken to construct a tunnel in solid rock with gunite lining received additional compensation because he was required to excavate the tunnel in loose rock and to place a monolithic concrete lining, City of Dallas *vs.* Shortall (34).

The cases in which the contractor has received additional compensation are not the only ones that show the importance of a clear and accurate description of the contract work. Other cases in which the owner has ultimately succeeded are equally persuasive because, if the engineer had been sufficiently careful in the preparation of the contract, the owner would have been saved the annoyance and expense of long litigation. In the case of Dean *vs.* Mayor of New York (4) the contract described the work as the grading of Dyckman Street from the Hudson River to Exterior Street. The drawings showed only a part of that work. It required litigation that went to the Court of Appeals to determine just what work the contractor was required to do. Dickinson *vs.* City of Poughkeepsie (3) is one of many cases that involve the question of whether a contractor for earth excavation is required to excavate hardpan. This difficulty may be overcome by taking bids for all excavation without classification, or by a sufficiently broad definition of "earth excavation," or by requiring that the contractor bid a separate unit price for hardpan excavation. In Barash *vs.* Board of Education (4) there would have been no lawsuit if the architect had been more careful to specify, fairly, the work that the contractor was required to do. The contract required the installation of Kalameined windows in all openings from a light shaft in an existing building. It was not clear whether this referred only to openings that the contractor was to make or to all openings, including those which already had windows.

Bidder Must Inspect Site.—The foregoing does not mean that a bidder may accept the description contained in the contract and "close his eyes" to facts that he knows or should know and which would demonstrate that the description is inaccurate. In Lentilhon *vs.* City of New York (4) the contractor undertook to remove the embankment forming the old reservoir at 42nd Street and Fifth Avenue, the site later occupied by the New York Public Library. He figured the contract on the basis of the slopes of the embankment shown on a plan which was not made from survey and not drawn to scale. He did not attempt to ascertain what the actual slope was although he might easily have done so. In Jahn Contracting Company *vs.* City of Seattle (34) a contractor for a sea-wall blindly relied upon a drawing made long before the grading of adjacent streets. A mere walking over the job would have shown that at the time of bidding the streets had been graded and that the line on the plan was wrong.

Representations Concerning Conditions Affecting Work.—After the contract provisions describing the work to be done by the contractor, the next subject is the information given to bidders concerning conditions that may affect the cost of the work. Such information may relate to subsoil conditions determined by soundings or borings, or to the sources of supply of materials, or to other conditions affecting the particular job.

It is recognized that, in general, the giving of information to bidders results in lower bids for the contract work because if bidders do not have the information, they are certain to make liberal allowances for contingencies. It is recognized, also, that engineers are subject to the temptation to make the contract appear attractive to bidders. In facing this situation the Courts have adopted certain rules which are undoubtedly fair and equitable. The difficulty arises in the application of these rules to the great variety of facts presented.

Borings.—It will probably be most helpful to discuss first the matter of borings. If borings have been made, bidders for the work should have the advantage of such information as those borings disclose. The question for the engineer is how this information should be given to bidders.

When engineers use borings, they draw lines from one boring to another and assume that between the borings the material will be substantially the same as that indicated by the borings. The Courts have recognized that this assumption is not always in accord with the fact. The Court ruled in Elkan *vs.* Sebastian Bridge District (25) that the giving of borings is not a warranty of the condition of the subsoil between the borings; and that all the owner warrants is that the borings have been correctly shown. In Groton Bridge and Manufacturing Company *vs.* Alabama and Vicksburg Railway Company (24) the contractor for the construction of bridge piers complained that the material encountered was unlike the material shown by borings which had been made near but outside the location of the piers. The borings had been plotted correctly, but there was a dip in the strata between the borings. The contractor did not recover.

In Detroit & I. Ry. Co. *vs.* A. Guthrie and Company, Inc. (28) the specifications stated that the borings showed blue clay in a borrow-pit 15 ft below the surface. The contractor found it much closer than that to the surface, but he failed to show that the borings were in error or that at the site of the borings the clay was less than 15 ft from the surface. The Court ruled that the fact that clay was found near the surface was some evidence that the borings were erroneous, but that, standing alone, such evidence was not sufficient.

The engineer may safely give to bidders all of the boring information in his possession, provided it is given accurately and without concealment. Among the cases cited by Mr. Herwitz in which this had been done and in which the contractors were not allowed to recover increased compensation are Foundation Company *vs.* State of New York (7); Pearson & Son Inc. *vs.* State of New York (8); and The Arthur A. Johnson Corporation *vs.* City of New York (10). A similar situation is presented in Murray *vs.* State of New York (8) but with the added fact that by reason of prior contracts in the same vicinity the contractor was entirely familiar with conditions and was not in any way misled by the borings. Irving Trust Company, as Trustee, *vs.* City of New York (21) presented an instance in which the boring information had been given to the bidder exactly as it was received from the boring contractor. The borings were inaccurate but the Court found that there was no proof that the city was aware of any condition different from what the borings showed, and disallowed the contractor's claim.

A clear illustration of the advantage of telling bidders just what preliminary investigation has been made is found in Kelly *vs.* City of New York (33) in which the contractor was denied a recovery notwithstanding the fact that the "probable surface of rock" shown on the drawings was incorrect. The contract described, correctly, the preliminary investigation that had been made, stating that some rod soundings had been made and the results were shown on the plan but were not warranted to be correct. This told the contractor exactly what had been done. The earth cover was not deep and he could have checked the soundings if he had cared to do so.

An engineer who furnishes borings or boring information to a bidder must exercise care that he furnish all of the information without distortion or concealment. If there has been distortion or concealment by which the contractor has been damaged, he is entitled to recover his damages. Cases of this kind are Christie *vs.* United States (13); United States *vs.* Atlantic Dredging Company (51); Jackson *vs.* State of New York (15); Stewart and Company, Inc. *vs.* State of New York (16). In the Christie case (13) the engineer who recorded the results of the borings had neglected to record logs and stumps encountered, which he considered of no importance. In the Jackson case (15) the State showed the results of one set of borings, but failed to show a set made some years earlier which indicated hard material. In the Atlantic Dredging Company case (51) the borings were given correctly but they did not penetrate to the full depth of the excavation required; and, although the engineer knew that the reason for not carrying the borings deeper was that they had encountered hard material, he nevertheless put in the contract the statement that the material was believed to be mainly mud, or mud with an admixture of fine sand. In the Stewart and Company case (16) the State gave information indicating soft material but withheld other information indicating hard material.

In these cases the giving of erroneous information or the withholding of information was not necessarily done intentionally or with fraudulent motive. In fact, in the Christie case (13) there was no question that the engineer honestly believed that the information concerning the logs and stumps was of no importance.

Statements Concerning Source of Supply of Materials.—The cases relating to the source of supply of materials for highway work show again that the decisions are based on the facts of each case rather than on exonerating clauses of the contract.

In Gross and Son *vs.* State of New York (49) the representation was made on the contract drawings that stone would be found in a ledge near a certain station. By a provision of the contract the contractor warranted that he had personally investigated the local sources of supply. He had made no such investigation and when he went to look for the stone after the contract was made he found a level field with no sign of stone. He wrote to the engineer that he had "assumed" that the information on the plan was correct. He had no right to make such assumption in the face of the provision which required that he personally investigate that particular matter.

In Atlanta Construction Company *vs.* State of New York (50) the contract contained a similar statement and the contractor recovered additional compensation because afterward it developed that the land on which the stone was located was in the possession of a life tenant who did not have the right to sell the stone. The difference between this case and the Gross case (49) is that in this case the bidder had done what the contract required; he had investigated and had found the stone. The Court found that there was a mutual mistake but one for which the State was responsible by reason of its direction and statement in the contract.

Semper *vs.* Duffy (23) and Brennan Construction Company *vs.* State of New York (49) are cases in which the State merely gave to bidders the reports of the probable location of materials, made by its field engineers to their superiors. The Court said that such reports were "suggestive merely, directing him (the contractor) for inquiries to what the State considered the nearest available source of supply."

An instance of a special provision that proved a bar to a recovery by a sub-contractor occurred in Kuhs *vs.* Flower City Tissue Mills Company (48). The contract provided that good gravel would be found in the excavation, but if sufficient gravel were not found, the sub-contractor should furnish gravel of equal quality. There was some gravel but not enough. In obtaining gravel from other sources the sub-contractor had done no more than he had agreed to do.

Other Cases Involving Representations.—Other cases in the group characterized by lawyers as "breach of warranty" cases show the necessity for both care and good faith in the making of statements or representations which may affect the cost of the work.

The leading case is Hollerbach *vs.* United States (45). The facts are stated in Mr. Herwitz's paper. The lesson for engineers is that no general exonerating clause will overcome the effect of a positive statement negligently or carelessly made in a contract. Another application of the rule is found in Delafield *vs.* Village of Westfield (46).

In the two foregoing cases the engineers did not know that their statements were wrong; but the statements had been made carelessly. Passaic Valley Sewerage Commissioners *vs.* Holbrook, Cabot and Rollins Corporation (17), the facts of which are stated by Mr. Herwitz, presents a situation in which the engineers did know that the borings did not disclose the true condition. They knew that the presence of the cemented formation was essential to the prosecution of the work; and, they knew that in the two prior attempts to build the tunnel this formation had not been found. Their attempt to shift the burden to the contractor received from the Courts the treatment which it deserved.

Representations as to Ground Surface Elevations.—There are two cases in which the facts are very similar; in one the contractor recovered; in the other the architect and Board of Education felt that the contractor should recover, but they were overruled by the Courts. The first of these cases is Pitt Construction Company *vs.* City of Alliance, Ohio (37) in which the Court held: That the contractor was entitled to use the ground-surface elevations given on a detailed plan for the work; that he was not required to make a survey to determine that those elevations were correct; and, when it was found that they were wrong, that he should have additional compensation. In the other case, T. J. W. Corporation *vs.* Board of Higher Education of the City of New York (52) there was a similar error in the elevation of ground surface given on the plans; but here the Court ruled that the contractor could not recover. The reason for the difference in the two decisions is that in the

Pitt Construction Company case the contract contained no reference about checking the elevations although it did contain the usual clause in regard to the contractor's inspection of the site. In the T. J. W. Corporation case there was an express provision in the contract which required the contractor to check the grades.

A similar result was reached in Conners *vs.* United States (35) where the contract showed approximate rock levels but contained a provision that the bidder must satisfy himself as to the accuracy of these levels and as to rock surfaces in general.

Equality of Opportunity.—Throughout many of the cases on this subject there runs the question of "equality of opportunity." If conditions are such that the bidder's opportunity to know them is as good as that of the owner, the Courts hold that the bidder should have taken advantage of that opportunity.

In Odell *vs.* City of New York (6) the contractor complained of an excess quantity of water encountered in the construction of a part of the New York City Aqueduct. The Court found that not only was there no representation of the quantity of water that would be encountered, but also that at the time of bidding, test pits along the line were open and the contractor had as much opportunity as the City to estimate the quantity of water. It did not appear that either party had made any attempt to measure the flow. In Faber *vs.* City of New York (22), on the other hand, the Court held the City responsible for an error in the elevation of rock, as shown by its borings, on the ground that the rock surface was far below the bottom of the river and that the contractor could not have been expected to make borings.

It should be noted, however, that a provision that the contractor must inspect the site does not require that he make borings or other investigations to determine that positive information given him in the contract is correct. For example, in the Hollerbach case (45) the court remarked that the contractor might have ascertained the true nature of the material back of the dam by the simple device of making rod soundings, but in view of the positive statement in the contract, he was not required to do so.

The rule seems to be that a contractor may rely upon the accuracy of the grades, including rock-surface elevations, given him in the contract; but if the contract expressly requires that he check such grades or elevations, and if the conditions are such that he might reasonably have done so before bidding but he has failed to do so, he should not complain if it is afterward found that the grades or elevations were in error. It is understood, of course, that this statement is not applicable in a case that involves deception. The author has called attention to the rule that no man may contract against his own fraud, City of Lima, Ohio *vs.* Farley (11), Bridger *vs.* Goldsmith (12).

Prior Work on Same Site.—The question sometimes arises as to the right of the contractor to rely upon information obtained from prior contracts. Dunn *vs.* City of New York (23) shows that a bidder, who looks at a prior contract and assumes without investigation that the work required by that contract has been fully performed, does so at his own risk. In that case a paving contractor assumed that the grading had been done under a prior contract. His own contract required that he do grading and he could easily have ascertained whether or not any grading was necessary. The Court held that the specifications of the prior contract, which were not made a part of his contract, did not constitute a representation on which he could rely.

In Cauldwell-Wingate Company *vs.* State of New York (35) the specifications for the prior contract were referred to in the claimant's contract. The contract was one to build the superstructure of the State Office Building in New York City. The prior contract was one to build the foundations. Because of unexpected sub-surface conditions, additional work requiring nearly a year for its performance was required of the foundation contractor, and this delay increased the cost of the superstructure. The contractor for the superstructure recovered, notwithstanding the contract provision that he should not be entitled to damages for any delay from any cause, including the act or neglect of the State.

Cases in Which There Was No Representation.—To establish a claim for increased compensation on account of an alleged breach of warranty the contractor must show that information has been given to him upon which he relied in making his bid and that such information was erroneous. He must show also that he had a right to rely on the information. The claims of the contractors in several of the cases cited by Mr. Herwitz failed because no representation had been made.

An example of this is Allen N. Spooner and Son Inc. *vs.* City of New York (21) in which the contractor complained that he had encountered loose rock in the construction of a pier. The City had shown two typical cross-sections, one for solid rock and one for loose rock, and had stated in the contract that no core borings had been made.

In another case, Maryland Casualty Company *vs.* Board of Water Commissioners (39), rock grades were shown on the plan but specifications required that the excavation be carried to the grade shown or to such increased depths as may be necessary to obtain a firm foundation. The contract contained a separate unit price for extra excavation below the grade shown, and this price had been paid to the contractor. The Court ruled that this negatived the idea of a representation that the rock would be at the grade shown, and that the contractor had received all of the compensation to which he was entitled.

In Simpson *vs.* United States (41) the work was done according to plans which the contractor had himself prepared without any information whatever from the United States as to sub-surface conditions. It was his responsibility, not that of the United States, that the design was not suited to the conditions which existed on the site.

Weston *vs.* State of New York (63) was a case in which the contractor claimed increased compensation because he was required to excavate more rock than was indicated in the engineer's estimate. He had been paid the rock-excavation price for all rock removed. The Court ruled that as there was no claim of deception, inequality, or inequity, the contractor was not entitled to increased compensation.

The Weston case (63) presents the question of the contractor's rights under a unit-price contract where there has been an increase in the quantity of one item. In the usual case such increase does not entitle the contractor to any payment beyond the unit price bid; but this rule is not applied if the increase is so great as to change, radically, the nature and cost of the contract as a whole.

Specifications Should Describe Finished Work, Not Methods of Construction.—The aforementioned cases have dealt with representations as to the work to be done or as to conditions affecting the work. There is another group of cases that should be helpful to engineers in the matter of drafting specifications. A contractor who has done the work according to the specifications is entitled to be paid the contract price although the result may not be what the owner anticipated. In three of the cases cited by Mr. Herwitz, MacKnight Flintic Stone Company *vs.* Mayor (65), Filbert *vs.* Philadelphia (65), and Kuhs *vs.* Flower City Tissue Mills Company (48), the owner sought to obtain watertight construction, and for this purpose gave to the contractors specifications which described in detail how the work should be done. In each case the contractor had done the work according to the specifications, and in each case the Court ruled that the contractor was entitled to be paid even if the structure leaked. In another case, Montrose Contracting Company, Inc., *vs.* County of Westchester (66), the owner had specified the methods to be used in the construction of a sewer tunnel. It was shown that those methods were appropriate for the construction of a tunnel in free air, but not for one in compressed air, and the Court held that the specification of those methods constituted a representation that the job was a free-air tunnel.

Mr. Herwitz has presented two cases which involved the same legal principle as the foregoing cases, but which relate to the plans rather than to the specifications. In United Construction Company *vs.* Town of Haverhill (38) the Court held that a contractor who had built bridge piers to solid ledge, at the depths indicated on the drawings, was entitled to be paid even if the piers were washed out in a subsequent storm. In United States *vs.* Spearin (65) the Court held that a contractor who had constructed a drain and had connected it with an existing drain, as required by the plans, was entitled to the damages suffered by reason of the fact that the existing drain was not sufficient to carry off the water which accumulated on the work during a storm.

The principle that underlies all of the cases in this group is that the owner is responsible for the sufficiency of the drawings and specifications. If, because of defects in the drawings or specifications, the completed work is defective, or the expense of doing the work has been increased, or the contractor has been induced to bid too low a price for the work, the responsibility is upon the owner, not upon the contractor.

Questions Arising During Performance of Work.—Mr. Herwitz has also cited a group of cases involving questions that have arisen during the performance of contract work. The questions presented do not relate to the assumption of risk of subsoil conditions, which is the subject of Mr. Herwitz's paper, but they are of such importance to engineers and contractors that it is fortunate that he has considered them.

The first of these questions relates to the engineer's final certificate. Nearly every construction contract contains provisions that the final certificate of the engineer is conclusive upon the parties and that such certificate must be made and filed before the contractor is entitled to receive his final payment. There is no doubt that if the certificate is made in good faith and in accordance with the terms of the contract, it is conclusive, and the contractor cannot recover as payment under the contract more than the sum certified, O'Brien *vs.* City of New York (5) and United Construction Company *vs.* St. Louis (32); but, if the Courts are convinced that an injustice has been done either because the engineer has been arbitrary in the making of his final certificate or because he has misconstrued the contract, they do not hesitate to set aside his certificate and to determine for themselves the sum due the contractor, White *vs.* United States (77), L. P. & J. A. Smith *vs.* United States (78), St. George Contracting Company *vs.* City of New York (79). The rule is firmly established that the final certificate is not a bar to an action for damages for breach of contract or breach of warranty in the making of a contract, Kuhs *vs.* Flower City Tissue Mills Company (48), Faber *vs.* City of New York (22), and Borough Constructor Company *vs.* City of New York (41).

Changes in Work.—Few large jobs are completed without changes in the drawings or specifications, additions to the work, or elimination of parts of the work. If the work is rendered more expensive, whether because of unforeseen conditions existing when the contract was let, as in Faber *vs.* City of New York (22), or because of any act or neglect of the owner after the making of the contract, as in Sundstrom *vs.* State of New York (9), Feeney and Sheehan Building Company *vs.* State of New York (9), Thileman *vs.* City of New York (55), the contractor is entitled to increased compensation. In such case no written order is necessary and the sum recovered by the contractor is technically recovered as damages, not as payment for extra work.

The elimination of work frequently results in hardship to the contractor because in making up his bid he has apportioned his indirect cost, overhead, and profit over the pay items of the contract. The elimination of work does not usually reduce the amount of the contractor's indirect cost and overhead, and it certainly deprives him of a part of his undistributed profit. Notwithstanding the hardship imposed, the Courts do enforce contract provisions permitting the elimination of work without liability for anticipated profits. Kinser Construction Company *vs.* State of New York (64), and in the case of New York City subway contracts, the Courts have enforced a provision that work may be eliminated without liability to the contractor for anticipated profits, overhead, or damages. In Del Balso Construction Corporation *vs.* City of New York (64) the Court said that the contractor must have had the contract provision permitting omissions in mind in calculating his overhead when preparing his bid. Even if the bidder does have it in mind, however, what can he do about it? He cannot foretell what items will be omitted, and certainly the City will not want bidders to add a contingent item to cover possible omissions of work. In view of this decision, the Board of Transportation, City of New York, should revise its omission clause, or, if it wishes to retain the unrestricted right to omit parts of the work, it should make provision for payment to contractors of the proportionate part of indirect cost and overhead applicable to the work omitted.

Conclusion.—A discussion of these cases would not be complete without a restatement of the rules to be observed by the engineer, and other rules that should be observed by the contractor. Among those rules the following stand out as most important:

For the engineer—

(1) The "Description of Work" in a proposed contract must describe, correctly, all of the work that the contractor will be required to do;

(2) All available sub-surface information should be given to bidders with complete information as to how, when, and by whom such information was obtained;

(3) A general statement concerning sub-surface conditions should not be made unless it is known that such statement is correct;

(4) An owner who has given the contractor inaccurate or incomplete boring information is responsible even if the boring information is not made part of the contract;

(5) A contractor is not required to make surveys, soundings, or borings to determine whether or not the information given him in the contract is correct;

(6) The Courts do not look with favor upon a one-sided contract, a contract that attempts to shift all of the risk to the contractor;

(7) If there is a subject as to which information is not complete and as to which it is intended that the contractor shall assume the risk, the contract should refer particularly to that question and the risk to be assumed should be clearly stated; and

(8) The specifications should tell a contractor what he is to do; they should not tell him how to do it.

For the contractor—

(9) The entire contract and specifications should be studied with particular reference to any provision directing attention to a special subject (it may be assumed that the engineer did not insert such provision unless the subject was one concerning which there was some uncertainty);

(10) A bidder should always inspect the site and is charged with knowledge of everything visible on such inspection;

(11) A bidder is not usually required to make borings or other sub-surface investigation, particularly if that work has been done by the owner; but the bidder is required to make any reasonable investigation which time and circumstances permit;

(12) Unless a prior contract for work on the site is referred to in the contract for which a bid is being made, the bidder is not justified in assuming that the work has been done according to the drawings and specifications of such prior contract; and,

(13) The owner is primarily responsible for the sufficiency of the drawings and specifications; but this will not protect the contractor if he knows, from facts presented to him or facts that were visible when he made the contract, that the drawings and specifications were not adequate.

David A. Molitor,[7] M. Am. Soc. C. E. (by letter).—Every sizable construction contract involves at least some features, the details of which are not predictable at the time of letting, and these must be appraised, when encountered, and allowed for in the final settlement.

Despite the most careful soil investigation and borings, made to obtain data for foundation design, there will usually develop, during construction, more or less material variations in the character and required depth of excavation, resulting in modifications from the design as originally prepared. To pass this risk on to the contractor, as is usually done, by introducing cleverly worded clauses into the specifications, is unwise and unbusinesslike, and generally leads to a case of arbitration or lawsuit.

If the designing engineer expects to deal honestly with the prospective contractor, he should write his specifications with that end in view. He should expect to pay for everything that is stipulated to be done or furnished without hoping to get something for nothing. Neither should the owner have to pay for anything not obtainable in fact.

Therefore, why not write a clearly worded specification covering everything, so far as knowable, in exact businesslike language, leaving the unforeseen eventualities to be dealt with under supplemental written agreements, and thus avoid serious disputes at the time of final settlement?

The wide variety of cases cited by the author illustrates the attitude of the Courts when called upon to clarify the business relations between owner and contractor, resulting from doubtful or contradictory requirements in the specifications with regard to the amount of work to be performed, usually under a lump-sum contract.

If the engineer cannot stipulate, clearly, the exact quantities of work which the contractor is to perform, and yet expects to hold the latter to a lump-sum price, he must also expect to obtain excessive bids covering the unreasonable, or inadequately described, features of the proposed work. Later, if the contractor finds himself a party to a losing contract, the case goes to Court for the final adjudication of disputes following in the wake of an ambiguous contract. As a matter of fact, lump-sum bids are generally unfair to both owners and contractors and should be avoided.

The practice of awarding contracts only to the lowest responsible bidder makes it mandatory to provide a method of obtaining comparable bids, but not necessarily lump-sum bids. This can best be accomplished by incorporating in the tender an approximate estimate of quantities of each kind of work or materials to be furnished, the bidder merely supplying the requisite unit prices to determine the aggregate amount of the proposal. The same unit prices should later apply to changes in the estimated quantities made necessary during prosecution of the work by additions to, or abatements from, the estimated quantities in arriving at the final estimate and settlement.

Extras, limited to work or materials not covered by the drawings and specifications, and hence not contemplated under the contract, are to be paid only in accordance with supplemental written agreements on a cost-plus basis.

---

[7] Cons. Engr., Harlingen, Tex. Mr. Molitor died September 8, 1939.

There will always be abundant evidence to cast doubt on the most carefully made soil investigation, and nothing further is necessary to start a controversy whenever the contractor sees his way clear to recover. One of the most fertile contentions for extra compensation is that not all the information was disclosed at the time of bidding.

The result obtained from the soil investigation should be accessible to all bidders merely for what it is worth, frankly recognizing the approximate, or even erroneous, character of the information, with a clear statement that all departures from the design data and changes in foundations as finally decided upon and built, shall be adjusted in the final estimate at the unit prices named in the bid.

If each intending bidder were to obtain his own information, this would necessitate duplicating the soil investigation, which is impractical. Any requirement whereby the bidder is to obtain technical data relative to depth and character of foundation, ground-water, pumping, sheet-piling, etc., and then be held responsible for the outcome, is manifestly unfair as it amounts to doing precisely what the designing engineer is supposed to do to obtain the required information for his own use and for all intending bidders. Contractors should not be penalized for shortcomings in the design. This is the responsibility of the engineer.

The very valuable advice contained in Manual No. 8 may be regarded as an enabling act to specification writing; but here again one finds a wide divergence in the quality of the product put out by different writers. It is sometimes easier to tell the other fellow how to do something than to do it oneself.

The author has presented innumerable cases which terminated in disputes in connection with "unexpected risks," and has cited the decisions rendered by the Courts on the basis of facts disclosed in each case. Although this information is very interesting and instructive, the matter of avoiding a recurrence of such cases is still an important issue. The writer contends that there is no justification for writing a contract based on a gamble.

F. B. Marsh,[8] M. Am. Soc. C. E. (by letter).—The conclusion that "a knowledge of what the Courts have done in similar circumstances lessens the element of uncertainty for the future" (see "Conclusion") is justified only for those types of cases in which the Courts agree among themselves. However, a reading of this paper confirms the previous feeling that in too many cases one Court decides in favor of the plaintiff and another Court, with a quite similar set of circumstances, decides the other way. One cannot escape the feeling that in one case the plaintiff's counsel or witnesses have appealed successfully to the Court's sympathy, or perhaps the weather or some other extraneous matter has given the Court a more generous disposition. Certainly the evidence alone does not account for many rulings, and precedents can be found for almost anything.

The one important fact that stands out is the obligation resting on the engineer to show the available sub-surface information not only accurately, but completely, including, for example, such data as the records of borings that had to be abandoned before the desired depth was reached. It may be that the reason for abandonment may, in itself, give a clue to sub-surface conditions not fully revealed by a subsequent deeper hole near-by. In any case, the listing of abandoned holes with the others is obvious evidence of the engineer's effort to make all information available.

---

[8] Div. Engr., Eastern Dept., New York Board of Water Supply, White Plains, N. Y.

Evan S. Martin,[9] Assoc. M. Am. Soc. C. E. (by letter).—The author views the problem and record from the legal standpoint. It has been common to insert, in construction contracts, general clauses that transfer the risks from the owner to the contractor. Acceptance of many of these risks may be optional with the owner, or he may pass them along; but certainly it is the duty of the owner to define the substance of the contract—to describe to the contractor what he is to do—if he invites a bid price.

The paper is predicated on the assumption that, in sub-surface construction, there are facts, essential to the completion of the contract, that have been overlooked in preparing plans and specifications. The writer maintains that a clause which places all risk of sub-surface conditions on the contractor renders the contract, basically, null and void. To cite an above-surface parallel: A contractor cannot be enjoined to construct a 12-room residence "of the best material and workmanship, to the satisfaction of the owner" without plans and specifications—that is, without knowledge of what is above the surface. The lawyers and Courts will not say so; they have not said so yet; but this continent has not emerged mentally and morally from wild free-shooting frontier conditions. When the people do emerge and really establish law and order, they shall be civilized because they will consider equity first and only. Whether the owner pays less for investigation than he does for contracting for something "in the dark" is irrelevant.

When the Construction Division was organized, in 1925, the writer stated that he thought that this section should consider the development of rules for the interpretation of contracts—an engineers' jurisprudence. There is no reason for engineers to defer to the legal profession in defining the ethics of their practice. Note the conflicting opinions cited in this paper; they are not much concerned with what is fair but, as a lawyer might say, what is fair "in the light of the contract." The writer's point is that the "light" is not in the contract but in equity behind the contract. What is fair is simple, in interpreting contracts; it is that neither party shall get something for nothing. In legal parlance "something for nothing" is "without consideration," and therefore null and void. It is not fair to force a contractor into the surety business; he is not organized to undertake risks, and he cannot know what to charge for them.

Engineers (and Courts, also) must delve behind the contract, "the law," and "the Constitution" to what is fair. Stated inversely, engineers must make ethics—equity—the sole guide in the interpretation and adjudication of contracts. Perhaps this is what is intended by Blackstone's British Common Law.

C. Maxwell Stanley,[10] Assoc. M. Am. Soc. C. E. (by letter).—The outline of the legal aspects of the risks involved in sub-surface construction, as presented by Mr. Herwitz, should be of help to every engineer who prepares specifications for such work. Although most engineers may have avoided litigation arising from unexpected conditions under the surface of the ground, few have avoided controversy with contractors, which has stopped short of litigation only because of compromise, extra compensation, or because the sum involved was too small to justify legal action.

---

[9] Secy.-Treas., Jas. A. Wickett, Ltd., Toronto, Ont., Canada.

[10] Cons. Engr. (Stanley Eng. Co.), Muscatine, Iowa.

As emphasized by Mr. Herwitz, there must be a complete "meeting of minds" between the parties to the contract if confusion and misunderstanding are to be avoided. Such a "meeting of minds" requires a full understanding, on the part of both parties to the contract, of the risks and the responsibilities to be assumed by each. Such an understanding can be obtained only through the contract documents, and, more particularly, through the specifications and the plans prepared by the engineer. Many of the cases cited by Mr. Herwitz obviously involve specifications in which the provisions regarding the risks and responsibilities have been vague or indefinite, thus preventing a "meeting of minds." In such cases, it is natural that disputes will arise which create controversy, and which may lead to litigation.

Every engineer should recognize that there are hazards involved in subsurface construction because experience has shown that no matter how carefully subsoil investigations are made, conditions may be encountered that are different from those anticipated. Likewise, every experienced engineer should realize that various types of subsoil require different methods of construction and types of equipment and, hence, result in different costs of construction. Furthermore, he should realize that if a contractor is required to assume the risk of unexpected subsoil conditions, that contractor is entitled to compensation for this risk and must include in his bid a sum sufficient to cover the item.

In preparing his specifications, the engineer has the three following choices of placing the responsibility for unexpected subsoil conditions:

(1) The entire responsibility may be placed on the contractor who must thus assume all risks and who will, of necessity, protect himself by including in his bid a sum which he estimates is equal to the risk that he must carry, plus the cost of any borings or investigations which he may make.

(2) The responsibility may be assumed by the owner by giving information on subsoil conditions and by guaranteeing this information. In this case, the risk is entirely upon the owner and the contractor need not protect himself by covering these items in his bid and, hence, may bid lower.

(3) The responsibility may be divided between the contractor and the owner. In such cases, the owner may provide available data on subsoil conditions but require the contractor to interpret them and be responsible for any misinterpretations or for any conditions which prove to be different from those indicated. Such a case saves the contractor the expense of borings but still makes him assume a risk for which he is entitled to proper compensation.

There are numerous circumstances which fully justify either of the first two choices, and there are undoubtedly some cases where the third choice is proper. It would seem, however, that the greatest difficulty has arisen in those cases in which the engineer has endeavored to "straddle the fence" and divide the responsibility between owner and contractor and has failed to state in the specifications, clearly and definitely, just what is intended. Sometimes the engineer has realized the shortcomings of the data he has available on subsoil conditions and, although he has used these to outline the general methods of construction and type of work on the project, he has endeavored to avoid the assumption of responsibility on the part of the owner by placing it upon the contractor through means of general or "catch-all" phrases in the specifications.

After the decision has been made as to who shall bear the responsibility for any unforeseen conditions, the specifications must then be prepared in such a manner that there will be a complete "meeting of minds." Vague and indefinite provisions and so-called "catch-all" phrases which are inserted in an effort to throw responsibility on the contractor while, at the same time, the engineer is endeavoring to specify methods and equipment, have no place in a specification. The engineer should state, fully, clearly, and fearlessly, who is to be responsible for unforeseen ground or subsoil conditions. If the owner is bearing the risk, the contractor should be instructed to bid upon the subsoil conditions as outlined in the data included in the specifications, and he should be compensated for unforeseen conditions which may change the cost of construction. Often, this can be taken care of by the use of unit prices on different types of material that may be encountered. On the other hand, if the contractor is to bear the responsibility, he should be made aware of it so that he may protect himself to whatever extent he deems necessary when he bids upon the work; and then, if unforeseen conditions are encountered, the specifications will be sufficiently clear and precise so that he has no valid claim for any additional compensation.

In all events, however, the specifications must be clear and precise and must deal fairly with both parties; and, they must bring about a complete "meeting of minds."

Francis J. Morgan,[11] **Esq., and Frederick C. Zeigler,**[12] **M. Am. Soc. C. E.** (by letter).—In the presentation of his paper, Mr. Herwitz has made a valuable contribution to engineers and others engaged in the preparation of construction contracts and their appurtenant specifications, plans, and documents where underground work is involved.

Sub-surface exploration work, in some cases (such as for tunnels and pipe lines) may be done primarily for the information of the engineer to guide him in establishing the location, grade, type, and shape of his structure. The character of materials and conditions encountered in borings may require radical changes in the engineer's preliminary designs so as to make these borings quite incomplete, if not unreliable, when used by contractors for bidding purposes. Sometimes complete deep borings are difficult to make especially in glacial drift where boulders abound and blasting is restricted in public thoroughfares. Drilling progress under such a condition is likely to be very slow with consequent long delay in letting the construction contract.

---

[11] Deputy, County Attorney, Westchester County, Yonkers, N. Y.

[12] Formerly Project Engr., Westchester County Sanitary Sewer Commission; and Project Engr., Bureau of Sewage Disposal, Dept. of Public Works, New York, N. Y.

Generally, the engineer on public work tries to obtain the best possible sub-surface data. If he employs a reliable and experienced boring contractor, it is obvious that the bidding contractors on the subsequent construction work have available as much information as to the nature of the sub-surface conditions as he has. Of course, the engineer is on "dangerous ground" when he attempts to interpret sub-surface conditions and draws on his own imagination as to the kind, form, and types of constructions or configurations of strata which the eye cannot see.

It should be the function of the engineer to protect, with great care, the interests of the owner or municipality for which construction work is to be done so that damage claims will be avoided or, if taken to Court, will not result in costly judgments. Waivers alone appear in certain cases to carry little weight in Court.

Since much of the public bidding on construction jobs is clearly a gamble on the part of many contractors, it behooves the engineer to use extreme care to make the provisions of his contract documents as tight as possible to protect the owner, especially if borings and other records are made available to bidders. It is unfortunate, but it must be admitted, that there are a few contractors who are quite ready to use, against the engineer, any honestly prepared informatory data that have been supplied to him in advance of bidding, especially where the contractor has been the loser financially through his own mismanagement or otherwise. The engineer must be prepared for that kind of a contractor.

It should be realized that on public work, the contractor usually can secure a Court order to make a "fishing expedition" to examine all the engineer's records for evidence to use against him in the Trial Court.

It is realized that it would have taken much space if the author of the paper had covered, in reasonable detail, the various Court Citations mentioned in the Appendix. Some of them, however, would probably be of considerable value to the engineer who is charged with the preparation of important contracts because the average engineer has little or no true conception of the judicial viewpoint.

In a 10-yr construction program in Westchester County, New York, the case of Montrose Contracting Company, Incorporated *vs.* County of Westchester (66) was the only one of several claims taken to Court by contractors, which finally resulted in a judgment in the contractor's favor. Bidders generally recognized that the work, a tunnel, was "a wet job," in a valley at considerable depth below the bed of a near-by stream. Sufficient information was available in the contract documents and borings to put the bidders on notice that the work was without doubt very wet. It is difficult, at least for a layman, to understand how a Court could be convinced that the contract documents, which clearly delineated wet conditions and required an examination of the site by bidders, meant anything else to experienced contractors than the necessity of using accepted methods under such circumstances to keep the work dry during construction.

In the first trial, the contractor contended that the contract provisions for under-drains, alignment holes, ventilation, care of water and placing of concrete tunnel lining in not less than 12-ft sections all indicated a free air job with the exception of about 600 ft of tunnel and that they could not be performed where compressed air was used. No defense was offered and the judge granted the County's motion for a directed verdict in its favor (80). The Circuit Court of Appeals on an appeal by the contractor reversed the judgment entered in favor of the County and ordered the second trial holding that as a matter of law the under-drains could not be built in compressed air; that the alignment holes required were used in free air but never in compressed air; that the specifications relating to ventilation, concreting and care of water were suitable for a "free-air" job and impossible of use in a compressed-air job.

Regardless of the fact that the County in the second trial offered expert evidence to show that the aforementioned provisions could be used in compressed air as well as in free-air tunneling the opinion of the Circuit Court prevailed and the contractor was successful in recovering a substantial judgment. The County then took an appeal to the Circuit Court of Appeals but was unsuccessful (81). The Supreme Court of the United States refused to grant Writs of Certiorari on both applications therefor made by the County, after each appeal had been decided by the Circuit Court of Appeals.

The lawsuit referred to herein presents a very interesting problem both for lawyers and engineers in that it relates to the interpretation of a contract, where mixed questions of law and fact are involved.

It is the duty of the Courts to interpret contracts and explain their meaning. The Trial Court on the first trial held in substance that the contract did not represent and impliedly warrant the tunnel job to be substantially a "free-air" job. That judge apparently interpreted the contract as it was written and especially those parts that referred to the use of compressed air. The words which related to compressed air, and the partial payment for 600 ft, specifically said "regardless of whether more be used or not" (82).

In its opinion on the first appeal the Circuit Court concluded that the compressed air provisions of the contract could not negate the representations in the plans that the tunnel was primarily one to be constructed in free air. It held further that the compressed air provisions covered a variation, but not a transformation, and that they would be effective if somewhat more than 600 ft of work were required in compressed air. It did not say how much more.

Furthermore, the Circuit Court took the evidence offered by the contractor and, with no evidence offered by the County, interpreted the contract to mean that the provisions of the contract relating to under-drains, alignment holes, ventilation, pouring of concrete in not less than 12-ft sections and the care of water applied to free air, and were impossible of use in compressed air.

If it is assumed that the trial judge on the first trial erred on the law in interpreting the contract, then questions of fact arose as to whether or not the aforementioned provisions of the contract applied both to free air and compressed air, and whether or not these things could be done only in free air and could not be done in compressed air.

As the case was tried before a jury, it seems that these questions, after the evidence thereon had been offered by both sides in the second trial, should have been given to the jury to determine as questions of fact. If this had been done and the jury had determined that these provisions of the contract could be performed only in free air, then the Court would have been in the position to apply these facts to the contract and hold that the contract represented, and impliedly warranted, the tunnel to have been substantially a "free-air" job.

Viewing it from the other side and assuming that the jury found these provisions of the contract could be performed in compressed air as well as in free air, the Court would have to say that there was no representation and implied warranty that the work was substantially a "free-air" job.

In his first opinion, did the Circuit Court judge stray from the beaten paths of the law and wander into the realms and fields of engineering and thereby become an engineer as well as a judge? Who determined the fact that the under-drains or the alignment holes could not be built in compressed air, or the ventilation provisions could not be used in compressed air, or that the care of water provisions was impossible of use in compressed air, or that concrete in at least 12-ft sections could not be poured in compressed air? It is a fundamental principle of law that the duty of a common-law jury is to determine and decide questions of fact. Furthermore, another question suggests itself and that is: Did the contract contain an "express warranty" that fixed the sum of money the County would pay specifically and directly for compressed-air work, and thereby require the contractor to include in Item 1 of the bid (this was a lump sum contract) any additional sum it needed to compensate itself for all compressed-air work costing more than $30 a foot, no matter how much was done under compressed air?

Different technicalities seem to arise in every Court case, which affects its outcome. Even the viewpoints of the Courts appear to differ; the Westchester case in point has no parallel. Care was taken to avoid specifying both the methods of construction and the results to be attained but the Court may have thought otherwise. A distinction has been drawn between constructional features which serve a useful purpose in the completed structure and those useful only during construction whether the engineer requires them for his use or not; such as, for instance, the alignment holes for checking tunnel lines from the ground surface.

Because of sub-surface conditions which a bidder seems to have disregarded this discussion shows what can happen in litigation regardless of the care in the preparation of borings and a seemingly fair and reasonable contract. It supports the author's conclusion that each case must rest upon its own distinctive facts. It appears, however, that the safest contract documents are those whose provisions are very general so far as possible; otherwise every conceivable contingency must be covered.

LAZARUS WHITE,[13] M. AM. SOC. C. E. (by letter).—The paper by Mr. Herwitz is very timely, of great value to the engineer, and is deserving of a full discussion. In the writer's experience very little on this subject is taught in engineering schools; yet contracts and specifications are primarily created by the engineer through his investigations, designs, and specifications, although the contract part of the documents may be the work of lawyers—and here "comes the rub." The engineer responsible for a sub-surface contract is often too hampered by a lack of time or funds to investigate sub-surface conditions properly; he then draws up designs, as nearly as possible fitting the conditions as known or assumed by him. He feels that if conditions are not actually encountered as assumed, claims for extra compensation will be filed by the contractor; and he relies upon the lawyer to protect the owner from such claims —hence, the "exonerating clauses" or "alibi clauses" found in many contracts. Thus, to the business risks and labor risks, which a contractor must assume, are added risks which do not properly belong to him. The owner should provide sufficient time and means for the engineer to ascertain the underground conditions properly; or, if this is impracticable, the owner should assume the extra cost of performing the work under conditions different from those upon which the contract was based. The engineer degrades his profession by hiding behind tricky legal defenses so that extra expenses (which, in fairness, the owner should assume) are thrown upon the contractor whose bid prices are usually narrowly competitive.

---

[13] Pres., Spencer, White & Prentis, Inc., New York, N. Y.

The engineer is properly the agent of the owner, and it is perfectly proper that as the owner should profit by the skill and economies of design of the engineer he should also assume the extra costs when the designs do not fit the assumed conditions, and when the conditions are different.

The "Synopsis" states that in the early days it was customary for the contract to provide that the risk of the unexpected should fall on the contractor. Broadly worded contract clauses took the place of scientific and careful investigation. Unfortunately this practice has persisted so that it is still common in contracts originating in certain organizations, as for instance, in the following note, attached to the principal sheet of an important contract.

"This record of borings is issued for the convenience of contractors. It is not a contract document. No figures nor notes are guaranteed. Sections and profiles are interpolated and are not exact."

Although Mr. Herwitz cites practically all the cases bearing upon sub-surface contracts and clearly defines the meaning of the decisions, there is no clear thread running through these cases and as he states in the last analysis "each case must rest upon its own distinctive facts."

The writer, from personal knowledge of several of the cases cited and with forty years experience in such sub-surface contracts, has sought to find some common logic in the series of apparently contradictory decisions, and has come to this conclusion that: (1) When the explorations, engineering and design, and the contract and specifications have been well done, the owner seldom loses; and (2) good and thorough engineering is the best protection and in most cases discourages litigation. Such contracts are consistent with the old definition of a contract being a "meeting of the minds." In other words a proper contract is that in which both parties agree upon a set of conditions upon which a price is based. Such contracts are not so uncommon; specifications of the U. S. Engineer Department contain clauses enabling adjustments and extra compensation to be made when "changed conditions" are encountered, or when designs must be altered because of "changed conditions."

Mr. Herwitz states that "reports of borings and similar data may return to plague both owner and contractor, even when given in the utmost good faith and with careful attention to the best modern practice." This seems to be apparent from the cases cited but again, in the writer's experience, the cases lost by the owners were not supported by well-prepared or well-explored contracts. The main difficulty with underground investigations is not that they are improperly made as much as improperly interpreted and classified—classifications made by a foreman rather than by a trained geologist. The Catskill Aqueduct crossed many buried gorges and difficult terrain; yet it was so well explored, and its geology so well investigated that little of the unexpected was encountered, and the few suits started were lost in the Courts. Of course the writer does not maintain that there is no residue of uncertainty in sub-surface work; but he believes that an arbitration clause is desirable to adjust claims arising from such uncertainties. A board of experts can come quickly to a just decision as compared to an ordinary jury. Almost manifest injustices will be avoided, as where public construction organizations use the sovereign power of the State to bar just claims or where clever lawyers write in trick clauses which serve as traps for the unwary.

In conclusion the writer would emphasize that the engineer prepare his contract thoroughly, act as a just judge in interpreting it, and forget that his judgment is "final and conclusive."

T. Kennard Thomson,[14] M. Am. Soc. C. E. (by letter).—Mr. Herwitz's very interesting paper draws attention to many cases in which Courts give diametrically different judgments on what, to the laymen, would appear to be identical cases. Most engineers have all had similar experiences.

The English Law Courts have the reputation (former Chief Justice Lord Reading has so stated within the writer's hearing) of trying to settle law suits on the equity of the case, instead of on technical legalities, as is so often done in the United States.

All that engineers can do is to try to be just to both client and contractor. In fifty-six years of experience the writer can recall only three or four cases in which the engineer deliberately tried to cheat the contractor. Few, if any, other bodies of men can equal or surpass such a record.

An engineer should give the bidders as much information as possible, and should explain, fully, the limitation of such information. It is neither wise nor pleasant to accept a bid when the "successful" bidder does not have a reasonable chance of making an honest profit. If the engineer furnishes the results of test "wash borings" he should explain that such borings give no indication of what kind of material lies under the bottom of such test borings.

In many cases the record states that the boring stopped on "rock or boulder." In some cases, such as Lower Manhattan, if the "wash boring" was made by an experienced engineer, he could tell which it was because he could wash up samples of New York bed-rock but could make no impression on a New York boulder.

---

[14] Cons. Engr., New York, N. Y.

With a sample smaller than a cherry he could state whether or not it was New York rock; but, a "core" or diamond-drill boring, penetrating from 3 ft to 5 ft into the rock, would be necessary for him to ascertain whether or not it was bed-rock in its original position, and not a misplaced or deposited piece of rock.

It is frequently difficult to interpret, correctly, the results of borings made by others. The writer once asked a contractor (who had, at that time, probably made more borings in New York than any other man) how much quicksand and hardpan he had found on a certain lot. The contractor replied, "none of either" and then showed the samples which confirmed what the writer already knew—that there was nothing but quicksand and hardpan above the rock at that place.

In some parts of Manhattan, an experienced man would know that he could reach hardpan with "wash borings" and find a fairly uniform level. At the same site the surface of the rock, even a foot or two away, might be entirely different. The writer recalls one case in which there was a drop of 6 ft in a horizontal distance of 2 ft. In such places the expense of many diamond-drill or "core" borings would scarcely be justified. At other points on Manhattan "wash borings" would be worthless, and nothing but "core borings" or test pits should be used.

The writer's practice for caisson work, where he is satisfied with the test borings, etc., is to call for a lump-sum bid to a certain depth, and for a price per cubic yard for work done below that depth. On one job he spent hours convincing the other members of the Board that they should make the estimated cost on the basis of $25 per cu yd to a stated depth and $50 per cu yd for all work done below that depth.

At first they could not see why so abrupt an increase should be made until it was explained that $25 was the average cost from zero to the stated depth, including plant, etc., and that $50 was a fair price for the greater depth. The irony was that, after convincing the Board and calling for bids on that basis, one contractor took a chance that he would not have to go below that level, and bid a flat $25 per cu yd. As an experienced contractor, he should have known better because the information was based upon "wash borings" only, which cannot be depended upon, with very rare exceptions, to reach bed-rock.

The call for bids asked for a price, per cubic yard, for 6 000 cu yd, more or less, below the stated depth. If he had bid $50 per cu yd for that item he would have received the contract anyway. The contractor lost money on that job, and would have lost much more if the Board had compelled him to go deeper as it had the right to do. It happened that the Board did not think it necessary in the interest of safety.

Experienced engineers have occasionally succeeded in carrying "wash borings" to bed-rock, through boulders and other hard material, by using dynamite or powder.

As an example of an unfair specification, a contractor once bid on some excavation work to be done by steam shovel at, say, 20 cents per cu yd. In that case the engineer had the privilege of changing the slope of the excavation if he considered it necessary, even after the excavation had been completed.

This meant that the contractor had to flatten the slope by pick and shovel at many times the contract price, and sometimes more than once at the same site. Although the engineer knew this was a rank injustice to the contractor (it was on public work), the engineer had no authority to pay more than the contract price.

Needless to say, books could be filled with descriptions of similar cases.

CARROLL A. FARWELL,[15] M. AM. SOC. C. E. (by letter).—A subject that has needed more study is presented in this paper, and its treatment by the author is of much value. The writer wishes to add emphasis to the importance of securing, in so far as possible, a definite "meeting of the minds" of the contracting parties. Each party must comprehend that there are risks to be taken and that the contract documents should allocate these risks clearly.

It appears that, in general, the Courts uphold the legality of placing a risk upon the contractor provided that the contract documents so state. Certain risks, which may not be mentioned specifically in the contract, are customarily assumed by the owner—such as, for example, extremely abnormal weather conditions or river floods, or hurricanes (in New England). Similarly, variations in the cost of materials and labor are customarily borne by the contractor. It would be better if the allocation of these risks were definitely specified instead of being left in the classification of "common practice." The risk of fire may be carried by either party and the allocation of this risk is often clearly defined in the contract through a specification that the contractor, or the owner, carry fire insurance. The risk involved in sub-surface conditions, to which the author's paper is limited, is one of the major items of construction work and, as such, needs accurate allocation.

Manual No. 8 states (1(*b*))

> "* * * The owner should provide the necessary funds for a complete preliminary investigation of the soil, its geological formation, and water conditions; and for the determination of all other facts needed to enable the engineer to design the work properly and the contractor to determine its cost accurately * * *. It is the contractor's duty to estimate only on work for which adequate preliminary information has been obtained, and for which adequate designs and specifications have been prepared by the engineer. He should not bid on the work until he has assured himself as to the adequacy of the information given him and the practicability of the type of construction shown in the engineer's design * * *.

The words "accurately," "adequacy," etc., are relative and their interpretation involves the personal equation. Regardless of how accurate and how adequate is the information given, there is a possibility of unforeseen conditions affecting the cost of the work, and the contract documents should specify which party is to bear, or is to be benefited by, the change in cost. Moreover, in order that the engineer may determine what are unforeseen conditions, the foreseen conditions must be shown or indicated definitely.

As has been stated, the contracting business is highly competitive and the writer agrees that the more the risks are carried by the owner, the lower will be the contract prices bid, and the more stable the contracting business. On the other hand, it is sometimes true that the owner is also engaged in a highly competitive undertaking, as for example, in the construction of an office building in which the cost of the building affects the rents that must be obtained; and, if the cost is greater than a certain limit, the project is not financially feasible.

---

[15] Cons. Engr. (Fay, Spofford and Thorndike), Boston, Mass.

Therefore, the owner may consider it desirable to carry as little risk as possible himself and to employ the contractor to carry all possible risks in order that he may know his upset cost as accurately as possible. This will usually cause the work of the contract to cost more than would otherwise be the case; but the additional cost may be well worth while. Moreover, the risk must be assumed by some one, even if it does not appear in the cost figures. If it were possible to arrange for some kind of insurance to cover such a risk (as is done by insurance against fire), the owner would undoubtedly be willing to pay for it, either directly or through the contractor.

If it were possible to include an item in the contract entitled "Assumption of Risk of Unexpected Sub-Surface Conditions," it would be highly desirable as it would call attention to the actual cost of carrying such a risk.

Public bodies, as well as private owners, often desire to know the upset cost of a project as nearly as possible in order to assure themselves that the cost will not exceed the funds available; and here again the contractor may properly be paid for the risk which he assumes.

The owner expects the engineer's estimate of cost to be reasonably accurate, and the Engineer should state (if it is a fact) that there are certain risks which may increase that cost over the estimated contract cost. This is often done under an item called "Contingencies" but many times this item is removed from the estimate when the contract documents are executed.

Under certain conditions, it has been the custom in the writer's office to arrange construction contracts for bridge foundations so that the contractor will be paid a lump-sum price for the foundations as built in accordance with the contract plans, and be paid under another item a unit price per vertical foot for the additional depth to which any pier may be carried below the depth indicated on the contract plans. This has been done on the theory that a decrease in depth was unlikely and, moreover, would have little effect upon the total contract cost, whereas each foot of additional depth would increase the cost materially.

As an example of an apparent lack of clarity in the "meeting of the minds" in a contract for foundations, the following, which came under the close observation of the writer, may be of interest.

The Contract Documents included the following:

> "ART. 3. *Changes.*—The contracting officer may at any time, by a written order, and without notice to the sureties, make changes in the drawings and (or) specifications of this contract and within the general scope thereof. If such changes cause an increase or decrease in the amount due under this contract, or in the time required for its performance, an equitable adjustment shall be made and the contract shall be modified in writing accordingly * * *.
>
> "ART. 4. *Changed conditions.*—Should the contractor encounter, or the Government discover during the progress of the work, sub-surface

> and (or) latent conditions at the site materially differing from those shown on the drawings or indicated in the specifications, the attention of the contracting officer shall be called immediately to such conditions before they are disturbed. The contracting officer shall thereupon promptly investigate the conditions, and if he finds that they materially differ from those shown on the drawings or indicated in the specifications, he shall at once, with the written approval of the head of the department or his representative, make such changes in the drawings and (or) specifications as he may find necessary, and any increase or decrease of cost and (or) difference in time resulting from such changes shall be adjusted as provided in article 3 of this contract."

The specifications included the following:

> "*Borings.*—Wash borings have been made under the direction of the contracting officer and of the engineer at various points at the sites of the work. These borings were made in the usual manner and with reasonable care; and their locations, depths, and the character of the material apparently encountered have been recorded in good faith on the contract plans. Samples of material obtained from the borings have been preserved and labeled and may be examined at * * * . There is no expressed or implied agreement that the depths or the character of the material have been correctly indicated and bidders should take into account the possibility that conditions affecting the cost or quantities of work to be done may differ from those indicated.
>
> "*Boulders.*—The borings apparently indicate that boulders may be encountered in varying numbers and sizes at any or all of the excavations to be made for the work."

During, and at the completion (1934) of, the work the contractor claimed, in effect: That the number of boulders and the quality of the sand encountered in the excavation was materially different from that shown in the drawings and indicated in the specifications; that, therefore, unforeseen and unanticipated sub-surface and latent conditions were actually encountered and it was necessary to change the method of work; and, that adjustments of the contract sum and of the contract time should be made to compensate for the extra work and time resulting from such changes. The controversy was finally taken to the Court of Claims and, in so far as is known to the writer, decision is still (1939) pending.

Oren Clive Herwitz,[16] Esq. (by letter).—Mr. Newton's discussion of the cases cited and analyzed in the paper was very helpful, for he refers in several instances to additional facts which the writer, because of space limitations, found it necessary to omit; but, with several of Mr. Newton's observations the writer is constrained to disagree.

Thus, Mr. Newton's statement that "an owner who has given the contractor inaccurate or incomplete boring information is responsible even if the boring information is not made part of the contract" is misleading and incorrect, for the Court of Appeals of the State of New York has held that in such a situation the owner is responsible only if guilty of bad faith. The Court said (42):

"If, however, notwithstanding the agreement as to honest mistake, damages might be recovered from the State for misrepresentations, upon which the bidder might rely, the boring sheet was not such a representation. It formed no part of the plans upon which the contract was based. It was not prepared or used for that purpose. It was an independent bit of information or supposed information in the possession of the State to which the bidder resorted in making the investigations which it was required to make. If it relied upon this paper, it did so at its own risk. The most it could ask for in regard to this information was good faith."

---

[16] Asst. Corporation Counsel, Div. of Contract Litigation, Law Dept., City of New York, New York, N. Y.

Mr. Newton's further statement that "a contractor is not required to make surveys, soundings, or borings to determine whether the information given him in the contract is correct" is equally misleading, especially when compared with his subsequent remark that "A bidder is not usually required to make borings or other sub-surface investigation, particularly if the work has been done by the owner." Mr. Newton's first statement is too broad, for the Court of Appeals of the State of New York has indicated that situations may arise where the contractor might be held responsible for what borings or soundings made by him would have disclosed. The Court said (60):

"He [the bidder] had the opportunity to physically examine the location of the work to be done, and if that called for extensive tests, by way of borings and soundings, that fact, however troublesome, was one he must take into consideration. He had presented to him an agreement, which was precise and strict in its requirements, and which cast upon him the possible risk that all might not be as it appeared, or as estimated by the surveyor. Admittedly, borings and soundings would have revealed the imperfect performance of the previous grading contract, in the leaving of native rock under the surface of the roadway."

Finally, Mr. Newton's discussion of the cases, in which he makes almost constant reference to the wording of particular contract clauses, is inconsistent with his earlier statement that "too much emphasis" can be placed on the wording of the contract, and that "the facts, not the contract clauses, form the real basis" of the Court's decision. It is the writer's belief that the language in the contract is of primary importance, for that is all that the Court may consider in determining whether owner or contractor was, by their agreement, intended for the rôle of risk-bearer. That intent, once determined, is controlling, and the Court is powerless to do anything but enforce it. As the Court said in Johnson vs. City of New York (83):

"In the absence of fraud or mistake, the contract made by the parties, however burdensome it may prove in execution, must govern. It is no excuse that a party to a contract did not foresee a hardship which its operation might entail. It is the function of courts to enforce contracts according to their true intent and meaning and neither to mitigate nor relieve against their operation."

The proposition that Courts of law, in deciding contract disputes, are doing nothing more than enforcing the intent of the parties, as expressed in their agreement, is one which laymen find difficult to comprehend. Judges cannot make a new contract for the parties; nor can they rewrite a contract so as to make it fairer to one party or the other. If (as so many of those taking part in the discussion seem to feel) it is unfair and inequitable to make the contractor assume the risk of unexpected sub-surface conditions, Court decisions which have that result are not the fault of judges but rather of the parties who willingly signed a contract containing language to that effect.

Mr. Martin states that the judicial opinions cited in the paper "are not much concerned with what is fair but, as a lawyer might say, what is fair 'in the light of the contract.'" He is quite right, if by that he means that the Courts cannot decide contract disputes upon general principles of ethics or morality. On the contrary, the Court's "hands are tied down" to the standard of ethics and morality previously fixed and determined by the parties themselves in the contract which they have signed. Although it is true that, in many cases, a contractor is compelled by the Courts to bear the loss resulting from an unexpected sub-surface condition, it is only because he assumed that risk in his contract; and the Courts, much as they might like to, are powerless to relieve him of the consequences of his unprofitable bargain.

When owners, contractors, and engineers realize that the solution of their difficulties lies not in catch-all contract clauses, but in a careful choice of unambiguous language to express their contractual obligations, there will be no occasion to criticize Courts for decisions which simply place the responsibility on the party who agreed to assume it.

Of course, the question as to whether or not it is sound, ethically or economically, for the contractor to assume the risk of unexpected sub-surface conditions (with which the discussion was almost exclusively concerned) is one which belongs more properly within the sphere of the engineer rather than that of the lawyer. There is much force in the arguments of Messrs. Hammond, Molitor, Marsh, Stanley, White, Thompson, and Farwell against making the contractor assume all the risks. On the other hand, there may be circumstances, as Mr. Farwell points out, under which the owner may feel compelled to require the contractor to do so, even if he thereby pays a higher upset price for the completed structure in order to cover the contractor's contingencies.

It is the writer's thesis that, whatever may be the decision reached in any particular case, the contract should express that decision, clearly and fairly, and in no uncertain terms. If a contractor is expected by the owner to assume all risks, he should be definitely advised of that fact so that appropriate allowances can be made in his bid. If he is not to bear the risk, it is to the owner's advantage that this also be made clear, for then the bids received will not be inflated by the contractors' fear of the unknown. In either event, neither party can have cause to complain or to litigate, for each will know the nature and extent of his contract obligations.

However, this ideal contract, in which there is a "clear meeting of the minds," cannot be wished into existence. The preparation of such a contract requires care and precision in the choice of language, a familiarity with the rules of contract interpretation customarily applied by the Courts, and a working knowledge of the fate that typical contract clauses have suffered in previous legal battles.

Realizing that in most instances construction contracts are drawn by engineers and not by lawyers, the writer hoped that his brief summary of the case law on this subject might prove helpful. If the necessity for careful, frank, and forthright contract draftsmanship has been emphasized, and the means to achieve it suggested, the paper may have served some useful purpose.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2066

DESIGN OF CIRCULAR CONCRETE TANKS

BY GEORGE S. SALTER,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. CARL A. ROSENGARTEN, I. K. SILVERMAN, FRANK J. MCCORMICK, H. B. MUCKLESTON, ROBERT B. B. MOORMAN, DANA YOUNG, L. J. MENSCH, JENS EGEDE NIELSEN, LLOYD S. DYSLAND, BASIL SOUROCHNIKOFF, REGIS F. FEY, AND GEORGE S. SALTER.

SYNOPSIS

The object of this paper is to present a rigorous mathematical design analysis of circular concrete tanks that is accurate for shallow tanks of rather large diameters, such as are used for settling purposes, as well as for deeper tanks of small diameter. It is shown that, by fixing the wall at the base so that the liquid pressure is divided between the circular rings and the vertical cantilevers, the magnitude of the maximum ring tension is greatly reduced. For tanks of proportions such as are ordinarily used for settling purposes the maximum ring tension amounts to only about one-quarter to one-third, and the total amount of wall reinforcement to less than one-half of that required were the wall free at the base. Diagrams are presented to facilitate design and an example is given showing their application.

INTRODUCTION

Theoretically, circular tanks having the least perimeter for a given area are the most economical that can be designed. If no restraint were offered at the junction of the wall with the base, the internal liquid pressure would resolve itself into pure tension and the ordinary cylinder theory would obtain for the full height of the wall. It has been quite common practice to design circular tanks for this condition and (although the wall is rigidly connected to the bottom slab) to provide ring reinforcement for the full liquid pressure. As concrete is a material that lends itself inherently to continuity, a logical method of design using this material is one that takes advantage of this action. Circular tanks built of concrete, however, have more or less restraint at the junction of the wall and the footing unless special precautions are taken to eliminate this action.

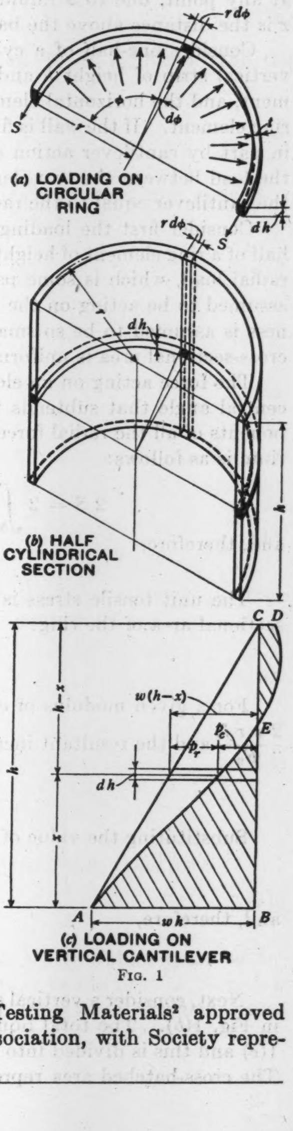
NOTE.—Published in March, 1939, *Proceedings*.

[1] Subway Structural Designer, Dept. of Subways and Super Highways, Chicago, Ill.

If the tank wall were rigidly fixed at the base, the ring action would be eliminated at the point where the pressure is the greatest, and thus the percentage of ring steel could be reduced; but this restraint of the wall at the base leads to rather complicated load distribution between the circular ring elements and the vertical cantilever elements. The division of the load between these two structural elements is emphasized by the disposition of the reinforcement in the wall. The size and spacing of the vertical bars is determined by the wall moments, due to that portion of the load carried vertically; and the horizontal ring steel is determined by the magnitude of the ring tension. Naturally, the sum of the loads carried vertically and horizontally must equal the total load and pressure on the wall.

Several methods have been presented in endeavors to evaluate the load distribution between the two structural elements but they have either been merely approximations or have been quite difficult to apply. Furthermore, most previous studies, apparently, were based on the condition of rather deep tanks or tanks of small diameter and thus, although they gave approximately correct results for tanks of those proportions, they were decidedly "out of line" for large-diameter, and rather shallow, tanks such as are ordinarily used for settling purposes for sewage, water purification, and certain metallurgical processes.

The purpose of this paper is to present a strictly rigorous analysis and yet one that is easy to apply and which yields results that are accurate, within the limits of the assumptions made, for tanks of any proportions. Although the analysis applies to tanks of all sizes, the diagrams presented herein apply only to tanks of proportions such as are ordinarily used for settling purposes. Of course, these diagrams can be extended to include tanks of all sizes.



(a) LOADING ON CIRCULAR RING

(b) HALF CYLINDRICAL SECTION

(c) LOADING ON VERTICAL CANTILEVER

FIG. 1

NOTATION

The letter symbols used in this paper are defined where they are first mentioned. An effort has been made to conform to the Symbols for Mechanics, Structural Engineering, and Testing Materials[2] approved by a committee of the American Standards Association, with Society representation, in 1932.

---

[2] A. S. A.—*Z10a*—1932.

ANALYSIS

The following is presented as a rigorous analysis of the problem when the wall is considered as an elastic cylinder, fixed at the base. Given a tank of radius r and height h. With origin at the base of the wall, the internal pressure at any point, due to a liquid whose unit weight is w, is $w(h - x)$, in which x is the distance above the base of the wall.

Consider one-half of a cylindrical tank as represented in Fig. 1(*b*). The vertical strip of height h and width $r\,d\phi$ represents a vertical cantilever element, and the horizontal element of height dh represents one-half of a circular ring element. If the wall is fixed at the base the liquid pressure will be resisted in part by cantilever action and in part by ring action. The distribution of the load between the two elements must be such as to make the deflection of the cantilever equal to the radial displacement of the ring.

Consider first the loading on the circular ring. Fig. 1(*a*) represents one-half of a ring element of height dh, and thickness t. It is subjected to a uniform radial load, which is some part of the liquid pressure, of intensity p_r, and is assumed to be acting on the center line of the ring. Furthermore, the thickness is assumed to be so small compared to the radius that the stress on the cross-sectional area is uniform.

The force acting on an element of the ring is $p_r\,r\,d\phi\,dh$, in which $d\phi$ is the central angle that subtends the vertical strip. Taking the sum of the components of all the radial forces acting on the half ring, the equation of equilibrium is as follows:

$$2\,S = 2\int_0^{0.5\pi} p_r \sin\phi\; r\,d\phi\,dh = 2\,p_r\,r\,dh \quad\ldots\ldots\ldots\ldots(1)$$

and, therefore,

$$S = p_r\,r\,dh \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The unit tensile stress is found by dividing the total stress by the cross-sectional area of the ring:

$$f_s = \frac{p_r\,r\,dh}{t\,dh} = \frac{p_r\,r}{t} \quad\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

For a given modulus of elasticity, E_0, of the shell, the elongation becomes $\frac{2\,\pi\,r\,f_s}{E_0}$ and the resultant increase in the radius, denoted by y, is

$$y = \frac{2\,\pi\,r\,f_s}{2\,E_0\,\pi} = \frac{r\,f_s}{E_0} \quad\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Substituting the value of f_s given in Equation (3)

$$y = \frac{p_r\,r^2}{E_0\,t} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

and, therefore,

$$p_r = \frac{E_0\,t}{r^2}\,y \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Next, consider a vertical strip, of width $r\,d\phi$ and height h, also as indicated in Fig. 1(*b*). The total liquid pressure is represented by the triangle in Fig. 1(*c*) and this is divided into two parts by the "load-distribution curve" *A E D*. The cross-hatched area represents that part of the load resisted by the cantilever, and the algebraic sum of the loads on the cantilever and the ring must equal the liquid pressure; that is,

$$p_c + p_r = w\,(h - x) \quad\text{(7a)}$$

or

$$p_c = w\,(h - x) - p_r \quad\text{(7b)}$$

Substituting the value of p_r, as given in Equation (6), in Equation (7*b*), the load on the cantilever may be expressed as

$$p_c = w\,(h - x) - \frac{E_0\,t}{r^2}\,y \quad\text{(8)}$$

The general differential equation of the elastic curve of a beam under any system of continuous loading is expressed[3] by

$$E\,I\,\frac{d^2y}{dx^2} = M \quad\text{(9a)}$$

in which M = moment at any point; also,

$$\frac{dy}{dx} = \theta \quad\text{(9b)}$$

$$E\,I\,\frac{d^3y}{dx^3} = V \quad\text{(9c)}$$

and

$$E\,I\,\frac{d^4y}{dx^4} = p_c \quad\text{(9d)}$$

in which θ, V, and p_c equal slope, shear, and unit load, respectively. Substituting in Equation (9*d*) the value of p_c given in Equation (8)

$$E\,I\,\frac{d^4y}{dx^4} = w\,(h - x) - \frac{E_0\,t}{r^2}\,y \quad\text{(10a)}$$

or

$$\frac{d^4y}{dx^4} + \frac{E_0\,t}{r^2\,E\,I}\,y = \frac{w\,(h - x)}{E\,I} \quad\text{(10b)}$$

The general solution of Equation (10*b*) is[4]

$$y = e^{\beta x}\,(A \cos \beta\, x + B \sin \beta\, x) + e^{-\beta x}\,(C \cos \beta\, x + D \sin \beta\, x) + \frac{w\,(h - x)\,r^2}{E_0\,t} \quad\text{(11)}$$

in which A, B, C, and D are arbitrary constants whose values must be determined from known conditions at certain points; e is the base of Naperian logarithms; E_0 = modulus of elasticity of the reinforced circular rings,

---

[3] "Strength of Materials," by S. Timoshenko, Part I, p. 182.

[4] *Loc. cit.*, Part II, p. 402.

E = modulus of elasticity of the reinforced vertical cantilevers; I = moment of inertia of the vertical section $= \frac{t^3}{12}$; and,

$$\beta = \sqrt[4]{\frac{E_0\, t}{4\, r^2\, E\, I}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

By successive differentiation of Equation (11) the following equations may be secured:

$$\frac{dy}{dx} = \beta\, e^{\beta x}\,[(A + B)\cos\beta\, x - (A - B)\sin\beta\, x]$$

$$- \beta\, e^{-\beta x}\,[(C - D)\cos\beta\, x + (C + D)\sin\beta\, x] - \frac{w\, r^2}{E_0\, t} \ldots\ldots (13a)$$

$$\frac{d^2y}{dx^2} = 2\,\beta^2\, e^{\beta x}\,(B\cos\beta\, x - A\sin\beta\, x)$$

$$- 2\,\beta^2\, e^{-\beta x}\,(D\cos\beta\, x - C\sin\beta\, x) \ldots\ldots\ldots\ldots (13b)$$

$$\frac{d^3y}{dx^3} = -\,2\,\beta^3\, e^{\beta x}\,[(A - B)\cos\beta\, x + (A + B)\sin\beta\, x]$$

$$+ 2\,\beta^3\, e^{-\beta x}\,[(C + D)\cos\beta\, x - (C - D)\sin\beta\, x] \ldots\ldots\ldots (13c)$$

and,

$$\frac{d^4y}{dx^4} = -\,4\,\beta^4\, e^{\beta x}\,(A\cos\beta\, x + B\sin\beta\, x)$$

$$- 4\,\beta^4\, e^{-\beta x}\,(C\cos\beta\, x + D\sin\beta\, x) \ldots\ldots\ldots\ldots (13d)$$

Under the condition of base fixed and top free, the slope and deflection are zero at the base of the wall where $x = 0$, whereas, at the top of the wall where $x = h$, the moment and shear are zero. Thus, by setting, for the proper values of x, the appropriate one of Equations (13) equal to zero, and solving, the values of the four arbitrary constants may be determined, and are expressed as follows:

$$A = \frac{w\, r^2}{\beta\, E_0\, t}\left\{\frac{e^{2\theta}\,[2\cos^2\theta + \theta\,(\sin 2\,\theta + \cos 2\,\theta - 4\cos^2\theta)] - \theta}{(1 + e^{2\theta})^2 + 4\, e^{2\theta}\cos^2\theta}\right\} = F\, C_1 \ldots (14a)$$

$$B = \frac{w\, r^2}{\beta\, E_0\, t}\left\{\frac{1 + \theta + e^{2\theta}\,[1 + \sin 2\,\theta - \theta\,(\cos 2\,\theta + \sin 2\,\theta)]}{(1 + e^{2\theta})^2 + 4\, e^{2\theta}\cos^2\theta}\right\} = F\, C_2 \ldots (14b)$$

$$C = -\,F\,(C_1 + \theta) \ldots\ldots\ldots\ldots\ldots\ldots (14c)$$

and,

$$D = F\,(1 - \theta - 2\, C_1 - C_2) \ldots\ldots\ldots\ldots (14d)$$

in which,

$$\theta = \beta\, h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15a)$$

and,

$$F = \frac{w\, r^2}{\beta\, E_0\, t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15b)$$

It may be readily seen that there is only one variable in Equations (14) so that for any given value of this term the expression in brackets has a definite value. Thus, by allotting values to θ at regular intervals, it is best to express these values in terms of π so as to facilitate computations; the values of the arbitrary constants may be determined, and thus by substitution in Equations (13*b*), (13*c*), and (13*d*), the moment, shear, and load, for any value of θ, at any wall height, may be determined. In Fig. 2, Functions C_1 and C_2 are shown graphically.

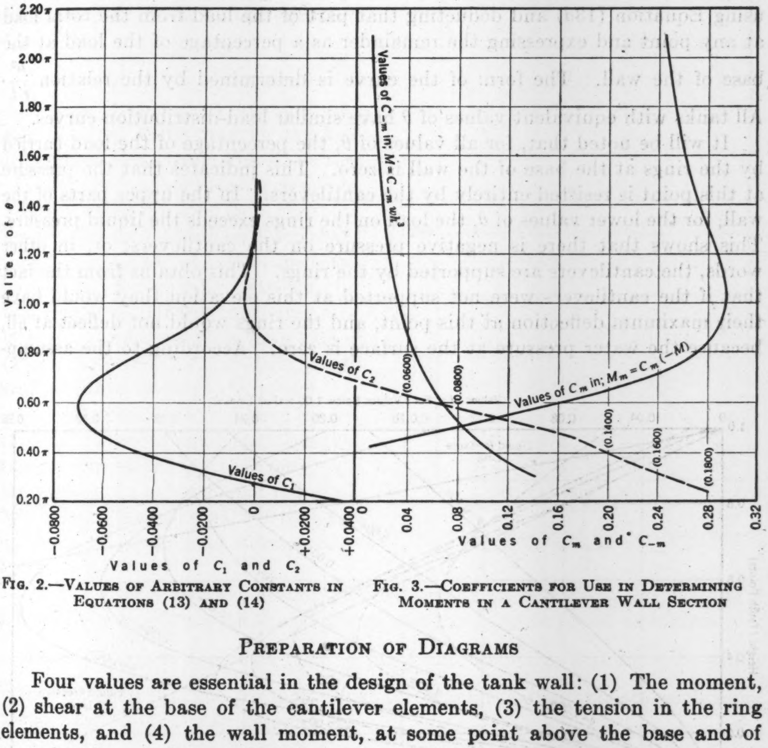


FIG. 2.—VALUES OF ARBITRARY CONSTANTS IN EQUATIONS (13) AND (14)

FIG. 3.—COEFFICIENTS FOR USE IN DETERMINING MOMENTS IN A CANTILEVER WALL SECTION

PREPARATION OF DIAGRAMS

Four values are essential in the design of the tank wall: (1) The moment, (2) shear at the base of the cantilever elements, (3) the tension in the ring elements, and (4) the wall moment, at some point above the base and of opposite sign to that at the base, due to the upper ring elements supporting the cantilevers.

By the use of Equation (13*b*) curves were plotted for the various values of θ, by which the moment at any height of the wall could be determined. However, as the moments at only two points are needed in the design, the negative moment at the base, and the positive moment at some point above the base, those curves were used to prepare the curves given in Fig. 3. In Fig. 3 the value of C_{-m}, cantilever negative moment coefficient, increases for diminishing values of θ, becoming 0.167 when θ becomes 0, the limiting case when the wall is a pure cantilever. The curve for C_m in Fig. 3 shows that the maximum value of the ratio of the positive moment to the negative moment occurs for a θ-value of approximately 1.0 π, and amounts to about 29.4% of the negative moment. This percentage decreases rapidly for θ-values below 1.0 π and less rapidly for θ-values above 1.0 π.

Fig. 4 gives the "load-distribution curves" for values of θ from 0.40 π to 2.00 π. The curves give the load carried by the circular rings, expressed as a percentage of the pressure at the base of the wall. These values were determined for the values of θ indicated by finding the load on the cantilever using Equation (13*d*) and deducting that part of the load from the total load at any point and expressing the remainder as a percentage of the load at the base of the wall. The form of the curve is determined by the relation $\frac{h^2}{r\,t}$. All tanks with equivalent values of θ have similar load-distribution curves.

It will be noted that, for all values of θ, the percentage of the load carried by the rings at the base of the wall is zero. This indicates that the pressure at this point is resisted entirely by the cantilevers. In the upper parts of the wall, for the lower values of θ, the load on the rings exceeds the liquid pressure. This shows that there is negative pressure on the cantilevers; or, in other words, the cantilevers are supported by the rings. This obtains from the fact that if the cantilevers were not supported at this elevation they would have their maximum deflection at this point, and the rings would not deflect at all, because the water pressure at the surface is zero. According to the assumptions of this paper the displacements of the rings and the cantilevers at all points are equal, and thus the rings at the higher elevations support not only the direct liquid pressure but also some load coming to them from the vertical cantilever elements. For the higher values of θ, the load on the rings in the upper parts of the wall is practically equal to the liquid pressure, as the curves

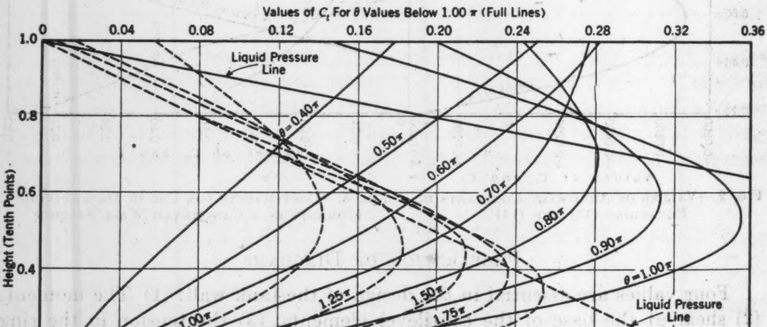


FIG. 4.—TENSION IN CIRCULAR RINGS ($T = C_t\,w\,h\,r$)

show. This indicates that for the deeper tanks, or for those of smaller diameter, restraint at the base has little or no effect at the upper parts of the wall, as would be expected.

The shear is determined by use of Equation (13c). The coefficient for determining its value has been plotted in Fig. 5 and for ease of application has been expressed in percentages of the total load; thus, the shear at the base of the wall is found by the equation

$$V = C_v \frac{w\,h^2}{2} \ldots\ldots (16)$$

Although computed by use of Equation (13c) its value may be checked by determining, from Fig. 4, values of C_t for the load on the ring elements and deducting this load from the total liquid pressure. It will be noted that, for a θ-value of about $1.08\,\pi$, the total load is divided equally between the ring elements and the cantilever elements. For θ-values less than $1.08\,\pi$ (which will include practically all shallow tanks of large diameter) the load on the cantilever will be in excess of 50% of the total load, and thus the ring tension will be less than 50% and the total ring reinforcement required will be less than one-half of that required if all of the load were carried by the rings.

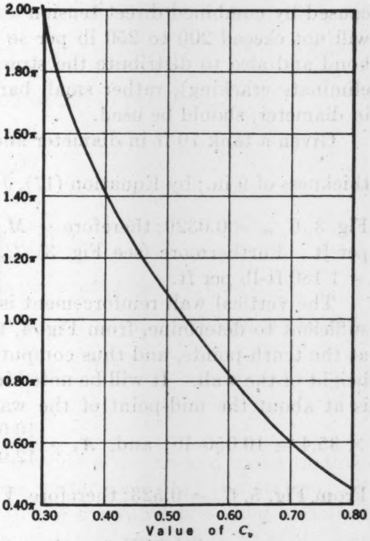


FIG. 5.—COEFFICIENTS FOR USE IN DETERMINING SHEAR AT BASE OF WALL

It is desirable to have a simplified expression to determine the value of θ for use in connection with the diagrams and this may be found as follows:

Substituting, in Equation (12), $\frac{t^3}{12}$ for I and assuming that $E_0 = E$, which is approximately correct for most tanks for which the diagrams are applicable, the expression for θ may be given as,

$$\theta = \frac{h}{\pi}\sqrt[4]{\frac{3}{r^2\,t^2}} = h\sqrt{\frac{0.176}{r\,t}} \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Note that the value of θ is expressed in terms of π and it is so indicated on the diagrams.

EXAMPLE

The use of the diagrams in design will be demonstrated by a numerical example. Certain conditions have been assumed in the analysis, and the designer should consider how closely the actual conditions fulfill these requirements. The most important are: The degree of restraint at the base of the wall; the moment of inertia of the reinforced section; the modulus of elasticity of the concrete; and the extent to which the concrete itself resists tension.

As in most structures where water-tightness is required, a rather rich concrete should be specified, and to reduce cracking, a low stress in the ring steel should be used; 12 000 lb per sq in. is recommended. The percentage of the ring reinforcement should be limited so that the tensile stress in the concrete, caused by combined direct tension and contraction due to curing and drying, will not exceed 200 to 250 lb per sq in. In order to secure better values in bond and also to distribute the stresses properly (both of which will help to eliminate cracking), rather small bars, preferably those not exceeding ⅝ in. in diameter, should be used.

Given a tank 70 ft in diameter and a water depth of 12.5 ft, assume a wall thickness of 9 in.; by Equation (17), $\theta = 12.5\sqrt{\dfrac{0.176}{35.4 \times 0.75}} = 1.018\,\pi$. From Fig. 3, $C_{-m} = 0.0329$; therefore $-M = 0.0329 \times 62.5 \times (12.5)^3 = 4\,020$ ft-lb per ft. Furthermore (see Fig. 3), $C_m = 0.294$; therefore $M = 0.294 \times 4\,020 = 1\,180$ ft-lb per ft.

The vertical wall reinforcement is determined from these moments. It is sufficient to determine, from Fig. 4, the values of the ring tension coefficients at the tenth-points, and thus compute the percentage of ring steel for the full height of the wall. It will be noted in Table 1 that the maximum ring tension is at about the mid-point of the wall; that is, $T_{\max} = 0.364 \times 62.5 \times 12.5 \times 35.4 = 10\,050$ lb; and, $A_s = \dfrac{10\,050}{12\,000} = 0.84$ sq in. per ft of wall height. From Fig. 5, $C_v = 0.523$; therefore, $V = 0.523\,\dfrac{62.5 \times (12.5)^2}{2} = 2\,560$ lb per ft.

TABLE 1.—COMPUTATION OF RING STEEL

Tenth-points with Point 1.0 at the top (1)	Coefficient C_t (2)	Ring tension, in pounds per foot of wall height (3)	Area of steel, A_s, in square inches per foot of height (4)	Tenth-points with Point 1.0 at the top (1)	Coefficient C_t (2)	Ring tension, in pounds per foot of wall height (3)	Area of steel, A_s, in square inches per foot of height (4)
1.0	0.137	3 780	0.32	0.5	0.364	10 050	0.84
0.9	0.204	5 640	0.47	0.4	0.334	9 220	0.77
0.8	0.266	7 350	0.62	0.3	0.262	7 240	0.61
0.7	0.322	8 890	0.74	0.2	0.158	4 360	0.37
0.6	0.356	9 820	0.84	0.1	0.053	1 460	0.12

The wall should be investigated for external pressures in the same manner. For earth loading this may be done by substituting for the earth pressure an equivalent fluid; and, if the external loading extends to either a greater or less height than the internal loading, also substitute the revised value for the height. For external loading the rings are in compression and thus do not require reinforcement.

The wall thickness should be checked to make certain that the tension stress in the concrete is not excessive. The curing stress may be determined from the usual formula.[5]

$$f_c = \frac{C\,E_c\,n\,p}{1 + n\,p} \quad \text{(18)}$$

in which C = coefficient of contraction of the concrete, assumed as 0.0003. Use the transformed section to find the stress in the concrete due to the ring tension:

$$f_t = \frac{T}{A_c\,(1 + n\,p)} \quad \text{(19)}$$

The sum of f_c and f_t should not exceed 200 to 250 lb per sq in.:

$$f_c = \frac{0.0003 \times 2\,500\,000 \times 12 \times 0.0077}{1 + 12 \times 0.0077} = 64$$

and

$$f_t = \frac{10\,050}{9 \times 12\,(1 + 12 \times 0.0077)} = \underline{85}$$

and, total stress, in pounds per square inch = 149.

The 9-in. wall is adequate. Note that 0.0077 is the maximum percentage of the ring reinforcement; that is, $p = \frac{0.84}{9 \times 12} = 0.0077$.

CONCLUSION

It is intended that this paper should provide a simple and direct method of designing circular concrete tanks. The writer feels that the accuracy of the rigorous analysis has been retained in the diagrams presented and that by their use circular concrete tanks can be designed, not only more easily, but also more economically, than with the approximate or rule-of-thumb methods now commonly in use.

ACKNOWLEDGMENT

The paper is based upon a thesis entitled "The Design of Circular Concrete Tanks," presented by the writer to the University of Nebraska in 1937, in partial fulfillment of the requirements for the professional degree of Civil Engineer.

---

[5] "Principles of Reinforced Concrete Construction," by F. E. Turneaure, Hon. M. Am. Soc. C. E., and Ward B. Maurer, Assoc. M. Am. Soc. C. E., Third Edition, p. 36.

DISCUSSION

Carl A. Rosengarten,[6] **Assoc. M. Am. Soc. C. E.** (by letter).—Although a circular concrete tank is economical so far as having the least perimeter for a given area, it would obviously be extravagant if it were designed only as a common cylinder with all stresses taken by the steel, and if a perfectly good structural material, such as concrete, were used merely as a filler. A design that utilizes the structural properties of the concrete more efficiently, therefore, should be more economical, as demonstrated in Mr. Salter's paper in which he considers the lower part of the wall to act as a cantilever.

However, if the wall were considered as bending vertically for its full height from a fixed-end condition at the base, to a supporting ring of tension steel at the top, the concrete would be utilized to a still greater extent as more of the wall would be stressed in bending. Water-tightness can be improved by pre-stressing the steel tension ring at the top which, again, is made possible by increasing the use of the concrete in compression.

In designing concrete tanks of various shapes, the writer has found that, wherever it is possible to secure a support at the top of the wall, it is generally more economical to design the wall spanning vertically. This is usually true because the triangular loading of liquid pressure is peculiarly adapted to this condition. The greater part of the load (two-thirds) goes to the bottom of the wall where the floor is already available as a support, leaving only one-third of the load to be taken care of by beams or ties at the top. A fixed-end condition at the base provides a still more favorable distribution of load, giving a reaction of four-fifths of the load at the bottom and one-fifth at the top.

Without attempting to check the rigorous mathematical analysis, the writer notes, in Mr. Salter's illustrative example, that he has determined the area of ring steel by dividing ring tension by a constant unit stress of 12 000 lb per sq in. The stretch or elongation of the steel in the tension rings varies from a maximum near the top of the wall to a theoretical zero at the bottom, according to the deflection of the cantilever wall. According to the fundamental theory that the stress is proportional to the strain or elongation, the unit stress in the ring steel must also vary from a maximum for the rings near the top of the wall to zero for a ring at the bottom.

Since the distribution of stress in any two-way system is complicated by many variables, it would seem advantageous to simplify designs wherever possible without sacrificing economy. In the case of relatively low tanks this can be accomplished by considering the wall as a one-way slab, spanning vertically from a fixed-end condition at the base to a support at the top consisting of a pure tension ring. If this ring is not pre-stressed, adjustment to the vertical steel can be made for the deflection (or, settlement of support) caused by the elongation of the top ring steel.

---

[6] Architectural Engr., The Ballinger Co., Philadelphia, Pa.

I. K. Silverman,[7] Assoc. M. Am. Soc. C. E. (by letter).—It is interesting to note that the problem discussed by Mr. Salter is a special case of a more general problem which includes the arch dam. The solution of the problem of the cylindrical tank has been known and used for many years, its first solution being given by H. Müller-Breslau.[8]

The author has neglected the Poisson effect which is equivalent to assuming that m (Poisson's number) is equal to infinity. Equation (10*b*), however, can be obtained from Müller-Breslau's solution by inserting $\mu = \frac{1}{m} = 0$. The type of formula exemplified by Equation (10*b*) appears throughout engineering literature in the discussion of problems of beams on elastic foundations, and therefore puts the problem of the circular tank in that category. Thus, the cantilever elements carry the difference between the water load and the reaction furnished by the ring elements. This reaction is proportional to the value of the deflection y. This statement is expressed by Equation (10*b*). The arch dam subjected to water load is amenable to the same method of attack, and the trial-load method for the analysis of arch dams is based on the equating of the deflections of the horizontal (arch) elements to those of the vertical (cantilever) elements. The loads on the cantilever elements are similar to that shown in Fig. 1.

Due to symmetry, the forces acting on an element in a cylindrical tank of varying wall thickness are as shown in Fig. 6. The equations of equilibrium are as follows:

Equilibrium against moment:

$$Q = -\frac{dM}{dz} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20a)$$

equilibrium in the radial direction:

$$-\frac{dQ}{dz} + \frac{T}{r} = w\,z \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20b)$$

and, equilibrium in the vertical direction:

$$\frac{dS}{dz} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20c)$$

The movement in the radial direction is denoted by y. The corresponding movement in the vertical direction is given by "u." The strains in the radial and vertical directions are accordingly defined as:

---

[7] Asst. Engr., U. S. Bureau of Reclamation, Denver, Colo.

[8] "Berechnung von Behältern," by Th. Pöschl, Berlin, 1926, Second Edition, p. 132 ff; "Beton und Eisen," Vol. 7, 1908, p. 150; "Concrete and Constructional Engineering," by H. Carpenter, April, 1927, and June, 1929; "Reinforced Concrete Reservoirs and Tanks," by W. S. Gray, London, pp. 11–15; and "Arched Dams," by B. A. Smith, *Transactions*, Am. Soc. C. E., Vol. 83 (1919–1920), p. 2027.

DISCUSSION

Carl A. Rosengarten,[6] Assoc. M. Am. Soc. C. E. (by letter).—Although a circular concrete tank is economical so far as having the least perimeter for a given area, it would obviously be extravagant if it were designed only as a common cylinder with all stresses taken by the steel, and if a perfectly good structural material, such as concrete, were used merely as a filler. A design that utilizes the structural properties of the concrete more efficiently, therefore, should be more economical, as demonstrated in Mr. Salter's paper in which he considers the lower part of the wall to act as a cantilever.

However, if the wall were considered as bending vertically for its full height from a fixed-end condition at the base, to a supporting ring of tension steel at the top, the concrete would be utilized to a still greater extent as more of the wall would be stressed in bending. Water-tightness can be improved by pre-stressing the steel tension ring at the top which, again, is made possible by increasing the use of the concrete in compression.

In designing concrete tanks of various shapes, the writer has found that, wherever it is possible to secure a support at the top of the wall, it is generally more economical to design the wall spanning vertically. This is usually true because the triangular loading of liquid pressure is peculiarly adapted to this condition. The greater part of the load (two-thirds) goes to the bottom of the wall where the floor is already available as a support, leaving only one-third of the load to be taken care of by beams or ties at the top. A fixed-end condition at the base provides a still more favorable distribution of load, giving a reaction of four-fifths of the load at the bottom and one-fifth at the top.

Without attempting to check the rigorous mathematical analysis, the writer notes, in Mr. Salter's illustrative example, that he has determined the area of ring steel by dividing ring tension by a constant unit stress of 12 000 lb per sq in. The stretch or elongation of the steel in the tension rings varies from a maximum near the top of the wall to a theoretical zero at the bottom, according to the deflection of the cantilever wall. According to the fundamental theory that the stress is proportional to the strain or elongation, the unit stress in the ring steel must also vary from a maximum for the rings near the top of the wall to zero for a ring at the bottom.

Since the distribution of stress in any two-way system is complicated by many variables, it would seem advantageous to simplify designs wherever possible without sacrificing economy. In the case of relatively low tanks this can be accomplished by considering the wall as a one-way slab, spanning vertically from a fixed-end condition at the base to a support at the top consisting of a pure tension ring. If this ring is not pre-stressed, adjustment to the vertical steel can be made for the deflection (or, settlement of support) caused by the elongation of the top ring steel.

---

[6] Architectural Engr., The Ballinger Co., Philadelphia, Pa.

I. K. SILVERMAN,[7] ASSOC. M. AM. SOC. C. E. (by letter).—It is interesting to note that the problem discussed by Mr. Salter is a special case of a more general problem which includes the arch dam. The solution of the problem of the cylindrical tank has been known and used for many years, its first solution being given by H. Müller-Breslau.[8]

The author has neglected the Poisson effect which is equivalent to assuming that m (Poisson's number) is equal to infinity. Equation (10*b*), however, can be obtained from Müller-Breslau's solution by inserting $\mu = \frac{1}{m} = 0$. The type of formula exemplified by Equation (10*b*) appears throughout engineering literature in the discussion of problems of beams on elastic foundations, and therefore puts the problem of the circular tank in that category. Thus, the cantilever elements carry the difference between the water load and the reaction furnished by the ring elements. This reaction is proportional to the value of the deflection y. This statement is expressed by Equation (10*b*). The arch dam subjected to water load is amenable to the same method of attack, and the trial-load method for the analysis of arch dams is based on the equating of the deflections of the horizontal (arch) elements to those of the vertical (cantilever) elements. The loads on the cantilever elements are similar to that shown in Fig. 1.

Due to symmetry, the forces acting on an element in a cylindrical tank of varying wall thickness are as shown in Fig. 6. The equations of equilibrium are as follows:

Equilibrium against moment:

$$Q = -\frac{dM}{dz} \quad \text{(20a)}$$

equilibrium in the radial direction:

$$-\frac{dQ}{dz} + \frac{T}{r} = w\,z \quad \text{(20b)}$$

and, equilibrium in the vertical direction:

$$\frac{dS}{dz} = 0 \quad \text{(20c)}$$

The movement in the radial direction is denoted by y. The corresponding movement in the vertical direction is given by "u." The strains in the radial and vertical directions are accordingly defined as:

---

[7] Asst. Engr., U. S. Bureau of Reclamation, Denver, Colo.

[8] "Berechnung von Behältern," by Th. Pöschl, Berlin, 1926, Second Edition, p. 132 ff; "Beton und Eisen," Vol. 7, 1908, p. 150; "Concrete and Constructional Engineering," by H. Carpenter, April, 1927, and June, 1929; "Reinforced Concrete Reservoirs and Tanks," by W. S. Gray, London, pp. 11–15; and "Arched Dams," by B. A. Smith, *Transactions*, Am. Soc. C. E., Vol. 83 (1919–1920), p. 2027.

Radial—

$$\epsilon_1 = \frac{y}{r} \dots\dots\dots\dots\dots\dots\dots\dots (21a)$$

and, vertical

$$\epsilon = \frac{du}{dz} \dots\dots\dots\dots\dots\dots\dots\dots (21b)$$

From the generalized Hooke's law the stress on the element in the vertical direction is

$$\frac{S}{t} = \frac{E}{(1 - \mu^2)} (\epsilon + \mu\, \epsilon_1) \dots (22a)$$

The tangential hoop stress is

$$\frac{T}{t} = \frac{E_0}{(1 - \mu^2)} (\epsilon_1 + \mu\, \epsilon) \dots (22b)$$

in which μ = Poisson's ratio = $\frac{1}{4}$ to $\frac{1}{6}$ for concrete. From considerations of curvature

$$M = \frac{E\, I}{(1 - \mu^2)} \frac{d^2y}{dz^2} \dots (23)$$

Integrating Equation (20c), using the free boundary condition at $z = 0$, together with Equations (21) and (22a), $S = 0$, or

$$\epsilon + \mu\, \epsilon_1 = \frac{du}{dz} + \mu \frac{y}{r} = 0 \;. (24)$$

Using Equations (20a), (20b), (23), and (24) the differential equation for the radial movement, y, is as follows:

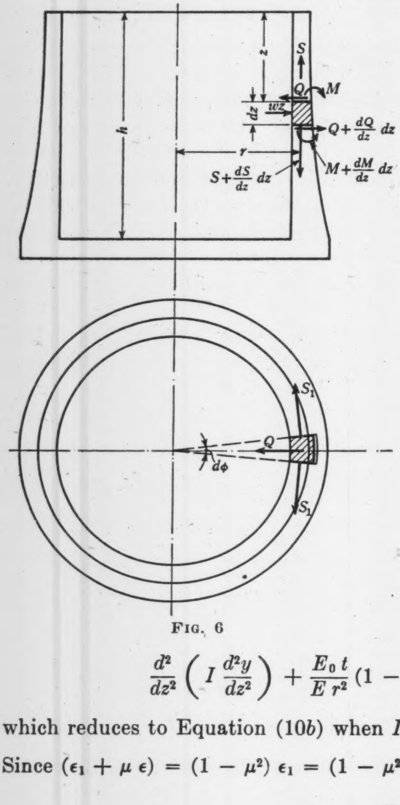


FIG. 6

$$\frac{d^2}{dz^2}\left(I \frac{d^2y}{dz^2}\right) + \frac{E_0\, t}{E\, r^2} (1 - \mu^2)\, y = \frac{w\, z\, (1 - \mu^2)}{E} \dots\dots\dots (25)$$

which reduces to Equation (10b) when I is constant and $\mu = 0$, $z = (h - x)$. Since $(\epsilon_1 + \mu\, \epsilon) = (1 - \mu^2)\, \epsilon_1 = (1 - \mu^2) \frac{y}{r}$ the hoop tension is

$$T = \frac{E_0\, t}{r}\, y \dots\dots\dots\dots\dots\dots\dots\dots (26)$$

so that the unit tensile hoop stress is proportional to the radial movement y.

When the walls of the tank have a triangular section, Equation (25) can be integrated in terms of Bessel functions as shown by F. Tölke.[9] For a trapezoidal section or one bounded by a parabola and a straight line approximate methods are available.[10]

---

[9] "Der Bauingenieur," by F. Tölke, January 8, 1937.

[10] "Berechnung von Behältern," by Th. Pöschl, p. 155 ff.

FRANK J. MCCORMICK,[11] ESQ. (by letter).—The author's repeated statement that he has presented a strictly rigorous analysis is open to question, because an analysis based on the assumption of ring stress uniformly distributed over the thickness of the wall is not strictly rigorous except for very thin walls. However, for a wall of homogeneous elastic material, it can be shown that his assumption is justified, in the design of tanks of usual proportions, by making use of Lamé's theory of thick-walled cylinders as described by Professor Timoshenko.[12, 13] He shows that the ring stress in the wall of a circular cylinder subjected to internal pressure is given by the expression

$$f_s = b \times \frac{a^2\,p}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right) \quad \text{(27)}$$

in which: a = the internal radius of the tank; b = the external radius of the tank; and r = the radius at which f_s is given by Equation (27). Equation (27) indicates maximum ring stress at the inner surface of the wall, and it is easy to determine that the ratio of the maximum stress to the minimum stress at any section is given by the expression

$$\frac{\text{Maximum}\, f_s}{\text{Minimum}\, f_s} = \frac{a^2 + b^2}{2\,a^2} \quad \text{(28)}$$

For the tank used in the author's example, $\frac{b}{a} = \frac{35.75}{35} = 1.0214$; and, $\frac{\text{Maximum}\, f_s}{\text{Minimum}\, f_s} = 1.021$, which indicates a variation of only 2.1% in the stress across any section. For $\frac{b}{a} = 1.1$ the variation is found to be only $10\frac{1}{2}$ per cent.

The position of the resultant, S, may be derived by writing the moment of f_s about the center of the tank,[14] integrating throughout the thickness of the wall, and dividing this quantity by the total stress, S. Thus,

$$r_s = \frac{\int f_s\, r\, dr}{\int f_s\, dr} = b \times \frac{\frac{1}{2}\left[1 - \left(\frac{a}{b}\right)^2\right] + \log_e \frac{b}{a}}{\frac{b}{a} - \frac{a}{b}} \quad \text{(29)}$$

in which r_s = the radius to the resultant total stress, S. In the author's example, $\frac{b}{a} = 1.0214$ and from Equation (29) $r_s = 34.3737$ ft. The distance from the center of the wall to the point of action of the resultant stress, S, is then only 0.0156 in., or 0.0017 t. The tacit assumption by the author, that there is no moment on a vertical section and that the stress is uniformly distributed across the section, may thus be justified.

For a tank wall the flexural rigidity is more accurately represented by the expression $\frac{E\,t^3}{12\,(1 - \mu^2)}$ (in which μ = Poisson's ratio) rather than by the author's[15] term $\frac{E\,t^3}{12}$. Because of the uncertainty involved in the evaluation of this quantity for reinforced concrete, and because the value usually chosen for μ is small (less than 0.25) probably no serious error results from the use of the simpler expression.

---

[11] Dept. of Applied Mechanics, Kansas State Coll., Manhattan, Kans.

[12] "Strength of Materials," by S. Timoshenko, Part II, p. 531.

[13] "Über die Spannungsverteilung in zylindrischen Behälterwänden," by H. Reissner, *Beton und Eisen*, B. 7, H. VI, 1908, p. 150.

[14] *Loc. cit.*, p. 151.

After evaluation of the constants in Equations (14) by the introduction of the boundary condition, $y = 0$ at $x = 0$, the author later discovers no ring stress at the base of the wall. This, he concludes, indicates that the pressure at this point is resisted entirely by the cantilevers. Any other conclusion would be much more alarming, as this result is implicit in the boundary condition just mentioned, and merely indicates the consistency of the intervening mathematics.

It should be mentioned that Equation (10), and its solution, Equation (11), were published by C. Runge in 1904.[16] H. Reissner extended the solution to include walls of variable thickness in 1908.[13] The latter work, reproduced and further extended by V. Lewe,[17] includes tables and curves to aid in the computation of the radial displacements, ring stresses, and cantilever moments. In part the tables and curves are equivalent to those given by the author in Figs. 3 and 4; but they also include data pertaining to tanks with walls of triangular section as well as to those with walls of constant thickness. In this work, furthermore, may be found a rather incomplete table of constants for use in the analysis of shallow circular tanks with walls of trapezoidal section.

H. B. MUCKLESTON,[18] M. AM. SOC. C. E. (by letter).—Stresses in a circular reinforced concrete tank are analyzed by an interesting method in this paper. In the course of this analysis, the author makes certain simplifying assumptions without which the resulting differential equation probably could not be integrated:

(1) Implicit in Equations (3) and (4), that the area of the ring steel is uniform from bottom to top of the wall;

(2) Implicit in Equations (9) and (17), that the moment of inertia of the elementary cantilevers is uniform from top to bottom (which means that the steel areas in both faces are uniform);

(3) Implicit in Equation (9), that deformation of the cantilever elements from shear may be neglected;

(4) That E and E_0 may be considered as equal; and,

(5) That Poisson's ratio need not be considered.

Although the last three of these may be of small importance, the first two are fundamental and must be closely realized in design or the entire analysis becomes null.

---

[15] "Strength of Materials," by S. Timoshenko, Part II, p. 475; see also "Applied Elasticity," by John Prescott, p. 391.

[16] "Über die Formänderung eines zylindrischen Wasserbehälters durch den Wasserdruck," by C. Runge, *Zeitschrift für Mathematik und Physik*, B. 51, 1904, p. 254; see also "Die graphische Statik der Baukonstruktionen," by H. Müller-Breslau, B. 2, II Abt., Aufl. 2, 1925, p. 207.

[17] "Die statische Berechnung der Flüssigkeitsbehälter," by V. Lewe, *Handbuch für Eisenbetonbau*, B. 5, 3 Aufl., 1923, pp. 87–96.

[18] Cons. Engr., Vancouver, B. C., Canada.

In Table 1, the author has adjusted the ring-steel areas to the computed ring stresses on a basis of 12 000 lb per sq in. If the tank were built to these areas, the computed ring stresses could not exist since they were computed on the hypothesis that the steel area was uniform. In fact if they did exist, the deflection y would be uniform from bottom to top, which is contrary to the hypothesis.

Either the ring steel and cantilever steel must be kept uniform (in which case the analysis is applicable), or they may be variable, in which case the analysis does not apply and a solution must be sought by some trial-load method.

If they are kept uniform, there is a question whether a tank so designed will show any economy over a tank of similar dimensions designed according to the simple cylinder hypothesis. In the example the maximum ring steel area is 0.77% of the concrete area. If the hypothesis is to apply, the same proportion must be used for the entire height. The ring steel will then be 0.77% of the entire volume of concrete in the wall. To this must be added the vertical steel which will scarcely be less than 0.75% of the volume. This is a total steel for the wall, amounting to 1.52% of the wall volume.

Using the same wall thickness and steel stress, the ring steel in a design by the simple cylinder hypothesis would require about 1.07% of the wall volume, to which about 0.18% must be added for vertical spacing rods. The difference, for a tank of these dimensions, is not far from 2 700 lb of steel. Moreover, the steel in the latter case would be cheaper to erect, and the wall thickness might well be somewhat less.

It might be argued that leaving the cantilever action out of consideration will result in ring cracks near the base of the wall. It is questionable if experience would bear out this contention. Many tanks have been built on that basis and it is not on record, so far as the writer knows, that even a large proportion of them show such cracks. If they are feared, the steel in the lowest foot might be computed on a basis of, say, 8 000 lb per sq in. That would reduce thc outward deflection to a point where the shear resistance of the concrete could take care of it easily. In the case in question, the outward movement, if there were a smooth, frictionless joint at the base, would be only about 0.0117 in.

ROBERT B. B. MOORMAN,[19] ASSOC. M. AM. SOC. C. E. (by letter).—This paper is a very worth while contribution to American engineering literature on the subject of circular concrete tanks. In England W. S. Gray has published two books[20, 21] on the design of reinforced concrete tanks and water-towers. The information on the analysis of circular tanks in these two books was obtained from an earlier publication[22] by H. Carpenter explaining the method by H. Reissner.[23]

---

[19] Associate Prof., Civ. Eng., Univ. of Missouri, Columbia, Mo.

[20] "Reinforced Concrete Reservoirs and Tanks," by W. S. Gray, Concrete Publications Limited, London, England, 1931.

[21] "Reinforced Concrete Water Towers, Bunkers, Silos and Gantries," by W. S. Gray, Concrete Publications Limited, London, England, 1933.

[22] *Concrete and Constructional Engineering*, Concrete Publications Limited, London, England, Vol. 22, April, 1927, p. 237, and Vol. 24, June, 1929, p. 345.

[23] "Handbuch für Eisenbetonbau," Vol. V.

Figs. 2, 3, and 4, however, are more comprehensive. According to Reissner's theory all tanks with equivalent K-values $\left(K = \frac{12\,h^4}{r^2\,t^2}\right.$, in which r = radius of inside of tank$\left.\right)$ have similar load-distribution curves. This K-value is similar to the author's θ^4. The author bases his curves on equivalent θ-values, whereas the curves presented by Gray are based on equivalent K-values.

Equation (17) would be understood more clearly if it were written

$$\theta = \frac{h}{\pi}\sqrt[4]{\frac{3}{r^2\,t^2}}\,\pi = h\sqrt{\frac{0.176}{r\,t}}\,\pi \;\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

This equation may be simplified further by letting $R\,h = r$ and $T\,h = t$ and writing the expression for θ as

$$\theta = \frac{0.419}{\sqrt{R\,T}}\,\pi \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

The work involved in computing θ by use of Equation (30) or (31) would probably be about the same when reviewing a tank of given dimensions. However, Equation (31) is more flexible in that only relative dimensions are needed to compute the value of θ.

Using the data of the author's example, the bending moment (both positive and negative) and shear are computed as follows:[20] Since the curves presented by Gray are for values of K equal to 0, 10, 100, 1 000, and 10 000, only a rough check can be expected. For this example

$$K = \frac{12\,(12.5)^4}{(35)^2\,(0.75)^2} = 425.$$

Upon sketching a curve for $K = 425$,[24] values are obtained for the coefficient, C_t, by use of which the ring tension can be computed in pounds per foot of wall height (see Table 2).

TABLE 2.—COMPUTATION OF RING TENSION

Description	TENTH-POINTS, WITH POINT 1.0 AT THE TOP										Average
	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	
Coefficient, C_t	0.13	0.18	0.27	0.31	0.36	0.37	0.33	0.25	0.15	0.06	0.241
Ring tension, in pounds per foot of wall height	3 560	4 920	7 390	8 480	9 850	10 120	9 030	6 840	4 100	1 640	....

A coefficient[25] $C_{-m} = 0.036$ for the negative moment gives $-M = 0.036 \times 62.5 \times (12.5)^3 = 4\,390$ ft-lb per ft. The value of the positive moment is taken as one-fourth to one-third of the negative moment or, using an average of this range, $M = 0.29 \times 4\,390 = 1\,273$ ft-lb per ft.

---

[24] From curves by Carpenter; "The Calculation of Cylindrical Tanks with Rectangular, Triangular or Trapezoidal Wall Sections," by H. Carpenter, *Concrete and Constructional Engineering*, Concrete Publications Limited, London, England, June, 1929, Fig. 2, p. 347.

[25] Selected from curves; *loc. cit.*, Fig. 3, p. 348.

The shear can be obtained by subtracting the portion of the horizontal load carried by the rings from the total horizontal load; or, $V = (0.5 - 0.241)$ $62.5 \times (12.5)^2 = 2\,539$ lb per ft. The use of the coefficient 0.241 is not quite proper since it is the average coefficient for the ring tension at the top of the tenth-points. These values of the positive and negative moments and shear agree fairly well with the author's values.

Although, in practically all cases, Equations (18) and (19) are satisfactory, the more exact expressions are

$$f_c = \frac{C\,E_c\,n\,p}{1 + (n - 1)\,p} \qquad (32)$$

and

$$f_s = \frac{T}{A_c\,[1 + (n - 1)\,p]} \qquad (33)$$

That is, $f_c = 65$, $f_s = 86$, and the total stress is 151 lb per sq in.

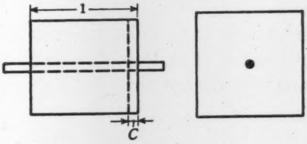


FIG. 7

A derivation of the expression for unit stress in the concrete due to contraction (Equation (32)), which is similar to Equation (19), follows: A steel bar, embedded in concrete, is shown in Fig. 7. The cross-section of the steel is $A\,p$ and of the concrete is $A\,(1 - p)$. In a unit length the concrete tends to shorten a distance C, which is the coefficient of contraction of the concrete. The deformation in the steel must equal the deformation in the concrete, or

$$\frac{f_s}{E_s} = C - \frac{f_c}{E_c} \qquad (34)$$

In order that the system shall be in equilibrium the summation of the horizontal stresses must equal zero, or

$$A\,p\,f_s = A\,(1 - p)\,f_c \qquad (35)$$

from which

$$f_s = \frac{(1 - p)}{p}\,f_c \qquad (36)$$

is obtained. From Equation (34):

$$f_s = C\,E_c\,n - f_c\,n \qquad (37)$$

Equation (32) is derived by combining Equations (36) and (37) and solving for f_c.

DANA YOUNG,[26] ASSOC. M. AM. SOC. C. E. (by letter).—The general analysis of the problem stated in this paper has been developed thoroughly by various European engineers and is presented in standard references such those of Pöschl,[27] Flügge,[28] Love,[29] and Löser and Lewe.[30] However, the work done by the author in determining the constants defined in Equations (14) is of great value in designing relatively shallow tanks.

---

[26] Assoc. Prof. of Civ. Eng., University of Connecticut, Storrs, Conn.

[27] "Berechnung von Behältern," by Th. Pöschl, Berlin, Germany, 1926.

[28] "Statik und Dynamik der Schalen," by W. Flügge, Berlin, Germany, 1934.

[29] "A Treatise on the Mathematical Theory of Elasticity," by A. E. H. Love, Cambridge, England, 1927.

[30] "Behälter," by B. Löser and V. Lewe, in Emperger's *Handbuch für Eisenbetonbau*, Third Edition, Vol. 9, Berlin, Germany, 1934.

Any method of analysis is based on various assumptions which the designer must bear in mind. In this regard it may be of interest to discuss several points in the paper. In Equation (3), the unit tensile stress in the concrete is found by dividing the total stress by the cross-sectional area of the ring. It would be more correct to divide by the area of the transformed section $(1 + n\,p)\,t\,dh$ as the author finally does in Equation (19). Equation (9), which is the formula of flexure for a beam, does not apply strictly to a strip of a plate or shell since the adjoining strips restrain the free transverse deformations. As shown in various references,[31] for the case of a plate or shell the equation of flexure is

$$\frac{E\,I}{1 - \nu^2}\frac{d^2y}{dx^2} = M \qquad (38)$$

in which ν is Poisson's ratio. Hence, using Equation (9) is equivalent to assuming that Poisson's ratio is zero. These corrections would not affect the analysis except that the value of the constant β would then be given by

$$\beta = \sqrt[4]{\frac{E_0\,t\,(1 + n\,p)\,(1 - \nu^2)}{4\,r^2\,E\,I}} \qquad (39)$$

The numerical effect of these corrections would be small except in unusual cases. Furthermore, E_0 will equal E in most cases.

The solution of Equation (10*b*) given by the author is based on the assumption that the tank is of constant cross-section and has the same reinforcement throughout its height; that is, that t and I are constant. Small variations in the percentage of reinforcement probably will not change the results seriously, but caution should be used in any design in which t and I vary considerably. The analysis for such conditions is more complicated, of course, but is well treated in the references of Pöschl[27] and Flügge.[28]

As is clearly stated by the author, the constants determined by Equation (14) are for the case in which the base is completely fixed. The designer must consider carefully whether or not this condition is realized in the tank being designed. If the base is not perfectly restrained against rotation of a vertical element, the negative moment at the base will be less than that calculated, but the values of the maximum ring tension and positive moment will be greater.

It may be well to note that the condition which the author assumes at the top where x equals h—namely, that the shear and moment are zero there—is true only if the water level is exactly at the top of the tank. If this is not so, the part of the tank above the water level exerts a restraint on the deformations at the water level. This effect can be taken care of in the evaluation of the constants, although the numerical work involved would be tedious.

The analysis of a circular tank is simplified when the depth is great, or more accurately, when θ is large (say, greater than π). In this case, it may be assumed that $A = B = 0$, or (what is the same thing) that $C_1 = C_2 = 0$.

---

[31] "Strength of Materials," by S. Timoshenko, New York, N. Y., Part II, 1930, pp. 475 and 517; see also Footnote References 27–30.

Mathematically this follows from the fact that the term in Equation (11) containing these constants represents a damped harmonic wave which starts at the upper boundary and is so rapidly damped that it has little effect on the conditions at the base. Physically, this may be seen from the fact that, in a deep tank, the conditions at the upper boundary will have little effect on those at the base. This is clearly shown in Fig. 2, in which, for values of θ greater than about $1.2\,\pi$, the values of the constants C_1 and C_2 are practically zero. Using this assumption, Equations (14) give $A = B = 0$; $C = -F\,\theta$; and, $D = F\,(1 - \theta)$. Then, further assuming a homogeneous tank of constant thickness and neglecting Poisson's ratio $\left(\text{in which case } \theta = h\sqrt[4]{\dfrac{3}{r^2\,t^2}}\right)$, the moment at the base is found by substituting the foregoing values for the constants in Equation (13*a*). After simplifying, this step reduces to

$$(M)_{x=0} = E\,I\left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{\theta - 1}{2\,\theta^3}\,w\,h^3 \ldots\ldots\ldots\ldots\ (40)$$

which checks closely with Fig. 3 for values of θ greater than π. Similarly, the shear at the base is found to be

$$(V)_{x=0} = E\,I\left(\frac{d^3y}{dx^3}\right)_{x=0} = \frac{1 - 2\,\theta}{\theta^2}\,\frac{w\,h^2}{2} \ldots\ldots\ldots\ldots\ (41)$$

which checks Fig. 5 for large values of θ. The ring tension for this case is given by

$$T = \frac{E\,t}{r}\,y = w\,h\,r\left\{\left(1 - \frac{x}{h}\right) - e^{-a}\left[\cos\theta\frac{x}{h} + \left(1 - \frac{1}{\theta}\right)\sin\theta\frac{x}{h}\right]\right\} \ldots (42)$$

in which $a = \theta\,\dfrac{x}{h}$. Numerical values computed from Equation (42) for the case, $\theta = 1.5\,\pi$, are given in Table 3.

TABLE 3.—RING TENSION COEFFICIENT C_t FOR $\theta = 1.5\,\pi$

$\frac{x}{h}$	FIXED BASE		PINNED BASE
	From Fig. 4	Computed from Equation (42)	Computed from Equation (46)
1.0	0.02	0.007	0
0.8	0.23	0.229	0.219
0.6	0.44	0.442	0.456
0.4	0.53	0.533	0.647
0.2	0.32	0.323	0.571

To show the effect when the wall at the base is not perfectly fixed, it is interesting to investigate the case in which the wall is pinned at the base. Then the results for partial restraint will lie between those for this case and those for the fixed base. Again assuming that $A = B = 0$, the constants C and D are determined from the condition that the deflection and moment at the base of the wall are zero. Substituting these conditions in Equations

(11) and (13*b*), the values of the constants C and D are found to be $C = -F\,\theta$; and, $D = 0$. Then the bending moment is given by

$$M = E\,I\,\frac{d^2y}{dx^2} = -\,2\,E\,I\,F\,\theta\,\beta^2\,e^{-\beta x}\sin\beta\,x \ldots\ldots\ldots\ldots (43)$$

in which the negative sign indicates that the bending produces compression on the inside fibers; that is, the moment is positive according to the convention used by the author. The maximum numerical value for this moment occurs where $x = \frac{\pi}{4\,\beta}$. Substituting this value of x in the moment equation gives the following for the maximum positive bending moment

$$M = \frac{0.161}{\theta^2}\,w\,h^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (44)$$

Values of this positive moment are greater than the positive moment in the case of a tank with a fixed base.

The shear at the base is given by

$$(V)_{x=0} = E\,I\left(\frac{d^3y}{dx^3}\right)_{x=0} = -\,2\,E\,I\,\beta^3\,F\,\theta = -\,\frac{1}{\theta}\,\frac{w\,h^2}{2} \ldots\ldots (45)$$

These shears are less than those for the case of a fixed base.

The ring tension for the case of the wall pinned at the base is given by

$$T = \frac{E\,t}{r}\,y = w\,h\,r\left[\left(1 - \frac{x}{h}\right) - e^{-a}\cos\theta\,\frac{x}{h}\right] \ldots\ldots\ldots (46)$$

The maximum ring tension for this case is greater than for a fixed base. For comparison, the ring tension coefficients as computed from Equations (42) and (46) for the case in which $\theta = 1.5\,\pi$ are shown in Table 3, together with the values read from the author's curves in Fig. 4.

L. J. MENSCH,[32] M. AM. SOC. C. E. (by letter).—Among the great number of similar problems occurring in the construction of tanks, grain elevators, ships and submarines, airplanes, arched dams, and beam and flat-slab foundations on yielding soil, the author's paper treats of the simplest case. By no means is it simple to the every-day engineer; very likely the author has spent years in developing the analysis and in computing the data on which Figs. 2, 3, and 4 are based.

This particular problem was solved in connection with the design of high-pressure boilers in 1878.[33] With the introduction of reinforced concrete, many papers appeared on secondary stresses in tanks, domes, and foundations.[34] After 1904 the Society published the most important analytical and experimental investigations ever made on arched dams, which is a vastly more difficult problem.

---

[32] Civ. Engr. and Constructor, Chicago, Ill.

[33] "Theorie der Elasticitaet und Festigkeit," by F. Grashof, Berlin, Germany, 1878, pp. 317–329.

[34] *Handbook on Reinforced Concrete,* by F. Emperger, Berlin, Germany, 1907, and succeeding editions; Volume Fluessigkeitsbehaelter and other volumes.

American designers will look with awe at the forbidding appearance of Equations (14) and will hesitate to use them because of possible typographical and other errors. Other authors have developed short-cuts and have constructed tables and diagrams.[35] Evidently, the author took cognizance only of the short-cuts published in American literature on the design of circular tanks with a characteristic, $\frac{h^2}{r\,t}$, greater than 10, and, for his own practice, developed a design method for tanks with lower characteristics.

How much confidence do practical engineers and owners have in such involved theories? They have very little, at least in the writer's experience. In all likelihood, the chief engineer or owner will direct the designer to use the same ring steel as if the wall were unrestrained at the bottom and, for greater safety, to use the vertical reinforcement required by this analysis.

Such practice prevailed 30 yr ago (1909) in the design and construction of arched bridges. Laboriously, by the elastic theory, using the summation method, designers determined the moments acting at critical points. They did not use these moments for the design of the arch, but drew the line of pressure for live loads on half of the span after many trials, and made the arch section deep enough so that the line of pressure did not fall outside of the middle third at any point. Many concrete bridges were built at that time which were heavier than stone bridges built many hundreds of years ago, and are still in use. When it became fashionable to use reinforced concrete, an ample supply of patented bars were "thrown in for good measure."

The same conditions may be found even to-day with respect to rigid-frame bridges. Trade associations scatter pamphlets by the ten thousands, praising the summation method. The final result is that such bridges have been built, according to the writer's opinion, for a moment of $\frac{W\,L}{8}$ at the haunches so that nothing serious could happen if the bridge cracked at the center; and sufficient concrete and steel were used in the center so that the bridge could not fail if the haunches or abutments were insufficiently designed.

The author's analysis is based on the assumption that the circular wall is fixed at the bottom, both for rotation and horizontal and vertical movement. This condition is obtainable only in the rare case in which the wall is anchored in a trench of solid rock; in most designs which came to the writer's notice, the designer tried to prevent rotation by a special wide footing beneath the wall and floor. Sometimes the cost of the footing was greater than the cost of the wall. In higher tanks the horizontal force from the water pressure at the bottom of the wall was sufficient to move the wall $\frac{1}{4}$ in., $\frac{1}{2}$ in., and in some known cases 1 in. and more, playing havoc with the ordinarily poorly designed joint between wall and floor, and this practice resulted in serious leakage. The ring stresses become greatly increased due to such movements. If, by some miracle, the wall does not move, it is a practical certainty that there will be a rotation of the wall at the bottom, especially if the footing was skimped in order to save money. Such a rotation will diminish the moment at the base, will increase the positive moment more rapidly, and will also increase ring stresses.

---

[35] *Handbook on Reinforced Concrete,* by F. Emperger, Berlin, Germany, 1907, Third Edition, Vol. 5, 1923, pp. 87–112.

Where a group of tanks is to be built, it is uneconomical to use circular tanks, and covered square tanks may be less costly than open round tanks built separately. In such tanks the walls are computed for top and bottom restraint, and the corner restraints are determined by an analysis similar to that presented by the author.

Domes of large span, covering circular tanks, will exert a large thrust at the top of the wall, resulting in a horizontal movement even at that point, where a heavy ring girder is used to take up the thrust, thereby increasing the ring stresses and causing secondary bending in the walls.

JENS EGEDE NIELSEN,[36] ASSOC. M. AM. SOC. C. E. (by letter).—The author has revived the problem of a cylindrical shell, closed at one end, in its simplest and most practical form where the thickness of the shell is uniform. The general problem was treated in detail by Professor Grashof [37] in 1874. However, his solution of the fourth-order differential equation is not directly applicable to practical use, and the author, therefore, is to be congratulated for offering a solution which embraces most considerations required by the designing engineer.

In order to render "unto Caesar the things which are Caesar's," it is only fair to state that it is an erroneous conclusion of the author that all concrete tanks constructed previous to the appearance of this paper had been designed haphazardly or inefficiently. It has long been known that the shape of the elastic curve is clearly a function of $\frac{h^2}{r\,t}$ as the author correctly states. Based on this fact, it has been possible to make a close approximation of the distribution of the load between the ring and the cantilever, and tables compiled on this basis may be found in offices where concrete water tanks are designed. These tables are probably as efficient as the author's diagrams, and naturally should be so, as they are fundamentally alike.

In water tanks it is important that no cracks develop; otherwise, due to the high internal pressure, water will penetrate and in time destroy the reinforcement and the wall. Experiments at the University of Illinois indicate that a stress of 6 000 lb per sq in. should not be exceeded in order to eliminate destructive hair cracks in the concrete. The author recommends 12 000 lb per sq in. as the proper working stress. Here, then, is a difference of opinion, and designers are again confronted with a method of design more refined than is warranted by other considerations.

If insufficient additional reinforcement is provided to take care of volumetric and temperature changes, as suggested by C. P. Vetter, Assoc. M. Am. Soc. C. E.,[38] and a stress in excess of 6 000 lb per sq in. is permitted, fine hair cracks will develop and, in time, the ring steel will be required to sustain all tension due to continuing diminished adhesion. The modulus of elasticity E_0, therefore, will not be equal to E when this stage has been reached, but will probably be nearer ten times E. Hence, the value of β and θ must be modified and the diagrams adjusted accordingly. In spite of the fact that this is one of the extremely rare cases in engineering where the exact live load is known, designers are confronted with a disturbing uncertainty which, when they are pressed for time, they overcome by "playing safe," providing for the most extreme possibility.

---

[36] Civ. Engr., C. B. & Q. R. R., Chicago, Ill.

[37] "Theory of Elasticity," by Franz Grashof, Berlin, Germany, 1874.

[38] *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1039.

It may be opportune here to draw the attention to the importance of continuous operation when placing concrete in the forms. If the operation is interrupted, the finished tank will develop a horizontal leak precisely at that level, and it requires considerable care and expense to correct it afterward. The writer recalls one tank for which the cost was increased several hundred dollars on account of a short lunch period.

This paper is highly meritorious in that it provides an avenue of approach which may be extended at leisure to conform to the individual designer's ideas.

LLOYD S. DYSLAND,[39] JUN. AM. SOC. C. E. (by letter).—This paper provides a distinct advance in accuracy and ease of design of an indeterminate problem of frequent occurrence, particularly by the inclusion of Fig. 4. Unfortunately the plotting of C_{-m} is not consistent with such accuracy. However, it can be calculated from the expression $M = \frac{w\,h^3}{2\,\theta^3}(2\,C_1 + 2\,C_2 + \theta - 1)$, which is derived from Equation (14*a*), by using a value of $\theta = \pi\,h\sqrt{\frac{0.176}{r\,t}}$ and the values of C_1 and C_2 in Fig. 2. A direct means of computing the height of the point of inflection of the wall moment is quite desirable but is not included.

The analysis presented provides for an inside fluid-pressure loading for the entire wall height and may, by a change of sign, be applied to exterior fluid-pressure loads for the entire wall height. However, it cannot be indiscriminately used for loading to any fraction of the wall height, although the author's statement following Table 1 would lead the reader to that conclusion.

From physical considerations it seems that with partial loading, when use of the loaded height gives values of θ less than about $1.0\,\pi$, the mathematical analysis may be seriously misleading. Fig. 4 shows that for values of θ less than $1.0\,\pi$ the "cantilever" effect (the effect more nearly resembles that of a beam fixed at one end and supported at the other than it does a cantilever) induces a considerable circumferential stress at the top of the loaded portion of the wall which is supposed to be the top of the tank. The analysis gives no stress where there is no load; therefore the wall above the loaded height would have no stress. Thus, for shallow tanks, the analysis would indicate that a highly stressed ring of wall existed adjacent to one which carried no load at all. Such a condition is unreasonable.

Actually, when the wall extends some distance above the height of fluid level, the upper part of the wall must provide (in shallow tanks) additional top restraint to the vertical staves. The result will be a redistribution of hoop stress and a decrease in moment at the base.

---

[39] Designer, Consoer, Townsend & Quinlan, Chicago, Ill.

Manning,[40] in an analysis based on the same fundamental equation (Equation (10*b*)), provides charts to give exact values of moment and shear at the base. However, he uses approximations to determine hoop reinforcement in deep tanks and gives no solution for finding hoop reinforcement in shallow tanks. Leeper[41] greatly simplifies his solution of the differential Equation (10*b*) by neglecting certain terms, and by further approximations derives simple algebraic expressions for the design factors.

For the height above the base at which the point of inflection of wall moment occurs, Manning[40] suggests $0.61\sqrt{r\,t}$ and Leeper[41] gives $0.56\sqrt{r\,t}$.

It frequently happens that greater economy can be realized by using a wall cross-section which tapers toward the top. Some conception of the effect of tapering walls can be gathered from an article by H. Carpenter[42] in which he gives load-distribution and base-moment curves for tanks with walls of triangular cross-section and of rectangular cross-section for several shallowness ratios.

To determine the relative economy of fixed-base design, compared with hoop-tension design, a careful estimate was made recently on two 75-ft diameter sewage digestion tanks, 24 ft deep, in which the chief difference (in addition to the redistribution of reinforcement) was a graphite painted copper joint at the base of the wall designed for hoop tension only. The estimate was $2 000 lower on the fixed-base tank.

BASIL SOUROCHNIKOFF,[43] ESQ. (by letter).—In Mr. Salter's numerical example only stresses due to hydrostatic pressure and curing are computed. As the structural stresses are low, the temperature stresses may be expected to be proportionally important. Assume, for instance, that, due to a decrease of temperature, the rings are shortened. The base, being restrained by the foundation, may not undergo the same shortening. For an effective difference of 10° F between the average temperature of the walls and that of the base, a ring, if not restrained by the cantilever, would have the radius decreased by $0.000006 \times 10 \times 35 \times 12 = 0.025$ in. Due to the hydrostatic pressure alone, a ring at mid-height of the tank, if not restrained by the cantilever, would have its radius increased by $\frac{62 \times 6.28 \times 35}{12 \times 9} \times \frac{1}{2\,500\,000} \times 35 \times 12 = 0.021$ in. Therefore, it may be expected that the temperature stress in the cantilever is of the same order of magnitude as the hydrostatic stress.

The stresses due to a difference of temperature between the walls assumed constant throughout, and the base, may be analyzed by the same method as the hydrostatic stresses are analyzed in the paper. For a temperature difference of $T°$ F, the rings, if not restrained by the base, would have the radius decreased by $y_0 = 0.00006\,T \times r$ ft. In this deformation they are restrained by the base. It is seen that the differential equations and their integration are exactly the same as for hydrostatic pressure, with $w = 0$. For the determination of arbitrary constants one may assume at the top that $\frac{d^2y}{dx^2} = 0$; and $\frac{d^3y}{dx^3} = 0$; and at the bottom $y = -y_0$, and a certain arbitrary angular deformation $\alpha_0 = \frac{dy}{dx}$.

---

[40] "Reinforced Concrete Design," by G. P. Manning, Second Edition, p. 331.

[41] "Design of Cylindrical Tanks," by L. D. Leeper, Jun. Am. Soc. C. E., *Civil Engineering*, Vol. 3, No. 11, November, 1933, pp. 598–600.

[42] "Calculation of Cylindrical Tanks with Rectangular, Triangular, or Trapezoidal Wall Sections," by H. Carpenter, *Concrete and Construction Engineering*, Vol. 24, No. 6, June, 1929, pp. 345–353.

[43] Structural Engr., St. Paul, Minn.

The stress distribution analyzed by Mr. Salter's method is certainly more exact than that obtained by other approximate methods. However, it seems that it can be extended with advantage to take into account the secondary stresses, which may be important.

Regis F. Fey,[44] Jun. Am. Soc. C. E. (by letter).—Progress in circular tank design is aided materially by Mr. Salter's paper. The effect of shell restraint or fixation at the base (a condition which tank designers realized existed, yet hesitated to consider in their designs) has been analyzed very clearly in this paper.

The importance of the type and degree of restraint of the tank wall at the base is somewhat difficult to determine. For a rational design it is reasonable to discount any appreciable horizontal or vertical movement at this point, but it is more difficult to conclude upon a reasonable assumption with respect to rotation about this point. The combined effect of the wall ring tension and the bottom restraint tends to cause a rotation. If the bottom is rigid enough to counteract any rotation, the cantilever effect of the vertical wall segments will come into full effect in carrying their share of the horizontal load; but if the bottom should deflect somewhat, due to the effect of rotation, the load-carrying capacity of the cantilever wall segments will be reduced somewhat. Therefore, considering the wall rigidly attached to the bottom, the degree of restraint may be considered as a function of the rigidity of the bottom.

When investigating a thin circular wall for external pressures, the resultant effects upon the wall are different from those caused by internal pressures. The stresses in the wall may be treated from the standpoint of the flexural rigidity of a ring. For stress investigation a nominal deformation from the true circle should be considered.

The deformation may be either elliptical or in the form of so-called "flat spots" in the wall (Fig. 8). For an elliptical deformation the maximum bending moment is at Point A. Since the ratio of the applied external pressure is small with respect to the critical external pressure at which the ring would collapse (that is, if no

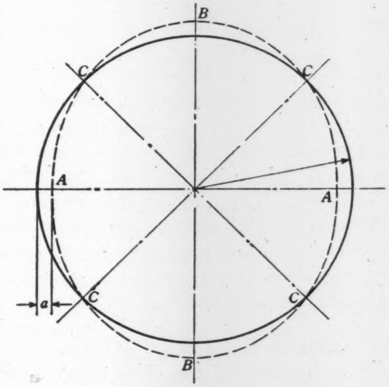


FIG. 8

---

[44] Engr., Pittsburgh Des Moines Steel Co., Pittsburgh, Pa.

further ellipticity would result from the applied external pressure), the moment at Point A is

$$M_A = h\,q\,a\left(r + \frac{a}{2}\right) \qquad (47a)$$

in which, in addition to the notation of the paper: q = the applied pressure; and, a = radial displacement = one-half "out-of-roundness"; that is, one-half the difference between the diameter of the true circle and the minimum diameter of the irregular section. The moment at Point B is in the opposite direction:

$$M_B = h\,q\,a\left(r - \frac{a}{2}\right) \qquad (47b)$$

Since a is small compared with r, the moments at Points A and B may be considered equal for all practical purposes, or:

$$M_A = M_B = h\,q\,a\,r \qquad (48)$$

The moments at Points C are zero.

So called "flat spots" in the circular wall are not plane surfaces but sections which have a lesser degree of curvature. For moment computations a section of wall including, and adjacent to, a flat spot may be considered elliptical in shape. Treating the section as before, the maximum moment at the center of the flat spot will be that given by Equation (48). In this case, however, the radial displacement, a, is equal to the "out-of-roundness." The designer should determine whether care will be taken to prevent any localized irregularities in building the wall section. If this cannot be ascertained, the moment due to a flat spot should be considered; otherwise, the moment due to a general ellipticity should be used. From several measurements, the writer has found that an "out-of-roundness" of 0.25% of the diameter may be anticipated in a circular concrete tank, considering average construction methods. These moments in the wall should be considered in conjunction with the compression in the wall caused by external forces acting on a ring considered circular in shape. These factors give a condition analogous to an eccentrically loaded column.

This treatment of external pressures on walls of circular tanks does not take into consideration the restraint of the bottom, the cantilever effect of the vertical wall segments, or any resistance to torque imposed by the moments due to irregularities, which are of minor importance except in very shallow tanks of large diameters.

GEORGE S. SALTER,[45] M. AM. SOC. C. E. (by letter).—It has often been noted that one of the chief values of a paper is that it brings out some interesting and instructive discussions. Several such discussions were presented on this paper.

Messrs. Silverman, McCormick, Moorman, and Young each brought out clearly, and correctly, that the solution of Equation (10) is nothing new. In this regard the writer wishes to stress the fact that the paper was conceived in the studies made on the design of a particular type of tank—settling tanks for sewage treatment which were 126 ft in diameter with water depth of only 12 to 13 ft. A study of the English literature on the subject gave very meager information on design for tanks of this proportion as, in most cases in the solution of Equation (10), the value of the arbitrary constants *A* and *B* had been assumed as equal to zero. Preliminary studies indicated that such an assumption led to results considerably in error and thus the primary purpose of the studies was to evaluate these constants for the conditions as stated in the paper. It was found that the values of these constants were of considerable importance for tanks in the lower range of values of $\frac{H^2}{r\,t}$. This is indicated clearly in Fig. 2. Since the publication of the paper it has been brought to the writer's attention that apparently a considerable part of the work had been done previously and that various graphs had been presented, chiefly in German publications, some of which are practically the same as some of those given in the paper.

---

[45] Subway Structural Designer, Dept. of Subways and Super Highways, Chicago, Ill.

Mr. Rosengarten suggests the use of pre-stressed rings at the top of the wall in order to simplify the design. This method, and also the method of using a pre-stressed band for the entire height of the wall, were studied; but it was found that they were patented features and also of doubtful savings for the conditions that prevailed.

Messrs. Muckleston, McCormick, Silverman, and Young have stated that the method is not strictly exact in that the value of Poisson's ratio has been neglected. The writer neglected this factor intentionally and made other assumptions as mentioned by Messrs. Muckleston and Young in order to simplify the mathematical work. It was not believed that any of these assumptions were of great importance. The word "exact" was used by the writer in contradistinction to the approximate methods used when the arbitrary constants *A* and *B* were assumed equal to zero. Mr. McCormick goes on to show that for all practical purposes the assumptions made, although not exact, may be justified.

The writer agrees with Professor Moorman that Equation (17) would be understood more clearly if written as he suggests in Equation (30). Equations (32) and (33) are more exact and should be used instead of Equations (18) and (19). Professor Moorman also shows that there is a close agreement between the results secured by the writer's method and that given by W. S. Gray. They should agree as the basis for the two methods appears to be the same.

Mr. Young reveals, clearly, the importance of considering how closely the actual conditions conform to those assumed in the analysis. Probably the most important of these conditions is the restraint at the base of the wall. If the base of the wall is not fixed the values of the moments, shears, and ring tension will be changed materially from those obtained under conditions of fixed base. He also mentions that the analysis may be simplified considerably when the value of θ is greater than π. This simplified analysis has been presented earlier by Smith, Rasmussen, Leeper, and others. The writer, as stated before, was interested primarily in tanks of proportions whose θ-values were close to, or less than, π.

Mr. Dysland's data on estimated savings for the fixed-base tank over the same tank designed for hoop tension only are similar to some studies made by the writer. As the studies made were preparatory to the design of two batteries of sixteen tanks each it will be seen that the total savings would be considerable if Mr. Dysland's data were even approached.

Previous to the plotting of Fig. 3 the writer made up diagrams showing the moment at any height of the wall for the same θ-values as given in Fig. 4. From these diagrams the point of inflection is found readily.

It was not intended to infer that the method could be used for either internal or external loading to any partial height of the wall. The conditions assumed in finding the values of the arbitrary constants state that the moment and shear at height H are zero. This condition obtains only when the loading extends to, or reasonably close to, the top of the wall.

Mr. Mensch states the problem and its answer, correctly, in that American literature contained very little on the subject of design of large-diameter shallow tanks and that the writer's method was developed to design a tank of these particular proportions. A difference of opinion appears to exist as to how much confidence designers will have in such theories. It was hoped that if the method met with the general approval of those who did follow the analysis the method would be of some value in the relatively limited field to which it particularly applies. It might be added that thirty-two tanks, 126 ft in diameter, with a water depth of 12.5 ft, and eight tanks, 75 ft in diameter, and water depth of 14 ft, were so designed and built and are now (1939) in successful operation.

In conclusion the writer wishes to thank all who have added to the value of the paper by their discussions.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2067

STATE-WIDE SURVEYING PRACTICE IN MASSACHUSETTS

A SYMPOSIUM

PAGE

Establishing a System of Rectangular Co-Ordinates.

By Elmer C. Houdlette, Esq. 534

The Land Court and Its Engineering Procedure.

By Clarence B. Humphrey, Esq. 554

With Discussion by Messrs. H. J. Shea, Philip Kissam, Lloyd G. Frost, William Bowie, R. M. Wilson, and Elliott B. Roberts.

Note.—This Symposium was presented at the meeting of the Surveying and Mapping Division, Boston, Mass., October 7, 1937, and published in November, 1938, *Proceedings*.

ESTABLISHING A SYSTEM OF RECTANGULAR CO-ORDINATES

By Elmer C. Houdlette,[1] Esq.

Synopsis

The story of the Massachusetts Geodetic Survey is that of the matching of two needs with one opportunity. This is not in its generality so very different from the ideal that underlies, in theory, the pattern of all projects to which the Federal Government lent support. Within the province of engineering there are certain special areas of knowledge in which advance has been halted and is awaiting some outside form of impulse. Endowed research may approach them, or a governmental agency, acting in the general interest, may sometimes undertake them; but in the main, they are not employment producing under ordinary conditions.

The Commonwealth of Massachusetts felt that there was a great need for a State-wide local control survey and sought to link it to a confronting human need.

Control surveys are necessary prerequisites to highway planning, Land Court surveys, map-making, flood control, water supply, sewage disposal, and perpetuation of private and public boundaries. They insure the accuracy of measurements incident to such projects, reduce the cost of maintaining the required degree of accuracy and provide means for co-ordinating adjacent projects executed concurrently or at different times.

Faithful use of the fixed points established by the Massachusetts Geodetic Survey will eliminate, for all time, the multiplicity of origins, the over-supply of level datum planes, and the stupendous economic loss of thousands of uncontrolled surveys.

Introduction

The representation of the face of Massachusetts, like the photography and portraiture of the face of Man, has undergone, in the course of the decades, certain easily discernible changes. Due partly to a growing naturalistic sense, partly to improved means of reproduction, it has all been in the direction of a more faithful copy. The story of these changes is one of gradual lifting of that face from its vague and indistinct contours to the more evenly defined surface recognized to-day.

The practice of accurate, up-to-the-minute map-making, courted by, and even indissolubly married to, highways, railroads, regional promotional enterprises, and travel bureaus—as well as sponsored by its traditional guardians, the Federal, State, and local governments—is subordinate to and consequent upon extended travel, broadened communications, and increased land use. As such the demands upon map construction through the years have been conditioned upon the size of those developments and the growth of the science of cartography itself. Its rise has been slow, steady, and rewarding.

---

[1] Associate Civ. Engr., Mass. Dept. of Public Works; Director, Mass. Geodetic Survey, WPA, Boston, Mass.

This is no less true of Massachusetts than of other regions. Thus, in 1829, the necessary preliminary step was taken by the Massachusetts House of Representatives, which then adopted an order appointing a committee "to consider the expediency of determining a map or maps of this Commonwealth as the public exigency requires and report thereon to the next session of the Legislature." Thus, having presumably established the necessity for such a map, the Legislature, on March 1, 1830, passed a resolve directing each city, town, and district to make a plan of its territory on a scale of 100 rods to an inch and to file such plan with the Secretary of the State before June 1, 1830.

Likewise, as a co-ordinating effort in the same direction, on March 30 of the same year, the Legislature directed the Governor to appoint a surveyor to make a skeleton map of the State, the intention being to use this survey as a basis for plotting the information furnished by the various towns. Accordingly, two months later, Robert Treat Paine, of Boston, Mass., was appointed as Chief Engineer, and James Stevens, of Newport, R. I., as an Assistant; Mr. Stevens undertook the triangulation and Mr. Paine the astronomical work necessary to determine the latitude and longitude of many points in the State. Simeon D. Borden, a mechanic employed by the Pocasset Manufacturing Company, of Fall River, Mass., designed and constructed a base-line measuring apparatus. He was persuaded to resign his position and undertake the measurement of a base line in Massachusetts. This line, 39 009.73 ft long, was laid out on the plain west of the Connecticut River in the Towns of Hatfield, Whately, and Deerfield. From it was developed a network of triangulation containing about 550 stations and these data were used to provide an outline map of the State, plotted on a polyconic projection and divided into five sections. In each section a central meridian was determined and from it rectangular co-ordinates were computed.

All town maps were reduced to a common scale and an attempt was made to co-ordinate them, along with the State lines and triangulation points, into a skeleton map that would use all data available at the time. In 1841, the map was completed and engraved in six plates; it was largely a one-man affair in its later stages by virtue of Mr. Borden's own re-surveying and fudging. In the end this map proved to be too thickly garbed with inaccuracies and mis-scalings to serve the increasing demands.

Beginning in 1844, the United States Coast Survey measured a base line along the railroad tangent of the Boston and Providence Railroad, running through the Towns of Attleboro and Mansfield and ending in the southeastern part of Foxboro. Upon this base line (10.75 miles long) there was developed a net of first-order triangulation as part of the general system of the eastern oblique arc, extending from the Epping Base, in Maine, through this Massachusetts Base, and thence to the Fire Island Base, in New York State. The precise stations of the Borden survey were included and adjusted as part of this general plan.

Under Chapter 72 of the Acts of 1884, the Massachusetts Legislature established the Massachusetts Topographical Survey Commission, consisting of three members. This body was delegated to co-operate with the U. S. Coast Survey in completing the necessary control for the preparation of a contour topographical survey and map of the Commonwealth. Later, under the Acts of 1888, Chapter 336, the duties of this Commission were broadened to provide for defining and preserving town boundary lines.

During the period, 1898–1900, the Topographical Survey Commission completed its assignment and published forty-two atlases for as many cities and towns in the eastern part of the State. With this task completed the Commission was dissolved and its duties transferred to the Harbor and Land Commission, which, in an effort to complete the assignment at cheaper cost, combined adjacent towns in one atlas.

By the end of 1912 all the atlases had been published for all the cities and towns east of the Connecticut River. During the period, 1913–1915, the atlases of the section west of the Connecticut River to the New York State line were published.

NEED OF CONTROL

However, in spite of the realization of this long-brewing task, the job, so to speak, was just beginning. Every one was still "in the dark" as to the exact location of a piece of property. True, there were maps showing every city and town in the State; true, by virtue of these maps, progress along the road of charting the State had been furthered; but people were still floundering in uncertainty.

They were still using assessors' maps based upon low standards of control and loose adjustment. The data for such maps had been acquired by closed traverses, supplemented by disconnected traverses, into which were fitted old surveys as described in deeds. These descriptions were rarely confirmed by a map, and no check had been made of the area said to be included by the bounds as given. Many towns still had failed to set the widths of their most important thoroughfares, thus creating a state of confusion for abutting owners who desired to make improvements on their land.

Engineers still were using about twenty-five or thirty different datum planes for level work throughout the State. Suggestive of the diversity that has existed is the variety of the following specimens: Mean low water at Boston Harbor; mean low water at Commonwealth Pier; Boston low-water datum; and mean tide level, Boston City Base, and Metropolitan Water-Works Base. In addition, there are many others.

The economic loss of thousands of uncontrolled transit surveys of all kinds was running into millions of dollars for the State as a whole. Such surveys were based on an unlimited assortment of assumed, magnetic, or true meridians. Plane co-ordinates, if used at all, were referred to an equally unlimited number of origins without regard of their geodetic positions. Levels were based on every conceivable kind of assumed datum plane. It all represented a mass of the wildest confusion.

Value of Control

How much more practicable and scientific, indeed, is a system of uniform and universal measurements used by and serving one and all communities of the State? There is a "hoary-haired truth" that runs to this effect, "You may get out of something what you put into it." This is particularly true of the earth. As the yield of a spring's plowing and sowing, and of a summer's care, protection, and re-plowing, is a fall crop paying triple or more the effort that went into it, so the yield of properly manipulated land control is a saving in money, time, and labor. Not only is this true, but it is also a fact that data gathered from controlled surveys afford a distinct advantage to those who come after.

Engineers are familiar with the advantages of geodetic co-ordination and through them all surveys are made permanent and indestructible, and are incorporated into one uniform system in which every survey is properly placed and related to all others. A controlled survey, being permanently fixed, may be drawn upon at any future time for many other purposes aside from that for which it was originally intended.

The first effort to place elevations in Massachusetts on a mean sea-level datum by means of spirit-leveling, was made under the auspices of the Massachusetts Topographical Commission in 1893. The late C. H. Van Orden, M. Am. Soc. C. E., an officer of the U. S. Coast and Geodetic Survey, was engaged to extend a line of precise levels from Boston, westward across Massachusetts into New York State, to a connection with the precise leveling which had been previously run along the east side of the Hudson River. His data,[2] published in 1894, included a detailed statement concerning instrumental equipment and field methods. In 1923 and 1927, several lines of precise levels were run by the U. S. Coast and Geodetic Survey in Massachusetts and, finally, in 1933, this organization re-ran the Van Orden line from Springfield, Mass., to Albany, N. Y.

In November, 1929, finally persuaded as to the importance of a mean sea-level datum for all surveys, the State Department of Public Works authorized the purchase of a precise level and two invar-faced rods. This marked a major and propitious development in the slow progress that was being made toward the adoption of mean sea-level datum for the Commonwealth as a unit.

Previous to the adoption of the Lambert projection, the increasing use by the Department of Public Works of the twelve Land Court systems of co-ordinates developed a need for simplification. Through conferences and co-operation between the Land Court and the Department of Public Works, on April 1, 1933, geodetic control was established on eight co-ordinate systems, six of which remained as before, while two systems replaced four and two were discontinued. As the need for trunk-line highways passing through many towns and counties increased, the use of eight origins became more or less complicated in that equations of co-ordinates and azimuths had to be applied every time a survey passed from one zone to another. This confusing situation has been clarified and difficulties incident to practical application have been greatly simplified by local control, with ample bench-marks, upon one standard basis.

---

[2] Rept., Commrs. of the Topographical Survey of Massachusetts, 1894.

PROJECTION SYSTEMS

Two methods of projection offer themselves for selection, the choice depending on the geographic figure of the State. A State with a narrow east-west width can be completely shown on a transverse Mercator projection, while another State with a narrow north-south width can be completely shown on a Lambert projection.

In the transverse Mercator projection, standard small circles are selected equi-distant from the meridian of longitude passing through the center of the figure. The development of the plane of projection and the transfer of terrestrial points to the plane is similar to that pursued in the Lambert conformal conic projection, to be described subsequently. Both methods approximately attain preservation of angles at a small sacrifice of accuracy in scale, provided the representation does not go beyond the critical dimension of the projection. The choice of projection depends upon the relative geographic dimensions of the area and the error in scale to be accepted.

Since the maximum error of closure accepted in second-order traverse is 1 : 10 000, it is natural to adopt this criterion as the maximum error in scale acceptable for distances on the projection. Applying this criterion it is found that the ratio will not be exceeded on a Lambert conformal conic projection when the north-south distance of the area to be represented does not exceed 158 miles. Likewise, on a transverse Mercator projection, the east-west dimension must also not exceed 158 miles. As the critical dimension is exceeded, the error in scale increases rapidly. If both dimensions of the area to be mapped exceed the critical dimension then two or more projections will be required to hold the scale ratio to an acceptable limit.

The adoption of either of these methods of projection provides a single system of plane co-ordinates for the entire area. After the geographic co-ordinates of the triangulation stations are converted to plane co-ordinates the computations to determine the position of all intermediate points established by traverse is simplified without any sacrifice in accuracy.

Previous limitations in the form of required use of geodetic formulas and methods, an impracticable expense in many cases, have been eliminated since the formulation of the State plane co-ordinate systems by the U. S. Coast and Geodetic Survey. Any engineer or surveyor in Massachusetts can now utilize the plane co-ordinate system of the Lambert conformal conic projection, and, at the same time, can conduct his work according to the standard methods of ordinary plane surveying.

DIFFICULTIES IN USING SPHERICAL CO-ORDINATES

As is well known the surface of the earth is not a plane but a curved and irregular surface. For purposes of surveying and mapping, however, an ideal surface is assumed which approaches very nearly the shape of the earth if the entire surface were composed of water. This assumed shape is not a sphere, but an ellipsoid of revolution.

Geodetic surveying comprises the extensive work of making observations and computations of positions on the surface of this assumed spheroid. It becomes extremely complicated when the surveys extend over large areas. Also, it requires an intimate knowledge of higher mathematics and astronomy, and the highest degree of precision in the field observations, using the most elaborate and expensive field equipment. All geodetic stations on the earth are expressed in terms of latitude and longitude; that is, each one is referred to a definite place in the network of meridians and parallels that are assumed to cover the surface of the earth in a definite and standardized manner.

It must be perfectly evident that geodetic surveying, although absolutely essential as a basis for precise control, is impracticable for use in ordinary survey work due to the rigorous technique and costly equipment required. Perhaps, for this reason, in spite of the fact that it reached a high stage of perfection and was taught in technical schools many years ago, engineers as a whole have not taken an extensive interest in the general subject, and, consequently, have not developed to the full any standard of practice which might enable them to utilize economically the great inherent possibilities to be found in the existing surveys of the U. S. Coast and Geodetic Survey.

Within these surveys lies a wealth of data, which can be made of tremendous value, not only to engineers and surveyors in every field of activity, but through them to every corporation, commonwealth, county, municipality, property owner, or individual who pays for any kind of instrumental surveying. This value lies in the use of the monumented stations of the national triangulation system as starting and ending points controlling all surveys. Since these points are accurately located on the spheroid, and in relation to one another, they furnish a means of checking the work and of adjusting the errors in the survey. The system of plane co-ordinates permits the use of these stations for control purposes without involving the complicated computations otherwise necessary. In a word, it constitutes a mathematically sound and accurate method of transposing work from geodetic or spherical to plane surveying methods.

The Plane Co-Ordinate System as Adopted for Massachusetts

The Lambert projection adopted for spreading local control stations throughout Massachusetts is based on a geographical frame selected with particular reference to the shape and area of the State. The frame consists of two standard parallels of latitude, held true to scale, and a central meridian of longitude. The north-south dimension of Continental Massachusetts is considerably less than the critical, with consequent reduction of error in scale distances.

In order that the scale ratio applied to distances at the middle of the State shall be equivalent, although of opposite sign, to that for the northern and southern boundaries, it is necessary to select the two standard parallels of latitude at distances from the most northerly and southerly points approximately equal to one-sixth of the north-south dimension of the State. Consequently, the north latitudes of 41° 43′ and 42° 41′ were selected, together with the meridian of west longitude of 71° 30′.

The geographic frame for the insular territory was similarly selected. The standard parallels of 41° 71′ and 41° 29′, together with the meridian of 70° 30′, form the frame. The latter projection excellently illustrates the decrease in scale ratio which accompanies a decrease in the area mapped.

The plane of projection is formed by a cone of revolution intercepting the earth's surface along the two standard parallels of latitude chosen for the frame. The apex of the cone is on the axis of the earth projected through the North Pole into space. The imaginary cone of projection passes beneath the earth's surface in the area between the parallels and above the earth's surface north and south of that area. Approximately two-thirds of the distance between the extreme northerly and southerly continental points in the State lie between the parallels. The remainder is equally divided and lies outside.

Distances between points on the plane between the standard parallels are shorter than those between the same points on the surface, while outside the standard parallels the reverse would hold. Only distances between points along the two standard parallels are true distances and, therefore, equal upon both the plane of projection and upon the surface of the earth. The angles between lines connecting points upon the plane are approximately equal to the angles between the lines connecting the identical points on the earth's surface. This equality of angles is denoted as "conformal."

The U. S. Coast and Geodetic Survey selected the State projection and prepared the plane co-ordinate projection tables to facilitate the computations necessary to transfer a point on the earth's surface, defined by geographic co-ordinates, to an equivalent position on the plane of projection defined by numerical co-ordinates developed on the single State system. The table for changes in latitude gives the numerical value of the radius and Y-co-ordinate on the plane of projection for each minute of change, together with the tabular difference per second for change in the Y-co-ordinate and correction for scale, with their respective logarithmic expressions. The table for changes in longitude gives the change in the divergent angle for each minute. These tables and an acceptable manual of logarithms, expressed to eight decimal places, equip the computer to convert geographic co-ordinates to plane co-ordinates.

The meridian through the origin might be used as the prime meridian, but to reduce the mechanical labor in applying interpolations in logarithmic tables it will facilitate computations to select, in this case, the meridian of west longitude, 71° 30′, just east of the central point of the State, and assign to it the X-co-ordinate value of 600 000 ft, this places the origin on a meridian west of the most westerly limits of the State. Selecting as the other base the parallel of north latitude of 41° 00′ (south of the most southerly point), places the origin of co-ordinates for the single State system at the intersection of these two lines.

History of the Massachusetts Geodetic Survey

The organization entrusted with the task of applying the Lambert conformal conic projection to Massachusetts is known as the Massachusetts Geodetic Survey. Established in November, 1933, as the Massachusetts Local Control Survey, its duty was primarily to make more widely accessible to local engineers the advantages of the Federal triangulation net already laid in the State. In general, its goal was to lessen throughout the State the distances between the marked points in the net by setting down intermediate points. At that time the spacing between marked points, approximately 25 miles apart, giving only $12\frac{1}{2}$ miles from any point to a triangulation point, was too wide for effective local use. The task and expense of running a 12-mile traverse or line of levels to the site of a project were more than burdensome for an engineer undertaking a small job.

The Massachusetts Local Control Survey began its assignment with the running of traverse and lines of levels along highways and railroads from one triangulation station or bench-mark to another, with monuments set at intervals of about two miles en route. The accuracy of the work was to be such as was appropriate to the short length of line and as could reasonably be attained with the instruments available. The error of closure allowed for traverse was 1 : 10 000; and for levels, it was according to the formula:

$$e = 0.4 \sqrt{s} \quad \text{(1)}$$

in which s is the distance, in miles. The plan of operation was as follows:

(1) The U. S. Coast and Geodetic Survey created a unit of its own trained personnel to have administrative and technical jurisdiction over the work.

(2) It appointed an engineer as Director of the work in each State.

(3) Through the State Director, it obtained the loan of surveying instruments, office space, drafting-tables, and other necessary equipment.

(4) The State Director closely supervised the surveying operations to assure economy and efficiency of performance and mathematical accuracy of results, in order that the prescribed standards of the U. S. Coast and Geodetic Survey might be met fully.

(5) The Director established a central office where the results of the field work could be adjusted and computed as the work progressed.

The early period of the project was attended with many of the uncertainties of adjustment and operation, characteristic of the emergency undertakings of the Government since the depression. The first limitation came in less than two months after operations had been begun. With thirty parties in the field in January, 1934, an executive order, reporting the insufficiency of funds to carry on the entire CWA (Civil Works Administration) project, called for a reduction of the personnel from 375 to 75 workers, by February 15. Accordingly, the field equipment was collected preparatory to tying up the project altogether.

Up to this time the office work consisted chiefly in computing and plotting on the Lambert projection the co-ordinates of the corners and mid-points of the fifty-four topographical sheets of the State for the Massachusetts Land Court. However, on February 23, efforts to have the project personnel approved for 182 employees were successful. Fourteen field parties and forty-one office men were re-hired forthwith; but in spite of this reinforcement the force was still insufficient to continue the traverse work west of the Connecticut River. This plan, for the time being at least, had to be abandoned.

The next major adjustment came on April 4, 1934, when the project was transferred to the ERA (Emergency Relief Administration), whose original approval of 177 workers was increased by August to 230. This provided twenty-one field parties. An order of September 24, 1934, reducing all salaries to a common basis of $16.80 per week, from Supervisor to Rodman, was rescinded within a week.

The first few months of this re-adjustment brought along anxiety and disturbing jolts. The accompanying loss in personnel definitely tended to obstruct the program. However, the spirit was unflagging and certain accomplishments stood out from the condition of disruption as indicated by the report for the year ending December 31, 1934:

(1) The project had completed 585 miles of traverse, including 1 170 monumented stations; and 343 miles of preliminary computations showed an error of approximately 1 : 18 400.

(2) The project had completed 304 miles of first and second-order levels, including the establishment of 679 bench-marks.

Meanwhile, by virtue of the triangulation and projection work, the demand for added service was increasing. One valuable use pointed in the direction of improved forest fire control. It happened that up to this time the "tying-in" of fire towers with triangulation was not complete. The Department of Conservation in the fall of 1934 requested the Survey to complete the "tying-in" process; likewise, in order that the Lambert projection might be used on the topographical sheets for the location of forest fires, it desired that the Lambert co-ordinates be computed for all the fire towers in the State. At about the same time the Massachusetts Land Court requested a computation of the Lambert co-ordinates for all the triangulation stations on Martha's Vineyard and Nantucket, as the triangulation stations on these two islands had been adjusted on the 1927 North American datum.

At the beginning of this project, the geographic positions of all the established triangulation stations in the State, with the exception of the seventeen precise stations, were based on the North American datum. Until such time as the remaining triangulation stations were either re-computed or re-adjusted on the 1927 North American datum it was necessary, for preliminary traverse computations, to apply to each triangulation station as it was used, a correction to the published position to place it upon this datum. The rapid determination of the amount of correction to be applied to a given point was facilitated by the construction of a correction chart based upon the seventeen precise stations furnished by the U. S. Coast and Geodetic Survey.

These stations were plotted in correct position on an accurate map of the State and the amounts of the differences in latitude and longitude between the North American datum and the 1927 North American datum were noted. These points were connected by direction lines which were divided into equivalent parts representing the units of difference in latitude between the stations. These subdivisions established points of equal correction which could be connected to equivalent points on adjacent direction lines. The space between the connecting lines defined an area within which all stations were to receive equivalent correction in latitude. The distribution of the corrections in longitude were similarly determined.

Thus, the map of the State was covered with a somewhat rectangular and more or less regular grid, the unit subdivisions being about a mile apart in the east and west direction and somewhat less north and south, corresponding to connections to the nearest 0.001 sec in longitude and latitude. This map became a chart showing the amount of correction to be applied to the geographical co-ordinates of points included between the parallels.

Plotting the position of a point (defined by geographical co-ordinates on the North American datum) places it in one of the numerous grid areas. Thus, the corresponding amount of correction is determined and the position of the point is adjusted to the 1927 North American datum.

This chart was found, by later triangulation, to give results in general only to the nearest foot, but it served the purpose of determining a preliminary adjustment of all the traverses and showed whether the work had been completed in accordance with the specifications of second-order accuracy. Its main purpose, however, was to give a quick solution of control points in order that this information might be available to the United States Geological Survey in re-mapping the State.

In 1935 the equipment and personnel of the project were strengthened to the extent that the original program could be more fully and thoroughly completed and new work could be handled in its course. In January, the quota of employees was increased to 300, including 26 field parties.

A further increase in personnel to a total of 367 was made upon the transfer on November 1, 1935, of this project from ERA to WPA (Works Progress Administration) control. However, the accompanying shortening of working time from 35 to 24 hr per week brought about a decided slowing down of the work which was ever increasing in volume. Prior to January, 1936, transits and levels used in all field work were loans to the organization, and, in general, they were only in fair condition. As the transits read merely to the nearest minute, about 15% of the traverses had to be rejected. In time, this state of affairs was brought to the attention of the local officials who obligingly agreed to the purchase on a rental basis, of twenty-four new 20″ and 30″ transits. With this improvement, errors of closure approached that of first-order accuracy and rejections were reduced to about 5 per cent.

Additional undertakings by the Survey call for special mention. During 1935 it started preparing maps for each city and town in the Commonwealth, showing the location of all traverse and triangulation stations and bench-marks, such maps to be used by the localities for local control surveys within their limits. In October, the project was requested by the Department of Public Health and the various municipal water supplies to establish bench-marks, determine the heights of high and low water, the crest of dams and spillways, and establish the elevation of stand-pipes and of the ground-water level in driven wells on a common mean sea level basis. Such a unified survey makes it possible to judge whether there is a presumption that a given city or town might find it physically feasible to obtain additional water from a neighboring source.

All systematic and reliable information about relative levels is certain to help in planning. It will be of potential assistance in times of emergency, which can partly be foreseen as to type but which can never be foreseen as to time or detail. There is a natural community of interest between adjoining towns and cities because of the reciprocal assistance they might afford each other in times of need, when one had been better able than the other to cope with drought conditions. Less spectacular is the opportunity for temporary tie-in with a neighboring supply in the event of temporary breaks or of breakdown of the servicing. A more glaring illustration is the assistance that sometimes can be rendered by a neighboring community in the event of large or protracted fire, or of suddenly threatening depletion of the ordinarily sufficient reserve because of fire.

During the same year (1935) requests for control were received from sixteen cities and towns around Boston. An additional request was received for a large amount of control for military purposes within the area of the newly established National Guard Camp at Bourne on Cape Cod. One encouraging feature about these requests was the marked inclination toward the adoption of the State co-ordinate system. By the end of that year 1 618 miles of first and second-order levels had been adjusted and 1 275 miles of traverse completed in the field with an average of error of closure of 1 : 23 100.

The supreme test of the strength and resources of the organization and of the fortitude of its engineers presented itself in March, 1936, when large areas of the State were engulfed by unprecedented conditions of flood. The Massachusetts Geodetic Survey was called upon to accept a task never before undertaken, namely, the collection, by specially directed and equipped eye-witnesses, of authentic high-water data. A plan to establish a systematic series of bench-marks at bridges and dams along the principal rivers in the State already had been partly executed. Thus, with a trained organization and a substantial vertical control base achieved, all was in readiness to launch the flood survey; and on March 18, supervisors were instructed to notify all party chiefs to prepare their men for quick action in the field, if and when the emergency arose.

The men were to reach the flooded area over as many roads as possible, in order to place the high-water marks at well distributed points along the entire course of streams where flooded conditions prevailed. An order was issued on March 19, and the full personnel of about 200 field men were thus assigned to placing suitable marks, recording high water at the site and at the very time it occurred. This work continued until the extreme flood stage had passed. As rapidly as the field reports were received at the Boston Office, the position of each mark was plotted on the town maps and these reports were grouped and dispatched to the several field supervisors for allotment to especially organized level parties.

During the fall of 1936, the U. S. Coast and Geodetic Survey asked the State Survey to determine the magnetic declination at about seventy stations scattered over the State, from the tip of Cape Cod to the New York State line, including all the islands. The results of 24-hr observations made at Mt. Wachusett for seven consecutive days, to determine diurnal variation, were compared with the observations at the Observatory in Cheltenham, Md.

In the beginning of 1937, also by request of the U. S. Coast and Geodetic Survey, two first-order arcs of triangulation across Massachusetts were started, together with the necessary second-order triangulation, to tie in all the completed traverses, in order that a final adjustment might be made on the 1927 North American datum.

An idea of the amount of Survey work accomplished may be gained from the following summary, through September, 1938:

Traverse:	
Extent, in miles	2 605
Number of stations established	4 700
Number of stations with preliminary co-ordinates	3 427
Number of stations with final co-ordinates	1 272
Average error of closure	1 : 35 400
Triangulation:	
Number of stations established	550
Number of first-order adjusted stations	55
Number of second-order adjusted stations	223
Bench-Marks:	
Extent of first-order levels, in miles	1 320
Extent of second-order levels, in miles	3 760
Number of first-order bench-marks	1 850
Number of second-order bench-marks	8 101
Magnetic Stations:	
Number of stations established and adjusted	68
City and Town Maps:	
Number of maps completed	385

Publication Program

One interesting feature of the Survey's work was the opportunity of printing its records, investigations, and standards. In June, 1935, the first publication of the Survey entitled "Leveling in Massachusetts" was issued. This volume contained the descriptions and elevations of all bench-marks, including those established by the U. S. Coast and Geodetic Survey and by the State project, together with a history of leveling in Massachusetts, specifications for first and second-order leveling, a discussion of the various datum planes, and a chart showing the relation of some of the principal datums in use and their relation to mean sea level.

Fortunately, the Traffic Survey was in the same building as the Survey. As its work was about to be terminated in August, the Geodetic Survey fell heir to a complete plant with all its divisions, including reproduction equipment, modern multilith and mimeograph machines, photostatic and reproducing cameras, dark room, and plate-making equipment.

Immediately the publication of a second volume entitled "Computations of Spherical and Grid Coordinates with Tables" was undertaken. This book contained instructions for solving geodetic positions, a description of the Lambert projection, and directions for computing co-ordinates, together with tables.

During the year, a request was received from the U. S. Geological Survey to undertake as part of the survey work in this State, the determination of the elevations above mean sea level datum, of the reference marks at the forty-two stream-gaging stations. In 1937, a report was issued entitled "Stream Gaging Stations in Massachusetts." This report contained notes on equipment and operation of stream-gaging stations, and stream-flow data as compiled by the U. S. Geological Survey, together with the descriptions and elevations of the reference marks at each of the forty-two stations.

Information gathered out of the flood-control activities formed one of several very significant books published by the project during 1936, "High Water Data: Flood of March 1936." This is a tome of about 350 pages, based upon about 1 400 observed points which required about 800 miles of levels to determine the elevations.

An April, 1936, release by the project was a 46-page, very readable book, entitled "Status of Land Surveying in Eastern Massachusetts." Another volume, "Technical Procedure for Geodetic Surveys," released the following month offered a manual of instruction in instrument operation, methods of procedure, and recording of data necessary for standard triangulation, traverse and leveling survey work. This publication has proved of immense value in standardizing survey work in the State. A year later a volume was published entitled, "Plane Coordinates of Minute Intersections of Meridians and Parallels of Latitude Based on the Massachusetts State System."

The various volumes mimeographed by the Reproduction Section testify to painstaking work on the part of the publishing unit of the project. The demand for these publications was more than flattering, especially in view of the lack of any provision for advertising or book promotion campaigns.

THE USEFULNESS OF THE PLANE CO-ORDINATE SYSTEM

What is the procedure of a surveyor who undertakes to make use of the plane co-ordinate system as devised for Massachusetts? He will have previously obtained from the agency the plane co-ordinates of the monuments at which he proposes to begin and to end his survey, and also the plane bearings or the plane azimuths, from these monuments to their respective azimuth marks. Then, setting up over the first monument with the vernier at zero, he sights to its azimuth mark. From this line of sight he turns the angle and measures the distance to the next point in his survey which the instrument is to occupy. He also computes the plane bearing of this line from the given plane bearing between monument and azimuth mark. He carries his calculated plane bearings, in the usual manner, right on to the end of the survey at the terminating monument, and on through the monument to its azimuth mark. Instantly, he has a check on all the instrument work, for the calculated bearing from monument to azimuth mark should be the same as that which was furnished by the agency.

After distributing his angular error, he computes the plane co-ordinates (latitudes and departures) of his survey, starting with those furnished him for the beginning monument. His co-ordinates for the terminating monument should agree with those previously furnished him. Thus, he not only has a check on both instrument work and chaining, but he has made a controlled survey which is permanent, and is also in its proper position with respect to all other surveys in the system so that it can be used in the future for many other purposes. Of course, he should correct the co-ordinates for his survey by distributing his error of closure in latitudes and departures.

If he prefers to use azimuths instead of computed bearings, he first sets the vernier to read the given azimuth from the beginning monument to its azimuth mark and sights to that mark. Thereafter, instead of measuring the subsequent angles, he simply reads the azimuths of the various lines directly from the vernier plate and closes on the previously given azimuth at the terminating monument. Because the magnetic bearings will vary from corresponding plane ones (often considerably), in various parts of the State, the magnetic bearings of all lines, and the date of the survey, should be recorded.

The surveyor finds not only that the system can be used without difficulty but also that it simplifies the use of control data and gives a permanent general grid for the entire State. County boundaries, township boundaries, property boundaries, intersections of roads and streets, and any prominent features of a region can be located accurately with definite X and Y-co-ordinates. These plane co-ordinates can be readily transformed into latitude and longitude, and the point can thus be definitely located in the network of meridians and parallels that serve to locate points in the earth's surface. If, in some manner, the marker at such a point should be destroyed, it could be relocated definitely on the ground from its relation to other marked points. This is an important characteristic and one that should be given due consideration by all engineers and surveyors.

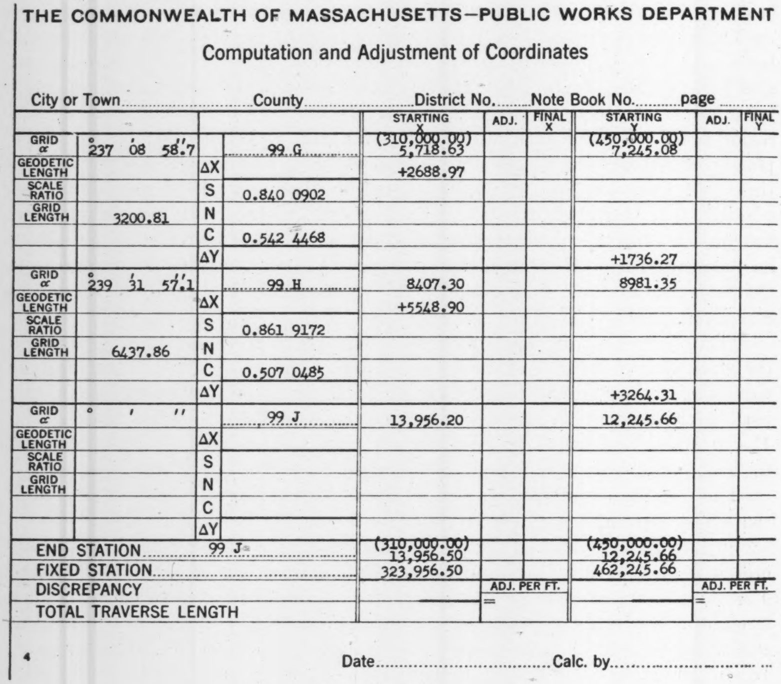
There have been strong objections to the use of grid co-ordinates based on the State system because of the large numbers involved. These large numbers were used in the State system only to avoid minus values in co-ordinates in any part of the State. In other words, the origin of the projection $X = 0$ and $Y = 0$ might be said to be an arbitrary point. Therefore, if the relation of the grid azimuth were maintained, one might use any set of X and Y-values.

The minimum requirements to compute an existing traverse, or tie it into the State system, are the grid co-ordinates of one marked station and grid azimuth from that marked station to another station visible from it. The co-ordinates are usually in hundred thousands, such as $X = 315\,718.63$ and $Y = 457\,245.08$, shown in Fig. 1. If the computer objects to carrying such large numbers in his computations, a value such as $X = 0$ and $Y = 0$ may be substituted for the known co-ordinate values of the marked starting station. In general practice, however, it is customary to assume some number sufficiently large so there will be no minus values such as 10 000 or 50 000. After computing the values of the traverse, the difference between the original co-ordinates and the starting co-ordinates should be added to the computed co-ordinates. This has been done in Fig. 1. The computations using natural lines and curves, designated by **S** and **C**, will be self-evident.

Co-Ordinates by Special Tables

A table of plane co-ordinates of minute intersections of meridians and parallels of latitude was prepared to enable engineers to compute the co-ordinates of any geographic position by interpolating rather than using the longer direct method involving 10-place natural or logarithmic tables. While especially valuable for plotting purposes, the accuracy obtained in most cases will be sufficient for any survey, giving results checking within 0.01 ft.

The basic values for the projection in Massachusetts are the radius, in feet, from the origin of the cone of the projection, of parallels of latitude, and the value known as *Y*, in feet, of the length along the central meridian, of the intercepts between minute parallels of latitude. The value, in seconds, of θ, the angle at the origin of the cone between the central meridian and any other meridian, is also given.



THE COMMONWEALTH OF MASSACHUSETTS—PUBLIC WORKS DEPARTMENT

Computation and Adjustment of Coordinates

City or Town.................................County.............................District No.........Note Book No...........page ....

				STARTING X	ADJ.	FINAL X	STARTING Y	ADJ.	FINAL Y
GRID α	237° 08′ 58″.7		99 G	(310,000.00) 5,718.63			(450,000.00) 7,245.08		
GEODETIC LENGTH		ΔX		+2688.97					
SCALE RATIO		S	0.840 0902						
GRID LENGTH	3200.81	N							
		C	0.542 4468						
		ΔY					+1736.27		
GRID α	239° 31′ 57″.1		99 H	8407.30			8981.35		
GEODETIC LENGTH		ΔX		+5548.90					
SCALE RATIO		S	0.861 9172						
GRID LENGTH	6437.86	N							
		C	0.507 0485						
		ΔY					+3264.31		
GRID α	° ′ ″		99 J	13,956.20			12,245.66		
GEODETIC LENGTH		ΔX							
SCALE RATIO		S							
GRID LENGTH		N							
		C							
		ΔY							
END STATION	99 J			(310,000.00) 13,956.50			(450,000.00) 12,245.66		
FIXED STATION				323,956.50			462,245.66		
DISCREPANCY					ADJ. PER FT.			ADJ. PER FT.	
TOTAL TRAVERSE LENGTH					=			=	

4

Date...................................Calc. by...............................

FIG. 1

Using these basic values, the co-ordinate values of the intersection of every minute of latitude and longitude, within the State boundaries, were computed. This necessitated approximately 25 000 separate computations for the *X* and *Y*-co-ordinates of these minute intersections. These computations were somewhat simplified by the fact that they were started at the 71° 30′ meridian, known in the projection system as the prime meridian, which has an *X*-value of 600 000. Thus, for about 100′ east and west of the prime meridian, the computed distances from this point were added to or subtracted from 600 000—added when east of the prime meridian and subtracted when west.

Lati-tude	71° 03′				71° 04′				71° 05′			
	X $\Delta' x$	ΔX $\Delta_2 x$	Y $\Delta' y$	ΔY $\Delta_2 y$	X $\Delta' x$	ΔX $\Delta_2 x$	Y $\Delta' y$	ΔY $\Delta_2 y$	X $\Delta' x$	ΔX $\Delta_2 x$	Y $\Delta' y$	ΔY $\Delta_2 y$
° ′ 41-45	722,798.61 -32.04	-4548.05 +1.18	273,620.06 +6,073.09	-23.55 0.00	718,250.56 -30.86	-4548.06 +1.19	273,596.51 +6,073.09	-22.66 0.00	713,702.50 -29.67	-4548.06 +1.19	273,573.85 +6,073.09	-21.78 +0.02
46	722,766.57 -32.04	-4546.87 +1.19	279,693.15 +6,073.08	-23.55 +0.01	718,219.70 -30.85	-4546.87 +1.19	279,669.60 +6,073.09	-22.66 +0.01	713,672.83 -29.66	-4546.87 +1.18	279,646.94 +6,073.10	-21.76 0.00
47	722,734.53 -32.04	-4545.68 +1.19	285,766.23 +6,073.10	-23.54 0.00	718,188.85 -30.85	-4545.68 +1.18	285,742.69 +6,073.10	-22.65 +0.01	713,643.17 -29.67	-4545.69 +1.19	285,720.04 +6,073.11	-21.76 0.00
48	722,702.49 -32.04	-4544.49 +1.18	291,839.33 +6,073.10	-23.54 +0.02	718,158.00 -30.86	-4544.50 +1.19	291,815.79 +6,073.12	-22.64 0.00	713,613.50 -29.67	-4544.50 +1.18	291,793.15 +6,073.12	-21.76 +0.01
49	722,670.45 -32.04	-4543.31 +1.19	297,912.43 +6,073.11	-23.52 0.00	718,127.14 -30.85	-4543.31 +1.18	297,888.91 +6,073.11	-22.64 0.00	713,583.83 -29.67	-4543.32 +1.19	297,866.27 +6,073.12	-21.75 0.00
41-50	722,638.41 -32.04	-4542.12 +1.19	303,985.54 +6,073.11	-23.52 +0.01	718,096.29 -30.85	-4542.13 +1.19	303,962.02 +6,073.12	-22.63 0.00	713,554.16 -29.66	-4542.13 +1.18	303,939.39 +6,073.12	-21.75 +0.01
51	722,606.37 -32.04	-4540.93 +1.18	310,058.65 +6,073.12	-23.51 0.00	718,065.44 -30.86	-4540.94 +1.19	310,035.14 +6,073.12	-22.63 +0.01	713,524.50 -29.67	-4540.95 +1.19	310,012.51 +6,073.13	-21.74 +0.01
52	722,574.33 -32.04	-4539.75 +1.19	316,131.77 +6,073.12	-23.51 +0.01	718,034.58 -30.85	-4539.75 +1.18	316,108.26 +6,073.13	-22.62 +0.01	713,494.83 -29.67	-4539.76 +1.19	316,085.64 +6,073.14	-21.73 0.00
53	722,542.29 -32.04	-4538.56 +1.18	322,204.89 +6,073.14	-23.50 0.00	718,003.73 -30.86	-4538.57 +1.20	322,181.39 +6,073.14	-22.61 +0.01	713,465.16 -29.66	-4538.57 +1.18	322,158.78 +6,073.15	-21.73 0.00
54	722,510.25 -32.04	-4537.38 +1.19	328,278.03 +6,073.14	-23.50 +0.02	717,972.87 -30.85	-4537.37 +1.18	328,254.53 +6,073.16	-22.60 -0.01	713,435.50 -29.67	-4537.39 +1.19	328,231.93 +6,073.15	-21.73 +0.02
41-55	722,478.21 -32.04	-4536.19 +1.19	334,351.17 +6,073.16	-23.48 0.00	717,942.02 -30.85	-4536.19 +1.18	334,327.69 +6,073.16	-22.61 +0.01	713,405.83 -29.67	-4536.20 +1.19	334,305.08 +6,073.17	-21.71 0.00
56	722,446.17 -32.04	-4535.00 +1.18	340,424.33 +6,073.17	-23.48 0.00	717,911.17 -30.86	-4535.01 +1.19	340,400.85 +6,073.17	-22.60 +0.01	713,376.16 -29.67	-4535.01 +1.19	340,378.25 +6,073.18	-21.71 0.00
57	722,414.13 -32.04	-4533.82 +1.19	346,497.50 +6,073.17	-23.48 +0.01	717,880.31 -30.85	-4533.82 +1.19	346,474.02 +6,073.18	-22.59 +0.01	713,346.49 -29.66	-4533.82 +1.18	346,451.43 +6,073.19	-21.71 +0.01
58	722,382.09 -32.05	-4532.63 +1.19	352,570.67 +6,073.19	-23.47 0.00	717,849.46 -30.86	-4532.63 +1.19	352,547.20 +6,073.19	-22.58 0.00	713,316.83 -29.67	-4532.64 +1.19	352,524.62 +6,073.19	-21.70 +0.01
59	722,350.04 -32.04	-4531.44 +1.19	358,643.86 +6,073.19	-23.47 +0.01	717,818.60 -30.85	-4531.44 +1.18	358,620.39 +6,073.20	-22.58 +0.01	713,287.16 -29.67	-4531.45 +1.19	358,597.81 +6,073.21	-21.69 0.00
42-00	722,318.00	-4530.25	364,717.05	-23.46	717,787.75	-4530.26	364,693.59	-22.57	713,257.49	-4530.26	364,671.02	-21.69

FIG. 2

When all the co-ordinate values for the minute intersections were computed, they were arranged in a tabular form of which Fig. 2 is a sample. A range of 15′ of latitude forms the side argument and 3′ of longitude forms the top argument. Each vertical column of 1′ of longitude is divided into four sections.

Section 1 gives the value of the co-ordinate, X, and between adjacent values, their increments, $\Delta' X$. Section 2 lists the difference, ΔX, in the value of X between each minute of longitude progressing easterly along a given parallel; and between these ΔX-values the differences, $\Delta_2 X$, of ΔX between successive minutes of latitude along a given meridian progressing northerly. Sections 3 and 4 are identical with Sections 1 and 2, except that the values listed are for the Y-co-ordinates. Thus, at a given latitude and longitude straight line differences can be found to the next larger minute of latitude and longitude. The complete table thus formed for the State of Massachusetts consisted of approximately 270 pages similar to Fig. 2, giving X and Y-values and their increments.

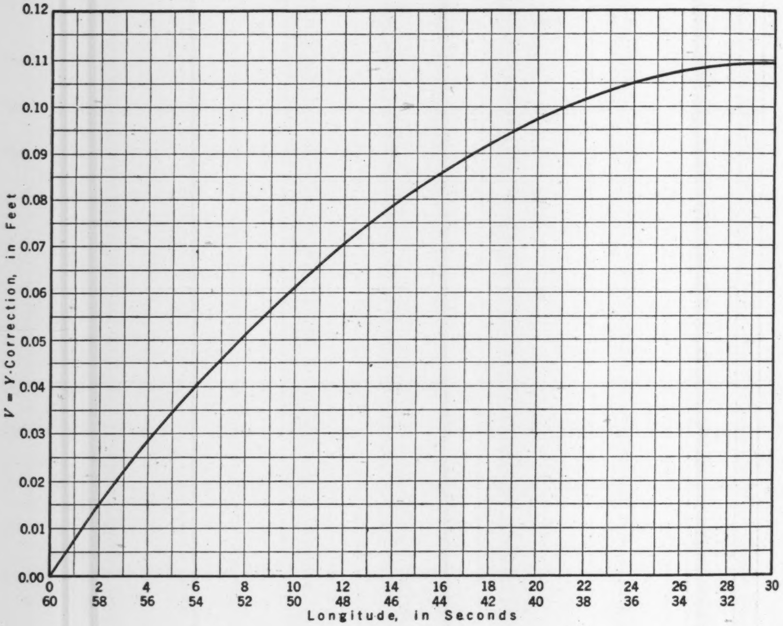


FIG. 3

To convert to grid co-ordinates a given latitude and longitude listed to hundredths or thousandths of a second, it is only necessary to locate the required page in the table similar to Fig. 2, by use of a tabulated key, which gives the X and Y-values of the intersection of the given latitude and longitude, rounded off to the next lower full minute. Then, by multiplying the seconds, divided by 60, into the Δ-values and their increments found in the tables, a straight line interpolation can be made.

The *Y*-co-ordinates, as determined from the tables, are slightly in error because, between minute intersections, the parallels of latitude are assumed to be straight lines and actually they are curved. A graph was constructed (Fig. 3) giving corrections to compensate for this discrepancy. The values of *X* and *Y* as determined from the tables and the *Y*-correction are within 0.01 ft of those obtained through the use of the regular computation, using logarithms for changing geodetic positions to grid co-ordinates. Also, by the use of these tables, the time required to compute a grid co-ordinate is shortened by 65 to 75 per cent.

SAMPLE COMPUTATIONS:

Station: SOMMINT

Latitude 41° 46′ 33″.877
Longitude 71° 03′ 15″.650

$\phi = \frac{33.877}{60} = 0.5646167$

$\lambda = \frac{15.650}{60} = 0.2608333$

At longitude 71° 03′ and latitude 41° 46′

$X_t = 722{,}766.57$; $\Delta' x_t = -32.04$; $\Delta X_t = -4546.87$; $\Delta_2 x_t = +1.19$
$Y_t = 279{,}693.15$; $\Delta' y_t = +6073.08$; $\Delta Y_t = -23.55$; $\Delta_2 y_t = +0.01$

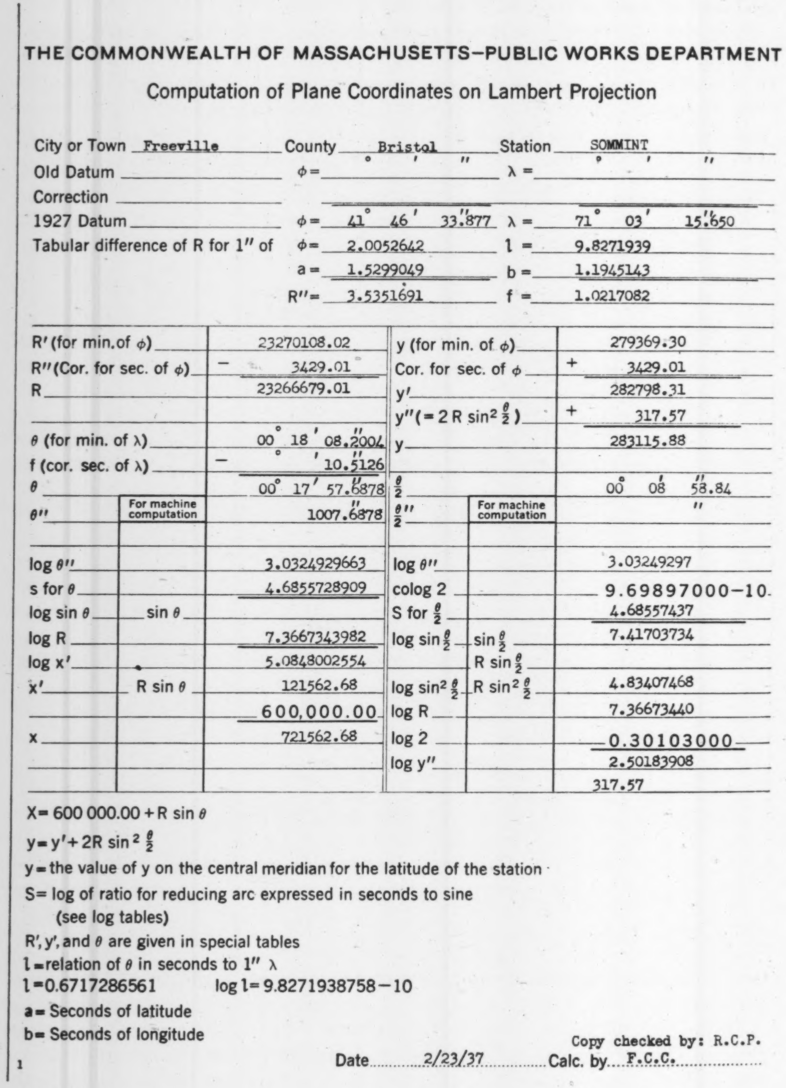
X:	722,766.57	
	-18.090	-32.04 x 0.5646167
	722,748.480	
	-1,185.800	$0.2608333[-4546.87 + (+1.19 \times 0.5646167)]$
	721,562.680	
Y:	279,693.15	
	+3,428.962	+6073.08 x 0.5646167
	283,122.112	
	-6.141	$0.2608333[-23.55 + (+0.01 \times 0.5646167)]$
	283,115.971	
	- .084	correction from graph
	283,115.887	

Adopted: X = 721,562.68
Y = 283,115.89

By Computation: X = 721,562.68
Y = 283,115.88

FIG. 4

If the intersection tables are used to obtain the co-ordinates when a geodetic position is given for the starting station, the other computations in a traverse need only 7-place logarithms or 7-place natural function tables, which is the same as required if the traverse were based on local co-ordinates. The only difference in computing traverse on the State system and on a local system is the starting azimuth. On the State system a simple computation, called a



THE COMMONWEALTH OF MASSACHUSETTS–PUBLIC WORKS DEPARTMENT

Computation of Plane Coordinates on Lambert Projection

City or Town Freeville County Bristol Station SOMMINT

Old Datum ____ ϕ = ° ′ ″ λ = ° ′ ″

Correction ____

1927 Datum ____ ϕ = 41° 46′ 33.″877 λ = 71° 03′ 15.″650

Tabular difference of R for 1″ of ϕ = 2.0052642 l = 9.8271939

a = 1.5299049 b = 1.1945143

R″ = 3.5351691 f = 1.0217082

R′ (for min. of ϕ)		23270108.02	y (for min. of ϕ)		279369.30
R″ (Cor. for sec. of ϕ)		− 3429.01	Cor. for sec. of ϕ		+ 3429.01
R		23266679.01	y′		282798.31
			y″ (= $2R\sin^2\frac{\theta}{2}$)		+ 317.57
θ (for min. of λ)		00° 18′ 08.″2004	y		283115.88
f (cor. sec. of λ)		− ° ′ 10.″5126			
θ		00° 17′ 57.″6878	$\frac{\theta}{2}$		00° 08′ 58.″84
θ''	For machine computation	1007.″6878	$\frac{\theta''}{2}$	For machine computation	″
log θ''		3.0324929663	log θ''		3.03249297
s for θ		4.6855728909	colog 2		9.69897000−10
log sin θ	sin θ		S for $\frac{\theta}{2}$		4.68557437
log R		7.3667343982	log sin $\frac{\theta}{2}$	sin $\frac{\theta}{2}$	7.41703734
log x′		5.0848002554		R sin $\frac{\theta}{2}$	
x′	R sin θ	121562.68	log sin² $\frac{\theta}{2}$	R sin² $\frac{\theta}{2}$	4.83407468
		600,000.00	log R		7.36673440
x		721562.68	log 2		0.30103000
			log y″		2.50183908
					317.57

X = 600 000.00 + R sin θ

y = y′ + 2R sin² $\frac{\theta}{2}$

y = the value of y on the central meridian for the latitude of the station

S = log of ratio for reducing arc expressed in seconds to sine (see log tables)

R′, y′, and θ are given in special tables

l = relation of θ in seconds to 1″ λ

l = 0.6717286561 log l = 9.8271938758 − 10

a = Seconds of latitude

b = Seconds of longitude

Copy checked by: R.C.P.

1

Date 2/23/37 Calc. by F.C.C.

FIG. 5

grid inverse, will give the azimuth and distance between two known stations. Using this inverse azimuth the work may proceed as with any other traverse.

A sample computation using the table of minute intersections (Fig. 2) and the graphic correction (Fig. 3) is given in Fig. 4. The chosen station, Sommint, is located so as to be covered by the sample page illustrated in Fig. 2. It is believed that the numerical work will be self-evident.

To check the results in Fig. 4, a computation of the co-ordinates for the same station by solution of the formula is given in Fig. 5, on the standard form of the Massachusetts survey. The detailed procedure needs no special comment. In particular, the close check with Fig. 3 is to be noted.

CONCLUSION

The value to all future land surveys of the rectangular system of plane coordinates will be in proportion to the local and community use made of Massachusetts Geodetic Survey work already completed and to be accomplished. Every effort is being advanced to provide each community in the State with fixed points of horizontal control together with the elevations above mean sea level.

Such stations are being located on individual town maps prepared for ready reference. Suitable symbols to identify the points are listed and explained in an accompanying legend. At each of these horizontal control stations the following data will be available: (1) A description of the general location; (2) a diagram showing distances and directions to station reference points; (3) plane co-ordinates on the State system; (4) direction angles, distances, and azimuths to adjacent stations; and (5) elevation above mean sea level.

Faithful use of this information will eliminate for all time multiplicity of origins, the over-supply of level datum planes, and the varied waste of land surveys, and will contribute to an enlarged network of fixed points whose permanence of location will rival that of the sun. It remains for the towns and small communities themselves to effect this realization. It remains for them to see to it that future public or private surveys are tied to and developed upon the data available at control stations; that the positions of all important points and of monuments are made permanently recoverable; that old established points are co-ordinated with the new control points and proper recomputations made; and that all future local work is tied to the State system. It likewise remains for these communities to appropriate sufficient funds to carry on this local work and to maintain complete records of procedure and results for future use.

Such is the story of the establishment of the system of rectangular coordinates in the State of Massachusetts and such is the path that must be followed for its complete extension and realization.

THE LAND COURT AND ITS ENGINEERING PROCEDURE

By Clarence B. Humphrey,[3] Esq.

Synopsis

The purpose of this paper is to place before the engineer the work done by the Land Court; to acquaint him with its practice and procedure; to provide a background for his examination of title deeds; and to encourage the use of the Massachusetts Geodetic Survey triangulation points and traverse stations in connection with the State-wide co-ordinate system.

It is hoped that engineers connected with State and town governmental agencies in Massachusetts, knowing the present situation in regard to independent surveys, will co-operate and place their survey projects on this system, thus making available valuable map data.

From its beginning the business of the Court has increased at a uniform rate, falling off slightly during the depression period. The number of registration petitions filed is about 18 000 and certificates issued about 150 000. The decree and subdivision plans on file number about 25 000 and the assessed value of land registered is about $200 000 000.

Early Grants

The original source of title to the soil in New England, as in other parts of the United States, was in the English Crown which claimed it by the right of discovery and possession. The King was the owner and thus all titles to landed property were derived from a grant of the English Crown.

First, there the Royal Grants, among which might be mentioned the Plymouth Company in 1606; the Massachusetts Bay Company in 1629; the Ferdinando Gorges grant in 1639, covering most of Maine; the River Towns, in Connecticut, in 1662; the Province of New Hampshire in 1679; and the Narragansett Bay settlement under Roger Williams in 1643. In 1691 the Massachusetts Bay Colony received a new charter, and its jurisdiction covered what is now Maine and Massachusetts. The Plymouth settlement was included in the Massachusetts Bay Province thus reconstructed.

Next came the Colonial Grants with the power vested in the General Court. These grants were to individuals and groups of individuals who became known as proprietors. The grants to individuals were not numerous and seldom contained more than 350 acres, while the grants to the proprietors for laying out of plantations would contain a tract about 6 miles square. These plantation lands were surveyed and divided; and at legally called proprietors' meetings, selections of the divided parcels were made by drawing lots. Although these deeds were properly recorded the title itself runs direct from the proprietors' records.

---

[3] Engr., Mass. Land Court, Boston, Mass.

RECORDING

Under the Massachusetts recording system [4] in vogue since 1634, the document recorded is a deed, which is operative without record, the title passing upon the delivery of the deed and before it is recorded.

According to the General Laws:[5]

> "A conveyance of an estate in fee simple, fee tail or for life, or a lease for more than seven years from the making thereof, shall not be valid as against any person, except the grantor or lessor, his heirs and devisees and persons having actual notice of it, unless it is recorded in the Registry of Deeds for the county or district in which the land to which it relates lies."

This Section although revised many times still reads very close to the original ordinance of 1640. The next paragraph, Section 5,[5] provides that:

> "The record of a deed, lease, power of attorney or other instrument, duly acknowledged or proved as provided in this chapter, and purporting to affect the title to land, shall be conclusive evidence of the delivery of such instrument, in favor of purchases for value without notice claiming thereunder."

These two Sections (Sections 4 and 5) are the very basis of the recording system, which not only preserves the evidence but gives a legal priority to the grantee of the recorded deed, insures a safe depository by the county, and makes it possible to investigate the title to property. First used in the Plymouth Colony in 1634, it remained in vogue until 1898 when it was modified by the adoption of the Land Registration Act.[6] The system proved well suited to the needs of the people and in a greater part of the Commonwealth it still would be satisfactory. However, with the passing of time, as cities became congested, as the records became exceedingly complicated, and as the methods of dealing with land have changed, real estate operators found it advisable to have their titles in such shape that they could be transferred speedily, the same as shares of stock, or other securities. For such purposes the existing system was not adequate. In the cities there was too much record title and in the country there was, at times, too little. Numerous other evils of the system needed correction.

Title to land is acquired in various ways, such as by inheritance, by will, by mortgage, and by direct purchase. During these changes over a period of years many things might occur to invalidate a title. Thus, no prudent man will purchase land until he has had the title studied by a competent examiner, as title on which money is loaned, or on which the issue of bonds is based, must be valid. If the land has repeatedly changed hands, it will be found that the title has been examined and re-examined by many separate interests with the resulting duplication of expense; and even then, the most exacting scrutiny might fail to disclose vital defects, such as prescriptive rights.

---

[4] "The Origin of the System of Recording Deeds in America," by Joseph H. Beale, Jr. Harvard Univ., 1907.

[5] General Laws, Tercentenary Edition, Chapter 183, Section 4.

[6] "Land Registration Act of Massachusetts," by Charles Thornton Davis, Judge of Land Court. (Read before the New York State Bar Assoc., 1908.)

It was said by the Hon. Fred E. Crawford,[7] of the Boston Bar, that "if a new beginning could be made" back of which it would be unnecessary to go—and if a title could be given new life and started on a new career—much would be accomplished. In theory, all titles come from the State and if a title might be considered as coming anew from the State, then from that day the State would stand behind the title. This is what the Land Court does.

To Australia—the home of that other great principle, the Australian ballot --goes the credit for breaking the precedent in passing title to land. Sir Richard Torrens was at one time Commissioner of Customs in South Australia. Apparently annoyed at the delay in transfer of titles in land, due to the time consumed in the examination and re-examination of back titles, he devised a system of registration of title to land incorporated in what became known as "The Torrens Act."[8] This furnished the idea for legislation adopted in various parts of the United States.

The first American State to adopt some system of conveyancing by the registration of titles, was Illinois in 1895; then came Ohio in 1896; California in 1897; and Massachusetts in 1898. Since then Acts have been adopted in some form or other by Colorado, Minnesota, New York, North Carolina, Oregon, Washington, Utah, Virginia, North Dakota, South Dakota, and Georgia; also Hawaii and the Philippine Islands. The Act adopted by Massachusetts was distinctly local—drawn to fit existing conditions and establishing a system designed to give relief from the defects of the old method.

THE LAND COURT

In the Commonwealth of Massachusetts there are fourteen separate and distinct counties and twenty-one Registries of Deeds. The Massachusetts Act (now the General Laws, Tercentenary Edition, Chapter 185)[5] provides for a separate body, called the Land Court,[9] having State-wide jurisdiction, with a judge, two associate judges, and a recorder. This new system was grafted on to the existing recording system by creating a Land Registration Section in each Registry of Deeds, and making each Register an Assistant Recorder of the Court, *ex officio*.

The Court is situated in Boston, Mass., and its decrees are sent out for recording to the Assistant Recorders. On the authority of these decrees certificates are issued and all subsequent transfers are made by certificate. Thus, the title is given a new lease of life and a day is set, after which an examination is unnecessary. The Act not only provides machinery whereby ownership of the premises is adjudicated, but boundary lines on the earth's surface are judicially determined, and remain definite and fixed. To register a title the procedure is as follows:

(1) The petitioner files a petition and a plan with the recorder and pays the statutory fee. The petition describes the land as claimed; states all known encumbrances and adverse claimants with their addresses, whether admitted or

---

[7] "The Massachusetts Land Court," by Fred E. Crawford, 1914.

[8] "The Torrens System," by Arnold Guyot Cameron, Houghton, Mifflin Co., Boston, Mass., 1915.

[9] "The Land Court: Being a Copy of the Land Registration Act, Chapter 185, General Laws (Tercentenary Edition) with Procedure and Practise," 1936.

denied; and gives the names and addresses of all adjoining owners. It is accompanied by an assessor's certificate signed by the local assessor verifying the adjoining owners as they appeared at the last assessment for municipal taxation. The plan is made from an actual ground survey and shows the land claimed with the location of the boundary lines. The petition is given a number, the first one filed being No. 1, and these numbers run consecutively throughout the State. All papers with the petition, as well as the plan itself, bear this same case number. At the same time, a notice of the action is filed in the registry of deeds.

(2) The petition is referred to a Court examiner to report on the state of the title; he is allowed two weeks for this work. He must be an attorney-at-law, appointed by the Court. When his report is filed, notice is given to all parties in interest by publication once each week for three successive weeks in a newspaper in the county where the land lies. It is expressly provided that by the description in the notice "to all whom it may concern," all the world are made parties defendant and are to be included by default, and that the registration proceeding is *in rem suam*. The return day of the notice must not be less than twenty nor more than sixty days from the date of issue, and within seven days the recorder sends an attested copy of the notice to each person named therein, by registered mail. An attested copy of the notice is also posted on the land itself by a deputy sheriff, not less than fourteen days before the return day. Such other and further notices are given as the Court may order.

(3) If on or before the return day no one appears to object to the registration of the title to the land in the name of the petitioner, the examiner's opinion with the complete abstract of the record title is carefully read and considered by the Judges. Even if no objections have been made, further service, or additional information, or both, are many times required. After all these precautions have been taken, an order for decree of confirmation and registration of the title is entered in the name of the petitioner.

(4) If on the return day appearances and answers have been filed objecting in part or in whole to the petitioner's claim for registration, then, on motion, the matters involved are set down for a formal hearing before the Court. If the objections are not disposed of in Court—by agreement, by stipulation, or by withdrawals—the case is heard and a decision filed by the Judge. This decision is final upon all questions of fact, but any questions of law arising in the Land Court may be taken directly to the Supreme Judicial Court within twenty days after the decision is filed. When remanded to the Land Court an order for decree is entered as in an uncontested case.

COST OF REGISTRATION

Upon filing a petition, the petitioner is required to deposit $25 plus one-tenth of 1% ($1 per $1 000) of the last assessment for taxation; and when the decree is issued he pays the further sum of $2 for the decree and copy, and one-fourth of 1% of the assessed valuation, but not less than $10, nor more than $1 000. At the same time, the recorder's office collects $5, which is transmitted to the registry with the decree, to pay for entering the papers and for preparing the certificate and owner's duplicate. In addition to these statutory charges, the petitioner pays his title examiner and also his engineer for survey work and plan. The $25 is for publication, sheriff's fees, etc., and if any surplus remains when the case is closed, it is returned to the petitioner. If such deposit is not sufficient, then the remainder is paid upon the request of the recorder and before any decree issues.

The one-tenth of 1% is paid as an assurance fund to be deposited with the State Treasurer who keeps it invested and adds the income to the principal; and when the fund amounts to $200 000, as it did in 1929, the income is used to help defray the expense of the Court. This fund is for the purpose of reimbursing the Commonwealth for any sum its Treasurer may be called upon to pay out because of negligence on the part of any officer under the Land Registration Act. The man given a title under a decree of the Court obtains an indefeasible title, one that is settled and that remains settled. This is most important.

SURVEYS AND PLAN WORK

The Land Registration Act does not provide for an engineer, but Judge Charles Thornton Davis, who has been with the Court from 1898 to 1936, early recognized the necessity for an engineering department. He established it as an adjunct to the Recorder's Department.[6]

Instructions as to surveys were issued to acquaint the engineers with the requirements of the Court in regard to surveys and plans, and to standardize Land Court surveys. This was necessary in order to obtain in the first instance full engineering data, showing field monuments, which would enable any engineer to reproduce the lines upon the ground. In the past the fault has been that plans filed in the local registries lacked both monuments and survey data and, at times, even distances on the property lines themselves. Plans showing property and subdivisions are still recorded which have little or no value because the lines shown on them are in dispute and the engineer who made them has long since been dead. It is a case in which the engineer considered the missing data as a part of his stock in trade. This condition, however, should not discourage the filing of such plans, because some deeds are so ambiguous that even a sketch of the land sought to be described would be of assistance.

What is a survey? What are the requirements for a model plan and what data should be shown thereon? To be of any value, a plan should show plotted lines drawn to scale, distances, angles or bearings, survey notes when necessary, and the character of existing physical boundaries and monuments. Thus, all lines may be reproduced accurately by another engineer many years hence should occasion require.

In making plans for the Land Court:

(1) The plan must be made on tracing cloth.

(2) The sheet may be 18 in. by 24 in., or 24 in. by 36 in., or any convenient larger size.

(3) The title must state the location, scale, date, name, and address of the surveyor.

(4) The plan must show the north point, town, or county lines, names of adjoining owners, passageways, street lines, boundary monuments, all fences,

walls, buildings on or adjacent to lines claimed (using proper conventional signs), and all other natural monuments pertinent to the purpose.

(5) The scale must be such as to show clearly all data necessary.

(6) The area may be by calculation or by scale; if by scale the sign, ±, is affixed.

(7) Such parts of buildings and other physical features on adjoining land as are within 10 ft of boundary lines are shown.

(8) Boundaries of the property, pertinent dimensions, and all physical features are shown in black; all traverse and surveying data in red.

(9) The plan must indicate plainly how each corner, property line, and reference point is marked on the ground. This information is essential, for the reason that respondents often object to a plan and survey because a stone bound, partition wall, retaining wall, or other object used to mark a boundary, has been erroneously included or excluded. On this account careful inquiry should be made to determine the ownership of such boundary monuments. If necessary a note is included describing what part of the boundary monument is used.

(10) The parcel sought to be registered must be enclosed within blue tinting.

(11) The plan should be oriented so that the top is in a general northerly direction.

Engineers are always reminded that the land may be bounded on ways, that is, streets, avenues, highways, boulevards, and parkways, also on railroad locations and public lands. Such ways are monuments, possibly referred to in the title deeds and as such the data are, or should be, deposited in the various public offices and the county registries. They are asked to examine these data and to place on the plan the initial data points used in making the survey.

In regard to bearings, engineers are asked to carry forward, if possible, one bearing used in a previous registration case; if not, to use a bearing found on an official street plan; or, as a last resort, to read the magnetic bearing of some one line of the new survey. All bearings on the survey must be relative, computed from measured angles.

A computation sheet is required, showing traverse on the lines claimed, or on lines from which the lines claimed are located. It gives computed latitude and departures and additional data, if necessary and not shown upon the plan, so that rectangular co-ordinates of all points on the plan can be computed.

On the plan is the endorsement: "I certify that this survey was made in accordance with Land Court instructions," with the surveyor's signature. Thus, the engineer must have all these requirements in mind when making his survey.

This plan is indexed and casually examined; if any data are missing or monuments should be set, the attorney of record is so notified. This procedure expedites matters when the case is ready for decree, which may be months later depending on the questions involved. The plan is used throughout the hearings. Usually, no changes are made; in some cases the lines are changed by agreements; and in still other cases, due to protracted hearings and formal decisions, many changes are made. When, however, an order for final decree is issued a plan called the decree plan is prepared in the Engineering Department. These plans are all of one uniform size, 10 in. by 15½ in.

This decree plan is sufficient in itself, with all survey data and monuments shown on it, to enable the established lines to be found on the ground. From this plan the decree is drafted describing the land as registered and setting forth all encumbrances, easements, or appurtenant rights, as set out in the order for decree. The Court early decided to use a bounding description rather than a so-called running description and, by the plan reference, to make the plan itself a part of the decree of the Court. The Supreme Court has often held that, by a well settled rule of construction where a plan is referred to in a deed, the lines, courses, distances, references to monuments, and other particulars appearing on the plan, are to be regarded as much as they would be if expressly recited in the deed.

The descriptions in the decrees, therefore, are brief, using a general bounding description, grouping together shorter lines running in the same general direction, but reciting, however, the adjacent owners and natural boundaries. The first boundary is usually a highway; thence the description reads clockwise. The plan reference most often used is as follows:

"All of said boundaries are determined by the Court to be located as shown upon plan numbered * * *, which will be filed with the original Certificate of Title issued on this decree, the same being compiled from a plan drawn by John Doe, Engineer, dated * * *, and additional data on file in the Land Registration Office, all as modified and approved by the Court."

The decree and plan are sent to the assistant recorder for the district within which the land is situated. He transcribes this decree in a registration book and this entry is the original certificate which is accompanied by the plan. The assistant recorder then makes an exact copy of the original certificate, adding to it the words, "Owner's Duplicate Certificate," and delivers this to the owner.

Thus, the title to the parcel of land may be considered as being born again. From this time it is guaranteed by the State. The parcel thus started on a new career may contain only a few hundred square feet, or it may comprise several square miles, as the case may be.

GEODETIC CONTROL

From the beginning in Massachusetts it has been the aim to have all Land Court surveys made so that co-ordinate values could be computed relative to an assumed origin which might be one corner of the locus. The early instructions asked to have all surveys connected with recognized triangulation points when within or near the locus. When not connected with any recognized origin, then an origin within the survey is assumed and contiguous surveys is connected with it. This is carried on until the group of surveys, in turn, is connected with some local origin connected with the triangulation system, and then when these lands are thereafter dealt with, the lines are recomputed, based on the local origin. No request has been made to have local engineers use geodetic computations. Very few are acquainted with the formulas and they do not associate values in degrees, minutes, and seconds with small surveys, but rather with the U. S. Coast and Geodetic Survey work. If they are to carry on this horizontal system of control it must be made easy for them. Thus, the calculations for all reductions from geodetic values to plane co-ordinates have been simplified by the methods of the Massachusetts Geodetic Survey, as explained in Mr. Houdlette's paper. In many cases the necessary co-ordinates are furnished them before they begin a survey, especially if it covers a large area.

With the State divided into twenty-one registry districts, the system first adopted was to have an origin in the center of each so that all values would be relative to land registered and recorded in the respective registry of deeds. This system of rectangular co-ordinates was based upon a plane tangent to the earth's surface at a point. The area of each district was such that no appreciable error would appear on the exterior perimeter. The origin being a part of the State Triangulation system, the reduction from geodetic co-ordinates to plane co-ordinates was first made using the L.M.Z. formula (L.M.Z. has been adopted as representing latitude, longitude, and azimuth) and, later, by using formulas in *Special Publication No. 71*, issued by the United States Department of Commerce in 1921. This system was gradually being utilized by local engineers and by engineers working on the State highways.

In April, 1933, working in conjunction with the Department of Public Works a new system was adopted, as explained by Mr. Houdlette, whereby the State was divided into eight zones, but still using the plane rectangular system. The adoption of these new zones made it easier for the State engineers in crossing from one system to the other, as the number of origins were reduced from 21 to 8. It was also a step forward in having the highway network placed on a definite system referred to a common meridian, and if co-ordinate values were carried forward on subsequent highway plans they would have an economic value, appreciated by the engineer, but little realized by the owner of the land.

In December, the U. S. Coast and Geodetic Survey reported that all future values for triangulation stations in Massachusetts would be computed and published, as based on a new datum known as the North American datum, 1927. In using Government maps and geographical positions given in terms of latitude and longitude, especially in research work, engineers should, therefore, have in mind the three main projections used in Massachusetts, namely the Bessel spheroid used previous to 1880; the North American datum based on the Clarke spheroid adopted in 1880; and the North American datum, 1927.

In consequence, it was decided to adopt one uniform system of co-ordinates with values expressed in feet and fractions thereof, extending across the State. The projection advised by Dr. O. S. Adams, U. S. Coast and Geodetic Survey, as approved by the Massachusetts Department of Public Works and the Land Court, is defined as the Lambert Conformal Conic Projection of Clarke's spheroid of 1866, with two standard parallels held true to scale. This system is fully described by Mr. Houdlette.

CADASTRAL MAPS

The need for key maps upon which to plot the location of registered parcels was early recognized. All the available commercial atlases of the towns, cities, and counties were purchased and upon these atlas sheets the lands sought to be registered were outlined in blue and the case number indicated. These atlases serve as a graphical index, not only for the use of the Court, but for the public as well, to ascertain whether any registered lands exist in any given locality.

In addition to these atlases, there have been made from time to time standard base maps, called cadastral plans. The basis for these plans is the geodetic control system and the fifty-four Topographical Sheets of the U. S. Geological Survey, which include the Commonwealth of Massachusetts.

These 15′ quadrangle sheets have been divided into nine lettered sections of 5′ each and each section is further divided into eight numbered plans. Thus, there are seventy-two plans to a topographical sheet, and the area covered by each plan is 2′ 30″ of longitude and 1′ 15″ of latitude. The scale for these plans is 300 ft to an inch which allows the use of standard paper, 27 in. by 40 in., with about 1-in. margin. On a decree plan the reference, "52-H-6," would indicate that the land as registered is plotted on Topographical Sheet 52, Section H, Plan 6.

These plans when first prepared show border lines of longitude and latitude, dimensions, in feet, on each side, and the rectangular co-ordinates of each corner intersection, as well as the sheet number, town names, and town boundaries. From these data, inside construction lines are plotted and the sheet is then ready for plotting any parcel or lines connected with the geodetic control. Existing maps upon which the geodetic lines appear may be easily enlarged and placed upon these plans.

SUBDIVISION PLANS

The Land Registration Act provides that a registered owner holding one certificate for several distinct parcels may surrender it and take out a certificate for each parcel; likewise, if he holds separate certificates for each parcel, he may surrender them and take out one certificate for all parcels or any number of them. The Act further provides that an owner who subdivides a tract of registered land into two or more lots shall file with the Court a plan thereof, when applying for a new certificate; that this plan shall be verified; and that all newly created boundaries and ways shall be distinctly and accurately shown thereon.

The original Certificate of Title describes with a bounding description the perimeter of the land registered, and this is true whether the decree plan shows one parcel containing 5 000 sq ft of land or 5 acres subdivided into, say, thirty numbered parcels. In the first case, if the parcel was conveyed, the owner would surrender his certificate; it would be canceled and a new certificate issued to the new owner, referring to the plan filed with the first certificate. This might happen many times.

In the second case, one or all of the thirty separate parcels as shown upon the decree plan might be conveyed to as many separate new owners. Each new owner would receive a certificate describing the parcel he had acquired and referring to it by Parcel No. * * *, Plan No. * * *, filed with Certificate of Title No. * * *. As each parcel was conveyed, a memorandum to that effect would be entered on the grantor's certificate giving the grantee's name, date, and other pertinent information. Likewise, all mortgages or other papers affecting the title would be entered, so that the state of the owner's title is always obtainable by first reading the owner's certificate and thus observing the nature of the encumbrances entered on its reverse. In many cases where mortgages have been registered, later discharged, and the parcels sold, the owner will surrender his certificate encumbered with all these references and ask that he have a new certificate, purged of all "dead wood," which after an examination by the Court is granted.

When the decree plan shows numbered parcels, or parcels and ways, separate Certificates of Title may be issued without any further Court order. The fact that the divisions are on the decree plan is the Assistant Recorder's authority. When, however, any parcel as registered and shown upon the decree plan is changed by creating new interior boundary lines, a new plan is required as described, and, at this time, the owner files with the Engineering Department of the Court his Certificate of Title, and the new plan. This certificate is examined to see if it covers the land shown on the plan as filed, and the plan itself is verified to see that it is consistent with the previous registration plans to which the certificate refers and that all new boundaries and ways are accurately shown with proper engineering data, the same as if a new petitioner's plan were being filed for original registration. Each new lot or parcel must be consistent in itself and the distances and angles indicated must be reasonable. If found correct in all these details it is approved and a plan made from it on standard-size white paper showing the new divisions with data appearing on the filed plan, and in many cases additional data appearing on the files of the Court. On this plan appears the legend:

> "Separate Certificates of Title may be issued for the numbered lots as shown hereon; or it may be for 'certain lots, *viz.*, Lots 24 and 25 as shown hereon'; as the case may be. This legend is attested by the Court."

This plan with the owner's Certificate of Title is sent to the Assistant Recorder for the district in which the land is situated with an order to cancel that part of any plan or plans upon which the land thus divided is shown. This process continues *ad infinitum*.

The subdivision plan work is an important and ever-increasing function of the Court. It is handled exclusively in the Engineering Department where in many instances complete descriptions based upon the new plan are furnished to the Assistant Recorders. The subdividing of land, the relocation of highways, the merging of contiguous parcels into one ownership, the re-arrangement of lots, the call for plans showing land remaining after a large tract has been actively dealt with, and the many other activities encountered in dealing with land and title create new plan work and the issuance of new Certificates of Title. This cancellation of the old or previous plans and the making of new plans is the means of continually keeping up to date the existing state of the title as well as the existing facts upon the ground, with newly acquired engineering data.

It is in dealing with subdivision plan work and the calculations incident thereto that the importance of a rectangular co-ordinate system, whether local or not, is impressed on the engineer. In many instances, the adoption by the State of one common projection to be used by all engineering agencies will prevent the duplication of survey work, which all engineers know exists at the present time.

Although a Land Court will quiet the title and make it indefeasible, a coordinate system such as is now recommended by the U. S. Coast and Geodetic Survey furnishes the foundation for all local and municipal surveys; eliminates the uncertainty of the magnetic meridian; provides a system for placing on plans the exact value of all corners, thus making property plans more valuable; provides a method of reproducing lines in areas devastated by fires; and, finally, tends to reduce materially the boundary line controversies which are so detrimental to the peace of a community, as a single point on the earth's surface furnishes a starting point for all surveys. Most disputes arise from a misinterpretation of the law controlling boundaries and a difference of opinion between engineers as to the starting point.

Doubtless problems of this general type were in the mind of Moses when he stated in "Deuteronomy" 27:17: "Cursed be he that removeth his neighbour's landmark, and all the people shall say, Amen."

DISCUSSION

H. J. Shea,[10] **Jun. Am. Soc. C. E.** (by letter).—In the face of such obstacles as had beset its path, the Massachusetts Geodetic Survey not only has triumphed, but has succeeded so well that the organization deserves a permanent place for the promulgation of engineering data in the Commonwealth of Massachusetts. Because of the haste with which the organization was instituted and the large number of parties operating in the field, the Home Office was shortly deluged with field observations. Recalling its early stages, it is not only a wonder that valuable data were forthcoming, but it is a marvel that the Massachusetts Geodetic Survey was able to extricate itself at all.

Massachusetts is indeed fortunate in having two excellent surveying agencies—the Massachusetts Geodetic Survey and the Land Court—both using the Lambert plane co-ordinates and advancing the engineering benefits of such a system. It is to be hoped that the Department of Public Works will be one of the first highway organizations to make widespread use of the Lambert co-ordinates. Such an application would obviate any duplication of data and would further efficient performance in this important public service.

However, the desired goal is not quite attained. Intelligent surveyors are after common applications of plane co-ordinates in point descriptions—those which concern the layman. The present problem is not one of "new surveying." No overhauling of specifications or methods of computations is necessary, but rather the problem appears to be the education of the every-day surveyor in the use of geodetic control through the translation of geodetic control into terms that are readily understood.

Perhaps the educational necessity involved may be inferred from a statement about the condition of control surveys in Great Britain. In a personal communication, the Brigadier of the Ordnance Survey states that geodetic control cannot be considered limited because it covers the entire country, as do the published plans. All the surveys of Great Britain are conducted from that office. Its large-scale property plans are published at the scale of 25 in. to the mile. Quoting the Brigadier:

> "They may need occasional addition and amplification by local authorities, but they are based on geodetic control and give graphically the positions of all important buildings and everything except the most recent developments in correct relative position. In England, the private surveyor is hardly ever called upon to take accurate measurements in the field."

Although the United States has no surveying unit similar in scope to the Ordnance Survey of Great Britain, the calculated effect of the present program should not be greatly different. Such a result may be obtained through unity of purpose in the various programs of surveying and the adoption of standard specifications.

---

[10] Instr. in Civ. Eng., Mass. Inst. Tech., Cambridge, Mass.

In modern precise surveying, the importance of accurate geodetic control has become increasingly great. Scientific and well-planned surveying units, municipal or private, are quick to recognize the advantages derived from a uniform surveying control. By geodetic control is meant the establishment of surveying data, obtained either by precise triangulation, traverse. or levels, adjusted to remove accidental errors and based on a uniform datum. The standards of such work are considered so high that subsequent and more divided surveys are based upon, and adjusted to, the fixed values of the control survey.

There is one point in the problem of establishing control surveys that should be clearly understood. The surveyor is in the nature of a public servant and is often forced to deal with people having less specialized training. Although geodetic methods of computation should be the foundation, because of the difficulty in handling such computations, the greater number of surveyors are unable to use geographic values efficiently. The impartial observer, at present, finds it difficult to agree with impatient statements relative to the inability of surveyors to cope with the specialized methods of geodesy. It must be remembered, however, that the results of surveying operations are not confined to specialists only, but to lawyers, land owners, and dealers—in short, every one connected with land is intimately concerned therewith. Through the winning of the layman's confidence, the success of the present program will be assured. Thus, for a widespread realization of the benefits of control surveys, it is important to present the subject on a sound common basis rather than await the training of specialized surveyors.

A plane co-ordinate system is the apparent answer to the problem. In evaluating the relative advantages of the various co-ordinate systems, primary consideration should be given to the ease of reference to basic geodetic values. Although various systems have been suggested adapting co-ordinates to surveying, at present the most widely used are the tangent plane, the Lambert conformal projection, and the transverse Mercator projection co-ordinates. Co-ordinates on the latter two systems are handled much the same way, one being adapted to States having greater east-and-west extent and the other to States having a greater north-and-south extent, respectively.

The characteristics of these systems may be compared in the following schedule, in which (*a*) refers to the use of the tangent plane, and (*b*) refers to the use of the Lambert and transverse Mercator projection:

Basic Origin.—

(*a*) One local origin in the tangent plane. For wide use several tangent points are necessary (three are needed for New York, N. Y.).

(*b*) The Lambert and transverse Mercator are based on two geodetic parallels (or a meridian). One State system covers States as large as North Carolina or New Jersey.

Comprehension.—

(*a*) The tangent plane system is very easy to understand.

(*b*) Somewhat more difficult than the tangent plane but can be grasped readily with some study.

Computation of Points and Traverses.—

(*a*) Points are computed easily from geographic co-ordinates, but difficulty is encountered in traverses, because of the varying scales for which there is no definite mathematical treatment.

(*b*) Points and traverses are computed simply. There is a definite relation between grid and geodetic lengths. Angles are well maintained through the rectangular grid.

Extent.—

(*a*) Definitely limited and local.

(*b*) State-wide; and basically national through ease of transfer from one system to another.

Basic Tables.—

(*a*) The basic tables are excellent through the use of U. S. Coast and Geodetic Survey *Special Publication No. 71.*

(*b*) Excellent through use of various State tables. Noteworthy of mention are the tables prepared in 1937 giving the plane co-ordinates of minute intersections of meridians of longitude and parallels of latitude published by the Massachusetts Geodetic Survey. This publication reduces the calculation of plane co-ordinates from geodetic points to mechanical interpolation.

Adjustment.—

(*a*) Well adapted to adjustment, but dependability of adjusted values varies because of poorly determined scale effect.

(*b*) Well adapted. Adjustments may be made directly with co-ordinate values on the grid. Subsequent geodetic values are easily found.

Publication of Co-Ordinates.—

(*a*) Comprehensive list may be formed, but strictly local in scope.

(*b*) State-wide publication possible.

At present, one disadvantage in the education of surveyors into the use of the Lambert and the transverse Mercator projection co-ordinates seems to be the necessity for the reduction of geodetic lengths to grid lengths. It should be emphasized that the scale factors represent, not errors, but scale variations—mathematically exact in their relation to the geodetic values, free from any such stigma as "errors," easily applied to measured distances. Any inference of the existence of errors in the Lambert and transverse Mercator projections must be strongly rejected.

The legal aspect of geodetic control is perhaps one of the most potent factors in its favor. Land, certainly a common article of transfer, is often sold and bought without definite knowledge of the area, the protection afforded, and the surety that the deed description entitles the grantee to the property sometimes so vaguely described. Without attempting to draw "hard and fast" lines, the types of deed descriptions of property may be classified into four categories (seemingly in order of diminishing frequency); namely, land descriptions which, when without the original monuments or ties to the original monuments, are: (1) Not retraceable; (2) replaceable if a sufficient number of original monuments are recovered; (3) replaceable if one property monument is found or is replaceable because of ties to some public monument; and, (4) replaceable because description gives co-ordinates of property corners in some comprehensive system based on geodetic control.

Existing land rights in Case (1) are based on the best evidence possible, and two different surveyors might give widely different interpretations. Cases (2) and (3) depend for their interpretation on the recognition of extant bounds. In Case (4) replacement is simple if horizontal control points are within reasonable distances.

An important benefit resulting from Case (4) would be an amelioration of the ethical standards of the Surveying Profession. Too little emphasis has been placed on the legal ownership of surveys, a fact considerably abused by practicing surveyors. Property corners described in a co-ordinate system based on geodetic control would certainly leave no doubt as to the meaning of the description and should increase the confidence of the property owner in the work of the surveyor.

Hitherto, monuments have been thought to be the ultimate in boundary and property demarcation; but has not surveying experience in every section of the United States demonstrated that loss or displacement of monuments through construction and natural causes is sufficiently frequent to instigate well-placed skepticism in the lasting properties of physical markers? With a universal and comprehensive system of property description based on the required use of State-wide co-ordinates, does it not seem possible that there is at last a definite goal—a surveying element "better than the monuments"?

Mr. Houdlette's paper is noteworthy as an outline of one of the best surveying units of its kind. It must be regarded, therefore, as a valuable paper and would serve well as a guide for similar State organizations.

Philip Kissam,[11] Assoc. M. Am. Soc. C. E. (by letter).—The importance to the public of closely spaced, previously determined survey control points, and definitely determined, mutually controlled, and permanently located boundary lines cannot be emphasized too strongly. This Symposium constitutes a description of an objective which must be ultimately attained throughout the United States. The advantages to be gained are obvious upon a moment's consideration.

It is evident that no survey is of use unless its precision is determined. Unless public survey control is available, individual control systems must be established for even the simplest project. If, on the other hand, standard, recognized control points of unimpeachable accuracy are established once and for all in a certain locality, all surveys made in that locality can be based on this control with the result that precision will be increased and survey costs reduced.

When once a survey has been completed, the data obtained cannot be combined with that of another survey unless both are based on the same system of control and preferably on the same datum. It is essential, therefore, that the control system, to be most effective, should be nation-wide so that in fact all surveys become part of one great survey. It will be noted that in the work that Mr. Houdlette describes, all control points, both vertical and horizontal, are connected with the national level, or triangulation net, so that the first requirement is abundantly fulfilled.

---

[11] Associate Prof., Civ. Eng., Princeton Univ., Princeton, N. J.

The utilization of the Massachusetts system of plane co-ordinates is an exceptionally important part of the project Mr. Houdlette describes. Practically all ordinary surveys are reduced to plane co-ordinates in order to determine their precision. It requires no more work or difficulty to use Massachusetts co-ordinates to make this reduction; but whenever Massachusetts co-ordinates are used for a survey, that survey is forever fixed in position and is correlated with every other survey so connected.

Perhaps the most important surveys of all are those made to define land boundaries. Property surveys are older than all others, and a large portion of the national wealth is dependent on their accuracy and permanence. Mr. Humphrey notes the set of rules he has worked out to be followed by land surveyors in making property surveys. Through these rules and by the able conduct of his office he has succeeded in developing a procedure and a type of land description which permanently locates land boundaries. Not only are the boundaries located definitely and permanently, but it is possible for any surveyor to re-mark the property corners by recourse to the description alone, and entirely without knowledge of local conditions. Those who are familiar with the archaic methods in common use to-day for making surveys and descriptions of land will recognize the singular importance of Mr. Humphrey's contribution.

One of the secrets of the success of the Massachusetts Land Court is the insistence of its engineer that durable monuments be established and that they be inter-connected by surveys so that they can be replaced when destroyed. Under his direction many local plane co-ordinate systems were developed, as noted by Mr. Houdlette. Quick to realize the importance of a universal datum, the Land Court now requires connection with the control points that the Massachusetts Geodetic Survey is establishing.

With this co-operation between the Land Court and the Massachusetts Geodetic Survey the cycle is complete. The Commonwealth of Massachusetts is being prepared for every type of survey requirement. It will be possible eventually to determine the relative position and elevation of every topographic feature or boundary line in the entire State with very little expenditure of time or money. It is evident that as these plans are developed, every survey made in the State will be correlated with the whole.

The Civil Engineering Profession cannot afford to overlook the advantages offered by this plan. If the profession is to take its proper place in public affairs every effort must be made to insure the establishment of these systems throughout the United States.

Lloyd G. Frost,[12] **Assoc. M. Am. Soc. C. E.** (by letter).—The splendid accomplishments of the Massachusetts Geodetic Survey, so ably set forth by the author, cannot be over-rated in their value to the Commonwealth of Massachusetts and to the long-neglected field of surveying and mapping, as well as to the profession at large. Mr. Houdlette has so clearly depicted the conditions that exist generally in the land-surveying field and the relation of a system of rectangular co-ordinates thereto that every engineer and surveyor, in the light of his own experience, will readily recognize the indisputable superiority of the co-ordinate method and look hopefully for extension of such State systems on a national scale.

---

[12] State Supervisor, Louisiana Geodetic Survey, New Orleans, La.

The Louisiana System of Plane Co-ordinates.—The Louisiana Geodetic Survey was established at the same time as the Massachusetts project and under like conditions. The same vicissitudes were met that visited themselves on the latter, and, in general, the details of organization and field procedure are parallel. Mr. Houdlette's description of the projection frame applies to that for Louisiana, except that the configuration of the State made two projections necessary—North and South. As with Massachusetts, the U. S. Coast and Geodetic Survey selected the projections and prepared the necessary tables of data for computations.

The difficulties encountered in securing competent personnel to perform the work adequately may have been somewhat greater in Louisiana because of the much larger area of the State, but the various obstacles were surmounted surprisingly well in the earlier stages of organization. The standards of accuracy maintained for the Louisiana project differ from those in Massachusetts and New Jersey,[13] as indicated by the following comparison (in which S is the distance run, in miles):

	Traverse	Levels
Louisiana	1 : 20 000	$0.25 \sqrt{S}$
Massachusetts	1 : 10 000	$0.4 \sqrt{S}$
New Jersey	1 : 10 000	$0.3 \sqrt{S}$

The interval between stations in the Louisiana network varied from 0.5 mile to 1 mile during the early years of the program. Experience showed that in many cases these distances were too great to permit maximum availability to the local user, and the optimum interval was set at 0.25 mile except in certain unusual cases. Wherever time and facilities permit, longer tangents are being broken down to this standard.

The strength of field forces in Louisiana varied from an average of fifteen parties to a maximum of thirty during a short period in 1936–1937. In February, 1938, a reduction was made to ten parties in a reorganization effected to increase efficiency.

The work accomplished by the Survey, through September, 1938, in comparison with that given by Mr. Houdlette for Massachusetts is as follows:

Traverse:	
Extent, in miles	5 964.82
Number of stations established	12 234
Miles of traverse with preliminary co-ordinates	531
Miles of traverse with final co-ordinates	3 754.72
Average closure error	1 : 35 000
Bench-Marks:	
Extent of second-order levels, in miles	5 964.82
Number of bench-marks	14 681

---

[13] *Bulletin No. 46*, New Jersey State Dept. of Conservation, 1938.

Protracted study of the system of field procedure led to the conviction in February, 1938, that maximum efficiency was not being attained. The field organization, unvaried since the beginning of the project, was as follows at that time:

Each party consisted of ten men—a chief of party, transitman, levelman, and seven rodmen and chainmen; transportation was by owner-operated truck, the driver working as chainman; and each party was provided with one transit and one level.

The methods of geodetic surveying necessary to insure the required standards of accuracy would not permit doing the various operations simultaneously with this number of men. There appeared to be a considerable waste of time that was unavoidable under the conditions. At no time since the inception of the project did the records of work performed indicate a rate of progress consistent with the number of men engaged on the work. The available funds did not permit any appreciable changes at that time, but it was decided to institute some investigations and experiments in anticipation of a new appropriation.

Two parties were chosen as "guinea pigs," and the personnel increased to seventeen men, classified as follows: Observation section—chief of party, recorder, and two rodmen; measurement section—transitman, recorder, two chainmen, and three rodmen; level section—levelman, and two rodmen; and monumenting section—truckman, and two laborers. It was believed that a party so organized could carry on the several operations simultaneously with a maximum of efficiency and accuracy. The results obtained more than realized expectations and all parties operating in the State (fifteen in number) were reorganized on this basis. Two division engineers were appointed with headquarters in the northern and southern parts of the State and intensive supervision of field operations instituted.

Each party was supplied with two transits and other necessary equipment to conduct the work with a minimum of lost motion and duplication of effort. Indications are that these parties, comprising a total of 255 men, will perform considerably more work than thirty parties of ten men each. The proportionate saving in cost is greater than indicated by the smaller personnel, due to reductions in the higher brackets of salaries, and in transportation costs.

Most important of all is the time factor—experience thus far (1938) leads to the conclusion that it will be reduced by one-half.

No triangulation has been attempted by the Louisiana Geodetic Survey. The 1938 winter program of the U. S. Coast and Geodetic Survey contemplates completing the Louisiana network. One party has been engaged in first-order traverse for about one year.

Publication of Geodetic Data.—A constantly increasing demand for station descriptions created such a burden of clerical work that it became necessary to establish some method of placing the accumulated data at the disposal of users in a compact and accessible form.

After a study of available means of distribution, it was decided to issue a series of geodetic quadrangles, enclosing 0° 15′ of latitude and longitude, identical in geographic position, nomenclature, and scale with those of the U. S. Corps of Engineers and the U. S. Geological Survey. A total of 208 such quadrangles was required to cover the State and, in addition, a series of three-minute quadrangles (6 in. to the mile), covering the more important municipal areas, will be published.

Quadrangles are printed in black and white with all water in blue. All control points are located and identified and sufficient topographic information, together with subdivision lines of the rectangular land office surveys, are shown to insure facility in selecting control points for any purpose. The reverse of each quadrangle bears complete geodetic data for each station.

The location of rectangular survey lines is from original township plats, adjusted to field ties made by the U. S. Geological Survey, the State Board of Engineers, the U. S. Corps of Engineers, and the Louisiana Geodetic Survey, and topography from controlled aerial survey data is from the same sources. In addition to the quadrangles an index or key map is published, bearing on its reverse side a brief explanation of the method of using the data. Mimeographed descriptions of reference marks are available with each quadrangle. The user thus has at hand a graphic portrayal of the available control points in the section where he proposes to work, together with ties to facilitate locating them in the field, and the data necessary for their use.

Land Ties.—The original rectangular surveys in Louisiana date back as far as 1806. Prior to this time innumerable grants had been made and surveyed under Spanish and French rule. Where the "vara" obtained in early Texas surveys, the "arpent" was the unit of measure used in Louisiana prior to the United States Land Office subdivision.

Because of the type of instruments available, and other factors, familiar to every engineer, that influenced the making of early surveys, the conditions existing in Louisiana with reference to identifiable land corners and markers were confusing and discouraging until the present co-ordinate system was established.

Wherever it is possible to locate and identify original corners authentically, they are tied to the control network. The special marker shown in Fig. 6(*c*) is used, and at a satisfactory distance (usually between 400 ft and 500 ft) a reference marker (Fig. 6(*d*)) is set. The co-ordinates of all such corners are included with the data on the corresponding quadrangles, together with the azimuth of the reference point. This enables the engineer or surveyor to start from any such corner with a known position and a true bearing.

Many amusing (and at times appalling) discrepancies are encountered in adjusting the original surveys to actual positions determined by geodetic methods. There have been instances, furthermore, in which the work of the early surveyor stood up amazingly well. Within the life of the Survey, all original corners that can be located will be thus included in the original network.

State Boundaries.—That part of the boundary between the Republic of Texas and Louisiana north of, and not formed by, the Sabine River, was surveyed prior to 1845. Earthen mounds about 5 ft high and 15 ft in diameter were established at intervals of 1 mile. A traverse of this boundary has been run by the Louisiana Geodetic Survey, and monuments bearing the names of the Governor of Texas or of Louisiana on traverse markers (Fig. 6(*b*)) are established in all existing mounds.

That there were markers used other than mounds is evidenced by the discovery of a marble monument, 12 in. by 12 in., projecting well above the ground, and marked on the east "Louisiana" and on the west "Republic of Texas." The location of this boundary by plane co-ordinates is important not only as a State line, but as a line of demarcation across the rectangular subdivisions, and great care is being observed to locate and connect all land corners near-by.

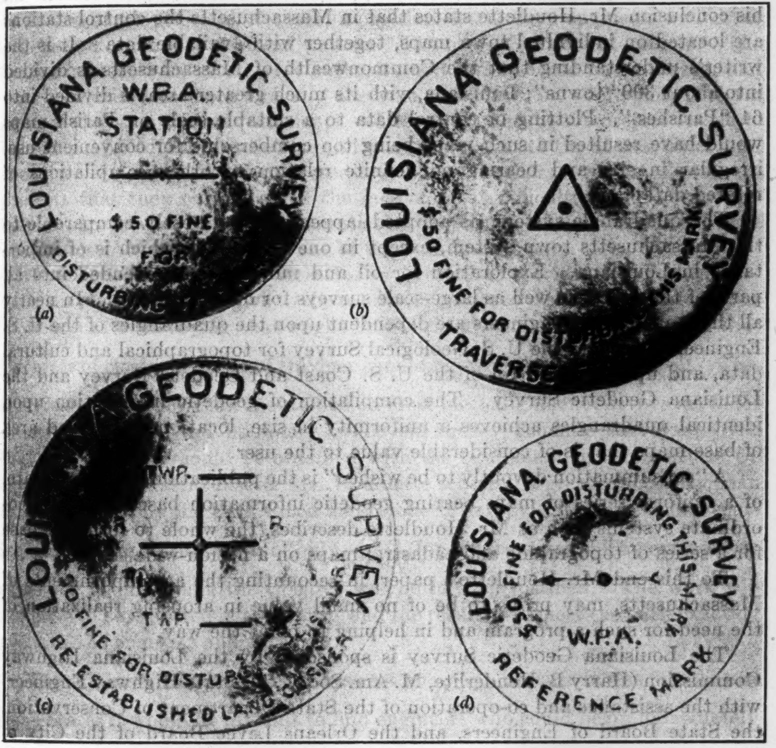

LOUISIANA GEODETIC SURVEY
W.P.A.
STATION
$50 FINE FOR DISTURBING
(a)
LOUISIANA GEODETIC SURVEY
$50 FINE FOR DISTURBING THIS MARK
TRAVERSE
(b)
LOUISIANA GEODETIC SURVEY
TWP.
R
R
TAP
$50 FINE FOR DISTURBING
RE-ESTABLISHED LAND CORNER
(c)
LOUISIANA GEODETIC SURVEY
$50 FINE FOR DISTURBING THIS MARK
W.P.A.
REFERENCE MARK
(d)

FIG. 6

The northern boundary of the State was included in this traverse. Although it was well monumented, there were no available data on geographic location of these points and their connection to the control network was important and necessary. That part of the northern boundary east of the Mississippi River is controlled as a line of latitude and it will be run and monumented as such.

Legal Recognition.—The New Jersey Legislature has enacted legislation establishing the plane co-ordinate system in that State as the base of all surveys.

Such an act was for presentation to the 1939 session of the Louisiana Legislature, incorporating provisions to guarantee work ability and penalties for non-observance. The legislature of 1918 enacted legislation which is applicable to field operations of the Louisiana Geodetic Survey in granting rights of entry and affording protection of monuments and structures.

Use of Information.—In Massachusetts, New Jersey, and elsewhere keen appreciation has been evidenced of the vital importance of making all accumulated data easily accessible so that its application can be made universal. In his conclusion Mr. Houdlette states that in Massachusetts the control stations are located on individual town maps, together with available data. It is the writer's understanding that the Commonwealth of Massachusetts is divided into about 300 "towns"; Louisiana, with its much greater area, is divided into 64 "Parishes." Plotting of control data to a suitable scale on Parish maps would have resulted in such maps being too cumbersome for convenient use, irregular in size and bearing no definite relation to other compilations of related data.

The quadrangle system, as adopted, appears to be loosely comparable to the Massachusetts town system, except in one particular, which is of importance in Louisiana. Exploration for oil and minerals has extended into all parts of the State, as well as large-scale surveys for other purposes. In nearly all these operations, engineers are dependent upon the quadrangles of the U. S. Engineer Corps and the U. S. Geological Survey for topographical and cultural data, and upon the control of the U. S. Coast and Geodetic Survey and the Louisiana Geodetic Survey. The compilation of geodetic information upon identical quadrangles achieves a uniformity in size, location, scale, and area of base maps that is of considerable value to the user.

A "consummation devoutly to be wished" is the publication, by every State, of a uniform series of maps bearing geodetic information based on plane coordinate systems such as Mr. Houdlette describes, the whole to form the base for a series of topographic and cadastral maps on a nation-wide scale.

To this end Mr. Houdlette's paper, in recounting the accomplishments of Massachusetts, may prove to be of no small value in arousing realization of the need for such a program and in helping to point the way.

The Louisiana Geodetic Survey is sponsored by the Louisiana Highway Commission (Harry B. Henderlite, M. Am. Soc. C. E., State Highway Engineer) with the assistance and co-operation of the State Department of Conservation, the State Board of Engineers, and the Orleans Levee Board of the City of New Orleans, La.

William Bowie,[14] M. Am. Soc. C. E. (by letter).—The story told by Mr. Houdlette should be given thoughtful consideration by all engineers and other users of land, and by the officials of States, counties, and cities, whose duties include the advancement of the welfare of their citizens. After many years of effort on the part of a few engineers and planners to arouse the governing bodies of the United States to the desperate mapping and surveying situation, at last something is being done about it in at least a few States and political subdivisions.

---

[14] Hydrographic and Geodetic Engr., U. S. Coast and Geodetic Survey (Retired), Washington, D. C.

The opportunity came when the Administration, late in 1933, sought work that could be done by engineers and others who were unemployed. Geodetic surveys were suggested as professional projects that would provide jobs for thousands of idle engineers. Work was organized in all of the States by the officials of the U. S. Coast and Geodetic Survey, with a local director in each State. Within less than two months after the start, more than 10 000 persons had been employed, and of these, more than 8 000 were engineers and other college and university men and women. The project was a Federal one. The State directors were given wide authority as to who should be employed, and were authorized to discharge any one who could not, or would not, perform services that met their requirements. After a few months the project ceased to be under the Federal jurisdiction, and was turned over to the States, cities, and counties. It is to the lasting credit of those in charge of the subsequent projects that they carried on in the face of great difficulties and discouragements. In several States and counties comprehensive geodetic surveys have been made. Massachusetts is one of the outstanding ones.

The U. S. Coast and Geodetic Survey prescribed specifications for the geodetic surveys that would give to the results an accuracy needed for all except the most accurate surveying that might be based on them. It was required that the error of closing of a line of traverse should not be greater than $\frac{1}{10\,000}$. This is a greater accuracy than is obtained on most local alignment surveys made for construction and land boundary. The emergency geodetic surveys proved that this accuracy could be obtained with the equipment in common use by engineers. In Massachusetts Mr. Houdlette required a greater accuracy than that prescribed by the U. S. Coast and Geodetic Survey. This led to his securing an average closing error of only $\frac{1}{35\,400}$ for his traverses. The extra effort expended in improving the closures was more than justified. The work will not have to be repeated in the future, and the results will meet the most exacting needs of the engineer and surveyor.

The minimum limit of closure was set at $\frac{1}{10\,000}$ because it was felt that in some States it would be difficult to find engineers who could begin immediately to produce surveys of higher accuracy. Even the $\frac{1}{10\,000}$ survey was a severe problem for some of the directors. One of them reported that he could not find a single engineer in his State, eligible for relief employment, who could run a traverse with the required accuracy. A few men had to be trained first to do the work in a satisfactory manner, and then used to train others. The writer cites this to show to what a low ebb the surveying of this country had fallen. Several millions of dollars have been spent on the relief geodetic surveys and valuable results have been secured in several States. The unit costs have been higher than if the work had been done under normal employment conditions, but the educational value alone of the emergency geodetic surveys more than justified the expenditure.

It cannot be expected that the States and their political subdivisions will continue indefinitely to receive relief funds for making geodetic surveys; but those surveys should be continued until there is a traverse or triangulation station within easy reach of the engineer or surveyor who may wish to have his work co-ordinated with surveys made by others in the same region. This means that in each State there should be an agency charged with the duties of making geodetic surveys, and furnishing information to those needing it. Such an agency should have authority to set minimum limits of accuracy for all agencies of the State Government that make surveys for engineering operations, and for the location or re-running of the boundaries of public and private property. If a few traverse parties in a State should operate continuously for five or ten years much could be accomplished. So many uses would be found for the results that there would be no difficulty in finding support for the completion of the work for the State.

One of the urgent needs of most of the States is the establishment of Land Courts. In a most interesting manner, Mr. Humphrey has described the Court dealing with title to land in Massachusetts, and the surveying that is done for the Court. Each State may well create a Land Court after the Massachusetts pattern. It would be a boon to engineers who have as their primary function the placing of valuable structures on the land. They should be sure that they have the boundaries accurately defined and located before starting their work.

Massachusetts is to be commended and congratulated for having sensed the importance of good land surveys, and for giving the effort necessary to make such work effective.

In recent years, the Society has published a number of papers that have told of the chaotic condition of land boundary surveys in such States as North Carolina,[15] Florida,[16] Massachusetts,[17] and Texas.[18] No doubt the conditions they portray are the same as in the other States. The writer knows of no State in which satisfactory surveys have been made on any comprehensive plan.

Mr. Houdlette has justly stressed the difficulty of using the national triangulation net as the base for local surveys on the system of spherical coordinates; but that difficulty has been overcome by devising State Plane Co-ordinate Systems. The engineer engaged on local surveys can use the plane co-ordinates of the triangulation and traverse stations, and needs to pay no attention to the computations that are involved in the transfer of spherical to plane co-ordinates. That work is done for him by the mathematicians of the U. S. Coast and Geodetic Survey or agencies like the Geodetic Survey of Massachusetts. There is really no excuse on the part of responsible officials of a State for delaying the adoption of plane co-ordinates, and the private engineer is doing himself a disservice if he does not use them.

---

[15] *Civil Engineering*, January, 1937, p. 33.

[16] *Loc. cit.*, July, 1938, p. 451; June, 1938, p. 386; and May, 1938, p. 331.

[17] *Loc. cit.*, December, 1937, p. 824.

[18] *Loc. cit.*, November, 1938, p. 722; and July, 1937, p. 507.

There is now widespread sentiment in favor of better survey methods, and this sentiment will soon be translated into action. One of the first steps to be taken to bring better conditions into being should be the completion of the National Triangulation net. In the past, the triangulation has been considered by some as merely the frame for the topographic mapping; but the triangulation has so many other uses that it should be finished as soon as possible and not merely meet the demands of the topographic engineers of the Federal Government. The plan for the triangulation calls for the spacing of the arcs of triangulation at intervals of about 25 miles. This is the immediate objective, but the ultimate one is that eventually there will be no place east of the Rocky Mountains more than 6 miles from a station, whereas this distance is increased to 12 miles in the mountainous regions. This spacing is considered as desirable for the control of the maps that will be compiled from photographs taken from the air. Such spacing of the triangulation stations will make it possible for the engineers of the State Governments to have traverses run between pairs of stations, along railroads and highways, for the purpose of setting many stations along the routes for use by engineers and surveyors engaged on local surveying for engineering and land boundary work.

Perhaps no one is to blame for the slip-shod surveying that has been done in the United States during the past hundred years, but the bad practice should not be continued. The Corps of Engineers of the U. S. Army has been making surveys for river and harbor improvement and flood protection for many years, and, although the surveys have met their immediate needs, in most cases they were of a low order of accuracy, and in most instances no permanent monuments were left on the ground. This triangulation or traverse cannot add much to the national horizontal control system of the country.

The Topographic Branch of the U. S. Geological Survey has done an enormous amount of control surveying for its topographic mapping; yet only a very small percentage of this has been of such a standard as to make the results of value to the engineer or surveyor who is making surveys for engineering work and land boundaries. In their old work they left few or no permanent monuments. They should not be blamed for doing work with no more accuracy than was just sufficient for their small-scale mapping. They received a pitifully small sum of money for their mapping, so they could not divert much of it to the control surveying. Since there are so many needs for the results of the control surveys, however, the Geological Survey should strengthen its work in this respect. This applies also to the control surveys made by the Corps of Engineers.

If the objectives outlined by Messrs. Houdlette and Humphrey are to be reached in each of the States, all State agencies, such as the Highway Department, should make their control surveys with the accuracy needed to meet future use of the data secured. To do this would cost the State very little additional. The first step would be to purchase good instruments. The day of the 1-min transit, the non-standardized tape, the old-fashioned level, and the wooden leveling rod, has passed just as the day of the wheelbarrow and the ox cart has passed. To continue to use the old types of instruments is bad practice and wasteful. Either poor results are obtained, or an undue length of time is taken to secure accurate results. In every branch of engineering except surveying and mapping the best available instruments, machines, and equipment are bought and used. It is costing money to continue the use of surveying instruments that were scarcely up to date even two generations ago.

Although standards of surveying have been low, there is no cause to be pessimistic over the mapping and surveying situation. There are dozens of engineers and surveyors to-day who are making strong efforts to put better methods into practice, whereas until quite recently there were few who even knew there were problems to be solved. This Symposium is an indication of awakening interest in about the oldest and the most neglected branch of engineering.

R. M. WILSON,[19] M. AM. SOC. C. E. (by letter).—One of the successful projects that grew from seeds sown in 1933, in each of the 48 States, by the U. S. Coast and Geodetic Survey under an appropriation by the former U. S. Civil Works Administration, is described by Mr. Houdlette. In several States these seeds fell upon fertile soil, as they did in Massachusetts. There they sprouted and were carefully cultivated, even without the continued support of funds from the Civil Works Administration, and they have lived to bear valuable fruit.

The Massachusetts Geodetic Survey is a credit to those who have supported and administered it not only because of its success as a relief project but also because of the quantity and high quality of the work done. The supplementary control surveys that have been completed will surely repay the State of Massachusetts for its support of the project. The uses of the new geodetic control in connection with cadastral surveys and land titles have been described by both authors of the Symposium; much of the valuable harvest that the State will reap will be through these uses.

At present (1939) there is in progress, also, a program of re-mapping by the Geological Survey of the United States Department of the Interior in co-operation with the State of Massachusetts for the purpose of making, by modern methods, up-to-date topographic maps of the State. Control surveys are needed for this new mapping, and ordinarily it would be necessary for the Geological Survey to spend considerable sums of money for the required preliminary surveys before the actual mapping could begin; but with the results of adequate control surveys made by the Massachusetts Geodetic Survey already available, a larger proportion of the funds appropriated for mapping can be applied toward the primary purpose of the program. In this way, also, the State is reaping a genuine harvest because it cultivated the project that has become the Massachusetts Geodetic Survey.

In his historical sketch of early surveys in Massachusetts, Mr. Houdlette makes a statement that may be misleading to those who are not familiar with the facts. In his reference to the mapping authorized in 1884, he conveys the impression that the co-operative agreement was between the State and the U. S. Coast and Geodetic Survey, then a bureau of the U. S. Treasury Department; but Chapter 72 of the Acts of 1884 of the Massachusetts Legislature provided for the mapping of the State in co-operation with the U. S. Geological Survey, not the U. S. Coast and Geodetic Survey.

---

[19] Chf., Section of Computing, U. S. Geological Survey, Washington, D. C.

That agreement, made in 1884, produced the original topographic "quadrangle" maps of Massachusetts which were published by the Geological Survey. They are known to all map users in the State, and their popularity is attested by the large number of copies that have been distributed since they first became available, shortly after the completion of the field surveys in 1888.

The co-operative agreement, defined by Chapter 72 of the Acts, is an important one in the history of the U. S. Geological Survey. It was the first of the agreements of its kind made with States wishing to expedite topographic mapping within their borders. The success of the program established it as a model for co-operative agreements with other States, and many similar agreements have since been made. At present the U. S. Geological Survey has co-operative agreements for the making of topographic maps with 17 States and with Puerto Rico. One of these has resulted in the new co-operative mapping program previously mentioned.

The new program in Massachusetts has been undertaken because the existing maps on a scale of about 1 mile to the inch, which were made between 1884 and 1888, do not meet the exacting modern requirements. The new maps, published on larger scales with smaller contour intervals, permit the showing of greater detail and are up to date in showing developments in the works of Man. Thus, they are more nearly consistent with the present economic importance of the areas they represent. The 192 separate sheets, whole and fractional, that will be required to show the entire State, are first reproduced as lithographs on a scale of 1 : 24 000. To the present time, 51 have been reproduced as unedited advance sheets in two colors. Beginning now (1939), however, the advance sheets will be edited and reproduced as three-color lithographs. The final publications are engraved maps on a scale of 1 : 31 680.

ELLIOTT B. ROBERTS,[20] M. AM. SOC. C. E. (by letter).—Added emphasis may well be given one aspect of the subject treated by Mr. Houdlette. Reference is made to the difficulties experienced in the past by those who have tried to popularize the use of geodetic control for the correlation of local surveys; namely (1) inadequate distribution of geodetic control points, and (2) a widespread reluctance of surveyors to undertake the somewhat specialized computation necessary in adapting geographic position data.

Mr. Houdlette's organization has gone far to remove the first of these difficulties in Massachusetts, and a few more years of such work may well see the State so well supplied with control points that few, if any, surveyors could conscientiously avoid controlling their work by use of the national geodetic datum. The U. S. Coast and Geodetic Survey, in designing adequate plane co-ordinate projections for the States, and publishing tables relating thereto, has largely resolved the second difficulty. When that Bureau completes its intended publication of the plane co-ordinates of all its triangulation stations, it will have done so completely. Naturally the well-known operations of so diligent an organization as the Massachusetts Geodetic Survey are a tremendous added force in popularizing the plane co-ordinate projection in Massachusetts.

---

[20] Hydrographic and Geodetic Engr., U. S. Coast and Geodetic Survey, Washington, D. C.

The emphasis intended to be given by this discussion is to the fact advanced by Mr. Houdlette that, with abundant local control points defined in plane co-ordinates, the local surveyor, to use them, has no specialized computations to make, no new surveying process to learn, and nothing unusual to do but to start from the nearest control points instead of at random. From there on he may make the usual survey, computing latitudes and departures in conventional manner, and adjusting for the closure errors as he has always done. He may make a perfectly traditional plane survey; yet there is the tremendously important difference that his work is on a standard grid rather than an arbitrary local one. His work is based on the national datum, and is therefore correlated with every other such controlled survey throughout the entire country. This simple innovation, therefore, can eliminate the multiplicity of origins, mostly unrelated, the over-supply of height datums, and the economic loss of thousands of uncontrolled surveys of only momentary value.

The foregoing is so simple that it may seem a trivial point. The writer has observed, however, that many engineers are preoccupied with technicalities and fail to "pound home" the simple fundamentals necessary to impress others. The simple fundamental here is that by accepting a standard origin and coordinate system instead of setting up local ones of their own, surveyors can reap great benefits. This truth should be shouted, figuratively, from the housetops, so that surveyors may be persuaded henceforth to work in harmony instead of at cross purposes.

Mr. Houdlette is to be congratulated for the untiring productive work he has done, and for his excellent presentation of the subject. It is to be hoped that similar programs may be adopted in all the States. Massachusetts is learning that planning pays, in surveying as well as otherwise, and all the States should become aware of the great unnecessary cost attending the present haphazard methods.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2068

ENGINEERING GEOLOGY PROBLEMS AT CONCHAS DAM, NEW MEXICO

By Irving B. Crosby,[1] Affiliate, Am. Soc. C. E.

With Discussion by Messrs. H. L. Johnson, and Irving B. Crosby.

Synopsis

None of the varied types of foundation conditions of large dams presents more difficult problems than those of dams on uncemented shale or siltstone, and perhaps in no other branch of Engineering Geology is there more need of additional research. The principal purpose of this paper is to describe some of these problems, using Conchas Dam as an example, to explain how they were investigated and met in this particular instance, and to outline the research necessary to solve similar problems more adequately in the future. In addition, this paper outlines some of the problems encountered with high dams on sandstone containing water under artesian pressure, since Conchas Dam rests on both shale and sandstone.

Introduction

Conchas Dam is a project of the United States Army Engineers on the South Canadian River 30 miles northwest of Tucumcari, N. Mex., designed to regulate and utilize the flood waters of the river. The dam is nearly a quarter of a mile below the mouth of the Conchas River, at a point where the South Canadian River turns from a southerly to an easterly course. The length of the river above its junction with the North Canadian River is 700 miles, of which only 150 miles are in New Mexico. Its basin is wider in the upper part, however, with the result that 51% of the drainage area and all the major tributaries are in New Mexico. Flash floods of great magnitude are common, but during much of the year the river is nearly dry. The greatest recorded flood had a peak flow of 279 000 cu ft per sec at the dam site. The climate is semi-arid, the annual rainfall being less than 15 in., but the river has its sources in high mountains where the precipitation is somewhat greater.

The main dam is a gravity, concrete structure 240 ft high and 1 250 ft long with the crest of the parapet at Elevation 4 240, and an uncontrolled

---

Note.—Published in January, 1939, *Proceedings*.

[1] Cons. Eng. Geologist, Boston, Mass.

over-fall spillway 340 ft long with a crest elevation of 4 201 ft. Earth wing-dams connect with both ends of the main dam and the South Dike extends 6 400 ft from the South Wing-Dam (see Fig. 1). In addition, north of the Main Dam there is a concrete Emergency Spillway 3 000 ft long with earth dikes at either end. At the south end of the South Dike are the Irrigation Headworks and beyond is a small dike in a saddle. The earth wing-dams and dikes, the highest of which will be 96 ft, total 3 miles in length.

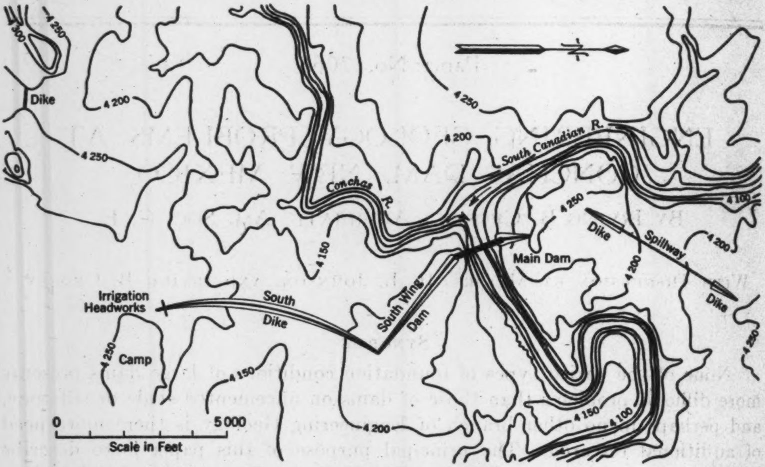

4 300
4 250
Dike
4 200
4 250
4 250
4 200
South Canadian R.
Conchas R.
4 100
4 250
Dike
Spillway
4 200
Main Dam
4 200
4 150
Dike
4 250
Irrigation Headworks
South Dike
South Wing Dam
Camp
4 250
4 150
0
5 000
Scale in Feet
4 200
4 150
4 100

FIG. 1.—CONCHAS DAM AND DIKES

FIG. 2.—VIEW OF MAIN DAM, FACING UP STREAM, DECEMBER 24, 1937

The main concrete dam crosses the canyon of the river and rises above the walls of the canyon, the wing-dams extending to higher ground on either side (see Figs. 2 and 3). The main dam contains approximately 750 000 cu yd of concrete and the emergency spillway and irrigation headworks contain nearly 80 000 cu yd of additional concrete, making a total of 830 000 cu yd. The wing-dams and dikes contain approximately 2 900 000 cu yd of earth-fill and 790 000 cu yd of rock-fill.

When full, the reservoir extends about 14 miles up the South Canadian River and 11 miles up the Conchas River, but the valley of the Conchas River provides the greater part of the storage capacity due to its greater width. Above the dam site the South Canadian River is generally in a narrow canyon from 100 to 200 ft deep. The total storage capacity will be approximately 600 000 acre-ft. Of this, 100 000 acre-ft is dead storage, 300 000 acre-ft is irrigation storage, and 200 000 acre-ft, flood control storage.

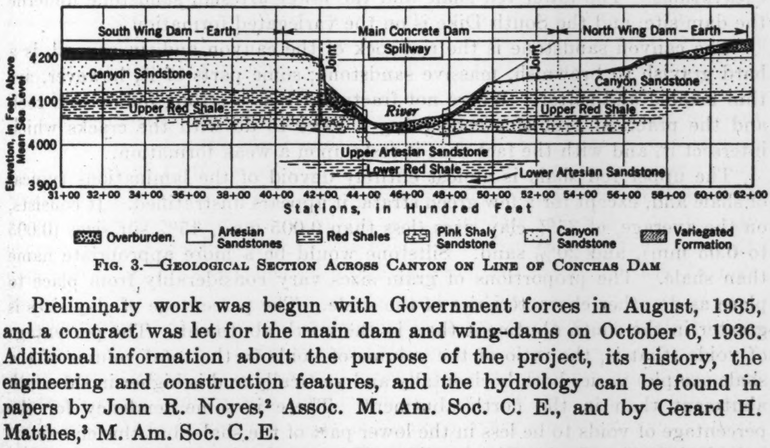

South Wing Dam – Earth
Main Concrete Dam
North Wing Dam – Earth
Spillway
Joint
Joint
Canyon Sandstone
Canyon Sandstone
Upper Red Shale
Upper Red Shale
River
Upper Artesian Sandstone
Lower Red Shale
Lower Artesian Sandstone
Elevation, in Feet, Above Mean Sea Level
4200
4100
4000
3900
31+00
32+00
34+00
36+00
38+00
40+00
42+00
44+00
46+00
48+00
50+00
52+00
54+00
56+00
58+00
60+00
62+00
Stations, in Hundred Feet
Overburden.
Artesian Sandstones
Red Shales
Pink Shaly Sandstone
Canyon Sandstone
Variegated Formation

FIG. 3.—GEOLOGICAL SECTION ACROSS CANYON ON LINE OF CONCHAS DAM

Preliminary work was begun with Government forces in August, 1935, and a contract was let for the main dam and wing-dams on October 6, 1936. Additional information about the purpose of the project, its history, the engineering and construction features, and the hydrology can be found in papers by John R. Noyes,[2] Assoc. M. Am. Soc. C. E., and by Gerard H. Matthes,[3] M. Am. Soc. C. E.

The project has been in charge of Hans Kramer, M. Am. Soc. C. E., District Engineer, Conchas District. James H. Stratton, Assoc. M. Am. Soc. C. E., was Chief of the Engineering Division until May 15, 1937, when he was succeeded by Captain L. H. Foote. J. B. Alexander, Assoc. M. Am. Soc. C. E., has been Planning Chief and H. V. Pittman, M. Am. Soc. C. E., Construction Chief. Joel D. Justin, W. H. McAlpine, William Gerig, H. T. Cory, Gerard H. Matthes, Members, Am. Soc. C. E., and the late Louis C. Hill, Past-President, Am. Soc. C. E., were Consulting Engineers, and the writer was Consulting Geologist.

GEOLOGY OF THE DAM SITE

Conchas Dam is in an extensive area of flat-lying sedimentary rocks known as the Red Beds. The rocks at the site are members of the Dockum

---

[2] "Conchas Dam and Reservoir Project," by John R. Noyes, *Engineering News-Record*, April 15, 1937, pp. 541–545.

[3] "Problems at Conchas Dam," by Gerard H. Matthes, *Civil Engineering*, July, 1936, pp. 437–441.

Group of Triassic Age. For convenience, the different members of this formation at the dam site were given local, descriptive names which are, beginning with the youngest and highest:

(1) Townsite Formation (sandstones and shales about 450 ft thick);
(2) Variegated Formation (shales with some sandstone about 300 ft thick);
(3) Canyon Sandstone (about 100 ft thick where not eroded);
(4) Upper Red Shale (average thickness, 64 ft);
(5) Pink Shaly Sandstone (average thickness, 25 ft);
(6) Upper Artesian Sandstone (average thickness, 64 ft);
(7) Lower Red Shale (average thickness, 30 ft); and,
(8) Lower Artesian Sandstone (thickness greater than 100 ft).

The canyon sandstone, upper red shale, and pink shaly sandstone outcrop at the main dam site, and the upper artesian sandstone was uncovered in the excavations. The lower red shale and the lower artesian sandstone underlie the dam site, and the South Dike is on the variegated formation.

The canyon sandstone is the rim rock of the canyon and, in general, is a hard grayish to brownish, massive sandstone; some parts of it, however, are thin bedded and softer. Where not fractured, it is a strong foundation rock and the practical problems concerning it have to do with the cracks which intersect it, and with the fact that it rests upon a weak formation.

The upper red shale is almost entirely devoid of the laminations typical of shale and, except for a few green strata, it appears unstratified. It consists, on the average, of 35% clay sizes (less than 0.005 mm), 45% silt sizes (0.005 to 0.05 mm), and 20% sand. Siltstone would be a more appropriate name than shale. The proportions of grain sizes vary considerably from place to place as do other characteristics of the shale. The percentage of clay sizes is greater in the south abutment than in the north abutment. The percentage of voids (that is, the ratio of the volume of voids to the total volume of the shale sample) varies both horizontally and vertically and is higher in the south abutment than in the north abutment. There is some tendency for the percentage of voids to be less in the lower part of the shale, but this characteristic is irregular in its distribution. The average percentage of voids in twenty-two samples of shale in the south abutment is 25.5; the maximum is 34.7, and the minimum, 12.6. The unit weight is 147 lb per cu ft in the natural condition. The average percentage of voids of forty-five samples from the north abutment is 17.6, the maximum being 26.0 and the minimum, 11.5. The specific gravity of the mineral grains averages 2.76. All the tests indicate that the shale is saturated with water. The red shale cracks upon exposure to air and crumbles to pieces. When re-wet, it breaks down to silt. If it is immersed in water in its natural condition without drying, however, it does not disintegrate. The most important foundation problems are concerned with this shale.

The pink shaly sandstone is an extremely variable formation ranging from a sandy shale to a fairly massive sandstone. It changes rapidly, both vertically and horizontally. Some parts disintegrate when dried and wet again, and other parts remain sound under such treatment. The practical problems are due to this variability. The shaly parts of it present problems similar to those of a typical laminated shale.

The upper artesian sandstone is a hard, gray, fine-to-medium grained, gritty sandstone with occasional flat shaly seams; but it is generally massively bedded. Shaly beds are more numerous in the upper part of the formation where they were a foundation problem. It was given its name because water under artesian pressure was encountered in the borings. This fact necessitated studies concerning uplift and seepage.

The lower red shale is similar to the upper red shale, and the lower artesian sandstone is similar to the upper artesian sandstone. The total thickness of this sandstone is unknown because none of the borings reached the bottom of it. Both these formations are at such depth that they cause no practical problems.

South and southeast of Conchas Dam the lower part of the Dockum Group and certain other formations contain gypsum, but no beds of gypsum were found at Conchas Dam. It is possible, however, that gypsum underlies the dam at great depth. The deepest boring reached 240 ft beneath the river without encountering gypsum, and it is certain that if any exists beneath the dam it is so deep that it will be harmless.

The formations at Conchas Dam were laid down essentially flat in water and have been slightly tilted and warped so that they now have a regional dip of about 2° to the southeast upon which have been superimposed gentle waves or folds. One of these folds brings soft rocks to the surface in the valley of the Conchas River, with the result that this valley is wider than the canyon of the Canadian River. No evidence of faulting was found near the dam site or in the reservoir basin, and the nearest known fault is 34 miles west.

The rocks at the dam site are intersected by fractures or joints running in various directions. These cracks are most numerous in the canyon sandstone and artesian sandstone and least developed in the red shale. The joints are most numerous in the directions N 50° W and N 40° E, and they are in general nearly vertical. The joints show every gradation from incipient planes of weakness to open cracks. In the canyon sandstone near the canyon wall where blocks of sandstone could move toward the unsupported edge, some of the cracks are gaping fissures.

There are comparatively few joints in the red shale, but there are numerous other fractures in various directions, and dipping at various angles. The characteristic fracture of the red shale is a short, curved, striated or slickensided fracture running in any direction and dipping at any angle. These fractures are generally from 1 in. to a few inches in length with a maximum length of about a foot. Similar slickensided fractures were produced in the Soils Laboratory at Conchas Dam by shearing a specimen of the red shale under a heavy transverse load. These fractures are evidently caused by very small movements.

The formations at the dam site were formerly buried by younger formations to a depth of at least 1 200 ft. Remnants of these younger and higher formations can be seen in the near-by mesas. Between the mesas is a broad undulating valley in which the river is entrenched in a canyon. Old river gravels give evidence that the river formerly flowed at the level of this old valley.

The dam crosses the canyon, rises above the canyon walls, and extends on to the cap rock on either side, as shown in Fig. 3. The central section of the high part of the dam is founded upon the upper artesian sandstone, but the two ends of the high section rest upon the pink shaly sandstone. The high section abuts against the canyon walls of red shale on either side of the canyon. The two ends of the dam which extend upon the cap rock will rest upon the canyon sandstone which overlies the red shale. From either end of the concrete dam, earth wing-dams extend to high land. The South Wing-Dam is on shale and sandstone of the variegated formation except for the northern end which rests upon the canyon sandstone. The North Wing-Dam is on the canyon sandstone, but the northern end will be against shale of the variegated formation.

The main part of the South Dike rests upon shale and sandstone of the variegated formation which provides a tight, satisfactory foundation for this structure. The southeasterly dip causes the top of the variegated formation to be lower at the south end of the South Dike, with the result that here the top is below the level of the crest of the dike and the south end of the dike rests upon sandstone of the townsite formation. The tunnel for the irrigation outlet is in shale of the variegated formation which is here overlain by a cap of townsite sandstone.

The Emergency Spillway rests upon the variegated shale and sandstone, but in places this is only a thin layer on the canyon sandstone and the foundation excavation was extended down to this sandstone. Where the spillway structure rests on shale the foundations were excavated to sufficient depth to remove the possibility of sliding of this low structure. These structures will not be discussed further since there are no interesting geological problems connected with them.

PRACTICAL PROBLEMS

The most important practical problems at the site of the main concrete structure were:

(*a*) The upper red shale as a foundation formation (its competency to support the design load, settlement of structures upon it, and the protection of the shale from disintegration);

(*b*) Seepage under the dam through the artesian sandstone, uplift pressure on the foundations of the dam from artesian water in this formation, and meeting these conditions by grouting and drainage; and the selection of foundation beds in this sandstone and in the overlying pink shaly sandstone, and the stability of structures resting upon them; and,

(*c*) Seepage under and around the dam through the canyon sandstone, and its prevention. Effect of the cracks in this formation on the stability of that part of the dam founded upon it.

Problems Connected with the Red Shale.—The red shale presented the most serious problems. It is an unlaminated siltstone which was deposited as a silty mud. Its consolidated condition is due to compaction and not to cementation. Following its deposition as mud other sediments were laid down upon it, and their weight compacted the mud, slowly squeezing the water out of the pores. As the compaction of the mud continued it became hard, silty clay and finally siltstone or shale. Erosion subsequently removed much of the load and the shale expanded somewhat but did not reach its original porosity. When load is again placed upon it there will be a renewal of compaction thus causing the settlement of any structure resting upon it. That part of the dam resting upon the shale will cause some compaction of the shale, with resultant slight settlement of the dam, even though the weight of the dam is much less than the weight of the overlying formations that have been removed.

Laboratory consolidation tests had been made on the shale from which estimates of settlement had been made, but there was some question as to the accuracy of these methods when applied to shale instead of to clay for which the tests had been designed; and it was desired to check the results.

From a consolidation test it is possible to estimate the load under which clay was originally consolidated.[4] Because of the close relationship between clay and uncemented shale it was possible to apply this method to this shale. In this case it was also possible to make an approximate estimate of the former load on the shale from field observations. Therefore, if these two estimates should agree there would be a check upon the laboratory method. The previous consolidation tests had been conducted with a maximum pressure of 25 tons per sq ft, and it was not possible to use higher pressures with the existing apparatus. It was evident that the original pressures were much higher and new apparatus was designed capable of testing the sample under a pressure greater than 100 tons per sq ft. The most reliable tests made with this apparatus indicated a former load of 64 to 70 tons per sq ft. Several miles south of the dam, Mesa Rica rises more than 1 200 ft above the river. The Ogallala Formation, the youngest known in this region, caps the highest part of the mesa. It is known that this high formation once extended unbroken to the Llano Estacado, about 15 miles to the south, and to the Canadian Escarpment 30 miles to the north. This high formation thus once overlay the dam site and it is possible to make an approximate estimate of the load that once rested upon the red shale. The difference of elevation between the summit of the mesa and the shale at the dam site is 1 250 ft, and this is the minimum over-burden on the red shale. Due to dip of the strata and other causes it may have been somewhat greater. Since the ground-water conditions at that time are unknown, the weight of this over-burden is not exactly known; but, using extreme assumptions gives values of 50 and 94 tons per sq ft for the two extreme conditions, with the most probable range between 60 and 70 tons per sq ft which checks with the laboratory estimate of 64 to 70 tons per sq ft and indicates that the laboratory tests are at least approximately correct.

Estimates of probable settlement of that part of the dam underlain by red shale were made by applying the laboratory results to the geological interpretation of the strata and gave a maximum settlement for extreme conditions of about 4 in. after a period of years. There will be differential settlement, therefore, between the parts of the dam underlain by the red shale and those parts resting directly upon the artesian sandstone. The condition being known, the dam was designed to meet them, and a special joint was devised between the different parts of the dam.[5] Differential movement between the monoliths underlain by the red shale and those resting upon the artesian sandstone can thus take place harmlessly.

---

[4] "The Structure of Clay and Its Importance in Foundation Engineering," by Arthur Casagrande, Assoc. M. Am. Soc. C. E., *Journal*, Boston Soc. C. E., April, 1932, Vol. 19, No. 4, pp. 176–179.

The strength of the shale—that is, its competency to support the load it will be required to carry—was studied by geological observations in the field and the conclusions were checked by laboratory tests and by computations. In the north abutment the red shale is about 70 ft thick, the sandstone cap rock averages 20 ft thick, and the dam rises 95 ft above this. Therefore, the load on the shale is 20 ft of sandstone and 95 ft of dam.

The canyon walls were studied for miles above the dam and places were found where the sandstone overlying the shale is more than 100 ft thick. Thus, there is proof that the shale is sustaining a load as great as, or greater than, that of the dam, without failure. The canyon walls generally break down by weathering of the shale. The shale cracks and disintegrates upon exposure to air, the sandstone cap is undermined, and large blocks tip and slide into the canyon. The only places seen where the walls had collapsed due to failure of the shale were where the stream had undercut the cliff, producing stresses greater than any that can exist in the shale under the dam. The field observations thus indicated that the red shale is strong enough to support the dam. This conclusion was checked by laboratory tests.

A series of shear tests and of unconfined compression tests on samples of the shale were made in the Soils Laboratory at the dam, with the results shown in Table 1. Compression tests were made on 2-in. cubes. The results for samples from different elevations, in both the north and south abutments, are given in Table 2.

TABLE 1.—SHEARING RESISTANCE, SAMPLES OF RED SHALE FROM THE NORTH ABUTMENT

(All Units Are Pounds per Square Inch)

Elevation, in feet, above mean sea level	Shearing Stress, for the Following Normal Loads:					
	50	75	100	125	150	157
4 110	218	253	263	287	290	...
4 084	202	260	220	292	...	337
4 065	185	242	242	310 327	...	...

A study was made of the stresses in the red shale under the action of the completed dam. The following assumptions and constants were used:

---

[5] For description of joint, see "Dam Building on Difficult Rock," *Engineering News-Record*, June 9, 1938, pp. 808–809.

(*a*) The net section of the non-overflow dam, without heel or toe fillets, was used; the foundation was assumed level at Elevation 4 140, which is the top of the canyon sandstone; and the bottom of sandstone was taken at Elevation 4 120, and the bottom of red clay shale at Elevation 4 050;

(*b*) The foundation materials were considered as homogeneous and isotropic, acting in agreement with Hooke's law;

(*c*) Five different classes of assumptions as to the distribution of stress at Elevation 4 140, and the effect of the sandstone in distributing the load at Elevation 4 120, were studied (see Cases I, II, III(*a*), III(*b*), and IV);

(*d*) The modulus of elasticity of the concrete, sandstone, and shale was assumed equal in Cases I, II, and III(*b*); and the modulus of elasticity of the shale was assumed to be one-third of the modulus of the sandstone or concrete in Cases III(*a*) and IV;

TABLE 2.—COMPRESSIVE STRENGTH OF RED SHALE, IN POUNDS PER SQUARE INCH

Elevation, in feet, above mean sea level	SAMPLE NOS.										
	1	2	3	4	5	6	7	8	9	10	Average
(*a*) NORTH ABUTMENT											
4 110	660	725	675	1 600*	600	675	625	650	1 300*	610	652
4 084	975	865	400	1 025	875	675	825	865	565	1 040	811
4 065	340	300	385	325	350	325	500*	350	300	570*	334
(*b*) SOUTH ABUTMENT (PILASTER SHAFTS)											
4 100	438	363	400	413	350	525	375	400	....	....	408
4 078	388	350	550	375	375	700*	463	475	....	....	444

* Omitted from average.

(*e*) A cross-section of the dam and foundation was studied in each case, and no forces acting at right angles with the plane of the section were considered;

(*f*) Uplift and internal pore pressure were neglected;

(*g*) Unit weights were assumed at 62.5 lb per cu ft for water, and 150 lb per cu ft for concrete, sandstone, and shale; and,

(*h*) The reservoir was assumed permanently filled to Elevation 4 230 for the full reservoir condition.

The following possible assumptions as to stress distributions were considered:

Case I.—Straight line distribution of stress at Elevation 4 140 was assumed and the sandstone was considered effective in spreading the load. This case represents the minimum stress concentration that can exist in the shale foundation.

Case II.—Non-linear distribution of stress at Elevation 4 140 was assumed and the sandstone was considered effective in spreading the load. This case gives stress concentration at the toe and heel in the plane of the base, but considerable distribution before the load is transmitted to the shale.

Case III(a).—Non-linear distribution of stress at Elevation 4 120 was assumed. The sandstone above Elevation 4 120 was assumed to act with the dam, but to be ineffective in spreading the load on to the shale.

Case III(b).—All assumptions were the same as in Case III(*a*), except for the substitution of linear distribution for non-linear at Elevation 4 120.

Case IV.—In this case, non-linear distribution of stress at Elevation 4 120 was assumed. The sandstone above Elevation 4 120 was assumed to act with the dam, and was considered to be effective in spreading the load at 45° with the horizontal.

Stresses were computed for all these cases. Case I is less severe than actual conditions; the assumptions in Cases II and IV may closely approximate actual conditions; but Case III(*a*) is the most severe condition possible. The results obtained are conservative and are the ones given herein. The maximum computed shear stress in the shale at Elevation 4 120 is 138 lb per sq in. The average shearing strength as computed from unconfined compression tests on twenty 2-in. cubes was 266 lb per sq in. This is for material unconfined, but at the point of maximum stress the shale will be confined by a monolith which will rise from the sandstone below the shale. The maximum stress for this condition will be about 35 lb per sq in., giving a factor of safety of about 7.6.

Laboratory tests and computations thus confirm the geological conclusions as to the competency of the shale to support the load of the dam. There are factors, however, which the laboratory tests and computations cannot take into consideration but which are included in the demonstration given by the canyon walls. The shale is intersected by cracks and planes of weakness along which the resistance to shearing is less. If only the laboratory results were considered there would remain the possibility that these fractures might so weaken the shale that it might fail under the increased load. The canyon walls, however, demonstrate that the shale is actually standing a similar or greater load without evidence of failure. Therefore, the field and laboratory methods together give greater assurance than either alone. The laboratory method gives definite values for certain cases and the field observations take into consideration factors that cannot be provided for in the laboratory. The conclusion must be, therefore, that the shale will not fail under the additional load to be imposed by the dam.

Another problem is caused by the cracking of the shale upon drying and its disintegration when wet again. Drying and disintegration of the shale proceed rapidly. Exposure of a few hours causes the formation of cracks, and every crack expedites drying by letting air into the mass of the shale. When the shale is again exposed to water it slakes and disintegrates rapidly to mud. As a result there are no natural exposures of shale, unless they have been formed very recently by the cutting of a stream, or otherwise. Where the red shale outcrops in the canyon wall the solid shale is concealed by many feet of cracked shale and chips.

Due to this rapid drying and disintegration of the shale it must be protected during excavation for foundations either by leaving several feet of shale in place until ready to pour concrete and then making the final excavation and placing concrete rapidly, or by painting the surface of the shale with an asphaltic or other protective preparation. The only places where this problem is important are at the two abutments where concrete is poured against vertical faces of shale, and the dam is stepped up from the lower sandstone to the canyon sandstone.

In order to avoid having any part of the dam rest directly upon the shale it was necessary to cut vertical faces in the shale which is 70 ft thick on the north side and 50 ft thick on the south side. On the south side the shale supports a load of cap rock 80 ft thick but on the north side the cap rock is only 20 ft thick. There was fear that on the south side a vertical face might not stand up under the heavy load of cap rock. To avoid the necessity of an unsupported face of shale at the south abutment, columns of reinforced concrete were constructed, extending from the sandstone below the shale to the canyon sandstone above the shale. These columns were designed to act as beams to give lateral support to the shale. Shafts were sunk in the shale under the edge of the cap rock, and concrete was poured in these shafts. Before the shale was excavated, a monolith 54 ft away had been brought up to the level of the top of the shale and then the columns were braced against it. The shale was then excavated without difficulty, concrete panels were placed between the columns, and the monolith was poured[6] (see Fig. 4). This method was adopted before the

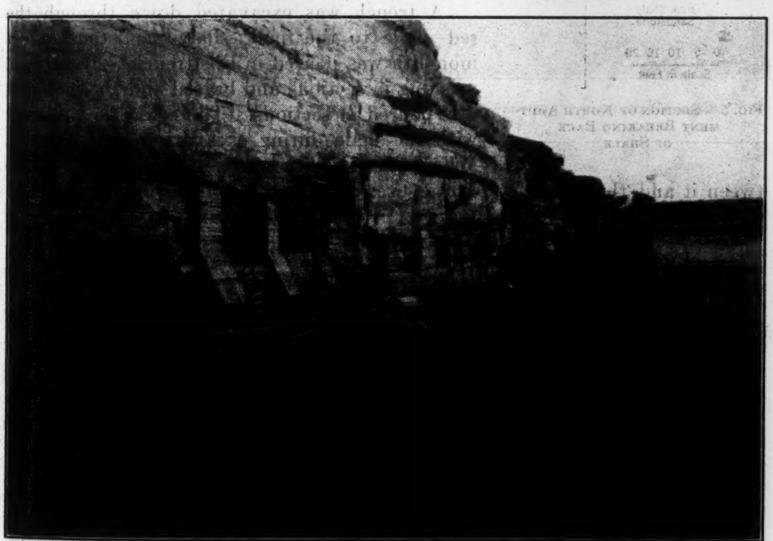


FIG. 4.—SOUTH ABUTMENT OF MAIN DAM, SHOWING SUPPORT OF SHALE, JANUARY 29, 1938

difficulties at the north abutment were encountered. This procedure was not considered feasible at the north abutment on account of the thinness and broken character of the cap rock. Since the load on the shale was much less here, an attempt was made to excavate a vertical face without any support and to pour the concrete against the shale; but when the face was about 50 ft high pieces of shale began to spall off and it was not safe to proceed. Spalling continued until the shale face had broken back an average of a few feet, with a maximum of 7 ft as determined by actual measurements from a skip suspended from the cableway. (See Figs. 5, 6, and 7.) The cap rock did not collapse and was left overhanging. The face remained in that condition for three months, until it was cut back. Several plans were studied and one was adopted which not only removed all question as to the stability of the shale face but solved other difficulties as well.

---

[6] For more complete description see "Dam Building on Difficult Rock," *Engineering News-Record*, June 9, 1938, p. 810.

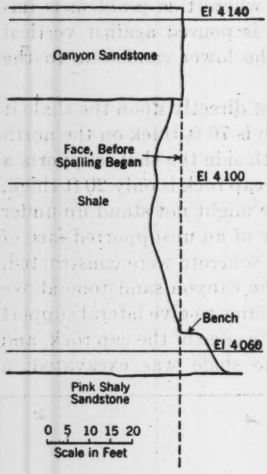


FIG. 5.—SECTION OF NORTH ABUTMENT BREAKING BACK OF SHALE

A trench was excavated down through the red shale to the pink shaly sandstone and a monolith was poured in this trench. The trench was dug in sections and braced and the monolith was poured in sections. There was no difficulty with the shale during excavation of the trench. When the monolith was complete, the shale between it and the exposed face was excavated and the intervening monoliths were poured.

PRACTICAL PROBLEMS OF THE ARTESIAN SANDSTONE AND THE PINK SHALY SANDSTONE

The artesian sandstone forms the foundations of the highest part of the dam and is a good rock for this purpose. It is moderately strong, its compressive strength as shown by eight tests ranging from 6 500 to more than 10 650 lb per sq in. There are problems, however, due to the fact that it is porous, is intersected by joint cracks, and contains water under artesian pressure.

The artesian sandstone was not exposed before excavations were begun, but it was investigated by numerous core borings and by two 30-in. borings, 76 and 81 ft deep, respectively, into which it was possible to descend and inspect the rock. Such large borings are the most satisfactory means of investigating foundation rocks. The smoothly cut surface of the holes shows up cracks, seams, and soft layers to much better advantage than the rough sides of shafts.

When the first holes were drilled into the artesian sandstone water flowed from them. The largest artesian flows were from the middle part of the upper artesian sandstone and from the lower artesian sandstone. The static head of water in the various holes was measured. The elevation of the river at the dam site is approximately 4 050 ft and the highest static head of water in the

FIG. 6.—NORTH ABUTMENT BEFORE SPALLING OF SHALE, SHOWING ASPHALTIC COATING

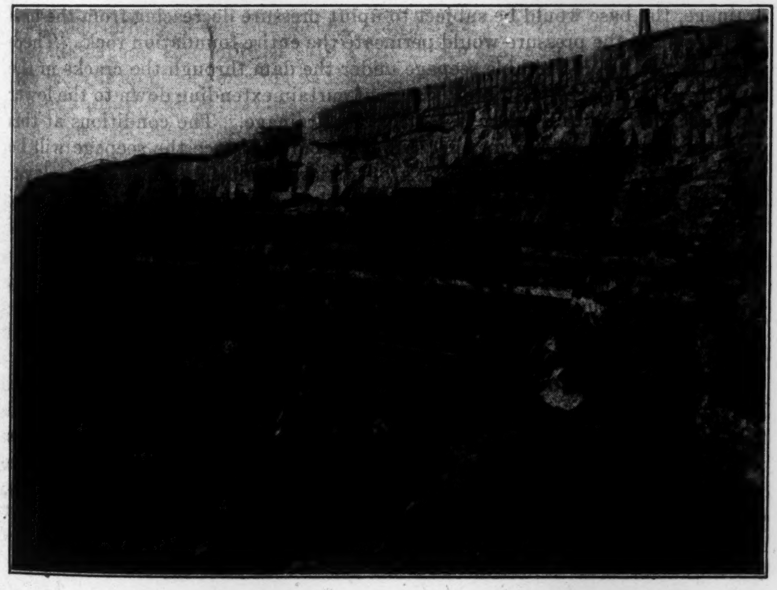


FIG. 7.—NORTH ABUTMENT AFTER SPALLING OF SHALE, SHOWING OVERHANGING SANDSTONE CAP ROCK

holes which penetrated the upper artesian sandstone only was 4 065 ft. The upper artesian sandstone outcrops in the South Canadian River two miles up stream from the dam at approximately the same elevation. The lower artesian sandstone outcrops farther up the river at a somewhat higher elevation and the highest static head of artesian water in holes penetrating the lower sandstone is 4 082 ft. Both these static heads agree with the elevations of the probable intake areas of the upper and lower artesian sandstones. Therefore, when the reservoir is full the water in these sandstones will be subject to the effect of this head.

The percentage of voids in the central part of the upper artesian sandstone varied from 6.4 to 22.7. The porosity of the upper 30 ft is comparatively low, and for the lower part, next to the lower red shale, it is also low. The permeability of the sandstone was tested both parallel and perpendicular to the bedding. The permeability parallel to the bedding is greater than it is perpendicular to the bedding, but even so it is low. The highest coefficient of permeability measured was 1.2 by 10^{-5} cm per sec. At this permeability water would flow through the sandstone under unit hydraulic gradient at the rate of 0.000024 ft per min. This is only 0.037 ft per day, an extremely slow flow, which could not account for a flow of 121 gal per min from some of the holes. This fact and others indicate that the artesian water moves principally through cracks rather than through the pores of the rock. Observations in the two 30-in. borings support this conclusion. Since the rock is porous and permeable, artesian pressure under static conditions will be equalized throughout it.

If the dam were built on this sandstone without a grout curtain and without drainage, the base would be subject to uplift pressure decreasing from the heel to the toe and the pressure would permeate the entire foundation rock. There might also be an undesirable seepage under the dam through the cracks in the sandstone. Seepage is reduced by a grout curtain extending down to the lower red shale, and uplift pressure is reduced by drainage. The conditions at this dam site are excellent for an effective grout curtain since the seepage will be principally through cracks, and the grout holes can reach an impervious layer at a moderate depth. Since seepage will be through cracks which are generally vertical, inclined holes are more effective in intersecting the greatest number of cracks.

The grout curtain consists of holes on 4-ft centers at the heel of the dam, dipping 75° S—that is, inclined 15° from the vertical. Every other hole extends down into the lower red shale and the alternate holes reach a vertical depth of 20 ft. Provision was made for closer grouting where necessary.

In rock with flat bedding seams, there is danger of opening the seams and lifting the rock by excessive grout pressure. Such uplift has been observed in some cases and since open bedding cracks were seen in the two large borings the possibility of this uplift was eliminated by restricting the pressure to the weight of the overlying material. Grouting was done after considerable concrete was poured and grout pressure was restricted to 1 lb per sq in. per ft of overlying rock and concrete. In order to use higher pressures in the lower part of the holes they were grouted in stages using stops at a vertical depth of 20 ft.

The average grout consumption of 122 deep holes was 62.9 sacks, the minimum, 0.1 sack, and the maximum, 3 683 sacks. The average per foot of hole was 1.05 sack. Omitting the 2 holes that required more than 600 sacks each the average is 24.9 sacks per hole, or 0.42 sack per ft of hole. The average grout consumption of 111 shallow holes was 15.5 sacks, the minimum was 0.1 sack, and the maximum was 433.2 sacks. The average per foot of hole was 0.78 sack. Subsequent boring has produced cores with cracks thoroughly filled and two parts of the core firmly cemented together. In addition to the line of grout holes at the heel of the dam, provision is made for a line of holes from a gallery in the dam. These holes can be grouted at higher pressures after the dam is completed. At the abutments where the foundation is stepped up to the canyon sandstone the grout holes were inclined under the cliff and higher pressures were used.

Relief of uplift pressure is provided by a line of 8-in. drain holes from a gallery down stream from the grouting gallery. The average spacing of drain holes approximated 15 ft on centers. They were drilled to the red shale after all grouting had been completed. This program of grouting and drainage should solve the practical problems of the artesian sandstone. The drain holes will relieve uplift pressure and the grout curtain will prevent undesirable seepage.

Artesian aquifers similar to this sandstone are believed to be subject to compression under increased load and to expansion under increased artesian pressure.[7] Since the artesian sandstone will be exposed to reservoir water it will be subject to the fluctuating head of the reservoir and, theoretically, it should expand and contract. It is not known that any such movement has been recognized at any existing dam and it would probably affect a considerable area uniformly and be harmless. Further research on this point is desirable.

As has been mentioned there are lenticular shaly beds in the upper part of the upper artesian sandstone. These beds are not continuous and, therefore, the foundations of the different monoliths were excavated to different depths in order to reach beds of massive sandstone.

On either side of the river near the abutments it was not considered necessary to go down to the artesian sandstone and the foundations were placed on massive beds in the pink shaly sandstone formation. The only possible danger from this rock is in regard to sliding on some of the shaly bedding seams, but the excavations were carried down to such depth that sliding would involve shearing overlying beds of massive sandstone, which would be practically impossible. Unconfined compressive tests on the pink shaly sandstone showed a minimum strength of 2 240 lb per sq in. when wet, and a maximum of 6 940 when dry. Shear tests parallel to the bedding on typical samples in natural condition under a transverse load of 100 lb per sq in. gave a minimum strength of 441 lb per sq in., a maximum of 2 035 lb, and an average of 971 lb per sq in.

---

[7] "Compressibility and Elasticity of Artesian Aquifers," by Oscar Edward Meinzer, *Economic Geology*, Vol. 23, No. 3, 1928, pp. 263–291.

It is believed that the foundation beds selected in both the artesian sandstone and the pink shaly sandstone are thoroughly competent to support the loads imposed upon them.

STABILITY AND TIGHTNESS OF THE CANYON SANDSTONE

The canyon sandstone at the north abutement was cut by many open cracks, and large blocks near the edge appeared to have moved. Therefore, questions were put to the geologist as to the cause of these cracks and their open condition, as to the cause of the movement of large blocks of sandstone, whether this movement would continue after the dam was built, and as to whether this rock was a satisfactory foundation for a dam 100 ft high. Furthermore, there was the problem of preventing seepage through cracks in the sandstone.

Most of the cracks in the sandstone are clearly joint cracks such as intersect the sandstones throughout this region. The two possible causes of opening of the cracks are solution and movement of the blocks. This sandstone is somewhat calcareous and in places shows signs of solution, but most of the cracks are free from such evidence. On the other hand it is obvious that many of the blocks near the canyon wall have moved, and since open cracks are restricted to the vicinity of the canyon, movement of the blocks is obviously the explanation. The cause of this movement is due partly to undermining by weathering and disintegration of the shale and partly to frost action and expansion from temperature changes. The blocks would move easily toward the canyon wall and there would be little tendency for them to move back.

The high part of the dam is extended back some distance from the canyon wall at the north abutment, resulting in the removal of the sandstone cap near the canyon with the consequent removal of all blocks that have moved. The problem, therefore, is to prevent future movement. When the dam is completed the outcrop of red shale in the canyon walls will be covered by an earth enbankment which will effectively prevent weathering of the shale near the dam and will thus prevent undermining of the sandstone. The earth embankment of the wing-dams will cover the surface of the sandstone to the canyon wall and will thus protect the sandstone from frost action and temperature changes. Therefore, the causes of movement of sandstone blocks will have been removed when the dam is completed, and there should be no future tendency for the blocks of the cap rock under or near the dam to move. Thus, by analysis of the causes of the unfavorable conditions, it was possible to remedy these conditions, prevent their future recurrence, and obtain assurance that the foundations will remain stable.

The canyon sandstone is a moderately strong rock, as is shown by the unconfined compression tests on six samples under different conditions, presented in Table 3. This formation varies from a hard massive sandstone to a softer thin-bedded sandstone, and the lower values were from samples of the softer sandstone.

From a foundation viewpoint the sandstone cannot be considered alone because it rests upon the red shale which is a much weaker rock. It has been shown that the red shale is strong enough to support the load of the dam but that there may be a total maximum settlement of 4 in. over a period of years. There may also be some adjustment due to plastic flow of the shale. The amount of settlement is not as important as its character (that is, whether it is irregular or even), and the way it is transmitted by the sandstone to the dam.

TABLE 3.—COMPRESSIVE STRENGTH OF CANYON SANDSTONE, IN POUNDS PER SQUARE INCH

Position, in respect to bedding	Strength of Sample Nos.:										
	1		2		3		4		5	8	
	Dry	Wet	Dry	Wet	Dry	Wet	Dry	Wet	Dry	Dry	Wet
Perpendicular	7 870	7 850	8 220	7 220	2 820	2 590	8 500	7 820	5 740	3 200	2 610
Parallel	8 550	7 030	4 130	4 070	2 310	2 150	7 670	6 410	8 120	6 020	3 280
45°	5 100	4 700	4 200	3 820	4 150	3 470	8 210	4 150	....	....	....

If the sandstone were a continuous, unbroken bed it would distribute any settlement effectively, but actually it consists of several beds intersected by numerous joint cracks. Since the cracks are offset from bed to bed, it may be likened to a mass of dry masonry with offset joints. It will thus be more effective in spreading the load than if the joints continued directly through all the beds, but less effective than an unbroken bed. The sandstone cap in the north abutment is only about 20 ft thick whereas at the south abutment it is about 80 ft thick, and, therefore, will spread the load much more effectively than at the north abutment. This was one reason for extending the high part of the dam farther back from the canyon wall at the north abutment. There will probably be some small uneven settlement of the different monoliths of the dam at the north abutment but provision has been made for it. The conclusion reached was that the canyon sandstone would provide satisfactory foundations for the dam.

There remains the problem of preventing seepage through the cracks of the sandstone. This is especially important because such seepage might tend to erode the top of the red shale. Consolidation grouting of the canyon sandstone to fill the cracks was required for the entire area of the concrete and earth dams; then a grout curtain was formed at the heel of the concrete dam by a line of inclined holes extending down to the red shale. This grouting was done after at least 25 ft of concrete had been placed, and grout pressures were restricted to 1 lb per sq in. per ft of overlying materials. Fifty-four shallow holes, each 20 ft deep, required 2.32 sacks of cement per foot, an average of 46.4 sacks per hole. The maximum was 935 sacks and the minimum was 0.0 sacks per hole. Twenty-six deep holes, averaging 7 ft, required 1.99 sacks per foot of hole or 77.4 sacks per hole. A comparison of inclined and vertical holes proves the greater effectiveness of the former. Thirteen deep inclined holes averaged 3.84 sacks per foot of hole, whereas ten vertical holes averaged only 0.31 sack per foot of hole. The permeability of the sandstone is very low, the highest coefficient of permeability found being 2.8 by 10^{-5} cm per sec, or 0.000056 ft per min. This is comparable with the artesian sandstone, and seepage through the unfractured rock will be negligible. Since seepage will be through cracks and not through the pores, grouting should prevent seepage, effectively, through the canyon sandstone under the dam.

CONCLUSIONS

The foregoing description of Conchas Dam illustrates the problems of constructing a high concrete dam on a soft, weak shale, or siltstone. Such rocks are common in the Southwest and similar problems occur in other regions; but high concrete gravity dams on such foundations are rare. There are certain similarities between the foundation problems of Conchas Dam and Tygart Dam on the Tygart River, near Grafton, W. Va. The fire-clay shale, or indurated clay at Tygart Dam, is not laminated and is somewhat similar to the red shale at Conchas Dam; but it occurs in thinner beds separated by beds of strong sandstone, and the practical problems are less serious. The same is true of the State Line or Lake Lynn Dam, on the Cheat River, in West Virginia, which has been in use for more than ten years.

The investigation of the foundations of Conchas Dam has involved extensive geological field studies, not restricted to the immediate dam site, an extensive drilling program, large drill holes, tunnels and test shafts, physical, and chemical tests, and tests adapted from soil mechanics; but, nevertheless, it has been essentially a geological investigation, using every means available to solve geological problems. Where precise laboratory methods could be used it was necessary to apply their results to the conditions in the ground, and it was realized that conclusions from the precise tests could be no more accurate than the geological interpretation of conditions in the ground to which they were applied. The application of precise methods to geological data, which from their very nature cannot be expressed in terms of precise measurement, involves the exercise of judgment lest the seemingly precise solution by laboratory methods of parts of the problem may seem to indicate that the solution of the entire problem is precise. Such a conclusion would lead to unwarranted confidence and might result in disaster.

Since at practically no dam site is it possible to obtain all the desirable information, the importance of geological judgment and familiarity with similar sites becomes clear. Great care must be exercised, however, so that "judgment" will not be carried to the dangerous extreme of assuming that, because a site is similar to another site, a similar dam will result in similar satisfactory results. The only safe course, therefore, is to obtain all the information possible, use the best precise methods applicable, compare details with similar details elsewhere, use the best geological judgment, and then provide for the worst possible conditions. The last provision will take care of the impossibility of obtaining all desirable information and the use of judgment based on experience will obviate resort to unnecessarily expensive designs.

The geological investigation of Conchas Dam site has emphasized the need of additional research upon several problems. A better understanding is desirable of the compaction of shale, and additional high-pressure "consolidation" tests on shales where the former load on them can be estimated should give valuable results.

Little is known about the slow plastic flow or creep of shale under pressures less than its shearing strength over a period of years, although there are many indications that such movement does take place. Laboratory tests, field tests on construction jobs, and systematic observations over a period of years on completed structures should give important information. Laboratory tests such as placing a specimen of shale under a constant load less than its shearing strength for months, and having provision for measuring shortening and lateral expansion of the specimen, should prove whether there is a slow deformation under moderate load. It is essential, however, to protect the specimen from drying during testing. If the elevation of a dam on uncemented shale were checked for a period of years against a datum well removed from the influence of the dam, and settlement of the dam were checked against the expected settlement as computed from consolidation tests, there would be an indication as to whether movement other than compaction were taking place in the shale.

It is known that artesian aquifers will expand when subject to artesian pressure and contract when that pressure is removed, but there is practically no information as to whether dams underlain by an artesian aquifer are subject to appreciable movement. Long-continued observations on dams, such as Conchas Dam, which rest upon an artesian aquifer that will be exposed to the fluctuating head of a flood control reservoir, should give valuable information. It would be necessary to refer the observations to a datum well down stream from the dam.

The problems of Conchas Dam are typical of those to be encountered with sites on the extensive red beds and similar formations. Since the most difficult problems—those concerned with settlement and plastic flow—may occur wherever uncemented shales occur, and since such formations are found throughout a vast area of the Southwestern, Western, and Northwestern plains, Conchas Dam illustrates some of the most important foundation problems of innumerable dam sites.

DISCUSSION

H. L. JOHNSON,[8] ESQ. (by letter).—The geological conditions at Conchas Dam and the major problems resulting from them, are presented in this excellent paper. The author demonstrates the application of geological knowledge to foundation and allied engineering problems. Mr. Crosby states (see heading "Geology of the Dam Site") that, in the upper red shale, the percentage of clay sizes and voids is higher in the south abutment than in the north abutment. This condition was indicated by the initial and incomplete tests run on the shales from both abutments; but a study of the complete data from all tests run on the shales indicates that the average composition and physical characteristics of the red shale in the north and in the south abutments are identical for all practical purposes. The average results for voids and density on a total of 114 samples are presented as Items 1 and 2 in Table 4. The voids in all samples were completely saturated. The average results of grain size determinations on a total of fifty-eight samples, with respect to clay sizes, are given in Item 3, Table 4.

TABLE 4.—CHARACTERISTICS OF RED SHALE

Item No.	Description	South abutment	North abutment
1	Percentage of voids	24.93	24.11
2	Density, in pounds per cubic foot	146.88	147.33
3	Clay sizes (percentages)	37.04	38.05

The author is correct in stating that the red shale is unstratified and that the proportions of grain sizes vary considerably from place to place, as do other characteristics of the shale. It is important to note that this variation is not regular, either horizontally or vertically, but is erratic in all directions. Thus, instead of regular zones, layers, or lenses of material having definite characteristics, the shale consists of irregular interlocked areas of varying characteristics which grade into each other without any definite line of demarcation other than an occasional color break.

The shear and compression values, used and cited by the author as a basis for determining the competency of the red shale, are confined to those obtained from two and three definite horizons in the south and north abutments, respectively. However, values actually used in determining the competency of the shale were representative of the full vertical and horizontal range of the shale at the abutments. In excavating the south and north abutments shale samples were taken, in so far as construction progress and facilities permitted, throughout the red shale exposed at the abutments so that the average characteristics could be determined for the full range of the material in question. In all, fifty-three shear tests and more than two hundred compression tests were made. In connection with the author's statement that, "The average shearing strength as computed from unconfined compression tests on twenty 2-in. cubes was 266 lb per sq in." (see heading "Practical Problems," following Case IV), it should be pointed out that all shear values entering into the design and stability computations were derived from direct shear tests on cylinders of shale 6 in. in diameter.

---

[8] Conchas Dam, N. Mex.

In the discussion of problems connected with the red shale Mr. Crosby refers to that part of the dam "resting upon the shale" and the parts of the dam "underlain by red shale." It should be made clear that no part of the Main Dam is founded directly on shale and that the entire dam is underlain by shale strata at depths of from 20 ft to 80 ft below the foundation sandstones. The terminal monoliths founded on the canyon sandstone are underlain by both the upper and lower red shales whereas the central monoliths, founded on pink shaly and artesian sandstone, are underlain by only the lower red shale.

That some settlement will occur because of the load of the dam and the presence of the underlying shales has been an accepted fact since the inception of the project. That there may be some differential settlement between those parts of the dam founded on the canyon sandstone and those founded on the artesian sandstone has been recognized and provided for in the design of the abutment monoliths. However, it is doubtful that the settlement differential will approach the 4 in. indicated by the author, whose computation is apparently based on the omission of the lower red shale from consideration or on the assumption that the lower red shale will be equally reconsolidated under the entire dam. This lower red shale has not been uniformly unloaded by erosion, that portion in the canyon being unloaded to a greater degree than those underlying the abutments or canyon walls. Hence, the degree of expansion due to erosional unloading (and, therefore, the resulting degree of reconsolidation possible in the lower red shale under the load imposed by the dam) must be greater in the canyon area than in the area underlying the canyon walls or abutments. Furthermore, the reconsolidation load imposed by the dam on the lower red shale will be much greater under the monoliths between the abutments, 200 to 235 ft high, than under the monoliths, 40 to 95 ft high, on top of the abutments. It is evident, therefore, that although the total thickness of shale underlying the high monoliths is only slightly more than one-third of the total that underlies the abutments it will unquestionably undergo materially greater reconsolidation. Thus the lower red shale is an important factor because it will tend to act as an equalizer of settlement between the high and low parts of the dam.

In addition to the construction problems involved in excavating the abutment faces in the upper red shale and constructing the abutting monoliths, there was also the problem of insuring a water-tight joint between the upper red shale and the adjacent monoliths. This problem arises from the fact that the shale shrinks appreciably upon exposure to air, with a resultant loss of moisture content. This drying and shrinking produces innumerable hair cracks and causes the opening of the old planes of movement (slickensided fracture planes) which traverse the shale in all directions and frequently are of considerable extent. Every effort was made to prevent drying of the shale by spraying it with a bituminous sealing compound as soon as possible after exposure. However, experience has shown that this treatment is not wholly effective and some drying and shrinkage, with the resultant development of cracks and opening of old fractures, is unavoidable. These cracks, especially those resulting from the opening of old fracture planes, are more or less inter-connected and provide a possible path of leakage around the abutments. In addition to the shale problem there was also the possibility that some differential settlement would occur between the abutments and the abutting monoliths and that shrinkage in the concrete of the abutting monoliths would tend to open the abutment joints. These conditions precluded the use of the conventional copper waterstops and indicated the need of a special material for grouting the shale and for use as waterstops, which would remain plastic and have the ability to come and go with any movement between the abutments and abutting monoliths. Three vertical sealing wells were installed, therefore, along and astraddle of each abutment joint. They were 15 in. in diameter, the two outside wells at each abutment being filled with an internal set-up bituminous material whereas the center well was filled with a mixture of sand and bentonite. At the south abutment the vertical joint between the shale and concrete, as well as the shale itself, was grouted with an internal set bituminous material that was mixed and pumped cold. This material possesses the property of setting up internally, several hours after mixing and injecting, to produce a plastic rubbery product. The shale was grouted with bitumen after the abutting monolith was constructed, the grout being injected by means of a system of piping embedded in the abutting monoliths during construction. Provision has been made for similar grouting at the north abutment should it become necessary.

In discussing the problems of the artesian sandstone upon which most of the high monoliths between the abutments are founded and which underlies the remainder of the dam at varying depths, the author states: "If the dam were built on this sandstone without a grout curtain and without drainage, the base would be subject to uplift pressure decreasing from the heel to the toe ***" (see heading "Practical Problems of the Artesian Sandstone and the Pink Shaly Sandstone"). This statement is obviously true of any gravity dam regardless of the kind and type of foundation material, unless the foundation materials were absolutely impermeable. The presence in the foundation of an artesian sandstone which outcrops or has intakes within the reservoir does give rise to an unusual uplift problem. With geological conditions, particularly those of artesian nature which occur at Conchas Dam, it is apparent that: (1) A grout curtain along the heel of the dam is wholly ineffective so far as uplift due to artesian pressure is concerned; and (2) without artificial drainage of the artesian aquifer at the dam, and in the absence of a natural outlet, the artesian uplift will be essentially uniform throughout the entire foundation area and will be equal to the hydrostatic head between pool level and foundation level. To relieve artesian uplift throughout the foundation area sixty-six 8-in. drain holes were drilled into the foundation, as mentioned by the author, at approximately 15-ft centers along the center line of the operating gallery, which is situated approximately 50 ft down stream of the grout curtain at the heel. These holes penetrate the upper artesian sandstone to within 2 to 5 ft of the underlying lower red shale. These drain holes and an inter-connecting tile and gravel drain laid upon the foundation will also serve to dispose of reservoir water passing the grout curtain at the heel of the dam, and will reduce uplift pressure resulting from it.

That artesian aquifers are compressible and elastic to varying degrees, dependent on the kind of rock or material comprising the aquifer, is known and generally accepted by authorities on ground water.[9] However, the nature of this phenomenon is such that any expansion or contraction, due to fluctuations in the artesian head resulting from variations in pool level in the artesian sandstone underlying Conchas Dam, should be fairly uniform throughout the area of the dam. The drainage system previously discussed will also be instrumental in reducing (if it does not wholly prevent) any tendency of the artesian aquifer to expand or contract by maintaining the artesian pressure under the dam at a uniform head.

In addition to the grout curtain which is continuous from the southeast end of the South Wing Dam, through the Main Dam, to the north end of the North Wing Dam, there was an extensive program of consolidation grouting in the canyon sandstone abutments. In this program every joint or crack of any appreciable size, exposed in the foundations or abutments, was cleaned out by washing and tapped with pipes to permit consolidation grouting after sufficient concrete or fill had been placed to permit the use of effective grouting pressures. Consolidation grouting was performed, whenever possible, in advance of the drilling and grouting of the grout curtain, in order that the cores obtained from the holes of the grout curtain might serve as a check on the effectiveness of the grouting. In addition to the consolidation grouting at the south abutment the vertical face of the canyon sandstone receives a protective coating of gunite for a distance of approximately 125 ft up stream from the dam. This is the only area adjacent to the dam where the highly jointed canyon sandstone is not to be covered by back-fill and would be exposed to the waters of the reservoir. In the remainder of the up-stream areas adjacent to the abutments the canyon sandstone, and the upper red shale, are protected from weathering, and the path of any possible seepage is lengthened in a somewhat unusual manner by virtue of extensive impervious back-fills. These back-fills extend approximately 160 ft up stream along the abutments from the dam and are themselves protected by pervious and rock-fill blankets.

In addition to past investigations of foundation conditions and allied problems, extensive preparations are being made to investigate: (1) The rate and distribution, as well as the ultimate future, of foundation settlement; (2) the quantitative effects of compressibility and elasticity of the artesian aquifer under operating conditions; (3) the deflection and tilting that will occur, and its relation to settlement; and (4), the effect of a full reservoir on artesian and ground-water conditions. To this end special equipment consisting of tiltmeters, plumb-bobs, underground bench-marks, and triangulation stations are installed in the Main Dam, and observations are made of artesian head at various core holes. The physical characteristics of the lower red shale are investigated by means of virgin samples obtained from several 8-in. drain holes which are extended into it. An extensive program of differential leveling and observing reference points set at the heel and toe of the various monoliths has been conducted continuously from October, 1937, to date (1939) in order to obtain foundation settlement data during construction.

---

[9] "Compressibility and Elasticity of Artesian Aquifers," by O. E. Meinzer, *Economic Geology*, Vol. 23, No. 3, May, 1928, pp. 263–291.

In closing this discussion the writer wishes to emphasize the fact that the difficulties and problems at Conchas Dam due to geological conditions were clearly recognized from the inception of the project and have been met by the continuous application of geological and engineering knowledge. Contributions to geological studies and to the solution of the various foundation problems at Conchas Dam have been made by E. G. Woodruff, who served as Geologist during the period of initial explorations; C. B. Theis, Associate Geologist, U. S. Geological Survey, Ground Water Division; W. M. Rau, Associate Geologist, U. S. Engineer Office, Little Rock, Ark., District; and the author. The writer has served as Foundation Engineer on the project since November, 1936.

IRVING B. CROSBY,[10] AFFILIATE, AM. SOC. C. E. (by letter).—Mr. Johnson has presented some additional data on the geological and foundation problems at Conchas Dam. He states that the conclusion that the percentage of voids was higher in the south abutment than in the north abutment was based upon initial and incomplete tests and that the complete data later available indicate that the physical characteristics of the shale in the south and north abutments are practically identical. However, the original tests cited in the paper were all that were available after investigation of the foundations had been under way for more than two years.

The writer stated, as information from official sources, that the average shearing strength of the red shale was determined from compression tests on twenty 2-in. cubes; but Mr. Johnson states that all shear values entering into the design and stability computations were derived from direct shear tests on cylinders of shale 6 in. in diameter. Since it is known that direct shear tests on shale give less reliable results than shearing values calculated from compression tests, the writer did not suppose that the results of the compression tests would be discarded at a late date in favor of results obtained by a less reliable method. The most accurate method of determining the shearing resistance of this shale would be by the triaxial compression tests.

In regard to possible settlement of that part of the dam underlain by red shale, the writer stated that consolidation tests had indicated that the maximum settlement under extreme conditions would be about 4 in. after a period of years. Mr. Johnson states: "However, it is doubtful that the settlement differential will approach the 4 in. indicated by the author, whose computation is apparently based on the omission of the lower red shale from consideration or on the assumption that the lower red shale will be equally reconsolidated under the entire dam." The writer did not omit the lower red shale from consideration, but the problem is extremely complicated. Certain facts essential for a rigorous solution were not obtainable, and simplification of the problem was necessary.

The formations at the dam site were laid down as flat beds in water and were buried by succeeding formations until there were at least 1 250 ft of sediments on top of the upper red shale. The lower red shale was buried approximately 150 ft deeper and had 1 400 ft of sediment on top of it. Therefore, other things being equal, the compaction of the two shales should vary as the loads upon them. About 1 225 ft of overlying rocks have been removed from above the upper red shale on the north abutment, reducing the load by 98% and allowing expansion of the shale. The dam on the north abutment will restore approximately 8% of this load and will cause some recompaction of the shale. In the canyon, erosion cut a maximum of 140 ft deeper, leaving only 70 ft of rock on the lower red shale and reducing the thickness of overlying rocks by 95 per cent. The highest part of the dam would restore about 18% of the original load. However, the lower red shale was unloaded only over a span of 300 ft, and it was covered by approximately 70 ft of strong artesian sandstone which would have some restraining effect. If the underlying shale had expanded appreciably, it should have uplifted the sandstone, with a tendency to open cracks in it. The fact that the sandstone contains water under artesian pressure indicates that the cracks have not been opened appreciably. Therefore, the amount of expansion and, consequently, the amount of possible recompaction of the lower red shale is uncertain. Since this shale is less than half as thick as the upper red shale, the possible settlement due to compaction of the lower red shale is less. Since undisturbed samples of the lower red shale were not available and, therefore, since it was impossible to estimate the compaction of this shale under a given load from a consolidation test, it was not possible to evaluate the settlement which might be caused by placing the dam on overlying rocks.

---

[10] Cons. Eng. Geologist, Boston, Mass.

It was desired, however, to estimate the effects of the worst possible conditions and, therefore, an estimate was made from consolidation tests of maximum possible settlement due to recompaction of the upper red shale from the load of the dam. The possible equalizing effect of differential recompaction of the lower red shale was disregarded purposely since it is uncertain and cannot be evaluated. It was realized that the estimates made gave an extreme value, probably considerably greater than could occur, as was stated in the writer's report, but that if the dam could be designed to meet this extreme condition it would be safe in any case. Since the dam could be designed to meet these extreme conditions without difficulty, this was done. An attempt to evaluate the uncertain effects of the lower red shale would not have made the dam any safer or have resulted in any important economy.

Mr. Johnson discusses the compressibility and elasticity of artesian aquifers without mentioning that these had been covered in the paper and that the conclusion had been reached that such movement would be uniform and harmless.

The special joint with sealing wells between the abutments and abutting monoliths is described in some detail by Mr. Johnson who thus supplements the paper on that subject referred to by the writer.[5] Mr. Johnson also gives additional information about the grouting program which was completed subsequent to the presentation of the paper. He also describes the most recent provisions made to study foundation settlement, and thus brings up to date (1939) the published information on Conchas Dam.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2069

LATERAL SPILLWAY CHANNELS

By Thomas R. Camp,[1] M. Am. Soc. C. E.

With Discussion by Messrs. W. E. Howland, Lewis V. Carpenter and John K. Vennard and Fenner H. Whitley, F. Knapp, Carl Rohwer, Harold Allen Thomas, Jr., and Thomas R. Camp.

Synopsis

The term "lateral spillway channel" is used in this paper to designate an open channel which receives inflow throughout its length, laterally from one or both sides, and discharges the accumulated flow at a point in the channel, usually one end. When used for reservoir spillways, these channels are sometimes referred to as "side channel spillways." On a smaller scale they are found in all water and sewage treatment plants. The best known example is the "wash-water gutter" of rapid filter plants. Other examples are the main wash gutter or "gullet" of a rapid filter, effluent flumes or "launders" of settling basins, and the underdrains of trickling filters. The term "lateral spillway channel" is used herein in order to avoid confusion with "side weirs," with which the flow is in the reverse direction or from the channel over the weir.

In this paper, equations for the water-surface profile are derived for channels of constant width which receive their inflow uniformly throughout their length. The equations hold for both level and sloping inverts and for bottoms of any regular shape. A term is included to account for friction, and the equations are put in simple form and are accompanied by a graph to facilitate their use in design. Some experimental measurements are presented for the purpose of verifying the theory, and establishing the approximate value of the friction factor for channels used in water and sewage treatment plants.

Introduction

A number of attempts have been made to develop equations for the flow in wash-water gutters. Some of these equations were semi-rational, containing arbitrary coefficients. One equation failed to account satisfactorily for a sloping invert. Another equation was based upon the assumption that the water surface draw-down is equal to the velocity head. It has since been

---

Note.—Published in February, 1939, *Proceedings*.

[1] Associate Prof. of San. Eng., Mass. Inst. Tech., Cambridge, Mass.

shown,[2] and will be made evident in this paper, that the draw-down is much greater than the velocity head, even if friction is neglected.

Julian Hinds,[2] M. Am. Soc. C. E., in working with spillways for dams, was probably the first to make a substantially correct statement of the fundamental differential equation for the water-surface profile of lateral spillway channels. Mr. Hinds' theory is based upon the momentum principle. The differential equation is for the case in which the increment of inflow per unit of channel length is constant; and it is applicable to channels of any shape. The momentum of the inflowing water and friction in the channel are neglected in Mr. Hinds' equation. It is not readily integrable for the general case, and solutions are obtained by successive approximations dealing with short lengths of the channel. Friction losses are computed separately.

Mr. Hinds' differential equation was amended by Mr. Henry Favre[3] to include a friction term and to account for a component of inflow velocity in the direction of the axis of the channel. Mr. Favre integrated the equation for short lengths of the channel in which average values of the cross-sectional area and hydraulic radius could be used without introducing appreciable error. Solutions by means of Mr. Favre's equation require successive approximations with short channel lengths, as is the case with Mr. Hinds' equation, but the computations are probably expedited because of the presence in the equation of a term for friction.

For channels of variable or irregular shape, such as are required for the economical design of dam spillways, a general solution probably necessitates the use of the Hinds or the Favre method. In water and sewage treatment plants, however, lateral spillway channels are usually of constant width, with inverts having no slope, or a constant slope. For such structures a simplification of the equations and methods is both possible and desirable.

The differential equation developed in this paper is identical with that used by Mr. Favre, except that the friction term includes the well-known Weisbach-Darcy friction factor. Because of the simplicity of the channel shapes considered, it is possible to integrate the equation approximately so that it may be used for the entire channel without serious error. For accurate solutions, and for the determination of several points on the water-surface curve, successive approximations are required; but the procedure is much simplified.

FUNDAMENTAL DIFFERENTIAL EQUATION

For a steady state, the flow of water in a lateral spillway channel is not only non-uniform but is complicated by the fact that both velocity and discharge increase from zero at the up-stream end to maximum values at the down-stream end. In the following development, a constant increment of discharge is assumed per unit length of channel, and the momentum of the inflow is neglected. For a changing velocity, the momentum principle states that the accelerating force is equal to the time rate of change of linear momentum; or, that is, to the mass of water flowing times the acceleration produced.

---

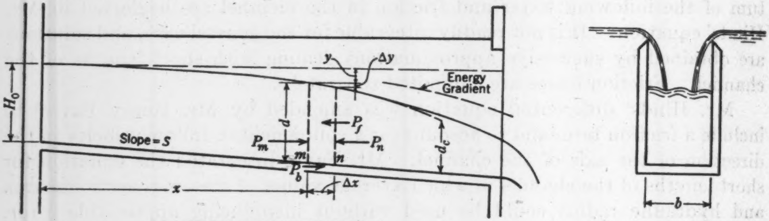
[2] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 885.

[3] *Engineering News-Record*, October 25, 1934, p. 520.

For a rectangular flume of width b (Fig. 1), consider two vertical cross-sections at Sections m and n, a distance Δx apart. The momentum at Section m, for the mass per second, is

$$M_m = \frac{Q\,w\,V}{g} \qquad (1)$$

in which Q = the discharge at Section $m = q\,x$; q = the constant increment of inflow per unit length of flume; w = the unit weight of water; g = the



(a) LONGITUDINAL SECTION (b) CROSS SECTION

FIG. 1

acceleration of gravity; and V = the horizontal component of the mean velocity at Section m. The momentum at Section n for the mass per second is

$$M_n = \frac{(Q + q\,\Delta x)\,w\,(V + \Delta V)}{g} \qquad (2)$$

The change in momentum in the distance Δx is

$$\Delta M = \frac{w}{g}\,[Q\,\Delta V + q\,\Delta x\,(V + \Delta V)] \qquad (3)$$

or, neglecting the product $\Delta x\,\Delta V$ and expressing Equation (3) in terms of differentials,

$$dM = \frac{w}{g}\,[Q\,dV + q\,V\,dx] \qquad (4)$$

Since $\dfrac{dM}{dt} = \dfrac{dM}{dx}\,\dfrac{dx}{dt} = V\,\dfrac{dM}{dx}$,

$$\frac{dM}{dt} = \frac{w}{g}\left[V\,Q\,\frac{dV}{dx} + q\,V^2\right] \qquad (5)$$

in which dM is the change in momentum in the time dt.

Equation (5) gives the time rate of change of linear momentum at any section, m, for the mass of water flowing per second. This is equal to the net force in the direction of x producing the change. The net force in the direction of x acting upon the volume of water between Sections m and n is the difference between the static pressures less the friction force. The static pressure at Section m is

$$P_m = \frac{b\,w\,d^2}{2} \qquad (6)$$

and at Section *n* is

$$P_n = \frac{b\,w}{2}(d - \Delta y + S\,\Delta x)^2 = \frac{b\,w}{2}(d^2 - 2\,d\,\Delta y + 2\,d\,S\,\Delta x) \ldots\ldots (7)$$

in which S = the slope of the bottom. The horizontal component of the static pressure on the bottom is

$$P_b = b\,w\left(\frac{2\,d - \Delta y + S\,\Delta x}{2}\right)S\,\Delta x = w\,b\,d\,S\,\Delta x \ldots\ldots\ldots (8)$$

The net static pressure on the water prism *mn* is the summation of Equations (6), (7), and (8), or

$$P_m - P_n + P_b = w\,b\,d\,\Delta y \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

It will be noted that the slope of the bottom has no effect upon the value of the net pressure. It may be shown also that a changing width b has no influence on the value of this force.

The friction drag is

$$P_F = F\,(b + 2\,d)\,\Delta x = \frac{F\,b\,d\,\Delta x}{R} \ldots\ldots\ldots\ldots\ldots (10)$$

in which F = the frictional drag per unit area of channel wall, and R = the hydraulic radius at Section *m*. The equivalent water head on the area *bd* corresponding to the friction drag P_F is:

$$\text{Head} = \frac{P_F}{w\,b\,d} = \frac{F\,b\,d}{w\,b\,d\,R}\,\Delta x = \frac{F}{w\,R}\,\Delta x \ldots\ldots\ldots\ldots (11a)$$

and from the Weisbach-Darcy formula this lost head is also given by

$$\text{Lost head} = \frac{f\,\Delta x\,V^2}{4\,R\,2\,g} \ldots\ldots\ldots\ldots\ldots\ldots (11b)$$

Hence, from Equations (11),

$$F = w\,\frac{f}{8\,g}\,V^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

in which f = the Weisbach-Darcy friction factor. From Equations (10) and (12),

$$P_F = \frac{w\,f\,V^2\,b\,d}{8\,g}\,\frac{}{R}\,\Delta x \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

The net accelerating force on the water prism *mn*, from Equations (9) and (13),

$$\alpha_{mn} = w\,b\,d\,\Delta y - w\,\frac{f\,V^2}{8\,g}\,\frac{b\,d}{R}\,\Delta x \ldots\ldots\ldots\ldots (14)$$

and since the volume of water between Sections *m* and *n* is

$$b\,d\,\Delta x = \frac{\Delta x}{V}\,Q \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

the accelerating force on the water flowing per second (that is, Q) is

$$\frac{V}{\Delta x}\left(w\,b\,d\,\Delta y - w\,\frac{f\,V^2}{8\,g}\,\frac{b\,d}{R}\,\Delta x\right) = w\,Q\,\frac{\Delta y}{\Delta x} - w\,Q\,\frac{f\,V^2}{8\,g\,R} \quad \ldots\ldots (16)$$

Expressing Equation (16) in terms of differentials and equating to Equation (5):

$$\frac{dy}{dx} - \frac{f\,V^2}{8\,g\,R} = \frac{1}{g}\left[V\,\frac{dV}{dx} + q\,\frac{V^2}{Q}\right] \quad \ldots\ldots\ldots\ldots\ldots (17)$$

Equation (17) is general for any shape of channel, and it is identical with the Hinds equation except for the friction term.

Application to Constant Width Flumes

In order to integrate Equation (17) it is convenient to express the velocity, discharge and draw-down, y, in terms of the dimensions of the channel. For this purpose, $V = \frac{q\,x}{b\,d}$; $\frac{dV}{dx} = \frac{q}{b}\left[\frac{1}{d} - \frac{x}{d^2}\,\frac{dd}{dx}\right]$; and (since $y = H_0 + S\,x - d$), $\frac{dy}{dx} = S - \frac{dd}{dx}$. Substituting these values in Equation (17) and simplifying, the equation becomes

$$\frac{dd}{dx} = \frac{2\,q^2\,x\,d - S\,g\,b^2\,d^3 + \frac{f}{8\,R}\,q^2\,x^2\,d}{q^2\,x^2 - g\,b^2\,d^3} \quad \ldots\ldots\ldots\ldots (18)$$

Equation (18) may be written as follows:

$$\frac{2\,x\,d\,dx - x^2\,dd}{d^2} + \frac{g\,b^2}{q^2}\,d\,dd = \frac{S\,g\,b^2}{q^2}\,d\,dx - \frac{f\,x^2}{8\,R\,d}\,dx \quad \ldots\ldots (19)$$

The terms on the left side of Equation (19) integrate directly, and the partially integrated equation is as follows:

$$\frac{x^2}{d} + \frac{g\,b^2\,d^2}{2\,q^2} = \frac{S\,g\,b^2}{q^2}\int d\,dx - \frac{f}{8}\int \frac{x^2}{R\,d}\,dx + C \quad \ldots\ldots\ldots (20)$$

When $x = 0$, $d = H_0$, from which

$$C = \frac{g\,b^2\,H_0^2}{2\,q^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

In the term including S in Equation (20) the expression under the integral is equal to the area in the distance x between the water-surface curve and the bottom. This area equals $x\bar{d}$, in which $\bar{d}$ = the average depth throughout the distance x. An approximate integration of the friction term may also be had by assuming both R and d constant and equal, respectively, to $\bar{R}$ and $\bar{d}$, where $\bar{R}$ = the average hydraulic radius throughout the distance x. With these approximations, the completed integration is

$$\frac{x^2}{d} - \frac{g\,b^2}{2\,q^2}\left(H_0^2 - d^2\right) = \frac{S\,x\,g\,b^2\,\bar{d}}{q^2} - \frac{f\,x^3}{24\,\bar{R}\,\bar{d}} \quad \ldots\ldots\ldots (22)$$

Solving for H_0, Equation (22) becomes

$$H_0 = \sqrt{d^2 + \frac{2\,Q^2}{g\,b^2\,d} - 2\,S\,x\,\bar{d} + \frac{f\,x\,Q^2}{12\,g\,b^2\,\bar{R}\,\bar{d}}} \qquad (23)$$

Equation (23) may be used to solve for the value of H_0 if the value of d at any point in the channel is known or fixed. The solution requires, however, that the average values of R and d throughout the length considered be assumed for substitution in the equation. If the invert is level and the friction loss is small, the last two terms may be omitted. For an accurate solution of the value of H_0, after the first approximation is made, the equation must be solved for d at several points along the channel in order to check the assumed values of $\bar{R}$ and $\bar{d}$. Adjustments are then made by trial and error.

Equation (22) is a cubical equation in terms of d, and its solution for d is simplified if the equation is rearranged in dimensionless form as follows:

$$\left(\frac{d}{H_0}\right)^3 + \left(\frac{f\,Q^2\,x}{12\,g\,b^2\,\bar{R}\,\bar{d}\,H_0^2} - \frac{2\,S\,x\,\bar{d}}{H_0^2} - 1\right)\frac{d}{H_0} + \frac{2\,Q^2}{g\,b^2\,H_0^3} = 0 \qquad (24)$$

or,

$$\left(\frac{d}{H_0}\right)^3 + A\left(\frac{d}{H_0}\right) + B = 0 \qquad (25)$$

in which

$$B = \frac{2\,Q^2}{g\,b^2\,H_0^3} \qquad (26)$$

and

$$A = \frac{f\,H_0\,x\,B}{24\,\bar{R}\,\bar{d}} - \frac{2\,S\,x\,\bar{d}}{H_0^2} - 1 \qquad (27)$$

The solution of Equation (25) for $\frac{d}{H_0}$ may be obtained graphically from Fig. 2 for various values of A and B.

CRITICAL DEPTH

The flow characterized by the foregoing equations is at the upper alternate stage. For most of the lateral spillway channels used in water and sewage plants, the depth at the lower end of the channel will be fixed at upper stage by the hydraulic conditions in the conduit down stream, or the flume will discharge freely. In the latter case, illustrated in Fig. 1, the pressure distribution near the end of the flume will be non-hydrostatic due both to aeration under the nappe and to the existence of appreciable vertical acceleration in this region. The region will extend for a distance three or four times the depth up stream from the lower end. For the design of freely discharging flumes, a close approximation to actual conditions will be obtained if a hydrostatic control section is assumed to exist at a distance up stream from the end equal to three or four times the critical depth. Since the value of the critical depth for a rectangular flume is

$$d_c = \sqrt[3]{\frac{Q^2}{g\,b^2}} \qquad (28)$$

Equation (23) may be written in terms of the critical depth as follows:

$$H_0 = \sqrt{3\, d_c^2 - 2\, S\, x_c\, \bar{d} + \frac{f\, x_c\, d_c^3}{12\, \bar{R}\, \bar{d}}} \ldots\ldots\ldots\ldots \quad (29)$$

For this case, the coefficients in the cubical formula, Equation (25), have values as follows:

$$B_c = 2\left(\frac{d_c}{H_0}\right)^3 \ldots\ldots\ldots\ldots \quad (30)$$

and

$$A_c = -\sqrt[3]{\frac{27}{4}\, B_c^2} = -3\left(\frac{d_c}{H_0}\right)^2 \ldots\ldots\ldots\ldots \quad (31)$$

The value of B_c in Equation (30) was obtained by combining Equations (26) and (28) to eliminate $\frac{Q^2}{g\, b^2}$. The value of A_c in Equation (31) was obtained from Equation (25) by setting the derivative of B with respect to $\frac{d}{H_0}$ equal to zero and solving for A. Values of $\frac{d_c}{H_0}$ for various values of B_c or A_c are shown on the "critical depth" line of Fig. 2.

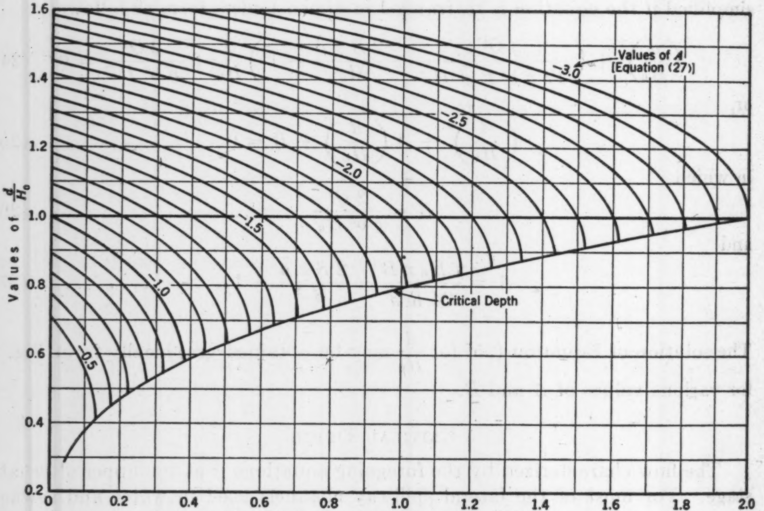


FIG. 2.—GRAPH FOR THE SOLUTION OF EQUATION (25)

CROSS-SECTIONS OTHER THAN RECTANGULAR

The foregoing equations were developed for rectangular cross-sections, but they are also applicable to channels with other bottom shapes if the depth at any section is taken as the average depth over the flume width. This may be demonstrated by consideration of Fig. 3 which shows a rectangular, a V-shaped and a semi-circular bottom. In all three cases, since the cross-sectional area is $b\,d$, the static pressure on the cross-section is

$$P = w\,b\,h_0\,d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

in which h_0 = the depth to the center of gravity of the cross-section. The value of h_0, however, is not the same for the three cases.

For a rectangular cross-section, $h_0 = \frac{d}{2}$. For the V-shaped bottom,

$$h_0 = \frac{d}{2} + \frac{z^2}{6\,d} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

and,

$$P = \frac{w\,b\,d^2}{2} + \frac{w\,b\,z^2}{6} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

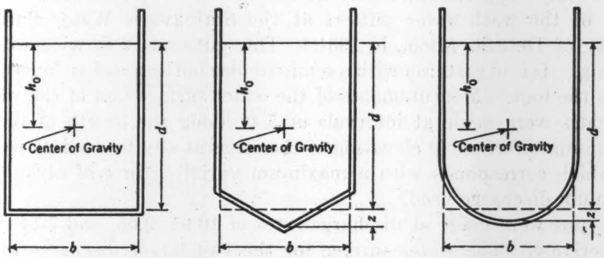


FIG. 3

The values of P_m and P_n are the same as those given by Equations (6) and (7) except that each contains the additional term $\frac{w\,b\,z^2}{6}$. The difference in pressures, therefore, is the same as for the rectangular flume and is given by Equation (9). For the semi-circular bottom,

$$z = \frac{4 - \pi}{8}\,b = 0.1073\,b \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35)$$

$$h_0 = \frac{d}{2} + \left(\frac{1}{12} - \frac{\pi^2}{128}\right)\frac{b^2}{d} = \frac{d}{2} + 0.0062\,\frac{b^2}{d} \ldots\ldots\ldots\ldots (36)$$

and

$$P = \frac{w\,b\,d^2}{2} + 0.0062\,w\,b^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

In this case, as in the case of the V-shaped bottom, the second term in the equation for P cancels when P_n is subtracted from P_m; and the net static pressure is given by Equation (9).

The critical depth is the same for any shape of bottom and is given by Equation (28), in which d_c is the average depth at the cross-section.

EXPERIMENTAL EVALUATION OF FRICTION FACTOR

Experimental measurements which are sufficiently complete and reliable for the evaluation of the friction factor are difficult to obtain. Since the theory is premised upon the assumption of a constant increment of inflow, it is imperative that the weir or weirs at the top of the flume walls be level. Moreover, because of the turbulence in the flume, an accurate measurement of the position of the water surface at any section is not possible. In order to counteract the effect of the errors in the measurement of the position of the water surface, the draw-down should be relatively great, and depth measurements should be made at three or more stations along the flume. The effects on the pressure distribution, of entrained air and of the momentum of the incoming water (both of which are of considerable importance for dam spillways), are negligible for the smaller channels used in water and sewage plants. The momentum of inflow results in increased friction factors due to increased turbulence.

The best set of experimental measurements available to the writer was made upon one of the wash-water gutters at the Springwells Water Purification Plant, City of Detroit, Mich., in 1931.[4] This gutter is 2 ft wide with weirs 40.25 ft long. It is of cast iron with a semi-circular bottom and an invert slope of 0.25 in. to the foot. Measurements of the water surface and of the weir crest on both sides were made at intervals of 5 ft along the length of the flume. The maximum difference in elevation of the weirs at any two points was about 0.01 ft, which corresponds with a maximum variation in q of about 15% at the maximum discharge used.

Three runs were made at discharge rates of 10.91, 9.05, and 7.14 cu ft per sec, respectively. The water surface for the two larger discharge rates was above the semi-circular bottom, so that b was constant along the flume. The friction factor was evaluated by means of Equation (23) at 0.055 for both of these runs. Fig. 4, with Table 1, shows the relative position of the measured water surface and of the profile computed by means of Equation (25) and Fig. 2.

Fig. 5, with Table 1, shows the measured and computed water surface curves for the central concrete gullets of one of the filters at each of the two water purification plants at Cleveland, Ohio. The measurements at the Division Avenue Plant were made in 1920, and at the Baldwin Filtration Plant in 1928, both sets being under the direction of G. W. Hamlin, M. Am. Soc. C. E. The values of f (0.10 and 0.11) obtained by these measurements on two different gutters of similar size tend to substantiate the reliability of both sets of measurements. The inflow to main gutters is at intervals along the length and not continuous, as called for by the theory. Notwithstanding this fact, these measurements indicate that the theory may be used with success for gullets in which the points of inflow are not too widely separated.

Measurements of the water surface at two points were also made in lateral gutters for the same runs. These gutters have weir lengths of about 15 ft at both plants. The computations for f in the lateral gutter at Division Avenue resulted in a negative value, doubtless due to the fact that the weirs were about

---

[4] *Transactions*, Am. Soc. C. E.. Vol. 101 (1936), Fig. 9, p. 362.

0.03 ft lower at the down-stream than at the upper end. The computed value of f for the lateral gutter at the Baldwin Plant was approximately 0.02 for a discharge of 4.42 cu ft per sec. This value is probably unreliable because it is based upon only two measured depths along the channel and because the condition of the weirs was not observed.

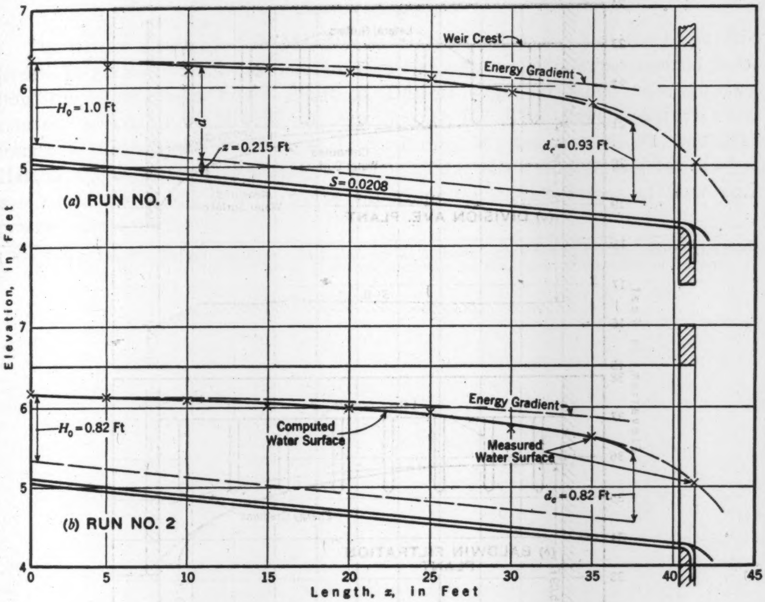


FIG. 4.—DETROIT (MICH.) EXPERIMENTS (SEE TABLE 1)

TABLE 1.—OPERATING DATA, TESTS ON LATERAL SPILLWAY CHANNELS

Figure No.	Test	Discharge, Q, in cubic feet per second	Increment of inflow, q, in cubic feet per second	Friction factor, f	Width, b, in feet
	Detroit, Michigan, Experiments:				
4(a)	Run No. 1	10.91	0.271	0.055	2.0*
4(b)	Run No. 2	9.05	0.225	0.055	2.0*
	Cleveland, Ohio, Experiments:				
5(a)	Division Avenue Plant	55.6	1.135	0.10	2.5
5(b)	Baldwin Filtration Plant	73.6	1.5	0.11	2.5
	Massachusetts Institute of Technology:				
6	Experimental Lateral Spillway Channel	1.237	0.0618	0.033	0.75

* Semi-circular bottom.

A number of experiments were run on a lateral spillway channel at the Massachusetts Institute of Technology under the direction of the writer.

This channel is of wood with rectangular cross-section, 9 in. wide, and with a level bottom. The inflow is from one side over a weir 20 ft long. Fig. 6 illustrates the results of one of the most reliable runs on this channel, made by

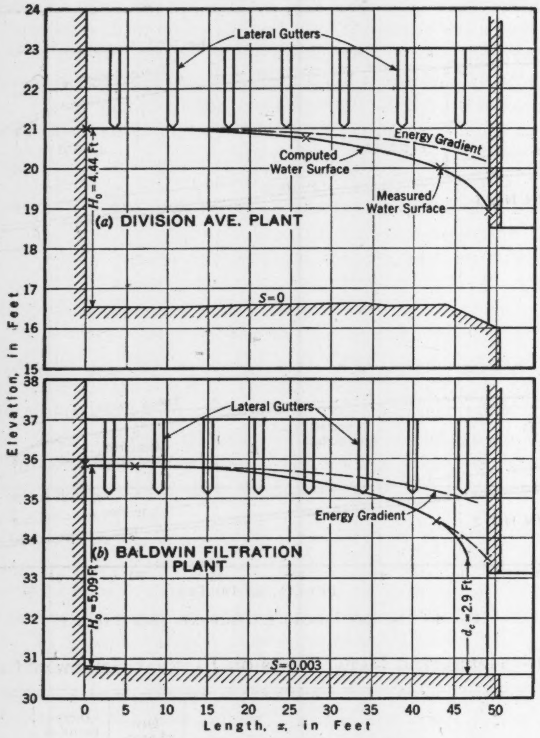


FIG. 5.—CLEVELAND (OHIO) EXPERIMENTS (SEE TABLE 1)

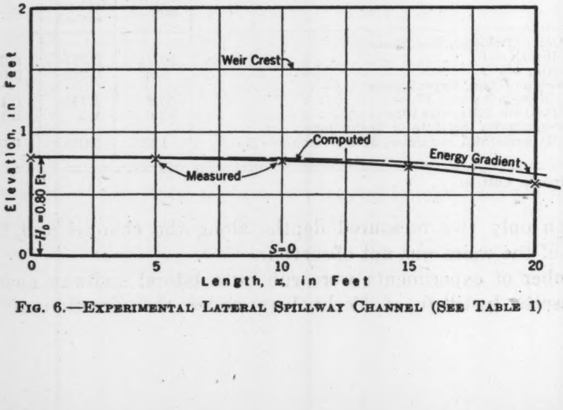


FIG. 6.—EXPERIMENTAL LATERAL SPILLWAY CHANNEL (SEE TABLE 1)

Mr. E. A. Kass.[5] The variation in level of the weir was only about 0.001 ft. The head on the weir was approximately 0.07 ft and the nappe fell freely. The computed value of *f* for this run was 0.033.

ACKNOWLEDGMENT

The writer is indebted to Mr. Hinds for the development of the basic differential equation which has been used in this paper. The experimental data from the Springwells Filter Plant in Detroit were furnished through the courtesy of Arthur B. Morrill, M. Am. Soc. C. E. The Cleveland data were furnished through the courtesy of J. W. Ellms, M. Am. Soc. C. E., and Mr. Hamlin. The writer also wishes to acknowledge the valuable assistance of a number of his students who made experimental investigations of flow and pressure characteristics.

---

[5] "Loss of Head in Lateral Spillway Channels," by E. A. Kass; presented to Massachusetts Institute of Technology in 1937, in partial fulfillment of the requirements for the degree of Bachelor of Science.

DISCUSSION

W. E. HOWLAND,[6] ASSOC. M. AM. SOC. C. E. (by letter).—The experimental data quoted in this paper clearly establish the validity of the basic differential equation used and the rational methods developed appear to the writer to be of great value in analyzing, completely, the behavior of lateral spillway channels. Because the integrated equations require adjustment by trial and error methods the writer has investigated the use of similar methods applied to the basic differential equation itself. Results indicate some value in this somewhat different method of approach.

Equation (17) may be rewritten as follows:

$$dy = \frac{1}{g}\left(\frac{f\,V^2\,dx}{8\,R} + 2\,V\,dV + \frac{dA}{A}\,V^2\right) \ldots\ldots\ldots\ldots (38)$$

since

$$q\,\frac{V^2}{Q} = \frac{dQ}{dx}\frac{V^2}{Q} = \frac{dQ}{dx}\frac{V^2}{A\,V} = \left(\frac{A\,dV + V\,dA}{dx}\right)\frac{V}{A} = \frac{V\,dV}{dx} + \frac{V^2}{dx}\frac{dA}{A} \ldots (39)$$

in which, in addition to the notation of the paper, A = area of cross-section of channel stream; otherwise symbols are those of the author. Equation (38) may be expressed as follows:

$$dy = 2\,d\left(\frac{V^2}{2\,g}\right) + 2\,\frac{V^2}{2\,g}\frac{dA}{A} + \frac{f}{4\,R}\frac{V^2}{2\,g}\,dx \ldots\ldots\ldots\ldots (40)$$

or, in words: The drop in the water surface between two sections a distance dx apart is twice the difference between the velocity heads of these sections, plus twice the product of the (average) velocity head in this length and the proportional change in area between the two sections, plus the allowance for friction head loss in this length between these sections.

An approximate method for calculating the difference in surface elevations along the channel might be to divide its entire length into a number of smaller lengths and then to apply Equation (40) to these separate lengths, treating the differentials that appear in the equation as actual assumed or calculated differences; thus:

$$\Delta y = 2\,\Delta\,\frac{V^2}{2\,g} + 2\,\frac{V_{av}^{\,2}}{2\,g}\frac{\Delta A}{A_{av}} + \frac{f}{4\,R}\frac{V_{av}^{\,2}}{2\,g}\,\Delta x \ldots\ldots\ldots\ldots (41)$$

in which V_{av} and A_{av} are average values of velocity and area. The burden of cutting and trying in applying Equation (41) progressively to a number of short sections of the channel, by the usual step method of calculation, has been found to be troublesome; nevertheless the equation has proved to be useful as will now be shown.

Calculations indicate that the first term to the right of the equation sign is of predominant importance in estimating the value of Δy (the vertical drop of water surface between sections). Thus, if the elevation of the water surface at the down-stream end of the channel is known, as well as the flows, the velocity head may be computed. Then the water-surface elevation at the upper end of the channel may be estimated approximately as two of these velocity heads higher than the down-stream water surface. This may be shown by a consideration of actual data in Table 2, obtained from the paper.

---

[6] Associate Prof., San. Eng., School of Civ. Eng., Purdue Univ., Lafayette, Ind.

TABLE 2.—OPERATING DATA, TESTS ON LATERAL SPILLWAY CHANNELS (Units Are in Feet)

Experiment	Depth of water in channel down stream, read from Figs. 4, 5, and 6	Distance from upper end	Values of H_0		Observed over-all difference of water-surface elevation in terms of velocity head at lower end
			Observed	Approximately calculated*	
Detroit, Mich.:					
Run No. 1	1.04	37.5	1.00	1.00	1.995
Run No. 2	0.91	37.5	0.82	0.80	2.06
Cleveland; Division Avenue	2.83	47.5	4.44	4.64	1.78
M.I.T.; Experimental Channel	0.58	20.0	0.80	0.83	1.76

* Two velocity heads added to down-stream water-surface elevation.

The same principle may be used in the reverse direction; that is, assuming the elevation at the upper end of the section, the elevation at the lower end may be estimated in this manner:

$$\Delta y = H_0 + S\,x - d = 2\,\frac{V^2}{2\,g} = 2\,\frac{q^2\,x^2}{b^2\,d^2}\,2\,g \qquad \text{.............}(42)$$

Knowing H_0, S, x, q, b and g, one may solve Equation (42) for d. Using this formula to estimate the depth at a distance of 37.5 ft down stream from the upper end, in the Detroit plant tests, one obtains, in Run No. 1, $d = 1.04$ ft and in Run No. 2, $d = 0.87$ ft.

In the other experiments Equation (42) results in no real roots owing to the slight errors in the theory evident from Table 2; but a slight readjustment of the values of H_0 will yield equations with real roots.

These approximate calculations—based upon the assumption that the drop in the water surface between the up-stream end of the channel and any other point is twice the velocity head at that point—will probably be satisfactory for most practical purposes. They also serve as a basis for a more exact method for the analysis of the channels of this study. The writer proposes the following procedure for this purpose:

Laboratory Study to Obtain f.—In a laboratory study to obtain f, from measurements of the elevation of the water surface in the channel at a number of points (such as are indicated by Professor Camp) determine depths, areas and velocity heads at these sections and average velocity heads and substitute these and other dependent calculable values in Equation (41) and then compute f therefrom for each separate length of channel. No "cuts" and "trys" are needed in this procedure.

Exact Analysis.—In an exact analysis of a design, compute the water-surface elevations and depths at a number of sections along the channel by means of the approximation already considered. When the depth of channel is given at the down-stream section, first compute the elevation at the up-stream section as two velocity heads higher. (If the elevation at this section is assumed this step would not be necessary.) Then establish approximate depths at a number of sections below by means of Equation (42), assuming x and solving for d by trial or, if it is preferred, assuming d and solving for x. The latter procedure is more simple because it involves a quadratic instead of a cubic and does not require a cut and try solution. In Equation (42) all other quantities would be known, of course. Use this approximate water surface to compute the second and third terms on the right side of Equation (41) for the several lengths into which the channel has been divided (that is, the lengths between sections at which depths have been computed). Add, algebraically, both sets of "minor" terms; that is, obtain $\sum 2 \frac{V_{av}^2}{2g} \frac{\Delta A}{A} + \sum \frac{f}{4R} \frac{V_{av}^2}{2g} \Delta x$ for the entire channel and add this to the original estimate of Δy $\left(\text{that is, to } 2 \frac{V^2}{2g}\right)$ to obtain the more exact difference between water surfaces at the extremities of the channel. Thus H_0 may be recomputed. If desired, all the intermediate elevations may then be recalculated from the following modification of Equation (41):

$$H_0 + s\,x - d - \sum_0^x 2 \frac{V_{av}^2}{2g} \frac{\Delta A}{A} - \sum_0^x \frac{f}{4R} \frac{V_{av}^2}{2g} \Delta x = \frac{2\,q^2\,x^2}{b^2\,d^2\,2\,g} \quad \ldots (43)$$

The entire process could be repeated to obtain still greater precision starting with the newly determined water depths from Equation (43); but, since it appears (from Table 2) that very close approximations to the water surface were obtained with the first crude or simple approximation there is good reason to suppose that the foregoing procedure would yield satisfactory precision without repetition.

It is interesting to consider why the simple approximation (which neglects the second and third terms in Equation (41)) yields results which agree so well with measurements and with the results of more exact and elaborate methods of analysis. Apparently the reason is that these terms are of small value and tend to compensate each other. As given in Equation (41), the second term on the right side is predominately negative and the third is always positive. The second term would disappear absolutely if the bottom of the channel were parallel to the water surface, a condition which is approximated with inclined channels. This term becomes of greater importance as the channel is made more level, in which case it may more than compensate for the effect of friction (see the last two tests in Table 2). From the data given the friction term, itself, appears to be of very small importance, amounting to about 8% of the first term, in the first experiment quoted.

This study calls attention to the very considerable loss of energy which occurs in these channels due, not to friction, but to the impact of the slow moving, and fast moving, water. It suggests that an increase in capacity might result from the use of devices to deflect the incoming water into the direction of the main stream before it enters the stream. The writer would like to see tried the simple device of placing vanes, curved only in one direction (see Fig. 7). Such vanes might extend from the crests of the weirs down to the anticipated elevation of the water surface in the channel. It appears to be possible, theoretically, to increase the capacity of a channel as much as 50% by such energy-saving devices. Although the use of such devices might not be economical in an original design, it might be helpful in increasing the capacity of old channels that have proved to be somewhat inadequate.

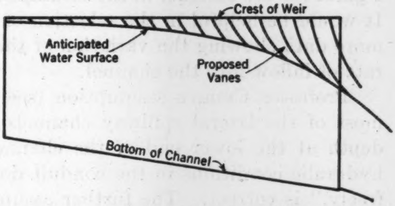


FIG. 7.—LONGITUDINAL CROSS-SECTION OF LATERAL SPILLWAY CHANNEL

To summarize: This study clearly establishes the validity of the basic differential equation employed. The writer suggests the use of the approximation—change in water-surface elevation in lateral spillway channels equals twice the difference in velocity heads between corresponding sections. This approximation may be used to advantage both for preliminary designs and also as the basis for exact analyses. Furthermore, the study emphasizes the great loss of energy that occurs from impact in these channels and suggests the possibility of improvements in design, to increase capacity.

LEWIS V. CARPENTER,[7] M. AM. SOC. C. E., JOHN K. VENNARD,[8] AND FENNER H. WHITLEY,[9] JUNIORS, AM. SOC. C. E. (by letter).—The problem treated in this paper has long been a vexing one for designers of certain types of sanitary engineering structures. The author is to be commended for the completeness of his solution. He has taken the well-known Hinds[2] analysis and adapted the equations so that they are available for the design of such appurtenances as wash-water gutters and trickling filter collection troughs.

The Weisbach-Darcy friction factor that Professor Camp uses in his mathematical analysis of flow is well known. As this friction factor is used in the equations of the paper, it is made to embody all of the various resistance phenomena that occur throughout the length of the channel. Resistance to flow in a lateral spillway channel comes from two fundamentally different sources: (1) That of skin friction at sides and bottom of the channel (which alone usually determines the size of f); and (2) that resulting from the interference of liquid entering the channel normal to its axis. Undoubtedly the latter effect is the predominant one in determining the size of the friction factor in lateral spillway channels. This fact is borne out by the trend in friction factors shown in Table 1 in which a channel having inflow over the entire length of only one side (M.I.T. tests) and having thus a minimum interference has the lowest friction factor (0.033). Where inflow is distributed over the entire length of the channel but enters from both sides (Detroit tests) a greater interference is produced and a greater friction factor (0.055) results. Where there is a concentrated inflow from lateral gutters (Cleveland tests) a maximum interference occurs and results in a still higher friction factor (0.10–0.11). Although the friction factor data in Table 1[9b] are meager, they indicate the effects of various types of inflow upon this factor, and thus provide a guide for the designer in the assumption of a friction factor for a given design. It would be helpful in the selection of values of f if the author could present more data showing the variation of the friction factor with a variation in the rate of inflow into the channel.

---

[7] Prof., San. Eng., Coll. of Eng., New York Univ., New York, N. Y.

[8] Asst. Prof., Fluid Mechanics, Coll. of Eng., New York Univ., New York, N. Y.

[9] Instr., San. Eng., Coll. of Eng., New York Univ., New York, N. Y.

Professor Camp's assumption (see heading "Critical Depth"), that "For most of the lateral spillway channels used in water and sewage plants, the depth at the lower end of the channel will be fixed at upper stage by the hydraulic conditions in the conduit down stream, or the flume will discharge freely," is correct. The further assumption that in the design of freely discharging flumes, a hydrostatic control section exists at a distance up stream from the end, equal to three or four times the critical depth, needs some qualifying statements.

Assume that in a rapid sand filter the lateral wash-water gutters are spaced 6 ft, center to center, and that they discharge freely. A vertical rise of 36 in. per min is used when washing the filter. The increment q will be 0.3 cu ft per sec per ft length of gutter, with one-half of the water coming in on each side. If the wash-water gutter is rectangular, the critical depth is given by Equation (28). In Table 3 the location of the hydrostatic control is calculated for lengths of 10, 20, and 40 ft and a breadth of gutter equal to 12, 15, 18, and 24 in., using the author's assumption. In the gutters at the Cleveland plant where the length is 49 ft, the author's assumption is fairly close because, at the hydrostatic control section, all of the water has entered the gutter.

TABLE 3.—EFFECT OF ASSUMING THAT THE HYDROSTATIC CONTROL SECTION IS UP STREAM FROM OUTLET AT A DISTANCE 3 TO 4 TIMES THE CRITICAL DEPTH

Length, l, in feet	Flow, Q, in cubic feet per second	Width, b, in inches	Depth, d_c, in feet	$3 \times d_c$, in feet	$4 \times d_c$, in feet	PERCENTAGE OF TOTAL Q NOT IN CHANNEL	
						Hydrostatic control up stream $3 \times d_c$	$4 \times d_c$
10	3.0	12	0.65	1.95	2.60	19.5	26.0
		15	0.56	1.68	2.24	16.8	22.4
		18	0.50	1.50	2.00	15.0	20.0
		24	0.41	1.23	1.64	12.3	16.4
20	6.0	12	1.04	3.12	4.16	15.6	20.8
		15	0.89	2.67	3.56	13.4	17.8
		18	0.79	2.37	3.16	11.9	15.8
		24	0.65	1.95	2.60	9.7	13.0
40	12.0	12	1.65	4.95	6.60	12.4	16.5
		15	1.42	4.26	5.68	10.7	14.2
		18	1.26	3.78	5.04	9.5	12.6
		24	1.04	3.12	4.16	7.8	10.4

However, if the more typical lateral wash-water gutter is considered, in which the water is coming in both sides in equal increments, for a 40-ft channel, 2 ft wide, between 7.8% and 10.4% of the wash water has not entered the gutter at the control. The two plants illustrated are among the larger plants in the United States. Many plants have lateral wash-water gutters 10 ft or less in length. If $l = 10$ ft and $b = 12$ in., with $q = 0.3$ cu ft per sec per foot of gutter, the quantity of water entering after the control is between 19.5 and 26.0 per cent. The greater the length for a given increment of flow, and the larger the value of b, the more nearly correct is the author's assumption. Naturally, the greater the rate of washing, the greater will be the value of d_c. Although the author's assumption is fundamentally unsound, it provides a means for calculating an artificial but apparently reliable starting point for the water-surface curve.

With the foregoing facts and a simplification of Equation (29), design computations may be simplified to a less general, but easier, method than that proposed by the author.

If

$$2\,S\,x_c\,\bar{d} = \frac{f\,x_c\,d_c^3}{12\,\bar{R}\,\bar{d}} \qquad (44)$$

Equation (29) becomes

$$H_0 = \sqrt{3}\,d_c \qquad (45)$$

and there appears to be no reason why this condition cannot be obtained for the usual channel, since physically it implies giving the channel slope enough to compensate for the assumed friction. From Equation (44) there results a unique and simple relation between S and f which permits the designer to observe the effect of the assumed f upon the slope of the channel. This relation is

$$\frac{S}{f} = \frac{d_c^3}{24}\,\frac{1}{\bar{R}\,\bar{d}^2} \qquad (46)$$

and may be placed in more practical form by the following relations

$$\bar{d} = \frac{H_0 + d_c}{2} \qquad (47a)$$

and

$$\bar{R} = \frac{\left(\dfrac{H_0 + d_c}{2}\right) b}{H_0 + d_c + b} \qquad (47b)$$

which are approximate, but permissible when the uncertainty in the selection of f is considered. Insertion of these values in Equation (46) results in

$$\frac{S}{f} = \frac{d_c^3}{3}\left[\frac{H_0 + d_c + b}{(H_0 + d_c)^3\,b}\right] \qquad (48)$$

which, with Equation (45), and the critical-depth formula, Equation (28), allows quick calculation of the principal dimensions of the spillway channel.

Equations (45), (48), and (28) may be used by the designer to give values for the size of lateral wash-water troughs which, while approximate, are as accurate as the value of "f" which must be assumed.

In the design of a system of wash-water troughs it is first necessary to assume a layout (as dictated by practice) and then investigate that particular layout. For any spacing of gutters there are an infinite number of breadths of gutter which can be used, but experience has shown that the wash-water gutters should not occupy more than one-third of the surface area. The applicability of Equations (45), (48), and (28) to design is given in the following illustrative problem.

The wash-water troughs are 20 ft long, spaced 6 ft on centers, and designed to carry a wash-water rate of 36 in. vertical rise per minute. In the illustrative computations a breadth of 12 in. was used, but Table 4 gives results for a breadth of 24 in. also. The inflow, q, per lineal foot of trough is given by $q = \frac{36}{60 \times 12} \times 6 = 0.3$ cu ft per sec per ft; the total flow Q is equal to $0.3 \times 20 = 6.0$ cu ft per sec; from Equation (28) $d_c = \sqrt[3]{\frac{(6.0)^2}{32.2 \times 1^2}} = 1.04$ ft; and, from Equation (45) $H_0 = \sqrt{3} \times 1.04 = 1.80$ ft. Substituting in Equation (48), $\frac{S}{f} = 0.063$. It is now necessary to assume a value of f. Many more experimental data are necessary before this value can be predicted accurately; but values of f of 0.03 and 0.10, from the author's data, will require respective slopes of 0.0019 and 0.0063.

TABLE 4.—COMPARISON OF APPROXIMATE DESIGN DATA FOR TWO, TWENTY-FOOT, WASH-WATER TROUGHS

Width of trough, b, in feet	Flow		Depth, d_c, in feet	Head, H_0, in feet	Ratio, $\frac{S}{f}$	Slope, S	
	q, in cubic feet per second per foot	Q, in cubic feet per second				For a friction factor, $f = 0.030$	For a friction factor, $f = 0.10$
1.0	0.30	6.0	1.04	1.80	0.063	0.0019	0.0063
2.0	0.30	6.0	0.653	1.13	0.0312	0.00094	0.00312

The foregoing computations are summarized in Table 4, which includes for comparison the results obtained from the same filter when a value of $b = 24$ in. is assumed. It is interesting to note that the value of S for a given value of b and f varies quite widely. A variation of f from 0.030 to 0.10 requires an increase of slope from 0.0019 to 0.0063 for a 12-in. width and from 0.00094 to 0.00312 for a 24-in. width. As would be expected from Equation (48), for all other items constant S varies directly with f; that is, the slope required for $f = 0.10$ for either a 12-in. or a 24-in. width is 3.33 times the slope required for $f = 0.03$. It is granted that these simplifications of the author's formula give only approximate results; but they are as accurate as the values of f. There is need for further research to determine the value of f for short channels, particularly on short wash-water lateral gutters and trickling filter collection troughs.

Presumably the experimental channel at Massachusetts Institute of Technology did not have free outfall according to the surface curve of Fig. 6.

F. KNAPP,[10] ESQ. (by letter).—This paper, treating a special chapter of applied hydraulics, should command the interest of all engineers interested in problems involving the flow of water in open channels. The author's treatment of the question is so complete that scarcely any improvement is possible. The same problem is being encountered in the design of tail-races for hydro-electric plants with laterals connecting the main channel with the turbine pits.

The author mentions a formula based upon the assumption that the draw-down of the water surface is equal to the velocity head. Rearranging Equation (17) and using the Manning flow formula instead of the Weisbach equation, the following is obtained:

$$dy = \frac{V^2}{g}\left(\frac{g}{K^2\,R^{\frac{4}{3}}}\,dx + \frac{dQ}{Q} + \frac{dV}{V}\right)\ldots\ldots\ldots\ldots\ldots(49)$$

in which $dQ = q\,dx$. The roughness factor K of Manning's formula is approximately equal to the reciprocal of Kutter's "n"; and R is the hydraulic radius of the spillway channel.

Equation (49) indicates the causes for the draw-down of the water level—namely, friction, increase of discharge and change of velocity. A comparison of this formula with the more general expression derived by H. Favre [11] shows that the derivation neglects the component of the velocity of the lateral discharge parallel to the axis of the flume, as well as the non-uniform velocity distribution of the flow in the channel. However, only in exceptional cases need these factors be taken into account and for this reason Equation (17) is perfectly general.

Equation (29) is only valid for the case with critical depth at the downstream end of the spillway channel. Neglecting, for the moment, the slope S of the channel and the friction, Equation (29) may be written:

$$H_0 = \sqrt{3\,d_c^2} = d_c\,\sqrt{3}\ldots\ldots\ldots\ldots\ldots\ldots(50a)$$

Note the comparison with the rearranged Equation (23):

$$H_0 = d\sqrt{2\left(\frac{d_c}{d}\right)^3 + 1}\ldots\ldots\ldots\ldots\ldots\ldots(50b)$$

Equation (50b) is valid in all cases, including the freely discharging flume.

The author calls attention to the difference between lateral spillway channels and side weirs. As shown by Mr. Favre,[12] Equation (17) is also valid for the case of a side weir provided the term $\frac{dQ}{Q}$ is disregarded. Therefore:

$$dy = \frac{V^2}{g}\left(\frac{g}{K^2\,R^{\frac{4}{3}}}\,dx + \frac{dV}{V}\right)\ldots\ldots\ldots\ldots\ldots(51)$$

This equation is exactly equal to the well-known expression for the computation of back-water curves. However, there is a fundamental difference between the two formulas. The condition of continuity in the case of the back-water curve is:

---

[10] São Paulo, Brazil.

[11] "Contribution a l'Étude des Eaux Courants," by H. Favre, Editor Rascher & Cie., Zurich, Leipzig and Stuttgart, 1933, Equation (14), p. 17.

[12] *Loc. cit.*, p. 60.

$$V_2 A_2 - V_1 A_1 = 0 \qquad (52a)$$

and in the case of the side weir:

$$V_2 A_2 - V_1 A_1 = \Delta Q \qquad (52b)$$

in which A is the cross-sectional area of the canal and ΔQ is the discharge over the side weir.

The model tests conducted by Mr. Favre[13] at the "Laboratoire des Recherches Hydrauliques Annexé à l'École Polytechnique Fédéral de Zurich" have shown that in the case of tail-race channels for hydro-electric power stations, it becomes necessary to apply Equation (17) neglecting the friction term. This procedure counterbalances, partly, the error due to the velocity component of the lateral flow, which is always present.

The smooth surfaces of the experimental profiles, given by the author, represent a most surprising fact. Observations made on lateral spillway channels for dams always indicate a very wavy profile, the theoretical surface of the water passing through the centers of these waves.

The author is to be commended for his most interesting, useful, and timely paper.

Carl Rohwer,[14] Assoc. M. Am. Soc. C. E. (by letter).—The applicability of the momentum principle to the general solution of the problems arising in the design of side channel spillways has been demonstrated by Mr. Hinds[2] and Mr. Favre.[3] The analysis presented in this paper shows the application of the principle to the special case of the design of wash-water gutters, with a consequent simplification in the procedure.

The experimental data presented by these investigators show that the calculated water-surface profiles are in reasonably close agreement with the observed profiles. This fact seems remarkable in view of the assumptions that must be made in order to simplify the theoretical analysis. For example, to compute the momentum at any section in the channel, it is necessary to know the average velocity of the water at the section. It is assumed to be equal to the discharge at the section divided by the area. Actually, the velocity in any section may vary through wide limits; some of the velocities may even be negative. It is evident, therefore, that the average velocity is greater than the mean computed in this manner. Since the average depth of the water in the channel is used in computing the velocity, entrained air which increases the indicated depth will tend to increase the difference between the average and the mean velocity still further. The total difference will depend on the depth of water flowing into the channel and on the turbulence produced.

The photographs presented by Messrs. Hinds and Favre show that the water flowing over the spillway crest creates a violent disturbance in the water, and may produce a definite transverse roll. The writer has observed spillway models in operation, and although the depth of water flowing over the weir into the channel did not exceed 6 in. in these model studies, the agitation of the water was pronounced. From the foregoing discussion, it is apparent that there are important differences between the assumed and observed conditions. There may be compensating factors, however, which tend to minimize the effect of the errors in these assumptions.

---

[13] "Contribution a l'Étude des Eaux Courants," by H. Favre, Editor Rascher & Cie., Zurich, Leipzig and Stuttgart, 1933, p. 51.

[14] Irrig. Engr., Div. of Irrig., SCS, U. S. Dept. of Agriculture, Colorado Experiment Station, Fort Collins, Colo.

In evaluating the friction factor "*f*" (see heading "Experimental Evaluation of Friction Factor") by substituting in Equation (23) the channel depths observed during tests made on several spillways, the author found values ranging from 0.033 to 0.11; and, in one case the value of "*f*" was negative. The wide range in the friction factors leads the writer to believe that part of the variation is due to the fact that the formula does not agree precisely with the experimental data and that, in effect, the friction factor becomes a coefficient that takes care of the difference between the observed and computed data as well as the friction in the channel.

The use of a friction factor derived in this manner in computing the depths for the profiles (Figs. 4, 5, and 6) by substituting in the formulas would tend to make the computed results agree more closely with the observed results than the accuracy of the formulas warrants. In designing a channel for a wash-water gutter, it would be necessary to assume a value of "*f*"; and consequently the computed profiles might not agree so closely with those observed in the actual structure. It would have been helpful if the author had included in this paper a table of values of "*f*" for use by any one having to design a spillway of this kind.

HAROLD ALLEN THOMAS, JR.,[15] ESQ. (by letter).—Professor Camp has successfully applied the momentum principle in the analysis of flow in lateral spillway channels of constant width and slope. His ingenious integration of the differential equation provides a general method of calculating the profile of the water surface in a wash-water gutter or gulley. The profiles obtained by his method coincide closely with the measured water surfaces over a large part of the length of the channel. It should be emphasized, however, that the hydraulic theory underlying his analysis does not apply to flow with sharply curving stream-lines and, as a consequence, the differential equation (Equation (17) or (18)) fails in the vicinity of the outfall, as he has shown. In the absence of actual measurements in this region the full success of his analysis is predicated upon the adequacy of empirical rules giving the position of such points as the critical depth in relation to the outfall.

The author's statement (see heading "Critical Depth") that "a close approximation to actual conditions will be obtained if a hydrostatic control section is assumed to exist at a distance up stream from the end equal to three or four times the critical depth" appears to be somewhat arbitrary. In working out a number of surface curves it has been found that it may make a considerable difference whether the "three" or the "four" in the above quotation is used. Moreover, there is no assurance that the effective control section will remain within these limits under all conditions of flow. The writer has experimental evidence that clearly indicates a somewhat wider variation in the position of the effective control section depending upon slope and discharge in the channel than that suggested by the author. Thus it would appear that, to attain the full degree of precision offered by the author's method, it is necessary to have more exact knowledge of the position of the effective control section. Indeed, it has been found that the uncertainty of the location of the effective control section frequently obscures the refinement of introducing the friction term into the formulas. Consequently, in the absence of more complete experimental information regarding the curve of the water surface near the outfall, the inclusion of the friction term, in many cases, is wasted effort.

---

[15] Instr., San. Eng., Graduate School of Engineering, Harvard Univ., Cambridge, Mass.

However, in the design of wash-water troughs there may be some question as to the necessity of a complete, meticulous analysis of the flow. From the standpoint of the designer, it usually suffices to know what depth of water to expect in the upper end of the channel under a particular set of conditions. It is seldom necessary in conventional design to compute the entire profile of the water surface. During the past two decades a great many wash-water gutters have been designed in accordance with equations giving the depth of water at the upper end of the channel. In spite of being only semi-rational and containing arbitrary coefficients, these formulas do possess the virtue of simplicity. It seems possible, given the differential equation developed by the author, that a simple formula might be derived for the depth of water at the upper end of the channel (H_0) in terms of the discharge, length, width, and slope, that would eliminate the necessity for successive approximations, as well as the element of trial and error, and yet provide sufficiently precise information for purposes of design.

The foregoing opinion was expressed to the writer by G. M. Fair, M. Am. Soc. C. E., in the hope that an investigation might reveal a suitable basis of simplification. Of course, such simplification, to be of any real value, would have to yield formulas capable of giving more reliable results than semi-empirical formulas used in the past. As a method of approach to the problem, Professor Fair suggested that the integrals in Equation (20) might be evaluated by replacing d by a second degree function of x. In checking various formulas developed according to this scheme with observed water surfaces in wash-water gutters, it was found that the friction term, involving f, made practically no contribution to the precision attained. Since the inclusion of friction forces complicated the formulas considerably, it was believed that a reasonable compromise between precision and simplicity resulted from the omission of the friction term entirely. Therefore, in the equations that follow, friction has been neglected.

The success of the analysis according to Professor Fair's scheme was found to depend upon the selection of the second degree function of x to be substituted for d in the remaining integral of Equation (20). The function finally chosen was a parabola that adheres rather closely to observed water surfaces. The arbitrary constants of the function were so chosen as to cause the parabola to coincide with the actual surface at $x = 0$ and $x = x_c$ and, further, to possess a slope of S when $x = 0$ as is required by Equation (18). This function of d in terms of x is as follows:

$$d = H_0 + S\,x + \frac{(d_c - H_0 - S\,x_c)\,x^2}{x_c^2} \qquad \text{.............(53)}$$

When this value is placed in Equation (20) and if $f = 0$, the following equation may be written:

$$\frac{x^2}{d} + \frac{g\,b^2\,d^2}{2\,q^2} = \frac{S\,g\,b^2}{q^2}\int\left[H_0 + S\,x + \frac{(d_c - H_0 - S\,x_c)}{x_c^2}\,x^2\right]dx + c \ldots \tag{54}$$

Since $\frac{g\,b^2}{q^2} = \frac{x_c^2}{d_c^3}$, and since $d = H_0$ when $x = 0$, the integrated equation may be written in the following form:

$$\frac{x^2}{d} + \frac{x_c^2}{2\,d_c^3}\,(d^2 - H_0^2) = \frac{S\,x_c^2}{d_c^3}\left[H_0\,x + \frac{S\,x^2}{2} + \frac{(d_c - H_0 - S\,x_c)}{3\,x_c^2}\,x^3\right] \ldots \tag{55}$$

Substituting $x = x_c$ and $d = d_c$ and solving for H_0, the final formula is

$$H_0 = \sqrt{2\,d_c^2 + \left(d_c - \frac{S\,x_c}{3}\right)^2} - \frac{2}{3}\,S\,x_c \ldots\ldots\ldots\ldots \tag{56a}$$

In comparing values of H_0 given by Equation (56*a*) with those obtained by measurement of the water surface over a wide variety of slopes and discharges, a rather close agreement was observed. In addition, it was found that the degree of precision attainable by this formula was not appreciably affected by taking x_c equal to the entire length of the channel (L), and d_c as calculated from the entire discharge by the formula $d_c = \sqrt[3]{\frac{Q^2}{g\,b^2}}$. Accordingly, Equation (56*a*) becomes

$$H_0 = \sqrt{2\,d_c^2 + \left(d_c - \frac{S\,L}{3}\right)^2} - \frac{2}{3}\,S\,L \ldots\ldots\ldots\ldots \tag{56b}$$

For $S = 0$, Equation (56*b*) reduces to

$$H_0 = 1.73\,d_c \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{57}$$

A number of rapid sand filter plants are designed or operated in such a manner that the wash-water does not discharge freely from the gutters to the central gulley. In this case the height of the water surface in the lower end of the channel exceeds that occurring with free discharge, and is determined by hydraulic conditions down stream. Therefore, if the hydraulic control lies beyond the gutter, the use of equations based on free discharge may involve considerable error. It is of interest to the designer to know what depth of water to expect in the upper end of the wash-water gutter for various depths in excess of the critical depth at the lower end of the channel. This depth of water (H_0), at the upper end of the channel, is given by the following formula, which was derived in the same manner as Equation (56*a*), for the condition $x = L$, $d = d_L$ (where d_L is the measured or expected depth of water at the lower end of the channel).

$$H_0 = \sqrt{\frac{2\,d_c^3}{d_L} + \left(d_L - \frac{S\,L}{3}\right)^2} - \frac{2}{3}\,S\,L \ldots\ldots\ldots\ldots \tag{58}$$

and for $S = 0$ this reduces to:

$$H_0 = \sqrt{\frac{2\,d_c^3 + d_L^3}{d_L}} \ldots\ldots\ldots\ldots\ldots\ldots \tag{59}$$

It is to be noted that Equation (56*b*) may be obtained from Equation (58) by making $d_L = d_c$.

For the convenience of the designer the relationship given by Equation (58) (including the special cases covered by Equations (56*b*), (57), and (59)) could be easily expressed in the form of a diagram.

The order of agreement obtained using Equations (56*b*) and (59) with the method of the author for the various sets of experimental measurements included in his investigation is indicated in Table 5. The comparison is based on the quantity $H_0 + L\,S$, since this represents the actual over-all head in the channels.

The average error in Table 5 is 1.5% and it may be noticed that the largest discrepancy, occurring in the Division Avenue Plant, is only 3.8 per cent. This should serve to demonstrate that the divergence between the two methods is quite small; in fact, it may actually fall within the limits of experimental error inherent in the measurement of water depths and discharge.

TABLE 5.—COMPARISON OF VALUES OF $H_0 + L\,S$ OBTAINED BY THE AUTHOR WITH THOSE CALCULATED USING EQUATIONS (56*b*) AND (59)

Test (1)	Discharge, Q, in cubic feet per second (2)	$H_0 + L\,S$ in feet given by the author (3)	$H_0 + L\,S$ in feet using Equations (56*b*) and (59) (4)	Percentage error‡ (5)
Detroit, Mich., Experiments—				
Run No. 1	10.91	1.84	1.82*	−1.1
Run No. 2	9.05	1.66	1.63*	−1.8
Cleveland, Ohio, Experiments Division Avenue Plant	55.6	4.44	4.25†	−3.8
Baldwin Filtration Plant	73.6	5.24	5.21*	−0.6
Massachusetts Institute of Technology Experimental Lateral Spillway Channel	1.237	0.80	0.794†	−0.7

* Computed by Equation (56*b*). † Computed by Equation (59). ‡ $\dfrac{\text{Column (4)} - \text{Column (3)}}{\text{Column (3)}} \times 100.$

In order to investigate further the precision offered by Equations (56*b*), (57), (58), and (59) under a wide range of channel slopes and discharges, a small lateral spillway channel was constructed and tested in the Gordon McKay Engineering Laboratory of Harvard University, at Cambridge, Mass. This channel, made of cypress, was 5 ft 11 in. long and 4 in. wide, with level top edges and bottom of adjustable slope. Experience has demonstrated that the results obtained from model channels of this size and type may be translated to pertain to structures of larger dimensions by application of the ordinary Froude law of hydraulic similitude. Investigations with this channel proved to be satisfactory in every respect.

A summary of the degree of accordance between Equations (56*b*) and (57) and experimental information acquired from measurement of the water surface at the upper end of the channel under various conditions of slope and flow with free discharge at the outfall is presented in Table 6. In Table 6, as before, the comparison is based upon the quantity $H_0 + L\,S$ for convenience in comparing tests of different slope.

In Table 6 the average absolute error is only 2.2%, and the largest error is 5.5%, occurring with a slope of 0.0313 and a relatively small discharge. Although these errors are small, it should be pointed out that, with increasing slopes, the velocities in the channel become larger and the friction term becomes of increasing importance. Hence, it is not expected that Equations (56*a*), (57), (58), and (59), in which this term is neglected, will apply indefinitely. Using these equations with slopes greater than 0.055, errors in excess of 15% have been found. Such slopes, however, involve an uneconomical channel design; it will be shown that Equations (56*a*), (57), (58), and (59) are valid for the range of slopes occurring in economically designed channels.

TABLE 6.—COMPARISON OF VALUES OF $H_0 + L\,S$ OBTAINED (1) BY MODEL EXPERIMENTS UNDER CONDITIONS OF FREE DISCHARGE AND (2) BY CALCULATION USING EQUATIONS (56*b*) AND (57)

Slope (1)	Discharge, Q, in cubic feet per second (2)	$H_0 + L\,S$ in inches measured in channel (3)	$H_0 + L\,S$ in inches, calculated using Equations (56*b*) and (57) (4)	Percentage error* (5)
0.0	0.055	2.04	1.97	−3.4
0.0	0.0703	2.39	2.32	−2.9
0.0	0.0876	2.70	2.68	−0.7
0.0208	0.0546	2.21	2.21	0.0
0.0208	0.0652	2.39	2.44	+2.1
0.0208	0.1033	3.19	3.23	+1.3
0.0313	0.072	2.92	2.76	−5.5
0.0313	0.0962	3.34	3.32	−0.6
0.0313	0.1097	3.63	3.50	−3.6

$* \, \frac{\text{Column (4)} - \text{Column (3)}}{\text{Column (3)}} \times 100.$

TABLE 7.—COMPARISON OF VALUES OF $H_0 + L\,S$ OBTAINED (1) BY MODEL EXPERIMENTS IN WHICH d_L EXCEEDED d_c, AND (2) BY CALCULATION USING EQUATIONS (58) AND (59)

Slope (1)	Discharge, Q, in cubic feet per second (2)	d_L in inches, measured (3)	d_c in inches, calculated (4)	$H_0 + L\,S$ in inches, measured (5)	$H_0 + L\,S$ in inches, calculated using Equations (58) and (59) (6)	Percentage error* (7)
0.0	0.0243	1.34	0.658	1.44	1.51	+4.9
0.0	0.0464	2.12	1.03	2.34	2.38	+1.7
0.0	0.0628	2.61	1.24	2.92	2.95	+1.0
0.0313	0.0123	2.45	1.36	3.14	2.99	−4.8
0.0313	0.1000	3.18	1.69	3.69	3.74	+1.4

$* \, \frac{\text{Column (6)} - \text{Column (5)}}{\text{Column (5)}} \times 100.$

In Table 7 is presented a summary of experiments in which the hydraulic control section did not occur in the channel. In these tests the depth of the water in the lower end of the gutter exceeded the critical depth. To show the extent to which the water was backed up in excess of the critical depth, the quantities d_c and d_L have been included in Table 7. The depths $H_0 + L\,S$ measured at the upper end of the channel are compared with corresponding depths in this region calculated with Equations (58) and (59).

The average absolute error in Table 7 is 2.8%, and the maximum error does not exceed 5 per cent. This is in agreement with the order of precision attained in the other experiments. Therefore it can be concluded that Equations (56*a*), (56*b*), (57), (58), and (59) give sufficiently precise information for practical purposes. Indeed, under many conditions, the error involved in using these equations is of a considerably smaller magnitude than the errors that result in practice from incorrectly estimated discharges and improperly leveled weir crests.

Economics of Wash-water Gutter Design.—An interesting application of Equation (56*b*) is the determination of the slope of the most economical wash-water gutter, assuming that Q, b, and L are specified by the rate of back-wash and filter-bed dimensions. The cost of a wash-water gutter may be taken as the cost of the sides and bottom and, in addition, certain fixed costs that do not depend upon the dimensions of the channel. This may be stated mathematically as follows:

$$C = 2\,K_s\,L\left[\frac{H_0 + (H_0 + L\,S)}{2}\right] + K_b\,b\,L + K_f \ldots\ldots\ldots (60)$$

in which C = cost of channel; K_s = cost of sides per unit area; K_b = cost of bottom per unit area; and K_f = fixed costs (assumed to be independent of dimensions of the channel). Substituting the value of H_0 from Equation (56*b*), Equation (60) becomes

$$C = K_s\,L\left[2\sqrt{2\,d_c^2 + \left(d_c - \frac{S\,L}{3}\right)^2} - \frac{S\,L}{3}\right] + K_b\,b\,L + K_f \ldots (61)$$

Since Q, b (hence d_c), L, K_s, K_b, and K_f are constants, the cost of the channel depends only upon the value of S. Differentiating Equation (61) with respect to S and setting the derivative equal to zero:

$$2\left(d_c - \frac{S'\,L}{3}\right) = -\sqrt{2\,d_c^2 + \left(d_c - \frac{S'\,L}{3}\right)^2} \ldots\ldots\ldots (62)$$

Solving for S', the slope corresponding to minimum cost of channel,

$$S' = 0.55\,\frac{d_c}{L} = \frac{0.55}{L}\sqrt[3]{\frac{Q^2}{g\,b^2}} \ldots\ldots\ldots (63)$$

From Equation (63) it may be seen that the optimum slope increases with discharge and decreases with the length and width of the channel. For average values of Q, L, and b used in the United States the most economical slope, according to Equation (63), usually falls between 0.0125 and 0.0175.

Conclusions.—It has been shown that the author's elaborate treatment of flow in lateral spillway channels, for practical purposes, may be simplified into elementary formulas or diagrams that show a remarkable agreement with experimental measurements. These formulas lend themselves to an analysis of the economics of wash-water gutter design in which a rule is developed for the determination of the optimum slope of the channel.

THOMAS R. CAMP,[16] M. AM. SOC. C. E. (by letter).—The discussion of this paper has been gratifying particularly because of its quality. Professor Howland presents a helpful rule for approximating the drop in water surface quickly. Professor Carpenter and Messrs. Vennard and Whitley suggest a unique procedure for simplifying design, whereby both the slope and friction terms in the equation are eliminated by equating them. Mr. Knapp rearranges the differential equation in a manner to indicate each factor which contributes to the draw-down of the water surface, and he also points out the applicability of the differential equation to side weirs. Mr. Rohwer rightfully questions the utility of the Hinds-Favre equation for large spillways where violent rollers and entrained air are of paramount importance. Mr. Thomas presents a useful development of the economics of wash-water gutter design. The writer wishes to express his appreciation for these helpful contributions.

Professor Howland's Equation (41) for computing the drop in water surface between two sections a distance Δx apart is of general applicability, regardless of the shape of the flume. To this extent it is more useful than the writer's integrated form (Equation (23)), which applies only to rectangular channels of constant width and slope. The use of Equation (41) for the more simple channels, however, is more laborious than the writer's method. Even for the evaluation of f from measurements of the water surface, Equation (23) is probably more direct and more simple than Equation (41). It possesses the added advantage of producing, immediately, the average value of f for the entire stretch of channel being investigated.

In a first approximation of the value of H_0 in channels where the depth d at a section at or near the lower end is known, Professor Howland neglects the second and third terms of Equation (41). The total drop is thus approximately equal to twice the velocity head at the section where d is known. This approximation is not a safe procedure, although in the examples cited it does produce results that are not widely in error. The last two terms in Equation (41) do tend to compensate for one another, but the assumption that they are equal may be too greatly in error for reliability. The second term depends upon the slope of the bottom, is negative for a level bottom, approaches zero as the bottom slope becomes nearly parallel with the water surface, and may become positive for bottoms with steep slopes. For a level bottom and a hydrostatic control section at the lower end, the sum of the first two terms is $1.46 \frac{V^2}{2g}$, as may be seen from Equation (45). The drop in the water surface due to the friction term bears little relation to the second term of Equation (41). In the writer's examples the friction drop is from 6,5% to 16% of the total drop.

Professor Howland's proposal to use vanes in order to utilize a part of the momentum of the inflowing water and thus increase the capacity of the channel is worthy of investigation for dam spillways. Some knowledge of the value of the friction losses in such large channels is necessary, however, before one can be sure that the vanes are worth their cost. Undoubtedly, the friction losses are large, but it is doubtful if they are so large as to permit of an increase in capacity of 50% by their reduction.

---

[16] Associate Prof. of San. Eng., Mass. Inst. Tech., Cambridge, Mass.

Messrs. Carpenter, Vennard, and Whitley note that the values of f as found by the writer for lateral spillway channels are greater than the usual values of 0.02 or 0.025 for uniform flow in pipes. The increase in the value of f is undoubtedly due to the greater turbulence resulting from the impact of the incoming water. The table of values of f as presented by the writer is indeed meager, but all data which were available were used. It is hoped that other data will become available in the future for the evaluation of f. It will be very helpful if values of f can be obtained from some full-scale dam spillways where air entrainment and rollers are pronounced. Until some such efforts are made, the validity of the Hinds equation for the design of such structures is in doubt. It will be noted from the data presented in Table 1 that the value of f increases as the increment of inflow increases. One may surmise that with dam spillway channels, where the inflow increment is much greater, the value of f may be very much larger than the values in Table 1.

The writer's statement (see heading "Critical Depth") that "For the design of freely discharging flumes, a close approximation to actual conditions will be obtained if a hydrostatic control section is assumed to exist at a distance up stream from the end equal to three or four times the critical depth" has been criticized by Messrs. Carpenter, Vennard, and Whitley and again by Mr. Thomas. The proper distance to take, of course, will be influenced by the shape of the outlet end of the flume, the discharge, the amount of submergence or degree of aeration of the discharging stream, and probably other factors. If a hydrostatic control section is assumed to exist at the end, the design will be on the safe side, but it may be uneconomical. The arbitrary figure of "three or four" times the critical depth should be tempered by the judgment of the designer. In the case of the Detroit experiments (Fig. 4), the distance was 3.8 and 4.3 times the critical depth for Run No. 1 and Run No. 2, respectively. In the case of the Baldwin Filtration Plant experiment (Fig. 5(b)), the distance was 1.25 times the critical depth. For the Division Avenue Plant experiment (Fig. 5(a)) the critical depth was substantially at the end of the channel; but this channel can scarcely be said to be discharging freely. The experimental channel at the Massachusetts Institute of Technology was provided with means below the outlet end for controlling the depth of flow, and for the run presented in Table 1 and Fig. 6 the outlet end was submerged.

In applying the arbitrary rule of three or four times the critical depth, the value of d_c should not be computed from the total channel discharge, but rather from the inflow up stream from the assumed position of the hydrostatic control section. As has been noted by Messrs. Carpenter, Vennard, and Whitley, a portion of the wash water will enter the channel below the control. Since this water is discharged at lower stage, it does not influence the shape of the water-surface curve above the control.

If a hydrostatic control is assumed to exist at the outlet end, the computed value of H_0 will be too great. If a hydrostatic control is assumed at the proper distance up stream but the total discharge is used for computing d_c and H_0, the value of H_0 will be too great again. Moreover, since q has thus been made too great, the water-surface curve will be distorted. If the hydrostatic control is assumed at the proper distance up stream and computations are based upon the inflow above this section, a correct value of H_0 will result, and the computed shape of the water surface curve will coincide with the actual almost down to the assumed position of the control. It will be noted from Figs. 4 and 5, however, that the actual depth at the assumed position of the control is likely to be greater than d_c. This is due to the prevalence of vertical accelerations in this region which result in pressures less than hydrostatic.

The approximate values of $\bar{d}$ and $\bar{R}$ as given in Equations (47*a*) and (47*b*) are based upon a straight water-surface profile. Since the profile is more nearly parabolic in shape, better approximations for $\bar{d}$ and $\bar{R}$ may be obtained from the following equations:

$$\bar{d} = \frac{2\,H_0 + d_c}{3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (64a)$$

and

$$\bar{R} = \frac{\left(\dfrac{2\,H_0 + d_c}{3}\right) b}{2\left(\dfrac{2\,H_0 + d_c}{3}\right) + b} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (64b)$$

If design data are computed for the two 20-ft wash-water troughs of Table 4 on the basis of these relations and Equation (46), the ratio $\frac{S}{f}$ becomes 0.052 for the trough 1 ft wide and 0.025 for the 2-ft trough.

The scheme proposed by Messrs. Carpenter, Vennard, and Whitley for simplifying the design of wash-water gutters by equating the slope term to the friction term, as shown by Equation (44), raises the question of the desirability of fixing the slope for the sole purpose of simplifying computations. If this is the aim, a simpler procedure is to make the bottom level. With a level bottom, H_0 may be approximated very quickly by computing the drop in the water surface from Equation (45) and increasing it by about 10% to allow for friction. There are advantages in a level bottom from both a construction and an operating viewpoint. However, neither a level bottom nor a bottom having a slope as obtained by Equation (44) is consistent with best economy of materials. Applying Mr. Thomas' Equation (63), one obtains an economical slope of 0.0286 for the 1-ft trough of Table 4 and 0.018 for the 2-ft trough. These slopes are much greater than those required by Equation (44).

Mr. Knapp points out that Equation (17) neglects the component of the velocity of the lateral inflow parallel to the axis of the flume, which component was considered in the Favre equation. In the horizontal plane no such component exists for the small channels dealt with by sanitary engineers. For such channels the inflow usually approaches at right angles to the axis of the flume. Such may not be the case, however, for dam spillways and for tail-race channels of hydro-electric power stations. Both Mr. Knapp and Mr. Rohwer suggest that an additional error is introduced by the use of the $\frac{Q}{A}$-velocity instead of the actual velocity, which varies widely over the cross-section. This appears to be the case since the actual momentum in an axial direction is somewhat greater than that computed by means of the mean velocity at the cross-section. This error is thrown into the value of f and tends to make it larger than it would be if the correction were made. Evidently, the error is small since the values of f obtained are not unreasonably large.

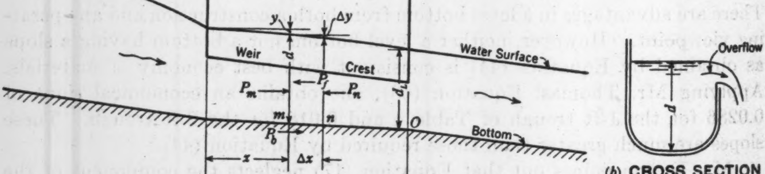
Both Mr. Knapp and Mr. Rohwer appear surprised at the apparent smoothness of the profiles of the water surface as shown in the illustrations of the paper. Although it is true that the water surface in these small channels was much smoother and had much less entrained air than is the case with dam spillway channels, the surface is always rough and transverse rollers are always present. Each point plotted on the curves to represent a measured point on the surface of the water is the average of at least two readings.

Mr. Thomas' suggested simplification of the writer's equations by the introduction of an empirical function for the depth seems quite as complicated as the writer's more rational development. The use of the empirical function, moreover, obscures the friction term in the equations. Since the friction term varies with the increment of inflow, large errors may result by the indiscriminate use of the empirical function.

It may be of interest to demonstrate how the differential equation (Equation (17)) may be applied to the analysis of the flow conditions for side weirs. The case is illustrated diagrammatically in Fig. 8. The development of Equation (17) follows in the same manner as previously presented except that for this case the lateral flow is outward and the increment (a decrement in this case) is not constant over the length of the side weir. Equation (17) becomes

$$\frac{dy}{dx} - \frac{f\,V^2}{8\,g\,R} = \frac{1}{g}\left(V\,\frac{dV}{dx} + \frac{V^2\,dQ}{Q\,dx}\right)\ldots\ldots\ldots\ldots\ldots(65)$$

in which $\frac{dQ}{dx} = q$. It is impractical because of the increased number of variables and the difficulty of integration to obtain an integral in terms of the depth for this equation. A "trial-and-error" solution may be obtained, however, in terms of the water-surface profile, in a manner similar to that suggested by Professor Howland for the lateral spillway channel.



(a) LONGITUDINAL SECTION SHOWING SIDE WEIR

(b) CROSS SECTION FACING UPSTREAM

FIG. 8

Equation (65) may be written

$$dy = \frac{f}{4\,R}\,\frac{V^2}{2\,g}\,dx + \frac{V}{g}\,dV + \frac{1}{g}\,\frac{Q}{A^2}\,dQ \ldots\ldots\ldots\ldots\ldots (66)$$

This equation may be integrated approximately over the short distance Δx as follows:

$$\Delta y = \frac{f}{4\,R_{av}}\left(\frac{V^2}{2\,g}\right)_{av}\Delta x + \frac{V_n^2 - V_m^2}{2\,g} + \frac{Q_n^2 - Q_m^2}{2\,g\,(A^2)_{av}} \ldots\ldots\ldots (67a)$$

or

$$\Delta y = \frac{f}{4\,R_{av}}\left(\frac{V^2}{2\,g}\right)_{av}\Delta x + \frac{V_n^2 - V_m^2}{2\,g} + \frac{\Delta Q(Q_n + Q_m)}{2\,g\,(A^2)_{av}} \ldots\ldots (67b)$$

The design may be started at the down-stream end (point o) where d_L, A, and Q are fixed to suit the requirements of the problem. For an assumed value of Δx and a corresponding value of ΔQ to suit the conditions of the problem, a trial position of the weir crest may be set for the down-stream end, and an estimate made of the value of the first Δy. After suitable adjustments are made, the process may be continued for succeeding values of Δx up stream until a length of weir is obtained which will deliver the required overflow.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2070

THEORY OF LIMIT DESIGN

By J. A. Van den Broek,[1] M. Am. Soc. C. E.

With Discussion by Messrs. John H. Meursinge, I. K. Silverman, Edward Godfrey, Basil Sourochnikoff, E. Mirabelli, C. M. Goodrich, George Winter, Francis E. Simpson, Joseph A. Wise, Alfred Freudenthal, Hans H. Bleich, Alfred S. Niles, A. Floris, L. H. Nishkian, F. G. Eric Peterson, E. S. Fabian, A. A. Eremin, L. H. Donnell, and J. A. Van den Broek.

Synopsis

This paper shows that the capacity load which a redundant structure can carry is not limited to the load that stresses one member to the elastic limit. In fact, the capacity load of a structure is reached only after as many members of a structure, corresponding in number to the structure's redundants, have all reached their elastic or buckling limit strength. As a load is gradually applied to a structure, its redundant members, in general, are successively stressed, one after another, until they reach their elastic or buckling limit strength. The remaining members may be designated as the primary system which still behaves elastically, and which can carry a still greater load, safely, until one of its members is likewise stressed to the elastic or buckling limit strength. This final load condition is defined as the "capacity load" upon which the design of the structure is based. Thus, if a structure is to have a factor of safety of two, the specified loading is doubled and the designer would find the "limit design" that would permit the redundant members in the structure to reach their elastic limit, or buckling strength. If so loaded some slight permanent set and some residual stresses might result, but the deformations of the structure would still be of the order of magnitude of elastic deformations. The structure could carry half the limit load safely, without any member being stressed to the elastic limit.

Introduction

Traditionally all theory pertaining to the strength of materials and to structural design is based on two major assumptions: (1) That the material is elastic; and (2) that the principle of superposition applies. To be sure, excep-

---

Note.—Published in February, 1939, *Proceedings*.

[1] Prof. of Eng. Mechanics, Univ. of Michigan, Ann Arbor, Mich.

tional problems, to which the principle of superposition does not apply, are treated by successive approximations, but even then the theory of elasticity is followed. In the common problem of designing rivet connections another assumption is introduced; namely, that the stresses are distributed equally over all rivets. Very little is said about that assumption other than that it represents established engineering practice. Recently, considerable attention has been paid to various theories of failure. However, the emphasis is placed upon the theory of the failure of materials rather than upon the theory of the failure of structures. The emphasis always has been, and continues to be, directed toward the analysis of stresses.

The man engaged in studies by photo-elasticity sees nothing but strains and talks about nothing but stress. If the theory of columns reveals anything, it is the critical, limit, load of the column, but designers are so much the creatures of habit that they must divide this load by area in order to get back again to talking about stresses. When a structural engineer adopts what is known as "good structural steel," he writes his specifications mostly with reference to stresses, and he expresses his theory in terms of stresses. Practically, however, no bridge or steel frame ever breaks; failure or collapse is the result of excessive deformation. To be sure, in a heap of wreckage it may be possible to find a ruptured steel bar; but there is not one chance in a million that this ruptured bar was the cause of the failure. Structural engineers wax enthusiastic about compound stresses, maximum stresses, working stresses, and critical stresses; in fact, they talk so much about stresses that it would seem that, if they only knew enough about them, they would not need to inquire into anything else. In this connection it is of interest to refer to an experiment[2] conducted in 1912 in which bars were broken as the result of excessive elongations. In every respect, the break resembled a tensile fracture with no stresses acting in the plane of fracture at the time of rupture. If any stresses were acting on this plane they were compression stresses as a result of friction in the packing, created as the bar was squeezed through (see Fig. 1).

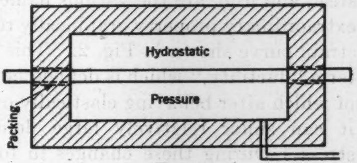


FIG. 1

This emphasis on stresses may have been acceptable in the past when civil engineering structures were statically determinate. In such structures each bar, each member, and each cross-section has a definite task to perform. There being no other member or restraint present to relieve conditions, the passing of the elastic limit stress means that excessive deformations of the order of magnitude of ductile deformations (say, from 5% to 10% of the length of the member) will take place without a corresponding increase in the carrying capacity of the structure; and that therefore the usefulness of the structure is brought to an end. Even in that case the excessive deformation is the primary, and the accompanying stress the secondary, consideration. So long as there is a direct relation between the two no practical harm results. However, as long as the engineer continues to talk about stresses when he means deformations, he violates sound engineering philosophy. In the consideration of compression members the logic is essentially the same as that applied to tension members. When a compression member is loaded to its capacity buckling load, although the deformations may continue for a time to be within the elastic range, excessive deformations take place, marking the limit capacity of the structure of which the column is a part.

---

[2] "Breaking Tests under Hydrostatic Pressure and Conditions of Rupture," by P. W. Bridgman, *Philosophical Magazine*, July, 1912.

In the study of statically indeterminate, redundant, structures it is proposed to continue to rely on the criterion that always was the primary one, the criterion of permissible deformations; the derived, or secondary, consideration (namely, the stresses involved) is relegated to its relative position of importance; it is treated as a matter of interest, as a matter that might well help to clarify questions, but not as a matter of primary importance.

A theory is of value in direct proportion to the extent to which it covers the pertinent factors that affect the problem to which it is applied. Any important factor not included within the scope of a theory limits that theory. The theory of elasticity does not give even a hint as to why a perfectly elastic material—a material that is elastic only—is unfit for use as a structural material. The application of the theory of elasticity is thus seriously circumscribed by rules of engineering practice and by stringent specifications. While thus restricted it is open to criticism as a sound and all-inclusive philosophy of structural design.[3]

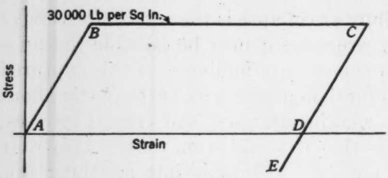


FIG. 2

Structural steel, mild steel, machine steel, open-hearth steel, low carbon steel, and iron, are the various names for the same substance. It possesses an extraordinary property graphically represented by Line *BC* of the partial stress-strain curve shown in Fig. 2. This property is designated by the engineering term "ductility," which is defined herein as the property of a material by virtue of which after behaving elastically over a considerable range of stress or strain, it can suffer relatively large deformation without appreciable change in stress. During these changes in form within the ductile range, the stresses remain essentially constant. It is important to realize that, after passing the elastic limit, the stresses are neither appreciably increased or decreased until very considerable deformation has occurred.

Although the curve of Fig. 2 may appear idealized, this is not necessarily the case to any great degree. To be sure, stress-strain curves frequently are recorded with a bulge instead of the sharp angle shown in Fig. 2 at Point *B*; or, they appear with a mild transition curve between the two straight lines meeting at Point *B*. It must be noted that the things tested are specimens and not filaments. For example, an ordinary steel bar suffers a differential stress between material in the outside and the center of the bar due to the cooling of the core (on the cooling bed) after the outside of the bar is already black and hardened. Furthermore, the rate of running the testing machine affects the curve. In other words, variations from stress-strain curves of mild steel, as shown in Fig. 2, may frequently be assigned to the test specimen rather than to the material. In several hundred tests reported in 1918[4] the writer found the horizontal part of the curve very pronounced and coincident with the proportional limit. Metallographically, an extraordinary thing happens when the elastic limit is exceeded and the ductile range begins, because it is unusual as judged from the behavior of other material. When the elastic limit is exceeded the crystal grains fail. However, in this connection failure is to be sharply distinguished from rupture. A cleavage is formed within the grain and the one part slips over the other. All this time, however, the two parts of the crystal grain cling to each other tenaciously as if connected by a very viscous glue. When a specimen of structural steel is tested in tension and strained beyond the elastic limit, to the extent indicated by Point *C*, Fig. 2, and subsequently examined, it is likely that the crystal grains in a part of the specimen will be found still intact, whereas in the other part they will be shown to have suffered permanent deformation.

---

[3] "Elastic Energy Theory," by J. A. Van den Broek, John Wiley & Son, 1931, p. 231.

In the training of engineers, the emphasis on the theory of elasticity leads to a respect for the elastic limit which sometimes borders on fear. To be sure, to exceed the elastic limit in a structure may mean disaster, but inherently it means no such thing. Only when several redundants, functioning to the same end, have each reached the elastic limit, or buckling strength, is failure imminent. Every inch of every wire in every strand of each cable of the Brooklyn Bridge in Brooklyn, N. Y., was strained cold, much beyond its elastic limit, before it was assembled and built into the bridge. This structure has stood up for more than 60 yr and is daily loaded well beyond what it was originally expected to carry. Cold-drawing, cold-rolling, and cold-stamping are processes involving the passing of the elastic limit of the material, sometimes resorted to in order to obtain a desirable shape, but more often done because of their beneficial effects on the properties of the material.

The theory of limit design is the instrument by means of which the capacity strength of a structure is computed. Its philosophy is predicated on the fact that a structure will fail only when as many members, or restraints, as the structure has redundants plus one, are loaded to their capacity strength. From this limit strength, the safe strength of the structure is derived by the introduction of an over-load factor or factor of safety.

It is proposed to present the theory by means of illustrative examples: Four examples in redundant beams referred to in order as Cases 1, 2, 3, and 4, and one example in redundant steel towers.

THEORY

In the theory of limit design the emphasis is shifted from the concept "stress" to that of "permissible deformation." In this connection three dicta of primary importance are proposed:

(1) A statically indeterminate, redundant, structure is one in which two or more members, reactions or restraints function to the same end;

---

[4] "The Effect of Cold Working on the Elastic Properties of Steel," Carnegie Scholarship *Memoirs*, Iron and Steel Inst., Vol. IX, 1918; see, also, *Engineering*, July, 1918.

(2) Sharp distinction is made between deformations of the order of magnitude of elastic deformations and deformations of the order of magnitude of ductile, or of buckling, deformations; and,

(3) When in a well designed, and especially a properly detailed, n-fold redundant structure, n redundants are stressed to their elastic limit, or up to their critical buckling load, the deformations involved are of the order of magnitude of elastic deformations until an $(n + 1)$th member has reached its elastic limit or its critical buckling capacity.

Case 1.—To find the maximum uniformly distributed load, applied in one direction and sense, that a beam built in at both ends can carry with a factor of safety of 2.—The elastic and ductile properties of the material of which the beam is made are assumed to be as represented graphically by Fig. 2. The beam is 6 ft long, fully restrained at both ends, and of rectangular cross-section, 6 in. × 2 in., as shown in Fig. 3.

The section modulus of this beam is $\frac{b\,h^2}{6} = \frac{2 \times 6^2}{6} = 12$. The maximum resisting moment that the beam can develop is:

$$M = \frac{s\,I}{c} \quad \text{..............................} \quad (1)$$

or, $M = 30\,000 \times 12$ in-lb $= 30\,000$ ft-lb.

The elastic behavior of this beam is represented by Fig. 3. From the theory of static equilibrium, the increment of bending moment between the end and middle of the beam is $\frac{w\,L^2}{8}$. The theory of elasticity is necessary, however, to tell that the ratio between M_1 and M_2 is as 1 : 2, or, in other words, that the bending moment at each end is $\frac{w\,L^2}{12}$, whereas that in the middle is $\frac{w\,L^2}{24}$.

The elastic curve presents essentially two cantilever beams, _A-B_ and _C-D_, and one simple beam, _B-C_.

To express the foregoing argument in physical terms, the beam fully restrained at both ends presents a case of two cantilever beams and one simple beam working together to the same end—that of carrying a uniformly distributed load w over the entire length of the beam. The combined task of the simple beam and the two cantilever beams is obvious. The theory of elasticity, however, is necessary to determine just how this task is distributed in simple beam action and cantilever beam action.

If no part of the beam is to be stressed beyond the elastic limit, the capacity load w that the beam can carry is $w = \frac{30\,000 \times 12}{6 \times 6} = 10\,000$ lb per ft; and the safe load, with a factor of safety equal to 2, is 5 000 lb per ft.

It is of interest to inquire into what would happen if this beam were subjected to a load greater than 10 000 lb per ft. As soon as this load is exceeded, the elastic limit in the outer fiber in a section at the wall is also exceeded. The steel at this point, which is ductile, flows somewhat in a manner indicated by Line _B-C_ in Fig. 2. The bending moment undergoes a change as indicated by the change from the dash line to the solid line in Fig. 4. The points of inflec-

tion, B and C, shift to Points B' and C' respectively (Fig. 4), and this process continues until the bending moment in the center has reached its critical value (equal to that at the wall) namely, 30 000 ft-lb, and the corresponding load on the beam has reached its critical value, w' lb per ft (Fig. 4). If loading beyond w' lb per ft is attempted, it appears that ductile deformation at both center and end of the beam continues unhindered. The deformations assume proportions of the order of magnitude of ductile deformations, and the end of the beam's usefulness is reached.

It has been stated that the theory of elasticity was necessary in order to determine the true ratio between M_1 and M_2 within the elastic range. In this instance, having determined by the foregoing argument that the capacity moments at the center and the end of the beam must be equal before disastrous results will take place, one may proceed with the arguments of static equilibrium to obtain the capacity carrying load w': $w'\dfrac{L}{2}\times\dfrac{L}{4} = 2\,M_1 = 60\,000$ ft-lb; and, $w' = 13\,333$ lb per ft.

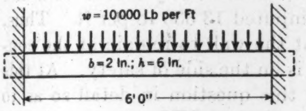

w = 10 000 Lb per Ft
b = 2 In.; h = 6 In.
6'0"
1.27 Ft
A
B
0.054 In.
C
D
ELASTIC CURVE
FREE BODY SKETCH
$\frac{wL^2}{12}$
15 000 Ft-Lb
30 000 Ft-Lb
MOMENT DIAGRAM
$\frac{wL^2}{24}$
$\frac{wL}{2}$
$\frac{wL^2}{12}$

FIG. 3.—BUILT-IN BEAM LOADED TO JUST WITHIN THE ELASTIC LIMIT

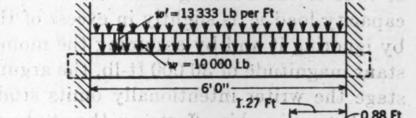

w' = 13 333 Lb per Ft
w = 10 000 Lb
6'0"
1.27 Ft
0.88 Ft
A
B'
B
0.144 In.
C
C'
D
0.004 Radians = 0° 13' 45"
ELASTIC CURVE
13 333 Lb per Ft
40 000 Lb
30 000 Ft-Lb
30 000 Ft-Lb
FREE BODY DIAGRAM
30 000 Ft-Lb
30 000 Ft-Lb
MOMENT DIAGRAM
$M_1 = 30\,000$
M_1

FIG. 4.—BUILT-IN BEAM LOADED WITH ITS CAPACITY LOAD

The application of the factor of safety again is contrary to the usual practice. Instead of applying it first in order to determine working stresses, it is applied last in order to determine safe load. Thus, if the capacity load is 13 333 lb per ft, and the factor of safety is 2, the safe load is 6 667 lb per ft. This change in the order of applying the factor of safety is not the trivial matter it may seem. By applying the factor of safety immediately, the designer finds the working stress in the middle of the elastic range and is helpless in extricating himself from the difficulty. By reserving the application of the factor of safety to the last and basing his analysis upon the elastic limit stress, he is in a position to study the effects of a slight ductile flow. For working loads, the ultimate safe load will fall back well within the elastic range. This means that ductile flow will most probably never occur in the life of the structure. It would occur only in case of a 50% over-load, and even then, if a slight permanent deformation is permissible, such permanent deformation would still be of the order of magnitude of elastic deformation up to a 100% over-load.

It may be argued that when a built-in beam is loaded to full ductile capacity, the magnitude of the bending moment at the walls would be somewhat in excess of that of the bending moment at the center. This merely means that the capacity load w' is slightly in excess of the computed 13 333 lb per ft. Thus, by ignoring it and by assuming the moment at the wall to remain at the constant magnitude of 30 000 ft-lb, the argument is on the side of safety. At this stage the writer intentionally omits studying this question in detail so as to avoid spending his efforts on the study of secondary considerations and thus diverting attention from the essential arguments.

When all the fibers in one cross-section of a beam are stressed to the elastic limit, the stress-distribution diagrams over the cross-section would appear as two rectangles, and not as two triangles. In that case the resisting moment is expressed by the formula:

$$M = 2\, s_1\, \bar{y}\, A \qquad \ldots\ldots\ldots\ldots (2)$$

in which s_1 is the elastic limit stress; A is the area on one side of the neutral axis; and $\bar{y}$ is the distance to the centroid of this area. Thus, the value of the limiting resisting moment becomes greater than that given by the usual formula, Equation (1). When all fibers along one section of the beam have reached the elastic limit this value would be 50% greater in a beam with a rectangular cross-section and 6.67% greater in a C.B. 12 in. by 12 in. by 65 lb beam.[5]

Case 2.—To find the maximum uniformly distributed load, applied an indefinite number of times in one direction and in one sense.—When a ductile steel is strained beyond the elastic limit (*B-C*, Fig. 2) and then the straining effect removed, the return path of the stress-strain curve (*C-D*, Fig. 2) is essentially parallel to the initial path *A-B*; that is, the material again behaves according to Hooke's law. If care is not taken at this point, one can easily become so lost in details that essentials are overlooked. As to whether or not Curve *C-D*, Fig. 2, is parallel to Line *A-B* depends primarily on the time that is allowed to elapse between the moment the over-strain is completed and the moment the straining effect is removed.[4] Furthermore, when the stress is completely released, and then reversed, the material continues to behave elastically as indicated by Line *D-E*, Fig. 2. To assume Line *C-E* in Fig. 2 to be a straight line is not exactly true, but by assuming conditions that would prevail ultimately if sufficient time elapses, conclusions are reached which, in certain cases, argue against the adoption of the theory of limit design (see Case 3). These conclusions, however, are on the safe side.

---

[5] See a dissertation, by E. O. Scott; presented to the University of Michigan in 1939, in partial fulfillment of the degree of Doctor of Philosophy.

If the load of 13 333 lb per ft, in Case 1, Fig. 3, is removed, the beam behaves elastically. The stresses return toward zero and in certain points may pass through zero and reverse. The elastic behavior of a beam under a negative load of 13 333 lb per ft is represented by Fig. 5. The final condition of the beam under a positive load of 13 333 lb per ft is represented by Fig. 4. When the free-body sketch of Fig. 5 is superimposed on the free-body sketch of Fig. 4, the result is a free-body sketch of the beam with the load removed (see Fig. 6). When the bending moment of Fig. 5 is superimposed on the bending moment of Fig. 4, the result is as demonstrated in Fig. 6, which shows the residual bending moment left in the beam after it had been loaded to full-capacity ductile load and after this load had been removed.

The conclusions to be drawn from the foregoing appear interesting and important. What would happen if a structure is loaded beyond the elastic limit? This question is introduced not because it is actually proposed ever, consciously, to permit it, but to find a better engineering philosophy than the one based on the consideration of working stresses. It must be kept in mind that after the capacity load is found the safe load is derived from it, and stipulated to be a safe margin below the capacity load. It is seen that a capacity ductile load induces deformations of the order of magnitude of elastic deformations. A removal of the capacity load leaves the structure with a slight permanent set, and, what is more important, with a residual stress condition set up in it. If the full capacity load is again applied in the same direction and sense, the behavior is completely elastic. For example, under completely elastic behavior the bending moment of a beam, loaded as shown in Fig. 7, develops bending moments of 20 000 ft-lb at the center and 40 000 ft-lb at the wall. When this loading and these moments are superimposed upon the residual loads and moments, represented by Fig. 6, the resultant loads and moments are those shown in Fig. 4, this time, however, obtained elastically. This means that for an application of one type of limit loading, up to ductile capacity, the

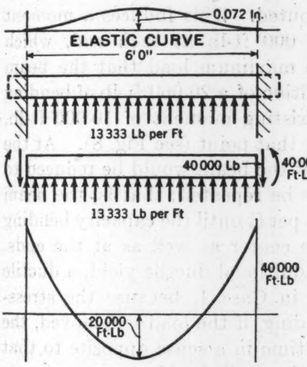


Fig. 5.—Restrained Beam Loaded Elastically in a Negative Sense

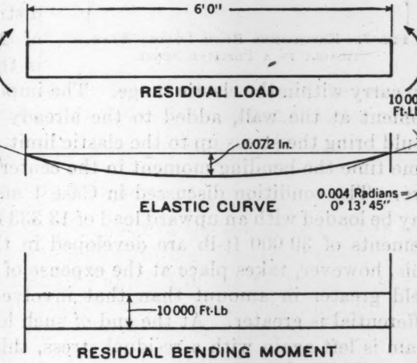


Fig. 6.—Built-in Beam After Removal of Limit Load

first application and removal of the load induce in the structure residual stresses of such nature and magnitude that subsequent application of loads does not call forth additional ductile deformation. After the first application of the loads the behavior of the structure thenceforth again becomes completely elastic.

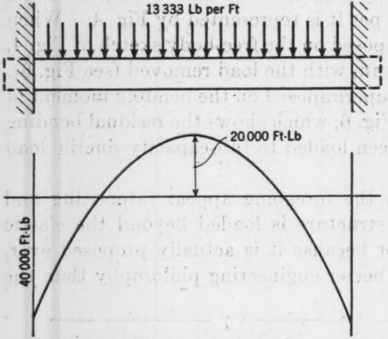


FIG. 7.—RESTRAINED BEAM LOADED ELASTICALLY IN A POSITIVE SENSE

Case 3.—To find the maximum uniformly distributed load applied an indefinite number of times in one direction, but in alternating sense (complete reversal of load), which the beam can carry with a factor of safety of 2.—Beginning with the residual stress condition of Fig. 6: A load of 6 667 lb per ft is applied in the upward direction, uniformly distributed. This induces a moment of 20 000 ft-lb at each end, which is the maximum load that the beam can carry within the elastic range. The imposition of a 20 000 ft-lb of bending moment at the wall, added to the already existing moment of 10 000 ft-lb, would bring the stress up to the elastic limit at that point (see Fig. 8). At the same time the bending moment in the center of the beam would be reduced to zero. The condition discussed in Case 1 may be repeated; that is, the beam may be loaded with an upward load of 13 333 lb per ft until the capacity bending moments of 30 000 ft-lb are developed in the center as well as at the ends. This, however, takes place at the expense of additional ductile yield, a ductile yield greater in amount than that involved in Case 1, because the stress-differential is greater. At the end of such loading, if the load is removed, the beam is left again with a residual stress, this time in a sense opposite to that shown in Fig. 6. If, next, a downward load is applied, this process repeats itself. Each time, however, additional ductile strain is involved. Each strain may be of the order of magnitude of elastic strains. A relatively few repetitions, however, will make the addition of these strains objectionable and dangerous. This phenomenon is not to be likened to what is commonly understood by fatigue. This is not a question of thousands or millions of repetitions of load but rather a question of, perhaps, a dozen repetitions or so. What happens may more aptly be likened to a wire nail held in a vise and bent back and forth, each time assuming a permanent set until, in comparatively short time, it breaks.

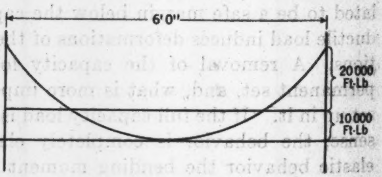


FIG. 8

If the foregoing arguments are applied to a reversal of load, smaller than the capacity ductile load but greater than the most severe load within the elastic range (say, a reversal of a load of 11 000 lb per ft), then it develops that each application of the load in opposite sense leaves residual stresses of a sense favorable to a load such as that last applied, but unfavorable to a load of a sense opposite to that last applied. The permanent sets would be smaller in magnitude, but would nevertheless be accumulative. There seems no other alternative, then, but to conclude that, if a complete reversal of stress is possible, the elastic limit is again the final criterion of strength. The saving feature of this argument lies in the circumstance that, in bridges and in steel frames, partial reversal of stress is comparatively rare, and complete reversal of stress is so rare as to be almost non-existent. (To avoid disastrous results under a complete reversal of loading the sum of the positive and negative loadings must not exceed an amount equal to twice the maximum elastic loading. Thus the beam in this example will sustain an indefinite application of an 11 000 lb per ft positive load and a 9 000 lb per ft negative load, or a 12 000 lb per ft positive and an 8 000 lb per ft negative loading.)

This phenomenon of residual stresses and accumulation of ductile deformations is of primary importance in the theory of limit design. Accumulation of ductile deformation may occur under moving loads even when no reversal of loads takes place.

Case 4.—To find the maximum (limit) value of two concentrated loads, P_5, spaced 10 ft apart, that can be moved an indefinite number of times along a beam of two spans, each 10 ft long, and of section modulus, $\frac{I}{c}$, equal to 30.—It is proposed to discuss this case in four steps:

(*a*) Find the single limit load P_1 applied in the center of the left span, and the residual moments and reactions resulting from that load;

(*b*) Find the double limit loads P_2 applied in the center of each span simultaneously, and the residual moments and reactions resulting from them;

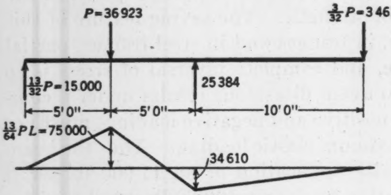
(*c*) Find the limit loads P_3 that may be applied alternately: Singly in the center of one span, or simultaneously in pairs, at the center of both spans; and,

(*d*) Find the limit loads P_4 that may be moved along the beam an indefinite number of times.

Step (*a*).—Fig. 9(*a*) shows the capacity load, concentrated in the center of one span, that the beam can carry without exceeding the elastic limit; Fig. 9(*b*) shows the limit load P_1 concentrated in the center of one span; and, Fig. 9(*c*) shows a negative loading of magnitude equal to P_1 when the beam behaves altogether elastically, the loads and reactions being in direct proportion to those of Fig. 9(*a*). As previously stated, if the limit load P_1 (Fig. 9(*b*)) is removed, even if the maximum concentrated loading within the elastic range is 36 923, the effect is the same as if a negative elastic loading (Fig. 9(*c*)) is superimposed upon the loading shown in Fig. 9(*b*). The material then behaves elastically as represented by Line *C D E* (Fig. 2). Fig. 9(*d*) represents the residual reactions and moments when the loading in Fig. 9(*c*) is superimposed upon the loading in Fig. 9(*b*); that is, when Load P_1 of Fig. 9(*b*) is removed.

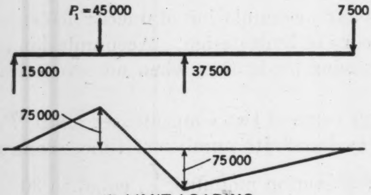
As a check of the loading and moment shown in Fig. 9(*b*), reverse the loading represented in Fig. 9(*c*) and superimpose it upon the beam in Fig. 9(*d*). This is a case similar to the one discussed in detail under Case 2.

Step (*b*).—The arguments for Step (*b*) on the basis of Fig. 10 are similar to those previously advanced in connection with the representations of Step (*a*) on basis of Fig. 9. The capacity limit loads P_1 and P_2, in each instance, are 45 000 lb. After they were once applied they could each be applied an indefinite number of times and the behavior of the structure would be perfectly elastic with each subsequent application of the loading.



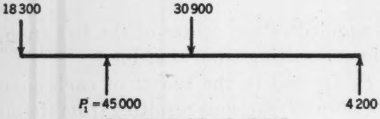
P=36 923
$\frac{3}{32}P$=3 461
$\frac{13}{32}P$=15 000
25 384
5'0"
5'0"
10'0"
$\frac{13}{64}PL$=75 000
34 610

(*a*) ELASTIC LOADING



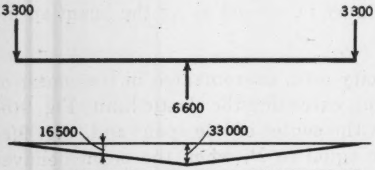
P_1=45 000
7 500
15 000
37 500
75 000
75 000

(*b*) LIMIT LOADING



18 300
30 900
P_1=45 000
4 200

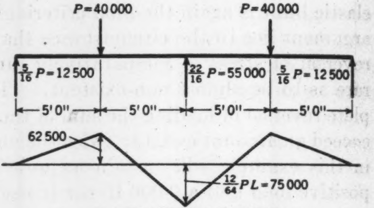
(*c*) NEGATIVE OF LOADING SHOWN IN (*b*)



3 300
3 300
6 600
16 500
33 000

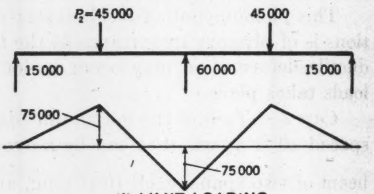
(*d*) RESIDUAL REACTIONS AND MOMENTS

FIG. 9



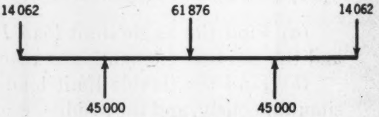
P=40 000
P=40 000
$\frac{5}{16}P$=12 500
$\frac{22}{16}P$=55 000
$\frac{5}{16}P$=12 500
5'0"
5'0"
5'0"
5'0"
62 500
$\frac{12}{64}PL$=75 000

(*a*) ELASTIC LOADING



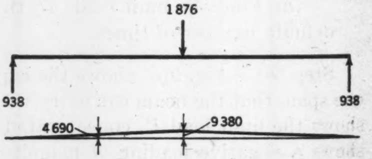
P_2=45 000
45 000
15 000
60 000
15 000
75 000
75 000

(*b*) LIMIT LOADING



14 062
61 876
14 062
45 000
45 000

(*c*) NEGATIVE OF LOADING SHOWN IN (*b*)



1 876
938
938
4 690
9 380

(*d*) RESIDUAL REACTIONS AND MOMENTS

FIG. 10

Step (*c*).—However, if the loading P_2 represented by Fig. 10(*b*) is applied after the loading P_1 (Fig. 9(*b*)) had been previously applied and removed, it

would meet with a residual loading condition (Fig. 9(*d*)) which is responsible for an initial bending moment over the middle support of 33 000 ft-lb. Thus the elastic limit over the middle support is reached sooner than it is in the case represented by Fig. 10(*b*), in which the loading presumably started from a zero base line. This would therefore result in ductile flow of somewhat greater intensity than that noted in the case given in Fig. 10(*b*). If subsequently the P_2-loading is removed and the P_1-loading is again applied, a similar situation results. As the P_1-loading is applied it meets with an initial positive bending moment of 4 690 ft-lb at the middle of the left span. This in turn requires more ductile flow before the full limit load of P_1 (Fig. 9(*b*)) can be carried. Each successive application of a P_2-loading, after a P_1-loading has first been applied and removed, is seen to require additional ductile flow which, in turn, would spell disaster.

The value of P_3, the limit load which can be applied singly at the center of one span, in pairs at the center of both spans, and applied alternately an indefinite number of times, may be computed as follows:

It will be recalled that, so far as repetitions of limiting loads are concerned, in accordance with the theory of limit design (Case 2), the structure is left with a residual load in a pre-stressed state, as the result of over-strain. This residual loading may be beneficial, if not too large, and if not of the wrong sign. In the case of repetitions of the same load (Case 2) the residual stresses are always the maximum and of the correct sign, thus making it possible for subsequent limit loading to take place entirely within the elastic range. When there is complete reversal of limit loading (Case 3) the residual loading is of the wrong sign, thus making the theory of limit design inapplicable for practical purposes. In the case of alternate loading (the loading shown in Fig. 9(*b*), alternating with the loading shown in Fig. 10(*b*)) the residual loading may well be too large. This does not mean, however, that a smaller residual loading could not be beneficial.

The largest possible values for the residual loading can be computed so that, with a design based upon that value, the beam will be stressed alternately to full capacity—first stressed to full capacity at the center of the span, and then stressed to full capacity over the middle support, without, however, resulting in further ductile flow, or over-strain. From the study of Fig. 9(*a*) the maximum moment in the center of the span is known to be $\frac{13}{64} P L$. Fig. 10(*a*) shows that the maximum moment over the middle support is $\frac{12}{64} P L$. The ideal residual bending moment would be the dash line in Fig. 11, so defined that the ordinate, measured from the dash line as a base to the point marking the maximum moment in the center of the span, shall equal the ordinate measured to the point marking the maximum moment over the middle support. If the distance to the vertex of the residual bending moment diagram is represented by x,

$$\frac{13}{64} P_3 L - \frac{x}{2} = \frac{12}{64} P_3 L + x \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

and, $x = \frac{1}{96} P_3 L$. The limiting bending moment is 75 000 ft-lb; and, therefore, $\frac{12}{64} P_3 L + \frac{1}{96} P_3 L = \frac{19}{96} P_3 L = 75\,000$ ft-lb; or, $P_3 = 37\,895$ lb. The answer is given in five significant figures, not because the writer is of the opinion such accuracy is justified in this instance, but because it may be of interest, if any one should be so inclined, to substitute the value of 37 895 lb for the 36 923 lb in Fig. 9 and compute the residual reactions and moments. Then repeat the argument of Step (*c*) with the value 37 895 lb substituted for 45 000 lb. It will be found that the alternate loading with this value of P_3 remains exactly within the elastic range.

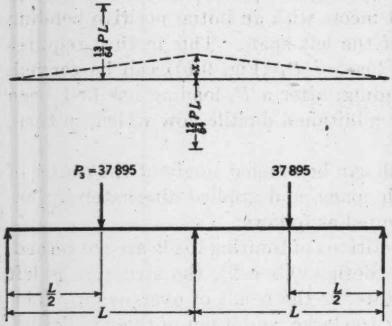


FIG. 11

In the case of moving loads, a similar procedure may be followed.[6] Suppose a single load, P_4, rolls along the beam. The maximum negative bending moment will be at the middle support. It will occur when Load P is at the distance 0.42265 L from the middle support and when it is of the magnitude 0.096225 $P L$. The maximum positive moment in both spans of the beam is expressed by the equation:

$$M = (2 L^3 x + L^2 x^2 - 4 L x^3 + x^4) \frac{P}{4 L^3} \ldots\ldots\ldots\ldots (4)$$

x being measured positive to the left from the middle support. The value of the maximum negative moment as well as the graph giving the maximum positive moment is represented in Fig. 12.

If it is assumed that the magnitude of P_4 is very small to begin with and that it becomes progressively larger with each passage along the beam, the elastic limit in positive bending will be reached and a residual stress built up favorable to the application of loads of the same magnitude subsequently applied. If the load P_4 is quite large the resulting over-strain will cause the residual loading, although favorable to the positive moments, to become unfavorable to the negative moment over the support. This is explained in Case 4, Step (*c*). The most favorable residual bending moment is the one represented by the dash line in Fig. 12. It may be found graphically by drawing the dash line so that the maximum positive ordinate shall equal the maximum negative ordinate. For the purpose of this paper the solution has been found analytically instead of graphically. It is found that the maximum moment equals 0.1744 $P_4 L$. Equating this moment to 75 000 ft-lb and solving for P_4, as was explained in Case 4, Step (*c*): $P_4 = 43\,000$ lb. It is to be noted that the maximum positive ordinate, measured from the dash line, does not occur in the center of the span. The physical meaning is as follows: When a load of 43 000 lb passes over the beam for the first time, it causes over-strain and builds up residual reactions and moments. These residual reactions and moments are of a magnitude such that, during subsequent passages of the load P_4 (43 000 lb), the beam behaves entirely elastically. The elastic limit stress will be reached at both the point of maximum positive bending and that of maximum negative bending; but this elastic limit stress is not exceeded. The safe load, on the basis of a safety factor of two, would thus be one-half of 43 000 lb, or 21 500 lb.

---

[6] "Über die Bemessung Statisch unbestimmter Stahltragwerke unter Berücksichtigung des Elastisch-Plastischen Verhaltens des Baustoffes," by Hans Bleich, *Bauingenieur*, 1932. Heft 19/20, p. 261.

Step (*d*).—Suppose that a number of loads, each of magnitude P_5, and 10 ft apart, roll along the beam. As the loads first come on, only one span will be loaded with a single load. The curve for maximum positive moment (Fig. 13), therefore, will be identical with the curve shown in Fig. 12. The maximum negative moment, however, will occur when the load P_5 is in the center of each span. This maximum negative moment will be $0.1875\ P_5\ L$.

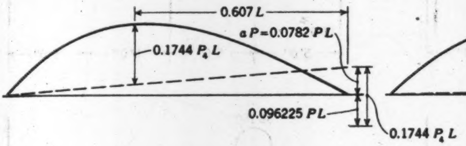


FIG. 12

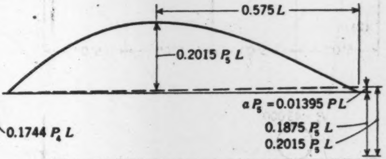


FIG. 13

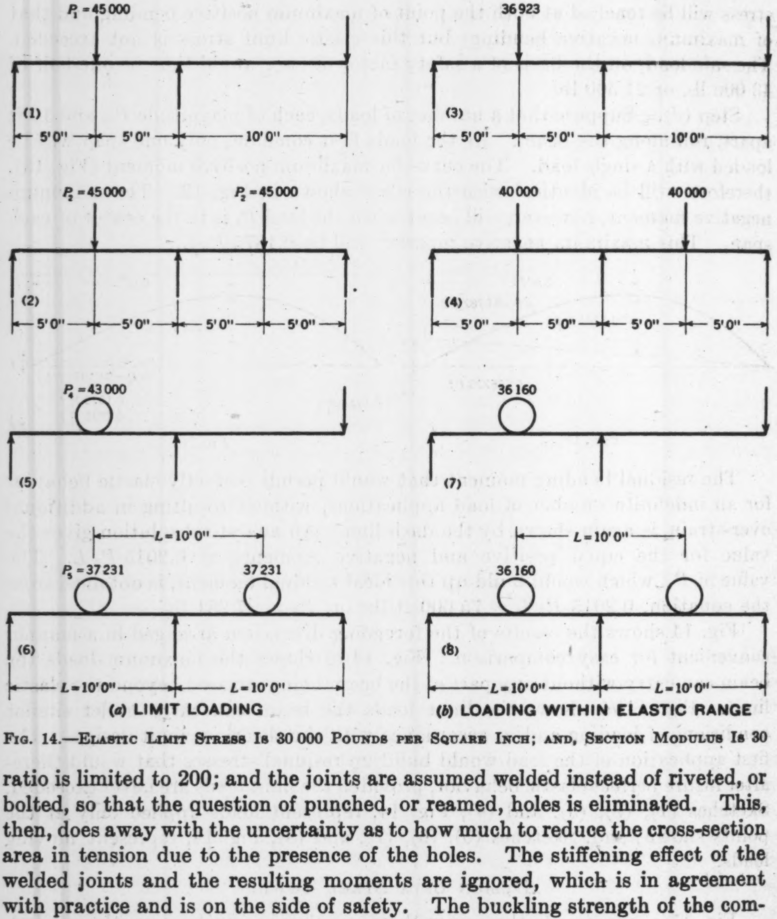
The residual bending moment that would permit perfectly elastic behavior for an indefinite number of load applications, without resulting in additional over-strain, is again shown by the dash line. An analytical solution gives the value for the equal positive and negative moments as $0.2015\ P_5\ L$. The value of P_5, which would build up this ideal residual moment, is obtained from the equation, $0.2015\ P_5\ L = 75\ 000$ ft-lb; or, $P_5 = 37\ 231$ lb.

Fig. 14 shows the results of the foregoing discussion arranged in a manner convenient for easy comparison. Fig. 14(*b*) shows the maximum loads the beam can carry without any part of the beam being stressed beyond the elastic limit. Fig. 14(*a*) shows the limit loads the beam can carry under similar conditions of loading and in accordance with the theory of limit design. The first application of the load would build up residual stresses that would thereafter insure perfect elastic behavior, provided the limit loads are never exceeded. Sketches (1), (2), (3), and (4), Fig. 14, represent loads applied only at the points shown; and, Sketches (5), (6), (7), and (8), Fig. 14, represent moving loads.

DESIGN OF A STEEL TOWER

Fig. 15(*a*) represents the upper three panels of a steel tower, the general, over-all, dimensions of which suggest a transmission tower. A transverse concentrated capacity load of 60 000 lb is assumed to be applied at Point *A* at the top of the tower. This load is purposely assumed rather large (about four times the value of a similar load on a representative transmission tower) because it is proposed to design this tower with only readily available data.

It is desirable to avoid the unusually small structural members commonly used in transmission tower design, the properties for which would not be easily available to any one inclined to check this design. Therefore, in this design, all structural members are limited to angle-irons of equal legs. The slenderness ratio is limited to 200; and the joints are assumed welded instead of riveted, or bolted, so that the question of punched, or reamed, holes is eliminated. This, then, does away with the uncertainty as to how much to reduce the cross-section area in tension due to the presence of the holes. The stiffening effect of the welded joints and the resulting moments are ignored, which is in agreement with practice and is on the side of safety. The buckling strength of the compression members is computed on the basis of Euler's formula,


$P_1 = 45000$
36923
(1)
(3)
5'0"
5'0"
10'0"
5'0"
5'0"
10'0"
$P_2 = 45000$
$P_2 = 45000$
40000
40000
(2)
(4)
5'0"
5'0"
5'0"
5'0"
5'0"
5'0"
5'0"
5'0"
$P_4 = 43000$
36160
(5)
(7)
L = 10'0"
L = 10'0"
$P_5 = 37231$
37231
36160
(6)
(8)
L = 10'0"
L = 10'0"
L = 10'0"
L = 10'0"

(*a*) LIMIT LOADING

(*b*) LOADING WITHIN ELASTIC RANGE

FIG. 14.—ELASTIC LIMIT STRESS IS 30 000 POUNDS PER SQUARE INCH; AND, SECTION MODULUS IS 30

$$P = \frac{\pi^2 EA}{\left(\frac{L}{r}\right)^2} \dots\dots\dots\dots\dots\dots\dots\dots (5)$$

The tower is to be designed symmetrically with reference to the center line.

It must be kept in mind that the purpose of this paper is to discuss and illustrate the theory of limit design. In listing the foregoing specifications for tower design the writer is guided solely by the one consideration, that his work shall lend itself to convenient and independent checks. It is not to be assumed, therefore, that he means to express a preference for Euler's formula over some other column formula; nor that he wants to be considered as committed to the feasibility of welding transmission towers in the field. The restriction to equal-leg angles is equally arbitrary and would of course have no merit in a practical case.

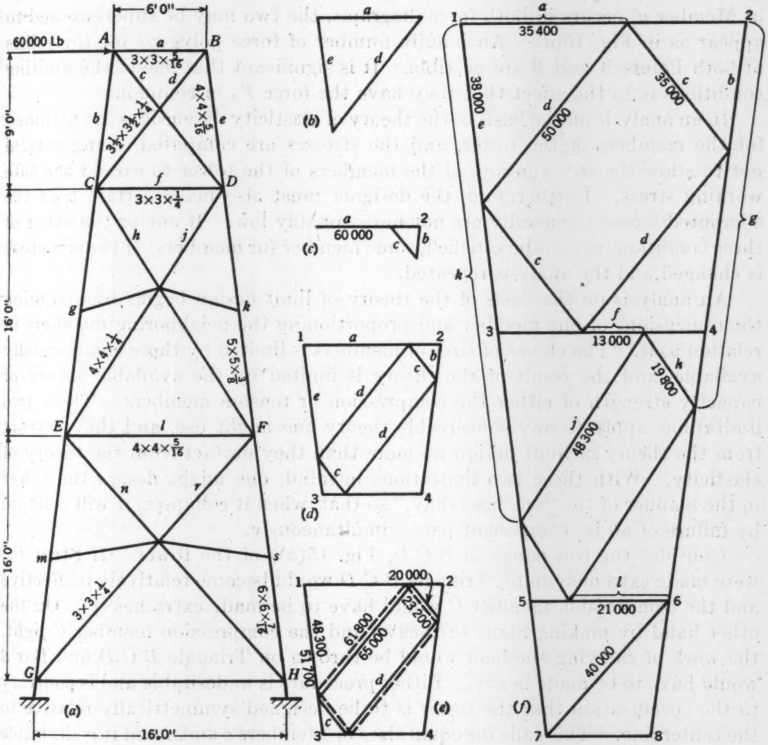

6'0"
60000 Lb
A
B
a
3×3×5/16
c
d
4×4×5/16
b
e
3½×3½×¼
9'0"
C
D
f
3×3×¼
h
j
g
k
16'0"
4×4×¼
5×5×3/8
E
F
l
4×4×5/16
n
r
m
s
16'0"
3×3×¼
6×6×7/16
G
H
(a)
16'.0"
a
e
d
(b)
a
2
60000
c
b
(c)
1
a
2
b
c
e
d
d
c
3
4
(d)
1
20000
2
48300
61800
23200
65000
51700
d
c
3
4
(e)
a
1
2
35400
38000
35000
c
b
d
50000
e
g
d
c
k
f
3
4
13000
h
19800
j
48300
l
5
6
21000
40000
(f)
7
8

FIG. 15.—DESIGN OF A STEEL TOWER

As indicated near the beginning of Case 1, the sum of the moments at the wall and in the center of the beam is $\frac{w\,L^2}{8}$. This conclusion was reached on the basis of the theory of static equilibrium. Any other theory applied to the problem is supplementary. On the basis of the theory of elasticity it was found that the ratio of the two moments is 2 : 1. On the basis of the theory of limit design it was held that, before failure could occur, the ratio between the two moments became 1 : 1.

Similarly, whether the theory of elasticity or the theory of limit design is used in the design of the tower represented by Fig. 15(*a*), either theory is only supplementary to the theory of static equilibrium. In any case the forces around Joint *A* are in equilibrium and their force diagram must close. The same is true about the forces around Joint *B*. Qualitatively, these respective force diagrams appear as shown in Fig. 15(*b*) and 15(*c*). Since F_a, the force in Member *a*, occurs in both force diagrams, the two may be superimposed to appear as in Fig. 15(*d*). An infinite number of force polygons for the joints at both Points *A* and *B* are possible. It is significant that one of the limiting conditions is to the effect that they have the force F_a in common.

In an analysis on the basis of the theory of elasticity dimensions are assumed for the members of the tower, and the stresses are computed, being careful not to allow the stress in any of the members of the tower to exceed the safe working stress. Furthermore, the designer must also make certain that the computed stresses generally are not unreasonably low. If one or the other of these conditions cannot be satisfied, some member (or members) of the structure is changed and the analysis repeated.

An analysis on the basis of the theory of limit design begins by assuming the dimensions of one member and proportioning the neighboring members in relation to it. The choice of sizes of members is limited by those commercially available, and the result of the theory is limited by the available theory of capacity strength of either the compression or tension members. These two limitations apply to any conceivable theory one might use, and they detract from the theory of limit design no more than they detract from the theory of elasticity. With these two limitations in mind, one might design the tower in the manner of the "one hoss shay," so that, when it collapses, it will collapse by failure of all its component parts simultaneously.

Consider the top panel (*A B C D*, Fig. 15(*a*)) of the tower. If Strut (*a*) were made extremely light, Triangle *B C D* would become relatively ineffective and the compression member *C* would have to be made extra heavy. On the other hand by making Strut (*a*) heavy and the compression member *C* light, the work of carrying the load would be thrown on Triangle *B C D* and Bar *d* would have to be made heavy. Either procedure is undesirable and is contrary to the specification that the tower is to be designed symmetrically relative to the center line. This calls for equal sizes of Members *c* and *d* and it is desirable that both be loaded as nearly as possible with their maximum safe load.

For Bar *a*, connecting Points *A* and *B* (Fig. 15(*a*)), select a $3\frac{1}{2}$ in. by $3\frac{1}{2}$ in. by $\frac{1}{4}$ in. angle-iron. Its buckling capacity strength would be (by Equation (6)): $\frac{9.87 \times 30\,000\,000 \times 1.69}{104.3^2} = 46\,000$ lb. It can develop this strength only on condition that the other members of the panel, of which Bar *a* is a part, are capable of developing proportional strength. That is, if (Fig. 15(*d*)) a force equal to 46 000 lb (see Fig. 15(*d*)) is laid off to represent the force in Bar *a*, then Bars *c* and *d* must be capable of developing forces at least equal to the magnitudes marked *c* and *d* in the force diagram (Fig. 15(*d*)).

The effective length of Diagonal *c* is measured from the point of intersection of the diagonals to Point *D*. While one of the diagonals is in compression the other is in tension, and since they are supposedly connected at the point of intersection, the tension diagonal restrains the compression diagonal. This effective length of Member *c* scales 80 in. A 3 in. by 3 in. by $\frac{1}{4}$ in. angle for Diagonal *c* would develop a capacity buckling strength of 23 200 lb. Since the force vector *c* (Fig. 15(*e*)) scales only 20 000 lb this angle would be satisfactory. However, the force vector *d* (Fig. 15(*e*)) scales 65 000 lb. The capacity strength of a 3 in. by 3 in. by $\frac{1}{4}$ in. angle in tension is only 1.44 × 30 000 lb = 43 200 lb. This size of angle for the diagonal *d* would thus be insufficient. A 3 in. by 3 in. by $\frac{3}{8}$ in. angle, with an area of 2.11 sq in., could develop a capacity tensile strength of 63 300 lb. Except for the stipulation of symmetry the use of this angle could be justified as follows: The force vector *c* could be moved parallel to itself until it reached a value equal to the 23 200-lb capacity strength of Diagonal *c*, as shown by the dash line in Fig. 15(*e*) (Diagonal *c* being 3 in. by 3 in. by $\frac{1}{4}$ in.). In that case it appears that the diagonal *d* would have to carry only 61 800 lb, and that the 3 in. by 3 in. by $\frac{3}{8}$ in. angle is large enough for that purpose. If there were a greater choice of commercial sizes one could select a still smaller angle for Diagonal *d*—one capable of developing a capacity tensile strength of only 61 800 lb. If the top panel of the tower had been thus designated, that panel would fail in the two diagonals before failing in Strut *a*.

Since the designer is restricted by a specification demanding a symmetrical structure he could, of course, also change the diagonal *c* to an angle, 3 in. by 3 in. by $\frac{3}{8}$ in. This appears wasteful, making some other solution desirable.

If, for Bar *a*, he selects an angle 3 in. by 3 in. by $\frac{5}{16}$ in., which has a capacity buckling strength of 35 400 lb, he obtains, for the forces around the Joints *A* and *B*, the force diagram shown in Fig. 15(*f*). A 3$\frac{1}{2}$ in. by 3$\frac{1}{2}$ in. by $\frac{1}{4}$ in. angle iron, instead of Bar *c*, with an effective column length of 80 in., will develop a buckling strength of 37 200 lb and a capacity tensile strength, instead of Bar *d*, of 50 700 lb. In each case this is a little more than required. The advantage in changing from a design based on Fig. 15(*e*) to one based on Fig. 15(*f*) is greater than the saving in weight of Bars *a*, *c*, and *d* would imply. For the stress distribution in Bars *a*, *c*, and *d*, as shown in Fig. 15(*f*), the compressive force to be carried by Member *e* (Fig. 15(*f*)) scales 38 000 lb. Its effective length is 109 in. A 4 in. by 4 in. by $\frac{5}{16}$ in. angle will just develop a buckling strength equal to 38 000 lb. The same angle in the place of Member *b* will develop a tensile capacity load equal to 72 000 lb, which is almost double the required capacity. However, the design of Bar *e* is the controlling consideration.

By strictly analogous processes, a force diagram may be drawn for the entire tower that will satisfy only the conditions of static equilibrium (Fig. 15(*f*)). Any continuous force diagram, and one that closes, may be regarded as satisfactory. The members of the tower are to be so selected that their capacity strength, either in compression or tension, as the case may be, will be greater than the value of the corresponding force indicated on the force diagram. In that manner the capacity load of the tower is obtained. If the tower is to be designed for an over-load capacity, say, of 100% (in other words, with a factor of safety of two), begin by multiplying the load for which the tower is to be designed by this over-load factor. Continuing thus, design the structure, and upon completion obtain the safe load by dividing the capacity load by this same over-load factor.

Comment on Steel Tower Design.—A few generalizations at this point would seem to be of interest. Figs. 15(*d*), 15(*e*), and 15(*f*) demonstrate that the forces in Members *c* and *d*, in each case, add up to the same value, 85 000 lb, which is equal to the distance graphically represented by the diagonal 2–3 (Fig. 15(*f*)). This is a geometric proposition, true for any parallelogram with sides parallel to the diagonal bracing *c* and *d* (Fig. 15(*a*)) that may be drawn within the isosceles trapezoid 1–2–3–4 (Fig. 15(*f*)). Taking advantage of this fact, one may scale the distance 4–5 (Fig. 15(*f*)) and find it equal to 68 300 lb, thus concluding that the diagonal-bracing in Panel C D E F (Fig. 15(*a*)) must carry this load. Next, select from available commercial stock, the smallest angle-irons, of which the combined capacity strength in tension and compression is at least equal to 68 300 lb. The effective length of the compression member *h* scales 135 in. The capacity buckling strength of a 4 in. by 4 in. by $\frac{1}{4}$ in. angle, 135 in. long, is 19 800 lb. Its capacity strength in tension is 58 200. The combined capacity of the two angles acting in the place of *h* and *j* (Fig. 15(*a*)), therefore, would be 78 000 lb. This is somewhat in excess of the required value, but, limited as he is by available commercial sizes, it is the closest approximation the designer can achieve. The next step is to lay off the full value of 19 800 lb (Fig. 15(*f*)), as this has a tendency to reduce the value of the compressive forces in Member *f* as well as in that of Member *k* (Fig. 15(*f*)).

The individual force polygon for the various members acting at each joint may be found in Fig. 15(*f*), which shows that the force required to be carried by the diagonal bracing *n* and *r*, in the bottom panel, scales 40 000 lb. The effective length of the compression diagonal *n* scales 144 in. A 3 in. by 3 in. by $\frac{1}{4}$ in. angle would develop a capacity tensile strength of 43 200 lb whereas its slenderness ratio is $\frac{144}{0.59} = 244$. According to the specifications a column with a slenderness ratio greater than 200 is not permitted; or, if used, is not to be considered as carrying any load. The tension diagonal *r*, therefore, is assumed to carry the full required load of 40 000 lb, which it is capable of doing. The functioning of the diagonal bracing in the bottom panel is in agreement with engineering practice as applied to the bracing of the center panel in highway bridges. In these bridges it has generally been the practice to brace the center panel with two light diagonals capable of carrying only tension. While one of the diagonals is loaded in tension the other is assumed to carry no load whatsoever. This is another instance of the theory of limit design being applied in an isolated case.

It may be observed (see Fig. 15(*f*)) that the higher the value one can assign to the compression members *c* and *h* the smaller will be the value of the forces in the compression members *e*, *f*, *l* and *k*.

If the values of the forces carried in Members *c* and *h* could be stipulated to be equal to those in Members *d* and *g*, respectively, the parallelograms *cd* and *hj* would appear as perfect "diamonds." They would be symmetrical to the center line of Fig. 15(*f*), and the force, Distance *f*, which now scales 13 000 lb, would be zero. In that case the cross brace *f*, between Points *C* and *D*, could be omitted.

There are two ways in which this object could be accomplished. Members *c* and *h* could either be designed heavy enough to carry one-half the diagonal bracing load, or more cross bracing could be introduced, thus reducing the effective lengths of *c* and *h*, so that their slenderness ratio would become less than 100. Either of these designs would be more costly than one including the cross braces *f* and *l*.

If the foregoing conclusions are compared with those dictated by the theory of elasticity, it is clear that, by no stretch of the imagination, can there be a larger force in Member *d* than in Member *c*. In order to avoid complications, consider only the top panel and assume the distance *C–D* to remain unaltered. Only if Member *a* were infinitely stiff would Points *C* and *D* carry the same load. With an angle-iron of ordinary proportions acting in the place of *a*, the load in Member *c* is certain to be larger than the load in Member *d*. The writer offers no objection to these conclusions; he objects only to making them a basis for design. Without computing exactly what the stress in Member *c* would be under the theory of elasticity, one needs to conclude only that it would be greater than half the sum of 50 000 lb and 35 000 lb—that is, greater than 42 500 lb. The stipulated angle ($3\frac{1}{2}$ in. by $3\frac{1}{2}$ in. by $\frac{1}{4}$ in.) would not carry this load; and thus, on the basis of the theory of elasticity, the member *c* would have to be made heavier, and therefore more costly. The theory of limit design takes account of the fact that, as the capacity load is applied to the tower, the member *c* bends out somewhat as it continues to carry load, but it refuses to carry more than its own capacity load. From the time that the capacity load in *c* is reached the member *d* assumes the burden of taking care of the load increments until its capacity strength also is reached. Only then may one look for serious results. Since the tower is supposed to be designed for an over-load capacity of, say, 50% or 100%, the working load permitted is 40 000 lb (or 30 000 lb instead of 60 000 lb) and thus limited this buckling does not occur although it would do no harm if it did.

The theory of elasticity, presumably, defines all stresses within the elastic limit and no stresses in the range of capacity loads. The theory of limit design defines no stresses within the elastic limit and is the only theory that offers some definition of conditions under capacity loads. Since, in reality, it is the capacity load that a structure can carry which should govern as a basis for design, the theory of elasticity fails where the theory of limit design serves as a sound philosophy and guide.

DESIGN OF RIVETED CONNECTIONS

The study of rivet connections is especially important in connection with a proper estimate of the theory of limit design. Consider for example Fig. 16. According to the theory of rivet design, in the space between the two rivets toward the right, the upper plate would carry one-fifth of the load, whereas the lower plate carries four times as much. It is to be remembered that, although the universal application of the theory of elasticity is seriously challenged, the assertion that a material like steel is almost ideally elastic, up to a certain stress, is not questioned at all. The ratio of 1 : 4 between the stresses in the upper and lower plates is obviously inconsistent with the fact that the distance between rivets in these plates is the same. According to the theory of elasticity the two outer rivets would carry the major part of the load and the three inner ones would practically be "duds." An interesting point is that up to a certain load unquestionably this circumstance would prevail. Nevertheless, the theory of elasticity fails as completely in the design of rivet connections as it does in the analysis of many other redundant structures.

Rivet connections appear to be "giving ground" to welded joints. It may be hoped that they soon will disappear entirely. Nevertheless, they have been designed in vast numbers for many years past and the simple rules of design have been proved very satisfactory. Eminently satisfactory results must be rooted in a satisfactory and sound theory. This theory is the theory of limit design. The pity is that it has been so extensively practiced, yet so little talked or written about. A rivet connection of purely elastic material is as impossible to conceive as the punching of holes into cold glass plates and riveting them together with glass rivets. If the material were purely elastic, and nothing but elastic, the theory of elasticity would apply. Provided a riveted joint, of purely elastic material, such as shown in Fig. 16, could be made, the outer rivets would carry nearly all the load. When they fail the next two inner rivets would take the load over, and finally the center rivet would be stressed with the full load. The entire process no doubt would be completed in a very small fraction of a second; but, the failure would nevertheless be progressive.

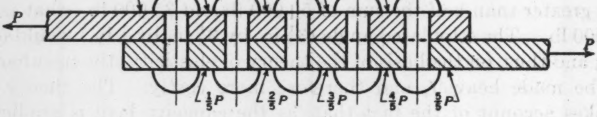


FIG. 16

The saving feature of rivet connections does not derive from the theory of elasticity, but is found in the most rigid specification which stipulates that the most ductile of steels be used. The philosophy which underlies and justifies the proved practice is the theory of limit design. According to this theory the distribution of stress in rivets is at first in agreement with that given by the theory of elasticity. As the load gradually increases, the outer rivets (Fig. 16) reach their elastic limit. They then yield slightly while they continue to carry a constant load, and only then are additional increments of the loads transmitted to the inner rivets. The structure as a whole will not fail until all rivets have passed their elastic limit. In other words, for ordinary loads, the stresses in rivets may not approach within a thousand per cent of what the theory of design indicates. The point is that the only stresses which really interest the designer are those under failing loads, and the theory of limit design teaches that for capacity loads, provided the material is ductile, the assumption of equal distribution of loads over all rivets is justified.

It is not to be thought that the problem of rivet connections is discussed at length because it offers an ideal example of the application of the theory of limit design. It is, in fact, a very poor illustration. The theory of limit design presupposes that the first redundant member in a structure that reaches its capacity strength will yield sufficiently, under a constant force, to permit other redundants to be taxed to their full capacity strength before complete failure can take place. The potential yielding of one rivet under capacity load is, after all, limited. The reason the practice in connection with the design of rivet connections is here submitted as a practical example of the application of the theory of limit design is that, for practical purposes, the theory of elasticity fails completely. The very simple rules that have governed rivet design for years have been proved to be reasonably satisfactory. If their use is to be philosophically justified, the arguments derived from the theory of limit design are the only ones that can do so. When it is argued (as is done with renewed emphasis since the appearance of the British Report of the Steel Structures Research Committee of 1931 to 1936[7]) that long rivet connections are to be avoided, this argument, again, follows directly from considerations of the principles of limit design.

PRACTICAL APPLICATION

In 1914 the writer designed a steel trestle for a three-track railway in Canada. The outbreak of the World War delayed the project and when it was reviewed again, other designers revised the plans in reinforced concrete, recommending a structure similar to that of Fig. 15(*a*) but, naturally, without diagonal bracing. The writer was told that his design of the structure was unquestionably correct, but that it was designed according to no theory at all. In fact, the structure was designed according to the best theory available. The capacity restraining moments at all the corners had been computed and from this the strength of the entire structure was determined. When this strength proved sufficient the design was approved. The interesting psychological point would seem to be the unwillingness to dignify as a theory a philosophy that ran counter to the established theory of elasticity. This attitude still appears to be very common and is giving way but slowly.

Another tale, for the truth of which the writer cannot vouch, but which sounds very plausible, is as follows: A final authority on office building design was asked what he did when a complicated analysis of wind stresses was submitted for his approval. His reply is reported to have been that he locked himself in his office, computed the safe resisting moments at top and bottom of columns and that of their connections to the floor beams in any one story, and compared them with the increment of moment from floor to floor (that is, the total wind load above the floor in question multiplied by the story height). If the former exceeded the latter, he would approve the design. This, to the writer, appears to be a sound procedure; but why lock the door? In fact, the writer questions whether there is any theory of wind stresses superior to this one.

---

[7] See, for example, Second Rept. of the Steel Structures Research Committee, Dept. of Scientific and Industrial Research; published by His Majesty's Stationery Office, London, England, 1934.

The theory of limit design has been applied to riveted connections as long as there have been rivets. The writer was aware of its being applied successfully to reinforced concrete structures on the Canadian Pacific Railway as long ago as 1914. In his own teaching of the elastic energy theory, the writer has always ended the course with a discussion of "The Limitations of the Theory of Elasticity."[8]

With reference to the application of the theory of limit design to towers it is of interest to cite two contradictory viewpoints. In 1932, Hans Bleich[9] stated that, in order to be effective, all compression members must be so proportioned that the stresses induced (including residual stresses) would never reach the buckling limit. Therefore, according to Bleich, the theory of limit design should never be applied to trusses.

On the other hand, Prof. N. C. Kist has held[10] that "every assumption of statically indeterminate values is correct if the dimensions of the structure are designed accordingly." Furthermore, he is undoubtedly responsible for the statement[11] that, "as a rule, the engineer need not apply the theory of elasticity in order to assign values for statically indeterminate quantities leading to an efficient and economical design."

The difference expressed in the foregoing viewpoints is significant. Contrary to Bleich's dictum, the writer has shown that a genuine economy, of time and material, is inherent in the theory of limit design as applied to trusses. The fear of buckling often appears deeply ingrained. This fear is fully justified from the standpoint of the theory of elasticity, or when statically determinate structures are involved. In the analysis of redundant structures a slight buckling (only enough to permit some other detail to function to its full capacity strength) need not be feared.

Professor Kist's dictum sounds sweeping but it is fully justified as long as complete reversals of capacity loads are guarded against.

CONCLUSION

The foregoing examples of the practical application of the principles of limit design are relatively few. It is believed, however, that practices essentially similar to the principles treated in this paper are commonly applied; yet the leaders in the profession, the writers of textbooks on strength of materials and theories of design, to the writer's knowledge, have consistently avoided the subject. In most of the writings on plasticity, even when the writer has ductility in mind, its application to the theory of design is ignored. Those who have written on the subject and shown its relation to problems in design are Kazinczy

---

[8] "Elastic Energy Theory," by J. A. Van den Broek, John Wiley & Sons, Inc., N. Y., 1931.

[9] *Bauingenieur*, 1932; Heft 19/20; p. 264.

[10] International Cong. for Metallic Structures, Liége, Belgium, 1930.

[11] Explanatory notes accompanying Specifications of the Royal Society of Engineers of the Netherlands, 1933, Art. 2, p. 20.

(1914),[12] Kist (1917),[13] and Bleich (1932), and, in the United States, Edward Godfrey,[14] M. Am. Soc. C. E. A number of papers on the subject appear in the report of the Second Congress of the International Association for Bridge and Structural Engineering, Berlin-München, October, 1936. Notable among these is a paper by H. Maier-Leibnitz.

The dominance of the theory of elasticity has resulted not only in an over-emphasis of the idea of "stress," but is also responsible for the fact that most published test data are given only in so far as stresses and deformations within the elastic limit are concerned. For example, numerous test results are available on columns. Such results give the buckling loads but very little or no information in regard to the load-deformation relationship after the buckling has begun. Since this is the type of information in greatest demand relative to the theory of limit design, a vast field for experimentation would be opened up if this theory should meet with favor.

In closing, it seems only fair to call attention to one more fact: The main part of the foregoing examples has been directed to the study of moving loads and reversal of loads. The exceptions to, and the limitations of, the theory of limit design have been emphasized to an extent that might well create a false perspective. Suppose a rivet connection, designed according to conventional rules, were subjected to reversal of capacity loads. Unquestionably, the connection would fail after a limited number of such full reversals. The point is that, although reversal of loads and stresses occurs, it seldom occurs for capacity loads. So seldom does it occur that the rules of procedure (that is, the application of the simpler rules of the theory of limit design) are either not affected by it, or affected very little.

Suppose a transmission line were to be covered with a heavy coating of sleet and subjected to a gale from the southwest, transverse to the direction of the line. Suppose, then, if supposition can be so fantastic, that the following year the same sleet condition occurred and the gale blew from the northeast. This is where the suppositions begin to appear absurd. Extreme over-load conditions may occur a few times during the life of the structure. That they occur with the same intensity, the same direction and complete reversal of sense, is against all probability. Even so, according to the theory of limit design they could so occur a few times without disastrous results.

It would seem that the structural engineer has more or less consciously designed in accordance with the principles of the theory of limit design, while paying "lip service" to the theory of elasticity. Is it not appropriate to take stock and develop the theory which most closely fits the facts and appears to have the largest range of application where redundant structures are concerned?

---

[12] *Betonszemle Nos. 4, 5* and *6*, 1914, Kisérletek Befalazott Tartokkod; "Bemessung von Statisch Unbestimmten Konstructionen unter Berucksichtigung der Bleibenden Formanderungen," International Congress for Metallic Structures, 1930; also "Versuche mit innerlich statisch unbestimmten Fachwerken," by Dr. Ing. G. von Kazinczy, *Der Bauingenieur*, 1938.

[13] "Does a Theory of Strength, which Is Based on the Assumption of Proportionality between Stress and Strain Lead to Good Construction of Steel Bridges and Buildings?" by N. C. Kist, Inaugural address, Technical University, Delft, Holland, 1917. Also, "Ductility as a Base for Design—Computation of Steel Bridges and Structures instead of Proportionality of Stress and Strain," by N. C. Kist, International Congress for Metallic Structures, Liége, Belgium, 1930.

[14] "Secondary Stresses in Bridges," by Cecil Vivian Von Abo, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 193.

DISCUSSION

John H. Meursinge,[15] Assoc. M. Am. Soc. C. E. (by letter).—Roughly, this paper can be divided into three parts: (1) A criticism of the structural engineers' conventional design methods; (2) a treatise on the theory of limit design, with examples; and (3) a review of the applications of the theory, with references to the proper authorities. All three parts will be discussed in the order mentioned.

(*1*).—In the light of present-day conditions, the harsh criticism of the structural engineers' "love" for stress analyses seems unwarranted because, since the discoveries of Galilei, "all" professions of the civilized world have adopted a number of more or less mathematical rules by means of which the object of their observation is tested. The structural engineer is no exception. Physicians satisfy their urge for numerical terminology by reading thermometers and blood pressure apparatus, and by counting heart beats and white and red blood corpuscles; and economists measure the economic health of the nation by the analysis of the balance sheet. Civilized Man has forgotten entirely that all other living creatures on earth live, multiply, do their engineering work, and die, without worrying about numbers. Their entire existence is based on the application of the laws of equilibrium, efficiency, and usefulness. Nevertheless, the status of their health, economics, and the strength of their structures (as, for instance, the bees) can be perfectly well expressed in terms other than mathematics. If their health is vigorous they carry capacity loads; if their economic system is functioning properly, each member of their society will receive capacity supplies; and, when their engineering structures are in good working order, they will carry capacity loads.

As soon as Mankind adopts such a system, in which his accomplishments are judged by tangible results, the engineer can begin to ignore mathematics. In the meantime, he had better follow the methods and conventions of the society in which he happens to live. Any other course would impair his usefulness seriously.

(*2*).—In the theory of limit design the investigator takes advantage of the ultimate strength of the full cross-section, even when a structure is subjected to bending stresses. There is no more efficient application of the material; and, as a result, structures are obtained, the strength of which can be expressed in capacity loads—that is, in terms of usefulness. Thus far structural designs have merely satisfied a number of mathematical requirements; hence, the theory advanced in this paper is a step in the right direction. The engineer should keep in mind, however, the fact that Hooke's law and the theory of limit design are both "simplifications of a very intricate reality—nothing more nor less. Both may be compared to cartoons, because they both 'express reality in a very primitive manner.'"[16]

---

[15] Huntington Park, Calif.

[16] "Ductility as a Base for Design Computations of Steel Bridges and Structures Instead of Proportionality of Stress and Strain," by N. C. Kist, International Congress for Metallic Structures, Liége, Belgium, 1930.

Professor Kist taught the writer how to apply the theory of limit design; and, although the examples in the paper had to be quite elaborate in order to clarify the principles for the benefit of new readers, the application can be simplified in practice for the greater part of the problems which the average designer has to solve. For instance, in Case 1 one needs simply to compute the maximum moment of resistance; then immediately draw the moment diagram of Fig. 4; and, apply the static equilibrium formula:

$$w' \frac{L}{2} \times \frac{L}{4} = 2\,M_1 = 60\,000$$

and solve: $w' = 13\,333$ lb.

For concentrated loads: Draw the moment line for capacity loads shown in Fig. 17. Considering the structure as a simple beam with maximum moments of $2\,M$, and calling the end reactions R_1 and R_2: $R_1 = \frac{2\,M_1'}{a}$; and, $R_2 = \frac{2\,M_1}{c}$. By assuming different values for P_1, one can determine the complementary capacity loads P_2.

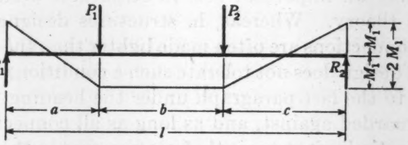


FIG. 17

Professor Van den Broek states that he acquires a factor of safety of 2; but in reality it can be called 3 because Fig. 4 represents a condition in which the cross-section at the middle of the beam still has a triangular stress-distribution diagram. Now, according to the ductility philosophy the beam cannot collapse until two sections have rectangular diagrams; and, as Professor Van den Broek demonstrates in Equation (2), this condition means a 50% increase in strength for rectangular beams.

This simplification of Case 1 is also helpful in solving Case 4, the continuous beams shown in Figs. 9 and 10. First, find the moment of resistance: $\frac{I}{c} = 30$; $s = 30\,000$; and, $M_1 = 30 \times 30\,000 = 900\,000$ in-lb $= 75\,000$ ft-lb. Draw the moment line of Fig. 9(b). Knowing the location of P_1, the computer finds for the left beam reaction: $\frac{75\,000}{5} = 15\,000$ lb. Taking moments about the center support: $15\,000 \times 10 - P_1 \times 5 = -75\,000$; or, $P_1 = 45\,000$ lb.

At first the application of the theory of limit design seems most simple as applied to structures that have no bending stresses, according to the elastic theory method (as, for instance, the tower in Fig. 15); but, if Professor Van den Broek had demonstrated the new method of detailing gusset plates, he would have revealed another radical departure from conventional methods.

(3).—The author considers that the problem of riveted connections is a poor illustration of the application of the theory of limit design (see "Design of Riveted Connections"). For engineers who have had little experience in modern design methods this section happens to be quite convincing. It reveals, plainly, the important fact that designers have to take advantage of ductility (but have not spoken or written about it) in many details, at the same time ignoring it in the design of the main members.

This same chapter, unfortunately, contains a statement from which the reader may infer that the theory of elasticity does not fail as readily in welded connections as in riveted connections. Professor Van den Broek states:

> "Nevertheless, the theory of elasticity fails as completely in the design of rivet connections as it does in the analysis of many other redundant structures.
>
> "Rivet connections appear to be 'giving ground' to welded joints. It may be hoped that they soon will disappear entirely."

H. M. Priest, M. Am. Soc. C. E., has presented[17] stress curves for both riveted and welded connections. In the specimens tested the ends of both connections were over-stressed, and the welded connection would not have held better than the riveted connection except for the ductility of the material.

Connections play an important rôle in structures designed in accordance with the author's theory. Whereas, in structures designed to conform with Hooke's law, the connections are often made lighter than the connected member, the theory of limit design does not tolerate such a condition. Consequently, the writer would add to the last paragraph under the heading "Practical Application": "* * * are guarded against, and as long as all connections are capable of transferring the elastic-limit stresses that may appear in the heaviest connected member."

What about the future of the theory of limit design? An interesting experience of the writer has a bearing on this question. In 1922, while he was a student at the Technical University of Delft, Netherlands, the writer was admonished by Professor Kist for using the elasticity theory of a continuous beam on three supports in order to determine the size of three rollers beneath a shallow roller bearing. Professor Kist demonstrated that the short distance between the actual bearing and the rollers did not give the bearing force in that type of support an opportunity to spread to the two outside rollers until the compression stresses in the middle roller had reached the elastic limit. In this case the theory of limit design offered a more logical solution.

In 1937, the writer, then in the employ of an oil company, was called upon to design a shallow roller bearing beneath a horizontal pressure tank. Remembering his college days, he began with the theory of limit design; but this computation was rejected by the engineer in charge of the design as being contrary to "standard practice." No other action was to have been expected. Hooke's law has been the conventional standard for 250 yr; the new theory is a radical departure from that method. Neither has the new philosophy either established a convenient inter-relationship with modern designers' general process of thinking. Furthermore, the structural engineer's brains resemble a carbureter. After it has been set for a certain mixture of assumptions, it is difficult to adjust it to a new recipe, even if the latter may prove to be more economical.

---

[17] "The Practical Design of Welded Steel Structures," by H. M. Priest, *The Welding Journal*, August, 1933, Fig. 8, p. 9.

Hence the writer has concluded that the theory of limit design will become accepted only gradually. Its extensive application will be postponed until the civilized world has learned that the health and happiness of mankind, as well as the strength of structures, can be expressed in terms of usefulness instead of in mathematical symbols.

I. K. Silverman,[18] Assoc. M. Am. Soc. C. E. (by letter).—The simplest structural form is a straight bar loaded axially. An example is the standard test piece used in determining such quantities as proportional limit, yield point, and ultimate stress. Numerical values of these quantities, obtained from tests on this type of structure, are the bases upon which the proportions of structural members, however they may occur, are estimated.

The proportional limit is defined as that stress at which stress and deformation are no longer connected linearly. The yield point has been defined as that stress at which there is a marked extension with very little increase in load. Sometimes this value is defined as the stress at which a certain permanent strain is obtained. The ultimate stress is determined by dividing the maximum loading recorded in a test by the cross-sectional area of the test piece.

When inserted in codes, these values (which, fundamentally, hold only for the simple case of an axially loaded bar) control the design of as simple a structure as a bar in tension or the relatively complex structure exemplified by a single-span beam.

Assume that instead of the standard test piece a single-span beam of given material, under a given type of loading, is taken as a standard. Then it is possible, in the light of the foregoing definitions, to obtain definite quantities corresponding to proportional limit, yield point, and ultimate strength. It will be found that these values are influenced slightly by the type of loading initially chosen (uniform or distributed) but are greatly influenced by the form of cross-section.[19] E. Mirabelli,[20] M. Am. Soc. C. E., has shown analytically, and has supported his results by citation of tests, that the factor of safety of a beam, based on its own yield point, may be as much as four, compared to a factor of safety of two, based upon the yield point of a simple tensile-stress test.

Although it seems impracticable to test every type of structure to obtain its proportional limit, yield point, etc., this point of view should lead to a new philosophy of design, as the author states.

The results of various tests on standard tensile specimens have led to far-reaching results in mechanical, structural design. It is well known that if a mild steel is loaded in tension above its yield point, unloaded, and then subsequently reloaded, a raising of the yield point is obtained. Along with this raising of the yield point in tension, there is a subsequent lowering of the yield point in compression. This result is known as the Bauschinger effect. In explaining these results, H. Hencky,[21] strangely enough, called upon the phenomena which the author has described. Mr. Hencky's analogy consisted of a three-membered symmetrical truss (Fig. 18(a)), statically indeterminate in the first degree. While the load is $O \leqq P \leqq 2\,s_1A$ (s_1 is the yield point stress in simple tension) an elastic state of stress exists. The load-deformation curve is Line *DF* (Fig. 18(b)). At the load $P = 2\,s_1A$ the vertical bar *DC* (Fig. 18 (a)) becomes plastic and, assuming that the material has a marked yield point, this vertical bar will not increase in stress as the load is increased to, say, $3\,s_1A$. The rate of deflection will be greater, as shown by the slope of *F F′* (Fig. 18(b)). If the structure is unloaded, a permanent deformation corresponding to *D D′* will occur, with residual stresses of tension in Bars *A D*, *B D* and compression in Bar *D C* (see Fig. 18 (a)). If the load is reversed the yield point will be reached at a lower load as shown by *F′F″* because of the

---

[18] Asst. Engr., U. S. Bureau of Reclamation, Denver, Colo.

[19] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 1397.

[20] *Loc. cit.*, p. 1400.

[21] "Zum Theorie Plastischer Deformationen und der hierdurch im Material Hervorgerufenen Nachspannungen," by H. Hencky, *Zeitschrift für angewandte Mathematik und Mechanik*, 1924, p. 323; also, "Strength of Materials," by S. Timoshenko, Part II, 1930, p. 668.

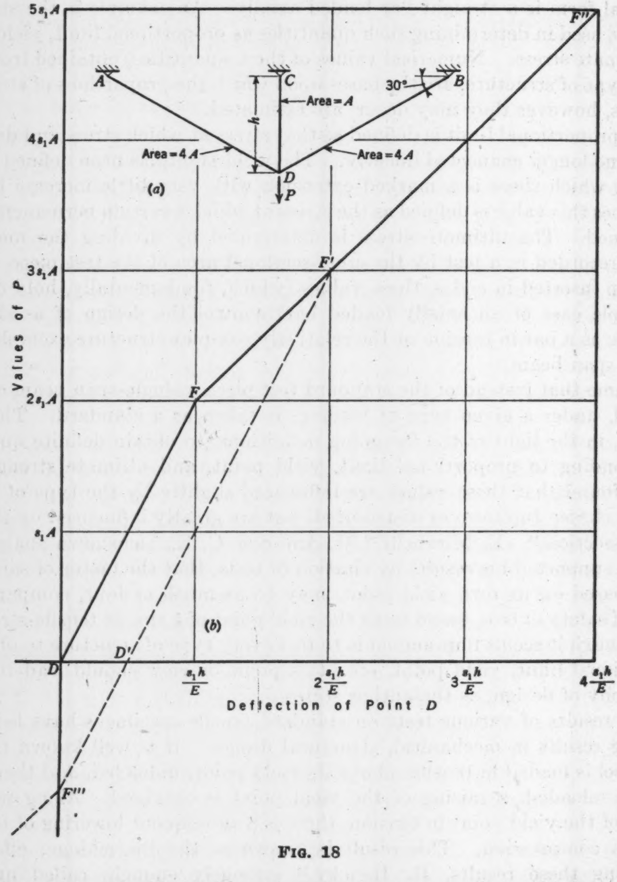


FIG. 18

residual compressive stress in Member $D\,C$. However, if the structure is reloaded in the same direction as the original force, the load will increase along $D'F'$ linearly and will follow along $F'F''$ until all the bars have reached the yield stress s_1. This occurs at a value of the load $P = 5\,s_1A$. Beyond this load, deflections outside the range of magnitude of elastic deformations are to be expected. From the point of view of the theory of elasticity the load capacity of the structure would be taken as $2\,s_1A$. Using the plastic properties of the material the load capacity could be taken as $5\,s_1A$.

The properties given by the simple tensile test, and which have their counterpart in the behavior of the simple structure in Fig. 18(a), have been used advantageously by mechanical engineers. Advantage has been taken of the yielding property of the material and its consequent higher yield point in the design of chains; the favorable residual stresses resulting from permanent deformation have been used in the design of rotors;[22] over-strain of metals has created residual stresses that are favorable during the firing of guns. It is to be noted that in all these cases the direction of the loads is fixed so that the question of reversal does not enter.

The preoccupation of engineers with the concept of stress, as the author states, can be traced to the definition of "factor of safety" as given in textbooks on strength of materials. There, the "factor of safety" is defined as the ratio of ultimate strength to working stress. The idea of stress is intimately connected with this definition. A "factor of safety" based on the ratio of yield stress to working stress is connected with the definition of yield stress which is itself defined in terms of deformation.

The author is to be commended for bringing this subject to the attention of the profession and for his success in clarifying many of the salient points.

EDWARD GODFREY,[23] M. AM. SOC. C. E. (by letter).—The subject of this excellent paper is one that has been neglected or ignored to a large degree by writers on the theory of elastic members. Much has been written on the behavior of materials that are perfectly elastic under the action of stresses, but as there are scarcely any materials that approach this quality there is a hiatus between the theory and the actual fact as represented by the materials under stress. A perfectly elastic material, of which glass is the best example, would be exceedingly dangerous, in many situations, as a structural material.

Professor Van den Broek has cited some cases where the hiatus referred to is most conspicuous. Riveted joints, for example, furnish an example in which the assumption of perfect elasticity would develop, theoretically, a case of certain failure; but failures in riveted joints, where proper rules of design are followed, are totally unknown, and the rules of design virtually ignore perfect elasticity. It is only because structural materials are ductile as well as elastic that they adjust themselves to conditions, and riveted joints designed accordingly, and according to good rules, are safe.

---

[22] "Zum Theorie Plastischer Deformationen und der hierdurch im Material Hervorgerufenen Nachspannungen," by H. Hencky, *Zeitschrift für angewandte Mathematik und Mechanik*, 1924, p. 331; also "Stress Distribution in Rotating Disks of Ductile Material After the Yield Point Has Been Reached," by A. Nádai and L. H. Donnell, *Transactions*, A. S. M. E., Vol. 51, 1929, APM-51-16, p. 173.

[23] Structural Engr., Pittsburgh, Pa.

The paper also refers to bends in parts that are under high stress, bends that are made on steel when it is cold and that stress the steel beyond the elastic limit. When the bend is not reversed, the steel is safe for unit stresses that would be considered proper in steel that is absolutely straight.

The entire fabric of the theory of secondary stresses, the "Theory of Limit" or, more properly, the "Limit of Theory," falls and fails when it is applied to such cases as stresses due to deflection of rigid structures, stresses due to rotation of members on pins, caused by deflection of a truss, and some other cases.

One of the safest structures in existence is the Firth of Forth tubular bridge, in Scotland, although computed secondary stresses, based on the elastic theory, would show disastrously high unit stresses.

BASIL SOUROCHNIKOFF,[24] ESQ. (by letter).—In the theory of limit design Professor Van den Broek assumes that immediately after a material has reached the elastic yield point, a perfect ductile flow occurs; that is, the material suffers considerable additional deformation, the stresses remaining constant. The stress-strain diagram is composed of two straight lines: Line *A B* corresponds to the elastic phase; and, Line *B C*, parallel to the strain, corresponds to the ductile phase (see Fig. 2). This hypothesis is very interesting. The actual material will probably follow a curve between Line *B C* and the extension of Line *A B* so that the designer can obtain a quite clear insight into the behavior of the structure. Line *B C* is probably limited by some Point *C* where actual rupture occurs after the material has undergone a certain ductile deformation.

The following analysis of a built-in beam, similar to the one analyzed in Case 1 by the writer, will serve to demonstrate the limitations of application of the theory. Suppose a straight beam *A B* of Span *L*, connected at Ends *A* and *B* into rigid supports, and carrying two symmetrical concentrated loads *P* at Points *C* and *D* at distances *a* from the ends *A* and *B* respectively. In this type of loading the moment curve is composed of straight lines, and therefore the algebraic calculations are quite simple.

Increase the loads *P* until the elastic yield point is reached at the supports. The bending moments at the supports (M_0) is then expressed by Equation (1) if s = the yield stress, I = moment of inertia, and c = the distance of the extreme fiber from the neutral axis. The moment at the center is less than M_0. If the loads *P* are further increased, the moment at the center increases. One must assume that the moments at supports also increase somewhat, and that the ductile phase is extended over lengths *A E* and $B F = l$ at each end of the beam. Otherwise, a finite angular deformation would have to occur in an infinitely small length of the beam, which is improbable (see text preceding Equation (2)).

The loads *P* are now increased until the moment in the central part almost reaches the value M_0 as defined herein. The moments at the supports are then κM_0, κ being a coefficient greater than unity. The lengths at each end of the beam where ductile flow occurs, can be calculated easily by noting that the moment diagram is a straight line, and that at Ends E and F of the lengths l the moment is M_0; thus:

---

[24] Structural Engr., St. Paul, Minn.

$$\overline{A\,E} = \overline{B\,F} = l = a\,\frac{\kappa - 1}{\kappa + 1} \dots\dots\dots\dots\dots\dots (6)$$

In the region $E\,F$ the material is completely in an elastic state and it is easy to find the slope α of the deformed axis of the beam with the horizontal at Points E and F; thus:

$$\alpha = \frac{M_0}{E\,I}\left(\frac{L}{2} - a\right) = \frac{s}{E}\,\frac{b}{c} \dots\dots\dots\dots\dots\dots (7)$$

in which $b = \frac{L}{2} - a$. The change of slope in each of the lengths l must be arithmetically equal to α as the beam is built into rigid supports. The average change of slope, per unit length, in the region where ductile flow occurs is, therefore,

$$\frac{\alpha}{l} = \frac{s}{E}\,\frac{b}{c\,a}\,\frac{\kappa + 1}{\kappa - 1} \dots\dots\dots\dots\dots\dots (8)$$

At the points when the unit change of slope is the greatest its value α_M is obviously greater than the average, so that $\alpha_M > \frac{s}{E}\,\frac{b}{c\,a}\,\frac{\kappa + 1}{\kappa - 1}$. Assuming that κ is greater than one, the minimum value of α_M is obtained by selecting the greatest possible value of κ. For a rolled beam, 12 in. by 12 in. by 65 lb, the greatest possible value of κ is 1.0662 (see last line of Case 1) and therefore the smallest value of $\frac{\kappa + 1}{\kappa - 1}$ is about 30. Consequently, $\alpha_M > 30\,\frac{s}{E}\,\frac{b}{c\,a}$. Now, independently of the system of loading, the unit change of slope at any point cannot be greater than a certain value α_x, which is dependent on the properties of the material and of the cross-section. The length of an extreme compressed fiber, originally equal to one, cannot be reduced to less than zero. The length of the extreme tension fiber, originally equal to one, cannot be stretched beyond a certain value u before a break occurs. Therefore, $\alpha_x < \frac{u}{2\,c}$. If, therefore, there is a ductile flow, but no actual break of fibers occurs in any place, one must have: $\alpha_M < \alpha_x$; therefore, $30\,\frac{s}{E}\,\frac{b}{c\,a} < \frac{u}{2\,c}$; and, $\frac{u\,E}{s} > 60\,\frac{b}{a}$.

Symbols u, E and s depend on the properties of the material only; b, a, c on the dimension of the beam and the positions of loads only. For a given material the value of $\frac{u\,E}{s}$ is given, and it seems, therefore, that (theoretically at least) one can determine a certain position of the loads for which, as the load is increased, a break at the supports will occur before the yield point is reached at the middle.

Reference to questions of this type was probably made when the author stated that he intentionally omitted studies of certain secondary considerations, thus diverting attention from the essential arguments (see text preceding Equation (2)).

Of course, if a few extreme fibers are broken at the supports, it does not mean that the entire beam will collapse; but permanent cracks will be formed and may be easily aggravated by frost, etc. The resistance to shear may also be impaired.

E. Mirabelli,[25] M. Am. Soc. C. E. (by letter).—As applied to flexural members, the author's theory of limit design appears to provide an ample degree of safety. The theory is based on the conservative assumption that the yield point in bending is reached when the outer fibers begin to yield. Actually, beam deflection does not increase rapidly with a small increment of load until there is a considerable excess in the loading beyond that which causes yielding in the outer fibers. The load-deflection curve for a simple beam continues as a straight line (or very nearly a straight line) within the order of magnitude of elastic deflections during the period of time that the plastically stressed region in the beam extends from the outer fibers toward the neutral axis. Eventually, a loading is reached at which the beam deflection increases rapidly. The amount of this loading depends on the shape and proportions of the beam. These facts have been demonstrated mathematically elsewhere.[26] For these reasons the actual amount of the margin of safety exceeds the assumed amount and the theory of limit design seems justifiable when applied to beams.

However, the new theory does not seem to be so easily acceptable when applied to frames which involve direct stress. In connection with indeterminate trusses, the theory is that any over-stressed member of the frame will continue to carry its share of the load at a constant capacity, while the member is being distorted until the capacity load is fully applied to the frame. A tension member may act in such a manner, but it is doubtful that a compression member is capable of carrying a constant amount of load after the yield point of the material is reached. An over-stressed compression member is likely to have a total resistance less than its capacity at the yield point (if it does not buckle and fail completely). This means that the other members of the frame must carry more than their design load. A reduction in the margin of safety is involved thereby.

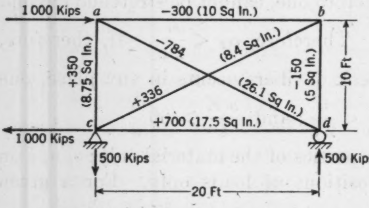


FIG. 19.—STRESSES BY LIMIT DESIGN THEORY OR BY ELASTIC THEORY

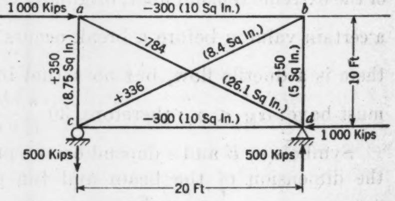


FIG. 20.—STRESSES BY LIMIT DESIGN

---

[25] Associate Prof. of Civ. Eng., Mass. Inst. Tech., Cambridge, Mass.

[26] "Structural Application of Steel and Light-Weight Alloys: A Symposium," *Transactions*, Am. Soc. C. E., Vol. 102 (1937), pp. 1397–1403.

In the type of indeterminate truss shown in Fig. 19 any stress may be assigned to any member (such as − 300 kips to Member *a b*) and the stresses in the remaining members may be determined by application of the static conditions of equilibrium at successive joints. This is essentially the procedure of the theory of limit design. If, now, the required cross-section areas are determined from these stresses and the truss is analyzed by an elastic theory, it is found that the bar stresses are the same as those determined by application of the limit design theory. For such a case the two theories lead to the same design. The only difference is that the use of the new theory involves some saving in the time required for making the analysis.

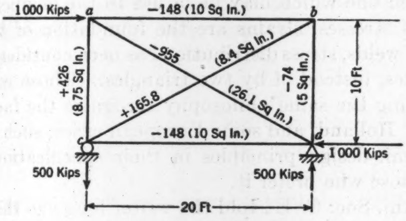


FIG. 21.—STRESSES BY ELASTIC THEORY

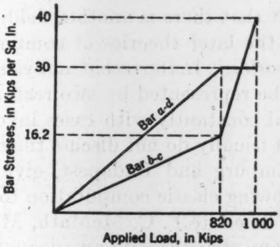


FIG. 22

Now, if the type of structure is changed by interchanging the hinged and roller supports, the elastic analysis and limit design analysis will not result in the same bar stresses. The stresses obtained by the limit-design method are shown in Fig. 20 and those found by the elastic method are shown in Fig. 21. The cross-section areas of members are assumed the same for the two cases. Diagonal *a d* will attain its capacity of − 784 kips when the applied load is $1\,000 \times \frac{784}{955} = 820$ kips. Until this loading the stresses are within the elastic range throughout the frame, and at this loading the stress in Diagonal *b c* is $165.8 \times \frac{820}{1\,000} = +\,136.0$ kips. According to the new theory, Member *a d* will continue to offer its capacity resistance of − 784 kips while the applied load is increased from 820 kips to 1 000 kips; and, the stress in Diagonal *b c* is increased from + 136.0 kips to + 336.0 kips. This theoretical action is shown graphically in Fig. 22, in which stress intensities are used instead of total stresses.

The writer feels that there is no justification for assuming that the *a d*-line (Fig. 22) will continue horizontally at the level of 30 000 lb per sq in. Probably, there is more reason to believe that at the applied load of 820 kips the *a d*-line will reverse its direction and dip, or that Member *a d* will be completely destroyed by buckling. As a consequence, the assumed margin of safety will be reduced, although it is true that in the majority of cases it is not likely that failure will occur at the working loads.

Possibly such objections could be overcome by use of a larger factor of safety for compression members than for tension members. However, if this were done, the economy in material effected by the application of the limit design method would be neutralized to some extent. The principal virtue of the new method, when applied to trusses, would then be in the simplification of stress analysis.

C. M. GOODRICH,[27] M. AM. SOC. C. E. (by letter).—Clearly and cogently, Professor Van den Broek has directed attention to the unfortunate change that the teachers, and the present generation in general, have made in Hooke's law, from *ut tensio sic vis* (as strain, so stress), to *ut vis sic tensio*. It is equally clear from his paper that no fault is found with any elastic theory in its proper field—namely, within elastic limits. He directs attention merely to the undoubted fact that there is another field, and one which may be of use to the engineer. In the later theories of combined stresses, strains are the foundation of the theories. In the recent analysis of welds, stress distribution has been considered to be represented by two rectangles, instead of by two triangles. Those who deal constantly with cases involving the same philosophy recognize the facts but usually do not discuss them. Holland, and some European cities, such as Hamburg and Budapest, give limit-design principles in their specifications, allowing elastic computation to those who prefer it.

The late F. C. McMath, M. Am. Soc. C. E., told the writer long ago that, in any structure, if any adequate path is provided to enable forces to reach the ground, any other path or paths taken will lessen work required of the structure as a whole. In one standard reference book[28] appears the statement, "These proximate solutions are offered as illustrative of simple approximate methods which may often be applied to very complicated problems of this class." In other words, this author recognizes the principles of what Professor Van den Broek has aptly called "limit design."

In a German work,[29] Föppl describes the treatment of structures over a street to protect traffic from falling objects, including this comment: "Accordingly, it is sufficient to equate the energy of the falling mass (after deduction of the impact-loss), with the form-changing work that must be expended in bending the beam severely, and it is sufficient to require that the impact loss does not exceed a reasonable fraction of the work expended." In addition, Föppl refers to A. Senft,[30] who apparently treats the subject more fully. He then cites the further case of bomb shelters, where also, clearly enough, limit-design principles were used. It is unfortunate that the logic of the case was not developed to show its many possibilities.

In 1910 designs for transmission towers were prepared by the writer's Company, using limit-design ideas freely. These designs, doubtless because of their unconventionality, were condemned by a succession of five different eminent authorities as inadequate to sustain the required load. Two sample structures, built to these designs and tested at the expense of the company, showed a load capacity about 50% in excess of that required as a minimum.

---

[27] Chf. Engr., The Canadian Bridge Co., Ltd., Walkerville, Ont., Canada.

[28] "Materials of Construction," by J. B. Johnson, Fifth Edition, John Wiley & Sons, 1918, p. 36.

[29] "Vorlesungen über die Mechanik," by August Föppl, Ninth Edition, Vol. III, 1921, p. 190.

[30] "Centralblatt der Bauverwaltung," by A. Senft, 1915, p. 233.

Structures containing more than 7 000 tons, built under one contract to these and to similar designs and employing such methods as are advocated by Professor Van den Broek, are in service to-day. Even yet such steel is often sold under a guarantee that the structures will carry the required loads under test, usually plus some fixed percentage. This guarantee obviates a not improbable condemnation should the designs themselves be submitted for approval.

The logic of limit design is inescapable; its application is simple; and it promotes economy. In such a field as that of rivet groups under alternating stresses it exposes a danger not discussed in current textbooks, and not at all adequately explored in testing laboratories. These principles are under a kind of tabu, apparently, although they are familiar enough in some of their practical expressions. To determine their field of usefulness makes it necessary to open the windows of the mind, and to encourage a frank discussion of "whens" and "whys."

George Winter,[31] Esq. (by letter).—The writer has known the "theory of limit design" ever since Bleich[32] in 1932 developed a method similar to that proposed by the author. The fundamental principles underlying both of these methods are indeed extremely appealing to engineering common sense. Professor Van den Broek is to be commended for introducing this topic for discussion and thus opening new prospects for a more exact theory of design based on the criterion of permissible deformation.

However, in the author's treatment (as well as in Bleich's) it is precisely the question of deformation which, in the last analysis, remains unsolved. Any design theory that does not provide the means of determining the deformations corresponding to any given load up to the ultimate one is incomplete and questionable. It is the indisputable strength of the theory of elasticity that it provides these means with a high degree of accuracy. Any design theory extending beyond the elastic range should attempt, at least, to satisfy this same condition, and even more so a theory which consciously assumes as its base the criterion of permissible deformation. This is not the case either in the author's theory or in Bleich's (the only one which formerly was known to the writer).

Realizing this deficiency the writer, in November, 1938, undertook to investigate the bending of steel beams beyond the elastic limit with the main purpose of its application to what the author, very exactly, calls "limit design." The writer intends to discuss herein only those aspects which bear a direct relation to the theory advanced for discussion by Professor Van den Broek. The most important question to be solved seems to be that of determining the deflection of beams, thus "over-stressed," in a manner similar to the determination of purely elastic deflections.

For his investigation the writer has used the same type of idealized stress-strain diagram Fig. 23(a) that underlies the author's and Bleich's theories as well as other treatments of plastic bending.[33] Fig. 24(b) represents the stress distribution over a cross-section after the elastic limit has been exceeded. At a distance 0.5 q above and below the neutral axis the magnitude of the unit fiber strain is ϵ_e. Hence the differential equation of the bent axis of such a beam is

---

[31] Research Investigator, School of Civ. Eng., Cornell Univ., Ithaca, N. Y.

[32] "Stahlhochbauten," by Hans Bleich, Berlin, 1932, Vol. 1, pp. 396–411.

[33] "Plasticity," by A. Nadai, McGraw-Hill Book Co., Inc., 1931; and "Strength of Materials," by S. Timoshenko, Vol. 1, D. Van Nostrand, 1930.

$$\frac{d^2y}{dx^2} = -\frac{2\,\epsilon_e}{q} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

in which $\frac{d^2y}{dx^2}$ is the curvature of the axis, ϵ_e the elastic limit strain (see Fig. 23(a)) and q the depth of the part of the cross-section within which the stresses are less than the yield point s_y—and, hence, where they obey the straight-line law (see Fig. 24(b)). As usual it is assumed that plane cross-sections remain plane if subject to this stress distribution, a fact which has been well confirmed experimentally in the plastic, as well as the elastic, range.[33] Since the moment of the external forces must be balanced by that produced by the stress distribution shown in Fig. 24(b), a simple equation of equilibrium determines q for any given moment. In this manner the writer has found for rectangular cross-sections

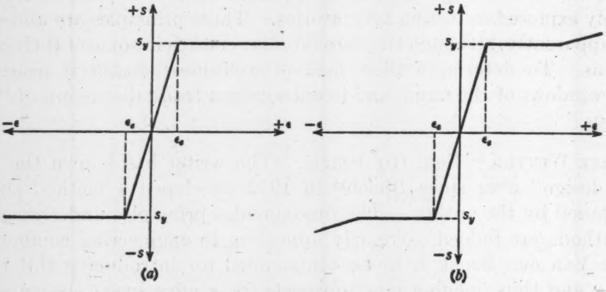


FIG. 23.—IDEALIZED STRESS-STRAIN CURVES

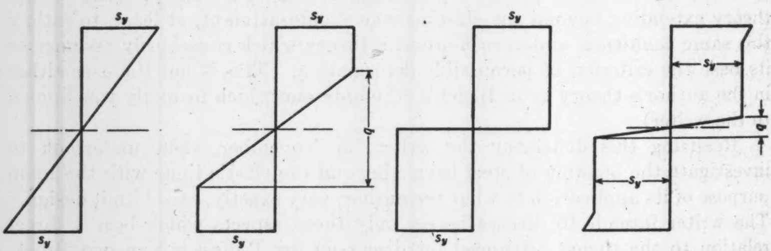


FIG. 24.—CONSECUTIVE STRESS DISTRIBUTIONS IN BENDING BEYOND THE YIELD POINT

$$M = \frac{s_y}{4}\,b\left(h^2 - \frac{q^2}{3}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

and for I-beams (see Fig. 25):

If $q \leqq h_1$,

$$M = s_y\left[b\,d_1h_2 + \frac{d_2}{4}(h_1^2 - q^2) + \frac{d_2q^2}{6}\right]\dots\dots\dots\dots(11a)$$

and, if $q \geqq h_1$,

$$M = s_y\left[b\,\frac{h^2 - q^2}{4} + \frac{b\,q^3 - (b - d_2)\,h_1^3}{6\,q}\right]\dots\dots\dots\dots(11b)$$

Obviously Equations (10) and (11) hold as long as the stress distribution of Fig. 24(b) holds and hence can be evaluated as long as $q \neq 0$. On the basis of the stress-strain curve of Fig. 23(a) q becomes zero when the moment of the external forces becomes equal to the internal moment corresponding to the stress distribution of Fig. 24(c). It is obvious that in this case

$$M = s_y\,S_p\dots\dots\dots\dots\dots\dots\dots\dots(12)$$

in which the section modulus S_p is twice the moment of the area of the upper or lower half of the cross-section about the neutral axis; S_p will be called the plastic section modulus as compared with the usual elastic section modulus S_e. The ratio $S_p : S_e$ depends largely upon the shape of the cross-section; it is 1.5 for rectangular beams, 1.136 to 1.195 for standard I-beams and 1.07 to 1.175 for wide flange sections. If the moment of the external forces at any section reaches the value given by Equation (12), this section has reached its ultimate bearing capacity. It is transformed into a "plastic hinge"; that is, the two parts of the beam at both sides of this cross-section can be rotated with respect to each other through a considerable angle without increasing the load, the internal moment remaining unchanged during this rotation.

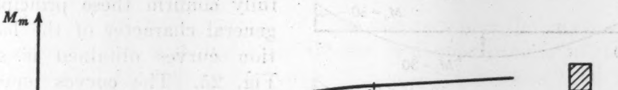


FIG. 25.—LOAD DEFLECTION CURVES OF A RECTANGULAR BEAM AND AN I-BEAM HAVING THE SAME SECTION MODULUS, S_e, AND THE SAME YIELD POINT, S_y

It may be noted that, actually, the state of an ideal "plastic hinge" can be approached but can never be reached completely, the reason being that the curve in Fig. 23(a) is idealized and for more accuracy should be replaced by Fig. 23(b) which includes work hardening. Then, instead of the stress distribution in Fig. 24(c), that in Fig. 24(d) will occur, in which q can never be zero, although it becomes very small for ordinary ductile structural steel. However, it may be shown from Equations (9) to (11) (as well as from tests performed by the writer) that the difference between the idealized and the actual state is negligible. In fact the difference between the moment of Distribution 24(c) and the actual moment at which Distribution 24(b) changes to Distribution 24(d) does not exceed a fraction of 1% and, from this load on the section, may be regarded as a "plastic hinge" with about the same degree of accuracy.

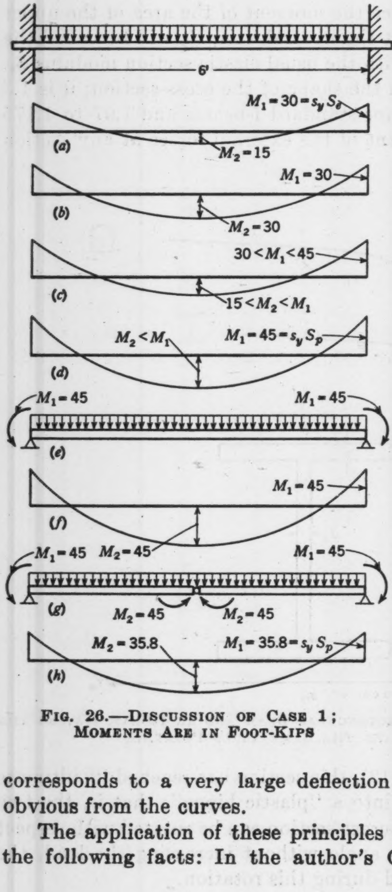


FIG. 26.—DISCUSSION OF CASE 1; MOMENTS ARE IN FOOT-KIPS

The writer has tested the foregoing ideas experimentally by means of bending tests performed on I-beams as well as rectangular beams. Although this investigation is not yet finished (1939), the results hitherto obtained fully confirm these principles. The general character of the load-deflection curves obtained is shown in Fig. 25. The curves represent the mid-span deflection of an I-beam and a rectangular beam both having the same section modulus S_e. Point a, Fig. 25, corresponds to the maximum moment $M_m = s_y S_e$—that is, to the ultimate elastic load, or moment, at which the transition from Stress Distribution 24(a) to Distribution 24(b) occurs; Points b correspond to $M_m = s_y S_p$—that is, to the transition from Distribution 24(b) to Distribution 24(d), or to the moment at which a "plastic hinge" is formed. The experimental curves between Points a and b show good coincidence with the ones computed from Equations (9) to (11). The parts of the curves above Point b, Fig. 25, show that the idea of a plastic hinge is justified because from that point a very small load increment corresponds to a very large deflection increment. The importance of S_p is obvious from the curves.

The application of these principles to the "theory of limit design" reveals the following facts: In the author's Case 1 (on which the other cases are based so far as bending is concerned), the moment distribution of Fig. 26(a) is the ultimate one which is governed by the usual principles of the theory of elasticity and ordinarily is regarded as the limit so far as bearing capacity is concerned. Professor Van den Broek proposes to take the moment distribution of Fig. 26(b) as the ultimate, and to base the safety factor on this distribution. However, Distribution 26(a) is defined exactly; it actually occurs whereas it is easy to show that Distribution 26(b) never can occur. If, after having reached Distribution 26(a), the load increases further, the end moments increase as well as the mid-span moments. Whereas the latter still remain in the elastic range, however, the end moments exceed $M = s_y S_e = 30\,000 \times \frac{12}{12} = 30\,000$ lb-ft—in other words, Distribution 24(b) takes place. Regarding Fig. 25 as the curve of relative rotation of two consecutive cross-sections as plotted against *M*, the parts of the beam near the ends are in the section *a-b* of the curve, whereas the mid-span part is still in section *o-a*. Hence the deformations of the former are governed by Equation (9), and those of the latter by the usual relation

$$\frac{d^2y}{dx^2} = -\frac{M}{EI} \qquad \text{(13)}$$

The essential point is that by means of these two relations the entire state of deformation as well as of moment distribution is defined exactly in the same manner as it is defined in the elastic range by Equation (13) alone. On further increasing the load, Moment M_1 finally reaches its ultimate value, $M = s_y S_p = 30\,000 \times \frac{18}{12} = 45\,000$ lb-ft (corresponding to Fig. 24(c) and Equation (12) and the end sections become transformed into "plastic hinges" (see Fig. 26(d)). The beam from now on acts as if it were freely hinged, with constant restraining moments at the supports (see Fig. 26(e)). In this state the moment distribution is determined by simple equilibrium relations and the deflections are still determinable by Equations (9) and (13). After a further increase of the load the mid-span section finally reaches its ultimate moment also, another plastic hinge forms at this point (see Figs. 26(f) and 26(g)) and, any slight further increase of the load results in excessive and uncontrollable deformation. In this entire process the deformations remain in the order of elastic deformations until shortly before the load corresponding to Fig. 26(f) is reached. (The foregoing idea is the one underlying Bleich's theory. The difference between his treatment and the one presented herein is that Bleich did not take into consideration the foregoing developed laws governing the deformations and moment distributions during the entire process. The method of integrating simultaneously the differential Equations (9) and (13) over the entire span is essentially the same as that used in the ordinary theory of elasticity for the integration of Equation (13) only. Moreover, the equations developed herein are still, fundamentally, elastic equations since the deformation is governed by the central part of the cross-section within *q* [see Fig. 24(b)] which remains elastic until the ultimate load is reached. However, the actual computation is somewhat cumbersome and exceeds the scope of this paper.)

As may now be seen, the moment distribution of Fig. 26(b), which underlies the author's concepts, actually cannot occur under any conditions. The author himself is well aware of this fact as may be concluded from his remarks at the end of the discussion of Case 1. However, the writer believes that, contrary to the author's opinion, these disagreements are far from being "secondary considerations." In principle it is scarcely possible to justify basing the safety factor on an entirely fictitious state such as Fig. 26(b), instead of on a clearly defined situation such as Fig. 26(a). The writer agrees fully with the author in that Fig. 26(a) by no means represents an ultimate load; but, if the safety factor or the working load is to be derived from the criterion of permissible deformation (as it should), the real and well-defined situation shown in Fig. 26(f) is by far preferable to the fictitious one in Fig. 26(b). In addition to these fundamental considerations important practical discrepancies result from the author's proposition, as may be shown by the following numerical example: Figs. 26(a), 26(b) and 26(f), respectively, correspond to loads of 10 000, 13 333 and 20 000 lb per ft. A safety factor of, say, 2 would bring the author's working load to 6 667, the writer's value to 10 000 lb per ft. (It should be emphasized that this value of the safety factor, the same as used by the author, is chosen purely for illustration and not as a practical design proposition, the question of finally deciding the actual value of the safety factor being rather complex. Offhand, a factor of 2 seems to be too low if applied to the ultimate load as defined in this paper. In the numerical example the factor of 2, however, had to be maintained in order to obtain results consistent with the author's numerical example.)

Now, suppose that this same beam, instead of consisting of the rectangular bar 6 in. by 2 in., as in the author's example, is made of a standard I-beam 7 in. by $3\frac{3}{4}$ in. by 20 lb, which has the same section modulus $S_e = 12$. According to the author's proposition this I-beam would be attributed the same working load as the rectangular beam, namely 6 667 lb per ft. However, the actual moment transforming the M_m-sections into "plastic hinges" is $M_m = s_y S_p = 30\,000 \times \frac{14.31}{12} = 35\,800$ lb-ft. (For this I-beam, $S_p = 1.193\, S_e = 14.31$.) Hence, the moment distribution at which excessive deformation actually occurs is that of Fig. 26(h). The corresponding uniform load amounts to 15 920 lb per ft and, applying again the same safety factor, the working load would be 7 960 lb per ft. Hence, by the author's method, an actual safety factor of $\frac{20\,000}{6\,667} = 3$ would result for the rectangular beam as compared with $\frac{15\,920}{6\,667} = 2.39$ for the I-beam instead of the nominal safety factor of 2; whereas by the method proposed in this paper the actual safety factor in both cases would be the same, namely the factor selected. It should be emphasized that this definition of the safety factor or working load as related to the load which actually results in excessive deformation has a clearly defined meaning which makes it subject to exact mathematical analysis. The difference in both these concepts follows clearly from Fig. 25(a) which is obtained from actual tests and is in agreement with the theoretical considerations outlined. The author proposes to let both beams, the deflection curves of which are given in Fig. 25(a), carry the same allowable load, disregarding their difference in bearing capacity which is rather obvious from the curves. The writer proposes to determine the working load as depending upon the maximum moment M_m at which excessive deformation begins—that is, essentially, the moments corresponding to Points *b*. The computing work is exactly the same in both cases the only difference being that, for the writer's method, S_p must be known. For standard beams this value, as well as the usual S_e, could be tabulated.

The foregoing discrepancy results from the fundamental fact that once the elastic range of loading is exceeded, the usual section modulus S_e, as based on Fig. 24(a) loses its significance entirely and the only expression that characterizes the further behavior of the beam is S_p, corresponding to Fig. 24(c). The author's method is based entirely on S_e despite the fact that he deals with loads beyond the elastic range; and hence the discrepancies follow logically. The difference between the author's nominal and the actual safety factor would have been even greater if, instead of the I-beam—$S_p : S_e = 19.3$—a beam with, say, $S_p : S_e = 1.10$, had been analyzed.

The writer does not recommend applying the proposed method prematurely to actual design. Many questions must be clarified before a solid basis for a new design theory can be advanced. A number of these problems have presented themselves in the writer's present experimental investigation. For example, the relation between the yield points in bending and in tension, and the influence of the load or strain speed on the yield point seem rather problematical. The questions of residual stresses in rolled sections and of dynamic loading are also so essential, with respect to "limit design," that they should be studied before a new design method is proposed for adoption. Wide possibilities for research are open in this field. It may be noted that similar tendencies toward limit design have gained ground in the field of reinforced concrete beginning from the time in 1931 when Emperger raised the question as to the actual meaning of the ratio *n*. Extensive experimental investigations have been conducted ever since that time and they have not begun to crystallize into a new design theory until now.

The author's important contribution is that he has clearly stated the problem of limit design, thus opening a discussion which, it is to be hoped, will result in useful new ideas in design. Decidedly, the writer shares the author's belief that, after sufficient additional knowledge has been obtained as to the fundamentals, the old design methods, considering the elastic range only, will indeed give way to a broader theory. Nevertheless, they will retain the methods of the theory of elasticity, but will be broadened to include the exact analysis of the actual ultimate strength as defined by permissible deformation.

Francis E. Simpson,[34] M. Am. Soc. C. E. (by letter).—For its pioneering properties, this paper is a valuable contribution to the profession. It should stir the practicing engineer into a realization that perhaps he has become a slave to thinking mechanically in terms of working stresses without giving much heed to either elastic or ductile performance for either structures or parts of structures.

---

[34] Associate Structural and Architectural Engr., Detroit Sewage Disposal Project, Detroit, Mich.

One cannot read this paper critically without becoming fully alive to the fact that, with shifting points of contraflexure and with fixity at the supports being altered in the process of ductile loadings, a beam loaded beyond its elastic limit becomes a most interesting subject of study with all kinds of possibilities in its elastic behavior.

The treatment of frames and of riveted joints brings out many points that are of prime importance, although some of these points are not given the weight they should be given by the majority of practicing engineers. The author states that the practicing engineer seems to have a dread of using stresses approaching the elastic limit, and possibly this is largely true. However, there are instances in which, during construction, parts of structures might necessarily be subjected to rather extreme conditions of loading, and in which designing on the basis of accepted working stresses would result in an uneconomical structure. It is fairly common in such instances for practicing engineers to use working stresses somewhat higher than those ordinarily prescribed. However, even if construction conditions would warrant the use of higher working stresses than is usual, the structure would be designed to carry its design loading with accepted working stresses.

In conclusion, it is appropriate to say that the author should be complimented not only for his splendid paper but for his daring in the suggestion of, what might be termed by some, a slight schism in the realm of thought concerning the theory of elasticity.

JOSEPH A. WISE,[35] M. AM. SOC. C. E. (by letter).—The philosophy upon which the theory of limit design is based appears to be a sound one, but before it can be accepted as a basis for the design of structures, it will require much clarification and perhaps amplification. Of the three dicta proposed, the first one is vague and fails to define a statically indeterminate structure. In one sense, all members, reactions, and restraints function to the same end—that of resisting the applied forces without failing or deforming excessively. The essential characteristic of a statically indeterminate structure is that it has redundant members, reactions, or restraints in excess of the number just sufficient to maintain static equilibrium. The third dictum is a proposition that is not correct. The tower shown in Fig. 15 is at least doubly indeterminate if Member $E\,H$ is considered not acting; yet the failure of two members, such as $E\,H$ and $F\,H$ or $C\,F$ and $D\,F$, would suffice to cause complete collapse instead of the three required by the third dictum. If it should be argued that, with reference to any one bay, this structure is singly indeterminate only, and therefore the failure of two members in one bay constitutes the failure of $n + 1$ members, one need only point out that Members $C\,D$ and $E\,F$ could be chosen as redundants. Thus, it would become necessary to define "bay" so as to include only one of these two members. For more complex structures any such definition would be difficult, if not impossible. As a matter of fact, the third dictum obscures, rather than clarifies, the manner in which structures may fail. In its place, the following is proposed:

---

[35] Associate Prof., Civ. Eng., Univ. of Minnesota, Minneapolis, Minn.

A framed structure will collapse completely when a sufficient number of members have buckled or yielded, so that the remaining part of the structure is statically unstable. The "remaining part" of the structure is defined as that part formed by omitting all members that cannot act to resist any increase in load, because they have buckled or yielded, but including any forces which they supply. It must be noted that even after a member has buckled, due to a compressive force, it may still be available to resist tensile forces, and therefore, whenever the increase in load tends to produce tension in a buckled member, that member should not be omitted in forming the remaining part of the structure.

In the paper, the tacit assumption is made that after a member has buckled it will continue to have the same axial stress until the structure collapses. This is not entirely correct. Fig. 27 shows a pin-connected member subjected to an axial load P. It will be assumed that the member is initially straight, and that the axial load is applied so that the member does not bend until the critical or buckling load is reached. If, now, the distance between the ends is decreased by a distance ΔL, the load P will not change as long as the member remains elastic; that is, no part of it is stressed beyond the yield point. The member will bend, however; the work done by the load will be stored as elastic strain energy of bending. Assuming that the elastic curve is a sine curve

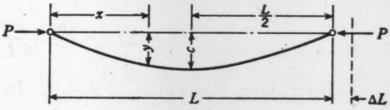


FIG. 27

$$y = c \sin \frac{\pi x}{L} \quad \ldots\ldots (14)$$

a close approximation for ΔL can be obtained as follows:

$$\Delta L = \tfrac{1}{2} \int \left(\frac{dy}{dx} \right)^2 dx = \frac{c^2 \pi^2}{4\,L} \quad \ldots\ldots (15)$$

and

$$c = \frac{2}{\pi} \sqrt{L\,\Delta L} \quad \ldots\ldots (16)$$

The moment at the center will then be

$$M = P\,c \quad \ldots\ldots (17)$$

As long as the material is not stressed beyond the yield point, the axial load will not change appreciably. The small change in axial load, which can be determined by using the exact expression for radius of curvature instead of the approximate value, $\frac{1}{\frac{d^2y}{dx^2}}$, can be neglected in this case.[36] When the member becomes stressed beyond the yield point at the center, however, the moment there cannot be appreciably increased unless the axial load P decreases. As fibers closer to the neutral axis than the extreme fiber reach the yield point, the resisting moment will increase slightly; but it will be sufficiently accurate for this analysis to assume that the moment, M', which exists when the extreme fiber reaches the yield point is the limiting bending moment. Thereafter the axial load will decrease as ΔL and c increase, since

---

[36] "Die Knickfestigkeit," by Rudolf Mayer, Section 4.

$$P = \frac{M'}{c} \qquad (18)$$

Furthermore, as P decreases, the axial length of the member will tend to increase, and this will tend to increase c still further. Calling ΔP the decrease in P, and Δc the increase in c, Equation (18) yields:

$$(P - \Delta P) = \frac{M'}{c + \Delta c} \qquad (19)$$

and including the axial change in length,

$$\Delta L = \frac{(c + \Delta c)^2 \pi^2}{4\,L} - \frac{\Delta P\,L}{A\,E} \qquad (20)$$

Substituting the value of $c + \Delta c$ from Equation (19) in Equation (20):

$$\Delta L = \frac{(M')^2 \pi^2}{4\,L\,(P - \Delta P)^2} - \frac{\Delta P\,L}{A\,E} \qquad (21)$$

Furthermore, as P decreases, the unit axial stress due to it will decrease and thus the section will be able to resist a greater moment when the extreme fiber stress is at the yield point. Calling s the unit stress, in extreme fiber due to moment M', the increased resisting moment will be $M'\left(1 + \frac{\Delta P}{s\,A}\right)$ and Equation (21) becomes:

$$\Delta L = \frac{(M')^2 \pi^2 \left(1 + \frac{\Delta P}{s\,A}\right)^2}{4\,L\,(P - \Delta P)^2} - \frac{\Delta P\,L}{A\,E} \qquad (22)$$

in which A is the area of member. Equation (22) is a cubic, ΔP being the unknown quantity to be determined. It can be solved easily by trial when numerical values are given. Although the axial deformations may be of the order of elastic deformations, the lateral deflections will be much larger. For example, a 3-in. by 3-in. by $\frac{1}{4}$-in. angle, 8 ft long, will have a buckling load (as a pin-ended column) of $P = 16\,100$ lb. It will shorten under this compression, $\frac{16\,100 \times 96}{1.44 \times 30 \times 10^6} = 0.0358$ in. If the ends are caused to move toward each other 0.01 in., the lateral deflection, c, will be $\frac{2}{\pi}\sqrt{96 \times 0.01} = 0.623$ in., or more than 60 times the axial change in length. The moment at the center will be $16\,100 \times 0.623 = 10\,040$ in.-lb. The extreme fiber stress due to this moment will be $\frac{10\,040 \times 1.19}{0.50} = 23\,900$ lb per sq in. at the heel. This stress, combined with the axial compressive stress of $\frac{16\,100}{1.44} = 11\,200$ lb per sq in., would produce a stress of 35 100 lb per sq in., a value close to the yield point of standard structural steel. Any further displacement of the ends of the member toward each other would be accompanied by a rapid increase in lateral deflection and a decrease in axial load. Thus a further decrease in length of 0.01 in. would make $\Delta L = 0.02$ in. Equation (22) becomes,

$$0.02 = \frac{10\,040^2 \times \pi^2 \left(1 + \dfrac{0.42 \times \Delta P}{1.44 \times 23\,900}\right)^2}{4 \times 96\,(16\,100 - \Delta P)^2} - \frac{\Delta P \times 96}{1.44 \times 30 \times 10^6}$$

By trial it will be found that $\Delta P = 5\,700$ lb and

$$c + \Delta c = \frac{M'\left(1 + \dfrac{\Delta P}{A\,s}\right)}{P - \Delta P} = \frac{11\,700}{10\,400} = 1.125 \text{ in.};$$

and $\Delta c = 1.125 - 0.623 = 0.502$ in. Thus, it is seen that, even for axial movements of the order of elastic deformations, the lateral deflections become very large, and the axial load decreases rapidly.

A comparatively simple structure (Fig. 28) will be used to illustrate these principles. Beginning with a load, P, of 200 kips, the stresses in the members, determined by the method of consistent deflections, are shown in Table 1(a), the final step being $X = -\frac{4\,940}{70.8} = -69.9$. The members are assumed to be built so that their slenderness ratios, $\frac{L}{r}$, are at least as small transversely as they are in the plane of the structure. Column (10), Table 1, gives the buckling stress, computed by Euler's formula. If the load is increased to 210 kips, Member $A\,D$ will buckle. At this load the stresses will be those shown in Fig. 29(a). The remaining part of the structure consists of Members $B\,D$ and $C\,D$ only. If a load $P = 1$ lb is applied, the stresses are as shown in Fig. 29(c). Member $C\,D$ has a margin of strength of $76.5 - 73.4 = 3.1$ kips before it

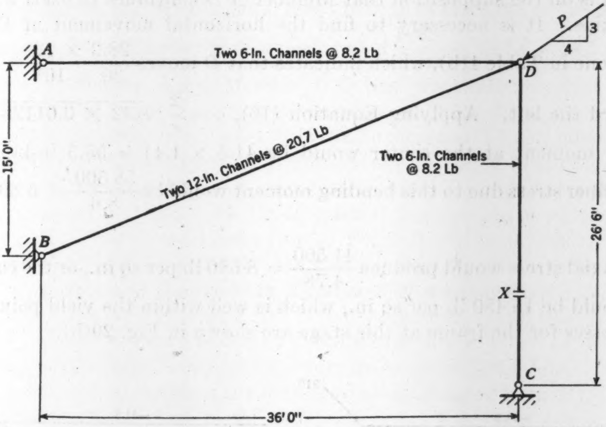

A
Two 6-In. Channels @ 8.2 Lb
P
3
4
D
15'0"
Two 12-In. Channels @ 20.7 Lb
Two 6-In. Channels @ 8.2 Lb
B
26'6"
X
C
36'0"

FIG. 28

TABLE 1.—ILLUSTRATIVE EXAMPLE

	(a) COMPUTATION OF STRESSES BY CONSISTENT DEFLECTIONS									(b) COMPUTATION, HORIZONTAL MOVEMENT OF POINT D			
Member (see Fig. 28) (1)	Length, L, in feet (2)	Area, A, in square inches (3)	$\frac{L}{A}$ (4)	Stress, S', in kips per square inch (5)	u (6)	$s'u\frac{L}{A}$ (7)	$u^2\frac{L}{A}$ (8)	Stress, S, in kips per square inch (9)	Buckling stress, in kips per square inch (10)	Stress, S, in kips per square inch (11)	$\frac{L}{A}$ (12)	u (13)	$su\frac{L}{A}$ (14)
AD	36.0	4.78	7.53	+128	+2.40	+2 310	+43.4	− 39.5	− 41.5	....	....	....	....
BD	39.0	12.03	3.24	−312	−2.60	+2 630	+21.9	−130.5	−346.0	−10.1	3.24	−1.083	+35.4
CD	26.5	4.78	5.54	0	+1.00	0	+ 5.5	− 69.9	− 76.5	− 3.1	5.54	+0.417	− 7.2
Total	....	....	....	....	....	+4 940	+70.8	....	....	....	....	....	+28.2

buckles. Thus, if a load of $\frac{3.1}{2.67} = 11.6$ kips is applied at Point *D*, Member *C D* will just buckle, and the added stresses will be as shown in Fig. 29(d). However, this is on the supposition that Member *A D* continues to exert a force of − 41.5 kips. It is necessary to find the horizontal movement of Point *D*. This is done in Table 1(b), which indicates that *D* moves $\frac{28.2 \times 12}{30 \times 10^3} = 0.01128$ in. toward the left. Applying Equation (16), $c = \frac{2}{\pi} \sqrt{432 \times 0.01128} = 1.41$ in. The moment at the center would be $41.5 \times 1.41 = 58.5$ in-kips. The extreme fiber stress due to this bending moment would be $\frac{58\,500}{8.6} = 6\,800$ lb per sq in.

The axial stress would produce $\frac{41\,500}{4.78} = 8\,680$ lb per sq in., or the combined stress would be 15 480 lb per sq in., which is well within the yield point. The total stresses for the frame at this stage are shown in Fig. 29(b).

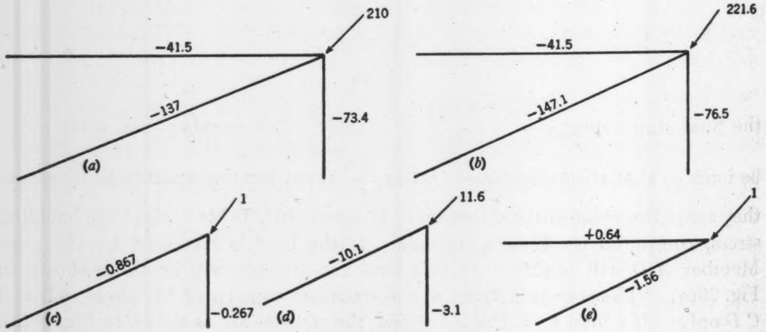


FIG. 29

The next step will be to consider Members $A\ D$ and $B\ D$ as the remaining structure after Member $C\ D$ buckles. Member $A\ D$ is included since an increase in load will tend to induce tension in that member. Fig. 29(e) shows the stresses due to $P = 1$ lb. The very smallest increase in load will permit Member $A\ D$ to transfer the elastic strain energy stored in it, due to the previous movement at Point D of 0.01128 in., into Member $C\ D$. Since no appreciable change in load or stress is assumed to occur during this transfer, Member $B\ D$ will merely rotate about Point B, guiding Point D along a line at right angles to Member $B\ D$. Therefore Point D moves vertically a distance equal to $\frac{12}{5} \times 0.01128 = 0.0271$ in. Again applying Equation (16):

$$c = \frac{2}{\pi} \sqrt{318 \times 0.0271} = 1.87 \text{ in.};$$

and, the moment at the center would be $76\,500 \times 1.87 = 143\,000$ in-lb. The maximum stress due to combined bending and axial stress would be $\frac{143\,000}{8.6} + \frac{76\,500}{4.78} = 32\,630$ lb per sq in., compression. Assuming a yield point of 33 000 lb per sq in., Member $B\ D$ has a margin of strength of 198.9 kips left at this stage. It would require an added load of $\frac{198.9}{1.56} = 127.6$ kips to produce this stress. However, since Member $C\ D$ has practically reached its yield point, the load in it will now tend to decrease as Point D moves downward, and therefore the stress in Member $B\ D$ will be increased by this effect. Thus, it is obvious that the structure will not sustain an added load of 127.6 kips.

Proceeding by trial, first add a load of 10 kips; the added stresses will be $+ 6.4$ in Member $A\ D$ and $- 15.6$ in Member $B\ D$, assuming no unloading from Member $C\ D$. The vertical deflection of Point D will be 0.1149 in. ($= \Delta L$). For this case, ΔP will be 41 500 lb, the stress in Member $A\ D$ is changed from $+ 99.5$ to $+ 58.0$, and the stress in Member $B\ D$ is changed to $- 254.9$ kips. This change, in turn, will cause Point D to deflect further, and the cycle will be repeated. It will be found that only about 5 kips can be added before collapse occurs; hence the limit load for this structure is about 226.6 kips. Thus, the structure will carry more than the limit load as determined from Members $A\ D$ and $C\ D$; and, also all three members buckle before final collapse occurs.

If a similar, step-by-step analysis is applied to the tower in Fig. 15, it will be found that Member $C\ F$ will buckle first, when the load at the top of the tower reaches 33.5 kips. It will be found that an additional load of 8 kips at the top will then cause the buckling of Member $D\ F$, and therefore the structure will collapse at a total load of 41.5 kips, instead of the 60 kips for which it was designed. Thus, it can be seen that the procedure proposed by the author is not valid unless checked by a step-by-step analysis of the behavior of the structure as a whole; and, that it is not correct to assume that a member will continue to have the same stress after it has buckled as it had at the buckling point.

One other phase of the subject should be emphasized. The dead load of a structure is not ordinarily likely to be increased appreciably, and only the increase in live load can lead to a failure due to over-load. By present methods of design, structures with large ratios of dead load to live load, such as reinforced concrete structures, are penalized in comparison with others (such as steel structures). Under limit design, concrete structures would probably have proportionately greater reductions in sizes of members than would steel structures.

There is also an inconsistency in the basic paper, in that the author states that, in the theory of limit design, the emphasis is to be shifted from "stress" to "permissible deformation." Nevertheless, the paper proceeds to discuss the stresses chiefly, a procedure which it had previously condemned. As a matter of fact it does not matter whether one uses the limiting deformation or the stress at which it occurs, as the criterion for the strength of a member, except in cases where two-dimensional or three-dimensional states of stress occur. Then the deformations depend on all stresses and not on the stress in the direction of the deformation alone. Therefore, the subject should be separated into two parts: (1) Structures with uni-axial states of stress; and, (2) structures with bi-axial or tri-axial states of stress. For the first type this discussion is pertinent; for the second type, the question of the controlling factor at failure will depend upon the material; for brittle materials generally, the deformations will control; and, for ductile materials, a shearing stress which is also a function of the maximum tensile stress, as indicated by the Guest-Hancock, or Mohr theory of failure, will control.

ALFRED FREUDENTHAL,[37] ASSOC. M. AM. SOC. C. E. (by letter).—Very ably the author has presented a picture of the behavior of steel structures stressed above the yield limit. A comprehensive treatment of the theory of limit design based on the ductility of steel is destined to meet with serious difficulties as, at the present time, the main features of the theory of plasticity are in a state of flux. Under the stipulated conditions and the assumptions made, the author's point of view may be justified. However, the writer believes that these conditions are encountered in only a restricted number of cases in practice and that, in general, the assumptions are not valid. Furthermore, a number of important influences have been insufficiently considered (or not at all); and this might involve fundamental changes in the conception of the theory of limit design.

(1) The author asserts that the capacity load of an n-time statically indeterminate structure is reached after $(n + 1)$ members have all reached their elastic or buckling limit strength. With regard to the danger of buckling, this statement, in its general form, is quite a dangerous one because the author's conviction that "in the consideration of compression members the logic is essentially the same as that applied to tension members" does not seem to be based on evidence. Furthermore, the author is not at liberty to choose a buckling formula. The introduction of Euler's formula is not subject to "preference"; its validity is purely a question of whether buckling occurs before the yield point is reached or after it has been exceeded.

---

[37] Res. Engr., Marine Trust, Tel-Aviv Port, Tel-Aviv, Palestine; Lecturer on Bridge Engineering, Hebrew Institute of Technology, Haifa, Palestine.

As the author states, the load-deformation relationship after the buckling has begun is of particular importance to the entire problem. This relationship (see Fig. 30) differs essentially in the cases of elastic and plastic buckling.[38]

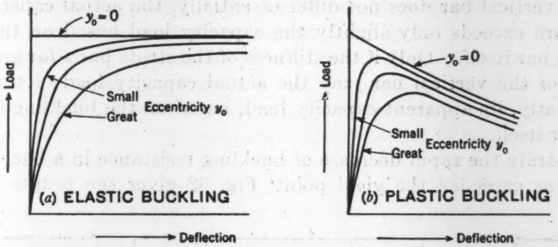


FIG. 30.—ELASTIC AND PLASTIC BUCKLING; LOAD-DEFLECTION CURVE FOR VARIOUS ECCENTRICITIES

Thus, the behavior of statically indeterminate structures is materially affected by the type of buckling involved.

To make this fact clear, the behavior of the structure in Fig. 31 may be considered. With an increasing load, P, the capacity load of the structure is reached in different ways, depending on the proportion of stresses in, and the buckling stiffness of, the struts and the vertical bar. According to the theory of elasticity the force acting in the vertical bar has the value

$$X = \frac{P}{1 + 2\,\dfrac{A_s}{A_v}\sin^3\phi} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

in which A_v and A_s, respectively, are the cross-section areas of the members indicated. For $\phi = 30°$ and $A_s = A_v$, the force in the vertical bar is $X_v = 0.8\,P$; and the force in the struts, $X_s = 0.2\,P$. Because of this difference between stresses in the elastic state, below the stability limit, the members of the structure cannot be stressed to capacity simultaneously. The following are the different possibilities of reaching the capacity load of the structure:

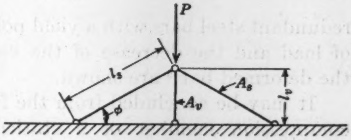


FIG. 31.—A STATICALLY INDETERMINATE SYSTEM

(a) The buckling limit of the vertical bar may be exceeded before the stress $X_s = \dfrac{X_v}{A_v}$ reaches the yield point. Under increasing lateral buckling deflection of the vertical bar and almost constant buckling resistance (see Fig. 30(a)), the forces in the struts increase gradually, until their elastic buckling limit (and, with it, the capacity load of the entire structure) is reached. Hence, the structure collapses under stresses which are, in all members, below the yield point.

(b) The buckling limit of the vertical bar may be reached after the stress has exceeded the yield point. Referring to Fig. 30(b), the resistance of the bar decreases considerably with increasing lateral deflection, thus involving a rapid increase of the forces acting in the struts. Subsequently, their buckling limit is reached and the structure collapses. If the buckling resistance of struts and vertical bar does not differ essentially, the actual capacity load of the structure exceeds only slightly the capacity load based on the buckling limit of the bar itself. Only if the stiffness of the struts has a far greater value than that of the vertical bar may the actual capacity load of the structure exceed, greatly, its apparent capacity load, based on the buckling limit of the vertical bar itself.

---

[38] "Plasticity," by A. Nadai, McGraw-Hill Book Co., Inc., New York, N. Y., 1931.

To illustrate the rapid decrease of buckling resistance in a bar,[39] the stress of which has exceeded the yield point, Fig. 32 gives the results of tests on

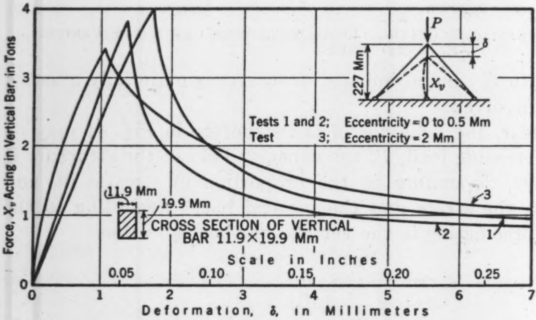


FIG. 32.—RELATION BETWEEN FORCE AND DEFORMATION FOR VARIOUS DEGREES OF ECCENTRICITY.

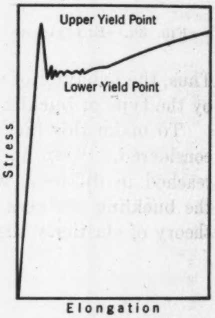


FIG. 33.—STRESS ELONGATION CURVE OF STEEL IN TENSION

redundant steel bars with a yield point of 33 500 lb per sq in. The relationship of load and the decrease of the vertical distance between the end points of the deformed bar[39] are shown.

It may be concluded from the foregoing that the behavior of a redundant compression member of a statically indeterminate structure, stressed to capacity under continually increasing load, differs essentially from the behavior of a tension member, under similar conditions. The "fear of buckling" which, as the author remarks, "appears deeply ingrained" is fully justified not only from the point of view of the theory of elasticity but even more from the point of view of the theory of (plastic) limit design. Hence, Bleich's statement[6] concerning compression members is correct: The theory of limit design must never be applied to trusses. The statement by Professor Kist,[13] however, that "the engineer need not apply the theory of elasticity in order to assign values for statically indeterminate quantities" is exaggerated; it applies only to a very limited number of structures; and, in its general form, it is open to strong criticism.

---

[39] "Three Contributions on the Loading Question of Statically Indeterminate Steel Trusses," by E. Chwalla, Publications of the International Assoc. for Bridge and Structural Engineering, Zurich, Switzerland, Vol. 2, 1933–1934.

Until now one fact has not been considered. In practice, compression members do not consist of bars of rectangular sections, but of angles, channels, and joists. The stability limit of such sections is generally not reached gradually as assumed theoretically for rectangular sections. The particular form of the section almost always causes elastic or plastic buckling, with simultaneous torsion, bending, or premature buckling of the flanges, and with an almost instantaneous, abrupt loss of the buckling resistance. This phenomenon leads to a load-deformation relationship of plastic buckling which is far more unfavorable with respect to the influence of the resistance of the compressed member on the capacity load of the structure than the load-deformation relationship assumed in the theory of limit design.

(2) The behavior of compression members in trusses, as assumed in the paper, is only one of the features that makes the general conclusions advanced by the author open to criticism. It will be shown that the assumption underlying the theory of limit design for members acting under bending stresses cannot be fully endorsed, either.

The assumption of an ideal plastic behavior of the material, according to Fig. 2, has proved to be valid only for special kinds of steel and under particular forms of loading. During the past few years the foregoing assumption has been strongly opposed by various writers. Their objections are based on a considerable number of tests which have shown that the existence of an upper yield point (Fig. 33), as well as the apparent rise of the yield stress, due to non-uniform distribution of stresses over the sections (as in the case of bending), causes an elastic behavior of the structure far above the limit assumed on the basis of a theory developed from the ideal plastic behavior of the material. The upper yield point is only partly influenced by the quality of the material and depends to a great extent on the velocity of the loading.[40] For slowly applied loads the ratio of upper and lower yield point will be greater than for rapidly applied loads. This is explained by the occurrence of the upper yield point, which may be considered as a "retardation of yield" similar to the phenomenon of the retardation of the boiling point of liquids. Instantaneously applied loads cause the upper yield point to drop.

Furthermore, it has been proved by tests that in many cases the maximum stresses in beams under loads have reached values considerably exceeding the yield point of the material established in the tension test. This "raised yield point" may amount to 1.2 to 1.7 of the yield point, in pure tension, depending on the cross-section of the beam. If the load is increased still further the cross-section reaches the plastic state instantaneously over its full height, and the stresses show a sharp drop from the upper yield point to the value of the lower yield point. Local instability (as, for example, buckling of the flanges, bending or torsional deformation) is often the result. The assumption of an ideal plastic behavior of the material, as proposed by the author, therefore, has only a restricted application.

(3) On the other hand in the design of bridges fatigue is far more important than plasticity. The occurrence of pulsating and reversed stresses will always

---

[40] "General Theory of Plasticity," by A. Freudenthal, Rept. I, 1, Second International Congress for Bridge and Structural Engineering, Berlin, Germany, 1936.

demand the design of bridges on the basis of the endurance limit of the material. Such a design will require a higher factor of safety than would be required by (plastic) limit design; therefore its use for such cases would always be imperative.

The endurance of a structure is particularly influenced by the form of the connections. It is diminished by notch action in gusset-plates as well as in welds, and by the thermal residual stresses in and around welds. One may conclude, therefore, that even under conditions favoring the application of the theory of limit design in the proposed form, the connections between the members will always represent weak and critical points where the validity of a design based on the ductility of the material alone may become questionable.

(4) The general conclusions to be reached on the basis of all the aforementioned facts are that the theory of limit design, in the form presented by the author, is applicable but to an extremely limited number of structures under permanent or slowly changing loads. It must not be applied to trusses even under fixed or slowly changing loads. In the design of bridges, where the endurance limit of the material, and the notch-action in the connections are the dominant factors, its application would not be justified.

Although the structural engineer is certain to be aware of the limits of the theory of elasticity, it does not seem justifiable to recommend its abandonment in favor of an unqualified acceptance of the theory of (plastic) limit design. It is certainly appropriate to endeavor to develop a theory which most closely fits the facts; but the theory of limit design, as presented by the author, can scarcely be regarded as fulfilling this requirement. Therefore, within certain limits, the theory of elasticity will remain the basic theory for the structural engineer, provided that the factor of safety is established on the basis of a thorough understanding of the actual behavior of the structure. This may involve the revision, and even the abandonment, of the current conception of stress.

The author is to be congratulated, sincerely, for having stressed this important fact.

HANS H. BLEICH,[41] ESQ. (by letter).—In his very instructive paper demonstrating the practical use of the theory of limit design the author extends its application to trusses. Such extension is not permissible because it is necessary that all compression members be so proportioned that the stresses induced will never reach the buckling limit. It is true that, if a compression member is loaded to its buckling capacity, a slight buckling will appear, the effect being to shorten the bar so that some other bar will begin to carry the load of the compression member. The extent of the buckling is limited, however, and must be very small. If the bar bends beyond a certain limit it can carry only a part of the full buckling load of the straight bar. An investigation of this permissible limit shows that, generally, compression members cannot be shortened as much as is required by the theory of limit design.

---

[41] Dr. Ing., Vienna, Germany.

For simplicity, consider only the top panel of Fig. 15(*a*). According to the force diagram (Fig. 15(*f*)), the forces in Bars *c* and *d* are assumed to be 35 000 lb and 50 000 lb, respectively. It is required to compute the shortening of Bar *c* in order to produce these total stresses in Bars *c* and *d*.

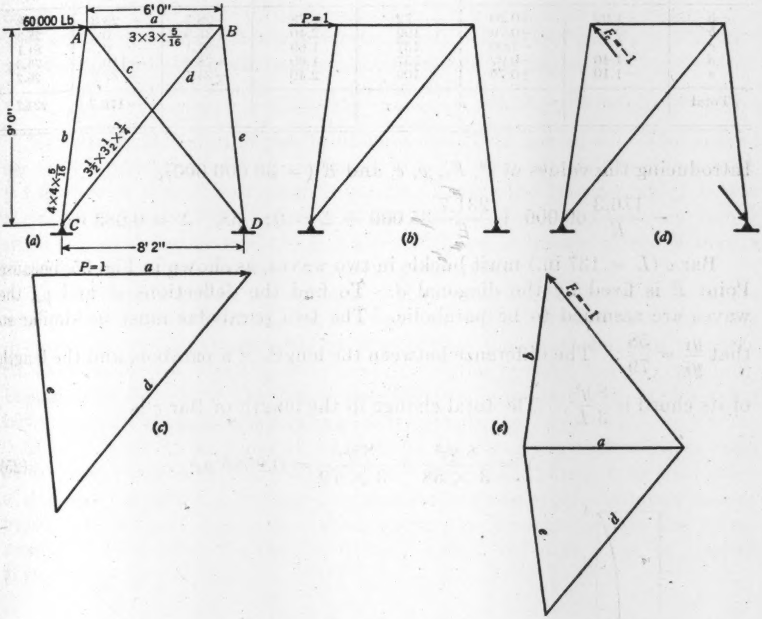


FIG. 34

Fig. 34(*c*) and 34(*e*) are the force diagrams for this panel, subjected to the loads $P = 1$ and $F_c = -1$. In Table 2 the total stresses induced by Load $P = 1$ are F_a' and F_b'; and, the total stresses induced by Load $F_c = -1$ are F_a'' and F_b''. The change in the length *A–D* caused by the force $P = 1$ is $\Delta_P = \sum \frac{F\,F''\,l}{E\,A}$; and, the change produced by the force $F_c = -1$ is $\Delta_F = \sum \frac{(F'')^2\,l}{E\,A}$ (see Columns (7) and (8), Table 2). Thus, it is found that $\Delta_P = \frac{176.3}{E}$ and $\Delta_F = \frac{231.7}{E}$. If the change in the length of Bar *c*, caused by the buckling of this bar, is called Δ, the general equation for the force F_c is

$$P\,\Delta_P + F_c\,\Delta_c + \Delta = 0 \quad \text{.................. (24)}$$

TABLE 2.—COMPUTATION OF STRESSES IN THE TOP PANEL OF FIG. 15(*a*)

Bar (see Fig. 34(*a*)) (1)	Total stress, F', due to a load $P=1$ (2)	Total stress, F'', due to a load $F_c=1$ (3)	Length of member, L, in inches (4)	Area, A, in square inches (5)	$\frac{F'' L}{A}$ (6)	$F' \frac{F'' L}{A}$ (7)	$F'' \frac{F'' L}{A}$ (8)
a	−1.00	+0.70	72	1.78	+28.3	− 28.3	19.8
b	0	+0.79	109	2.40	+35.9	0	28.3
c	0	−1.00	137	1.69	−81.1	0	81.1
d	+1.40	−0.97	137	1.69	−78.6	−110.1	76.3
e	−1.10	+0.76	109	2.40	+34.5	− 37.9	26.2
Total	....	....	....	....	....	−176.3	231.7

Introducing the values of P, F_c, p, c, and E (= 30 000 000),

$$-\frac{176.3}{E}\,60\,000 + \frac{231.7}{E}\,35\,000 + \Delta = 0; \quad \text{or,} \quad \Delta = 0.083 \text{ in.}$$

Bar c (L = 137 in.) must buckle in two waves, as shown in Fig. 35, because Point E is fixed by the diagonal d. To find the deflections y_1 and y_2, the waves are assumed to be parabolic. The two parabolas must be similar so that $\frac{y_1}{y_2} = \frac{58}{79}$. The difference between the length of a parabola and the length of its chord is $\frac{8\,y^2}{3\,L}$. The total change in the length of Bar c is:

$$\Delta = \frac{8\,y_1^2}{3 \times 58} + \frac{8\,y_2^2}{3 \times 79} = 0.0586\,y_2^2 \ldots\ldots\ldots\ldots\ldots (25)$$

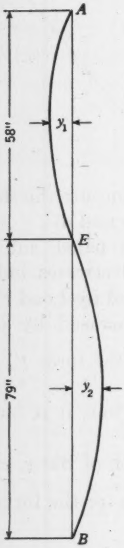


FIG. 35

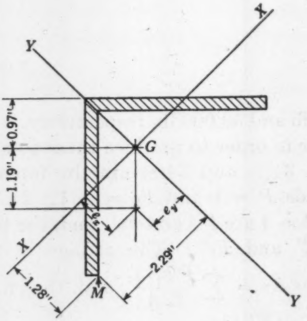


FIG. 36

The value of Δ necessary to produce the given stresses was found to be 0.083 in.; and the corresponding deflection was $y_2 = \sqrt{\frac{0.083}{0.0586}} = 1.19$ in.

Fig. 36 shows the cross-section of Bar *c* with its center of gravity, *G*, and the eccentricities, e_x and e_y, of the load with respect to the *X* and *Y* axes. The eccentricity is $e_x = e_y = 0.707\ y_2 = 0.83$ in.; the area of the cross-section is $A = 1.69$ sq in.; the moments of inertia are $I_x = 0.83$ in.^{4} and $I_y = 3.22$ in.^{4}; and, the greatest stress will occur in the corner of the angle, *M*:

$$s = \frac{35\,000}{1.69} + \frac{35\,000 \times 0.83}{0.83} 1.28 + \frac{35\,000 \times 0.83}{3.22} 2.29 = 86\,000 \text{ lb per sq in.}$$

This stress exceeds the elastic limit of 30 000 lb per sq in. The curved bar with a deflection of 1.19 in. is not able to carry the full buckling load. It follows that the entire system is unable to carry the load for which it was originally designed. The assumed redistribution of the load between Bars *c* and *d* can never occur because Bar *c* would have to buckle so much that it would lose about two-thirds of its buckling strength.

This example leads to the conclusion that either all compression members must be safe against buckling or, if a bar can be loaded beyond its buckling strength, the truss system without this critical bar must be able to carry the entire load (the small loading capacity of the buckled bar is neglected). Engineers made use of the latter possibility a long time ago in designing trusses with crossed diagonals in the center panels.

If the theory of limit design, limited by the foregoing statement, is used for the design of trusses, there will be no economical advantage over the theory of elasticity, except in a few special cases such as the aforementioned case of trusses with crossed diagonals. It was with this conviction that the writer stated,[9] in 1932, that the theory of limit design had better not be applied to trusses.

ALFRED S. NILES,[42] ASSOC. M. AM. SOC. C. E. (by letter).—The interesting question as to the proper method of determining the factor of safety of an engineering structure is reviewed by the publication of this paper. In practice the engineer has to choose between two classes of design methods. At present many structural engineers use what may be termed "working stress methods" in which they compute the stresses that would be developed by the maximum probable loads and so proportion the structure that these stresses do not exceed the "allowable working stresses" of the materials used. These allowable working stresses are obtained by dividing the pertinent "ultimate stresses" by the desired factor of safety. The alternative is to use what may be termed an "ultimate stress method" in which the maximum probable load is multiplied by the desired factor of safety to obtain the "ultimate load." The stresses that would be produced by this ultimate load are then computed and the structure so proportioned that these stresses do not exceed the appropriate ultimate stresses. The method of "limit design" described in the paper is obviously in the latter class. The author may protest against describing it as a method of comparing stresses, since he claims that emphasis should be placed on deformations rather than on stresses; but it is difficult for the writer to understand how the deformations can be determined without first computing the stresses that cause them.

---

[42] Prof., Aeronautic Eng., Leland Stanford Junior Univ., Aero Laboratory, Stanford Univ., Stanford University, Calif.

The essential difference between the two basic design methods can be shown most clearly by the aid of a numerical example. Suppose it is desired to determine the adequacy of a 1-in. square bar of steel, 40 in. long, to carry simultaneously an axial compression of 6 250 lb and a uniformly distributed transverse load of 5.5 lb per in. It will be assumed that the bending moment at each end is zero, that Young's modulus is 30 000 000 lb per sq in., and that the compression yield point of the material, 36 000 lb per sq in., is the appropriate ultimate stress. For the 1-in. square cross-section the moment of inertia, I, is $\frac{1}{12}$ and the section modulus, $\frac{I}{y}$, is $\frac{1}{6}$. The formula[43] for the maximum bending moment resulting from the assumed loading is

$$M_{\max} = w j^2 \left[1 - \sec\left(\frac{L}{2j}\right) \right] \quad\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

in which: w is the transverse load; L is the length of the member, $j = \sqrt{\frac{EI}{P}}$; E is Young's modulus; I is the moment of inertia of the cross-section; and, P is the axial compression.

If the desired factor of safety is 2.0 and the working stress method is used, the maximum permissible value for the total unit stress, $f = \frac{P}{A} + \frac{My}{I}$, will be $\frac{36\,000}{2} = 18\,000$ lb per sq in. Using Equation (26) to determine the imposed stress, $j^2 = \frac{EI}{P} = \frac{30\,000\,000}{12 \times 6\,250} = 400$; $j = 20$ in.; $\frac{L}{2j} = 1.00$; $\cos\left(\frac{L}{2j}\right) = 0.5403$; and, $1 - \sec\left(\frac{L}{2j}\right) = 0.8508$. Then $M_{\max} = 5.5 \times 400 \times 0.8508 = 1\,872$ in-lb. The total imposed stress is therefore $f = \frac{P}{A} + \frac{My}{I} = \frac{6\,250}{1} + 1\,872 \times 6 = 17\,482$ lb per sq in. Thus, according to this method of computation the member in question would be adequate, the desired factor of safety having been provided with a margin of about 3 per cent.

Suppose, however, the investigation were made by the ultimate stress method. The analysis would then be made for an axial load of 12 500 lb acting in conjunction with a transverse load of 11.0 lb per in., whereas the allowable total stress would be 36 000 lb per sq in. From these data, $j^2 = 200$; $j = 14.14$ in.; $\cos\left(\frac{L}{2j}\right) = 0.15615$; and, $1 - \sec\left(\frac{L}{2j}\right) = 5.4041$. Then $M_{\max}$

---

[43] "Airplane Structures," by Alfred S. Niles and Joseph S. Newell, John Wiley & Sons, Inc., New York, N. Y., 1938, Vol. II, Second Edition, p. 98.

= 11 889 in-lb, and the maximum unit stress becomes $f = 83\ 834$ lb per sq in., which is more than twice the allowable value of 36 000 lb per sq in.

This example is an extreme one, but it illustrates the difference between the two basic methods and shows that the decision as to whether to use a working stress or an ultimate stress method of analysis may have a great influence on the adopted design sizes. Using the working stress method the 1-in. square rod was found adequate by a small margin; using the ultimate stress method, however, it was found inadequate by a large margin.

The ultimate stresses that may be used in either type of procedure are of various kinds. Some, like the ultimate tensile strength, are the maximum true stresses that can be resisted without causing fracture of the material. Others, like the yield point, are stresses at which plastic flow and permanent set begin to be excessive. Still others, like the stresses used in column design, are those at which a member becomes unstable, due either to elastic deformation, plastic flow, or a combination of the two. Finally, there are ultimate stresses in common use, like the modulus of rupture, which are not true stresses but merely convenient numbers which represent, indirectly, the maximum load that a member can carry.

Before an intelligent selection can be made of a general method of design and the ultimate stresses to be utilized, it is necessary to consider in more detail what is to be recognized as "failure." The most serious type of failure is that which occurs when a structure becomes unstable and collapses. Simple examples of this are failures of a truss due to the elastic buckling of a long slender strut or to a tension member being pulled away from its anchorage. In some structures, however, instability and collapse may be preceded by excessive deformations due to plastic flow, and such conditions are properly to be regarded as constituting a type of failure. The elongation of a bolt hole due to over-load is an example of this type, and may be termed "plastic failure" to distinguish it from the previously mentioned "instability failure."

In order to obtain the most economical designs consistent with safety, both types of failure must be considered, and it is advisable to use different ultimate stresses and factors of safety for the two types. This has been the normal practice in airplane design for several years, and it has proved to be quite satisfactory.

The factor of safety that should be provided for any specific part of a structure depends on both the type of failure it is intended to prevent and the type of ultimate stress used. For plastic failure, the yield point is the most appropriate ultimate stress, and since experience has indicated that a factor of safety of 1.00 is sufficient, that is the value specified by the Civil Aeronautics Authority. This requirement is governing only where it would take a much larger load to cause collapse than that needed to produce excessive plastic flow. It is also equivalent to the author's proposal that a design be checked to insure that the maximum probable load will not cause stresses in excess of the yield point. The justification for such a low factor of safety against plastic failure is that, even if the assumed maximum probable load should be exceeded in service, the result would not be disastrous but would merely cause some deformations of the structure which normally could be repaired satisfactorily.

The factor of safety to be provided against instability failure, however, should exceed 1.00 since otherwise a load in excess of the assumed maximum that is probable would be likely to cause a disastrous collapse. The desirable magnitude of this factor of safety depends on many influences, including the importance of weight economy, the reliability and precision of the methods of analysis, the uniformity of materials used, and the quality of workmanship to be expected. In aeronautical work the importance of weight economy is so great that a factor of 1.50 is the usual value, but larger values are specified for members and types of construction for which experience has shown it to be difficult to predict the strength with precision.

For the benefit of the engineer who may consider these aeronautical factors of safety to be dangerously low, the writer believes a few explanatory remarks are in order. It must first be remembered that these factors are not applied to the loads encountered in normal flying, but only to the most severe loads likely to be encountered in strenuous maneuvers or very rough air. The factors provided against the loads encountered in normal flight are several times the foregoing values. Secondly, the maximum load to which an airplane can be subjected in flight can be predicted with more confidence than the weight of the heaviest truck that may try to cross a new highway bridge. In the third place, airplane designers have the benefit of many static tests to destruction to guide them, but full-scale destruction tests of bridges and buildings are most unusual. As a result, the airplane designer can be more confident that he actually has provided a factor of safety of 1.5 than many designers of other structures can be that they have provided their nominal 2.0 or more. Finally, the fact should be mentioned that major structural failures of airplanes are very rare, and aeronautical engineers therefore consider that, although the factors of safety they use are very low, they are adequate if proper care and skill are used in developing their designs.

The factor of safety may be defined as either the ratio of the ultimate stress to the stress produced by the maximum probable load, or as the ratio of the load that would cause failure to the maximum probable load. The use of a working-stress method of design implies the use of the former definition, whereas the use of an ultimate-stress method implies the latter. If internal stresses remained strictly proportional to external loads right up to the point of failure, it would make no difference which definition and method of design were used. In actual structures, however, this proportionality up to failure is the exception rather than the rule, and usually the second definition results in a lower value for the factor of safety for a given design than the former.

A common type of structural member in which the internal stresses are not directly proportional to the external loads is the "beam-column," or member subjected to combined axial compression and transverse bending loads. If such a member is subjected to a loading that produces a maximum internal stress equal to half the ultimate, the external loads can be increased until the maximum unit stress is doubled before failure takes place. This result will be obtained, however, before the external loads have been doubled. Thus, when the factor of safety in terms of unit stresses is 2.00, it would be less than 2.00 in terms of external loads. In situations of this character it appears much more reasonable to define the factor of safety in terms of load than in terms of stress. A logical consequence is that an ultimate-stress method rather than a working-stress method should be used in design.

An additional reason for this practice is that the use of an ultimate-stress method permits a designer to check the reliability of his methods of analysis by static tests to destruction. If a structure is designed to a factor of safety of 1.50 by an ultimate-stress method, the static test will show, clearly and directly, whether or not the desired end has been achieved. However, if it is designed to a factor of safety of 1.50 by a working-stress method, it is practically impossible to use destruction tests to determine whether the desired end has been achieved. In such a case it may be that the tension members should not fail until the structure is subjected to 1.50 times the design load, whereas some compression members may be expected to fail under 1.35 times the design load. If the compression members failed under 1.38 times the design load, their adequacy would have been established, but the adequacy of the tension members would remain undetermined. Any attempt to check designs made by a working-stress method by means of static tests to destruction involves a mass of interpretative computations which are of doubtful validity. Thus, if tests are to be made (and they are clearly desirable if economically feasible), the designs must be made by an ultimate-stress method.

In much structural design the deviation of the internal stresses from strict proportionality to the external loads is small, the factors of safety provided are relatively large, and weight saving is of minor importance. As a result, working-stress methods of design have given satisfactory results. In airplane design these conditions have not obtained. Types of construction are in common use in which the deviations of the internal stresses from strict proportionality to external loads are large, factors of safety are relatively small, and weight saving is of the utmost importance. As a consequence of these conditions the airplane designers have, for a long time, been accustomed to using ultimate-stress methods of design.

Some types of structural members, notably struts of moderate slenderness, become unstable as soon as the yield point stress is reached, or the load that can be added after the yield point has been reached is negligible. For the design of such members against instability failure the yield point of the material is a logical ultimate stress to be used in practice. Other types of members, such as rectangular beams, must be subjected to considerable increments of load after the yield point has been reached before they collapse. The writer is in complete accord with the author in protesting against the use of the yield point as the ultimate stress in the design of such parts.

In designing to obtain a desired factor of safety against collapse, particularly when it is desired to take advantage of any capacity of the structure to carry a larger load than the minimum necessary to produce elastic buckling or plastic flow in some of its elements, the methods used to compute the imposed stresses must be considered in the selection of the corresponding ultimate stresses. The most common practice is to compute the stresses that would be caused by the ultimate design load under the assumption that the stress distribution would be the same as if the material remained elastic, even though it is recognized that the resulting computed stresses are fictitious. This is the sensible thing to do whenever the corresponding ultimate stresses can be suitably determined. An example is a beam subjected to simple bending. At maximum load the actual maximum unit stress in such a beam is difficult to determine, but it is relatively easy to determine a "modulus of rupture," assuming that the formula $f = \frac{M\,y}{I}$ remains valid.

With some other types of structure this procedure is inapplicable, as some parts will buckle elastically under a fraction of the load needed to cause failure of the whole. In most cases of elastic buckling of a redundant member, the buckled member continues to carry the load which caused the buckling but refuses to carry any more when additional external load is imposed on the structure. Thus, when a plate girder with a very thin web is subjected to simple bending under low loads, the shear on the girder is transmitted primarily by shear stresses in the web. These shear stresses may be resolved into diagonal tensile and compressive stresses acting at right angles to each other. As the external load is increased, the diagonal compressions cause the web to buckle into a series of waves or wrinkles parallel to the diagonal tensile stresses. If the web were not provided with suitable vertical stiffeners, this buckling would result in instability and collapse of the entire beam. When the stiffeners are provided, however, increases in shear can be carried by increased diagonal tensions in the web, compression in the vertical stiffeners, and suitable stresses in the chord members, and that is just what takes place. If the buckling of the web into wrinkles were considered to represent failure, the ultimate strength of the girder would often be only a small fraction of the load actually required to cause collapse. The airplane designer recognizes this fact and uses the actual load-carrying capacity of the girder as its ultimate strength for use in design. In computing the stresses present when the girder becomes unstable the more conservative method is to disregard the diagonal compressive stresses that are actually carried across the wrinkles. Some airplane designers, however, prefer to include these compressive stresses as part of the resistance of the girder to load, and to count on this resistance in design.

A similar situation is found in the panels of thin sheet metal with longitudinal stiffeners that are frequently used in the construction of airplane fuselages. In this construction the parts of the sheet midway between stiffeners buckle elastically under relatively low loads. As the loads increase, the width of the buckled sheet also increases, increments of load being carried by the stiffeners and the parts of sheet adjacent to them. Eventually only the stiffeners are able to provide resistance to increments of load, and when the critical stresses are reached in the stiffeners, the entire structure collapses. One might say that the proper method of design would be to consider only the strength of those parts of the combination that are still accepting increases in load when the ultimate load is approached in computing the strength of the panel. This is not the practice, however, as the parts that have buckled elastically are still carrying some of the load, and the carrying capacity of those parts is included in computing the strength of the unit. The magnitude of this supporting effect of elements that will have buckled elastically is often difficult to predict, and the question is the subject of much research, both theoretical and experimental.

Although it is common practice in airplane design to take cognizance of the changes in stress distribution due to elastic buckling in computing ultimate loads, it is not usual to do the same with respect to changes in stress distribution caused by plastic flow. There would appear to be no valid objection to doing this if the detailed procedure employed were logically sound. The author offers certain suggestions along this line, but some of his detailed proposals are open to serious question.

In his first case the author assumes that after the elastic-limit stress has been reached at the supports of a beam fixed at both ends, the section at each support acts as a pin joint. Thus, if the load of 10 000 lb per ft produces a moment of 30 000 ft-lb and that moment produces the yield-point stress in the outer fibers, the resisting moment at the support under a load of 13 333 lb per ft will also be only 30 000 ft-lb. This is far from being true. Under 10 000 lb per ft there will be linear variation of stress over the cross-section, and only the outermost fibers will be subjected to the yield point stress. Under any additional load the deformation of the end sections will increase, more and more fibers will be subjected to the yield point stress, and the resisting moment will increase. The result will be that the movement of the points of inflection toward the supports will be less than that estimated by the author. From this it might be argued that the author's method is conservative, and that may be true, but before applying it in practice a more careful study of all the factors involved should be made.

In his second to fourth cases the author makes a study of the effect of unloading and repetition of load, but omits any consideration of the distribution over the cross-section of the residual strains and internal stresses that he assumes to exist. His conclusions may be useful in stimulating research but should not be used as a basis of design until the distribution of such strains and stresses has been more thoroughly investigated.

Another factor that should be given careful study before adopting the author's detailed procedure for cases involving plastic flow is the shape of the stress-strain diagram of the material used. Ordinary structural steel has a definite yield point, but this is not true of structural aluminum alloys, stainless steel, or many of the high-strength steels that are coming into use. This means that an important factor underlying the author's proposals is lacking when applied to the materials used where economy of weight is an important consideration. His theory may be modified to apply to these materials without definite yield points, but for these, as well as for structural steel, it requires more extensive digestion than is indicated in the paper.

A. Floris,[44] Esq. (by letter).—When the stresses in some parts of the statically indeterminate structure reach the plastic limit of the material, other less stressed parts come into play, thus causing a better distribution of stress. This equalization, commonly called "self help" of the redundant structure, is the thesis of the author's paper on limit design.

In some instances, this equalization occurs quite naturally, without the aid of the plastic property of the material. In a beam fixed at both ends and carrying a vertical concentrated load in the middle of the span, for example, the bending moments at both ends and in the middle are equal and of opposite signs. Within the elastic limit of the material, therefore, the ratio of positive and negative moments is 1 : 1. Such cases are rare, however, and, in general, recourse must be taken to the plastic behavior of the material, if an equalization of stress is to be achieved.

In this thought provoking paper the author treats the fixed-end beam exclusively. Unfortunately, the use of this structural element is rather restricted in practice. Continuous structures—beams (which are very common) as well as rigid frames—have been ignored. No doubt, this is a very serious omission.

Steel structures of these types can be analyzed conveniently by means of factors that take care of the moment reduction caused by the plastic flow of the material. As soon as this stage is reached, imperfect hinges are formed in places where the bending moments have their greatest value, thus relieving the strains by redistribution. The ensuing greater freedom of motion in the structure can easily be counterbalanced by the remaining numerous constraints of the manifold redundant structures.[45] In the light of the theory of plasticity, therefore, highly redundant systems are safer against the effect of unforeseen emergencies, including settlements of supports and destructive earthquakes. This is quite the opposite of the results obtained by the elastic theory.

Although the theory of plasticity is readily applicable to steel structures, in the case of reinforced concrete the problem is more complicated. In this composite material, only the steel reinforcement has a well-defined yield point; the concrete is a brittle material. However, as G. von Kazinczy has shown experimentally,[46] the compressive strength of concrete remains constant during the initial yielding of steel. On the basis of this fact it is possible to develop a theory of stress analysis and design.

In statically determinate framed structures it is always possible to stress all bars to the limit; but it is never possible in internally redundant frames, if the dimensioning is done in accordance with the elastic theory. In such structures of n-fold static indeterminacy, it is possible to stress all except n bars to the limit. These n bars constitute, so to speak, the reserve strength of the frame. Consequently, the greater the number of redundants in relation to the remaining bars, the greater will be the economic advantages by applying the theory of plasticity.[47]

---

[44] Dipl.-Ing., Los Angeles, Calif.

[45] "Der Momentenausgleich durlaufender Traggebilde im Stahlbau," by Felix Kann, Berlin, Germany, 1932.

[46] "Die Plastizität des Eisenbetons," by G. von Kazinczy, *Beton u. Eisen*, 1933, p. 74.

Framed structures designed either by the elastic or plastic theory are perfectly safe. The advantages gained by the use of the theory of plasticity are economy of material and simplicity of analysis. Thus, the most economical solution can be found with the least effort.

Redundant compression bars cannot be included in the analysis of the structure for the following reasons: When the buckling strength is reached, the resistance of the bar ceases and the supporting capacity of the redundant frame is reached almost instantaneously. This has been demonstrated by several tests.[48] The factor of safety is much larger for compression than for tension bars. The latter can sustain considerably plastic deformations without greatly endangering the safety of the structure. However, as soon as the former reach their buckling strength, the frame will collapse immediately. The redundant tension bars having a smaller factor of safety will reach the plastic stage earlier than the redundant compression bars with a greater factor of safety.

For example, by removing the redundant bar of a frame, statically indeterminate in the first degree, in which the tension reaches the permissible stress first, a force is applied in the direction of this bar equal to the allowable unit tensile stress multiplied by an arbitrarily chosen cross-section. This force, combined with the external loads, determines the axial forces of the bars, which later are dimensioned on the basis of the allowable unit stress. In designing a framed structure in accordance with the theory of plasticity, therefore, the permissible unit stress is not eliminated. The method proposed in this paper, to design framed structures for an over-load capacity in order to avoid the use of the allowable unit stresses, is objectionable because of the foregoing reason concerning factors of safety.

In the following a simple example will illustrate the points in question: The cantilever frame composed of Bars *ab*, *ac*, and *ad*, Fig. 37 has a span of 6.5 ft, carries a load of 75.0 kips and is statically indeterminate in the first degree. All double angles are $\frac{3}{8}$ in., back to back, and all bars have welded connections.

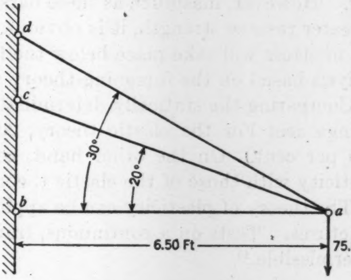


FIG. 37.—CANTILEVER FRAME

By using the elastic theory, the axial force in any of the three bars can be chosen as the redundant quantity. In the present case the axial force of Bar *ad* is taken as the redundant. Inasmuch as the frame has one degree of statical indeterminacy, the stresses in one bar only cannot be utilized completely. This is Bar *ac*, as the results of the elastic theory have shown. After repeated trials, the most economical solution has been determined, as given in Table 3, in which the solution of the statically determinate frame is included for the purpose of comparison.

---

[47] "Zuschrift," by E. Melan, *Der Bauingenieur*, 1938, p. 488.

[48] "Drei Beiträge zur Frage des Tragvermögens statisch unbestimmter Stabwerke," by E. Chwalla, International Assoc. for Bridge and Structural Eng., 1933–1934, Vol. II, p. 96; see also "Versuche mit innerlich statisch unbestimmten Fachwerken," by G. von Kazinczy, *Der Bauingenieur*, 1938, p. 236.

According to the results of the elastic theory, Bar *ad* is the only tension member in the frame. Even in a different combination of sections and possible reversal of sign in Bar *ac*, the tension in Bar *ad* will always reach the permissible stress first. In all cases Bar *ab* is under compression and consequently is excluded from consideration. For this reason only Bar *ad* can be subjected to plastic deformation.

TABLE 3.—VARIOUS SOLUTIONS OF THE CANTILEVER FRAME IN FIGURE 37

Bar	STATICS			ELASTICITY			PLASTICITY		
	Dimensions, in inches	Axial force, in kips*	Unit stress, in kips per square inch*	Dimensions, in inches	Axial force, in kips*	Unit stress, in kips per square inch*	Dimensions, in inches	Axial force, in kips*	Unit stress, in kips per square inch*
ad	....	....	....	2—4×3½×⅝	+154.0	+17.9	2—4×4×½	+135.0	+18.0
ac	2—7×3½×⅝	+219.0	+17.8	2—2½×2½×½	− 8.0	− 3.4	2½×½	+ 22.0	+17.6
ab	2—6×6×⅝	−206.0	−14.5	2—4×4×⅝	−125.0	−13.6	2—4×4×⅝	−137.0	−14.8

* Plus denotes tension and minus denotes compression.

By removing and replacing Bar *ad* by a tensile force, equal to the product of allowable tensile stress by an arbitrarily chosen sectional area, the axial forces in the other bars are determined by statics. Selecting for Bar *ad* a smaller section than that given by the elastic theory, it is obvious that the stress in this bar will be nearer to the plastic deformation than in all other bars. However, inasmuch as these bars, in accordance with the design, possess a greater reserve strength, it is obvious that the force distribution or equalization of stress will take place below the limit of safety. The data of the frame analysis based on the foregoing theory are also given in Table 3.

Comparing the statically determinate frame with the redundant frame, the savings are: For the elastic theory, 20.3%, and for the theory of plasticity, 29.5 per cent. On the other hand, comparing the results of the theory of plasticity with those of the elastic theory, the saving is 11.5 per cent.

The theory of plasticity can be applied also to externally redundant framed structures. Tests on a continuous, framed beam have demonstrated that this is permissible.[49]

L. H. NISHKIAN,[50] M. AM. SOC. C. E. (by letter).—Many experienced structural engineers have long recognized that, generally, in a redundant structure those structural elements best able to resist any given set of forces will ultimately carry the major part of the load if the structure is sufficiently loaded. (The term "redundant structure" as used herein does not necessarily mean a structure with more than the minimum number of members. A continuous beam or other indeterminate structure where all members or all

---

[49] "Tragfähigkeitsversuche an einem durchlaufenden Fachwerkbalken aus Stahl," by G. Grüning and E. Kohl, *Der Bauingenieur*, 1933, p. 67.

[50] Cons. Engr., San Francisco, Calif.

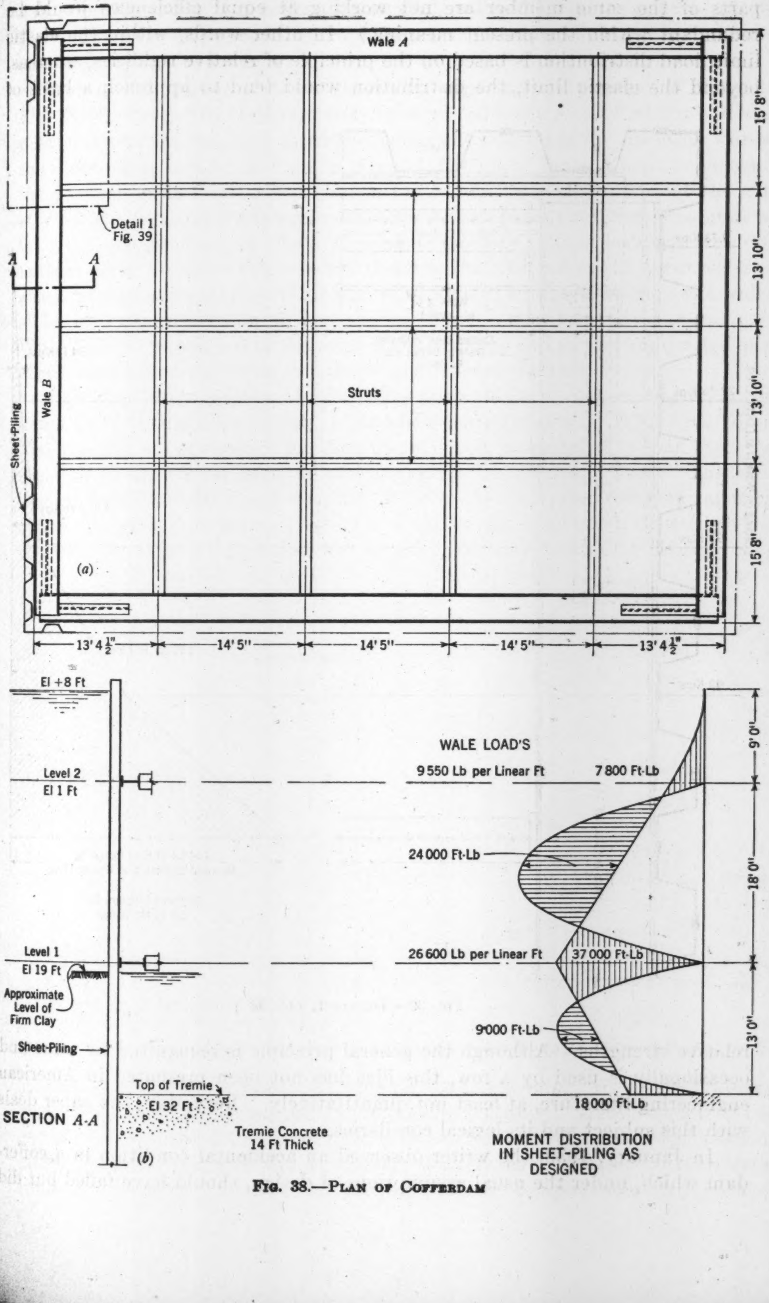

Wale A
Detail 1
Fig. 39
A
A
Wale B
Sheet-Piling
Struts
(a)
15' 8"
13' 10"
13' 10"
15' 8"
13' 4 1/2"
14' 5"
14' 5"
14' 5"
13' 4 1/2"
El +8 Ft
Level 2
El 1 Ft
Level 1
El 19 Ft
Approximate Level of Firm Clay
Sheet-Piling
Top of Tremie
El 32 Ft
SECTION A-A
Tremie Concrete 14 Ft Thick
(b)
WALE LOAD'S
9 550 Lb per Linear Ft
7 800 Ft-Lb
24 000 Ft-Lb
26 600 Lb per Linear Ft
37 000 Ft-Lb
9 000 Ft-Lb
18 000 Ft-Lb
9' 0"
18' 0"
13' 0"
MOMENT DISTRIBUTION IN SHEET-PILING AS DESIGNED

FIG. 38.—PLAN OF COFFERDAM

parts of the same member are not working at equal efficiencies would be redundant within the present meaning.) In other words, within the elastic limit, load distribution is based on the principle of relative rigidities, whereas, beyond the elastic limit, the distribution would tend to approach a basis of relative strengths. Although the general principle is recognized by some and occasionally is used by a few, this idea has not been presented in American engineering literature, at least not quantitatively. Very ably, the paper deals with this subject and its logical corollaries.

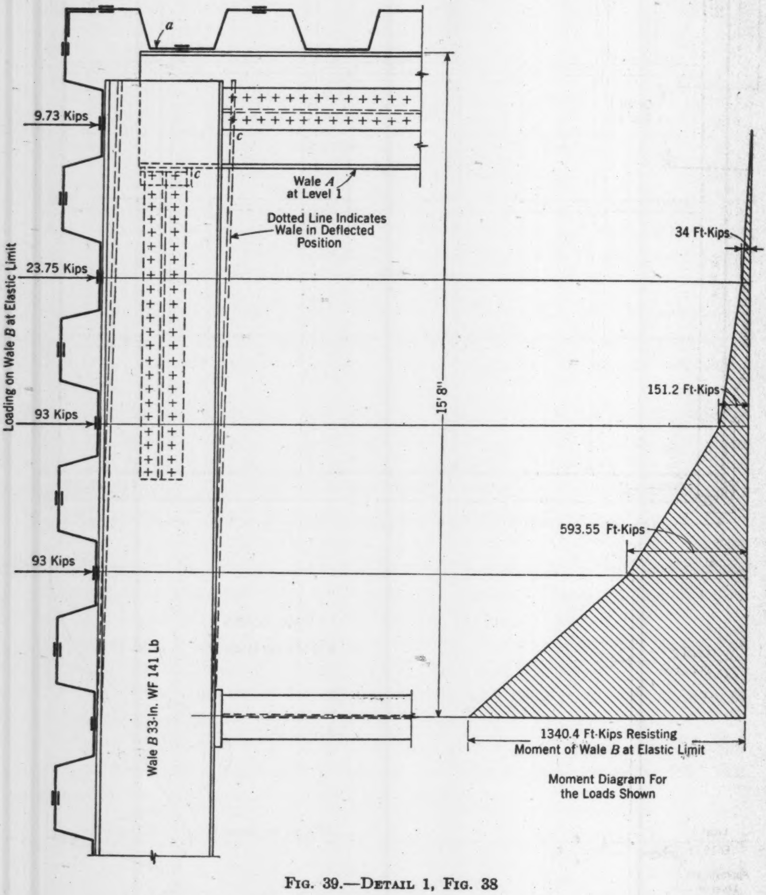
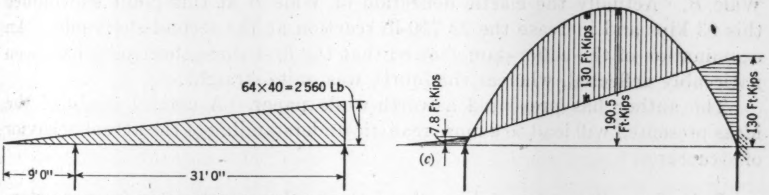

a
9.73 Kips
c
c
Wale A at Level 1
Dotted Line Indicates Wale in Deflected Position
Loading on Wale B at Elastic Limit
23.75 Kips
93 Kips
93 Kips
15'8"
Wale B 33-In. WF 141 Lb
34 Ft-Kips
151.2 Ft-Kips
593.55 Ft-Kips
1340.4 Ft-Kips Resisting Moment of Wale B at Elastic Limit
Moment Diagram For the Loads Shown

FIG. 39.—DETAIL 1, FIG. 38

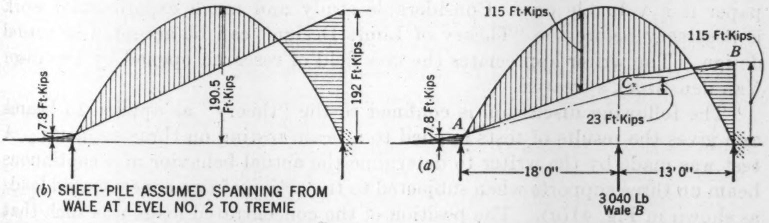
In January, 1939, the writer observed an accidental condition in a cofferdam which, under the usual assumptions of design, should have failed but did not. Fig. 38 shows this coffer-dam as designed and built. Fig. 38(b) shows the loads on the wales resulting from the water pressure when pumped out to a level just below the lower wale.

During the driving of the sheet-piling, apparently the corner of Wale *A*, Fig. 38, was caught by the lower end of the pile with the result that one end was pushed down, twisted, and entirely disengaged from Wale *B*. In other words, the sheet-piling caught at Point *a*, Fig. 39; the lower Wale *A* bent down; and the connection at Point *c* was broken loose, removing all support at the end of Wale *B*. Wale *A* was so connected to Wale *B* that it acted as a column to take the end reaction of Wale *B*—about 185 kips. This accident was not noticed until the water was pumped down to the level below the lower wales—about Elevation − 20 ft. With the water at this level, the load on the wales at Level 1 was more than 95% of the maximum load when fully pumped out. With Wale *A* pushed out of the way, Wale *B* had no end support and acted as a cantilever from the next strut—about 15.5 ft from the end.

Checking the capability of Wale *B* to carry this load as a cantilever, it was found to be entirely inadequate, even when using stresses of 36 kips per sq in. The capacity of the sheet-piling then was checked, assuming it to span from the top wale to the tremie concrete and assuming the lower end fixed. This also was inadequate at 36 kips per sq in. Adding the maximum carrying capacity of the sheet-pile and Wale *B*, acting as a cantilever, both being stressed to the elastic limit, it was found that the resulting combined carrying capacity was still less than the water pressure acting on the coffer-dam. By all the ordinary rules, Wale *B* and the sheet-piling should have collapsed. Measurements at the end of Wale *B* showed a horizontal deflection in excess of the elastic deflection of either the sheet-piling or the cantilever, Wale *B*, each stressed to the elastic limit of the steel. This corroborated the previous computation that the steel



(*a*) LOADING ON SHEET-PILING, IN POUNDS PER FOOT OF WIDTH



(*b*) SHEET-PILE ASSUMED SPANNING FROM WALE AT LEVEL NO. 2 TO TREMIE

Fig. 40.—Loading and Moment Diagrams (Back Pressure of Water in Cofferdam Neglected)

would be stressed beyond its elastic limit, but did not explain why it did not fail.

An analysis was then made along lines similar to those suggested by the author. If the first two sheet-piles continued to yield until the negative moment (never more than the resisting moment of the section at the elastic limit) at the tremie support became equal to the positive moment in the sheet-piling, and the remaining load was taken by Wale *B* as a cantilever, it was found that a balance would be struck although both the sheet-piling and the wale would be strained beyond their elastic limits. Figs. 39 and 40 show the moments resulting under this assumption. Fig. 40(*b*) demonstrates that a resisting moment of 192 ft-kips, within the elastic limit, is necessary to resist the loading given in Fig. 40(*a*). Even if a greater efficiency is effected by exceeding the elastic limit (see Fig. 40(*c*)), it is still necessary that the sheet-piling have a resisting moment of 130 ft-kips at the elastic limit to avoid failure. Since the resisting moment at the elastic limit is only 115 ft-kips, a reaction of 3 040 lb (as indicated in Fig. 40(*d*)) must be developed at Wale *B*, Fig. 38, to prevent deflection to the point of failure. Line *AB*, Fig. 40(*d*), was determined by the resisting moment at the elastic limit of the sheet-piling (115 ft-kips). Line *AC* was drawn in such a manner that the moment intercepted at the center was also 115 ft-kips. This can be produced only by a reaction of 3 040 lb at Wale *B*.

In Fig. 39 the third sheet-pile from the end is shown reacting 93 kips on Wale *B*. Actually the elastic deflection of Wale *B* at this point will reduce this 93 kips and increase the 23 750-lb reaction at the second sheet-pile. An examination of the coffer-dam showed that the first three sheet-piles had been noticeably deflected, whereas the fourth was quite straight.

The author has presented a worth-while paper. A careful study of the ideas presented will lead to a more realistic understanding of the actual behavior of structures.

F. G. ERIC PETERSON,[51] ESQ. (by letter).—Inasmuch as it focuses attention on possible revisions of design theory now used in engineering practice, this paper is a valuable one. Considerable study and much experimental work is necessary before the "Theory of Limit Design" can be accepted in actual design. The writer appreciates the new field of research opened by Professor Van den Broek's treatise.

The following discussion is confined to the "theory" as applied to beams and gives the results of tests applied to a beam resting on three supports. A test was made by the writer to determine the actual behavior of a continuous beam on three supports when subjected to the action of two concentrated loads as shown in Fig. 41(*a*). The position of the concentrated loads was such that the moment over the center support, as determined by the principle of continuity applied to an elastic structure, was a maximum. Before cutting the beam to the desired length a short piece was sawed out and submitted to the usual tensile test, the results of which were 38 600 lb per sq in. for the yield point and 61 700 lb per sq in. for the ultimate strength.

---

[51] Instructor in Mathematics and Mechanics, Univ. of Minnesota, Minneapolis, Minn.

Referring to Fig. 42: For stresses below the elastic limit the moment over the center support and the moment under the concentrated loads will equal, respectively:

$$M_1 = -0.192\frac{PL}{2}\ldots\ldots\ldots\ldots\ldots\ldots(27a)$$

and

$$M_2 = 0.133\frac{PL}{2}\ldots\ldots\ldots\ldots\ldots\ldots(27b)$$

From the foregoing yield point stress, and the structural properties of the beam, it is found that the maximum resisting moment of the beam with no stresses exceeding the yield point is equal to 7 720 ft-lb, corresponding to a total load of 13 400 lb, or 6 700 lb on each span.

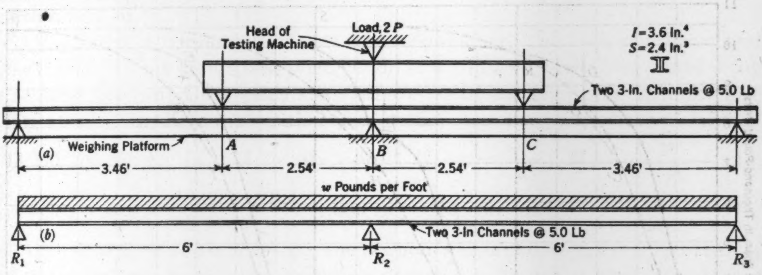


FIG. 41

According to Professor Van den Broek's theory the moment over the center support would reach a certain maximum and then remain constant at this value with increasing load until the moment under the loads reached this same value, at which time the beam would fail. From the test made by the writer this does not seem to be the case. Professor Van den Broek acknowledged the fact that the negative moments at the supports of his fixed beam increased somewhat as the load is increased. The writer believes from his test that as the load is increased the positive moment under the load and the negative moment at the supports increases appreciably until the failure load is reached. Of course, after the extreme fiber at a section has reached the yield point the resisting moment at this section increases more slowly but will not reach a constant value until failure defined as continued deflection without increase in load occurs. At this point the critical sections act as elastic hinges.

A separate test was made on a simple beam 5 ft long of the same cross-section as the beam continuous over three supports. When a single concentrated load at mid-span reached a value of 7 800 lb the beam continued to deflect without appreciable increase in load. This gives a resisting moment of $M = \frac{PL}{4} = 9\,750$ ft-lb and would seem to indicate a stress of $s = \frac{9\,750 \times 12}{2.4} = 48\,750$ lb. Of course, this is not a true stress since the outer fibers were stressed beyond the yield point somewhat before failure occurred; but it is comparable to the usual modulus of rupture. Almost all of the cross-section is at the yield point stress but has not begun to approach the ultimate strength.

Assuming that the moment over the center support and also under the loads could reach this value of 9 750 ft-lb before deflection without increase in load, the load on each span could be 10 500 lb or the total load 21 000 lb. This value is in close agreement with the test made as the maximum load was found to be somewhat more than 20 000 lb.

The beam was loaded as indicated in Fig. 41(*a*). At each load increment strain gage readings were taken, four at each of the three sections *A*, *B*, and *C*, using a 2-in. gage. The graphs of Fig. 42 show the strains at the extreme fibers corresponding to the loads. The solid lines show the measured strains; and they appear to be somewhat smaller than those computed from the loads indicated by the testing machine, probably attributable to the fact that the strain gage records average strains over a distance which has a varying bending moment. Furthermore, the gage points were placed at the edge of the flanges and therefore a little closer to the neutral axis than the extreme fibers. The vertical lines at *D*, *E*, and *F*, Fig. 42, indicate the yield point strain (approximately). The line *L H* is the computed load strain line at Section *B* up to a load of 6 700 lb. At Point *H* the extreme fibers at Section *B* have reached the yield point. Lines *K G* and *M I* represent the load strain line for Sections *A* and *C*, respectively. If the moment at Section *B* remained constant when the extreme fiber reached the yield point the load strain curve at Points *A* and *C*, Fig. 41(*a*), would be as *G D* and *I Q*, respectively, up to the yield point strain of the extreme fibers at these sections. However, since Section *B* gives an increasing resisting moment beyond this point the strains at Sections *A* and *C* would increase less rapidly and would follow curves similar to *G R S* and *I T V* instead. Points *S* and *V*, Fig. 42, are on a horizontal line at the theoretical maximum load and are obtained by neglecting a section 0.5 in. above and below

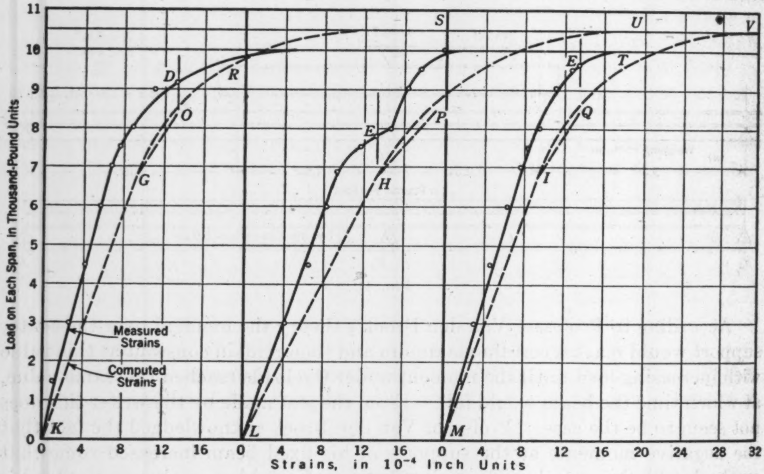


FIG. 42

the neutral axis. This section has a moment of inertia about one one-hundredth of the total section.

Applying a factor of safety of 2, the maximum total load would be 10 500 lb (calculated) or, the load P on each span would be 5 250 lb. Note that the load necessary to cause a yield-point fiber stress at the center support is 13 400 lb or about one-third higher. Allowing a maximum fiber stress of 18 000 lb per sq in. (which is common practice) the allowable total load would be $P = \frac{2 M_1}{0.192 L} = 6\,250$ lb, or 3 125 lb on each span. This load allows a factor of safety of about 2 for a yield point stress of 38 600 lb but a factor of safety of 3.68 for the structure as a whole. It would thus seem that for statically indeterminate structures that the theory of limit design shows that higher loads can be used with safety.

The Case of a Uniform Load Over Three Supports.—The ordinary theory and the theory of limit design is applied herein to a uniformly distributed load using the same beam and the same span lengths as in the preceding example (see Fig. 41(*b*)). In the ordinary theory of design, assuming a maximum stress of 18 000 lb per sq in.: $M_1 = M_3 = 0$; $4 M_2 = -\frac{w l^2}{2}$; $M_2 = -\frac{w l^2}{8}$; and, $w = \frac{8 M_2}{l^2} = \frac{8 \times 18\,000 \times 2.4}{36 \times 12} = 800$ lb per ft, safe load.

In the theory of limit design, the section was found to resist 9 750 ft-lb of moment before appreciable deflection occurred. Assuming that M_2 reaches this value, the minimum load per foot necessary to cause the same positive moment at some point in the beam, is obtained as follows: $R_1 = \frac{\frac{1}{2} w l^2 - 9\,750}{l} = 3 w - 1\,625$. The moment at any point in the beam is

$$M = R_1 x - \frac{w x^2}{2} = (3 w - 1\,625)\, x - \frac{w x^2}{2} \ldots\ldots\ldots (27c)$$

Setting Equation (27*c*) equal to 9 750 ft-lb, $w = \frac{19\,500 + 3\,250\, x}{6 x - x^2}$; $\frac{dw}{dx} = \frac{3\,250\,(6 x - x^2) - (19\,500 + 3\,250\, x)\,(6 - 2 x)}{(6 x - x^2)^2} = 0$; and, $x = 6\,(\sqrt{2} - 1)$ gives a minimum value of $w = 3\,160$ lb. The value of $x = l\,(\sqrt{2} - 1)$ found herein holds for any maximum resisting moment of any beam of two spans of equal length on three supports.

Applying a factor of safety of 2 to this load results in a safe load of 1 580 lb per ft. The maximum deflection with this load is approximately 0.12 in. Since this load is almost sufficient to cause yield-point stress over the center support the practicing engineer might wish to reduce it. However, even if it is reduced to 1 200 lb it is still 50% greater than the value obtained by the customary method. Note that, in this method, the concept of stress did not enter.

The writer wishes to acknowledge the suggestions and assistance of Paul Andersen. Assoc. M. Am. Soc. C. E., in the preparation of this discussion.

E. S. Fabian,[52] Assoc. M. Am. Soc. C. E. (by letter).—The essential functional requirements of any structure are: (1) That it be strong enough to withstand the application of the loads of specified magnitude; and (2) that under the application of these loads it exhibit a certain degree of resistance to deformation and thus maintain its geometric integrity. Strength and stiffness within the prescribed limits are the basic considerations of any design theory whether applied to individual members or to entire structures. Since the indices of both the strength and the stiffness of any structural member are expressed in terms of its dimensions and the physical properties of the material of which it is made, strength and stiffness of a structural member are definitely related. Parametric equations for strength and for stiffness can be written in each case and, by varying the parameters, simultaneous values can be obtained from which curves can be drawn. From these curves the functional relation of strength and stiffness can be ascertained conveniently. It is the character of this function that determines whether the principle of superposition (see heading "Introduction") applies.

To admit the application of the principle of superposition to a structure, the following conditions must be postulated:

(1) The character of the impressed load must not change as the load is increased;

(2) The geometric configuration of the structure must not appreciably change with the increase in the load;

(3) As the Load P changes, the axial (and bending) loads on each member of the structure must change proportionally to the change in Load P;

(4) As the loads on the members change, with the change in Load P, the unit stresses and total deformations must remain proportional to the loads; and,

(5) The total deformations of the various members—tensile, flexural, and compressive—must be of the same order.

Conditions (1) and (2) must also apply to each member in the structure.

The author's predication (see heading "Introduction") "that a structure will fail only when as many members, or restraints, as the structure has redundants plus one, are loaded to their capacity strength" and his third dictum (heading "Theory") are unquestionably valid within the limits of the operation of the principle of superposition; so is his design procedure as illustrated by the application to the design of a tower. As the writer understands it, this design procedure consists of three main steps which can be outlined as follows: (*a*) Apply an over-stress factor, n, to the design load of a structure; (*b*) on the basis of this "capacity" load, draw a continuous force diagram, and if it closes, it may be regarded as satisfactory; and (*c*) assume the dimensions of one member, in conformance to the force diagram, and proportion the neighboring members in relation to it as indicated by the force diagram.

To design a structure in this manner on the basis of the configuration of the structure when it is unloaded, is to assume that this configuration will not change appreciably while the capacity load is gradually diminished to zero value and then gradually increased back to the $\frac{1}{n}$th portion of its capacity value. To assume that when this is accomplished each member (which was designed for its capacity load) will be subjected to $\frac{1}{n}$th portion of its capacity load, is to postulate that throughout the process of unloading and partial loading the loads on each member, its unit-stresses and its total deformations, are constantly proportional to Load P. It must be assumed that

---

[52] Asst. Prof. of Civ. Eng., Univ. of Tennessee, Knoxville, Tenn.

$$\frac{dP'}{dP} = \frac{ds}{dP} = \frac{d\Delta}{dP} = \text{a constant} \ldots\ldots\ldots\ldots\ldots (28)$$

in which P = the total load on the structure; P' = the load on a member; s = the unit stress in the member; and Δ = the total deformation of the member. This assumption is equivalent to stating that the structure must act as a "unit"; that is, the structure acts as a whole and is analogous to a single member of like character. However, a condition of this nature can be approximated only in extremely rigid structures, in which the members at each joint can be assumed to be of virtually equivalent rigidities, giving the structure a character of homogeneity. Their axial deformations, whether tensile, compressive, or flexural, can be assumed to be very small, and the resultant displacements of the joints are negligible. The application of the principle of superposition can be admitted and the design can be made on the basis of static equilibrium. However, due attention to permissible joint displacements and consequent deformations has resulted, in the case of suspension bridges, in considerable economy of design.[53] Similar economy can undoubtedly be shown for other structures.

The stiffness of a structure as a whole is a function of the ratio of the loads (dead and live) that the structure must carry to its over-all dimensions. In certain types of structures this ratio is rather small and, if the deflections are of no consequence and are permitted to be large, the design results in a "light" type of structure. The behavior of these structures under applied loads is very dissimilar to the behavior of structures of a heavy or rigid type. This latter type will display a degree of elastic stability under any condition of stress intensity in its members up to the yield point. (In certain cases elastic stability may persist when the stresses in some members are beyond the yield point.) The light type of structure, however, may reach a condition of neutral equilibrium, presenting the danger of failure by buckling, when the magnitude of stresses in its members is considerably below the elastic limit. This dissimilarity of action can be explained only by the supposition that the ratios of stresses to deformations are not the same in the two types. In a light structure the deformations proceed at a greater rate than the stresses, or, stated briefly, the principle of superposition does not apply. Such light structures are very sensitive to the effects of the eccentricities and nonsymmetries at the joints, and their light members will often be subjected to a combination of axial and lateral loads.

---

[53] *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 894.

When applied to whole structures, the principle of superposition must be postulated from the proportionality of total deformations to the applied loads. In the manner indicated in the first paragraph of this discussion, it can be shown that, in general, the relation between the capacity loads of various structural members and their stiffnesses is not a straight-line function. It is an asymptotic curve, similar in many respects to Euler's curve, the asymptotic part of which is in the range of low slenderness ratios (that is, high stiffness) of the members. Assuming this part of the curve to be a straight line it can be seen that the axial deformations of heavy, stiff members, whether they are subjected to tensile, flexural, or compressive loads, will be of the same order. Experiments validate this assumption. As the curve recedes from its asymptote in the direction of higher slenderness ratios, the rapid change in the slope of tangents to the curve indicates that such similarity in axial deformations in slender members cannot be assumed. In other words, the principle of superposition cannot be applied to structures joined by slender members of heterogeneous character. For similar reasons the principle of superposition cannot be applied to slender members subjected to the simultaneous action of heterogeneous loads such as compressive and bending loads. It can be shown that under these conditions the load-resisting function of a member, as regards lateral loads, is approximately proportional to 1 minus the ratio of the applied axial load to the capacity axial load.[54] By analogy a similar proportionality can be postulated for the structure as a whole.

Where the conditions are such that the law of superposition cannot be applied, the distribution of loads to various members cannot be made on the basis of simple statical conditions—namely, the force diagram—but it must be made on the basis of Maxwell's equations—that is, by the "influence-line" method. Herein, to the writer's mind, lies the anomaly of the author's thesis. Although Professor Van den Broek emphasizes the importance of deformations, the force diagram is the key to his analysis; although he holds the deformations of a structure to be of prime importance, he criticizes, as unsound, the very methods of analysis which are based on considerations of deflections.

Just how far a theory postulated upon the law of superposition applies to structures to which the law does not apply, and within what limits of deviation from this law, is not a problem pertinent to this discussion of limit design. A theory pushed beyond its rational limitations becomes an hypothesis. That the standard specifications, formulas and methods used in structural design are open to questioning cannot be denied. They have been developed with reference to certain types of structures and to meet specific problems. Their application to the light type of structures, particularly of the highly indeterminate character, may be questioned on the grounds of economy. To this end, the author has rendered a valuable service in raising the problem. The theory of limit design is undoubtedly applicable in certain cases. With modifications, particularly with reference to compression members and the factor of safety, perhaps its application could be extended. The limits of its application and the extent of the necessary modifications can be ascertained, however, only by considerable additional analysis and by checks against the methods that experience has shown to be reliable and economical.

---

[54] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 882.

A. A. Eremin,[55] Assoc. M. Am. Soc. C. E. (by letter).—In the last half of the Nineteenth Century various theories of failures in structural materials were developed. However an extensive theoretical and experimental study is needed to apply them to the computation of the stability of structures. Development of the limit-design method based on the theory of failure may be found in various practical rules applied to the design of structures.

The most common application of the limit-design method is in the design of foundations for statically indeterminate structures. It is well known that foundation deformations do not follow Hooke's law. Therefore, in each case the designer establishes the maximum possible displacement of the footings, and computes the factor of stability of the structures in carrying a given loading. An interesting illustration of the practical application of limit-design computation is in determination of the critical loading carried by a group of piles under spread footings. The piles with the least bearing resistance settle faster and approach a uniform distribution of loading on all piles in the group, which determines the limit-design loading. Computation of the critical loading carried by structural details, such as hinges or pins, is often based on practical rules established by the principles of the limit-design method. Likewise, in the design of rivet connections the principles of the limit design may be considered. Interesting computations of the distribution of deformations in rivets were shown by the late G. F. Swain, Past-President and Hon. M. Am. Soc. C. E.[56] An application of the limit-design method to the computation of the critical loading in statically indeterminate structures has been demonstrated by Professor Timoshenko.[57] Professor Timoshenko has shown that strains in a member with stresses above the elastic limit may be expressed by the average modulus of elasticity, and a critical loading carried by the structure may be computed with the equations based on the elastic theory.

It is unfortunate that the author, in Cases 1 and 3, has considered the beams with constant moments of inertia. In important srtuctures the moment of inertia is varied according to the maximum bending moment stresses. Evidently the economic relation of the limit-design and elastic methods in the design of beams with curved haunches will not be the same as that shown by the author.

The elastic theory is advantageous in that it may readily be verified by tests, whereas the limit-design method will require greater experimental and theoretical study before conclusive results will be obtained.

L. H. Donnell,[58] Esq. (by letter).—This paper is an important contribution not only to the field of structural analysis, but to fundamental engineering philosophy.

---

[55] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[56] "Structural Engineering: Strength of Materials," by G. F. Swain, 1924, McGraw-Hill Book Co., Inc., p. 408.

[57] "Strength of Materials," by S. Timoshenko, Part II, 1930, p. 668.

[58] Associate Prof. of Mechanical Eng., Armour Inst. of Technology (formerly in chg. of Stress Analysis and Structural Research, Goodyear-Zeppelin Corporation), Chicago, Ill.

Every one knows, but sometimes needs to be reminded, that the real object of engineering design is not to insure that stresses remain below a certain limit, but to build the most efficient structures and machines that will perform, satisfactorily, the functions for which they are intended. The method of computing stress distributions under purely elastic conditions, and comparing these with the stresses that will cause some elastic buckling, or other type of "failure," has been, and remains, a most useful tool for this purpose, but designers have long been familiar with cases in which its strict application would be not only very difficult but also highly misleading. This paper demonstrates that in other cases where designers have considered the elastic approach the most advanced method possible, another method may be simpler and may give a more accurate measure of the capacity of structures to perform their real functions satisfactorily.

Undoubtedly, this entire subject has been studied too little, with the result that there are few who have a clear, well-thought-out understanding of it. Designers should not be afraid of such questioning and reviewing of engineering fundamentals, and surely there can be no better place for this than the national forums of the great engineering societies.

The method in question is one which all designers have long accepted in certain fields of engineering, and which in essence has been accepted in some other countries in the structural field. Professor Van den Broek has made important contributions to its development, particularly in generalizing it to include compression members of trusses and other parts subject to buckling. His most important contribution, however, is in bringing to the attention of the profession, and helping to clarify, some of the gaps and inconsistencies in engineering reasoning.

It is quite possible, of course, that he goes too far in some ways in his exposition of the method, as is natural and perhaps necessary when one is combating ingrained habits. Thus it may be, as he states, that it is excessive deformation which really marks the limit of usefulness of structures. What the designer is really interested in, however, is practical means for predicting this limit of usefulness; and it is to be noted that the author does not propose to calculate this limit (and would indeed find it very inconvenient to so calculate it) by a study of the deformations of the structure. Rather he calculates it by considering the way in which the forces distribute themselves through the structure, just as in any elastic theory.

It would seem that the author does demonstrate, beyond a doubt, the following: He shows that the forces that first produce yielding or buckling in certain parts of a structure, while they may indicate the limit of strength of these parts, do not necessarily indicate the limit of strength of the structure as a whole, when these parts are redundant. He shows that when these parts are capable of yielding sufficiently, while still maintaining more or less undiminished resistance, the limit of strength of the structure itself is not reached until all the redundant parts have reached their individual limits. He shows also that this limit of strength of the structure is easily found and in many important cases differs an important amount from the strength usually calculated by much more laborious methods. He shows, furthermore, that the deformations of the parts and of the structure as a whole remain "of the order of magnitude of elastic deformations" until the limit of strength of the structure is reached.

Two things remain which do not seem to be clearly demonstrated—namely, that such deformations of the structure as a whole may not be great enough to destroy its usefulness, and that such deformations can be borne by the parts involved while still maintaining more or less constant resistance. It is no criticism of the paper to say that these questions are not adequately covered, because obviously one paper cannot be expected to cover all sides of such a subject; but it seems evident that these questions must be studied thoroughly before the method can be acceptable for application in cases where proof loading is impossible.

For most types of structures the possibility of having permanent deformations of this order of magnitude under "once-in-a-life-time" load conditions is probably not serious, and a decision as to whether this is permissible can be given offhand. In this connection it should be noted that the large deformations of steel buildings, bridges, etc., which have sometimes been produced by hurricanes or floods, are undoubtedly produced after all redundant members have reached their limits and not before.

The question of whether individual members of structures can "give" the amounts required without weakening is more difficult and requires quantitative studies of (1) the amount of giving which would be required in different types of structures, and (2) the amount which various types of members can give without weakening. Considering the first question, it is obvious that the amount of giving required will depend very much upon the type of structure, and in many cases may be very much more than maximum elastic deformations. For example, in Fig. 43, the amount bar *a* would have to stretch plastically before bars *b* would reach their yield point would depend on the angle of bars *b*, and would be $\left(\frac{b}{a}\right)^2 - 1$ times the maximum elastic strain, if the bars are all of the same material; or, in Fig. 44, the plastic strain in the tie bar *a* must be large before the stresses in the truss reach the yield point if the dimensions *a* and *b* are small compared to *c*.

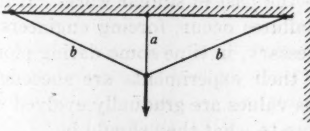


FIG. 43

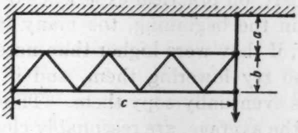


FIG. 44

Considering question (2), studies must be made of the relation between the longitudinal resistance of struts and the longitudinal deformation during local or Euler buckling, and of the relation between the bending resistance of deep beams and the bending deformation when local or lateral buckling of the compression flange occurs. There have been many theoretical and experimental studies of buckling problems in both the elastic and plastic range; but, heretofore, this particular phase of the question has not been considered important and has apparently escaped investigation.

As to members under tension, although it is true that plastic strains can be a hundred times or more as large as elastic strains in a mild steel tensile test specimen, it does not necessarily follow that this is true in structural tension members as they are commonly made, which frequently includes reductions of area and stress concentrations at the ends. To be sure, there is a powerful antidote against the obvious tendency in such cases for most of the strain to concentrate in these short, weakened sections—namely, the phenomenon of strain hardening, which stiffens up such weak points after they have yielded somewhat. However, obviously there are limits to the extent to which this favorable tendency can overcome the unfavorable, as is observed in tensile test specimens after the necking-down stage is reached. Hence some quantitative studies of this question are needed. This is especially important in view of the present tendency toward the use, for greater efficiency, of some of the higher strength materials, which almost always have lower ductility.

It should be emphasized that such studies are important not merely in order to make some new method of analysis possible, but chiefly because of the possibility which they present for making improvements in the actual ultimate strength of present types of structures through minor modification of details. Thus it might be found that certain types of tension members, although strong enough elastically, will not stand the plastic strain required to realize the full ultimate strength of the structure, and that this ultimate strength can be obtained by changing to tie-rods with enlarged ends, such as are frequently used now for economy of material and resistance to shocks. Such possibilities for increasing efficiency would not be suggested by present elastic methods of analysis.

If such studies show that no modifications are necessary in order to realize the increased strength indicated by the new method of analysis—that is, if present-type structures are really stronger than designers have been computing them to be—then the factors of safety to be used in applying the new method require discussion.

The factors of safety used in many branches of engineering are based, at least in part, on practical experience of the past in similar work. If they were too low in the beginning, too many failures occur, forcing engineers to raise them; or, if they were higher than necessary, in time some daring pioneers are certain to try lowering them, and if their experiments are successful other engineers eventually copy them. Thus values are gradually evolved which, at least on the average, are reasonably close to what they should be.

Obviously, this process is valid only when the same methods of analysis are used on the new and old designs. If engineers do not improve the type of design, and introduce a new method which gives more optimistic estimates of the strengths of structures, factors of safety based on past experience should presumably be raised enough so that structures designed by the new method will be as strong, on the average, as those designed by the old methods.

This does not at all mean that the new method would give the same results in individual cases as the old, and hence that it has no advantage other than its greater simplicity. On the contrary, a design method which gives an imperfect measure of the real useful strength of a structure, as elastic methods evidently may, can obviously result in too low a real strength in some cases and too high a real strength in others, even if it is regulated to give correct results on the average. Conversely, any method which gives a better measure of the real useful strength, as the one under discussion evidently may, will always give results closer to what they should be in individual cases; and, of course, in so far as this represents decreased uncertainty in design, the factor of safety can be reduced somewhat, counterbalancing, at least partly, the necessity for raising it.

One other point is suggested by this paper: If there is any real objection to relying on the yielding of some members of a structure under very extreme conditions, the same ultimate strengths could be obtained, without the necessity for such yielding, by the expedient of introducing initial stresses into the structure when it is built, such that under the worst conditions all redundant members would reach their ultimate loads at the same instant. Practical means for introducing such initial stresses could probably be devised if it proved worth while. Such an expedient, of course, could be fully applied only in cases where it can be assumed that loadings are predominantly in one direction.

J. A. Van den Broek,[59] M. Am. Soc. C. E. (by letter).—It is gratifying indeed to note the large number of men who, by their generous discussions, have manifested their interest in the "Theory of Limit Design." As was to be expected, the views expressed reflected unfavorable as well as favorable opinions. The unfavorable opinions are the more valuable since they are likely to lead to the greatest clarification of the subject. Of these unfavorable opinions, some are the result of misunderstanding, and others of sincere disagreement. First, an attempt will be made to clear up possible misunderstandings, after which the disagreements will be discussed.

Mr. Sourochnikoff states: "Of course, if a few extreme fibers are broken at the supports, it does not mean that the entire beam will collapse." The theory of limit design does not envisage the breaking of any fibers anywhere at any time. In the realm of structural engineering, failure is the result of excessive deformations, and rupture is practically non-existent. Where non-ductile materials are used, the principles of limit design are non-operative.

Professor Fabian devotes his major attention to the opening sentence of the "Introduction"—namely, the statement that, "Traditionally all theory pertaining to the strength of materials and to structural design is based on two major assumptions: (1) That the material is elastic; and (2) that the principle of superposition applies." The theory of limit design breaks completely with these traditional assumptions. It would seem evident, from what has gone before, that as soon as the elastic limit at any point is exceeded the principle of superposition is violated. Professor Fabian, furthermore, claims that, "Where the conditions are such that the law of superposition cannot be applied, the distribution of loads to various members cannot be made on the basis of simple statical conditions—namely, the force diagram—but it must be made on the basis of Maxwell's equations—that is, by the 'influence-line' method." No doubt it is agreed that Maxwell's equations are strictly within the limits of the theory of elasticity and are themselves predicated on the assumptions of elasticity and superposition. It is not clear, then, how they can be suggested as a tool in a field where the principle of superposition is non-operative and where the stresses exceed the elastic limit.

---

[59] Prof. of Eng. Mechanics, Univ. of Michigan, Ann Arbor, Mich.

The principle of superposition, in the theory of elasticity, is generally assumed, except in case of buckling. It is practically never rigorously satisfied. The question as to whether or not the violation of the principle invalidates the theory built on it is one of degree. It has nothing whatever to do with the theory of equilibrium (statics), which is always true.

In the example of the steel tower (Fig. 15), the force diagrams were drawn on the assumption that the geometric proportions of the structure remained substantially unaltered. In this the practice usually followed is applied. It is justified on the basis of dictum (3), which states that deformations are of the order of magnitude of elastic deformations (that is, of secondary order of magnitude), under all loads up to the limit loads, but cease to be so when the limit load is exceeded.

Mr. Floris regrets the fact that only the fixed-end beam was analyzed; and, quite properly, Mr. Meursinge and Mr. Winter emphasize the fact that the capacity resisting moment of a beam is reached when the stress distribution is rectangular rather than triangular (Fig. 24(*c*) rather than Fig. 24(*a*)). Mr. Winter, furthermore, insists that deformation analysis on the basis of theory of limit design is essential, whereas Professors Wise and Niles deny that the strength criterion in structural design is deformation rather than stress.

The writer has the impression that most of the discussers had some familiarity with theories allied to the theory of limit design, if not with the latter theory itself. His paper, however, was directed primarily to the reader with little or no familiarity with the principles of limit design. For that reason, his main purpose was to present the major idea involved with the least possible mathematical or other complexities. A fixed-end beam is a theoretical abstraction rather than a reality. It was chosen to serve as an example simply because all its properties are thoroughly well known. For the same reason, the capacity moment was assumed to be the one obtained from the familiar formula, $M = \frac{S\,I}{c}$, rather than the more logical one obtained by Equation (2), which is strictly applicable only in cases of beams of symmetrical cross-section. In the case of a beam of irregular cross-section, the neutral axis shifts as soon as the elastic limit is exceeded, and Equation (2) should be written:

$$M = S_1 (\bar{y}_1 A_1 + \bar{y}_2 A_2) \ldots\ldots\ldots\ldots\ldots\ldots (2a)$$

in which $A_1 = A_2$ (see Fig. 45). It was the desire to postpone considerations of detail such as these which lead to the assumption that the capacity-bending moment might be expressed by Equation (2). The writer stands corrected. He realizes that this is a consideration of insufficient merit and recommends that capacity moments be computed by Equation (2), or, in the case of beams with unsymmetrical cross-section, by Equation (2*a*).

As to deformations of beams subject to ductile yielding, Mr. Winter supplies some theory and certain very interesting test results. A very close agreement between theoretical and test results involving the ductile behavior of beams may be observed in Fig. 46.[60] If the deflection under a limit load (when the stress distribution resembles that shown in Fig. 24(*c*)) is compared with the deflection obtained when the beam behaves completely elastically under the same load, the ratios of these deformations are found to be: For the rectangular beam (Fig. 46(*a*)), 24/16 = 1.50; for the circular beam (Fig. 46(*b*)), 23/17 = 1.35; and for the wide-flange beam (Fig. 46(*c*)), 38/31 = 1.25. This evidence serves as proof of the claim that the deformations are of the order of magnitude of elastic deformations until the last redundant (the fibers at the neutral axis) members have reached their elastic limit.

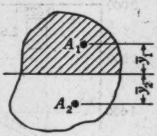


FIG. 45

Professor Wise is the only reviewer who questions the validity of the three dicta under the heading "Theory." It is conceded that without these dicta the entire theory of limit design collapses. As to the first dictum, Professor Wise calls it vague. The writer merely wanted to emphasize what he regarded as a well-known special aspect of engineering structures. The third dictum Professor Wise dismisses with a single word "incorrect." On the contrary, the writer regrets that he could not emphasize it more strongly by means of italics or capital letters, as he regards it as the very cornerstone of the theory of limit design. To test dictum (3) by Professor Wise's definition of statically indeterminate structures: He states, "The essential characteristic of a statically indeterminate structure is that it has redundant members, reactions, or restraints in excess of the number just sufficient to maintain static equilibrium." Suppose, then, that such superfluous redundant members are all stressed beyond the elastic limit. By Professor Wise's definition, sufficient members are left to form an elastic system in equilibrium. This system, as stated in the "Synopsis," behaves elastically. Hence the statement in dictum (3) that one member in the primary system (that is, one member in addition to the redundant members) must exceed the elastic limit, or buckling strength, before deformation in excess of elastic deformation will take place. It would involve extra diagrams to prove the point mathematically, but the writer suggests that the reader take any redundant structure, beam or truss, or a combination of both, and proceed to find the displacement at some point. He will then probably apply an auxiliary load at this point in the direction of the desired displacement and compute the resulting auxiliary forces in the structure. When he does so, however, he will, no doubt, in perfect conformity with the rules of the theory of elasticity, remove as many reactions, restraints, or members (in other words, redundants) as is consistent with stability. In his computations for the desired displacement, the redundants do not appear, since the forces therein, induced by the auxiliary load, are all zero. The displacement of a point in a redundant structure thus appears as the displacement of a point in a statically determinate elastic system. Hence, so long as the primary system is intact, deformations are of the order of magnitude of elastic deformations, which is all that dictum (3) states.

---

[60] "Deformations of Beams Involving Ductile Behavior," by E. O. Scott; thesis presented to the University of Michigan in 1939 in partial fulfillment of the requirements for the degree of Master of Science in Engineering.

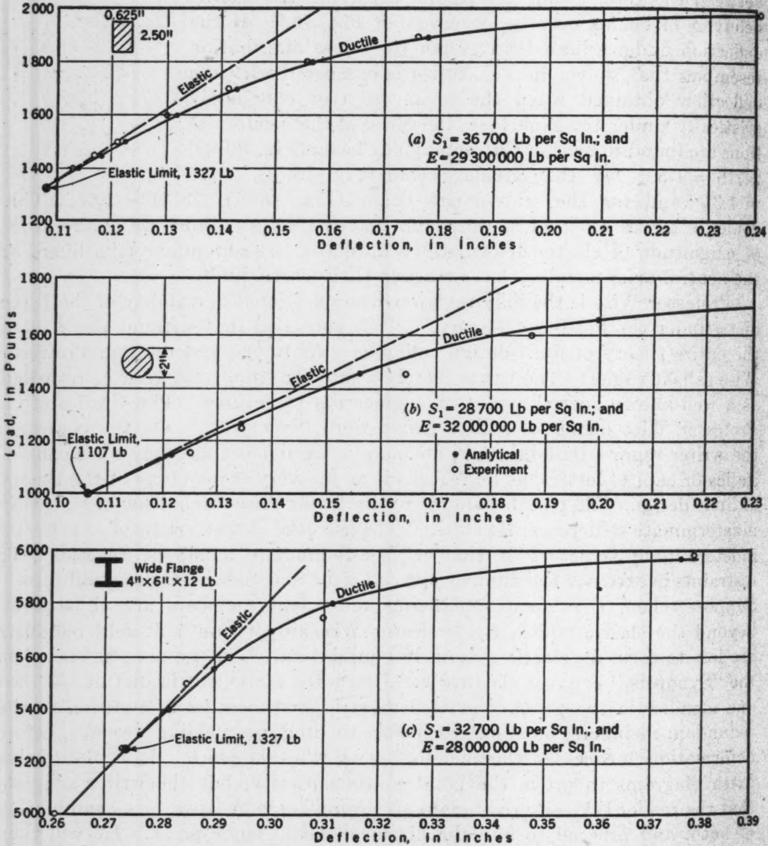


FIG. 46.—COMPARISON BETWEEN THEORETICAL AND TEST RESULTS INVOLVING THE DUCTILE BEHAVIOR OF BEAMS

There are shelves full of treatises on the theory of elasticity. The striking revelation to the practicing engineer is that the elastic deformations of a structure, in themselves, are only rarely of interest, and then generally only of secondary interest. In structural engineering the theory of elasticity is a supplementary theory. It serves the purpose (area moments, slope deflection, conjugate beams, least work, elastic energy, etc.) of writing independent equations to supplement those obtained from statics. Ask any engineer the probable stress in a modern railroad bridge, and he will give an immediate and correct answer. Ask him the probable deflection of the same bridge under load, and he is likely to want paper, pencil, dimensions, etc. The point is that, for structures of normal proportions, deflections are safe, so long as they are of the order of magnitude of elastic deflections. Failures, generally, are not the result of rupture but of excessive deformations—deformations of an order of magnitude greater than elastic deformations. In the laboratory specimens are tested and actually broken. A structural steel beam is never broken either in the laboratory or in the field. Under load it may be distorted out of shape to the point where it becomes a mass of wreckage, but in spite of the designer's emphasis on strength and on stresses, in the last instance, it is only deformation that really counts.

The writer did not state, as Professors Wise and Niles infer, that the theory of limit design is predicated upon permissible deformations, as distinct from the theory of elasticity, which is supposed to be predicated upon permissible stresses. In the "Introduction," prior to the introduction of Fig. 2, it is stated that "* * * it is proposed to continue to rely on the criterion that always was the primary one, the criterion of permissible deformations." In a statically determinate beam in a building, the designer generally is not concerned about deformations so long as the stresses remain within the elastic limit. Dictum (3) maintains that, in a redundant beam, even when the redundants have become completely ductile, the deformations are not of a magnitude that would give the structural engineer especial concern, because the primary system still remaining behaves in the manner of a statically determinate, purely elastic, beam with deformations of relatively small order of magnitude.

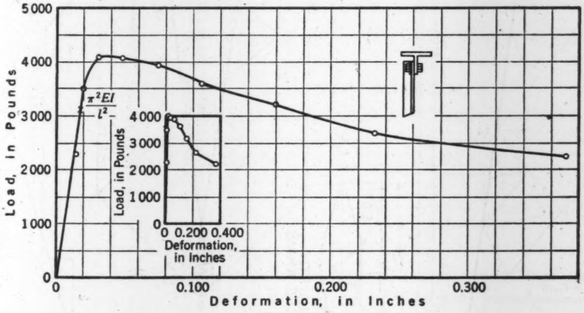


FIG. 47

Mr. Meursinge calls attention to the importance of connections. Specifications emphasize the importance of the property of ductility, but in order that this property may be made effective, the connections must be so designed that the yield point in the main part of the member is exceeded before the connection fails. In the discussion of the tower (Fig. 15) the connections were assumed to be welded in order to keep the argument simple and postpone the consideration of complexities.

Elsewhere[61] the writer has shown the results of a series of tests on 1¾-in. by 1¾-in. by ⅛-in. angle irons such as are commonly used as diagonal bracing in transmission towers (see Fig. 15). Bolt connection of an angle iron through one leg weakens the iron on two counts—the reduction of net area and the eccentricity of the applied load. By perforating both legs of the iron with extra holes, the strength of the iron is somewhat increased and the ductility of the member is augmented in proportion to the number of extra holes. Different types of end connections of angle irons and different degrees of perforation show marked differences in strength and even greater variations

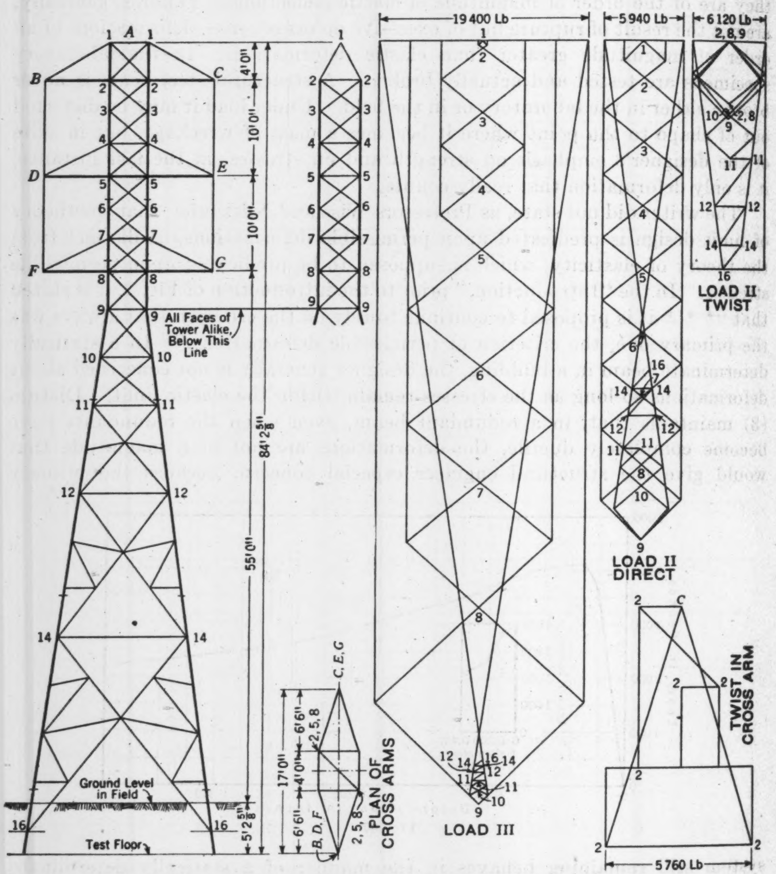

A
B
C
D
E
F
G
1
2
3
4
5
6
7
8
9
10
11
12
14
16
4'0"
10'0"
10'0"
84'2⅝"
55'0"
5'2⅝"
All Faces of Tower Alike, Below This Line
Ground Level in Field
Test Floor
C, E, G
B, D, F
2, 5, 8
17'0"
6'6"
4'0"
PLAN OF CROSS ARMS
19 400 Lb
LOAD III
5 940 Lb
LOAD II DIRECT
6 120 Lb
2, 8, 9
LOAD II TWIST
TWIST IN CROSS ARM
5 760 Lb

FIG. 48.—STRESS ANALYSIS OF A FULL-SIZE, 55-FOOT TRANSMISSION TOWER

---

[61] "Effect of Connections and Rivet Holes on Ductility and Strength of Steel Angles," by J. A. Van den Broek, *Civil Engineering*, February, 1940, Figs. 1 to 6, p. 83 (see also Table 1).

TABLE 4.—STRESS TABLE CORRESPONDING TO FIGURE 48

(Stresses in Pounds per Square Inch)

Member	Description	Dimensions, one angle	Maximum working stress, S	ULTIMATE TEST STRESS, U, FOR THE FOLLOWING ULTIMATE LOADS (SEE TEXT)						Ratio, $\frac{U}{S}$
				I	II Direct	II Twist	III	IV	Maximum	
1–1	Guy-wire support	6 by 6 by ⅜	± 300	....	....	....	± 700	....	± 700	2.34
1–2	Peak angle	2½ by 2½ by 3/16	± 3 600	+2 500	± 3 600	....	± 400	+ 100	+ 6 600	1.83
1–2	Peak angle (diagonal)	1¾ by 1¾ by 3/16	± 300	....	....	....	± 600	....	± 600	2.00
1(*A* and *B*)	Vertical hanger	1¾ by 1¾ by 3/16	− 3 500	−6 300	....	....	....	....	− 6 300	1.80
B(6–2), *DE*5, and *FG*8	Cross arm	3½ by 3 by ¼	+ 7 600	+3 300	± 9 900	....	± 3 800	....	+ 17 000	2.14
2–2, 5–5, and 8–8	Normal face: Strut	3½ by 3 by ¼	+ 5 500	+3 300	....	+6 600	....	....	+ 9 900	1.80
2–2, 5–5, and 8–8	Parallel face: Strut	3 by 2½ by ¼	+ 1 900	+1 100	....	+1 600	± 1 100	....	+ 3 800	2.00
2–2, 5–5, and 8–8	Horizontal, diagonal	1¾ by 1¾ by 3/16	± 2 600	....	....	±4 700	....	....	± 4 700	1.80
2–5	Leg angle	3 by 3 by ¼	+14 900	+3 800	+15 400	....	+17 000	+ 400	+ 36 600	2.46
2–3, 3–4, and 4–5	Normal face: Diagonal	1¾ by 1¾ by 3/16	± 3 400	....	....	±3 900	± 5 100	....	± 9 000	2.65
2–3, 3–4, and 4–5	Parallel face: Diagonal	1¾ by 1¾ by 3/16	± 4 800	....	± 4 000	±3 900	....	....	± 7 900	1.65
4–4, 7–7	Normal face: Strut	1¾ by 1¾ by 3/16	− 1 800	−3 300	....	....	....	....	− 3 300	1.84
4–4, 7–7	Parallel face: Strut	1¾ by 1¾ by 3/16	+ 600	+1 100	....	....	....	....	+ 1 100	1.84
4*D*, 4*E*, 7*F*, 7*G*	Vertical hanger	1¾ by 1¾ by 3/16	− 2 000	−3 600	....	....	....	....	− 3 600	1.80
5–8	Leg angle	4 by 4 by 5/16	+32 200	+5 400	+30 400	....	+48 600	+ 700	+ 85 100	2.64
5–6, 6–7, and 7–8	Normal face: Diagonal	1¾ by 1¾ by 3/16	± 4 900	....	....	±3 900	± 8 800	....	± 13 200	2.69
5–6, 6–7, and 7–8	Parallel face: Diagonal	1¾ by 1¾ by 3/16	± 5 100	....	± 4 000	±3 900	....	....	± 7 900	1.55
8–11	Leg angle	5 by 5 by 3/16	+41 300	+5 400	+33 700	....	+73 200	+1 000	+113 300	2.75
8– 9	Normal face: Diagonal	2 by 2 by 3/16	± 6 100	....	....	±4 300	±13 560	....	± 17 800	2.92
8– 9	Parallel face: Diagonal	1¾ by 1¾ by 3/16	± 5 600	....	± 4 300	±4 300	....	....	± 8 600	1.54
9–10	Diagonal	1¾ by 1¾ by 3/16	± 4 000	....	± 2 700	±3 900	± 1 000	....	± 6 600	1.65
10–11	Diagonal	1¾ by 1¾ by 3/16	± 2 900	....	± 2 000	±2 800	± 700	....	± 4 800	1.65
11–11	Strut	2 by 2 by 3/16	+ 2 100	....	+ 1 500	+1 800	+ 500	....	+ 3 300	1.57
11–14	Leg angle	5 by 5 by 3/16	+43 000	+5 400	+32 200	+3 800	+72 000	+1 500	+114 900	2.67
11–12	Diagonal	1¾ by 1¾ by 3/16	− 5 400	....	− 3 500	−5 000	− 1 200	....	− 8 500	1.57
12–12	Strut	2¼ by 2¼ by 3/16	+ 3 400	....	+ 1 900	+2 700	+ 800	....	+ 4 600	1.35
12–14	Diagonal	2 by 2 by 3/16	− 4 800	....	− 2 600	−3 900	− 900	....	− 6 500	1.35
14–14	Strut	3 by 2 by 3/16	+ 3 200	....	+ 1 500	+1 800	+ 500	....	+ 3 300	1.03
14–16	Leg angle	5 by 5 by 3/16	+40 600	+5 400	+27 600	+1 900	+71 300	+2 400	+108 600	2.67
14–16	Diagonal	2½ by 2 by 3/16	− 4 300	....	− 2 000	−2 600	− 700	....	− 4 600	1.07

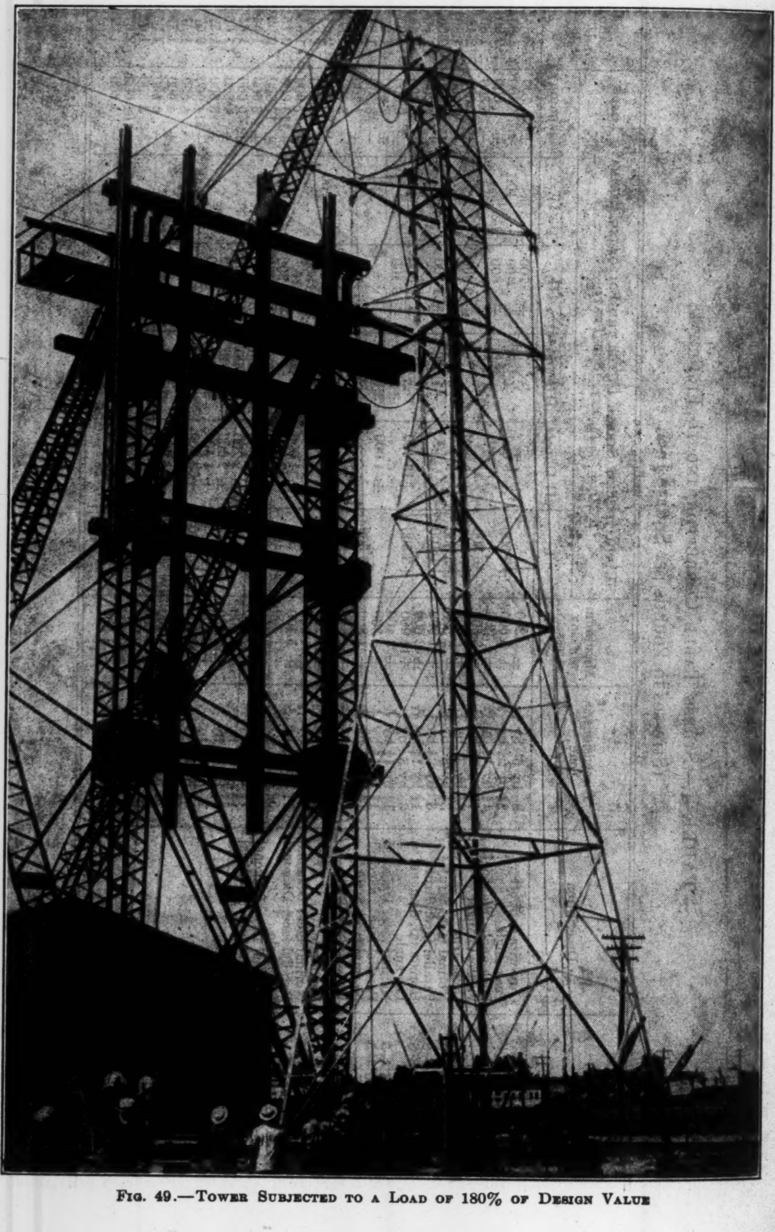


FIG. 49.—TOWER SUBJECTED TO A LOAD OF 180% OF DESIGN VALUE

in ductility. Fig. 47 shows the load-deformation relationship of a 1¾-in. by 1¾-in. by ⅛-in. angle with a slenderness ratio of 200 in compression. Figs. 48, 49, and 50 illustrate the results of a test of a full-size transmission tower, 55 ft high.

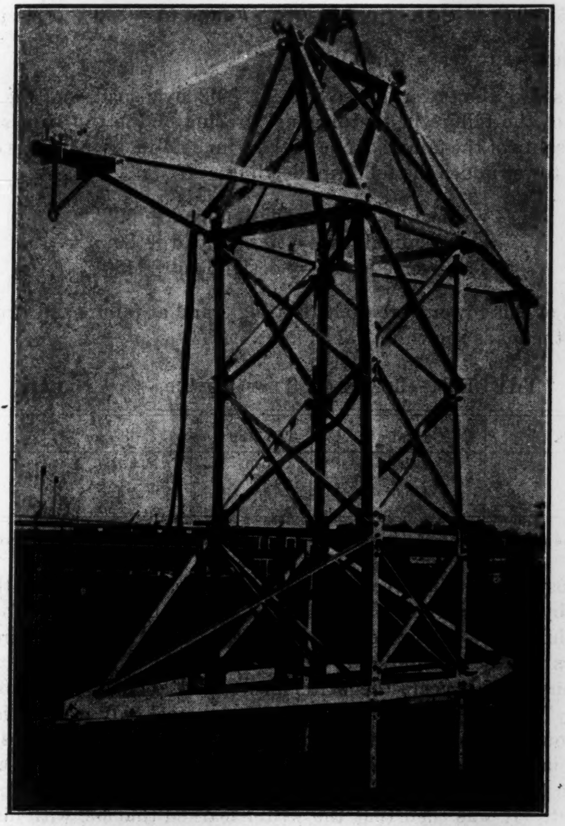


FIG. 50.—ENLARGED VIEW OF THE FAILURE (SEE FIG. 49)

Stress diagrams are shown in Fig. 48, with corresponding stresses in Table 4, the ultimate loads being identified as follows:

I. Vertically downward
  (*a*) 2 160 lb at Point A = 180% of design load
  (*b*) 3 240 lb at each of points B, C, D, E, F, and G = 180% design

II. Horizontal parallel
  (*a*) 6 120 lb at Point A = 180% of design load
  (*b*) 5 760 lb at Point C = 180% of design load

III. Horizontal normal

(*a*) 1 300 lb at Point A = 306% of design load
(*b*) 7 290 lb at Point B = 424% of design load
(*c*) 7 290 lb at Point C = 610% of design load
(*d*) 5 730 lb at each of points D, E, F, and G = 333% of design load

IV. Weight of tower = 9 600 lb

Fig. 49 shows the tower loaded with an 80% over-load, and Fig. 50 shows a close-up of the failure in the diagonal bracing of the top panels. Fig. 51 shows a close-up of the failure of one of the 4-in. by 4-in. by ⅜-in. legs of a heavy suspension tower under the following loading:

Vertical load = design load + 80%
Longitudinal load = design load + 80%
Transverse load = design load + 180%

The panels of this tower were each 10 ft high, and at failure the structure was sustaining the loads shown in Table 5.

TABLE 5.—LOADS ON TEST TOWER, AT FAILURE

Loads	Guy wire	Top arm	Mid-arm	Bottom arm
Vertical...............	1 620	3 060	4 500	4 500
Longitudinal...........	4 680	5 400	....	....
Transverse*...........	4 600	5 460	7 160	7 160

* The wind load was compensated by an addition of 750 lb to the transverse load at each of the four points.

The most serious disagreement revealed by the discussion is the opinion advanced by Messrs. Bleich, Freudenthal, Mirabelli, and Wise to the effect that the philosophy of limit design is not applicable in cases where buckling of members may be encountered.

For some twenty years the writer has talked about a theory of ductility ("plasticity" is a term applicable to structural steel only when it is red hot). In every course he gives on the theory of elastic energy the lectures are closed with a discussion of the limitations of the theory of elasticity. On one such occasion, in 1924, at the termination of a course in Detroit, Mich., Mr. Goodrich was present. It was then that the writer learned that he, with his theory of ductility, and Mr. Goodrich with his design practices, talked essentially the same language. After innumerable discussions with Mr. Goodrich, the term theory of ductility was abandoned and the term "theory of limit design" was substituted, so as to make the title applicable to trusses involving members subject to buckling. Whereas some men (Professors Mirabelli and Wise) merely question the application of the theory of limit design in cases where buckling may occur, others (Messrs. Bleich and Freudenthal) conclude definitely that such application should not be permitted.

The writer's reply is two-fold. The first is that Fig. 47 shows that although the load-deformation relationship for a member in compression is not the

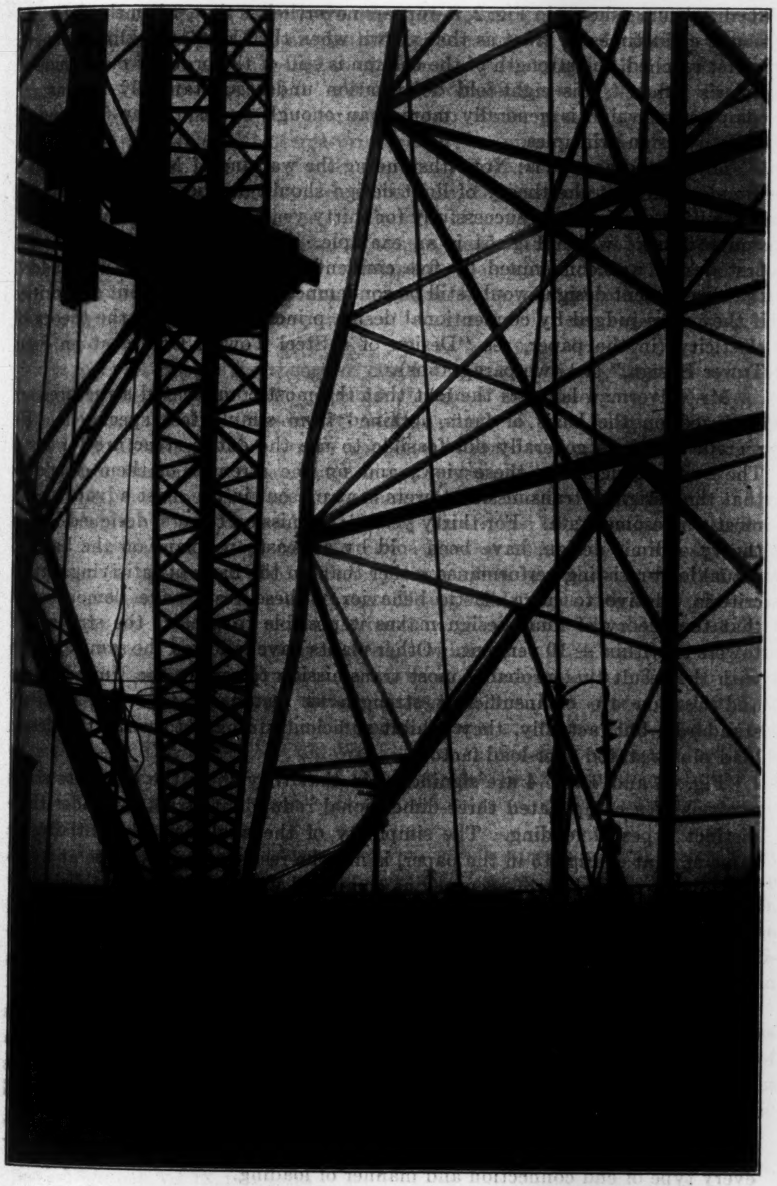


FIG. 51.—FAILURE OF ONE OF THE LEGS OF A HEAVY SUSPENSION TOWER

idealized curve shown in Fig. 2, it appears nevertheless that, when the deformation is eight times as great as that shown when the Euler's buckling strength is first reached, the strength of the column is still of the order of magnitude of Euler's value. This eight-fold deformation under substantially a constant compressive value is generally more than enough to insure the functioning of limit-design principles.

The second answer is: Notwithstanding the warning of Messrs. Bleich and Freudenthal that the theory of limit design should not be applied to trusses, it has been applied very successfully for thirty years to the design of transmission towers of which Fig. 51 is an example. Mr. Goodrich states that his first design was condemned by five eminent authorities. It may be added that his present designs would still be condemned by other eminent authorities if they were judged by conventional design principles based on the theory of elasticity (in the paper, see "Design of a Steel Tower: Comment on Steel Tower Design," last two paragraphs).

Mr. Silverman laments the fact that the most complicated structures are analyzed on the basis of facts obtained from simple test specimens. He regrets that it is generally not feasible to use the entire structure for tests. The writer concurs in these views and on the strength of them concludes that the design of transmission towers is at present in the most advanced and most wholesome state. For thirty years transmission towers, designed by the theory of limit design, have been sold by at least one plant on the basis of actual load-resisting performance rather than on the basis of satisfying certain criteria relative to mere elastic behavior. These tests have demonstrated that the theory of limit design makes it possible to predict the strength of towers to within ± 10 per cent. Other plants have followed the same example with the result that probably most transmission towers in the United States and Canada are of insufficient strength as tested by theory-of-elasticity standards; but, actually, they exhibit sufficient strength to carry the design load plus a stated over-load factor.

Fig. 48 and Table 4 are significant in that they show the complete stress analysis of a complicated three-dimensional redundant structure under three distinct types of loading. The simplicity of the analysis, substantially the same as that of Fig. 15 in the paper, is not the result of compromise and does not involve a sacrifice of accuracy as compared with standards established by stress analysis based on Hooke's law. On the contrary, it has a two-fold merit: First, that of great simplicity; and second, the merit of very satisfactory agreement with observed facts, as established by innumerable tests. It is conceded that the load-deformation relationship of columns beyond the buckling load is an involved one, affected as it is by end-connection, slenderness ratio, and eccentricity of applied load. In spite of innumerable tests on columns, which have been recorded, this particular relationship appears practically never to have been measured. Before limit-design principles will be applied with confidence to any type of truss, other than transmission towers, such data should be obtained for every type of compression member under every type of end connection and manner of loading.

Mr. Meursinge, very aptly, quotes Professor Kist to the effect that Hooke's law and theory of ductility are both "simplifications of a very intricate reality * * *. Both may be compared to cartoons, because they both 'express reality in a very primitive manner.' " The writer's only hope is that the theory of limit design is not too much of a caricature. Quite properly, Professor Niles warns against using limit-design principles where materials are involved which do not behave in agreement with Fig. 2. Wood, concrete, and earth do not obey Hooke's law rigorously. Nevertheless, the designer commonly applies strength-of-materials formulas to them, which are based on Hooke's law. This procedure is fully justified, of course, when a proper factor of safety is used. Similarly, limit-design principles may be applied, with reservations, to structures built of materials that are imperfectly ductile. The reservations naturally become prohibitive when the material is completely lacking in ductile properties.

It was not due to the theory of elasticity, but rather to the good sense of structural engineers (possibly without the aid of any theory to which a label could be attached, but nevertheless the result of sound logic and correct observation) that such materials, completely devoid of ductile properties, were never used for structures such as bridges, towers, buildings, or airplanes.

Mr. Silverman refers to Timoshenko,[21] even as does Mr. Eremin. Mr. Silverman, however, correctly attributes the original contribution not to Professor Timoshenko but to Mr. Hencky. Furthermore, Mr. Silverman points out that the agreement of Mr. Hencky's contribution with the theory of limit design is incidental, which is very gratifying. Mr. Hencky's contribution is offered as an attempt to explain what Mr. Silverman calls the Bauschinger effect—namely, that the effect of cold working on steel in tension is to raise the elastic limit in tension and to lower it in compression. This so-called Bauschinger effect has two serious flaws. In the first place, Bauschinger never made this claim; and in the second place, the claim when made is not true. (See "The Effect of Cold Working on Elastic Properties of Steel,"[62] in which an entire chapter—entitled "Cold Stretching Steel Does not Lower Its Elastic Limit in Compression"—is devoted to refuting this commonly found erroneous statement.)

Professor Donnell's discussion is very much appreciated as being most constructive. He "hits the nail on the head" when he states: "There have been many theoretical and experimental studies of buckling problems in both the elastic and plastic range; but, heretofore, this particular phase of the question [the relation between longitudinal resistance of struts and the longitudinal deformation during buckling] has not been considered important and has apparently escaped investigation."

The theory of elasticity has not only dominated methods of design but has controlled research as well. To return to the original thesis: If the theory of elasticity is so excellent, why not specify purely elastic material; and, *vice versa*, if the engineer realizes that he cannot possibly build with purely elastic material, then why not include the property of ductility in design theory?

---

[62] "The Effect of Cold Working on the Elastic Properties of Steel," Carnegie Scholarship *Memoirs*, Iron and Steel Inst., Vol. IX, 1918, p. 138.

The writer is unable to accept two alleged statements of fact made by Mr. Freudenthal. In Section (2) of his discussion Mr. Freudenthal states that "Instantaneously applied loads cause the upper yield point to drop." In the writer's opinion the reverse is the fact. In structural steel, the corner at Point *B* as shown in Fig. 2 is the sharpest when loads are applied slowly. In Section (3) Mr. Freudenthal states: "The occurrence of pulsating and reversed stresses will always demand the design of bridges on the basis of the endurance limit of the material." Reversal of stresses in bridges is so rare as to constitute the exception rather than the rule. Furthermore, although capacity loads for certain details[63] (for example, stringer connections to floor beams) may occur each time a heavy locomotive passes over the bridge, capacity loads for the entire truss may not occur oftener than once or twice in the life of the bridge. Therefore, although fatigue must be considered in connection with bridge design, it is of secondary and not of primary importance.

The writer desires to express his indebtedness to The Canadian Bridge Company, Ltd., for permission to reproduce illustrations, to Mr. Goodrich for his unfailing interest, and to the Horace H. Rackham School of Graduate Studies of the University of Michigan for aid received in providing assistance and in furthering tests herein recorded.

---

[63] "Fatigue Tests on Connection Angles," by W. M. Wilson, M. Am. Soc. C. E., and John V. Coombe, Jun. Am. Soc. C. E., *Bulletin*, Univ. of Illinois, Vol. XXXVII, No. 6, Series No. 317.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2071

PROPOSED IMPROVEMENTS FOR LAND SURVEYS AND TITLE TRANSFERS

BY PHILIP KISSAM,[1] ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. GEORGE D. WHITMORE, AND PHILIP KISSAM.

SYNOPSIS

The transfer of real estate in the United States is surrounded by difficulties out of all proportion to the values involved. This is particularly obvious when a comparison is made with the transfer of any other form of wealth. The unnecessary costs and delays, the investigations, surveys, and uncertainties of boundary location, the prolonged procedure used in proving title—all tend to reduce the negotiability of real property and to damage its value.

Surveying equipment, skill, and procedure have been developed which give results far beyond the accuracy required to eliminate boundary difficulties, and methods of recording title have already been successfully employed which eliminate delays in title search. It appears to the writer that the time has come for surveyors and title examiner to combine in a joint effort to find ways and means for employing these excellent tools already at hand.

The uncertainties of boundary location are caused basically by: (*a*) A lack of permanent, recognized, and correlated monuments from which surveys may originate; (*b*) increasing land values with no provision for more accurate surveys; (*c*) faulty descriptions (not erroneous); (*d*) blunders in description and surveys; and (*e*) the rules of adverse possession in force where accurate surveys and proper title records eliminate their usefulness.

The difficulties of title search are caused by: (*a*) The many and ever-increasing sources of information and records that must be studied; and (*b*) recurrent study of the same information and records.

To eliminate these difficulties the writer recommends:

(1) The establishment of State Systems of plane co-ordinates;

(2) A public officer to aid in, and permanently record, the determination of boundary lines for properties above an assessed unit value;

NOTE.—Published in April, 1939, *Proceedings*.

[1] Associate Prof. of Civ. Eng., Princeton Univ., Princeton, N. J.

(3) The assignment of certain members of the judiciary to a Land Court where title can be confirmed at the cost of only a small fee for legal service; and

(4) Simplification of title records by making a court decree final, with only a limited time for appeal, and by requiring all liens to be recorded with the evidence of title.

Introduction

One of the primary elements of civilization is the dedication of land areas to definite purposes. Devices for maintaining life, reducing labor, regulating men's relationships, providing enjoyment, or protecting spiritual values cannot be developed or used without the assignment of land. When the purpose of the dedication becomes clouded, or when the delineation of areas becomes vague, such confusion arises that progress is threatened. These truths are so fundamental that they are accepted without question. They are so well recognized that the first step upon the occupation of new territory is the assignment or division of land. They are so immutably fixed in men's minds that wholesale disregard of them is never contemplated.

Wherever the principle of private ownership obtains, possession of land carries with it the right to direct the use of the land under certain restrictions. It is therefore important, if progress is to be expected, that ownership and the limits of ownership shall be clearly defined.

It follows that land ownership must be a matter of public record, and that therefore it must be a function of government to maintain these public records. Not only must the records show who are the owners of the various land parcels and the conditions of ownership, but they must also clearly define the boundaries of the holdings. If land is to have its greatest value, it must be readily negotiable. Therefore, the public records must be quickly available and of unimpeachable accuracy. Each State must insure through legislative enactment that such conditions shall obtain, if confusion and serious economic loss are to be avoided.

How far has the United States progressed toward this goal? Unfortunately, in all the States, records of title and of liens affecting title are not only difficult of access but often require an expert to find them. They are frequently inconsistent with each other so that unless the title examiner's opinion is accepted, a court decree is necessary to bring them into agreement temporarily.

Are the conditions of the records of location any better? On the contrary, they are frequently worse. Recognition of position is usually based on hearsay, while size and shape may rest on the integrated opinion of many individuals. Under these conditions it can hardly be supposed that the parcels presumably owned will fit the existing ground, or that the location of a single parcel can be independently determined. Every land surveyor realizes this. Furthermore, the systems of recording are archaic, inadequate, and impractical. Obviously, therefore, the time has come for a change.

THE JOINT COMMITTEE

As civil engineers are continuously engaged in the design of structures and projects necessitating the acquirement and use of land, and as land surveying is also one of their functions, they should take the lead in initiating steps to bring about betterments. Furthermore, the locating of property is so intimately connected with its title that changes in procedure in regard to location must be accompanied by changes of procedure with regard to title. At the request of the Society, acting through its Surveying and Mapping Division, a Joint Committee has been formed with the American Bar Association, represented by the Real Property Division of its Section of Real Property, Probate and Trust Law. Presumably it will require some time for this Committee to formulate recommendations and make a final report.[2] Certain ideas brought out in Committee discussion are here described, although none of the statements in this paper represent action of the Committee, nor will they necessarily be included in the Committee's report.

BOUNDARY LOCATION

Whatever function a land surveyor is called upon to perform, he must first locate the boundaries of a parcel of land, even if a subdivision is required. If he can locate one corner and the direction of one line extending from that corner, a large portion of his difficulties are overcome. To aid him, the description of the parcel recorded in the deed probably is available. If this description is not independent, but is based on some other division of the land, he must resort to the earlier description. Sooner or later he must find mention of a point of beginning and some description of it. If this point is a natural or artificial monument, he is fortunate, provided check measurements made from other points he believes to be correct prove that the monument is in its original position.

Frequently the point of beginning is not marked, but is described as being so many feet along the center line or boundary of a public highway from some other public line. It is unusual if either of these lines can be found. Moreover, the surveyor never is sure that the present locations of these public lines, or his best judgment of where they should be located, will agree with the location assumed by the person who first described the point of beginning. In other words, more evidence must be brought to bear besides that found in the deed itself; and so many elements must be considered in each case that more judgment than skill is often required to establish even the initial point and direction.

To overcome this difficulty, recognized official survey positions must be provided by which property can be described and relocated. Systems of official monuments must be established and inter-connected by accurate surveys so that each monument is witnessed by all the others. When five or six monuments constitute the entire system, a balanced traverse loop will tie them together satisfactorily, and a plat showing the various adjusted traverse distances and angles will serve as a record of their positions. Any municipality can safeguard the property in an important section of town by establishing a small system of this kind, recording the plat at the registry office, and maintaining the integrity of the monuments by re-surveys or check witness measurements at intervals. A purchaser of real estate in this area should be advised by his title examiner and his surveyor to demand a description in his deed based on these monuments.

---

[2] "Land Surveys and Titles," First Progress Report of the Joint Committee of the Real Property Division, American Bar Association, and the Surveying and Mapping Division, American Society of Civil Engineers, *Proceedings*, Am. Soc. C. E., November, 1938, pp. 1879–1884.

It is better to substitute for the adjusted traverse data a rectangular co-ordinate system. The land surveyor need then only state the co-ordinates of each point mentioned in the description and name the co-ordinate system used. If possible, the co-ordinates of street intersections should be given on the plat. The title examiner will then have more information for tracing the title, and the public property will be better protected.

But when more than one co-ordinate system is established, it is inevitable that the systems will be extended until they meet, and then confusion will result. It is therefore desirable to have a few large systems rather than many small ones.

STATE PLANE CO-ORDINATE SYSTEMS

The United States Coast and Geodetic Survey has established a system of plane rectangular co-ordinates for every State in the Union.[3] Each system is composed of one or more definite mathematical projections which are so well designed that they can be 150 miles wide and of any length desired without introducing material divergences, due to the earth's curvature, between actual measured distances or angles and those given by the co-ordinate positions. The usual maximum divergence is one part in 10 000, and for the greater part of each system the divergence is far less. The adaptation of these projections to the various States is chiefly the work of O. S. Adams of the Coast and Geodetic Survey, and the transverse Mercator projection was especially developed by him for this purpose.

HORIZONTAL CONTROL SURVEYS TO SUPPLEMENT THE FUNDAMENTAL NET

To use these systems it is necessary only to connect a survey with the Fundamental Triangulation Net of the United States. Wherever Coast and Geodetic Survey first- or second-order triangulation exists, this can be done immediately. The geographic positions of the triangulation stations can be reduced to plane co-ordinates by a comparatively simple computation. For many triangulation stations the Coast and Geodetic Survey has already made these computations. Of course it is better to amplify the triangulation net by horizontal control surveys supplemental to the Fundamental Net. This can be done by second-order traverse and, if plane co-ordinates are used, all reductions can be made by the usual plane surveying methods. Latitudes and departures are determined and the adjustment is made proportional to the lengths of the courses.

---

[3] See "Development of State Grid Systems," by O. S. Adams, *Civil Engineering*, January, 1937, pp. 33–37.

When control of this kind is developed, a sufficient number of control points presumably will be established in each town or locality so that surveys can be extended throughout the streets by local authorities.

ENABLING ACTS

Each State legislature should pass an enabling act, naming and describing the State system, so that State co-ordinates can be used in descriptions, and can be identified simply by naming the system. In New Jersey, Pennsylvania, and New York this has already been accomplished. Also, as soon as possible, in each State a survey bureau should be established to extend the control or supervise the establishment of control in various localities. Such bureaus should be the repositories of control survey data, should disseminate these data to the public, and should maintain the monuments. Pennsylvania has already established such a bureau by law; and in certain other States, control survey organizations act in this capacity.

With a State co-ordinate system, or other legally recorded co-ordinate system, the land surveyor has at his disposal a method of establishing a point of beginning and an initial direction which can always be precisely re-established. Such a system should not be confused with the Public Land System, for which the monuments (except in Alaska) are not inter-connected by precise surveys, and therefore are not susceptible of accurate replacement.

The method of describing property under the Public Land System has many advantages, particularly in tracing title. From the description alone previous title holders can be identified immediately, a procedure requiring infinite time and pains for rural districts in the East. The description gives the approximate location of the property, the description of the adjoining tracts, the approximate area of the parcel and a means of indexing the deeds by locality. However, it is impossible to define, in the description, the precise location of the property without resort to the marks originally established on the ground. They bear no mathematical relation to each other except in the roughest sense. When lost they can seldom be replaced in their original position and without the introduction of another survey it is impossible to determine rectangular co-ordinates for them. The State Plane Co-ordinate Systems provide all the advantages of the Public Land System and in addition provide a means of defining, precisely, the location of the land on the ground.

DESCRIPTIONS INCONSISTENT WITH EACH OTHER

There is a further difficulty, however. Should the surveyor try to mark a land parcel according to its description, it is very likely that he will encroach upon or fail to connect with abutting property. The shapes of the parcels described in the deeds seldom fit together to form the original tract from which they were separated. The blocks are not of the recorded length; the same line will have different lengths in adjacent lots; old boundary lines long established at inconsistent positions have become fixed by the rules of adverse possession, and the angles in possession are seldom as found in the deeds. Moreover, the adjacent properties are usually indifferently marked on the ground so that the surveyor is completely in the dark if he attempts to disregard these inconsistent deeds.

There are five causes for these inconsistencies: (1) Lack of marked, recognized, and recorded positions, as already discussed; (2) variation in land values; (3) faulty descriptions (not erroneous); (4) blunders in descriptions and surveys; and (5) adverse possession (corollary of inconsistencies).

Variation in land value is a very natural reason for inconsistency. Survey accuracy is directly proportional to survey costs. Therefore, farm lands should not be surveyed with the same accuracy as business lots. Thus, precise measurements made in business areas usually disclose discrepancies in original surveys.

Although there is much diverse opinion relative to what constitutes a proper description, from the surveyor's point of view it should completely describe the size, shape, location, and orientation of the actual holdings. As such determination is next to impossible, many palliatives have been used, none of which has been satisfactory. Little reform can be expected until the causes of inconsistencies are removed.

Further, there exists a definite disagreement of purpose in descriptions. The title examiner needs a description which may be followed back accurately through previous transfers. The surveyor needs a description that can be used to mark accurately the present holding. The repetition of an inaccurate description copied from deed to deed is a boon to the title examiner, but is fraught with difficulties for the surveyor. A new and accurate description presents difficulties for the examiner. Mention of the abutters is good policy and aids both the examiner and surveyor, but without dimensions it is useful only to the examiner. It seems evident, therefore, that the solution of these difficulties must satisfy the needs of both examiner and surveyor.

How are these difficulties handled at present? If the surveyor is to make a proper location he must make a complete survey in the field of all the existing landmarks in the original tract or city block. He must abstract all the deeds for this area and, using his best judgment, work out a consistent solution which will satisfy the deeds, the physical location, and the requirements of geometry. If no objections are raised, his location usually stands, and, if court action ensues, he is well prepared for defense.

In such a procedure, however, the survey work is costly indeed. It cannot all be charged against the one parcel to be located. In substance the surveyor has worked out the solution for all the lots in the tract. Thus he has been obliged to invest a considerable fund in the area and he must protect his investment. He is forced to use secret marks so that he alone can profit by additional surveys in the same area, and he must maintain a complete office record of his solution and of the marks which show it on the ground. Actually, he has created a public benefit by his work, but this is available only when he is employed.

As long as surveys are made according to his solution and based on his system of marks, no difficulties will arise; but this is obviously impossible. The general public is unaware of the importance of his marks and records and will employ any surveyor offering a lower bid. The public streets and other properties will not necessarily be made to agree with his solution and there is every likelihood that his office will not continue long enough to make his solution fully effective.

This leads to the second step in the reconstruction of land surveying methods. A public office must be maintained under the direction of an able land surveyor whose duty it is to see that when a deed is recorded the description is based on an accurate survey; that an adequate system of enduring monuments exists to mark the property permanently; and, finally, that the location is consistent with the surrounding holdings. This may appear to be an impossibility, but such is not the case. In Massachusetts, C. B. Humphrey, Land Court Engineer, performs all of these functions for land to be registered in the State.[4] After a land court decree, an official plat is drawn, so complete that even many years thereafter a surveyor entirely unacquainted with the area could mark the property.

The establishment of a system of this kind would be an immediate and important improvement. It would necessitate the appointment of a State surveyor-general with a deputy at each registry office. An unnecessary hardship would be created if it were made obligatory to bring all land under the system at once, as a certain fee would be required to cover the costs to the State. It would be better to require that, in order to record a deed to property assessed at perhaps $75 per front foot, there must be attached, in lieu of a description, a plat of the property issued by the deputy surveyor-general. This would stabilize property lines where the need was greatest, and require by law only an expenditure for surveys which should be made in any event under present conditions. Under this arrangement, the surveys would be made by private surveyors to whom the records of the deputy surveyor-general would be available. Their work would be checked, as is done in Massachusetts, and any control surveys necessary for solution where inconsistencies developed would be made by the surveyor-general's staff. Should suit be brought threatening any location prescribed, the surveyor-general would be required to defend the owner. As his records would probably be very complete, unless a blunder had been made, his evidence should be sufficient to bring a favorable decision.

ADVERSE POSSESSION

With State co-ordinate systems, or even local co-ordinate systems, officially marked, recorded, and maintained by a proper bureau, the first difficulty would be eliminated, namely, indefinite initial position and direction. With an able surveyor-general there would be a gradual elimination of the inconsistencies due to: (1) Previous indefinite points of beginning; (2) old surveys made when the land had less value; (3) faulty descriptions; and (4) blunders in surveys and descriptions.

Adverse possession still remains, however, as a factor with disruptive possibilities. It is based on a principle of major importance. Possession is said to be nine-tenths of the law; and it is very proper that it should be so considered, as its effect is to stabilize ownership. It protects the owner against the innumerable possibilities of loss of title to all or part of his land which may arise under the present system; but it is an admission of the failure of the system. If a proper system is established, adverse possession becomes a threat rather than a stabilizing influence. It can be eliminated only when a system is established which is so nearly perfect that there can be no fear of unknown claims arising. Such a system must require very careful and complete study of conditions affecting the title, together with the citation and hearing of all persons interested. A court decree would also be required, which, when issued, would constitute a barrier to any inquiry into title prior to that time; but the effect of the decree would not be lasting and a new or supplemental decree would be required whenever the property changed hands. To overcome this defect and reduce the difficulties connected with title search, the Torrens System was devised, and in some form has been adopted in a number of States.

---

[4] "State-Wide Surveying Practice in Massachusetts: A Symposium," by Elmer C. Houdlette and Clarence B. Humphrey, *Proceedings*, Am. Soc. C. E., November, 1938, pp. 1847–1878.

THE TORRENS SYSTEM

Briefly, the Torrens System provides the means of obtaining a certificate of ownership when a petition is presented in court. The original certificate is kept at the office of registry, and a duplicate is given to the owner. This certificate is complete evidence of title. On it must appear a record of all liens against the property, except Federal liens and recent tax liens. References are given to the proper documents. Liens not appearing are invalid. When the property changes hands, a new certificate is issued upon presentation of the owner's certificate and the filing of the deed. In order to procure the first certificate, the owner must apply to the court. The court appoints an examiner who makes a thorough examination. Upon receiving the examiner's report, all persons possibly interested are cited, and after a short interval the court holds a hearing and issues a decree. The recorder immediately issues the certificate according to the decree.

The weakness of the system is twofold. Usually no attempt is made to find the location of the property, nor to register its location. Also the petitioner bears the cost of registration, which is of no more advantage at the time than an ordinary court decree. Although forms of the Torrens System have been adopted in many States, the extent of their use is negligible in spite of the fact that all of the serious difficulties of search are eliminated, as is also the fear of unknown claims arising. Most of the Torrens Acts also establish an assurance fund to protect the petitioner if loss is incurred. It is questionable whether this is a proper governmental function or should not be left to private title companies.

THE TITLE COMPANIES

A properly operated title company offers considerably more security than the Torrens System. It brings about, unofficially, the elimination of location inconsistencies and arduous title search. Thus the owner is usually protected against the threat of subsequent redetermination of his boundaries as well as against any unknown claims arising. Such companies should maintain complete survey records of all solutions, as well as complete abstracts of title for the tracts of land where they operate, although maintenance of the survey records is sometimes left to a practising surveyor. With these records, location and title search are quickly and accurately performed.

There is no assurance, however, that any company will be the sole repository of all the survey records for any tract. Inconsistencies may arise between the solutions of two companies. Also, the title company system provides no means of clearing a clouded title, and frequently where location solutions, or title difficulties, require a court decree, the costs of both the insurance and the decree prohibit clearing the title.

In Massachusetts a splendid system has been in operation since 1898, with a special court, called the Land Court.[4] Under this plan are included all the elements necessary to the successful administration of land ownership.

CONCLUSION

It appears that certain elements are necessary to improve land surveys and title transfers. These elements can be obtained gradually by progressive steps, or they can be created simultaneously:

(*a*) Of first importance are the State Systems of Plane Co-ordinates. They provide precise initial positions and directions so as to give descriptions definite meaning and property definite position. These systems require enabling laws, and the establishment of supplemental control surveys and a State survey bureau to administer them.

(*b*) Inconsistencies between deed descriptions may be eliminated by the appointment of a surveyor-general to supervise the surveys and descriptions for properties above a certain unit value, when the deeds are to be recorded. He must make solutions for these inconsistencies in each tract required, maintain public records of his solutions, and defend his solutions in court.

(*c*) A land court should be established; after proper citation of interested parties, and a review of examination of title, it will issue a decree which will include a description approved by the surveyor-general, will constitute a barrier to any prior inquiry into title, and will be filed at the office of the register of deeds for public reference.

(*d*) Finally, there is the question of simplification of current records. Whether this can best be handled by title registration, or by title companies, appears to be a question. Certainly a title company can render more varied and comprehensive service. Probably the answer lies in the ability, strength, and administration of the title companies themselves.

None of these essential elements is revolutionary or especially costly. State co-ordinates provide a basis for the accurate maps and plans so necessary for engineering progress, afford a method of compiling map data for planning in detail and over large areas, and justify their cost even when used only for control for tax maps.

A surveyor-general would perform a service which is required at present. He would function only by bringing under one head all the unco-ordinated efforts of many land surveyors privately attempting to make solutions for boundary tangles, which, to be effective, must be of public record.

The land court is only an assignment of certain of the present judiciary to a court which will handle all real estate title matters. It has been estimated that a decree to confirm title would cost the State little, or nothing, and the petitioner about $25 for legal fees.

Title companies under such a system could insure with less risk and therefore for lower fees. The difficulty of maintaining their records would be reduced to a minimum.

With these objectives as a goal and with the concentrated effort of all who understand these problems and realize their importance, the attainment of proper methods for land surveys and title transfers cannot be far in the future and the people can look forward with considerable hope that the present inadequate, archaic, costly, and wasteful methods of determining land ownership have not long to last.

DISCUSSION

George D. Whitmore,[5] M. Am. Soc. C. E. (by letter).—The author has presented the case for better land surveys and title transfers quite clearly and convincingly. The writer, a practicing land surveyor, whose area of activity extends into several States, finds little to question or dispute in Professor Kissam's summary of the situation. Changes in land-surveying and title-proving procedures must come, and are in fact already slowly beginning, along lines suggested by the author.

There seems to be no question but that surveying equipment, skill, and techniques have been developed which, if applied, would largely eliminate land-boundary difficulties. The real problem seems to be that of bringing the equipment, skill, and techniques into general use. The Joint Committee of which the author is a member may find that there is relatively little difference of opinion as to what should be done; but they may find a discouraging amount of inertia in trying to get the recommended procedures actually used. It would seem that the Committee should organize itself for a long-range program of two parts: (1) Relatively simple (and apparently well advanced) to get agreement on sound basic procedures between the legal and engineering professions; and (2) (and much more difficult) to get these procedures adopted and into general practice.

There is one matter which the author's summary does not seem to cover, and which seems to the writer to be of greatest importance in the matter of improved land surveys—that is, improvement in the accuracy of the surveys and resulting descriptions or plats. Accurate co-ordinates will permit the accurate replacement of any lost corner by means of routine surveys; but they are a result of, and not a cause of, accurate surveys. Hence, it seems to the writer that more emphasis needs to be placed on accuracy of the surveys, whether these be control surveys, boundary traverses, or traverses connecting control surveys to the boundary surveys. It is the writer's belief that 80% to 90% of the trouble which surveyors now have in trying to re-establish lost or obliterated property corners would be eliminated if the original boundary surveys and deed descriptions had been accurate and free of blunders. If, also, all the accurate surveys had been tied together and computed on a State-wide co-ordinate system, there would be practically no difficulty of any kind in re-establishing lost corners. In other words, the simple requirement that boundary surveys shall be co-ordinated through traverse connections to co-ordinated control points, although of the greatest importance, is not in itself sufficient—there must be the requirement that these surveys shall be of a certain degree of accuracy and blunder-free.

The outstanding proof of this contention is the difficulty, almost everywhere encountered, of re-establishing in their original locations the township, range, section, and quarter-section corners of the older public-land surveys of the United States. These old subdivision surveys practically constitute in themselves a series of co-ordinate systems. Had the original surveys all been made with the same degree of accuracy, there would be relatively little difficulty in re-establishing the original locations of lost corners from the bearings and distances alone. The principal difficulty is that the old surveys, many of them handled by incapable contractors without proper supervision or inspection, were not of uniform accuracy, and also that they had more than their share of blunders. It is obvious that, co-ordinated or not, accurate original land subdivision surveys would eliminate most of the modern surveyor's difficulties in re-establishing lost section corners.

---

[5] Chf. of Surveys, Maps and Surveys Div., TVA, Chattanooga, Tenn.

Another important point, from a public relations standpoint, is that surveyors should be careful to see that all published measurements and co-ordinates of the same course or point are in agreement. This has been stressed by A. H. Holt, M. Am. Soc. C. E.[6] To illustrate by a simple case, the writer saw considerable ridicule heaped on two reputable surveyors several years ago because they showed different distances for the same course on two adjoining record plats. The course in question was about 900 ft long; it was flat, open, and in every way conducive to accurate taping, and was the dividing line between two new building-lot subdivisions in a large Ohio city. Later investigation revealed that Surveyor A, whose tract was subdivided first, measured this course on a hot summer day, with his tape supported flat on the ground, and stretched under high tension. Surveyor B, whose tract was subdivided the following winter, measured the same course, at a temperature just a little above zero, with the tape suspended 2 or 3 ft above the ground, and stretched under relatively low tension. The two distances thus determined by the two surveyors differed by nearly 1 ft; yet both did accurate field taping. If the original field-computed distances had been corrected in the office for temperature, pull, and sag, the two final distances as determined by A and B would have agreed within a few hundredths of a foot. Publishing of the two uncorrected, different distances caused considerable confusion among, and ridicule from, public officials and others who had to use the two plats.

Philip Kissam,[7] **Assoc. M. Am. Soc. C. E.** (by letter).—There seems to be no question but that the methods involved in land transfer and land surveying records are in need of thorough revision. Co-operation between the engineering profession and the legal profession is essential if this objective is to be obtained. As Mr. Whitmore has stated, the Joint Committee may find that there is relatively little difference of opinion as to what should be done.

It may be remembered, however, that lawyers and engineers look at the problem from slightly different points of view. This results in a wide difference of opinion in regard to the purpose of land surveys and the requirements for land descriptions. The lawyer must have a survey and a description which identify the land. The engineer requires positive determination of the boundaries. At first glance these objectives may seem to be identical.

Suppose, for example, a residential lot were located in an area where the streets were not monumented but where there was a pair of arbitrarily placed survey control monuments for which the State plane co-ordinates had been determined accurately. To safeguard the title properly, the identity of the lot must be established. If street lines and distances from street intersections are mentioned (even if these lines are unmarked and quite indefinite), the lot is specifically identified. The streets mentioned and the rough location of the lots in the neighborhood are a matter of common knowledge; but, with such a description alone, the land surveyor cannot mark the boundaries. Thus such a description will satisfy the lawyer but not the engineer.

---

[6] "Trends in Boundary Surveying," by A. H. Holt, *Civil Engineering*, June, 1939, p. 359.

[7] Associate Prof. of Civ. Eng., Princeton Univ., Princeton, N. J.

To locate the property properly, the best procedure would be to include, in the description, ties to the control monuments. These ties alone, with the dimensions of the lot, would locate it specifically. However, the title examiner would be unable to assure himself that the lot so described was, in fact, the lot for which he was searching the title, and such a description, although satisfying the engineer, would not be acceptable to the lawyer.

From these divergent points of view, in the opinion of the writer, spring the chief difficulties encountered in land transfer to-day and the abortive attempts to rectify them. Aside from these considerations, however, basic agreement between the two professions is not too difficult to obtain.

Mr. Whitmore emphasizes the fact that an important difficulty preventing the improvement in land-transfer procedure is the inertia that must be faced which tends to maintain the same systems hitherto utilized. Difficult as it is to institute change, a beginning has certainly been made. The work under Mr. Whitmore's direction in the Tennessee Valley is one of the beginnings of such a change. His work is based on sound principles which will pay dividends in the future. Again, Carl M. Berry,[8] Assoc. M. Am. Soc. C. E., has described another cadastral survey at the Grand Coulee Dam, based on these sound principles. Naturally, the first changes toward better methods will be found in the work of larger organizations and governmental units. State highway departments and county and municipal governments will utilize better methods for property surveys. On the other hand, it is expected that the general adoption of sound methods will not occur very rapidly but rather will grow outward from the centers established by the larger organizations, both public and private.

The great difficulty which prevents the more rapid adoption of good methods of property transfer, in addition to the inertia mentioned by Mr. Whitmore, is the lack of machinery for recording, properly, the results of land surveyors' work in the areas where conflicting survey evidence exists. Frequently a land surveyor must, of necessity, develop a solution for a certain area containing many parcels in order to establish the location of one particular parcel. There is no governmental agency which records the results of his work; nor is there simple machinery to obtain adjudication of his conclusions so that his work may not be lost nor later challenged by another surveyor. With public recognition of his work properly recorded, there would be a permanence of location and a reduction of survey repetition. It will be necessary, therefore, not only to overcome inertia, and divergent points of view between lawyers and engineers, but also to develop methods for recording land surveying data.

---

[8] "Combining Geodetic Survey Methods with Cadastral Surveys," by Carl M. Berry, *Proceedings*, Am. Soc. C. E., September, 1939, p. 1159.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2072

DESIGN OF AN OPEN-CHANNEL CONTROL SECTION

BY KARL R. KENNISON,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. V. L. STREETER, EMERY H. WILLES, ROBERT O. THOMAS, FRANK S. BAILEY, GEORGE E. BARNES, AND KARL R. KENNISON.

SYNOPSIS

In designing an open channel to measure the flow of water, the shape of the controlling section to produce any desired rating curve can be determined by a definite mathematical relationship, the expression of which is claimed to be new, and to have possibilities in the application to practical problems. An analysis of the problem is contained in this paper. For example, if it is desired to modify a Venturi flume so that the rating curve at the given piezometer section immediately up stream from the controlling throat will be a predetermined convenient straight line, it can be done readily by the method presented herein. The writer first outlines the principles involved and arrives at mathematical expressions for the shape of the controlling section in terms of the desired rating, and then illustrates the application by the solution of specific problems.

INTRODUCTION

In any weir, flume, open nozzle, etc., it does not appear to be difficult to fit the dimensions of the controlling section to the desired rating curve by cut-and-try computations. However, the writer has frequently made designs of this character and has been intrigued by the observation that, even with precision so extreme that it seems unjustifiable, such cut-and-try computations do not give the true theoretical dimensions of the control. For example, at any depth of flow there may be identical velocities, with a great variety of section shapes, as long as the section areas are the same, excess width at one point being offset by decreased width at another. In fact, two controlling sections can be quite different in general appearance; that is, they can have a different relationship between depth and width and yet their two curves of

---

NOTE.—Published in May, 1939, *Proceedings*.

[1] Chf. Engr., Met. Dist. Water Supply Comm., Commonwealth of Massachusetts, Boston, Mass.

area, plotted against depth, can be so nearly alike that the resulting rating curves are the same within the precision expected in hydraulic computations. For this reason, cut-and-try computations fail to reveal the true theoretical shape of a controlling section required to produce a given rating curve, although they may solve any single specific problem with entire satisfaction so far as practical results are concerned.

The writer has long felt the need of a better understanding of the principles involved. His earlier attempts to obtain a correct analytic solution of the problem were discouraged by the mathematical complications encountered. A solution is presented herein which obtains the desired result—namely, simple expressions for the width and depth of the controlling section in terms of the given rating.

THEORETICAL SOLUTION

Referring to Fig. 1, it is required to find d_c and B_c in terms of the desired rating curve—that is, in terms of Q, d_p, S and the elements of the given piezometer section, or combinations of them, B_p, A_p, and H_p. The frictional loss, h_f, is relatively quite small and the first assumption in the interest of practical simplicity is to neglect it and to assume H_p equal to H_c in finding the expressions for d_c and B_c. It is practical as well as accurate and convenient to do this and to take the relatively small amount of friction into account later when finding the actual dimensions in any specific problem.

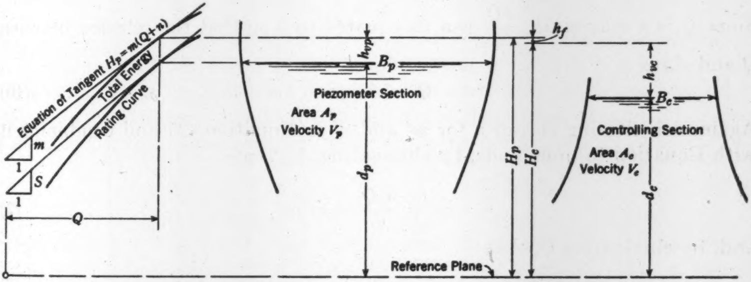


FIG. 1

Three general equations, applicable to any conditions and variations of flow, through any type of section, even through a section whose dimensions themselves are variable, are as follows:

$$Q = A_c V_c \quad \text{(1a)}$$

$$V_c = \sqrt{2 g h_{vc}} \quad \text{(1b)}$$

and,

$$h_{vc} = H_p - d_c \quad \text{(1c)}$$

A fourth general formula—

$$H_p = m (Q + n) \quad \text{(2)}$$

—requires the condition of critical flow through a given controlling section. Equation (2) also becomes general if m' and n' are substituted for m and n, such that

$$H_p = m' (Q + n') \quad \text{(3)}$$

Combining Equations (1) and (3):

$$\frac{Q^2}{2\,g\,A_c^2} = m'\,Q + m'\,n' - d_c \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

There are five variables in this general equation. To expedite the analysis, consider a continuous variation of flow, in a section of given dimensions, under a constant head H_p. When the desired critical depth d_c is reached, the discharge Q is at its maximum rate; and, m' and n' become m and n, which are constant because H_p is constant. Furthermore, in any given section the area varies with the depth and *vice versa*, and for present purposes one may assume d_c to be some function of A_c. Hence dd_c = (function of A_c) dA_c.

Accordingly, to determine the desired relationships in a controlling section, differentiate Equation (4), substituting m and n as constants instead of m' and n' and (function of A_c) dA_c instead of dd_c. As a result:

$$\frac{dQ}{dA_c} = \frac{Q^2 - f(A_c)}{A_c\,(Q - A_c^2\,g\,m)} \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Since Q is a maximum, $\frac{dQ}{dA_c}$ can be equated to 0 so that the relation between Q and A_c is

$$Q = A_c^2\,g\,m \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Again, substituting m and n for m' and n' in Equation (4) and combining it with Equation (6), one finds, by eliminating A_c, that

$$d_c = \frac{m\,Q}{2} + m\,n \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

and, by eliminating Q, that

$$A_c^2\,m^2\,g = 2\,(d_c - m\,n) \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Since B_c is the differential of A_c with respect to d_c, one can differentiate Equation (8), m and n continuing constant as before; thus:

$$\frac{dA_c}{dd_c} = B_c = \frac{1}{A_c\,m^2\,g} \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Combining Equation (9) with Equation (6) to eliminate A_c:

$$B_c = \frac{1}{\sqrt{m^3\,Q\,g}} \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Thus Equations (7) and (10) are simple expressions for the depth and width at the controlling section, in terms of m and n; and the latter may be expressed in terms of the given rating curve and the elements of the given piezometer section as follows:

Since velocity head varies as the square of velocity, its rate of change is twice that of the velocity; and, since velocity varies as quantity discharged and inversely as area, its rate of change is the difference between those of the quantity and of the area. In other words

$$\frac{dh_{vp}}{h_{vp}} = \frac{2\,dV_p}{V_p} = 2\left(\frac{dQ}{Q} - \frac{dA_p}{A_p}\right) \dots\dots\dots\dots (11)$$

It is obvious that $dA_p = B_p\, dd_p$, and hence

$$\frac{dA_p}{dQ} = B_p \frac{dd_p}{dQ} = B_p\, S \dots\dots\dots\dots (12)$$

Since, by definition, $H_p = d_p + h_{vp}$; $\frac{dH_p}{dQ} = m$; and, $\frac{dd_p}{dQ} = S$:

$$m = S + \frac{dh_{vp}}{dQ} \dots\dots\dots\dots (13)$$

Combining Equations (11), (12), and (13) to eliminate the differentials:

$$m = S + 2\, h_{vp}\left(\frac{1}{Q} - \frac{B_p\, S}{A_p}\right) \dots\dots\dots\dots (14)$$

Also, by definition:

$$n = \frac{H_p}{m} - Q \dots\dots\dots\dots (15)$$

Thus, the values of the convenient intermediaries m and n are given in Equations (14) and (15), in terms of the desired rating curve and the known elements of the given piezometer section, which values of m and n may be substituted in Equations (7) and (10) to determine the precise theoretical shape of the controlling section necessary to produce the desired rating.

APPLICATION OF THEORY TO PRACTICE

It is obvious that the foregoing computations are based on a theoretically perfect condition, free from cross currents and eddies, in which the filaments of flow are horizontal, and the pressure proportional to the depth. A satisfactory condition in this respect will not be obtained at the throat of the controlling section if the latter is too short or its immediate approach too suddenly contracted. It is not the purpose of this paper to discuss the proper design of a contraction to produce a smooth horizontal flow. A sufficiently gradual section change to a throat or crest which is not too short in the direction of flow, together with a sufficient length of the critical throat section to straighten the filaments, will result in a condition of flow past the controlling section which approaches remarkably close to the perfection sought. For example, in the writer's experience, a properly designed, flat-crested, submerged weir is an extremely accurate measuring device and one which shows little if any difference between theoretical and actual rates of discharge. However, it is not the intention to disclaim the practical necessity of modifying the theoretical relationships by an empirical coefficient, since the principal purpose of this paper is to analyze and present these relationships.

CORRECTION FOR FRICTION

This practical necessity for an appreciable length of water travel between the piezometer section and the controlling section results in a certain frictional loss between the two. Such loss is relatively small and may be compensated for by constructing the entire measuring device on a slight slope. It is generally so small that rough computations and estimates of the weighted average hydraulic elements and coefficient of roughness will indicate the appropriate slope to be given to the invert or reference plane of the measuring device.

Although such a fixed slope will generally be sufficient to compensate for frictional losses, throughout the desired range of discharge rates, without material sacrifice of precision, a more complete and very precise compensation can best be made as follows: The foregoing rough computations and estimates will give the frictional loss or slope of the total-energy gradient between the piezometer section and the controlling section for different rates of discharge. In a device of the type under discussion, this slope will be fairly constant throughout a considerable range of the lower rates of discharge but will increase noticeably at the higher rates. Hence one may select, arbitrarily, a fixed slope for the invert or reference plane which will compensate for the frictional loss at the lower rates of discharge, so that only at the higher rates of discharge must the extra head loss, h_f, or difference between H_p and H_c, be accounted for. The easiest and most satisfactory way to do this is to plot a curve of the roughly computed values of h_f for different values of Q from which the slope, $\frac{dh_f}{dQ}$, as well as h_f, can be read for any value of Q. To make the correction for excess frictional loss, subtract h_f from H_p and $\frac{dh_f}{dQ}$ from m. Equation (14) then becomes

$$m = S + 2\,h_{vp}\left(\frac{1}{Q} - \frac{B_p\,S}{A_p}\right) - \frac{dh_f}{dQ} \quad \text{.............} (16)$$

and Equation (15) becomes

$$n = \frac{H_p - h_f}{m} - Q \quad \text{....................} (17)$$

If friction is neglected (that is, if one does not wish to use expressions to which corrections for excess frictional loss at the higher rates of flow can be applied as described herein) Equations (7), (10), (14), and (15) can be combined to obtain the dimensions of the critical section in terms of the desired rating curve and of the known elements of the given piezometer section, without using the values of m and n. Thus, for the drop in water level between the piezometer section and the controlling section:

$$d_p - d_c = Q\,S\left(\frac{1}{2} - \frac{h_{vp}\,B_p}{A_p}\right) \quad \text{...............} (18)$$

The width of the controlling section can be expressed similarly, but the equation would be too complicated to warrant its use instead of Equations (10) and (14).

ILLUSTRATIVE EXAMPLES

Example 1.—In Fig. 2, a piezometer section is chosen arbitrarily, as an example, with a 3-ft bottom width, a 3-ft depth and side slopes of 4 on 1.

The rating curve is also chosen arbitrarily as a straight line, the maximum discharge being 28.3 cu ft per sec. Of course, such a line must show, at all points, a definitely greater depth, less discharge, than could pass the piezometer section itself as a controlling section. Otherwise, because of the well-understood tendency toward the propagation of surface waves, a definite relation between rate of discharge and depth at the piezometer section would not be assured at the lesser depths. To insure this condition the invert, in this example, is arbitrarily raised 0.1 ft at the controlling section (see Fig. 2), and the straight-line rating is drawn tangent to the rating curve which this raised invert causes at these lesser depths.

TABLE 1.—SUMMARY OF COMPUTATIONS TO DETERMINE DIMENSIONS OF CONTROLLING SECTION

(Ten-Inch Slide-Rule Computations)

Rate of discharge, Q, in cubic feet per second (1)	PIEZOMETER SECTION					Slope, m, of the total-energy rating curve (7)	Values of n† (8)	CONTROLLING SECTION	
	Depth, d_p, in feet* (2)	Area, A_p, in square feet* (3)	Surface width, B_p, in feet (4)	Velocity head, h_{vp}, in feet (5)	Energy head, H_p; total energy gradient* (6)			Depth, d_c, in feet* (9)	Surface width, B_c, in feet (10)
(a) S = 0.0982; STRAIGHT-SIDED CHANNEL SHOWN IN FIG. 2									
28.30	3.00	11.25	4.50	0.098	3.098	0.0974	3.51	1.720	1.092
23.21	2.50	9.06	4.25	0.102	2.602	0.0976	3.47	1.470	1.200
18.12	2.00	7.00	4.00	0.104	2.104	0.0980	3.35	1.216	1.350
14.04	1.60	5.44	3.80	0.103	1.703	0.0988	3.20	1.009	1.519
9.97	1.20	3.96	3.60	0.098	1.298	0.1004	2.95	0.797	1.757
5.89	0.80	2.56	3.40	0.082	0.882	0.1047	2.53	0.574	2.146
2.00	0.418	1.298	3.209	0.0368	0.455	0.1171	1.88	0.337	3.112
(b) S = 0.0924; CIRCULAR OPEN NOZZLE SHOWN IN FIG. 3									
28.30	3.00	7.069	0	0.248	3.248 3.233‡	0.1099	1.275	1.694	0.909
25.05	2.70	6.701	1.800	0.216	2.916 2.906‡	0.0990 0.0980‡	4.42 4.60‡	1.677 1.678‡	1.131 1.150‡
20.18	2.25	5.687	2.598	0.195	2.445 2.439‡	0.0952 0.0945‡	5.50 5.62‡	1.484 1.485‡	1.335 1.350‡
15.30	1.80	4.428	2.939	0.185	1.985 1.982‡	0.0939 0.0934‡	5.85 5.92‡	1.267 1.268‡	1.566 1.580‡
10.43	1.35	3.085	2.985	0.177	1.527 1.526‡	0.0947 0.0944‡	5.70 5.74‡	1.033 1.034‡	1.872 1.880‡
7.18	1.05	2.205	2.861	0.164	1.214	0.0987	5.11	0.859	2.118
4.03	0.759	1.405	2.508	0.128	0.887	0.1137	3.77	0.658	2.290

* Measured above the fixed reference plane.
† Intercept in the equation of the tangent to the total-energy rating curve, such that $H_p = m(Q + n)$.
‡ Includes correction for excess friction.

Table 1 is a summary of computations made to determine the shape of the controlling section, m being computed from Equation (14), n from Equation (15), d_c from Equation (7) or directly from Equation (18), and B_c from Equation (10). The shape of the controlling section is plotted from the computed values of d_c and B_c, with the results shown in Fig. 2. (It should be noted that the Sutro weir[2] is a special case of the general case presented in this paper.)

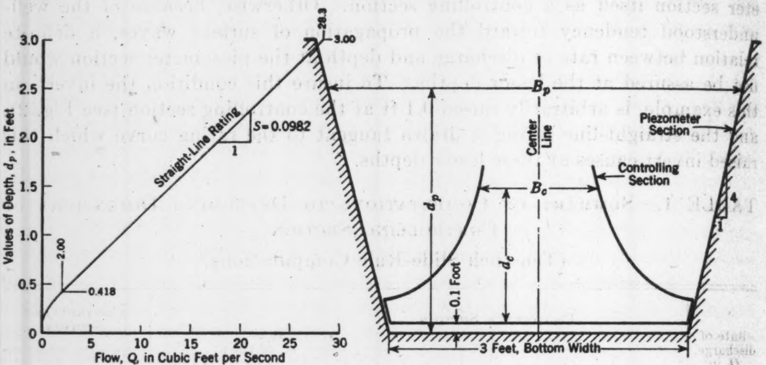

3.0
2.5
2.0
1.5
1.0
0.5
0
Values of Depth, d_p, in Feet
28.3
3.00
Straight-Line Rating
$S=0.0982$
1
2.00
0.418
0
5
10
15
20
25
30
Flow, Q, in Cubic Feet per Second
B_p
Center Line
Piezometer Section
Controlling Section
d_p
B_c
d_c
0.1 Foot
4
1
3 Feet, Bottom Width

FIG. 2

Example 2.—Another example is modeled after the so-called open nozzle[3] for measuring the flow of sewage. As shown in Fig. 3(*b*), the piezometer section is circular, 3 ft in diameter. The contraction of the invert of the controlling section is made arbitrarily by using a center offset of 0.075 ft (see Fig. 3). As in Example 1, the rating curve is an arbitrary straight line, the maximum discharge being 28.3 cu ft per sec.

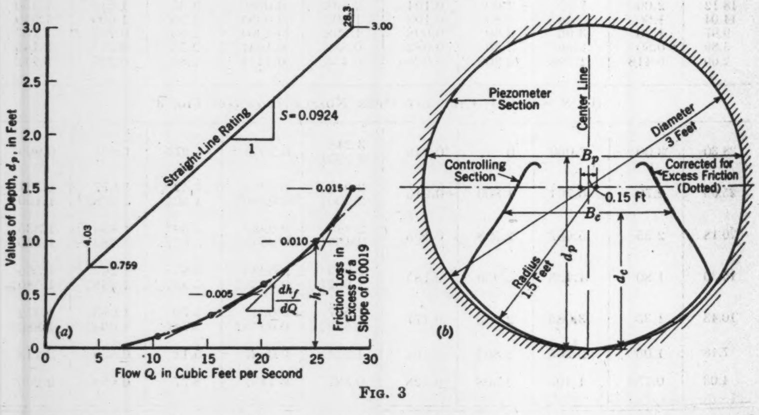

3.0
2.5
2.0
1.5
1.0
0.5
0
Values of Depth, d_p, in Feet
28.3
3.00
Straight-Line Rating
$S=0.0924$
1
4.03
0.759
0.015
0.010
0.005
$\frac{dh_f}{dQ}$
1
h_f
Friction Loss in Excess of a Slope of 0.0019
(a)
0
5
10
15
20
25
30
Flow Q, in Cubic Feet per Second
Piezometer Section
Center Line
Diameter 3 Feet
Controlling Section
B_p
Corrected for Excess Friction (Dotted)
0.15 Ft
B_c
Radius 1.5 Feet
d_p
d_c
(b)

FIG. 3

To illustrate the application of the correction for excess friction, as well as its smallness, the velocities and hydraulic elements were computed at different depths in the case of this open nozzle. They indicated that the invert or reference plane should be on a slope of about 0.0019 to compensate for the frictional losses at the lesser depths, and that at the greater depths the excess friction, h_f, to be subtracted from H_p, and its rate of change, $\frac{dh_f}{dQ}$, to be subtracted from m, are approximately as shown by the supplementary graph in Fig. 3(*a*). These corrections have been applied as shown in the parentheses in Table 1 and result only in a slight widening of the upper part of the controlling section, as shown by the dotted line in Fig. 3(*b*).

---

[2] "Sutro Weir Investigations Furnish Discharge Coefficients," by F. T. Mavis, M. Am. Soc. C. E., E. Soucek, Jun. Am. Soc. C. E., and H. E. Howe, *Engineering News-Record*, November 12, 1936, p. 679.

[3] *Journal*, Boston Soc. of Civ. Engrs., January, 1934.

The exact shape of the controlling section becomes relatively less important as the depth d_c increases. The result is that, although a circular piezometer section, closed at the top, results in the peculiar theoretical shape of the top of the controlling section shown in Fig. 3(*b*), the omission of this sharp irregularity, and the substitution of any convenient arbitrary extension of the sides of the controlling section, merely results in a negligible gradual departure from the straight-line rating at depths greater than 95% of the diameter. The maximum departure at full pipe is only about one-third of 1% increase in the rate of discharge.

CONCLUSION

The depth and width at the controlling section given by Equations (7), (10), and (18) are those which, if adhered to throughout the entire range of discharge, or depth, will result in the desired rating. This does not mean that in any particular limited case another set of dimensions determined, for example, by an approximate cut-and-try computation, will not fill the specifications satisfactorily. However, the section dimensioned by these formulas is the only one that will be theoretically correct throughout the entire range of depths; and, a knowledge of the theoretically correct shape cannot fail to contribute to good design in the selection of dimensions to fit the conditions in any case. There are undoubtedly other adaptations than those illustrated herein.

DISCUSSION

V. L. Streeter,[4] **Assoc. M. Am. Soc. C. E. (by letter).**—The theoretical solution for an open-channel control section, presented in this paper, will be of value to the designing engineer. The writer, however, found the derivation difficult to understand, particularly as to how Equation (2) requires the condition of critical flow, and why the denominator, instead of the numerator, of Equation (5) is set equal to zero for maximum discharge. Although the theoretical solution given by the author results in correct final equations, the two foregoing points so obscured the derivation that the writer attempted to find a more simple solution. For example, combining Equations (1),

$$\frac{Q^2}{2\,g\,A_c^2} = H_p - d_c \qquad (19)$$

As critical flow requires maximum discharge for a given available energy head, Equation (19) will be differentiated with respect to A_c, considering H_p a constant; thus

$$\frac{Q\,dQ}{g\,A_c^2\,dA_c} - \frac{Q^2}{g\,A_c^3} = -\frac{dd_c}{dA_c} \qquad (20)$$

Setting $\frac{dQ}{dA_c}$ equal to zero for maximum Q, a critical flow relation results; thus:

$$\frac{dd_c}{dA_c} = \frac{Q^2}{g\,A_c^3} \qquad (21)$$

As $B_c\,dd_c = dA_c$, Equation (21) contains the two unknowns B_c and A_c. To determine another relationship involving the two unknowns, differentiate Equation (19) with respect to Q, with H_p a variable. Then:

$$\frac{dH_p}{dQ} = \frac{Q}{g\,A_c^2} - \frac{Q^2}{g\,A_c^3}\,\frac{dA_c}{dQ} + \frac{dd_c}{dQ} \qquad (22)$$

Eliminating dd_c in Equations (21) and (22):

$$\frac{dH_p}{dQ} = \frac{Q}{g\,A_c^2} \qquad (23)$$

which contains one unknown, A_c. Equations (21) and (23) are equivalent to Equations (10) and (6), as $\frac{dH_p}{dQ} = m$. When the desired rating curve is given in the form:

$$d_p = f_1(Q) \qquad (24)$$

and

$$A_p = f_2(d_p) = f_2[f_1(Q)] \qquad (25)$$

---

[4] Associate Engr., U. S. Section, International Boundary Comm., U. S. and Mexico, El Paso, Tex.

(in which f_1 and f_2 are known functions) $\frac{dH_p}{dQ}$ may be determined from:

$$\frac{dH_p}{dQ} = \frac{Q}{g\,A_p^2} - \frac{Q^2}{g\,A_p^3}\,\frac{dA_p}{dQ} + \frac{dd_p}{dQ} \ldots\ldots\ldots\ldots\ldots (26)$$

for any value of Q. Equation (26) is Equation (22) with Subscript c replaced by Subscript p.

In designing the control, Equations (19), (21), (23), and (26) are needed. From Equation (26) $\frac{dH_p}{dQ}$ is determined for several values of Q throughout the range of capacity of the structure. With these values of $\frac{dH_p}{dQ}$, A_c is found from Equation (23), and with A_c known, values of $\frac{dd_c}{dA_c} = \frac{1}{B_c}$ are found from Equation (21). Corresponding values of d_c are obtained from Equation (19).

If friction is to be taken into account, it may be estimated for the length of travel between the piezometer section and control section, for various discharges, thus:

$$h_f = f_3(Q) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

Equation (23) then becomes:

$$\frac{dH_p}{dQ} - \frac{dh_f}{dQ} = \frac{Q}{g\,A_c^2} \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

and Equation (19):

$$\frac{Q^2}{2\,g\,A_c^2} = H_p - h_f - d_c \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

Equations (21) and (26) remain the same.

EMERY H. WILLES,[5] JUN. AM. SOC. C. E. (by letter).—The author needs to be commended on his unique approach to a problem, which, due to the mathematical complicity at first encountered, leads the pioneer investigator to despair of finding a simple theoretical solution. However, by expressing the width B_c and the depth d_c of the controlling section in terms of the slope m and intercept n of the tangent to the total energy-curve, followed, in turn, by expressing m and n in terms of the slope S of the rating curve and the known elements of the piezometer section, the author has presented a simple and effective solution to this seemingly non-solvable problem.

The writer, however, is unable to follow the author from Equation (5) to Equation (6). It is obvious from Equation (5) that $Q^2 - f(A_c) = 0$; also from Equation (6) that $Q - A_c^2\,g\,m = 0$. Then, by substitution, Equation (5) becomes the indefinite relationship $\frac{0}{0} = 0$. Equation (6) may also be obtained if $\frac{dQ}{dA_c} = \infty$ instead of 0. Thus, since $\frac{1}{\infty} = 0$, $\frac{A_c\,(Q - A_c^2\,g\,m)}{Q^2 - f(A_c)} = 0$; and, $Q = A_c^2\,g\,m$. This is very perplexing and led the writer to investigate $f(A_c)$

---

[5] Hydrographer, Utah Copper Co., Salt Lake City, Utah.

in Equation (5) as follows: Referring to Equation (4)

$$\frac{1}{2\,g}\left(\frac{2\,A_c^2\,Q\,dQ - 2\,Q^2\,A_c\,dA_c}{A_c^4}\right) = m\,dQ - dd_c = \frac{A_c\,Q\,dQ - Q^2\,dA_c}{g\,A_c^3} \quad \ldots (30)$$

Simplifying, and solving for $\frac{dQ}{dA_c}$, $\frac{dQ}{dA_c} = \frac{Q^2 - g\,A_c^3\,\frac{dd_c}{dA_c}}{A_c\,(Q - A_c^2\,g\,m)} = 0$; and,

$$Q = \pm\sqrt{g\,A_c^3\,\frac{dd_c}{dA_c}} \quad \ldots\ldots (31)$$

In any section $dA_c = B_c\,dd_c$; hence

$$\frac{dd_c}{dA_c} = \frac{1}{B_c} \quad \ldots\ldots (32)$$

and

$$Q = \sqrt{\frac{g\,A_c^3}{B_c}} \quad \ldots\ldots (33)$$

The foregoing investigation is leading the writer no nearer Equation (6) or the final solution. Therefore, since the author's solution revolves around Equation (6), the writer presents the following derivation: From Equations (1*c*) and (2)

$$Q = \frac{H_p}{m} - n = \frac{h_{vc} + d_c}{m} - n \quad \ldots\ldots (34)$$

and, because m and n are constant at the critical section, $dQ = \frac{dh_{vc} + dd_c}{m}$; and,

$$\frac{dQ}{dA_c} = \frac{1}{m}\left(\frac{dh_{vc}}{dA_c} + \frac{dd_c}{dA_c}\right) \quad \ldots\ldots (35)$$

From Equations (1*a*) and (1*b*): $h_{vc} = \frac{V_c^2}{2\,g} = \frac{Q^2}{2\,g\,A_c^2}$; and, $Q^2 = 2\,g\,h_{vc}\,A_c^2$. Furthermore, $2\,Q\,dQ = 4\,g\,h_{vc}\,A_c\,dA_c + 2\,g\,A_c^2\,dh_{vc}$; and,

$$\frac{dQ}{dA_c} = \frac{g\,A_c\left(2\,h_{vc} + A_c\,\frac{dh_{vc}}{dA_c}\right)}{Q} \quad \ldots\ldots (36)$$

Combining Equations (35) and (36) and solving for Q:

$$Q = \frac{m\,g\,A_c\,(2\,h_{vc}\,dA_c + A_c\,dh_{vc})}{dh_{vc} + dd_c} \quad \ldots\ldots (37)$$

From Equations (1*a*), (1*b*), and (1*c*) $H_p = d_c + h_{vc} = d_c\,\frac{V_c^2}{2\,g} = d_c + \frac{Q^2}{2\,g\,A_c^2}$; $Q^2 = 2\,g\,(H_p\,A_c^2 - A_c^2\,d_c)$; and, since H_p is assumed constant, $2\,Q\,dQ = 2\,g\,(2\,H_p\,A_c\,dA_c - 2\,A_c\,d_c\,dA_c - A_c^2\,dd_c)$. Finally, at the critical section, $\frac{dQ}{dd_c} = \frac{g}{Q}\left(2\,H_p\,A_c\,\frac{dA_c}{dd_c} - 2\,A_c\,d_c\,\frac{dA_c}{dd_c} - A_c^2\right) = 0$; and, solving for H_p:

$$H_p = A_c\left(\frac{2\,d_c\,\frac{dA_c}{dd_c} + A_c}{2\,A_c\,\frac{dA_c}{dd_c}}\right) = d_c + \frac{A_c}{2}\,\frac{dd_c}{dA_c} \quad \ldots\ldots (38)$$

From Equation (32) $\frac{dd_c}{dA_c} = \frac{1}{B_c}$; and, $H_p = d_c + \frac{A_c}{2\,B_c}$. Combining with Equation (1*c*):

$$h_{vc} = \frac{A_c}{2\,B_c} \quad \text{.......................} (39)$$

Writing the differential equation of Equation (39):

$$dh_{vc} = \frac{1}{2}\left(\frac{B_c\,dA_c - A_c\,dB_c}{B_c^2}\right) = \frac{1}{B_c}\left(\frac{dA_c}{2} - \frac{A_c}{2\,B_c}\,dB_c\right)$$

$$= \frac{1}{B_c}\left(\frac{dA_c}{2} - h_{vc}\,dB_c\right) \quad \text{.......................} (40)$$

Combining Equations (32), (37), (39), and (40), and solving algebraically for Q:

$$Q = \frac{m\,g\,A_c^2\left(\frac{3}{2}\,dA_c - h_{vc}\,dB_c\right)}{\frac{3}{2}\,dA_c - h_{vc}\,dB_c} = m\,g\,A_c^2 \quad \text{..........} (41)$$

ROBERT O. THOMAS,[6] JUN. AM. SOC. C. E. (by letter).—Considerable credit is due the admirable analysis made by the author. The writer is constrained, however, to offer a few criticisms and suggestions with a view to a more complete understanding of the hydraulic action of a control such as has been proposed. To that end the following equations are presented, using only one intermediary, *m*, to determine the elements of the control section:

$$d_c = H - \frac{m\,Q}{2} \quad \text{.......................} (42)$$

$$B_c = \frac{0.249\,\sqrt{H_p - d_c}}{Q\,m^2} \quad \text{.......................} (43)$$

$$m = \frac{S\,(A_p - 2\,B_p\,h_{vp})}{A} + \frac{V}{A\,g} \quad \text{.................} (44)$$

or,

$$m = S + \frac{2\,h_{vp}}{Q}\,(1.00 - V_p\,B_p\,S) \quad \text{..............} (45)$$

The use of Equations (42) to (44) will require less computation than those given in the paper.

The mathematical correctness of the method proposed by the author is beyond question, but the writer entertains doubts as to the practical feasibility in the use of such a control. Analysis of the flow through the control and in the channel above the control leads to the conclusion that the incorporation of such a design would seriously affect the conditions of flow in the canal. It is a condition of the design that the total energy head on the moving water prism shall be the same, neglecting friction, at the piezometer section and at the control. Since the flow through the control is at critical depth, this condition definitely establishes the water surface at both points. It is also a condition of the design that the canal shall carry less water than it otherwise would, if the control station were omitted. This condition alone leads to the conclusion that the channel could not be designed economically, because the cross-section would, of necessity, be larger than would ordinarily be required to convey the given flow.

---

[6] Junior Engr., Rapid Transit Design, City Engr. Office, Los Angeles, Calif.

However, the principal difficulties in the design arise in the section of channel which lies above the rating station. The elevation of the water surface at the piezometer section is not that which can be maintained in the channel above, due to the fact that the flows and depths at this section are not those which would result from uniform flow. Assuming a concrete-lined channel, with "n" equal to 0.014, Table 2 (Column (3)) gives the slopes necessary to maintain a uniform flow in the channel above the piezometer section.

TABLE 2.—CHANNEL SLOPE NECESSARY TO MAINTAIN UNIFORM FLOW

Depth, in feet	Flow, Q, in cubic feet per second	Channel slope
3.00	28.30	0.00043
2.50	23.21	0.00058
2.00	18.12	0.00061
1.60	14.04	0.00072
1.20	9.97	0.00087
0.80	5.89	0.00104
0.418	2.00	0.00090

TABLE 3.—FLOW IN A TRAPEZOIDAL CHANNEL (TEN-INCH SLIDE-RULE COMPUTATIONS)

Depth d_n, in feet	Velocity V_n, in feet per second	HEAD,* IN FEET		
		H_{vn}	H_n	H_p
2.28	3.51	0.192	2.472	3.098
2.00	3.34	0.173	2.173	2.602
1.68	3.11	0.150	1.830	2.104
1.41	2.98	0.138	1.548	1.703
1.13	2.64	0.108	1.238	1.298
0.78	2.27	0.080	0.860	0.882
0.40	1.65	0.042	0.442	0.455

* n denotes uniform flow, and p denotes flow at the piezometer section.

Since it is a requisite, for the proper functioning of the control, that the channel be built so that the depth for any flow, at the piezometer section, will be greater than the depth for uniform flow, it is obvious that the slope must be greater than any calculated slope in Table 2. The effect of this will be to cause a backwater curve extending up stream from the piezometer section to the section of uniform flow.

The writer has assumed a slope of 0.0012, with n equal to 0.014, for the purposes of illustration. Table 3 gives the hydraulic elements as computed with these constants for the flows in Table 2.

For the larger flows there will be a considerable drop in the water surface in passing up stream from the piezometer section. The maximum flow that can pass the control section utilizes only a portion of the channel capacity after uniform flow is attained. The channel, as designed on this slope, would carry a maximum flow of 43.5 cu ft per sec, in uniform flow with a depth of 3 ft. The efficiency of the channel, therefore, is only $\frac{28.3}{43.5}$, or 65 per cent. It is the writer's contention that the construction of a channel that can only be utilized for a portion of its design capacity, due to the existence of a bottleneck at the point of flow measurement, is not economical.

Inasmuch as such devices as the Parshall flume[7] and the control meter developed by Julian Hinds,[8] M. Am. Soc. C. E., will accomplish the same purpose with little interference, if any, with the flow of water in the channel above the control, the writer favors their use, especially so because of the ease and accuracy with which flow conditions in the main channel may be predicted.

The writer is of the opinion that the convenience of a straight-line relationship between depth and discharge is not sufficient to justify the uneconomic over-design of the channel section which is necessitated by its use.

Frank S. Bailey,[9] Assoc. M. Am. Soc. C. E. (by letter).—The author seems to have assigned himself a difficult task and performed it very ingeniously. The writer's first impression of this paper was that it was too complicated for his understanding and possibly in this case the first impression was the best. However, during a leisure hour he began trying to follow the author's reasoning and to check the equations he has derived.

Becoming puzzled over Equation (6) he wrote a letter of inquiry to the author, who replied courteously:

> "* * * if (5) is equated to 0, we find that this equation is satisfied if any of the factors are equated to 0. In other words, $Q - A_c^2\,g\,m = 0$. I did not think it necessary to call attention to the other mathematical possibilities which have no practical significance to our problem, for example when $A_c = 0$."

Although it is probably true, as Mr. Kennison states, that $Q - A_c^2\,g\,m = 0$, the writer has not found why it must be so. He believes that

$$Q^2 - g\,A_c^3\,f'(A_c) = 0 \qquad (46a)$$

and that

$$Q^2 = \frac{g\,A_c^3}{B_c} \qquad (46b)$$

Equation (5), obtained by differentiating Equation (4), could read

$$\frac{dQ}{dA_c} = \frac{Q^2 - g\,A_c^3\,f'(A_c)}{A_c\,(Q - A_c^2\,g\,m)} \qquad (47)$$

in which $f'(A_c)$ signifies the derivative of $f(A_c)$ with respect to A_c. Perhaps Equation (5) is just as valid; but Equation (47) seems more complete. Referring[10] to Fig. 4, it is evident that $\Delta A_c = B_c\,\Delta d_c$; and, in the limit, $\frac{dd_c}{dA_c} = \frac{1}{B_c}$, which is the derivative of $f(A_c)$. Therefore, placing the numerator of Equation (47) equal to zero, the relation between Q and A_c is as given by Equation (46b).

As to the propriety of having a factor in the denominator which equals zero: As the numerator must equal zero to satisfy Equation (5) if the denominator is also zero it seems that the value of the fraction becomes $\frac{0}{0}$ which is generally indeterminate, although not always. In any event no factors in the denominator must, of necessity, be equal to zero, as the denominator may have almost any value and still satisfy the requirement for a maximum value of Q that $\frac{dQ}{dA_c} = 0$, if the numerator is zero.

---

[7] "The Improved Venturi Flume," by R. L. Parshall, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 841.

[8] *Loc. cit.*, p. 859.

[9] East Braintree, Mass.

[10] "Handbook of Hydraulics," by Horace Williams King, McGraw-Hill Book Company, Inc., New York and London, Third Ed. (1939), p. 373.

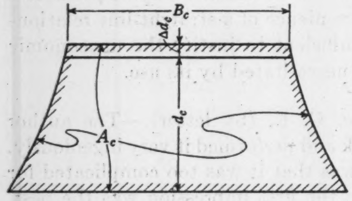


FIG. 4

Horace W. King, M. Am. Soc. C. E., describes the formula

$$\frac{a^3}{T} = \frac{Q^2}{g} \qquad (48)$$

as "a general criterion, applicable to all channels, indicating the relation between cross-section and discharge which must exist in order that flow may occur at critical depth."[10] Equation (48) resolves to

$$Q^2 = \frac{g\,A_c^3}{T_c} \qquad (49)$$

and is identical with Equation (46*b*) if $f'(A_c) = \frac{1}{T_c}$, T_c being the top width of the cross-section of the stream at the control section.

The writer has made a rough calculation of the area A_c, corresponding to a flow Q of 28.3 cu ft per sec, by using values given for d_c and B_c in Table 1(*a*). In this calculation it was assumed that the sides of the section in Fig. 2 were straight between the d_c-intervals. As the sides are curved, the value of 3.34 sq ft, obtained for A_c, is a little too large.

If this value is used in Equation (48), the resulting numerical expression is $\frac{(3.34)^3}{1.092} = \frac{(28.3)^2}{32.2}$, or $34.1 = 24.9$, which does not fulfil the requirements of the formula, as the two sides should be equal. If the precise value of A_c is found to be very close to 3.00 (which is probably the case), Equation (48) would check exactly and would be a proof of the correctness of Mr. Kennison's results.

GEORGE E. BARNES,[11] M. AM. SOC. C. E. (by letter).—On the subject of open-channel control sections, the author is to be credited with: (1) Definition of the problem in general terms amenable to analysis, (2) development of the topic by several original steps, and (3) valid conclusions and results, including certain derived equations of useful form.

Unfortunately, the validity of the author's derived equations is not established by the methods followed in the paper. Equations (7), (10), (14), and (15) depend on Equation (6). The writer challenges the author to defend his derivation of Equation (6) on the basis of the reasoning given in the paper—

---

[11] Prof., Hydr. and San. Eng., and Head, Dept. of Civ. Eng., Case School of Applied Science, Cleveland, Ohio.

namely, "Since Q is a maximum, $\frac{dQ}{dA_c}$ can be equated to 0 so that the relation between Q and A_c is $Q = A_c^2\, g\, m$."

Relative to the foregoing, the following comments are pertinent: If, as the author states, Q is made to vary under constant H_p, the resulting curve (see Fig. 5) is of such form that Q becomes a maximum at some particular value of d, designated as the critical depth. Since $\frac{dQ}{dd}$ or $\frac{dQ}{dA}$ is known to be continuous and is known to change in sign from plus to minus at the point of critical depth, the slope of the tangent to the curve at that point is zero. Therefore, the numerator of the right-hand side of Equation (5) is equal to zero when the denominator is not zero. However, if both numerator and denominator are zero at d_c, further investigation of the nature of Q as a function of A_c is necessary. The author's assumption that the denominator is equal to zero cannot be accepted without adequate proof. Lacking such proof, the correctness of the succeeding equations is open to question.

The method used by the writer to determine the author's equations is as follows: In any open channel of whatever cross-section, the elevation of the energy gradient above the channel bed is $H = h_v + d$ (see Fig. 6). For any particular value of Q the channel proportions for which H is a minimum (flow at critical depth) are found thus:

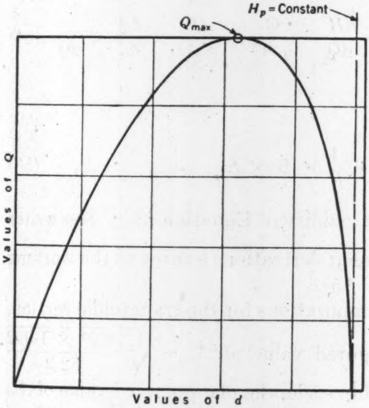


FIG. 5

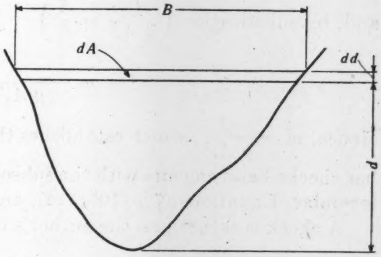


FIG. 6

$$H = \frac{Q^2}{2\,g\,A^2} + d \quad \text{.......................} (50)$$

$$\frac{dH}{dd} = -\frac{Q^2}{g\,A^3} \times \frac{dA}{dd} + 1 = 0 \quad \text{.................} (51)$$

but $\frac{dA}{dd} = B$; hence

$$Q^2 = \frac{g\,A_c{}^3}{B} \qquad \text{(52}a\text{)}$$

and

$$\frac{Q^2}{2\,g\,A_c{}^2} = \frac{A_c}{2\,B} \qquad \text{(52}b\text{)}$$

Referring to Fig. 1—by construction

$$d = H - \frac{Q^2}{2\,g\,A^2} = m'\,Q + m'\,n' - \frac{Q^2}{2\,g\,A^2} \qquad \text{(53}a\text{)}$$

By definition, the slope of the rating curve is

$$S = \frac{dd}{dQ} \qquad \text{(53}b\text{)}$$

and, by definition, the slope of the depth versus energy-head curve is

$$m' = \frac{dH}{dQ} \qquad \text{(53}c\text{)}$$

Then, differentiating Equation (53*a*),

$$\frac{dd}{dQ} = \frac{dH}{dQ} - \frac{Q}{g\,A^2} + \frac{Q^2}{g\,A^3} \times \frac{dA}{dQ} = \frac{dH}{dQ} - \frac{Q}{g\,A^2} + \frac{Q^2}{g\,A^3} \times \frac{dA}{dd} \times \frac{dd}{dQ} \qquad \text{(54)}$$

and, by substitution $\left(\frac{Q^2}{g\,A_c{}^3} = \frac{1}{B}\right)$:

$$S = m - \frac{Q}{g\,A_c{}^2} + \frac{1}{B} \times B \times S \qquad \text{(55)}$$

Hence, $m = \frac{Q}{g\,A_c{}^2}$, which establishes the validity of Equation (6). The writer has checked and concurs with the subsequent derivations leading to the working formulas, Equations (7), (10), (14), and (15).

A check is secured on the author's computations for the trapezoidal section, at peak discharge, as follows: The required value of $A_c = \sqrt[3]{\frac{28.3^2 \times 1.092}{32.2}}$ $= 3.01$. If the control section is plotted to scale, the planimetered value of A_c, as drawn, will be found to agree. It follows also that the computed value of $h_{vc} + d_c = H_p$ (Equation (1*c*)), which proves to be the case, provided that the reference plane for both control and piezometer sections is taken at the bottom of the piezometer section for the values shown in Table 1.

The writer would also like to comment on certain matters touched upon in the paper, bearing on applications of the method and practical considerations.

A control section is one at which the minimum energy head for any discharge is independent of the energy head down stream. The particular control section to be adopted for any given case depends first on the function it is to perform. One common function of a control section is the measurement of discharge;

another, perhaps less common, is that of regulating velocities up stream. With intelligent design, it is possible to combine the two functions in one structure, although the requirements for one do not necessarily meet the requirements for the other. In the matter of measurement, sensitivity of the water-surface level with respect to discharge is important. In the matter of velocity regulation, not only is the average velocity up stream of importance, but the effect of the control section on velocity distribution up stream may be important as well. Frequently, the necessity of conserving head has considerable influence on the selection of a suitable control section. The author's statement that his method "is claimed * * * to have possibilities in the application to practical problems" is a modest one. The writer believes that the author's paper, if read with understanding, will do much to offset a tendency, sometimes noted, toward the adoption of accepted types of control sections, after only superficial investigation, in situations where an original design is much to be preferred.

In the general case, when flow is at critical depth, the velocity head is half the average depth (see Equation (52*b*)), and the energy head at the control section is $1.5\, d_{\text{average}}$. Of course, this is H_p minus losses. Where both the control section and the up-stream sections are of regular geometric shape, the equation, $H_p = H_c +$ losses, expressed in terms of the variables including d_c, may be set up without great difficulty. However, except for the very simplest cases, its solution may be too complicated for handling except by "cut-and-try" methods. The writer has had occasion to design control sections for grit

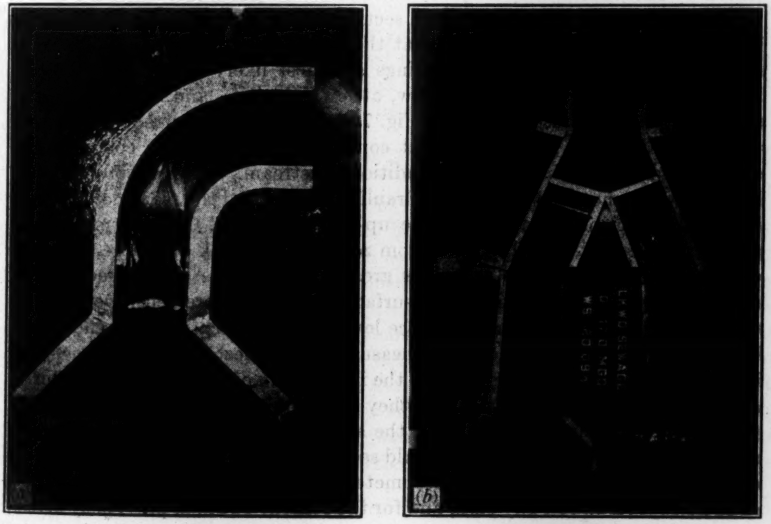


Fig. 7.—Flow in Control Sections Designed for Grit Chambers (Model Scale 1 : 6)— (a) Ward's Island, New York, N. Y. (Flow Equivalent to 86 Cu Ft per Sec); (b) Lakewood, Ohio (Flow Equivalent to 26 Cu Ft per Sec)

chambers for the purpose of measuring discharge, or for regulating velocities, or both (see Fig. 7). In such cases, the velocity head in the grit chamber is so small as to be practically negligible, and the difference in water-surface elevation between the control section and the up-stream section may be taken as the velocity head at critical depth in the control section, plus the head loss in the transition. The head loss in the transition may be quite closely estimated, or, if necessary, determined by experiment, as a function of this velocity head. The following simplified equation may then be written:

$$Z_g = Z_c + d_c + (1 + k)\, h_{vc} \ldots\ldots\ldots\ldots\ldots\ldots (56)$$

in which Z_g = the water-surface elevation in the grit chamber, and Z_c = invert elevation in the control section. It is important to note in this connection that conditions may permit designing the up-stream section to fit a suitable control section, as an alternate to the more obvious procedure of designing the control section with respect to the up-stream section. By arbitrary proportioning of the control section and the up-stream section, suitable design for a given range of flows often may be secured with readiness.[12] Such cut-and-try methods, as stated by the author, may be just as useful and important as others.

If the performance of the control section is to be measured, it will probably be with piezometers located either at the up-stream section or at the control section, or both. Piezometer readings are most likely to be accurate when taken in sections of undisturbed flow, at low velocities, and in the absence of standing waves or cross currents. Fig. 7 illustrates the occurrence of standing waves in the throat of two different control sections, and the comparatively smooth and undisturbed surface condition up stream. Thus there is likely to be better agreement between the hydraulic gradient and the piezometer reading when observations are taken at the up-stream section. Furthermore, for a given rating curve with Q varying from zero to some peak discharge, the range of depth in the up-stream section is greater than at the control section, and consequently the up-stream water-surface level is a more sensitive indicator than the control section water-surface level. In some cases, adherence to the belief that critical depth can be measured with piezometers at some point in the control section has resulted in the installation of open-channel measuring devices which are less sensitive than they might be with the piezometers located up stream. Throughout his paper, the author refers to the up-stream section as the piezometer section. This would seem to imply, although it is not specifically stated, that the location of piezometers in the up-stream section is desirable or to be expected. In any case, and for the reasons outlined previously herein, the writer would recommend strongly that, in the usual situation, piezometer readings be taken up stream rather than at the control section.

The writer is not quite clear as to the author's statement that the reason for raising the invert of the channel at the control section by some arbitrary amount is to offset any tendency toward surface waves. Of course, it is necessary to raise the invert, or otherwise to constrict the control section, to insure that there is a localized break in the hydraulic gradient. Otherwise the true location of the control section would not be known so accurately. The author does not point out other means for constricting the control section which are perhaps equally suitable. In the writer's experience, it has often been preferable to reduce the width of the control section all the way from floor level to free-board level with, or without, raising the invert.

---

[12] "Unique Solution for Grit Chamber Design," by George E. Barnes, *Waterworks and Sewerage*, September, 1939.

The author limits the scope of his paper to a precise statement of the theory involved and a short discussion of some practical applications. Any further discussion by the writer of practical considerations would be beyond the limits of the paper as written. The paper is remarkably comprehensive, considering its brevity, and is written with an accurate conception of the practical aspects of the subject.

KARL R. KENNISON,[13] M. AM. SOC. C. E. (by letter).—There is a reasonable unanimity among the various discussions as to the correctness of the theory advanced and conclusions reached. It is evident that the writer's aversion to the publication, in *Proceedings*, of lengthy details of mathematical equations has gotten him into difficulties and led to the expenditure of much time and effort by those discussing the paper which might have been avoided.

Prior to its last-minute hasty revision, the paper contained the following explanation of the method of finding Equation (6), which should meet, satisfactorily, the criticism properly aimed at the explanation as published: Combining Equations (1) and (2), Equation (4), adapted to the condition of critical flow through the controlling section, becomes

$$\frac{Q^2}{2\,g\,A_c^2} = m\,Q + m\,n - d_c \ldots\ldots\ldots\ldots\ldots\ldots (57)$$

There are five variables in Equation (57), but m and n are constant for small or differential changes in the others and hence do not affect, other than as constants, the relation which it is desired to find between the other three. One of the conditions that makes the section a controlling or critical section is that, for any one value of the variable depth d_c and with the same relation between all the variables and this tentatively constant depth, the area A_c must be the minimum area that will satisfy the equation; because if any smaller area at such constant depth could discharge as much as the rating curve calls for, then A_c is too large. That is, some other smaller critical section must be holding the water back. Accordingly, for present purposes, one may: Tentatively consider m, n, and d_c as constants; differentiate Equation (57) with respect to the variables Q and A_c; and—since A_c is a minimum and $\frac{dA_c}{dQ}$ equal to 0—write:

$$\frac{dA_c}{dQ} = \frac{A_c}{Q} - \frac{A_c^3\,g\,m}{Q^2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (58)$$

Hence (see Equation (6)) $Q = A_c^2\,g\,m$. The independent proofs developed in the discussion, such as that of Mr. Willes and Mr. Barnes, are very gratifying.

Mr. Streeter questions why Equation (2) requires the condition of critical flow. This is because it involves m and n which, by definition, are tied to the rating curve of the controlling section. In other words, they apply to the condition of critical flow through the controlling section.

---

[13] Chf. Engr., Met. Dist. Water Supply Comm., Commonwealth of Massachusetts, Boston, Mass.

It is impossible for the writer to follow the reasoning of Mr. Thomas in his criticism that the use of such a controlling section precludes an economical approach channel. Whatever the maximum discharge of the economically designed channel is—whether 28.3 cu ft per sec or 43.5 cu ft per sec—this is, of course, what determines the maximum capacity of the controlling section, or the upper end of the desired rating curve. The slope of the rating curve must fit the conditions of available head. If there is a dearth of available head for the measurement, then the rating curve must be correspondingly flat. Within the range of desired flows there will be times when there is some non-uniform flow or back-water curve; but this should not affect the economical design of the main channel in any way, and any special modification of this channel that might be needed would be confined to a very short length immediately up stream from the piezometer section.

Answering some of the inquiries by Mr. Barnes: It is true, and should be stated definitely, that the location of the piezometers is in the up-stream section. If they were near the control section, recording would be erratic and unpredictable, because, when flow is nearly at the critical depth, the upper and lower alternative stages are so close together that the surface jumps easily from one to the other in waves. For the lower rates of flow, in the practical examples given for illustration, this tendency would extend up stream to the piezometer section if the invert at the control section were not raised to prevent it. The accurate recording of a water surface depends on forcing the alternative stages apart from each other. As Mr. Barnes points out the same thing could be done by reducing the width instead of raising the invert. However, the latter was better adapted to the examples given which sought a straight-line rating curve of maximum steepness.

In conclusion the author is gratified to note an appreciation of the comprehensive character of this study and of its adaptation to practical aspects.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2073

GRAPHICAL ARCH ANALYSIS APPLICABLE TO ARCH DAMS

BY CARL H. HEILBRON, JR.,[1] ASSOC. M. AM. SOC. C. E., AND
WILLIAM H. SAYLOR,[2] JUN. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. I. M. NELIDOV, AND CARL H. HEILBRON, JR. AND WILLIAM H. SAYLOR.

SYNOPSIS

In this paper is presented a system, mostly graphical, for analyzing unsymmetrical fixed-end arches of variable thickness and radius under loads of any type. The general equations of arch analysis are first written, from which the deflections and stresses in the arch may be obtained. These equations are then modified to include the concept of a "trial crown thrust," this concept making it possible to obtain arch stresses and deflections with all necessary accuracy when the greater part of the computations are performed graphically. The graphical constructions used in evaluating the equations are developed, the procedure is outlined in detail, and an illustrative example is given.

INTRODUCTION

In 1936 the Design Division of The Metropolitan Water District of Southern California undertook the design of the Gene Wash and Copper Basin Dams,[3] arch structures 156 and 210 ft high which form reservoirs near the easterly end of the Colorado River Aqueduct. In analyzing the designs for these dams, a system of trial load analysis was developed and successfully applied, following the principles presented by C. H. Howell, M. Am. Soc. C. E., and the late Mr. A. C. Jaquith.[4] A most important part of this system was the analysis of the arches. Disregarding the broader aspects of the analysis of arch dams, the details of the method of analyzing arches, as applied to this problem, are herein outlined.

NOTE.—Published in January, 1939, *Proceedings*.

[1] **Associate Engr., U. S. Engr. Office, Los Angeles, Calif.**

[2] **Asst. Engr., U. S. Engr. Office, Los Angeles, Calif.**

[3] **"High Arch Dams to Be Built on Colorado River Aqueduct," *Western Construction News*, March, 1937.**

[4] ***Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1191.**

Although, to date, the procedure presented has been used only for dam arches, it is equally applicable to arches of other types. It covers any fixed-end arch, symmetrical or otherwise, under any loading or temperature change. Deformations due to moment, thrust, shear, and uniform temperature change are included. Complications such as foundation deformation, interior arch sections, moment loads, and non-uniform temperature change, may be introduced.

The general theory of arch analysis will be reviewed briefly. The usual assumptions as to homogeneity of material in the arch and the application of Hooke's law are made. The arch is considered as cut at some point known as the crown. The effect of either half of the arch upon the other is then replaced by three unknown forces acting at the crown, a moment, M_c, and two direct forces, H and V. These three forces are applied to both halves of the arch, but in opposite directions, and each half of the arch is considered as a cantilever acted upon by its share of the external load, the temperature change, and the unknown crown forces. Equations are then written which give the three cantilever deflections at the crown (rotation, β_c, and translations, Δ_x and Δ_y). As the crown deflections are the same for each half of the arch, corresponding expressions for the two halves are equated, giving three formulas that may be solved simultaneously for the values of the crown forces. With the crown forces determined, each half of the arch is analyzed as a cantilever under a known system of loads.

The foregoing general approach to arch analysis has been presented in detail elsewhere[3] and in the following explanation it is assumed to be familiar to the reader. The equations for the crown deflections of an arch are given without derivation. Starting from this point, necessary changes are made in the equations, and the graphical construction used in this method of arch analysis is developed.

Notation.—The symbols used in this paper are defined where they first appear and are assembled for reference in the Appendix. The sign conventions adopted for the paper are either given in the definitions or illustrated by Fig. 1.

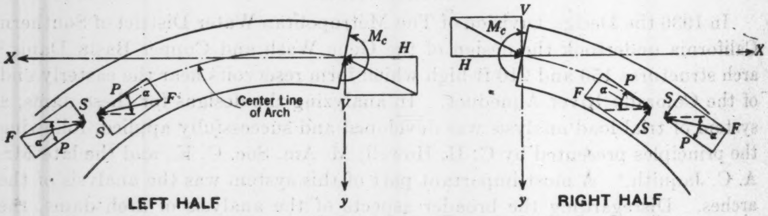


FIG. 1.—SIGN CONVENTIONS FOR ARCH ANALYSIS; POSITIVE QUANTITIES ILLUSTRATED

EQUATIONS

General equations for the crown deformations of an arch, including the effects of moment, thrust, shear, and temperature strain, were given by George R. Rich,[5] M. Am. Soc. C. E., in 1929. The crown deformations for the left half of the arch are:

---

[5] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1230.

Rotation at the crown:

$$\beta_c = +\sum_l \frac{M_e\,s}{E\,I} + \sum_l \frac{V\,x\,s}{E\,I} + \sum_l \frac{H\,y\,s}{E\,I} + \sum_l \frac{M_c\,s}{E\,I} \quad \ldots\ldots\ldots (1)$$

vertical deflection at the crown (positive upward):

$$\begin{aligned}\Delta_y = &+\sum_l \frac{M_e\,x\,s}{E\,I} + \sum_l \frac{V\,x^2\,s}{E\,I} + \sum_l \frac{H\,x\,y\,s}{E\,I} + \sum_l \frac{M_c\,x\,s}{E\,I} \\ &-\sum_l \frac{P_e\,s\sin\alpha}{E\,t} + \sum_l \frac{V\,s\sin^2\alpha}{E\,t} - \sum_l \frac{H\,s\sin\alpha\cos\alpha}{E\,t} \\ &+ y_{al}\,\epsilon\,T + n\sum_l \frac{S_e\,s\cos\alpha}{E\,t} + n\sum_l \frac{V\,s\cos^2\alpha}{E\,t} + n\sum_l \frac{H\,s\sin\alpha\cos\alpha}{E\,t} \quad \ldots (2)\end{aligned}$$

and, horizontal deflection at the crown (positive leftward):

$$\begin{aligned}\Delta_x = &+\sum_l \frac{M_e\,y\,s}{E\,I} + \sum_l \frac{V\,x\,y\,s}{E\,I} + \sum_l \frac{H\,y^2\,s}{E\,I} + \sum_l \frac{M_c\,y\,s}{E\,I} \\ &+\sum_l \frac{P_e\,s\cos\alpha}{E\,t} - \sum_l \frac{V\,s\sin\alpha\cos\alpha}{E\,t} + \sum_l \frac{H\,s\cos^2\alpha}{E\,t} \\ &- x_{al}\,\epsilon\,T + n\sum_l \frac{S_e\,s\sin\alpha}{E\,t} + n\sum_l \frac{V\,s\sin\alpha\cos\alpha}{E\,t} + n\sum_l \frac{H\,s\sin^2\alpha}{E\,t} \quad \ldots (3)\end{aligned}$$

in which: β_c = the slope or rotation of the arch center line, at the crown (positive counter-clockwise); l = a subscript that refers to the left half of the arch; M_e = moment in the arch, taken about the center line, due to external loads; x and y = rectangular co-ordinates with their origin at the crown center line; s = length of a voussoir along the center line; E = modulus of elasticity in direct stress; I = moment of inertia of a radial section of arch; V = vertical force at the crown; H = horizontal force at the crown; M_c = moment in the arch at the crown; P_e = thrust in the arch parallel to the center line, due to external loads; α = angle between the X-axis and the center line of the arch; t = thickness of the arch, normal to the center line; x_a and y_a = values of x and y, respectively, for the abutment; T = temperature change (rise positive); S_e = shearing force due to external loads only; n = ratio of E to effective modulus in shear $\left(\text{that is, } \frac{E}{\frac{5}{6}E_s}\right)$; and, ϵ = thermal coefficient of expansion.

Similar equations may be written for the right side of the arch, with appropriate changes in signs.

It is convenient to write Equations (1), (2), and (3) for the left side of the arch in the following form:

$$E\,\beta_c = +A_{1l}\,M_c + B_{1l}\,H + C_{1l}\,V + D_{1l} \quad \ldots\ldots\ldots\ldots (4a)$$

$$E\,\Delta_y = +C_{1l}\,M_c + B_{2l}\,H + C_{2l}\,V + D_{2l} \quad \ldots\ldots\ldots\ldots (4b)$$

and,

$$E\,\Delta_x = +B_{1l}\,M_c + B_{3l}\,H + B_{2l}\,V + D_{3l} \quad \ldots\ldots\ldots\ldots (4c)$$

For the right side of the arch the equations are similar but with negative signs prefixed to the left sides of the equations.

TABLE. 1.—VALUES OF THE COEFFICIENTS A, B, C, D, AND G

Coefficient	Quantity No.	Definition	SIGNS, AT RIGHT AND LEFT HALVES	
			Left	Right
A_1	1	$\Sigma \frac{s}{I}$	+	+
B_1	2	$\Sigma \frac{y\,s}{I}$	+	+
B_2	3	$\Sigma \frac{x\,y\,s}{I}$	+	−
	4	$(n-1)\,\Sigma \frac{s \sin\alpha \cos\alpha}{t}$	+	−
B_3	5	$\Sigma \frac{y^2 s}{I}$	+	+
	6	$\Sigma \frac{s}{t}$	+	+
	7	$(n-1)\,\Sigma \frac{s \sin^2\alpha}{t}$	+	+
C_1	8	$\Sigma \frac{x\,s}{I}$	+	−
C_2	9	$\Sigma \frac{x^2 s}{I}$	+	+
	10	$n\,\Sigma \frac{s}{t}$	+	+
	11	$(n-1)\,\Sigma \frac{s \sin^2\alpha}{t}$	−	−
D_1	12	$\Sigma \frac{M_e\,s}{I}$	+	+
D_2	13	$\Sigma \frac{M_e\,x\,s}{I}$	+	−
	14	$\Sigma \frac{P_e\,s \sin\alpha}{t}$	−	+
	15	$n\,\Sigma \frac{S_e\,s \cos\alpha}{t}$	+	−
	16	$y_a\,\epsilon\,T\,E$	+	−
D_3	17	$\Sigma \frac{M_e\,y\,s}{I}$	+	+
	18	$\Sigma \frac{P_e\,s \cos\alpha}{t}$	+	+
	19	$n\,\Sigma \frac{S_e\,s \sin\alpha}{t}$	+	+
	20	$x_a\,\epsilon\,T\,E$	−	−
G_1	21	$\Sigma \frac{M_t\,s}{I}$	+	+
G_2	22	$\Sigma \frac{M_t\,x\,s}{I}$	+	−
	23	$\Sigma \left(\frac{P_t\,s \sin\alpha}{t} - n \frac{S_t\,s \cos\alpha}{t} \right)$	−	+
	24	$y_a\,\epsilon\,T\,E$	+	−
G_3	25	$n\,\Sigma \frac{M_t\,y\,s}{I}$	+	+
	26	$\Sigma \left(\frac{P_t\,s \cos\alpha}{t} + n \frac{S_t\,s \sin\alpha}{t} \right)$	+	+
	27	$x_a\,\epsilon\,T\,E$	−	−

Upon substituting $(1 - \sin^2 \alpha)$ for $\cos^2 \alpha$, and combining terms, the values of the coefficients, A, B, C, D, etc., are brought into the forms given in Table 1. As an example of the interpretation of this table, the B_2-coefficients are written out in the usual algebraic form:

$$B_{2l} = + \sum \frac{x\, y\, s}{I} + (n - 1) \sum \frac{s \sin \alpha \cos \alpha}{t} \quad \text{..........(5a)}$$

and,

$$B_{2r} = - \sum \frac{x\, y\, s}{I} - (n - 1) \sum \frac{s \sin \alpha \cos \alpha}{t} \quad \text{..........(5b)}$$

The corresponding crown deflections for the two sides of the arch as given by Equations (4) are now equated. In the resulting expressions the coefficients for the left and right sides are combined to give new coefficients as follows:

$$A_1 = A_{1l} + A_{1r} \quad \text{......................(6a)}$$

$$B_1 = B_{1l} + B_{1r} \quad \text{......................(6b)}$$

$$C_1 = C_{1l} + C_{1r} \quad \text{......................(6c)}$$

$$D_1 = D_{1l} + D_{1r} \quad \text{......................(6d)}$$

and, similarly, for coefficients with the subscripts, 2 and 3. Using these new coefficients the three formulas resulting from equating the crown deflections are,

$$A_1\, M_c + B_1\, H + C_1\, V + D_1 = 0 \quad \text{..............(7a)}$$

$$C_1\, M_c + B_2\, H + C_2\, V + D_2 = 0 \quad \text{..............(7b)}$$

and,

$$B_1\, M_c + B_3\, H + B_2\, V + D_3 = 0 \quad \text{..............(7c)}$$

When the A, B, C, and D coefficients are determined for any arch, their values may be substituted in Equations (7). Simultaneous solution of these three equations then gives the values of the crown forces, M_c, H, and V.

TRIAL CROWN FORCE

In the system of analysis presented herein, Equations (7) are replaced by another similar set of formulas.

To show the need for this substitution, it will first be assumed that Equations (7) are being used. After the values of the crown forces have been determined from them, each half of the arch is considered as a cantilever acting under a known system of loads. From these loads, the moments, thrusts, and shears in the arch are obtained, and from the latter, in turn, the deflections are computed. It is found that in a comparatively thin arch most of the deflections are moment deflections. Thus, the accuracy with which the moments in the arch are found may largely determine the accuracy with which the deflections are found. The general equation for moment in the left half of an arch is,

$$M_l = M_c + H\, y + V\, x + M_{el} \quad \text{................(8)}$$

For the right half of the arch a similar equation holds, with a minus sign before the term, $V\, x$. With ordinary loads on a thin arch the value of M_{el} at any point will be negative and may be ten or a hundred times as large as the value of M_l. If the external load and the arch are reasonably symmetrical the values of M_c and $V x$ will be small, but the value of $H y$ will be positive and numerically about the same as the value of M_l. Thus, the value of M_l depends upon the difference of two relatively large values, M_{el} and $H y$; and a small percentage error in either of these values will make a large percentage error in M_l. Using graphical procedures it has been found to be impracticable to evaluate these functions with enough accuracy to obtain a satisfactory determination of deflections for an arch.

To avoid this difficulty the concept of the "trial crown force" is introduced. The value of H is considered as divided into two parts,

$$H = H_t + H_c \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

in which H_t is the trial crown force. Its value is chosen at the beginning of an analysis, as close to H as it may be estimated; thereafter it is handled as if it were a part of the external load. Equations (7) may now be solved exactly as previously explained except that the value of H that is determined is not the true horizontal crown thrust but is H_c, that part of the crown thrust that was not included in the trial crown thrust, H_t. To avoid confusion, however, Equations (4), (7), and (8) are rewritten using a notation which includes the use of a trial crown thrust, H_t, as an external load. First Equation (8) becomes,

$$M_l = M_c + H_c y + V x + M_{tl} \ldots\ldots\ldots\ldots\ldots (10)$$

for the left half, and with $(-\,V x)$ for the right half. Symbol M_t is the moment at any point on the center line of the arch, due to the external loads and the trial crown force.

If H_t has been chosen close to the value of H, H_c will be small and M_{tl} will be of the same order as M_l. Thus, the accuracy obtainable with graphical procedure is sufficient to give a satisfactory determination of deflections in an arch, as will be shown subsequently.

Using the notation which includes the trial crown force, H_t, as an external load, Equations (4) become,

$$E\,\beta_c = +\,A_{1l}\,M_c + B_{1l}\,H_c + C_{1l}\,V + G_{1l} \ldots\ldots\ldots (11a)$$

$$E\,\Delta_y = +\,C_{1l}\,M_c + B_{2l}\,H_c + C_{2l}\,V + G_{2l} \ldots\ldots\ldots (11b)$$

and,

$$E\,\Delta_x = +\,B_{1l}\,M_c + B_{3l}\,H_c + B_{2l}\,V + G_{3l} \ldots\ldots\ldots (11c)$$

and, similarly, with minus signs on the left, for the right side of the arch. The G-coefficients are the same as the D-coefficients except that M_e, P_e, and S_e are replaced by M_t, P_t, and S_t, in which P_t and S_t are thrust and shear due to external loads and trial crown force. The G-values are given in Table 1. They may be combined similarly to Equations (6), thus:

$$G_1 = G_{1l} + G_{1r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

and, similarly, for G_2 and G_3. Using the concepts and notation of Equations (11), Equations (7) become,

$$A_1\,M_c + B_1\,H_c + C_1\,V + G_1 = 0 \ldots\ldots\ldots\ldots (13a)$$

$$C_1\,M_c + B_2\,H_c + C_2\,V + G_2 = 0 \ldots\ldots\ldots\ldots (13b)$$

and,

$$B_1 M_c + B_3 H_c + B_2 V + G_3 = 0 \quad \ldots\ldots\ldots\ldots (13c)$$

Equations (13) are the formulas that are used in determining the values of the crown forces, by a procedure which will be explained subsequently. The trial crown force need not be restricted to H. Under certain conditions the same concept may be applied beneficially to V or to M_c. The procedure is similar to that shown for H. Generally speaking, however, unless the arch shape or the load is very unsymmetrical, the use of a trial force for H is all that is desirable.

DEFLECTION EQUATIONS

With the values of the crown forces, M_c, H_t, and V, known, the deflections in each half of the arch are calculated. The deflections of the arch crown may be determined by substituting the values of the crown forces into Equations (11); but the deflections at other points are also desired. These deflections are determined by means of the following relations: The rotation at any point of the arch is given by,

$$E\,\beta = \sum_a \frac{M\,s}{I} \quad \ldots\ldots\ldots\ldots (14)$$

In any individual voussoir, the displacement of the end farther from the abutment with respect to the nearer end is composed of four parts, as follows:

Displacement due to moment (at right angles to the center line of the arch) is:

$$\Delta_M = \beta\, s \quad \ldots\ldots\ldots\ldots (15a)$$

displacement due to thrust (parallel to the center line of the arch) is:

$$\Delta_P = \frac{P\,s}{E\,t} \quad \ldots\ldots\ldots\ldots (15b)$$

in which P = the thrust parallel to the arch center line; displacement due to shear (at right angles to the center line of the arch) is:

$$\Delta_S = n\,\frac{S\,s}{E\,t} \quad \ldots\ldots\ldots\ldots (15c)$$

in which S = the shearing force normal to the arch center line; and, displacement due to temperature (parallel to the center line of the arch) is:

$$\Delta_T = \epsilon\, T\, s \quad \ldots\ldots\ldots\ldots (15d)$$

The displacement at any point may be expressed as the vector sum of a series of terms such as Equations (15). In a commonly used system of notation, a vector of length, r, in a direction making an angle, θ, with the positive X-axis is denoted by $r\,e^{i\theta}$, and the vector sum of two such vectors is indicated simply by using a plus sign.[6]

---

[6] See, for instance, "Engineer's Manual," by Ralph G. Hudson (1917), pp. 61–62.

In this notation, the deflection at any point on the left side of the arch can be found in magnitude and direction from,

$$E\,\Delta = \sum_{a}\left[E\,\beta\,s\,e^{i(\alpha+0.5\pi)} + \frac{P\,s}{t}\,e^{i(\alpha+\pi)} + n\,\frac{S\,s}{t}\,e^{i(\alpha+0.5\pi)} + \epsilon\,T\,E\,s\,e^{i\alpha}\right]\,.\,.(16)$$

and, similarly, for the right side, with appropriate changes in sign.

FINITE VOUSSOIRS

The terms in Table 1 have been written as summations of finite units. This is necessary as graphical constructions are to be used in the evaluation of the terms. Thus it is that the arch is considered divided into a number of finite lengths called voussoirs, each voussoir having a length, s. Any number of voussoirs may be chosen, but it is found that using from six to eight, of about equal length in each half of the arch, gives satisfactory results. The ends of the voussoirs are referred to as joints and are numbered from the crown to the abutments, left and right.

As finite voussoirs are used it is necessary to determine, in respect to each function, such as M, P, S, t, and I, whether it is to be measured at the joints or at the mid-points of the voussoirs. In this analysis the following convention is used: That part of the load on the arch which lies between the centers of adjacent voussoirs is considered as acting as a concentrated load at the joint. Thus, the thrust and shear are considered constant through the length of a voussoir. The length, s, entering into the $\frac{s}{t}$-terms, or thrust and shear terms, is measured from joint to joint, and the t-values are measured at the centers of the voussoirs. Increments of thrust and shear deflection are thus found from joint to joint.

As the change in moment values is obtained by multiplying the shear values by the length of the voussoir, the moments are found at the joints, and are considered as constant from center to center of adjacent voussoirs. In finding the $\frac{s}{I}$-values entering into the moment deflection terms, the s-values, therefore, are measured between voussoir centers, and the I-values are computed from t-values measured at the joints.

The $\frac{M\,s}{I}$-values, or curvature values, like the M-values, are considered as constant from center to center of voussoirs, giving increments of slope or rotation from center to center. Then rotations are found at the voussoir centers, and are considered as constant from joint to joint. Increments of moment deflections are thus found from joint to joint, just as for the thrust and shear deflections. All deflections, then, are found at the joints.

GRAPHICAL MULTIPLICATION AND INTEGRATION

Several of the functions listed in Table 1 are evaluated by a method of graphical multiplication and integration. An example of this procedure is given in Fig. 2, as taken from Figs. 3 and 4. The values of y and $\frac{s}{I}$ being given for each joint, it is desired to determine the values of $\sum \frac{y\,s}{I}$ and $\sum \frac{y^2\,s}{I}$ for the left half of an arch. On a vertical line called the "y-base" (see Fig. 3(c)), the values of y are laid off to a given scale, each value being measured from the zero point. From an origin on a horizontal line called the "$\sum \frac{s}{I}$-base," values of $\frac{s}{I}$ (Fig. 3(d)) are laid off, successively, to another arbitrary scale. The abscissa of each point thus established then represents the value of $\sum \frac{s}{I}$ taken from the crown to some voussoir center, whose number is given to the point. Vertical lines are drawn through these $\sum \frac{s}{I}$-points, and are called by the same numbers.

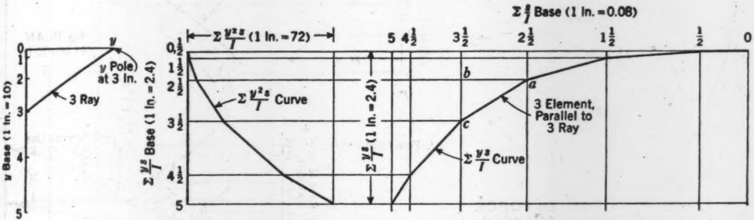


FIG. 2.—GRAPHICAL MULTIPLICATION AND INTEGRATION

A "y-pole" is established at a point an arbitrary horizontal distance ("pole distance") from the zero point of the y-base (Fig. 3(c)). Rays are considered as drawn from the y-pole to each of the points on the y-base. A broken line called the "$\sum \frac{y\,s}{I}$-curve" (Fig. 3(e)), is drawn as follows: Parallel to a ray from the y-pole to Point 1 on the y-base a line is drawn, called the "1 element," beginning at a point on the $\frac{1}{2}$ line of the $\sum \frac{s}{I}$-base and continuing to the $1\frac{1}{2}$ line. Parallel to a ray from the y-pole to the 2 point on the y-base is drawn a "2-element," beginning at the end of the 1-element and continuing to the $2\frac{1}{2}$-line of the $\sum \frac{s}{I}$-base. The procedure is continued to the abutment, the y-value at the abutment being used with the $\frac{s}{I}$-value for the last half voussoir. The projection on a vertical line of the jointed line thus drawn is the value of $\sum \frac{y\,s}{I}$ to a certain scale. The proof is as follows: Consider (Fig. 2) the 3-ray and the 3-element. Then, in the similar triangles (y, 0, 3), and (a, b, c), $\frac{(y,\,0)}{(0,\,3)} = \frac{(a,\,b)}{(b,\,c)}$; whence,

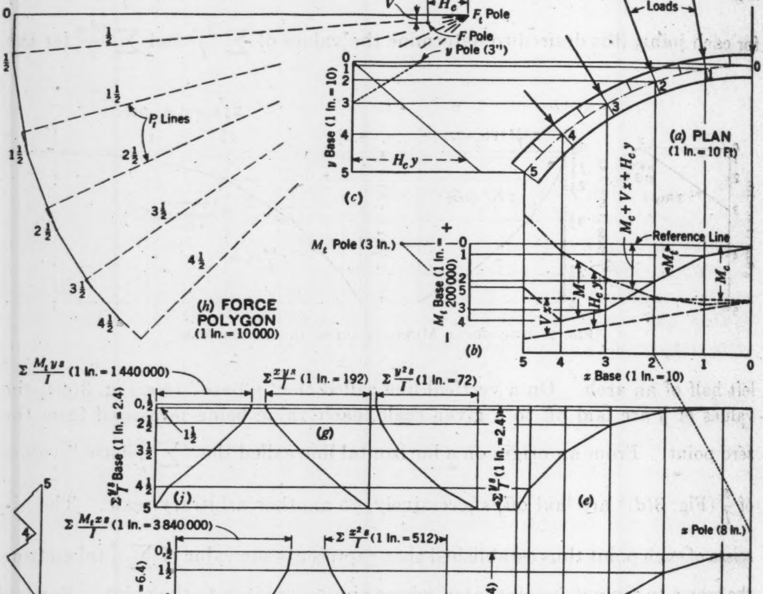

V
H_c
F_t Pole
F Pole
y Pole (3")
Loads
P_t Lines
(h) FORCE POLYGON (1 In. = 10 000)
y Base (1 In. = 10)
$H_c y$
(c)
(a) PLAN (1 In. = 10 Ft)
$M_c + Vx + H_c y$
Reference Line
M_t Pole (3 In.)
M_t Base (1 In. = 200 000)
M_t
M_c
Vx
$H_c y$
(b)
x Base (1 In. = 10)
$\Sigma \frac{M_t y s}{I}$ (1 In. = 1 440 000)
$\Sigma \frac{x y s}{I}$ (1 In. = 192)
$\Sigma \frac{y^2 s}{I}$ (1 In. = 72)
$\Sigma \frac{y s}{I}$ Base (1 In. = 2.4)
(g)
(j)
(e)
$\Sigma \frac{y s}{I}$ (1 In. = 2.4)
x Pole (8 In.)
$\Sigma \frac{M_t x s}{I}$ (1 In. = 3 840 000)
$\Sigma \frac{x^2 s}{I}$ (1 In. = 512)
$\Sigma \frac{x s}{I}$ Base (1 In. = 6.4)
(i)
$\Sigma \frac{x s}{I}$ (1 In. = 6.4)
(S)
$\Sigma \frac{M_t s}{I}$ (1 In. = 48 000)
(k)
(l)
$\Sigma\left(\frac{P_t\, s \operatorname{Cos} \alpha}{t} + 3 \frac{S_t\, s \operatorname{Sin} \alpha}{t}\right)$
$\Sigma\left(\frac{P_t\, s \operatorname{Sin} \alpha}{t} - 3 \frac{S_t\, s \operatorname{Cos} \alpha}{t}\right)$
F_t Deflections (1 In. = 200 000)
$\Sigma \frac{s \operatorname{Sin} \alpha \operatorname{Cos} \alpha}{t}$
$\Sigma \frac{s \operatorname{Sin}^2 \alpha}{t}$ (1 In. = 1)
(m)
Origin
M Pole (2 In.)
− M Base (1 In. = 200 000) +
(n)
$\Sigma \frac{M s}{I} - E\beta$ (1 In. = 32 000)
(o)
Graphical
Computed
E × Deflections (1 In. = 500 000)
(p)
$\Sigma \frac{s}{I}$ Base (1 In. = 0.08)
(d)
0
$\frac{1}{2}$
$1\frac{1}{2}$
$2\frac{1}{2}$
$3\frac{1}{2}$
$4\frac{1}{2}$
1
2
3
4
5

FIG. 3.—ANALYSIS OF LEFT HALF OF ARCH

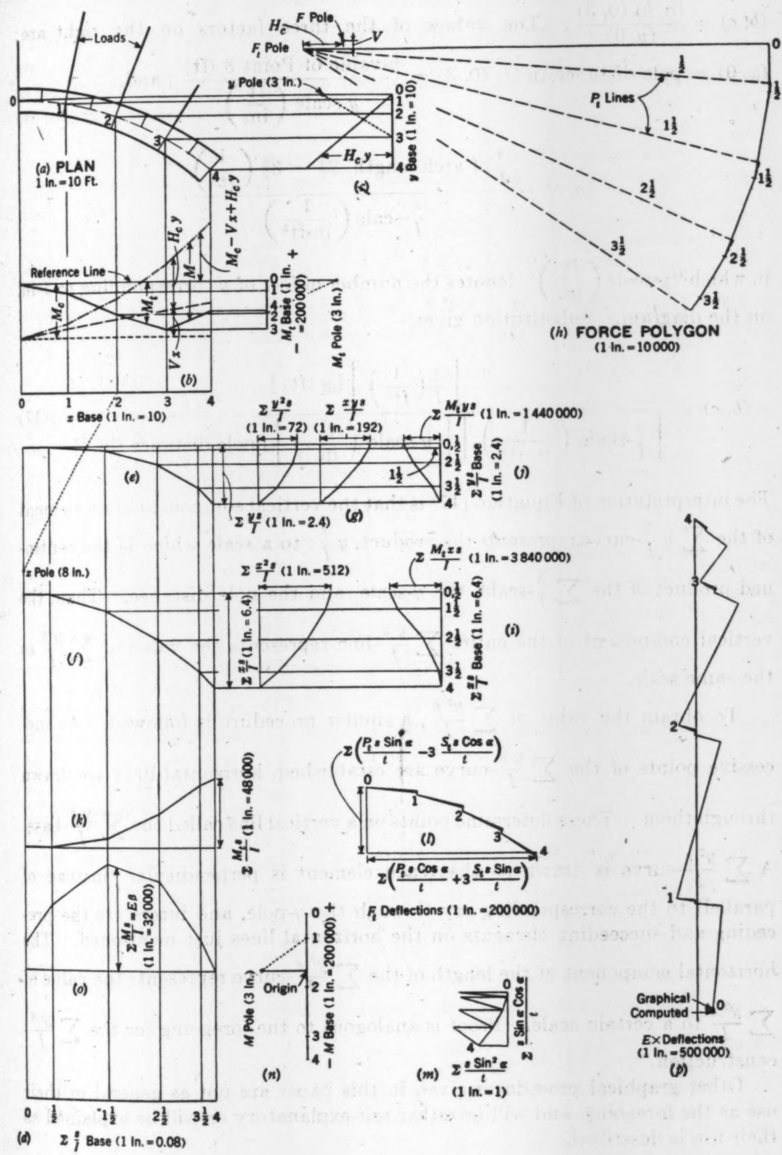

(a) PLAN
1 In. = 10 Ft.
Loads
F_t Pole
H_c F Pole
V
y Pole (3 In.)
$H_c y$
y Base (1 In. = 10)
(c)
P_t Lines
(h) FORCE POLYGON
(1 In. = 10000)
Reference Line
M_c
M_t
M
$H_c y$
$M_c - Vx + H_c y$
Vx
(b)
M_t Base (1 In. = 200000)
M_t Pole (3 In.)
x Base (1 In. = 10)
$\Sigma \frac{y^2 s}{I}$ (1 In. = 72)
$\Sigma \frac{xys}{I}$ (1 In. = 192)
$\Sigma \frac{M_t ys}{I}$ (1 In. = 1 440 000)
$\Sigma \frac{ys}{I}$ Base (1 In. = 2.4)
$\Sigma \frac{ys}{I}$ (1 In. = 2.4)
(e)
(g)
(j)
x Pole (8 In.)
$\Sigma \frac{x^2 s}{I}$ (1 In. = 512)
$\Sigma \frac{M_t xs}{I}$ (1 In. = 3 840 000)
$\Sigma \frac{xs}{I}$ (1 In. = 6.4)
$\Sigma \frac{xs}{I}$ Base (1 In. = 6.4)
(f)
(i)
$\Sigma \frac{M_t s}{I}$ (1 In. = 48000)
(k)
$\Sigma \frac{M s}{I} = E\beta$ (1 In. = 32000)
(o)
$\Sigma\left(\frac{P_t s \sin\alpha}{t} - 3\frac{S_t s \cos\alpha}{t}\right)$
(l)
$\Sigma\left(\frac{P_t s \cos\alpha}{t} + 3\frac{S_t s \sin\alpha}{t}\right)$
F_t Deflections (1 In. = 200000)
M Pole (3 In.)
Origin
− M Base (1 In. = 200000) +
(n)
$\Sigma \frac{s \sin\alpha \cos\alpha}{t}$
(m) $\Sigma \frac{s \sin^2\alpha}{t}$
(1 In. = 1)
Graphical
Computed
E× Deflections
(1 In. = 500000)
(p)
(d) $\Sigma \frac{s}{I}$ Base (1 In. = 0.08)
0
$\frac{1}{2}$
1
$1\frac{1}{2}$
2
$2\frac{1}{2}$
3
$3\frac{1}{2}$
4

FIG. 4.—ANALYSIS OF RIGHT HALF OF ARCH

$(b, c) = \dfrac{(a, b)\ (0, 3)}{(y, 0)}$. The values of the three factors on the right are: $(y, 0)$ = pole distance (in.); $(0, 3) = \dfrac{y\text{-value of Point 3 (ft)}}{y\text{-scale}\left(\dfrac{\text{ft}}{\text{in.}}\right)}$; and,

$$(a, b) = \frac{\dfrac{s}{I}\text{ of arch length, } 2\tfrac{1}{2} - 3\tfrac{1}{2}\left(\dfrac{1}{\text{ft}^3}\right)}{\dfrac{s}{I}\text{-scale}\left(\dfrac{1}{\text{in-ft}^3}\right)},$$

in which "y-scale $\left(\dfrac{\text{ft}}{\text{in.}}\right)$" denotes the number of feet of y corresponding to 1-in. on the diagram. Substitution gives,

$$(b, c) = \frac{\left[\dfrac{s}{I}\left(\dfrac{1}{\text{ft}^3}\right)\right][y\ (\text{ft})]}{\left[\dfrac{s}{I}\text{-scale}\left(\dfrac{1}{\text{in-ft}^3}\right)\right]\left[y\text{-scale}\left(\dfrac{\text{ft}}{\text{in.}}\right)\right][\text{pole distance (in.)}]} \qquad \text{..(17)}$$

The interpretation of Equation (17) is that the vertical component of an element of the $\sum y \dfrac{s}{I}$-curve represents the product, $y \dfrac{s}{I}$, to a scale which is the continued product of the $\sum \dfrac{s}{I}$-scale, the y-scale, and the pole distance. Then, the vertical component of the entire $\sum \dfrac{y\,s}{I}$-line represents the value of $\sum \dfrac{y\,s}{I}$ to the same scale.

To obtain the value of $\sum \dfrac{y^2\,s}{I}$, a similar procedure is followed. As successive points of the $\sum \dfrac{y\,s}{I}$-curve are established, horizontal lines are drawn through them. These determine points on a vertical line called the $\sum \dfrac{y\,s}{I}$-base. A $\sum \dfrac{y^2\,s}{I}$-curve is drawn so that each element is perpendicular (instead of parallel) to the corresponding ray through the y-pole, and intersects the preceding and succeeding elements on the horizontal lines just mentioned. The horizontal component of the length of the $\sum \dfrac{y^2\,s}{I}$-curve represents the value of $\sum \dfrac{y^2\,s}{I}$ to a certain scale. Proof is analogous to the foregoing for the $\sum \dfrac{y\,s}{I}$-construction.

Other graphical procedures given in this paper are not as general in their use as the foregoing, and will be either self-explanatory or will be explained as their use is described.

METHOD OF ANALYSIS

In starting an analysis the loads on the arch, the temperature change in the arch, and the physical dimensions and characteristics of the arch, are assumed to be known. Furthermore, the factors governing foundation deformations are assumed to be known.

TABLE 2.—COMPUTATIONS FOR ILLUSTRATIVE EXAMPLE

Joint (see Figs. 3(a) and 4(a)) (1)	Thickness, t, in feet (2)	Length, s, in feet (3)	$\frac{s}{t}$ (4)	Length, s, in feet (5)	$\frac{1}{I} = \frac{12}{t^3}$ (6)	$\frac{s}{I}$ (7)	Cumulative: $\sum \frac{s}{I}$ (8)	Load in pounds (9)	Shear force, S_t, in pounds (10)	Moments in Foot-Pounds: $S_t s$ (11)	Moments in Foot-Pounds: M_t (12)
0 *l*	10.8	....	....	12.0	0.00953	0.114	....	22 000	....	....	0
	10.8	24.0	2.22	....	....	....	0.114	....	−3 900	−93 600	....
1 *l*	10.9	....	....	24.0	0.00927	0.222	....	42 700	....	....	−93 600
	11.1	24.0	2.16	....	....	....	0.336	....	−10 200	−245 000	....
2 *l*	11.4	....	....	24.0	0.00810	0.194	....	37 700	....	....	−338 600
	11.8	24.0	2.03	....	....	....	0.530	....	−11 100	−266 500	....
3 *l*	12.2	....	....	24.0	0.00661	0.159	....	30 000	....	....	−605 100
	12.7	24.0	1.89	....	....	....	0.689	....	−4 100	−98 400	....
4 *l*	13.4	....	....	24.0	0.00499	0.120	....	20 000	....	....	−703 500
	14.1	24.0	1.70	....	....	....	0.809	....	+13 000	+312 000	....
5 *l*	14.9	....	....	12.0	0.00363	0.044	....	....	....	....	−391 500
			$\Sigma = 10.00$				0.853				
0 *r*	10.8	....	....	12.0	0.00953	0.114	....	22 000	....	....	0
	10.9	24.0	2.20	....	....	....	0.114	....	−3 900	−93 600	....
1 *r*	11.1	....	....	24.0	0.00877	0.210	....	41 000	....	....	−93 600
	11.4	24.0	2.11	....	....	....	0.324	....	−8 600	−206 500	....
2 *r*	11.7	....	....	24.0	0.00749	0.180	....	34 300	....	....	−300 100
	12.2	24.0	1.97	....	....	....	0.504	....	−6 100	−146 500	....
3 *r*	12.7	....	....	24.0	0.00586	0.141	....	23 500	....	....	−446 600
	13.3	24.0	1.80	....	....	....	0.645	....	+7 000	+168 000	....
4 *r*	14.1	....	....	12.0	0.00428	0.051	....	....	....	....	−278 600
			$\Sigma = 8.08$				0.696				

TABLE 2.—(*Continued*)

Joint (see Figs. 3(a) and 4(a)) (1)	Resultant force, F_t, in pounds (13)	$\frac{F_t s}{t}$ (14)	$E \beta$ (15)	$E \beta s$ (16)	Resultant force, F, in pounds (17)	$\frac{F s}{t}$ (18)	$\epsilon T E s$ (19)	Moment, M, in foot-pounds (20)
0 *l*	....	....	....	....	....	....	....	+503 000
	200 000	444 000	−22 000	−528 000	183 000	406 000	−581 000	....
1 *l*	....	....	....	....	....	....	....	+450 000
	201 000	434 000	−121 000	−2 904 000	184 500	399 000	−581 000	....
2 *l*	....	....	....	....	....	....	....	+145 000
	202 500	411 000	−149 000	−3 576 000	187 500	381 000	−581 000	....
3 *l*	....	....	....	....	....	....	....	−284 000
	204 500	387 000	−105 000	−2 520 000	189 500	358 000	−581 000	....
4 *l*	....	....	....	....	....	....	....	−626 000
	205 500	349 000	−29 000	−696 000	190 000	323 000	−581 000	....
5 *l*	....	....	....	....	....	....	....	−651 000
0 *r*	....	....	....	....	....	....	....	+503 000
	200 000	440 000	+93 000	+2 232 000	184 000	405 000	−581 000	....
1 *r*	....	....	....	....	....	....	....	+282 000
	201 000	424 000	+153 000	+3 672 000	187 000	395 000	−581 000	....
2 *r*	....	....	....	....	....	....	....	−145 000
	202 500	399 000	+127 000	+3 048 000	190 000	374 000	−581 000	....
3 *r*	....	....	....	....	....	....	....	−598 000
	203 500	366 000	+43 000	+1 032 000	191 000	344 000	−581 000	....
4 *r*	....	....	....	....	....	....	....	−812 000

To illustrate the procedure used in this method of arch analysis an example will be given in detail. For simplicity the illustration is confined to an unsymmetrical fixed-end arch without foundation deformations. The loads considered are restricted to direct forces in the plane of the arch, and a uniform temperature change is assumed. Without appreciable error, n is assumed equal to 3. At the end of this explanation, the manner in which other factors may be introduced will be indicated. It should also be noted that, in order to simplify the example, a smaller number of voussoirs are used than is generally desirable.

The arch chosen for this example is a section of the Gene Wash Dam, in California, at Elevation 710, and the loading is approximately that used at one stage of the trial-load analysis of that dam. All the graphical work of arch analysis is done on one large sheet (Figs. 3 and 4). The numerical work is contained in Tables 2 and 3 and in the solutions of the equations in Steps (29) and (30) presented subsequently herein. All quantities pertaining to the structure are given in units of feet and pounds and combinations of those units, exclusively, so that no further notation of dimensions is necessary. In the explanation that follows the graphical constructions referred to as *a*, *b*, *c*, etc., are to be found on Figs. 3 and 4. Steps that are followed separately for the two sides of the arch are described for the left side of the arch only.

PROCEDURE

(1) In Construction (*a*) lay out the arch to a convenient scale and locate the center line. Establish a crown point.

(2) Along the center line of the arch, lay off voussoir lengths starting at the crown. Locate the centers of the voussoirs.

(3) Number the ends of the voussoirs along the center line, starting with 0 at the crown and numbering to the abutment. Assign whole numbers, 1 *l*, 2 *l*, etc., to the joints (ends of voussoirs) and half numbers, ½ *l*, 1½ *l*, etc., to the voussoirs themselves or to their centers. List the numbers in Column (1), Table 2.

(4) Scale the thickness of the arch at the center and ends of each voussoir and list in Column (2), Table 2.

(5) In Column (3), Table 2, list the voussoir lengths as measured along the center line of the arch.

(6) Calculate values of $\frac{s}{t}$ in Column (4), Table 2. Add these values to obtain the value of $\sum \frac{s}{t}$ and list this at the bottom of the column.

(7) Scale the distances along the center line of the arch between the centers of the voussoirs. At the crown and abutment, use the distance from crown or abutment to the center of the adjacent voussoir. List the distances in Column (5), Table 2.

(8) Calculate values of $\frac{1}{I} = \frac{12}{t^3}$ and $\frac{s}{I}$ in Columns (6) and (7), Table 2.

TABLE 3.—COMPUTATION OF COEFFICIENTS; ILLUSTRATIVE EXAMPLE (ϵ = 0.0000056 per Degree Fahrenheit; T = − 10.0° F; E = 432 000 000; $\epsilon T E$ = − 24 200; n = 3)

Quantity No. (see Table 1, Column (3))	Scale	Measured		Value		Coefficients	
		Left (inches)	Right (inches)	Left	Right	Symbol	Value
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	....	....	....	+0.853	+0.696	A_{1l} = A_{1r} = A_1 =	+0.853 +0.696 +1.549
2	2.4	+4.69	+2.64	+11.26	+6.34	B_{1l} = B_{1r} = B_1 =	+11.26 + 6.34 +17.60
3	192	+4.53	+2.15	+870	−413	B_{2l} = B_{2r} =	+877 −418
4	1	+3.36	+2.44	+7	−5	B_2 =	+459
5	72	+4.34	+1.65	+312.5	+118.8	B_{3l} =	+326.8
6	....	....	....	+10.0	+8.1	B_{3r} =	+129.3
7	1	+2.16	+1.22	+4.3	+2.4	B_3 =	+456.1
8	6.4	+6.33	+4.30	+40.5	−27.5	C_{1l} = C_{1r} = C_1 =	+40.5 −27.5 +13.0
9	512	+5.31	+3.08	+2 720	+1 577	C_{2l} =	+2 746
10	....	....	....	+30	+24	C_{2r} =	+1 599
11	1	+2.16	+1.22	−4	−2	C_2 =	+4 345
21	48 000	−5 92	−3.11	−284 000	−149 000	G_{1l} = G_{1r} = G_1 =	−284 000 −149 000 −433 000
22	3 840 000	−5.02	−2.23	−19 280 000	+8 565 000	G_{2l} =	−21 446 000
23	200 000	+4.72	+3.04	−944 000	+608 000	G_{2r} =	+9 976 000
24	....	+50.5	+33.2	−1 222 000	+803 000	G_2 =	−11 470 000
25	1 440 000	−4.20	−1.48	−6 050 000	−2 131 000	G_{3l} =	−1 768 000
26	200 000	+8.76	+7.40	+1 752 000	+1 480 000	G_{3r} =	+1 474 000
27	....	+104.5	+87.8	+2 530 000	+2 125 000	G_3 =	−294 000

(9) Starting at the crown, add progressively values of $\frac{s}{I}$ to obtain values of $\sum \frac{s}{I}$ in Column (8), Table 2.

(10) In Construction (*b*), establish a horizontal line, or "*x*-base," and in Construction (*c*), a vertical line, or "*y*-base." Through the ends of the voussoirs on the center line, draw vertical and horizontal lines, known as *x*-lines and *y*-lines. Number the intersections of these lines with the *x*-base and *y*-base to correspond with the numbers of the ends of the voussoirs.

(11) Locate x-poles and y-poles on perpendiculars through the zero points of the x-base and y-base, at distances that will give convenient sizes for the curves described subsequently. Record the pole distances, in inches.

(12) In Construction (*d*), plot a $\sum \frac{s}{I}$-base as follows: From a zero point lay off, horizontally to some convenient scale, values of $\sum \frac{s}{I}$ from Column (8), Table 2. Record the scale and number the points established with the corresponding voussoir numbers. Draw through each point a vertical line.

(13) In Construction (*e*), use the y-pole and y-base to plot a $\sum \frac{y\,s}{I}$-curve on the $\sum \frac{s}{I}$-base, and a $\sum \frac{y^2\,s}{I}$-curve on the $\sum \frac{y\,s}{I}$-base. This procedure is one of graphical multiplication and integration, and has been described previously.

(14) In Construction (*f*), use the x-pole and the x-base to plot $\sum \frac{x\,s}{I}$ on the $\sum \frac{s}{I}$-base, and $\sum \frac{x^2\,s}{I}$ on the $\sum \frac{x\,s}{I}$-base; and in Construction (*g*) plot $\sum \frac{x y s}{I}$ on the $\sum \frac{y\,s}{I}$-base. This procedure is similar to that of Step (13).

(15) In Column (9), Table 2, list the load considered to act at each joint. Take this as the total load actually acting between centers of the voussoirs; except that at the crown, take the load actually applied in the adjacent half-voussoir.

(16) Estimate a trial horizontal crown force, H_t, as by multiplying the average unit load on the arch by the estimated average radius of the thrust line. Apply the same crown thrust to each half of the arch.

(17) In Construction (*h*), draw a force polygon to a convenient scale. Include H_t and the loads of Column (9), Table 2. Start the polygon from a point called the F_t-pole. From this pole plot, horizontally, a vector representing H_t, and from the end of H_t plot, successively, vectors representing in magnitude and direction all the loads on the half-arch, beginning at the crown. Mark the points on this polygon with half-numbers, Point $1\frac{1}{2}$ being between the loads at Joints 1 and 2, etc.

(18) Through the F_t-pole draw a ray, or P_t-line, for each voussoir, in the direction of the thrust, P_t, in the voussoir. Take this direction as parallel to a line through the two center-line points at the ends of the voussoir.

(19) Obtain the average shear, S_t, in each voussoir by scaling the perpendicular distance from its P_t-line to its force polygon point. List these values in Column (10), Table 2. If the diagram is drawn as shown, call the shear, S_t, positive if the P_t-line is below the force polygon point.

(20) Multiply S_t by s of Column (3), Table 2, to obtain, in Column (11), the change in moment between the ends of each voussoir. Sum up from the crown toward the abutment to obtain values of M_t in Column (12).

(21) In Construction (*b*), draw a horizontal reference line intersecting the x-lines. From this line plot, to a convenient scale, values of M_t to form an M_t

versus x-curve. Plot positive values upward. To form an M_t-base, project the points on this curve horizontally on to a vertical line.

(22) Locate an M_t-pole on a perpendicular through the zero point of the M_t-base, at a convenient distance. Record this pole distance, in inches.

(23) Using the M_t-base and M_t-pole, plot: $\sum \frac{M_t\,x\,s}{I}$ on the $\sum \frac{x\,s}{I}$-base (Construction (*i*)); $\sum \frac{M_t\,y\,s}{I}$ on the $\sum \frac{y\,s}{I}$-base (Construction (*j*)); and, $\sum \frac{M_t\,s}{I}$ on the $\sum \frac{s}{I}$-base (Construction (*k*))—all by the method of "graphical multiplication and integration."

(24) Find values of F_t, the resultant force at any point due to external loads and the trial crown force, by scaling, on the force polygon (Construction (*h*)), from the F_t-pole to each numbered point ($\frac{1}{2}$, $1\frac{1}{2}$, etc.). List the values in Column (13), Table 2. Multiply by $\frac{s}{t}$ of Column (4) and list the products, $\frac{F_t\,s}{t}$, in Column (14).

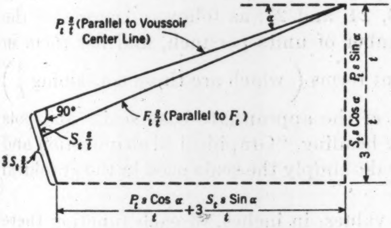

$P_t \frac{s}{t}$ (Parallel to Voussoir Center Line)
α
90°
$F_t \frac{s}{t}$ (Parallel to F_t)
$S_t \frac{s}{t}$
$3S_t \frac{s}{t}$
$\frac{P_t\,s\,\text{Sin}\,\alpha}{t}$
$3\frac{S_t\,s\,\text{Cos}\,\alpha}{t}$
$\frac{P_t\,s\,\text{Cos}\,\alpha}{t}+3\frac{S_t\,s\,\text{Sin}\,\alpha}{t}$

FIG. 5.—F_t DEFLECTION DIAGRAM

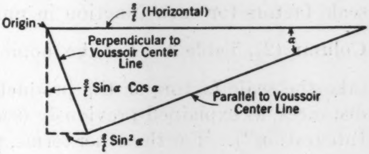

Origin
$\frac{s}{t}$ (Horizontal)
α
Perpendicular to Voussoir Center Line
$\frac{s}{t}$ Sin α Cos α
Parallel to Voussoir Center Line
$\frac{s}{t}$ Sin^2 α

FIG. 6.—CONSTRUCTION FOR FUNCTIONS, $\Sigma\,\frac{s}{t}\sin\alpha\cos\alpha$ and $\Sigma\,\frac{s}{t}\sin^2\alpha$

(25) In Construction (*l*), draw an "F_t-deflection diagram," an element of which is illustrated in Fig. 5. Proceed as follows: To some convenient scale plot the value of $\frac{F_t\,s}{t}$ for the first voussoir in the direction of F_t as taken from the force polygon; that is, from the F_t-pole to the point on the polygon corresponding to the voussoir. Resolve this vector into its $\frac{P_t\,s}{t}$ and $\frac{S_t\,s}{t}$-components, the first parallel, and the second normal, to the direction of the center line of the voussoir, as taken from the corresponding P_t-line on the force polygon. Increase the $\frac{S_t\,s}{t}$-component to n, or three times its length, the increase being away from the $\frac{P_t\,s}{t}$-component. From the end of this increased normal, draw the $\frac{F_t\,s}{t}$-line for the next voussoir in the same manner, and repeat the procedure. Continue to the abutment. The horizontal and vertical distances from the

beginning to the end of this construction give, respectively, the values of Quantities Nos. 26 and 23, Table 1, as required for the coefficients, G_3 and G_2. This may be verified by a consideration of Fig. 5.

(26) To obtain the values of Quantities Nos. 4 and 7, Table 1, make Construction (*m*), an element of which is illustrated in Fig. 6. Proceed as follows: From Column (4), Table 2, take the value of $\frac{s}{t}$ for the first voussoir and plot it as a horizontal line on a convenient scale to the right from the point of origin. From the right end of this $\frac{s}{t}$-line draw a line parallel to the center line of the voussoir, and from the origin end draw a line perpendicular to the center line of the voussoir. Using the intersection of the two lines as the origin, repeat the construction for the next voussoir, and continue to the abutment. Take the vertical and horizontal components of the distance from beginning to end of this construction as the required summations. Proof is evident on inspection of the diagram.

(27) At the top of Table 3 list the values of ϵ, T, E, and the value of the product, $\epsilon\, T\, E$, with the same sign as T.

(28) Having now made all the graphical constructions for evaluating the terms listed in Table 1, determine their values and assemble them into the coefficients of Equations (11) and (13) by filling out Table 3. Fill out all lines of the table except Quantities Nos. 1, 6, 10, 24, and 27, as follows: Determine the scale factors for each function in number of units per inch, and list them in Column (2), Table 3. For the moment terms $\left(\text{which are those containing } \frac{s}{I}\right)$ take the scale factors as the product of the appropriate base scales and pole distances, as explained previously (see heading, "Graphical Multiplication and Integration"). For the other terms, take simply the scale used in the graphical work.

Measure off on Figs. 3 and 4 the values, in inches, of each function there determined and list it in the appropriate line of Columns (3) and (4), in Table 3. Prefix to each value, as measured, its proper sign. If, in Figs. 3 and 4, the positive direction of each term introduced has been taken as shown or specified, and if the relative positions of poles and bases are as shown, determine the signs as follows: Take the sign of any function, except that of Quantity No. 26, Table 3, as plus if its value is measured from the abutment end of the curve up or to the left for the left half of the arch, and up or to the right for the right half of the arch. For the function of Quantity No. 26, take the reverse.

In Columns (5) and (6), Table 3, place the product of the "scale" values and the "measured" values, and in Quantities Nos. 4, 11, and 7, also multiply by the factor, $(n - 1)$, or by 2. Determine the sign of each term by multiplying the sign in the "measured" column by the sign in Table 1.

In Quantity No. 1, Columns (5) and (6), Table 3, place the values of $\sum \frac{s}{I}$ as taken from Column (8), Table 2. In Quantity No. 6, Columns (5) and (6), place the values of $\sum \frac{s}{t}$ as taken from Column (4), Table 2; and in Quantity

No. 10, Columns (5) and (6), place the same values multiplied by n, or by 3. The signs are always positive.

From Construction (*a*) scale the values of x_a and y_a, and list them in Columns (3) and (4) of Quantities Nos. 27 and 24, respectively. Prefix the proper signs. Place the products of these quantities and $\epsilon\, T\, E$ in Columns (5) and (6), Table 3. Prefix the sign determined by multiplying the signs of the two factors and the sign given in Table 1. In Column (8), Table 3, add algebraically the proper terms of Columns (5) and (6) to determine the coefficients of Equations (11) and (13).

(29) Substitute the values of the coefficients just found into Equations (13). Solve the three equations simultaneously for M_c, H_c, and V:

$$\begin{aligned} &+ 1.55\, M_c + 17.60\, H_c + 13\, V - 433\,000 = 0 \\ &+ 13.0\, M_c + 459.0\, H_c + 4\,345\, V - 11\,470\,000 = 0 \\ &+ 17.6\, M_c + 456.1\, H_c + 459\, V - 294\,000 = 0 \end{aligned}$$

The solution is: $M_c = +\,503\,050$; $H_c = -\,22\,277$; and $V = +\,3\,488$.

(30) Substitute the values of the coefficients for the left half of the arch from Table 3 into Equations (11) to obtain the values of the crown deflections: $E\,\beta_c = +\,36\,000$; $E\,\Delta_y = -\,11\,031\,000$; $E\,\Delta_x = -\,325\,000$. The same deflections may be computed for the right half of the arch.

This completes the first part of the arch analysis. The following procedure evaluates the deflections at all points in the arch and determines the arch stresses.

(31) On the y-base (Construction (c)), plot values of $H_c\, y$ to the scale used for M_t in Step (21), as follows: Compute $H_c\, y$ for any point; plot its value horizontally from the corresponding point on the y-base to the scale of M_t; and through this point and the zero point of the y-base draw a straight line. The intercepts between this line and the y-base are the required $H_c\, y$-values at all points.

(32) Construct a curve of values of $(M_c \pm V\,x + H_c\, y)$, *versus* x, with the same scale and the same reference or zero line as the M_t-curve drawn in Step (21), but with reversed sign, as follows: From the reference line measure, vertically, the quantity, M_c, and draw a horizontal line. This is the M_c-line. Through the intersection of the M_c-line with the line, $x = 0$, draw a line with slopes such that intercepts on the x-line between it and the M_c-line give the values of $V\,x$. The procedure is similar to that used in Step (31) for finding $H_c\, y$. From the diagram of Step (31), take off values of $H_c\, y$ with dividers and lay them off along the corresponding x-lines from the line just drawn. Connect the points thus found, forming the desired curve.

In the foregoing procedure some confusion may arise as to the direction in which the constructions are made. As stated previously, the signs of the construction are reversed from that of Step (21). Thus, a positive M_c is plotted down, a positive $V\,x$ is plotted down on the left and up on the right, and a positive $H_c\, y$ is plotted down. By plotting the values of $M_c + H_c\, y \pm V\,x$ to the same scale and from the same reference line as M_t, but in the opposite direction, the intercepts on the x-lines between the two curves give the value,

$M = M_t + M_c + H_c\,y \pm V\,x$ (Equation (10)). The value, M, is positive if measured up from the last constructed curve.

(33) Establish a vertical line as an M-base (Construction (n)). Take off values of M with dividers from the construction of Step (32), and lay them off on the M-base from some reference point or origin, plotting positive values upward. Also, scale off values of M and list them in Column (20), Table 2.

(34) Locate an M-pole on a perpendicular through the origin of the M-base, at a convenient distance. Record this pole distance.

(35) In Construction (o), use the M-pole and M-base, to plot $\sum \frac{M\,s}{I} = E\,\beta$ on the $\sum \frac{s}{I}$-base, using the method of graphical multiplication and integration previously explained. Start the diagram at both abutments and work toward the crown, where it should close. With the sign convention on the M-base as stated and the relative pole positions as shown, the value of $\sum \frac{M\,s}{I}$ or $E\,\beta$ will be positive upward. Determine the scale of $E\,\beta$ from the product of the base scales and pole distance.

(36) Scale the values of $E\,\beta$ at each mid-voussoir point on the diagram just drawn and enter them in Column (15), Table 2. Multiply by values of s from Column (3) and enter the product $E\,\beta\,s$ in Column (16).

(37) Add, vectorially, the values of H_c and V, determined in Step (29), into the force polygon (Construction (*h*)). To do this, plot, successively, from the F_t-pole, vectors representing in magnitude and direction the values of H_c and V but with reversed sign. That is, plot positive values of H_c horizontally to the right and positive values of V vertically downward on the left-hand polygon, and in the reverse directions on the right-hand polygon. The new pole thus established is the F-pole, and the force polygon from this pole represents the true forces existing in the arch.

(38) Find values of F, the resultant force at any point in the arch, by scaling, on the force polygon, from the F-pole to the numbered points ($\frac{1}{2}$, $1\frac{1}{2}$, etc.) and list them in Column (17), Table 2. Multiply by $\frac{s}{t}$ from Column (4) and enter Product $\frac{F\,s}{t}$ in Column (18).

(39) Multiply s, Column (3), by $\epsilon\,T\,E$ of Step (27) and enter the product, $\epsilon\,T\,E\,s$, in Column (19).

(40) In Construction (p), plot a deflection diagram to a convenient scale, using the vectors indicated in Equation (16). Beginning at some origin plot successively, for each voussoir from abutment to crown, its four increments of deflection. This procedure is illustrated in Fig. 7 for Voussoir $2\frac{1}{2}\,l$. First, plot a vector representing the value, $\frac{F\,s}{t}$, taking the value from Column (18), Table 2, and the direction the same as F, taken from the force polygon. Resolve this vector into its $\frac{P\,s}{t}$ and $\frac{S\,s}{t}$-components parallel and perpendicular to

the voussoir center line, and increase the vector, $\frac{S\,s}{t}$, by three times in the direction away from $\frac{P\,s}{t}$. This procedure is similar to that described in Step (25). From the end of the foregoing construction plot a vector representing, in magnitude and direction, the function, $\epsilon\,T\,E\,s$. This vector is plotted parallel to the arch center line, in the direction away from the abutment if it is positive. Magnitudes are given in Column (19), Table 2. Finally, plot a vector representing, in magnitude and direction, the value of the moment deflection, $E\,\beta\,s$. This vector is plotted perpendicular to the arch center line, a positive value being toward the arch extrados on the left and toward the arch intrados on the right. Magnitudes are given in Column (16), Table 2. Number the points on this diagram the same as the corresponding joint numbers. Plot the crown displacements as computed in Step (30) to check the results of this step.

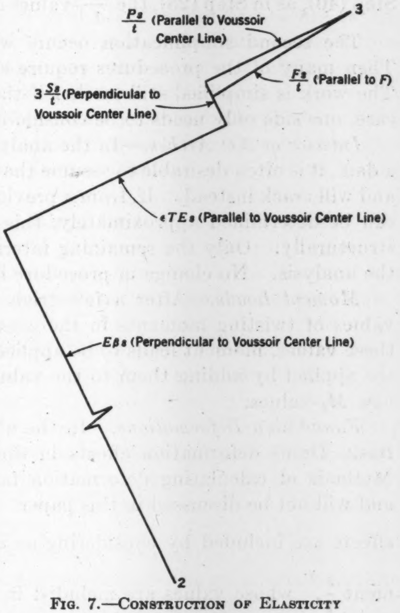

$\frac{Ps}{t}$ (Parallel to Voussoir Center Line)
3
$\frac{Fs}{t}$ (Parallel to F)
$3\,\frac{Ss}{t}$ (Perpendicular to Voussoir Center Line)
ϵTEs (Parallel to Voussoir Center Line)
$E\beta s$ (Perpendicular to Voussoir Center Line)
2

FIG. 7.—CONSTRUCTION OF ELASTICITY DEFLECTION DIAGRAM

(41) Calculate stresses. Obtain values of P from the force polygon as follows: For the abutment point take the component in the direction of the arch axis of the distance from the F-pole to the abutment point. For any other joint, obtain P-components for adjacent mid-voussoir points on either side, and interpolate. Obtain M-values from Column (20) and t-values from Column 2, Table 2. The stresses are computed at the desired points with the familiar equations. No example is given.

ADDITIONAL FACTORS

The foregoing simplified example illustrates all the fundamental points of this system of analysis. In order to demonstrate the wide range of applicability of the method, the procedures used in allowing for several other factors will be described briefly. However, before proceeding to these complicating factors, two possible simplifications will be presented. The first concerns the treatment of shear deformations. In thin arches, these deformations are negligible and may very well be omitted. Instead of omitting them entirely, however, it is simpler to assume that the effective shear modulus is equal to E, or that n equals 1. Then, the total deformation due to thrust and shear is always in the direction of the resultant force producing it. In Table 1 the terms with coefficients, $(n - 1)$, drop out. In Step (25), since the $\frac{S_t s}{t}$-components are not to be multiplied by 3, it is unnecessary to resolve the $\frac{F_t s}{t}$-values into components; they are simply plotted, successively, and added vectorially. Since the terms obtained in Step (26) do not appear, this step is omitted. Finally, in Step (40), as in Step (25), the $\frac{F s}{t}$-values are not resolved into components.

The second simplification occurs where the arch shape is symmetrical. Then many of the procedures require solution for one side of the arch only. The work is simplified still further if the loads are also symmetrical. In this case, one side only needs to be considered in all the procedures.

Interior or Net Arches.—In the analysis of a plain concrete arch, such as in a dam, it is often desirable to assume that the concrete is unable to take tension, and will crack instead. If, from a previous analysis, the area subject to tension can be determined approximately, this area can be considered as not acting structurally. Only the remaining interior arch or net section is then used in the analysis. No change in procedure is involved.

Moment Loads.—After a few trials in a trial-load analysis, approximate values of twisting moments in the cantilevers can be determined, and, from these values, moment loads to be applied to arches can be found. These loads are applied by adding them to the values of Column (12), Table 2, to obtain new M_t-values.

Foundation Deformations.—In the analysis of the Gene Wash and Copper Basin Dams deformation effects in the foundation were generally included. Methods of calculating deformation factors have been presented elsewhere[7] and will not be discussed in this paper. However, it may be stated that these effects are included by considering an effective "abutment $\frac{s}{I}$" and an "abutment $\frac{s}{t}$," whose values are included in Columns (7) and (4), respectively, of Table 2, and are handled similarly to the other values in the same columns.

Non-Uniform Temperature.—A dam with its reservoir full of water will have a relatively constant temperature at the up-stream face and may have a quite different temperature at the down-stream face. This results in a temperature gradient, $\frac{dT}{dt}$, in the up-stream-down-stream direction, which may be quite important. To include this effect in the arch analysis the quantity, $\frac{dT}{dt} \epsilon E I$, is computed at each joint and added to M_t of Column (12), Table 2, to obtain a new M_t. These corrected M_t-values are used to determine crown forces and all deflections. In determining stresses the $\frac{dT}{dt} \epsilon E I$-corrections are subtracted from the final M-values.

The foregoing discussion of the inclusion of additional factors into the procedure shows that this method of analysis is readily adaptable to various conditions. In general, it may be stated that any effect that can be included in any type of arch analysis may be included in this procedure. In many details of the procedure, alternative methods are available. Many of these methods have been tried, and some have been found equally as convenient as those adopted, but none was more so.

---

[7] "Ueber die Berechnung der Fundamentdeformation," by Fredrik Vogt, Assoc. M. Am. Soc. C. E., Math. Naturv. Klasse 1925, No. 2.

CONCLUSION

The procedure described herein was used about a hundred times in the course of the analyses of the Gene Wash and Copper Basin Dams in California. When first developed it was used at the same time as an entirely numerical method of computation, and a comparison of the two forms of analysis was possible. The graphical procedure gave accuracy comparable to that of the numerical method, and in both cases some of the work was done by men who followed the mechanical procedure given them, with little or no concern for the principles involved.

Several definite advantages were apparent for the graphical method. In the hands of an experienced operator the graphical procedure has some advantage as to time required. More important, however, is the fact that the graphical procedure presents to the calculator a picture of what has been done. When errors are made they often appear as discontinuities in curves, or as curves of disproportionate size or shape, and, therefore, are discovered much more readily than in the arithmetical method. The picture which the graphical work presents to the calculator also has another value. In an arch which is repeatedly analyzed for varying conditions, all the changes that occur in the component curves of the analysis have a significance to the analyst. For instance, in a case where changes in loads are being made to obtain, in successive analyses, closer approach to desired deflections, he is aided in judging as to whether the changes in assumptions are being chosen properly or what further changes should be made. Thus, whereas a numerical procedure tends to become an endless accumulation of meaningless data, the graphical procedure gives a concise picture of all the component parts of the work.

If different physical characteristics are used for each trial analysis, as when using net sections, a numerical procedure is likely to be very ponderous. The tendency of designers, therefore, has been to avoid the use of net sections or to make simplifying assumptions so as to bring the amount of work within practical limits. As demonstrated in this paper, however, new physical properties of the arch may be accepted, with this method, for each new arch analysis without unduly increasing the work. Thus, this graphical procedure may make practical an exact analysis which by other methods would be too cumbersome.

The concept of the trial crown force is the one important feature that has permitted the development of this method in which satisfactory accuracy is obtained with the extensive use of graphics. It should be noted that this concept would also allow the use of a procedure closely paralleling that presented, in which graphical work would be replaced by arithmetic, and slide-rule accuracy would be sufficient. For a person preferring numerical work to graphical work such a simplification would be of great help.

ACKNOWLEDGMENT

The procedures described in this paper were developed and used in the Design Office of The Metropolitan Water District of Southern California, under the direction of Julian Hinds, M. Am. Soc. C. E., Assistant Chief Engineer. All engineering and construction activities of the District are under the direction of Frank E. Weymouth, Hon. M. Am. Soc. C. E., General Manager and Chief Engineer.

APPENDIX

NOTATION

The following notation conforms essentially with American Standard Symbols for Mechanics, Structural Engineering, and Testing Materials[8] compiled by a Committee of the American Standards Association with Society representation, and approved by the Association in 1932. The sign conventions are either given in the definitions or illustrated by Fig. 1.

A = coefficients in the general arch equations (see Coefficients A, B, C, D, and G, in Table 1);

a = a symbol referring to the abutments; generally used as a subscript;

B = coefficients (see Symbol A);

C = coefficients (see Symbol A);

D = coefficients (see Symbol A);

E = modulus of elasticity in direct stress; E_s = modulus of elasticity in shear;

e = a subscript denoting "due to external loads";

F = a resultant force at any point in an arch; F_t = a resultant force at any point in an arch due to external loads and a trial crown force;

G = coefficients (see Symbol A);

H = a "horizontal" force at the crown; that is, a force parallel to the X-axis; H_c = difference between the estimated and actual values of H at the crown; H_t = the estimated, trial, horizontal crown force;

I = moment of inertia of radial section of an arch;

l = a subscript denoting "the left half of the arch";

M = the moment in an arch at any point, taken about the center line of the arch; M_c = value of M at the crown; M_e = value of M at any point due to external loads; M_t = value of M at any point due to external loads and a trial crown force;

n = ratio of E to effective modulus in shear: $\frac{E}{\frac{5}{6} E_s}$;

P = thrust acting on an arch section, parallel to the center line; P_e = value of P due to external loads; P_t = value of P due to external loads and a trial crown force;

r = length of a vector; as a subscript, r denotes "the right half of the arch";

---

[8] A. S. A.-Z 10A-1932.

S = a shearing force in a plane normal to the center line; S_e = value of S due to external loads only; S_t = value of S due to external loads and a trial crown force;

s = length of a voussoir, measured along the center line;

T = temperature change (rise positive);

t = thickness of the arch, normal to the center line; as a subscript, t denotes "due to external loads and a trial crown force";

V = "vertical" force at the crown; that is, a force parallel to the Y-axis;

x = a distance measured parallel to the X-axis, beginning at the crown center line; the X-axis is called "horizontal" and is approximately parallel to the arch center line at the crown; x_a = values of x for the abutment;

y = a distance measured parallel to the Y-axis, beginning at the crown center line; the Y-axis is called "vertical" and the positive y-direction is called "downward"; y_a = values of y for the abutment;

α = angle between the X-axis and the arch center line;

β = slope, or rotation, of the arch center line (counter-clockwise positive); β_c = the value of β at the crown;

Δ = vector displacement at any point on the arch center line; Δ_x = value of Δ at the crown in the X-direction (positive leftward); Δ_y = value of Δ at the crown in the Y-direction (positive upward);

ϵ = coefficient of thermal expansion;

θ = angular distance; angle between the X-axis and the direction of any vector (sign convention same as for α).

DISCUSSION

I. M. NELIDOV,[9] ASSOC. M. AM. SOC. C. E. (by letter).—The method of graphical analysis given in this paper represents a development over the similar methods in existence, in that it offers, in one complete diagram, the graphical determination not only of forces but also of deformations.

In so far as they refer to Equations (1) to (13) the formulas and coefficients in Table 1 are correct. If a more simple case of a circular arch of constant cross-section under uniform load is taken, the same expression for crown thrust is obtained as is given by the late William Cain,[10] M. Am. Soc. C. E. The derivations, being somewhat voluminous, are not given herein. In reference to the finite voussoirs the writer wishes to emphasize that the selection of the center of the voussoir, at one time for one purpose, and of the ends of the voussoir, at another time for another purpose, does not help to mechanize the process, which in other respects the authors succeeded in doing. The primary cause of this selection is the small number of the voussoirs and the desire to maintain accuracy. It might be better to double the number of voussoirs and thus avoid the complicated assumptions of centers and sides.

It is evident that the authors were greatly interested in evolving the graphical multiplication and integration. The subject of graphical operations of mathematical functions has attracted investigators in the past.[11] Various operations can be performed, such as addition and subtraction, multiplication and division, raising to a given power, evaluation of radicals, operations with trigonometric functions, integration, etc. It is commendable that the authors offered a complete explanation of the graphical processes involved in this method.

As to the "trial force," the writer wishes to remark that this is partly a misnomer and that it is not presented with sufficient clarity. It has been known in the theory of arches[12] that all the forces in an elastic arch can be divided into two classes, the first resulting from the rigid arch and the second resulting from its elastic action. The forces then are called "main" or "original," and "secondary." Using the notation of the paper with the subscript t to indicate the original forces, the expressions for any section of the arch are:

$$P = P_t + P_0 \quad \text{(18a)}$$

$$S = S_t + S_0 \quad \text{(18b)}$$

and

$$M = M_t' + M_0 \quad \text{(18c)}$$

in which the subscript 0 refers to the forces caused by the elastic action, and, M_t' is accented in order to distinguish it from M_t as shown subsequently in Equation (21).

---

[9] Senior Engr. of Hydr. Structure Design, State Div. of Water Resources, Sacramento, Calif.

[10] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 522.

[11] "Die Graphische Methode," by C. Runge, Berlin, Germany; also, "Manual for Graphical Computations," by V. P. Pharmakovsky (in Russian).

[12] "Solution of Circular Arch Under Water Load," by Emil Mörsch, Zurich, Switzerland, *Schweizerische Bauzeitung*, Vol. 51, 1908.

For the crown section Equations (18) become:

$$H = H_t + H_0 \tag{19a}$$

$$V = V_t + V_0 \tag{19b}$$

and

$$M_c = M_t' + M_0 \tag{19c}$$

In Equations (18) and (19) P_t, H_t, S_t, V_t, and M_t' are known, being the forces in the rigid arch resulting from the external forces. Thus:

$$P_t = P_e + H_t \cos \alpha \tag{20a}$$

$$S_t = S_e + H_t \sin \alpha \tag{20b}$$

and

$$M_t' = M_e + H_t\, y + V_t\, x \tag{20c}$$

However, the authors defined M_t as:

$$M_t = M_e + H_t\, y + V\, x \tag{21}$$

thus including, in V, both V_t and V_0. For illustration, in applying Equations (18) to (21) to the case of a circular arch with neutral radius r, constant thickness t, half of the central angle α_1, and uniform radial load p:

$$H_t = \frac{p\, 2\, r^2 \sin^2 (0.5\, \alpha_1)}{r\, (1 - \cos\, (0.5\, \alpha_1))} = p\, r \tag{22a}$$

$$V_t = 0 \tag{22b}$$

$$P_t = 2\, p\, r \sin^2 (0.5\, \alpha) + p\, r \cos \alpha = p\, r \tag{22c}$$

$$S_t = 2\, p\, r \sin\, (0.5\, \alpha) \cos\, (0.5\, \alpha) - p\, r \sin \alpha = 0 \tag{22d}$$

and,

$$M_t' = -\, 2\, p\, r^2 \sin^2 (0.5\, \alpha) + p\, r^2\, (1 - \cos \alpha) = 0 \tag{22e}$$

In other words, the forces with the subscript t are always known because they are obtained from ordinary statics and no "trials" are needed.

In the case of irregular loading and of the arch, Equations (22) may be obtained graphically from the force diagrams of statics. Referring to the method of obtaining the moment M_t by a summation of the values of shears, as shown in Table 2, Columns (11) and (12), it may be noted that the moments M_t' may be obtained directly by graphics, by constructing a force polygon and an equilibrium polygon for the forces acting and superimposing the latter on the neutral line of the arch.

In conclusion, what interests the writer is the statement by the authors that, to include the effect of the non-uniform temperature, one must add to M_t, due to the external forces, the expression $\frac{dT}{dt}\, \epsilon\, E\, I$. The writer suggests that the authors verify it by applying their formulas in Table 1 to the simple case of a circular arch of neutral radius r, uniform thickness t, half the central angle α_1, and finding a total crown thrust H due to this change of temperature. The expression for H is as follows:[13]

$$H = \left[\Delta x - \theta\, r \left(1 - \frac{\sin \alpha_1}{\alpha_1} \right) \right] \frac{2\, E\, I}{D\, r^3} \tag{23}$$

---

[13] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), pp. 492 and 558.

in which the horizontal displacement of the crown section, due to the effect of non-uniform temperature, is

$$\Delta x = r^2 \epsilon \frac{dT}{dt} (\alpha_1 - \sin \alpha_1) + r \epsilon \left(t_i + \frac{t}{2} \frac{dT}{dt} \right) \sin \alpha_1 \ldots\ldots (24a)$$

The rotation of the crown due to the same effect is

$$\theta = \frac{dT}{dt} r \epsilon \alpha_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24b)$$

Coefficient D is

$$D = \left(1 + \frac{I}{t\, r^2} \right) \left(\alpha_1 + \frac{\sin 2\, \alpha_1}{2} \right) - \frac{2 \sin^2 \alpha_1}{\alpha_1} + n \frac{I}{t\, r^2} \left(\alpha_1 - \frac{\sin 2\, \alpha_1}{2} \right) \ldots\ldots\ldots\ldots\ldots\ldots (24c)$$

and, t_i is the temperature at the intrados, in degrees Fahrenheit.

CARL H. HEILBRON, JR.,[14] ASSOC. M. AM. SOC. C. E., AND WILLIAM H. SAYLOR,[15] JUN. AM. SOC. C. E. (by letter).—The comments of Mr. Nelidov indicate that, in several instances, the writers have failed to offer sufficiently clear explanations. It is hoped that the questions raised will be answered satisfactorily herein.

The writers cannot agree with the suggestion that the convention whereby the centers of voussoirs and ends of voussoirs are used alternately "does not help to mechanize the process." In any method wherein all functions are determined at a single set of points it is necessary to average values at adjacent points in every step where integration is to be performed, and it is only by means of a convention such as that adopted that this averaging is avoided. Although the assumptions sound "complicated" when described, any one using them will realize that they result in the simplest possible set of operations.

In this connection Mr. Nelidov suggests the use of a larger number of voussoirs. It may be well to reiterate that in their example the writers, for simplicity, used a smaller number of voussoirs than they would recommend generally.

Mr. Nelidov identifies the "trial" forces with a similar set of forces which he calls "main" or "original." The latter, he says, are "known," meaning, it is presumed, that they can be obtained by processes not involving trial. Using his concept of "original" forces he arrives at a value of M_t' as (see Equation 20(c)) $M_t' = M_e + H_t\, y + V_t\, x$. He then mistakenly states that the writers define M_t (see Equation (21)) as $M_t = M_e + H_t\, y + V\, x$. Actually, the writers define M_t as

$$M_t = M_e + H_t\, y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

Thus, M_t is identical with M_t' except that V_t is taken as zero. That it is possible to let V_t be zero is due to the fact that the "trial" forces used by the writers are arbitrary values which are used as an aid in improving the accuracy of subsequent calculations, whereas the "original" forces considered by Mr. Nelidov are actual "known" forces which must be calculated. The advantage of the writers' concept of these forces is that they need not be calculated exactly but may be chosen arbitrarily as best suits the needs of the problem. In the case of V_t, it has been found practical to choose a value of zero so that this quantity is eliminated from the procedure.

---

[14] Associate Engr., U. S. Engr. Office, Los Angeles, Calif.

[15] Asst. Engr., U. S. Engr. Office, Los Angeles, Calif.

Mr. Nelidov's question in regard to the effects of non-uniform temperature change is due, perhaps, to the writers' failure to explain that, in addition to taking the temperature gradient into account, of course, the average change of temperature must be used just as if the change were uniform.

The writers appreciate the fact that Mr. Nelidov brought these points to their attention.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2074

HYDROLOGY OF THE GREAT LAKES
A SYMPOSIUM

PAGE

Typical Quantitative Analysis as Applied to Lake Superior.
By C. R. Pettis, M. Am. Soc. C. E. 795

Evaporation Experiments.
By Harold C. Hickman, Jun. Am. Soc. C. E. 807

With Discussion by Messrs. A. A. Young, Alfred J. Cooper, Jr., Adolph F. Meyer, S. T. Harding, O. E. Meinzer, R. W. Davenport, G. H. Hickox, and C. R. Pettis, and Harold C. Hickman.

Note.—Published in April, 1939, *Proceedings*.

TYPICAL QUANTITATIVE ANALYSIS AS APPLIED TO LAKE SUPERIOR

By C. R. Pettis,[1] M. Am. Soc. C. E.

Synopsis

A method of making a quantitative analysis of the hydrology of the Great Lakes, as illustrated by a detailed study of the hydrology of Lake Superior, is presented in this paper. Certain deductions are made, based on evidence from available data. By means of these deductions, values are obtained for the evaporation from the lake surface. Knowing the evaporation, the underground flow to the lake and the land losses can be computed.

The validity of the method has been checked by evaporation experiments which are described in the companion paper of this Symposium.

The conclusion reached is that the evaporation and the underground flow are greater, and the land losses are less, than the values that have been generally accepted.

Introduction

The hydrology of the Great Lakes has been the subject of several studies: Thomas Russell, of the United States Lake Survey (1906); the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E. (1926); Robert E. Horton, M. Am. Soc. C. E., and the late Carl E. Grunsky, Past-President, Am. Soc. C. E. (1927); and John F. Hayford and J. A. Folse (1930). Most of them assumed a value for evaporation determined from ordinary land pans; Mr. Freeman assumed that the underground flow to the lake was zero; all of their results differed considerably from those obtained by the present analysis.

A quantitative analysis of the hydrology of Lake Superior must account for the disposal of all the water that enters the main body of the lake, for such periods of time as may be considered. In any given period of time, the total quantity of water that enters the lake, as precipitation on the surface of the lake, river flow and underground flow, will be equal to the evaporation from the lake surface plus the flow through St. Marys River, plus the change in storage, as indicated by lake levels in the same period of time. If the river flow to the lake is known, and the underground flow to the lake can be determined, the tributary land drainage basin can be included in the analysis by using the relationship: Over a period of several years, the rainfall on the land basin is equal to the river flow to the lake, plus the underground flow to the lake, plus the land losses. The term "land losses" includes all land precipitation that does not reach the lake; presumably the land losses consist primarily of land evaporation and transpiration, or other vegetative losses.

---

[1] Colonel, Corps of Engrs., U. S. Army (*Retired*), Head, Dept. of Math., Mississippi State Coll., State College, Miss.

ANALYSIS

Formulas.—The following formulas apply to any given period of time:

$$P + R + U = O + S + E \quad \text{.................. (1)}$$

and

$$B = R + U + L \quad \text{........................ (2)}$$

All of the terms in the equations represent volumes of water in the period of time under consideration. They are defined as follows:

P = the rainfall on the surface of the lake, and becomes an immediate addition to the body of water in the lake as soon as it falls. It is assumed that the depth in inches of rainfall on the lake is the same as the depth of rainfall on the land; it was decided that this assumption would give better results in this analysis, because the evidence of a slight difference between P and B is conflicting.

B = the rainfall on the tributary land area. This value was computed from records of 59 stations of the United States Weather Bureau in the area, by taking a weighted average for the sections into which the area was divided.

R = the run-off into the lake from the surface streams as determined from stream-flow records of the United States Geological Survey and other sources, using a method of weighted averages, based on a personal reconnaissance of the area, 49% of which is covered by stream-flow records. The value of R, as a whole, is considered fairly accurate.

U = the net underground flow into the lake, which is not known, and is to be determined.

O = the outflow from the St. Marys River, which is based on careful records kept by the U. S. Engineer Department.

S = the change in storage in the lake, as indicated by gage records, being plus for a rising lake and minus for a falling lake. The gage records are kept by the United States Lake Survey.

E = the net evaporation from the surface of the lake, and is considered in the usual sense of evaporation minus condensation.

L = the difference between the rainfall B and the surface and underground flow into the lake $(R + U)$, for the period under consideration, as given by a transposition of Equation (2).

The period of time used in most of this analysis is one calendar month. Symbols P, R, U, O, S, and E all denote quantities of water that enter or leave the main body of the lake during the month in question. Symbol B is the precipitation on the land area for the same month. In winter much of the precipitation remains on the surface of the ground and does not enter the lake, either as surface flow or underground flow, until the spring thaw. Throughout the year, the land precipitation, B, is subject to a delay before entering the lake, due to both surface and underground storage effects. For this reason, the value of L for any particular month, as obtained by Equation (2), will not represent the land losses (evaporation and transpiration) which pertain to the precipitation of that particular month, the land losses being that portion of the precipitation that does not ultimately enter the lake. Over a number of years, the values of L for the period will represent the land rainfall that does not reach the lake; that is, the land losses, primarily land evaporation and transpiration.

There are two equations and three unknowns, E, U, and L. A rigid solution is impossible; but by making certain assumptions in accord with the present knowledge of the subject (and which, in most cases, seem to be the only reasonable assumptions possible, in view of the evidence) it is possible to arrive at an approximate solution. If one unknown is determined approximately, a rigid solution is possible for the other two unknowns, although the results necessarily cannot be more accurate than the approximate determination of the first unknown.

As far as is known, none of the lakes freezes over entirely in the winter. "Open-water" evaporation refers to the rate of evaporation from the part of the lake that is not covered by ice. The actual evaporation from a lake in the winter will be less than the "open-water" evaporation by an amount that depends on the extent of ice that is formed.

Since January, 1937, evaporation stations have been maintained by the Lake Survey at Detroit, Mich., Kewaunee, Wis., Duluth, Minn., and Buffalo, N. Y. An effort is made to keep the water temperature the same as the temperature of the water in the open lake, which makes them different from ordinary land-pan experiments.

Results reported by Mr. Hickman in this Symposium are based fundamentally on the evaporation experiments. In this paper, a mathematical analysis is made of data which is entirely different from that used by Mr. Hickman, and monthly values of the evaporation on Lake Superior are obtained. The results of the two independent solutions check rather closely; the results obtained by investigators who neglected underground flow do not check with the evaporation experiments. When the evaporation has been determined, by any method, the values of U and L can be determined mathematically from Equations (1) and (2). The analysis and solution given in this paper were made before the evaporation experiments were begun. It is evident from Fig. 1 that the remainder of this paper, with slight modifications, might be considered to be a solution and discussion based on the evaporation experiments.

From the data and Equations (1) and (2), monthly values of $(E - U)$, $(E + L)$, and $(U + L)$ can be obtained which will be as accurate as the data upon which they are based.

As a matter of convenience, in all computations, the water that will cover the land area tributary to Lake Superior to a depth of 1 in., will be used as a unit of volume. Unless otherwise indicated, all data will be given in the same unit. The land area of Lake Superior is 49 078 sq miles. One unit, or 1.0 in. of water on this area, will represent a certain definite quantity, which is practically 114 billion cu ft. The water area of Lake Superior is 31 817 sq miles. The ratio of land area to water area is 1.54. If a 1-in. depth of water on the land were placed in the lake, the lake level would be raised 1.54 in. Inches of depth on the land can be converted to inches of depth on the lake by multiplying by 1.54; and, conversely, inches of depth on the lake can be converted to inches of depth on the land by dividing by 1.54.

In this analysis, the following assumptions are made:

(*a*) It is assumed that the net underground flow into the lake, *U*, is always positive; that is, there will be no monthly period in which there is a net flow of underground water away from the lake. This assumption is definitely indicated by the analysis of curves in this study. The flow of underground water even in the winter has been indicated by Mr. Horton who states:[2] "In case of sandy soils with low field moisture capacity, infiltration takes place at a relatively high rate even when the ground is frozen." Messrs. H. K. Burton

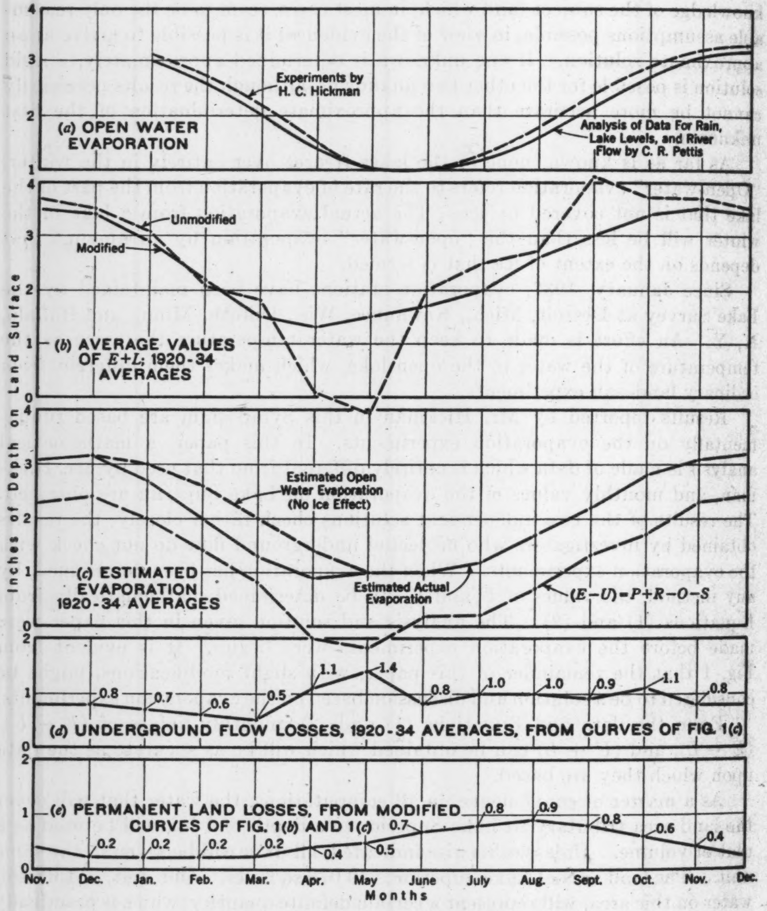


FIG. 1.—HYDROLOGICAL DATA APPLYING TO LAKE SUPERIOR

and J. Cecil Alter state:[3] "The mountain snow-layer melts continually from the bottom, the moisture leaching steadily into the soil."

---

[2] National Research Council Report, 1936, p. 351.

(*b*) It is assumed that the normal open-water evaporation curve for monthly periods will be approximately a sine curve with a periodicity of one year. The curves for air and water temperatures, relative humidity and wind, which are the factors that are supposed to affect evaporation, are approximate sine curves with periodicities of one year. There is some variation from normal in the evaporation for particular months; but, in general, the variation in evaporation from the normal for a particular month is not large. According to H. K. Barrows,[4] M. Am. Soc. C. E., for any given month there is, in general, a greater range in the rate of precipitation than in the rate of evaporation. For that reason less time is required to obtain data of average monthly evaporation sufficiently accurate for use in storage estimates. Other engineers have noted the same condition.

(*c*) The $(E + L)$-curve in Fig. 1(*b*), with obvious modifications, is approximately a sine curve. The curve representing the permanent land losses, when distributed over the months in which the losses occur, should be something like a sine curve; the numerical variations in the permanent (distributed) L-curve are probably less than the variations in any of the other curves.

If the $(E - U)$-curve is plotted by months (see Fig. 2, with Tables 1 and 2), and compared with the rainfall B-curve, it is apparent at once that, in practically every case when the weather is not freezing, a high value of rainfall in one month will be followed by a low value $(E - U)$ in the following month, or sometimes in the second month. This is true for each of the Great Lakes. This relation between B and $(E - U)$ must be accounted for by one of two hypotheses:

(1) The high rainfall in one month causes a radical change in the evaporation for the following month. This assumption is directly contrary to present theories on the subject of evaporation; it is inconsistent with the previous assumption in regard to the regularity of evaporation by monthly periods; and, therefore, this assumption is rejected as being untrue.

(2) High rainfall in one month will be followed by an increase in underground flow which will be most noticeable in the lake in the following month or, say, five weeks later. There is nothing in this assumption contrary to the present knowledge of the subject; it seems reasonable and, therefore, it is adopted for the purpose of this analysis. It may be noted that a high rainfall in the summer will have a greater effect on the underground flow in the next month than the same rainfall in the late fall or winter, which is consistent with the present engineering opinion that underground water flows more freely when the temperatures are warm than when the temperatures are cold (see Table 1).

There is a considerable flow, R, from the surface streams through the winter; November, 0.9 in.; December, 0.7 in.; January, 0.6 in.; and February, 0.5 in. Since the ground in the Superior basin is ordinarily covered with snow during these months, it is assumed that most of this winter flow comes from underground sources. The surface streams can tap only the portion of the underground water that is above certain levels; the underground water below these levels can reach the lake only through underground channels.

---

[3] National Research Council Report, 1936, p. 534.

[4] "Water Power Engineering," by H. K. Barrows, p. 83.

The winter underground flow is deduced by the following analysis: The 6-yr period 1920–1925 was a period of relatively small annual rainfall (average 26.6 in.); and, the 6-yr period 1926–1931 had a greater annual rainfall (average 29.8 in.). The average annual temperature for the two periods varied by less than 0.5° F. If the winter evaporation, December to March, for the two periods is assumed to average the same, the values of $(E - U)$ indicate that the underground flow from December to March for the second period was greater than that for the first period by 0.9 in.

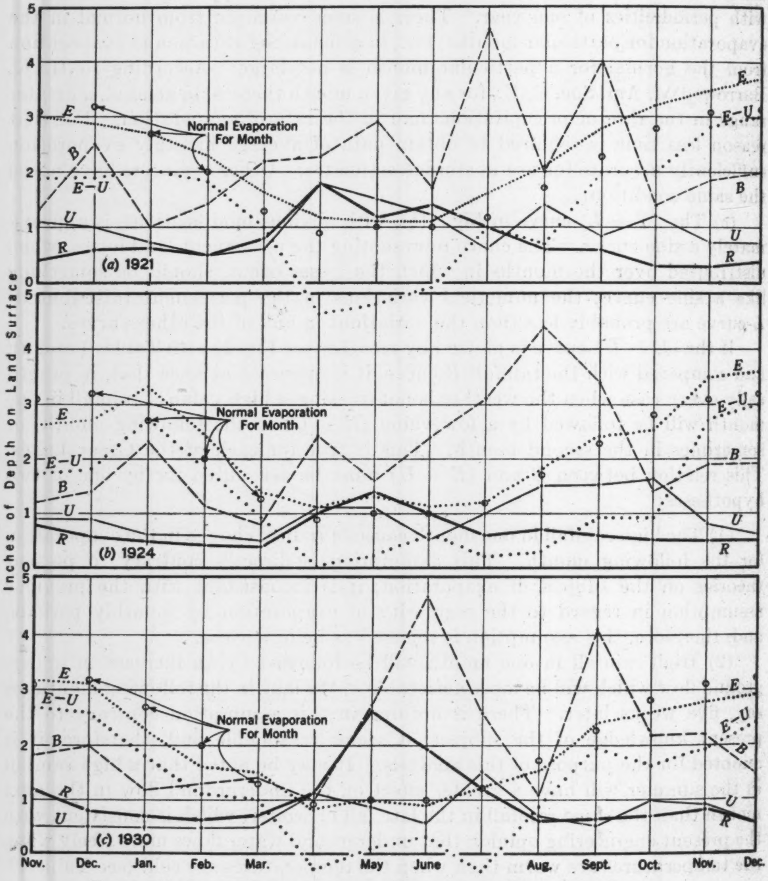


FIG. 2.—ESTIMATED EVAPORATION AND UNDERGROUND FLOW; LAKE SUPERIOR

TABLE 1.—Temperature and Wind Records; Lake Superior; Corresponding to Fig. 2.

Description	November	December	January	February	March	April	May	June	July	August	September	October	November	December
(*a*) 1921; a Warm Year; Winter Warm, with Little Ice														
Average 15-year normal air temperature, in degrees Fahrenheit	31	18	13	15	24	37	50	60	65	63	57	45	31	18
Actual monthly temperature, 1921, in degrees Fahrenheit...	31	23	20	20	26	42	51	64	72	64	60	47	27	19
Wind, percentage of normal over a 15-year period............	110	120	100	100	120	140	100	100	100	100	120	100	82	110
Evaporation (+ = above normal)...................	...	...	...	...	...	...	...	...	...	...	...	...	...	...
(*b*) 1924														
Average 15-year normal air temperature, in degrees Fahrenheit	31	18	13	15	24	37	50	60	65	63	57	45	31	18
Actual monthly temperature, 1924, in degrees Fahrenheit...	36	28	5	17	26	36	45	57	62	62	53	51	31	10
Wind, percentage of normal over a 15-year period............	90	100	110	100	90	120	120	100	100	112	100	90	110	120
Evaporation (+ = above normal)...................	−	−	+	...	...	+	+	...	...	+	+	−	+	+
(*c*) 1930														
Average 15-year normal air temperature, in degrees Fahrenheit	31	18	13	15	24	37	50	60	65	63	57	45	31	18
Actual monthly temperature, 1930, in degrees Fahrenheit...	28	17	8	18	24	39	51	62	65	67	56	44	34	21
Wind, percentage of normal over a 15-year period............	90	90	90	90	110	90	100	90	100	90	110	90	100	80
Evaporation (+ = above normal)...................	...	...	...	−	+	−	...	−	...	−	+	−	−	−

TABLE 2.—Hydrological Data; Lake Superior, Corresponding to Fig. 2.

Description	1921	1924	1930
Net Evaporation, *E*, from the Surface of the Lake:			
November to October, inclusive..................	25.6	23.8	22.1
Run-off, *R*, into the Lake from Surface Streams:			
January of following year..................	0.5	0.5	0.3
February of following year..................	0.5	0.5	0.3
March to February, inclusive..................	9.4	8.2	11.3
Net Underground Flow, *U*, into the Lake:			
January of following year..................	0.7	0.6	0.5
February of following year..................	0.6	0.6	0.5
March to February, inclusive..................	11.8	11.4	9.2
Rainfall, *B*, on Tributary Land Surface:			
November to October, inclusive..................	27.3	25.5	26.1
$R + U$: March to February, inclusive..................	21.2	19.6	20.5
$L = B - (R + U)$: From two items above..................	6.1	5.9	5.6

The rainfall in August, September, and October is a major (but not the sole) factor affecting winter underground flow. From Equation (2), the rainfall available for underground flow is $(B - R - L)$. Taking an average value of L for the given months from Fig. 1(*d*), and using known data, $(B - R - L)$ August to October, for the first period, was 3.4 in., and for the second period, 4.8 in.; the average value was 4.1 in. for both periods combined; and, the difference between the low period and high period was 1.4 in. Assuming that the difference in available rainfall (1.4 in.) is to the difference in winter underground flow (0.9 in.) as the average available rainfall (4.1 in.) is to the average underground flow, then the average underground flow, December to March, is 2.6 in. This was distributed to months as follows: December, 0.8; January, 0.7; February, 0.6; and March, 0.5.

The average monthly values of $(E + L)$ for the 15-yr period, as computed by the formula:

$$E + L = P + B - O - S \dots\dots\dots\dots\dots\dots (3)$$

are shown by the curve in Fig. 1(*b*). For any period long enough to equalize the underground-storage and lake-storage effects, $(E + L)$ will represent the quantity of water that falls on the total area and which has not passed out of the area through St. Marys River.

In the $(E + L)$-curve shown in Fig. 1(*b*), if more water than usual goes into underground (or snow) storage in any particular month, then L for that month —and also $(E + L)$—will contain not only the permanent land loss for that month, but also a temporary loss, which represents water that has gone into storage, and which will decrease the formula values of (E + L) for the later months in which it comes out of storage. It is evident that certain adjustments may be made in the $(E + L)$-curve to obtain a modified $(E + L)$-curve which will represent the values of $(E + L)$ with land losses distributed to the months in which they actually occur. It will be noted that there is a considerable sag in the unmodified $(E + L)$-curve in April and May (see Fig. 1(*b*)). This sag is due to the fact that, in these months, considerable water that had fallen during the winter months, in the form of snow, becomes available for surface flow and underground flow, causing a negative value of L.

The years 1924 and 1931 had the smallest winter precipitation of any years in the 15-yr period under study. The April value of $(E + L)$ for 1924 was 1.1, and the May value of $(E + L)$ for 1931 was 1.4. These values are only slightly less than those adopted for the modified $(E + L)$-curve for these two months. In comparing the $(E + L)$-curve and the modified $(E + L)$-curve, it will be noted that 3.0 in. is gained in the spring thaw. This represents the water that has gone into storage in the preceding months, and which comes out of storage at the time of the thaw. The difference between the $(E + L)$-curve and the modified $(E + L)$-curve (Fig. 1(*b*)) is caused by the varying effects of storage on the values of L.

If the modified $(E + L)$-curve is compared with a sine curve, it will be noted that there is a slight sag in the winter months, ending about May. This is caused by a certain area of ice forming near the shores of the lake in the winter, which reduces the evaporation. Lake Superior very seldom, if ever, freezes over entirely. It is doubtful if as much as half of the lake has frozen over during the period 1920–1934.

Before an attempt is made to analyze the curves for individual years, it is desirable to ascertain the approximate shape of the E-curve for an average year. The average monthly values of $(E - U)$ for the 15-yr period are plotted in Fig. 1(c). Since it is assumed that U is positive at all times, all points of the E-curve must lie above the corresponding points of the $(E - U)$-curve; in each case it is by an amount that is equal to the underground flow for the particular month. Previously, herein, average values of U were assumed for December to March. This assumption yields points on the E-curve for December to March, with the maximum point of the E-curve in December. For open-water evaporation, the minimum point of the curve can be assumed to occur about six months from the maximum point, or about June. The evaporation experiments, reported in this Symposium by Mr. Hickman, indicate that the minimum monthly evaporation to be expected on Lake Superior will not be less than 1 in. (land area unit). It will be assumed that the minimum evaporation in May and June is 1.0 in. per month. The open-water evaporation curve can now be obtained by drawing an approximate sine curve between the high points and the low points, and through the other points determined. The actual evaporation curve, or the full-line curve in Fig. 1(c), is obtained for the month of April by considering that a reduction in evaporation, due to ice, continues through this month.

From the E-curve (Fig. 1(c)), the $(E - U)$-curve (Fig. 1(c)), and the modified $(E + L)$-curve (Fig. 1(b)), the average monthly values for U and the permanent distributed values of L can be obtained. The curves for U and L are shown in Fig. 1(d).

The U-curve is higher in the warm months, and lower in the winter. The highest point is in the spring thaw, when the melting snow is added to the rainfall. The curve reflects the higher rainfalls in July and September, by increased flow in August and October.

The L-curve (Fig. 1(d)) may be considered to be caused primarily by land evaporation and transpiration. The shape of the L-curve suggests that the land-evaporation curve is somewhat similar to that of the ordinary pan-evaporation experiment, being higher in the summer, and, that transpiration losses vary with the activity of vegetable growth. Attention is called to the fact that "land evaporation," as the term is used in this paper, means the evaporation proper (water vapor leaving the land surface), minus condensation (water vapor absorbed from air by soil). According to some investigators, the rate of condensation is considerable.

An estimate has been made of the monthly evaporation for each one of the fifteen years in the period 1920–1934; the result of this detailed analysis for three typical years is shown in Fig. 2, with Tables 1 and 2. The methods used in the analysis have been evolved as the result of an extended study.

The average air temperature and relative wind velocities were computed for each month from data supplied by the U. S. Weather Bureau (see Table 1); a decided departure of either one of these factors from normal for any month will generally have a noticeable effect on the evaporation for that month.

Relative humidity was computed for each month; an effort was made to relate the monthly values of relative humidity to the corresponding data for evaporation, but the conclusion was reached that the monthly relative humidity is more the result of the evaporation than it is the cause of evaporation. For this reason, relative humidity was not used in the analysis; which is equivalent to making the assumption that, for the 15-yr period, the values of relative humidity remain normal according to the period of the year considered.

In the analysis, the first step is to plot monthly values of the $(E - U)$-curve; then values of E are assumed for each month, which best comply with the following conditions:

(1) If air temperature is normal for the month, and has not been abnormal for the months immediately preceding, and if the wind is normal for the month, E should be nearly normal for the month, as shown by the E-curve in Fig. 1(*c*).

(2) A sudden lowering of the temperature for one month will tend to cause a higher value of E for this month; this accords with Dalton's law, since water temperature tends to follow the air temperature, but with a considerable lag.

(3) If air temperatures are above normal for a few months (say, three or more), water temperature will probably be above normal, and E will be increased.

(4) In a cold winter, E will tend to decrease, due primarily to a larger area of ice formed on the lake.

(5) A relatively high wind will cause a higher value of E.

(6) The normal winter values of U previously deduced are assumed to apply, except where modification may be indicated by some of the other factors.

(7) The values of E in the spring months should be such as to reflect the spring thaw in the values of U; and, for different years, the values of U in the spring thaw should have some relation to the winter precipitation.

(8) There should be a relation between the precipitation and the values of U. Abnormally high or low values of B, except in the winter, should be followed in general by high or low values of U within a month or two.

(9) In some cases, the conditions will be conflicting; a few minor discrepancies are to be expected, due to slight errors in the data, or to the method of treating relative humidity.

The application of the foregoing principles to three typical years is shown in Fig. 2, with Tables 1 and 2. The climatic data for each month are plotted on the vertical line beneath the given month. In Table 1, the first line of each sub-table is the average (15-yr) air temperature for the corresponding month. The second line gives the average air temperatures for the months in question. The effect of the air temperature is judged by a comparison of the first two lines. The third line gives the wind velocity for each month as compared with the average (15-yr) wind velocity for that month. The fourth line shows by $(E +)$ or $(E -)$ certain months in which the temperature and wind indicate that the evaporation for the particular months will be appreciably above or below normal for those months.

The $(E - U)$-curve is computed by a transposition of Equation (1). The R-curve is drawn from the tabulated values obtained from the Geological

Survey data. The small circles in the figures indicate the normal evaporation for respective months, as previously determined. The E-curve indicates the estimated monthly evaporation, giving consideration to climatic factors, and to the conditions previously enumerated. The value of U for each month, as shown by the U-curve, is determined by taking the distance between the E-curve and the $(E - U)$-curve.

For convenience in making comparisons, the B-curve, for rainfall on the land, is plotted on the same sheet.

Values of E, R, U, $(R + U)$, B, and L, are presented in Table 2, L being computed from a transposition of Equation (2), using different periods for the several factors, in order to reduce the error in land losses, L, due to temporary storage effect. The twelve-month period, November to October, inclusive, was used for B in this formula; and, March to February was used for R and U, for the following reasons: The precipitation (in the form of snow), from November to February, has a much greater effect on R and U in the spring thaw after March than it does on the winter flow of R and U. The winter flow is affected principally by the rainfall in October and the preceding months, so that the rainfall during the period used in the formula for B reaches the lake principally in the period used in the formula for R and U; and the values of L computed by this method are fairly uniform for most years, since the storage effect will be small unless conditions are somewhat abnormal. The correlation between climatic factors and evaporation, and between rainfall and underground flow can be observed in Fig. 2.

The year 1921 was an unusually warm year, and the evaporation was high (25.6 in.). The high evaporation for this year is further substantiated by a correspondingly high value of $(E + L)$, which can be computed from known factors by Equation (3). The winter of 1930 apparently had more ice than either 1921 or 1924. It was the only year of the three with a November-to-March temperature below normal. The year 1921 had the highest winter precipitation, from November to March, and it also had the largest underground flow in the spring thaw in April and May. The curves in Fig. 2 indicate that there is a fairly close correlation between rainfall, B, and the total flow to the lake, $(R + U)$. The distribution of B between R and U, evidently, depends on other factors than those that have been shown.

Next, consider the possibility of errors in the data, and the effect that such errors might have on the values of E, L, and U. For the purposes of this analysis, it is believed that the values of O and S may be accepted as correct, and that minor errors are more probable in the values of P, B, and R. The following statements are believed to represent the principal effects of possible errors:

From Equation (1), $(R + U) = O + S + E - P$. If the values on the right are assumed to be correct, an error in R will cause an equal error in the opposite direction in U; and to a considerable extent, errors in R will be compensated by opposite errors in U, without affecting the values of E and L.

From Equation (1), $E = P + (R + U) - O - S$. If O and S are assumed correct, and errors in R are compensated in the determined values of U, an error in P will cause an equal error in the same direction in E.

From Equation (2), $L = B - (R + U)$. If errors in R are compensated in the determined values of U, an error in B will cause an equal error in the same direction in L.

The collection of water in rain gages is subject to wind effects, and the water in most gages is subject to some slight evaporation. It is believed that both B and P may be slightly greater than the Weather Bureau data used, and that the values of E and L may be slightly larger than the determined amounts.

Summary and Acknowledgment

The results of this analysis for Lake Superior are as follows:

Land area		Inches of depth on land area
Normal annual rainfall on land is		28.2
Of which land losses are		5.9
The remainder		22.3
Reaches the lake as:		
Surface run-off	11.6	
Underground flow	10.7	

Lake area		Inches of depth on lake area
Normal annual rainfall on lake is		28.2
Annual surface run-off to lake is		17.9
Annual underground flow to lake is		16.5
Total addition to lake in one year is		62.6
Of which annual evaporation is	35.6	
And outflow from St. Marys River is	27.0	

The writer has received assistance from a number of sources, especially from Messrs. Sherman Moore, H. C. Hickman, Jun. Am. Soc. C. E., L. D. Kirshner, and F. W. Townsend, of the Lake Survey, and Professors C. O. Wisler and W. C. Hoad, Members, Am. Soc. C. E., and E. F. Brater, Jun. Am. Soc. C. E., of the University of Michigan, at Ann Arbor, Mich. The data form a part of a thesis[5] by the writer, entitled "Hydrology of the Great Lakes," presented to the University of Michigan in 1938, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

---

[5] A copy of the thesis, including all observations and other data, has been placed on file in the Engineering Societies' Library, 33 West 39th Street, New York, N. Y.

EVAPORATION EXPERIMENTS

BY HAROLD C. HICKMAN,[6] JUN. AM. SOC. C. E.

SYNOPSIS

The evaporation experiments reported in this paper were conducted at Duluth, Kewaunee, Detroit, and Buffalo, for the purpose of providing a basis for estimating the evaporation from the water surfaces of the Great Lakes. In order to be applicable to local conditions, an effort was made to keep the temperature of the water in the pans approximately the same as the temperature of the water in the open lakes. Data were secured which cover a greater variety of combinations of air and water temperatures and wind velocities at the evaporation pans, than any known heretofore. In applying these data to the Great Lakes it was assumed that the evaporation from the open lakes, for a given air temperature, water temperature, and wind velocity, would be the same as the observed evaporation from the experimental pans for the same combination of air temperature, water temperature, and wind velocity.

The readings were analyzed and plotted on two charts, from which an estimate of the monthly (or daily) evaporation from the Great Lakes can be made. In these charts it has been assumed that the relative humidity and barometric pressure correspond to the average conditions that existed on the shores of the lakes, at all four stations, during the particular time of the year in which the data were being obtained. The experiments indicate that the effect of the wind on evaporation is fundamentally different from that indicated by the common evaporation formula. The rate of evaporation from open-water surfaces when the air temperature is below freezing has been investigated for the first time.

INTRODUCTION

The physical conditions that affect evaporation on the Great Lakes are quite different from conditions in comparatively shallow lakes. Because of the great depths involved (Lake Superior being as much as 1 300 ft deep in parts, Lake Michigan 920 ft, Lake Huron 750 ft, Lake Erie 210 ft, and Lake Ontario 760 ft), the water temperatures tend to remain relatively constant. Measurements of water temperatures in Lake Superior indicate that the entire body, below a depth of 180 ft, remains at a temperature of approximately 39.4° F throughout the year, which is the temperature of maximum density. In summer, the upper water is at a higher temperature than the lower body of water; and, in general, the temperature becomes increasingly warmer from a depth of 180 ft to the surface. In winter, the upper water is colder than the lower body;

---

[6] Junior Engr., U. S. Lake Survey, Detroit, Mich.

and, in general, the temperature becomes increasingly colder from the lower depths to the surface.

The average wind velocity on the lakes is about 10 miles per hr, which is the 15 yr average for four stations located around Lake Superior, the stations being about 100 ft above the lake surface where the wind velocity is comparable to that on the lake. There is considerable evidence to indicate that the effect of the wind is to cause a circulation and interchange of water between the upper and lower layers of the lake, and that there is some interchange most of the time. The effect of this interchange is to keep the surface temperature, which affects evaporation, cooler in spring and summer than would be the case in a shallow body of water; and, the water temperature lags considerably behind the air temperature in the spring and summer when the air temperature is rising. There is a similar lag in the water temperature in the fall and early winter when the air temperature is decreasing. As far as is known, none of the lakes freezes entirely over in the winter, although such a condition might be possible for a short time.

John R. Freeman, Past-President and Hon. M. Am. Soc. C. E., and others who have studied the subject have made attempts to deduce the lake evaporation from the standard evaporation formulas. Realizing that his results were not satisfactory, Mr. Freeman stated[7] that, "Evaporation is at least a matter of great scientific interest, and a precise knowledge of the causes leading to change in its amount could be of use * * *. The collection of accurate statistics can not be begun too soon." After considerable study, he recommended the following procedure for the collection of data:

(1) A few months of experimental work, with relatively inexpensive equipment, for obtaining accurate values of constants in the formulas, by means of which evaporation can be estimated from standard records of the U. S. Weather Bureau Stations, which give the temperature, dew point, and velocity movement of the air.

(2) At each lake, one, or preferably two, evaporation tanks, on opposite sides (one in Canada and one in the United States) each preferably 6 ft or 8 ft in diameter by 4 ft to 5 ft in depth, of rigid shape, set on a rigid support high enough above the lakes to escape waves, and located on some projecting pier or on the shore. This type of tank should have provision for warming the water in winter barely enough to keep the ice from freezing on the surface, should have a device for agitating the surface slightly, and an anemometer should be set beside it.

(3) One daily temperature measurement shown by the maximum-minimum thermometers should be made along with observations for all the meteorological factors affecting the evaporation.

(4) Comparison should be made between the wind velocities at the pan and those recorded at the regular Weather Bureau Station in the same vicinity.

Because present formulas were not satisfactory for the required determination, new experiments which were more or less in accord with Mr. Freeman's recommendations were undertaken.

---

[7] "Regulation of the Great Lakes," by J. R. Freeman, p. 421.

Description of Equipment

All stations were located near the lake shore away from any buildings or land obstructions which might create atmospheric disturbances. The design and arrangement of the equipment approximated that used by the Weather Bureau as nearly as possible in order to facilitate comparison of data. The dimensions of the pan and relative positions of the instruments were identical. However, the pan had several new features which heretofore had never been used.

Design of Pan.—In order to control and regulate the temperature of the water in the pan, heat had to be supplied in winter and refrigeration in summer. The original design, shown in Fig. 3, called for a water-jacket around the pan where warm or cool water could be circulated. This design was modified in two cases because of local conditions. At Duluth, enclosed electrical heating elements were submerged in the pan water; and at Detroit, electrical heating elements were placed in the de-watered jacket. The supply of heat was regulated by thermostatic control. All units were cooled in summer by circulating cold water around the tank. The supply of cooling water was regulated periodically, by hand, as the need arose. The source of supply was either cold well-water or water pumped from the lake at a depth of 20 ft or more. The pans were placed in large insulated boxes which aided in the regulation of the temperature.

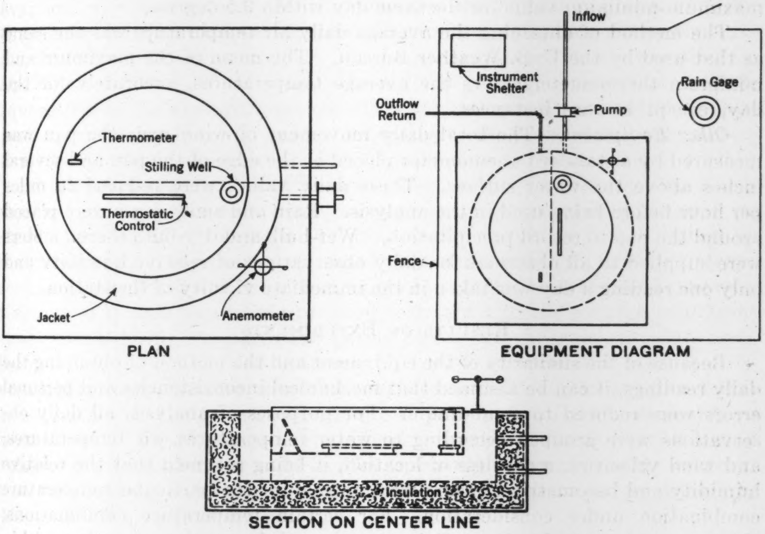

Thermometer
Stilling Well
Thermostatic Control
Jacket
Anemometer
PLAN
Inflow
Instrument Shelter
Rain Gage
Outflow Return
Pump
Fence
EQUIPMENT DIAGRAM
Insulation
SECTION ON CENTER LINE

Fig. 3.—Design of Detroit Evaporation Unit

It has been mentioned that an attempt was made to keep the temperature of the pan water as nearly as possible to mid-lake temperatures at all times.

Due to mechanical difficulties, the control was far from perfect; but the results were entirely satisfactory when analyzed by the method which was used and which does not require that the pan water be at the same temperature as the lake water.

Average Daily Temperature.—A maximum-minimum thermometer was placed in the pan water to obtain the mean daily temperature. It was assumed that for all practical purposes the mean of the maximum and the minimum water temperature for the 24-hr period represented the average temperature for the period. As a check on this assumption, observations every half hour for 24 consecutive hours were made on a number of different days. The thermometer was also read for each of these different days. The average of the 48 half-hour readings for each day, in all cases, checked the mean of the maximum-minimum value for the same day within 0.5 degree.

The method of obtaining the average daily air temperature was the same as that used by the U. S. Weather Bureau. The mean of the maximum and minimum thermometers gives the average temperatures, accurately for the day, except in rare instances.

Other Equipment.—The total daily movement of wind over the pan was measured by a standard anemometer placed at the edge of the pan and several inches above the water surface. These daily values were reduced to miles per hour before being used in the analysis. Rain and snow gages were placed around the pan to record precipitation. Wet-bulb and dry-bulb thermometers were supplied to all observers for daily observations of relative humidity and only one reading a day was taken in the immediate vicinity of the station.

Results of Experiments

Because of the similarity of the equipment and the method of obtaining the daily readings, it can be assumed that mechanical inconsistencies and personal errors were reduced to a minimum. For purposes of analysis, all daily observations were grouped according to water temperatures, air temperatures, and wind velocities, regardless of location, it being assumed that the relative humidity and barometric pressure were normal for the particular temperature combination under consideration. For certain temperature combinations, this may not be true; but if a sufficient number of observations are taken within any one range, the average relative humidity value for that range will approach normal. The accuracy of a statistical analysis of this nature depends on the number of observations.

Master Chart.—The daily readings, listed according to temperatures and wind velocities, were arranged again into groups with a water and air temperature range of 5 degrees. For purposes of illustration, one group is described briefly: 44 daily readings—11 from the Duluth station, 14 from Kewaunee, 10 from Detroit, and 9 from the Buffalo station—where the average daily air temperature of each reading was between 40° and 45° and where the average daily water temperature was between 45° and 50°, were included in this group. The centroid of these 44 readings was found and plotted, and the average of these readings was assigned this point. The average wind velocity of the group of readings was found to be 5 miles per hr. Approximately 300 daily readings from the four stations were grouped, averaged, and plotted in this manner. An additional 300 observations daily, scattered throughout the period of the experiments, were not used. These 300 readings represented days during which there was considerable rain or snow accompanied by high winds. Under these conditions, the evaporation pan did not collect the same amount of precipitation as the rain gage, and although individual readings of the evaporation pan "catch" were sometimes high and at other times low, it was decided not to include these irregular observations in the averages because of these discrepancies.

Some groups had as many as eighty individual readings and others had only five. The average wind velocities in the various groups or squares ranged from 4 to 6 miles per hr, with most of the squares averaging 5 miles per hr. Each individual square was studied in order to determine the effect of the wind on evaporation in that square. It was found that the increase in the amount, due to an increase in the wind velocity, did not vary in the same proportion throughout the range of the experiments. The proportion in each square was different and followed a well-defined plan. The value of average evaporation, in those squares where the average wind velocity was either greater or less than 5 miles per hr, were decreased or increased, according to the proportion for the respective square, to correspond to a value of evaporation based on a 5-mile wind.

Contours representing equal lines of evaporation were drawn. Fig. 4(*a*) is the final result of this work representing inches of evaporation per month instead of inches of evaporation per day. Daily values may be obtained by dividing by 30. The curves are based on a wind velocity, measured at one foot above the water surface, of 5 miles per hr. It has been assumed that relative humidity and barometric pressure correspond to the average conditions that existed during the particular period of the year in which the data were being obtained at the four stations. In order to determine whether discrepancies, due to location or methods, existed in the data, charts for each station were made. No irregularities in the slope or position of the contour lines were evident. Fig. 4(*b*) represents values of monthly evaporation with a wind velocity of 10 miles per hr, and Fig. 4(*c*) represents values of monthly evaporation with zero wind velocity according to the formula[8] proposed by Adolph F. Meyer, M. Am. Soc. C. E.

$$E = 30\,(V - v)(0.5 + 0.05\,W) \qquad (4)$$

Temperature Loop.—The range of the two master charts (Figs. 4(*a*) and 4(*b*)) is sufficient to cover average conditions on the Great Lakes, and monthly values of evaporation may be obtained directly from them if the respective mean monthly air and water temperature and wind velocity data are available. It is known that the air temperature for individual days drops down well below the low point of the charts. Should it be necessary to obtain values for extreme ranges in temperature or wind velocities, the curves may be extended; daily observations under extreme conditions have been plotted, and they checked the extended contours very well.

To obtain the monthly values of open-water evaporation for any lake, it is necessary to have accurate information as to the air temperatures over the lake,

---

[8] "Elements of Hydrology," by A. F. Meyer, John Wiley & Sons, Inc., New York, N. Y., 1917, p. 238.

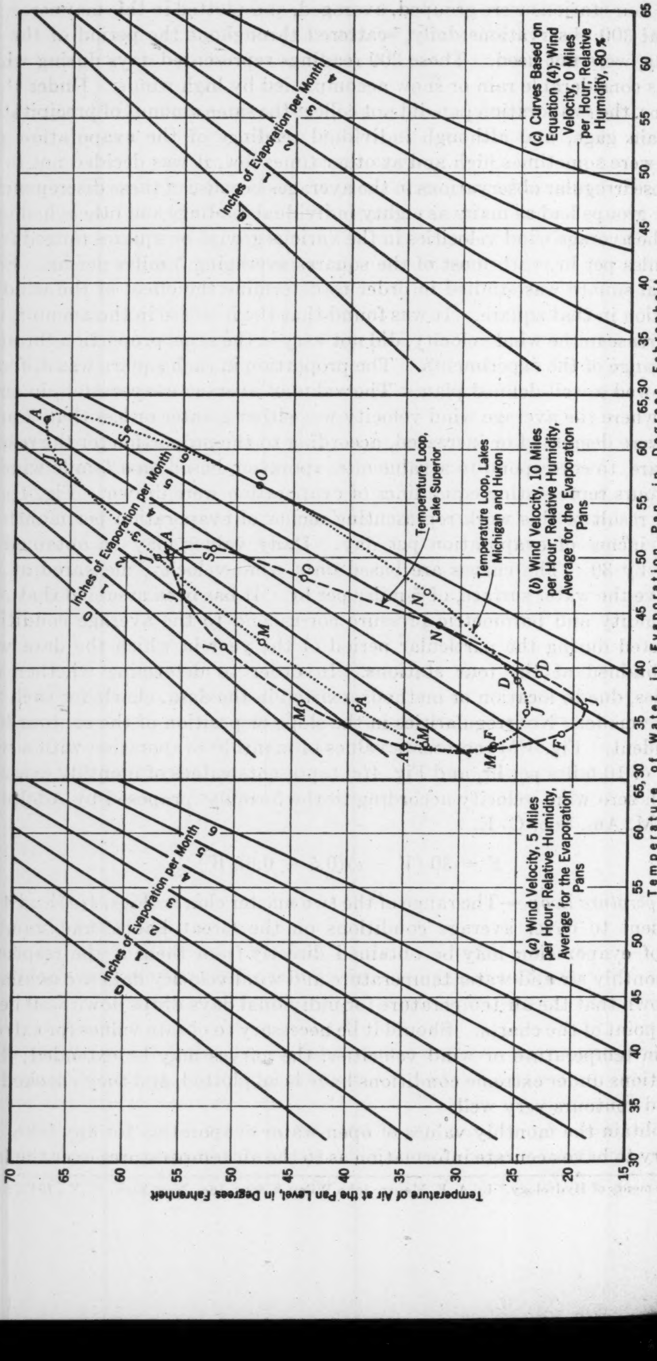


FIG. 4.—MONTHLY EVAPORATION CHARTS

the average water temperatures, and the average wind velocity over the lake. Considerable attention was given to these factors. Mean monthly air temperatures obtained from the records of the U. S. Weather Bureau Stations along the shore of the lakes do not represent mid-lake conditions. Readings observed out on the lake differ materially from those taken on shore because the air is influenced by the temperature of the water, both in summer and in winter.

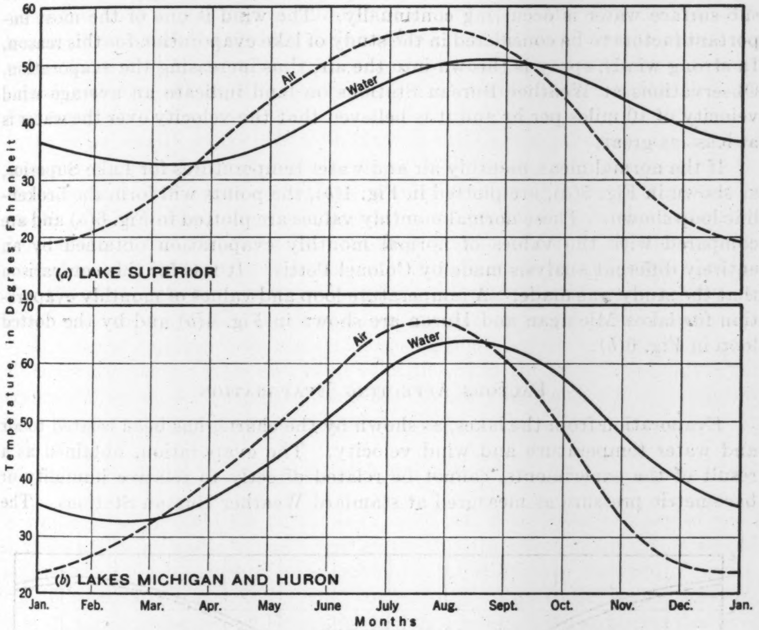


FIG. 5.—AVERAGE MEAN MONTHLY VALUES OF AIR AND WATER TEMPERATURES, WITH OPEN-WATER CONDITIONS

The mean monthly air temperatures as shown in Fig. 5 represent average values deduced from readings observed at the middle of the lake. Information pertaining to temperatures, both water and air, was obtained from the records of Lake Survey steamers, lake freighters, ferry boats, lighthouses, and municipal water-supply departments.

Seasonal air temperature variations in the lake region are considerable, and although these variations are somewhat decreased by the lakes, they do affect the temperature of the water. Large bodies of water, and particularly Lake Superior, react slowly to climatic variations in temperature. In fact, the yearly range of the average mean water temperature as shown in Fig. 5(*a*) is only 18 degrees. Due to the size and depth of the lakes, it can readily be seen that a large amount of heat is necessary to produce temperature changes in the water. The lake is never at the same temperature throughout, at any one time, and the temperature in the middle may be 20° lower than that recorded near the shore. No matter how the air temperature varies, and at what rate, the temperature of the surface water lags behind that of the air. The mean monthly values for lakes Superior, Michigan, and Huron, shown in Fig. 5, represent average open-water conditions.

The surface water is subject to frequent changes in temperature when there is no wind; but since this condition seldom exists, interchange of surface and sub-surface water is occurring continually. The wind is one of the most important factors to be considered in the study of lake evaporation for this reason. In strong winds, spray is thrown into the air, thus increasing the evaporation. Observations at Weather Bureau Stations on land indicate an average wind velocity of 10 miles per hr and it is believed that the velocity over the water is at least as great.

If the normal mean monthly air and water temperatures for Lake Superior, as shown in Fig. 5(*a*), are plotted in Fig. 4(*b*), the points will form the broken-line loop shown. These normal monthly values are plotted in Fig. 6(*a*) and are compared with the values of normal monthly evaporation obtained by an entirely different analysis made by Colonel Pettis. It was for this comparison that the study was made. A temperature loop and values of monthly evaporation for lakes Michigan and Huron are shown in Fig. 4(*b*) and by the dotted loop in Fig. 6(*b*).

FACTORS AFFECTING EVAPORATION

Evaporation from the lakes, as shown by the charts, has been related to air and water temperature and wind velocity. The evaporation, obtained as a result of the experiments, cannot be related directly to relative humidity or barometric pressure as measured at standard Weather Bureau Stations. The

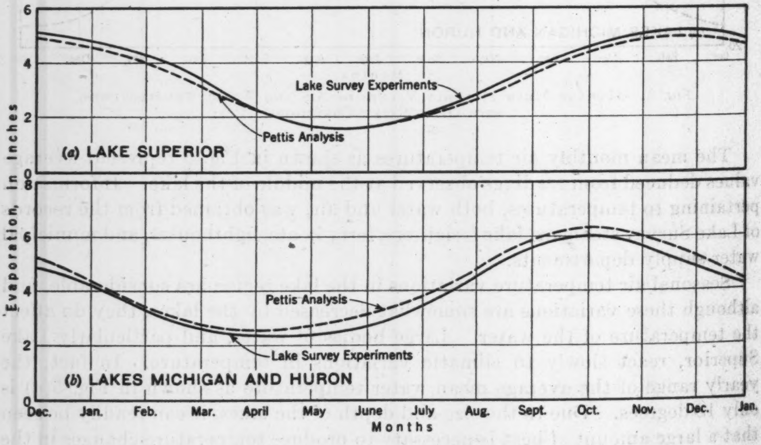


FIG. 6.—EVAPORATION FROM THE LAKE SURFACE

temperature of the air and water influencing evaporation, and likewise the velocity of the wind, may be used with considerable accuracy, but no correlation could be found between evaporation and relative humidity. No evidence could be found that the amount of evaporation varied with either the relative humidity as read at the station or as determined at neighboring Weather Bureau Stations. Although both relative humidity and barometric pressure have not been related directly to evaporation, they have been considered. It is logical to assume that if a sufficient number of observations are made in the same locality, over a sufficiently long period of time, using a particular air-and-water temperature combination, the resulting average relative humidity and barometric pressure will be characteristic for that temperature combination, for that locality.

In other words, evaporation for any month, say September, as obtained from Fig. 4(*b*), is the most probable value of the evaporation for that month, under normal or average conditions of relative humidity for that particular month. This is true since the evaporation curves for this particular locality were obtained from numerous daily readings of evaporation, taken under conditions of relative humidity which existed in this region, in this particular season. Fig. 4(*b*) thus gives the results in the most desirable form to use in any study, either general or specific, which involves average or probable conditions.

In order to emphasize the reason for basing this analysis on the three factors, air and water temperature and wind velocities, it might be well to analyze the ordinary evaporation formula that has been in use for some time.[9] Considering the equation as a modification of Dalton's Law

$$E = K\,(V - v)\left(1 + \frac{w}{c}\right) \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

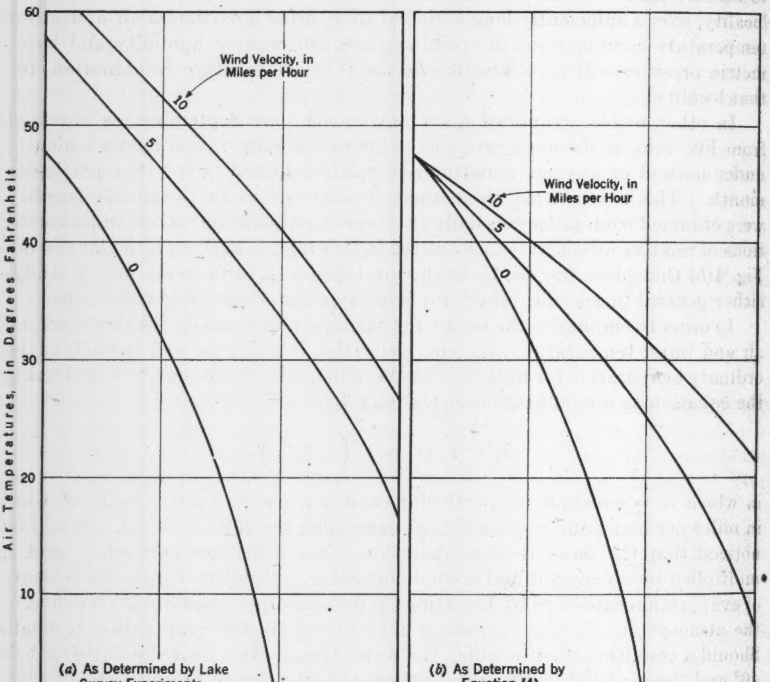
in which K = constant for particular conditions; w = mean velocity of wind in miles per hour; and c = coefficient expressing the effect of wind. It will be noticed that the measure of evaporation is the difference between V and v, multiplied by a constant and a wind correction. In other words, the measure of evaporation expressed by Equation (5) depends upon the saturation deficit of the atmosphere. If the saturation deficit is small, the evaporation is small. Should a condition exist in which the water temperature on Lake Superior was 40° and the air 42.4°, with a wind velocity of 10 miles an hr and a relative humidity of 80%, as measured by the Weather Bureau, a condition quite common for this lake, the evaporation for the day would be zero, according to Equation (5). The same temperature and wind combination on the master chart (Fig. 4(*b*)) would give 0.09 in. of evaporation for the day.

In order to use Equation (5), the temperature of the water and air must be known. After reducing the air temperature to the dew-point temperature, dependent on the relative humidity, each is converted into pressures. It is assumed that the moisture in the air influences the rate of evaporation. This cannot be denied, but experiments indicate that under the influence of wind, evaporation occurs consistently, although Equation (5), used with the proper climatic factor, indicates that the evaporation would be zero. Evaporation increases relative humidity, and it is believed that relative humidity, as indicated by the experiments, is a result of evaporation more than a cause of evaporation.

---

[9] "Regulation of the Great Lakes," by J. R. Freeman, 1926 Edition, p. 135.

Experiments conducted in different parts of the United States by the U. S. Weather Bureau show evidence that a humidity gradient exists, varying inversely as the elevation above the earth's surface. Even if the relative humidity is 100% at the water surface, there is atmosphere directly above where the percentage is less, and there is a continual escape of vapor upwards. It seems logical to assume, for the purposes of this presentation, that evaporation is dependent on temperatures and wind velocities, and that relative humidity on the Great Lakes tends to follow the seasonal averages.



(*a*) As Determined by Lake Survey Experiments (*b*) As Determined by Equation (4)

FIG. 7.—VALUES OF MONTHLY EVAPORATION FOR A CONSTANT WATER TEMPERATURE OF 45 DEGREES FAHRENHEIT

If the wind factor in Equation (5) is considered, it will be noticed at once that E approaches zero as the difference between the vapor tension and the vapor pressure becomes less. When the values of V and v are equal, evaporation ceases, regardless of the wind velocity. Experiments described in this paper show that this is not true. The air temperature is higher for zero evaporation, assuming a constant water temperature, as the wind velocity is increased (see Fig. 7), and similarly the water temperature is lower for zero evaporation, assuming a constant air temperature, as the wind velocity is increased. According to Mr. Meyer,[8] an increase in the wind velocity from zero miles per hr to 10 miles per hr, assuming that other factors in Equation (4) remain constant, doubles the evaporation. With a constant water temperature of 45° F and an air temperature of 32° F, the monthly value of evaporation increases from 3 in. to 4 in., as the wind velocities increase from 5 to 10 miles per hr, relative humidity remaining constant. Similarly at 25° F, the monthly value of evaporation increases from 4 in. to 5.3 in., or an increase in both cases of 33% (see Fig. 7(*b*)). If the same temperature combinations are used with Fig. 7(*a*), it will be noticed that the increase of monthly evaporation, due to an increase in the wind velocity from 5 to 10 miles per hr, for an air temperature of 32° F, will be from 4.2 in. to 5.3 in., or an increase of 26%, whereas the increase at an air temperature of 25° F, for the same conditions, will be from 5.0 in. to 6.1 in., or an average increase of 22 per cent.

At 48° F, the increase, due to an increase in the wind velocity from 5 to 10 miles per hr, according to Fig. 7(*b*), is zero. At this temperature, Fig. 7(*a*) gives an increase in the values of monthly evaporation, with an increase in wind velocity from 5 to 10 miles per hr, of 80 per cent. The wind has been found to have a decided effect on evaporation, particularly when the air and water temperatures are nearly equal. The comparison with the Meyer formula is as accurate as could be made with the data available. The evidence indicates that current opinion as to the effect of wind on evaporation is in need of revision.

With present-day mechanical equipment, much could be learned about the laws of evaporation if tests could be made in a wind tunnel under laboratory conditions. With little effort and expense, all factors governing evaporation could be controlled, each could be isolated and its effect determined.

ACKNOWLEDGMENTS

The data from which the air and water temperature curves were prepared were obtained from the files of the U. S. Lake Survey Office and from information personally obtained by the writer from a number of different sources. Copies of the monthly evaporation reports are on file at the various sectional headquarters of the U. S. Weather Bureau. Mr. Freeman's report[7] on "The Hydrology of the Great Lakes" was consulted frequently.

The data form a part of a thesis[10] by the writer, entitled "Evaporation from the Great Lakes," presented to Lawrence Institute of Technology, Highland Park, Mich., in 1938, in partial fulfillment of the requirements for the degree of Doctor of Engineering. The writer wishes to acknowledge the invaluable assistance of Colonel Pettis and Mr. Sherman Moore, of the Lake Survey, Professors H. L. Woolfenden and G. P. Brewington of Lawrence Institute of Technology; Professor C. W. Chamberlin of Michigan State College, at East Lansing, Mich.; and valuable constructive criticism of Professors C. O. Wisler and W. C. Hoad of the University of Michigan, at Ann Arbor.

---

[10] A copy of the thesis, containing all observational data, has been filed for reference in the Engineering Societies Library, 33 W. 39th Street, New York, N. Y.

DISCUSSION

A. A. Young,[11] Assoc. M. Am. Soc. C. E. (by letter).—The Symposium on Hydrology of the Great Lakes is an excellent discussion of evaporation from large bodies of water under unusual conditions. The quantitative analysis so closely supported by experimental data is proof of the value of the methods involved. An interesting feature is the shape of the evaporation curve which, contrary to evaporation losses from smaller lakes, shows higher rates of loss in winter than in summer.

This agrees with the fundamental law developed by Dalton (see Equation (5)) which, in brief, states that if all other factors are constant the rate of evaporation is nearly proportional to differences in vapor pressure at the temperature of the water surface and vapor pressure of saturated air at the temperature of the dew-point. When water is warmer than air at less than the saturation point, the difference in vapor pressure is favorable for evaporation, which accounts for the greater losses during winter months. Negative vapor pressures may cause moisture held in the air to return to the water surface although the actual contribution will probably be negligible.

The same conditions were observed by the writer in connection with evaporation studies of the Division of Irrigation of the United States Department of Agriculture in a small covered reservoir in southern California to which relatively warm water was pumped from a near-by well. A narrow belt of screen extending entirely around the superstructure of the reservoir permitted some degree of ventilation but prevented marked air currents at the water surface. Average temperatures of water and of air at the water surface were almost exactly the same during August, with the result that evaporation was only 0.9 in. for the month. Contrasting with this was a January loss of 2.50 in. under conditions for which the air temperature was 8° less than water temperature. The monthly evaporation curve for the reservoir was an approximate sine curve only from June to November; the next 6 months plotted as an approximate straight line.

Fig. 1(*e*) in the paper by Colonel Pettis shows permanent land losses of 5.9 in. annually, with a maximum monthly loss of 0.9 in. in August. Definition of land losses is given as "the difference between the rainfall B and the surface and underground flow into the lake * * *," with B indicating precipitation on tributary land areas. Land losses thus include transpiration from all classes of vegetation plus evaporation from moist surfaces.

It would be interesting to know whether the value given to land losses is supported by results of investigations of consumptive use of water by native vegetation and agricultural crops for the area involved. The writer has no data for comparison except estimates by A. F. Meyer[12] of 4 to 12 in. of normal seasonal transpiration for such growth as coniferous and deciduous trees, brush, grains, grass, and agricultural crops. However, investigations of the effect of drought on farm-shelter belts and tree plantations of the northern prairie region in Minnesota, to determine the survival of the principal trees native to the region, following the severe drought of 1934, give an indication of moisture requirements of several tree species.[13] Climatic differences between Minnesota and the Lakes region would appear to be confined principally to differences in humidity which may be favorable to greater consumptive use in the prairie district.

---

[11] Associate Irrig. Engr., Div. of Irrig., Bureau of Agri. Eng., U. S. Dept. of Agriculture, Pomona, Calif.

[12] "Elements of Hydrology," by A. F. Meyer, John Wiley & Sons, Inc., New York, N. Y., 1917.

Conditions leading up to the drought of 1934 were in themselves unfavorable to soil moisture. Tree plantings were in relatively good condition prior to 1930. From this time on, subnormal precipitation, coupled with depleted soil moisture, induced severe losses for all tree species. During this period a large percentage of the tree population died for lack of moisture. Much of the area is on the critical line of 20 in. of rainfall. When less rain occurred, tree losses ranged from 22% to 73% of the total number of trees investigated. From this it appears that the minimum water requirement of tree growth on the Minnesota prairies is not far from 20 in. annually. This is considerably different from the land losses for the Lakes area and, despite the difference in localities, leads to some speculation as to whether 5.9 in. annually is the total loss.

Evaporation experiments made by Mr. Hickman as a basis for estimating actual losses from the Great Lakes involved much conscientious work. The evaporation pans were set in water baths in which attempts were made to maintain water temperatures the same as mid-lake temperatures. The close comparison of the pan experiments and the Pettis analysis is remarkable when the difficulties inherent to each method are considered.

Some of the drawbacks of the Weather Bureau type of pan used were overcome largely by the temperature control. The principal objection to the Weather Bureau pan lies in rapid changes of the water temperatures, which follow air temperatures more closely than in deeper pans set in the ground. The principal advantage of the Weather Bureau pan is in the opportunity of comparing the long series of records obtained from it under a variety of climatic conditions. Records from the pan used in the experiments cannot be so compared because, in effect, it is a floating, rather than a land pan. Records[14] show the relation of evaporation from small reservoirs or large tanks to be about 0.70 of the Weather Bureau pan loss. Under conditions of temperature control, and with the pan surrounded by water, this value would increase and probably approach unity. It would be a matter of interest if Mr. Hickman had stated whether actual pan evaporation was assigned as direct lake evaporation without use of a conversion coefficient.

Since March, 1936, the Division of Irrigation, Bureau of Agricultural Engineering, U. S. Department of Agriculture, has been experimenting with a specially designed ground pan, 2 ft in diameter, with the object of reducing evaporation losses to a point that will not require use of a coefficient. A new principle is introduced in evaporation experiments. By using a screen to intercept a portion of the rays of the sun, comparisons have been made, since 1936, with other pans of which the most important are the Weather Bureau pan and a ground tank, 12 ft in diameter. Evaporation from the screen-covered pan for the 3 yr of record averaged 80% of that from the Weather Bureau pan and 103% of the 12-ft tank loss. The close agreement between losses from the screen-covered pan and the 12-ft tank demonstrates its value as representing lake or reservoir evaporation more closely than any other known type of small evaporation pan. It has been tested variously from coastal to desert climates with close agreement in results under different climatic conditions. It is inexpensive and easily transported and installed. It also has the advantage of using less water than is required ordinarily, and consequently does not require such frequent attention as other pans.

---

[13] "Drouth Damage to Prairie Shelterbelts in Minnesota," by M. E. Deters and Henry Schmitz, *Bulletin No. 329*, Univ. of Minnesota Agr. Experiment Station, 1936.

[14] "Evaporation from Different Types of Pans," by Carl Rohwer, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 673.

Mr. Hickman's statement (see heading, "Results of Experiments: Temperature Loop") that: "The lake is never at the same temperature throughout, at any one time, and the temperature in the middle may be 20° lower than that recorded near the shore," calls attention to a condition that perhaps has not been given due consideration heretofore. In estimating lake evaporation through use of a conversion coefficient no attention has been paid to the relation of the water surface in the pan to that in the lake. Contrary to lake surfaces, water surfaces in pans have approximately the same temperature for each unit of water area. Pan surfaces are not greatly disturbed by wind, and losses are not increased by waves throwing spray into the air. This all reduces to the fact that evaporation from pans is practically uniform over the pan area, whereas evaporation from lakes is influenced by many factors that have been given little attention in the past and about which relatively little is known.

For some time the writer has been considering factors that influence evaporation from reservoir surfaces. In the West, water is at a premium and its value is increasing. There are many evaporation pan records but few tests of the value of such records when applied to large water surfaces. Through the courtesy of the Water Department of the City of Pasadena, Calif., a preliminary investigation was undertaken by the Division of Irrigation, on Morris reservoir, to determine evaporative influences for different parts of the water surface. Instruments placed upon barges recorded air temperature, humidity, and wind movement for different sections of the reservoir surface. Evaporation was measured from a small pan on each barge, and lake temperatures were taken. The results indicated wide differences in evaporation and wind movement and significant differences in temperature and humidity. They were thought to be of sufficient value to justify a more extended study over a much longer period to determine further the distribution of those factors affecting evaporation from the larger water surfaces.

Morris reservoir is a narrow, irregularly shaped body of water, surrounded by abruptly rising mountains that influence wind movement and direction, a condition that also influences temperature and humidity. It is representative of many western mountain reservoirs which are different in shape from those in more open country. There is little doubt that size, shape, depth, and surroundings affect evaporation from such water bodies to a considerable degree, and that such factors at present are not included when reservoir losses are estimated.

ALFRED J. COOPER, JR.,[15] JUN. AM. SOC. C. E. (by letter).—Mr. Hickman's paper is an interesting treatise on the subject, and it is with some reluctance that an attempt is made to add to, or detract from, the value of his conclusions. However, the impression is given that some of his statements are based either upon conditions which are peculiar to the Great Lakes region where the experiments were conducted, or upon insufficient data of relative humidity. The purpose of this discussion is to show that, in the Tennessee River basin, there is a marked variation of relative humidity with evaporation (or *vice versa*), as well as the variation of evaporation with wind velocity.

The Hydraulic Data Division of the Tennessee Valley Authority has collected daily evaporation data at four stations in the Tennessee River basin from the fall of 1934 to the present date, 1939. These stations contain equipment in accordance with U. S. Weather Bureau Class "A" evaporation station specifications. A brief description of the two stations from which the data used in the discussion were obtained follows:

Norris, Tenn.—The station is situated at an elevation of 1 000 ft above mean sea level, approximately 20 miles, air-line, northwest of Knoxville, Tenn. It is about 4 000 ft south of the Norris Dam and Reservoir but only 500 ft south of the Clinch River. Observations were started on October 23, 1934, and additional instruments, other than standard equipment, were installed subsequently. These additions are: A weighing evaporation pan, two sets of maximum and minimum water thermometers (one set for each pan), a recording hygrothermograph, a water-temperature recorder (in the standard pan), a recording anemometer, and a barograph.

Murphy, N. C.—The station is approximately 70 miles east of Chattanooga, Tenn., in the town of Murphy, N. C., at an elevation of 1 575 ft above mean sea level. It is about 700 ft north of the Hiwassee River, and at the extreme upper end of the pool to be formed behind the Hiwassee Dam. Observations were begun on November 17, 1934, and since that date the following instruments were added: A recording hygrothermograph, maximum and minimum water thermometers, and a recording rain gage.

The other two stations are at Beetree Dam, N. C., and Savannah, Tenn. Prior to January, 1939, the latter station was situated at Pickwick Landing Dam, Tenn. Data from these stations are in accord with the trend of Fig. 8(*a*) but are not plotted in detail on that diagram.

To confine the study, the range of air temperature was selected arbitrarily as 40° to 46° F, and that of the water temperature as 44 to 50 degrees. A wide variation in relative humidity over this range was found, and a total of 78 points was used. Data of days on which rain and snow occurred were not used, in order to eliminate any possible errors due to differences of rainfall catch between the evaporation pan and the rain gage.

In the Tennessee Valley experiments, wind velocities are measured by an anemometer beside the pan, with the cups about 1 ft above the water surface, in accordance with U. S. Weather Bureau recommendations. As a result, these velocities are smaller than those at the same instant several feet above the ground. In the particular data used, the maximum average daily ground velocity of the wind was 5.01 miles per hr.

---

[15] Hydraulic Engr., TVA, Knoxville, Tenn.

The integrated averages of the daily air temperatures and relative humidities were determined from the hygrothermograph charts. At the Norris station, the integrated daily averages of the water-temperature recorder charts were used, but it was necessary to utilize the daily average of the maximum and minimum water thermometers at the Murphy station. It is believed that no correlation of evaporation and relative humidity is possible when only one daily observation is made of relative humidity unless the relative-humidity curve for the entire day has little or no variation. For any other condition it is necessary to obtain a true daily average by frequent psychrometer or wet-bulb and dry-bulb thermometer readings over 24-hr periods or from a relative-humidity recorder.

Mr. Hickman states (see heading, "Factors Affecting Evaporation"): "The evaporation, obtained as a result of the experiments, cannot be related directly to relative humidity or barometric pressure as measured at standard Weather Bureau Stations," and "No evidence could be found that the amount of evaporation varied with either the relative humidity as read at the station or as determined at neighboring Weather Bureau Stations." The latter statement may be true if it is interpreted exactly as it is written because the average daily relative humidity is obtained only by many readings throughout the 24 hr—not by one or two readings. In the Tennessee River basin it is not uncommon for relative humidities to vary from a minimum of 25% to a maximum of 99% or 100% within 24 hr. With such a variation an observation of relative humidity taken at any time of the day could not represent a 24-hr average, except by chance.

A plot was made of daily evaporation *versus* average daily relative humidity for the data selected. Fig. 8(*a*) shows the points as well as the most likely straight line to indicate the trend. The average wind velocity, in miles per hour, is noted beside each point. For large rates of evaporation there are correspondingly large wind velocities; but for small rates there is a heterogeneous mixture of large and small velocities, in many of which the large velocities approach or exceed those in the higher evaporation "region" of the diagram. It is apparent from Fig. 8(*a*) that evaporation does depend definitely upon relative humidity. The trend toward decrease in evaporation with increase in humidity is clearly shown by the slope of the trend line. It is not intended that this line be construed as representing the equation of evaporation, but rather that it be considered as showing merely the distinct trend of the plotted points. Were there no connection between evaporation and relative humidity, such a scattering of the points would be anticipated that the eye could not envision any trend. For a given evaporation the maximum variation in humidity from the line is 20%, and the average variation of all the points from the line is only 5 per cent. It must be remembered that the other factors influencing evaporation are concurrently active in these field observations and affect the foregoing relationship.

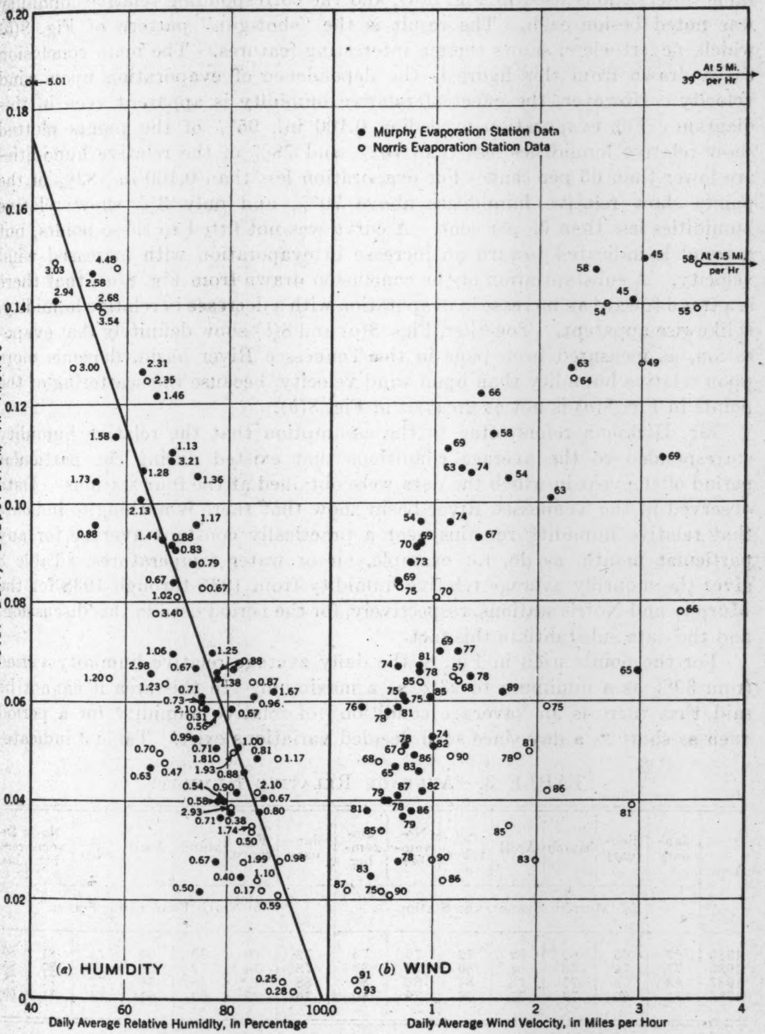


Fig. 8.—Variation of Evaporation with Relative Humidity and Wind

Another plot was made of daily evaporation *versus* wind velocity for the same observations used in Fig. 8(*a*), and the corresponding relative humidity was noted beside each. The result is the "shot-gun" pattern of Fig. 8(*b*) which, nevertheless, shows certain interesting features. The main conclusion to be drawn from this figure is the dependence of evaporation upon wind velocity. However, the effect of relative humidity is apparent even in this diagram. For evaporation exceeding 0.100 in., 95% of the points plotted show relative humidities less than 70%, and 78% of the relative humidities are lower than 65 per cent. For evaporation less than 0.100 in., 82% of the points show relative humidities above 70%, and only 3% show relative humidities less than 65 per cent. A curve was not fitted to these points, but a trend is indicated toward an increase in evaporation with increased wind velocity. A substantiation of the conclusion drawn from Fig. 8(*a*), that there is a trend toward an increase in evaporation with a decrease in relative humidity, is likewise apparent. Together, Figs. 8(*a*) and 8(*b*) show definitely that evaporation, as measured from pans in the Tennessee River basin, depends more upon relative humidity than upon wind velocity, because the scattering of the points in Fig. 8(*a*) is not as great as in Fig. 8(*b*).

Mr. Hickman refers often to the assumption that the relative humidity corresponded to the average conditions that existed during the particular period of the year in which the data were obtained at the four stations. Data observed in the Tennessee River basin show that there is nothing to indicate that relative humidity remains near a practically constant average for any particular month, as do, for example, air or water temperatures. Table 3 gives the monthly average relative humidity from 1935 through 1938 for the Murphy and Norris stations, respectively, for the period used in this discussion, and the data substantiate this fact.

For the points used in Fig. 8, the daily average relative humidity varied from 39% as a minimum to 93% as a maximum. In this area it cannot be said that there is an "average condition" of relative humidity for a period even as short as a day since such decided variations exist. Table 3 indicates such changes from year to year in monthly average relative humidity that it is difficult to consider an average value to be used with measurements of evaporation in a study of field conditions. One wonders whether relative humidity is not one quantity of the atmosphere of which the seasonal trend from year

TABLE 3.—AVERAGE RELATIVE HUMIDITY

Year	January	February	March	April	October	November	December	January	February	March	April	October	November	December
	(*a*) Murphy Evaporation Station							(*b*) Norris Evaporation Station						
1935	67	65	61	62	72	76	73	72	69	69	66	71	81	80
1936	77	74	68	61	86	79	83	78	74	72	68	90	80	85
1937	83	72	71	73	84	80	88	88	..*	56	56	72	71	78
1938	81	80	77	77	76	79	80	76	77	69	61	64	71	69

* Report not complete.

to year is erratic and unpredictable, in so far as estimating closely what can be expected for a certain month or season. This variation is dependent largely upon the frequency of rainfall, wind conditions, and air temperatures. These factors then influence the rate of evaporation through the relative humidity they produce.

No definite relationship was found from the barograph charts of the Norris station which would show a correlation between evaporation and barometric pressure. However, daily pressure variations at this station are small—rarely more than 0.5 in. of mercury, which is less than 2% of the mean barometer reading. Results of the evaporation experiments at Fort Collins, Colo.,[16] indicate an approximate increase of 2% in evaporation for each inch of decrease in barometric pressure. Since the maximum variation in pressure at the Norris station is only 0.5 in., the maximum variation that could be attributed to barometric pressure change is about 1 per cent. This variation would be so small that it could not be detected in an observation involving such small magnitudes.

In conclusion it may be stated that the evidence presented indicates that evaporation is influenced by both relative humidity and wind velocity, but with the greater effect being due to relative humidity. It is believed that Mr. Hickman is correct in suggesting that current opinion as to the effect of wind on evaporation is in need of revision—but this revision should be toward giving less weight to its effect and more to that of relative humidity.

ADOLPH F. MEYER,[17] M. AM. SOC. C. E. (by letter).—The subject matter of this Symposium is of recognized importance. There is great need for observations of evaporation from cold water during the summer and from warm water during the winter. The following discussion relates to the Duluth evaporation records and to evaporation from Lake Superior, without reference to conditions on the remainder of the Great Lakes.

In his paper, Mr. Hickman states (see "Synopsis") that "an effort was made to keep the temperature of the water in the pans approximately the same as the temperature of the water in the open lakes"; and (see "Description of Equipment: Design of Pan") that "Due to mechanical difficulties, the control was far from perfect; but the results were entirely satisfactory when analyzed by the method which was used and which does not require that the pan water be at the same temperature as the lake water." It may be quite true that the pan water need not be at the same temperature as the lake water at any given time, but the range of evaporation observations should certainly cover the general range of conditions prevailing on Lake Superior.

An examination of the records filed in the Engineering Societies Library[10] reveals the fact that, at the Duluth station, only about 75 days' records are available which are reasonably applicable to fall and winter evaporation from Lake Superior, including the months of September to February, inclusive. The arrangement for heating the evaporation pan during the winter apparently functioned satisfactorily. In general, the results secured indicate high winter evaporation of approximately the same magnitude determined by the late John R. Freeman,[9] Past-President and Hon. M. Am. Soc. C. E., through the use of the evaporation formula (Equation (5)), utilizing the writer's wind factor.[8]

---

[16] "Evaporation from Free Water Surfaces," by Carl Rohwer, Assoc. M. Am. Soc. C. E., *Technical Bulletin* No. 271, U. S. Department of Agriculture, December, 1931.

[17] Cons. Hydr. Engr., Minneapolis, Minn.

The situation respecting summer evaporation from Lake Superior is quite another matter. No records whatsoever are on file which even reasonably simulate the conditions prevailing on Lake Superior from March to August. At this time, the Lake Superior water is cold and the air is warm.

In Table 4 the lake temperature given in Fig. 5 is compared with the actual recorded temperature of the water in the evaporation pan.

TABLE 4.—COMPARISON OF OPEN-LAKE TEMPERATURES

Month	MEAN WATER TEMPERATURE		Difference, in degrees Fahrenheit	Month	MEAN WATER TEMPERATURE		Difference, in degrees Fahrenheit
	Open Lake	Evaporation Pan			Open Lake	Evaporation Pan	
April	33	41.8	8.8	July	47	68.1	21.1
May	37	53.5	16.5	August	50	69.3	19.3
June	42	62.9	20.9	September	51	57.6	6.6

Observations of pan evaporation, with water temperatures ranging 20° higher in the pan than in the open lake, can scarcely be accepted as a measure of summer evaporation from Lake Superior. This phase of the subject is of the utmost importance, since the main difference between the conclusions of both papers and those of some of the earlier investigators results from differences in summer evaporation, and the assumption of summer underground flow and evaporation being somewhere near equal. This also has a bearing on the conclusion that land losses are only 5 in. or 6 in. a year, as against the normally assumed losses of from 14 in. to 22 in., depending upon precipitation and temperature.

In the paragraph preceding Equation (4), Mr. Hickman states, "In order to determine whether discrepancies, due to location or methods, existed in the data, charts for each station were made. No irregularities in the slope or position of the contour lines were evident."

The writer plotted on a large-scale chart, corresponding to an enlargement of Fig. 4(*a*), all the available daily records of evaporation for Duluth when the wind velocity was between 2.5 and 7.5 miles per hr at the pan level; and there was either no precipitation or less than 0.02 in. The average wind velocity for the plotted points was found to be substantially 4 miles per hr.

The writer then drew on this chart the evaporation lines shown on Mr. Hickman's master chart, Fig. 4(*a*). It appears that about one-third of the daily evaporation observations at the Duluth station plot outside of the limits of Mr. Hickman's master chart. No data whatsoever are available for the Duluth station to form the basis for either the zero or the 1-in. line shown in Fig. 4(*a*). Only a few points are available for the 2-in. line.

A study was made of the 4-in.-per-month evaporation line along which the greater number of the daily evaporation points fell. All of the points lying in a belt extending from half-way between the 3-in. and 4-in. lines to half-way between the 4-in. and 5-in.-per-month evaporation lines were assumed to control the 4-in. line. The average of these points was found to be 0.123 in., or 3.7 in. of evaporation per month. The points ranged from 0.04 in. on March 31 (representing 1.2 in. monthly evaporation, with a southwest wind of 3.0 miles per hr and 92% relative humidity at the U. S. Weather Bureau Station) to 0.28 in. on May 17 (representing 8.4 in. monthly evaporation, with a northwest wind of 7.0 miles per hr and 49% relative humidity).

The second most erratic points ranged from 0.05 in. on March 15 (representing 1.5 in. monthly evaporation, with a northwest wind of 3.6 miles per hr and 72% relative humidity at the Weather Bureau Station) to 0.22 in. on March 9 (representing 6.6 in. monthly evaporation, with a southeast wind of 3.0 miles per hr and 53% relative humidity at the Weather Bureau Station). Another typical low reading was 0.09 in. on April 14 with a northeast wind at 3.9 miles per hr and 95% relative humidity at the Weather Bureau Station. Surely these records indicate the effect of large differences in relative humidity in so far as they reflect changes in vapor pressure in the air.

The points controlling the upper and the lower portions of the 4-in. line were considered separately. The average of those points controlling the upper half of the curve represented 5.0 in. of evaporation per month, ranging from 2.4 to 8.4 in. per month. The average of the points controlling the lower half of the 4-in. line represented only 3.2 in. per month, varying from 1.2 in. to 5.4 in. per month. The lower half represented conditions prevailing during the part of the year when the relative humidity was high.

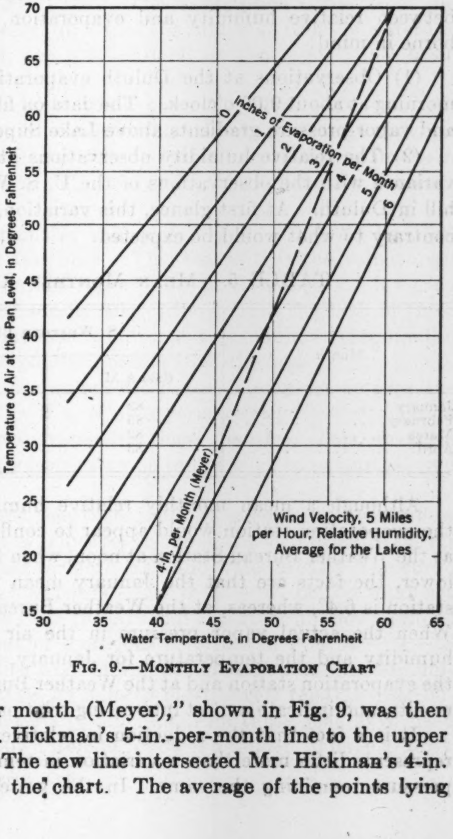


Fig. 9.—Monthly Evaporation Chart

A trial line marked "4 in. per month (Meyer)," shown in Fig. 9, was then drawn from the lower end of Mr. Hickman's 5-in.-per-month line to the upper end of his 2-in.-per-month line. The new line intersected Mr. Hickman's 4-in. line practically in the center of the chart. The average of the points lying within a belt of the same width as that previously used on the author's 4-in. line gave an average of 0.138, or 4.1 in. of monthly evaporation. This is a better average than the author's 3.7 in. for the 4-in. line. Furthermore, the points controlling the upper half of the writer's line averaged 4.5 in. per month, and those controlling the lower half of the line averaged 3.9 in.

A second trial 4-in. line was drawn, starting at the same point at the bottom of the chart, but ending at the 1-in. line instead of the 2-in. line. The average of the points controlling the upper part of this line represented 3.5 in. of monthly evaporation, whereas those controlling the lower part represented an average of 3.9 in. of monthly evaporation. Evidently, if the 4-in. line started at Mr. Hickman's 5-in. line at the bottom of the chart and ended at his 1.5-in. line at the top of the chart, it would probably come nearest to conforming to the data for the Lake Superior observation station. The fact remains, however, that the daily readings are so erratic as to constitute an unsatisfactory basis for any conclusion.

In connection with Mr. Hickman's statement that no relation was found between relative humidity and evaporation, the following facts should be borne in mind:

(1) Observations at the Duluth evaporation pan were made only in the morning at about 9:00 o'clock. The data on file do not cover relative humidity and vapor-pressure gradients above Lake Superior.

(2) The relative humidity observations at the evaporation station are at variance with the observations of the U. S. Weather Bureau situated on the hill in Duluth. At first glance, this variation, illustrated by Table 5, appears contrary to what would be expected.

TABLE 5.—Mean Monthly Relative Humidity

Month	U. S. Weather Bureau		Evaporation Station; 9:00 A.M.
	6:30 A.M.	12:30 P.M.	
January	82	73	63
February	85	70	60
March	82	60	58
April	83	72	64

Although a mean monthly relative humidity of 63% at 9:00 A.M. at the evaporation station would appear to conflict with a reading of 73% made at the Weather Bureau Station at noon, when the relative humidity is normally lower, the facts are that the January mean temperature at the evaporation station is 6.4°, whereas, at the Weather Bureau Station, it is only 3.3 degrees. When the actual vapor pressure in the air is computed from the relative humidity and the temperature for January, it appears that the readings at the evaporation station and at the Weather Bureau Station give nearly identical results, and indicate a most interesting relationship worthy of further study.

It is a fact that the reduction in relative humidity during the day often represents little more than a variation in temperature, with the actual vapor pressure remaining the same. In the writer's judgment, temperature and relative humidity, considered together, as they are in the Meyer evaporation formula (Equation (5)), are a much better indication of the actual vapor pressure in the air which controls evaporation than temperature alone.

S. T. HARDING,[18] M. AM. SOC. C. E. (by letter).—The two papers in this Symposium represent a hydrologic balance sheet for the Great Lakes. The method used is illustrated by its application to Lake Superior. The resulting values found for the individual items in such a balance have values which differ for some items from those more generally considered applicable to similar conditions. The authors' results can be reviewed both from the point of view of their detail procedure and from general comparisons of their results.

As Colonel Pettis states, a rigid solution is impossible where there are three unknowns and only two equations. He meets this condition by means of assumptions which provide the equivalent of a third equation. The correctness of his conclusions thus depends on the correctness of these assumptions.

Colonel Pettis assumes that the area supplying water to Lake Superior is limited to its topographic drainage basin. This assumption is the usual and natural one for this area. Its use requires that the sum of all items of water use must equal, and cannot exceed, the total rainfall on the topographic basin tributary to the lake.

The known items are combined in the expression $P + R - O - S$. Although all of these items cannot be measured completely or exactly they are determinable within reasonable limits. The greatest uncertainty is in the R item as Colonel Pettis states the run-off of only 49% of the tributary drainage has been measured.

The plotting of the average monthly values of $P + R - O - S$ (Fig. 1(*c*)) shows negative values for April and May. This means that the values used for the rainfall on the lake, the surface run-off into the lake and the lowering of the lake are not sufficient to supply the outflow from the lake in these months. This, in turn, means that some additional item of inflow is required to supply the outflow even if no evaporation occurs from the lake.

Although the uncertainty in the values for P and R may account for the negative values found for $P + R - O - S$, all experience requires the inclusion of some material amount of evaporation in these months. This, in turn, results in requiring an additionally larger item of inflow from some unmeasured source.

Proceeding with the assumption that there is unmeasured ground-water flow into the lake, Colonel Pettis derives values for its amount which produce the results shown in his summary balance for both land and water areas. The curve for $P + R - O - S$, in Fig. 1(*c*) which is equal to $E - U$, or the total net value of the unknown evaporation from the lake and any other unknown inflow, indicates that either there are major omissions or errors in $P + R - O - S$ or that other sources of inflow must exist of appreciable amount. It also indicates that the values for evaporation by months must have a relative distribution through the year that is materially different from those that have been usually considered. If the unknown item of inflow is assumed to have a fairly constant distribution through the year, the resulting values for evaporation from the lake must have winter values that are larger than the summer values.

---

[18] Prof., Irrig., Univ. of California, Berkeley, Calif.

Using his assumptions Colonel Pettis completes the analysis to secure the resulting values for land losses, ground-water flow into the lake and evaporation from the lake. Just as Colonel Pettis used his results for $P + R - O - S$ to judge the amount of distribution by months of his unknown items, his conclusions can be used to judge the reasonableness of his results.

The rainfall on the drainage area must supply the land losses, the rainfall penetration which becomes the source of the ground-water and underground flow into the streams and the lake as well as the surface run-off. For Lake Superior alone the items of water supply must balance similarly. If one item is over-estimated some other item must be reduced correspondingly in the land balance. If inflow to the lake is over-estimated, evaporation from the lake will have to be increased.

Some of the limitations on the values that may be secured for evaporation from Lake Superior can be shown by varying the assumed value of the land losses for consumptive use. The total rainfall of 28.2 in. may be distributed in various amounts as illustrated in Table 6. These results illustrate the limitations on the different items. If 16.6 in. of the rainfall on the land are used in soil-moisture evaporation and plant transpiration, including rainfall interception by vegetation, and Colonel Pettis' estimate of stream flow is correct, there is no remaining supply available for ground-water flow into the lake. To meet Colonel Pettis' conclusions land losses can only be 5.9 in. and rainfall penetration would have to be 10.7 in. plus the ground-water portion of the stream flow.

TABLE 6.—DISTRIBUTION OF TOTAL RAINFALL

Description	Pettis	For Various Assumed Values		
Consumptive use, L, in inches of depth on land area	5.9	10.0	15.0	16.6
Inches of Depth on Land Area:				
Surface run-off, R	11.6	11.6	11.6	11.6
Underground flow into lake, U	10.7	6.6	1.6	0
Inches of Depth on Lake Area:				
Surface inflow to lake, R	17.9	17.9	17.9	17.9
Underground flow into lake, U	16.5	10.2	2.5	0
Resulting evaporation from lake, E	35.6	29.3	21.6	19.1

Although no exact values can be set for these items, Colonel Pettis' conclusions differ materially from most of the present values used for these items. He secures a land loss of 5.9 in. where water surface evaporation is 35.6 in. or land losses are only one-sixth of water losses. The drainage area of Lake Superior contains many other lakes. Their total area is not available; if they represent one-sixth of the drainage area, their evaporation would require all of the water assigned by Colonel Pettis to land losses, leaving nothing for loss from the five-sixths of the area in land. For the vegetative conditions of this area, consumptive uses should exceed 5.9 in. The rainfall is sufficient to support active vegetation. Cropped areas would require more than 6-in. loss for usual yields; forest areas would consume in excess of this amount. Colonel Pettis finds a maximum monthly land loss of less than 1 in. per month. General values shown by available records are well summarized by Joseph Kittridge, Jr.,[19] who places the area surrounding Lake Superior in the group of annual water losses between 10 and 20 in. Mr. Kittridge found such losses to be less than 10 in. only in sagebrush and desert shrub areas. Although his method of deriving results includes unmeasured deep ground-water movement with the land losses, his values are derived from areas adjacent to Lake Superior where such deep ground-water movement is not a factor.

The reasonableness of the land-area balance can also be approached from another point of view. All the underground flow to the lake, plus at least part of the surface run-off, must represent rainfall that has penetrated below the plant roots and reached the water table. The portion of the surface run-off that represents ground-water flow into the streams is not discussed in numerical terms in the Symposium and would be difficult to estimate closely. As the stream flow is fairly well sustained throughout the year such ground-water flow appears to be a material part of the measured stream flow.

The quantity of the derived underground flow into the lake represents 37% of the precipitation. With ground-water movement into the surface streams, the summary presented by Colonel Pettis would require that more than one-half of the rainfall penetrate to the ground-water. This is a much larger proportion of rainfall penetration than has been found for other similar areas. The general results of observations of rainfall penetration in the Great Plains area[20] would indicate little penetration from the 28.2 in. of rainfall on heavy soils, to perhaps 6 in. on sandy lands if conditions are as favorable as on the drainage of the Loup River in Nebraska. The probable amount of rainfall penetration on the drainage area of Lake Superior would not appear to be sufficient to account for more than the ground-water flow into the streams which is included in the stream item of Colonel Pettis' water supply balance.

The small value found for consumptive use requires a correspondingly high value for the total run-off. For both surface and underground run-off, the total run-off derived is 22.3 in. from a rainfall of 28.2 in. This exceeds the quantity of run-off obtained from adjacent similar areas. Areas farther west in the same latitude, with about 20 in. rainfall, have a surface run-off of only 2 or 3 in., and have little rainfall penetration.

Evaporation.—The Symposium presents two determinations of evaporation from Lake Superior, separately derived. Their close agreement does not necessarily represent a proof of the correctness of either.

Mr. Hickman's method of observation of evaporation represents a commendable effort to eliminate one of the factors which causes observations with pans to differ from the loss from large water areas. The method used, however, raises additional questions regarding the interpretation of the pan results in terms of lake evaporation.

The maintenance of the temperature of the water in the pans at the same value as the temperature of the surface water in the lake does not of itself assure the same rate of evaporation. Although it may reduce the extent of the difference, comparisons between the temperature-controlled pans and measured evaporation from large areas are needed before the assumption that the controlled pan has a coefficient of unity in all months can be justified. Although the difference in temperature between the water in the pans and in lakes is one of the important factors in their different rates of evaporation, other factors such as rim effects of the pan, differences of exposure and differences in pan size still need to be considered.

---

[19] "The Magnitude and Regional Distribution of Water Losses Influenced by Vegetation," by Joseph Kittridge, Jr., *Journal of Forestry*, Vol. XXXVI, No. 8, August, 1938, pp. 775–778.

[20] "Direct Accretions to Ground-Water from Rainfall," by S. T. Harding, M. Am. Soc. C. E., *Transactions*, Am. Geophysical Union, 18th Annual Meeting, 1937, pp. 368–371.

Evaporation is dependent on the available heat supply. On the lake the heat supply is primarily that received from solar radiation. The evaporation will be less than the quantity of water that the total solar radiation can convert into vapor because of back radiation and any loss in heat to surrounding bodies. Heat is stored in the lake during months of rising water temperatures and withdrawn during those of lowering temperatures. The authors state that temperature variations extend to a depth of 180 ft. They present the monthly mean surface temperatures but do not show the monthly temperature profiles in the lake.

If the heat added to the pans during the cooling months, to maintain the same water temperature in the pan as in the surface water in the lake, is equal to the heat obtained at the lake surface by the cooling of the lake water, then the loss from the pan would represent the probable evaporation from the lake as far as the heat balance items are involved. In the summer months the heat removed from the water in the pan would need to be equal to the heat stored in the water in the lake. Without records of the heat added or removed from the pan, or of temperature profiles in the lake, comparisons on the basis of a heat balance cannot be made.

Mr. Hickman's results differ from direct observations on other lakes which do not freeze during the winter. On some western lakes conditions permit the same type of analysis used by Colonel Pettis where ground-water inflow or outflow is negligible in amount. Among such lakes in areas of fairly low winter temperatures are Pyramid and Walker lakes in Nevada. These lakes also do not freeze during the winter; both have sufficient depth to be comparable, in this respect, to Lake Superior. Results to 1935[21] show much less relative evaporation during the winter months than are found by the authors. The evaporation and air temperatures of the three lakes are compared in Table 7. For the air temperatures the U. S. Weather Bureau records at the nearest station are used, Lahontan being used for Pyramid Lake and Schurz for Walker Lake.

Until the heat-controlled pan can be tested adjacent to a large water surface for which the evaporation can be determined, the coefficient that needs to be applied to the results with the heat-controlled pan must remain unknown. Although the coefficient might prove to be unity, both for annual and for monthly periods, the writer will venture the prediction that the loss from the heat-controlled pan will be found to exceed that from the adjacent large water area and that the monthly ratios of the pan to the large area will be variable.

---

[21] "Evaporation from Large Water-Surfaces Based on Records in California and Nevada," by S. T. Harding, *Transactions*, Am. Geophysical Union, 16th Annual Meeting, 1935, pp. 507–511.

The size of the heat-controlled pan is also a factor in its amount of evaporation if its performance is similar to other types of pan. Comparisons with different sizes of heat-controlled pans would not be difficult to make; if size were found to be a factor it would demonstrate that the assumption of a value of unity for the coefficient of their pan as used by the authors was not justified.

TABLE 7.—COMPARISON OF EVAPORATION AND AIR TEMPERATURES; PYRAMID AND WALKER LAKES IN NEVADA AND LAKE SUPERIOR

	Pyramid Lake		Walker Lake		Lake Superior		Percent of the Annual Total		
	Evaporation, in feet	Mean air temperature, in degrees Fahrenheit	Evaporation, in feet	Mean air temperature, in degrees Fahrenheit	Evaporation, in inches	Mean air temperature, in degrees Fahrenheit	Pyramid Lake	Walker Lake	Lake Superior
January	0.25	31.7	0.20	31.7	4.2	26	6.0	4.8	11.8
February	0.25	37.9	0.15	37.4	3.1	24	6.0	3.6	8.7
March	0.30	44.8	0.20	44.2	2.0	27	7.25	4.8	5.6
April	0.30	51.7	0.20	50.4	1.4	32	7.25	4.8	3.9
May	0.35	60.2	0.25	58.5	1.5	40	8.4	5.9	4.2
June	0.40	69.1	0.40	66.2	1.5	48	9.65	9.5	4.2
July	0.40	78.2	0.50	73.1	1.9	58	9.65	11.9	5.3
August	0.40	76.1	0.55	71.1	2.6	64	9.65	13.1	7.3
September	0.45	65.7	0.65	62.0	3.4	66	10.85	15.5	9.6
October	0.40	54.1	0.45	52.4	4.3	60	9.65	10.7	12.1
November	0.35	42.3	0.40	40.6	4.8	48	8.4	9.5	13.5
December	0.30	32.9	0.25	32.8	4.9	34	7.25	5.9	13.8
Total	4.15	53.7	4.20	51.7	35.6	...	100.0	100.0	100.0

Conclusions.—Colonel Pettis meets the apparent contradiction of negative evaporation in April and May, if the ground-water inflow to Lake Superior is taken as zero, by assumptions which remove that difficulty but lead to improbable values for land losses and rainfall penetration. Mr. Hickman's heat-controlled evaporation pans give values for evaporation which agree with Colonel Pettis' results but, if applied to Lake Superior without correction, lead to the same difficulties in land losses that result from Colonel Pettis' analysis.

Although these two papers represent a commendable effort to determine the hydrologic balance for the drainage area of Lake Superior, the problem does not appear to be solved. Further work needed appears to include: (1) A more direct study of land losses to determine what part of the rainfall may be available for ground-water movement into the lake; (2) perhaps a review and possible revision of the estimated surface flow from the unmeasured portions of the drainage area; (3) comparisons of the heat-controlled pans with large water areas whose evaporation can be measured; and (4) monthly temperature profiles in the lake to determine if cooling in the late fall and winter months can supply the amount of heat required for the estimated amount of evaporation in these months. Although previous estimates of evaporation from Lake Superior may have been too small, Mr. Hickman's estimate, when all factors are considered, appears to be too high. Actually the evaporation from the lake must have a value that can be supplied from the rainfall and leave a consistent water supply available for land losses. Mr. Hickman's results do not appear to meet this standard.

O. E. MEINZER,[22] ESQ. (by letter).—The paper by Colonel Pettis does not discuss the geology and ground-water conditions of the drainage basin tributary to Lake Superior. The area is known to be underlain, for the most part, by pre-Cambrian crystalline rocks which in general are impervious or have only low permeability. In most places these hard old rocks are overlain by a thin mantle of glacial drift which is, in part, water-bearing.

So far as is known, the rocks of this basin do not, except locally, have any artesian structure or other structural features for conveying the ground-water laterally toward the lake. The ground-water is believed to be essentially under water-table conditions, with the water-table sloping toward the streams and branches and not generally toward the lake. If these conditions prevail, a large part of the stream flow is doubtless derived from ground-water, but the quantity of water percolating directly into the lake can be only a small fraction of the 10.7 in. postulated in the paper.

On the basis of the great mass of information that is available in regard to the geology and ground-water conditions of the United States, it can be stated confidently that, for most of the large drainage basins in the United States, the loss or gain of water by subterranean flow is relatively small, and the discharge by transpiration and evaporation approximates the difference between precipitation and stream discharge. In general, the rate of discharge by transpiration and evaporation changes gradually and systematically from one region to another with changes in climate.

Colonel Pettis gives the land losses for the drainage basin of Lake Superior as only 5.9 in. If a larger rate of loss is conceded for the extensive swamp areas and numerous minor lakes within the basin, the rate of loss from the actual land areas would be still smaller. If 10.7 in., the purported value of U, is added to 5.9 in., the purported value of L, the total is 16.6 in., which is approximately the discharge by transpiration and evaporation that would be expected in the Lake Superior basin from the known regional loss.

The writer has not been able to get a clear understanding of the process by which the evaporation from the lake (E) has been estimated or computed in this paper. It is apparently quite conjectural. The value of 10.7 in. for ground-water inflow (U) has been derived merely by subtracting $P + R$ from $O + S + E$. Furthermore, the value of 5.9 in. for the land loss (L) has been derived merely by subtracting $R + U$ from B. Apparently no effort was made to study the actual ground-water conditions.

It would seem that, before the results of 10.7 in. and 5.9 in. can be given favorable consideration, a systematic ground-water survey of the drainage basin should be made, including a quantitative ground-water study of at least one locality that appears relatively favorable for percolation of ground-water into the lake. The quantitative study should include such items as a contour map of the water-table and a determination of the extent, thickness, and coefficient of permeability of the water-bearing beds.

---

[22] Geologist in charge Division of Ground Water, Water Resources Branch, U. S. Geological Survey, Washington, D. C.

R. W. DAVENPORT,[23] M. AM. SOC. C. E. (by letter).—From his analyses, Colonel Pettis reaches the conclusion that, for the drainage area of the Great Lakes, the evaporation and underground flow are greater, and the land losses are less, than the values that have been generally accepted.

The mean annual evaporation from Lake Superior has been estimated in notable investigations of recent years as follows: 22 in. by Mr. Freeman, 15 in. by Messrs. Horton and Grunsky, and 23 in. by Messrs. Hayford and Folse. These results contrast with 35.6 in. obtained by the authors of the Symposium. The substantial excess of the latter value of estimated evaporation over the others is impressive.

Unquestionably, the mean annual rainfall and the mean annual run-off from the land area, 28.2 and 11.6 in., respectively, as obtained by Colonel Pettis, may be accepted as reasonably reliable. The results for outflow from Lake Superior and changes in storage in the lake are determined by methods which are reasonably precise and should be correspondingly reliable. Consequently, Colonel Pettis' further conclusion that underground flow is more and losses are less than generally accepted seems to rest fundamentally on the validity of the values adopted by him for evaporation from the lake.

The writer has participated in rather extensive studies of so-called land (or water) losses, or the difference between rainfall and run-off, in Eastern United States. (The term "rainfall" as used herein includes all forms of precipitation.) The studies were supervised by Gordon R. Williams, Assoc. M. Am. Soc. C. E., under the general direction of the writer.[24]

They were limited to streams in the humid or semi-arid regions of the United States east of the Rocky Mountains. The basic period for which the computations were made is the water year, or year ending September 30. Only records which had continued without interruption for three years or more were used. The basins were also selected with a view to: (*a*) The avoidance of complications due to underground flow, as indicated by large springs or other evidence of material ground-water influence; (*b*) the avoidance of artificial regulation by storage; and (*c*) the availability of a sufficient number of precipitation stations for the reliable determination of the average precipitation in the respective basins. The report[24] gives results for about 200 areas with an aggregate period of record of more than 2 000 years, or an average of about ten years per record.

Table 8 shows a summary of precipitation, run-off, and water losses for areas in the western St. Lawrence River Basin and the adjoining region, as they appear in the report. The water-loss results are plotted in Fig. 10 in order to facilitate consideration in relation to their several geographical positions. The requirements of the study for continuity and duration of the records, freedom from complications caused by artificial regulation of stream flow, and adequacy of the rainfall data caused the distribution of selected basins to be sparse in the region indicated.

As used in these studies, the run-off is the surface stream flow measured at a gaging station, and the rainfall is the computed average precipitation over the drainage area tributary to the gaging station. The difference between these two factors, of course, represents the evaporation and transpiration from the basin plus ground-water outflow from the basin that is not measured at the gaging station, minus ground-water accretion from sources outside the basin which appears in the measured surface flow at the gaging station. The common designation of this difference as water losses, as representing the evaporation and transpiration, is a misnomer to the extent that underground flow is involved in the ways indicated. Adjustments to the water-loss results as computed would be subtractive for ground-water outflow that does not appear at the gaging station and additive for ground-water accretion from outside the basin that appears at the gaging station. Variations in evaporation, transpiration, and ground-water effects, both additive and subtractive, would be reflected in variations in the computed difference between rainfall and run-off.

---

[23] Hydr. Engr., U. S. Geological Survey, Washington, D. C.

[24] "Water Losses in the United States," by Gordon R. Williams and others, *Water-Supply Paper No. 846*, U. S. Geological Survey (publication pending).

TABLE 8.—SUMMARY OF PRECIPITATION, RUN-OFF, AND WATER LOSS IN WESTERN ST. LAWRENCE RIVER BASIN AND ADJOINING REGION

Item No.	Basin above the indicated gaging station	Period studied, in water years	Drainage area, in square miles	MEAN ANNUAL QUANTITIES, IN INCHES		
				Precipitation	Run-off	Water loss
	(a) ST. LAWRENCE RIVER BASIN					
	Streams Tributary to Lake Michigan:					
1	Thornapple River near Caledonia, Mich..	1932–1934	773	32.0	9.3	22.7
2	Muskegon River at Newaygo, Mich.....	1932–1934	1 450	30.1	10.2	19.9
	Streams Tributary to Lake Huron:					
3	Tittabawassee River at Freeland, Mich...	1913–1914 1916–1920 1932–1934	2 530	29.7	9.3	20.4
	Streams Tributary to Lake Erie:					
4	River Rouge at Detroit, Mich..........	1932–1934	194	28.6	6.0	22.6
5	Huron River at Barton, Mich..........	1915–1920	723	31.7	9.2	22.5
	(b) MISSISSIPPI RIVER BASIN					
	Chippewa River Basin:					
6	Jump River at Sheldon, Wis...........	1916–1934	510	30.5	12.9	17.6
	Trempealeau River Basin:					
7	Trempealeau River at Dodge, Wis......	1915–1919	633	29.5	8.3	21.2
	Black River Basin:					
8	Black River at Neillsville, Wis..........	1915–1934	756	31.1	9.6	21.5
	La Crosse River Basin:					
9	La Crosse River near West Salem, Wis.	1915–1934	412	30.3	10.0	20.4
	Wisconsin River Basin:					
10	Rib River at Rib Falls, Wis............	1926–1934	309	30.1	12.4	17.7
11	Yellow River at Sprague, Wis..........	1927–1934	420	28.8	6.3	22.5
12	Kickapoo River at Gays Mills, Wis.....	1915–1933	.....	31.6	9.3	22.3
	Rock River Basin:					
13	Sugar River near Brodhead, Wis.......	1915–1934	529	32.5	9.2	23.3
	(c) HUDSON RIVER BASIN					
14	Red River at Fargo, N. Dak.................	1919–1923 1925–1933	6 420	20.8	0.6	20.3
15	Red River at Grand Forks, N. Dak..........	1882–1934	25 500	20.9	1.2	19.7
16	Red Lake River at Crookston, Minn........	1922–1924 1926–1933	5 320	19.5	1.6	17.9

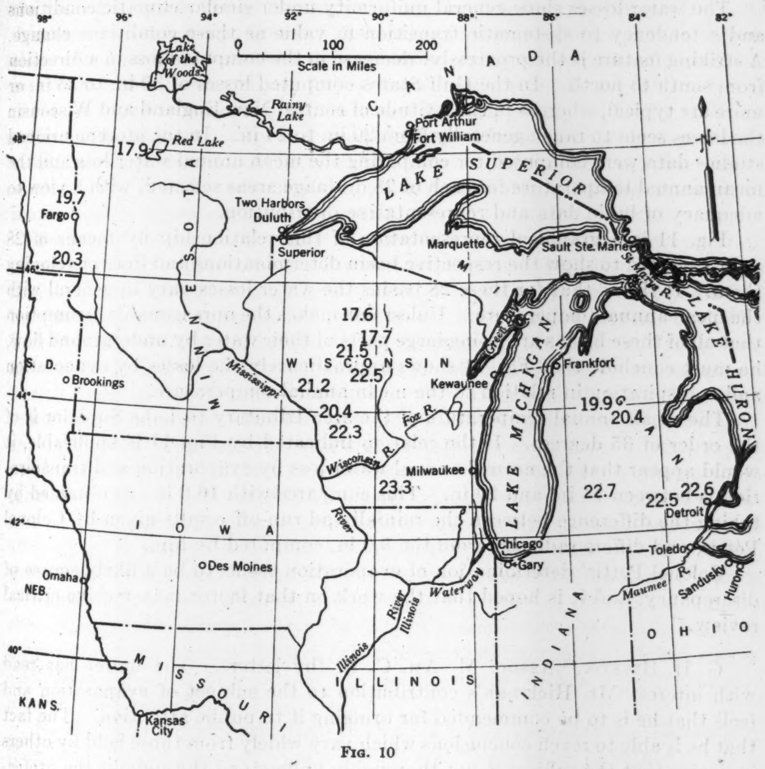

98°
96°
94°
92°
90°
88°
86°
84°
82°
48°
46°
44°
42°
40°
0 100 200
Scale in Miles
Lake of the Woods
Rainy Lake
Red Lake
C A N A D A
Port Arthur
Fort William
LAKE SUPERIOR
Two Harbors
Duluth
Superior
Marquette
Sault Ste. Marie
St. Marys R.
LAKE HURON
Green Bay
Frankfort
Kewaunee
LAKE MICHIGAN
M I C H I G A N
17.9
19.7
Fargo
20.3
17.6
17.7
21.5
22.5
21.2
20.4
22.3
23.3
19.9
20.4
22.7
22.5
22.6
M I N N E S O T A
W I S C O N S I N
S. D.
Brookings
Mississippi
Fox R.
Wisconsin R.
Milwaukee
Detroit
Toledo
Sandusky
Huron
Maumee R.
I O W A
River
Chicago
Gary
Des Moines
Omaha
NEB.
Waterway
Illinois River
Illinois
I N D I A N A
O H I O
I L L I N O I S
M I S S O U R I
KANS.
Kansas City

FIG. 10

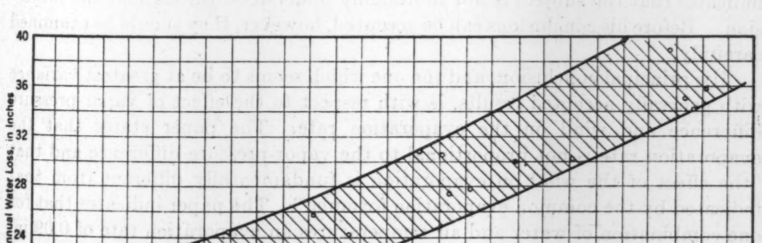


FIG. 11.—COMPARISON OF MEAN ANNUAL WATER LOSS AND MEAN ANNUAL TEMPERATURE FOR SELECTED BASINS WITH MEAN ANNUAL PRECIPITATION IN EXCESS OF 20 INCHES

The water losses show general uniformity under similar climatic conditions and a tendency to systematic transition in value as these conditions change. A striking feature is the progressive decrease in the computed loss in a direction from south to north. In the Gulf States computed losses of 30 in. to 35 in. or more are typical, whereas in the latitude of central New England and Wisconsin the losses seem to range generally from 20 in. to 24 in. In the aforementioned studies data were computed for comparing the mean annual water loss and the mean annual temperature for each of 28 drainage areas selected, with a view to adequacy of basic data and representative distribution.

Fig. 11 is a graphical representation of this relationship by means of 28 points plotted to show the respective basin determinations and lines enveloping them. It shows that for these 28 basins the water losses vary in general with the mean annual temperature. Unless one makes the unreasonable assumption that all of these basins are losing large parts of their water by underground flow, he must conclude that Fig. 11 shows approximately the losses by evaporation and transpiration in relation to the mean annual temperature.

The mean annual temperature of the area tributary to Lake Superior is of the order of 35 degrees. If the relation indicated by Fig. 11 is applicable, it would appear that the normal annual water loss by evaporation and transpiration is between 14 in. and 20 in. This compares with 16.6 in., as obtained by taking the difference between the rainfall and run-off results given by Colonel Pettis, and differs radically from the 5.9 in. computed by him.

Colonel Pettis' determination of evaporation seems to be a likely source of discrepancy, and it is hoped that the work on that factor may receive critical review.

G. H. HICKOX,[25] ASSOC. M. AM. C. E. (by letter).—The writer has read with interest Mr. Hickman's contribution to the subject of evaporation and feels that he is to be commended for bringing it to public attention. The fact that he is able to reach conclusions which vary widely from those held by others indicates that the subject is not thoroughly understood throughout the profession. Before his conclusions can be accepted, however, they should be examined carefully.

His principal conclusion, and the one which seems to be at greatest variance with generally accepted results, is with respect to the effect of vapor-pressure difference and wind on the evaporation rate. The paper states that the evaporation rate is not proportional to the vapor-pressure difference and that "the effect of the wind on evaporation is fundamentally different from that indicated by the common evaporation formula." The paper indicates that for one combination of water and air temperatures an evaporation rate of 0.09 in. per day might occur with zero vapor-pressure difference, the evaporation being due solely to the effect of wind at 10 miles per hr. It is entirely possible that water may be removed from a free water surface by wind when the vapor-pressure difference is zero. Such removal could be accomplished mechanically, the water being carried away from the evaporation pan or reservoir surface in the form of spray. This might easily occur when waves break, for example, the resulting spray being carried away by the wind on the form of minute droplets, without evaporation. There is evidence that such action occurs along the Pacific coast where the accumulation of salt on transmission line insulators several miles inland has become a serious problem. The writer is unable to conceive, however, of 0.09 in. per day being removed bodily from the surface of Lake Superior by this means. Atmospheric turbulence would seem to be the only means of holding the droplets in suspension, and any lessening of wind velocity would result in a noticeable precipitation, it being assumed, of course, that evaporation could not occur.

---

[25] Senior Hydr. Engr., TVA, Hydraulic Laboratory, Norris, Tenn.

Such speculation seems rather idle since the proportionality of evaporation to vapor-pressure difference has been well established. Dalton's Law has been accepted as a basis for correlating evaporation experiments for many years and is apparently sound theoretically. It has also been verified experimentally by Carl Rohwer,[26] F. Graham Millar,[27] B. C. Shepherd, C. Hadlock, R. C. Brewer,[28] and others. The three sets of experiments specifically referred to were conducted under the conditions suggested by Mr. Hickman; that is, "in a wind tunnel under laboratory conditions." The results were in agreement that, for any given wind velocity, the evaporation rate is proportional to the vapor-pressure difference. In these experiments, the vapor pressures were defined in the customary manner—as the pressure of saturated water vapor at the temperature of the water surface, and as the partial pressure of the water vapor in the atmosphere at a considerable distance from the water surface. C. W. Thornthwaite and Benjamin Holzman[29] have computed evaporation rates on the basis of Dalton's Law from observations of vapor pressures at different elevations above the ground surface. They have been successful not only in calculating evaporation rates when a positive vapor-pressure difference existed, but also in calculating the rate of condensation that occurred with a negative vapor-pressure difference.

No theory is any stronger than the evidence that supports it. Before Mr. Hickman's conclusions can be accepted, it will be necessary to refute the contrary results obtained in many carefully conducted experiments, or to explain them in terms of the new theory. It is believed that Mr. Hickman's conclusion regarding evaporation into a wind at a zero vapor pressure can be explained in terms of the accepted theory. His conclusion is apparently based on the improper method of using one daily observation of relative humidity to determine an average vapor pressure for the day. The variation of relative humidity in the vicinity of the Great Lakes is not known, but the point may be illustrated by reference to conditions in the Tennessee Valley. Figs. 12(*a*) and 12(*b*) show a typical example of the variation of relative humidity and air temperature throughout the day at a point in the vicinity of Norris Lake. The corresponding vapor pressure is shown in Fig. 12(*c*). It will be noted that in spite of considerable variation in temperature and relative humidity, the vapor pressure is relatively constant. In general, the actual change of the water vapor content of the air proceeds slowly. At night, when air temperatures are low, the water vapor content may be near saturation. During the day, when the temperature is high, the relative humidity necessarily drops, although the total water vapor (and hence its pressure) may not change greatly. The errors involved in applying a single relative humidity determination to the average air temperature in order to calculate an average vapor pressure for the day are obvious. In the case illustrated, the average air temperature is 63.5° F. Mr. Hickman does not state when the relative humidity was determined, but according to the source material[10] it was 9:00 a.m. Using this time for the data of Fig. 12, the relative humidity is 52% and the vapor pressure for a temperature of 63.5° is 0.77 cm of mercury. The actual average vapor pressure is 0.87 cm of mercury.

---

[26] "Evaporation from Free Water Surfaces," by Carl Rohwer, Assoc. M. Am. Soc. C. E., *Technical Bulletin* No. 271, U. S. Department of Agriculture, December, 1931.

[27] "Evaporation from Free Water Surfaces," by F. Graham Millar, *Canadian Meteorological Memoirs*, Vol. 1, 1937, pp. 43 to 65.

[28] "Drying Materials in Trays," by B. C. Shepherd, C. Hadlock, and R. C. Brewer, *Industrial and Engineering Chemistry*, April, 1938, pp. 388 to 397.

[29] "The Determination of Evaporation from Land and Water Surfaces," by C. W. Thornthwaite and Benjamin Holzman, *Monthly Weather Review*, January, 1939, pp. 4 to 11.

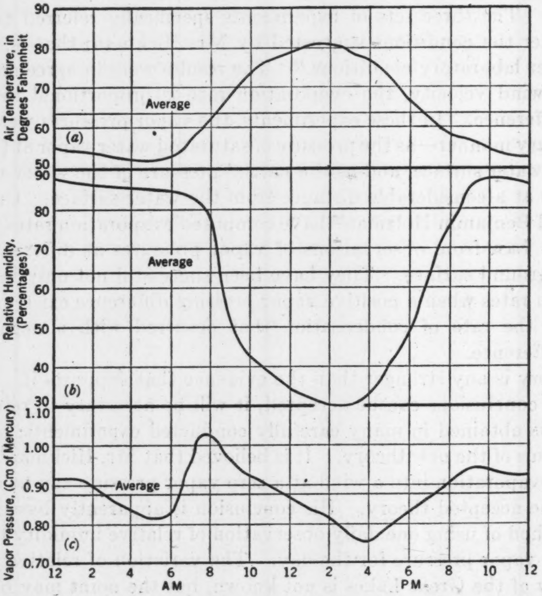


FIG. 12.—TYPICAL AIR TEMPERATURE, RELATIVE HUMIDITY, AND VAPOR PRESSURE NEAR NORRIS LAKE

Even the use of average air temperature and average relative humidity does not give the average vapor pressure, due to the non-linear relationship between vapor pressure and temperature. Referring to Fig. 12 again, the average air temperature of 63.5° F and average relative humidity of 64% give an apparent average vapor pressure of 0.95 cm of mercury, as compared with the true average of 0.87 cm. Similar reasoning applies to the use of average water temperatures to determine the vapor pressure at the water surface.

Dalton's Law is strictly applicable only when the true values of vapor pressure are used. The success of Meyer's formula (Equation (4)) is due to the empirical evaluation of the constants which takes care of the manner in which V and v are calculated. An equation containing constants derived in this manner may give absurd results when applied outside the range of conditions which were used in its derivation. This would not disturb the validity of the fundamental laws on which it was based, and this is where Mr. Hickman "goes astray." It is quite possible to have a definite evaporation rate when the apparent vapor-pressure difference, determined from average air and water temperatures and one observation of relative humidity, is zero. It does not follow, however, that Dalton's Law is thereby proved to be in error.

The foregoing constitutes the chief objection to Mr. Hickman's thesis. The writer disagrees on a number of minor points which will be enumerated in the following paragraphs.

The assumption that "the evaporation from the open lakes, for a given air temperature, water temperature, and wind velocity, would be the same as the observed evaporation from the experimental pans for the same combination of air temperature, water temperature, and wind velocity" (see "Synopsis") is open to serious question. The data of the late Reuben B. Sleight,[30] Assoc. M. Am. Soc. C. E., and Mr. Rohwer[14] indicate that the rate of evaporation is reduced with an increase in the diameter of the evaporating water surface for diameters up to 12 ft. It does not follow that data from a pan 4 ft in diameter can be applied directly to a large lake without correction. This is to be expected from a consideration of the edge effects. Dry air passing over a water surface will take up water vapor more rapidly at the point where it first passes over the water than at some distance farther on. For this reason, the rate of evaporation from a small pan will be greater than that from a larger area.

Mr. Hickman states (see "Synopsis") that "The rate of evaporation from open-water surfaces when the air temperature is below freezing has been investigated for the first time." This statement overlooks Mr. Rohwer's work,[26] published in 1931, in which experiments made under these conditions were described. Mr. Millar's experiments[27] also apparently cover this range. A comparison of the data obtained in these experiments with the records obtained by the Weather Bureau with their standard evaporation equipment would have been greatly facilitated if the same type of equipment had been used. The standard Weather Bureau pan is so mounted as to be exposed fully to sunshine and wind. The desirability of this exposure is open to criticism; but the provision of a water jacket or an insulated shell interferes materially with the similarity of the installation by reducing the transfer of heat through the walls of the pan both by solar radiation and by convection.

The writer is unable to see the reason for omitting relative humidity or vapor pressure as a basis of correlating the experimental results. Would it not be as logical to assume that September evaporation records would be grouped around a certain average value as to assume that any particular daily temperature in September should be accompanied by a certain relative humidity? Certainly it would make the work much easier if all the September evaporation records could be grouped and averaged. Such a value would probably be satisfactory for average conditions.

---

[30] Discussion by the late Reuben B. Sleight, Assoc. M. Am. Soc. C. E., of paper "Evaporation on United States Reclamation Projects," by Ivan E. Houk, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 90 (1927), pp. 303 to 316.

Correlation between the evaporation rate and relative humidity as such is not to be expected in any case. The proper basis for correlation, as predicted from physical theory and verified by experiment, is between the evaporation rate and the difference of vapor pressure. Relative humidity, as implied by the name, is purely a relative term and has no absolute value.

Mr. Hickman's expressed desire for tests made in a wind tunnel under laboratory conditions has already been fulfilled by experiments made by workers in various fields, including the Canadian Meteorological Service, which is also studying the hydrology of the Great Lakes. It is unfortunate that the results of these experiments are not more generally known to the civil engineering profession.

C. R. PETTIS,[31] M. AM. SOC. C. E. (by letter).—Since 1936 the writer has searched for quantitative information about underground water. Harold Conkling, M. Am. Soc. C. E., estimated that 40% or 50% of the rainfall on Long Island reaches the water table; and he stated that probably most of the underground water reaches the sea without coming to the surface.[32] A few engineers in California seem to realize that, at times, ground-water movement may involve appreciable quantities; but engineers in general seem to be almost unanimously of the opinion that ground-water movements are negligible, and can be neglected entirely in the study of the hydrology of any region. Messrs. Harding, Meinzer, and Davenport state the generally accepted opinion clearly and emphatically. Mr. Freeman[7] and Robert E. Horton,[3] M. Am. Soc. C. E., started their studies on the hydrology of the Great Lakes with the assumption that ground-water flow is negligible, and the assumption is carried over into their conclusions. From comments by Mr. Harding it seems that the same was probably done in the study of Pyramid Lake and Walker Lake.

The writer tried to outline the method so that it could be followed by any one who wished to make a similar study; and, in the paper, he gave all of his conclusions that seemed to be of general interest. The statistical studies upon which the conclusions were based[5] were voluminous, and it was not thought that they would be of sufficient general interest to justify publication.

The writer attempted to reach his solution without being unduly influenced by preconceived opinions; but he recognizes that it is fair and logical to ask the question, "Is the value of U too high?" A few remarks will be offered on this question. Necessarily, most of the evidence is circumstantial, but it is cumulative, and indicates that the underground flow, U, is appreciable, and that its numerical value is not much less than the value of the surface flow R.

The underground flow into the lake in the period from April 1 to July 31, inclusive, is greater than the evaporation from the lake in the same period of time. The evidence to support this statement is direct, and is based on only one geometrical assumption—namely, that water which appears in the lake, but which neither falls on its surface nor flows into it from surface streams, must be classed as underground flow. From Fig. 1(*c*), it may be noted that the average value of $(E - U)$ for the period April 1 to July 31, inclusive, is negative (-0.2). This means that for the stated 4-month period the underground flow is greater numerically than the evaporation from the lake surface, both being measured in land units. In other words, if there is any net evaporation from the lake in the average period from April 1 to July 31, there must have been at least an equal quantity of underground flow coming into the lake in the same period, in order to make the water accounts balance.

---

[31] Colonel, Corps of Engrs., U. S. Army (*Retired*), Head, Dept. of Math., Mississippi State Coll., State College, Miss.

[32] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 759.

It is believed that the data may be considered to be excellent and accurate for the purpose of the foregoing statements, and that the statements are true without appreciable error.

The solution in the text indicates that the evaporation from the lake, in lake units, from April 1 to July 31, inclusive, is 6.4 in.; and the underground flow for the same period is 6.6 in., in lake units. This value of evaporation for the stated period checks very well with the results obtained by Mr. Hickman in his analysis of the evaporation experiments.

If an effort is made to check the values of evaporation from standard textbooks, difficulty is encountered. The evaporation experiments which have been made in this latitude[12] (and which have been mentioned elsewhere by Mr. Meyer) indicate an evaporation from April 1 to July 31 (which would mean a corresponding underground flow) almost three times the values given in the preceding paragraph. Mr. Meyer there[33] indicates about 80% more evaporation than in Fig. 1, and Mr. Meyer's chart indicates at another place[34] about 36% more evaporation. The writer's values for evaporation and underground flow for the stated period are the smallest that are consistent with any known published data on the subject.

An abnormally high rainfall, *B*, in any one month will ordinarily increase the surface run-off *R* in the same month, and to some extent in the following months; the effect of the high rainfall on *R* can be computed directly from the data. The effect of a high rainfall on the underground flow *U* can likewise be computed by a method which will be described. The relative effects of the high rainfall on *R* and *U* can be compared therefore.

An abnormally high rainfall in any given month has no appreciable effect on the value of $(U - E)$ for the given month; but there is a strong tendency for the value of $(U - E)$ to be above normal for both the second month and the third month, and this tendency is quite pronounced not only on Lake Superior, but on each of the other Great Lakes.

An analysis of the data with respect to variations in evaporation *E* indicates that temperature is the main factor affecting *E*. Similarly it is evident that rainfall *B* is the principal factor affecting underground flow *U*. Furthermore, over longer periods of time, such as one year, there is no apparent correlation between temperature and rainfall; U and E are independent variables. If a group of values of $(U - E)$ are selected where there was high rainfall in some preceding month, and a similar group following low rainfall in the same preceding month, the difference in values of $(U - E)$ will tend to represent the difference in values of U, since the values of E, being independent, will tend to cancel. The greater the number of items, the more accurate the results from the assumption will be. This method has been used with many different combinations, and the indication is always the same—that high underground flow tends to follow high rainfall, nearly all of the effect being in succeeding months.

---

[33] "Elements of Hydrology," by A. F. Meyer, John Wiley & Sons, Inc., New York, N. Y., 1928, p. 235.

[34] *Loc. cit.*, p. 236.

To illustrate, consider the month of September for the 15-yr period 1920 to 1934. For the years 1921, 1926, 1928, 1929, 1930, 1931, and 1933, the average value of B for September was 4.3 in., the average value of R for October was 0.8 in., and the average value of $(U - E)$ for October was (-1.5) in. For the years 1920, 1922, 1923, 1924, 1925, 1927, and 1932, the average value of B for September was 2.7 in., the average value of R for October was 0.7 in., and the average value of $(U - E)$ for October was (-1.9) in. Subtracting the different items, it appears that in the first period an average monthly excess of 1.6 in. of rainfall, B, produced an increase of 0.1 in. in surface run-off R in October, and an increase of 0.4 in. in underground flow U, for October. Since the value of U was computed for seven values of $(U - E)$ in years of high rainfall, and seven values in years of low rainfall, it is possible that the value of U contains some slight error because the values of E have not balanced. However, if the foregoing process is repeated a number of times, and the results combined for a final conclusion, the probability is that the final results will contain no appreciable error.

The foregoing method was used for various periods of months, and throughout the 15-yr period. For example, high and low rainfall for corresponding periods were compared, for periods varying from one month to twelve months, and the indicated effects on underground flow U were studied for the following months, up to twelve months. The indication is definite and consistent, that underground flow to the Lake increases after high rainfalls. This indication is clear in all of the Great Lakes, for the 15-yr period. It seems that the peak of underground flow into the Lake follows the high rainfall by about six weeks. It then decreases. The study indicated that the effect of a high rainfall on underground flow will not vanish completely for several months.

One calendar month was the unit of time used for all the basic data. Fifteen years was the total period considered for the four Great Lakes. The number of months considered was 720, or 180 months for each Lake. Considering the statistical methods used, the writer is of the opinion that his results cannot be considered to be absolutely accurate; but that they are mathematically excellent approximations, and the error of current engineering opinion about ground-water has been proved beyond a doubt.

In the Freeman report[35] certain reasons are given for believing that the actual rainfall may be greater than that indicated by U. S. Weather Bureau records. More than 100 stations were considered in the Lake Superior area, and most of them are not operated by Weather Bureau personnel. It is believed that most of the errors due to location and other causes would tend to decrease the quantity of rain reported. The writer has seen a rain gage that was dry after a light summer shower. The gage was warm, and the water evaporated almost as soon as it fell. The writer is of the opinion that the actual annual rainfall may have been 3 in. more than the value that was used in his study. The effect of this on the computations would be to add 3 in. to the land losses, without affecting the underground flow.

---

[35] "Regulation of the Great Lakes," by John R. Freeman, 1926.

Mr. Horton has published a summary[36] of engineering opinion about land losses. It is thought that the writer's conclusions about land losses are not inconsistent with Mr. Horton's statements and tables, if local conditions are considered; although the writer's data are slightly on the low side.

The area of Lake Nipigon, on the north shore of Lake Superior, estimated from a small-scale map, is about 1,600 sq miles. All other lakes in the Lake Superior drainage area are relatively small. It is thought that the water surface in the land-drainage area is not more than about 5 per cent. As far as is known, Lake Nipigon and the other lakes freeze in the winter and remain frozen about five months, so that evaporation from them is necessarily much less than from Lake Superior. The writer's data apply to Lake Superior, and they will not apply to localities where conditions are different.

The entire Great Lakes region was covered by glacial drift. Considerable old beach sand is in evidence in places. The writer has made numerous automobile trips covering areas around the Great Lakes; he has talked to geologists and mine operators; and he has found nothing to indicate the impossibility of a large underground flow.

The writer does not claim that he has said the final word, even about Lake Superior. He only hopes that he has offered evidence about a subject on which evidence is needed.

HAROLD C. HICKMAN,[37] JUN. AM. SOC. C. E. (by letter).—During the early stages of the investigation of the hydrology of the Great Lakes certain unexpected relations of negative yield became apparent, which, after making due allowances for errors of observations, could be explained only by unexpectedly large evaporation in winter and correspondingly smaller evaporation in summer. Mr. Freeman had given careful consideration to all available data on evaporation in his study of the hydrology of the Great Lakes, and advised further study of the subject.

Before experiments were begun the writer reviewed most of the work that had been done on evaporation to familiarize himself with conditions under which this work had been conducted, to determine if these conditions were comparable to those in the Great Lakes, and also to determine whether the results of this work could be used in arriving at a suitable and accurate value of evaporation from the lake surface. It was found that conditions affecting evaporation from the lake were quite different from those under which all available accurate data on evaporation from other large bodies of water had been obtained, and very different from the conditions around the small experimental tanks from which formulas and constants for estimating evaporation had been derived.

---

[36] "Hydrology of the Great Lakes," by Robert E. Horton.

[37] Junior Engr., U. S. Lake Survey, Detroit, Mich.

The areas of the water surfaces of the lakes are so large, and the distances across in the direction of the wind so great, that factors affecting evaporation under these conditions must be considered separately. The comparatively mild winters, cold damp springs, cool mild summers, and warm prolonged autumns, characteristic of the lake region, are evidence of the influence of the broad, deep expanse of these waters on climatic conditions. The climate and wind movement, the amount of moisture in the air (which largely affects both radiation loss by night and absorption of heat during the day), the depth of the water which tends to keep the water temperature (as in the case of Lake Superior) between 33° F in winter and 51° F in summer—all are factors in a particular evaporation problem.

The experiments reported in the original thesis[10] were not undertaken to present a new formula, nor to detract in any way whatsoever from the work that had been done previously in this field. They were conducted to determine the best available value for monthly evaporation from the surface of the Great Lakes, using only data which the writer considered as accurate and as having a direct bearing on the rate of evaporation, and excluding from consideration data over which controversy might arise. In the discussions certain objections were raised to the methods used in arriving at a monthly value of evaporation based on water temperature, air temperature, and wind velocity, to the exclusion of relative humidity. Mr. Cooper presents data which show that, apparently, evaporation in the Tennessee Valley was influenced by both relative humidity and wind velocity, but with greater effect being due to relative humidity. He also states that, in his opinion, any revision of current opinion as to the effect of wind on evaporation should be toward giving less weight to wind and more to relative humidity. The writer spent considerable time studying meteorological conditions in the locality in which Mr. Cooper's experiments were conducted and found that, in no degree, did they approach conditions existing in the lake region.

The Tennessee stations were equipped in accordance with U. S. Weather Bureau "Class A" evaporation specifications. The principal objection to this type of installation lies in the rapid changes of pan temperatures. The water temperature tends to follow that of the air and for the purpose of an investigation of evaporation from large cold bodies of water is entirely unsatisfactory. Data obtained with equipment of this type do not make allowance for the large differences between water temperature and air temperature so characteristic in the lake region. It is quite conceivable that the Tennessee experiments would show that evaporation is influenced more by changes in relative humidity than wind velocity when the equipment used and the conditions under which the experiments were conducted are considered. An air mass passing over a body of water at approximately the same temperature as the air will absorb water vapor in quantities depending upon the saturation deficit of that air. At low wind velocities it is evident that the quantity of water evaporated under these conditions would be influenced more by relative humidity than by the velocity of the wind.

In the lake region the air temperature is influenced by the temperature of the lake water at all times. The temperature on shore may be 80° and 5 miles out in the lake it may be 60°. During a large part of the year "thermal" winds blow off the lake. In the winter months a cold dry mass of air, passing across a lake that does not freeze over, tends to "warm up" and consequently the carrying capacity is increased. Under these conditions evaporation would go on during the winter months contrary to all preconceived ideas. That this carrying capacity does exist in winter is evident in the snow belt on the south shore of Lake Superior and Lake Ontario. It is likely that evaporation under these conditions is influenced more by the wind than by relative humidity, as reported by the Weather Bureau Stations around the lakes, and that these relative humidity data cannot be used as a means of determining evaporation from the Great Lakes.

Mr. Cooper states that a few relative humidity readings during the day will not give the average value for that day except in isolated instances. It will be noted that the writer stated clearly that no evidence could be found wherein the amount of evaporation varied with either the relative humidity as read at the station or as determined at Weather Bureau Stations in the vicinity. Mr. Cooper suggests that, because the daily average value was difficult to obtain, it was neglected. It might be well to point out that tests to determine the effect of this factor were conducted extensively before actual experiments began. The average value for the day was obtained by averaging the hourly readings throughout each 24-hr period. Although a number of daily tests were made, four typical separate days may be noted to show the effect of humidity. The first set of two days had an average water temperature for the day of 52° and an average air temperature of 50°. The second set had an average daily water temperature of 56° and an air temperature of 64°. The daily wind velocity for the four days was 5 miles per hr. The evaporation for the days, under the first set, was 0.124 and 0.128 in. with average daily relative humidities of 67 and 85 per cent. The evaporation for the second set was 0.058 and 0.055 in. with respective relative humidities of 73 and 81 per cent. In a number of other instances evaporation continued during periods of complete saturation under a strong wind. Evidence of this kind was found throughout the study and it seems, in the light of these facts, that relative humidity is not as important as hydrologists have generally believed.

The data presented by Mr. Cooper are confined to air temperatures between 40° and 46° and water temperatures between 44° and 50° with wind velocities ranging from 0.5 to 1.5 miles per hr. It would be interesting to see what results could be obtained by including the data from a number of stations with air temperatures ranging from 10° to 95° and with water temperatures from 33° to 75° with wind velocity up to 22 miles per hr.

Although evaporation was found not to be related to relative humidity as reported at the U. S. Weather Bureau Stations, these stations, when the average monthly values were compared from year to year, did show a certain degree of consistency. The average monthly relative humidity for the years from 1920 to 1934 for the Superior Basin was computed by averaging the monthly means as reported by the Weather Bureau Stations at Sault Ste. Marie, Mich.;

Marquette, Mich.; Houghton, Mich.; Duluth, Minn.; and Port Arthur, Ont., Canada.

Table 9 shows the averages of relative humidity for the Superior Basin. Although these data should not be construed as representing open lake conditions they do indicate what changes occur from season to season. Note the regularity of descent from a high in January to a low in May and the progressive increase back to a high in December. If relative humidity does influence evaporation it must vary directly with evaporation in the lake regions as indicated in Table 9 and inversely as shown by Fig. 8 in Mr. Cooper's discussion of the Tennessee Valley.

TABLE 9.—RELATIVE HUMIDITY, LAKE SUPERIOR

Year	January	February	March	April	May	June	July	August	September	October	November	December	Mean
1920	92	86	82	78	64	76	74	76	78	78	85	92	80
1921	91	90	85	72	72	75	76	79	79	81	86	87	81
1922	89	87	82	80	76	75	76	78	81	80	86	86	81
1923	87	84	84	73	66	72	76	77	82	76	80	83	78
1924	87	85	80	74	62	70	76	80	83	82	82	86	79
1925	86	86	79	64	64	75	77	78	82	80	81	86	78
1926	87	86	82	70	66	70	76	80	86	81	85	89	80
1927	87	85	79	71	76	73	77	75	79	79	86	88	80
1928	85	85	78	73	66	74	81	83	84	84	78	83	80
1929	85	86	82	72	68	71	72	73	77	76	80	88	77
1930	86	88	81	71	71	72	77	64	77	84	81	88	79
1931	87	84	82	70	72	76	74	74	81	79	80	82	78
1932	85	85	82	69	65	74	74	78	75	80	83	86	78
1933	84	86	82	72	68	69	73	70	74	80	86	89	77
1934	89	87	82	80	62	72	70	70	83	..	..	..	..
Average	87	86	81	73	68	73	75	76	80	80	83	87	79

It is believed that a seasonal variation in relative humidity, which is rather uniform from year to year, does exist in the lake region. It does not follow, however, that because of this fact evaporation is the same from year to year.

The objections stated by Mr. Meyer were confined to the data upon which the final conclusions were based. He states that no records were on file[10] which even reasonably simulated conditions prevailing on Lake Superior from March to August and proceeds to show that for the summer months of April to September the average daily water temperatures in the pan were from 6.6° to 21.1° higher than those shown by the open lake water temperatures curve and upon which the values of monthly evaporation in the Pettis analysis were based.

The paper describes clearly the procedure used in analyzing the data from the various stations and, as stated, these data are on file in the Engineering Societies Library. The thesis states that[10]

> "one group of 44 readings—11 from Duluth, 14 from Kewaunee, 10 from Detroit, and 9 from Buffalo,—where the average daily air temperature of each reading was between 40° and 45° and where the average daily water temperature was between 45° and 50°, were included in this group. The centroid of these 44 readings was found and plotted, and the average of these readings assigned this point. The average wind velocity of the group was found to be five miles per hour."

When a method of analyzing data of this kind is used, any departure of one station from the average water temperature curve for any one month does not exclude the station entirely from consideration.

In the original analysis, 185 observations, a good portion of which were obtained from the Kewaunee Station, were used, where the water temperatures ranged from 35° to 40°, and air temperatures from 10° to 45°; 193 readings with the water from 40° to 45° and the air from 10° to 50°; 151 readings where the water temperature was between 45° and 50° and the air from 20° to 50°; and 100 readings ranging from 50° to 55° for the water and 35° to 70° for the air. It is believed that this range of air and water temperature does simulate conditions prevailing on Lake Superior.

It must be admitted that the control of the pan water temperature was not all that could be desired. It was extremely difficult to obtain this control, particularly during the summer months, as funds for mechanical refrigeration were not available. The various stations had to depend upon cold well water which did not provide sufficient cooling effect except at Kewaunee. The experiments have been continued with better efficiency and the indications are that the ever-increasing number of daily observations will strengthen the curves materially and add to their range.

It is believed that the analyses of these experiments in conjunction with the air-temperature and water-temperature curves in the paper do give a much more satisfactory value for monthly evaporation from the Great Lakes than any other method or formula presented heretofore.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

TRANSACTIONS

Paper No. 2075

LARGE CORE DRILLS AID CONSTRUCTION AT CHICKAMAUGA DAM

By James S. Lewis, Jr.,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. J. B. Newsom, J. G. Tripp, James B. Hays, O. N. Floyd, and James S. Lewis, Jr.

Synopsis

The use of large core drills for the purpose of investigating foundations, and for supplementing and verifying the information obtained with small borings, is becoming increasingly common in conjunction with the construction of heavy hydraulic structures. However, the use of these large drills as a direct means of furthering the ends of construction is somewhat unusual, and this paper contains a description of the methods that were successfully used, under difficult working conditions, to obtain a foundation for the upper guard wall of the Chickamauga (Tenn.) Project navigation lock. The equipment and its operation are described in detail, and cost and progress information is included.

Introduction

The preliminary borings, made for the purpose of investigating the site of Chickamauga Dam, near Chattanooga, Tenn., indicated that unusual foundation difficulties would probably be encountered in part of the area covered by the structure. The subsequent excavation in the lock area revealed, however, that with the exception of the up-stream end of the river guard wall, the lock walls could be rested upon ledge rock without unusual difficulty. Under the upper end of this wall, the rock was covered by a heavy layer of over-burden through which it was both unsafe and uneconomical to attempt to excavate, and it was decided to support the piers carrying the wall upon reinforced concrete columns extending through the over-burden and weathered limestone to sound rock.

The guard wall (see Fig. 1) consists essentially of a number of cantilever sections that are supported by piers. Each pier supports a section 42 ft long, 20 ft deep, and 8 ft thick that spans half the distance to the adjacent piers.

Note.—Published in June, 1939, *Proceedings*.

[1] Asst. Constr. Supt., Watts Bar Dam, TVA, Spring City, Tenn.

From its junction with the lock wall, the guard wall is 264 ft long. Each pier is 14 ft by 40 ft in plan at the base, and the side away from the navigation channel is battered so that the top width of the pier equals the thickness of the cantilever wall. This description does not apply to the nose pier, which is rounded on the up-stream side in a conventional manner.

Fig. 2 shows the general plan of the lock, and this structure, with the exception of Piers 27 through 32, was founded directly upon the Chickamauga limestone that composes the massive geologic formation of the region. The information that had been obtained from the early borings caused considerable suspicion with regard to the area to be covered by the guard wall, and additional borings were made when the lock coffer-dam was unwatered. Holes were drilled at the four corners of each of the proposed pier locations, and it was learned that the apprehension that had been felt was fully justified. The over-burden in the area of the piers was found to be more than 60 ft thick in places and was composed of soft clay containing numerous boulders of varying size. Heavy flows of water were not generally encountered in the over-burden, but it was not entirely impervious, and the saturated condition of the material resulted in a lack of stability that would have threatened the safety of any open excavation that might have been attempted.

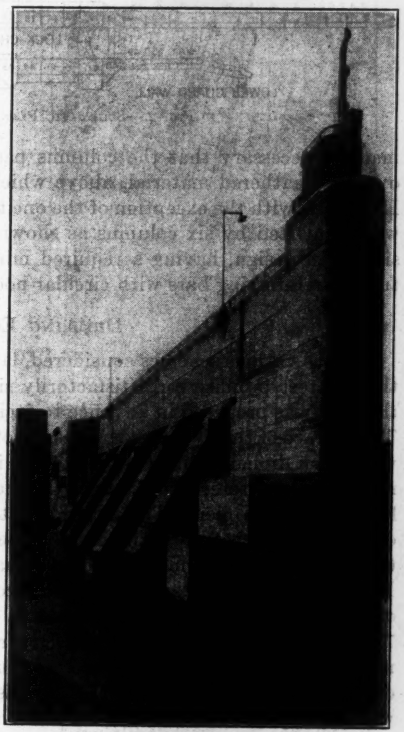


Fig. 1.—Completed Guard Wall

The surface of the underlying ledge rock was of the extremely irregular contour that is typical of weathered limestone and was characterized by frequent pinnacles, pot-holes, and deep, clay-filled crevices. The ledge rock itself was characterized by the existence of extremely sharp folds and numerous faults, and contained well-defined layers of shale and bentonite. The latter was comparatively soft, having little bearing strength when unrestrained, and was frequently found in contact with the shale. Numerous seams and weathered joints and fissures existed to afford passage for the ground-water.

In consideration of the uncertainties and difficulties attendant upon securing a satisfactory foundation of a conventional type under the conditions that existed, reinforced concrete columns, extending through the over-burden and weathered limestone to bear upon sound rock, were chosen as the most economical and satisfactory method of supporting the piers. Sound rock was usually found below a generally prevalent stratum of shale and bentonite that formed a very irregular bed below which the ledge rock was comparatively free from large seams and openings. However, the sharp folding in some areas made it necessary that the columns pass through layers of sound rock that overlay weathered material, above which it was not considered wise to stop. Each pier, with the exception of the one that formed the nose of the guide wall, was supported by six columns as shown in Fig. 3. All of the columns were similar in design, having a required minimum diameter of 36 in., and longitudinal reinforcing bars with circular hoops.

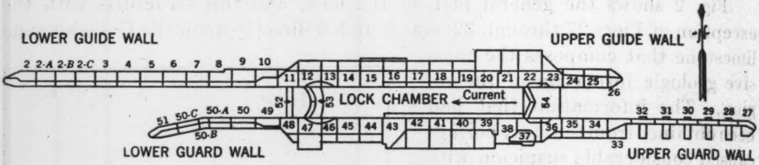


Fig. 2.—Schematic Plan; Chickamauga Lock

Drilling Equipment

Of the various schemes considered, large core drills were selected as offering the greatest promise for satisfactorily sinking the holes for the columns, and the area was prepared in advance to facilitate the operation of the equipment. The surface of the over-burden was so soft and uneven that working conditions were very difficult and, before the drilling was started, a rectangular slab of reinforced concrete, of the size of the pier and 3 ft thick, was poured over the location of each pier. This slab was intended to afford a firm support for the drills as well as better footing for the operators and to prevent the loss of small tools and parts that might otherwise have become covered by the soft mud. Four large drills were used during the course of the work, three of which were originally designed for 36-in. holes, and one of which was designed for drilling 72-in. holes. In order to drill the larger holes required for obtaining the minimum diameter at the bottom of the columns, it was necessary to install 40-hp drive motors to replace the 15-hp units with which two of the smaller drills were equipped. The third 36-in. drill was driven by a gasoline motor. It was found that neither type of driving power possessed any decided advantages under ordinary circumstances. The 36-in. drills were mounted on skids and weighed approximately 3 tons each when fully equipped. They could be moved with the hoist on the drill or with a crane or tractor.

The fourth machine was designed for drilling 72-in. holes and was of much larger size and heavier construction throughout. Instead of being mounted on skids, it was equipped with six wheels designed to roll on rails 8 ft apart, and it was necessary to place track over the location of any hole to be drilled. The spindle was in the center of the machine instead of in the customary position at the end and, as a result, the hole was entirely covered by the drill when it was in the operating position. To remove the tools and core, it was necessary to

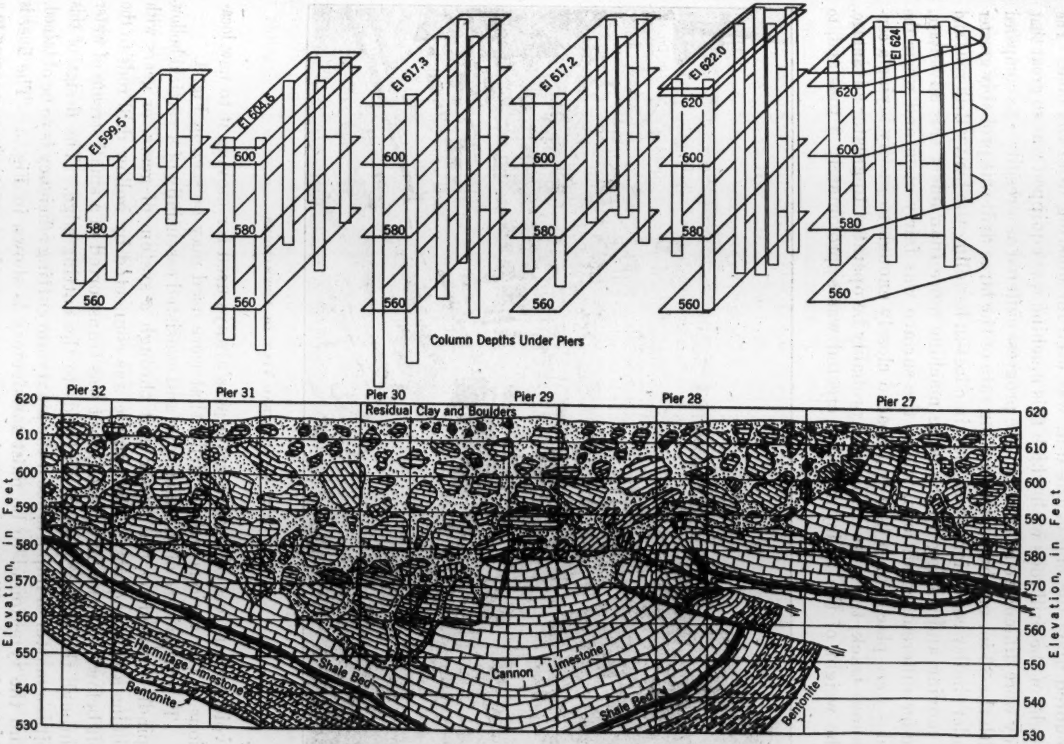

El 599.5
El 604.5
El 617.3
El 617.2
El 622.0
El 624
620
600
580
560
Column Depths Under Piers
Pier 32
Pier 31
Pier 30
Pier 29
Pier 28
Pier 27
Residual Clay and Boulders
Hermitage Limestone
Bentonite
Shale Bed
Cannon Limestone
Shale Bed
Bentonite
620
610
600
590
580
570
560
550
540
530
Elevation, in Feet

FIG. 3—GENERALIZED GEOLOGIC SECTION UNDER UPPER GUARD WALL

roll the drill back upon the track until the hole was clear, when the derrick mounted upon the end of the unit was used for hoisting from the hole. The weight of the drill rods with which this machine was equipped was so great that they were difficult to handle and the progress suffered as a result. As compared with the 36-in. machines, the cutting rate of the large drill was probably greater under similar conditions; but, the time lost in handling the heavy drill rods and in moving and setting up the large machine more than offset this advantage. Under different conditions, where the surface was fairly even and where the holes were placed on a line and spaced closely enough to justify laying a continuous track, the large drill could probably be operated to greater advantage. The weight of the 72-in. machine erected was approximately 9 tons and, to avoid serious loss of time, a crane was always used for moving it to new locations. The various sizes of drills that were used may be seen in Fig. 4.

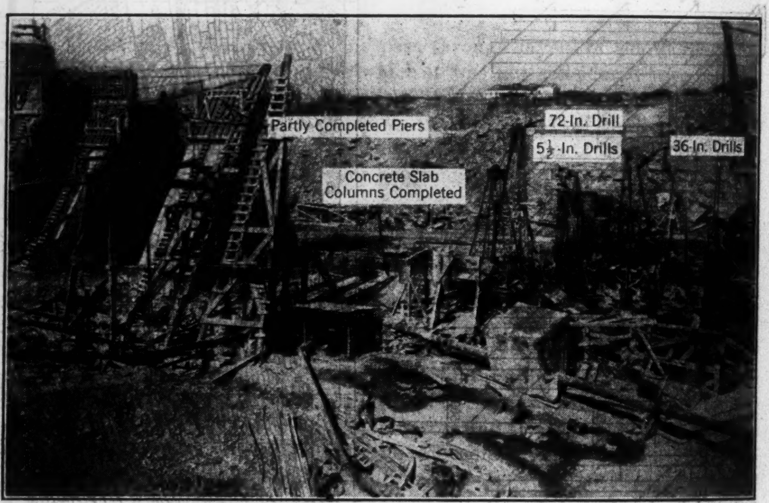


Fig. 4.—General View of Operations

The drills were of the shot type and consisted essentially of a vertical hollow spindle rotated by the power unit through a system of gears, or gears with chains and sprockets. The spindle was connected by hollow drill rods to the bit below. As the bit rotated, shot was transported in a small stream of water through the hollow spindle and rods to the cutting edge. The design of this bit is a matter of importance if the maximum cutting efficiency is to be obtained, and the type found to be most satisfactory is shown in Fig. 5. The liner is desirable for drilling in deep holes, as it prevents the bottom from wearing to a sharp edge under which shot will not remain. Furthermore, as the holes become deeper and the weight of the tools increases, the additional width of the cutting edge prevents the shot from being forced aside, where they are ineffective. The core is cut smaller, giving it less tendency to bind and block in the barrel, and the offset formed by the top of the liner occasionally grips pieces of loose core which are brought to the surface with the bit. The liner is not needed when the holes are shallow. The shot-spreader plate deflects the shot outward so that they fall under the cutting edge rather than on top of the core projecting upward in the bit.

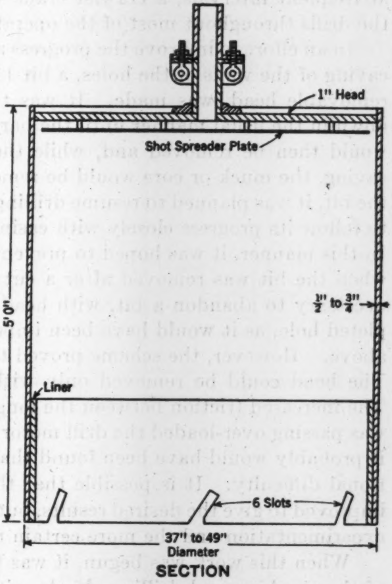


Fig. 5.—Typical Shot Bit for Large Holes

It was required that the columns should have a minimum diameter of 36 in. and, to insure that this size would be obtained at the bottom, it was necessary to begin the holes much larger at the top. The bits varied in outside diameter from 49 in. to 37 in., decreasing in 2-in. increments, and the casing varied from 48 in. to 36 in. inside diameter, also decreasing in 2-in. increments. As shown in Fig. 5, the thickness of the plate from which the bits were rolled varied according to the diameter. The casing was rolled from ¼-in. plate and was purchased in sections 10 and 15 ft long, with a few 5-ft sections. By standardizing upon the sizes indicated, the bit in use always had a clearance of ½ in. when centered in the smallest casing through which it was necessary for it to pass. A clearance of ¼ in. was found to be insufficient, as it was impractical to keep either bit or casing perfectly round, and trouble frequently resulted when the bit could not be passed through the casing.

Operation of Drills

When a bit full of core had been cut, or conditions in the hole made it desirable to stop drilling, the tools were removed so that a man could be lowered to the bottom. Ordinarily, to cut more than 3 ft without removing the bit was considered fair; and a run of more than 4 ft was unusual. After removing the tools, a jackhammer hole was drilled in the center of the core and a pin with hoisting ring was wedged tightly in the small hole. A light charge of dynamite was then placed in the groove on one side of the core and the man was removed from the hole; a strain was taken upon a hoisting line fastened to the pin wedged in the core and the dynamite was discharged. The core could then frequently be raised to the surface with the hoist. If the core did not break at the bottom, so that all of it could be removed in one piece, it was necessary to fire another charge of dynamite beside the remaining core. It was found that a light charge that just sufficed to crack the core gave better results than a charge heavy enough to shatter the rock and make its removal in fragments necessary. Since it was necessary to handle the drills, casing, bits, and cores at frequent intervals, a crawler crane was kept in constant attendance upon the drills throughout most of the operation.

In an effort to improve the progress and to reduce the trouble resulting from caving of the walls of the holes, a bit 11 ft long and 42 in. in diameter, with a removable head, was made. It was thought that this bit could be rotated down in the usual manner until the barrel was filled. The head and drill rods would then be removed and, while the bit supported the side walls against caving, the muck or core would be removed from the inside. After emptying the bit, it was planned to resume drilling without removing it from the hole and to follow its progress closely with casing to insure against caving from above. In this manner, it was hoped to prevent the caving which frequently occurred when the bit was removed after a cut had been made. It would have been necessary to abandon a bit, with head removed, in the bottom of each completed hole, as it would have been impossible to remove it through the casing above. However, the scheme proved to be unsatisfactory for several reasons. The head could be removed only with the greatest of difficulty and delay. The increased friction between the long bit and the material through which it was passing over-loaded the drill motor. If sufficient progress had been made, it probably would have been found that the setting of the casing caused additional difficulty. It is possible that this idea might have been modified and improved to give the desired results, but the pressing schedule precluded further experimentation and the more certain methods were again adopted.

When this work was begun, it was thought that, by following the progress of the mucking and drilling closely with the casing, difficulties resulting from caving of the walls of the holes might be avoided. However, soon after operations were begun, it became evident that the persistent caving of the walls, where the saturated and unstable over-burden was penetrated, was delaying progress seriously, and the heavy flows of water that were encountered in the rock made a complete cessation of drilling necessary at times. The nature of the material composing the over-burden made it impracticable to lower the water-table in the area by pumping from deep holes drilled for that purpose, and it was necessary to take measures to exclude the water in the immediate vicinity of each hole.

ADVANCE GROUTING TO EXCLUDE WATER

Advance grouting in stages was chosen as the system offering the greatest promise of consolidating the over-burden and sealing the openings in the rock satisfactorily, and 5.5-in. shot core holes were drilled and grouted in the center of each column location before the large holes were started. These small holes were grouted at the completion of each stage of drilling so that every stage except the last was subjected to repeated injections of grout under pressure. The depth of the various stages was not determined in advance but depended upon the nature of the material encountered. When it was necessary to penetrate a considerable thickness of over-burden to reach rock, the grouting was accomplished in several stages. The first would be drilled down probably 10 ft and, if the material penetrated was soft, loose, caving, or otherwise unstable, the drill tools were withdrawn, and a 1-in. pipe was inserted to the bottom of the hole. The hole was then grouted through the circulating arrangement shown in Fig. 6. By grouting through the pipe extending to the bottom, the grout in the hole was kept alive, or liquid, at all times, and material that caved from the sides was prevented from permanently blocking off any part of the hole. The pressure in the hole was regulated by the valve in the return line, and a stuffing box in the top of each header allowed the 1-in. pipe to be kept free by raising and lowering it at frequent intervals while grouting. The pipe was composed of 3-ft sections so that, when sticking was threatened, it could be raised and a section removed with only a monentary interruption of the flow. In this manner, the pipe was removed slowly, 3 ft at a time, until the hole refused to take more grout. An effort was made to grout to refusal at a pressure

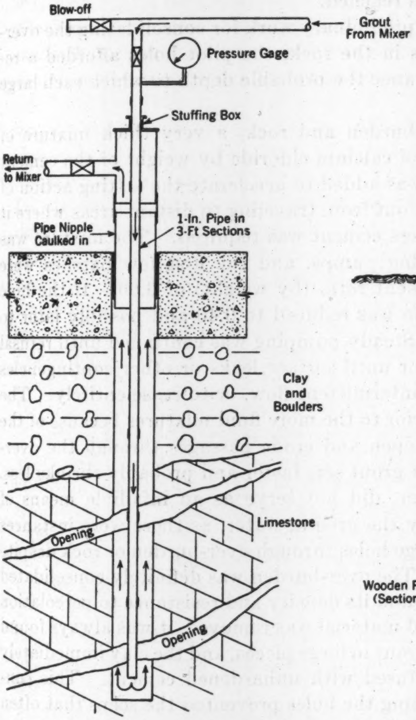

Blow-off
Grout From Mixer
Pressure Gage
Stuffing Box
Return to Mixer
1-In. Pipe in 3-Ft Sections
Pipe Nipple Caulked in
Clay and Boulders
Limestone
Opening
Opening

FIG. 6.—FULL CIRCULATING ARRANGEMENT FOR STAGE GROUTING OF OVER-BURDEN AND ROCK

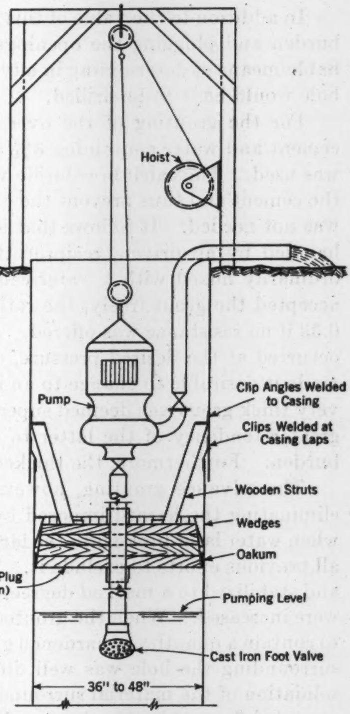

Hoist
Pump
Clip Angles Welded to Casing
Clips Welded at Casing Laps
Wooden Struts
Wedges
Oakum
Wooden Plug (Section)
Pumping Level
Cast Iron Foot Valve
36" to 48"

FIG. 7.—INSTALLATION OF WOODEN PLUG FOR GROUTING THROUGH LARGE HOLES

which varied between 25 and 50 lb per sq in. in the over-burden and as much as 100 lb per sq in. in rock. When surface leaks occurred, the rate of pumping was reduced; empty sacks were packed into any large openings and covered with sand bags. This usually retarded the flow sufficiently to allow the grout to harden and plug the leak. When the first stage was grouted, drilling was resumed through the same hole and extended to such depth as was determined to be suitable for the second stage. The advance grouting was thus carried down in each column location until the cores recovered from the pilot holes disclosed that sound rock had been reached.

In addition to the value of this preliminary work for consolidating the over-burden and plugging the openings in the rock, the pilot holes afforded a reliable means of determining in advance the probable depth to which each large hole would have to be drilled.

For the grouting of the over-burden and rock, a very thick mixture of cement and water containing 3% of calcium chloride by weight of the cement was used. The calcium chloride was added to accelerate the setting action of the cement and thus prevent the grout from traveling to distant areas where it was not needed. It follows that less cement was required. The mixture was handled by air-driven, reciprocating pumps, and the first few batches were ordinarily mixed with a water-cement ratio (by weight) of 0.66. If the hole accepted the grout freely, the ratio was reduced to 0.40 and possibly later to 0.33 if no resistance was offered. Steady pumping was continued until refusal occurred at the desired pressure, or until surface leaks or other contingencies made it desirable to change to an intermittent flow, or to cease entirely. The very thick grout was deemed superior to the more fluid mixtures because of the greater tendency of the latter to open and erode passages through the over-burden. Furthermore, the thicker grout sets faster and probably shrinks less.

The advance grouting, however, did not serve as an infallible means of eliminating the hazard imposed by the ground-water, as there were instances when water later broke into the large holes through over-burden or rock despite all previous efforts to exclude it. The over-burden was definitely consolidated and stabilized to a marked degree, and its density and resistance to percolation were increased. When the grouted material was removed, it was always found to contain a quantity of hardened grout in large pieces, and the clay immediately surrounding the hole was well diffused with unhardened cement. This consolidation of the material surrounding the holes prevented the seeps that often preceded flows of large volume. Following the advance grouting, it was generally true that less trouble from water was encountered in the over-burden than in the rock. This is probably best explained by the fact that in the rock the water followed well-defined channels, whereas in the over-burden it simply caused a generally saturated condition. Thus, when the water followed a narrow vertical fissure or a channel that passed just within the boundary of the large hole, the small hole for advance grouting might fail to intercept the water bearing opening. When the large drill cut into this ungrouted source, of course, the hole was flooded quickly. It was then necessary to regrout the hole by means of a wooden plug which will be described subsequently. Judged as a whole, the advance grouting was quite successful and improved conditions to an extent that made the work possible with the methods used.

DRILLING AND MUCKING

Following the completion of the grouting of any location, a large drill was placed upon the concrete working slab and the hole was started with a 49-in. bit. To avoid confusion, bits were designated by the outside diameter, and casing was designated by the inside diameter. Although all bits were 5 ft long, it was only with rare good luck that sufficient core could be cut to fill the barrel. Blocking of the core, caving of the wall of the hole, or other trouble, usually made it necessary to remove the tools before a full barrel had been cut. A sound, homogeneous rock would no doubt make the use of a longer bit desirable and would result in improved progress by reducing the frequency with which it was necessary to remove the core. The hole was drilled through the concrete slab and into the over-burden with the 49-in. bit until the walls of the hole appeared to be in danger of caving. This distance was ordinarily 10 ft or less. In the boulder-strewn over-burden, the bit would cut a clear opening of the desired size until the 5-ft length was full, or until blocking or binding made it necessary to stop. The bit was then removed and the muck was excavated by hand and brought to the surface by the air hoist on the drill. The removal of the muck ordinarily required more time than the actual drilling as the limited space allowed only one man to work in the hole at any one time. If, after removing the muck to the depth to which the bit had cut, an examination disclosed that the walls were in danger of caving, the exposed part was cased and the size of the bit was reduced for the next cut.

Walls that were apparently safe against caving would sometimes fall in when drilling was resumed and swirling drill water eroded and saturated the standing material. As it was necessary to pump the water from the hole each time that core or muck was to be removed, any seepage through the over-burden tended to increase when the head opposing it was removed. Two-stage, air-driven, centrifugal pumps were used for unwatering the holes as their light weight and compact construction gave them the advantage over other types. The leakage tended to increase as the passages through which the water was flowing were eroded to a larger size, and sudden blow-ins of sufficient volume to interrupt the drilling seriously were not infrequent during the early stages of the program.

In an effort to prevent the blow-ins, the space between the casing and the wall of the hole was well packed with dry cement or dry mortar as each piece of casing was set. The size of the space to be filled varied from practically nothing to several inches, and the type of material that was to be used for packing was determined by the size of the opening. Quick-setting cement was used when the volume of leakage indicated its desirability. It was found that the tendency of small leaks to increase to damaging proportions was eliminated if this space behind the casing was properly packed. When the drilling was in sound rock, the hole was not cased and no unusual precautions were necessary.

Frequently the drill was removed as soon as the bit had cut through the concrete slab, and a light bent, on which an air hoist was mounted, was then placed over the hole. The over-burden was excavated by hand and hoisted to the surface in buckets, casing being set as required. When ledge rock was reached, or when large boulders were encountered, excavation was stopped and it was necessary to wait until a drill could be replaced upon the hole. The progress made by this method, especially when numerous boulders were encountered, was not equal to that obtained with the core drills but, without increasing the unit cost, it permitted the work to proceed simultaneously on a larger number of holes than would have been possible if only the drills had been used. By constantly shifting the drills to the places where they could operate to best advantage, the progress was aided materially.

HANDLING HEAVY FLOWS OF WATER

When heavy flows of water were encountered, despite the precautionary measures taken to prevent them, it was necessary to stop drilling and to move the drill immediately to another location to avoid loss of time. The volume of flow, as judged by the pumping capacity required to handle it, varied from several hundred to several thousand gallons per minute, and prolonged pumping usually resulted in appreciably increased flow. Of the various methods that were tried for stopping these flows so that drilling might be resumed, only one proved entirely satisfactory. First, after striking a heavy flow, a 4-in. centrifugal pump was placed in the hole to learn the depth to which the surface of the water could be lowered by pumping. The size of the hole at this depth was then ascertained and the pump was removed. A laminated, circular, wooden plug, about 12 in. thick, and tapered from botton to top, was made of 3-in. boards, with tar-paper between the layers to reduce leakage. The plug was tapered in order to form a groove for calking against the steel casing, and the diameter was such as to permit the plug to just pass into the hole at the depth at which it was to be set. A 4-in. pipe passing through the center of the plug was connected to the suction of a motor-driven centrifugal pump to form the assembly shown in Fig. 7. This assembly was suspended from a small bent and lowered into the hole with the pump running. A valve in the discharge of the pump permitted regulation of the flow to avoid loss of prime, and this valve was gradually opened as the volume of the incoming water increased with the lowering of the level in the hole. When the plug had reached the intended position, the V-shaped groove formed between the plug and the casing was calked tightly with oakum backed up with wooden wedges. During the calking, and until the plug could be firmly braced against uplift, it was necessary to keep the pump in operation. The total static pressure acting to lift the plug was frequently in excess of 15 tons, and it was necessary to brace to the casing and to weld together, at the laps, the individual pieces of casing above the plug to help carry the load.

When the calking and bracing were completed, the valve in the suction was closed and the pump was removed. The pipe passing through the center of the plug was then extended to the surface and the valve was opened so that the cast-iron foot valve could be broken. A few blows with a heavy rod of reinforcing steel inserted through the suction pipe usually shattered it so that it would offer no obstruction to the passage of grout. A pump was then connected to the pipe so that grout could be forced below the wooden plug and back into the passages through which the water had entered. A small, air-driven, centrifugal pump was always placed on top of the plug while grouting to remove any water or grout that might leak from below and to permit any additional calking that became necessary.

For grouting below the plug, an extra thick mixture with a water-cement ratio varying from 0.40 to 0.33, and containing 3% of calcium chloride by weight of the cement, was used. Once started, the grout was pumped continuously and slowly until no more could be injected at a pressure that exceeded the static head by approximately 20 lb per sq in. The pumping rate and pressure were held down in an effort to avoid wasting the grout by forcing it to distant areas where it was not needed. The calcium chloride, by accelerating the set of the cement, reduced the quantity of grout required to plug the water passages and permitted a resumption of drilling almost immediately upon the completion of the grouting. It was necessary to remove the plug by hand, and the grout filling the hole below it was either mucked out by hand, using paving breakers, or was cored out with a drill. Better progress resulted from the use of the core drills. In every case where the methods that have been described were used, the water was stopped successfully so that no further trouble was experienced in the hole, unless other water-bearing passages were intercepted at a lower elevation. In a number of cases, the water (as evidenced by the quantities of leaves and trash brought in) was entering through passages that extended to the river-bed outside of the coffer-dam. With the normal seasonal flow, the river water stood about 25 ft above the average elevation of the tops of the holes, which meant that it was necessary to work against an appreciable head when water was struck in the lower part of a hole.

It has probably become apparent that the greatest difficulties that arose in sinking the holes were encountered in the over-burden and in unsound, water-bearing rock. Where the rock was sound, and excess water was not encountered, the work became more or less routine. Sound rock was cut smoothly, with a minimum of trouble from blocking and binding, and the core was usually removed in a single piece without difficulty.

When the shale and bentonite bed previously described was passed, or sound rock was apparently reached, a jackhammer hole was drilled 8 or 10 ft into the bottom of the hole to learn whether any openings existed in this zone. A rod of 0.5-in. tool steel, with a 90° hook on the lower end, was used to feel for seams in the small hole. If the rock below was sound or contained only small seams, the large hole was extended no deeper and a pipe was calked into the jackhammer hole to permit the grouting of any small seams later. If the dip or folding of the rock caused the bottom of a hole to slope steeply, it was leveled roughly with a jackhammer.

COLUMN CONSTRUCTION AND TOLERANCES

Upon the completion of drilling, a hole was always filled with concrete as quickly as possible to avoid the constant threat of increasing leakage.

The reinforcement consisted of sixteen parallel steel bars 1 in. sq, pre-fabricated into 32-in. cages not more than 40 ft long. These cages were placed in the holes with a crane and then carefully braced in a position as nearly plumb as possible. A variation not exceeding 1.5 in. from the vertical was allowed when necessary. It was practically impossible to keep the large holes plumb while they were being drilled, as any number of conditions may cause them to turn, and it was necessary to check at frequent intervals so that any deviation might be corrected before it was too late.

A hole could sometimes be plumbed by bracing the drill rods to one side, well above the bit, so that they were thrown into a bow. This forced the bit to lean in the desired direction and the hole could occasionally be righted. More certain results were obtained by reducing the size of the bit sufficiently so that a smaller hole could be started properly. This method was found to be the best if the reduction left enough margin to insure that it would be possible to complete the hole at the required size.

After the reinforcing steel was plumbed and fastened rigidly in position by short steel bars welded to the cage and bearing against the casing, it served as a useful ladder extending the full depth of the hole. This ladder was used to advantage by the workmen while calking the numerous small leaks that usually spouted through the casing joints and for running grout pipes to any openings known to exist behind the casing. These pipes were either connected to nipples welded over holes burned through the casing, or were flattened and forced into any space left where the sections of casing lapped. Despite the efforts made to fill the space behind each section of casing, the material was occasionally washed out by the inflowing water. The grout pipes were placed in order that these spaces might be filled after the columns were poured, affording all possible lateral support for the full length of the column.

As wide intervals of time usually elapsed between the pouring of the first and the last of the columns supporting any one pier, it was feared that the varying degrees of shrinkage, which would have resulted when the columns were loaded, would result in an unequal distribution of stresses in the pier, and cooling pipes were installed to dissipate the heat generated by the setting concrete. The cooling system consisted simply of two vertical pipes connected at the bottom to form a U, through which cool water was forced after the concrete had attained its final set. Resistance thermometers embedded in the columns indicated that a decided drop in temperature resulted when water was circulated.

To fill the holes, concrete was dumped from 3-yd buckets at the surface into a small, steel hopper, with a short guide chute, mounted directly over the hole. Vibrators were used to obtain a proper density of the concrete and to insure that the steel would be well covered for the full length of the column.

Although the columns were 49 in. in diameter at the top where they passed through the slab, the diameter of the upper 18 in. was reduced to 36 in. with a pipe form. Before the first lift of the pier was poured, a cushion of bags was placed upon the offset formed at the reduction. This was done to avoid loading the columns eccentrically, as it was rarely possible to plumb the steel reinforcement so that it was concentric with the upper part of the hole. It was not intended that the slab that served as a working platform for the drills should carry any of the load imposed by the structure.

SPECIAL TREATMENT OF NOSE PIER

The methods that have been described are those which, in general, were used for sinking the columns that support the piers carrying the guard wall. In the case of the nose pier (Pier 27, Fig. 3) it was necessary to vary the procedure that was followed in pouring the working slab for the drills. The excavation of the over-burden at this pier was begun before it was known that it would be necessary to support all of the piers upon columns, and it was soon learned that great irregularity of the rock surface characterized the area. The surface of the rock was found only a few feet below the surface over part of the area of the pier; later it was found to lie as much as 50 ft below the surface only a short distance away. Removal of the first few feet of over-burden released a number of heavy flows of water in the area of the pier, and it was impossible to determine whether it issued from passages in the rock or was traveling in a bed of gravel overlying the rock. Although steel sheet-piling was driven around the pier, the flow continued undiminished, and the great irregularity of the depths at which the piling stopped indicated that the water was probably issuing from crevices and joints through the ledge rock. The total flow into the sheet-pile enclosure was approximately 12 000 gal per min, and it was obviously not feasible to attempt to rest the pier directly upon rock.

In order to obtain a concrete slab upon which the drills could be operated, the area of the nose pier was cleaned up with clam shell and hand labor, and boxes were built around the suctions of three pumps that were left in the enclosure to handle the water. One other pump removed water from a sump excavated just outside of the enclosure. From one particularly bad boil, a wooden-box culvert was carried across the bottom to one of the pumps. Cobbles were then dumped into the water, covering the bottom of the enclosure and surrounding the boxed-in pump suctions until they rose above the water to form a level surface. The interstices between the cobbles formed passages through which the water traveled to the various pump suctions. Vertical grout pipes were installed, leading into the culvert before it was covered, and at random in the cobbles as they were placed. The surface of the cobbles was covered with small stone and the first lift of concrete was poured, using the steel pile enclosure for a form. In order to improve working conditions by bringing the level of the concrete up to that of the surrounding area, another lift was poured later, through which holes were formed for the columns.

Before the drilling for the supporting columns was begun, the pumps were removed and grout was forced into the cobble bed through the pipes that had been provided for that purpose, the wooden culvert being filled at the same time. Subsequent inspection, made when the large holes were drilled, revealed that the cobbles had been well stabilized by the grout. Each column location in the nose pier was grouted in advance in the same manner as has been described for the other piers.

CONCLUSION

In selecting the methods that were chosen for obtaining a foundation for the wall, full cognizance was taken of the fact that much was unknown concerning the practicability of drilling large holes in the type of material that was to be encountered and some doubts were felt as to the economy of the choice. However, cost and progress records which have been compiled since the work was completed indicate that, of the methods considered feasible, none would have proved as economical or as expeditious as the one selected.

Using these methods, 39 holes of an average depth of 55.1 ft and a maximum depth of 82 ft were completed. A total of 2 147.2 ft was drilled of which 1 039.9 ft was in over-burden and 1 107.3 ft was in rock. During a typical operating period of one month, when the obstacles and difficulties encountered were about average, the progress amounted to 10 ft per drill per 24-hr day, or 0.417 ft per hr. At this rate of drilling, the direct labor charge for a crew consisting of driller with two helpers, mechanic with two helpers covering all drills, and foreman is $8 per lin ft. Hourly rates of pay were as follows: Foreman $1.25, drillers $1, mechanics $1.10, and helpers 75 cents and 60 cents. Power, water, air, shot, casing, shop repairs, other services, depreciation of drilling equipment, and overhead should exceed these values somewhat, and a total cost of between $18 and $20 per lin ft is approximately correct. Under different conditions, where the drilling could be done from the surface of the rock, and where the hazard of excessive flows of water did not exist to make casing necessary, it should be possible to reduce the unit cost appreciably.

Observations made since the piers were fully loaded have not disclosed any evidence of settlement of the foundations or of deflection of the guard wall that is carried by the piers.

The operations described herein were conducted 24 hr per day in four 6-hr labor shifts and, despite the numerous delays that were suffered when water broke into the holes and made a cessation of drilling necessary, the work was completed in time to permit the lock to be opened for navigation when scheduled.

ACKNOWLEDGMENTS

The Chickamauga Project Navigation lock was built by the Tennessee Valley Authority with its own forces: T. B. Parker, M. Am. Soc. C. E., Chief Engineer of the Tennessee Valley Authority, Lee G. Warren, M. Am. Soc. C. E., Project Engineer, with James B. Hays, M. Am. Soc. C. E., Construction Engineer, and Mr. F. C. Schlemmer, Construction Superintendent. Mr. H. R. Johnston was in charge of the work described at its inception and was succeeded by the writer. Jack C. Evans, Jun. Am. Soc. C. E., Mr. G. A. Carlson, and Mr. H. E. Murray each supervised the operations on 8-hr shifts.

DISCUSSION

J. B. Newsom,[2] **Esq. (by letter).—Most engineers do not understand that drills which will bore holes large enough for a man to enter can be used in any ground, from the loose water-bearing ground containing some boulders described in the paper, to the hardest rocks known to the mining industry. In the work described, it was necessary to use casing. Diameter was lost every time it was placed, so that a hole which started 4 ft in diameter might be 3 ft in diameter at the bottom. In boring a hole 1 125 ft deep in California in 1935,**[3] **the hole went through heavy ground and gouge which had to be supported without losing size. This was accomplished satisfactorily by using an expanding steel form, behind which quick setting concrete was packed. These large drills are capable of higher development and have a much wider field of possible usefulness than is generally recognized.**

J. G. Tripp,[4] **M. Am. Soc. C. E. (by letter).—The difficulties encountered in sinking relatively small shafts in the over-burden, broken, and solid sedimentary rocks at Chickamauga Dam are outlined clearly in this paper. The methods chosen by the engineers, and the costs achieved, deserve commendation.**

There were probably three methods open to the builders to complete satisfactory foundations under the lock guide wall:

(1) The shot-drill and casing method (that chosen);

(2) The standard "air" method; and

(3) The rotation water method, more or less successfully used in very deep foundations in several difficult locations in and around New York, N. Y.

It appears clear that method (1) was chosen on account of a desire to explore the ground, equipment availability, and operating personnel. If the costs given are the actual totals, there is little doubt that the choice was wise. Under standard "air" conditions, costs would have approached twice those achieved. The rotation water method, if available, would have cost about $20 per ft.

However, the third method deserves mention. Shafts averaging 55 ft with a maximum depth of 82 ft have been driven successfully in Manhattan, where broken rock, boulders, quicksand, and rip-rap were the materials encountered. Also caissons 8.5 ft by 110 ft were sunk successfully across the Connecticut River.

This method consists of a caisson rotating in a column of water. The water enters the caisson under controlled pressure and the lighter and smaller materials pass under the bottom of the cutting edge of the steel shell and appear at the surface annularly about the caisson. Those elements too large

---

[2] With Idaho Maryland Mines Corp., San Francisco, Calif.

[3] "Shaft Sinking with a Shot Drill, Idaho Maryland Mine, Grass Valley, Calif.," by J. B. Newsom and C. F. Jackson, U. S. Bureau of Mines *Information Circular 6923*.

[4] Pres., Tripp Constr. Corporation, New York, N. Y.

to pass are left in the caisson and removed from time to time, as required. A properly designed cutting edge cuts into the solid rock for any desired distance, say, to 4 ft, when rotation is stopped; the materials surrounding the caisson are then allowed to settle and consolidate, and the caisson becomes ready for unwatering and for concrete. This method obviates the necessity of grouting except in the bottom rock should conditions require. Standard methods accomplish such grouting and a safe steel-cased shaft is ready for steel reinforcement and concrete.

In conclusion, if relatively solid rock had existed from the grass roots down, no shafts would have been needed. In the case at Chickamauga Dam the engineers were faced with the common ailments of all limestone dam sites; that is, corrosion, eroded contacts, and redeposited, tilted, folded, and faulted strata with consequent water problems. In such ground little useful knowledge can be obtained from ordinary core borings. In such cases, the large core drill is indispensable, because an inspection well that the geologist or foundation designer can actually go down into gives all necessary predesign information with certainty. Finally, it would appear that the difficulty of ascertaining foundation conditions under the guide wall at Chickamauga lock led inexorably to the choice of method described.

James B. Hays,[5] M. Am. Soc. C. E. (by letter).—One of the most difficult operations necessary to secure a satisfactory foundation at the Chickamauga Dam is described in this paper. The author demonstrates the fact that exploration in limestone rock must be thorough.

The original borings were not spaced closely, and in the area covered by the work described by the author there was only one hole directly under the wall, with two or three close by. It was realized that the foundation was not particularly good; but it was not until work had been started and excavation for the lock walls was well under way that the real situation developed. Preliminary borings were put down at the corner of each pier, and the detailed information, with other experience on the project, indicated that open excavation to rock would be practically impossible. The coffer-dam was too close to allow open cuts without timbering. Timbered shafts would have exposed too much surface to the mud and clay seams which, in time, would have required excessive bracing and involved considerable leakage. As a result the design was revised, as shown, to support the piers on reinforced concrete columns on rock, since most of the equipment necessary was already available.

Mr. Lewis was placed in charge of this particular feature which was one continuous battle from start to finish against water and mud, and the danger of blows. However, when finally ready for concrete, the holes were in excellent shape so that the leakage caused little, if any, trouble in concreting.

Evidently the cooling of concrete placed in the columns was successful. The writer recalls that one column was shortened more than $\frac{1}{8}$ in. by the circulating cold water. This work was done during the late winter months; hence the river water was used for cooling.

---

[5] Constr. Engr., Kentucky Dam, TVA, Gilbertsville, Ky.

One lesson that was demonstrated on this project was the fact that exploration holes in limestone formation should be spaced closely and drilled accurately; and they should be carefully "logged" by a competent geologist. A second lesson was that grouting of coffer-dams in such a formation was necessary. The work described by the author was in the first of three stages of river coffer-dams. Coffer-dam grouting was started too late in the first stage to be a complete success, although it did benefit to some extent. The second and third stages were grouted thoroughly and, as a result, were remarkably dry. During the construction period, the geologist, Portland P. Fox, Jun. Am. Soc. C. E., was of great assistance in defining the detailed rock structure. Fig. 3 is one of many that were used.

The large core drills were useful for checking the results of grouting and foundation treatment for shafts and pump sumps.

O. N. Floyd,[6] M. Am. Soc. C. E. (by letter).—A rather complete account of a difficult piece of work, planned and executed with very limited guidance from precedent, is presented in this paper. The frankness with which the limitations of the equipment and the failure of certain attempted methods are treated adds materially to its value.

The writer wishes to call particular attention to the added value of these large holes to the engineer, in that they make it possible to see the rock in place. The bedding planes, thickness of cored strata, and any bentonite or clay seams or open cavities are all there to be examined and probed. It was quite helpful, in the work described by the author, to be able to see the rock on which the columns were to rest and also to judge better the lateral support the columns would have because of the developed thickness and probable extent of the large blocks of limestone penetrated on the way down.

The 36-in. holes were helpful in determining the extent and nature of some bentonite seams under parts of the main walls of the lock at this same job. Furthermore, in the cut-off under the earth wings of this dam, where much bad rock had to be removed and a few deep caverns had to be cleaned out and filled with concrete, the 36-in. holes were used for working shafts during the cleaning operations and later for refilling with concrete. Both the cost and the time for sinking shafts in this manner were very favorable.

Another place illustrating the superior value of the larger holes was at one of the early small core drill holes under the south earth embankment of the Chickamauga Dam which indicated a rather deep nest of boulders, clay, and gravel. A 36-in. hole sunk to include this hole showed that the small drill had gone down about a foot away from the almost vertical face of a solution channel in the rock and, since several of the more resistant strata were protruding from this face, they had been cored or broken off and classed as boulders in the record of the first hole.

---

[6] Cons. Engr., Dallas, Tex.

James S. Lewis, Jr.,[7] **Assoc. M. Am. Soc. C. E.** (by letter).—The discussions have revealed an appreciation of the difficulties often encountered in obtaining satisfactory foundations in limestone. Extensive weathering, solution, erosion, and geologic disturbance frequently have combined to produce a condition which is not disclosed by generally accepted methods of exploration. The erratic results obtained from small borings may prove very misleading and difficult to interpret, and even large borings will leave much essential information to be revealed by the excavation.

As emphasized by Mr. Hays, the services of a geologist skilled in interpreting the information obtained from borings were used to excellent advantage. The aid rendered by Mr. Fox proved invaluable. Mr. Floyd indicates the value of large core holes as shafts for various purposes, and their usefulness also has been demonstrated in mining operations where it was necessary to attain great depths.

There is no gainsaying Mr. Newsom's statement to the effect that large drills are capable of higher development. This is true of shot drills of all sizes, as they have not enjoyed the mechanical refinements that have been applied to the best diamond drills. A reduction in weight would afford greater ease of handling over the rough terrain that must frequently be traversed, and a closed type of construction would prevent the damage that follows when shot get into the moving parts. An automatic shot feed, powered by a standard water meter, was developed at Norris Dam. The immediate effect was a reduction in shot consumption to 25% of the former quantity, a smooth core and hole of uniform size, and less wear on the drilling tools. There is still opportunity for great improvement.

Different methods of obtaining a foundation might have been chosen, as suggested by Mr. Tripp. The writer doubts, however, that the caisson used in the rotation water method could have been used economically to cut the limestone encountered for the distances required.

In describing the use of large core drills for sinking shafts under extremely adverse conditions, the writer's chief desire was to reveal the adaptability of the equipment when the drilling was supplemented by grouting. It is obvious that the methods described would have been impractical without sealing the rock and overburden against excessive leakage.

---

[7] Asst. Constr. Supt., Watts Bar Dam, TVA, Spring City, Tenn.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2076

BEACH EROSION STUDIES

BY EARL I. BROWN,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. MORRIS N. LIPP, GEORGE A. SOPER, JAMES J. O'ROURKE, ELLIOTT J. DENT, RALPH F. RHODES, CHARLES T. LEEDS, AND EARL I. BROWN.

SYNOPSIS

Items of information considered necessary in a comprehensive beach erosion study, the reasons for desiring each particular item, and some general observations on design of protective works are offered, in some detail, in this paper. The presentation is made for two reasons: (1) That engineers interested in beach protection may have the advantage of experience gained by the Beach Erosion Board, United States War Department (under the Chief of Engineers, U. S. Army), as to factors involved in a study of beach erosion; and (2) that the items of information now believed best suited to the purpose by the Board may be the subject of full and free discussion by all engineers interested.

BEACH PROTECTION STUDIES

The science of beach protection is comparatively new in the United States. The rapid growth of seashore resorts and the immense investment therein since automobiles and good roads have made them easily accessible have resulted in a demand for improvement and stabilization of old beaches, and development of new ones. In the past, as each seaside resort developed, local judgment determined the proper method of improving and stabilizing its beach. There resulted a wide difference in character and design of protective structures. Some have proved effective, others have been useless, and still others have done more harm than good.

Each locality presents a different problem. In all cases there is need for detailed and accurate information on local conditions. It is inadvisable to prescribe a given plan of protection for a beach simply because it has proved effective elsewhere, unless conditions can be shown to be similar. If engineers along the coasts can be encouraged to develop a general understanding of the character of information necessary for designing protective systems, and the

---

NOTE.—This paper was presented at the meeting of the Waterways Division at Jacksonville, Fla., on April 21, 1938, and published in January, 1939, *Proceedings*.

[1] Col., Corps of Engrs., U. S. Army (*Retired*), Wilmington, N. C.

general principles of design, a large field opens for them in undertaking beach erosion studies and designing protective works for private property owners. A suggested outline for a report on beach erosion studies is given in the Appendix.

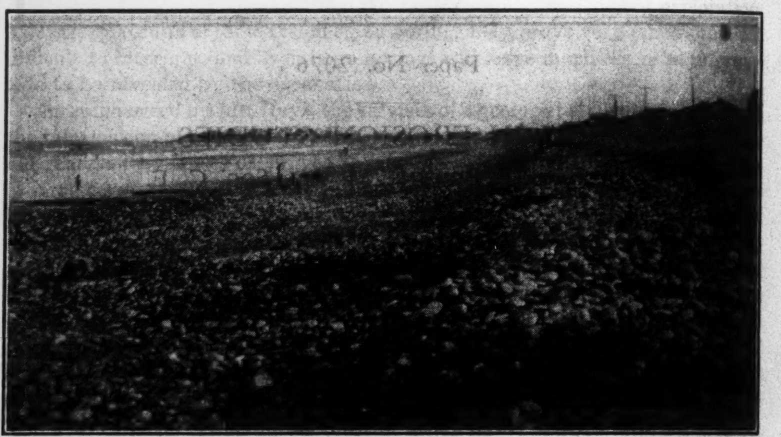


FIG. 1.—TYPICAL SHINGLE BEACH ALONG THE NEW HAMPSHIRE SHORE, NORTH OF BOAR'S HEAD

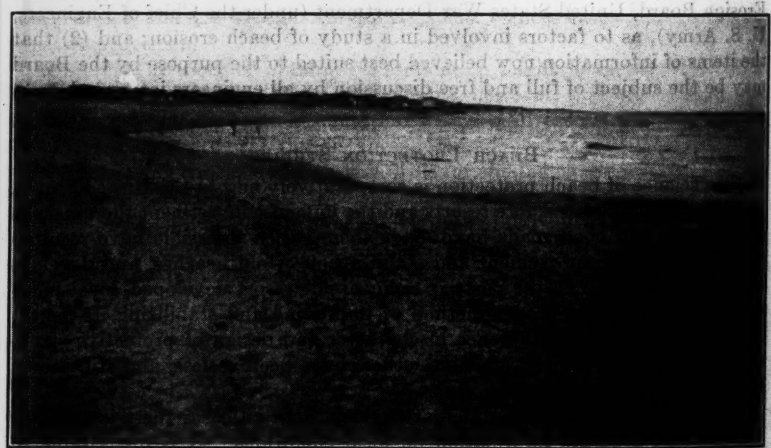


FIG. 2.—POCKET BEACH FORMED AGAINST A JETTY, AT HAMPTON BEACH SOUTH OF BOAR'S HEAD, N. H.

The Fifteenth International Congress of Navigation which met in Venice, Italy, in 1931, discussed the question of protection of coasts against the sea, and adopted a comprehensive report drawn up by its General Reporter for this subject, expressing its matured general conclusions. This report contains the following conclusions as to preliminary studies:

"Any plan for protecting coasts against the sea must be preceded by a careful study of the locality and all the factors acting in the formation of the coast, such as the nature and shape of the coast, and as to how much it is exposed; the action set up by waves, by currents of various kinds, by atmospherical water and by ice; the origin and nature of the materials constituting beaches; the flow of water courses which empty themselves onto the beaches and the regimen of the mouths of such water courses; the situation and regimen of the aquiferous sheets of fresh water flowing towards the sea; the influence of new constructions on the sea, when they project from the sea."

Characteristics of Beaches

The type of information required in a study is based on these general principles, and will depend for detail on the location of the area. Conditions vary somewhat for each location. The coastal area of the United States may be divided roughly into subdivisions in each of which general conditions are quite similar: North Atlantic, Central Atlantic, South Atlantic, Florida, Gulf of Mexico, South Pacific, North Pacific, and Great Lakes.

For purposes of beach erosion studies, the North Atlantic area may be considered as ending at Montauk Point, Long Island, N. Y., but including the north shore of Long Island Sound. The coast in this area consists of rocky or boulder-strewn headlands, and pocket or crescent beaches of sand or pebbles. Illustrative of these two types, Fig. 1 shows a steep shingle slope of the shore, with a narrow strip of sand at the water's edge; and Fig. 2 shows a pocket

Fig. 3.—Formation of Moriches Inlet, Long Island, N. Y., March 10, 1931

beach formed against a jetty. There are a few exceptions to this rule, such as at Salisbury Beach, and beaches along Cape Cod, in Massachusetts.

The south shore of Long Island, formed by the outwash of a terminal glacial moraine, with lagoon areas in the rear, is quite similar to the New Jersey shore and may be classed in the Central Atlantic area. This Central Atlantic type of beach (a comparatively narrow sandy barrier, broken at intervals by inlets communicating with lagoon areas in the rear) extends as far south as Cape Romain, in South Carolina. Fig. 3 is an aerial photograph of Moriches Inlet, Long Island, N. Y., as it was in the process of cutting through a sand barrier. South of this point, to the Florida line, the coast is intersected by innumerable tidal streams resulting in a chain of islands with sandy beaches on the ocean side and occasional marsh areas behind them. Still farther south lie the Florida beaches which superficially resemble those of New Jersey, but which, from the standpoint of beach erosion studies, are distinctly different because of a difference in climate and exposure to wave action, and the presence of limestone rock.

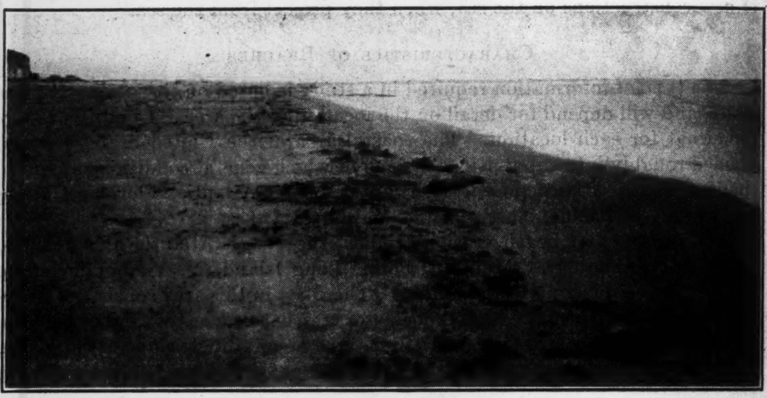


FIG. 4.—TYPICAL LOUISIANA BEACH; GRAND ISLE, LA.

The beaches along the Gulf of Mexico vary considerably among themselves. They are subjected to the same type of wind, weather, and wave conditions, and, therefore, are given a separate classification. From the southern tip of Florida northward there is the inaccessible and greatly intersected Everglade area, with a total absence of beaches. Northward from Naples, Fla., to Tarpon Springs, Fla., there are barrier islands with excellent sandy beaches. North of Tarpon Springs the shore is so well sheltered from wave action because of its location and the presence of shallow limestone ledges that mangrove swamps grow to the water's edge. This condition extends to Apalachee Bay, Florida. West of Apalachee Bay to the vicinity of the Mississippi River and from Sabine Pass, Tex., westward and southward to the Mexican border there are sandy barriers with good beaches, although the island barriers of the section east of the Mississippi Delta are so far offshore that each of the two parallel beach lines presents a set of problems of its own. This is particularly true of the mainland and island beaches of the Mississippi Sound area. In this vicinity the coast is broken by the low marshy islands of the delta, with some sandy beaches on the Gulf face. Fig. 4 shows a typical Louisiana beach on the Gulf side of a barrier island. The sand is very fine and both the foreshore and offshore are very flat. There are few dunes behind the water-line and the barrier is topped by hurricane tides.

For the purpose of beach erosion studies, the beaches in the South Pacific Ocean may be considered as those lying in the reach from Point Conception, Calif., southward to the Mexican border. The coast is comparatively young geologically, and the shores are narrow with comparatively high hills close to shore and deep water close offshore. This results in headlands between crescent beaches (see Fig. 5). The distance between headlands is comparatively great. Annual rainfall is light. During the rainy season, however, there are periods of heavy rain, the run-off from which carries sediment to the coast. This sediment forms much of the beach material.

North of Point Conception, in the North Pacific, the headlands become more frequent and the intervening beaches shorter. There is comparatively little sea-bathing in this area because of the coldness of the water, but the beaches are used as recreational areas during the summer months.

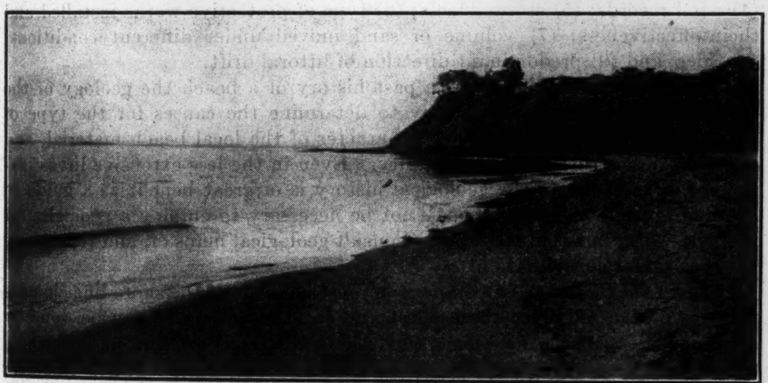


FIG. 5.—CABRILLO BEACH, SAN PEDRO, CALIF., FACING NORTH FROM THE BREAKWATER

The Great Lakes area offers a different set of conditions. The Lakes are tideless, but have seiches and seasonal variations in level. They are frozen near the shore during the winter and are subject to violent storms. The bottom materials, the fetch, the slope of the bottom, currents, and ice action, differ from those on the open ocean. The Lake beaches are generally composed of a layer of very fine sand over clay or mud. The offshore is flat, and some distance out from shore the bottom changes from sand to fine clay or mud. In some sections, such as the southern end of Lake Michigan around Chicago, Ill., there are extensive sand dune areas, but in most parts the Lake shores end against bluffs of clay of varying heights.

Scope of a Beach Erosion Study

Within each area local conditions will dictate to a large degree the extent of a study and the data required. Studies on a long unbroken beach will require different information from those in the vicinity of an inlet; and a study involving a highly developed resort, such as Atlantic City, N. J., would justify a much more searching investigation than a study of a similar area which is entirely undeveloped. In determining the data to be required, the location and the economic justification for the degree of detail are first considered.

The usual requirements for a beach erosion study are an investigation of the past history of the area from all available records, and an investigation of the present conditions by means of local surveys and observations. After this information is assembled a comparison with other similar areas where there are protective works is possible, and a proper design for the protection of the area under investigation can be made. With these requirements in mind, a brief description is given of information generally secured in a study, with the reasons for, and the method of, securing it.

There are nine principal items to be investigated in considering the past history of a beach: (1) Geology; (2) shore-line changes; (3) offshore changes; (4) wind records; (5) storm effects; (6) type of protective works installed and their effectiveness; (7) volume of sand moved under different conditions; (8) tides; and (9) predominant direction of littoral drift.

Geology.—In investigating the past history of a beach the geology of the area should be reviewed primarily to determine the causes for the type of coast encountered, the source and character of the local beach material, and the geological structure in the vicinity. Even in the less extensive investigations, a brief study of the geological history is of great benefit as a guide to the more recent history. It may not be necessary to employ a geologist for the field work, but it is desirable to consult geological maps of, and reports on, the area under consideration.

Shore-Line Changes.—A comparison must be made of the shore-line changes that have occurred since the earliest authentic maps. This can be done by studying all available field sheets of the United States Coast and Geodetic Survey, local surveys, and maps of surveys of the United States Engineer Department. These data are reduced to a common scale and the shore lines, both mean high water and mean low water, for all dates are plotted so that they may be superposed on one sheet. If sufficient surveys are available, covering a long period of time, a "good picture" is afforded of the progressive changes that have occurred. Along many parts of the New England and New Jersey coast excellent surveys have been made by the U. S. Coast and Geodetic Survey, extending back to 1839. Unfortunately, this is not true for all the seacoast. At some places the available surveys are irregularly or remotely spaced in point of time. For example, in a study at Daytona Beach, Fla., the earliest accurate survey that could be relied upon for shore-line changes was that of 1872–1874; the next one was made in 1928, a lapse of 54 yr. An office comparison of shore lines as outlined is made for all studies. Where a study warrants more detail, a survey showing the present location of mean high and low-water shore lines is made, and the results are superposed on a map of the old shore lines to show the changes to date.

Offshore Changes.—A history of offshore changes is secured in much the same way as that of shore-line changes. Copies of hydrographic field sheets of the offshore area, obtained from the U. S. Coast and Geodetic Survey and the U. S. Engineer Department, are reduced to a common scale and curves for 6-ft, 12-ft, and 18-ft depths superposed on one sheet for comparison. Where the changes between surveys are considerable, separate work sheets on a relatively large scale for each depth-curve comparison are prepared and then consolidated on one sheet for the report. In case of an extensive study, the 24-ft, 30-ft, 36-ft, 42-ft, and 48-ft curves are sometimes added. Generally, an offshore survey is required for the study in order to determine the present position of the curves to be compared.

The purpose of comparison of offshore depth curves is to determine if progressive changes are occurring. Past changes will have a bearing on probable changes to be expected along the shore in the future and will thus affect the design of proposed protective works. If the changes show a progressive and rapid movement shoreward of the depth curves, indicating a deepening of the offshore area, it is evident that severe wave action close to shore may be expected, which may cause increasingly rapid erosion of the beach. Conversely, a progressive shoaling of the offshore area will indicate a probable lessening of destructive wave action.

In studies of larger scope, it is desirable at times to study general trends on the bottom for distances of 5 to 10 miles offshore. This study is made from hydrographic field sheets of the U. S. Coast and Geodetic Survey. Ranges are selected along the area to be investigated on which soundings were made on several of the surveys. Composite profiles along these ranges are plotted for each survey and compared on each range. On the Atlantic Coast it has been possible, in several instances, to follow the seaward or shoreward trend of the depth curves to depths of 70 ft. On the California coast where depths increase rapidly offshore, there were bottom changes, in 300-ft depths, of as much as 18 ft between the 1869 and 1930 surveys. Such a large difference can scarcely be attributed to errors in soundings or movement of littoral drift, but may well have been caused by slipping or faulting during an earthquake. Investigation of this kind is most effective and profitable of results when done on a large scale.

Wind Records.—The next step in investigating the past history of an area is a collection of wind and weather records. There are three available sources for past weather records: The United States Weather Bureau, the Coast Guard Stations in the area under investigation, and airports. Generally, the nearest first-order station of the Weather Bureau can furnish quite accurate records since 1923. Prior to that time the instruments used were not standard and there is some doubt as to the accuracy of the records. Where the nearest Weather Bureau Station is some distance from the area under investigation it is well to take the records from the nearest Coast Guard Station. These generally consist of readings every four hours on the wind direction and velocity. The readings are approximate and must be reduced to a usable form for a study. This involves considerable office work, but affords a clear idea of general wind conditions in the area, particularly the duration and intensity of storms and of prevailing winds.

Some difficulty may be encountered in interpreting and using wind diagrams furnished by outside sources. Two types of wind-roses have been developed for beach erosion studies (see Fig. 6), which make it possible to determine the prevailing wind movement, the average velocities from each direction, and the direction of predominant storm winds. If, from an examination of the data, it is apparent that there is a great seasonal variation in wind direction and intensity, and if the detail of the study warrants, such wind-roses should be plotted for each month in the year over the entire period covered, and a second series should be made showing the average for each month, and, finally, the average for the entire period. Another source of wind information is the Hydrographic Office, U. S. Navy Department. Its records show prevailing winds and their intensities in 5° areas offshore as reported by ships.

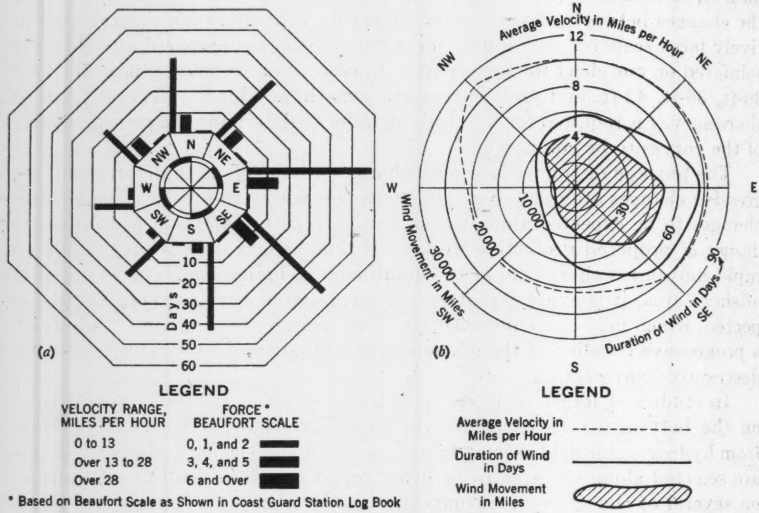


FIG. 6.—TYPICAL WIND DIAGRAMS

The study of past wind and weather conditions is important when correlated with the shore-line and offshore changes and reports of the effects of storms on the beach. By affecting the direction of wave approach, winds play an important part in determining the direction of predominant littoral sand movement. Wind-blown sand carried ashore plays an important rôle in dune building and lateral sand movement along the shore. By examining the history of winds and storms over a considerable number of years, it is possible to obtain some idea as to why certain changes have occurred and what to expect in the future, particularly with respect to sand movement, the direction, duration, and intensity of storms, and the time of year when storms may be expected. The extent to which the investigation of wind and weather is carried will be determined by the general detail warranted for the study. It should not be ignored in any study.

Storm Effects.—In addition to the study of winds, a history of the general effect of intense storms in the area has been found of value. The information may be obtained from long-time residents, from newspapers and other records, and from photographs. Particularly desirable is the extent of erosion or accretion caused, the apparent effect of any protective works or other structures in the area, and the height to which the tides rose during storms. Since there is danger of exaggeration, in any eye-witness description of storm action, such description must not be relied upon too heavily. Photographs showing conditions before and after a storm are among the most reliable sources of information.

Existing Protective Works.—In the history of a beach area, a complete and accurate record of any protective works which have been installed must be secured. This should cover the location, profile, and date of construction of each, with a history of its effect upon the beach and its length of life. It is usually possible to get the plans and specifications if the structures were built by a municipality, State agency, or the Federal Government. In all cases, it is possible to note the present condition, the effect on the beach, and the probable life. Where long structures such as jetties have been in place a considerable period of time, changes in the offshore areas as well as those at the shore line should be considered.

In a detailed study, the effect of all structures projecting into the sea on the shore line over a number of years should be followed. Changes in the location of the shore line and the depth curves are valuable indications. Frequent or yearly hydrographic surveys should be available with sounding ranges fairly close together and profiles extended back from these ranges over the area of dune sand. From such surveys it is possible, by using a similar grid for each year, to compute the volumetric sand changes in the shore and offshore area to the limit of the surveys. This is being done at several places along the Atlantic Coast with very interesting results.

In evaluating the effect of groins or jetties it is necessary to discriminate between the effects of a single structure and that of a system. The supply of sand that may be trapped along a given shore may have been sufficient to show satisfactory results with a single structure, but may not suffice for prompt results for a system of structures. An artificial supply of sand may be required in such a case.

Tidal Data.—It is advisable to learn about the tides in the area under investigation. The time and height of tides, the mean range, and spring and neap ranges may be obtained from tide tables published by the U. S. Coast and Geodetic Survey. In addition to these mean values, it is important to secure information on the extremes under storm conditions, since any design of protective works must consider these extremes. This information can be obtained from the U. S. Coast and Geodetic Survey for points where tide gages have been established. In other cases it is necessary to refer to the U. S. Coast Guard or other local records.

There are no tides on the Great Lakes but there are minor irregular variations in level known as seiches and fairly regular seasonal variations between winter and summer, averaging between 1 ft and 2 ft. During the winter the Lake levels are low and during spring and summer they are high. In addition to the seasonal variations there are long-time cycles of higher waters and lower waters. In the case of Lake Huron, for example, the seasonal variation since 1860 averages 1.2 ft, the cycle of higher waters averages 14 yr and the maximum difference between higher high waters and lower low waters has been 6.30 ft. It is apparent that variations of this nature must be considered in designing any protective works on the Lakes. The United States Lake Survey has kept accurate records of seasonal variations in all the Lakes since 1860.

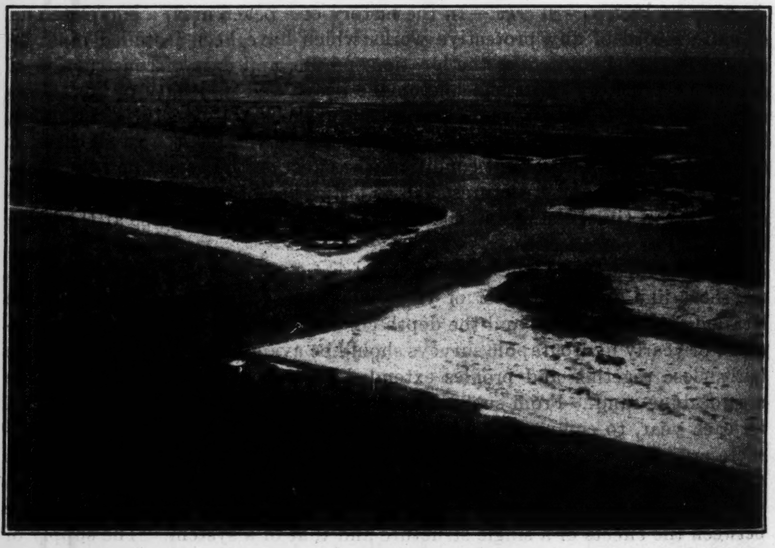


Fig. 7.—A Typical Florida Beach; Lake Worth Inlet, Fla.

Littoral Drift.—The last item to be considered under the past history of an area is the direction of predominant littoral drift. This information is most important in the design of a protective system. From the shore-line changes, particularly near an inlet or jetty, and from changes in offshore depths, the direction of preponderant movement of the littorally drifting sand may be determined. The direction of growth of a point, the migration of an unimproved inlet, or the accretion on one side of a jetty, will give good evidence of the direction of predominant sand movement. In Fig. 7, for example, littoral drifting sand is piled against one side of a jetty entrance to the detriment of the beach on the leeward side. Inferences from an accumulation of sand against either side of a short groin at any particular time of year may be erroneous since, at many places, there are seasonal reversals of the direction of sand movement. The direction of wind and of wave approach plays an important rôle in directing sand movement along the shore and in seasonal reversals. If the detail of the study permits, observations as to the direction of littoral drift should be continued over a period of a year to determine these seasonal changes.

Comment.—In a beach erosion study, the foregoing nine items dealing with the history of the beach are all considered. This is relatively expensive, since considerable office work is involved. Caution is given against singling out one of the items of investigation and basing conclusions and recommendations on it. For example, the history of offshore changes may show a considerable movement seaward of the curves for 6-ft, 12-ft, and 18-ft depths indicating shoaling of the offshore area. Normally this would afford greater protection to the beach. However, there might be a near-by longshore channel with comparatively high current velocities which could cause a serious erosion of the beach. In all studies it is important to complete all the items of investigation, to correlate them, and to give due consideration to their accuracy before arriving at conclusions and prescribing a remedy.

FIELD WORK

After having decided upon the items involving historical phases of the study, a plan of field work should be drawn up to ascertain present conditions. The location of the beach, the problem to be solved, and the availability of funds will dictate what field work should be included.

Eleven major items of field work must generally be considered, many of them being closely related and all of them involving varying degrees of detail, namely: (1) Character of the beach; (2) character of the offshore area; (3) currents; (4) tidal prism of inlets; (5) tides during the field work; (6) littoral movement of sand; (7) present condition of protective and other works, and their apparent effectiveness; (8) wave direction, velocity, and height during current and sand movement observations; (9) wind conditions during the course of field observations; (10) sand analyses; and (11) periodic aerial photographs.

Character of Beach.—The usual procedure for small studies, in determining the characteristics of the beach, is to measure profiles extending across the dune line to wading depths beyond mean low water. These profiles are spaced from 200 to 500 ft apart along the entire front under investigation and are perpendicular to the shore. When plotted they furnish information on the height of dunes, the general slope and shape of the area above high water, the location of mean high-water and mean low-water lines, and the slope of the foreshore. These are all factors in the design of protective works and should be included in all studies. Sand samples are taken at the mid-tide elevation on the beach at each profile. These samples are analyzed for median diameter, specific gravity, porosity, and shell content. Since the degree of wetness of beach sand greatly affects its physical properties, a determination of its moisture content, and of the source of the water, whether the sea or ground-water, is coming to be considered a matter of some importance.

There is a possibility of a relationship between the slope of a beach and the characteristics of the sand composing it. In general, it appears that the greater the median diameter the steeper will be the beach slope; and the smaller and more uniformly graded the sand, the flatter the slope will be. Although it is not of great importance to any particular study this information may be included for statistical purposes.

The analyses of the sand are of value in determining the source of supply of the beach sand. Where this is of material interest to the study, samples are secured from all suspected sources for comparison with the beach samples. Complete petrographic studies are usually made only in special cases.

In the case of extensive and detailed studies, topographic maps of the area should be made to show dune location and height, and the character of the country some distance landward of the beach. Aerial photographs are useful in connection with these maps to show the extent of developed areas and buildings.

Character of Offshore Area.—The under-water topography can be obtained from a general hydrographic survey of the area or by extending the shore profiles to depths of 18 ft to 30 ft below mean low water. For smaller studies it is usually not necessary to extend these lines beyond 1 500 ft from shore, or to more than a depth of 18 ft below mean low water. These profiles or hydrographic surveys are important in any study to show the offshore slopes, and the presence of offshore bars and reefs, and to afford a comparison of present conditions with those shown in previous surveys. The existence of an offshore bar or reef, if close to shore, will affect the design of groins.

In more detailed studies it is necessary not only to know the under-water topography, but also the character of the bottom materials, since the bottom may be the immediate source of material reaching the beach. Bottom samples should be dredged on the intersections of a grid system and then mechanically analyzed and leached with hydrochloric acid. The results will show the character of bottom material and shell content of the samples. Borings may be used, if desired.

In cases involving inlets the system of profiles is usually not sufficient because, in addition to the profiles, it is desirable to have a sufficiently detailed hydrographic survey of the inlet to develop the entire outer bar, the inlet channel, and the inner bar.

Currents.—A study of currents in the ocean comparatively close to shore is of importance in determining the direction of sand movement along the beach and bottom. In measuring currents offshore, it has been found convenient to use floats covering a complete tidal cycle, to determine any changes caused by difference in the phase of tide. Except immediately adjacent to the shore, and at inlets, such currents are usually moderate or slow.

At the same time that the offshore currents are being measured by means of sub-surface floats, the currents on the same lines close to shore should be measured. From the limit of wave up-rush to wading depths this can be done conveniently by timing the longshore movement of the water by surface floats or by the use of colored liquid. The latter is preferable since it registers the actual movement of the water through the wave up-rush and back-wash and is not affected by wind. Potassium permanganate or washing blueing have been satisfactory for this work. The measurements of current near the shore must continue over the entire time during which the offshore currents are being measured—one full tidal cycle.

For more detailed studies warranting a close determination of the bottom currents a specially designed current meter[2] has been developed which can be operated at any reasonable depth from a boat offshore. These meters are very satisfactory and are much to be preferred to sub-surface floats since they register the actual bottom currents—those that move the sand. Measurements must be made at a reasonable number of points along each range line to be covered, and, at each point, must cover a complete tidal cycle.

In offshore current work, with the present equipment, it is impracticable to make measurements except during fairly calm conditions. It would be a decided advantage to know the relation between calm-weather currents and sand movement offshore and those prevailing during storms, and also the depth in which sand movement occurs during storms.

Tidal Prism of Inlets.—If the study involves the fixation or improvement of a tidal inlet, additional current measurements of a somewhat different type are required. A careful cross-section of the gorge is made and currents are measured at sufficient points along its line to enable making an accurate computation of the tidal prism. This requires that the measurements at each point extend over a complete tidal cycle. A tide-gage should be established at the ocean entrance to the inlet and another at the lagoon end. Where staff-gages are used they should be read every 15 min during the period of current measurements. If the study warrants, staff-gages should be established and read at critical points in the rear areas of the lagoon to determine the propagation of the tidal wave in the lagoon. Since the currents in the inlet are tidal and persist longer than oscillations due to wave action, any good type of current meter will be found satisfactory. The data are used, and conclusions drawn therefrom, as described by the writer in a paper published in 1928.[3]

Tidal Data.—Many of the items of field work are closely related to others. One of these items is the securing of accurate tidal data. The times of high and low waters and the average ranges of mean tides and spring tides, as given in tide tables, cannot be used for the reduction of soundings made along profile lines in the ocean, nor for correlation with the current measurements. They are means of long-time observations. Wind and weather have such an effect that the actual tide rarely, if ever, follows exactly the mean, as given in the tables. Therefore, whenever a study involves ocean profiles, ocean currents, a study of sand in suspension, or of inlets, it is important to establish a tide-gage to register ocean tides at the area under investigation. These observations must be made continuously during the time of field work.

Littoral Drift.—Direction of sand movement is determined, if possible, from historical evidence. It should also be studied in the field. If the importance of the study warrants it, a field determination of the volume of sand movement in the water under calm-weather conditions at the time of the study may be made with a fair degree of accuracy. This is done at the same time and on the same lines as the current measurements. Samples of water with the sand in suspension are taken from the bottom upward to a point where there is none. Fig. 8 shows a deep-water sampler in a partly open position. If currents are being measured by means of sub-surface floats, the boat releasing the floats takes several samples at each point and additional samples each time the floats are set on the range. Samples are taken in 1-ft increments above the bottom point until no sand is noted in the sample. This process must be continued throughout the full tidal cycle as in the case with the current measurements. If bottom currents are being measured with the current meters, samples are taken periodically throughout the time of meter readings at each station. The samples of sand in suspension should be secured for each line upon which currents are being measured.

---

[2] "Effect of Turbulence on the Registration of Current Meters," by the late David L. Yarnell and the late Floyd A. Nagler, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 95 (1931), pp. 800–806.

[3] "Inlets on Sandy Coasts," by Earl I. Brown, *Proceedings*, Am. Soc. C. E., February, 1928, pp. 505–553.

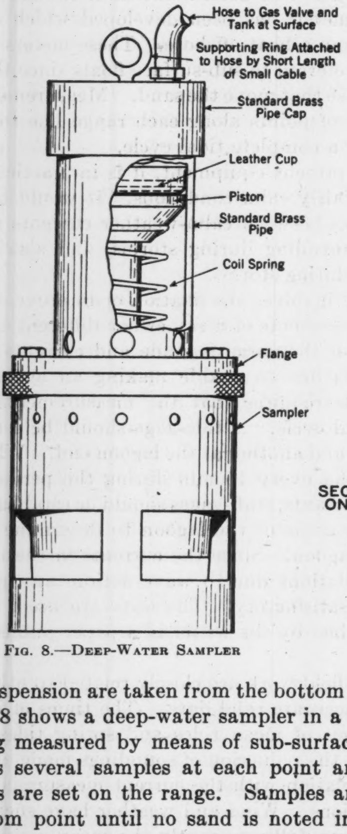

Hose to Gas Valve and Tank at Surface
Supporting Ring Attached to Hose by Short Length of Small Cable
Standard Brass Pipe Cap
Leather Cup
Piston
Standard Brass Pipe
Coil Spring
Flange
Sampler

FIG. 8.—DEEP-WATER SAMPLER

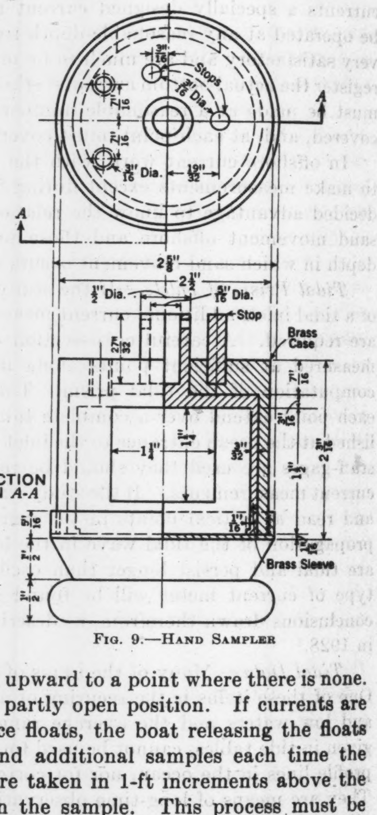

3"/16
Stops
3"/16 Dia.
A
7"/16
7"/16
3"/16 Dia.
19"/32
A
2 5/8"
2 1/2"
1/2" Dia.
5"/16 Dia.
Stop
Brass Case
27"/32
7"/16
3"/16
1 1/4"
1 1/4"
3"/32
1 1/2"
2 3"/16
SECTION ON A-A
5"/16
1"/16
1"/16
1"/2
1"/2
Brass Sleeve

FIG. 9.—HAND SAMPLER

In addition to the aforementioned offshore samples, samples to show percentage of sand in suspension must be secured from the limit of wave up-rush to the limit of wading depths on the same lines and covering the same period as the offshore samples. A hand sampler, similar in general principle to the offshore sampler, is used for this work (see Fig. 9).

The sand is filtered from all samples on the site and sent to a laboratory where the percentage of sand in suspension is determined. From the results thus obtained, curves showing the average percentage of sand in suspension at each point and the upper limit of suspended sand may be plotted. These data, combined with the results of the current measurements, permit rough computations of the volume of sand in suspension moving across a given line perpendicular to the beach from the limit of up-rush to the profiles. This method necessarily neglects the sand that is moving along the bottom by saltation, but there will be no such movement unless the current velocity is greater than the critical. As in the case of the current measurements it is impracticable to secure these samples except under fairly calm conditions, and, therefore, no comparison has been made between sand movement during normal and storm conditions. The investigation of the Beach Erosion Board of this sand movement thus far indicates that during calm weather approximately 80% of the sand movement is in depths of water less than 6 ft.

On many beaches the movement of sand by the wind may be of importance. Offshore winds have little dry sand-moving capacity, but winds on shore or parallel with it may move it in large volumes. Observation has shown that dry sand is not moved by wind having a velocity less than 11 to 12 miles per hr, but above that rate movement increases rapidly with wind velocity. The rate of wind-blown sand movement, when desired, may be measured in a suitably designed trap.

Observation of Protective Works.—In order to determine the effectiveness of protective works a close examination should be made to compare their present condition with the plans and specifications. It is well to check whether they were built in accordance with the specifications or modified. Careful notes should be made as to the spacing, length, and height of groins, and the apparent effect on the beach of these factors. Photographs of the beach for some distance on either side of the protective works are of value in formulating conclusions and in planning new protective works. Especial care should be taken to observe the effects of existing protective works which project from the coast line.

Wave Data.—Whenever observations on currents in the ocean or sand in suspension are made, careful readings of wave data should be made and recorded. The following data should be observed and recorded both for the shore and offshore stations: Wave height, period, length, velocity, and direction of approach. Accurate determination of these five factors is difficult, but a close approximation may be made by observing and averaging ten waves at intervals of 15 to 30 min. Offshore the wave height may be measured by observing the rise and fall on a lead line held with the lead just resting on the bottom. The period may be clocked, of course, and the velocity timed from bow to stern of an anchored boat. The length may be computed from the period and velocity, and the direction of approach read by compass. Near the shore the measurements are easily obtained if there is a pier close by. If no pier is available, piles may be driven at known intervals perpendicular to shore. This is comparatively expensive and is not warranted in a small study. Wave height, direction of approach, and period, which are the most important factors, can always be read from shore without the pier or piles.

Waves are a most active agency in moving sand along the beach. The data obtained from the aforementioned observations should be scrutinized carefully in conjunction with the currents, the sand in suspension, and the wind and weather conditions for some time prior to the observations. An examination of the wind-rose and the history of direction of predominant sand movement correlated with the wave data, currents, and sand in suspension, will assist in forming an opinion of the erosive forces and in designing a system of protective works.

Wind Data.—As accurate a record as possible should be kept of the wind direction and intensity during the course of the field work. The cost of this item is small, and it is useful in determining the effect of wind on the direction and characteristics of the waves.

Sand Analyses.—In the design of a bulkhead or groin, or for the dredging of a channel through a spit or outside bar, it is important to know the character of the sub-surface materials. In the normal beach-erosion study it is only necessary to know whether there is any rock or hard material that will make the driving of piles difficult or impossible. This information may be obtained by probing or by jetting if the piles are to extend to a considerable depth. To secure a more accurate idea of the character of the material, test pits above the high-water line may be dug at intervals along the beach and a log made of the materials.

To determine the character of the offshore sub-surface, wash borings may be made below the bottom from the shore out to depths of 40 ft, or more, below mean low water.

Aerial Photographs.—Fig. 10, a view of Moriches Inlet, Long Island, may be compared with Fig. 3, taken one month earlier. In Fig. 3, the rapid on-rush of water from the bays in the rear shows the inlet in process of forming. Fig. 10 shows the effect of tide action in forming a beautiful delta inside the inlet. Such aerial photographs are excellent for a continuing study, such as when it is desired to follow and record progressive changes like the migration of an unimproved inlet on a sandy coast, or the effect of accretion or erosion near jetties. For any study, they have an advantage in showing the general character of the shore. Unless they are already available the cost of securing them is prohibitive for a small study. Through the co-operation of the United States Army Air Corps, the Coast and Geodetic Survey, and the Geological Survey, the Beach Erosion Board listed the areas of the coast that have been photographed.

Comment.—In outlining the field work for a beach erosion study, all the items mentioned should be considered and evaluated with regard to the particular problem to be solved and to the extent of detail justified by the funds available. Some items may be eliminated and others may be greatly curtailed. In this case as in many other engineering problems, the experience of the investigator and his judgment play an important part since it is impossible to prescribe fixed rules. One general rule may be given: An engineer laying out a beach erosion study must not allow his desire for quick results to curtail the work necessary to secure sufficient data upon which to base a plan of protection.

After the data decided upon have been secured, each item should be developed separately and then correlated with the others to obtain a true picture of the forces at work and their effect. The wind-roses should be examined carefully and compared with the wave data to determine the possible effect of the winds upon the height and direction of approach of the waves. Study the current measurements, the sand in suspension, and the tides, to note the respective effects on heights and time. The long-time wind-rose should be studied in connection with the history of beach and offshore changes and the effect of storm action. From past beach changes as effected by past weather conditions and past storms an idea may be formed as to the future changes to be expected.

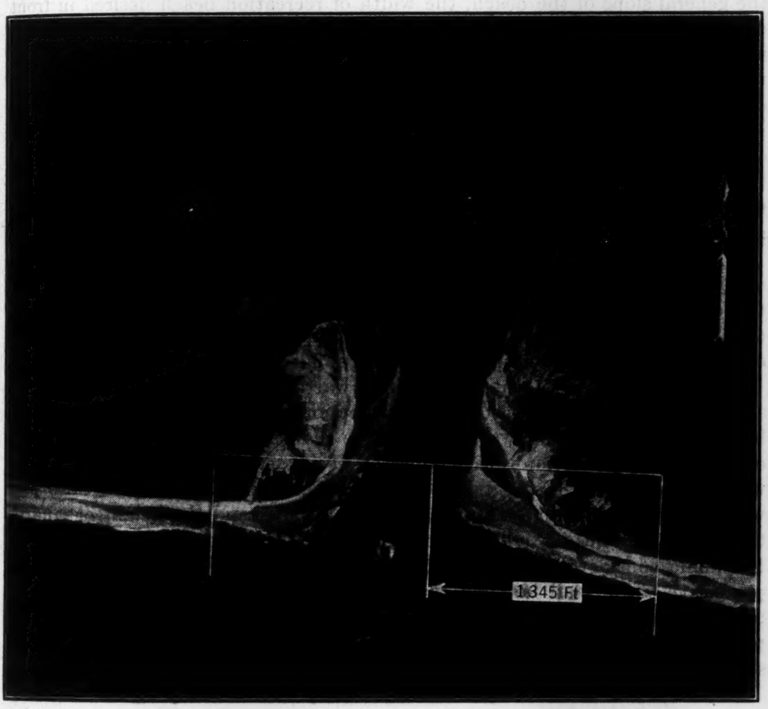


FIG. 10.—MORICHES INLET, LONG ISLAND, N. Y., SHOWING DELTA FORMED BY ONE MONTH OF TIDE ACTION

OBSERVATIONS ON DESIGN OF PROTECTIVE WORKS

Bulkheads.—The problems to be considered in preparing a design for a simple bulkhead and groin system may be summarized briefly. Assume the case of an area to be protected on a long straight stretch of beach distant from the influence of an inlet or change in direction of the shore. Assume that the solution may call for both bulkhead and groins. First, consider the placing of a bulkhead. The factors governing this problem are the location and character of the developed area in the rear, the presence or absence of dunes, the general slope of the beach, the width of recreation beach desired in front of the bulkhead, the character of wave action, and the character of the subsurface material. This information can be obtained from the topographic map or the profiles, the wave observations, history of storms, and the probings or borings.

The line of the bulkhead should be kept as straight as possible. Where slight changes in direction of the beach require a change in direction of the bulkhead, this should be accomplished by a gradual smooth curve, not by sharp angles. Sharp angles or set-backs in a bulkhead system generally cause a concentration of the wave forces with resulting erosion.

Determination of the proper height for a bulkhead requires the consideration of four factors: (1) The character and extent of the development in the area to be protected; (2) the elevation and character of the ground in the rear; (3) the maximum heights and frequency of storm tides; and (4) the probable effect of the groin system in accumulating sand.

Where the development is extensive, warranting the expenditure of considerable sums to afford complete protection against over-topping by the highest storm tides and waves, a massive sea-wall is suitable. An example of this type of protection is the Galveston (Tex.) sea-wall. At other locations, with extensive developments, due to the character of the storm tides and wave action, the volume of sand on the foreshore, and the height of land in the rear, a bulkhead would answer the purpose at a considerably lower cost. Economic considerations determine whether the height and strength of the protective work shall be such as to afford complete protection or whether protection should be afforded against ordinary storm tides only. The lesser protection against ordinary storm tides and waves may result in some destruction during exceptional storms but will prevent complete disaster.

Assume that the development warranted protection only against ordinary storm tides, and that the land in the rear was comparatively low. In determining the proper bulkhead height the probable effect of the groin system in accumulating sand to act as a buffer to storm waves should be considered. If a large quantity of sand is moving along the shore, to be trapped by the groins, this accumulation will act as a buffer to trip the storm waves and prevent their breaking over the bulkhead. The average frequency, duration, and direction of storms should be studied to determine whether one storm might be expected to remove all the sand accumulated during normal conditions, and whether recurrence of storm conditions might be expected before normal conditions have restored a sand supply between the groins. Where conditions are bad, the sand supply cannot be counted upon to act as a buffer to the storm waves, and the top of the bulkhead should be placed at an elevation equal to the average height of the highest yearly storm tides plus wave heights, omitting from consideration extremely infrequent hurricane tides. If the sand accumulation can be counted on to trip the waves before they reach the bulkhead, the bulkhead height may be reduced by the average wave height.

The details of design of the bulkhead will depend upon the character of the sub-surface material and on the probability of the bulkhead being subjected to the direct force of the breaking wave. The length of piles should be such that they will have about two-thirds penetration when the beach is cut down to the point where the low water is at the foot of the wall. Piles should be tied back so as to insure against failure by pressure on the land side when exposed to low water on the ocean side and should be braced to resist wave action on the ocean face. It is important in the design to insure imperviousness to protect the back-fill. The leaching of the back-fill through a bulkhead is a common cause of failure. Consideration should be given to providing release for the water topping the bulkhead during storms so that property in the rear is not damaged.

Where groins may be expected to trap sand to form a beach, the bulkhead is simply a last line of defense and an insurance against serious property loss during exceptional storms. Where the land in the rear is fairly high and well consolidated, the average storm action is not severe, and there is a sufficient volume of littoral drift to insure filling the groins, the bulkhead may be omitted. The groins should then be well keyed back into the high land in the rear. In omitting the bulkhead, it must be realized that since there is no insurance against damage by flanking, immediate and costly repairs may become necessary.

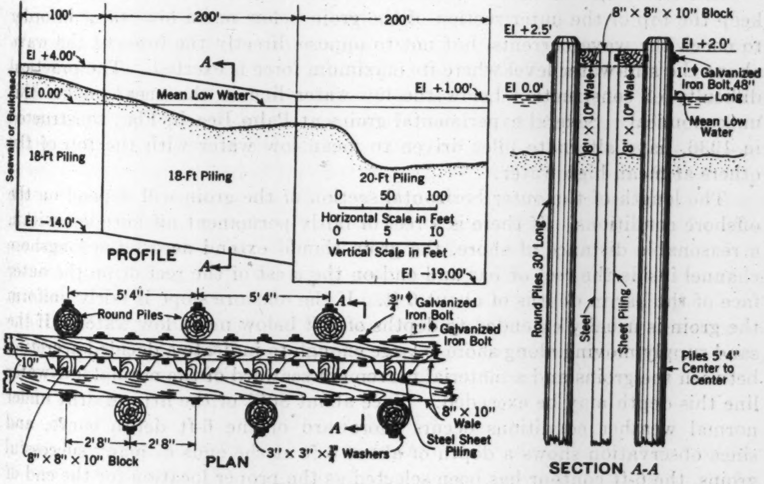

100'
200'
200'
El +4.00'
A
El 0.00'
Mean Low Water
El +1.00'
Seawall or Bulkhead
18-Ft Piling
18-Ft Piling
20-Ft Piling
0 50 100
Horizontal Scale in Feet
0 5 10
Vertical Scale in Feet
El -14.0'
PROFILE
A
El -19.00'
5' 4"
5' 4"
A
3/4" Galvanized Iron Bolt
Round Piles
1" Galvanized Iron Bolt
10"
8" × 10" Wale
A
Steel Sheet Piling
2' 8"
2' 8"
8" × 8" × 10" Block
PLAN
3" × 3" × 3/8" Washers
El +2.5'
8" × 8" × 10" Block
El +2.0'
1" Galvanized Iron Bolt, 48" Long
El 0.0'
Mean Low Water
8" × 10" Wale
8" × 10" Wale
Round Piles 30' Long
Steel
Sheet Piling
Piles 5' 4" Center to Center
SECTION A-A

FIG. 11.—TYPICAL GROIN

Groins.—In the design of a groin the profile should be based as nearly as possible on maintaining or restoring natural conditions (see Fig. 11). An examination of profiles of beaches measured after a considerable period of normal conditions shows, in general, a flat horizontal berm above the high-water line; then there is a sloped section extending from this upper berm to a point below low-water line; and, finally, a more gentle under-water slope. The profile should follow these variations with modifications made necessary by the difficulty of construction in the ocean. Fig. 11 shows an inner horizontal section, a connecting slope, and an outer horizontal section.

The inner horizontal section must be securely fastened to the bulkhead or must be well keyed into the high land in the rear. The top of this inner section is at the average elevation of the natural berm over the entire area to be protected. The length of this horizontal inner section is determined by the requirement of providing an adequate recreation area for the number of people expected to use the beach. Here, economics influences the "picture." If the demand is for a wider upper beach or berm than may be expected from natural accumulation, the groins may be filled artificially by sand pumped from areas in the rear or from the sea offshore. Sometimes, sand is transported from a selected source.

The angle of slope of the intermediate section is made slightly less than the average natural slope of the beach in order to encourage as flat a beach as possible. Since this section joins the inner and outer horizontal sections its length will be fixed by its slope and the difference in elevation between the two horizontal sections.

The top of the outer section is usually fixed at the elevation of mean sea level. This is a compromise with the ocean forces and the difficulties of construction. As far as the ocean forces are concerned it would be well to keep the top of the outer section of the groin below mean low water in order to check the wave currents, but not to oppose directly the force of the wave above the still-water level where its maximum force is exerted. The practical difficulty of construction below the low-water line in the ocean makes this uneconomical. Several experimental groins at Palm Beach, Fla., constructed in 1936, have alternate piles driven to mean low water with the top of the others at mean high water.

The length of the outer horizontal section of the groin will depend on the offshore conditions. If there is a reef or fairly permanent offshore bar within a reasonable distance of shore, the groin should extend across the longshore channel inside the reef or bar and end on the crest of the reef or on the outer face of the bar in depths of about 6 ft. If the offshore slope is fairly uniform the groin is usually extended to depths of 6 ft below mean low water. If the sand supply moving along shore is large enough to indicate a decided accretion between the groins and a material movement seaward of the normal low-water line this depth may be exceeded. Since about 80% of the littoral drift under normal weather conditions occurs shoreward of the 6-ft depth curve, and since observation shows a depth of about 6 ft at the ends of many successful groins, the 6-ft contour has been selected as the proper location for the end of the groins.

The detail of design of the groin will depend on the exposure to wave action and on the material used. Groins should be made sand tight with sheet-piles well bound together by wales and supported by long piles to resist the wave action against them. The sheet-piles should have approximately two-thirds penetration under the most severe erosion expected to occur. In some locations it was unnecessary to use long supporting piles but their omission is not recommended unless good penetration in hard material affords a natural anchorage below which erosion cannot go.

In determining the spacing between groins there is a general rule, obtained from many observations of groin system: The ratio between the length of groin and the distance to the next groin should be between 1 to 1 and 1 to 3. Spacing closer than 1 to 1 does not injure the beach, but is never economical. Spacing greater than 1 to 3 appears to be ineffective in holding a good beach. Within these limits the spacing is determined by the direction of approach of the storms causing the most severe erosion on the beach. After the length of groin has been fixed by offshore conditions, draw a line through the end of the groin parallel with the direction of the storm approach. The projection of this line on the line of the bulkhead will determine the proper space to be allowed between groins. The spacing thus determined, of course, will not always fall within the limits previously set. If not, the nearest limit should be used. This method of determining spacing is not so haphazard as it may seem from a brief description. The more directly onshore the wave action is, the less will be the littoral movement of sand under the influence of the wave forces, and the less serious the erosion. In the case of storm action parallel to the coast the resulting waves do not travel parallel, but the shoreward ends are retarded so that the wave reaches the shore at an angle of about 16 degrees.

The bulkhead must be protected securely at each end against flanking by providing substantial wing-walls into solid land. The danger of increased, or of continued, erosion beyond protective works must be recognized. Numerous methods have been tried both in Europe and in the United States to prevent this cutting back beyond the ends of protective systems. In some cases a progressive reduction in length of groins toward the ends has proved successful whereas in other cases it has not been effective at all. Normally, the greatest erosion occurs at the end of the system away from the direction of sand drift. In some of its designs of protective works, where the littoral sand movement is small or where immediate results are desired, without waiting for slow natural accumulation, an artificial supply of sand can be provided to the beach by dredging from the open sea, or from near-by sheltered waters, if available, or by other means, if required. In many cases dredging offers a quick and inexpensive method of compensating for the normal losses of sand from beaches. Artificial supplies of sand must be renewed from time to time, but the method is worthy of consideration in preparing most designs.

CONCLUSION

Considerable research and study remain to be done before the problem of beach protection can be reduced to a definite science. There is further need for experiments in the ocean on currents, waves, and sand movement; by model experiments in its wave tank; by investigation of materials used in protective works; by periodic aerial photographs of certain areas; and by continued examination of the effect of existing structures.

APPENDIX

OUTLINE FOR REPORTS ON BEACH EROSION STUDIES

I.—Scope of Investigation.—

- (*a*) Authority.
- (*b*) Purpose.
- (*c*) Outline of Study:
    - (1)
    - (2) etc.
- (*d*) Inspection by Board.

II.—Location and Description.—

- (*a*) Location.
- (*b*) Geology.
- (*c*) Land areas.
- (*d*) Existing projects or improvements.

III.—Field and Office Studies.—

- (*a*) Surveys.
- (*b*) Comparison of Maps:
    - (1) Shoreline Changes:
        - (*A*) Present topography.
        - (*B*) High water.
        - (*C*) Low water.
    - (2) Depth Contour Changes:
        - (*A*) Present hydrography.
        - (*B*) 6 ft.
        - (*C*) 12 ft.
        - (*D*) 18 ft.
- (*c*) Composite profiles.
- (*d*) Beach profiles.
- (*e*) Current observations.
- (*f*) Tides.
- (*g*) Sand in suspension.
- (*h*) Sand Analysis:
    - (1) Beach.
    - (2) Offshore bottom.
- (*i*) Borings:
    - (1) Beach.
    - (2) Offshore.
- (*j*) Winds:
    - (1) Diagrams.
    - (2) Analysis.
    - (3) Storm conditions.

IV.—Improvement Desired.—

V.—Discussion.—

VI.—Plans of Improvement.—

- (*a*) Plans.
- (*b*) Specifications.
- (*c*) Estimate of cost.
- (*d*) Comparison of plans.

VII.—Recommendations.

DISCUSSION

Morris N. Lipp,[4] **M. Am. Soc. C. E.** (by letter).—The mention, in this discussion, of certain phases of the beach erosion problem which Colonel Brown did not set forth in his comprehensive paper, is not intended as a criticism of his failure to discuss these matters. They are presented in accordance with his request that his paper be the subject of full and free discussion by all interested engineers.

With full realization of the correctness of his assertion that every locality presents a different problem, a discussion should essentially be confined within the limits of an engineer's own experiences and observations, in order that an accumulation of discussions will establish some definite fundamentals. It is first necessary to know why certain problems exist, whether they are Man made or Nature made. If caused by the activities of Man, the engineer in charge of such activities should be fully cognizant of the future problems that are created.

Two of the factors that contribute to the problem of beach protection are the desire to erect buildings on ocean-front property, and the construction of inlets for sanitation and navigation. It is not within the province of the civil engineer to protest these desirable and necessary activities when kept within reasonable limits, but it is his duty to help solve the problems they produce.

Colonel Brown cites the necessity for geological studies and establishes vital points in his discussion of littoral drift. Once the direction of drift is determined an important barrier is hurdled. It is now accepted as true that the general movement of sand along the Florida east coast is in a southerly direction. This principle was established by the Florida State Geological Division. Analyzing sand at various points along the coast, and tracing the sand to its source of supply, entered into its studies. The engineer, as Colonel Brown points out, sees evidence of the direction of drift at jetties.

At inlet jetties on the Florida east coast there is accretion to the north and erosion to the south. In certain instances sand tending to pass by an inlet is carried from the ocean through the inlet and deposited in the bay to the rear. This latter phenomenon tends to destroy the purpose for which the inlet was constructed. An attempt toward the solution of the beach erosion problem in the vicinity of an inlet has been attempted at the Boynton Inlet, where the sand is pumped from the north side to a system of protective works on the south side. Conditions to the south have been considerably improved since pumping began. No doubt general practice of this kind will provoke objections from property owners on the north side of the inlet. In certain instances it may be feasible to pump sand that has drifted from the ocean into the bay back into the ocean to the south of the inlet.

It should be emphasized that in many instances a beach erosion problem would not exist had inlets not been constructed. Therefore, serious consideration must be given by the civil engineer, during the design of an inlet, to its ultimate effects on adjacent property.

---

[4] City Engr., Miami Beach, Fla.

Colonel Brown states that the progressive reduction in length of groins toward the ends of a protective system has proved successful in some instances, whereas in other cases it has not been effective at all. In South Florida attempts have been made toward progressive reduction, but they have not been successful. It should be realized that after a system of protective works has been constructed on the east coast of Florida its extension to the south will be required eventually.

The requirement advocated by Colonel Brown that a groin be sand tight is correct. This type has proved to be the most successful in Florida. Permeable groins have been constructed, but there have been instances in which they have been converted to solid ones.

To adopt a ratio between the length of the groin and spacing of 1 to 3, which is one limit of Colonel Brown's suggested spacing, would be a radical change from the ratio now used in southeast Florida where an economical length of groin is approximately two-thirds of the spacing.

The length of a groin plays a large part in the extent to which a beach will build up. At Miami Beach, where approximately 9 000 ft of ocean front are protected by city-constructed works, groins in one system have a length of 170 ft, in a second a length of 200 ft, and in a third a length of 250 ft. More extensive lengths of beach have been built up in the areas having longer groins. In all these systems the two-thirds rule has been followed. Naturally, in many instances, such considerations as depth of water, wave action, and structural stability will determine the length.

Other factors such as wave height and life of the structure should be considered, in addition to those mentioned by the author, to determine the profile of a groin. The inshore end of a groin should be above normal wave height, not only because it is desirable to have an available beach at high tides, but so that such a beach will be formed that ordinary waves will break on it, and not against the bulkhead. The back-lash of waves breaking against a sea-wall and meeting the incoming waves will not only retard the building up of a beach, but may prevent the making of one. Where it is desired that the beach be made naturally, it would not be necessary to construct the inshore groin elevation above a point where occasional heavy storms would deposit sand.

Because of the expense involved in protecting beaches it is necessary, as Colonel Brown states, to study the history of existing protective works. In Florida, problems of deterioration and structural stability enter into the economics of groin design. In localities such as Palm Beach, where there is a strong surf, steel-piling has been badly eroded by sand action. At Miami Beach, where there is comparatively light surf, there are only a few signs of steel erosion. When difficulties of this kind are anticipated creosoted timber sheeting, so designed that it can be renewed readily, should partly encase the steel groins. Corrosion of steel in groins appears to be of minor importance. The sand covering at the inshore end, and the continuous wetting and accumulation of barnacles on the outer end, are factors that retard corrosion.

In southern waters, toredo attack creosoted timber through field cuts and drill holes. The life of creosoted timber can be increased by requiring all cutting and drilling to be done before treatment. Special care during construction has proved this method to be feasible. At Miami Beach, where there is a 2.5 ft mean tidal range, steel groins braced by creosoted timber wales and piles have been built at such an elevation, in some instances, that normal water does not touch the timber wales. Groins of this design, with an inshore elevation of 6.5 ft and an offshore elevation of 3.5 ft above mean low water, have functioned successfully.

The forming of deep water at the end of a groin, by eddies, makes special attention necessary for structural stability. Longer sheet-piles and heavier braces are advisable in the affected areas. The placing of granite blocks, if such blocks are available at a reasonable cost, should be effective in strengthening the ends.

Colonel Brown sets forth four important factors that should be given consideration in determining the proper height for a bulkhead. The problem of determining the height of a bulkhead has been comparatively simple at Miami Beach. The top coincides approximately with the backshore terrace. The top elevation is 8.5 ft above mean low water, which is above the ocean water at all times except during severe hurricane periods. To build a bulkhead to hold back hurricane seas would not only be unjustifiable economically, but would impair the appearance of the beach and the view of the ocean.

At Palm Beach the problem is more difficult. In that town there is a highway along the backshore terrace at 16 ft above mean low water. Generally the bulkhead has been constructed close to the highway at approximately the elevation of the highway. In 1937, a bulkhead was constructed to the seaward of the customary location, at an elevation of about 9 ft above mean low water. Naturally, this method of construction is cheaper than the usual type. The height is sufficient to withstand ordinary seas. There is not only an expansive beach in front, built up by the groins, but there is also considerable sand space between the bulkhead and the highway. During hurricane seas there would possibly be some erosion to the rear of the bulkhead. However, if the slope of the sand between the bulkhead and the highway were constructed at a grade not much steeper than the slope of the beach seaward from the bulkhead the tendency toward erosion could be diminished. By virtue of the economies involved and the additional available beach space, construction of this type is worthy of special attention and study.

Corrosion of a steel piling bulkhead does not appear to present a serious problem as only the exposed portions are affected. Since the exposed portions are of shallow depth in a well-designed system an inexpensive concrete cap can be placed to replace a corroded portion.

Colonel Brown has made an excellent contribution to the science of beach protection and his excellent paper deserves a place in the library of every interested civil engineer.

George A. Soper,[5] M. Am. Soc. C. E. (by letter).—Engineers will be indebted to the author for setting forth with such admirable clearness the resources which have been developed by the Beach Erosion Board for the investigation of beach erosion problems along the sandy coasts of the United States, and for the recommendations he has made for the construction of bulkheads and groins.

---

[5] Cons. Engr., New York, N. Y.

The point of view of the engineer who is called upon to protect a piece of property is not always one which is favorable to exhaustive investigation. He must be satisfied with what is necessary and sufficient; for the engineer has a specific, and usually a limited and pressing, problem that he must solve, and he must solve it at the least expenditure of time and money. His investigations must be restricted to what is absolutely required. He cannot carry on elaborate studies, some of which have comparatively little bearing on the particular problem in hand, but tend to build up the science and art of beach protection.

The writer strongly indorses what the author says about the value of inquiries into the history of the particular locality in question, with reference to destructive storms and the fate of structures that have been built at this place. Of the highest importance, also, is the necessity of assembling at the earliest moment such evidence as already exists concerning the probable effects of changes, natural and otherwise, that have been made in the vicinity of the place where the protective works are to be located. As early as possible a rapid survey should be made to see what information may be available and how much reliance may be placed upon it. It will sometimes be surprising to find what useful leads personal testimony may supply. Often this is all the evidence procurable. The opening of the Moriches Inlet and, in fact, the curious behavior of the entire beach at Fire Island, Long Island, N. Y., since the earliest times, have much light thrown upon them when such information is taken into consideration.

Human testimony sometimes affords the only clews which the engineer can obtain concerning storms; and, it should be emphasized that it is storms, and not average weather conditions, in which he should be particularly interested. Unfortunately, lists of storms are not kept by the U. S. Weather Bureau or any other branch of the Government. Nor is the severity or damage done by them often made the subject of an engineering report.

An investigation of the subject of destructive storms along the Atlantic Coast—their frequency, duration, damage done, and the effectiveness of the measures taken to protect property against them, with the practical lessons to be derived therefrom—would be a valuable contribution for the Federal Government to make.

The writer is in favor of inducing the U. S. Coast Guard to make more definite and accurate records of the weather, and he would have observations of high and low tide, the angle of approach of the waves, the height of the tide, and the configuration of the beach included in the log.

More information is needed concerning the general movements of the coast line than can always be had from the maps and surveys that are available. Is the south side of Long Island retreating, for example, and if so, to what extent? Is it subsiding and, if so, at what rate? To what extent does the formation of bays and points along the shore represent continuous tendencies, and how can this be determined when a system of bulkheads and groins, say, 1 000 ft long, is to be constructed?

The author has said little about the movement of sand under the influence of wind and in the regions above the reach of the ocean; and yet this would seem to be an important part of the beach erosion investigations. He has stated that sand traps and their proper use might give considerable helpful information; but, will he not suggest how they should be constructed and used? The quantity of sand moved by the wind toward, and away from, a beach is at times very great. The writer has seen, from a distance, the atmosphere for a height of more than 200 ft look as clouded by sand as from smoke where a fire was raging. More often the sand is swept in drifting currents along the surface of the beach. The writer knows a situation in which the movement of dry sand has considerable effect in determining the proper means of beach protection. The question of sand fixation is closely related to that of beach erosion and is by no means a simple matter.

The writer is in hearty accord with Colonel Brown as to the need of placing as long a stretch of beach as practicable under the protection of a single comprehensive scheme, but he does not see how an engineer who is called on to build a structure in front of a house or along any other limited water front can control this very important matter. No matter how well his work is done, if the scope is not adequate, the result will be more costly and less effective than it should be.

Among the elements of design mentioned by the author one which seems especially worthy of endorsement is the need of keeping the water from getting behind a bulkhead; this is imperative, whether it gets there over the top or around the ends. It is almost impossible to get such water out, once it gets in; but if the author has some simple way of accomplishing it, it is hoped he will describe it in his closing discussion. It seems much better to keep the water from getting behind the bulkhead in the first place. It may be prevented from getting around the ends by bringing them back to high land or to another bulkhead, if there is a good one at hand; but against water coming over the top the best plan seems to be to back up the bulkhead to the top with sand or other solid filling, and keep it filled. The author seems to think this is not a very expensive procedure, but the writer believes it is generally so. Suction dredging is, by no means, always available. It is not practicable on the open seashore unless there is a plentiful source of supply in the rear, and any other method of handling sand is expensive.

JAMES J. O'ROURKE,[6] ESQ. (by letter).—Under the heading "Observations on Design of Protective Works" Colonel Brown suggests a "bulkhead and groin system" or a "groin" system. These will be successful where there is a considerable quantity of sand carried by the littoral drift. Otherwise, such groins are unsuccessful and Colonel Brown recommends that an artificial supply of sand be pumped between the groins, but admits that this supply may have to be renewed from time to time. This requires considerable money for maintenance.

---

[6] General Eng. Designer, Chicago Park District, Chicago, Ill.

The Outer Drive in Chicago, Ill., through Lincoln Park, extends parallel to Lake Michigan from North Avenue to Fullerton Avenue. The east curb (that is, the one nearest the Lake) is separated from it by a paved concrete beach about 60 ft wide. The elevation of the drive is about + 10. The paved beach slopes from the roadway at Elevation + 10 to the old sea-wall at Elevation + 2. The elevation of Lake Michigan since 1919 has averaged about 1 ft below datum. In 1929 there was an unprecedented rise in the waters of Lake Michigan to an average level of above + 2 during the summer months, which is the approximate elevation of the top of the old sea-wall. This rise, accompanied by strong winds off the Lake, destroyed part of the drive.

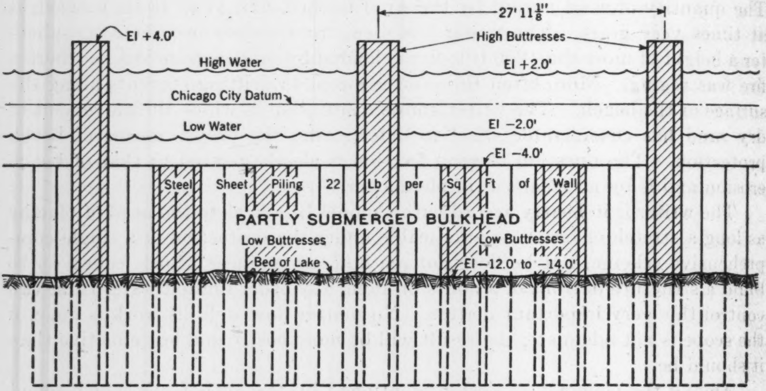

27' 11⅛"
El +4.0'
High Buttresses
High Water
El +2.0'
Chicago City Datum
Low Water
El −2.0'
El −4.0'
Steel Sheet Piling 22 Lb per Sq Ft of Wall
PARTLY SUBMERGED BULKHEAD
Low Buttresses
Low Buttresses
Bed of Lake
El −12.0' to −14.0'

FIG. 12.—ELEVATION OF PARTLY SUBMERGED SEA-WALL

In 1930 a series of groins each about 500 ft long and spaced 600 ft apart were built along this stretch of roadway and shore line, inclined at an angle of about 70° from the north, the idea being that a considerable quantity of sand would be accumulated between the groins and thus protect the roadway from future storms. These jetties did not accumulate any sand in the 9-yr period since that time because there is practically no littoral drift at this point. Fortunately, however, there has been no high water since 1929 and no further damage has resulted on account of storms.

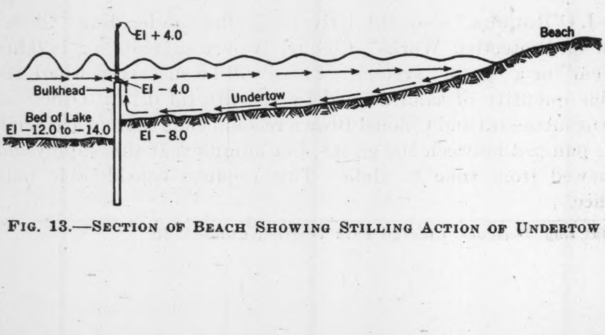

El + 4.0
Beach
Bulkhead
El − 4.0
Undertow
Bed of Lake
El −12.0 to −14.0
El − 8.0

FIG. 13.—SECTION OF BEACH SHOWING STILLING ACTION OF UNDERTOW

In 1939 the Commissioners of the Chicago Park District, who control the shore line of Lake Michigan at this point, decided to build a bathing beach with the two-fold purpose of protecting the roadway from future storms and creating much needed bathing facilities for the people of Chicago. In 1938, 1 800 ft of bathing beach had been completed.

A partly submerged bulkhead (Fig. 12) was constructed, joining the lakeward ends of four adjoining jetties. This formed three bays, each 500 ft long and 600 ft wide, into which sand was pumped to an elevation of + 5 at the old sea-wall and + 8 at the new, partly submerged bulkhead. This created a sloping beach from a point about half-way up on the old paved beach to the new bulkhead, the old sea-wall being covered up in the operation and no longer serving any purpose.

This method of shore protection differs from that recommended by Colonel Brown in that the bulkhead is eliminated at the shore end of the groins and the partly submerged bulkhead substituted at the lakeward end. The advantages of the latter over the former are:

(1) It is much cheaper; the partly submerged bulkhead costs from one-third to one-quarter as much as a sea-wall along the shore; and,

(2) It effectively holds the sand behind it, whereas with the former the danger of sand being carried away is always present.

In operation, the partly submerged sea-wall breaks up the sea in this manner. The portion of the wall above the water partly impedes the waves. All the water that passes over it must return to sea in some way. It goes back in the form of an undertow along the bottom. When this undertow reaches the submerged part of the sea-wall it cannot travel any farther along the bottom. In order to reach sea it must rise to the surface of the water and then, in trying to escape, it encounters the sea coming in, thus causing a stilling action at the wall. This is shown in diagram in Figs. 13 and 14.

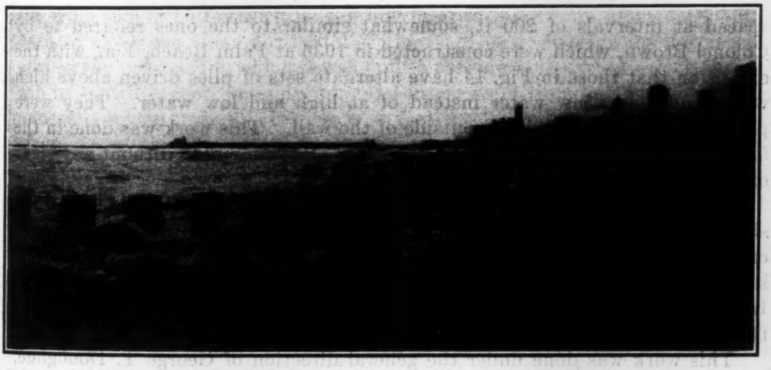


FIG. 14.—SHORE-LINE WAVES AT SEA-WALL; NEW NORTH AVENUE BEACH, LINCOLN PARK, CHICAGO, ILL.

Fig. 14 is a view photographed on March 27, 1939, with a wind of 30 miles per hr blowing from the northeast at about right angles to the shore. It was taken at low water, approximately Elevation − 1. Just to the left of the partially submerged sea-wall are partially submerged jetties 75 ft long and spaced at intervals of 200 ft, somewhat similar to the ones referred to by Colonel Brown, which were constructed in 1936 at Palm Beach, Fla., with the exception that those in Fig. 14 have alternate sets of piles driven above high water and below low water instead of at high and low water. They were placed to reduce scour on the outside of the wall. This work was done in the fall of 1938 and has withstood the storms of one full winter without any signs of movement.

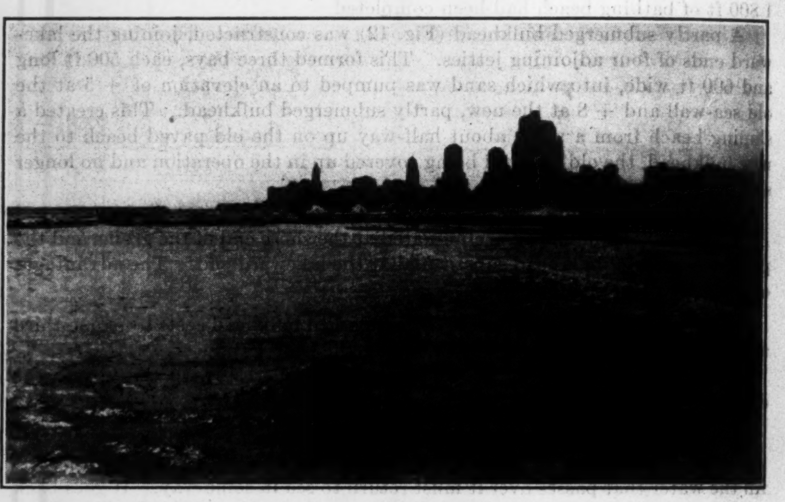


FIG. 15.—SHORE-LINE WAVES BEHIND SEA-WALL; NEW NORTH AVENUE BEACH, LINCOLN PARK, CHICAGO, ILL.

Fig. 15 shows the beach behind the wall, the waves gently lapping on the shore, their force dissipated by it. In contrast the waves from the open sea can be seen pounding against the old sea-wall in the distance. The partly submerged sea-wall has the high buttresses spaced at intervals of 27 ft 11½ in. In future installations this interval will be reduced to 14 ft to assist in breaking the waves at high water.

This work was done under the general direction of George T. Donoghue, M. Am. Soc. C. E.

ELLIOTT J. DENT,[7] M. AM. SOC. C. E. (by letter).—The information that should be secured before undertaking the design of shore-protection works is described fully in this paper. The list of items appears formidable, but the full investigation of one site will ordinarily supply many of the data needed for other sites in the same general vicinity, and the assembly of basic regional data in the files of the Beach Erosion Board often reduces the time, expense, and effort that might otherwise be required when a particular project is under consideration.

---

[7] Col., U. S. Army (*Retired*), Washington, D. C.

The author states that: (*a*) The ratio of length of groin to interval between groins should be between 1 to 1 and 1 to 3; (*b*) spacing closer than 1 to 1 does not injure the beach, but is never economical; and (*c*) spacing greater than 1 to 3 appears to be ineffective. He also states that, subject to the above limits, the correct spacing may be found by drawing through the outer end of a groin a line parallel to the direction of wave approach. The projection of this line upon the beach will give the proper interval.

Along a straight section of beach the axis of wave approach rarely makes an angle of as much as 20° with the perpendicular to the shore. If a line parallel to this axis is drawn through the outer end of a groin, the point of intersection with the beach will be less than half a groin length to leeward of the base of the structure. The use of this rule for the spacing of groins would indicate that, except for economic reasons, the structures should be more than twice as long as the interval between them. This is more than twice as many groins as the author advocates for the closest spacing, and a strict application of the rules would seldom, if ever, permit a spacing of more than 1 to 1.

Fig. 11 shows the profile of a typical groin of the form advocated in 1933 by the Beach Erosion Board for normal use. Experience has since shown that the top of the inner horizontal section may be lowered to at least 1 ft below the normal berm level without affecting the height or width of the finished berm. The sand level shown in Fig. 11 gives conditions before the beach has been built up to the desired form; when the beach has assumed the desired section the groin should be buried from the bulkhead to some point along the sloping part, or along the outer horizontal section. After a set of groins has accomplished the purpose for which it was built a considerable part of each structure will be buried except, possibly, for brief intervals of time following periods of unusually severe erosion. If there is, in fact, a relation between the best length of groin and the best interval between structures, the question arises as to whether the length should be measured from the outer end to the bulkhead, or only to the point where the buried section begins after the beach has assumed the desired form. The latter would seem to be the more logical choice.

It cannot be too strongly emphasized that groins are, in no sense of the word, breakwaters serving to protect the intervening beaches from direct wave attack. However, they may be designed to control the wave-generated currents in contact with the foreshore, and the distance required by diagonal wave attack to create currents of destructive strength is, therefore, an important item in the determination of the proper interval; this distance is vitally affected by exposure, but only slightly so by groin length.

The adequacy of the sand supply is also an important item in the selection of the proper spacing between groins. With a liberal supply of littoral drift the spacing may be much greater than when a scarcity exists.

Each groin inevitably reduces the volume of sand available for the nourishment of the leeward beaches. During the period of accretion along the windward beach the quantity of sand impounded is definitely deducted from the supply of drift. When the impounding capacity of a groin has been exhausted, the quantity of sand passing the site will be unchanged but, unfortunately, part of the sand stream will be detoured around the outer end of the structure in deeper water and at a greater distance from the leeward shore than would be the case on an unobstructed beach. This results in a wastage of sand, and, to minimize this loss, the number of groins should be held to a minimum and the interval between structures should be correspondingly increased. Economy is not the only reason for using the minimum number. The inadequacy of the supply of beach nourishment to the leeward of well-built sets of groins is a phenomenon that has been observed often and has been the subject of much comment.

The proper length of groin for a given location will depend, in part, upon such factors as the width of berm desired, the slope of the foreshore, the tendency for offshore bars to form, and their location. The proper interval between groins will depend largely upon the adequacy of the supply of littoral drift, and the distance required by nature for the formation of destructive wave-generated currents. There appears to be little or no relation between the best length and the best interval.

The art of shore protection has not yet progressed to a state in which past practice can be safely accepted as correct practice, and existing works should be studied judiciously with a view to the development of better practices. In promoting this principle the Beach Erosion Board has made a photographic and written description of a multitude of existing shore protection structures. A careful examination of this record, including a study of the local conditions, the results accomplished, and the spacing, fails to show the necessity for groins as closely spaced as indicated by the rules presented by the author.

In an address before the American Shore and Beach Preservation Association,[8] the writer advocated the creation of improvement districts, each of which would be responsible for the maintenance of an entire natural subdivision of the shore, usually extending from inlet to inlet. It was assumed that, in most cases, an artificial supply of littoral drift or nourishment would be called for and that it would be to the best interests of the districts to conserve that nourishment after having gone to the expense of placing it upon the beach. It was also assumed that there would be an economic balance between the supply of more sand and the cost of other methods of protection. The writer stated that the best spacing of the groins depends upon a number of factors. The district will be called upon to supply the necessary amount of nourishment; in the immediate vicinity of the point of supply no groins will be required to regulate the movement of the littoral drift; as the distance from the point of supply increases irregularities in the beach will develop and groins may be necessary to prevent excessive local erosion. Groins are expensive to build, expensive to maintain, unsightly, and cause a wastage of sand. The best spacing will depend upon a balance in the cost of a more generous supply of nourishment, more frequent points of supply, stronger bulkheads, more groins, and the general utility of the improvement finally adopted.

---

[8] "The Next Step in Shore Protection," by Elliott J. Dent, *Shore and Beach*, Vol. VI, No. 3, July, 1938.

Ralph F. Rhodes,[9] M. Am. Soc. C. E. (by letter).—As stated by the author, the science of beach erosion is new in the United States and comparatively little information dealing directly with the subject is familiar to the average engineer. This paper, therefore, will be a valuable guide to less experienced engineers in the study of the subject, particularly when a problem for determining the cause of beach erosion and making plans for its arrest or prevention at a particular locality is taken up. The author covers his subject comprehensively, and very properly warns the inexperienced engineer against adopting a plan for a particular locality without full consideration of all factors contributing to the destruction of the beach or simply because such a plan has proved effective elsewhere. All plans made for the protection of a beach are, to a certain extent, experimental; but they should be based on the scientific application of all pertinent available data to the problem involved. In the writer's experience, when a beach erosion problem arises and a system of protection works is being planned, usually the plans recommended locally as "certain to do the work best and cheapest" are many and varied in character.

The writer's studies of problems of beach erosion and protection have been confined principally to the coast of Georgia in what the author calls the "South Atlantic Subdivision of the Atlantic Coast." The Georgia coast is part of a long swinging curve extending from the North Carolina capes to the central part of the east coast of Florida. This curve so affects tidal ranges that, while mean ranges are about 3.5 ft and 4.0 ft at the northerly and southerly end of the curve, the value is increased to about 7 ft at Tybee Light, Georgia. The corresponding spring ranges of tide are 4.2 ft at Cape Hatteras, North Carolina, 4.7 ft at Daytona Beach, Fla., and 8.0 ft at Tybee Light. At Cape Hatteras, high water occurs 6 hr and 56 min after the moon's meridian passage, while at Tybee Light it is 15 min later. In general, this tends toward the generation, during a rising tide, of a southerly current along the shore between the two points.

In Georgia, the principal beach erosion problems occur on the ocean shore of the string of islands which form the coast line. These islands are separated from each other by deep sounds scoured out by the swift tidal currents generated by the large tidal ranges on this section of the coast. Some of these sounds are at the mouths of large inland river systems, while the others are the outer ends of large estuaries which project far into the mainland.

The large tidal volumes moving through these sounds necessarily affect the currents along the ocean beaches for a considerable distance on each side of the inlets and, consequently, the sand movement along these beaches. The inlet currents interfere with the general movement of littoral drift toward the south, building it into bars outside of the sounds and pushing the longshore drift out to sea. This robs beaches immediately to the south, of a supply of sand which otherwise they would normally have. These conditions are affected still further by the winds, which vary considerably in direction and intensity. The problems in a beach erosion study in this section are, therefore, vastly more complicated than they are where the beaches are continuous for long distances, the inlets small, and the tidal currents through them moderate.

---

[9] Prin. Engr., U. S. Engr. Dept., Savannah, Ga.

The author calls attention to the necessity for a study of the changes in the offshore depths. Such a study is important in the vicinity of large inlets, especially where these have built bars immediately to the seaward of the mouth. The extent of such bars affects the location and intensity of the tidal currents entering and leaving the inlet, frequently in such a way as to crowd them in close to the shore line. Under these conditions, rapid erosion of the beaches, especially in the immediate vicinity of the inlet, may be expected due to the fact that all beach material broken down by wave action is carried away rapidly by the accelerated currents along the shore.

As pointed out by Colonel Brown, a knowledge of the geology of the beaches is helpful in solving their erosion problems and designing the protection works. The islands of the Georgia coast were once offshore bars and probably very much more extensive than they are now. Before they became covered with vegetation and trees, sand blew over them and covered extensive areas of marsh between them and the mainland. Since this time, the sea has recaptured a large part, if not all, of the areas occupied by the original islands so that, in some localities, the present islands consist only of the sand that was carried landward over the bars during prehistoric times. As the ocean shores of these islands erode, a chocolate colored "rock," consisting of a hard mixture of marsh mud and sand, is exposed on the beaches. Although such material may not materially affect the rate of erosion of the beaches, it does affect the design of protection works, at least as to their foundations.

In the study of the shore-line changes and changes in offshore depths, which is an important part of a beach erosion study, a most valuable source of data is found in the U. S. Coast and Geodetic Survey charts. These charts, however, must be used with caution when comparing those of different dates. They usually contain a note such as "Surveys to 1900." This does not necessarily mean that the whole chart was corrected to show conditions as they existed in 1900. What is meant is that the 1900 chart has been revised in certain portions to incorporate data obtained in 1900. Other portions of the chart may show shore lines and offshore depths based on data of a much earlier date. Without knowledge of this fact and just where on the chart the revisions were made, the student may come to the erroneous conclusion, from comparison of charts of different dates, that shore lines and offshore depths in certain areas have remained stable. In such studies it is necessary, therefore, to obtain from the U. S. Coast and Geodetic Survey copies of the field sheets which show the particular area in which revisions have been made, and use them instead of the whole chart. In 1934 the South Atlantic coast line was mapped by aerial methods, and new general surveys were made of offshore depths. The latest editions of these charts are based on these data and show throughout the chart area conditions as they existed in 1934.

As stated by the author, the cost of securing aerial photographs for a section of beach for a small study is prohibitive. This probably refers to photographs made with regular aerial mapping equipment. It may be found, however, that aerial photographs taken by an amateur with ordinary photographic equipment are valuable in supplementing other data. Small planes are often available at reasonable hourly charges, so that flights over a section of beach at relatively frequent intervals may be made at small expense. Photographs so made show changes produced by periods of good and bad weather, and even if the pictures are taken at an angle with the vertical which is not too large, they may be corrected in an ordinary machine during enlargements. The degree of enlargement may also be adjusted, so that their scales are convenient for use in comparing them with maps made by ground methods.

Beach protection works may be divided into two general classes: (*a*) Sea-walls, and (*b*) groins. In certain cases, the two classes may be combined to form a system with a bulkhead parallel to the shore and connected groins perpendicular thereto, extending below the low-water level. Generally speaking, a sea-wall alone is not expected to recapture lost beach area, but only to hold the shore line against further erosion and to enable property in the rear to be developed. The details of construction depend on whether beach area is being lost and deep water is moving shoreward or whether the low-water line is stationary and the high-water line is moving landward. In the former case the sea-wall must be of heavier construction and its seaward toe well protected from undermining.

The degree of protection ordinarily afforded by a system consisting of a bulkhead and groins is measured, to a large extent, by the amount of sand entrapped and held between the groins and in front of the bulkhead. The groins in themselves offer little protection to the shore line. Their office is to collect and hold sand against the bulkhead, leaving the remainder of the protection to be furnished by the bulkhead structure. Although this structure must be made strong enough to withstand the force of the seas while the sand is collecting in front, it ultimately gives protection only against waves which reach higher elevations than the accumulated sand between the groins.

In designing a system of protective works for a beach which is eroding, full consideration should be given to the length of shore line which should be covered by the protective works. If this is not done, it may be found, after the works are erected, that adjacent sections are beginning to erode which have never before showed signs of cutting back. It may also be found that, as the works are being constructed, the process of erosion is accelerated in adjacent locations where works have not been started. This is because the protective works have trapped sand which was the usual supply of beach building material for the other sections of the beach and the damage has simply been transferred from one locality to another. The susceptibility to such occurrences is greatest in localities where the supply of moving sand for maintaining the beaches is limited. Therefore, the construction program for such conditions should be based on full consideration of probable changes in weather in that part of the beach where no works are yet erected as construction proceeds in the other part. Similarly, consideration should be given to the effect of groins partly or wholly completed on other parts of the beach during sudden storms or other radical changes in the weather.

The author has gone very thoroughly into the details of design of the works for beach protection, and his criteria should not be ignored unless conditions at the locality under study show good and proven reasons therefor. As stated in his first paragraph on the design of protective works, he has assumed shore conditions along straight stretches of beach distant from the influence of an inlet or change in the direction of the shore. For protection against ordinary storm tides and with the land in the rear comparatively low, he specifies that the top of the bulkhead should be placed at an elevation equal to the average height of the highest yearly storm tides plus wave heights. In the vicinity of the mouth of the Savannah River, this would be about 9.5 ft above mean low water, the height of the storm tide, plus 3.5 ft for the wave heights. These two added together give a total of 13 ft, and if the bulkhead consisted of piles with lengths three times this total, 40-ft piles should be used.

In view of Colonel Brown's requirement that the bulkhead should be tied back, so as to insure against failure from pressure from the fill on the back side of the bulkhead, and braced against impact of the breaking waves on the front side, a penetration of 26 ft below low water may be excessive. A bulkhead so designed does not depend for stability on the structural strength of the sheet-piles at the ground line, the length of these piles below ground being for the most part protection against the wall being undermined. If the worst condition as to scour in front of the bulkhead is assumed, below which no erosion can take place, then the length of piles below this level need be only sufficient to prevent blowing out beneath them from hydrostatic pressure at the rear. A similar analysis for length of pile beneath the level of most severe erosion should be made for groin structures as well. Where brace piles are used, these members are designed for, and intended to take up, all stress due to shock of wave action, and the sheet-piling below the level of most severe erosion is only for the purpose of preventing a blowout beneath the groin due to hydrostatic pressure. In a groin where no brace piles are used, sheeting must take up wave shock also; and in determining its penetration, its length in undisturbed material and the nature of the material must be considered.

On a curving shore approaching a large inlet, the rule laid down by the author for the location and spacing of groins on a straight beach cannot be applied near the inlet. No exact rule which can be used as a guide is known to the writer, but data should be available as to directions and velocities of the currents in and near the inlet at various stages of the tide and at several distances from the shore. How these currents are affected by winds from several directions should also be known. From consideration of these data, it will probably be found that the groin spacing should be less near the inlet than that found correct for a straight section of the beach at a greater distance from the inlet. It will also be found that the pointing of the groins should be about normal to the shore line, the last groin being about at right angles to the axis of the inlet and as far inside as erosion from waves may occur. After reaching this point, it may be found necessary to extend the groin system along the shore into the gorge as far as erosion from inlet currents is taking place. The design for such a groin system, therefore, gradually changes from one based on the specifications laid down by the author for a straight beach to one based on specifications for bank protection against lateral currents.

As Colonel Brown states, there is still need for additional research and study. Well-directed inquiry, however, will disclose considerable useful information of which little is known to the engineer who is engaged in general practice only and to whom the science of beach protection is a comparatively new subject. Such information or references to it can be had for the asking by applying to the Beach Erosion Board mentioned in this paper. However, the engineer who expects to have beach erosion problems presented to him for solution should be continually on the lookout for other information on the subject, especially as to the history of structures already built in other localities where beach, weather, and current conditions are similar to his own.

CHARLES T. LEEDS,[10] M. AM. SOC. C. E. (by letter).—The author has given an excellent syllabus of beach erosion study with an enlightening commentary thereon. Beach protection is a most fascinating and a most tantalizing field of engineering, because of its many elusive and often indeterminable factors. In a field of science which is so largely empirical the wise engineer will search all possible sources of information. Therefore, such a general outline as the author gives of a typical study, together with an explanation of the need of each item, is a valuable contribution. Because this field of engineering is so largely based on experience, however, one of the greatest needs is a more extended and detailed record of results under varying conditions, but perhaps such record should be made the subject of another paper.

The engineer who attempts to diagnose the symptoms of an inadequate or menaced beach, and to prescribe remedial measures, is in much the same position as a doctor prescribing for a human patient; for a beach, like a human being, is ever-changing—continually wasting and rebuilding. The first requisite is the past physical record (geologic as well as historic) and all available past surveys and photos should be obtained and supplemented by personal observations, past and present. Even then the records are often too meager and intermittent to afford other than a very broken story of the changes that have taken place in the shore line and in the offshore conditions.

Just here a word of caution is important. Not only the year of the survey, but also the season, should be learned, if possible, for the direction of littoral drift may vary with the time of year. Unfortunately there probably will be no record of the weather at the time of these old surveys; yet this is important, for the beach may recede a considerable distance in a single storm, and in a succeeding period of fair weather it may undergo a more or less complete restoration by natural accretion. Therefore, a given survey or photograph may or may not be representative of the average shore conditions. This is especially worthy of remembrance in case of a dispute regarding title to lands bordering the ocean, for in some instances the land values in question may reach huge proportions.

Where there has been a considerable lapse of time between surveys it is highly important to use these with great care. It is by no means certain that the movement of the shore line between the locations shown on different dates has been either continuous or all in the same direction.

---

[10] Maj., Corps of Engrs., U. S. Army (*Retired*); Cons. Engr. (Leeds, Hill, Barnard & Jewett), Los Angeles, Calif.; State Consulting Seacoast Engr.

The writer recalls an engineer friend who compared two surveys dated, say, 1860 and 1890, and because the river mouth in 1890 was 1 000 ft westward of its location in 1860, he assumed a progressive westward littoral migration of 30 to 35 ft per yr, notwithstanding that the present prevailing littoral drift is in the opposite direction. He assumed, from recorded statements of "old-timers," that southeasterly storms were more frequent and more severe in the Nineteenth Century than they have been in the Twentieth Century, and therefore, that the predominant littoral drift has been reversed. Such a theory of meteorological cycles is not believed to be tenable. It is more probable that the river mouth actually migrated eastward under the influence of prevailing waves from the southwest, until a heavy flood broke out a new mouth well toward the west, when it again resumed its eastward travel. Such a migratory cycle is well known to those familiar with inlets on sandy coasts where there is a predominant littoral drift. The survey of 1860 probably showed the river mouth at nearly the eastward limit of its migration during a dry period, whereas the survey of 1890 probably showed its position just after a heavy flood, at the beginning of its migratory cycle.

In discussing the need for study of past wind and weather conditions, the author might well have given even greater emphasis to the statement that "If it is apparent that there is a great seasonal variation in wind direction and intensity, and if the detail of the study warrants, such wind-roses should be plotted for each month." The writer has seen (as have doubtless many others) a single storm, in a short time, undo the work of the prevailing wind movement over a long period. Such information will not receive adequate attention in a wind-rose showing the aggregate of all observations over an extended period.

A very important item of information, of which there is seldom adequate accurate record, is that of wave data. It is true that the author states (see heading "Field Work: Wave Data"): "Whenever observations on currents in the ocean or sand in suspension are made, careful readings of wave data should be made and recorded." However, such records can be only fragmentary and are pertinent only for the times of the "observations on currents in the ocean or sand in suspension." Wave action undoubtedly constitutes the major factor in littoral sand movement; therefore, the wave direction, velocity, height, and especially duration are even more important than the wind direction and velocity. Indeed, serious beach erosion is sometimes caused by severe wave action occurring in periods when there is no appreciable local wind, these waves being the result of distant storms.

As the author states (see heading "Scope of a Beach Erosion Study: Littoral Drift"): "* * * the direction of predominant littoral drift * * * is most important in the design of a protective system," and "If the detail of the study permits, observations as to the direction of littoral drift should be continued over a period of a year to determine these seasonal changes." Yet current measurements and sand samplings are admitted to be impracticable, except under relatively calm conditions, and to continue them over an extended period, such as a year, is often unduly expensive.

What is needed is the summation of all movement of the sand, landward or seaward, and upcoast or downcoast. By placing, on the beach, some material of approximately the same specific gravity as the beach sand or gravel, but easily distinguishable therefrom, and by observing the location of this material periodically, the aggregate movement can be recorded. The writer feels that inadequate efforts have been made in this direction, although he recognizes the difficulties. For instance, in one case, an engineer had small piles of brickbats placed on the beach at about mid-tide, only to find later that children and picnic parties had used them to build picnic fireplaces and toy castles on the beach beyond reach of the waves. Notwithstanding this discouraging experience, it is still believed that some modified application of this procedure is practicable. Pea coal or any obtainable sand and gravel clearly distinguishable from the local beach material would be satisfactory.

One pertinent factor not mentioned by the author, which has much to do with beach maintenance in regions where detritus from the upland constitutes the major source of beach material, is the magnitude and frequency of flood flows. It has been noticeable in California that during years of little rainfall complaints of beach erosion become very prevalent, whereas in years following heavy rainfall and floods shore communities are wont to boast of their wonderful beaches.

Flood control measures which include long jetties at the river mouths, designed to stabilize and maintain these outlets by delivering the flood-borne sand into deep water, are likely thereby to result in beach starvation by depositing the major part of the sand beyond the reach of ordinary wave action instead of near the shore line as in a state of nature. Similarly, upland debris control measures, although most meritorious in preventing land erosion, must inevitably deprive the beaches of much of their natural replenishment. As a consequence, engineers must look forward to an increasing demand for beach protection.

In all these preliminary investigations, it is believed that the chief danger lies in attempting, or being forced, to draw conclusions from data which are necessarily, or otherwise, inadequate. This inadequacy may be the fault of the engineer through his lack of experience; or it may be beyond his control, due to lack of available funds or non-existent historical records.

The effort of an individual to protect his property from marine inroads is not unlike that of a property owner on a river to protect himself from flood damage. Not only is it far more expensive if done individually than if done in cooperation, but it is far less effective and sometimes almost hopeless. Just as flood control, to be effective, must be designed with the entire drainage basin in mind, so shore protection, to be most effective and economical, must be designed with due thought for the influence from and upon a very considerable extent of shore line. Wherever feasible the coast line should be divided into such sections or stretches as can be considered independently of each other. Then for each section a protection system, such as a "groin field," should be planned before undertaking the construction of any individual part.

The only adequate solution of a beach improvement problem lies in such "coastal planning" considering a stretch of beach of sufficient magnitude to be treated as an independent and complete unit. By thus envisaging a future comprehensive system, the engineer can lay out the most suitable ultimate location for high-water mark and determine the length, number, and spacing of groins which will best produce this result. Parts of this system can then be constructed from time to time with the assurance of an ultimate co-ordinated whole. However, if each property owner is concerned only with the protection or improvement of his own property, he will devise a plan to fit his own selfish ends, which may (but probably will not) harmonize with a general plan which would be best for the public good. The writer realizes fully that this ideal policy is usually practicable only for the publicly employed engineer, and that the engineer employed by an individual owner can seldom consider aught but the interests of his own client.

Furthermore, in most States, the individual riparian owner is legally limited by the "common enemy" doctrine of law. He may erect structures of only such nature and extent as will protect his own property from inroads of the sea. If he limits himself carefully by this requirement, any consequent damage to his neighbor's property is "damnum absque injuria," or injury without legal wrong. However, if he goes beyond work purely for protection and endeavors to improve his property by widening his beach, then he renders himself legally liable for any injury which may result to his neighbor's property.

When all the property owners join in an improvement or protection district, however, they are then mutually responsible for the work of the district, and moreover, the engineer then has a better chance to so design his work and plan his construction (normally constructing the leeward groin first) as not to cause injury to neighboring property.

Not only is co-operation between beach owners essential to secure the best design of a shore protection project, but it is even more so on the grounds of equity and economics. Very often the cost to an upland owner of providing protection for his property alone is out of proportion to the value of the land, whereas his proportional share of a complete shore protection system would be much more reasonable. Even in the latter case, however, it is often inequitable to expect the upland owners to pay the entire cost of this shore protection. The indirect value to the community of an attractive beach frontage, as well as the direct return to the community in increased taxable value, makes it justifiable for the public to bear some proportion of the total cost.

Much has been written of the unsightliness of groins and the saw-toothed beach line which they produce. It is the high, solid groin which is chiefly responsible for this unwarranted criticism. The remedy lies in low groins, built up gradually as the beach builds up, the portion of the groin shoreward of ultimate high water line being kept at or below the natural beach level. Examples of this may be seen in Fig. 16, which shows low groins, the inshore ends of which have become buried by natural accretion, and which have produced a smooth shore line.

It is also well known among engineers experienced in shore protection that the ideal method of developing a smooth beach is to keep the groins only a few feet above the beach level, and if possible to raise, gradually, the part of the groins seaward of high-water mark as the beach builds up. This ideal is not always feasible, but the writer has used a very effective modification of this system where the groin is of steel sheet piling. Every fourth pile is driven a few feet lower than the remainder, thus leaving "windows" in the groin. At first these "windows" permit the upper part of the wave (which, of course, is carrying the least sand) to pass on through the groin, thereby decreasing the volume of return flow down the beach slope parallel to the groin and increasing the tendency to sand deposit. Later, when the beach has built up to the bottom of the "windows," the sand passes on through them to the leeward side, thereby fostering a more smooth and uniform beach outline. Should it be desired to build the beach level still higher, these "windows" can be closed by short pieces of sheet pile. The beach then builds up to the top of the groin and buries it to a point seaward of the break in the profile. Fig. 17 shows such a groin under construction, several of which have been built by the State of California, and which have functioned very satisfactorily. The shoreward end of this groin was constructed of wood sheet piling for economy, as it later became buried by natural accretions.

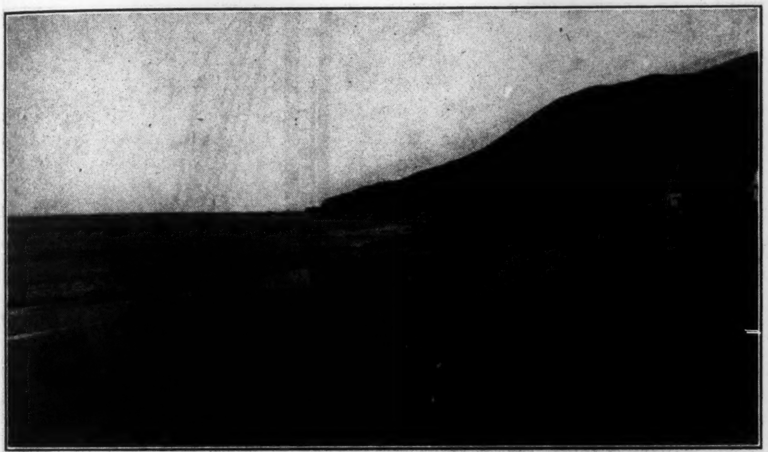


FIG. 16.—GROINS AT LAS TUNAS BEACH, CALIF., SHOWING INSHORE ENDS BURIED BY NATURAL ACCRETION AND RESULTANT SMOOTH SHORE LINE

It is understood that on the Atlantic Coast certain groins have been constructed with these "windows" extending out to the seaward end of the groin and having the bottoms of the "windows" approximately at beach level. It is believed preferable to keep the seaward end of the groin solid, in order that it may tend to divert the littoral drift shoreward where it will be either deposited on the windward side of the groin or pass through the "windows" and be deposited on the leeward side. Every effort must be directed to minimize the amount of sand being diverted seaward around the outer end of the groin where much of it may be lost.

The author touches only lightly on materials of construction, implying that groins will be built of sheet piling. This type of construction is usually the simplest, and steel is ordinarily preferable to wood, unless the cost differential is too great. Steel is particularly suitable where the beach is underlain by material that is difficult to penetrate. Indeed, steel master piles are often preferable to the round wooden piles indicated in Fig. 11.

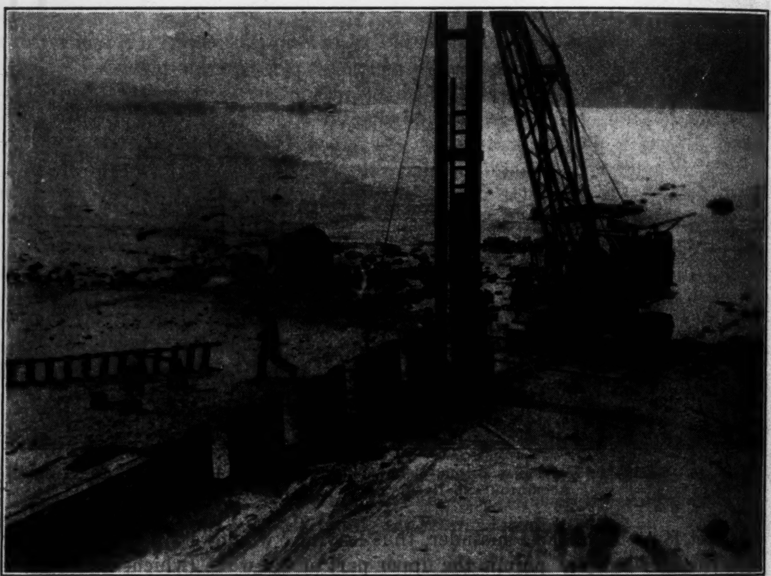


FIG. 17.—STEEL SHEET PILE GROIN, WITH "WINDOWS," UNDER CONSTRUCTION NEAR SANTA MONICA, CALIF.

A strong argument against wood piling is its buoyancy. The writer has seen wood groins fail by becoming loosened in a storm and then "popping" out, without being broken off. Perhaps the penetration was inadequate in these cases, but steel piling would have required less penetration for safety. Sand erosion of the steel is more of a problem than corrosion. Asphalt and sand coating has been found more effective protection than most of the commercial paint coverings.

Sometimes it becomes necessary to construct groins where no type of piling is practicable. Of course, the rock mound type familiar on the New Jersey coast can be used where the groin (or "jetty," as it is called in that locality) is large enough to justify use of rock sizes capable of withstanding wave action; but such structures are likely to be lacking in sand-tightness. Solid concrete

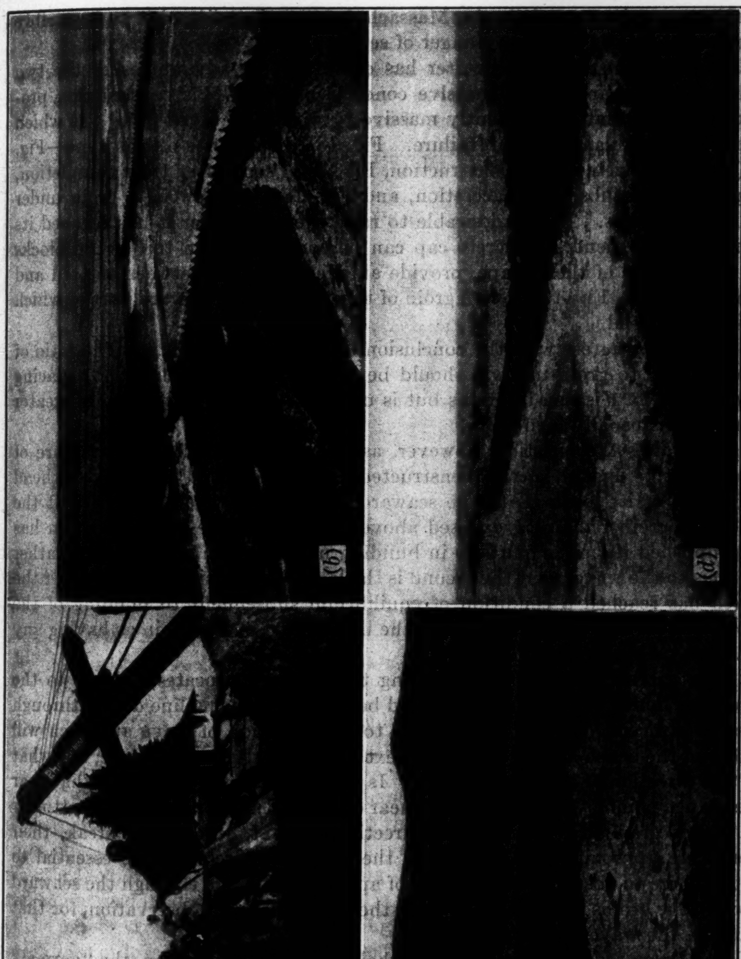


FIG. 18.—CONCRETE BLOCK GROINS, SANTA BARBARA COUNTY, CALIFORNIA

has been used successfully in Massachusetts and California, and possibly elsewhere, where there is no danger of settlement.

In certain instances the writer has combined the advantages of the two types by using articulated massive concrete blocks. This construction produces a groin that is sufficiently massive to withstand storms and yet which can settle irregularly without failure. Fig. 18 shows groins of this type—Fig. 18(*a*) being a view during construction, Fig. 18(*b*) soon after their completion, Fig. 18(*c*) the subsequent accretion, and Fig. 18(*d*) a view of a groin under storm conditions. If it is desirable to raise the groin after it has reached its ultimate settlement, a concrete cap can be constructed on top of the blocks which, because of their shape, provide an effective bond between the old and the new work. Fig. 19 shows a groin of this type, the shoreward part of which has been capped.

The writer agrees with the conclusions of the author that: (*a*) The ratio of groin length to groin interval should be between 1 : 1 and 1 : 3; (*b*) spacing closer than 1 : 1 is non-injurious but is uneconomical; and (*c*) spacing greater than 1 : 3 is usually ineffective.

This raises the question, however, as to what is the effective measure of groin length. Is it the actual constructed length, measured from the bulkhead regardless of its location, to the seaward extremity of the groin, or is it the length of groin remaining exposed above the beach slope after the groin has accomplished its major purpose in building up the beach? It appears rather obvious to the writer that the second is the logical conclusion because, after the beach has reached a condition of equilibrium subsequent to building of the groin, one certainly cannot consider the buried part of the groin as having any influence on wave action.

As a further rule for groin spacing the author advocates, subject to the foregoing limits, that the groins should be so spaced that a line drawn through the seaward end of one groin parallel to the direction of wave approach will intersect the beach at the root of the next groin. This sounds simple, but what is "the direction of wave approach"? Is it that in deep water or is it that near the plunge point? On approaching near enough to shore to "feel the bottom," all waves gradually change their direction so that, when they break, their fronts are approximately parallel to the shore. Therefore, it is essential to know where to observe the direction of approach. A line through the seaward ends of the groins would appear to be the logical point of observation, for that is where the waves are intercepted.

Furthermore, which kind of wave shall be used as a criterion, the "normal" waves, the direction of which is most predominant through the year, or the storm waves, which, although less frequent, are much more powerful? In Southern California the fair weather waves approach generally from the southwest or west and predominate through two-thirds to three-fourths of the year, whereas the southeasterly or southerly storm waves occur only in the three or four winter months. There is little question here in adopting the prevailing fair weather direction. In other localities, however, the problem may be less simple, for the wind and wave direction may be much more variable.

The author's rule for groin spacing, based on direction of wave approach, does not seem entirely clear. He states that "In the case of storm action parallel to the coast the resulting waves do not travel parallel, but the shore ends are retarded so that the wave reaches the shore at an angle of about 16 degrees." Applying the author's own rule to this case would appear to justify a groin interval equal to only approximately one-fourth the groin length, if the angle is measured from the perpendicular, or four times the groin length, if the angle is measured from the shore. It is hoped that the author will amplify his rule.

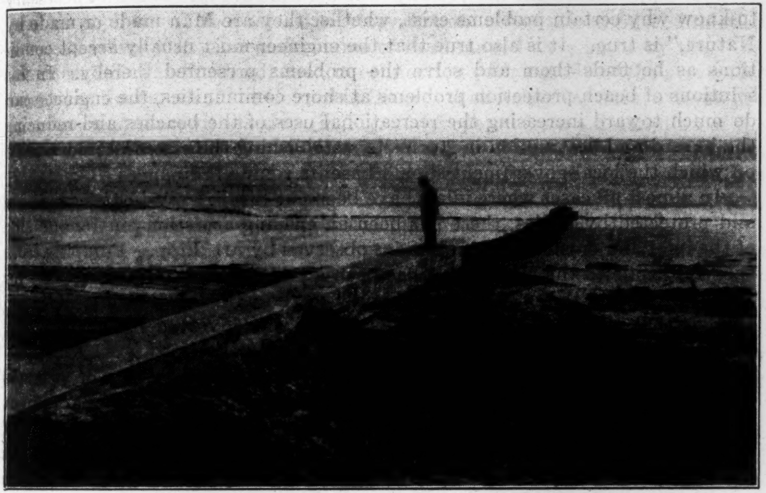


FIG. 19.—CONCRETE BLOCK GROIN, SANTA BARBARA, CALIF. INSHORE END RAISED BY MONOLITHIC CAP

In California the State Division of Lands is gradually accumulating a record of seacoast structures within the State, their behavior, and their effect on the coast line. Since 1931 State permits have been required for construction and maintenance of all structures extending seaward across the State-owned tide and submerged lands. Each applicant is required to submit for approval plans of the proposed structure, a map showing the present (and past, so far as available) lines of low tide, high tide, and extreme reach of the sea in the vicinity, accompanied by photographs. From time to time after completion of a structure, inspections are made and photographs obtained to record the effect produced by the structure on the shore line. By this procedure it is planned to accumulate a reservoir of information that should be of service, not only to the State, but to the engineering profession in future shore protection and development.

EARL I. BROWN,[11] M. AM. SOC. C. E. (by letter).—In general, the publication of this paper has resulted in beneficial discussion.

---

[11] Col., Corps of Engrs., U. S. Army (*Retired*), Wilmington, N. C.

Mr. Lipp discusses the paper from the standpoint of his Florida experience, and gives the reasons for his practices on that shore. He stresses the difficulties encountered at inlets, those arising from heavy surf action, and the other local conditions to which his discussion calls attention. He finds, in general, that his experience corroborates and confirms the views expressed in the paper.

Mr. Lipp's statement that, in any beach erosion study, "It is first necessary to know why certain problems exist, whether they are Man made or made by Nature," is true. It is also true that the engineer must usually accept conditions as he finds them and solve the problems presented thereby. In his solutions of beach protection problems at shore communities, the engineer can do much toward increasing the recreational uses of the beaches and reducing the hazard and destruction in storms by establishing the essential information on which the local government should base its zoning ordinances.

In almost all cases where inlets have been cut through sand barrier beaches and protected by jetties, there has been an ensuing accretion on the one side of the inlet and erosion on the other, as observed by Mr. Lipp. Pumping from the "full" side to the eroded side has been advocated by some individuals for many years, but the only location known to the writer at which it has been tried is that cited by Mr. Lipp. The possibilities involved merit additional study and experimentation.

The starving of beaches to the leeward of a well-built system of groins is familiar to all students of the subject of beach erosion and is another argument in favor of comprehensive planning for the development and protection of entire beach areas. To take care of the original filling of groin bays, it is advisable, of course, to start the construction of a system of groins with the most leeward. Unless there is known to be a large littoral drift, the necessity of maintaining the beach by artificial filling between groins must always be considered when developing a plan of protection.

Mr. Soper expresses opinions probably held by many practicing engineers concerning the elaborateness of the investigations recommended in connection with beach erosion studies. It is probably true that many problems in connection with shore protection have been solved successfully "at the least expenditure of time and money." On the other hand, the practice of designing and constructing beach protective works on little or no real information is dangerous and has been the cause of the wasting of millions of dollars. It is the writer's opinion that much of this loss might have been saved if the investigations had been more complete. The recommended investigations prior to construction (particularly as to geology, shore-line and offshore changes, wave action and storm effect, tidal and other current conditions, character and extent of littoral drift) are as necessary as foundation investigations for buildings. If direct data are not obtained, practicing engineers are forced to base decisions on criteria taken from existing construction which has proved successful. The weaknesses of this condition are well known to the engineering world. It is to be understood that the scope of the investigations to be made in each particular study will be based on local conditions, available information, and the problem involved. Each of the different investigations should be made only when it is known that the results will have an important bearing on the problem.

Mr. Soper, very properly, emphasizes the desirability of inquiries into the history of destructive storms and their effects on structures. He also suggests the value of personal history as to local conditions. In these views the writer concurs fully. He states that the paper does not discuss, adequately, the movement of sand under the influence of the wind. This omission is acknowledged, and is to be attributed to the fact that few observations are available on this subject. The most satisfactory known observations on the relation between velocity of wind and volume of sand moved were made by the U. S. Engineer Office at Portland, Ore., in connection with a study of the movement of sand at the mouth of the Columbia River. A sketch of sand traps developed in connection therewith is given in Fig. 20 (prepared from a description of the trap used by Morrough P. O'Brien, M. Am. Soc. C. E., and B. D. Rindlaub, Lieutenant, Corps of Engineers, in tests at Clatsop Beach, Ore.[12]). When it is recalled that the sand dunes along the coast are formed from wind-blown sand, its importance in the study of coastal phenomena may be appreciated. Although the removal of sand from the beach by wind undoubtedly affects the rate of erosion, statistical data are meager. Mr. Soper also queries whether any method of removing the water behind a bulkhead can be suggested. The writer believes that every effort should be made to prevent water from seeping behind a sea-wall or bulkhead, but, when once there, no satisfactory way to remove it is known except to provide a suitable opening for drainage in the wall, so constructed as to prevent the escape of back-fill material. The writer deems that a bulkhead that is not completely back-filled is not a complete structure, and that if the economics of the problem do not justify the cost of a back-fill, some other solution of the problem should be sought.

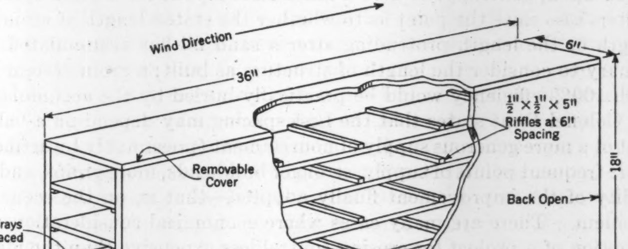

Wind Direction
6''
36''
$\frac{1}{2}'' \times \frac{1}{2}'' \times 5''$ Riffles at 6'' Spacing
18''
Removable Cover
Back Open
5 Sliding Trays Equally Spaced
Lip to Prevent Scour
40''

FIG. 20

---

[12] An unpublished report to the District Engineer, Portland, Ore.

Mr. O'Rourke has made a considerable contribution to the art of shore protection in designing his submerged bulkhead. This structure appears to be well suited for conditions on the Great Lakes where it was constructed. There may be some question as to its effectiveness on an ocean beach where tidal variations may reduce its beneficial effects at the higher stages of the tide. An attempt to accomplish a similar result on the ocean beach has been devised experimentally by Mr. Richard Arpen, in the form of a triangular concrete block termed a "submerged breakwater." It appears to have had some beneficial effect in trials.

Mr. O'Rourke's description of the partly submerged bulkhead is of great interest. In several instances known to the writer, the construction of offshore breakwaters has been followed by a building out of the beach behind it. As a general rule, the high cost of offshore structures precludes their use solely for the purpose of beach protection. An experimental study of such structures has been made by the Beach Erosion Board in its experimental laboratory at Washington. The preliminary results appear to justify further study, both in the laboratory and the field.

Colonel Dent and Major Leeds question the rules used by the Beach Erosion Board in determining the ratio of length of groin to interval between groins. These rules are entirely empirical and cannot withstand logical analysis. However, observation of installed groin systems indicates that the rules may be relied upon, generally, to give a reasonable layout for a groin system. These writers also raise the point as to whether the stated length of groin is the total length or the length protruding after a sand fill has accumulated. It is customary to consider the length of structure as built; a groin system functioning with 100% efficiency would be practically buried by the accumulated sand.

Colonel Dent states that the best spacing may depend on a balance of the cost of a more generous supply of nourishment (presumably by artificial means), more frequent points of supply, stronger bulkheads, more groins, and the general utility of the improvement finally adopted—that is, on the economics of the problem. There are many cases where economical considerations require the adoption of a project for erosion control less expensive than the ideal solution. Such a case has arisen in the writer's experience in designing a beach-erosion control project for the Town of Wrightsville Beach, N. C.

The frontage to be controlled was about 12 000 ft long, and the cost had to be kept within the sum of $250 000. The ocean was rapidly encroaching on, and endangering, existing structures, and it was desired to provide a safe margin of protection as rapidly as possible. The design made by the Beach Erosion Board for the locality comprised a groin and bulkhead system estimated to cost about $750 000, but such a sum was entirely beyond the capacity of the locality to finance. Its legal bonding limit confined its efforts to the aforementioned sum. The design adopted to meet these limitations consisted of an extension of the berm seaward for distances of from 100 to 200 ft by sand artificially supplied by dredging from the lagoon in the rear of the island on which the town is located, and the fixation of the artificially created beach by a system of sixteen groins. The latter are of creosoted timber constructed in general accordance with the Beach Erosion Board's typical design, with some modification, particularly at the outer end, to prevent erosion by eddies around the ends. The inner part of the groins is not horizontal but slopes from Elevation 9.5 ft (M.L.W.) at the landward end to Elevation 8.5 ft at a point 90 ft seaward; thence it slopes for 140 ft down to Elevation 2.0 ft; and then extends seaward horizontally for 95 ft, making the total length of groin 325 ft. It was possible to provide only sixteen groins to cover the distance of 12 000 ft; hence the spacing was about 800 ft. The volume of sand supplied to extend the berm seaward as desired, and to fill the intervals between groins substantially to the tops of their inner and middle sections, was 700 000 cu yd. The groins were practically buried within the fill since their function was to stabilize the berm and not to create one by accumulating drift. This project is functioning satisfactorily.

Mr. Rhodes discusses the paper in the light of his experience on the Georgia coast where he finds beach problems very intimately involved with those of currents at inlets, and their subsidiary effects. He finds that, in general, the experience and practices described in the paper correspond with his experience. He points out that at, and in, inlets the rules for spacing groins should be modified to require quite close spacing, as the problem there becomes one of bank protection rather than one of shore protection.

Major Leeds also finds it necessary to make an extended research into the past history of a beach under study in formulating a design for protective works, and he stresses the necessity for the use of good judgment in interpreting the available data. He stresses in particular the desirability of the fullest and most complete data possible as to winds and waves. He considers wave data more important than wind data.

Major Leeds advocates the study of the residual movement of some material placed on the beach, of approximately the same specific gravity as the beach sand or gravel, but readily distinguishable therefrom, and a periodic observation of its aggregate movement. The Beach Erosion Board has attempted to make such studies,[13] using a red sand in lots of 16 to 20 cu yd each, placed in a trench extending from the crest of the berm to mean sea level. It found that this material became so diluted by mixture with other materials that its travel could not be followed along the beach for any considerable distance, and that in a short time it could not be identified at all. Further experiments with larger volumes of foreign materials might yield further information.

Major Leeds calls attention to a matter of grave concern to beaches on deeply indented and rocky shores, such as those of California. Many of these beaches are fed by detritus brought down to the sea by mountain torrents. The improvement of the valleys of these torrents in the way of flood-control and erosion-control projects is likely to reduce or eliminate such supplies, resulting in great damage to the beaches.

Major Leeds makes many valuable suggestions on groin design, some of which do not agree in detail with the recommended design. These comments are appreciated, although it was conceded in the paper that many variations in design of groins are permissible under certain conditions.

---

[13] Interim Report, Beach Erosion Board, April 15, 1933, paragraphs 3/1 to 3/6, inclusive.

Regardless of the methods adopted, the cost of beach protection is high. Engineers studying beach problems should take full cognizance of the necessary relationship between the cost of protection and the value of the property to be protected. The general public readily endorses the construction of a shore boulevard or boardwalk at costs of \$25, \$50, \$100, or possibly more per running foot. The necessity of proper expenditures for the protection of the beach itself should be apparent but it is often disregarded. There are more than sufficient examples to warrant the advocated principle of the Beach Erosion Board that upon decision to build, it should be axiomatic to build carefully and strongly. This does not prevent the possibility of savings in construction costs based on intimate and detailed knowledge of local conditions probably beyond the scope of a study by the Board.

As stated in the original paper, the science or art of shore protection is comparatively new. Sound judgment based on careful observation and critical analysis of existing beaches and structures is, and will remain for many years to come, the principal tool of the beach protection engineer. Quantitative methods of predicting any aspect of shore-line behavior are almost nonexistent and the subject is of such nature that it will not soon be presented in handbook form.

Shore-line behavior, however, is uniquely related to the hydraulic phenomena at work and the difficulty lies not in mysterious phenomena or insuperable difficulties in observation and interpretation but rather in the extreme variability and uncertainty regarding the sequence and magnitude of the winds, tides, currents, and sand supply. Hydraulic models of typical or specific situations, if built to sufficiently large scale, would undoubtedly yield reliable data concerning major phenomena but one is always uncertain about the manner in which the controlled variables are to be introduced.

The large number of variables involved in every study of the forces acting upon the shore line emphasizes the necessity for comprehensive planning after thorough consideration of all detailed information that can be secured, in order that there will be no possibility of overlooking any clue to the best possible solution.

In conclusion the writer wishes to express his appreciation to those who have discussed the paper, and his indebtedness to the Beach Erosion Board for valuable assistance.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2077

SETTLEMENT STUDIES OF STRUCTURES IN EGYPT

By Gregory P. Tschebotareff,[1] M. Am. Soc. C. E.

With Discussion by Messrs. L. C. Wilcoxen, Trent R. Dames, Edwin J. Beugler, W. S. Hanna, D. P. Krynine, and Gregory P. Tschebotareff.

Synopsis

Systematic settlement observations are described, in this paper, of a number of new buildings—public and private—constructed within a radius of several hundred miles. Foundations of every conceivable type were included, and the observations were combined with comparative investigations in the laboratory—all based on the principles of modern soil mechanics.

The organization of these studies is outlined herein, as conducted by the Foundation Soils Research Laboratory of the Egyptian University, at Cairo, Egypt, with the support of the Egyptian Government and the co-operation of local consulting engineers, architects, and contractors. Emphasis is placed on the methods of overcoming the prejudice, ordinarily encountered by practising engineers and architects of all countries, against settlement observations of their structures by independent third parties.

Some of the most important results of studies on eleven of the new structures are presented in graphical form. A comparison is made between modern theories and the results of laboratory tests. The effects of these studies on new trends of local engineering practice are outlined. Conclusions are presented concerning the features considered as being essential for the successful organizations of such studies in any other locality.

Beginning of Settlement Studies in Egypt

Evidence of settlement is to be found on numerous structures erected on the clays and silts of the Nile Valley and Delta. Unforeseen and particularly great difficulties were encountered by the settlements of a large public building erected in Cairo about 1928. Its completion had to be delayed for more than

---

Note.—Published in October, 1938, *Proceedings*.

[1] Asst. Prof. of Civ. Eng., School of Eng., Princeton Univ., Princeton, N. J.; formerly, Research Engr., Foundation Soils Research Laboratory, Egyptian Univ., Cairo, Egypt.

three years until continued observations showed that the rate of settlement had decreased to a reassuring extent. Tests on clay samples and a settlement analysis made by Professor Dr. F. Kögler at his laboratory, in Freiberg, Germany, also showed that the period of consolidation was nearly ended. Some of the settlement observations and other data pertaining to this building (identified herein as Building *T*) are shown subsequently on Figs. 13 and 14. The case attracted attention to the numerous "gaps" in the engineering knowledge concerning the behavior of foundations and to the methods recommended by the new science of Soil Mechanics for the purpose of closing these gaps.

An extensive study of settlement problems was decided upon and the writer was engaged to organize a Foundation Soils Research Laboratory.

Purpose of the Settlement Studies

An investigation was planned of all problems connected with settlements. In particular, the studies undertaken were:

(*a*) To provide the local Engineering Profession with accurate knowledge concerning the final value, the rate and the distribution, in plan, of settlements as observed on actual structures erected on different types of foundations and on different local soil deposits (the factors influencing cracks and other damages to the superstructure were to be established); and,

(*b*) To allow comparisons between the settlement values as actually observed in the field and as forecast on the basis of computations and of laboratory tests; the theoretical methods of analysis were to be corrected and developed by means of such comparisons.

Organization of the Settlement Studies

The following principles guided the organization of the settlement studies:

(1) The studies were classified as a research in the field of public utility. As such, they were financed by the Egyptian Government and the Laboratory was equipped to handle every phase of the investigations—that is, the borings, the extraction and the testing of the samples, and the levelings themselves. The Survey of Egypt assisted in some of the phases of the leveling work.

(2) It was recognized that the usual opposition of practising engineers against settlement observations of their structures might be justified in some cases. This opposition was generally due to fear of damages to the business interests concerned, should settlement observations show that the foundation design had not been fully effective. A number of special measures were necessary, therefore, to safeguard the legitimate business interests involved. These measures were:

(3) A guaranty was given that all data would be kept confidential unless the written approval of those responsible for it was first obtained. The small staff of the Laboratory was made personally responsible for the secrecy of the data in its possession. The staff was authorized, in all cases, to refuse requests for information that were inconsistent with the foregoing provisions, even if the request came officially from the Law Courts.

(4) No charges were to be made in the early stages of the investigations until the degree of accuracy of the methods used had been verified and generally acknowledged.

In this manner the initial reluctance to allow settlement observations was gradually overcome. In December, 1936 (that is, at the end of the first three years of the existence of the Laboratory) twenty-four structures were under observation, of which only three were public buildings and twenty-one were privately owned. The writer understands that this total number was doubled a year later.

RESULTS OF THE SETTLEMENT STUDIES

Conditions in the Nile Valley and Delta proved to be particularly favorable to the studies. A large number of new buildings were being erected every year. Furthermore, the compressible water-logged clay and silt deposits seldom exceeded a thickness of 30 ft and often averaged only 10 and 15 ft, being underlaid by sand which extended to considerable depths. Since the time of consolidation of a clay layer increases with the square of its thickness, the settlement studies on the relatively thin compressible layers met with in Egypt could give conclusive results much sooner than would be the case in localities with thicker deposits. The results, therefore, may be of more than purely local interest.

Naturally, only an outline of the most pertinent data is possible within the limited space of this paper. This will be realized, if one considers that a report prepared by the writer to cover, fully, the case of only one building (T) consisted of 288 pages of typescript with 72 illustrations.

The liquid and plastic limit, the natural water content, and the laterally unconfined compressive strength test were adopted as routine soil tests for descriptive and classification purposes. The advantage of these tests is that they can be performed rapidly. The liquid (w_e) and plastic (w_p) limit tests illustrate the potential properties of the basic soil material in a disturbed state. The density in the natural state is shown by the natural water content (w_n), the cohesion by the ultimate compressive strength (q_d) and the brittleness or plasticity by the strain at failure.

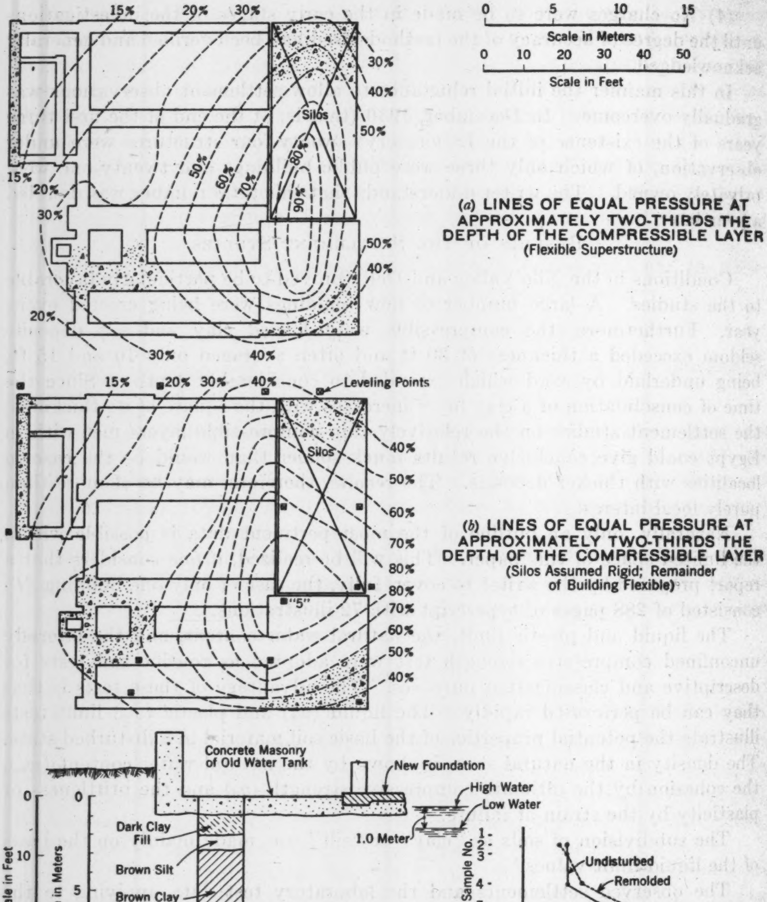
The subdivision of soils as "clay" or "silt" was made mainly on the basis of the liquid limit values.[2]

The observed settlements and the laboratory test data applying to the first ten buildings studied by the Laboratory are presented graphically on Figs. 1 to 12, which diagrams are intended to be self-explanatory.

Data Concerning the Final Value of Settlements.—It was found that buildings erected on clay soils could settle appreciably, the settlements increasing in direct proportion to the weight of the structure, to the thickness of the stressed clay deposit, and to the value of the compressibility coefficient of the clay type of that deposit. Driving piles into the clay by no means prevented settlements, unless the piles penetrated into the underlying sand. The intensity of stress on the clay layer beneath the pile-bearings and the thickness of that layer appeared to be the governing factors. Thus, the buildings erected on piles,

---

[2] See *Paper C-1*, Vol. I, *Proceedings*, International Conference on Soil Mechanics and Foundation Engineering.


15%
20%
30%
30%
40%
50%
Silos
50%
60%
70%
80%
90%
15%
20%
30%
50%
40%
20%
30%
40%
0 5 10 15
Scale in Meters
0 10 20 30 40 50
Scale in Feet
15%
20%
30%
40%
Leveling Points
Silos
40%
50%
60%
70%
80%
80%
"5"
70%
60%
50%
40%
Concrete Masonry of Old Water Tank
New Foundation
High Water
Low Water
1.0 Meter
Dark Clay Fill
Brown Silt
Brown Clay
Sand
Scale in Feet
Scale in Meters
Sample No.
Undisturbed
Remolded
Kg per Sq Cm
Water Level
$W_{(\text{Plastic Limit})}$
$W_{(\text{Liquid Limit})}$
$W_{(\text{Natural})}$
Percentage Weight of Solid Matter
Undisturbed
Remolded
At Failure of Undisturbed Soil

(*a*) LINES OF EQUAL PRESSURE AT APPROXIMATELY TWO-THIRDS THE DEPTH OF THE COMPRESSIBLE LAYER (Flexible Superstructure)

(*b*) LINES OF EQUAL PRESSURE AT APPROXIMATELY TWO-THIRDS THE DEPTH OF THE COMPRESSIBLE LAYER (Silos Assumed Rigid; Remainder of Building Flexible)

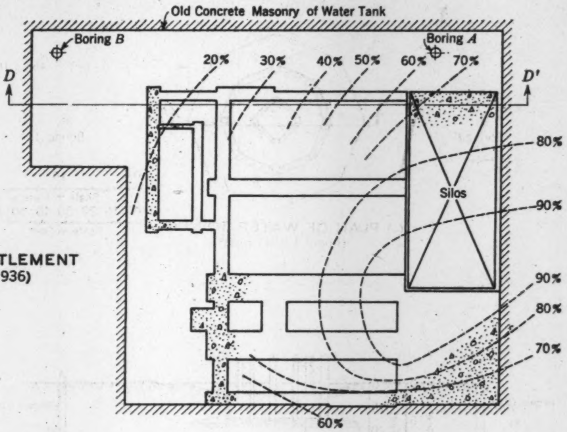
(*c*) RESULTS OF BORING *B*

ULTIMATE COMPRESSIVE STRESS

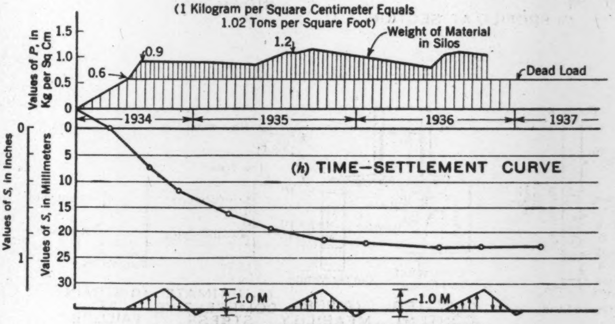
(*d*) WATER CONTENT

(*e*) STRAIN AT FAILURE

FIG. 1.—


Old Concrete Masonry of Water Tank
Boring B
Boring A
20%
30%
40%
50%
60%
70%
D
D'
80%
Silos
90%
90%
80%
70%
60%

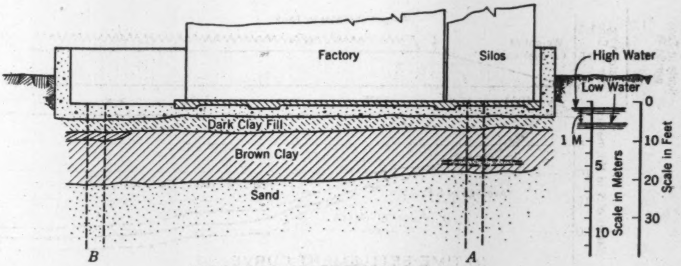
(f) LINES OF EQUAL SETTLEMENT
(Observations to Jan. 1936)



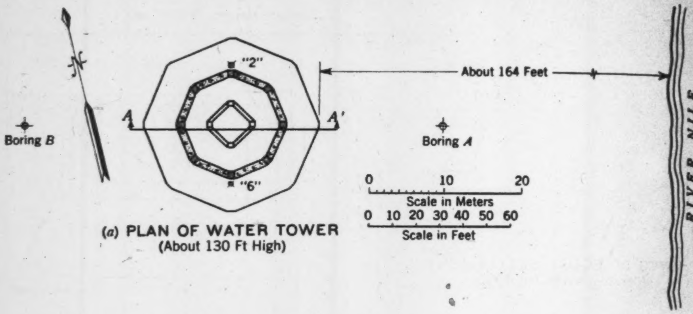
(h) TIME–SETTLEMENT CURVE

VARIATIONS OF GROUNDWATER LEVEL

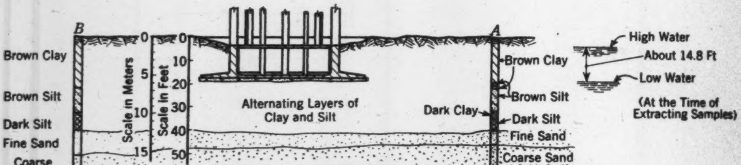
(g) LEVELING POINT "5" MAXIMUM
See Fig 1 (b)


Factory
Silos
High Water
Low Water
Dark Clay Fill
Brown Clay
Sand
1 M
B
A
0
5
10
Scale in Meters
0
10
20
30
Scale in Feet

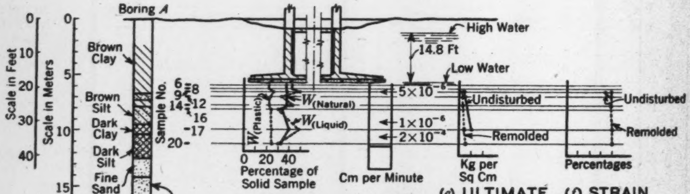
(i) PROFILE AT SECTION D-D', Fig 1 (f)


N
"2"
"6"
A
A'
Boring B
Boring A
About 164 Feet
RIVER NILE
0 10 20
Scale in Meters
0 10 20 30 40 50 60
Scale in Feet

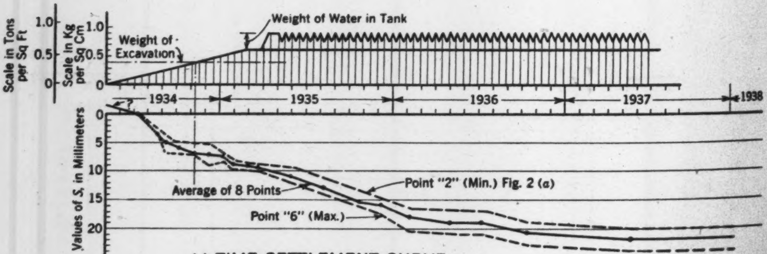
(a) PLAN OF WATER TOWER
(About 130 Ft High)


B
A
Brown Clay
Brown Silt
Dark Silt
Fine Sand
Coarse Sand
Scale in Meters
0 5 10 15
Scale in Feet
0 10 20 30 40 50
Alternating Layers of Clay and Silt
Dark Clay
Brown Clay
Brown Silt
Dark Silt
Fine Sand
Coarse Sand
High Water
About 14.8 Ft
Low Water
(At the Time of Extracting Samples)

(b) PROFILE AT SECTION A-A'



(c) WATER CONTENT (d) PERMEABILITY (e) ULTIMATE COMPRESSIVE STRESS (f) STRAIN AT FAILURE



(g) TIME-SETTLEMENT CURVE

FIG. 2.—BUILDING II

embedded in the clay only, settled: Building *T* (Figs. 13 and 14), more than 10 in.; Buildings V and X (Figs. 7 and 12), more than 2 in.; and Building VIII (Figs. 9 and 10), more than 4 in. Building VIII, although only seven stories high, had about 15 ft of clay beneath its pile-bearings and settled five times as much as the heavier ten-story Building VI (in which the settlement was about 1 in., see Fig. 8) with exactly the same system, length, and loading of the piles. In the case of Building VI, the piles penetrated into sand. A five-story structure (Building VII, Fig. 9), on a lighter type of piles just touching the sand surface, had an intermediate value of settlement (slightly greater than 1 in. at one end and almost zero at the other).

Light buildings, weighing less than 1 ton per sq ft of bearing area, erected on spread footings or rafts resting on a stiff variety of local clay of a brown color (Buildings I and III, Figs. 1, 3, and 4), or on compact old silts (Building II, Fig. 2) showed small settlements of less than 1 in. The stiff brown clay and the old silt of the latter two buildings were underlaid by a thick, silty, dark clay layer about 3 ft thick.

When the stiff brown clay was underlaid by a thicker layer of a local clay, about 10 ft thick, of the same dark color but more plastic, equally light structures on spread footings or rafts settled substantially: The three-story structure (Building IV, Figs. 5 and 6) settled more than 3 in. and a light four-story villa (Building IX, Fig. 11) settled more than 6 in. In this latter case a recent silt deposit overlying the dark clay appears to have contributed to part of the settlement.

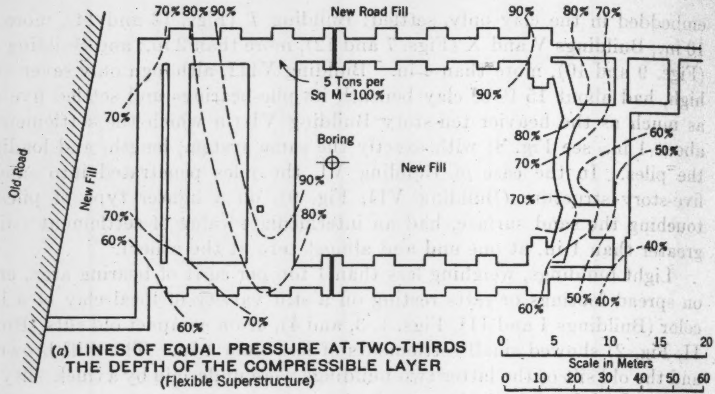
Comparison Between Load-Test Results and the Settlements of the Full-Sized Structures.—No useful relation could be established between load-test results and the settlements of full-sized structures. Thus, in the case of Building II (Fig. 2), a load test made on an area of about 10 sq ft did not show any measurable settlement under a load of 1 ton per sq ft (accuracy of reading about $\frac{1}{32}$ in.), under which load the structure settled about 1 in.

In another similar case (Building IX), the dry clay soil at the level of the foundation was so hard that no impression was left by the heel of a man spinning round on it. Obviously, a load test would have been useless under such conditions, yet the entire structure settled 6 in., due to the compression of deeper lying layers below subsoil water level.

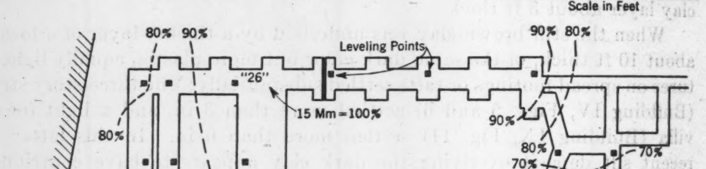
Load tests on such layers in bore-holes gave much too high settlement values due to the inevitable loosening of the wet exposed soil surface. Under similar conditions load tests, therefore, appear inadvisable for any purposes.

In the case of Building *T*, test-piles observed for periods of several months until complete stabilization showed settlements about one-twentieth of the actual settlement of the entire structure under the same load per pile.

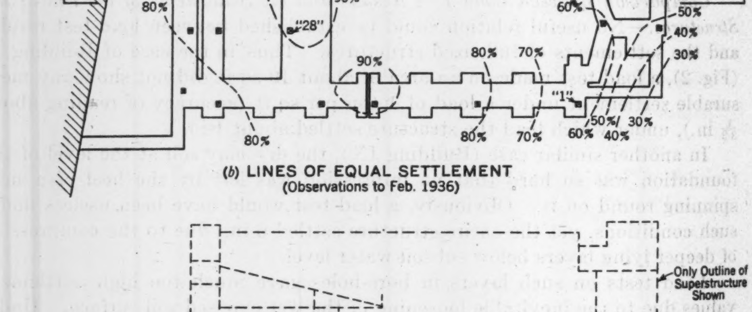
All piles of the buildings described in this paper are concrete piles cast in place. The steel tube driven into the ground is withdrawn and the concrete rammed with a heavy tamper. This tamper weighs about 1 500 lb for the type of piles used in Buildings V, VII, X and *T* (the usual working load allowed locally on such piles varies from 30 to 45 tons per pile). For the type of piles used in Buildings VI and VIII the tamper weighs about 4 500 lb (usual working load, 60 to 80 tons per pile).


70% 80% 90%
New Road Fill
90%
80% 70%
5 Tons per
Sq M = 100%
90%
70%
Boring
New Fill
80%
60%
50%
70%
90%
80%
70%
Old Road
New Fill
70%
60%
40%
50% 40%
60%
70%
0 5 10 15 20
Scale in Meters
0 10 20 30 40 50 60
Scale in Feet

(a) LINES OF EQUAL PRESSURE AT TWO-THIRDS THE DEPTH OF THE COMPRESSIBLE LAYER
(Flexible Superstructure)

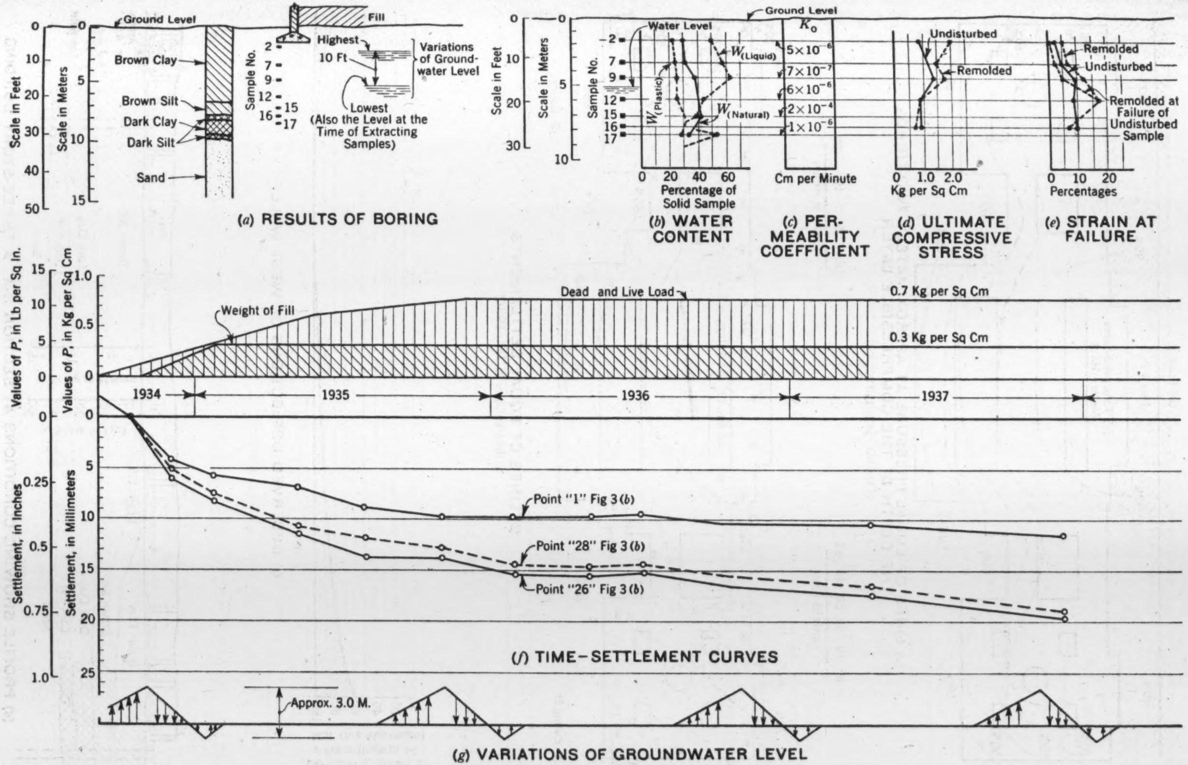

80% 90%
Leveling Points
90% 80%
"26"
15 Mm = 100%
90%
80%
80%
70%
70%
60%
A'
-60%
A
90%
80%
60%
50%
"28"
40%
90%
80%
70%
30%
"1"
50%/ 30%
80%
80%
70%
60%
40%

(b) LINES OF EQUAL SETTLEMENT
(Observations to Feb. 1936)


Only Outline of
Superstructure
Shown
+5
Fill
Ground Surface
0
0
High Water
10
Low Water
Brown Clay
20
(Also at Extraction
of Samples)
Brown Silt
30
Dark Clay
Dark Silt
Sand
40
15
50
Scale in Feet
Scale in Meters

(c) PROFILE THROUGH A'-A'

FIG. 3.—BUILDING III



(a) RESULTS OF BORING

(b) WATER CONTENT

(c) PERMEABILITY COEFFICIENT

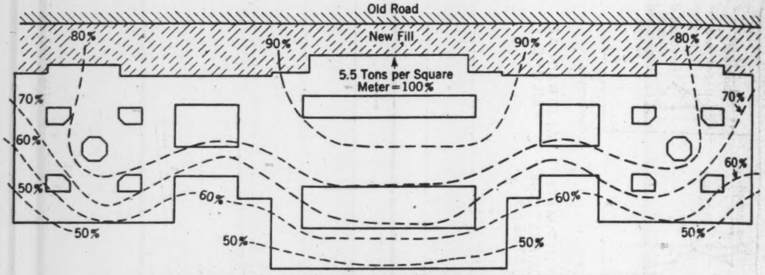
(d) ULTIMATE COMPRESSIVE STRESS

(e) STRAIN AT FAILURE

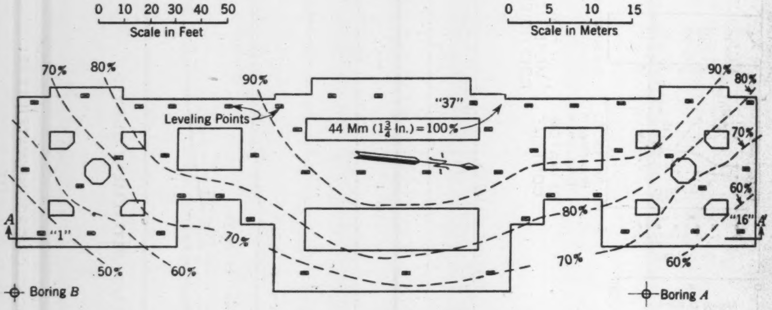
(f) TIME–SETTLEMENT CURVES

(g) VARIATIONS OF GROUNDWATER LEVEL

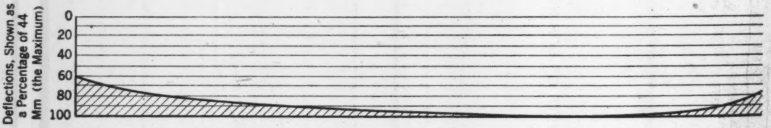
FIG. 4.—BUILDING III


Old Road
New Fill
5.5 Tons per Square Meter = 100%
80%
90%
90%
80%
70%
60%
50%
60%
60%
50%
50%
50%
50%
70%
60%

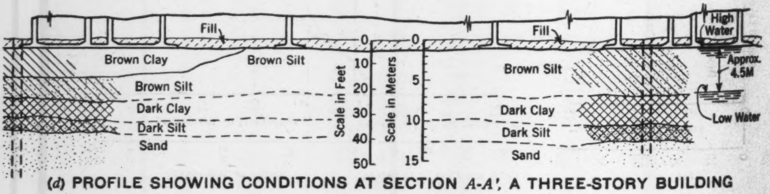
(a) LINES OF EQUAL PRESSURE AT APPROXIMATELY TWO-THIRDS THE DEPTH OF THE COMPRESSIBLE LAYER
(Flexible Superstructure)


0 10 20 30 40 50
Scale in Feet
0 5 10 15
Scale in Meters
70%
80%
90%
90%
80%
"37"
Leveling Points
44 Mm (1¾ In.) = 100%
70%
60%
80%
A
"1"
70%
"16"
A'
50%
60%
70%
60%
Boring B
Boring A

(b) LINES OF EQUAL SETTLEMENTS
(Observations to Jan. 1936)

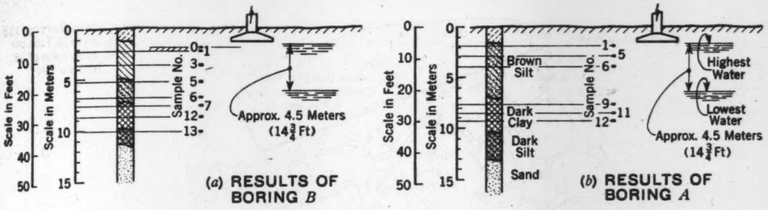


(c) OBSERVED DEFLECTIONS OF WEST WALL


Fill
Fill
High Water
Approx. 4.5M
Low Water
Brown Clay
Brown Silt
Brown Silt
Dark Clay
Dark Silt
Sand
Brown Silt
Dark Clay
Dark Silt
Sand
Scale in Feet
0
10
20
30
40
50
Scale in Meters
0
5
10
15

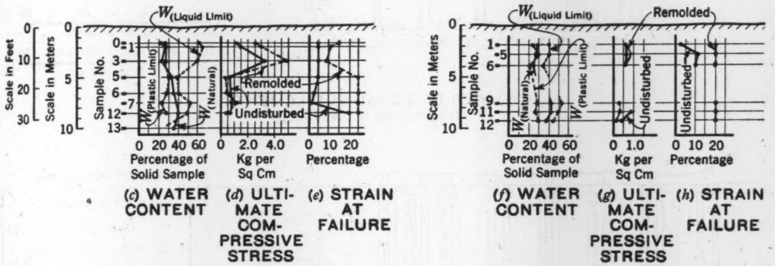
(d) PROFILE SHOWING CONDITIONS AT SECTION *A-A'*, A THREE-STORY BUILDING

FIG. 5.—BUILDING IV


Scale in Feet
Scale in Meters
0
10
20
30
40
50
5
15
Sample No.
0
1
3
5
6
7
12
13
Approx. 4.5 Meters
(14¾ Ft)
Brown Silt
Dark Clay
Dark Silt
Sand
9
11
Highest Water
Lowest Water

(a) RESULTS OF BORING *B*

(b) RESULTS OF BORING *A*



(c) WATER CONTENT

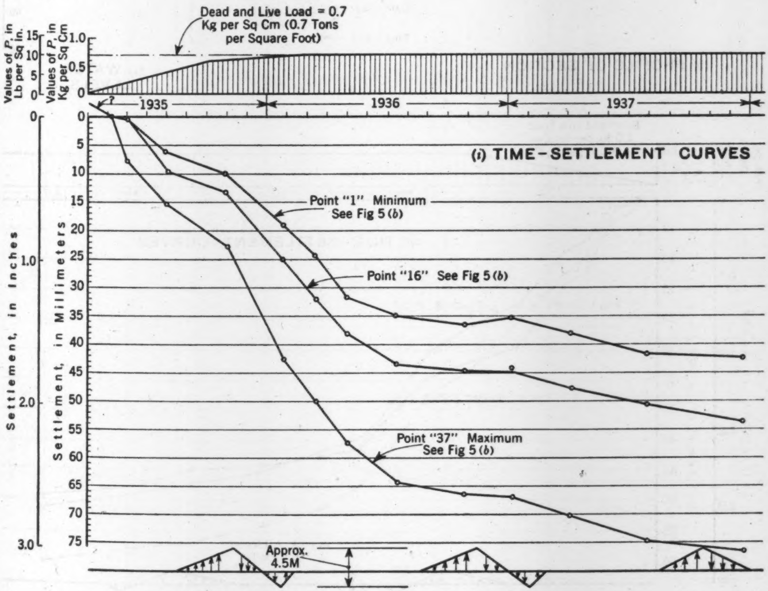
(d) ULTIMATE COMPRESSIVE STRESS

(e) STRAIN AT FAILURE

(f) WATER CONTENT

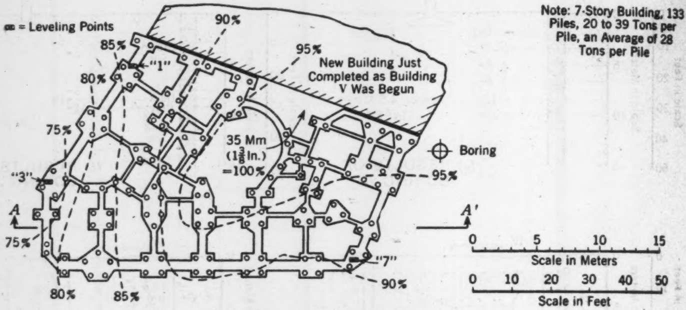
(g) ULTIMATE COMPRESSIVE STRESS

(h) STRAIN AT FAILURE



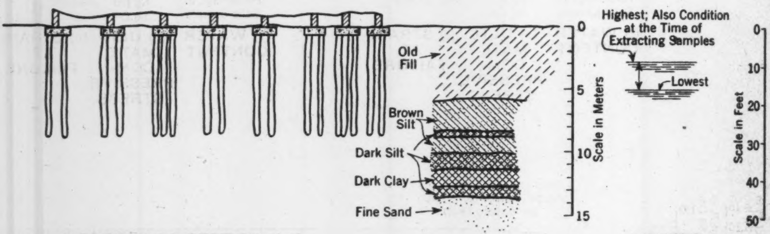
(j) VARIATIONS IN SUBSURFACE WATER LEVEL

Fig. 6.—Building IV


= Leveling Points
90%
85%
"1"
95%
80%
New Building Just Completed as Building V Was Begun
75%
35 Mm ($1\frac{3}{8}$ In.) =100%
Boring
"3"
95%
A
A'
75%
"7"
90%
80%
85%
0 5 10 15
Scale in Meters
0 10 20 30 40 50
Scale in Feet

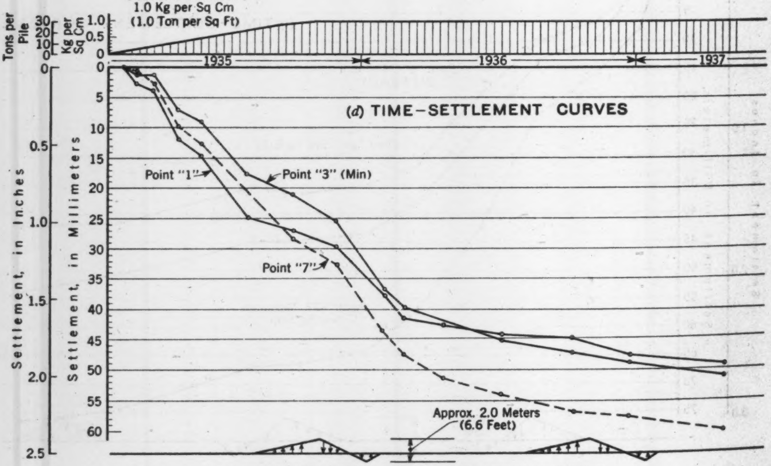
Note: 7-Story Building, 133 Piles, 20 to 39 Tons per Pile, an Average of 28 Tons per Pile

(*a*) LINES OF EQUAL SETTLEMENTS
(Observations to Dec. 1935)


Old Fill
Brown Silt
Dark Silt
Dark Clay
Fine Sand
0
5
10
15
Scale in Meters
Highest; Also Condition at the Time of Extracting Samples
Lowest
0
10
20
30
40
50
Scale in Feet

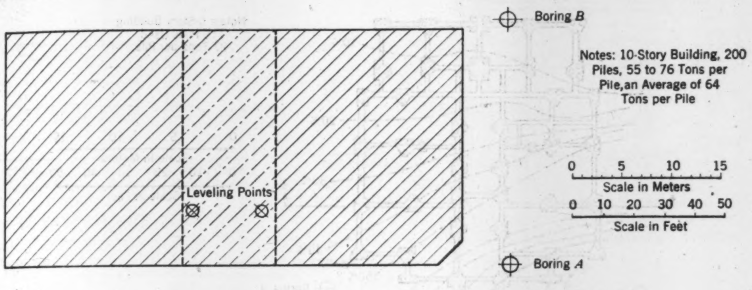
(*b*) PROFILE OF SECTION *A-A'*

(*c*) WATER LEVELS

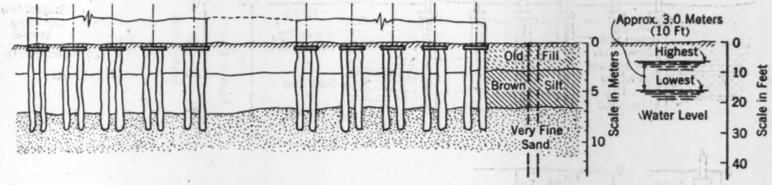


(*e*) VARIATIONS OF SUBSOIL WATER LEVEL

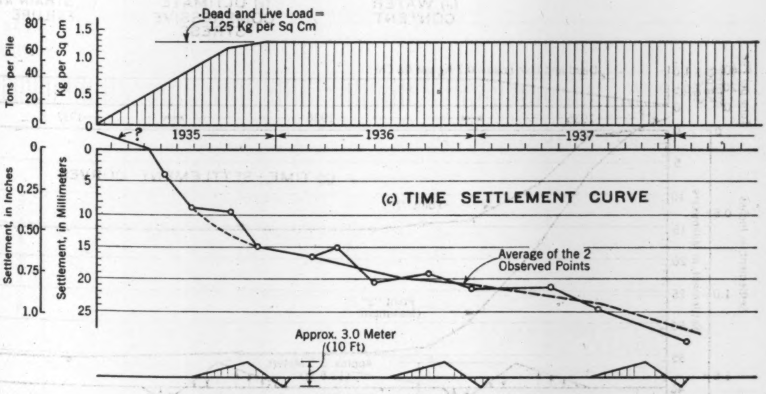
FIG. 7.—BUILDING V


Boring B
Notes: 10-Story Building, 200 Piles, 55 to 76 Tons per Pile, an Average of 64 Tons per Pile
0 5 10 15
Scale in Meters
0 10 20 30 40 50
Scale in Feet
Leveling Points
Boring A

(a) PLAN OF TEN-STORY BUILDING


Old Fill
Brown Silt
Very Fine Sand
0
5
10
Scale in Meters
Approx. 3.0 Meters (10 Ft)
Highest
Lowest
Water Level
0
10
20
30
40
Scale in Feet

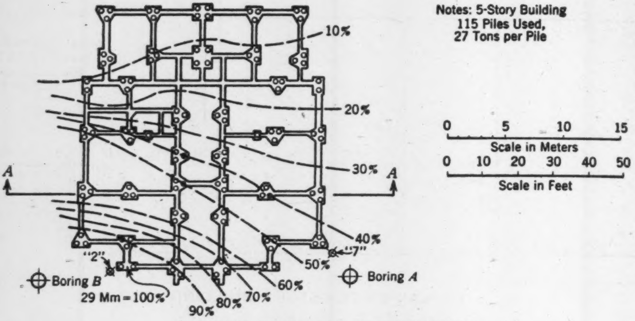
(b) PROFILE SHOWING SUBSOIL CONDITIONS



(c) TIME SETTLEMENT CURVE

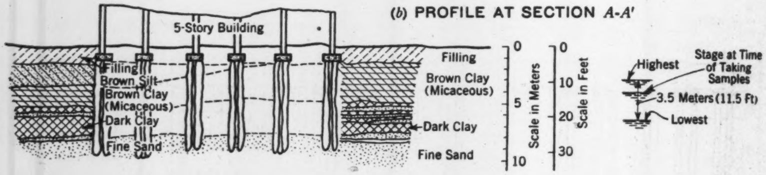
(d) VARIATION OF GROUNDWATER LEVEL

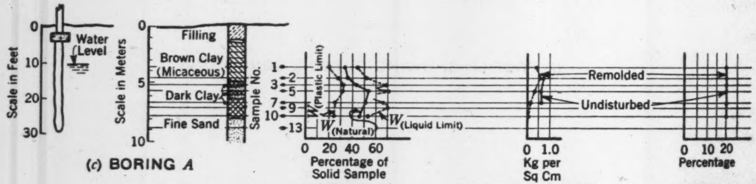
Fig. 8.—Building VI


Notes: 5-Story Building
115 Piles Used,
27 Tons per Pile
10%
20%
30%
40%
50%
60%
70%
80%
90%
29 Mm = 100%
A
A
"2"
"7"
Boring B
Boring A
0 5 10 15
Scale in Meters
0 10 20 30 40 50
Scale in Feet

(*a*) LINES OF EQUAL SETTLEMENTS
(Observations to Jan. 1936)

(*b*) PROFILE AT SECTION *A-A'*


5-Story Building
Filling
Brown Silt
Brown Clay (Micaceous)
Dark Clay
Fine Sand
Filling
Brown Clay (Micaceous)
Dark Clay
Fine Sand
0
5
10
Scale in Meters
0
10
20
30
Scale in Feet
Highest
Stage at Time of Taking Samples
3.5 Meters (11.5 Ft)
Lowest

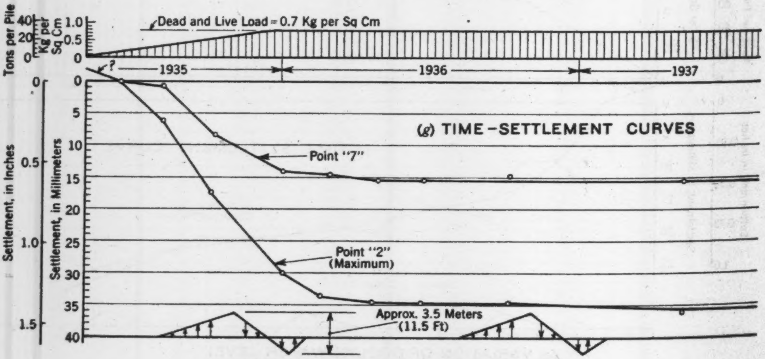


(*c*) BORING *A*

(*d*) WATER CONTENT

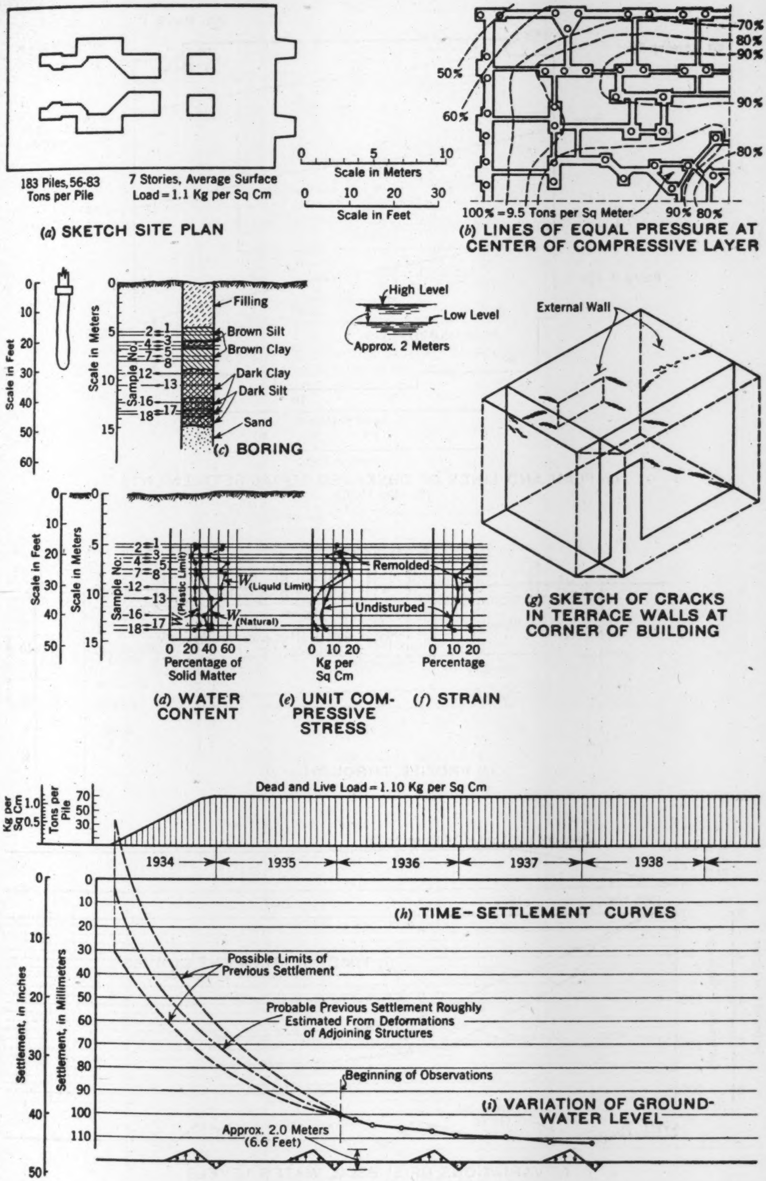
(*e*) ULTIMATE COMPRESSIVE STRESS

(*f*) STRAIN AT FAILURE



(*h*) VARIATIONS OF SUBSOIL WATER LEVELS

FIG. 9.—BUILDING VII



183 Piles, 56-83 Tons per Pile
7 Stories, Average Surface Load = 1.1 Kg per Sq Cm
0 5 10
Scale in Meters
0 10 20 30
Scale in Feet
70%
80%
90%
50%
60%
90%
80%
100% = 9.5 Tons per Sq Meter
90% 80%
Scale in Feet
0 10 20 30 40 50 60
Scale in Meters
0 5 10 15
Sample No.
1 2 3 4 5 7 8 12 13 16 17 18
Filling
Brown Silt
Brown Clay
Dark Clay
Dark Silt
Sand
High Level
Low Level
Approx. 2 Meters
External Wall
Remolded
Undisturbed
$W_{(Plastic\ Limit)}$
$W_{(Liquid\ Limit)}$
$W_{(Natural)}$
0 20 40 60
Percentage of Solid Matter
0 10 20
Kg per Sq Cm
0 10 20
Percentage
Kg per Sq Cm
0.5 1.0
Tons per Pile
30 50 70
Dead and Live Load = 1.10 Kg per Sq Cm
1934
1935
1936
1937
1938
Settlement, in Inches
0 10 20 30 40 50
Settlement, in Millimeters
0 10 20 30 40 50 60 70 80 90 100 110
Possible Limits of Previous Settlement
Probable Previous Settlement Roughly Estimated From Deformations of Adjoining Structures
Beginning of Observations
Approx. 2.0 Meters (6.6 Feet)

(a) SKETCH SITE PLAN

(b) LINES OF EQUAL PRESSURE AT CENTER OF COMPRESSIVE LAYER

(c) BORING

(d) WATER CONTENT

(e) UNIT COMPRESSIVE STRESS

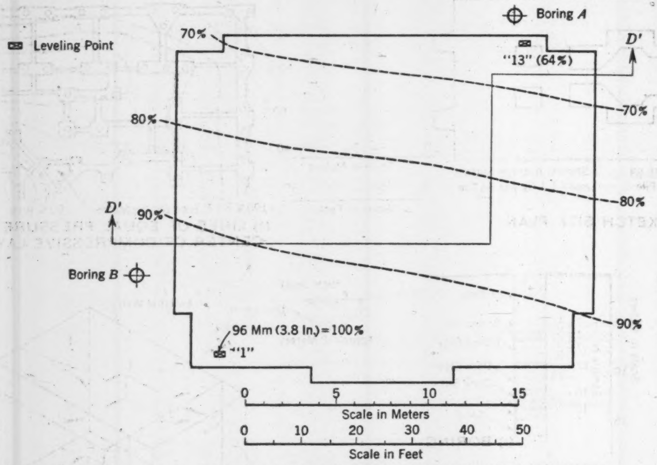
(f) STRAIN

(g) SKETCH OF CRACKS IN TERRACE WALLS AT CORNER OF BUILDING

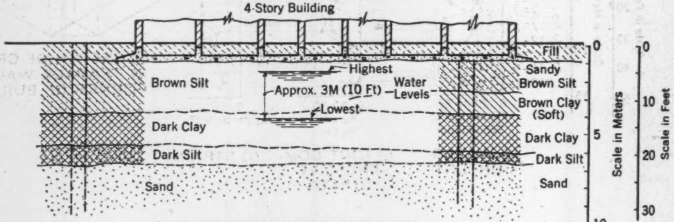
(h) TIME-SETTLEMENT CURVES

(i) VARIATION OF GROUND-WATER LEVEL

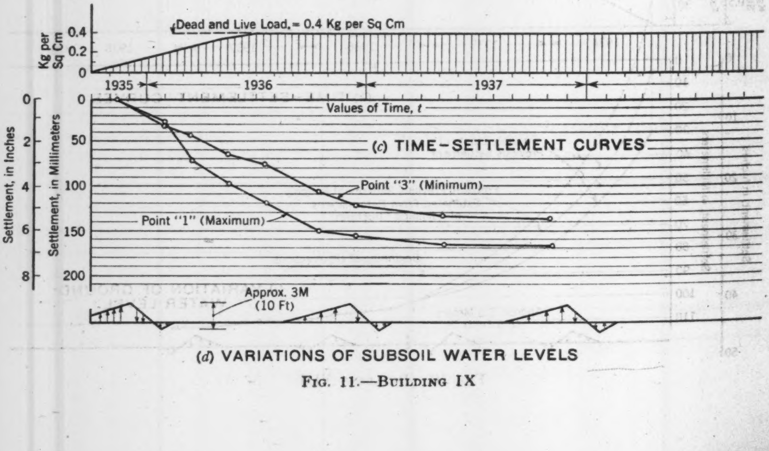
FIG. 10.—BUILDING VIII


Leveling Point
Boring A
Boring B
D'
D'
70%
70%
80%
80%
90%
90%
"13" (64%)
96 Mm (3.8 In.) = 100%
"1"
0
5
10
15
Scale in Meters
0
10
20
30
40
50
Scale in Feet

(*a*) PLAN AND LINES OF OBSERVED EQUAL SETTLEMENTS
(To May 1936)

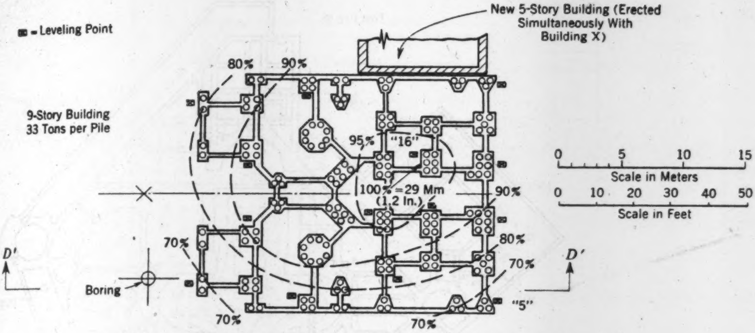

4-Story Building
Fill
Sandy Brown Silt
Brown Silt
Highest
Approx. 3M (10 Ft)
Water Levels
Lowest
Brown Clay (Soft)
Dark Clay
Dark Clay
Dark Silt
Dark Silt
Sand
Sand
0
5
10
Scale in Meters
0
10
20
30
Scale in Feet

(*b*) PROFILE THROUGH *D'-D'*

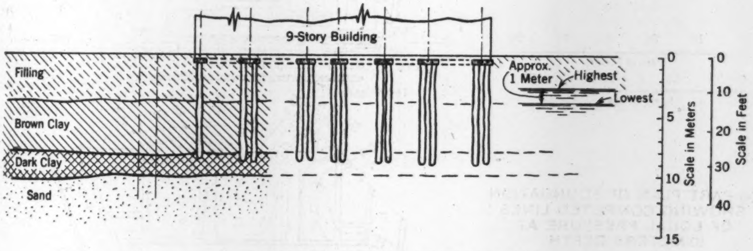


(*d*) VARIATIONS OF SUBSOIL WATER LEVELS

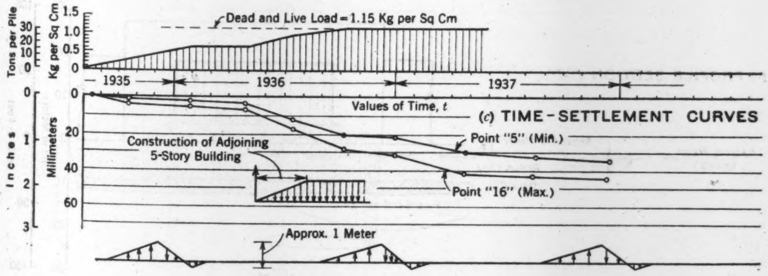
FIG. 11.—BUILDING IX


▪ = Leveling Point
New 5-Story Building (Erected Simultaneously With Building X)
9-Story Building
33 Tons per Pile
80%
90%
95%
"16"
100% = 29 Mm (1.2 In.)
90%
80%
70%
70%
70%
70%
"5"
D'
D'
Boring
0 5 10 15
Scale in Meters
0 10 20 30 40 50
Scale in Feet

(a) LINES OF OBSERVED EQUAL SETTLEMENTS
(Observations to Dec. 1936)

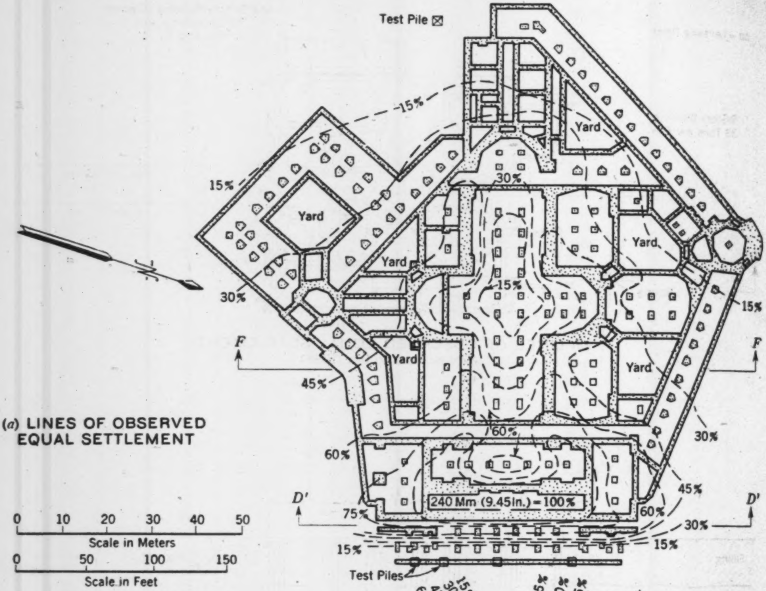

9-Story Building
Filling
Brown Clay
Dark Clay
Sand
Approx. 1 Meter
Highest
Lowest
0
5
10
15
Scale in Meters
0
10
20
30
40
Scale in Feet

(b) PROFILE THROUGH SECTION D'-D'

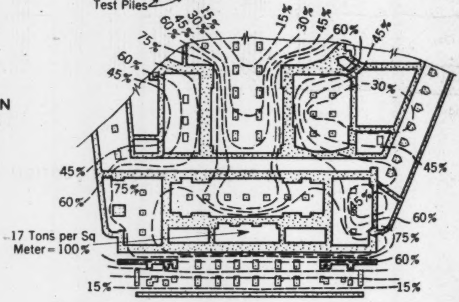


(d) VARIATIONS OF GROUNDWATER LEVEL

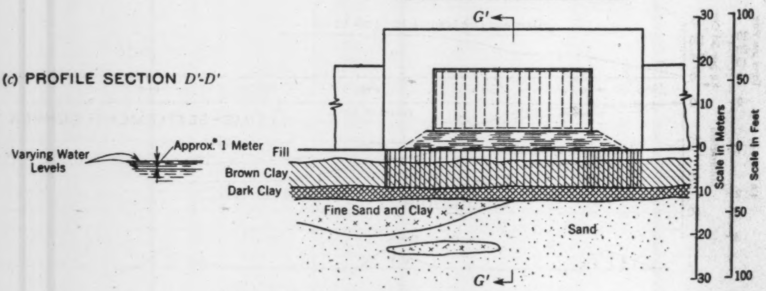
FIG. 12.—BUILDING X


Test Pile ☒
15%
Yard
15%
30%
Yard
Yard
15%
Yard
30%
15%
F
F
Yard
Yard
45%
60%
60%
30%
D'
D'
45%
240 Mm (9.45 In.) = 100%
75%
60%
30%
15%
15%
Test Piles
0 10 20 30 40 50
Scale in Meters
0 50 100 150
Scale in Feet

(a) LINES OF OBSERVED EQUAL SETTLEMENT

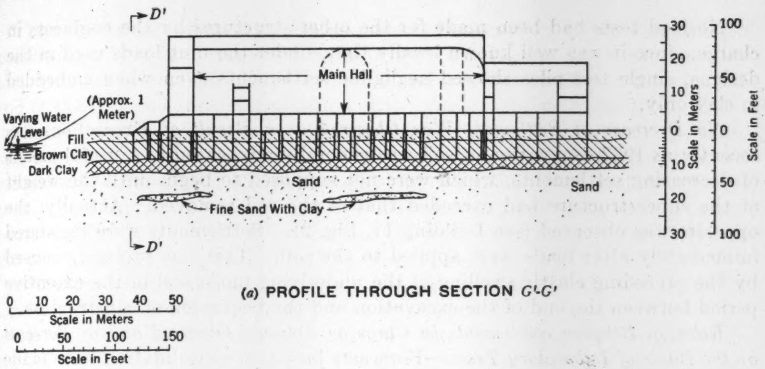

15%
30%
45%
60%
75%
15%
30%
45%
60%
45%
60%
45%
30%
45%
45%
75%
60%
45%
60%
75%
17 Tons per Sq Meter = 100%
60%
15%
15%

(b) PART PLAN OF FOUNDATION SHOWING COMPUTED LINES OF EQUAL PRESSURE AT 10-METERS DEPTH (Center of Compressible Layer)

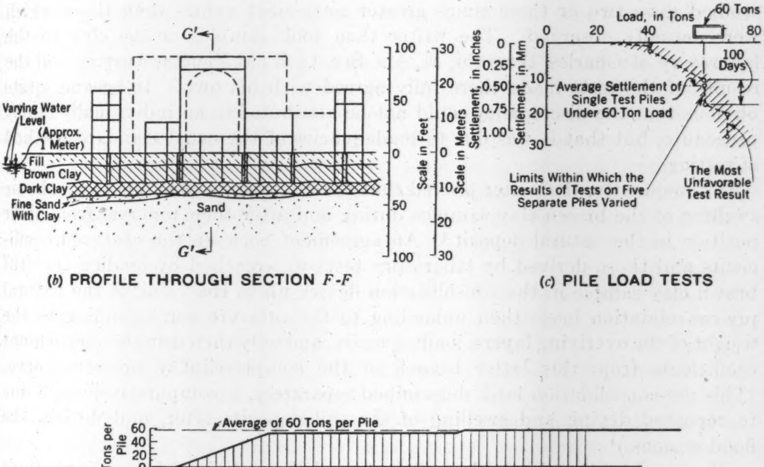

G'
Varying Water Levels
Approx. 1 Meter
Fill
Brown Clay
Dark Clay
Fine Sand and Clay
Sand
G'
30
20
10
0
10
20
30
Scale in Meters
100
50
0
50
100
Scale in Feet

(c) PROFILE SECTION D'-D'

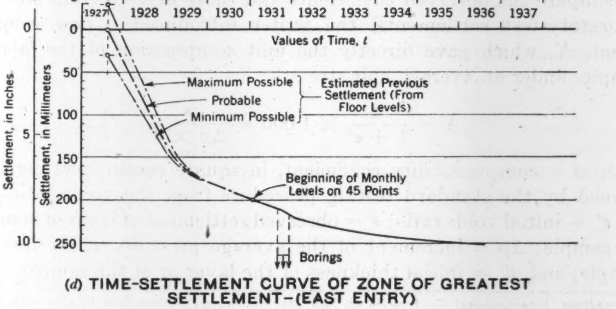
FIG. 13.—BUILDING T


D'
Main Hall
Varying Water Level
(Approx. 1 Meter)
Fill
Brown Clay
Dark Clay
Sand
Sand
Fine Sand With Clay
D'
Scale in Meters
Scale in Feet
30
20
10
0
10
20
30
100
50
0
50
100
0 10 20 30 40 50
Scale in Meters
0 50 100 150
Scale in Feet

(a) PROFILE THROUGH SECTION *G'G'*


G'
G'
Varying Water Level
(Approx. 1 Meter)
Fill
Brown Clay
Dark Clay
Fine Sand With Clay
Sand
Scale in Feet
Scale in Meters
100
50
0
50
100
30
20
10
0
10
20
30
Load, in Tons
60 Tons
0 20 40 60 80
Settlement, in Inches
Settlement, in Mm
0
0.25
0.50
0.75
1.00
0
10
20
30
100 Days
Average Settlement of Single Test Piles Under 60-Ton Load
Limits Within Which the Results of Tests on Five Separate Piles Varied
The Most Unfavorable Test Result

(b) PROFILE THROUGH SECTION *F-F* (c) PILE LOAD TESTS



(d) TIME-SETTLEMENT CURVE OF ZONE OF GREATEST SETTLEMENT—(EAST ENTRY)

FIG. 14.—BUILDING *T*

No load tests had been made for the other structures by the engineers in charge, since it was well known locally that, under the unit loads used in the designs, single test-piles showed negligible settlements even when embedded in clay only.

No Decrease of Settlement Was Observed as a Result of Excavation.—As recently as 1933 excavation of soil was generally advocated as a useful means of decreasing settlements, which were not supposed to begin until the weight of the superstructure had exceeded that of the soil removed. Actually, the opposite was observed (see Building II, Fig. 2). Settlements were registered immediately after loads were applied to the soil. This was probably caused by the preceding elastic swelling of the underlying moist soil in the extensive period between the end of the excavation and the beginning of construction.

Relation Between Settlements in Clays as Actually Observed and as Forecast on the Basis of Laboratory Tests.—Forecasts based on consolidation tests made on the stiff and easily swelling brown clays from the first three buildings studied gave two or three times greater settlement values than those which were actually observed. The writer then took samples of the clay to the laboratory of Charles Terzaghi, M. Am. Soc. C. E., at Vienna, Austria, and the results of the tests made there fully agreed with his own. It became quite obvious that the discrepancy could not be attributed to an individually faulty procedure, but that it was due to inadequacies of the usual standard method of testing.

Subsequently, the writer proved that the discrepancy was due to an elastic swelling of the brown clay samples during and after their removal from their position in the natural deposit.[3] An agreement between the observed coefficients and those derived by laboratory tests was reached by loading the stiff brown clay sample in the consolidation device up to the value of the natural pre-consolidation load; then unloading to the pressure corresponding to the weight of the overlying layers, loading again, and only then using the settlement coefficients from this latter branch of the compressibility pressure curve. (This pre-consolidation load, determined separately, is comparatively high due to repeated drying and swelling of the soil deposits after, and during, the flood seasons.)[4]

To compare the observed coefficients and those derived from observations on laboratory-test settlements, the writer introduced a new compression coefficient, X, which gave directly the unit compression of the layer, or of the sample, under an average unit stress:

$$X = \frac{a}{1 + e'} \times 1\,000 = \frac{s}{\Delta p \times d'} \ldots\ldots\ldots\ldots\ldots (1)$$

in which: a = compressibility coefficient, in square centimeters per gram, as determined by the standard testing procedure from the voids ratio-pressure curves; e' = initial voids ratio; s = observed settlement of layer or compression of test sample; Δp = increment of the average pressure on the layer, or on the sample; and, d' = initial thickness of the layer or of the sample.

---

[3] *Proceedings*, International Conference on Soil Mechanics and Foundation Engineering, *Paper D-1*, Vol. I, 1936.

[4] *Proceedings*, Am. Soc. C. E., October, 1937, p. 1621.

The use of Equation (1) permitted the determination of the compression coefficient, X, at any stage of loading during the laboratory compressibility test with not more than 3% or 4% error and without having to wait for the end of the test and the determination of the voids ratio of the sample.

Subsequent observations on buildings underlaid by the non-swelling plastic and weaker dark clays showed that a fairly good agreement with the observed values was obtained for these clays by the usual standard test procedure; that is, by taking the settlement coefficients from the voids ratio-pressure curve for the first loading after the test pressure had exceeded the weight of the over-burden.

Therefore, the method of taking a sample, combined with the potential capacity of a soil to swell, or to fall apart, appears to be among the factors primarily affecting the accuracy of settlement forecasts and the advisable method for the interpretation of test results. Another important factor is the original stiffness of the material because the same disturbance while removing a sample will cause a relatively greater error in the settlement forecast for a stiff clay than for a weak one.

This is a probable explanation of why most of the earlier laboratory settlement analyses gave such good agreement with the observed field values; the first investigations were made where some foundation trouble had occurred, which may be taken as an indication in itself of the presence of weak soil material. The entire question, however, takes on a totally different aspect if, as a matter of routine, one is called upon to make settlement forecasts for varying types of soil on which no previous settlement observations have been made. One often hears about the necessity of "experience" in the interpretation of soil test results. Generally, this means the number of tests and analyses performed within the four walls of a laboratory. The writer wishes to emphasize that such experience is of little value unless it is accompanied by numerous comparisons with field observations on full-scale structures. Furthermore, the experience derived from such comparisons in one locality cannot always be applied with equal success in another locality with a different geological history of its soil deposits. Hence, it follows that settlement should be studied in all localities if indications of local and general practical value are to be obtained.

The following are some average values of the compression coefficient, X, for Egyptian soils, as approximately determined from the observations on actual structures:

Material	Values of X
Plastic dark clay	0.070 to 0.030
Loosely deposited recent silt	0.070 to 0.030
Stiff brown clay	0.007
Compact old silt	0.003

These values were determined for metric linear dimensions and for stresses expressed in kilograms per square centimeter (equivalent to about 2 044 lb per sq ft); that is, $X = 0.010$, would mean that a layer 1 m thick would compress 1 cm (0.4 in.) under an average vertical unit stress of 1 kg per sq cm (14.5 lb per sq in.) throughout its entire thickness. Since this stress is approximately equal to a stress of 1 ton per sq ft, the foregoing values of X remain unchanged if the thickness of the layer is expressed in feet and the stress in tons per square foot. In all cases the underlying sand deposits were assumed to be relatively incompressible.

Efficiency of Devices for Taking "Undisturbed Samplers."—The so-called "undisturbed samplers" used had a cutting-edge of a slightly smaller diameter than the diameter of the sample-containing tube. They were similar to the so-called M.I.T. type and were found to yield reliable information as to the composition, the structure, and the approximate stiffness of the layers encountered. They preserved, intact, and revealed, important details of the layer formation none of which could be noticed if and when ordinary augers were used; the latter disturbed the structure of the soil completely and mixed its various constituents.

Thus, the "undisturbed samplers" were found to be quite efficient for purposes of soil identification and to be definitely preferable in this respect to the augers and to other more primitive methods of soil exploration. However, samplers of this type could not be considered as giving entirely undisturbed samples to the extent required for laboratory compressibility investigations, because they allowed the expansion and the swelling of the soil within the sample-containing tube. This swelling caused errors in the settlement forecasts of more than 100% and, sometimes, even 200 per cent. The narrower cutter appears to have been originally introduced by the designers of this type of sampler for the purpose of decreasing friction along the walls of the container tube, this friction having been considered as the source of greatest possible disturbance. In view of the facts observed, it appears likely that the unfavorable effects of this lateral friction have been greatly over-estimated, especially if compared to the effects of the dislocation of the structure of the soil due to lateral expansion in the container following the use of narrower cutters.

In the writer's opinion, constant-diameter samplers, at least 4 in. wide, should be tried for soils in which lateral expansion is likely to cause considerable disturbance—for instance, for pre-compressed clays of a swelling type. In such clays a state of considerable horizontal stress is likely to exist,[4] and an uninterrupted lateral confinement becomes essential for all stages during the transfer of the sample from its position in the natural deposit to the compressibility testing apparatus in the laboratory. Friction along the walls of the container is likely to be detrimental only to clays the structure of which is broken by remolding.

Not All Clays Are Unfavorably Affected by Remolding.—Compressive strength tests on numerous clay samples, both in the undisturbed and in the remolded states, showed that the dark clay samples were very unfavorably affected by remolding. This was not the case for the stiffer brown clay samples, which were only slightly affected, or not at all. The types of brown clay that were likely to swell more appeared to be least affected by remolding. The fundamental causes of this difference in behavior are not quite clear and require investigation.

Estimation of the Compressibility of Silts.—Comparison of the settlement records of Buildings II (Fig. 2), III and IV on one hand and of Building IX on the other (Fig. 11) showed that the compressibility of loose, recently deposited silts could be more than ten times as great as that of compact old silts and could produce appreciable settlements (Building IX, Fig. 11). At the same time, there are no simple means of establishing, rapidly and beforehand, whether one is dealing with a compact, or with a loose, silt. Such silts with little cohesion cannot be extracted from bore-holes below ground-water level in a really undisturbed state as required for laboratory testing. Furthermore, there is very little hope that this will ever be attained by any new system of samplers.

Nevertheless, for practical purposes, it is essential to be able to estimate the degree of compressibility of silts on new building sites. It seems to the writer that the only method which may give satisfactory results is the use of small cones slowly jacked into and through the silt or sand deposit. The pressure applied in relation to the penetration registered gives comparative empirical coefficients of the density and of the compressibility of the soil penetrated. Such methods were tried first by Professor Terzaghi in the United States and then by Dutch engineers in Holland.[5] They appear to be in need of correlation to observations and of further development, but they form what seems to be a promising innovation.

Rate of Settlement.—The rate of settlement observed was found to be in good agreement with the theoretical rate based on the permeability of the water-logged deposits stressed by the weight of the buildings when they were founded on mat or strip-footing foundations. The settlements of Building II (Fig. 2) continued for a much longer time than could be expected from the permeability tests. This continued settlement appears to be due to the constant variations of 20% of the total load twice weekly before and after the refilling of the water tank.

The settlements of light structures founded on piles resting on sand (Building VII) stopped almost immediately after construction. On the other hand, the settlements of similar but heavier structures (Building VI) continued long afterward. The causes of this latter phenomenon are not clear but may be attributed to the effect of gradual consolidation of the overlying clay and silt layers and the resulting decrease of the frictional support they temporarily provided, combined with lateral and upward flow of the stressed sand beneath their bearings.

Ground-Water Level.—The effects of ground-water level variations were found to be much less important than was generally believed. These variations depend on the Nile flood which occurs yearly at almost exactly the same periods and with very much the same amplitude. The variation in the ground-water level ranges from 1 ft to about 18 ft, according to the distance of the site from the river and to the permeability of the adjoining soil layers. Before the settlement studies were undertaken, many engineers believed that these yearly variations should be held responsible for a dislocation of the soil structure which then caused any of the settlements noticed. An examination of the time-settlement curves of the observed structures shows that this is not the case.

---

[5] *Proceedings*, International Conference on Soil Mechanics and Foundation Engineering, Rept. from the Delft Laboratory, *Paper I-I*, Vol. I, p. 181.

A pronounced effect on the time-settlement curve was observed only when the rise of the ground-water level coincided with the period of construction and when the seat of settlement was located in a deep, dark, clay layer (Buildings IV and V, Figs. 5, 6, and 7). This may be explained as follows: A rise of the ground-water level decreases, by the amount of buoyancy, the pressure component on the deep, dark clay due to the weight of the overlying layers within the height of the water rise. This dark clay has been fully consolidated under the maximum pressure from the overlying soil corresponding to the lowest ground-water level. When the water rises at the same time as the building is constructed, not all, if any, of the newly applied load of the superstructure creates stresses in the dark clay layer in excess of the previous maximum. This newly applied surface load is compensated by the decrease in the weight of the newly saturated soil layers. The rate of settlement decreases and the time-settlement curve flattens out. When, subsequently, the ground-water level falls, all the surface load of the structure erected during the period of the water rise is suddenly applied to the deeper layer of dark clay. It creates stresses in excess of the preceding maximum and a sudden and rapid increase in the rate of settlement results (Buildings IV and V, Figs. 5, 6 and 7). The same process may be repeated on a much smaller scale in succeeding years until consolidation under the effect of the load of the structure is completed.

This shows that the variations in ground-water level, in soils of the type studied, only slow down the rate of settlement during the rise of the water level and accelerate it during its fall. The final value of the settlements does not appear to be affected.

Lines of Equal Settlement.—The distribution of the settlements in plan seemed to be governed in most cases by the intensity of stress at different points of the same planes within the clay layers, which seemed, therefore, to have a horizontally uniform degree of compressibility. Lines of equal pressure, computed from the Boussinesq formulas at about two-thirds the depth of the compressible layer, or at its center, showed an agreement of shape and of relative magnitude which had to be considered as fairly good if the distributive action of the semi-rigid superstructure is taken into account as demonstrated by Buildings I (Fig. 1), III (Figs. 3 and 4), IV (Figs. 5 and 6), V (Fig. 7), X (Fig. 12), and *T* (Figs. 13 and 14). The inclination of cracks in Building VIII (Fig. 10) furnished evidence in the same direction; the horizontally uniform compressibility of the soil was also evidenced by the uniform settlement of the perfectly rigid water tower in Building II (Fig. 2), which had only a slight tilt in the direction of the prevailing winds.

The remaining two structures, Buildings VII and IX, in Figs. 9 and 11 (Building VI could be observed only on two points) unexpectedly showed irregular settlements which obviously were not governed by stress distribution but which were primarily influenced by irregularities in the compressibility of the soil. Examination of old maps showed that both buildings were situated on very recent clay, and on silt deposits of an age of less than 100 yr; they were both found to be built on sites upon which the main branch of the Nile flowed only a century ago. These recent deposits apparently had not been compacted and consolidated by repeated drying and flooding, which may account for their irregular nature.

Structures with a theoretically regular distribution of settlements were founded on older clay and silt deposits more than 800 yr old for Buildings III, IV, V, VII, X, and *T*, and deposits more than 2 000 yr for Buildings I and II. They were subjected to repeated drying and flooding during this period, which presumably had a consolidating effect on the degree of compressibility over large areas.[4]

Thus, on a total number of eleven structures discussed in this paper two were an exception to what appears to be a local rule concerning the horizontal uniformity of soil compressibility. The writer wishes to emphasize that, as yet, very little is known concerning the degree of this uniformity for different types of soil deposits in various localities. Furthermore, records from three or four, or even from a dozen, buildings in one locality cannot be considered as sufficient for the purpose of establishing rules for actual designs. Much more extensive information is required; and it can be obtained only by numerous and systematic observations in the respective localities. Until such information becomes available, foundation designs should not be based exclusively on any laws of pressure distribution but should also take into account the most unfavorable conditions that may be created by any assumed possible irregular variations in the compressibility of the soil over the area of the structure.

The settlements described in this paper were observed mostly by means of optical levelings and observation bench-marks systems devised by Professor Terzaghi,[6] several hundred of which were built in, and were found to be most satisfactory in use. Their number generally varied from ten to sixty per building, depending on its area. Bench-marks of the network of the Survey of Egypt served as datum in Cairo. The accuracy of the leveling results reported is about 0.1 in.

Damages.—The damages caused to the buildings by the settlements were found to depend mainly on the radius of curvature of the superstructure, created by its adaptation to the surface of the settlement crater. This confirmed previous investigations of the subject.[7] Only two buildings (Building *T* and VIII) suffered damages. The remainder did not show any cracks, although some settled considerably.

Thus, Building IX settled more than 6 in. with a uniform tilt to one side but without any damage. Buildings I, III, V, VII, and X had differential settlements within their small area which varied from ½ in. to 1 in.; the long Building IV had a differential settlement, between its center and its ends, of about 1.5 in.; the radius of curvature created by these differential settlements was not sufficient to crack the superstructure which deformed without external signs of damage. The maximum settlement in all these cases varied from 1 in. to 3 in.

The radius of curvature and the differential settlement of Building VIII are not known, but they must have been considerably more than those of the preceding cases in proportion to the greater maximum settlement of this building of more than 4 in.

---

[6] *Proceedings*, Am. Soc. C. E., September, 1937, Fig. 42(*a*) and Fig. 42(*b*), p. 1360.

[7] *The Structural Engineer*, Lond., March, 1935, p. 148.

Numerous cracks were observed, mainly in partition walls. These cracks were particularly pronounced at the corners of the building, which, in accordance with the theoretical stress distribution, should have a tendency to settle less than the center of the building, severe stresses being created thereby at the corners in the superstructure. The inclination of the cracks supported this conclusion.

The old rule-of-thumb—"strengthen the corners"—should be recalled in this connection and, if correctly interpreted, should refer only to the superstructure. Some engineers understand it as meaning that more piles should be driven or that smaller unit pressures on the soil should be used at the corners. This would have an opposite effect to the one desired, by still further increasing the difference of settlements when structures are founded on horizontally homogeneous soil.

Building *T* was also strongly cracked as a result of its large maximum settlement (10 in.) and of large differential settlements. The separation of its parts of different heights and weights, by means of expansion joints, remained totally ineffective. In this connection it should be noted that many older buildings have been damaged in Cairo when situated partly within the limits of the settlement crater of the adjoining new structures. Harmful effects of the settlement crater were noticeable even when the older structures were separated from the new ones by a free space of 6 to 12 ft.

Future Trends.—The influence of the settlement studies on the new trends of local engineering practice was considerable. In 1934, previous to beginning these studies, the height of buildings in Cairo did not exceed eight or nine stories. Some of them suffered damages, but opinions were strongly divided as to the causes. The observations of the Laboratory provided logical explanations for most of the known facts, disclosed many new ones, and dispelled many erroneous conceptions. In particular, the observations showed how strongly settlements were reduced when the loads of the structure could be transferred to deeper sand layers in a manner not endangering their state of lateral confinement. As a result, the construction of buildings (on long piles driven into the sand) of ten, twelve, and fourteen stories was started in rapid succession, and investigations for an eighteen-story building have been begun.

A few engineers now go to the other extreme and maintain that the loads of all structures should be transferred to sand by means of long piles. This view ignores considerations of economy and, therefore, is just as exaggerated as the previous local antagonism to long piles because of their excess cost.

The observations disclosed the total weight of different types of structures which could be safely supported by various successive soil layers. As a result smaller structures also benefited from these observations, greater assurance and, therefore, more justified economy becoming possible for their designs.

CONCLUSIONS

Systematic settlement observations of full-sized structures are the only reliable means for obtaining accurate data on the actual behavior of these structures and also concerning the soil on which they were erected. In many respects present-day laboratory testing methods and procedures of foundation design are still in need of correction and of development through comparisons with the results of such full-scale observations. Results obtained from observations in one locality are not necessarily valid in other localities. Therefore, such settlement studies should be undertaken in all large cities. This necessity has been recognized generally for some time, but little has been done largely because individual efforts are inadequate as compared to the scope and magnitude of the work to be performed.

The required large scale of such studies necessarily calls for a co-operative effort in their organization. To be scientifically effective these studies should be made by independent organizations not having any business connections with the structures investigated. Special guaranties to the business interests concerned are then necessary, safeguarding them against damages and thereby enlisting their co-operation. The studies in Egypt described in this paper show that such a plan is feasible and can be made successful in practice.

ACKNOWLEDGMENTS

The writer wishes to emphasize the generous support given by the Egyptian Government to the investigations described. He also wishes to express acknowledgments to the persons who have facilitated the development of these studies: To Hussein Sirry Pasha, Minister of Public Works, Egyptian Government, under whose instructions and direction the first scientific settlement investigation in Egypt was made; to Professor Terzaghi who had first proposed systematic settlement studies on full-sized structures and who, through personal contacts, contributed many valuable new ideas and inspiration; to Dr. C. Andreae, former Dean, Faculty of Engineering, Egyptian University, for his support in the organization of the new laboratories; to Dr. W. S. Hanna, Head of the Foundation Soils Research Laboratory, Egyptian University, in close collaboration with whom most of the work described was done[8] and to whose courtesy the writer is indebted for data concerning the continued settlements observed during 1937; to Dr. F. E. Relton, Professor of Mathematics, Faculty of Engineering, Egyptian University, for his help in the treatment of some mathematical problems; to Kamal Khalifa Effendi, former Research Assistant, Foundation Soils Research Laboratory, Egyptian University, for his assistance in making some of the borings, tests, and computations; and to many engineers and architects of Cairo, and of the Provinces, without whose co-operation this work could not have been successfully developed.

---

[8] *Bulletin No. 1*, Soil Mechanics and Foundation Research Laboratory, Egyptian Univ., Faculty of Eng., October, 1938.

DISCUSSION

L. C. Wilcoxen,[9] **Assoc. M. Am. Soc. C. E.** (by letter).—The last decade of research on the problem of the stresses and strains occurring beneath foundations has done much to consolidate the current theoretical knowledge of the subject. In the matter of large non-rigid foundations on certain types of soils, for instance, it is now quite generally recognized that differential settlements must occur.

Professor Tschebotareff's contribution is to put in the record important differential settlements of a number of structures built on clay and silt. His chief conclusion is to emphasize the necessity for settlement studies in all large cities. This conclusion deserves wide recognition.

It should be understood, however, that such knowledge will be of practical value only when methods for preventing such settlements have been developed. Two methods for doing this may be used: (1) By designing the structure with sufficient rigidity to sustain the stresses induced by the unbalanced footing loads; and, (2) by treating the foundation soils in a manner to offset their movements due to compression, consolidation, and plastic flow. The former has been consciously done in at least one case.[10]

The latter could undoubtedly be done by the principle of "mudjacking," now used in raising sunken highway pavements. It may be that at some future time "mudjacking" pipes will be extended deep into the foundation soil during construction and specifications will demand that, over a period of a year or two, the contractor restrict the building settlements to certain limits.

Trent R. Dames,[11] **Jun. Am. Soc. C. E.** (by letter).—The most significant single statement in Professor Tschebotareff's paper is his conclusion that "Results obtained from observations in one locality are not necessarily valid in other localities" (see heading, "Conclusions"). This truth deserves more emphasis than it has yet received and is a fact of which any one engaged in developing general theories in the new field of Soil Mechanics should be always mindful.

Most of the new theories are the result of studies made largely on the glacial clays of Northeastern United States and Europe. Several years of effort on the part of the writer and an associate, W. W. Moore, Jun. Am. Soc. C. E., to apply many of them without modification to problems involving the foundation and embankment soils of the semi-arid regions of Southwestern United States have, in general, met with failure. Different factors assume importance; different soil tests become desirable; and different methods of analysis seem necessary.

Believing the soils of the Nile Valley to be somewhat akin in character and behavior to certain of those with which he has dealt, the writer is encouraged to enter this discussion and contribute what he has been able to learn of the behavior of soils of reasonably similar geologic background. This discussion will take the form of certain comments on the testing methods and the advancing of alternate interpretations of such settlement data as the author has found it possible to include.

---

[9] Asst. Civ. Engr., City Engr.'s Office, Detroit, Mich.

[10] "Economics of Rigid Frame for Building Foundations," by Kimball R. Garland, *Engineering News-Record*, September 26, 1935, p. 427.

[11] Dames and Moore, Los Angeles, Calif.

Building I, Fig. 1.—In common with all of the other structures cited, Building I raises some questions as to testing procedure. The liquid and plastic limit tests admittedly "illustrate the potential properties of the basic soil material in a disturbed state" (see heading, "Results of the Settlement Studies"), and accordingly have value for such purposes as routine tests for highway subgrades. Their value is questionable, however, when they are applied to foundation soils which, in most cases, are used only in the undisturbed state. The use of these tests to subdivide "silt" and "clay" is novel to the United States where a division based on mechanical analysis is popular. Both methods are cumbersome and difficult of ready application; and a flexible classification based on appearance, texture, and local behavior has been found equally, if not more, effective. For foundation purposes, the effort spent on these and similar tests might better be applied to more thorough field explorations and on tests which describe the behavior of undisturbed soil under load, as, for example, shear and consolidation tests on core samples extracted from the borings.

Building II, Fig. 2.—In discussing Building II, Professor Tschebotareff, in common with nearly all modern writers, condemns the load test. In this case the accuracy of the readings taken is not sufficient to "close the case" and justify this condemnation. Admittedly the load test, as generally performed, clouds the picture and frequently breeds false security, but when properly conducted, it is a valuable aid to the foundation engineer. In the case in question, if the test area and water tower can be assumed to settle roughly in proportion to their diameter (no data on which to base an opinion being available), the settlement of the test area during the probable time of test (not stated) very possibly would not have been detected.

To get results from a load test, the soil and location at which it is made must be chosen carefully, not necessarily at foundation depth. The increments of load should be known to within 5%; the settlements should be measured (for test areas of, say, 2 sq ft to 10 sq ft) to 0.001 in.; and sufficient time should be allowed for the test footing under each increment of loading to achieve equilibrium. The shape of the curve is frequently of more importance than the magnitude of the settlements, and it indicates to the engineer who is familiar with its interpretation many characteristics that no laboratory test will describe.

The author describes the errors in computed settlements resulting from an analysis dependent on compression tests made on swelling clays and silts that are sampled with difficulty. The load test can be, and has been, used to obtain an experimental ratio by which the settlement computations may be multiplied to predict the settlements of a structure more accurately. (The adjustment of settlement computations by means of a ratio obtained in this manner frequently formed part of the settlement computations reported by Mr. Moore and the writer.)[12]

---

[12] *Civil Engineering*, January, 1939, p. 41.

The record of Building II is used to justify the statement: "No Decrease of Settlement Was Observed as a Result of Excavation" (see heading, "Results of Settlement Studies"). It is true that Building II has settled about 1 in. during the three years of record, but no data are introduced to disprove the logical premise that this structure would have settled more than 1 in. if founded at, or near, the surface of the ground. Comparison with the only other building founded on soil 2 000 yr old (Building I) is futile because, in this case also, moderate excavation was resorted to and more particularly the soil had undoubtedly been pre-stressed by the considerable load of the old water tank, the bottom of which was utilized for the foundation of the new structure.

Of the buildings on soil 800 yr old, Building III is on spread footings and is not comparable as to load over an extended area. The spread footings of Building IV so nearly cover the area as to approach a mat in behavior. A similar sequence of soils both as to classification and description by the physical tests given exists under both of Sites II and IV. In each case the sand is at 40 ft; and the unit loads on the foundations are the same. The settlements of Building IV, founded near the ground surface, are more than three times those of Building II, founded at a depth of 18 ft, and occur during a somewhat shorter period of record.

Another mat of similar size, that of Building IX, founded near the surface and underlain by the same quantity of compressible soil as Building II, settled six times as much under one-half the unit load, or a factor of 12 to 1. The uppermost silt underlying Building IX is stated to be about 100 yr old, and the lower soils are of more recent age than those under Building II. However, in the absence of tests of the compressibility of these recent soils, it would seem unwarranted to assume an average compressibility coefficient in one case of twelve times the other, particularly when the proof of the case against excavation as a means of reducing settlements rests on such an assumption.

Building III, Figs. 3 and 4.—The settlement of Building III at an undiminished rate from the middle of 1935 through 1937 would seem to warrant an attempt at explanation. On many California soils such a settlement curve would probably indicate that incipient shearing failure has occurred, and that a moderate degree of plastic flow of the over-loaded clay from beneath the spread foundations of this structure is occurring.

Building IV, Figs. 5 and 6.—This building was discussed under Building II as an example of lack of excavation. As in Building III, the continued, undiminished rate of settlement since the middle of 1936 could be interpreted as caused, at least in part, by shearing failure and the resulting plastic flow of over-loaded soil from beneath the foundations of this building.

Building T, Figs. 13 and 14.—For convenience of discussion, Building *T* will be considered before any of the other buildings on pile foundations. The writer is unfamiliar with the type of piling used in these Egyptian structures as it is seldom, if ever, used on the Pacific Coast of the United States. For this reason he will have to assume that the diameters of these piles (which are not given by the author) are nearly comparable to the diameters of the driven or cast-in-place piles of his experience.

The individual piles of all of the buildings described in this paper seem more heavily loaded than the soil conditions warrant. Tests of the shearing strength of the soil which would permit a determination of the load at which individual piles would begin to penetrate the soil about them are apparently not available. The load tests presented in Fig. 14(*c*), however, most certainly indicate that this penetration is occurring at loads in excess of 40 tons, possibly even at lower loads, the settlement scale being too small to describe the shape of the curve below this point. On several occasions the writer has found that the point at which the curve of a pile load test departs from an approximate straight line agrees remarkably well with a determination based on a summation of the circumferential area of the pile times the shearing strength of the soil penetrated and the bearing on the pile point. The spacing of the piles when grouped in clusters is not stated, but if it is at all similar to the practice in California they are so close that the circumference of the group times the shearing strength of the soil will give a lower value than 40 tons per pile for the load at which penetration of the group occurs.

Additional support for the belief that the settlement of Building *T* has been largely the result of individual pile failure and not due to the compression of the 10 ft of dark clay between the pile points and the sand is furnished by the following reasoning: This dark clay is stated to be more than 800 yr old and to have been subjected to compaction and consolidation by the repeated drying and flooding by the River Nile through the intervening centuries. Accordingly, it is reasonable to assume, from the tabulation following Equation (1), that the compression coefficient for stiff brown clay (0.007) is reasonably representative of this material and not the value given for plastic dark clay (0.070), which, in the absence of test data on either Fig. 13 or 14, was presumably obtained by working backward from the settlements of the structure. Using the value of the compression coefficient (0.007) the maximum load of 17 tons per sq m (= 1.6 tons per sq ft), and the thickness of the dark clay (10 ft), the compression in the clay layer becomes $1\frac{3}{8}$ in. leaving the remainder of the 10 in. of settlement recorded to date to be attributed to individual or group pile penetration.

Building V, Fig. 7.—The piles in Building V are more moderately loaded than those in Building *T*, but in view of the 800-yr age of the soils beneath the piling and the magnitude of the settlement (2.5 in.) to the middle of 1937, about one-half the present settlement may need to be explained by penetration of the piling. If this is true, this building may continue to settle appreciably for some years to come.

Building VI, Fig. 8.—The average load of 64 tons per pile seems excessive, even when the 6-ft to 8-ft penetration of the 27-ft piles into the very fine sand is considered. The shape of the time-settlement curve and its failure to quickly become horizontally asymptotic would indicate that shearing failure and flow of the soil is occurring. The attribution of any appreciable part of this settlement to the effect of gradual consolidation of the overlying clay and silt layers and the resulting decrease of the frictional support they temporarily provided does not seem reasonable. In view of its shearing rigidity being necessarily considerably less than that of the sand, the brown silt layer could scarcely have been carrying any large proportion of the total load at any time; and, in the three years of record, this 12-ft or 13-ft layer could scarcely have consolidated under the weight of the superimposed silt and "old" fill as much as the 1.25 in. settlement recorded.

Lacking data as to the occupancy and location of this building the writer would hazard a guess that the continuation of the settlements may be aided by elevator or traffic vibrations transmitted by the piling to the over-stressed sand. Similar examples have come to his notice in the case of buildings founded on sand which has been over-loaded by the foundations. A building occupied by a publishing establishment showed visible distress only in the vicinity of a heavy press. In some buildings cracks have been observed only in the bays immediately adjacent to the elevator shafts.

Building VII, Fig. 9.—Inasmuch as the piles seem primarily end-bearing piles and apparently would not receive any large proportion of their support from the less than 100-yr old silt and clay deposits above the sand, the pronounced irregularities of settlement cannot be primarily the result of irregularities in the compressibility of the soil. Vibrations are again hazarded as a possible explanation. Included in this category must be considered any vibrations that occur during the placing of the piling and during construction. For example, the state of stress in the compacted and densified sand under the first piles to be placed might have been released by vibrations occasioned during the placing of subsequent piles. The remaining three buildings are similar to those previously discussed.

Although the writer has taken a different point of view than the author and has taken every opportunity to present and advance a different interpretation of the data, he appreciates, thoroughly, having had the opportunity to study such a comprehensive paper and believes that Professor Tschebotareff deserves the highest praise for having so completely made the results of his observations available to the profession.

Edwin J. Beugler,[13] M. Am. Soc. C. E. (by letter).—A distinct service has been performed by Professor Tschebotareff in presenting unusually complete data as to settlements occurring with various types of foundations on sedimentary formations under an imposed unit loading. The data, largely in graphic form, afford a clear picture of cause and effect. Several types of footings are shown with a diversity of soils and intermittent ground-water levels.

Of prime importance is a distinction between the relative settlement during construction and that occurring later under the completed structure. The former is due to compression, shrinkage, and more or less elastic reaction, and ceases when the building is finished, or shortly thereafter, provided the unit loading of the soil has been selected conservatively. Conversely, continued progressive settlement of the completed structure indicates over-loading and ultimate failure. If the rate of progressive settlement is decreasing it is possible, in most cases, to predict the time of ultimate cessation.

The time-settlement curves given in Fig. 9 (Building VII) shows a well-designed foundation for the situation involved, and illustrates the foregoing statements. Loading as construction proceeded caused a maximum compression of about 1.3 in. After the structure was completed no appreciable settlement occurred. The action may be likened to placing a load on a spring scale which is compressed proportionately to the applied load and thereafter remains stationary under a constant load.

---

[13] Cons. Engr., Cheshire, Conn.

Much of the confusing information concerning settlements has been due to overlooking this important phase. For example, the progress report of the Special Committee on Earths and Foundations, presented in 1933, pictures a settlement record of the Washington Monument[14] from 1879 to that date, amounting to 5.5 in. and "probably still slowly settling." Analysis of the record shows that 4.5 in. of this settlement occurred between 1879 and 1885 during construction, which added 400 ft to the height of the shaft as completed in 1885. The remaining settlement of only 1 in. under a constant load during nearly 50 yr, it must be granted, indicates a fairly stable structure, and a monument to the engineers who so skillfully underpinned the old foundation, as well as a monument to the hero of 1776.

The self-explanatory diagrams may enable one to determine offhand the cause for failure or inadequacy. From the time-settlement curves for the various buildings it may be concluded that over-loading is responsible for most of the progressive settlement. Some was due to the fact that piles did not penetrate to firm material, or that they failed to distribute the load in other respects.

The author states that no decrease of settlement was observed as a result of excavation (heading "Results of the Settlement Studies"). This was doubtless true for the case cited. On the other hand, one instance in the writer's experience is worthy of note. In 1919 an important structure was settling progressively at one end and remaining stationary at the other. It was supported on piles driven to refusal. Rock was 40 ft below grade at the undisturbed section and 170 ft below with intervening sedimentary strata, including a band of soft clay, at the depressed end. There were no cracks in the structure. The settlement of 18 in. was stopped by excavating earth within and around the building, the outside area being replaced by light reinforced cellular construction. The load on the area was reduced about one-third before the settlement ceased. The structure has since remained stationary.

The author states that no useful relation could be established between load-test results and the settlement of full-sized structures. The meager data given regarding the method of making such tests precludes comment as to other methods and deductions therefrom. Reference to this and other matters has been made by the writer elsewhere.[15]

W. S. HANNA,[16] ESQ. (by letter).—The method applied to Buildings I, II, and III in this paper gave a fairly satisfactory forecast of settlements, based on laboratory tests (see heading, "Results of the Settlement Studies: Relation Between Settlements in Clays as Actually Observed and as Forecast on the Basis of Laboratory Tests"). Briefly, the process involves loading the sample until it has reached its preconsolidation pressure, and then unloading it until it sustains only the weight of the upper layers. The resulting branch of the compression curve is taken as the basis for computing the calculated settlement forecast.

---

[14] *Proceedings*, Am. Soc. C. E., May, 1933, Fig. 34, p. 813.

[15] *Loc. cit.*, October, 1933, p. 1358; also, *The Military Engineer*, July–August, 1933, pp. 326–327.

[16] Head of Soil Mechanics and Foundation Research Laboratory, Egyptian Univ., Cairo, Egypt.

Buildings investigated subsequently by the Laboratory at Egyptian University gave different results; in a few cases the predicted settlements were less than the actual settlements; whereas in the cases reported by the author the forecast settlements were higher than the actual. Professor Tschebotareff tried several methods, of which the foregoing seemed to fit most closely. Contradictory results observed subsequently led the writer to investigate the entire problem again. There is no doubt that the samples of brown clay expand while being removed. On the other hand, it seems improbable that the resulting strains can be avoided by the process described in the paper, which amounts to remaking the sample in the odometer until a state similar to its condition in the soil is reached. This procedure ignores all the plastic strains produced and the effect of confining the sample in the odometer. Furthermore, there is considerable difference in the compression curve of undisturbed local clays and the curves derived from clay samples, and this is observed in both the swelling and the non-swelling types. Therefore, the writer undertook to study the entire problem from a new angle. Preliminary results of these studies indicate that the author's statement that the method described in the paper has proved a success cannot be considered final.

The writer does not agree with Professor Tschebotareff's conclusions regarding the pronounced effect on some time-settlement curves of the annual rise and fall of ground-water level as a result of the flood. He states (see heading "Results of the Settlement Studies: Ground-Water Level") that "A pronounced effect on the time-settlement curve [due to the variation of the subsoil water level during the yearly flood season] was observed only when the rise of the ground-water level coincided with the period of construction and when the seat of settlement was located in a deep, dark, clay layer" (words in brackets are by the writer). On the contrary, the writer believes that the effect is mainly due to the upper layers traversed by the rising water, which consists generally of the brown clay. The reasons are briefly as follows:

(*a*) The decrease of pressure on the deep, dark clay, caused by the buoyancy of the upper layers affected by the rising water, is quite small. It is true that buoyancy decreases the specific weight of the layer through which the water flows; but the specific weight is increased as a result of the increase in the water content. The actual weight of 1 cu m of brown clay, below normal ground-water level, is 1.65 tons; but it weighs 2.08 tons during the flood season, when weighed after it has been disturbed. Except in its undisturbed position in the immediate vicinity of the river, the maximum rise is 2 m. In such cases, therefore, the maximum reduction in pressure as a result of buoyancy is $2 \times 0.057 = 0.114$ kg per sq cm (233 lb per sq ft). When part of the period of construction coincides with the flood period, the increase in pressure due to the weight of the buildings ranges from 0.4 to 0.8 kg per sq cm of the area of the building (817 to 1 635 lb per sq ft). Of course, these values do not represent the component of the increased pressure on the deep layers, but they furnish a basis of comparison. In computing the buoyancy, furthermore, full upward hydrostatic pressure has been assumed, which is impossible in the case of plastic clay layers.

(*b*) In most cases the layers through which the flood water passes are the brown clays, which are either stiff or soft and silty. The stiff clay, if permeable, exhibits strong swelling tendencies when placed in the odometer. This is not elastic swelling, because the sample has had sufficient freedom to swell completely during the shaping and after leaving the sampler. It is due to the absorption of water from the porous stone when it is placed in the odometer.

The writer conducted several tests to study the effect of the swelling due to absorption. The pressures required to return the 12-mm sample to normal, when swelling occurs under the weight of the piston alone, vary between 0.18 and 0.30 kg per sq cm (368 and 613 lb per sq ft, respectively). Therefore, it is safe to assume that, where such brown clays are reached by rising water, the swelling of these layers reduces the pressure on the layer above them and, probably, to a slight degree, on the lower layers (this is a mere assumption). When the water recedes and the soil retains its normal water content, the swelling resistance disappears and the resultant pressure on the upper layers is suddenly increased. This explains the reduced rate of the time-settlement curve during the rise, and the increased rate, during the fall, of water level. Furthermore, it is a known fact that most old houses settle soon after the ground-water level beneath them has been lowered. The slight movement of the building when it responds to the swelling of the soil seems sufficient to crack its walls. Most of these houses have bearing pressures not exceeding 0.3 kg per sq cm (613 lb per sq ft) and the mortar, which was originally of a poor quality, has deteriorated with age.

As compared with dark clays the brown clays are relatively little affected by remolding when tested in the unconfined state (see heading "Results of the Settlement Studies: Not All Clays Are Unfavorably Affected by Remolding"); and brown clays, of the same origin as those found in the interior, have been found near the sea. This brown clay is very remoldable. Furthermore, extremely silty types of it, which are very stiff in the natural state, crumble under the smallest pressure when remolded; and, both the brown and dark clays are strongly affected by remolding when tested in the confined state, a case which is not observed in samples of brown and dark clays found in the interior. It would be quite interesting, therefore, to learn whether similar behavior in clays deposited under saline water has been observed elsewhere.

The writer agrees entirely with the author's statement (see "Results of the Settlement Studies: Efficiency of Devices for Taking 'Undisturbed Samplers' ") that in the swelling types of clays, uninterrupted lateral confinement becomes essential for all stages until it is placed in the odometer. Some time ago the writer experimented with a special ring that can be fitted both to the samples and to the odometer, so that the soil sample can be tested without removing it from the sampler. It was tested in connection with the different samplers and also was used alone to extract the sample after the hole was cleared directly under the bore tubes. Some interesting, and some rather contradictory, results were obtained. On the whole, however, the effect of lateral swelling seemed to be about 100% in the clays most seriously affected. This means that the investigator must search elsewhere for the explanations of the difference (in some cases of more than 100%) between the actual values of settlements and the values based on the odometer tests.

D. P. KRYNINE,[17] M. AM. SOC. C. E. (by letter).—The importance of settlement observations on full-sized structures has often been emphasized in technical literature. The paper by Professor Tschebotareff is a valuable contribution in this respect.

Two Cases of Settlement Discussed.—Before examining the interesting collection of settlement contour lines presented by Professor Tschebotareff, the writer wishes to express some general thoughts on the shape of such lines. Two different cases will be distinguished: Case A, in which the structure, or the tips of the piles supporting it, reaches the top (or nearly the top) of a compressible layer; and Case B, in which the compressible layer is at a considerable depth. Most of the structures discussed in the paper are examples of Case A.

Settlement in Case A.—When the compressible layer is more or less uniform and is sufficiently thick, it appears first that it may be considered as a semi-infinite, elastically isotropic body. In such a case settlements could be computed according to elastic theories:

$$s_0 = \frac{P}{r} \frac{1 - \mu^2}{\pi E} \quad \text{.......................(2)}$$

in which: s_0 = settlement at a given point; P = load; r = horizontal distance from the load P to the given point; μ = Poisson's ratio; and E = modulus of elasticity of the earth material.

However, it is difficult, and perhaps impossible, to determine the elastic constants of the given earth material in the field. Furthermore, the settlement of a clay deposit is due also to the load squeezing moisture from its pores so that the actual settlement is many times greater than the computed value. An assumption can be made, however, which is of the same order of accuracy as assumptions made in the theory of consolidation; namely, that the actual value of the settlement to be expected is proportional to the value of s_0, as computed using Equation (2). In other words, the settlement is proportional to the ratio $\frac{P}{r}$. In Equation (2) substituting

$$C = \frac{1 - \mu^2}{\pi E} \quad \text{.......................(3)}$$

relative values of the expected settlement can be computed easily. Let s_0' and s_0'' represent settlements at two different points of the ground surface, as caused by load P; and, let r' and r'' represent respective horizontal distances of these points from load P. Then:

---

[17] Research Associate in Soil Mechanics, Dept. of Civ. Eng., Yale Univ., New Haven, Conn.

$$s_0' = C \frac{P}{r'} \qquad (4a)$$

and

$$s_0'' = C \frac{P}{r''} \qquad (4b)$$

from which:

$$s_0' : s_0'' = r'' : r' \qquad (5)$$

that is, elastic settlements at two different points of the ground surface are inversely proportional to their horizontal distances from the load. Relative values of elastic settlement at any point of the ground surface, as caused by a uniformly loaded area of arbitrary shape, can be computed, even in complicated cases, by using the method of graphic integration. W. Steinbrenner has given analytical formulas and has prepared tables of such settlements in the simplest case of rectangular loaded areas.[18]

It should be noted, however, that Professor Terzaghi advises that the settlement of shallow layers (such as described under "Case A") be computed in the same manner as that of deep layers[19]; and Professor Tschebotareff follows that procedure.

Settlement in Case B.—Should the compressible layer occur at a considerable depth, its consolidation is attributed to the action of the vertical pressure, p_z, which is computed using the well-known Boussinesq formula. The distribution of the vertical pressure at the top surface, $a\ a_0\ a'$, of the compressible layer is given in the form of a curve, $a\ a_0\ a'$, in Fig. 15. A similar distribution occurs along any other horizontal plane of the compressible layer; as, for instance, at the bottom (Section $b\ b'$, Fig. 15). All ordinates of the distribution curve traced at the bottom of the layer would be smaller than those at the top. Consider a vertical section ($o\ r\ u\ m\ n$, Fig. 15) of the earth mass. Plotting vertical pressures, p_z, at different depths of this section as horizontal ordinates, a curve, $o\ s\ w\ p\ q$, would be obtained. The shaded area, $m\ p\ q\ n$, multiplied by a coefficient depending on the compressibility of the layer in question ("specific loss of moisture" as discussed hereafter), furnishes the value of the final settlement at Point o. It is assumed in this connection that a column of earth such as $o\ m$ or $t\ t'$ settles independently of the remainder of the earth mass, and that, in sinking down, it does not undergo any change of volume or shape. In other words, soil deformations above the compressible layer $a\ a'\ b'\ b$, Fig. 15, are neglected. The soil above the compressible layer is thus assumed to be deprived of shearing resistance.

It should be noted that the Boussinesq formula for the vertical pressure does not contain the elastic constants E and μ; hence, it is valid for any homogeneous, elastically isotropic material. Were the entire mass made of clay, or of steel, or of some other homogeneous material, which could be considered as elastically isotropic, Boussinesq's formula for the vertical pressure would furnish fairly satisfactory results, regardless of the type of material. Its use becomes questionable, however, in the case of a compressible layer, since there is discontinuity at the top of the latter, and the material is no longer homogeneous.[20] This phenomenon has been explained also by the "bridging (or arching) effect" of a sand or gravel layer which overlies a compressible clay layer. According to the latter view, the load applied at the earth's surface is spread by the intermediate sand or gravel layer so that the magnitude of the stresses and their distribution in the lower compressible layer are thus modified.

---

[18] "Tafeln zur Setzungsberechung," by W. Steinbrenner, *Die Strasse*, Vol. 1 (1934), p. 121.

[19] "Theorie der Setzungen von Tonschichten," by Charles Terzaghi and O. K. Fröhlich, Leipzig and Vienna (1936), p. 14.

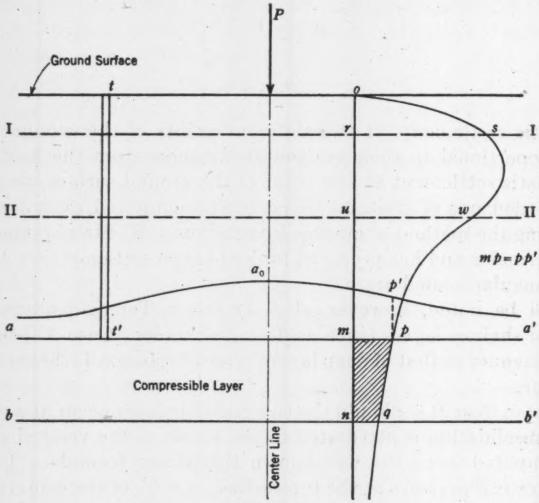


FIG. 15

Settlement at a Considerable Depth as Caused by a Loaded Area.—According to Saint Venant's principle, stresses at a point within a body remote from the surface where a system of forces is applied are practically the same as those caused by the resultant of that system. Hence, stresses and strains in a deep compressible layer are practically the same as those caused by the resultant P of the forces acting on that area. It is evident that stresses thrown to the compressible layer in question should be practically symmetrical with respect to the vertical line of action of force, P. In other words, the set of lines of equal settlements at the earth's surface should approach a set of concentric circles if the seat of settlement is at a considerable depth. Professor Tschebotareff's contour lines are far from being concentric circles, however; this is because, in reality, the settlements in question have their seats in rather shallow layers.

Influence of the Rigidity of the Superstructure.—Professor Tschebotareff presents interesting comparisons between theoretical settlements in the case of rigid and non-rigid (flexible) superstructure, or more accurately, of the rigid and non-rigid base of the superstructure. Quite striking is the case of

---

[20] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 885.

Fig. 1 in which lines of actual settlement follow rather closely the lines of equal pressure shown in that figure, which is assumed to be rigid. It is important to notice that, in this case, the seat of settlement is in a rather shallow layer. The writer believes that the influence of rigidity on the pressure distribution at a horizontal plane decreases with the depth. This is in accord with Saint Venant's principle since at a greater depth the values of the stresses are controlled merely by the total weight of the entire structure (assuming that this weight is the same in the case of both rigid and non-rigid superstructure). In the same degree, the size of a rigid structure may have a certain influence on the settlement; apparently, the larger the structure, the less will be the influence of the rigidity on the stresses underneath the central part of that structure. In the limiting case in which both rigid and non-rigid structures are infinitely long and infinitely wide, there should be no difference whatsoever between a rigid and a non-rigid structure, since in such a case the value of the settlement would depend entirely on the properties of the earth material.

Depth at Which the Vertical Pressure Should Be Computed.—Professor Tschebotareff computes the vertical pressure "at approximately two-thirds the depth of the compressible layer" (in some occasions one-half that depth).

This approximation is not always warranted. In the case of a deep layer, such as that bounded by Plane $a\,a'$ (Fig. 15) and Plane $b\,b'$, the area controlling the settlement is $m\,p\,q\,n$, and its average ordinate is approximately at the middle of the layer. Should the layer be close to the earth's surface (for instance, the layer bounded by Planes I–I and II–II) the situation is entirely different; and the average ordinate may or may not be at the middle or at two-thirds of the height of area $r\,s\,w\,u$. Professor Tschebotareff's assumption is practically correct in the case of a shallow layer between the ground surface and Plane I–I (Fig. 15).

Coefficient of Compressibility, X.—Professor Tschebotareff is to be commended for introducing the coefficient of compressibility, X, as defined by Equation (1), and in comparing the laboratory value of this coefficient with the corresponding field value. Since the value of $\frac{a}{1 + e'}$ in Equation (1) is the reciprocal of what is analogous to the modulus of elasticity, that value is measured by a reciprocal of stress (in square centimeters per gram, as Professor Tschebotareff states). The second part of Equation (1) proves it, since $\frac{s}{d'}$ is a strain and Δp is a stress, so that X is a strain-stress ratio. For practical purposes it is convenient, however, to express X in units of length as is done by the author as well as by Terzaghi and Fröhlich.[21] The latter expresses the value of $v = \frac{a}{1 + e'}$ as "specific loss of moisture." If a prism of arbitrary horizontal cross-section is imagined to be cut from a compressible layer 1 m thick, and is still confined laterally and subjected to the action of a compressive stress of 1 gram per cm², v will be the height (in meters) of the column of water squeezed from that prism. If the stress is increased 1 000 times and equals 1 kg per cm² (1 ton per sq ft, approximately), the value of *v* is to be multiplied by 1 000, thus becoming Professor Tschebotareff's coefficient:

---

[21] "Theorie der Setzungen von Tonschichten," by Charles Terzaghi and O. K. Fröhlich, Leipzig and Vienna (1936), p. 24 *et seq.*

$$X = \frac{a}{1 + e'} \times 1\,000 \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

The value of X is still in meters; for instance, $X = 0.010$ m, as in Professor Tschebotareff's example tabulated in the text following Equation (1). The expression "specific loss of moisture" has been introduced as an analogy of the conception "specific heat" in physics and thermodynamics.

Quite ingeniously, Professor Tschebotareff has shown that it is not necessary to make a complete consolidation test if an error of 3% or 4% is tolerable. He thus obtains the valuable part of the test without wasting time and energy on non-essential phases of it.

Conclusions.—In the five years, 1934 to 1939, the literature on soil mechanics has been substantially increased by observations of settlement in different parts of the world. To the original observations in Western Europe and the eastern sections of the United States, observations in the Western United States, in the South (Texas and Louisiana), in Mexico, and now in Egypt, are added. Numerous interesting observations in Russia have been published in that language. All this material needs to be digested and classified. The writer believes that a new chapter in the study of soil mechanics is opening which may be called "geography of settlement observations," and that its adequate development is of great importance for foundation engineers.

GREGORY P. TSCHEBOTAREFF,[22] M. AM. SOC. C. E. (by letter)—The contributions made to the problem of settlement studies by the discussers of the paper are appreciated sincerely. The writer is glad to have the opportunity to clarify the problems discussed by stressing some of the data contained in the plans and diagrams, which were overlooked by the discussers, and by supplying additional data concerning the studies reported in the paper in a necessarily condensed and summarized form.

The Design of Structures to Withstand Differential Settlements Is Quite Practicable.—It was gratifying to note that Mr. Wilcoxen recommended wide recognition of the writer's chief conclusion concerning the necessity for settlement studies in all large cities. Mr. Wilcoxen suggests that two methods may be "developed" for the purpose of preventing detrimental effects of differential settlements.

The first of these two methods—that is, a suitable preventive design of the structure—is quite practicable at present. However, it does not always imply merely the provision of sufficient rigidity for the combined superstructure and foundation. In some special cases (such as steel tank foundations) a flexible foundation, matching the flexible superstructure, will give better results.[23]

Further, a suitable architectural design is important for all buildings on soft ground. The omission of high towers and low annexes, and the provision of deep basements, as well as other measures permitting the use of reinforced concrete walls several stories high, are among the features that help to equalize settlements.

---

[22] Asst. Prof., Civ. Eng., School of Eng., Princeton Univ., Princeton, N. J.; formerly, Research Engr., Foundation Soils Research Laboratory, Egyptian Univ., Cairo, Egypt.

[23] "Progress Report of Special Committee on Earths and Foundations," *Proceedings*, Am. Soc. C. E., May, 1933, p. 812.

The construction procedure followed may also be of importance. For instance, if a low annex is essential, settlement stresses in its framework may be reduced considerably by postponing the erection of the annex as long as possible. Its simultaneous construction with the upper, and not the lower, stories of the adjoining high part of the building may suffice in the numerous cases when a large percentage of the final settlement will have occurred during construction.

The practical importance of the knowledge of the settlements to be expected is not limited to cases of excessive settlement. Any definite knowledge in the matter reduces guesswork and permits more economical and rational designs.

"Mudjacking" of Compressible Soils for the Purpose of Increasing Their Resistance Is Impracticable.—Mr. Wilcoxen further suggests treating foundation soils to prevent differential settlements by using "mudjacking" pipes extended deep into the foundation soil.

This suggestion does not appear at all promising. "Mudjacking," as successfully used in highway construction, serves to raise sunken pavements by injecting, under pressure, a soil-water slurry between the relatively light concrete pavement slab and the surface of the soil. The entire operation is similar in principle to the frequent and successful use of jacks for the purpose of raising a bridge bearing and then wedging it up the distance that the pier beneath it had settled.

The problem is different, however, if one tries to inject anything into the ground itself for the purpose of preventing its compression or deformation under load. The injected liquid tends to flow in the direction of least resistance. This circumstance may be utilized to fill, with cement grout, or chemicals, cracks in rocks or in dry clays, or cavities beneath foundations in order to prevent seepage. In plastic materials, and even in sands, this very circumstance means that only some irregularly distributed and more permeable, but not necessarily more compressible, veins of the soil may be given rock-like hardness. Obviously, this is of no advantage, and may be directly detrimental where differential settlements are concerned.

In 1932 a European firm specializing in this kind of work attempted to consolidate the weak, dark clay beneath the pile bearings of Building *T* by successively injecting into that clay separate solutions of two different chemicals. This attempt failed, although the two solutions formed a "gel" when coming into contact with each other and gave a rock-like hardness to that clay when mechanically mixed with it in the laboratory. A further test was made with the chemical-consolidation method in a dry, silty sand layer. A subsequent excavation of an injection revealed what resembled an inverted sandstone tree. The trunk was only about 10 in. in diameter. Four or five branches stretched out horizontally, one of the branches being almost 5 ft long. Therefore, in order to create a continuous mass of solidified, silty sand, the injection pipes would have to be driven at not more than 10 in., center to center, and much closer in less pervious soils. This is obviously impracticable in most cases.

The General Principles of Soil Mechanics Are Valid in Any Locality.—Mr. Dames considers the author's conclusion that "Results obtained from observations in one locality are not necessarily valid in other localities" to be the most significant single statement of the paper. The writer agrees that this conclusion is very important and should be emphasized. At the same time he wishes, emphatically, to warn against the inference contained in further statements by Mr. Dames that most of the new soil mechanics theories (unspecified) are seldom valid except for glacial clays of Northeastern United States and Europe.

The general principles of soil mechanics are valid everywhere. It is a frequent but regrettable practice to apply a theory, superficially, to local conditions which do not correspond to its underlying assumptions. After the inevitable failure the entire theory is then often proclaimed invalid without further analysis. The writer cannot judge the nature of the "different" soil tests and methods advocated for Southwestern United States by Mr. Dames since their nature is not specified; but whatever their nature or their merits they cannot warrent arbitrarily ignoring conclusions based on a theoretical analysis of observations. This is especially true when the observations are made under field conditions closely approximating the assumptions on which a widely known theory[24] is based, whereby both the field and the laboratory results agree as well as agreement can be expected, theoretically. To illustrate these remarks the writer will examine some of the discussions of the time-settlement curves, which ignored the effect of the permeability of a stressed clay layer on the rate of its compression.

The Rate of Settlement of a Fully Saturated Clay Layer Is Governed Primarily by Its Permeability and Thickness.—In discussing Building IV, Figs. 5 and 6, Mr. Dames states: "* * * the continued, undiminished rate of settlement since the middle of 1936 could be interpreted as caused, at least in part, by shearing failure and the resulting plastic flow of over-loaded soil from beneath the foundations of this building." An examination of Fig. 6 shows that the rate of settlement had not been constant since the middle of 1936. Seasonal variations of subsoil water levels provide definite breaks in the time-settlement curve; the rate of settlement is slowest when the subsoil water level is highest, almost reaching to the elevation of the foundations; then the rate of settlement increases as the subsoil water level drops about 14 ft. The reverse should have been the case if Mr. Dames' interpretation were correct, because a decrease in the moisture of a cohesive clay increases its cohesion and therefore its shearing strength, simultaneously decreasing its tendency to undergo plastic deformations. In the case of Building IV the borings were made at the time of lowest water level and the chart of laboratory test results (Fig. 6) shows that at that time the clay soil immediately beneath the foundation was very stiff and was of a definitely brittle type least likely to undergo plastic deformations.

Thus all available facts contradict the aforementioned interpretation by Mr. Dames. On the other hand, computations based on the rate of compression of the deeper lying, soft, dark clay layer following the expulsion of surplus moisture show that settlements of Building IV due to that cause were still to be expected during 1936–1937. This therefore provides reasonable explanation for them. Nevertheless, the writer does not deny that plastic shearing deformations may have some influence on the rate of settlement and has recognized this fact in the past.[24] On deep lying, fully water-logged, clay soils, however, this cause is likely to be of any relative importance only after the settlements due to the expulsion of surplus moisture have slowed down almost entirely. The results of further-continued observations may disclose whether this is the case for any of the buildings under discussion. The only case reported in this paper where plastic shearing deformations may be at all reasonably assumed to be of importance thus far is Building VI (see Fig. 8). The writer stated that the cause of the continued settlement was not clear and suggested tentatively that it might be attributed to the effect of gradual consolidation of the overlying silt layers and to the resulting decrease in the frictional support they temporarily provided. Mr. Dames misunderstood this to mean that the silt layer was believed to be consolidating under its own weight. Undoubtedly this is most unlikely. The writer referred to consolidation under pressures from the building transmitted by the footings which capped the piles and by the relatively high skin friction of the piles themselves.

---

[24] "Factors Affecting the Accuracy of Settlement Rate Forecasts," by Gregory P. Tschebotareff, *Proceedings*, 18th Annual Meeting, Highway Research Board, Part II, December, 1938.

The writer agrees that gradual plastic shearing deformations of both the silt and the sand (as suggested by Mr. Dames) may have been equally or even more important than consolidation in this one case of Building VI. Furthermore, the possibility of some small, plastic, shearing deformations is not entirely excluded in the cases of Buildings II and III, but a much more likely explanation is provided by the action of the repeated variations of the live load. The total pressure on the ground surface beneath Building II is decreased by 20% and then increased by the same amount twice every week. A similar but daily fluctuation of about 10% of the total pressure on the ground surface occurred beneath Building III in the zone of Points 26 and 28 under the balcony of that movie house. It was only these points which showed any noticeable continuation of the settlements, whereas Point 1, with the same unit pressure on the surface of the ground, but so located that the effect of the live load variations would be relatively small, showed only a negligible continued settlement of about $\frac{1}{16}$ in. during 1.5 years.

Mr. Dames mentions vibrations as a possible cause for continued settlements; but they could not have been of importance in this case. No machinery was present. None of the lifts of Building *T* functioned until 1934, none of them was located in the zone of continued greatest settlements, and no cracks were to be found near them. Street-car lines and an otherwise busy thoroughfare passed next to Building *T*; but this thoroughfare ran east and west along the north side of the building, which side settled slightly less than the south side of the building (Fig. 13) where it adjoined a small street with much less traffic and no street-car lines. The reverse should have been the case if traffic vibrations were influencing the settlements.

Mr. Beugler states that: "From the time-settlement curves for the various buildings it may be concluded that over-loading is responsible for most of the progressive settlement." This statement appears to be due to his belief that settlement "due to compression, shrinkage, and more or less elastic reaction, * * * ceases when the building is finished, or shortly thereafter, provided the unit loading of the soil has been selected conservatively."

It can be demonstrated easily in any soil mechanics laboratory that such belief is entirely without foundation, in so far as the compression of clay soils is concerned. The compression of a fully saturated, confined, clay sample never occurs instantaneously after application of the load, no matter how small the load increment is; and it can be extremely slow for very impermeable clays. Clays and silts from the northeastern, southern, and southwestern regions of the United States have been tested by the writer at Princeton University and have not shown any behavior, in this respect, different from the soils of Central Europe or Northern Africa. In all cases the permeability of a saturated clay exercised the most important influence on the rate of its compression. The relative influence of other factors may vary and will increase for more pervious or only partly saturated soils; but for fully saturated clays this influence always remained secondary as compared to the permeability of such clays.

An illustration of the contradictions in the aforementioned reasoning of Mr. Beugler is provided by Building IX (Fig. 11) which exerted a unit pressure of only 0.4 tons per sq ft on the surface of the ground. This pressure cannot be called anything but a conservative value for any soil, and it could not have been reduced further for the continuous mat of the building. The phenomenon of continued and gradually decreasing settlement of this building is fully explained by the gradual expulsion of water from the compressed voids of the silt and clay beneath the building, whereas "overloading" certainly does not offer any explanation. The settlements of this case almost ceased a year after the end of the construction, as should be expected because of the relatively thin compressible layer (20 ft). With a thicker layer they might have lasted much longer.

Soil Exploration for Foundation Work and Soil Testing Procedure for Engineering Classification Purposes.—Mr. Dames questions the value of the plastic and liquid limit tests when they are related to foundation soils in the undisturbed state, but overlooks the fact that nowhere in the paper were the plastic and liquid limits reported alone. In all cases a curve for each of these limits was plotted along the entire depth of the boring, side by side with a curve for the natural water content (in the undisturbed state). The curves for the compressive strength and the strains at failure, both in the undisturbed and in the remolded state, were plotted also. Any one of these tests is inadequate, if taken by itself; but it supplements the others so that, taken together, they give a good idea of the undisturbed nature of the soil profile.

The writer has a strong distrust of the sufficiency of descriptions of "appearance, texture, and local behavior," as suggested by Mr. Dames, unless they are accompanied by reliable data fixing the definite soil properties numerically. Such terms as "hard" and "soft" or "plastic" and "semi-plastic" may mean entirely different things to different people.

Shear and consolidation tests are undoubtedly among the most important, but they require so much time that in most cases it is not possible to perform them as routine classification and exploration tests. They then must be performed on a few selected samples only, as was done in Egypt. It should be mentioned here that, tested after complete disturbance, followed by full consolidation, at a rate of application of the shearing force equal to one-twentieth of the normal load per minute, both the dark and the brown clay had approximately the same value of the angle of internal friction (20 degrees). Only the cohesion varied.

The unconfined compressive-strength test on undisturbed samples extracted from borings was chosen as a routine test because it gives a direct measure of the cohesion of the soil and therefore of its shearing strength in the undisturbed state; at the same time, it can be performed much more rapidly than a properly conducted shear test. The strain at failure was given because it showed whether the undisturbed material was plastic (20% strain being taken as failure by plastic flow), semi-plastic, or brittle. The natural water content, given together with the plastic and the liquid limit, also illustrates the state of plasticity of the undisturbed soil.

In itself, an indication of the compressive strength in the undisturbed state is insufficient to identify a soil unless supplemented by an indication of the clay content or of the liquid limit. If the compressive strength has a low value, the liquid limit serves to show whether this is due to a low clay and silt content and to the resulting low cohesion (low liquid limit and low natural water content) or to an unconsolidated state of the undisturbed clay material (high liquid limit and high natural water content).

Some modifications of this method of soil exploration may naturally become desirable when dealing with other soil conditions. For instance, for soils that are permanently above the ground-water table, the indication of the volume, in place, may become desirable in addition to the water content. For sands and sandy silts, all laboratory tests could be replaced, usefully, by indications of the number of standard blows per foot required to force down a standard sampling spoon.

The method which the writer developed for work in Egypt is particularly adapted to conditions similar to the ones encountered there—that is, for the exploration of foundation conditions on cohesive soils beneath subsoil water level. Its practical advantages for such soil conditions appear to be recognized in the United States, since a similar method was used for the classification of soils along the Chicago subway.[25]

Limitations in the Value of Field Load Test.—Mr. Dames considers that the data given by the writer is not sufficient to "close the case" and to condemn load tests. No indiscriminate condemnation of such tests was contained in the paper; but, in suggesting load tests "not necessarily at foundation depth," Mr. Dames overlooks the fact that load tests are more unreliable than ever at elevations below subsoil water level. The excavation of a sheeted test pit to reach the layers likely to provide the "seat of settlement" (for instance, the dark clay) would have been entirely impracticable for the cases reported. It may be seen from Figs. 2, 4, 6, and 10 that the natural water content of the dark clay approaches very closely, and sometimes even exceeds, the water content at the liquid limit. When a boring was stopped for the night in a layer of dark clay or silt, one would very often find, in the morning, that it had been squeezed 2 or 3 ft up inside of the 6-in. steel casing of the bore hole. This would happen despite the fact that the casing had been kept filled with water to the very top in order to more than counterbalance any possible hydrostatic uplift pressure on the soil at the bottom of the bore-hole casing. The squeezing, therefore, was caused only by the weight of the overlying soil layers and of buildings in the vicinity. Short of using pneumatic caissons, in most cases there would be no way to make a load test on the dark clay in its really undisturbed state. Because of their high cost, pneumatic caissons have been used in Egypt only for some of the largest bridges.

---

[25] "Soil Tests Check Chicago Subway Work," by Ralph B. Peck, Jun. Am. Soc. C. E., *Engineering News-Record*, December 7, 1939, Fig. 1.

Mr. Dames advocates tests on footings of different sizes. Several semi-empirical theories have been developed for the purpose of estimating the settlement of a full-sized structure from data supplied by several test footings of different sizes. However, such estimates may be successful only when all the compressible layers can be reached by excavation and tested, and when each of these layers is homogeneous, horizontally, beneath the entire area occupied by the building. The numerous records from different localities examined by the writer tend to show that a naturally deposited soil layer with such a uniform degree of compressibility may be more in the nature of an exception than a rule and that non-uniform soil deposits near the surface are at least equally frequent.

Fig. 16 illustrates an extreme case in point and gives the results of load tests made on a weak surface layer, presumably an old fill, near Building *T*. Complete stabilization after each load increment was permitted, with at least 24 hr elapsing between the increments; the settlements were measured to about 1.0 mm (= 0.04-in.) accuracy, which is more than sufficient for all practical purposes. The writer wishes to emphasize that settlements cannot be measured in the open field to the accuracy of 0.001 in., as suggested by Mr. Dames. It is possible to provide dial gages reading to such accuracy, but thermal deformations of the necessarily long cantilevered supports, as well as other factors, preclude an actual accuracy exceeding 0.01 in. when observations are made over a period of several days.

It may be seen from Fig. 16 that the results would have been very embarrassing to any one who intended to use them for any estimates of the type suggested by Mr. Dames. The footing (10.8 sq ft) settled slightly more than the footing that was four times larger (43.2 sq ft). The reverse relation of the settlements should have been observed in homogeneous soil.

There was no opportunity for the advantageous use of soil load tests in any of the buildings described in the paper. Such tests can be applied only for testing surface layers above subsoil water level, thereby providing information that is seldom even of a limited practical value. In the few cases when it does have some practical value, the writer believes it preferable not to use footings of different sizes, but to make more numerous tests over the entire area of the future building with a small footing of some standard size—say, 1 sq ft. The results could then be compared, both individually and as averages, to a set of load-settlement curves for a footing of that same standard size, typical of the settlement of very hard and of very soft soils, and of intermediate types.

Effect of Excavation.—Mr. Beugler agrees with the writer that no decrease of settlement was observed as a result of excavation and states that this was doubtless true for the case cited (Building II). He then cites another case in which settlements were reduced by excavation after construction. This is quite understandable, since such excavation definitely reduces the pressures on compressible layers without allowing any dislocation of their structure. The writer has also reported a similar case.[26]

If the excavation is done before construction, however, everything will depend upon whether the deeper lying soil is dislocated as a result of the excavation. In the case of naturally stiff and impervious clays, any appreciable swelling may sometimes be prevented by draining the foundation pit properly and by rapid construction. The observations made under similar conditions at the Telephone Building at Albany, N. Y., led, at first, to unrestricted recommendations concerning the utility of excavation as the only reliable method for reducing settlements on clays.[27] Later observations disclosed limitations of the method in the case of weak plastic clays squeezed into the foundation pit. In extreme cases (Mexico, D. F., Mexico) the recorded rise of the bottom of the

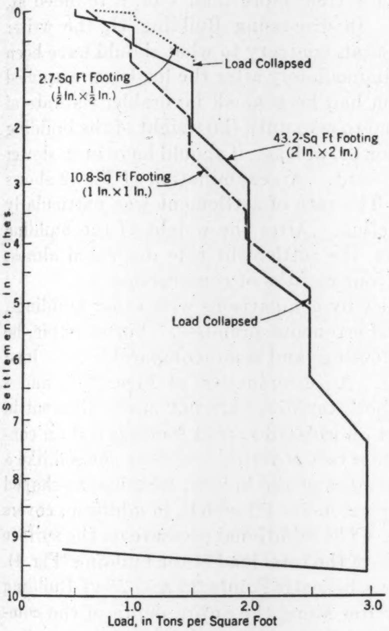


FIG. 16.—RESULTS OF LOAD TESTS ON WEAK LAYER OF IRREGULAR COMPRESSIBILITY

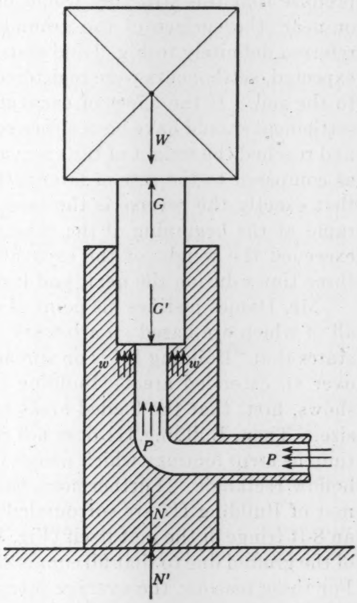


FIG. 17.—MECHANICAL EQUIVALENT OF INCORRECT ASSUMPTION CONCERNING THE SWELLING OF SOILS

pit sometimes reached 4 ft.[28] Special measures must then be adopted to counteract this rise and the accompanying weakening of the surrounding clay.[29]

---

[26] "Settlement Studies of Structures," by Gregory P. Tschebotareff, *Civil Engineering*, November, 1938, p. 751.

[27] "The Structure of Clay and Its Importance in Foundation Engineering," by A. Casagrande, Assoc. M. Am. Soc. C. E., *Journal*, Boston Soc. of Civ. Engrs., April, 1932, Fig. 28, pp. 191–193, and 203.

Observations on Building II have disclosed a further limitation when fairly stiff swelling clays and silts are permitted to absorb moisture after removal of the over-burden. No drainage could have helped here since excavation was extended to the lowest water level; furthermore, the foundation pit was left open for some time.

Mr. Dames states that "no data are introduced to disprove the logical premise that this structure would have settled more than 1 in. if founded at, or near, the surface of the ground." In discussing Building II, the writer referred definitely to Fig. 2 and stated that, contrary to what should have been expected, settlements were registered immediately after the loads were applied to the soil. If the effect of excavation had been at all favorable, the rate of settlement should have been either equal to zero until the weight of the building had reached the weight of the excavation or, at least, it should have been slower as compared to the rate of later settlement. An examination of Fig. 2 shows that exactly the reverse is the case. The rate of settlement was particularly rapid at the beginning of the construction. After the weight of the building exceeded the weight of the excavation, the settlement rate decreased almost three times during the next, and last, four months of construction.

Mr. Dames justifies his point of view by comparisons with other buildings, all of which are based on arbitrary and erroneous premises. For example, he states that "Building III is on spread footings and is not comparable as to load over an extended area" (Building II). An examination of Figs. 2, 3, and 4 shows, first, that the loaded areas of both buildings are not much different in size. Then, Building III does not rest on isolated spread footings but on continuous strip footings which merge into a raft covering a surface shaped like a hollow rectangle. Furthermore, three sides of the hollow, rectangular-shaped mat of Building III are surrounded by extensive fill which, in addition, covers an 8-ft fringe of the mat itself (Fig. 3). The additional pressure on the surface of the ground due to that fill equals 38% of the total load of the building (Fig. 4). For these reasons, the average pressures beneath Points 26 and 28 of Building III (Fig. 3(*a*)) as computed by the writer along the entire depth of the compressible layers above the sand are slightly greater than the corresponding pressures beneath Building II, even if these latter pressures are computed under the assumption that they have not been reduced by excavation. The type of soil formation is similar beneath both buildings. The thickness of the stressed compressible soil layers is slightly greater in the case of Building III (27 ft) than in the case of Building II (22 ft); nevertheless, over the same period of three years Building III has settled slightly less (20 mm) than Building II (22 mm). The comparison between these two buildings, therefore, is entirely possible and fully substantiates the statement of the writer that the excavation did not decrease the settlements of Building II.

---

[28] "Discussion of the Movements Within Foundation Pits During Construction," by J. A. Cuevas, Lazarus White, M. Am. Soc. C. E., and Karl Terzaghi, M. Am. Soc. C. E., *Proceedings*, International Conference on Soil Mechanics and Foundation Engineering, Vol. III, *Paper N-15*.

[29] "Modern Methods of Design and Construction of Foundations," by Lazarus White, *loc. cit.*, *Paper N-14*.

Buildings II and IV cannot be compared as suggested by Mr. Dames because his statement, "A similar sequence of soils both as to classification and description by the physical tests given exists under both of Sites II and IV," is incorrect. He overlooked the fact that beneath Building IV the dark clay was five times thicker than under Building II and later made an incorrect assumption concerning the compressibility of the dark clay.

Compressibility of Soils.—Mr. Dames assumed the dark clay of Building *T* to be just as compressible as the brown clay above it, whereas in all the cases reported in the paper the brown clay formed, on an average, a much more compact and stiffer material than the dark clay. This can be seen by examining Building IV in Fig. 6 where this difference was particularly pronounced. The brown clay had a natural water content of about 25% and a compressive strength varying from 1.3 to 3.2 tons per sq ft, whereas the deeper lying dark clay had a greater natural water content varying from 35% to 40% and a smaller compressive strength of about 0.7 ton per sq ft.

As stated in the paper, laterally confined, combined compressibility and permeability tests were also made on both types of clays and served to interpret the field observations. About forty such tests were actually performed. The "X" coefficients for these laboratory compressibility tests, performed in the usual standard manner, varied from 0.012 to 0.018 for different brown clays and from 0.024 to 0.045 for different dark clays.

Building I provided a key for an attempt to interpret and correlate the results of some of the field observations and laboratory tests. This was possible because the settlements of that building were caused almost entirely by the compression of a stiff brown clay layer (Fig. 1). The writer found the value of $X = 0.007$ to be approximately correct for the very stiff brown clay beneath Building I.[3] This value may be considered as not being too high for the still stiffer and more compact brown clay immediately beneath one of the wings of Building IV (Fig. 6). The compressibility of the compact brown silt beneath the rest of that building cannot exceed, to any extent, that of the brown clay. This follows from a comparison of the symmetrical computed lines of equal pressure and of the very slightly unsymmetrical lines of observed equal settlements (Fig. 5). The slightly greater settlement of the right wing of Building IV is directly proportional to the slightly greater depth of the compressible layers there.

Taking a value of $X = 0.007$ for the upper 18 ft of brown clay and silt, with an average pressure of 0.6 ton per sq ft along their depth, one obtains a value for their compression equal to: $0.007 \times 0.6 \times 18 \times 12 = 0.90$ in. Thus far, the total settlement has been reported as slightly exceeding 3 in. Thus, at least 2.1 in. of settlement must have been caused by the compression of the dark clay and silt. The dark silt is much less compressible than the dark clay, as demonstrated by tests, and a coefficient of $X = 0.007$ similar to that of the upper brown silt represents a liberal value for it. This gives a total compression of the 6-ft layer of dark silt equal to: $0.007 \times 0.5 \times 6 \times 12 = 0.25$ in. and leaves $2.10 - 0.25 = 1.85$ in. compression for the 10-ft dark clay layer. This figure, in turn, leads to the following average value of the compressibility coefficient for that dark clay: $X = \frac{1.85}{0.5 \times 10 \times 12} = 0.031$, which agrees well with laboratory test values for dark clays from different localities (ranging from 0.024 to 0.045). The clay is of the non-swelling type so that disturbances due to this cause cannot be considerable. Therefore, the value of $X = 0.030$ was reported as representing an average lower limit approximately determined from observations; the writer considers it to be reasonable. The approximations involved should not produce a cumulative error of much more than 50%; greater accuracies can never be reached for this type of work except accidentally.

The upper limit of $X = 0.070$ (a minimum average value) was reported for the dark clay on the basis of the field observations of Building IX only. The extremely high soil compressibility in this one case was very puzzling to the writer at first, until the examination of an old map made by the surveyors of Napoleon revealed that in 1798 the main channel of the Nile River flowed through what is now the site of Building IX. This gave a reasonable explanation for the abnormal compressibility since the river did not change its course until about 1850 when the present site was silted rapidly. The resulting soil formation, although covered by a relatively thin, dry, and hard crust, may be likened, therefore, to a hydraulic fill rapidly deposited over a horizontal surface of large area. Instances of the high compressibility of such fills have come to the notice of the writer.

The foregoing explanation appeared all the more reasonable since a further study of maps, based on medieval data,[30] showed that the sites of all other buildings were much more ancient and that the main channels of the Nile River near Cairo, Egypt, had not undergone any other appreciable changes since the Fourteenth Century.

Professor Hanna points out that in addition to swelling there must be also other sources of error, such as plastic deformations during sampling and friction along the sides of the testing ring. The writer has always fully endorsed similar views. Professor Hanna mentions some new studies he is undertaking and reports that "Preliminary results of these studies indicate that the author's statement that the method described in the paper has proved a success cannot be considered final." No such statement is contained in the paper; nor is it implied directly or indirectly. It would be entirely contrary to the views of the writer to suggest that any one method could take care of the numerous possible variables affecting the engineering properties in general, and the compressibility in particular, of different types of soils. The writer merely reported a method which disclosed the relatively great importance of the swelling of some compact clays as a source of error; this method also gave a procedure to counteract this swelling which checked with the field observations. Far from recommending the unrestricted use of this procedure, the writer presented a special discussion to the International Conference on Soil Mechanics and Foundation Engineering in 1936[31] for the purpose of stressing, on the basis of the observations of Building IV, the limitations when applied to non-swelling clays of the method described earlier.[3] In the present paper the writer has again stressed the limitations of this and other methods of settlement forecast and has emphasized the fact that they depended on the type of soil and on the method of sample extraction.

---

[30] "Cairo—Origin and Development," by C. R. Haswell, *Bulletin* de la Société Royale de Géographie d'Egypte, Tome XI, Decembre, 1922, Plate II.

[31] *Proceedings*, International Conference on Soil Mechanics and Foundation Engineering, Vol. III, *Discussion No. D-19.*

Pressure Distribution Computations.—Professor Krynine has stressed an important point by emphasizing the fact that the Boussinesq formula cannot be applied, very well, for estimating vertical pressures on deep lying compressible layers if these pressures are transmitted to the layers through an overlying, more rigid layer on which the foundation rests. In the writer's opinion, this "arching effect" of the upper layers, which relieves the pressures on the lower, weaker layers by distributing them over a greater area, should decrease in importance (1) with a decrease in the difference of the shearing rigidity of the two layers, (2) with a decrease in the thickness of the upper layer, and (3) with an increase of the area of the building. Therefore, in the case of the Egyptian buildings reported, this factor cannot have been very important. Any considerable excess in the assumed values of the actual pressures would mean that the assumed "X" values for the stiff brown clay ($X = 0.007$) would have to be increased and those of the dark clay ($X = 0.030$) decreased.

With reference to Fig. 15, the writer agrees that in some cases the average pressure in the ground may be neither at one-half nor at two-thirds of the depth of the compressible layer, but may have an intermediate value. However, for the ratio between the depth of the compressible layers and the foundation area of the cases reported, the foregoing assumptions do not involve an error exceeding 10%,[32] which is negligible as compared with the possible degree of accuracy of the settlement analyses as a whole.

Settlement of Pile Foundations.—Arbitrarily, Mr. Dames assumed that the compressibility of the dark clay is equal to that of the stiff brown clay ($X = 0.007$) and expressed the "belief that the settlement of Building *T* has been largely the result of individual pile failure and not due to the compression of the 10 ft of dark clay between the pile points and the sand."

Under the heading "Compressibility of Soils" herein the writer has explained that the foregoing assumption is entirely unjustified and that the estimated average compressibility of the dark clay under Building IV was $X = 0.030$. Earlier laboratory tests performed by the late Professor Kögler at Freiberg on soils from Building *T* had given approximate values of $X = 0.017$ for the brown clay and $X = 0.026$ for the dark clay.

Computations similar to the one performed for Building IV give the values of 5.3 in., 1.0 in., and 0.5 in. for the respective compressions of the dark clay, the brown clay, and the silty sand. Since the total observed settlement is 10.0 in., this leaves only 3.2 in. to be attributed either to too small values of X (taken at 0.026, 0.007, and 0.007 respectively) or to additional effects of plastic shearing deformations reaching to some distance around the piles. Such shearing deformations are likely to be relatively important only along the outside perimeter of the building. The term "group penetration" used by Mr.

---

[32] *Proceedings*, International Conference on Soil Mechanics and Foundation Engineering, *Paper No. E-1*, Fig. 18, pp. 57 and 58.

Dames is misleading since it implies the formation of a definite, vertical, shearing surface of failure in the soil around the pile groups and their "slipping" past the surrounding ground, together with the footings which cap them. No evidence of any such "slipping" could be obtained in this building. The very argument, correctly used by Mr. Dames to explain larger "group penetration" as compared to smaller single pile penetration, testifies against this conclusion in this case. The pile groups are so close to each other in the eastern part of the building that the shearing resistance between the stiff brown clay prisms confined by these groups and the piles themselves should be of the same order of dimension as the resistance to compression of the weaker dark clay supporting these prisms. Thus the piles supporting the building, on the whole, must have settled with the surrounding soil, except possibly along the outside perimeter of the building; and most of the surface settlement must have been caused by the compression of the lower lying dark clay layers.

To study this question further, in 1931 the writer introduced into the soil beneath Building *T* nine underground bench-marks almost identical to a type designed by Professor Terzaghi.[33, 34] They were placed in three groups at three different elevations. The results were satisfactory in a qualitative sense, showing that a large part of the settlement was due to the compression of the layers beneath the pile bearings, perhaps even including, to a slight extent, the deeper sand layers which have been assumed incompressible in this paper. However, the results from the separate bench-marks were unsatisfactory in a quantitative sense, since they were not in a logical numerical agreement with each other. It is impossible to say whether these numerical discrepancies were due to the fact that some of these bench-marks may have sunk slightly deeper into the ground under their own weight, thus registering too great settlements of deeper lying layers, or because other bench-marks may have been slightly pushed up as a result of swelling of the clay immediately adjoining them.

Under different conditions such bench-marks may give more consistent results. It would appear advisable to place them mainly in compact layers or relatively rigid soils, such as compact sands, and to place such underground bench-marks in several symmetrically located groups as could provide a check on each other through comparison with the performance of the foundation at the surface.

Effect of Ground-Water Variations.—Professor Hanna attempts to show that the buoyant effect of rising subsoil water is so small that it cannot account for the observed corresponding slowing down of settlements as suggested by the writer. The premises on which Professor Hanna bases this attempt are incorrect. Thus he states that when the period of construction coincides with the flood period, the corresponding increase of pressures due to the continued construction of the buildings ranges from 0.4 to 0.8 kg per sq cm. Only Buildings IV and V show any definite change in the rate of settlement during flood. An examination of Figs. 6 and 7 illustrating these buildings confirms the writer's previous statements and shows that in the three months during which the water rises the increase in their surface unit pressure was only 0.2 kg per sq cm or a

---

[33] "Die Tragfaehigkeit von Pfahlgruendungen," by Karl Terzaghi, *Die Bautechnik*, 1930, No. 34.

[34] "Settlement of Structures in Europe and Methods of Observations," by Karl Terzaghi, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), Fig. 52(*a*), p. 1446.

value two to four times smaller than the one given by Professor Hanna. The corresponding increase in the pressures on the dark clay is only about 0.160 kg per sq cm. Thus, the value of 0.114 kg per sq cm, representing the maximum reduction of pressures on the dark clay due to buoyancy as computed by Professor Hanna, cumulatively counteracts 70% of the corresponding increase in the pressure of the building up to the moment when the peak of the subsoil-water rise is reached. This is more than sufficient to explain the observed discontinuities in the time-settlement curves of the two buildings, as has been done by the writer. Furthermore, the foregoing value of 0.114 kg per sq cm, which expresses the effect of buoyancy, is far from being excessive, as suggested by Professor Hanna, especially for Building IV. The subsoil-water rise at Building IV is much larger than the 2 m assumed by him (Fig. 5); and the coefficient 0.057 which he used (and which corresponds to 1 m water rise, of 35-lb-per-cu-ft buoyancy) is not too small since it does not take into account the capillary and entrapped moisture contained in the clay before the water rose. This should have been done for Building IV (see Fig. 5).

Professor Hanna further states that during laboratory compressibility tests which he performed "The pressures required to return a 12-mm sample to normal, when swelling occurs under the weight of the piston alone, vary between 0.18 and 0.30 kg per sq cm." The writer observed similar values during his own tests; but then Professor Hanna proceeds to attribute to these swelling pressures the observed changes in the rate of settlement of buildings and states: "Therefore, it is safe to assume that * * * the swelling of these layers reduces the pressure on the layer above them and, probably, to a slight degree, on the lower layers." The foregoing, he states, "is a mere assumption," and, the writer adds, an unwarrantable one. The pressure on the layer either immediately above or below the swelling layer will always remain equal to the superimposed weights and loads. The assumption made by Professor Hanna is equivalent to saying that an attempt to raise a load by a jack appreciably reduces both the weight of the load itself and the pressure of the jack on its support! That is (Fig. 17), if $p > 0$, then $G < W$ and $N' < N$. Obviously, this is impossible. If $p > 0$, then both $G = W$ and $N' = N$. Only "w" would decrease correspondingly—that is, the stresses in the swelling layer. This would reduce the component of the settlements due to the compression of the swelling layer itself; but, as discussed under the heading "Compressibility of Soils" herein, this settlement component is relatively small.

Conclusive evidence that the changes in the rate of settlement could not be primarily affected by swelling is provided by Building IV. Only part of that building rests on a strongly swelling brown clay, the remainder being supported by a silt which scarcely swells at all. Nevertheless, both the part supported by the brown silt (Point 16) and the part supported by the brown clay (Point 1) have shown exactly the same changes in the rate of settlement during changes in the subsoil-water level (Figs. 5 and 6). They should have shown a marked difference if Professor Hanna's assumption concerning the primary importance of swelling were correct for the cases reported in the paper.

On the other hand, the writer agrees fully with the explanation of the fact that the cracking of ancient houses in Cairo is particularly pronounced during the yearly flood. Not only do most cases of collapses of such houses occur during the seasonal flood, but during the same period many breaks occur in the water mains of Cairo. This must be due to the light weight of such old houses and water mains and to their incapacity to resist stresses induced by unequal swelling of the soil on which they rest.

Remolding of Clays.—Professor Hanna mentions some interesting and promising recent studies he made near the sea-coast on both brown and dark clays deposited in salt water which were very unfavorably affected by remolding, whereas similar fresh-water clays were not. It is difficult to decide at present whether the obvious conclusion from these studies could be generalized, since many glacial clays, presumably deposited in fresh water, are nevertheless found to be very unfavorably affected by remolding.

Conclusions.—The studies reported in the paper have provided much data of practical importance for local foundation designs. They have further served to prove the qualitative correctness of the general theories developed by the modern science of soil mechanics, which provide a surer basis for the exercise of sound engineering judgment. At the same time they have shown the quantitative limitations attached to numerical forecasts of the behavior of undisturbed soils.

Although much of the data given in the closing discussion has served to show the errors in some of the assumptions made by the discussers, the writer is glad that these assumptions were advanced, because he believes that the exchange of views has served to emphasize the complexity of the problems connected with settlement analyses and the necessity of having numerous co-ordinated observations on different structures before conclusions may be reached even for one locality.

The writer's work in this field constantly strengthens his conviction that further progress in the knowledge concerning the actual behavior, under stress, of undisturbed soils, is not possible on the basis of fragmentary data from isolated observations. The time involved in making systematic observations, as well as other reasons, precludes the possibility of their satisfactory execution by any single person or group of individuals unless this work is assigned to them as a regular, full-time job. This consideration, as well as the large investments depending on the satisfactory and economic design of foundations, coupled with the large degree of guesswork on which present codes and design methods are based, should justify the creation of special offices in most large cities for the purpose of making control observations, of the type described, on new structures.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2078

POLLUTION OF BOSTON HARBOR

By Arthur D. Weston[1] and Gail P. Edwards,[2]
Members, Am. Soc. C. E.

With Discussion by Messrs. E. Sherman Chase, F. E. DeMartini, A. M. Rawn, Harrison P. Eddy, Jr., and Samuel A. Greeley, George A. Soper, and Arthur D. Weston and Gail P. Edwards.

Synopsis

The sewerage systems serving Metropolitan Boston are among the oldest in the United States, and the disposal works used in connection with these systems represent methods which require modifications to meet public demand for improved conditions. The discharge of sewage into Boston Harbor from these three systems has never resulted in depletion of oxygen in the tidal waters sufficient to cause a nuisance from odors but there have been increasing esthetic demands from the population residing in the vicinity of Boston Harbor for improved conditions because of sleek areas and pollution of beaches by floating matter. These increasing demands for improvements have been made regardless of the fact that the quantity of sewage discharged has not greatly increased in recent years.

The paper is framed chiefly around the results of a legislative investigation made in the years 1935 and 1936 under the general direction of the authors. It includes descriptions of the existing works and conditions of sewage disposal in harbor waters, presents results of chemical analyses and bacterial examinations sufficient in detail to permit comparison with data relative to the disposal of sewage in other tidal waters, discusses experimental work undertaken, and presents conclusions as to existing conditions and the best methods of meeting the demands for improvement.

Introduction

Under the provisions of Chapter 42 of the Resolves of the Massachusetts Legislature of 1935, a special commission[3] was appointed to investigate the

---

Note.—Published in March, 1939, *Proceedings*.

[1] Chf. Engr., Dept. of Public Health, State of Massachusetts, Boston, Mass.

[2] Prin. San. Chemist, Wards Island Sewage Treatment Plant, Dept. of Public Works, City of New York, Wards Island, N. Y.

[3] See "Report of the Special Commission on the Investigation of the Discharge of Sewage into Boston Harbor and Its Tributaries," House Doc. No. 1600, December, 1936.

discharge of sewage into Boston Harbor and to consider what change, if any, might be necessary in any of the present systems of sewerage or sewage disposal in the territory tributary to the harbor in order to prevent nuisances or to remove objectionable conditions. The investigation was conducted during 1935 and 1936.

Boston Harbor is usually defined as that body of water inside or west of a line drawn from Deer Island to Pemberton in the town of Hull and extending to the lowest bridge on each of the tributary rivers and estuaries. It consists of four bays, Dorchester, Winthrop, Quincy, and Hingham, and an inner and outer harbor. The Mystic, Charles, and Chelsea rivers and Fort Point and Reserve channels are estuaries of the inner harbor. The total area of Boston Harbor is about 27 700 acres or 43.3 sq miles at high water. The volumes of the tidal prism of the average tide which enters and leaves Boston Harbor is 76 086 million gal. The quantity of water remaining at mean low tide is 108 305 million gal, and the total quantity of water at mean high tide is 184 391 million gal.

Drainage Area of Boston Harbor.—The tributary water-shed to Boston Harbor is about 645 sq miles. Included in this area, in whole or in part, are 53 cities and towns having a total population, according to the State Census of 1935, of 1 957 311.

Metropolitan Boston is served by three main sewerage systems (see Fig. 1), the Boston Main Drainage System, the North Metropolitan System and the South Metropolitan System. Included in this area are 32 towns and cities having a total population of 1 872 400 (or 95% of the population of the drainage area of Boston Harbor). The population contributing sewage in 1936 was estimated at 1 800 000.

The Boston Main Drainage System.—The Boston Main Drainage System was constructed during the period from 1877 to 1884 to intercept the sewage discharged at that time along the water-front from some 70 sewer outlets. It was designed to provide ultimately for sewage from 20 sq miles. The system was extended in 1892 and 1897 to intercept sewage from Waltham, Newton, Watertown, Brookline, the Brighton and Hyde Park section of Boston, Dedham, and Milton. Upon completion of the high-level sewer of the South Metropolitan Sewerage District in 1904 the system served only an area of 18.7 sq miles, consisting of Boston proper, South Boston, parts of Back Bay, the Roxbury, Dorchester, and West Roxbury sections of Boston, and an area of Squantum and Milton.

The system was designed to care for sewage at an average rate of 75, and a maximum rate of 112.5, gal per capita per day from a population of 800 000. In addition, it was designed for about 0.25 in. of rainfall upon the 20 sq miles in 24 hr, which was assumed to be at the rate of 0.01 in. per hr. All computations were based on the sewers running about half full and for the mingled storm water and sewage, in excess of the capacity of the sewer, to discharge through the old sewers as overflows. The resident population served by the district probably does not exceed 493 000 persons, but the per capita daily rate of sewage flow during the six months from June to November, in the period 1932 to 1935, inclusive, was 154 gal per capita instead of 75 gal per capita as originally contemplated. Estimates of flow on the basis of permanent population may be misleading in a municipality such as Boston, which has a large transient population in manufacturing and mercantile establishments, offices, and schools. In 1927 the number of people entering and leaving the business sections each day was estimated to be 825 000.

The trunk sewer of this system has a maximum diameter of 10.5 ft and terminates at a pumping station at Calf Pasture or Old Harbor Point where, after passing through cage screens of iron bars set with an opening of 1 in., the sewage is lifted about 35 ft and is discharged through force mains into two deposit sewers, each 8 ft wide, 16 ft deep, and having a total length of 1 260 ft. The velocity in these deposit sewers is about 1 ft per sec, and at their lower end the sewage flows over a weir into a tunnel to Moon Island off Squantum. The tunnel (see Fig. 1) is 7 160 ft long and 7.5 ft in diameter and is located about 143 ft below mean low water. At Moon Island the sewage is stored in four large reservoirs having a total effective capacity of approximately 35 400 000 gal. As far as practicable, sewage is discharged from the storage tanks on the second and third hours of the outgoing tide, and it enters the harbor at mean low water. The outfall sewer provided for this purpose is divided into two parts and is 10 ft 10 in. high and 12 ft wide on the inside. It extends 600 ft from the tanks to the tide water where there is a depth at mean low water of approximately 5 ft.

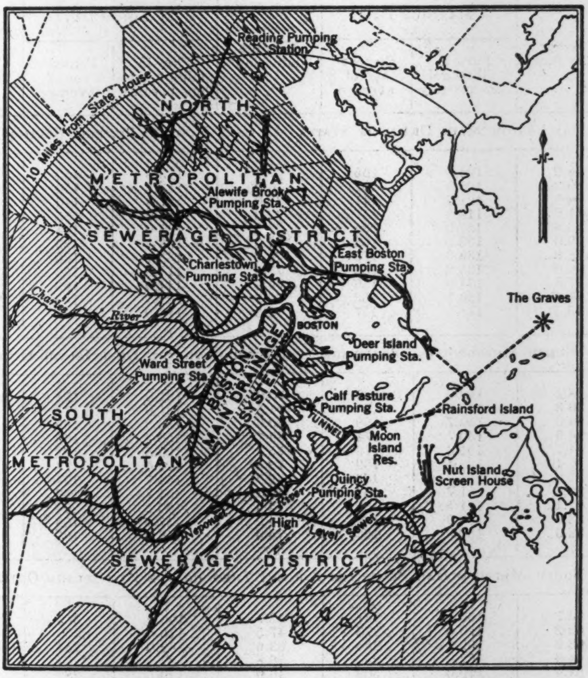

Reading Pumping Station
10 Miles from State House
NORTH
METROPOLITAN
Alewife Brook Pumping Sta.
SEWERAGE DISTRICT
East Boston Pumping Sta.
Charlestown Pumping Sta.
The Graves
Charles River
BOSTON
Deer Island Pumping Sta.
Ward Street Pumping Sta.
BOSTON MAIN DRAINAGE SYSTEM
Calf Pasture Pumping Sta.
Rainsford Island
TUNNEL
Moon Island Res.
SOUTH
METROPOLITAN
Nut Island Screen House
Quincy Pumping Sta.
Neponset River
High Level Sewer
SEWERAGE DISTRICT

FIG. 1.—SEWERAGE SYSTEMS IN METROPOLITAN BOSTON

Table 1 shows the quantity of sewage pumped at the Calf Pasture Pumping Station and represents the quantity discharged into Boston Harbor from the Boston Main Drainage System. It is interesting to note that, whereas the maximum daily pumpage throughout each year has varied from 141% to more than 200% of the average daily value, the average daily amount pumped during June to November, inclusive, has varied little from that pumped on the average throughout the entire year.

TABLE 1.—ESTIMATED QUANTITY OF SEWAGE PUMPED IN MILLION GALLONS PER DAY

Year	For Entire Year			June to November, Inclusive			Ratio; dry weather average to daily average (percentages)
	Average daily flow	Maximum Day		Average daily flow	Maximum Day		
		Flow in million gallons per day	Percentage of average		Flow in million gallons per day	Percentage of average	
(a) Boston Main Drainage System; at Calf Pasture Pumping Station							
1925	76.2	125.8	165	77.7	115.8	150	102
1926	80.3	126.0	157	76.0	113.5	150	95
1927	86.6	121.5	141	87.0	121.3	140	101
1928	71.7	113.0	157	72.6	111.9	155	101
1929	76.8	157.0	205	73.2	120.0	164	95
1930	79.0	133.5	169	84.0	133.1	159	106
1931	78.3	133.5	170	74.0	133.1	180	94
1932*	76.0	125.8	166	77.3	118.0	153	102
1933*	77.8	144.0	186	74.8	144.0	193	96
1934*	79.4	129.1	163	79.8	129.1	162	100
1935*	75.3	118.0	157	72.3	118.0	164	96
(b) North Metropolitan Sewerage System through the Deer Island Outlet							
1925	78.1	148.0	189	70.9	121.0	171	91
1926	79.3	143.8	181	74.1	131.0	177	94
1927	84.0	150.7	180	80.9	136.3	169	96
1928	86.9	150.7	174	89.1	150.7	169	103
1929	84.7	151.6	179	75.7	129.5	171	89
1930	77.1	130.4	169	72.8	129.0	177	94
1931	84.2	154.7	184	79.7	154.7	194	95
1932	82.4	151.7	184	80.1	151.7	189	97
1933	82.1	151.5	185	72.4	150.2	208	88
1934	83.6	150.9	180	74.2	117.4	158	89
1935	82.9	150.0	181	74.4	146.6	197	90
(c) South Metropolitan Sewerage System; Through the Nut Island Outfall							
		‡	‡		‡	‡	
1925	63.7	166.0	261	52.7	166.0	315	83
1926	62.2	170.0	273	47.5	113.5	239	76
1927	68.2	162.0	238	63.6	162.0	254	93
1928	64.9	162.0	250	58.8	137.5	234	91
1929	64.4	199.0	309	46.0	129.5	282	71
1930	61.0	166.0	272	51.7	144.5	280	85
1931	76.3	220.5	289	66.5	220.5	332	87
1932	68.4	205.0	300	48.7†	162.01†	333	71
1933	84.1	227.0	270	70.8	227.0	820	84
1934	74.2	215.0	290	62.3	150.0	241	84
1935	86.5	230.0	266	70.5	145.0	206	82

* Pump slippage and other losses estimated at 20% because of new equipment instead of 30% of previous years.

† June to September, inclusive.

‡ Represent maximum rate during any period; gravity flow.

OVERFLOW OF SEWAGE FROM BOSTON MAIN DRAINAGE SYSTEM

The Boston Main Drainage System was designed to take care of some surface run-off in addition to the sanitary sewage; but for a number of years it has been considerably surcharged during storms because of the much larger amount of surface drainage that actually enters the system. At present (1939), during certain storms large quantities of mingled storm water and sewage enter the Charles River Basin and cause objectionable conditions. The Special Commission recommended[3] that legislation be provided to compel separation of storm water from domestic sewage in the city of Boston and that funds be appropriated to permit the construction of adequate works for removing storm water from the sewers in this district. Because of the impracticability of separating storm water from domestic sewage in the business sections of Boston, the ultimate solution of the problem probably would be to provide relief sewers of large diameter to divert the overflow of mingled storm water and sewage to parts of Boston Harbor where the least objectionable conditions may result so that the waters of the Charles River Basin and at certain beaches may not be polluted.

QUALITY OF SEWAGE FROM BOSTON MAIN DRAINAGE DISTRICT

Chloride determinations indicate that about 23% of the volume of sewage pumped in recent years has consisted of salt water which entered the system through leaks, or through tide gates, or by other means.

Table 2(*a*) gives the results of the analyses of the samples collected during the 1935–1936 investigation[3] and during the investigations of 1913 and 1929.

TABLE 2.—SEWAGE ANALYSES

Year	Number of samples	Average flow in million gallons daily	FREE AMMONIA		TOTAL ALBUMINOID AMMONIA		OXYGEN CONSUMED		Chlorides in parts per million	Five day 20° Centigrade B.O.D. in parts per million	KJELDAHL NITROGEN		Total suspended solids in parts per million
			In parts per million	In pounds per day	In parts per million	In pounds per day	In parts per million	In pounds per day			In parts per million	In pounds per day	
(*a*) BOSTON MAIN DRAINAGE SYSTEM													
1913	24	67.1	27.9	15 500	10.0	5 580	77.4	43 200	2 226	..	20.8	11 000	..
1929	25	76.5	22.7	14 450	4.2	2 680	34.5	22 000	4 142	..	..	..	..
1935	230	75.4	21.5	13 400	4.1	2 560	53.0	33 100	4 080	157	9.05	5 650	162
1936	72	86.0	18.1	12 990	3.4	2 440	49.6	35 600	3 478	144	8.6	6 170	119
(*b*) NORTH METROPOLITAN SEWERAGE SYSTEM													
1913	41	56.5	25.0	11 800	13.4	6 300	106.0	50 000	2 290	..	25.5	12 000	..
1929	49	84.7	25.7	18 050	6.36	4 470	86.2	60 600	3 856	..	..	..	..
1935	81	82.9	34.1	23 500	7.52	5 180	81.0	55 800	3 280	249	16.4	11 300	256
1936	105	86.4	26.2	18 890	..	..	69.0	49 750	3 151	175	11.4	8 220	152
(*c*) SOUTH METROPOLITAN SEWERAGE SYSTEM													
1912	6	53.0	16.1	7 080	4.6	2 030	35.5	15 700	204	..	9.6	4 200	..
1929	24	64.4	21.2	11 350	3.45	1 845	30.9	16 550	134	..	..	..	..
1936	64	96.5	20.3	16 350	3.71	3 096	50.0	40 240	198	119	9.6	7 730	156

The material removed from the bar cage screens at the Calf Pasture Pumping Station is deposited on an adjacent dump to be drained and then burned.

ADEQUACY OF THE BOSTON MAIN DRAINAGE SYSTEM

At times of heavy run-off the storm water entering the main trunk sewers of the Boston Main Drainage System exceeds the capacity of the tunnel from Calf Pasture to Moon Island and the capacity at which it is practicable to operate the pumps at the Calf Pasture Pumping Station. At such times, large quantities of mingled storm water and sewage must discharge through the overflows provided for the purpose. If the full capacity of this main drainage system were utilized by increasing the pumping, the system could be made to deliver about 244 million gal in 24 hr; but this quantity is about 56% in excess of the designed capacity of the tunnel from Calf Pasture to Moon Island. Moreover, if it were possible to pass this quantity of sewage through the tunnel to Moon Island it would be necessary to increase the tank storage considerably at Moon Island if the sewage is to be discharged only on the second and third hours of the outgoing tide. Under present conditions these tanks are not allowed to fill above Grade 20, to prevent crowding of the tunnel and overflow of sewage from the deposit sewers at Calf Pasture.

It was the original intention that the stored sewage would be discharged only on the second and third hours of the outgoing tide, but from June to November, inclusive, during the eight years 1928 to 1935, the maximum average daily quantity of sewage pumped was about 84 million gal. Accordingly, it has been necessary, at times, to discharge the sewage on other than the second and third hours of the outgoing tide. The 1936 investigation showed that during the period from 1928 to 1936, inclusive, there have been 1 034 times when sewage was discharged either before or after the period in which the sewage was intended to be discharged, about 97 of which were made necessary by experimental work during the investigation.

POLLUTION OF BOSTON HARBOR BY THE NORTH METROPOLITAN SEWERAGE DISTRICT

The Metropolitan Sewerage District was created by acts of the Massachusetts Legislature in 1889 to provide sewerage facilities in the drainage areas of the Charles and Mystic rivers as recommended by the State Board of Health in the same year. The proposed Metropolitan District was to be divided into two parts, one part for the district north of the Charles River to be known as the North Metropolitan Sewerage District, and the other to provide for the diversion of sewage from the Charles River Valley into the Boston Main Drainage System. The North Metropolitan Sewerage District was to include the Charlestown and East Boston sections of Boston and the cities of Cambridge, Somerville, Malden, Chelsea, Medford, Melrose, Everett and Woburn, and the towns of Stoneham, Winchester, Arlington, Belmont, and Winthrop. Additional legislation has been provided from time to time authorizing extensions of the North Metropolitan Sewerage District, and in 1935 the size of the system and district[4] was substantially as shown in Table 3.

---

[4] "Report of the Special Commission on the Investigation of the Discharge of Sewage into Boston Harbor and Its Tributaries," House Doc. No. 1600, December, 1936, p. 95.

TABLE 3.—AREAS AND POPULATIONS SERVED BY VARIOUS SEWERAGE DISTRICTS

Cities and towns (1)	Miles of local sewers connected (2)	Number of connections with local sewers (3)	Estimated number of persons served by each house connection (4)	Estimated population contributing sewage in 1936 (5)	Estimated present total population (6)	Estimated area contributing sewage in 1936 in square miles (7)	Area ultimately to contribute sewage in 1936 in square miles (8)	Ratio of contributing population to present total population (percentages) (9)	Ratio of contributing area to ultimate area (percentages) (10)
(*a*) NORTH METROPOLITAN SEWERAGE SYSTEM (Populations Estimated as of December 31, 1935)									
Arlington	64.56†	6 329	5.60	35 440	39 060	3.02	5.20	90.7	58.1
Belmont	51.26†	3 628	7.00	25 400§	26 280	2.40	4.66	96.7	51.5
Boston:									
Deer Island	0.70†	....	....	930‖	930	....	....	....	....
East Boston	35.96‡	5 549	11.20	62 150	64 870	1.27	2.18	95.8	58.3
Charlestown	22.04‡	5 614	5.18	29 080	29 200	0.67	1.27	99.6	52.8
Cambridge	165.17‡	19 213	6.17	118 540	119 000	5.18	6.11	99.6	84.8
Chelsea	33.04‡	4 865	8.50	41 350	42 010	1.23	2.24	98.4	54.9
Everett	53.46‡	6 698	6.90	46 220	46 980	2.15	3.34	98.4	64.4
Lexington	17.44†	801	4.30	3 440	11 090	0.97	16.20	31.0	6.0
Malden	81.41†	9 581	5.90	56 530	57 120	3.59	5.07	99.0	70.8
Medford	97.16†	10 975	5.60	61 460	61 800	4.47	8.35	99.4	53.5
Melrose	51.89†	5 175	4.70	24 320	24 490	2.29	3.73	99.3	61.4
Reading	11.15†	539	4.20	2 260	10 900	0.54	9.82	20.7	5.5
Revere	54.50†	5 403	6.30	34 040	35 250	2.54	5.86	96.6	43.3
Somerville	106.78‡	17 976	5.55	99 770	100 110	3.68	3.96	99.7	92.9
Stoneham	20.74†	1 616	4.70	7 600	11 000	1.04	5.50	69.1	18.9
Wakefield	27.06†	1 784	4.50	8 030	16 530	1.15	7.65	48.6	15.0
Winchester	43.12†	3 025	4.40	13 310	13 510	2.02	5.95	98.5	33.9
Winthrop	33.81†	3 896	4.35	16 950	17 030	1.43	1.61	99.5	88.8
Woburn	28.27†	1 755	5.50	9 650	19 750	1.35	12.71	48.9	10.6
Totals	999.52	114 422	6.10	696 470	746 910	40.99	111.41	93.2	36.8
(*b*) SOUTH METROPOLITAN SEWERAGE SYSTEM									
Boston:									
West Roxbury	98.41‡	7 904	8.30	65 600¶**	86 230¶	3.86	8.92	76.1	43.3
Roxbury*	....	....	....	....	60 500¶	....	1.23	....	....
Back Bay	27.84‡	2 251	22.20	49 970	50 190	1.17	1.61	99.6	72.7
Brighton	74.85‡	6 024	11.65	70 180	70 360	3.41	3.74	99.7	91.2
Dorchester	74.00‡	8 414	12.50	105 175¶	150 140¶	2.97	4.89	70.1	60.7
Hyde Park	44.85†	3 475	9.60	33 360	33 620	1.99	4.57	99.2	43.5
Braintree	16.23†	167	4.30	720	17 410	0.76	13.44	4.1	5.6
Brookline	94.71‡	7 423	6.80	50 480	50 910	4.28	6.81	99.2	62.8
Canton	3.11†	275	4.90	1 350	6 650	0.15	17.84	20.3	0.8
Dedham	23.56†	1 494	4.65	6 950	15 420	1.13	9.66	45.1	11.7
Milton	35.69‡	2 817	4.75	13 380¶	18 510¶	1.55	12.59	72.3	12.3
Needham	18.55†	696	4.80	3 340	12 040	0.87	12.50	27.7	7.0
Newton	188.17†	13 246	4.95	65 570	66 320	9.41	16.88	98.9	55.7
Norwood	32.10†	2 141	6.10	13 060	15 680	1.70	10.16	83.3	16.7
Quincy	151.87†	13 176	5.85	77 080	77 950	5.82	12.56	98.9	46.3
Stoughton	.... †	....	....	....	8 540	....	16.23	....	....
Walpole	5.79†	53	4.70	1 515	7 490	0.18	20.54	20.2	0.9
Waltham	64.89†	5 380	7.90	42 500††	42 920††	3.62	13.63	99.0	26.6
Watertown	63.56†	6 096	5.80	35 360	36 020	2.95	4.04	98.2	73.0
Wellesley	38.83†	1 737	5.90	8 100	13 790	2.17	9.89	58.7	21.9
Weymouth	.... †	....	....	....	21 930	....	16.46	....	....
Totals	1 057.01	82 769	7.8	643 690	862 620	47.99	218.19	74.6	22.0

* At present connected with Boston Main Drainage System.

† Separate. ‡ Separate and combined. § Including two connections with McLean Hospital, having an estimated population of 803. ‖ Estimated by Superintendent of the Institution on Deer Island.

¶ Parts of Dorchester, West Roxbury, Roxbury and Milton, which are situated within the South Metropolitan Sewerage District limits, are tributary to the Boston Main Drainage System.

** Including connection with the Boston State Hospital, having an estimated population of 2 873.

†† Including connections with the Metropolitan State Hospital and the Middlesex County Tuberculosis Hospital, authorized by Chapter 372 of the Acts of 1928 and Chapter 373 of the Acts of 1929, having an estimated population of 2 078.

The North Metropolitan Sewerage System was designed to exclude storm water except in the cities of Cambridge and Somerville; and, to provide for an ultimate population of 571 000 in 1930, and for a flow of 210 cu ft per sec, or about 136 million gal per day. It was assumed that the volume of sewage to be cared for would be 30 cu ft per person in 24 hr except in the cities of Cambridge and Somerville where the sewers are on the combined plan and where an allowance of 35 cu ft per person per 24 hr was made.

There are approximately 74 miles of Metropolitan sewers in the North Metropolitan Sewerage District which vary in diameter from 10 to 109 in. The main sewer terminates at a pumping station at Deer Island where the sewage is screened and, by means of low-lift pumps, raised to an elevation that will permit it to discharge into the harbor by gravity through a system of multiple outlets. The discharge of sewage through the original outlet at this site caused complaints from the keeper of the Deer Island Light, and in 1917 the multiple outlets were provided. The present outfall, at its lower end, consists of 9-ft lengths of cast-iron pipe varying in diameter from 84 in. to 48 in. This pipe is open at the end and the thirteen lengths of pipe near the end have openings on the top. It is through these thirteen openings and the one at the end that sewage is discharged. The thirteen openings are practically elliptical in cross-section and vary in size from 44 in. by 25 in. to 23 in. by 13 in. The depth of water in the vicinity of these outlets is from 30 to 50 ft.

All the sewage from the district except Winthrop is pumped at a station at East Boston before reaching the Deer Island Station. The district also provides stations to pump the sewage from a section of Charlestown and from the Alewife Brook sewer which receives sewage from sections of Cambridge, Somerville, and Belmont; and a station is provided by the town of Winthrop to pump the sewage from that town into the North Metropolitan Sewerage System. Sewage at all of the Metropolitan pumping stations is screened through wrought-iron barred cage screens provided in duplicate.

QUANTITY OF SEWAGE FROM NORTH METROPOLITAN SEWERAGE DISTRICT

The estimate of sewage flow for the North Metropolitan District does not include the large quantities of mingled storm water and sewage entering Boston Harbor or its tributaries through storm overflows; and, accordingly, it cannot be considered alone in determining the adequacy of the North Metropolitan Sewerage System. Fig. 2 shows the estimates of flow of sewage from the North Metropolitan Sewerage District made by the Metropolitan District Commission. Table 1(*b*), for the period 1925 to 1935, includes maximum flow as a percentage of the average daily flow. The comparison of Table 1(*b*) with Table 1(*a*) shows that the flow during the maximum day is considerably greater from the North Metropolitan Sewerage District than from the Boston Main Drainage District.

ADEQUACY OF THE NORTH METROPOLITAN SEWERAGE SYSTEM

A comparison of the average daily flow for 1935 (82.9 mgd) with a flow of 136 mgd, which is the capacity for which the system was designed, would indicate that the capacity of this system in the maximum section is approximately 64% in excess of the average daily flow. The population served by this system in 1935 (696 470) was about 22% in excess of the 571 000 for which the system was designed. If storm water were excluded from the system except in the cities of Somerville and Cambridge, as originally proposed, many of the main trunk sewers in the lower part of the system would have adequate capacity to care for sanitary sewage. It should also be stated that, although the total population served by the system is only 22% in excess of that for which the system was designed, actually in some parts (namely, in the towns of Arlington and Belmont and Winchester) the 1935 population is from 3 to 12 times what it was when the system was designed.

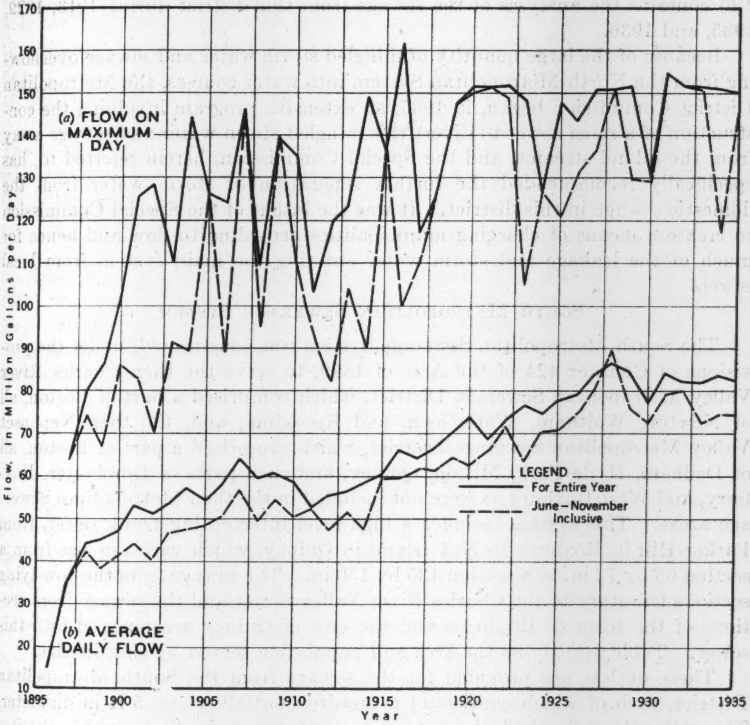


FIG. 2.—SEWAGE FLOW, NORTH METROPOLITAN SYSTEM

CHARACTER OF THE SEWAGE OF THE NORTH METROPOLITAN DISTRICT

Samples collected by the Massachusetts Department of Public Health during the investigations of 1929–1930 and 1935–1936 show that the sewage of the North Metropolitan Sewerage District is somewhat stronger than that of either the South Metropolitan or the Boston Main Drainage Districts. Approximately 19% of the sewage pumped at Deer Island consists of salt water which is believed to have entered the system through gates or leaks. Table 2(*b*) contains the analyses of the sewage from this district during 1913, 1929, 1935, and 1936.

Because of the large quantity of mingled storm water and sewage overflowing from the North Metropolitan System into water courses, the Metropolitan District Commission began, in 1935, an extensive program involving the construction of a relief sewer to divert this mingled storm water and sewage away from the inland streams, and the Special Commission, herein referred to, has specifically recommended the further separation of storm water from the domestic sewage in this district. It was the intent of the Special Commission to create a means of charging municipalities according to flow and hence for much of the leakage and storm water entering the main system from local sewers.

SOUTH METROPOLITAN SEWERAGE SYSTEM

The South Metropolitan Sewerage System was constructed, under the provisions of Chapter 424 of the Acts of 1899, to serve the then Charles River Valley Metropolitan Sewerage District, which comprised a part of Boston, all of Newton, Waltham, Watertown, and Brookline; and, the then Neponset Valley Metropolitan Sewerage District, which comprised a part of Boston, all of Dedham, Hyde Park, Milton, Quincy, and such parts of Dorchester, Roxbury, and West Roxbury as were not included in the then Metropolitan Sewerage areas. This system includes a high-level intercepting trunk sewer, from Parker Hill in Roxbury to Nut Island in Quincy, which varies in size from a section 68 by 72 in. to a section 135 by 150 in. The sewage from the low-lying sections tributary to the Charles River Valley Sewer and the sewage from sections of the town of Braintree and the city of Quincy are pumped into this sewer. Table 3(*b*) shows the area and population served by this district.

Three outlets are provided for the sewage from the South Metropolitan District, each of which consists of a cast-iron outfall sewer, 5 ft in diameter. The outlet through which the sewage is most commonly discharged is about a mile north of the screen house and about 1 200 ft from the westerly end of Peddocks Island. The second outlet, which is used at times in common with the first, is about the same distance from Nut Island and about 1 300 ft west of the first outlet, whereas the third outlet which is about 1 400 ft from the screen house has a point of discharge about 2 300 ft from the westerly shore of Peddocks Island. This latter outlet is used only in cases of emergency or at times of heavy run-off. At mean low tide the water over the two main outlets is about 30 ft deep and over the emergency outlet, 20 ft deep. These outlet pipes are laid in a trench at an average depth of about 9 ft below the bottom of the harbor. The outlets are encased in concrete and consist of a special casting in the form of a quadrant bend, 11 ft long weighing about 11 tons. Each pipe is surrounded by a rectangular timber casing 13 ft sq and 10 ft deep, resting upon 12 piles, 22 ft long. These structures are surmounted and surrounded by cut granite rings in six pieces, 30 in. deep.

The South Metropolitan Sewerage System was designed to serve an area of approximately 109 sq miles, but ultimately 218.19 sq miles will be tributary to this system. Except for the Charles River Valley Sewer (which receives the sewage from certain combined sewers), the South System is intended to receive only sanitary sewage. It was designed to take care of the sewage of a population of approximately one million persons on a basis of 300 gal per person per day in 1940. Records of the flow in this system, recorded at the screen house at Nut Island, show that the flow in 1935 averaged about 86.5 mgd.

Table 1(*c*) shows the flow in the South Metropolitan Sewerage System. The average daily flow of sewage from June to November, inclusive, was considerably less than that during the entire year, a condition contrary to that in the North Metropolitan Sewerage System and the Boston Drainage System.

Quality of Sewage Reaching Nut Island Screen House

Analyses of sewage collected by the Department of Public Health during 1913, 1929, and 1936 at the Nut Island screen house are as reported in Table 2(*c*). The chloride content of the sewage from the South District seldom exceeds 250 ppm indicating that only a small quantity of salt water leaks into the system of this district.

Effect of the Discharge of Sewage from Boston Main Drainage District and the North and South Metropolitan Sewerage Districts

For a number of years the discharge of sewage at Moon Island from the Boston Main Drainage District has caused objectionable conditions in the vicinity of the outfall because the discharged sewage enters the harbor at considerable velocity on the outgoing tide, at approximately the surface of the water, and spreads over a very large area, causing extensive sleek areas.

The sewage discharged from the North Metropolitan Sewerage District near Deer Island has never caused objectionable conditions except from odor in the vicinity of the Deer Island Light and from floating matter reaching nearby shores. Extensive sleek areas have seldom been noticed in connection with the discharge of sewage from this district. The limited size of the sleek areas is probably due to the rough water in that vicinity.

Although the sewage discharged from the South Metropolitan District does not result in as objectionable conditions as that from the Boston Main Drainage District, it does cause large sleek areas at times, especially in the vicinity of Nut Island on the incoming tide.

In order to ascertain the general condition of the harbor water resulting from the discharge of sewage from these three main outlets, an extensive program of sampling was conducted during the investigation by the Special Commission in 1935 and 1936 and, as far as practicable, these sampling stations conformed with those of previous investigations. Samples of the water were

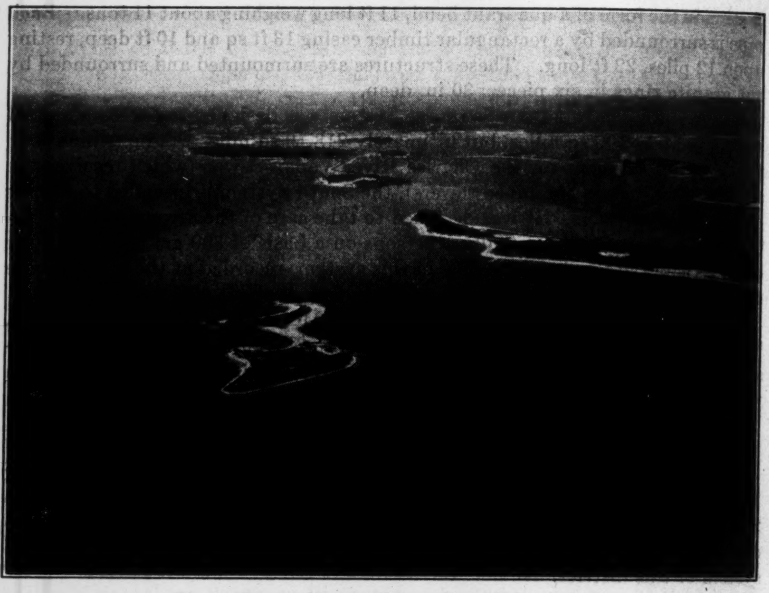


FIG. 3.—BOSTON HARBOR, FACING MOON HEAD OUTLET

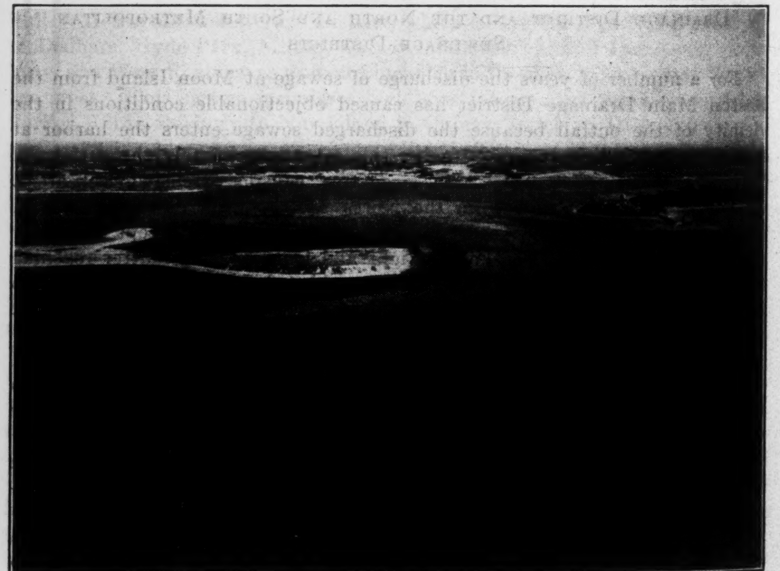


FIG. 4.—NUT ISLAND; 1.5 HOURS AFTER LOW WATER; NORTHWEST WIND; VELOCITY, 16 MILES PER HOUR

collected for chemical analysis and bacterial examination, and field tests were made to determine the quantity of dissolved oxygen in the water.

In order to interpret more intelligently the results of the chemical analyses and bacterial examinations, and in order to determine the extent to which sewage matters reach the shores of Boston Harbor and vicinity, extensive float tests were made to compare with the result of previous tests. To determine the extent to which floating matter, especially grease, might travel from these outlets, observations were made of sleek areas from boats, and a number of aerial photographs were taken (see Figs. 3 and 4).

Surface and depth samples were collected along the courses of floats for chemical analysis and bacterial examination; and samples were collected at regular intervals along the shores, especially in the vicinity of bathing beaches.

Tidal Currents in the Vicinity of the Outfall Sewers

The velocity and direction of the tidal currents were determined by observing the courses taken by floats released at the various outfalls and at other points throughout the harbor. Floats were also used to determine locations better adapted for points of discharge of the sewage. Three types of floats were used for this work—floats of deep submergence, surface floats of shallow submergence, and chips. The depth floats were constructed of 2 in. by 4 in. spruce timbers, 6 ft long, weighted at the bottom to hold the float in a vertical position and to provide submergence, except for 2 in. which extended above the surface of the water, and which was surmounted with a small signal flag. Each of the floats was equipped with vanes nailed to the sides to prevent whirling. The surface floats were similarly constructed of timber but were only 3 ft to 4 ft long. The chips consisted of small pieces of lath appropriately painted and marked by branding iron with the name and address of the Department of Public Health with the request that they be returned to the Department.

In general, the float tests confirmed the results of previous tests and showed that: (a) The velocity of the tidal flow in Boston Harbor has not changed considerably over a number of years, regardless of certain changes in and about the harbor due to the dredging operations and the filling of large areas for the construction of the Boston Airport, a bulkhead to Castle Island, and the filling in of Shirley Gut; (b) regardless of where sewage is discharged in Boston Harbor it may be expected that floating matter in the sewage will be washed on to the shores in the harbor or on adjacent shores. The courses taken by some of the floats released in various parts of the harbor are shown in Fig. 5, and Figs. 6 and 7 indicate the places where some of the chips were retrieved.

Results of Chemical Analyses

During the investigations of 1929 and 1930 the Department of Public Health made numerous tests on both the incoming and outgoing tides to determine the dissolved oxygen content in various parts of Boston Harbor, and during the joint investigation of 1935–1936 analyses and tests were made on samples from these same stations and at many additional ones.[3] The dissolved oxygen present in the water during the later investigation was found to

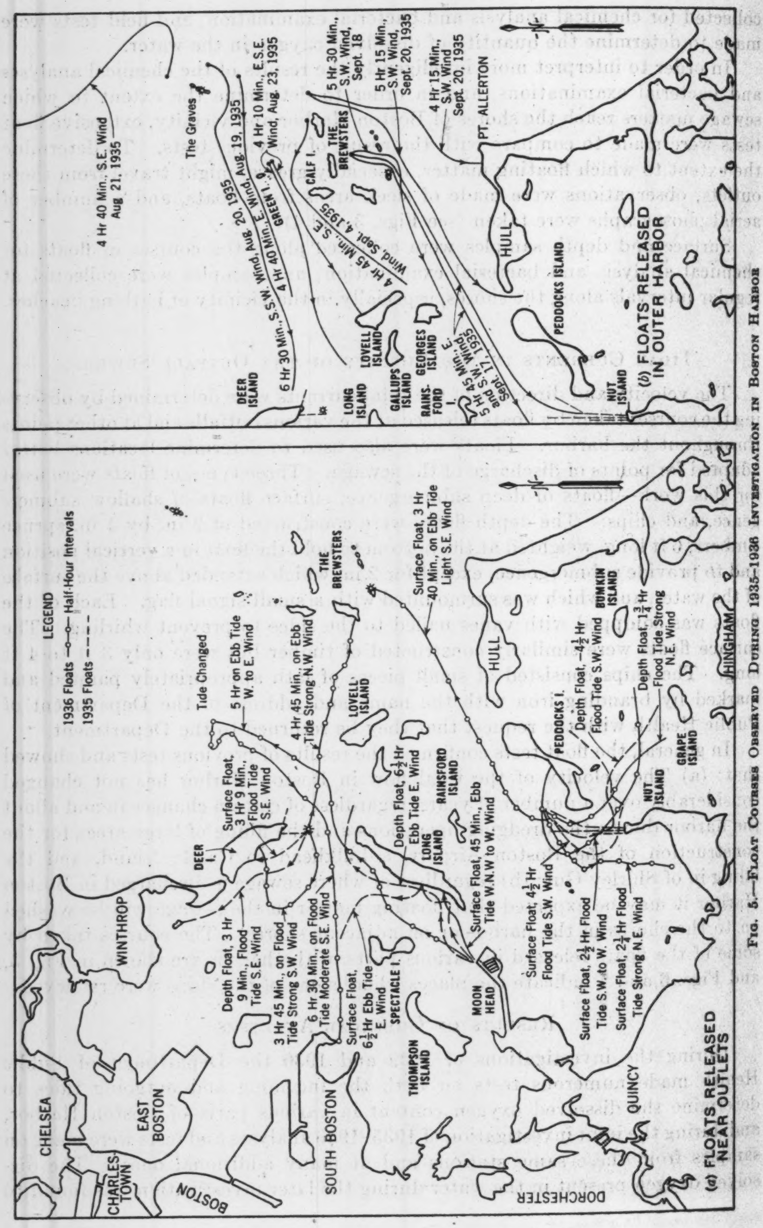

(a) FLOATS RELEASED NEAR OUTLETS
LEGEND
1936 Floats —o—o— Half-Hour Intervals
1935 Floats — — —
CHELSEA
CHARLES-TOWN
BOSTON
EAST BOSTON
WINTHROP
SOUTH BOSTON
DORCHESTER
QUINCY
THOMPSON ISLAND
SPECTACLE I.
MOON HEAD
DEER I.
LONG ISLAND
RAINSFORD ISLAND
LOVELL ISLAND
THE BREWSTERS
PEDDOCKS I.
HULL
BUMKIN ISLAND
NUT ISLAND
GRAPE ISLAND
HINGHAM
Tide Changed
5 Hr on Ebb Tide W. to E. Wind
Surface Float, 3 Hr 9 Min., Flood Tide S.E. Wind
4 Hr 45 Min., on Ebb Tide Strong N.W. Wind
Depth Float, 3 Hr 9 Min., Flood Tide S.E. Wind
3 Hr 45 Min., on Flood Tide Strong S.W. Wind
6 Hr 30 Min., on Flood Tide Moderate S.E. Wind
Surface Float, 6½ Hr Ebb Tide E. Wind
Depth Float, 6½ Hr Ebb Tide E. Wind
Surface Float, 3 Hr 40 Min., on Ebb Tide Light S.E. Wind
Surface Float, 45 Min., Ebb Tide W.N.W. to W. Wind
Surface Float, 4½ Hr Flood Tide S.W. Wind
Depth Float, –4½ Hr Flood Tide S.W. Wind
Surface Float, 2½ Hr Flood Tide S.W. to W. Wind
Surface Float, 2¼ Hr Flood Tide Strong N.E. Wind
Depth Float, ¾ Hr Flood Tide Strong N.E. Wind
(b) FLOATS RELEASED IN OUTER HARBOR
4 Hr 40 Min. S.E. Wind Aug 21, 1935
The Graves
6 Hr 30 Min., S.S.W. Wind, Aug. 20, 1935
4 Hr 40 Min., E. Wind, Aug. 19, 1935
4 Hr 10 Min., E.S.E. Wind, Aug. 23, 1935
GREEN I.
CALF I.
THE BREWSTERS
5 Hr 30 Min., S.W. Wind, Sept. 18
5 Hr 15 Min., S.W. Wind, Sept. 20, 1935
4 Hr 15 Min., S.W. Wind Sept. 20, 1935
PT. ALLERTON
4 Hr 45 Min., S.E. Wind, Sept. 4, 1935
DEER ISLAND
LONG ISLAND
LOVELL ISLAND
GALLUPS ISLAND
GEORGES ISLAND
RAINS-FORD I.
5 Hr 45 Min., E. Wind 5 Hr and S.W. Sept. 17, 1935
PEDDOCKS ISLAND
HULL
NUT ISLAND

FIG. 5.—FLOAT COURSES OBSERVED DURING 1935–1936 INVESTIGATION IN BOSTON HARBOR

be in excess of that found in 1929 at all stations, and on the incoming tide it was in excess of that found during the 1930 investigation. At no time during the 1935–1936 investigation was the harbor water found to be less than 50% saturated, even in the vicinity of the main sewer outlets, and in the Inner Harbor opposite Fort Point Channel the dissolved oxygen content of the water averaged about 75% saturated. Except for the Inner Harbor the lowest

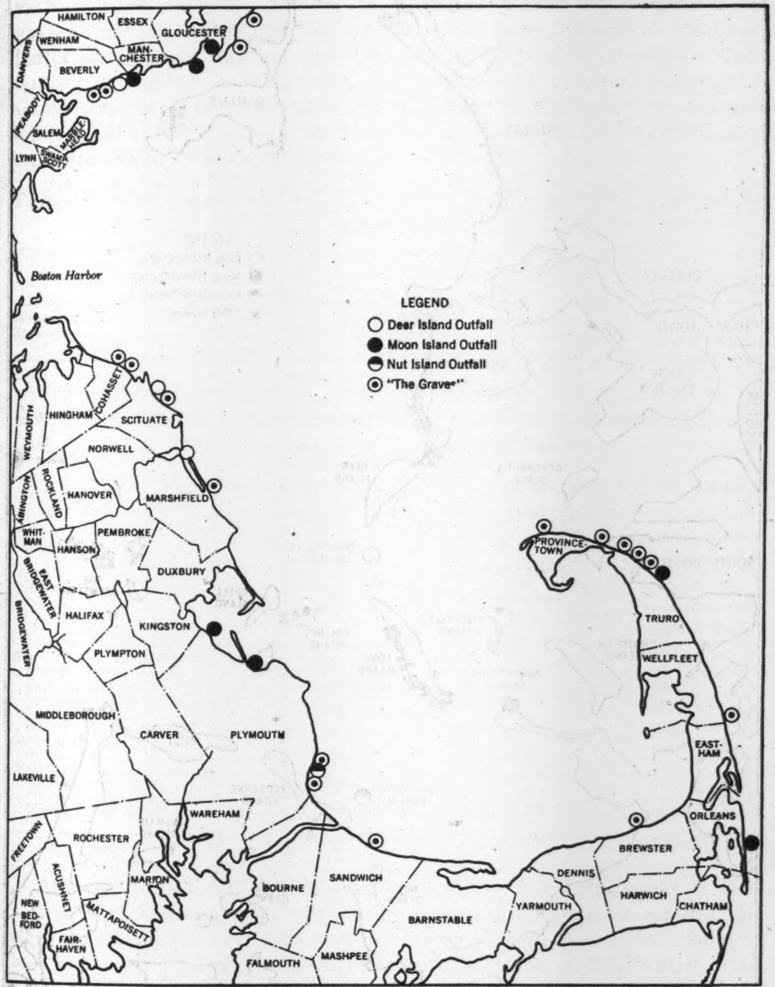

HAMILTON
ESSEX
GLOUCESTER
WENHAM
MAN-CHESTER
DANVERS
BEVERLY
PEABODY
SALEM
MARBLEHEAD
LYNN
SWAMPSCOTT
Boston Harbor
LEGEND
○ Deer Island Outfall
● Moon Island Outfall
◓ Nut Island Outfall
◉ "The Graves"
COHASSET
HINGHAM
WEYMOUTH
SCITUATE
NORWELL
ABINGTON
ROCKLAND
HANOVER
MARSHFIELD
WHIT-MAN
HANSON
PEMBROKE
EAST BRIDGEWATER
DUXBURY
BRIDGEWATER
HALIFAX
KINGSTON
PLYMPTON
MIDDLEBOROUGH
CARVER
PLYMOUTH
LAKEVILLE
WAREHAM
FREETOWN
ROCHESTER
MARION
ACUSHNET
MATTAPOISETT
NEW BED-FORD
FAIR-HAVEN
BOURNE
SANDWICH
FALMOUTH
MASHPEE
BARNSTABLE
YARMOUTH
DENNIS
BREWSTER
HARWICH
CHATHAM
ORLEANS
EAST-HAM
WELLFLEET
TRURO
PROVINCE-TOWN

Fig. 6.—Distant Points Where Chips Released at Outfalls, in October, 1935, Were Found

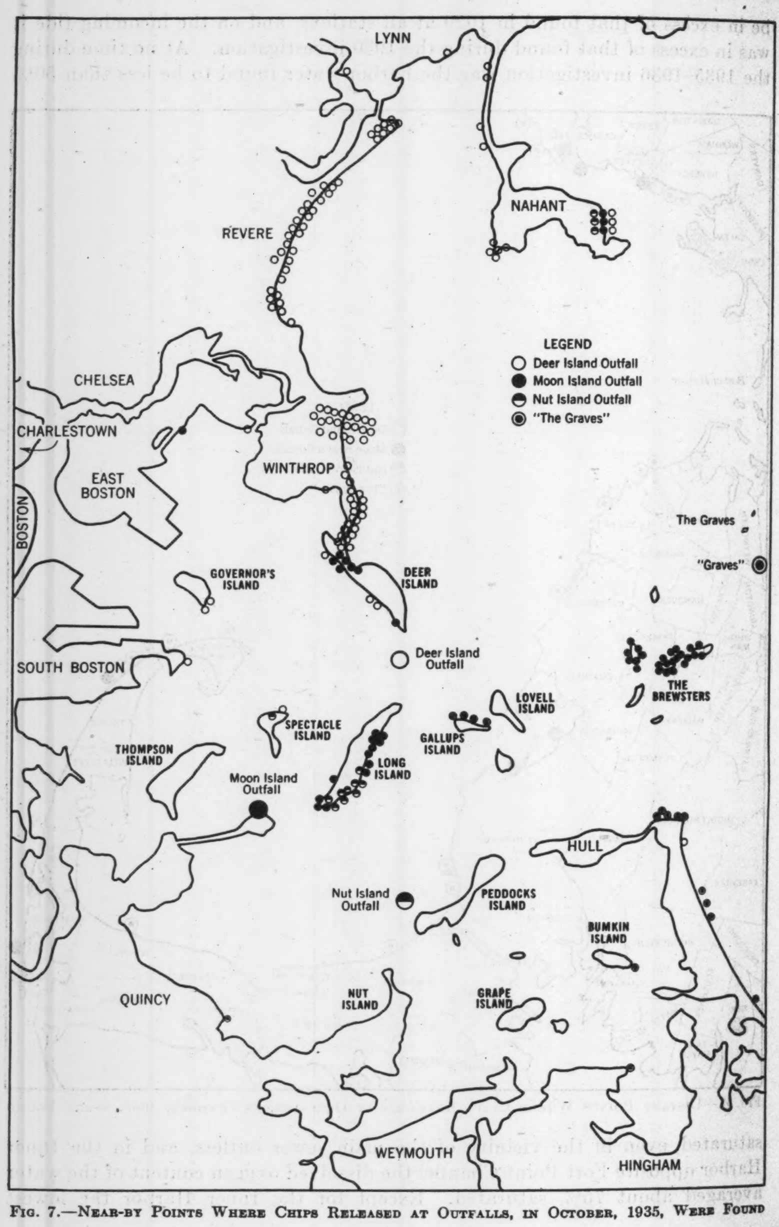

LYNN
NAHANT
REVERE
LEGEND
○ Deer Island Outfall
● Moon Island Outfall
◒ Nut Island Outfall
◉ "The Graves"
CHELSEA
CHARLESTOWN
EAST BOSTON
WINTHROP
BOSTON
The Graves
"Graves"
GOVERNOR'S ISLAND
DEER ISLAND
SOUTH BOSTON
Deer Island Outfall
THE BREWSTERS
LOVELL ISLAND
SPECTACLE ISLAND
GALLUPS ISLAND
THOMPSON ISLAND
LONG ISLAND
Moon Island Outfall
HULL
Nut Island Outfall
PEDDOCKS ISLAND
BUMKIN ISLAND
QUINCY
NUT ISLAND
GRAPE ISLAND
WEYMOUTH
HINGHAM

FIG. 7.—NEAR-BY POINTS WHERE CHIPS RELEASED AT OUTFALLS, IN OCTOBER, 1935, WERE FOUND

average dissolved oxygen saturation was 78.3%, which was found at a point a short distance northwest of the pier on Long Island. Determinations also were made[3] of the bio-chemical oxygen demand at various sampling points in the harbor (see Table 4). On the incoming tide in 1935 the oxygen demand was low in all sections of the harbor, the highest being 1.3 ppm in the Outer Harbor and Quincy Bay and the lowest being 0.3 ppm in Dorchester Bay. On the outgoing tide in 1936, the oxygen demand was somewhat higher, with a maximum of 4.1 ppm in Dorchester Bay. At no station where samples were collected did the results show that the dissolved oxygen in the water had been depleted sufficiently to cause a nuisance.

TABLE 4.—Bio-Chemical Oxygen Demand, Five Days at 20° Centigrade
(Parts per Million)

Tide	Year	Inner Harbor	Outer Harbor	Dorchester Bay	Winthrop Bay	Quincy Bay	Hingham Bay	Harbor entrances
Surface flood	1935	0.8	1.3	0.3	1.2	1.3	1.0	1.0
Surface ebb	1936	2.0	0.9	4.1	1.0	....	....	....

For purposes of comparison, results of the dissolved oxygen tests of the waters of New York Harbor and estuaries, and certain estuaries of Boston Harbor, and the North River in Salem are shown in Table 5.

TABLE 5.—Dissolved Oxygen; Various Harbors and Tributaries

Place	Station	Number of samples	Percentage Saturation		
			Minimum	Average	
				Ebb	Flood
New York, N. Y. (1933):					
Hudson River	42nd Street*	20	20	48	36
Harlem River	155th Street	24	0	12	26
Lower East River	42nd Street	32	0	16	21
Upper Bay	Bell Buoy No. 2G	30	21	38	35
Narrows	....	26	29	49	51
Salem, Mass. (1936):					
North River	Mouth	6	0	0.7	....
Boston, Mass.:					
Chelsea River (1935)	Lower Bridge	1	....	63.3	....
Fort Point Channel (1936)	....	6	55.9	63.7	87.5
Charles River (1935)	....	1	....	62.2	....

* North River.

Although tests for B.O.D. have been made by the Department of Public Health of Massachusetts only during recent years, analyses for free and albuminoid ammonia have been made for many years, in some instances dating back to the early nineties in connection with pollution of tidal waters. Accordingly, it has been the practice of the Department to continue these analyses in order that comparisons may be made between these results and the old results where practicable. They show that, in most cases, the organic matter in the harbor waters during the 1935–1936 investigation[3] was about the same as in the 1929–1930 investigation. A slightly greater degree of contamination was found in 1936 at the entrance of Hingham Bay near Windmill Point, at the entrance of the Harbor north of Point Allerton, in the North Channel northeast of Deer Island, and in Winthrop Bay near Shirley Gut. The condition of the water in the Inner Harbor, in the vicinity of Boston proper, was somewhat better in 1935–1936—especially on the outgoing tide—than in 1929–1930. On the incoming tide in 1935 and 1936, the content of organic matter was greater, in Dorchester Bay, than in 1929 and 1930; and, on the outgoing tide, it was less. Because of the large concentration of dissolved oxygen in the harbor waters, the ammonia results were of greater assistance in determining the extent of pollution throughout the harbor than were the results of the tests of dissolved oxygen. In Fig. 8 the 1935–1936 results are shown in diagrammatic form as a means of comparison with the dissolved oxygen determinations.

BACTERIAL EXAMINATIONS

Earlier investigations as well as those of 1935–1936[5] show that the chief nuisances resulting from the disposal of sewage in Boston Harbor are those created by bacterial pollution and by floating matter from the sewage reaching the shores, especially the shores used for bathing.

The 1935 bacterial examinations showed that on the incoming tide the water at the surface of the Outer Harbor in the vicinity of Deer Island and Nut Island outlets contained large numbers of bacteria characteristic of sewage, and that the contaminated areas extended from the harbor entrance to the westerly part of Hingham Bay and into the easterly section of Winthrop Bay in the vicinity of Deer Island and Point Shirley. They also showed the presence of a large number of such bacteria in that section of the Charles River between the Inner Harbor and the Charles River Dam and, on the incoming tide, in the waters of the Outer Harbor entrances in the vicinity of Lovell, Gallups, and Georges islands; and, in a small area east of Deer Island. The samples collected on the outgoing tide contained large numbers of bacteria characteristic of sewage between the Inner Harbor and the Charles River Dam, in Fort Point Channel and along the Boston water-front. The waters of Dorchester Bay, in the vicinity of the Naval Air Base at Squantum, and in the vicinity of Commercial Point, contained large numbers of bacteria characteristic of pollution, a condition probably due to local contamination. Quincy Bay in the vicinity of Moon Island contained large numbers of bacteria on the outgoing tide, a condition due to the discharge of sewage from the outfall works of the Boston Main Drainage system. In the vicinity of Deer Island, water on the outgoing tide contained large numbers of bacteria in the northeasterly section of the Outer Harbor and at the entrances to the harbor. The results of samples collected in 1935 and 1936 show that on the outgoing and incoming tides the water, at a 20-ft depth in the vicinity of the outlets, contained a larger number of bacteria than at the surface; but at points close to the shore the number present was about the same in the surface and depth samples. On the incoming tide large areas of the Outer Harbor, Winthrop Bay, and parts of the Inner Harbor are contaminated by the discharge of sewage from Deer Island, and the entrance to the harbor is polluted, in addition, by the discharge of sewage from the Moon Island outlet. The 1936 samples showed that, on the incoming tide, the waters in the vicinity of the Deer Island and Nut Island sewer outlets, and in the vicinity of Long and Rainsford islands, are grossly polluted. Pollution in the vicinity of Long and Rainsford islands may be caused partly by the discharge of sewage from Long Island and from the return of sewage which had previously been discharged from Moon Island on the outgoing tide. This area appeared to be highly polluted on both the outgoing and incoming tides. The depth samples collected on the incoming tide showed a polluted area off the northwesterly shore of Thompson Island and between Gallups and Long islands and that under certain conditions the sewage from the Nut Island outfall sewer approaches the shore along Houghs Neck in Quincy. In general, on the outgoing tides the polluted areas were greater in 1936 than in 1935; but this condition may be explained in part by the fact that samples from a number of new sampling points between Long and Rainsford islands, between Long and Gallups islands, and north of Thompson Island, collected during the 1936 investigation, were not collected during the 1935 investigation.

---

[5] "Report of the Special Commission on the Investigation of the Discharge of Sewage into Boston Harbor and Its Tributaries," House Doc. No. 1600, December, 1936, p. 221.

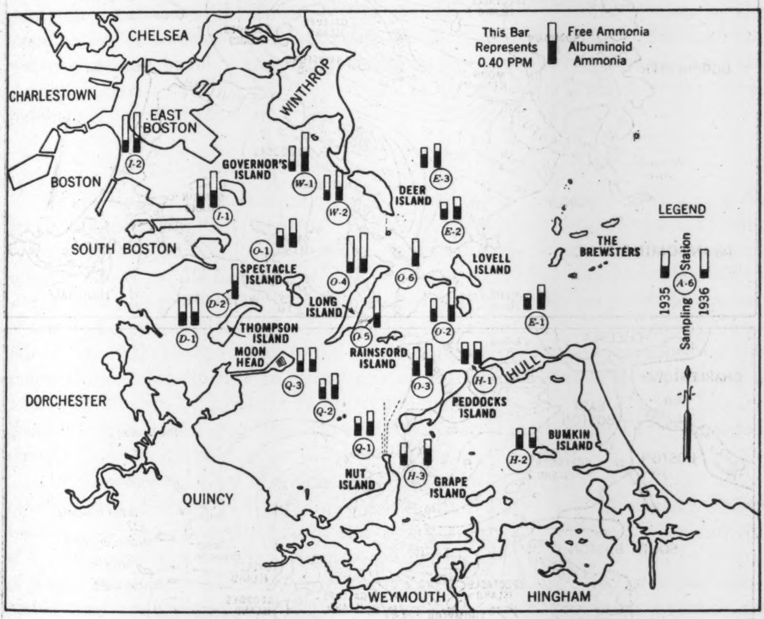

This Bar Represents 0.40 PPM
Free Ammonia
Albuminoid Ammonia
CHELSEA
CHARLESTOWN
EAST BOSTON
WINTHROP
BOSTON
SOUTH BOSTON
GOVERNOR'S ISLAND
DEER ISLAND
SPECTACLE ISLAND
LONG ISLAND
LOVELL ISLAND
THE BREWSTERS
THOMPSON ISLAND
MOON HEAD
RAINSFORD ISLAND
HULL
PEDDOCKS ISLAND
DORCHESTER
BUMKIN ISLAND
NUT ISLAND
GRAPE ISLAND
QUINCY
WEYMOUTH
HINGHAM
LEGEND
1935
Sampling Station
A-6
1936
I-2
I-1
W-1
W-2
E-3
E-2
E-1
O-1
O-2
O-3
O-4
O-5
O-6
D-1
D-2
Q-1
Q-2
Q-3
H-1
H-2
H-3

FIG. 8.—AVERAGE CONTENT OF ALBUMINOID AND FREE AMMONIA DURING FLOOD AND EBB TIDES

Fig. 9 shows the percentage of the 0.1-cu cm portions of the samples collected during the investigation of 1936 containing bacteria of the *Coli-Aerogenes*

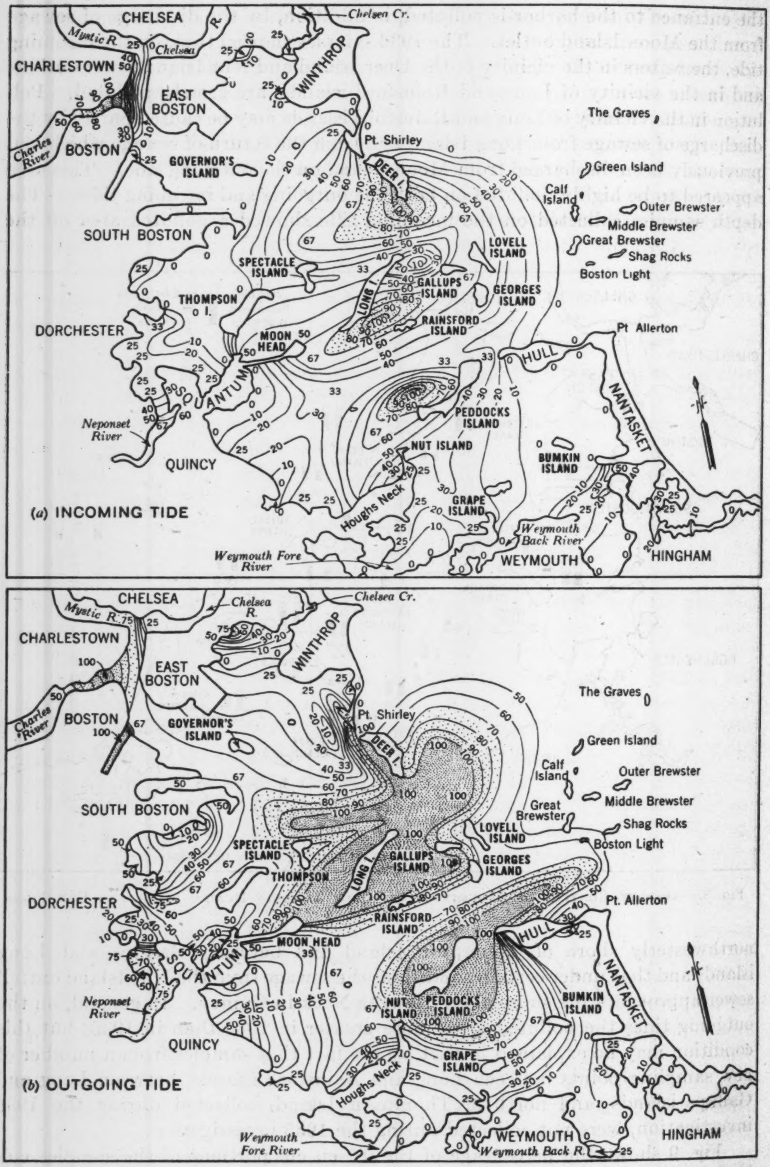

(a) INCOMING TIDE
(b) OUTGOING TIDE
CHELSEA
Chelsea Cr.
Chelsea R.
Mystic R.
CHARLESTOWN
EAST BOSTON
WINTHROP
BOSTON
Charles River
GOVERNOR'S ISLAND
Pt. Shirley
DEER I.
The Graves
Green Island
Calf Island
Outer Brewster
Middle Brewster
Great Brewster
Shag Rocks
Boston Light
SOUTH BOSTON
SPECTACLE ISLAND
THOMPSON I.
LOVELL ISLAND
GALLUPS ISLAND
GEORGES ISLAND
LONG I.
RAINSFORD ISLAND
DORCHESTER
MOON HEAD
SQUANTUM
Pt Allerton
HULL
NANTASKET
PEDDOCKS ISLAND
Neponset River
NUT ISLAND
BUMKIN ISLAND
QUINCY
Houghs Neck
GRAPE ISLAND
Weymouth Back River
Weymouth Back R.
Weymouth Fore River
WEYMOUTH
HINGHAM

FIG. 9.—DETERMINATIONS OF MEMBERS OF THE *COLI-AEROGENES* GROUP FROM SURFACE SAMPLE

groups. For conveniences, the lines connecting zones of equal *Coli-Aerogenes* density have been termed "isocols."

BACTERIAL CONDITION OF THE HARBOR BOTTOM

In connection with the bacterial investigation of the waters of Boston Harbor, samples were collected of the mud and silt along the bottom of the harbor at the regular sampling stations. In practically all cases the samples of mud contained many more bacteria, characteristic of pollution, than the waters at the surface, indicating that bacteria are deposited in the mud with sewage solids and are not washed out to sea with the outgoing tide. These results verify previous ones obtained by the Department of Public Health in the examination of certain bathing beaches where it was found that, although the waters over the beaches may sometimes be free from organisms of the *Coli-Aerogenes* group, the silt on the bottom at these beaches may contain large numbers of these organisms.

RECREATIONAL AND OTHER USES OF BOSTON HARBOR

Boston Harbor is used extensively for recreation and, to a limited degree, for the taking of shellfish for food. Although the harbor waters were not found to be so grossly polluted that nuisance from odor resulted, it is important to consider the effect of this pollution on waters used for bathing and from which shellfish are taken for food. Certain consideration should be given to the use of these waters for yachting. During the investigation by the Special Commission in 1935–1936, it was found that there were 21 yacht clubs along the shores from Point Shirley in Winthrop to Windmill Point in Hull, having a total membership of approximately 3 200 persons, and some 1 370 pleasure boats. It is interesting to note that at 6 of these clubs sewage was discharged directly into the harbor.

The sanitary surveys of 1935–1936 also showed that there were 29 areas between Point Shirley in Winthrop and Windmill Point in Hull where bathing beaches had been established. Of this number only 12 were under the control of the Metropolitan District Commission or the City of Boston, and at many of the areas no bath-houses or close supervision were provided. It was estimated that approximately 100 000 persons frequent the popular beaches on some pleasant Sundays and that 15 000 additional persons use those beaches where no record of attendance is kept.

In order to compare the results of the bacterial examination of the samples of the water collected at the beaches of Boston Harbor and some of those outside the harbor, Table 6 is presented. It is apparent that the beaches in Boston Harbor are more highly polluted than those outside but not as grossly polluted as at some beaches on streams tributary to the harbor. Some of the areas in Boston Harbor, now used for bathing, must be abandoned for that purpose, or a considerable sum of money must be expended to treat the sewage, or to remove it by extending the present sewer outlets to points more remote from the beaches. The investigation failed to disclose any illness due to bathing in these waters; but, regardless of the lack of such evidence, it is obvious that there must be always a danger to public health where bathing is permitted in the vicinity of sewer outlets. Accordingly, the Special Commission stated in its report to the Legislature[6] that in view of the numerous clean beaches available to the public there was no good reason to permit bathing near sewer outlets, and that public bathing along the water-front of Boston Harbor and its estuaries and tributaries should be restricted to such areas as may meet with the approval of the Board of Health of the city or town in which such bathing site is situated, or with the approval of the State Department of Public Health.

TABLE 6.—AVERAGE BACTERIAL ANALYSIS OF PUBLIC BEACHES IN BOSTON HARBOR COMPARED WITH THOSE OUTSIDE THE HARBOR

Location*	BACTERIA PER CUBIC CENTIMETER			*COLI-AEROGENES*					
		24 Hours 37° Centigrade		Percentage Positive in:					
								10 Cubic Centimeters	
	Four days 20° Centigrade	Total	Red†	0.001 cubic centimeters	0.01 cubic centimeters	0.1 cubic centimeters	1.0 cubic centimeter	Positive	Negative
1936:									
Wood Island Park, public, Boston	421	20	4	0	0	0	37.5	87.5	12.5
Marine Park, public, Boston	741	31	8	0	0	25	62.5	100	0
Carson Beach, public, Boston	1 004	137	84	12.5‡	12.5	37.5	67.5	87.5	12.5
Savin Hill, public	2 501	31	16	0	0	25	87.5	100	0
Tenean Beach, public, Boston	780	49	21	0	0	37.5	75	100	0
Wollaston Beach, North Beach (Atlantic Street)	1 210	105	57	0	12.5	12.5	37.5	100	0
Wollaston Center Beach (Wollaston Yacht Club)	1 966	407	300	12.5‡	25	37.5	50	100	0
Wollaston South Beach (Fenno Street)	911	16	3	0	0	0	12.5	87.5	12.5
Merrymount Beach, private	1 411	218	125	0	0	25	37.5	75	25
Weymouth Municipal, public	596	56	10	0	0	0	50	87.5	12.5
Hingham Beach, public (outside beaches)	1 059	22	4	0	0	12.5	25	87.5	12.5
Salem, salt water pool (6)	2 030	5	0.3	0	0	0	0	33.3	66.7
Winthrop, outside beach	1 186	11	3	0	0	0	12.5	87.5	12.5
Swansea, Long Beach (5)	884	5	0	0	0	0	0	100	0
Marblehead, Preston Beach (3)	2 507	5	0.3	0	0	0	0	33.3	66.7
1933:									
Revere Beach, South Beach (14)	2 063	32	4	0	0	14.3	28.6	78.5	21.5
Revere Beach, North Beach (13)	6 212	101	7	0	0	7.7	23.1	84.6	15.4
Revere Beach, Center Beach (14)	1 662	227	4	0	0	7.1	28.6	78.5	21.5
1931:									
Barnstable-Hyannis (4)	5 730	279	35	0	0	0	0	100	0
Fairhaven (6)	2 147	15	3	0	0	0	0	83.7	16.3
1928:									
Nantasket Beach (3)	1 260	43	8	0	0	0	33	100	0

* Eight samples, except as shown in parentheses. † "Red" refers to the number of acid-forming bacteria. ‡ 0 in 0.0001 cubic centimeter.

---

[6] "Report of the Special Commission on the Investigation of the Discharge of Sewage into Boston Harbor and Its Tributaries," House Doc. No. 1600, December, 1936, p. 318.

TAKING OF SHELLFISH FROM BOSTON HARBOR

The taking of shellfish from Boston Harbor is regulated by acts of the Massachusetts Legislature. Also in accordance with Massachusetts laws, the Department of Public Health has permitted master diggers, under permits issued by the Supervisor of Marine Fisheries, to take shellfish from certain areas in Boston Harbor for treatment purposes at an approved shellfish purification plant. Fig. 10 shows the areas in Boston Harbor where shellfishing was permissible as of July 1, 1937, either for treatment purposes or for use as food without treatment.

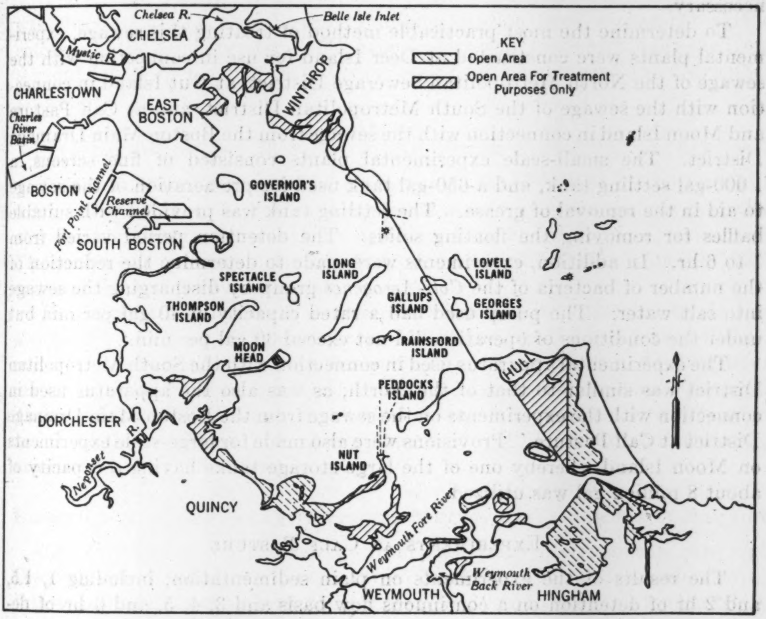

Chelsea R.
Belle Isle Inlet
CHELSEA
Mystic R.
KEY
Open Area
Open Area For Treatment Purposes Only
CHARLESTOWN
EAST BOSTON
WINTHROP
Charles River Basin
BOSTON
Fort Point Channel
Reserve Channel
GOVERNOR'S ISLAND
SOUTH BOSTON
LONG ISLAND
LOVELL ISLAND
SPECTACLE ISLAND
GALLUPS ISLAND
GEORGES ISLAND
THOMPSON ISLAND
RAINSFORD ISLAND
MOON HEAD
HULL
PEDDOCKS ISLAND
DORCHESTER
NUT ISLAND
Neponset R.
QUINCY
Weymouth Fore River
Weymouth Back River
WEYMOUTH
HINGHAM

FIG. 10.—SHELLFISH AREAS IN BOSTON HARBOR AS OF JULY 1, 1937

EXPERIMENTAL SEWAGE TREATMENT WORKS

The ordinary routine investigation to determine the most appropriate method of sewage treatment would consider the capacity of the water in which the effluent is to be discharged to replenish the oxygen that would be consumed by the bio-chemical processes of the purification of the sewage. There is no question of the capability of the tidal prism of Boston Harbor on the outgoing and incoming tide, and the waters of the streams tributary to this harbor, to supply sufficient oxygen to replenish that consumed by the disposal of sewage in the harbor.

Bacterial examinations, tests for dissolved oxygen, and chemical analyses, were made of the waters of Boston Harbor, and of sewage discharged from the North and South Metropolitan Sewerage districts and the Boston Main Drainage District. They indicated that objectionable conditions can be removed if works are provided for the removal of certain suspended matters and grease from the sewage before it is discharged. The settled sewage may be chlorinated should further reduction of bacteria of the *Coli-Aerogenes* group become necessary.

To determine the most practicable method of treating this sewage, experimental plants were constructed at Deer Island for use in connection with the sewage of the North Metropolitan Sewerage District, at Nut Island in connection with the sewage of the South Metropolitan District, and at Calf Pasture and Moon Island in connection with the sewage from the Boston Main Drainage District. The small-scale experimental plants consisted of fine screens, a 1 000-gal settling tank, and a 650-gal tank used for pre-aeration of the sewage to aid in the removal of grease. The settling tank was provided with suitable baffles for removing the floating solids. The detention period varied from 1 to 6 hr. In addition, experiments were made to determine the reduction of the number of bacteria of the *Coli-Aerogenes* group by discharging the sewage into salt water. The pump used had a rated capacity of 40 gal per min but under the conditions of operation did not exceed 30 gal per min.

The experimental apparatus used in connection with the South Metropolitan District was similar to that of the North, as was also the apparatus used in connection with the experiments on the sewage from the Boston Main Drainage District at Calf Pasture. Provisions were also made for large-scale experiments on Moon Island whereby one of the large storage tanks having a capacity of about 8 million gal was utilized.

EXPERIMENTS AT CALF PASTURE

The results of the experiments on plain sedimentation, including 1, 1.5, and 2 hr of detention on a continuous flow basis and 3, 4, 5, and 6 hr of detention on an intermittent basis are shown in Table 7(*a*).

As was expected the suspended solid removal in these experiments was about 30% in 1 hr and 50% in 2 hr. The additional removal after detention periods greater than 2 hr was small, being only a total of 62.5% in 6 hr. Removal of the organic matter as represented by the bio-chemical oxygen demand (about 19% with a 2-hr detention period) was relatively low. The removal of the nitrogenous matter as represented by the albuminoid ammonia and Kjeldahl nitrogen determinations was 21% and 21.9%, respectively, for a 2-hr detention period. In these experiments there was little relation between the grease removal and the detention period but the results indicate that about 40% of the grease can be removed by skimming during a plain sedimentation period of 2 hr.

The experiments made to determine the quantity of grease that could be skimmed from the surface of the tanks with aeration prior to sedimentation showed that a slightly greater proportion of grease could be removed with pre-aeration. The additional detention period resulting from aeration was about one-third of the total aeration period. Pre-aeration, however, resulted in a greater reduction in the percentage of suspended solids and nitrogenous matter. The percentage of removal of the suspended solids by plain sedimentation was about 50% for a 2-hr detention period, whereas with pre-aeration, the removal was about 57% for 2.25-hr periods. The results of the sedimentation tests, with pre-aeration, are shown in Table 7(*b*).

The sewage used in the experiments for the Boston Main Drainage System was obtained at the Calf Pasture Pumping Station from a chamber at the entrance to the deposit sewer. The experiments were made on the sewage from this district 24 hr per day for a 7-day week from September 26 to November 21, 1935.

EXPERIMENTS ON THE SEWAGE OF THE NORTH METROPOLITAN SYSTEM

The Deer Island experimental plant was operated for the most part 16 hr a day, from 7 A.M. to 11 P.M. Early in the morning the sewage was generally at such a low level that it was impracticable to raise it with the equipment used. The period of experimentation was from September 30 to November 9, 1935, and from May 15 to July 31, 1936.

In the plain settling tests the average reduction in suspended solids was about 45% in 1.5 hr, 51% in 2 hr, and 57% in 3 hr. The average B.O.D. reduction was about 25% during these periods and the reduction of nitrogenous matter, as represented by the Kjeldahl nitrogen determination, was 33 per cent. Approximately 40% of the grease in the sewage was removed in a detention period of 2 hr. Table 7(*d*) shows the results of the plain sedimentation experiments.

EXPERIMENTS ON SEWAGE OF THE SOUTH METROPOLITAN SEWERAGE DISTRICT

The experiments on the sewage from the South Metropolitan Sewerage District were extended over a period from 7 A.M. to 11 P.M. from Monday to Friday, inclusive, from August 25 to September 19 and October 2 to October 9, 1936. Because of the short period of operation a few of the samples were 4-hr composites instead of the usual 8-hr composites. The sewage for these experiments was taken from the main outfall sewer through a manhole to the grit-chamber at the entrance to the screen house at Nut Island. The efficiency of settling was much greater in connection with the sewage of the South Metropolitan System than from the Boston Main Drainage System although the analyses showed that the sewage of these two systems was similar. The reduction in suspended solids during a 2-hr detention period was 69% as compared with 50% ordinarily obtained. The reduction in the oxygen demand, organic nitrogen, and oxygen consumed, was 40%, 40%, and 36%, respectively. It is probable that if the experiments had been extended over a sufficient period, lower reductions, more consistent with the results of the experiments on the sewage of the Boston Main Drainage System, would have been obtained. Table 7(*e*) shows the results of plain sedimentation experiments on the sewage of the South Metropolitan Sewerage System.

TABLE 7.—Observations of Plain Sedimentation

Detention period, in hours	Number of samples*	Suspended Solids			Bio-chemical Oxygen Demand				Kjeldahl Nitrogen			Total Albuminoid Ammonia			Oxygen Consumed			Fats			
		Parts per Million		Percentage reduction	Number of samples*	Parts per Million		Percentage reduction	Parts per Million		Percentage reduction	Parts per Million		Percentage reduction	Parts per Million		Percentage reduction	Number of samples*	Parts per Million		Percentage reduction
		Raw sewage	Effluent			Raw sewage	Effluent		Raw sewage	Effluent		Raw sewage	Effluent		Raw sewage	Effluent			Raw sewage	Effluent	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
(*a*) Calf Pasture; without Pre-Aeration																					
1	19	153	108	29.4	17	173	143	17.3	8.9	7.4	16.9	4.21	3.40	19.3	56	48	14.3	18	54	37	31.5
1.5	9	159	92	42.2	....	173	143	17.3	8.7	7.7	11.5	4.41	3.68	16.5	61	46	24.6	8	39	22	43.6
2	9	190	93	50.9	7	161	130	19.3	8.7	6.8	21.9	4.23	3.34	21.0	56	42	25.0	....	46	28	39.1
3	18	156	77	50.6	17	192	142	26.1	9.6	6.6	31.3	4.40	3.14	28.6	58	47	19.0	17	52	29	44.2
4	12	136	70	48.5	10	186	136	26.9	8.7	6.2	28.8	4.28	2.78	35.0	53	46	13.2	....	45	28	37.8
5	11	172	82	52.3	....	203	139	31.5	9.3	6.9	25.8	4.47	3.18	26.6	59	48	18.7	....	57	34	40.3
6	9	173	65	62.5	....	174	115	33.9	9.0	5.3	41.2	4.27	2.66	37.7	56	42	25.0	....	62	26	58.0
(*b*) Calf Pasture; with Pre-Aeration																					
0.75	6	132	85	35.6	....	163	146	10.4	8.7	8.1	6.9	3.60	3.32	7.8	52	44	15.4	....	41	31	24.4
1.5	4	169	88	48.0	....	203	160	21.2	13.1	9.0	31.3	4.85	3.42	29.5	56	48	14.3	....	61	41	32.8
2.25	8	169	72	57.3	....	165	121	26.7	11.9	6.9	42.0	5.43	3.42	37.0	56	44	21.4	....	58	31	46.6
3	6	168	75	55.3	....	174	114	35.5	10.7	6.9	35.5	4.83	2.95	38.9	56	43	23.2	....	50	36	28.0
(*c*) Deer Island (1935); with Pre-Aeration																					
1	3	224	164	26.8	....	269	244	9.3	12.9	10.5	18.9	7.30	6.43	11.9	80	74	7.5	....	67	56	16.4
1.5	9	238	126	47.1	....	260	190	26.9	17.4	13.6	35.0	8.03	5.70	29.0	79	64	19.0	....	75	38	49.3
2.25	3	233	123	47.2	....	234	171	31.2	17.5	10.3	41.2	7.17	4.93	31.2	81	71	12.4	....	41	33	19.5
3	5	238	106	55.5	....	250	168	32.8	16.5	11.1	32.7	7.56	4.84	36.0	80	66	17.5	....	66	31	53.0

TABLE 7.—(*Continued*)

Detention period, in hours	Number of samples*	SUSPENDED SOLIDS			BIO-CHEMICAL OXYGEN DEMAND				KJELDAHL NITROGEN			TOTAL ALBUMINOID AMMONIA			OXYGEN CONSUMED			FATS			
		Parts per Million		Percentage reduction	Number of samples*	Parts per Million		Percentage reduction	Parts per Million		Percentage reduction	Parts per Million		Percentage reduction	Parts per Million		Percentage reduction	Number of samples*	Parts per Million		Percentage reduction
		Raw sewage	Effluent			Raw sewage	Effluent		Raw sewage	Effluent		Raw sewage	Effluent		Raw sewage	Effluent			Raw sewage	Effluent	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
(*d*) DEER ISLAND (1935 AND 1936); WITHOUT PRE-AERATION																					
1	14	170	97	42.9	....	218	181	17.0	13.0	9.8	24.6	....	....	....	72	62	13.9	....	49	30	38.8
1.5	16	176	97	44.9	....	224	167	25.4	12.5	8.5	32.0	....	....	....	73	61	16.5	....	40	28	30.0
2	15	171	83	51.4	....	183	138	24.6	11.9	8.0	32.8	....	....	....	71	58	18.3	....	49	28	42.9
3	16	187	80	57.2	....	250	181	27.6	13.3	8.8	33.8	....	....	....	73	58	20.6	....	58	36	38.0
4	16	166	66	60.3	....	198	148	25.3	12.5	8.2	34.4	....	....	....	75	58	22.7	....	36	17	52.8
5	10	192	58	69.9	....	178	120	32.6	11.5	7.4	25.6	....	....	....	66	49	25.8	....	41	21	48.8
6	10	147	55	62.6	....	185	139	24.9	12.1	8.2	32.2	....	....	....	76	61	19.8	....	42	21	50.0
(*e*) NUT ISLAND (1936); WITHOUT PRE-AERATION																					
1	10	195	73	63	....	160	117	27	11.4	6.9	40	4.98	2.99	40	54.0	38.7	28	....	59	38	36
1.5	6	168	62	63	....	118	81	31	10.6	6.9	35	3.63	2.70	26	54.0	34.0	37	....	46	28	39
2	10	190	59	69	....	134	80	40	10.7	6.4	40	4.00	1.98	51	58.8	37.6	36	....	48	24	50
3	9	150	55	63	....	126	76	40	9.2	5.8	37	3.64	2.28	37	51.3	33.0	36	....	57	30	47
4	7	136	51	63	....	115	73	36	9.1	6.0	34	3.27	2.04	38	47.3	32.0	32	....	51	31	39
5	2	121	28	77	....	84	42	50	6.0	3.8	37	2.15	1.00	54	41.0	28.0	32	....	25	15	40
6	5	145	58	60	....	114	71	38	8.3	5.8	30	2.74	2.18	20	43.0	31.0	28	....	51	36	29

* For number of samples in fat determinations see Columns (6) and (19).

Summary of the Results of Sedimentation Experiments

Pre-aeration of the sewage to aid in grease removal was not successful in connection with sewage of the Boston Main Drainage System and the North Metropolitan Sewerage System. It is probable that the aeration does help to separate the grease from the settleable solids; but it is doubtful if, in these experiments, it was of sufficient assistance in increasing the over-all removal of grease to warrant the additional expense.

These experiments, confirming the result of other similar experiments, show that the greatest reduction of suspended solids occurs during the first hour and that with increasing detention periods the increments of removal become less and less.

In general, the results obtained from all three stations were similar and indicated that about 50% of the suspended solids could be removed in a 2-hr detention period. Calculated from the removal of suspended solids, the daily quantities of sludge that would be removed from the sewage of the three sewerage systems on the basis of 1935 flows is shown in Table 8.

TABLE 8.—Quantity of Sludge Expected per Day

System	Dry solids, in tons	Sludge (95% moisture), in gallons	Sludge (80% moisture), in cubic feet
Boston Main Drainage	25	114 000	3 800
North Metropolitan Sewerage	33	153 000	5 120
South Metropolitan Sewerage	30	138 000	4 610
Totals	88	405 000	13 530

Experiments with Fine Screens

The solids obtained by passing the sewage through fine screens, 18 in. by 21 in., which had openings of $\frac{1}{16}$ in., $\frac{3}{32}$ in., and $\frac{1}{8}$ in., by 2 in., is shown in Table 9. The nature of the screenings is described as follows:

Experiment station	Screenings
Calf Pasture	Garbage and paper, with leaves after a heavy rain
Deer Island	Garbage and paper with some hair and bristle
Nut Island	Garbage and paper

Because the Boston Main Drainage System is relatively short, more screenings were obtained from sewage of this system than from that of the larger North and South Metropolitan Systems where the larger particles in the sewage have an opportunity to become well disintegrated before reaching the screen chambers.

The weather during the tests was about normal except during the day of September 18, 1936, when the $\frac{1}{16}$-in. screens were in use at Nut Island. At this time 4.64 in. of rain fell and screenings were obtained at the rate of 143 lb of dry material per million gallons of sewage.

Large particles of floating and suspended solids were removed with the use of fine screens. Only a small quantity of the grease (one of the main offenders in rendering the harbor waters unsightly) was retained on the screens, and it is apparent that fine screens without additional help are insufficient to improve the objectionable appearance of the harbor satisfactorily. Only about 5% to 8% of suspended solids were removed by the fine screens as compared with about 50% removal by plain sedimentation.

TABLE 9.—EXPERIMENTAL RESULTS, FINE SCREEN TESTS

Experiment station	Year	(a) $\frac{1}{16}$-INCH SCREEN				(b) $\frac{3}{32}$-INCH SCREEN				(c) $\frac{1}{8}$-INCH SCREEN			
		Total hours of test	DRY SCREENINGS		Percentage reduction in suspended solids	Total hours of test	DRY SCREENINGS		Percentage reduction in suspended solids	Total hours of test	DRY SCREENINGS		Percentage reduction in suspended solids
			Pounds	Pounds per million gallons			Pounds	Pounds per million gallons			Pounds	Pounds per million gallons	
Calf Pasture	1935	167.5	21.23	106.5	7.5	131.5	14.46	92.0	7.1	102.5	10.24	83.3	5.7
Deer Island	1935	60.0	5.92	74.9	3.8	55.8	5.32	59.3	2.7	....	....	....	....
Deer Island	1936	60.9	9.16	108.0	7.4	77.5	5.39	52.0	3.3	80.9	4.04	32.9	3.2
Nut Island	1936	33.5	4.70	75.0	5.5	27.5	2.40	52.0	5.9	33.7	1.90	47.7	3.6

LARGE SCALE EXPERIMENTS, BOSTON MAIN DRAINAGE SYSTEM, 1936

Mention has been made herein of the use of one of the large storage tanks at Moon Island for continuous flow experiments to determine the removal of suspended solids and grease from the sewage of the Boston Main Drainage System. These experiments were conducted in the first place to compare, so far as practicable, the results of the small scale experiments with those of the large scale, and to give some indication of the possible use of the present equipment of the Boston Main Drainage System for purposes of sedimentation and grease removal. The tank used was trapezoidal in shape, 162 ft wide, and had an average length of 612 ft. The floor at the outer end, which is slightly lower than that of the inner end, was at Elevation 9 (mean sea level). The tank was equipped with a dike, with weir and skimming baffle and gates to prevent the sewage from flowing out of the bottom. The skimming baffle consisted of a floating, vertical, wooden structure about 3 ft deep, 2 ft of which were submerged, held 6 ft out in front and parallel to the sides of the dike by spreaders. The spreaders were connected in such a way as to allow the baffle to rise and fall with the level of the sewage. It was the purpose of the baffle to prevent the passage of this floating matter over the weir. During the first experiment, from August 11 to September 4, the settled sewage was stored in the three remaining tanks and discharged to sea in the regular manner on the second and third hours of the outgoing tide. Sewage from the experimental tank was discharged to sea for two hours longer to compensate for the loss of one tank from the total of the storage capacity. The average detention period in the tank was estimated to be about three hours. Beginning September 8, experiments were made in the same manner except that the sewage was discharged to sea continuously; that is, the three storage tanks were not used, sewage passing through the experimental tank only. This part of the experiment was discontinued on September 18. At the conclusion of each week's run on Friday evening, the sluice gates were opened to allow the basin to drain and the lower section of each bulkhead was removed and the basin flushed out. Frequent cleaning of the tank prevented the occurrence of septic conditions with the formation of gas which would interfere with sedimentation. The scum was discharged to sea on the outgoing tide, as necessary. Samples of the raw sewage and the settled sewage were collected each half-hour and composited over an 8-hr period. Determinations for dissolved oxygen and pH were made in the field and the regular chemical determinations were made in the laboratory. The experiments were run continuously from Monday morning through Friday evening. Because the weir was submerged much of the time, accurate weir measurements of the flow could not be made, but estimates were made from the level of sewage in the basins before discharge. Considerable sewage, perhaps 5 to 10 mgd, escaped through the stop planks in the boat chamber, or through the numerous gates, and were not treated in the settling tank.

During continuous discharge, the weir readings were made every two hours and these measurements should show rather accurately the quantity of sewage treated in the settling tank; but they did not necessarily show the total flow to Moon Island, because of the losses mentioned. The large scale settling and skimming tests at Moon Island were made over a period of 28 operating days. During these experiments, harbor and shore samples were collected in addition to the regular composites of raw and settled sewage. Samples of the scum and sludge were also collected to determine their value for grease recovery and for fertilizer or fill.

The detention period varied from 1.7 to 5.8 hr and averaged 2.8 hr. During the test the average daily flow through the tank was about 60 million gal. The average results of the settling tests for the entire period are shown in Table 10.

TABLE 10.—SETTLING TESTS, MOON ISLAND; BOSTON MAIN DRAINAGE SYSTEM

Average sewage concentration, in parts per million (1)	Free ammonia (2)	Chlorides (3)	Oxygen consumed (4)	Kjeldahl nitrogen (5)	Fats (6)	Bio-chemical oxygen demand (7)	Suspended solids (8)
Raw sewage	18.1	3 478	49.6	8.58	34.2	144	119
Settled sewage	18.8	3 437	39.7	5.94	20.0	107	51.5
Percentage reduction	....	....	19.9	30.8	41.5	25.6	56.7

The raw sewage from the Boston Main Drainage System, during the tests of 1936, was somewhat weaker than during the experimental work in 1935 at Calf Pasture. The concentration of suspended solids was particularly less in 1936, averaging about 162 ppm in 1935 and 119 in 1936. The strength of the sewage varied from hour to hour and from day to day. The strongest sewage was received during the afternoon and early evening, and the weakest sewage arrived in the early morning. The suspended solids in the raw sewage varied during the tests from 50 to 302 ppm. It is interesting to note that the removal of the material by sedimentation, in the large tank experiments, compared favorably with those obtained in the small unit on the sewage from this district at the Calf Pasture Pumping Station.

In connection with these experiments, it should be remembered that the large basin used for the experiment was designed as a storage tank and not as a settling tank, and accordingly the distribution of the sewage at the inlet was poor. The length of the effluent weir was small for the large volume of sewage, and the velocity of approach to the weir was relatively great. When a large volume of scum accumulated in the tank, the velocity of the sewage carried some of the scum under the baffle and over the weir. Considering the fact that the large chamber was not well fitted for settling and skimming of sewage solids, the results obtained were very satisfactory and indicated that better results might be expected with a tank properly designed and operated.

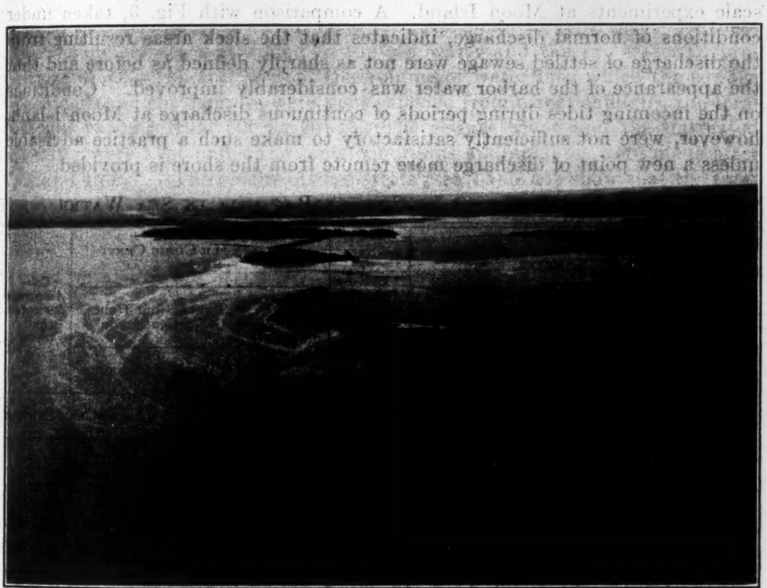


FIG. 11.—VIEW OF MOON ISLAND, SHOWING IMPROVEMENT OVER CONDITIONS IN FIG. 3

It was impracticable to measure the sludge removed as it was distributed over the bottom of the tank in a thin layer, and it was impossible to draw down the tank without disturbing and losing some of the solids. However, calculations made from the reduction of the suspended solids indicate that, in all probability, an average of 21.2 tons of dry solids were removed daily, which compares favorably with the estimates obtained during the Calf Pasture Pumping Station experiments.

The quantity of floating matter held back on the surface of the sewage by the skimmers was much greater than expected, and rough estimations indicate that as much as 40 cu ft per million gal of sewage may be removed in this manner. The grease content of the sewage varied considerably from day to day. The average results of the analyses of the scum showed that it contained about 89% moisture and 77.5% volatile matter. An average of 42% of the dry solids was fat and 2.8% was Kjeldahl nitrogen.

The raw sewage had an average of 34.2 ppm of fat which was reduced to 20.0 ppm by sedimentation, a reduction of 41.5 per cent.

In order to determine the effect on the harbor waters, of the discharge of settled sewage, aerial photographs such as Fig. 11 were taken during the large-scale experiments at Moon Island. A comparison with Fig. 3, taken under conditions of normal discharge, indicates that the sleek areas resulting from the discharge of settled sewage were not as sharply defined as before and that the appearance of the harbor water was considerably improved. Conditions on the incoming tides during periods of continuous discharge at Moon Island, however, were not sufficiently satisfactory to make such a practice advisable unless a new point of discharge more remote from the shore is provided.

TABLE 11.—VIABILITY OF SEWAGE BACTERIA IN SEA WATER

Time, in hours from the start	Bacteria per Cubic Centimeter			*Coli-Aerogenes* per cubic centimeter	Bacteria per Cubic Centimeter			*Coli-Aerogenes* per cubic centimeter
	Four days, 20° Centigrade	Twenty-four Hours, 37° Centigrade			Four days, 20° Centigrade	Twenty-four Hours, 37° Centigrade		
		Total	Red*			Total	Reduction	
	(*a*) Sewage				(*e*) Tap Water, with 1½ Per Cent Sewage			
0	1 200 000	270 000	130 000	20 000	120 000	3 800	2 400	2 000
24	2 200 000	340 000	195 000	10 000	230 000	3 100	1 900	1 000
48	1 000 000	120 000	50 000	10 000	260 000	2 800	1 200	1 000
96	1 800 000	22 000	9 000	5 000	16 000	1 100	630	1 000
	(*b*) Sea Water				(*f*) Sea Water, with ½ Per Cent Sewage			
0	340	1	0	0	12 000	1 400	400	100
24	400	1	0	0	2 500	110	40	100
48	590	1	0	0	2 400	110	50	20
96	480	1	0	0	2 000	8	1	0†
	(*c*) Tap Water, with ½ Per Cent Sewage				(*g*) Sea Water, with 1 Per Cent Sewage			
0	48 000	1 800	800	500	22 000	2 100	700	500
24	380 000	1 400	600	500	4 000	140	70	100
48	180 000	1 600	1 000	1 000	3 300	150	60	20
96	26 000	850	420	500	1 800	5	0	0†
	(*d*) Tap Water, with 1 Per Cent Sewage				(*h*) Sea Water, with 1½ Per Cent Sewage			
0	84 000	2 700	1 200	1 000	36 000	4 000	1 400	1 000
24	480 000	2 500	1 400	500	3 200	180	90	100
48	270 000	1 800	1 100	1 000	3 900	130	60	50
96	21 000	1 300	900	1 000	2 300	6	1	0*

* "Red" refers to the number of acid-forming bacteria. † *Coli-Aerogenes* present in 10 cubic centimeters.

VIABILITY OF SEWAGE ORGANISMS IN SALT WATER

Studies made by the Massachusetts Department of Public Health to determine the effect of polluted water on quahaugs, a mollusk of the hard-shell variety, showed that quahaugs obtained from polluted waters were comparatively free from organisms of the *Coli-Aerogenes* group after immersion for 48 hr in clean sea water. In order to ascertain if the disappearance of organisms of this group was due to the shellfish themselves or to the sea water, or both, experiments were conducted at the Lawrence Experiment Station of the Department of Public Health to determine the reduction of bacteria of the *Coli-Aerogenes* group in sewage when discharged into sea water. Similar experiments also were made to show the effect on these bacteria when discharged into the water drawn from a tap of the city of Lawrence. In these experiments, sewage was added to the sea water and the tap water in proportions of ½%, 1% and 1½% of the volume of water into which the sewage was added, and the samples were kept shielded from direct sunlight, the temperature varying from 50° to 70° F. The sewage alone and the untreated sea water were kept under the same conditions and analyses were made after 1, 2, and 4 days. The results of these tests (see Table 11) show that the total bacteria in the samples incubated at 20° and 37° C, as well as the organisms of the *Coli-Aerogenes* group, decreased very slowly in the sewage and the tap-water mixture, but very rapidly in the sewage and sea-water mixture. Because of this very considerable reduction in the numbers of bacteria in some instances, in the first 24 hr, the experiments were later continued by providing shorter periods of contact of the sewage with both the fresh and salt water. The latter results are shown on Fig. 12, a comparison between regular sewage and Nahant sea water being as follows:

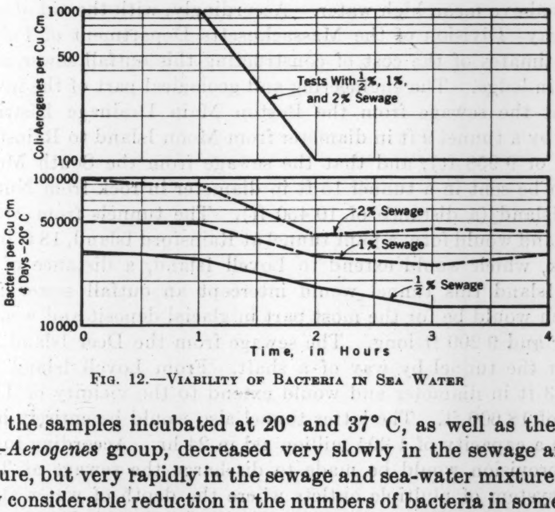


FIG. 12.—VIABILITY OF BACTERIA IN SEA WATER

Sample	Number of bacteria per cubic centimeter	Number of *Coli-Aerogenes* per cubic centimeter
Regular sewage............	5 300 000	100 000
Nahant sea water.........	770	1

REMEDIAL MEASURES CONSIDERED FOR ALLEVIATING POLLUTION OF BOSTON HARBOR

Although the discharge of sewage into Boston Harbor has not as yet caused conditions dangerous to the public health, objectionable floating matters characteristic of sewage may sometimes reach certain shores and, in the future, the demands of the public may require improvement.

To eliminate the pollution of Boston Harbor, either the sewage may be: (1) Discharged farther out to sea; (2) treated for the removal of suspended solids and grease before its discharge into the harbor; or, (3) treated to remove the solids and then conveyed by tunnel and discharged outside the harbor. Consideration was given to the construction of an outfall sewer which would intercept the sewage from the existing main sewer outlets—namely, the North Metropolitan outlet at Deer Island, the outlet of the Boston Main Drainage District at Moon Island, and that of the South Metropolitan outlet at Nut Island—and which would have a point of outfall in the Outer Harbor in the vicinity of The Graves. It was found to be impracticable to construct such an intercepting outfall sewer unless it was in the form of a tunnel, because Boston Harbor consists of a number of deep valleys filled with clay and other glacial deposits. Between these valleys are peaks of ledge which reach in some places above mean high water. Accordingly, with the aid of a geologist, the Waterways Division of the Massachusetts Department of Public Works prepared estimates of the cost of constructing this outfall sewer in the form of a tunnel in ledge. The engineering and geological part of the investigation showed that the sewage from the Boston Main Drainage District can be disposed of by a tunnel 9 ft in diameter from Moon Island to Rainsford Island (a distance of 9 950 ft); and that the sewage from the South Metropolitan District can be sent in a tunnel 15 ft in diameter in rock from Nut Island to Rainsford Island (a distance of 10 450 ft). The tunnels from Moon Island and Nut Island would form a joint tunnel at Rainsford Island, 18 ft in diameter, also in rock, which would extend to Lovell Island, a distance of 10 000 ft. At Lovell Island this tunnel would intercept an outfall sewer from Deer Island which would be for the most part in glacial deposit and would be 14 ft in diameter and 9 200 ft long. The sewage from the Deer Island connection would enter the tunnel by way of a shaft. From Lovell Island the tunnel would be 23 ft in diameter and would extend to the vicinity of The Graves, a distance of 18 600 ft. The latter tunnel also would be entirely in rock and would have a capacity of 1 205 million gal in 24 hr. According to the design proposed, provision would be made to discharge the sewage at The Graves through a system of multiple outlets where the depth of water at mean low tide is 30 to 35 ft. The estimated capitalized costs of these outlet works, including the radial outlets from the terminal chamber and all tunnel construction, but not including the cost of new pumping facilities, were $26 100 000. The capacity of these works which was for a flow of sewage amounting to 1 205 mgd is approximately five times the average daily flow of sewage from these districts in 1935.

Chips that were released in the vicinity of The Graves reached the shores of Boston Harbor and various points along the coast remote from The Graves, proving rather conclusively that, if the sewage is to be prevented from reaching the shores and causing objectionable conditions among bathers, it will be necessary, even with the outlet as remote as at The Graves, to treat it for the removal of grease and floating matter. Under the circumstances, the plan of extending the sewer outlets to the vicinity of The Graves would not appear to be justified until it is necessary to provide such works in connection with the disposal of treated sewage. Fig. 1 shows the location of the suggested outfall works to The Graves.

The most economical plan appeared to be to provide treatment works in the vicinity of the main sewer outlets at Deer Island, Moon Island, and Nut Island. When the treated sewage discharged from such works causes objectionable conditions in the harbor a program of progressive tunnel development can be provided which will permit the treated sewage to be discharged, in the beginning, in the vicinity of Rainsford Island. Float tests indicated that, near this island, sewage treated for the removal of grease and suspended wastes could be dissipated on either the incoming or outgoing tides before reaching any of the main ship channels.

Treatment works for the removal of suspended solids and grease would consist of bar screens and sedimentation tanks with skimming and sludge removal equipment. The skimming and sludge would be disposed of by barging to sea. The first cost of such works for the three districts, including the cost of operation and the cost of disposing of the sludge, grease, and other floating matter, together with the cost of additional pumping, is shown in Table 12.

TABLE 12.—FIRST COST OF TREATMENT WORKS

Description	Boston Main Drainage System	North Metropolitan Sewerage System	South Metropolitan Sewerage System
First cost of works consisting of bar screen with mechanical cleaning devices, sedimentation tanks with skimming and sludge removal equipment	$2 907 000	$ 5 539 850	$ 6 023 100
Capitalized cost of operation and disposal of skimmings, screenings, and sludge by barging to sea, to 1955*	3 300 000	7 190 000	7 560 000
Annual charges	(123 630)	(269 910)	(283 290)
Total cost of partial treatment to 1955	$6 207 000	$12 729 850	$13 583 000

* Interest at 3¾ per cent.

SLUDGE DISPOSAL

The problem of sludge disposal is usually the most difficult one to be solved in the treatment of sewage. The location of the proposed plants on islands in Boston Harbor points at once to the disposal of sewage sludge by barging to sea; and this would undoubtedly be the most economical method, especially if the sludge could be carried to sea as cheaply as is done by the Passaic Valley (N. J.) Sewerage District where the cost is only 14.7 cents per wet ton. In Boston, operation of sludge steamers or barges by the authorities in charge of sewage disposal would probably be less expensive than a contract, as sea-going ships of the type necessary for this work are not now available in Boston.

Representative samples of sludge from the three systems were analyzed by the Massachusetts Agricultural Experiment Station and the organization reported that the fat content is rather high and that the product is rather poorly balanced as a fertilizer for most crops, as it contains too much nitrogen as compared with the phosphoric acid and potash. For many hoed crops the product used at the rate of 1.5 tons per acre would be properly balanced by the use of about 1 200 to 1 400 lb of 16% super-phosphate and 240 to 300 lb of muriate of potash per acre.

Although the park departments of the city of Boston and the Metropolitan District Commission were not interested in the use of sewage sludge as fertilizer, provision should be made in designing the treatment plants for proper elevations and sufficient area for the preparation of sludge as fertilizer if it should become desirable.

SUMMARY

This study shows that the quantity of putrescible matter discharged into the harbor is not sufficient as yet to cause a nuisance resulting from the depletion of oxygen. Objectionable sleek areas, however, are often present over extensive portions of the harbor and, under certain tide and wind conditions, grease and other floating matters, characteristic of sewage, reach the shores. At times of considerable run-off, sewage overflows from the main trunk and lateral sewers and reaches shores used for bathing.

Bacterial examinations do not show the presence of bacteria characteristic of sewage pollution in the waters of the bathing beaches in such numbers as would warrant the prevention of bathing except in the vicinity of sewer overflows and at a few places where sewage is discharged directly into the harbor.

To correct these unsatisfactory conditions it will be necessary to discontinue the use of certain areas as bathing beaches or to make changes in the sewerage systems by moving the sewer overflows to points remote from the shore and by treating the sewage reaching the main outlets for the removal of matters that might reach the bathing places.

Although the discharge of sewage into Boston Harbor from the main sewer outlets may not as yet cause conditions dangerous to the public health, the nuisance from sleek and floating solids makes a change in the present methods of sewage disposal very desirable.

The most economical method seems to be the removal of suspended and floating solids in sedimentation tanks provided with skimming devices and the transportation of the solids and grease to sea in barges. More complete treatment and the extension of the present outlets at some future time, when larger sewage flow makes it necessary, are the logical steps in the disposal of the sewage of Metropolitan Boston.

The extension of the outlet systems by a tunnel to a site in the vicinity of The Graves without preliminary treatment of the sewage seems impracticable because of the high cost and because floating solids would probably still reach the neighboring shores.

DISCUSSION

E. SHERMAN CHASE,[7] M. AM. SOC. C. E. (by letter).—A number of important points relating to the sewerage and drainage of the Boston Metropolitan Area are presented in this excellent paper. Among these points are the distinct limits of capacity life of the drainage systems, in the increasing frequency of overflows as time goes on, the effect of suburban growth and transient populations upon the sewerage problems, the limitations of analytical data, standards of cleanliness for bathing beach waters, the relative economic merits of the proposed extension of the outlets, and the treatment of the sewage without the extension. The writer will limit himself to certain general points of public interest instead of discussing the more technical features of the paper.

From the very beginning of its organization, the Massachusetts State Department of Public Health (and its predecessor, the State Board of Health) has conducted intensive and noteworthy investigations into matters of pollution by sewage, and into methods of sewage treatment. The second annual report of the old State Board of Health, covering the year 1870, contains a most interesting letter from the chairman of the Board, Dr. H. I. Bowditch, entitled "Houses for the People, Convalescent Homes and the Sewage Question"—a somewhat curious combination. In this letter, in which several references are made to his visit to England and the continent in 1870, considerable space is devoted to the controversy, then raging abroad, over the earth closet system *versus* the water carriage system for removal and disposal of excretal matters—a controversy now forgotten.

It is also interesting to note, in light of the consideration given by the authors to the possible production of fertilizer from sludge obtained by treatment of the Boston sewage, the following paragraph from Dr. Bowditch's letter:

> "But there is one fatal defect of London and of all American sewage, and that is its waste.. Probably there is no such widespread recklessness of spendthrift prodigality anywhere so noticeable among civilized nations as this throwing away of such vast amounts of this most excellent of manures. We take thousands of tons from the earth annually, and totally ignoring Nature's law of economy, which declares that what has been once taken away must be returned again to earth, otherwise the earth itself will become improverished and will refuse to labor for us, I say totally ignoring this law, we squander an immense amount of really valuable property."

Dr. Bowditch was not the first nor the last to feel that way; but after sixty-nine years sanitary engineers have not yet reached the goal of economic use of sewage for its manurial value, as is evidenced by the very tentative way in which the authors discuss the possibility of utilizing sludge for fertilizer.

The interest displayed in the sewage problem by the State Board of Health undoubtedly was one of the reasons, at least, for the construction of the Main Drainage System of Boston and of the Metropolitan Systems. With the introduction of the Cochituate supply of water into Boston in 1848 there had been a marked improvement in sanitary conditions within the city; but objectionable conditions in the adjacent waters resulted from the discharge of sewage from sewers and drains at many points along the water-front. As time went on, the progressive improvements in the collection, interception, and discharge of sewage from the city and the metropolitan districts eliminated the more objectionable conditions, irrespective of the steady growth of population and industry. In fact (as the authors show), based on the criterion of the oxygen balance, the waters of Boston Harbor have ample capacity to assimilate much more sewage than now reaches them.

---

[7] Cons. Engr. (Metcalf & Eddy), Boston, Mass.

On the other hand, the fact that solids from the sewers can reach the shores, and the striking evidence afforded by Figs. 3, 4, and 11, show clearly that filthy conditions exist in parts of Boston Harbor. Furthermore, in spite of the difficulty in proving a specific health hazard from polluted waters, other than those used for water supplies, there is a definite trend of public opinion which is resulting in a demand for a standard of cleanliness consistent with ordinary good housekeeping. A similar trend has already been noted for some years in the water-supply field where the public now insists upon supplies of attractive qualities as well as safe from the standpoint of health. The authors have recognized this trend in their conclusions with respect to the undesirability of grease balls and other materials of sewage origin reaching the bathing beaches.

It must be remembered, however, that the correction of conditions of pollution costs substantial sums. In the last analysis the people can have just what they are willing to pay for, and when demands for improved conditions are made, those who make these demands must be prepared and willing to pay their share of the costs. The sanitary improvement of Boston Harbor is a type of public works project that might better be undertaken than many for which the taxpayers' money has been spent in recent years.

F. E. DeMartini,[8] Assoc. M. Am. Soc. C. E. (by letter).—In many respects this excellent paper could apply as a description of pollution in San Francisco Bay, in California. It may be of interest to outline briefly the situation there.

Until February, 1939, the untreated sewage of San Francisco was discharged to San Francisco Bay through twenty-eight outlets. Approximately half of these outlets discharged sanitary sewage and the others discharged storm water and varying quantities of sanitary sewage, depending upon the extent and intensity of rains. Three of the outlets—North Point, Bakers Beach, and Mile Rock—discharged the sanitary sewage from the largest proportion of the city's population. Local nuisances at some of the smaller outlets are critical, particularly in the ship channel parallel to Channel Street, and in Islais Creek. Sludge deposits have been extensive here in the past, resulting in extensive odors at low tide.

In 1932 a 1.0 mgd activated sludge sewage treatment plant was constructed by the Park Department, in Golden Gate Park, to reclaim sewage for irrigation of park lands and sludge for use as fertilizer. This was the first treatment plant of any importance built in the city.

---

[8] Associate Public Health Engr., U. S. Public Health Service, Cincinnati, Ohio.

In February, 1939, the 15.0 mgd Richmond-Sunset Sewage Treatment Plant started operation. At times of heavy rains and diluted sewage the plant is designed to handle peak flows of 55.0 mgd. This is a primary treatment plant for removal of grit, grease, and sludge. Chlorinating equipment is provided for the control of odors and disinfection of plant effluent. Briefly the factors leading to the construction of this first major treatment plant were as follows:

Disposal of raw sewage direct to the Bay waters resulted in (1) heavy pollution of a number of bathing beaches, (2) unsightly sleek areas, and (3) floating sewage solids and odors at many points along the bulkhead line on the water front. Practically all areas from which clams and oysters had been taken on the San Francisco side of the Bay were closed by the California State Department of Public Health in recent years due to increasing pollution of the Bay waters.

The complacent attitude of the city toward this pollution of San Francisco Bay underwent a gradual change starting about 1931. Invest gations by the city and State health departments were the principal stimuli to the new viewpoint. These investigations definitely indicated the gross pollution at Bakers Beach, Aquatic Park, and Yacht Harbor in the Marina. These recreational areas are used extensively by large numbers of people.

Conditions at China Beach were studied by the California State Department of Public Health in connection with a project to make the beach and adjacent lands a State and city park. Shortly thereafter a bacteriological study of the bay waters along the northerly and westerly shore line of the city (from pier 45 to the Funston Life Guard Station) was undertaken for one year by the San Francisco Department of Public Health. Both of these investigations indicated the serious extent of the pollution and the magnitude of the problem involved in its elimination. By 1934 interest had been aroused to such an extent that a board of consulting sanitary engineers was appointed by the city to study the problem of treatment and disposal of sewage from the city and county of San Francisco. In May, 1935, this board submitted a report and recommendations to the city.

A comprehensive plan was presented providing for the concentration of the city's sewage at a few points and for treatment of sewage before disposal into San Francisco Bay and the Pacific Ocean. In accordance with this plan sewage from the Marina District has been pumped to the North Point Outlet since 1937, and the Richmond Sunset Sewage Treatment Plant was completed in February, 1939. The sewage from Bakers Beach District, Richmond District, and Sunset District is treated at the new Richmond-Sunset Sewage Treatment Plant, prior to disposal into the Pacific Ocean at the Mile Rock Outlet. These developments will greatly improve conditions at a large number of small beaches and particularly at the Marina, Bakers Beach, and China Beach.

In addition to San Francisco there are a large number of communities discharging raw sewage to San Francisco Bay. A few places, among which are Burlingame, San Mateo, Palo Alto, and Petaluma, have built treatment plants, but there remain a large number of places that are seriously polluting the bay waters. There is no doubt that the program undertaken by the city of San Francisco, involving the expenditure of several million dollars, will have a salutary effect on the attitude of these communities toward elimination of their pollution load from the Bay.

Many details of the studies reported by the authors for Boston Harbor and the general recommendations for treatment and improvements are strikingly similar to those of the San Francisco Bay area.

A. M. RAWN,[9] M. AM. SOC. C. E. (by letter).—The authors' description of the investigation of the pollution of Boston Harbor and of corrective measures proposed raises a point or two in the writer's mind which, in connection with a problem of this nature, may merit some discussion.

It is not stated in the narrative whether the tubes from which data were gathered for preparation of the "isocols" in Fig. 9 were set immediately after the samples of mixed sewage and sea water were taken, or were iced and set later. The procedure here may have an important bearing upon conclusions because of the fact that sea-water plankton destroy bacteria quite rapidly, and time becomes an important factor in arriving at the number of tubes positive. This is well illustrated in Fig. 12. If the samples were set immediately after they were taken, the "isocols" shown in Fig. 9 may be said to represent a fairly true degree of pollution. However, if they were iced and set some hours later, a situation more aggravated than that shown in Fig. 9 might exist.

As an alternative to discharging sewage at the Graves, the authors propose plain subsidence works for the Boston Main and the North and South Metropolitan Sewerage Systems. It is not clear to the writer that construction of plain subsidence works at the points indicated by the authors, followed by the discharge of settling-tank effluent at present outfalls, will have any great effect upon the spread of contamination over the surface of Boston Harbor waters. In all probability, it will prevent the washing of objectionable material identified as of sewage origin on to the beaches; but the sewage discharged at or near the surface or through the sea outfalls, at present locations, will probably spread over and pollute as great an area within the harbor as is now the case with discharge of raw sewage. The fact that the harbor mud is polluted to the extent indicated in the authors' description seems to testify that a large proportion of the suspended material in the sewage settles quite promptly in the comparatively still harbor waters, as is to be expected. Such being the case, surface pollution, even at the present time, must be due largely to the spread of mixed sewage and sea water from which much of the suspended matter has settled. The writer's criticism is entirely invalid if plans contemplate the sterilization of the effluent prior to final disposal.

HARRISON P. EDDY, JR.,[10] AND SAMUEL A. GREELEY,[11] MEMBERS, AM. SOC. C. E. (by letter).—An excellent description of one of the earliest comprehensive sewerage systems, and of conditions in Boston Harbor after these works had been in service many years, is presented in this paper. The authors are to be complimented on the completeness and thoroughness of their work.

---

[9] Asst. Chf. Engr., Los Angeles County Sanitation Dists., Los Angeles, Calif.

[10] Cons. Engr. (Metcalf & Eddy), Boston, Mass.

[11] (Greeley & Hansen), Chicago, Ill.

This area has been most fortunate in having available favorable tidal currents. Although the photographs in the paper give the appearance of an extremely unclean condition on the surface of the harbor waters from the Moon Head outlet, the intermittent discharge and the prompt removal and dispersion of the sewage by the currents have made possible the operation of this system for about fifty years without creating intolerable conditions. At the Nut Island outlet, the submergence of the outlets tends to offset the continuous discharge; and at Deer Island submergence, strong tidal currents, and proximity to the ocean are factors that have helped to reduce the unfavorable effects of pollution.

Under the provisions of a resolve of the Massachusetts Legislature of 1938, a special commission was appointed to make an investigation relative to the systems of sewerage and sewage disposal in metropolitan Boston. As a part of its investigation, this commission retained the writers' respective offices as consulting engineers to investigate and report upon the following two major subjects:

(*a*) Pollution in the Mystic River, and its major tributaries, and in the Charles River, and methods of correction; and

(*b*) Proposed methods of sewage treatment for the main outlets in Boston Harbor of the North and South Metropolitan Sewerage Districts and the Boston Main Drainage System.

In making their investigation and report, the engineers reviewed House Document No. 1600, December, 1936,[3] the testimony gathered by the special commission, and many other documents, and made personal observations of metropolitan Boston and the harbor, and of the main structures of the sewerage systems. Conclusions pertinent to the paper under discussion are as follows:

Mystic and Charles Rivers.—Objectionable conditions exist, at times, in the Mystic River, and its tributaries, and in the Charles River, due to pollution from sewage. This pollution occurs because, in times of storm, the capacity of the existing intercepting sewers is insufficient to carry away the sewage and storm water that are delivered to them, and the excess beyond their capacity is discharged into the streams through overflow channels provided for this purpose.

Correction of the objectionable conditions in the Mystic River, and its tributaries, and in the Charles River can be obtained best and most economically by the construction of storm overflow conduits of sufficient capacity so that no overflows into the streams will occur until the volume of storm water flowing in the sewers has provided sufficiently great dilution of the domestic sewage to prevent objectionable conditions from such overflows. If these recommendations are followed, it should no longer be necessary for Boston and a few other municipalities to separate storm water and sewage in areas tributary to the Charles River, as now required, when new sewers are built or old sewers are replaced.

The existing Mystic Valley sewers were relieved by the construction, in 1936 and 1937, of the North Metropolitan relief sewer from Reading, Mass., to a point a short distance below Craddock Dam in Medford, Mass. This relief sewer is now (1940) being extended to the East Boston pumping station; and it is proposed by the consulting engineers that the North Metropolitan relief sewer be extended from East Boston to Deer Island. The estimated cost of construction of the storm overflow conduits, pumping stations, and force mains recommended, including the extension of the North Metropolitan relief sewer to Deer Island, is $14 695 000.

Pollution of Boston Harbor.—To aid them in reaching a conclusion as to the limiting effect of sewage in Boston Harbor upon the use of the waters, and as to the need for sewage treatment, the consulting engineers have adopted a standard for the desirable condition of the harbor waters, after consideration of standards proposed and adopted elsewhere and of the existing sanitary conditions. They have assumed that, for the bathing beach waters of the harbor, a single *B. coli* index of 1 000 per 100 cc is a warning that excessive contamination may be present at times, and that an average index in excess of 3 000 *B. coli* per 100 cc shows that the waters may be dangerous to the public health.

In examining the bacteriological analyses made by the Massachusetts Department of Public Health during the investigations of Boston Harbor in 1929–1930, and 1935–1936, the consulting engineers made a large number of charts showing the distribution of the *B. coli* index throughout the harbor. One of these, Fig. 13, has been selected as giving a representative picture of the average or general bacteriological condition of the harbor. It shows the average and maximum *B. coli* indices of all samples collected at each sampling station during the 1935 investigation, as well as the areas where the *B. coli* index was probably in excess of 3 000 per 100 cc and those where there was a probable *B. coli* index between 1 000 and 3 000 per 100 cc.

It is of interest to note that, in general, the number of days on which samples were taken was five to ten at each station during each of the years 1935 and 1936. During 1935, the prevailing winds on the sampling days were from the west and southwest, and during 1936 they were from the east and southwest. The higher *B. coli* indices occurred when the wind was blowing from the north and east. There is a possibility, therefore, that if all samples had been taken on days when the wind was blowing from the north or northeast, the average of the analyses would have shown much higher *B. coli* indices for the shore waters.

The consulting engineers have come to the conclusion that the bacterial content of the shore waters of the harbor, as evidenced by the *B. coli* index, in some places is at times higher than what is generally considered a reasonable upper limit for the safety of bathing beach waters. Furthermore, the proximity to the shore, of harbor waters that are highly contaminated with raw sewage from the outlets, is a potential source of excessive bacterial pollution which is certain to increase in the future. In addition, the stirring up of the mud at the bottom of the harbor, which was found to contain many more bacteria characteristic of pollution than did the water at the surface, by strong onshore winds, is a potential source of excessive bacterial pollution. Present (1940)

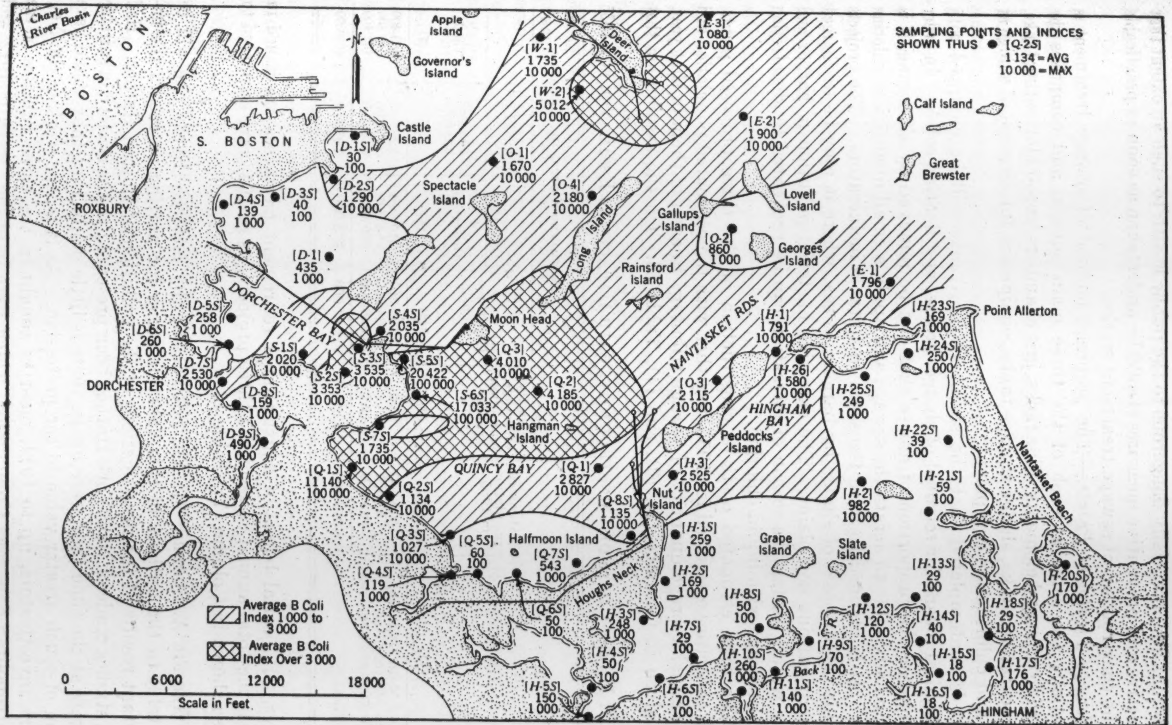

Charles River Basin
BOSTON
S. BOSTON
ROXBURY
DORCHESTER
DORCHESTER BAY
Apple Island
Governor's Island
Castle Island
Spectacle Island
Long Island
Moon Head
Deer Island
Rainsford Island
Gallups Island
Lovell Island
Georges Island
Calf Island
Great Brewster
NANTASKET RDS.
HINGHAM BAY
Peddocks Island
Hangman Island
QUINCY BAY
Nut Island
Halfmoon Island
Houghs Neck
Grape Island
Slate Island
Back R
Point Allerton
Nantasket Beach
HINGHAM
SAMPLING POINTS AND INDICES SHOWN THUS ● [Q-2S] 1 134 = AVG 10 000 = MAX
[D-1S] 30 100
[D-2S] 1 290 10 000
[D-3S] 40 100
[D-4S] 139 1 000
[D-1] 435 1 000
[D-5S] 258 1 000
[D-6S] 260 1 000
[D-7S] 2 530 10 000
[D-8S] 159 1 000
[D-9S] 490 1 000
[S-1S] 2 020 10 000
[S-2S] 3 353 10 000
[S-3S] 3 535 10 000
[S-4S] 2 035 10 000
[S-5S] 20 422 100 000
[S-6S] 17 033 100 000
[S-7S] 1 735 10 000
[W-1] 1 735 10 000
[W-2] 5 012 10 000
[E-3] 1 080 10 000
[E-2] 1 900 10 000
[E-1] 1 796 10 000
[O-1] 1 670 10 000
[O-4] 2 180 10 000
[O-2] 860 1 000
[O-3] 2 115 10 000
[Q-3] 4 010 10 000
[Q-2] 4 185 10 000
[Q-1] 2 827 10 000
[Q-1S] 11 140 100 000
[Q-2S] 1 134 10 000
[Q-3S] 1 027 10 000
[Q-4S] 119 1 000
[Q-5S] 60 100
[Q-6S] 50 100
[Q-7S] 543 1 000
[Q-8S] 1 135 10 000
[H-1] 1 791 10 000
[H-2] 982 10 000
[H-3] 2 525 10 000
[H-26] 1 580 10 000
[H-1S] 259 1 000
[H-2S] 169 1 000
[H-3S] 248 1 000
[H-4S] 50 100
[H-5S] 150 1 000
[H-6S] 70 100
[H-7S] 29 100
[H-8S] 50 100
[H-9S] 70 100
[H-10S] 260 1 000
[H-11S] 140 1 000
[H-12S] 120 1 000
[H-13S] 29 100
[H-14S] 40 100
[H-15S] 18 100
[H-16S] 18 100
[H-17S] 176 1 000
[H-18S] 29 100
[H-20S] 170 1 000
[H-21S] 59 100
[H-22S] 39 100
[H-23S] 169 1 000
[H-24S] 250 1 000
[H-25S] 249 1 000
▨ Average B Coli Index 1 000 to 3 000
▩ Average B Coli Index Over 3 000
0 6 000 12 000 18 000
Scale in Feet
N

FIG. 13.—AVERAGE AND MAXIMUM *B. COLI* INDICES OF ALL SAMPLES; 1935 SERIES

conditions are potentially dangerous to the public health to such an extent that corrective and preventive measures should be undertaken as soon as practicable by the construction of sewage treatment works.

Sewage Treatment Plants.—The proposed method of sewage treatment is essentially the same at each of the three main outlets, and comprises the following principal elements: Fine racks, grit chambers, aero-chlorinating tanks, sedimentation tanks, sludge storage tanks, equipment for disposal of sludge at sea, and chlorination equipment.

It is contemplated that the sedimentation tanks will be equipped with mechanical means for removal of sludge, grease, and floating solids. In order to increase the proportion of grease that can be removed by skimming, it is deemed advisable to aerate the sewage and apply a small quantity of chlorine to it. It is planned to burn the scum in suitable incinerators. The projects include the disposal of sludge by barging to sea, where it may be discharged without causing complaint. The excessive bacterial pollution of harbor and shore waters during the season of recreation can be prevented by disinfection of the sewage with liquid chlorine.

The total estimated costs of construction of the three proposed plants, with capacities estimated for 1955, together with the estimated population and the annual average quantity of sewage tributary to each of them in that year, are given in Columns (2), (3), and (4), Table 13. On the assumption that 20-yr, 2¼% serial bonds of equal annual retirement payments are issued in 1941, the total annual costs of the three projects, as of 1943, are estimated to be as shown in Column (7), Table 13.

TABLE 13.—COSTS OF CONSTRUCTION, WITH CAPACITIES ESTIMATED FOR 1955

Outlet	Estimated population in 1955	Average quantity of sewage, in million gallons daily	ESTIMATED COSTS, IN DOLLARS			
			Construction	Annual Costs		
				Operation	Capital charges	Total
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Boston Main Drainage....	400 000	85.0	3 180 000	240 000	223 400	463 400
North Metropolitan......	750 000	125.0	3 784 000	284 000	265 800	549 800
South Metropolitan.......	900 000	112.5	3 058 000	249 000	214 400	463 400
Total...............	2 050 000	322.5	10 022 000	773 000	703 600	1 476 600

GEORGE A. SOPER,[12] M. AM. SOC. C. E. (by letter).—Every one will be grateful to the authors of this valuable paper for putting on record the important results of the investigations of the pollution of Boston Harbor. Had the studies made of New York Harbor been presented to the Society in suitable papers at the time they were made, there is little doubt that they would have been of much greater value to the profession. As it is, the work of the New York investigators, up to 1914, must be sought in the three volumes of the Metropolitan Sewerage Commission, which were published in 1910, 1912, and

---

[12] Cons. Engr., New York, N. Y.

1914, and as these editions were limited, the information is not always to be found even in important libraries.

The conditions attending the discharge of sewage into Boston Harbor have been of much interest in connection with the investigations which have been made of New York Harbor.

The New York Bay Pollution Commission, which existed between 1903 and 1906, took careful account of the conditions as they were reported in the official reports and looked upon them as a warning of conditions which should never be allowed to occur in the harbor of New York.

Some years later the members of the Metropolitan Sewerage Commission of New York went to Boston and visited the Deer Island, Moon Island, and Peddocks Island outlets, to see for themselves the results of discharging sewage under the three methods used. The reasons for the works and what they were intended to accomplish had been made familiar to them through reports and correspondence. The chief engineer of the Massachusetts State Board of Health was invited to consult on the New York work, because of his familiarity with the disposal of the sewage of Boston and vicinity.

Not being able otherwise to obtain all the information wanted, the Metropolitan Sewerage Commission sent its floating laboratory, *Quadrant*, to Boston to make studies, especially of dissolved oxygen, by the identical method which had proved successful in the Commission's hands in studying the digestive capacity of New York Harbor for sewage.

Throughout its eight years of existence, the Metropolitan Sewerage Commission recognized that it would not be necessary nor practicable to keep all sewage out of New York Harbor, but that the harbor had a large capacity for assimilating sewage, harmlessly and inoffensively, provided the principles under which the digestive capacity was utilized were thoroughly understood and skilfully employed.

Proper diffusion was indispensable to digestion and many experiments designed to "throw light" upon this subject were made in the laboratory, in large tanks courteously set aside in the New York Aquarium by the New York Zoological Society, and by means of properly equipped boats, and pontoons filled with sewage on Long Island Sound and in various parts of New York Harbor. There were parts of the harbor itself which were looked upon as affording exceptional opportunities for investigating the phenomena of diffusion and digestion, and as these had the merit of practical application, they received very careful attention. Before the investigations of the Metropolitan Sewerage Commission were concluded, the entire harbor of New York came to be looked upon in the light of a great laboratory where experiments on a huge scale were being conducted.

ARTHUR D. WESTON,[13] AND GAIL P. EDWARDS,[14] MEMBERS, AM. SOC. C. E. (by letter).—In closing the discussion, attention is called to the remarks by Mr. Rawn, in which the question has been raised as to the effect on the bacterial counts of the length of time which elapsed between the collection of the samples and the "setting" of the samples, the results of which are shown in Fig. 9.

---

[13] Chf. Engr., Dept. of Public Health, State of Massachusetts, Boston, Mass.

[14] Prin. San. Chemist, Wards Island Sewage Treatment Plant, Dept. of Public Works, City of New York, Wards Island, N. Y.

Laboratory records show that, of all the samples collected in 1936, 47% of the fermentation tubes were set the same day that the samples were collected, 50% were set the next day (usually in the forenoon), and only 3% of the samples were more than 24 hr old before setting. Confirmed tests were made on all positive fermentations.

The close correlation between the results of the bacterial examinations and the sanitary surveys would lead one to believe that any error from delay in setting has not affected the findings seriously, and to all practicable purposes these results are a fair indication of the coliform density of the waters in various parts of Boston Harbor. Undoubtedly, the bacteria density of tidal waters is affected by the plankton, but the laboratory of the Massachusetts State Department of Public Health has no definite information as to such relationship. Accordingly, without supporting data the effect of the plankton is speculative. Regardless of the killing, by plankton or other causes, one would have to assume that, if the killing were as Mr. Rawn states, the harbor waters inside of the point of outfall on the outgoing tide would be nearly free from coliform organisms. Fig. 12 shows that most of the killing is done in the first 2 hr and that there is little effect after that period of time. An examination of Table 11 shows that in accordance with laboratory experiments conducted with sea water and 0.5% of sewage the same number of coliform organisms was present in the sample at the end of 24 hr as at the start. This number diminished to practically 0 at the end of 96 hr; but in the case of the 1% sewage with sea water, the reduction was from 500 to 100 in 24 hr and again to about 0 at the end of 96 hr. With 1.5% sewage added to sea water the reduction was from 1,000 to 100 in the first 24 hr, 50 at the end of 48 hr and 0 at the end of 96 hr. The results of tests made in 1935 by Paul J. Beard and Niel F. Meadowcroft of Stanford University, at Stanford University, Calif., show that 60% of *Escherichia Coli* survive for a period of one day and at the end of ten days more than 3% are still present in the water.[15] Undoubtedly there is a need for further research in the matter of the effect of plankton or other causes on coliform organisms in sea water.

It should be borne in mind that, in conducting any studies such as that in Boston Harbor, many inaccuracies can enter the study. For example, with the use of one boat a number of hours may elapse between the collection of some samples; and during this period of collection change of winds and variations in tidal conditions may play an important part. Furthermore, it should be stated that the results shown illustrate only the conditions found at the time the samples were collected; undoubtedly at other times objectionable conditions may be more pronounced. Taking all of these things into consideration, the writers feel, therefore, that the results illustrate, unusually well, the conditions disclosed by the sanitary survey.

Relative to the effect of partial treatment of the sewage before it is discharged into the harbor, on the bacterial content of the water of the harbor, it should have been stated that the recommendations do provide for chlorination of the settled sewage.

The various discussions by Messrs. Chase, Eddy and Greeley, DeMartini, Rawn, and Soper are greatly appreciated.

---

[15] *American Journal of Public Health*, Vol. 25, No. 9.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2079

SIMPLIFIED WIND-STRESS ANALYSIS OF TALL BUILDINGS

BY OTTO GOTTSCHALK,[1] ESQ.

WITH DISCUSSION BY MESSRS. SAMUEL T. CARPENTER, CHARLES B. WINICK, AND OTTO GOTTSCHALK.

SYNOPSIS

The simple method of analysis explained herein is based on experiments to determine visible model deformations[2] and on studies leading to a purely geometrical interpretation of them.[3] The subject has been outlined by the writer several times since 1934[4] in computing inflection points under vertical loads. A structural frame is analyzed by subjecting it to given displacements. The resulting deformations are multiplied by the given loads in order to obtain the stresses.[5] This process of interpreting the model experiment directly avoids the need of solving innumerable abstract equations, by introducing only the simple expressions for the deformation curves. For one-story portal frames, with relatively stiff beams and fixed or hinged column supports, simple formulas were developed by the writer in 1932.[6] The method presented herein applies to frames of any number of stories with any number of bays and with beams and columns having any constant section.

NOTATION

The letter symbols used in this paper are defined herein when first introduced in the text. They are consistent with those introduced in a prior paper[3] and conform essentially with American Standard Symbols for Mechanics, Structural Engineering, and Testing Materials,[7] compiled by a committee of the American Standards Association, with Society representation and approved by the Association in 1932.

NOTE.—Published in December, 1938, *Proceedings*.

[1] Buenos Aires, Argentine Republic.

[2] *Journal*, Franklin Inst., July, 1926, and February, 1929.

[3] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1019.

[4] *Civil Engineering*, May, 1934, p. 264, and February, 1935, p. 101; see, also, *Civil Engineers' Digest*, 1935, No. 1, pp. 1 and 4.

[5] *Engineering News-Record*, February 27, 1936, p. 325.

[6] *Beton u. Eisen*, 1932, No. 16, p. 252.

[7] A. S. A.—*Z* 10*a*—1932.

THEORY

In order to compute stresses due to wind, earthquake, or other lateral loads, the structures must be observed to determine the lateral displacements of the floors. The resistance of a column to that displacement (that is, the share of the total horizontal loads that it will absorb) is proportional to its stiffness multiplied by the sum of the relative resistance to rotation of all the members meeting at a joint. For lateral loads geometrical analysis may be greatly simplified because any approximation can be introduced that will affect, almost equally, all the members of the part of the frame under consideration.

Let k = the stiffness, $\frac{I}{h}$, of Column $A\,B$; I = the moment of inertia of the column, of height, h; m = a ratio of end rotations such that[8] $m = \frac{f}{f'} = \frac{0.5\,k}{S_B + k}$ and $m' = \frac{f'}{f} = \frac{0.5\,k}{S_A + k}$; S = a relative stiffness value such that the resistance to rotation of End A of a column is:

$$S = (1 - 0.5\,m)\,k \qquad (1a)$$

and, the resistance to rotation of End B, is:

$$S' = (1 - 0.5\,m')\,k \qquad (1b)$$

f = the intercept at End A of the tangent to the elastic curve at End B; thus,

$$f = \theta_A\,h \qquad (2a)$$

and,

$$f' = \theta_B\,h \qquad (2b)$$

θ = the angular rotation of the column relative to the final direction of its axis; thus θ_A is the rotation at End A (Fig. 1), with respect to Line $A\,B'$, and θ_B is the rotation at End B' with respect to Line $B'\,A$; and the height of the point of contraflexure is:

$$h_A = \frac{h\,(2 - m)}{3\,(1 - m)} \qquad (3)$$

Furthermore:[9]

$$\frac{f}{\Delta} = \frac{-\,S_B}{S_B + k} + \frac{m\,S_A}{S_A + k} \qquad (4a)$$

and,

$$\frac{f'}{\Delta} = \frac{S_A}{S_A + k} - \frac{m'\,S_B}{S_B + k} \qquad (4b)$$

and the force, P, necessary to produce Displacement $\Delta = 1$, is:

$$P = \frac{6\,E\,k\,(f' - f)}{h^2} \qquad (5)$$

---

[8] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1022, Equations (1) and (2).

[9] *Loc. cit.*, Equations (11), (12), and (15), pp. 1025, 1027.

In Equation (5) $\frac{6E}{h^2}$ is equal for all the columns of any one story. The horizontal resistance, therefore, against a unit side-sway (that is, the percentage that each column of a story absorbs of a horizontal force acting upon the floor above) is in proportion to $k\,(f' - f)$. None of the many approximate methods of wind-stress analysis of frames proposed heretofore yields safe results, unless it takes that quantity into consideration. The complexities introduced by poor proportioning of columns were emphasized by George E. Large, Assoc. M. Am. Soc. C. E., and Samuel T. Carpenter, Jun. Am. Soc. C. E., in 1934.[10]

Equations (4) may be considerably simplified for typical stories. In Fig. 1, when Column $A\,B$ is fixed at one end (that is, when $S_B = \infty$) $m = 0$ and $S = (1 - 0.5\,m)\,k = k$ at the other end.[3] As an opposite extreme assumption, that is, when $S_B = k$, the corresponding values are: $m = 0.25$ and $S = (1 - 0.5 \times 0.25)\,k = 0.875\,k$. The range of values of m and $\frac{S}{k}$, then, is small. For typical stories, therefore, it may safely be assumed that the conditions of rigidity at the tops of columns are sufficiently similar to those at the bottom so that, in all structural frames, for lateral movements of floors, $m = 0$ and $S = k$. Simplifying Equations (4) accordingly:

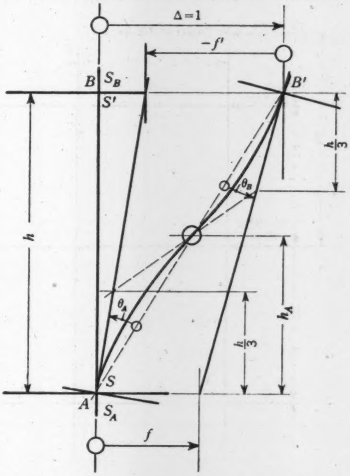


FIG. 1.—DEFORMATION OF A COLUMN DUE TO UNIT DISPLACEMENT OR SIDE-SWAY, AT TOP.

$$\frac{f}{\Delta} = \frac{-\,k_B}{k_B + k} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6a)$$

and,

$$\frac{f'}{\Delta} = \frac{k_A}{k_A + k} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6b)$$

in which k_A and k_B, instead of S_A and S_B are the sum of the stiffness, $k = \frac{I}{h}$, of the structural members joining Line $A\,B$ at Points A and B, respectively. For the purpose of the present analysis, therefore, one may read the assumed k-values directly instead of computing the value of S first.

Note that, in Equations (6), f and f' have different signs and, therefore, they must be added in Equation (5).

As a further admissible simplification, when analyzing any one typical story, the designer may disregard the columns above and below that story.

---

[10] *Engineering News-Record*, May 17, 1934, p. 637.

With all those simplifications, which affect the deformations of all columns in nearly equal proportion, results have proved to be quite close to those found by the "exact" abstract analysis—safely within 5% for shears and reactions, and little more for bending moments when assuming inflection points at the middle of the column heights of typical stories.

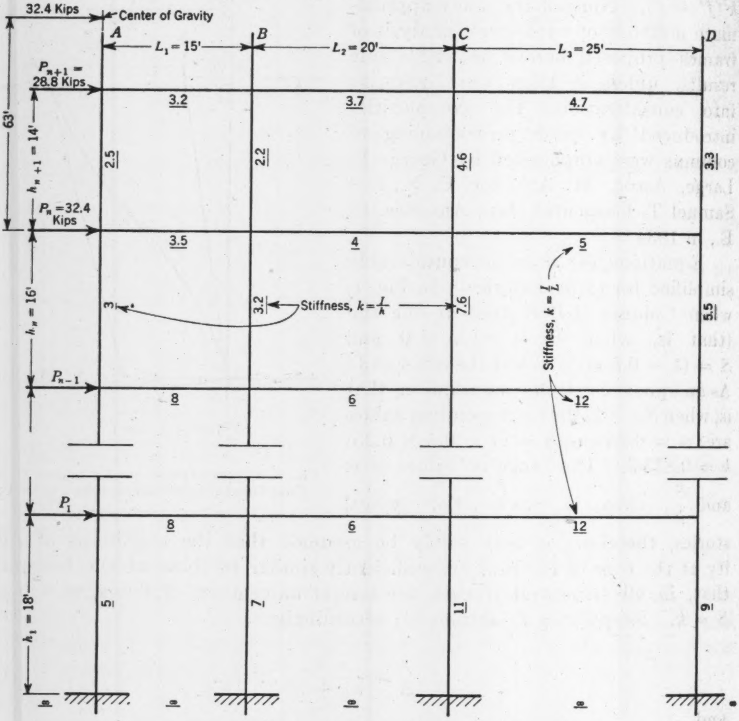
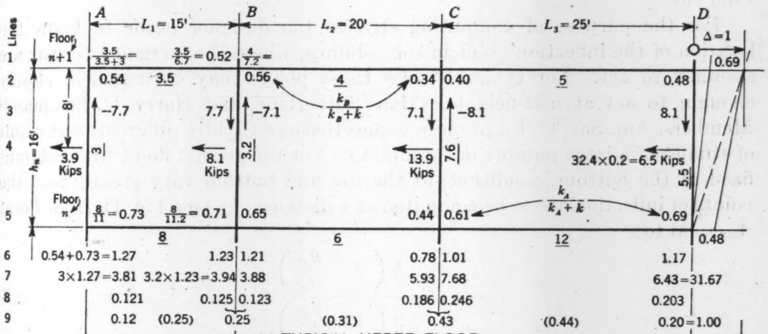

32.4 Kips
Center of Gravity
A
B
C
D
$L_1 = 15'$
$L_2 = 20'$
$L_3 = 25'$
$P_{n+1} =$ 28.8 Kips
63'
$h_{n+1} = 14'$
$P_n = 32.4$ Kips
$h_n = 16'$
P_{n-1}
P_1
$h_1 = 18'$
3.2
3.7
4.7
2.5
2.2
4.6
3.3
3.5
4
5
3
3.2
Stiffness, $k = \frac{I}{h}$
7.6
5.5
Stiffness, $k = \frac{I}{L}$
8
6
12
8
6
12
5
7
11
9
∞
∞
∞
∞
∞

FIG. 2.—DIAGRAM OF TYPICAL STRUCTURAL FRAME

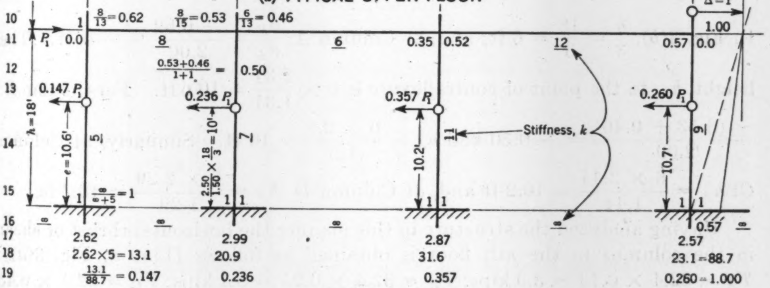
Fig. 2 is the diagram of the ground floor and two typical upper stories of a structural frame, showing the assumed k-values $\left(\frac{I}{L}\right.$ in beams and $\frac{I}{h}$ in columns$\left.\right)$, all of which are different. At the ground floor, on both sides of the fixed base of the interior and exterior columns $k_A = \infty$, and, therefore, $\frac{k_A}{k_A + k} = 1$. At the nth and $(n + 1)$th floor horizontal wind loads, $P_n = 32.4$ kips and $P_{n+1} = 28.8$ kips, may act. (The kip, or "kilo-pound" equals 1 000 lb.)

Each story and, within each story, each bay are next analyzed separately by subjecting the upper beams of the nth story (see Fig. 3(a)) to a side-sway,

$\Delta = 1$. Interior columns always form part of two adjacent bays, and interior beams, part of two stories. The height, h, is constant for all columns and, as the designer is only concerned with relations between the deformations of the several columns, he need simply write at every angle the $\dfrac{k_B}{k_B + k}$-values at the upper beams and the $\dfrac{k_A}{k_A + k}$-values at the lower beams. For example, at Point D at the top floor $(n + 1)$, $\dfrac{5}{5 + 5.5} = 0.48$, and at the next floor below (n), $\dfrac{12}{12 + 5.5} = 0.69$; and for Column D the sum of resistance at the nth story, $0.48 + 0.69 = 1.17$.


Lines
A
B
C
D
L₁=15'
L₂=20'
L₃=25'
Floor n+1
Δ=1
1
2
3
4
5
6
7
8
9
3.5/(3.5+3) =
3.5/6.7 = 0.52
4/7.2 =
0.69
0.54
3.5
0.56
4
0.34
0.40
5
0.48
8'
h_n=16'
−7.7
7.7
−7.1
k_B/(k_B+k)
7.1
−8.1
8.1
3
3.9 Kips
8.1 Kips
3.2
13.9 Kips
7.6
32.4×0.2=6.5 Kips
5.5
Floor n
k_A/(k_A+k)
8/11 = 0.73
8/11.2 = 0.71
0.65
0.44
0.61
0.69
8
6
12
0.48
0.54+0.73=1.27
1.23
1.21
0.78
1.01
1.17
3×1.27=3.81
3.2×1.23=3.94
3.88
5.93
7.68
6.43=31.67
0.121
0.125
0.123
0.186
0.246
0.203
0.12
(0.25)
0.25
(0.31)
0.43
(0.44)
0.20=1.00

(a) TYPICAL UPPER FLOOR


10
11
12
13
14
15
16
17
18
19
Δ=1
1
8/13 = 0.62
8/15 = 0.53
6/13 = 0.46
1.00
P₁
0.0
8
6
0.35
0.52
12
0.57
0.0
(0.53+0.46)/(1+1) =
0.50
0.147 P₁
0.236 P₁
0.357 P₁
0.260 P₁
h=18'
e=10.6'
5
1.50/2.50 × 18/3 = 10'
7
11
Stiffness, k
10.2'
10.7'
9
1
∞/(∞+5) = 1
1 1
∞
0.57
2.62
2.99
2.87
2.57
2.62×5=13.1
20.9
31.6
23.1=88.7
13.1/88.7 = 0.147
0.236
0.357
0.260=1.000

(b) GROUND FLOOR

FIG. 3.—RELATIVE RESISTANCE OF BUILDING COLUMNS TO UNIT LATERAL DISPLACEMENT (SIDE-SWAY), BY THE COLUMN SHEAR METHOD

Line 6, Fig. 3(a), gives the total rotations $(\theta_A - \theta_B)$ of the columns at the left and at the right sides of the given bay, and Line 7 gives the values of $k\,(\theta_A - \theta_B)$. The sum of the values in Line 7 is 31.67 and, consequently, the relative resistance values, shown in Line 8, are $\dfrac{3.81}{31.67} = 0.121$; $\dfrac{3.94}{31.67} = 0.125$; $\dfrac{3.88}{31.67} = 0.123$; etc. In Line 9 the relative resistance of the columns of the nth story to the lateral load, $P_n = 1$, at the top, equals 0.121, $0.125 + 0.123 = 0.25$, etc. Similar values for the columns of the $(n + 1)$ story, for a lateral load of $P_{n+1} = 1$ at the top, have been computed as 0.135, $0.125 + 0.140 = 0.265$, $0.198 + 0.217 = 0.415$, and 0.185.

At the ground floor (Fig. 3(*b*)) all the columns are fixed (that is, $k_A = \infty$). Therefore, $\theta_A = \dfrac{k_A}{k_A + k} = \dfrac{\infty}{\infty + k} = 1$ (Line 15, Fig. 3(*b*)). In Line 17, $(\theta_A - \theta_B) = 2.62$, 2.99, 2.87, and 2.57; and in Line 18, $k\,(\theta_A - \theta_B) = 2.62 \times 5 = 13.1$; $2.99 \times 7 = 20.9$; $2.87 \times 11 = 31.6$; and, $2.57 \times 9 = 23.1$—total 88.7. Every column then absorbs of a lateral load, $P_1 = 1$, at the top in the proportion: $\dfrac{13.1}{88.7} = 0.147$; $\dfrac{20.9}{88.7} = 0.236$; $\dfrac{31.6}{88.7} = 0.357$; and, $\dfrac{23.1}{88.7} = 0.260$ (see Line 19).

For the purpose of computing stresses the designer needs to know the location of the inflection point in the columns, where the foregoing thrusts are assumed to act. For typical stories those points may be assumed, closely enough, to act at mid-height. (Hale Sutherland and Harry L. Bowman,[11] Members, Am. Soc. C. E., propose values that are slightly different, as a result of studies of a large number of examples.) For the ground floor, with columns fixed at the bottom, conditions at the top and bottom vary greatly and the points of inflection are to be computed at a distance, h_A (see Fig. 1), from Point *A*, equal to:

$$h_A = \frac{h\left(2 - \dfrac{\theta_B}{\theta_A}\right)}{3\left(1 - \dfrac{\theta_B}{\theta_A}\right)} \qquad \text{.......................(7)}$$

In Fig. 3(*b*), $\dfrac{h}{3} = \dfrac{18}{3} = 6$ ft; and, at Column *A*: $\dfrac{\theta_B}{\theta_A} = \dfrac{-\,0.62}{2.00} = -\,0.31$. The height, h_A, to the point of contraflexure is $6 \times \dfrac{2.31}{1.31} = 10.6$ ft. For Column *B*: $\dfrac{-\,(0.53 + 0.46)}{2.00} = -\,0.50$ and $h_A = \dfrac{6 \times 2.5}{1.5} = 10$ ft. Similarly, at Column *C*: $h_A = \dfrac{6 \times 2.44}{1.44} = 10.2$ ft and, at Column *D*: $h_A = \dfrac{6 \times 2.29}{1.29} = 10.7$ ft.

Having analyzed the structure in this manner the horizontal thrust or shear in the columns in the *n*th floor is obtained as follows (Line 4, Fig. 3(*a*)): $T_A = 32.4 \times 0.12 = 3.9$ kips; $T_B = 32.4 \times 0.25 = 8.1$ kips; $T_C = 32.4 \times 0.43 = 13.9$ kips; and, $T_D = 32.4 \times 0.20 = 6.5$ kips. Assuming that the inflection points are at the mid-height of the columns, the bending moments at the tops will be $8.0 \times 3.9 = 31.2$ kip-ft; $8.0 \times 8.1 = 64.8$ kip-ft; $8.0 \times 13.9 = 111.2$ kip-ft; and, $8.0 \times 6.5 = 52.0$ kip-ft, respectively.

The vertical reactions in the columns of the *n*th floor, produced by the wind load at the *n*th floor, may be obtained, assuming that the several bays of the story must resist the overturning wind moment of $28.8 \times 7.0 + 32.4 \times 8.0 = 460.8$ kip-ft. As indicated in parentheses in Line 9, Fig. 3(*a*), the part of the horizontal thrust absorbed by the three bays, respectively, is 0.25, 0.31, and 0.44; and, consequently, the shear within the bays is $\frac{0.25 \times 460.8}{15} = 7.7$ kips; $\frac{0.31 \times 460.8}{20} = 7.1$ kips; and, $\frac{0.44 \times 460.8}{25} = 8.1$ kips. The reaction at each of the four columns is: $R_A = -7.7$ kips; $R_B = 7.7 - 7.1 = 0.6$ kips; $R_C = 7.1 - 8.1 = -1.0$ kips; and, $R_D = 8.1$ kips.

---

[11] "Structural Theory and Design," by Hale Sutherland and Harry Lake Bowman, N. Y., John Wiley & Sons, Inc., 1930, Method C, p. 207.

When only column reactions or beam moments are wanted it may be preferable and sufficiently accurate to analyze the beams directly for shear as demonstrated in Fig. 4. The top beam of the *n*th story is assumed to be deflected laterally a distance, Δ, assuming for convenience that $\Delta = h$, with respect to the beam below. The approximate relative rotations at the four columns are thus determined as shown. In Line 2, Fig. 4: $\theta_A = 0.46$; $\theta_B = 0.30$; $\theta_C = 0.46$; and, $\theta_D = 0.53$. These values may be assumed positive on the right and, therefore, negative on the left of the columns. Note that those rotations are only relative values and must not be used separately for the analysis of any beam.

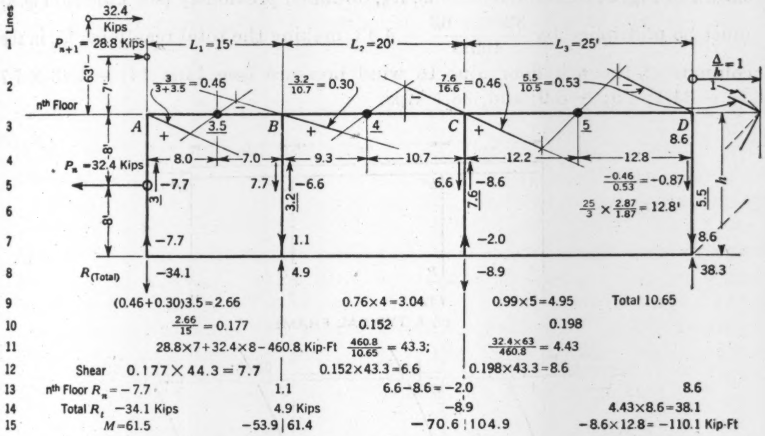
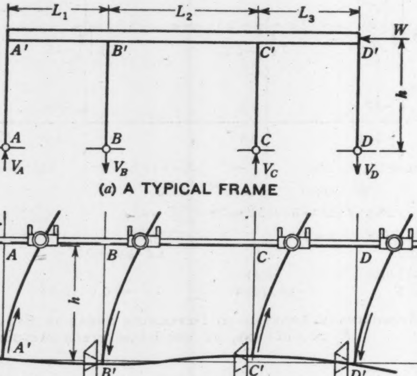

Lines
32.4 Kips
1 P_{n+1} = 28.8 Kips
L_1=15'
L_2=20'
L_3=25'
63'
7'
2 $\frac{3}{3+3.5}$=0.46
$\frac{3.2}{10.7}$=0.30
$\frac{7.6}{16.6}$=0.46
$\frac{5.5}{10.5}$=0.53
$\frac{\Delta}{h}$=1
1
nth Floor
3 A
3.5
B
4
C
5
D
8.6
+
−
4 8.0
7.0
9.3
10.7
12.2
12.8
8'
P_n =32.4 Kips
5 −7.7
7.7
−6.6
6.6
−8.6
$\frac{-0.46}{0.53}$=−0.87
h
3
3.2
7.6
5.5
6 8'
$\frac{25}{3} \times \frac{2.87}{1.87}$= 12.8'
7 −7.7
1.1
−2.0
8.6
8 $R_{(Total)}$
−34.1
4.9
−8.9
38.3
9 (0.46+0.30)3.5=2.66
0.76×4=3.04
0.99×5=4.95
Total 10.65
10 $\frac{2.66}{15}$ = 0.177
0.152
0.198
11 28.8×7+32.4×8−460.8 Kip-Ft $\frac{460.8}{10.65}$ = 43.3;
$\frac{32.4 \times 63}{460.8}$ = 4.43
12 Shear 0.177 × 44.3 = 7.7
0.152×43.3=6.6
0.198×43.3=8.6
13 nth Floor R_n = −7.7
1.1
6.6−8.6=−2.0
8.6
14 Total R_t −34.1 Kips
4.9 Kips
−8.9
4.43×8.6=38.1
15 M=61.5
−53.9 | 61.4
−70.6 | 104.9
−8.6×12.8= −110.1 Kip-Ft

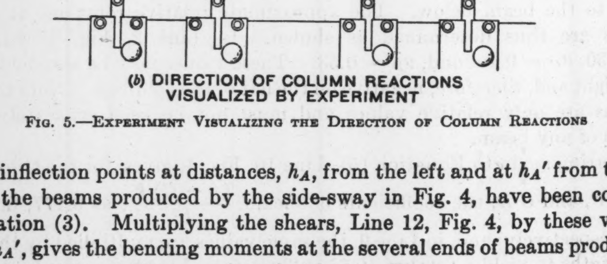
FIG. 4.—APPROXIMATE ANALYSIS TO DETERMINE SHEAR IN BEAMS AND REACTIONS, *R*, IN COLUMNS, BY THE BEAM SHEAR METHOD

Referring again to Equation (5), Line 10, Fig. 4, gives, for the three beams, *A B*, *B C*, and *C D*, the coefficients of shears, $\frac{(f - f')\,k}{L^2}$, as 0.177, 0.152, and 0.198, respectively; and, in Line 9, the same values are multiplied by the several span lengths to yield a total of 10.65. Therefore, Line 9 represents the sum of the moments of the shear at the ends of the beams, equal to the moment of wind loads with respect to the Axis *A B C D*, which (as computed previously and entered in Line 11, Fig. 4) is 460.8 kip-ft. Since $\frac{460.8}{10.65} = 43.3$, the shear in the several openings is, as indicated in Line 12: 43.3 × 0.177 = 7.7; 43.3 × 0.152 = 6.6; and, 43.3 × 0.198 = 8.6 kips.

The reactions at Columns A, B, C, and D, corresponding to the nth floor, are as shown in Line 13, Fig. 4, as follows: $R_n = -7.7$; $7.7 - 6.6 = 1.1$; $6.6 - 8.6 = -2.0$; and, 8.6 kips, respectively. These reactions would have to be computed in a similar manner for all the floors above, and added from the top floor down to the nth floor to obtain the total column pressure, but generally it will be close enough to increase the reactions obtained for the nth floor in relation to total moments, as follows: Suppose that the center of gravity of the wind load (32.4 kips) at the nth floor is at 63 ft above the nth floor, as shown in Fig. 2; then the reactions, R_n, obtained previously (see Line 13, Fig. 4) must be multiplied by $\frac{32.4 \times 63}{460.8} = 4.43$, making the total reactions, R_t, in the columns of the nth floor, due to wind pressure (see Line 14) $= 4.43 \times 7.7 = -34.1$; 4.9; -8.9; and, 38.1 kips.



(a) A TYPICAL FRAME



(b) DIRECTION OF COLUMN REACTIONS VISUALIZED BY EXPERIMENT

FIG. 5.—EXPERIMENT VISUALIZING THE DIRECTION OF COLUMN REACTIONS

The inflection points at distances, h_A, from the left and at h_A' from the right ends of the beams produced by the side-sway in Fig. 4, have been computed by Equation (3). Multiplying the shears, Line 12, Fig. 4, by these values of h_A and h_A', gives the bending moments at the several ends of beams produced by wind load at the nth floor, as given in Line 15, Fig. 4.

It will be noted that the reactions in the columns are generally positive and negative, alternately, and not (as frequently assumed in wind-stress analysis) positive at one side of the center of the frame and negative on the other side. Such arbitrary assumptions may lead to considerable error, especially when the lengthening and shortening of columns is being taken into consideration.

The simple experiment in Fig. 5 will illustrate how the signs for column pressures may alternate. Fig. 5(*a*) shows a typical portal frame and Fig. 5(*b*) is the model reproducing it, held upside down at the top of the columns. The column bases are held at their original distance from each other by means of clamps fixed on a sliding rod. When the rod is pushed to the right the first and the third columns of the model are being pulled upward and the second and fourth columns are pushed down, indicating pull and compression in alternate columns, respectively.

For very tall buildings the secondary stresses from the shortening of compressed columns and the lengthening of expanded columns, may be of interest. Suppose that there is a possible average of 6 000 lb per sq in. computed stress due to wind loads alone in exterior steel columns (modulus of elasticity = 30×10^6), the column would shorten or lengthen $\frac{2}{10.000}$ or, 0.24 in. in every 100 ft. Eventually, part of this deformation will be absorbed by the masonry.

When designing very tall frame structures, therefore, it may be convenient to compute column stresses by either one of the methods shown in Figs. 3 and 4 and correct the column loads according to the change of length to be expected, applying Fig. 1 and Equation (5). As an example, assume that, at a great height, in a tall building the leeward column shortens 1 in. with respect to the nearest interior columns; and, assume a restraint equivalent to 50% fixation at both ends of the adjoining beam: $f - f' = 1$ in. If that beam has a length of $L = 200$ in. and a moment of inertia of $I = 1\,000$ in.^{4}, making $k = \frac{1\,000}{200} = 5$ in.^{3}, Equation (5) yields:

$$P = 6 \times 30\,000\,000 \times 5 \times \frac{1}{200^2} = 22\,500 \text{ lb} = 22.5 \text{ kips}$$

which is the amount by which the load on the exterior column must be decreased and the load on the interior column increased for a 1-in. difference of level.

CONCLUSION

The elementary simple methods, derived from the visible model experiment, which are presented for immediate application in Figs. 3 and 4, should prove a convenient tool in structural design. By merely starting from actual $k\left(=\frac{I}{L}\right)$-values of any frame the designer can take into account the practical peculiarities encountered in the field. That is not possible with prior methods of approximation, nor with the abstract mathematical treatments, except, perhaps, for the simplest of structures. Fig. 3 especially presents the only procedure known to the writer of analyzing, conveniently, those frequent cases of application in the field, where the stiffness value, $k = \frac{I}{L}$, of some of the sustaining members of a skeleton is relatively infinite, such as reinforced concrete stair-cases and elevator shafts.

The "column shear method," demonstrated in Fig. 3, may be applied to advantage generally. The "beam shear method" in Fig. 4 requires even less work and is recommended for quick calculation of reactions in columns subjected to wind pressure and for revising the design of structural frames to take account of possible differences of levels produced in tall columns by horizontal loads, impacts, or unequal changes of temperature.

DISCUSSION

Samuel T. Carpenter,[12] Assoc. M. Am. Soc. C. E. (by letter).—The statement in the "Synopsis" that the author has presented a simple method of analysis is true, the writer having applied the method satisfactorily to several cases with a great deal of ease, but not without first studying, thoroughly, the principles[3] upon which it was based.

Equation (5) expresses the horizontal resistance of a column for a total lateral displacement of $\Delta = 1$. Not only is this equation important in the analysis, but it may be of equal importance in making the structural design, if the designer places any importance in the final elastic action of his structure and the achievement of initially assigned column and girder shears. Although it is impracticable always to provide the required value of k for the columns of a bent, it is usually practicable to provide approximately the correct stiffness for the girders framing to the columns. For example if, by the cantilever method, the designer has assigned a value for shear equal to P to Column A of the bent shown in Fig. 3, and has provided a column of a stiffness k, then $(f' - f)$ should equal $\frac{h^2 P}{6 E k}$. This criterion, plus Equations (6*a*) and (6*b*), when used in conjunction with a desired story deflection equal to Δ, should yield approximately correct values of stiffness for the adjacent girders.

For a case of building design Δ may be controlled by the designer to be $0.002\,h$, or any other desired value. In most cases of regular story framing, and for an exterior column, k_A could be assumed equal to k_B, making $(f' - f)$—from a simplification of Equations (6*a*) and (6*b*)—equal to $\frac{2\,k_A\,\Delta}{k_A + k}$. Then

$$P = \frac{6\,E\,k\,\Delta}{h^2}\left(\frac{2\,k_A}{k_A + k}\right) \qquad (8)$$

from which k_A could be found and would be approximately the desired stiffness of the girders connecting to the exterior column. A similar treatment could be used in designing the girders framing to the interior columns. It is apparent, of course, that the girder selections must also depend on maintaining a safe combined unit stress.

The designing of a tall building to achieve a certain elastic action was pointed out and described by H. V. Spurr,[13] M. Am. Soc. C. E. Later, in a paper[14] describing the analysis of a model, designed to check experimentally the action of a structure elastically designed, it was shown that such a design satisfied not only the required deflection but yielded experimental shears equal to previously assigned values. This latter paper also deals with the practical problems of correct designing in the lower stories and around points of structural irregularity. The writer believes that the author's method would be thoroughly applicable in design, but would be somewhat dependent upon the choice of correct lengths of columns and girders for the calculation of stiffness.

---

[12] Asst. Prof., Civ. Eng., Swarthmore Coll., Swarthmore, Pa.

[13] "Wind Bracing," by H. V. Spurr, McGraw-Hill Book Co.

[14] "Tests and Design of Steel Wind Bents for Tall Buildings," by George E. Large, Assoc. M. Am. Soc. C. E., Samuel T. Carpenter, Jun. Am. Soc. C. E., and Clyde T. Morris, M. Am. Soc. C. E., *Bulletin No. 93*, The Engineering Experiment Station, Ohio State University, 1936.

The author shows that the secondary stresses, from the shortening and lengthening of columns by the wind stresses beyond the points allowing planarity of the floor joints, are of importance. It has been shown elsewhere[15] that in a tall building, the adjustment of column stresses is not serious in the upper stories, but of considerable importance in the lower stories, due to the accumulation of girder shear corrections. The method presented by the author would appear to be applicable in the adjustment of column stresses and the calculation of the secondary girder shears. However, it should be the duty of the designer, particularly in the taller building, to assign correct values of direct stress and shear in his original design, where possible, and then to design the structure so that they are obtained, thus eliminating this adjustment.

CHARLES B. WINICK,[16] ASSOC. M. AM. SOC. C. E. (by letter).—An able and simplified method of computing wind stresses is presented in this paper. Designers of buildings have long been in search of a simple method of obtaining shears, reactions, and bending moments due to wind loads for the various members of a building frame. Since the development of the Hardy Cross method[17] of moment distribution, extensive literature has been written on the subject. The application of the Cross method to obtain shear, reaction, and moment distribution due to wind loads on a building frame is relatively simple, especially if it is not extended to a high degree of accuracy. Numerous attempts have been made, more or less successfully, to obtain "accurate analyses" by some one or other "exact method." The writer is cognizant of the fact that the problem of stress distribution due to wind loads, even in a relatively simple building, does not lend itself readily to an exact solution. Flooring, partitions, stair openings, elevator shafts, concrete slabs, and numerous other factors invariably affect the stiffness of the structure. It is questionable, therefore, whether the time consumed in arriving at a so-called accurate solution, through the application of some "exact method," is justifiable and consistent with the accuracy required in a problem of such uncertain nature. The method as outlined by Mr. Gottschalk is sufficiently accurate to serve the purposes of almost any type of building when the moments of inertia of the several members involved are constant. A designer familiar with building structures can readily apply this method, and it is questionable whether results obtained by any other complicated "exact method" are superior to it.

The writer has undertaken to recompute the values given by the author, as shown in Figs. 2 and 3, with the aim of applying some "short cuts" wherever possible, and has found some interesting results. For this purpose, all computations are referred to Fig. 2. The writer has found it difficult to refer to Fig. 3 because it is not consistent with Fig. 2. Thus, the $(n + 1)$th story of Fig. 3 corresponds to the nth story of Fig. 2, etc. Reference to both Figs. 2 and 3 leads to considerable confusion and, as a substitute, the writer developed

---

[15] "Wind Stress Investigation Method Considering Direct Deformation of Columns," a Report by George E. Large, Ohio State University Experiment Station, 1937.

[16] Engr., Queens Midtown Tunnel, PWA, New York, N. Y.

[17] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

TABLE 1.—RELATIVE RESISTANCE VALUES OF BUILDING COLUMNS TO UNIT LATERAL DISPLACEMENT, USING UPPER FLOOR BEAMS ONLY

Nomenclature	Columns Between Floors $(n+1)$ and (n)				Beam *AB*; $L_1 = 15$ ft		Beam *BC*; $L_2 = 20$ ft		Beam *CD*; $L_3 = 25$ ft	
	Column A	Column B	Column C	Column D	at *A*	at *B*	at *B*	at *C*	at *C*	at *D*
$k = \frac{I}{h}$ or $k = \frac{I}{l}$	2.5	2.2	4.6	3.3	3.2		3.7		4.7	
$\frac{k_B}{k_B + k}$ Upper beams equal to $\theta_A - \theta_B$	....	....	....	....	$\frac{3.2}{3.2+2.5} = 0.562$	$\frac{3.2}{3.2+2.2} = 0.592$	$\frac{3.7}{3.7+2.2} = 0.627$	$\frac{3.7}{3.7+4.6} = 0.446$	$\frac{4.7}{4.7+4.6} = 0.505$	$\frac{4.7}{4.7+3.3} = 0.588$
$k(\theta_A - \theta_B)$	....	....	....	....	$0.562 \times 2.5 = 1.405$	$0.592 \times 2.2 = 1.302$	$0.627 \times 2.2 = 1.379$	$0.446 \times 4.6 = 2.051$	$0.505 \times 4.6 = 2.323$	$0.588 \times 3.3 = 1.940$ Total = 10.40
Relative resistance values	....	....	....	....	$\frac{1.405}{10.4} = 0.135$	$\frac{1.302}{10.4} = 0.125$	$\frac{1.379}{10.4} = 0.133$	$\frac{2.051}{10.4} = 0.197$	$\frac{2.323}{10.4} = 0.224$	$\frac{1.940}{10.4} = 0.186$ Total = 1.00
Relative resistance of columns between (n) and $(n+1)$ stories to lateral load $P_{(n+1)} = 1$	0.135	$0.125 + 0.133 = 0.258$	$0.197 + 0.224 = 0.421$	0.186 Total = 1.00	....	....	....	....	....	....
Horizontal shear *T* between (n) and $(n+1)$ floors in kips	$28.8 \times 0.135 = 3.9$	$28.8 \times 0.258 = 7.4$	$28.8 \times 0.421 = 12.1$	$28.8 \times 0.186 = 5.4$ Total = 28.8	....	....	....	....	....	....
Bending moments *M* at top in kip-feet; inflection points at mid-heights	$3.9 \times 7 = 27.3$	$7.4 \times 7 = 51.8$	$12.1 \times 7 = 84.7$	$5.4 \times 7 = 37.8$	....	....	....	....	....	....
Total bay resistance	....	....	....	....	$0.135 + 0.125 = 0.260$		$0.133 + 0.197 = 0.330$		$0.224 + 0.186 = 0.410$ Total = 1.00	
Overturning wind moment $28.8 \times 7 = 201.6$ kip-ft. Therefore, shear of bays in kips	....	....	....	....	$\frac{201.6 \times 0.260}{15} = 3.5$		$\frac{201.6 \times 0.33}{20} = 3.3$		$\frac{201.6 \times 0.410}{25} = 3.3$	
Reaction *R* in kips	−3.5	$3.5 - 3.3 = 0.2$	$3.3 - 3.3 = 0$	3.3	....		....		....	

separate tables for each analysis of each floor shown in Fig. 2. All values obtained and their methods of computation were arranged as shown, for example, in Table 1. The complete computations were made in logical order, with painstaking care. Before discussing some of the results it is well to emphasize the fact that the method is only approximate but sufficiently accurate for all practical purposes.

As the author states, when $S_B = \infty$, $m = 0$ and $S = k$; and, when $S_B = k$, $m = 0.25$ and $S = 0.875$. The range of values of m, therefore, is between 0 and 0.25; that of $\frac{S}{k}$ is between 0.875 and 1.0; and, therefore, the assumptions that—(*a*) The conditions of rigidity at the tops of columns are sufficiently similar to those at the bottom for a typical story, and (*b*) that, in all structural frames, for lateral movements of floors, $m = 0$ and $S = k$—will produce a small error in the final computation of the column shears, moments, and reactions.

As shown in Fig. 3, the relative resistances of building columns of a typical floor, to unit lateral displacement, were computed by utilizing both the upper and lower floor beams which the column spans. The writer computed similar values by using either the upper or lower floor beams and also the beams of both floors, with remarkable agreement in results.

Table 1 shows the relative resistance values of building columns for the $(n + 1)$th story to a lateral load of $P_{n+1} = 1$ at the top as shown in Fig. 2 using upper floor beams only. The first line shows the k-values equal to $\frac{I}{h}$ (or $\frac{I}{l}$, as the case may be). The second line gives the $(\theta_A - \theta_B)$-values at the various points. Thus, starting with Point A, $\frac{k_B}{k_B + k} = \frac{3.2}{3.2 + 2.5} = 0.562$; $\frac{3.2}{5.4} = 0.592$; $\frac{3.7}{5.9} = 0.627$, etc. Next, the $k(\theta_A - \theta_B)$-values are given which are equal to $0.562 \times 2.5 = 1.405$; $0.592 \times 2.2 = 1.302$; $0.627 \times 2.2 = 1.379$ etc., the total being equal to 10.40. The relative resistances were computed as shown on the next line in the table. Then the relative column resistances to the lateral load $P_{n+1} = 1$ were computed; and, in a similar manner, these values were computed using the lower floor beams, the results being 0.135; $0.126 + 0.132 = 0.258$; $0.199 + 0.223 = 0.422$; and, 0.185. Finally, using both floors as the author does, the corresponding values were: 0.135; $0.126 + 0.132 = 0.258$; $0.198 + 0.223 = 0.421$; and, 0.186. Table 2 (*a*) shows the comparison of these relative resistance values as obtained for each case and further compares them with the same values as obtained by the author. The disagreement between the writer's and author's values is probably due to the fact that Mr. Gottschalk has used the slide-rule. In order to make certain and to ascertain the probable percentage of error by comparing each case, all computations were extended at least three decimal places and in some instances even five places.

Table 2 (*a*) shows a close agreement in the first three columns, which would lead to the definite conclusion that the relative resistance values of columns may be obtained by taking into account either the upper floor beams, the lower floor beams, or the beams of both floors. It is interesting to compare the author's results with those of the writer as shown in Columns (3) and (4) of Table 2 (*a*) (since both have been developed under the same assumptions) in order to ascertain the probable error caused by the use of the slide-rule. Thus, at Column B $\frac{0.265}{0.258} = 1.027$; and, at $\frac{415}{421} = 98.6$; or, 2.7% at Column A and 1.4% at Column B. In other words, this comparison shows that the slide-rule may cause an error of as much as 3 per cent.

TABLE 2.—COMPARISON OF RESISTANCE VALUES OF BUILDING COLUMNS

Column	Case I, using the upper floor beams (1)	Case II, using the lower floor beams (2)	Case III, using both floors (3)	Gottschalk analysis (4)
	(*a*) FOR A LATERAL LOAD $P_{n+1} = 1$			
A	0.135	0.135	0.135	0.135
B	0.125 + 0.133 = 0.258	0.126 + 0.132 = 0.258	0.126 + 0.132 = 0.258	0.125 + 0.140 = 0.265
C	0.197 + 0.224 = 0.421	0.199 + 0.223 = 0.422	0.198 + 0.223 = 0.421	0.198 + 0.217 = 0.415
D	0.186	0.185	0.186	0.185
Total	1.000	1.000	1.000	1.000
	(*b*) FOR A LATERAL LOAD $P_n = 1$			
A	0.121	0.119	0.120	0.121
B	0.126 + 0.133 = 0.259	0.124 + 0.114 = 0.238	0.125 + 0.122 = 0.247	0.248
C	0.197 + 0.226 = 0.423	0.183 + 0.254 = 0.437	0.189 + 0.242 = 0.431	0.432
D	0.197	0.206	0.202	0.203
Total	1.00	1.00	1.00	1.00

Again referring to Table 1, the horizontal shears, T, in kips, were computed; and, since they are a direct function of the relative column resistances which are the same in all three cases, the results are: 3.9, 7.4, 12.1, and 5.4; and, if bending moments are computed with inflection points assumed at mid-height of columns, the corresponding values (in kip-feet) are: 27.3, 51.8, 84.7, and 37.8. In order to obtain the vertical reactions, the shears at the bays were computed as shown on the next to last line in Table 1. The last line, completing the table, gives the vertical reactions at the four columns of the structure.

In a similar manner, the writer computed the relative resistance values of building columns for the *n*th story, due to the lateral load $P_n = 1$ at the top, as shown in Fig. 2. These values were computed, first taking into account the upper floor beams only, as in Table 1. The same values utilizing the lower floor beams, and the beams of both upper and lower floors, were derived as in Table 1.

Table 2 (*b*) affords a comparison of the relative resistance values of columns as obtained in each case and also compares it with the values obtained by the author.

A comparison of values in Table 2 (*b*) (see Table 3) shows a maximum error of only 4.86% in the extreme case, which would again lead to the conclusion that the relative resistance values of columns may be obtained by taking into account either the upper floor beams or the lower floor beams or the beams of both floors.

TABLE 3.—Comparison of Percentage Errors in Relative Resistance Values

For column:	Cases I and II, Respectively, with Case III, Yield:			
	Case I		Case II	
	Computation	Percentage error	Computation	Percentage error
B	$\frac{12}{247} \times 100$	+ 4.86	$\frac{9}{247} \times 100$	− 3.65
C	$\frac{8}{431} \times 100$	− 1.86	$\frac{6}{431} \times 100$	+ 1.39
D	$\frac{5}{202} \times 100$	− 2.48	$\frac{4}{202} \times 100$	+ 1.98

Table 4 gives a comparison of the horizontal shears T, the moments M, and the vertical reactions R, for all columns of the structural bay, for each case, with the probable percentages of error. The maximum probable error in computations of bending moments and shears is only 5% and is sufficiently accurate for all practical purposes. The percentages for vertical reactions were neglected since the reactions are so small that an error of even 100% would not affect the design materially.

TABLE 4.—Comparison of Horizontal Shears, Bending Moments and Vertical Reactions

Column	(a) Horizontal Shear, T						(b) Bending Moment, M						(c) Vertical Reactions, R			
	Case I		Case II		Case III; kips	Gottschalk; percentage error	Case I		Case II		Case III; kip-feet	Gottschalk; kip-feet	Case I; kips	Case II; kips	Case III; kips	Gottschalk; kips
	Kips	Percentage error	Kips	Percentage error			Kip-feet	Percentage error	Kip-feet	Percentage error						
A	3.9	0.0	3.9	0.0	3.9	3.9	31.2	0.0	31.2	0.0	31.2	31.2	−7.6	−7.5	−7.5	−7.7
B	8.4	+5.0	7.7	−3.8	8.0	8.1	67.2	+5.0	61.6	−3.8	64.0	64.8	+0.0	+0.7	+0.3	+0.6
C	13.7	−2.1	14.1	+0.7	14.0	13.9	109.6	−2.1	112.8	+0.7	112.0	111.2	−0.2	−1.7	−1.0	−1.0
D	6.4	−0.2	6.7	+0.3	6.5	6.5	51.2	−0.2	53.6	+0.3	52.0	52.0	+7.8	+8.5	+8.2	+8.1

An attempt was also made to obtain similar values for the three cases for the ground floor, but since the columns at this point are assumed to be entirely fixed, it can readily be seen that in order to obtain the relative resistances, both ends of columns must be considered. However, the computations for the ground floor were tabulated as in the other cases, taking into consideration: (1) The upper beams only, (2) the lower end of columns only, and (3) both ends as solved by the author. The results (not included herein) showed sufficient disagreement to emphasize the necessity of ascertaining the condition of column fixity at both ends before analysis is made.

The beam shear method of obtaining the approximate shear, T, in beams and reactions, R, in columns, as outlined by Mr. Gottschalk, is rather ingenious. This method is relatively simple, and with little experience an analysis can be made in a relatively short time. The writer has checked the computations, extending them to a greater degree of accuracy and finds no serious discrepancies.

In conclusion, it may be added that the very simplicity and brevity of the theoretical analysis and derivation of formulas, as well as the ready applicability of the method to practical problems, are indeed a worthy contribution to such a highly controversial subject, and the author is to be congratulated for his effort to lighten the burden of engineers and designers engaged in this field of endeavor. Undoubtedly many short cuts will be found as familiarity with either the "column-shear" or "beam-shear" method is gained. Thus, since nearly the same relative resistance of the columns to unit lateral displacement was obtained for a given set of columns, in a particular bay, by utilizing the beams of the upper, lower, or both floors, it can be seen readily that intelligent application of the method will lead the experienced engineer to choose that set of k-values in the beam system which will result in simple computations, thereby saving considerable time and minimizing the chances of errors.

OTTO GOTTSCHALK,[18] ESQ. (by letter).—The discussions by Professor Carpenter and Mr. Winick are to be highly commended. Equation (8) will serve as a guide toward the proper dimensioning of structural parts to account for the final elastic action of the structure, especially for earthquake design; and Mr. Winick's suggestion that the analysis of stiffness of a story be based on either lower or upper beams only is a further great simplification and convenience, especially for preliminary and tentative studies. Both upper and lower beams must be considered when k-values are not proportional; for instance, in Fig. 3(a), when the k-values of the beams in the top floor (see Line 2) are 3.5, 4, and 6, and in the bottom floor (see Line 5) 12, 6, and 4 in.^{3}, the differences between results from either upper or lower beams are only about 30% and may create a wrong picture of the distribution and sign of stresses. The upper and lower floor beams must also be given equal weight when the joint stiffnesses given in Line 7 of Fig. 3(a) are to be used for the calculation of deflection and the period of oscillation of the skeleton.

A report of exceptional value, concerning the Empire State Building, in New York, N. Y., has been presented by J. Charles Rathbun, M. Am. Soc. C. E.,[19] and also in the discussions of this paper such as that by Colin Skinner, Assoc. M. Am. Soc. C. E.,[20] who present fundamental observations not previously available to the profession. In the writer's opinion, no work on the subject of wind bracing can claim to be complete in the future without referring to that paper. It certainly proves the need for some simplified method of wind-stress analysis such as the one presented by the writer.

---

[18] Buenos Aires, Argentine Republic.

[19] "Wind Forces on a Tall Building," by J. Charles Rathbun, *Proceedings*, Am. Soc. C. E., September, 1938, p. 1335, and December, 1939, p. 1771.

[20] *Loc. cit.*, June, 1939, p. 1024.

Even in the greatest of buildings the distribution of the many varieties of material cannot be expected to be perfect at the beginning of its life. Nature must work with irresistible force to bring about that perfection after, say, ten or fifteen years, so that finally every kind of material will work as a part of the structure, in harmony with its physical properties. Thus, one more new and interesting responsibility rests upon the engineer to design and balance the resisting structure so that the building may be perfect as soon as possible after its completion. He must provide appropriate joints in the masonry work, from the first, in such a way as to assure its permanent co-operation in the maintenance of joint stiffness.

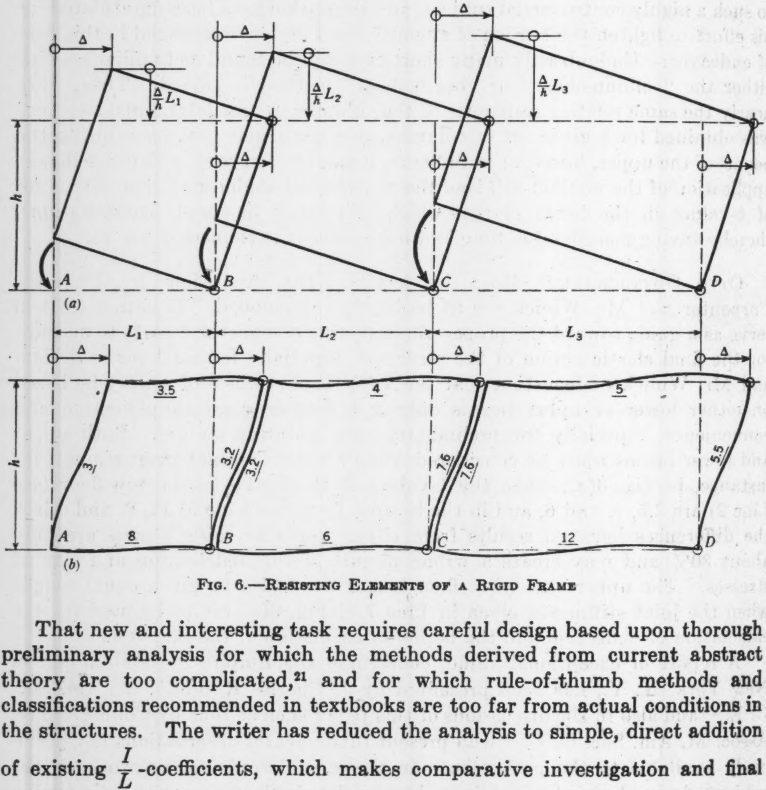


FIG. 6.—RESISTING ELEMENTS OF A RIGID FRAME

That new and interesting task requires careful design based upon thorough preliminary analysis for which the methods derived from current abstract theory are too complicated,[21] and for which rule-of-thumb methods and classifications recommended in textbooks are too far from actual conditions in the structures. The writer has reduced the analysis to simple, direct addition of existing $\frac{I}{L}$-coefficients, which makes comparative investigation and final design easy, rapid, and interesting. Incidentally, the method presented affords a better distribution and a considerable economy of steel.

Fundamental Conceptions.—As stated in the "Synopsis," the analysis presented is an application of the natural geometrical conception[3] based upon model statics.[2] Although the experimental or geometrical method of analysis simplifies, equally, all other comparatively intricate statical problems, the analysis of wind stresses is possibly one of the fields most in need of perfecting, and therefore it has been emphasized in the paper.

---

[21] "Wind Bracing in Steel Buildings," Sixth Progress Report of Sub-Committee No. 31, Committee on Steel of the Structural Division, *Proceedings*, Am. Soc. C. E., June, 1939, p. 969.

In principle, it should be remembered that, whereas the methods based on current abstract statics tend to anticipate complications over the entire structure, in the way of redundants, etc., the experiments and theory derived from the writer's method consider and deform only the stressed element that is to be analyzed and take into account only those of the surrounding structural members which are visibly affected by that deformation. When it seems necessary to refine the results, appropriate corrections are applied rather than recomputations that complicate the procedure fundamentally. In wind-stress analysis, moreover, the complications of "exact" methods are the less justified because an unknown part of the stresses, estimated as between one-half and nine-tenths, is being absorbed by brick work.

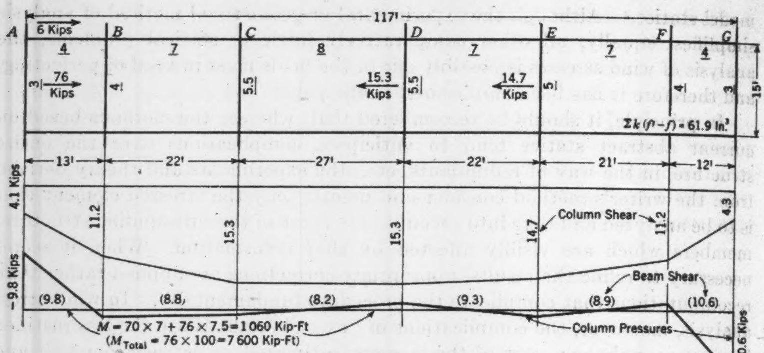
The theory explained in the paper may be further illustrated by reference to Fig. 6. A floor of the structure shown in Fig. 3(*a*) may be assumed to consist of three resisting elements or independent frames formed by the columns and beams of the three bays. First rotate each frame about Points *B*, *C*, and *D* (Fig. 6(*a*)) through an angle $\tan^{-1} \dfrac{\Delta}{h}$; fix each joint against rotation between the adjacent members meeting at that joint; and restore the vertical displacement of the two joints that were elevated in the first step. The resultant deformation is shown diagrammatically in Fig. 6(*b*).

The conclusions of the Committee on Wind Bracing[21] confirm the results of the writer's investigations. One of the conclusions may be quoted as a welcome, progressive step: "The wind-reaction ratio—that is, the ratio of vertical wind reaction of inner column to that of outer column—was seen to be wholly determined by the relative sizes of members and to be entirely under control by properly varying these relative sizes."[22] This conclusion is the more valuable because the Committee arrived at the same results along entirely different lines of approach. At the same time, the report emphasizes the necessity of an analysis, such as the one presented herein, which may be applied quickly and easily to frames of any number and length of bays. The writer has applied the simplified analysis to all beams and columns of the interesting three-bay, ten-story bent of the University of Pennsylvania tests, the bending moments of which were given in the report. His analysis showed a close agreement with those obtained by the Cross method—within 0 to 2% in most cases and as much as 5% in a few exceptional cases. The same stiffness values calculated for the bending moments served to obtain the comparative deflection immediately.

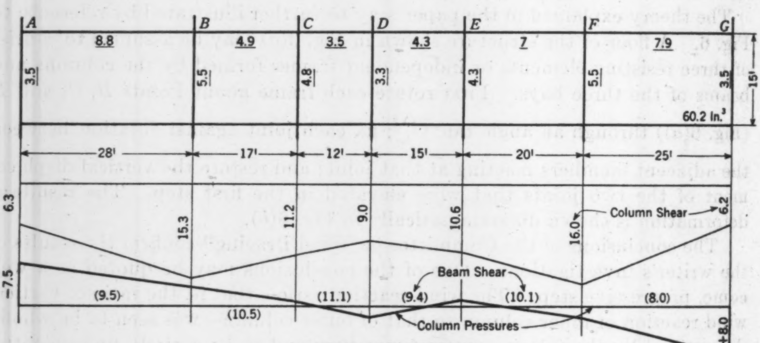
Tentative Designs.—As an example of an unsymmetrical frame of six bays, consider Fig. 7. In Fig. 7(*a*) the four central spans are long in relation to the two exterior spans; in Fig. 7(*b*) they are short in the center but long at the ends; and in Fig. 7(*c*) the three left spans are long and the remaining ones are

---

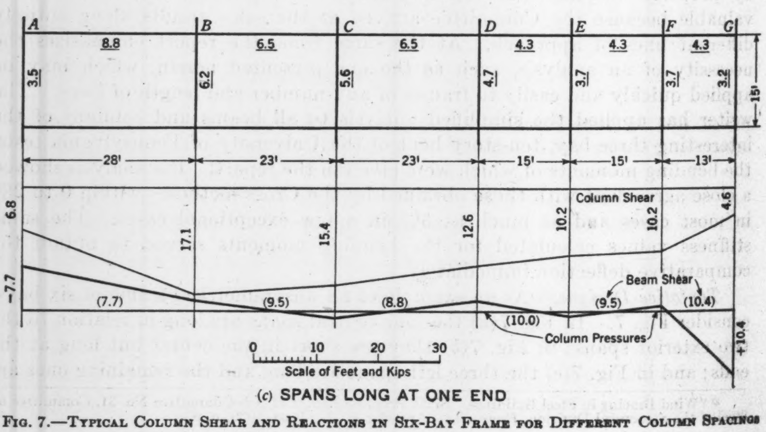
[22] "Wind Bracing in Steel Buildings," Sixth Progress Report of Sub-Committee No. 31, Committee on Steel of the Structural Division, *Proceedings*, Am. Soc. C. E., June, 1939, p. 970.


117'
6 Kips
A
B
C
D
E
F
G
4
7
8
7
7
4
3
76 Kips
4
5.5
15.3 Kips
5.5
14.7 Kips
5
4
3
15'
Σk(f' − f) = 61.9 In.³
13'
22'
27'
22'
21'
12'
4.1 Kips
11.2
15.3
15.3
14.7
11.2
4.2
Column Shear
Beam Shear
−9.8 Kips
(9.8)
(8.8)
(8.2)
(9.3)
(8.9)
(10.6)
M = 70 × 7 + 76 × 7.5 = 1 060 Kip-Ft
(M_Total = 76 × 100 = 7 600 Kip-Ft)
Column Pressures
+10.6 Kips

(*a*) LONG CENTRAL SPANS


A
B
C
D
E
F
G
8.8
4.9
3.5
4.3
7
7.9
3.5
5.5
4.8
3.3
4.3
5.5
3.5
15'
60.2 In.³
28'
17'
12'
15'
20'
25'
6.3
15.3
11.2
9.0
10.6
16.0
6.2
Column Shear
Beam Shear
−7.5
(9.5)
(10.5)
(11.1)
(9.4)
(10.1)
(8.0)
Column Pressures
+8.0

(*b*) LONG SIDE SPANS


A
B
C
D
E
F
G
8.8
6.5
6.5
4.3
4.3
4.3
3.5
6.2
5.6
4.7
3.7
3.7
3.2
15'
28'
23'
23'
15'
15'
13'
6.8
17.1
15.4
12.6
10.2
10.2
4.6
Column Shear
Beam Shear
−7.7
(7.7)
(9.5)
(8.8)
(10.0)
(9.5)
(10.4)
Column Pressures
+10.4
0
10
20
30
Scale of Feet and Kips

(*c*) SPANS LONG AT ONE END

FIG. 7.—TYPICAL COLUMN SHEAR AND REACTIONS IN SIX-BAY FRAME FOR DIFFERENT COLUMN SPACINGS

short. To obtain a uniform criterion the k-values of beams in the three designs have been assumed to vary in relation to spans and the k-values of the columns in relation to half the lengths of adjoining bays. The total floor loading has been assumed at 150 lb per sq ft and the load of exterior walls at 1 kip per front ft of each story. Under each of the three arrangements in Fig. 7 there is a diagram of column shear and one of the column reactions, with the shears in the beams indicated in parentheses.

These diagrams are most instructive and their convenience can scarcely be over-estimated. Wide spacing of end spans (see Fig. 7(*b*)) make the reactions approach a planar distribution, whereas short exterior spans, such as in Fig. 7(*a*), produce the most marked differences between the reactions of exterior and the nearest interior column. This would agree with Mr. Skinner's observation:[23] "Cracks were also observed in the concrete floors at the first row of interior columns, parallel to the exterior walls." Consequently, where diagrams of reactions, such as those of Fig. 7, indicate abrupt differences between reactions at exterior spans and where prevention of cracks is considered important, joints or dummy joints are advisable at the first row of columns unless, in the architectural design, exterior spans may be widened until the reaction diagram (Fig. 7) shows a more planar distribution, although the cost of the steel frame may be higher.

Note that at reactions of the two exterior columns at each side of the structures a slight correction may be applied, as shown at the end of the paper.

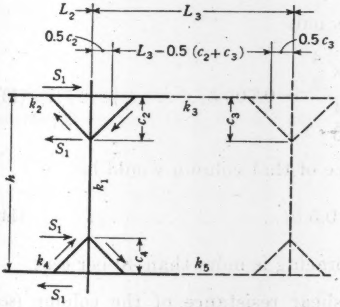


FIG. 8.—WIND BRACING

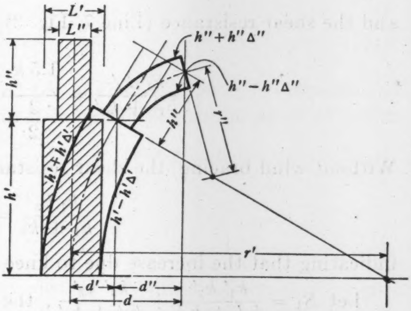


FIG. 9.—DEFLECTION OF BENT FROM UNIFORM CHANGE OF LENGTH OF EXTERIOR COLUMNS

Wind Bracing.—Knee bracing at the ends of columns has the effect of shortening the free length of both columns and beams, thus increasing the stiffness values from k to k'. For conditions shown in Fig. 8, the k'-values may be computed as follows:

$$k_1' = \frac{h\,k_1}{h - b_2 - b_4} \dots\dots\dots\dots\dots\dots (9)$$

in which

$$b_2 = c_2\,\frac{k_2 + k_3}{k_1 + k_2 + k_3} \dots\dots\dots\dots\dots\dots (10a)$$

---

[23] *Proceedings*, Am. Soc. C. E., June, 1939, p. 1025.

and

$$b_4 = c_4 \frac{k_4 + k_5}{k_1 + k_4 + k_5} \ldots\ldots\ldots\ldots\ldots\ldots (10b)$$

Furthermore,

$$k_2' = \frac{L_2\, k_2}{L_2 - 0.5\,(c_1 + c_2)} \ldots\ldots\ldots\ldots\ldots\ldots (11a)$$

and

$$k_3' = \frac{L_3\, k_3}{L_3 - 0.5\,(c_2 + c_3)} \ldots\ldots\ldots\ldots\ldots\ldots (11b)$$

Then, the stiffness quotients corresponding to Line 2, Fig. 3, are $\frac{k_2'}{k_1' + k_2'}$ and $\frac{k_3'}{k_1' + k_3'}$, making the shear resistance of column (Line 7, Fig. 3) $\frac{k_1'\, k_2'}{k_1' + k_2'}$ and $\frac{k_1'\, k_3'}{k_1' + k_3'}$.

For example, let $k_1 = k_2 = k_3 = k_4 = k_5$; $c_2 = c_3 = c_4 = 0.25\,h$; and $L_3 = h$. Then $b_2 = 0.25\,h \times \frac{2}{3} = \frac{h}{6} = b_3$; and

$$k_1' = \frac{k_1}{1 - \frac{1}{3}} = 1.5\,k_1 \ldots\ldots\ldots\ldots\ldots\ldots (12a)$$

and

$$k_3' = \frac{k_3}{1 - 0.25} = 1.33\,k_3 \ldots\ldots\ldots\ldots\ldots\ldots (12b)$$

and the shear resistance (Line 7, Fig. 3) equals

$$\frac{k_1'\, k_3'}{k_1' + k_3'} = \frac{1.5\,k_1 \times \frac{4}{3}}{\frac{3}{2} + \frac{4}{3}} = 0.706\,k_1 \ldots\ldots\ldots\ldots\ldots (13)$$

Without wind bracing, the shear resistance of that column would be:

$$\frac{k_1\, k_3}{k_1 + k_3} = 0.5\,k_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

indicating that the increase due to knee bracing is more than 40 per cent.

Let $S_1 = \frac{k_1'\, k_2'}{k_2' + k_3'} + \frac{k_1'\, k_3'}{k_2' + k_3'}$, the shear resistance of the column (see Fig. 8); then the 45° knee bracing will receive a stress at the upper left and right sides, respectively, of $\frac{1.4\, S_1\, k_2}{k_2 + k_3}$ and $\frac{1.4\, S_1\, k_3}{k_2 + k_3}$. In steel structures the heights c_2 and c_4 of the knee bracing should be measured from the outside rivets, and in reinforced concrete structures from the most remote steel bars crossing the column.

Deflection from Wind Stresses.—A further interesting application of the simplified analysis is the calculation of deflection or sway of tall buildings by means of the shear resistance at the floors (shown at Line 7, Fig. 3), applying the actual values of $k = \frac{I}{L}$ in beams and $k = \frac{I}{h}$ in columns.

The total deflection of a structure may be analyzed as the sum of three distinct stress actions which, in the inverse order of their numerical importance, are as follows:

1. Direct Shear at Columns.—The direct shear at columns, due to wind pressure, without discounting the large portion actually absorbed by the walls, will generally not exceed 0.26 kips for every square inch of steel section of columns. For the steel, the modulus of elasticity in shear is assumed at $E_s = 13\,000$ kips per sq in. The deflection from direct shear will not exceed 0.24 in. for every 1 000 ft of height of the building and therefore, generally, may be neglected.

2. Reactions in Exterior Columns.—The pull at the left and the compression at the right exterior columns, resulting from reaction diagrams as shown in Fig. 7, lengthens the first and shortens the latter. A prism of width L' and height h' (Fig. 9), one side of which is uniformly lengthened by Δ' and the opposite side shortened by Δ' per unit of length, forms a circular arch of radius r', such that: $\dfrac{r' + \dfrac{L'}{2}}{r' - \dfrac{L'}{2}} = \dfrac{h' + h'\,\Delta'}{h' - h'\,\Delta'}$; and

$$r' = \frac{L'}{2\,\Delta'} \ldots\ldots\ldots\ldots (15)$$

At the height of h' the "geometrical" deflection d' (see Fig. 9) is $(r' - d')^2 = (r')^2 - (h')^2$; or

$$d' = \frac{(h')^2}{2\,r'} = \frac{(h')^2\,\Delta'}{L'} \ldots\ldots\ldots\ldots (16)$$

A prism of base L'' and height h'' on top of the base prism, lengthened and shortened respectively by Δ'' per unit of height, will form a curve of radius $r'' = \dfrac{L''}{2\,\Delta''}$ and deflect $\dfrac{(h'')^2\,\Delta''}{L''} + \dfrac{2\,h'\,h''\,\Delta'}{r'}$, so that the total deflection d'' on top of the upper prism is:

$$d'' = \frac{h'\,\Delta'\,(h' + 2\,h'')}{L'} + \frac{(h'')^2\,\Delta''}{L''} \ldots\ldots\ldots\ldots (17)$$

Equations (15), (16), and (17) apply also for any other difference of height, $2\,h'\,\Delta'$ and $2\,h''\,\Delta''$, caused by uniform extension or compression of both exterior columns, or in one column only with respect to the other—as, for instance, from change of temperature along one side of the building.

In the field it is not likely that exterior columns will be required to sustain an average wind stress over their total length of more than 2 kips per sq in.; assuming $E = 30\,000$ kips per sq in. (see Fig. 9), this would make $\Delta' = \Delta'' = \dfrac{2}{30\,000} = \dfrac{1}{15\,000}$. As an example, let: $h' = 400$ ft, $L' = 200$ ft, $h'' = 200$ ft, and $L'' = 100$ ft, making $h'\,\Delta' = \dfrac{4}{150}$ ft and $h''\,\Delta'' = \dfrac{2}{150}$ ft. Then, by Equation (16), $d' = \dfrac{4 \times 400}{200 \times 150} = 0.0533$ ft; by Equation (17), $d'' = \dfrac{2 \times 200}{100 \times 150}$

$+ \frac{2 \times 4 \times 200}{200 \times 150} = 0.0800$ ft; and (see Fig. 9) $d = d' + d'' = 0.1333$ ft $= 1.6$ in. This displacement may be termed a geometrical effect of uniform alteration at exterior columns.

3. Aggregate Sway of Floors.—Referring to Fig. 1, Equation (5), for $\Delta \leqq 1$, may be written as follows:

$$P = \frac{6\,E\,k\,(\theta\,\Delta - \theta'\,\Delta)}{h^2} \quad \text{.....................(18)}$$

If the deflection of any one story of height, h, being $\theta = \frac{f'}{h}$ and $\theta' = \frac{f}{h}$:

$$\Delta = \frac{P\,h^2}{6\,E\,[\Sigma k\,(\theta - \theta')]} \quad \text{.....................(19)}$$

For example, at the *n*th floor of the frame in Fig. 3, according to Line 7: $\Sigma k\,(\theta - \theta') = 31.67$ in.³ Furthermore: $P = 32.4$ kips; $6\,E = 180\,000$ kips per sq in.; $h = 16 \times 12 = 192$ in.; and $d = \frac{32.4 \times 192^2}{31.67 \times 180\,000} = 0.2095$ in. In other words, the deflection is $\frac{100 \times 0.2095}{16} = 1.31$ in. per 100-ft height of building. However, if the height of the floors was only 12 ft, the horizontal deflection would be only 0.72 in. per 100 ft. Although Equation (19) does not take into account the reciprocal influences of stiffness and displacements between floors, experience indicates that it presents a sound basis for calculating deflections. After more observations on existing buildings become available, it may possibly be completely corroborated.

In Fig. 10 an analysis of this part of the deflection of a structure has been illustrated. One bent only has been shown as an example. In practical application, however, all bents joined by rigid floors must be assumed to resist, jointly, the total wind pressure upon the building, taking into account possible effects of eccentricity of the wind force. Walls anchored within a bent will be treated separately.

Walls in Skeleton.—Walls tied to columns within a bent increase the rigidity and reduce the deflection of that bent. The simplified wind-stress analysis may be applied to analyze and judge the possibilities and limitations of that ever-present structural element.

A 25-story bent with a distribution of bays similar to Fig. 7(*b*) but with a rigid wall joining Columns *B* and *C*, as would be required for an elevator shaft or staircase, has been illustrated in Fig. 10. It is assumed that cross braces or some other form of reinforcement tie the wall firmly to Columns *B* and *C* (at least in the lower floors) so that these columns act as chords of a vertical cantilever beam fixed at the ground floor. At the same time Columns *B* and *C* act, at all the floors, as members of the frame, the same as all other columns, and they contribute to the joint wind force a resistance, $\Sigma k\,(\theta - \theta')$, at the several floors, as analyzed in Line 7 of Fig. 3.

This joint resistance may be called the "section modulus" of the bent or frame at the several floors, in the same sense as $\frac{I}{c}$ is defined as the section

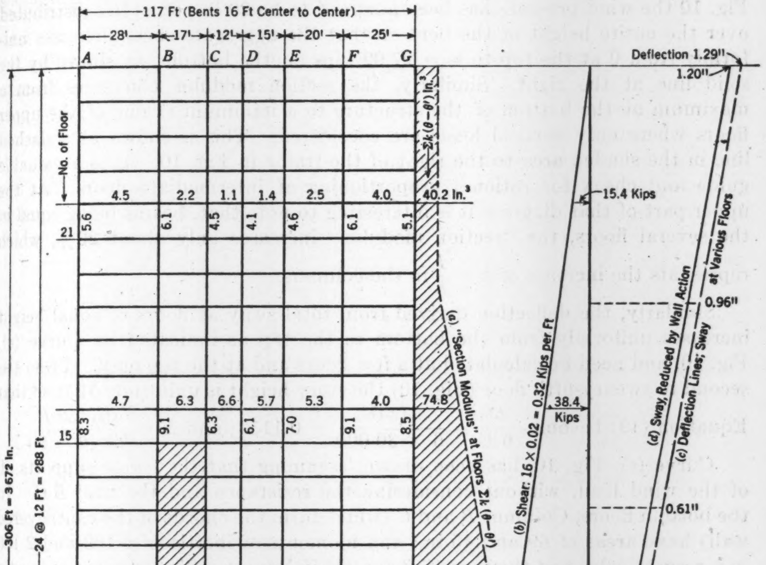

117 Ft (Bents 16 Ft Center to Center)
28'
17'
12'
15'
20'
25'
A
B
C
D
E
F
G
Deflection 1.29"
1.20"
No. of Floor
$2k(\theta-\theta')$ In.^{3}
4.5
2.2
1.1
1.4
2.5
4.0
40.2 In.^{3}
15.4 Kips
5.6
6.1
4.5
4.1
4.6
6.1
5.1
21
(d) Sway Reduced by Wall Action
at Various Floors
0.96"
(a) "Section Modulus" at Floors $2k(\theta-\theta')$
(b) Shear: 16 × 0.02 = 0.32 Kips per Ft
(c) Deflection Lines; Sway
306 Ft = 3 672 In.
24 @ 12 Ft = 288 Ft
4.7
6.3
6.6
5.7
5.3
4.0
74.8
38.4 Kips
8.3
9.1
6.3
6.1
7.0
9.1
8.5
15
0.61"
17.0
20.6
18.4
13.4
13.2
10.3
144.0 In.^{3}
61.4 Kips
11.3
11.9
9.1
7.3
9.0
12.0
10.8
9
2.18
14.90 Kips
0.31"
27.70 Kips
40.45 Kips
0.185"
a = 92.0'
84.5 Kips
53.22 Kips
(e) Wall BC Alone
31.6
39.0
42.2
31.0
31.0
23.3
202.0 In.^{3}
14.2
16.6
10.4
9.3
11.0
16.6
13.2
3
65.99 Kips
78.76 Kips
46'
18'
-647 Kips
+647 Kips
306×16×0.02 = 97.92 Kips
Scale of Length, in Feet
0
10
20
30
40
50
60
0
0.5
1.0
1.5
Scale of Deflection, in Inches

FIG. 10.—DIAGRAMS OF "SECTION MODULUS" AND DEFLECTION FOR A TWENTY-FIVE STORY BENT WITH A "WALL" BAY

modulus of an ordinary beam; and, like $\frac{I}{c}$, its dimension is in inches³. In Fig. 10 the wind pressure has been assumed to be 20 lb per sq in., distributed over the entire height of the bent so that the aggregate shear increases uniformly from 0 at the top to $w = 97.92$ kips on the bottom, as shown by the solid line at the right. Similarly, the section modulus converges from a maximum at the bottom of the structure to a minimum at one of the upper floors where only vertical loads are considered. This is shown as a dashed line in the shaded area to the right of the frame in Fig. 10. It is a valuable guide and check for rational proportioning at intermediate floors. At the upper part of that diagram it is interesting to note that, beams being equal in the several floors, the "section modulus" increases only about 20%, which represents the increase of $k = \frac{I}{h}$ in the columns.

Similarly, the deflection derived from total sway at floors of equal height increases uniformly from the bottom to the top as indicated by Curve (*c*), Fig. 10, and need be calculated at a few floors and at the top only. From the second to twenty-fifth floor (Fig. 10) the story height is uniformly 12 ft so that Equation (19) becomes $\frac{h^2}{6\,E} = \frac{144^2}{6 \times 30\,000} = 0.1152$; and $\Delta = \frac{0.1152\,P}{\Sigma k\,(\theta - \theta')}$.

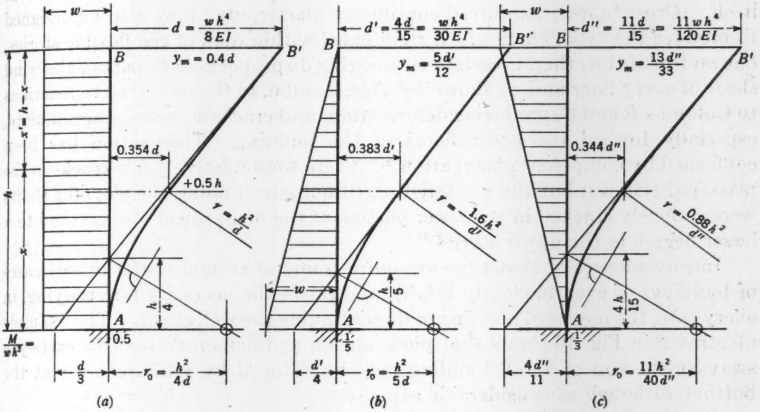
Curve (*c*), Fig. 10, has been drawn, assuming that the frame supports all of the wind load, without considering the resistance and the wall *B C*. In the bottom floors, Columns *B* and *C* (which form the chords of the cantilevered wall) have areas of 69 and 45 in.² and moments of inertia of 5 100 and 2 100 in.⁴, respectively; and they are 17 ft (or 204 in.) apart. The moment of inertia of the wall, therefore, is: $I = 5\,100 + 2\,100 + \frac{69 \times 45 \times 204^2}{69 + 45} = 114.06 \times 10^4$ in.⁴ and $E\,I = 342.18 \times 10^8$ kip-in.²

The wall alone, as a free cantilever 306 ft (or 3 672 in.) long, loaded uniformly by $16 \times \frac{0.02}{12} = 0.0267$ kip per in. deflects as shown by Curve (*e*), Fig. 10, and would neither contribute to stiffness nor reduce deflection. However, since Columns *B* and *C* are part of the frame, one may assume that the wall *B C* acts as a cantilever fixed at the bottom and loaded at the several floors. The shear decreases uniformly from 97.92 kips at the bottom to 0 at some level, distant x above the ground, so that at $\frac{x}{2}$ the deflection of the cantilever would be equal to the sway of the skeleton without the wall *B C*.

TABLE 5.—ANALYSIS TO DETERMINE THE DEFLECTION OF A TWENTY-FIVE-STORY BENT (SEE FIGURE 10)

Floor (1)	External load, P, in kips (2)	$\Sigma k(\theta - \theta')$, in inches³ (3)	Deflection Δ, in inches per 12 ft of height (4)	Number of stories included in section (5)	Deflection Δ, in inches per section (6)	Total deflection, in inches (7)
3	81.5	202.0	0.0482	6.5	0.312	0.312
9	61.4	144.0	0.0491	6.0	0.295	0.607
15	38.4	74.8	0.0592	6.0	0.355	0.962
21	15.4	40.2	0.0442	7.5	0.332	1.294

According to Table 5 the sway line of the lowest section starts from 0 at the bottom and increases at the rate of $\frac{0.0482}{144} = 3.347 \times 10^{-4}$ in. per in.



w
$d = \frac{w h^4}{8 E I}$
B
B'
$y_m = 0.4\,d$
$x' = h - x$
h
x
$0.354\,d$
$+0.5\,h$
$r = \frac{h^2}{d}$
$\frac{h}{4}$
A
0.5
$\frac{M}{w h^2} =$
$\frac{d}{3}$
$r_0 = \frac{h^2}{4 d}$
(a)
$d' = \frac{4 d}{15} = \frac{w h^4}{30 E I}$
B
B''
$y_m = \frac{5 d'}{12}$
$0.383\,d'$
$r = \frac{1.6 h^2}{d'}$
w
$\frac{h}{5}$
A
$\frac{1}{5}$
$\frac{d'}{4}$
$r_0 = \frac{h^2}{5 d}$
(b)
w
$d'' = \frac{11 d}{15} = \frac{11 w h^4}{120 E I}$
B
B'
$y_m = \frac{13 d''}{33}$
$0.344\,d''$
$r = \frac{0.88 h^2}{d''}$
$\frac{4 h}{15}$
A
$\frac{1}{3}$
$\frac{4 d''}{11}$
$r_0 = \frac{11 h^2}{40 d''}$
(c)

FIG. 11.—DEFLECTION CURVES OF CANTILEVER BEAMS FOR (a) UNIFORM LOAD, AND (b) AND (c) TRIANGULAR LOADS

Substituting, for the pressures at the several floors, a distributed load which varies uniformly from $\frac{97.92}{144} = 0.68$ kip per in. at the bottom to 0 at a distance x, the value of x (Fig. 10) can be obtained from the following equation (see Fig. 11(b)):

$$0.383 \times 0.68 \times \frac{x^4}{30\,E\,I} = 3.347 \times 10^{-4}\,x \ldots\ldots\ldots\ldots (20)$$

Therefore, $x^3 = 1.31924 \times 10^9 = 1\,096.8$ in. $= 91.4$ ft—that is, approximately 92 ft. The deflection at $\frac{x}{2}$ is $d = 46.0 \times 12 \times 3.347 \times 10^{-4} = 0.1848$ in. and therefore the reduced deflection of the bent, due to the wall, is $0.5\,d = 0.0924$ in.

The bending moment of the wall, at the bottom (see Fig. 10), is $M_0 = 2.13 \times 90 + 14.9 \times 78 + 27.7 \times 66 + 40.45 \times 54 + 53.22 \times 42 + 65.99 \times 30 + 78.76 \times 18 = 10\,999.00$ kip-ft, and therefore the additional reaction at Columns B and C to be resisted is $\pm \frac{10\,999.00}{17} = \pm 647$ kips.

On the other hand, the section modulus at the floors below x may remain constant, thus saving the additional steel in beams which the increase of "section modulus," according to the shaded area in Fig. 10, would require without the wall action.

Cantilever walls may start at the ground floor or at any upper floor at which fixation is assured by three horizontally rigid floors capable of transmitting the fixing stresses to other rigid walls below. This facilitates the design of offsets in walls without lessening the wind stiffness of the structure and may also be of temporary value to make up for local lack of rigidity, especially during construction and before the exterior walls have been placed in all of the floors.

To produce the "wall effect" the "wall" need not be resistant and rigid in itself. Cross braces, reinforced continuous plaster, etc., may also be designed to produce the "wall" action. A rigid panel within each of the flexible stories, one on top of the other, is certain to absorb a disproportionate part of the wind shear of every floor and, as shown by Figs. 3 and 6, to throw excessive reactions to Columns B and C, producing deformations and cracks as stresses accumulate, especially toward the lower floors of the building. This action has been confirmed by competent observation:[24] "There were relatively few cracks in the plastered ceilings; but the plastered partitions surrounding the elevator shafts were seriously cracked in the lower portion of the main tower and cracked to a lesser degree in the upper stories."

Inconveniences of that type are quite common around shafts or staircases of buildings of even moderate height, and prove the necessity for studying, in every case, the merits of utilizing or discontinuing the wall effect. The example illustrated in Fig. 10 shows that phenomenon and demonstrates how excessive sway at the top of a tall building may be reduced by reinforcement at the bottom, although at considerable expense.

Bending Lines of Cantilevers.—For possible combinations of cantilever walls the following data referring to a beam $A\,B$ of constant section, fixed at Point A (Fig. 11), will be of interest. When the moment of inertia I is decreasing uniformly from the bottom (A) to the top (B), it may be taken as the I-value at about $x = \frac{h}{4}$ above the bottom. A fairly close approximation is obtained by substituting tangents to the curves at Points A and B'. The last three lines of Table 6 are based on that assumption, with $y = 0$ for the lower heights, $\frac{h}{4}$, $\frac{h}{5}$, and $\frac{4\,h}{15}$, respectively.

Earthquake Stresses.—In territories subject to earthquake shocks, it now seems customary to specify, for the statical computation, a horizontal force at each floor equal to one-tenth of the overlying vertical load. This will make the horizontal force in normal buildings six to twelve times the usual wind pressure, and the additional shears and reactions will become really important even for low buildings; and the simplified analysis will be so much more necessary. The following definition by Mr. Jacob J. Creskoff[25] may be quoted:

> "The seismic force, $F = W\,K$, acts against the mass of a building. Assuming that the mass is concentrated at the floor levels, and that floors are horizontal girders that deflect as units, then the columns and beams between two consecutive floors may be regarded as vertical beams—deflecting equally because of a concentrated horizontal force acting against the upper floor—and with end fixities determined by the degree of fixity between the vertical beams and the floors. Under these assumptions, the seismic force is distributed to all of the vertical beams, and to each in proportion to its relative rigidity."

---

[24] *Proceedings*, Am. Soc. C. E., June, 1939, p. 1026.

[25] "Earthquakes and Structures," by the late Leander M. Hoskins, Esq., and John D. Galloway, M. Am. Soc. C. E.; discussion by Jacob J. Creskoff, *loc. cit.*, April, 1939, p. 725.

To take care of these elementary forces, the simplified wind-stress analysis will apply directly, especially Equation (8); beyond that it will facilitate the distribution of columns along the lines suggested in Fig. 7 and contribute to the formation of a resisting nucleus within the structure to cope best with the effects of oscillation.[26]

TABLE 6.—DEFLECTION CURVES FOR A CANTILEVER BEAM, $A\ B$, FIXED AT A POINT, A, OF CONSTANT SECTION

Description (see Fig. 11)	DISTRIBUTION OF LOAD		
	Uniformly distributed w	Decreasing from w at the bottom to zero at the top	Decreasing from w at the top to zero at the bottom
Moment, M_0	$\frac{w\,h^2}{2}$	$\frac{w\,h^2}{6}$	$\frac{w\,h^2}{3}$
Deflection at Point B	$d = \frac{w\,h^4}{8\,E\,I}$	$d' = \frac{w\,h^4}{30\,E\,I}$	$d'' = \frac{11\,w\,h^4}{120\,E\,I}$
Average deflection, y_m	$\frac{2\,d}{5}$	$\frac{5\,d'}{12}$	$\frac{13\,d''}{33}$
Deflection, y	$\frac{x^2\,d}{3\,h^2}\left(6 - \frac{4\,x}{h} + \frac{x^2}{h^2}\right)$	$\frac{5\,x^2\,d'}{4\,h^2}\left(\frac{2\,x'}{h} + \frac{x^2}{h^2} - \frac{x^3}{5\,h^3}\right)$	$\frac{5\,x^2\,d''}{11\,h^2}\left(4 - \frac{2\,x}{h} + \frac{x^3}{5\,h^3}\right)$
Tangent, $\frac{dy}{dx}$	$\frac{4\,x\,d}{h^2}\left(\frac{x'}{h} + \frac{x^2}{3\,h^2}\right)$	$\frac{5\,x^2\,d'}{4\,h^3}\left(\frac{4\,h}{x} - 6 + \frac{4\,x}{h} - \frac{x^2}{h^2}\right)$	$\frac{5\,x\,d''}{11\,h^2}\left(8 - \frac{6\,x}{h} + \frac{x^2}{h^2}\right)$
Curve $= \frac{1}{r} = \frac{d^2y}{dx^2}$ ...	$\frac{4\,d}{h^2}\left(1 - \frac{2\,x}{h} + \frac{x^2}{h^2}\right)$ $\left(\text{in which } h^2 = \frac{4\,(x')^2\,d}{h^4}\right)$	$\frac{5\,x\,d'}{16\,h^3}\left(\frac{h}{x} - \frac{3\,x'}{h} - \frac{x^2}{h^2}\right)$	$\frac{20\,d''}{11\,h^2}\left(2 - \frac{3\,x}{h} + \frac{x^3}{h^3}\right)$
	$x > \frac{h}{4}$	$x > \frac{h}{5}$	$x > \frac{4\,h}{15}$
Approximate, y	$\frac{(4\,x - h)\,d}{3\,h}$	$\frac{(5\,x - h)\,d'}{4\,h}$	$\frac{(15\,x - 4\,h)\,d''}{11\,h}$
Approximate, $\frac{dy}{dx}$	$\frac{4\,d}{3\,h}$	$\frac{5\,d}{4\,h}$	$\frac{15\,d}{11\,h}$

Stresses in Existing Buildings.—The simplified wind-stress analysis may be used to advantage in determining stresses of skeletons in existing buildings, the investigation of which should be stimulated to arrive at a better knowledge of the permanent stiffness conditions of structures and of the portion of stresses definitely absorbed by masonry work.[27]

Unless perfect measuring and recording instruments are available in abundance, it is convenient,[27] and economical, to place small, flat, metal boxes in the base layer of existing floors, next to the columns, so that they follow safely the vertical oscillations of the latter. The boxes should be sealed hermetically and should contain an easy flowing liquid. Two boxes communicate by means of small rubber or metal tubes placed out of the way along partitions, etc. The slightest lifting of one of the boxes will cause at the communicating box an overflow into an attached receptacle, or glass tube, which may be weighed, measured, or read annually or semi-annually; the sum of the overflow of each pair of boxes indicates the maximum change in level between the two respective columns. If the boxes are $2\frac{1}{2}$ in. by 4 in. by 8 in., every 0.01 in. change of level shows a column of liquid 6.52 in. long in a $\frac{1}{4}$-in. glass tube, so that even $\frac{1}{1\,000}$ in. can be estimated.

---

[26] *Proceedings*, Am. Soc. C. E., December, 1938, p. 1993.

[27] "Wind Forces on a Tall Building," by J. Charles Rathbun; discussion by Otto Gottschalk, *loc. cit.*, May, 1939, p. 880.

It will be advisable, if possible, to set a series of pairs of boxes at two or three different floors in tall buildings; and, at one or two floors below or above each series, one pair should be set at each of the exterior panels (see Fig. 12). Assuming a story height of 12 ft, a change of level of 0.01 in. between two floors represents an additional column pressure of 2 kips per sq in. of column section.

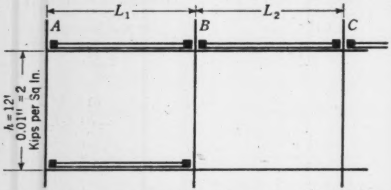


FIG. 12.—ARRANGEMENT OF BOXES FOR RECORDING RECIPROCAL CHANGES OF LEVEL AT COLUMNS

For example, consider the floor in Fig. 7(*c*) to be subjected to a total wind moment of 7 600 kip-ft. The first step is to analyze the frame according to Fig. 3, obtaining the diagram of reactions shown in Fig. 7(*c*). This supplies the sign of column pressures and, after some practice, permits the designer to estimate the reactions of most of the interior columns, thus simplifying readings at the building. Suppose the readings are as follows, in kips: $AB = -42.2$; $BC = -17.9$; $CD = +13.2$; $DE = -12.3$; $EF = +10.7$; and $FG = -81.6$. The seven unknown column reactions $A, B, \cdots G$ are calculated as follows:

$$\begin{array}{lr}
 & \text{Kips} \\
A - B \qquad\qquad\qquad\qquad = & -\ 42.2 \\
A - C = -42.2 - 17.9 = & -\ 60.1 \\
A - D = -60.1 + 13.2 = & -\ 46.9 \\
A - E = -46.9 - 12.3 = & -\ 59.2 \\
A - F = -59.2 + 10.7 = & -\ 48.5 \\
A - G = -48.5 - 81.6 = & -130.1 \\
A+B+C+D+E+F+G = & 0.0 \\
\hline
7A \qquad\qquad\qquad\qquad\qquad = & -387.0
\end{array}
\qquad
\begin{array}{lr}
 & \text{Kips} \\
B = 42.2 - 55.3 = & -13.1 \\
C = 60.1 - 55.3 = & +\ 4.8 \\
D = 46.9 - 55.3 = & -\ 8.4 \\
E = 59.2 - 55.3 = & +\ 3.9 \\
F = 48.5 - 55.3 = & -\ 6.8 \\
G = 130.1 - 55.3 = & +74.8 \\
A = -387.0 \div 7 = & -53.3 \\
\hline
A+B+C+D+E+F+G = & 0.0
\end{array}$$

Inverting the process illustrated in Fig. 3(*a*), one may obtain from those column reactions all other stresses of columns and beams of the structure which are desired. The differences between reactions analyzed and recorded will indicate the proportion absorbed by masonry work.

In conclusion the writer hopes that he has shown an efficient method of analyzing wind stresses in tall buildings. Since the inherited abstract conception of structural statics has failed to provide engineers with adequate methods of analysis in this field of design as well as in others, it must be replaced by more natural devices and lines of reasoning. Apparently, skyscrapers are neither uneconomical nor harmful to street traffic,[28] and the engineer can contribute much to encourage that most interesting field of technical activity. By wise distribution of materials the proper analysis of the steel frame will insure against annoying vibrations and unsightly deteriorations, and at the same time reduce the cost.

---

[28] Editorially, citing Henry Wright before the American Institute of Steel Construction, *Engineering News-Record*, October 26, 1939, p. 40 (Vol. p. 526).

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2080

DESIGN OF A HIGH-HEAD SIPHON SPILLWAY

By Elmer Rock,[1] Jun. Am. Soc. C. E.

With Discussion by Messrs. J. C. Stevens, B. E. Torpen, G. E. Hyde, and R. B. Cochrane, I. M. Nelidov, and Elmer Rock.

Synopsis

A direct and clear-cut method of analysis, for use in determining the proportions of a high-head siphon spillway, is presented in this paper. The manner in which well-established principles of hydraulics are applied is outlined, general equations for the determination of throat area and outlet area are derived, and the application of the method as presented is illustrated by the solution of a specific problem.

Introduction

The siphon spillway has become increasingly popular as a device for the automatic control of water-surface elevations. In reservoirs maintained for recreational purposes, in canals, in head-water bays of hydro-electric power plants or, in general, wherever close regulation of pool elevation is necessary, such a device can be used to distinct advantage. Numerous installations in the United States and elsewhere have proved their practicability and adaptability, and they indicate that the siphon spillway occupies a definite place among worthy hydraulic structures.

This paper applies, in the main, to problems arising in connection with the determination of the proportions of a high-head siphon spillway. The term "high-head siphon" is used by the writer to denote siphons that operate under heads in excess of 34 ft, or the barometric height. The head in this case is the difference in elevation of the head-water and tail-water surfaces, or, if the outlet discharges into atmosphere, it is the difference in elevation of the head-water surface and the outlet. Siphons operating under heads less than 34 feet are termed "low-head siphons." The barometric height in feet of water represents the magnitude of atmospheric pressure and is of paramount importance when the action of high-head siphons is considered. The manner

---

Note.—Published in April, 1939, *Proceedings*.

[1] Hydr. Engr., U. S. Engr. Dept., U. S. Engr. Office, Providence, R. I.

in which its value enters into design computations and the effect it has upon siphonic action will be treated subsequently herein.

Strictly, the head at which a siphon becomes a high-head siphon should include some mention of head losses within the siphon tube as well as practical limitations on working pressures. For example, let it be assumed that the total available head at a site is 32 ft, the head lost in the entire structure is 6 ft, and the allowable negative pressure is 24 ft of water This would be a high-head siphon because there is an excess of head (2 ft) over and above the amount it is possible to utilize for producing flow. Cognizance of this excess head comprises one of the special considerations dealt with in this paper. The evaluation of head losses and the selection of an allowable negative pressure are given further mention in a later paragraph. It is believed that, because of its common usage and its vital significance, 34 ft serves, appropriately, to designate the dividing line between a high-head and a low-head siphon. Many of the principles contained in this paper may be applied to low-head siphons with equal effectiveness. The conditions dealt with in the design of low-head siphons, however, present separate problems of a different nature and no attempt will be made to include siphons in that classification.

In the past there has been some discussion concerning the feasibility and the performance of high-head siphon spillways. In general, it is now definitely established that heads far in excess of 34 ft may be utilized, and the action of spillways designed to function under such heads can be predicted with a reasonable degree of accuracy. It is probable that, in some cases, the tendency to over-design a siphon spillway is present. In other words, a factor of ignorance is included to offset the lack of a dependable basis for design. This is true because access to a definite rational method of analysis, carried through from beginning to end, and pertaining to the case at hand, is difficult to find in American literature. A review of the published articles dealing directly with this subject will reveal that methods presented are generally indirect and clumsy.

Some methods are based on the formula

$$Q = C\,A\,\sqrt{2\,g\,H} \quad \text{.......................} \quad (1)$$

in which: Q = the discharge; C = a coefficient of flow; A = area of cross-section; g = acceleration due to gravity; and, H = total hydraulic head available. Experiments and prototype measurement generally establish the value of C between 50 and 80 per cent. Equation (1) serves well to establish a basis for comparison of siphon spillways. It is inadequate for design purposes, however. In any case, it necessitates a cut-and-try procedure and does not take into account local conditions at critical sections. Particularly in the design of high-head siphons, it is inadequate in that no provision is made to convert the excess head into some form of energy such as heat or kinetic energy. The writer proposes to show how excess head is of utmost importance, and how it is provided for by proper dimensioning of the lower leg. In view of these shortcomings it may be said that, as it stands, Equation (1) does not represent a direct and thorough approach. Methods based on a study of the hydraulic gradient alone fail to take into account the nature of the flow in the upper bend where velocity distribution is entirely different from that in a straight pipe. The velocities used in the computation of this gradient are mean values and the pressures at the throat section, as measured from the gradient, do not represent actual localized pressure intensities. The writer does not wish to minimize the value of a plot showing the hydraulic gradient; certainly, it can be used to a great advantage, but its limitations must be recognized.

UPPER BEND AND THROAT

At the outset it must be understood that when a siphon is operating under a head greater than 34 ft, the excess head cannot be used to increase flow through the spillway. If two siphons were constructed, each having the same throat area and designed for maximum discharge, one operating under a 34-ft head and the other operating under an 80 ft head, the latter would not have a greater capacity. This fact is easily understood when the fundamental principle of siphonic action is considered. The lower leg of a siphon makes possible the existence of extremely low absolute pressures in the upper bend, but the only force being exerted to produce flow is the pressure of atmosphere on the head-water surface. Obviously the magnitude of this force is the limitation imposed by nature on all siphons.

Generally, the critical section of any siphon is at the crest. At this section the negative pressures are of greater intensity than at any other section in the siphon tube. Hence it may be concluded that the dimensional characteristics of the structure at its summit determine its capacity. In design work, therefore, it is only natural for initial studies to be confined to the throat of the siphon.

The upper bend is usually formed with regular curves of moderate radii. Almost always the tube is rectangular in cross-section at the summit. The manner in which the water flows—that is, its velocity distribution when passing through this portion of the siphon tube—is the basis for determining the size of the throat opening. The most recent experimental data show conclusively that when water flows through a bend in a conduit the velocity distribution is not symmetrical about the axis of the conduit cross-section. Furthermore, they indicate that velocities are greatest at the inside of the bend and decrease in magnitude toward the outside of the bend. The condition of flow is comparable to that in a free vortex. In the case of a siphon where the upper bend is a rectangular conduit it may be said that, for all practical purposes, the condition is analogous to free vortex flow. It is possible then to utilize the laws governing vortex action in the design of the throat section.[3]

Before any determination of proportions can proceed it is necessary to adopt a minimum absolute pressure that can be tolerated in practice. Theoretically, a high-head siphon could be designed to develop a vacuum of 34 ft of water at its summit, and this would represent the greatest possible efficiency. In other words the absolute pressure would be zero, or the lower theoretical limit. In practice, such degree of rarefaction can never be attained; vapor pressure and elevation above sea level decrease this figure appreciably. Water in its natural state has air in solution and, as the absolute pressure decreases, the quantity of air drawn from the solution increases. In the case of the siphon, this action may cause sufficient air to collect at the summit to interrupt the flow. Furthermore, with negative pressures approaching 34 ft of water, it would not be unreasonable to expect parting of the water from the crest, with its accompanying vibration and cavitation. As a general rule experience with siphon spillways has established 24 ft of water as a maximum negative pressure to be allowed at any point in the structure.[3] In other words the absolute pressure should not be permitted to be reduced below 10 ft of water.

---

[3] "Siphon Spillways," by A. H. Naylor, published in London, England, 1935, by Edward Arnold.

Frequently many of the dimensional characteristics of a siphon spillway are fixed by physical limitations of the structure in which it is incorporated. Thus the radii of curves which form the upper bend may not be dictated by consideration of hydraulic performance. If such limitation does not exist, then two general rules may be applied: (1) As the radius of curvature is increased the cross-sectional area to be provided at the throat decreases; hence it is obvious that excessively sharp curves should be avoided (the logic in this statement is demonstrated subsequently herein); and (2) as the radius of curvature decreases the priming qualities are enhanced. Thus to forestall difficulties from objectionable priming action it is desirable to avoid unduly large radii.

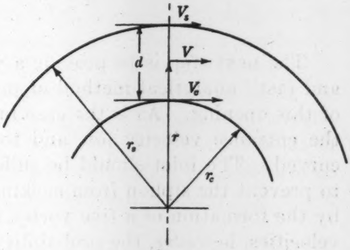


FIG. 1

Having thus fixed the minimum permissible absolute pressure and the upper bend radii, which tie in the depth of the throat section, it is possible to determine the discharge per foot of spillway crest. The maximum head to be utilized for producing velocity is 24 ft. Then the velocity in the upper bend cannot exceed $\sqrt{2\,g\,H} = \sqrt{2 \times 32.2 \times 24} = 39.3$ ft per sec. Referring to Fig. (1), the following procedure is established:

In free vortex flow. . . . . $V \times r =$ constant

Hence. . . . . $V \times r = V_c \times r_c \qquad = V_s \times r_s$

Or. . . . . $V \qquad = \frac{V_c\,r_c}{r}$

From the foregoing computation. . . . $V_c \qquad = 39.3$ ft per sec

Then. . . . . $V \qquad = \frac{39.3\,r_c}{r}$

The discharge for a small element of depth dr is

$$dQ_1 = V\,dr \qquad (2)$$

Substituting for V

$$dQ_1 = \frac{39.3\,r_c}{r}\,dr \qquad (3)$$

---

[3] *Proceedings*, Inst. C. E., Vol. 231 (1930–31), p. 201.

Integrating between the limits of r_c and r_s

$$Q_1 = 39.3\, r_c \int_{r_c}^{r_s} \frac{1}{r}\, dr = 39.3\, r_c \log_e \frac{r_s}{r_c} \quad \ldots\ldots\ldots\ldots\ldots (4)$$

Equation (4) is an expression, in terms of the upper-bend radii, for the maximum discharge per foot of crest that can obtain without exceeding the criterion for minimum absolute pressure. If Q represents the total discharge for which the siphon is being designed, the length of crest, or width of throat section, is given by

$$B = \frac{Q}{Q_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The throat area to be provided is thus determined and equal to the width times the depth. Equations (4) and (5) are perfectly general and can be applied to the upper bend of any spillway whether it is a high-head or a low-head siphon. They indicate the discharge that can pass safely through the upper bend. The remainder of the siphon tube must be proportioned in such a manner that this discharge is not exceeded.

INLET

The next step is to provide a well-proportioned inlet. There is no "hard and fast" analytical method of procedure that can be followed in the design of this opening. As is the case in any conduit intake, it is desirable to keep the entrance velocity low and to avoid sharp corners where flow lines are curved. The inlet should be sufficiently removed from the head-water level to prevent the siphon from sucking air through the surface depression caused by the formation of a free vortex in the head-water bay. With low entrance velocities, however, the probability of having a pronounced vortex is lessened. Hydraulic considerations and experience with siphons have indicated that entrance losses are minimized when the inlet area is two or three times the area of the throat and is formed with well-rounded corners. To prevent the entrance of ice and drift, and to guard against surface disturbances, it is desirable to locate the opening in such a manner that it is at all times well below the water surface. Usually the inlet is protected by rack bars spaced quite far apart.

UPPER LEG

The design of this part of the siphon tube generally presents little difficulty. Probably the only requirement is that the upper leg should taper gradually from the inlet to the throat, thereby providing a smooth transition in which acceleration is affected.

LOWER LEG

Primarily the function of the lower leg of a siphon is to make possible the existence of the partial vacuum that must occur at the summit, and to provide a channel through which the water may escape and be directed to the point of discharge. In the case of a low-head siphon no especial difficulty need be experienced with the lower leg. It is simply a matter of making the tube uniform and equal in area to the throat section, and positive action can be expected. Frequently the lower leg is constructed to incorporate a gradual expansion in cross-sectional area thereby making use of draft action to increase the discharge. When the capacity of the spillway has been determined, the pressures developed at the summit should be investigated by application of Equation (4). If such analysis indicates that the absolute pressure is below the practical limit (say 10 ft), the upper bend should be redesigned to prevent the existence of undesirable negative pressures. For a high-head siphon, however, the problem becomes more involved and warrants a careful analysis. In a high-head siphon, if the lower leg were to be made equal in cross-sectional area to the throat section, and made uniform throughout, excessive vacuums would prevail. Air would be drawn from solution, cavitation would occur at points of reduced pressure, and parting of the water from the siphon tube could be expected. As a result siphonic action would be interrupted intermittently, with accompanying vibration and violent shock.

As has been explained previously, the maximum head that can be utilized to produce flow is 24 ft, plus the head lost due to friction, bends, entrance, etc. Consequently the total head available in excess of that amount cannot be used to increase the discharge of the siphon. That excess head may be expended in two ways: (1) A part of it can be dissipated by introducing resistances such as a bend and friction; and (2) the remainder can be converted into kinetic energy by increasing the velocity of flow at the outlet; that is, by reducing the cross-sectional area of the siphon tube near the outlet by means of a reducer. In effect these resistances provide sufficient back pressure in the lower leg of the siphon to keep the tube flowing full. The problem then is to determine the necessary reduction in outlet size to accomplish this without restricting the capacity of the throat opening.

The cross-sectional area of the throat, the upper bend radii, and the upper leg proportions have already been fixed. To facilitate the following derivation the lower leg is assumed to be equal in area to the throat opening and of uniform cross-section throughout. In this manner the problem is reduced to one that lends itself to analytical solution.

It is possible, now, to compute the head loss in the siphon tube for the discharge Q. The losses should include those due to entrance, friction, bends, and the discharge on exit loss. This determination involves merely a selection of head-loss coefficients and does not warrant further comment. All of the losses can be expressed as a percentage of the velocity head (h_v) in the throat section. Hence, if $K_1 h_v$ is the head loss due to the entrance, friction, bends, etc., and, $K_e h_v$ is the head converted into kinetic energy, then the total head lost is $K_1 h_v + K_e h_v$ or, $h_v (K_1 + K_e)$. It must be remembered that in this expression if no reducer were provided the value of K_e would be equal to unity. Assuming the siphon to discharge into atmosphere, it is evident that the total head lost must equal the total head available. In other words the hydraulic gradient must intersect the siphon tube at its outlet; thus:

$$H = (K_1 + K_e) h_v \qquad \text{(6)}$$

The exit loss coefficient may be expressed in terms of the reference area (the area A_t, at the throat) and the area of the reduced outlet, A_r; that is:

$$K_e = \left(\frac{A_t}{A_r}\right)^2 \quad \text{.......................(7)}$$

Substituting the value for K_e (Equation (7)) in Equation (6):

$$H = \left[K_1 + \left(\frac{A_t}{A_r}\right)^2\right] h_v \quad \text{.......................(8)}$$

and, solving for A_r:

$$A_r = A_t \sqrt{\frac{h_v}{(H - h_v K_1)}} \quad \text{.......................(9)}$$

Thus, the area to which it is necessary to reduce the siphon tube at the outlet can be expressed in terms of quantities given in the basic data or determined from the foregoing calculation. By use of Equations (4), (5), and (9) the proportions of a high-head siphon spillway can be determined.

SOLUTION OF A SPECIFIC PROBLEM

In order to clarify the application of the method presented herein the following concrete example is solved completely. The basic data chosen are: (1) The discharge for which the siphon is to be designed is 350 cu ft per sec; (2) the outlet is to discharge into atmosphere; (3) the head available is 80 ft; (4) the permissible negative pressure is 24 ft of water; and (5) to compute the head lost due to frictional resistance a value of 0.012 is used for Kutter's *n* (this is a fair value in practice for a tube lined with cast iron). Detailed mention of the individual head losses is believed unnecessary. In the selection of head-loss coefficients, it is interesting to note that even sizable errors (for instance, using an *n* value of 0.020 instead of 0.012, for friction loss) have but little effect upon the hydraulic performance. If the coefficients used are too large, the actual discharge and velocities realized will be slightly greater than computed, thus increasing the total head lost. These effects are interrelated and tend to offset each other, with the result that a balance is reached at a discharge not far in excess of the original design value. A similar readjustment will occur if the coefficients used in the design computations are smaller than the actual values in the finished structure. Let it be assumed that consideration of the priming qualities desired and the physical limitations of the structure, as well as other factors, have fixed the radii of the upper bend as 4 ft and 7 ft. The problem is to determine the proportions of the siphon spillway so that the required capacity is obtained with satisfactory hydraulic performance. Fig. 2 is a graphical representation of the basic data selected. The form of the siphon tube was chosen at random and does not follow the pattern of any existing structure.

The first step is to determine the size of throat opening necessary so as not to exceed practical limitations on the minimum permissible absolute pressure at the crest. Equations (4) and (5) are derived in such a manner that the throat dimensions obtained by their application will conform to the fore-

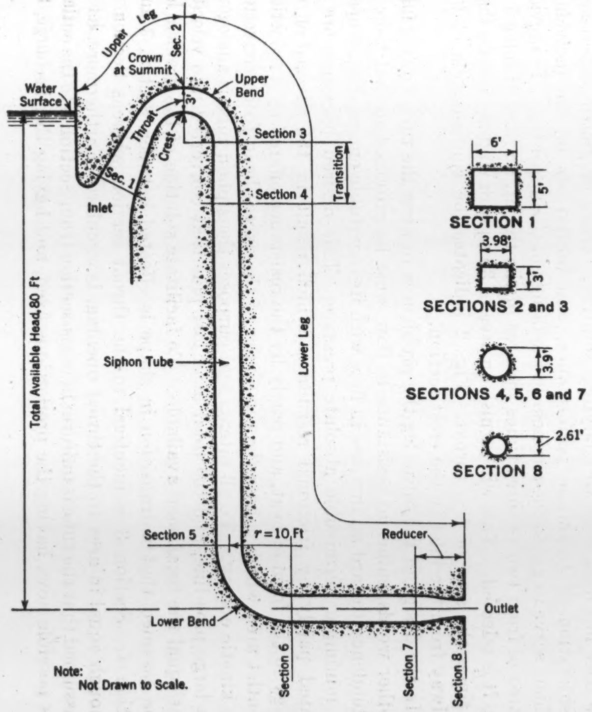

Upper Leg
Sec. 2
Crown at Summit
Water Surface
Upper Bend
3'
Throat
Crest
Section 3
Sec. 1
Inlet
Section 4
Transition
6'
5'
SECTION 1
3.98'
3'
SECTIONS 2 and 3
Total Available Head, 80 Ft
Siphon Tube
Lower Leg
3.9'
SECTIONS 4, 5, 6 and 7
2.61'
SECTION 8
Section 5
r = 10 Ft
Reducer
Outlet
Lower Bend
Note:
Not Drawn to Scale.
Section 6
Section 7
Section 8

FIG. 2.—GENERAL LAYOUT SHOWING PROPORTIONS AND TERMINOLOGY

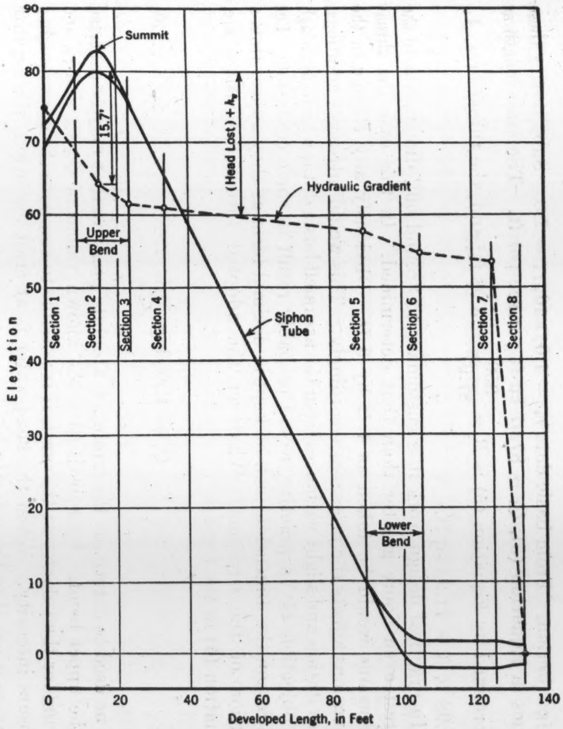


FIG. 3.—PLOT OF COMPUTED HYDRAULIC GRADIENT (DISCHARGE, 350 CUBIC FEET PER SECOND; AND POOL ELEVATION 80 FEET)

going criterion. From basic data $r_c = 4.0$; and, $r_s = 7.0$. Substituting these values in Equation (5), $Q_1 = 87.97$ cu ft per sec per ft. The crest length as determined by Equation (5) is $B = \frac{350}{87.97} = 3.98$ ft; and, the throat area, A_t, is $3.98 \times 3 = 11.94$ sq ft.

In practical design work it is customary to "round off" dimensions to the nearest even figure greater than that determined. In this case the throat dimensions would be taken as 4 ft by 3 ft. Obviously any change in the dimensions affects the pressures accordingly. This would introduce additional computation and slight variances from the assumed basic data, which, although not objectionable in practice, would probably result in some confusion. For this reason the dimensions will be kept in decimal form as they appear from a solution of the equation. Whatever the adopted throat dimensions are, Equation (4) in the form

$$Q_1 = V_c \, r_c \log_e \frac{r_s}{r_c} \qquad (10)$$

can be used to determine the minimum absolute pressure that may be expected in the upper bend. By substituting the known values of Q_1, r_c, and r_s, and solving for the velocity at the crest, it is possible to compute the corresponding pressure intensity. When the head-water is at crest elevation, the negative pressure is equal to the velocity head.

The next step is to provide some form of inlet and to fix the proportions of the upper leg. As stated previously, the design of this part of the siphon tube does not lend itself to analytical solution. Good judgment, experience, consideration of hydraulic performance, and limitations of the individual problem serve to guide the engineer in the design of the upper leg. To provide an area of from two to three times the area of the throat, an inlet opening 5 ft by 6 ft is selected. This will furnish an area of 30 sq ft or slightly greater than 2.5 times the area of the throat. Fig. 2 indicates the configuration of the spillway from the inlet to the crest section.

In the design of the lower leg the problem is to keep the tube flowing full; in other words, sufficient resistance to flow must be provided so that there is no tendency toward an increase in flow with its inevitable encroachment upon the minimum permissible absolute pressure. This desired back-pressure is created partly by frictional resistances which result in the dissipation of energy in the form of heat, and partly by the resistance offered by a reduction in outlet area which causes the remainder of the excess head to be converted into kinetic energy. To all intents and purposes the siphon tube is analogous to a large pipe line, and the total head lost plus that converted into velocity must equal the total head available. To facilitate solution of the problem let it be assumed that a transition in shape is effected between Section 3 and Section 4. Section 3 is identical to the throat section whereas Section 4, although equal in area to the throat opening, is circular. Furthermore, let it be assumed that the tube is uniform in cross-section from Section 4 to the outlet. It is possible now, having the proportions fixed, and having the discharge, to compute the losses that occur in the entire structure. The order and magnitude of these losses, as computed for the problem at hand, are recorded in Table 1.

TABLE 1.—TABULATION OF LOSSES

Section (see Fig. 2)	LOSSES EXPRESSED AS A DECIMAL FRACTION OF THE VELOCITY HEAD IN THE THROAT SECTION					Loss, in feet	Velocity, in feet per sec	h_v, in feet	Accumulated losses plus h_v	Elevation of hydraulic gradient
	Entrance	Contraction	Bends	Friction	Total					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	0.032	....	....	....	0.032					
1						0.4	16.7	4.3	4.7	75.3
	....	0.084	0.052	0.018	0.154					
2						2.0	29.3	13.3	15.7	64.3
	....	....	0.160	0.044	0.204					
3						2.7	29.3	13.3	18.4	61.6
	....	....	....	0.048	0.048					
4						0.6	29.3	13.3	19.0	61.0
	....	....	....	0.245	0.245					
5						3.2	29.3	13.3	22.2	57.8
	....	....	0.160	0.069	0.229					
6						3.0	29.3	13.3	25.2	54.8
	....	....	....	0.088	0.088					
7						1.2	29.3	13.3	26.4	53.6
	....	....	....	0.035	0.035					
8						0.5	65.4*	66.4	80.0	0

* Velocity in the reduced outlet area.

The values shown in Column (6), Table 1, represent the total head lost between sections expressed as a percentage of the velocity head in the throat section. A summation of the quantities in Column (6), Table 1, gives the value of K_1 in Equation (9) when $A_t = 11.94$ sq ft; $H = 80.0$ ft; $K_1 = 103.5\%$ (the summation of Column (6)); and, $h_v = 13.3$ ft. Substituting these values in Equation (9), it is found that the area to which the outlet must be reduced is 5.35 sq ft, or the diameter of the outlet will be 2.61 ft. Thus with the determination of the outlet size the problem is solved. Usually the outlet piece is a special casting and can be made to furnish the exact area required. The purpose of the reducer or choke at the outlet is to avoid excessive negative pressures, which result in unsatisfactory hydraulic performance. If an area greater than that determined herein is selected, smooth action in the operation of the spillway would be jeopardized. On the other hand, if greater constriction is provided, other conditions being equal, the only adverse effect would be a reduction in the spillway capacity. This is undesirable economically since the remainder of the siphon tube is not being used to the greatest advantage.

In the computation of the head losses appearing in Columns (2), (3), (4), and (5), Table 1, the writer used standard forms accepted in practice. The actual coefficients used are not shown since it is believed that a discussion relative to the evaluation of head losses might detract from the original purpose of the paper. Other ramifications might be introduced such as moving the reducer nearer to the throat or providing a gradual taper instead of a relatively sudden contraction. The writer is confident that the problem, as solved herein, together with the simplifying assumptions introduced, serves adequately to elucidate the fundamental principles involved. The engineer frequently finds it necessary to use the standard pipe available in stock sizes. This may be done, but caution must always be used and the final layout must be investigated to be sure that the revised proportions do not induce pressures below the minimum permissible absolute pressure.

Columns (7) to (11), Table 1, are self-explanatory and were computed so that a plot of the hydraulic gradient could be drawn. Since Column (6) represents the total head lost between sections, expressed as a decimal fraction of the velocity head in the throat section, Column (7) is obtained simply by multiplying the values in Column (6) by 13.3 ft, or the velocity head in the throat section. The drop in the hydraulic gradient from the level of the headwater surface at any section is equal to the sum of the accumulated losses plus the velocity head at that section. Using the center line of the outlet as datum, Column (11) shows the elevation of the hydraulic gradient at any section. A graphical plot of the gradient is shown in Fig. 3 and illustrates quite clearly its limitations as well as its application. The condition of flow in the upper and lower bends is such as to invalidate the pressures shown by the computed hydraulic gradient in those regions. For the sake of illustration the siphon tube is distorted by plotting length along the spillway axis as the abscissa against elevation as the ordinate. With such an arrangement it is possible to measure pressures at any point along a vertical line. The preceding text has demonstrated that a computed hydraulic gradient could not be utilized to predict localized pressure intensities at the crest. In Fig. 3 it would seem that the negative pressures to expect at the crest and the crown at the summit would be 15.7 ft and 18.7 ft of water, respectively. Actual prototype measurements and model experiments prove conclusively that such deduction is erroneous. The order of magnitude of pressure variations in the upper bend, as computed by assuming free vortex flow, gives results closely in accordance with actual flow conditions. In the basic data for the problem under consideration the outlet was assumed to be discharging into atmosphere; hence the hydraulic gradient must intersect the siphon tube at its outlet. Fig. 3 indicates how the formula derived and the procedure outlined so adjusts the proportions of the spillway that for the design discharge of 350 cu ft per sec the head losses sustained through the entire structure, plus the exit velocity head, bring the gradient down to the proper elevation at the outlet.

In this paper the writer has intentionally made little or no reference to details such as priming, air vents, ejectors, sealing basins, etc. (A. H. Naylor in 1935[2] and the late G. F. Stickney, M. Am. Soc. C. E., in 1922,[4] have offered excellent discussions of these features.) This was done, not because they are unimportant, but because the phase of siphon design dealt with is complete without them. Naturally each high-head siphon spillway that is designed presents individual problems. The writer does not wish to create the impression that the simple application of the specialized formulas constitutes an "iron-clad," fool-proof method of solution. It is necessary to have a clear conception of the laws governing siphonic action as well as an understanding knowledge of basic hydraulic principles involved.

---

[4] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1098.

CONCLUSION

In the design of a high-head siphon spillway for which it is desired to fix the proportions so that a definite capacity can be provided, there are two considerations of paramount importance: (1) The provision of adequate cross-sectional area at the throat, thereby preventing the occurrence of excessive negative pressures; and (2) the provision of a reduced outlet area to prevent excessive pressures and consequent intermittent and highly unsatisfactory operation.

The first requirement can be met by assuming free vortex flow in the upper bend. Such an assumption very closely approximates actual flow conditions in that part of the siphon tube. A study of the hydraulic gradient will not reveal true localized pressure variations at the summit.

To determine the proper degree of constriction at the outlet without encroaching upon the full capacity of the throat opening it is necessary to analyze the head losses in the siphon and so adjust the outlet area that the hydraulic gradient intersects the lower leg at its extremity. If the siphon does not discharge into atmosphere the gradient must indicate the pressure at the outlet.

ACKNOWLEDGMENT

The writer is indebted to John B. Drisko, Jun. Am. Soc. C. E., for his constructive criticism and helpful suggestions.

DISCUSSION

J. C. Stevens,[5] M. Am. Soc. C. E. (by letter).—Two outstanding features of this paper are its clarity and brevity. May this discussion be as concise.

It will be of interest to compare the ideological design presented by Mr. Rock with the behavior of the Leaburg[6] and Walterville[7] siphons on which field tests have been made and fully described in the Society's publications.

The writer has set forth the basic principles of siphon behavior in a paper published in 1933.[6] The author's "high-head" siphon corresponds to the case in Fig. 10 of that paper when Section 3 coincides with or lies below Section 2, and the "low-head" siphon when Section 3 lies above (submerges) Section 2.

An important fact in siphon behavior which the author has failed to take into consideration is that the siphon barrel is occupied only in part by water moving down stream and in part by eddies. Fig. 3 of the writer's paper[6] illustrates an idealized concept of the effective and non-effective areas of the Leaburg siphons.

In 1938[8] data were presented to show the eddy area at each cross-section indicating a maximum of 28% of non-effective area. The effect of this phenomenon is to nullify, largely, the author's calculations as to a limiting velocity within the barrel.

The writer will hazard the guess that, if the siphon illustrated by Mr. Rock in Fig. 2 were in use and under observation, the maximum mean velocity would be found, not at the summit section, but somewhere in the lower leg and that its value would be 40 ft per sec or more. Table 2 shows that the maximum mean velocities in the Leaburg and Walterville siphons were from 23% to 55% greater than the mean velocities in the summit sections and that they occurred well down in the lower leg. During this acceleration the jet must suffer a contraction and leave eddying water around it. The continuity law "$Q = A V$" holds only if A is the area of the jet and not that of the barrel.

The author's assumption that the velocity distribution through the summit section follows the free vortex law, wherein the radial acceleration $v r$ is constant, is not substantiated by the tests. Table 2 also gives the velocities at the crowns and inverts of the five siphons tested, determined from the piezometer readings at these points. The data may be verified from Table 2 and Figs. 7, 8, and 9 of reference given in Footnote 6, and Table 1 and Fig. 7, of reference given in Footnote 7. Note that the radial acceleration at the crowns is from 14% to 35% greater than at the inverts.

The necessity of admitting water to the summit section through a priming conduit is one factor that may nullify the application of the vortex principle.

The author is correct in stating that the head on a siphon may exceed that of an atmosphere. The highest head that can be utilized is one atmosphere minus vapor pressure of the water plus all entrance, friction, bend, eddy losses, and exit velocity head. On the Walterville siphons the head is nominally 46 ft. There is no shock or vibration or other evidence of a disconnected water column.

---

[5] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[6] "On the Behavior of Siphons," by J. C. Stevens, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 986.

[7] "Siphons as Water-Level Regulators," by J. C. Stevens, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 1787.

[8] *Loc. cit.*, Table 4, p. 1638.

TABLE 2.—DATA FROM TESTS OF LEABURG AND WALTERVILLE SIPHONS

	LEABURG SIPHON			WALTERVILLE SIPHON	
	No. 7	No. 6	No. 5	No. 1	No. 2
(*a*) Basic Data:					
Discharge, in cubic feet per second	119	250	470	700	350
Total head, in feet	28.4	27.4	26.4	45.9	45.7
Mean velocity at summit, in feet per second	34.6	28.4	28.5	27.0	29.5
Maximum mean velocity, in feet per second	42.6	44.0	43.0	36.0	41.2
Distance below summit, in feet	7.3	7.2	8.8	29.5	26.2
Ratio of maximum velocity to summit velocity	1.23	1.55	1.51	1.33	1.39
Forebay level, in feet	734.3	734.3	734.6	595.9	595.8
(*b*) For the Crown at the Summit Section:					
Elevation, in feet	735.2	735.8	736.8	598.7	597.4
Pressure head, in feet	−17.0	− 9.6	−10.9	− 9.8	−11.1
Elevation pressure line, in feet	718.2	726.2	725.9	588.9	586.3
Pressure drop from forebay, in feet	16.1	8.1	8.7	7.0	9.5
Energy loss, in feet*	1.6	0.8	0.9	0.7	1.0
Velocity head, in feet	14.5	7.3	7.8	6.3	8.5
Velocity V_1	30.5	21.7	22.4	20.1	23.4
Radius r_1, in feet	3.5	4.0	5.0	5.7	4.5
Radial acceleration $V_1 r_1$	117	87	112	115	105
(*c*) For the Invert of the Summit Section:					
Elevation	734.0	734.0	734.0	595.1	595.0
Pressure head	−30.9	−24.3	−28.8	−30.2	−32.3
Elevation pressure line	703.1	709.7	705.2	564.9	562.7
Pressure drop from forebay	31.2	24.6	29.4	31.0	33.1
Energy loss*	3.1	2.4	2.9	3.1	3.3
Velocity head	28.1	22.2	26.5	27.9	29.8
Velocity V_2	42.5	37.8	41.3	42.4	43.8
Radius r_2	2.0	2.0	2.0	2.0	2.0
Radial acceleration $V_2 r_2$	85	76	83	85	88
Ratio $\frac{V_1 r_1}{V_2 r_2}$	1.35	1.14	1.35	1.35	1.20

*Estimated at $\frac{0.1\ V^2}{2\ g}$; includes entrance, friction, and bend losses, entrance to summit section.

The writer can see no value in contracting the outlet to consume head. If 80 ft of head were available it would be far more practicable to discharge the siphon into an open chute after using 40 ft or so for the siphon head. Open conduits are much cheaper than siphon barrels. This plan was followed at Walterville. In the final stage of reconstruction the tail-water is to be lowered about 7 ft and a chute built from the end of the siphon barrel,[9] to discharge the water well below the water surface, absorbing the energy by an hydraulic jump.

B. E. TORPEN,[10] M. AM. SOC. C. E., G. E. HYDE,[11] JUN. AM. SOC. C. E., and R. B. COCHRANE,[12] ASSOC. M. AM. SOC. C. E. (by letter).—The approach to the problem of siphon spillway design presented in this paper may prove meritorious. Equation (4), based on the assumption that free vortex flow prevails in the bend of the throat section of a siphon spillway is a simplifying step, and

---

[9] "Siphons as Water-Level Regulators," by J. C. Stevens, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), Fig. 6, p. 1794.

[10] Prin. Engr., Corps of Engrs., U. S. A., Portland, Ore.

[11] Junior Engr., Bonneville Hydr. Laboratory, U. S. Engr. Office, Bonneville, Ore.

[12] Assoc. Engr., War Dept., U. S. Engr. Office, Bonneville, Ore.

TABLE 3.—COMPARISON BETWEEN VELOCITIES COMPUTED BY THE AUTHOR'S FORMULA (EQUATION (11)) AND ACTUAL VELOCITIES

Item	Description	SIPHON NUMBER*			Average
		5	6	7	
	(*a*) DATA ON SELECTED SIPHONS*				
1	Discharge Q, in cubic feet per second	470.0	250.0	119.0	....
2	Operating head H, in feet	26.4	27.4	28.4	....
	Dimensions of Throat Section, in Feet:				
3	Width, b	6.03	4.52	2.68	....
4	Height, d	2.75	1.77	1.26	....
	Radius at the:				
5	Crest, r_c	2.12†	2.12†	2.12†	....
6	Mid-section, r_{av}	3.50	3.00	2.75	....
7	Summit, r_s	4.88†	3.88†	3.38†	....
	(*b*) VELOCITIES COMPUTED BY THE ROCK FORMULA (EQUATION (11))				
8	$\log_e r_s/r_c$	0.833	0.605	0.466	....
9	Discharge per unit width ($Q_1 = Q/b$), in cubic feet per second	78.0	55.3	44.4	....
	Computed Velocities in the Throat Section, in feet per second:				
10	Crest, V_c	44.1	43.2	45.0	44.1
11	Mid-section, V_{av}	26.7	30.6	34.6	30.6
12	Summit, V_s	19.2	23.6	28.2	23.7
	(*c*) ACTUAL VELOCITIES (SEE EQUATION (12))				
	Hydraulic Heads, in Feet:				
13	Elevation of energy line, B‡	764.8	765.0	760.6	....
14	H_c = Item 13 − 734.0§	30.8	31.0	26.6	....
	Elevation of points above the crest, at the:				
15	Crest, Z_c	0	0	0	....
16	Mid-section, Z_{av}	1.375	0.885	0.630	....
17	Summit, Z_s	2.75	1.77	1.26	....
	The ratio p/w,‖ at the:				
18	Crest	−28.8	−24.3	−30.9	....
19	Mid-section	−14.3	−14.2	−24.6	....
20	Summit	−10.7	−9.6	−17.0	....
	Velocity Heads,¶ at the:				
21	Crest	26.6	22.3	24.5	....
22	Mid-section	10.7	11.3	17.6	....
23	Summit	5.8	5.8	9.3	....
	Actual Throat Velocities, in Feet Per Second:				
24	Crest, V_c	41.4	37.9	39.7	40.0
25	Mid-section, V_{av}	26.2	27.0	33.7	29.0
26	Summit, V_s	19.3	19.3	24.5	21.0
	(*d*) COMPARISON OF ACTUAL VELOCITIES WITH THOSE COMPUTED BY EQUATION (11)				
	Percentage Differences by Equation (13):				
27	Crest = [(Item 10 − Item 24)/Item 24] 100	+6.5	+14.0	+13.4	+10.2
28	Mid-section = [(Item 11 − Item 25)/Item 25] 100	+1.9	+13.3	+2.7	+5.5
29	Summit = [(Item 12 − Item 26)/Item 26] 100	−0.5	+22.3	+15.1	+12.9

---

* See *Transactions*, Am. Soc. C. E., Vol. 99 (1934), Fig. 3, p. 989.

† Assumed average values in order to start the radii at a common center.

‡ *Transactions*, Am. Soc. C. E., Vol. 99 (1934), Column (8), Table 3, p. 995.

§ Assume the datum at Elevation 734.0 to equal the elevation of the crest of all three spillways.

‖ *Transactions*, Am. Soc. C. E., Vol. 99 (1934), Columns (4), (7), and (10), Table 2, p. 994.

¶ Velocity head computed as the difference between the energy gradient and the hydraulic gradient.

results in doing away with the necessity of selecting a "coefficient of ignorance" masked under the name of Flow Coefficient. Therefore, his attempt to rationalize the common procedure of design for throat sections of siphon spillways is worthy of commendation.

Vortex Flow Assumption.—Certain questions and doubts arise, however, concerning the soundness of the author's assumption of vortex flow in siphon throat sections, and concerning the practical application of his formula in the design of siphon spillways. In an attempt to answer these questions and to eliminate these doubts, the velocity distributions, as they actually exist in the throat sections of three different siphon spillways[13] were compared, respectively, to the velocity distributions as computed from Mr. Rock's formula (see Equation (10)):

$$V = \frac{Q}{r \log_e \frac{r_s}{r_c}} \quad \text{.........................} (11)$$

The numerical calculations involved in this comparison are given in Table 3 and the results are portrayed graphically in Fig. 4. Table 3 first lists the actual operating conditions of the three selected siphons. These data were used in Mr. Rock's formula to compute the velocities at the crest, mid-section, and summit of each siphon throat section. A second set of similar velocities at these points was then computed by subtracting the actually measured hydraulic gradient from the energy gradient. That is,

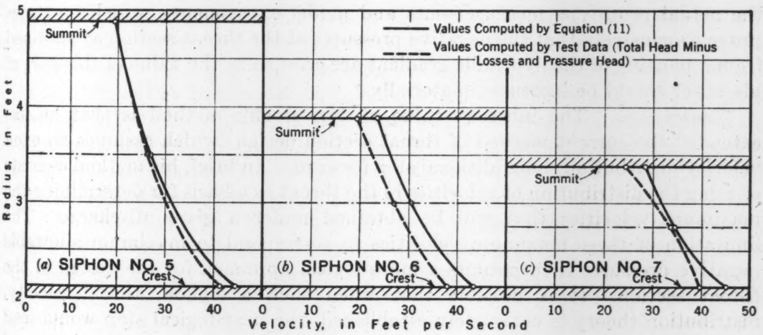


FIG. 4

$$\frac{V^2}{2g} = H_e - \left(Z + 33 + \frac{p}{w}\right) \quad \text{.................} (12)$$

Comparison was finally made between the two sets of velocities by percentage difference in order to obtain an index as to the validity of Mr. Rock's formula.

---

[13] "On the Behavior of Siphons," by J. C. Stevens, M. Am. Soc. C. E., *Transactions*, Vol. 99 (1934), p. 986.

Arithmetically:

$$\text{Percentage difference} = \frac{V_{\text{Rock}} - V_{\text{actual}}}{V_{\text{actual}}} \dots\dots\dots\dots (13)$$

It will be noted from Fig. 4 that in all three cases, the actual velocity values, as derived from the test data, are greater at the crest than at the summit, and that the velocity values as computed by the author's formula (Equation (11)) conform fairly well with the so-called actual values. When the number of ways in which experimental and personal error may have entered into the test data are considered, the comparison in Fig. 4 is believed to be surprisingly good; and, furthermore, this comparison appears to be a strong indication that the author's assumption of vortex flow is justified. In addition to the foregoing comparison, the practical application of Mr. Rock's formula was tested by checking the throat sections of several other existing siphon spillways. It was found that the computed and actual values checked quite satisfactorily.

Further Considerations.—The author's treatment of the importance of proper lower leg design in high-head siphons is considered good. The mathematics involved in this phase, as well as those concerning the derivation of his vortex flow formula, were found to be valid. The numerical solution of the example siphon, as given by Mr. Rock, aptly illustrates his point that a plot of the hydraulic gradient does not result in true negative pressures at the crest and summit of the throat section. However, if the author had included in his text the actual prototype measurements and model experiments, which he states prove conclusively that the negative pressures at the throat section as obtained from a plotting of the hydraulic gradient are erroneous, the value of this part of his paper would be increased materially.

Conclusions.—The inherent value of Mr. Rock's method is that he has extended the current method of throat section design (which assumes an even velocity distribution) an additional step forward. In brief, his method consists of using the distribution of velocities in the throat as a basis for determining the maximum velocities that will be obtained under a given discharge. The limitation of these maximum velocities to correspond to maximum allowable negative pressures then permits a more logical approach for the design of the throat section. It is possible that when such a siphon, throat-velocity, distribution theory is completely established, the next logical step would lead to an attempt to increase the efficiency of siphon operation by redesigning the throat section into other than a rectangular or circular section. Such a design, of course, would have to result in a gain in efficiency of operation sufficient to overcome increased construction costs.

I. M. Nelidov,[14] Assoc. M. Am. Soc. C. E. (by letter).—Not only does the paper demonstrate the design of a particular siphon, but it also contains a theory with general rules intended for the design. The author has based his statements on an ideal case of a frictionless siphon, consisting only of the throat section. For this case he determines the theoretical velocity based on full vacuum head, and the resulting discharge. He also classifies as "high-head" siphons those which have operating heads greater than 34 ft. Furthermore, he has selected a siphon with an 80-ft head of fairly uniform cross-sections, with the exception of the outlet section, which he throttled in order to fill the outlet leg; and he classifies it as a "high-head" siphon.

---

[14] Senior Engr. of Hydr. Structure Design, State Div. of Water Resources, Sacramento, Calif.

Under "Introduction" the author states: "The term 'high-head siphon' is used by the writer to denote siphons that operate under heads in excess of 34 ft, or the barometric height"; and, under "Upper Bend and Throat": "As a general rule experience with siphon spillways has established 24 ft of water as a maximum negative pressure to be allowed at any point in the structure."[3] It is not clear from the foregoing which of the two heads (24 ft or 34 ft) should be used in defining the high-head and low-head siphon.

Mr. Rock qualifies the conditions for an ideal frictionless siphon without outlet, as stated herein. This may be satisfactory for the purpose of design but is not acceptable for the purpose of checking when one must classify an existing siphon and obtain the discharge on the basis of this classification.

Referring strictly to terminology, one may ask why, for instance, a 50-ft head should be called a "high head." In reference to dams at which siphon spillways mostly occur it cannot be called a high head. Perhaps heads beginning with 100 ft or 150 ft should be called high heads.

The dividing line, in the opinion of the writer, lies not in the head but in the functioning of a siphon. To illustrate, a siphon with an operating head of 10 ft and in which there is a considerable contraction at the summit will create a limiting velocity; and a further increase of head more than 10 ft will not produce any increase of flow. Under the same head, if the maximum contraction is made at the outlet, the maximum velocity will not be developed. On the other hand, a siphon under 100 ft of head and with a maximum contraction at the outlet may not produce the limiting velocity; and, further increase or decrease of the operating head will produce a change in the discharge. If the outlet is sufficiently constricted this siphon will develop the maximum velocity.

Consequently, independently of value of the operating head, a siphon may flow with a limiting velocity and without an increase of discharge due to further increase of the head; or, it may flow with a velocity below the limiting value, discharge being affected by increase of the head. The writer proposes to divide the siphons into two classes depending upon whether the operating head is greater or less than the limiting head at which the siphon produces limiting velocity. Thus, siphons may be classed according to whether their discharge is equal to or lower than the maximum discharge. The head at which this division occurs may be called the "limiting head." It should be made clear that the limiting head should not be related to any definite value of 24 ft or 34 ft, but that it should be taken as the head producing the limiting velocity. As mentioned previously herein, the classification should be made general both from the standpoint of design and checking, so that by reviewing an existing siphon one may also be able to classify it.

As to the theory proper, it may be noted that if the postulates of the author about the position and magnitude of vacuum are temporarily accepted the following derivations will be produced.

The limiting head will be (see Equation (6)):

$$H_{\text{lim}} = (K_e + K_1\, n)\, h_v \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

in which: K_1 and K_e are as denoted by the author; h_v is the velocity head at the throat corresponding to mean velocity, V_m; and n is a coefficient in the expression:

$$E_k = n\, h_v \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

in which E_k is the average kinetic energy of the curvilinear flow per unit of flow, such that[15]

$$E_k = \frac{\displaystyle\int_0^d \frac{V^3}{2\,g}\,dy}{\displaystyle\int_0^d V\,dy} \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

In order to understand, better, the conditions existing at the throat, and using the criterion that the absolute pressure at crest of the throat should not be less than zero, one may draw Fig. 5. The notation is by Boris A. Bakhmeteff,[15] M. Am. Soc. C. E., and the author.

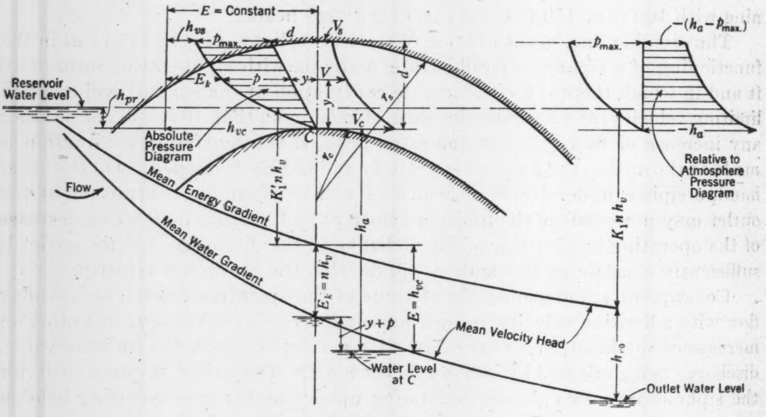

E = Constant
h_{vs}
$p_{max.}$
d
V_s
$-(h_a - p_{max.})$
$p_{max.}$
Reservoir Water Level
h_{pr}
E_k
p
y
V
y
d
r_s
h_{vc}
V_c
C
$-h_a$
Absolute Pressure Diagram
r_c
Relative to Atmosphere Pressure Diagram
Flow
Mean Energy Gradient
$K_1' n\, h_v$
h_a
$K_1 n\, h_v$
Mean Water Gradient
$E_k = n\, h_v$
$E = h_{vc}$
$y + p$
Mean Velocity Head
h_{r0}
Water Level at C
Outlet Water Level

FIG. 5

Then, denoting $m = \frac{r_s}{r_c}$ and

$$\beta = \frac{V_s}{V_m} = \frac{1}{\frac{r_s}{d}\log_e \frac{r_s}{r_c}} \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

integration of Equation (16) yields:

$$n = \frac{\beta^2}{2} \times \frac{m^2 - 1}{\log_e \frac{r_s}{r_c}} \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

---

[15] Discussion by Boris A. Bakhmeteff on "Tests of Broad-Crested Weirs," by James G. Woodburn, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 423.

The criterion for zero absolute pressure at the crest of the throat is:

$$h_a + h_{pr} = h_{vc} + n\, h_v\, K_1' \qquad (19)$$

in which: h_a is the atmospheric head; h_{pr} is the priming head; $h_{vc} = m^2 h_{vs} = m^2 \beta^2 h_v$; and K_1' is the sum of the coefficients of loss from inlet to crest, referred to mean velocity of the throat. Substituting

$$h_v = \frac{h_a + h_{pr}}{m^2 \beta^2 + n\, K_1'} \qquad (20)$$

from Equation (19) into Equation (14):

$$H_{\text{lim}} = (K_e + K_1\, n)\, \frac{h_a + h_{pr}}{\beta^2 m^2 + n\, K_1'} \qquad (21)$$

The discharge per linear foot of the crest, with velocity $V_m = \sqrt{2\, g\, h_v}$, will be:

$$Q = d V_m = d \sqrt{\frac{2\, g\, (h_a + h_{pr})}{\beta^2 m^2 + n\, K_1'}} \qquad (22)$$

which reduces to Equation (4) if $h_{pr} = K_1' = 0$. Applying Equations (14) to (22) for the particular siphon in consideration: $K_1 = 1.035$; $K_1' = 0.154$; $K_e = 4.980$; $m = \frac{7}{4} = 1.75$; $d = 3$ ft; $h_a = 24$ ft; and $h_{pr} = 0$. Then, by Equation (17), $\beta = 0.770$; $\beta^2 = 0.593$; and $m^2 = 3.060$. Furthermore, by Equation (18): $n = 1.090$; and, by Equation (21), $H_{\text{lim}} = 78.5$ ft < 80 ft.

Consequently, the siphon as designed by the author is a "high-head" siphon by only a very small margin. The means by which the limiting head was raised to 78.5 ft is somewhat artificial and consists in throttling of the outlet section only. Otherwise, this siphon would definitely fall into the "high-head" class. In this case, with $K_e = 1$, the limiting head will be $H_{\text{lim}} = 26$ ft.

It would be better to designate this siphon as one belonging to a class with maximum discharge. The limiting discharge corresponding to a head of 78.5 ft, by Equation (22), will be: $Q = 3 \sqrt{2\, g\, \frac{24}{3.050 \times 0.593 + 1.090}} = 3 \times 29.9 = 84$ cu ft per sec per ft. This is the maximum discharge that this siphon will produce, disregarding the further increase of the operating head.

In regard to the limits of applicability of Equation (4), it may be noted that with a constant depth of the throat "d" the discharge Q_1 decreases with the decrease of r_c, and does so especially rapidly for the ratio $\frac{r_s}{r_c} < 2$.

With a sharp crest such as may occur in a siphon with a vertical barrel, $\frac{r_s}{r_c} = \infty$ and, theoretically, the discharge approaches zero. Actually, however, there will be some flow through the siphon. A cavity will form above the sharp crest, producing some minimum radius r_c, depending on the conditions of the inlet. Consequently, if a siphon under consideration has an exceedingly small radius, r_c, it should be verified that the latter is greater than its allowable minimum if an expression for flow similar to Equation (4) is to be applied safely. The value of r_e (min) may be determined with the aid of the momentum principle used for entrance and throat sections.

The ratio of radii selected by the author is: $k = \frac{r_c}{r_s} = \frac{4}{7} = 0.570$. This is not the ratio, however, that gives the maximum discharge. From Equation (4) it follows that the discharge reaches its maximum value with a certain value of ratio k, r_s being constant. Representing Q_1 as a function of k:

$$Q_1 = 39.5\, r_s\, F(k) \qquad (23)$$

in which $F(k) = k \log_e \frac{1}{k}$. Differentiating F by k and equating it to zero, the value $k = 0.367$ produces the maximum value of $F_{max} = 0.368$; and Q_1 (max) $= 103$ cu ft per sec per ft. In this case the radius at the crest should be $r_c = 0.367 \times r_s = 2.57$ ft, and the depth of the throat should be $d = r_s - r_c = 4.43$ ft, against $d = 3$ ft, selected by the author. It would be interesting to compare the author's basis for selecting the radii.

The position of the maximum vacuum within the siphon, so far as it may be determined for siphons with straight outlet leg, is at the crest of the throat section.[3, 16] In the siphons with another convex-upward curvature, besides the throat proper, situated down stream from the latter and without throttling at the outlet end maximum vacuum is developed at the section somewhat down stream from the throat section.[7] This is due to an unrestricted increase in velocity and could be relieved by throttling the outlet section. Therefore, apparently the position of maximum vacuum at the crest is justified for a siphon with straight outlet leg.

The magnitude of the maximum relative negative pressure is limited, by the author, to 24 ft. This value acquires significance for computing the "limiting head" and the maximum discharge. It becomes especially important for siphons at high elevations. For instance, at an elevation of 6 000 ft the vacuum head is normally reduced from 34 ft to 27 ft. If, then, 10 ft should be deduced from it for vapor pressure and other imperfections, it will leave only 17 ft as actual vacuum head. It is of importance, therefore, to make the recommendation as to the limit of the vacuum head to be used at different elevations more specific.

Prof. A. N. Gibson[3] states: "With reference to the limit of velocity at the throat of the siphon the maximum velocity should not be higher than would cause a negative pressure greater than about 24 feet of water." Mr. A. H. Naylor, one of the discussers of Professor Gibson's paper, states: "The pressure was lowest on the inside radius and that minimum pressure must not be less than a vacuum of about 28 feet of water." B. A. Etcheverry,[17] M. Am. Soc. C. E., states: "In practice on account of the air entrained in the water, the maximum suction lift is often taken as about 28 feet."

It follows from the foregoing citations that recommendations for vacuum head vary from 24 ft to 28 ft. On the other hand, during experiments made by

---

[16] "Hydraulic Laboratory Practice," by the late J. R. Freeman, Hon. M. Am. Soc. C. E., pp. 515 and 610.

[17] "Irrigation Practice and Engineering," by B. A. Etcheverry, and S. T. Harding, Members, Am. Soc. C. E., 2d Ed., McGraw-Hill Book Co. Inc., New York, N. Y., 1933.

J. C. Stevens,[7, 13] M. Am. Soc. C. E., negative pressures of about 32.5 ft were actually observed at Elevation 700 above sea level, all of which shows the variety of numerical values suggested for vacuum head. There is a need for a more precise definition of the vacuum head based more on experimental data than has been produced to date. Consequently, it would be valuable to receive from the author a more complete description of data upon which he recommended 24 ft of vacuum head with such confidence.

In conclusion, the writer wishes to say that the value of the proposed theory should be proved in checking the computed discharges with the discharges actually observed. He has selected the papers[7, 13] by Mr. Stevens for this purpose, using the observed maximum negative pressure at the crest as the velocity head h_{vc} (omitting losses from the entrance to the throat).

Siphons Nos. 5, 6, and 7, reported by Mr. Stevens, gave the discrepancies from the observed discharges as 0, −2, and 12 per cent. Siphons Nos. 1 and 2 reported in a second paper indicated a discrepancy of −6% and 0 per cent. This indicates that, at least for this type of siphon, the agreement of theory and practice is fairly satisfactory.

ELMER ROCK,[18] JUN. AM. SOC. C. E. (by letter).—The discussions are of a high caliber and were written by engineers well qualified to criticize, constructively, siphon design procedure. Unquestionably the value of the paper has been greatly enhanced by these contributions.

Mr. Stevens has selected Siphons Nos. 5, 6, and 7 at Leaburg, and the two siphons at Walterville for the purpose of comparing prototype measurements with predicted velocities based on assuming free vortex flow in the upper bend. He emphasizes the fact that in these particular siphons the barrel is occupied only in part by water moving downstream and in part by eddies. It is doubtful that the designer of these siphons intended to have a portion of the tube noneffective. This phenomenon is not characteristic of all siphons. At Leaburg the total head is approximately 27 ft and one would not expect separation of the jet from the siphon tube. However, upon investigation it is found that the cross-sectional area of the tube at the outlet is 1.5 times the throat area. The jet issuing from the throat section is unable to follow the expanding barrel, under those conditions, with the result that part of the area is rendered noneffective, velocities are increased, and extremely low absolute pressures are developed. At Walterville the total head is approximately 46 ft and the lower leg has a constant area throughout its length. In this case separation of the jet from the barrel of the siphon could be prevented by tapering the lower leg so that the outlet area is reduced. In the opinion of the writer a siphon, in order to operate smoothly and without vibration, should be designed to eliminate parting of the water from the siphon tube, cavitation, and the presence of eddies or turbulent water pockets.

When the water leaves the invert of the upper bend, as it does at Walterville and Leaburg, a new boundary is formed and its radius is no longer equal to the radius of the invert, but somewhat greater. It is difficult to determine the exact value of this increased radius. Accordingly, the values of r_2 that Mr. Stevens used in Table 2 are not correct and the values for radial acceleration at the crest are misleading. It is obvious that if r_2 is increased the radial acceleration ($V_2\ r_2$) will increase, resulting in better agreement with the radial acceleration at the crown. Aside from this there are other variations that should be considered. For instance, it may not be justifiable to place full trust in prototype measurements where near absolute zero pressures exist and flow conditions are extremely disturbed. In his tests[19] Mr. Stevens has found that a piezometer connection registering very low absolute pressure would suddenly spout water up in to the mercury gage. This indicates that conditions are highly unstable in the upper bend of a siphon in which the jet is allowed to separate from the walls, and measurements made under such circumstances might not be entirely reliable. From the foregoing it follows that Mr. Stevens has not shown conclusively that the assumption of free vortex flow in the upper bend of a siphon is invalid.

---

[18] Hydr. Engr., U. S. Engr. Dept., U. S. Engr. Office, Providence, R. I.

Flow conditions in a pipe bend (such as the upper bend of a siphon) are very complex and are not susceptible of exact mathematical analysis. The problem is made even more complex when the velocity distribution in the conduit, as it approaches the bend, is unbalanced. The shape and alinement of the conduit immediately down stream from the bend likewise influence the flow around the bend. Mr. Stevens states: "The necessity of admitting water to the summit section through a priming conduit is one factor that may nullify the application of the vortex principle." Not all siphons have priming conduits which permit water to enter in this manner. Naturally, in those that do have such provision, there is a tendency further to complicate the velocity distribution in the upper bend. According to the analysis made by Messrs. Torpen, Hyde, and Cochrane, shown graphically in Fig. 4, the effect of such admission of water in the Leaburg siphons does not influence the velocity distribution enough to nullify the application of the vortex principle.

The writer disagrees with Mr. Stevens in the contention that a mean velocity of 40 ft per sec or more would be found in the lower leg if the siphon used for illustration were in use and under observation. Perhaps this would be true if the outlet were not reduced in area. To omit the reducer would result in some increase in discharge which in turn would be accompanied by pressures close to absolute zero and separation of the water from the walls of the siphon barrel. In this case a contracted effective area might well give mean velocities as high as 40 ft per sec. The siphon designed in the problem selected for illustration, however, will discharge no more than 350 cu ft per sec, and it is the writer's contention that such a discharge (in an upper bend of the dimensions given) will not produce the phenomenon discovered at the Walterville and Leaburg siphons.

It has been reported that the Walterville siphons vibrate to a certain extent at full flows.[20] It is reasonable to attribute such vibration to the unstable condition of flow mentioned in the foregoing paragraph.

It is true, as Mr. Stevens states, that open channels are cheaper than siphon barrels. There are many instances, however, in which the flow must be carried to the point of discharge in a closed conduit. For example, consider the case of a dual-purpose reservoir built primarily for flood control, but which at the same time is to create a lake for recreational purposes. The dam is an earth-fill embankment with the conventional flood outlet consisting of a tunnel with a gate control-tower located approximately at its mid-point. A siphon spillway, to regulate the level of the recreation pool, is to be incorporated in the tower. It draws water from the emergency gate wells and discharges into the tunnel down stream of the service gates. For a project of this description there is no alternative; the flow must be carried in a closed conduit.

---

[19] "On the Behavior of Siphons," by J. C. Stevens, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 998.

[20] *Loc. cit.*, Vol. 104 (1939), p. 1805.

When the writer first used the free vortex theory in analyzing flow conditions in the upper bend of a siphon, he too was skeptical as to its usefulness. After making several comparisons with actual test data there remained no question as to its value. The reaction on Messrs. Torpen, Hyde, and Cochrane apparently was similar. It is undoubtedly true that the paper would have been more convincing if comparison with actual measurements had been presented. It should be remembered, however, that the upper bend of a siphon is only a bend in a closed conduit, and that free vortex flow has been shown to approximate, closely, the actual velocity distribution in bends by others. Nevertheless the comparison given by the authors will be of value to those who doubt the application of the vortex theory. Once it has been shown that the velocity distribution at the summit section is far from uniform (and this is now definitely established) it follows that a plot of a computed hydraulic gradient based on mean velocities will not reveal the true diminution of absolute pressure intensities in the upper bend.

Mr. Nelidov disagrees with the application of the term "high-head siphon." He proposes to apply this term only to those siphons in which the operating head exceeds the "limiting head." As defined, the limiting head is that which develops the maximum discharge capacity of the siphon. In order to compute it the dimensions of the siphon to be classified must be known. In other words, a siphon would have to be designed first and then classified. According to this reasoning any siphon could be either put into, or taken out of, the high-head class by making some small dimensional changes. For instance, the siphon used for illustration by the writer would cease to be a high-head siphon if the outlet area were reduced slightly. This makes Mr. Nelidov's definition seem irrational. The classification according to head as interpreted by the writer is by no means new. Perhaps to restate the definition in a little different manner will serve to clarify its meaning—a siphon naturally falls into the high-head class where the operating head is so great that, to get good performance (and this means that a break in the water column or its separation from the conduit walls is prevented), special provision must be made to consume excess head. Throttling the outlet is a convenient way to expend excess head.

The diagrammatic sketch in Fig. 5 is very comprehensive and will aid the designer in visualizing flow conditions in the upper bend. The writer has followed through Mr. Nelidov's derivation and finds that it is mathematically correct. It is important to note that the entire procedure is predicated on the same assumptions relative to flow in the upper bend as made by the writer; namely, that velocity distribution is according to the free vortex theory. Except for a slight rearrangement of terms, Equation (17) is the same as Equation

(10). Where the variation in velocity in curvilinear flow can be expressed as an equation it is possible to determine, mathematically, the average kinetic energy per unit of flow. In this manner Equation (16) is useful for the case at hand. Mr. Nelidov illustrates how the priming head and the head losses from the inlet to the throat can be included in the computations. There is some question as to whether it is justifiable to include such small forces when the basic assumptions regarding flow are only approximate, and the head losses involved are difficult to evaluate. Actually the elevation of the forebay and the head lost in the siphon are directly related to the pressure at the summit section. It may not be commensurate, however, with the general accuracy of the method to compute the effect of priming head and head losses in the upper leg which, incidentally, act to offset one another. In the problem used for illustration the only effect that neglect of these factors has is to increase the negative pressure at the throat slightly. Since the permissible negative pressure of 24 ft of water has a factor of safety in it, this is inconsequential. The capacity remains unchanged.

The value K_1' from Table 1 is the sum of the losses between Section 1 and Section 2, and should be 0.186 instead of the 0.154 used by Mr. Nelidov. In checking through the arithmetic the writer finds that Equation (21) yields a value for H_{llm} of 73.9 instead of 78.5. The substitution in Equation (22) as shown is incomplete, but the answer is correct.

Mr. Nelidov calls attention to the limitations of Equation (4). Naturally when r_c becomes zero (that is, the crest is a knife-edge) Equation (4) gives a discharge of zero. This is true because, theoretically, water moving at any finite velocity, no matter how low, cannot change its direction without some curvature, however small the radius may be. In the opinion of the writer, except where hydraulic performance is of no consequence, the upper bend of a siphon should be formed with regular curves of moderate radii so that the formation of cavities is prevented. It is now a matter of common knowledge that the discharge increases when a cavity (space filled with eddying water) of this nature is filled with masonry, in spite of the fact that the flow cross-section is contracted. Equation (4) will be valid under these conditions. At the same time the writer recognizes that siphons have been built, and will be built, with exceedingly sharp crests, and that in checking the performance of these siphons the minimum value of r_c, as mentioned by the writer, is of paramount importance.

The question of selecting upper bend radii and throat dimensions arises. Many considerations enter into this problem. Intentionally avoiding these details, because it was feared they might detract from the original purpose of the paper, the writer selected radii which, in his opinion, gave satisfactory proportions. Mr. Nelidov points out that if a value for r_s of 7.0 is used Equation (4) reads $Q_1 = 39.3\, r_c \log_e \frac{7.0}{r_c}$, and that if the first derivative of Q_1 with respect to r_c is set equal to zero, a value of r_c can be determined which will give a maximum value for Q_1. For the case at hand, this procedure produces throat dimensions of $d = 4.43$ ft and $B = 3.40$, giving a throat area of 15.06 sq ft. Comparison with the throat dimensions of the siphon illustrated in Fig. 2 indicates that the proportions selected by the writer are superior on three counts:

(1) The throat area is considerably smaller, thus decreasing the cost; (2) the priming qualities are better since the length of the crest is greater; and (3) the transition from the rectangular throat to the circular lower leg is easier. The variation of throat dimensions with the upper bend radii can best be represented graphically. In this connection it can be shown that Equation (4), having a given value of r_c inserted, and plotted as a curve (r_c against A_t, in which A_t is equal to Q_{design} × throat depth ÷ Q_1) for a definite siphon capacity, will give smaller throat areas for greater values of r_c, but the throat width (or crest length) will increase rapidly. Obviously there is a practical limit depending upon such considerations as priming qualities, transition in section, and structural requirements.

Perhaps it is well that Mr. Nelidov holds open for discussion the matter of maximum permissible negative pressure. A vacuum head of 24 ft was recommended because it is a reasonable value and has been used in the past with success. It was not the intention of the writer to suggest that hydraulic engineers use the value blindly and without regard for special considerations—for example, operation at extremely high altitudes, or where the quantity of air dissolved in the water is excessive. Each case should be analyzed individually by the designer. If all influencing factors were susceptible to exact analysis it would be perfectly proper to use a vacuum head closer to the maximum of 34 ft. Owing to unforeseen disturbances and unpredictable behavior, the value used should contain a factor of safety. It is common practice in structural design to use a working stress for steel of 16 000 or 18 000 lb per sq in. Actually steel will withstand stresses up to the elastic limit of 32 000 lb per sq in. before it fails. One reason for using the lower stress is to allow for unpredictable secondary stresses.

One method of procedure sometimes resorted to for determining velocity distribution, that has not been discussed, but which warrants mention, is the utilization of the flow net or streamlines. For example, it may be necessary to find the distribution of velocity in the upper leg of a siphon. This can be done by drawing out the streamlines to a large scale.

It is often true in a paper of this nature that there is a tendency on the part of those participating in the discussion to single out certain points and so concentrate their criticism that the broad aspects of the problem as a whole are lost from sight. The writer's aim was to answer certain questions arising in the mind of the engineer confronted with the task of designing a high-head siphon spillway. That atmospheric pressure is a governing force in simple siphonic action is not difficult to understand. On the other hand, the design of a siphon spillway where the operating head is much greater than 34 ft requires a rigorous analysis to be able to guarantee, within practical limits, a fixed capacity and smooth hydraulic performance without vibration and pulsation.

In concluding this discussion the writer wishes to emphasize two points: (1) The assumption of free vortex flow in the upper bend is a valuable aid in designing a siphon; and (2) there is no limit to the operating head for which a siphon can be designed. If the head is greater than 34 ft the only special requirement is that the siphon conduit be formed in such a manner that a break in the water column or its separation from the conduit wall is impossible.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2081

DESIGN OF DOWELS IN TRANSVERSE JOINTS OF CONCRETE PAVEMENTS

By Bengt F. Friberg,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. L. E. Grinter, W. O. Fremont, and Bengt F. Friberg.

Synopsis

Technical literature lacks a rational solution for the deflections and stresses in and around dowels through transverse joints in concrete pavements. The action of the dowel may be established according to general elastic formulas governing for an elastic structure in a yielding mass. These elastic equations are presented herein in simple form and are applied to round dowels such as may be used in road joints under assumed, approximately correct, conditions of yield in the concrete around the dowels. The economical length of dowels is indicated.

A reasonable rule for the rapid appraisal of the effect of dowel spacing is stated and applied to varying dowel spacing. The concrete stress relief furnished by dowels, being a function of dowel deflection and pavement deflection, has been established in practical form based upon the theoretical deflection conditions. The use in design is indicated, and experimental values of stress relief are checked with those obtained with this method of design.

Knowledge of deflections in and around dowels makes it possible to estimate the loads and stresses resulting from inaccurate dowel installation. The values have been developed and applied to experiments of record, giving some indication of the strength of the concrete. The variation in dowel alignment that may be permitted in actual construction, can be gaged to varying joint widths, movements, and dowel sizes.

Present Construction Practice

Transverse joints in concrete pavements are necessary to make longitudinal movements of the pavement possible, and to relieve shrinkage and temperature stresses. In most highway designs round steel dowels are placed across these transverse joints, consisting of $\frac{3}{4}$-in. or $\frac{7}{8}$-in. round plain bars, 2 ft long, spaced

---

Note.—Published in November, 1938, *Proceedings*.

[1] Research Engr., Laclede Steel Co., St. Louis, Mo.

from 12 in. to 20 in. on centers. One end of the dowel is capped and painted or greased to present the least possible resistance to movement of the pavement. The dowels are most frequently in direct contact with the surrounding concrete. In some designs, however, they project into closely fitted heavy sleeves, which, in turn, bear in the concrete. Practical construction considerations, observation of performance, and experimental data rather than design analysis, have led to the construction practices in common use.

The dowels should be required to keep the road surfaces on each side of the joint in alignment even if subgrade support is lacking. They are also required to relieve the concrete stresses due to wheel loads in the immediate vicinity of the joints by transferring a part of these wheel loads to the adjacent pavement. To perform these functions the dowels themselves must be strong enough to carry a reasonable part of load across the joint; and, further, they must be dimensioned so that the bearing pressure between the dowel and the concrete surrounding it does not exceed permissible values. They must transfer their portion of the wheel loads with a sufficiently small deflection to offer a measure of side support to the loaded side of the joint even if the pavement rests on a relatively non-yielding subgrade.

A general mathematical solution of the design problem with specific application to dowels through joints has not appeared in technical literature, although the methods have been referred to by investigators and solutions have been provided for application in specific cases, or under assumed distribution of concrete pressure along the dowel.[2,3] The exact mathematical solution of the problem pertaining to a dowel structure of infinite length has been presented by Professor S. Timoshenko and Mr. J. M. Lessels.[4]

As the deflection of a long dowel in concrete, and the resultant pressure distribution between the dowel and the concrete, are not difficult to establish mathematically, these solutions are presented in the following analysis and from them the dowel performance and dowel effectiveness under common construction conditions have been determined. It has been necessary to use material constants with regard to the deflection of the dowel in the concrete as well as with regard to the deflection of the pavement on the subgrade, which have not been experimentally determined with accuracy. Numerical values included in the paper, therefore, are not intended to be used other than for illustration of the method presented, or with respect to specific experimental evidence.

Notation.—The letter symbols in this paper are defined where they first appear and are arranged, for convenient reference, in the Appendix.

GENERAL SOLUTION

For an elastic structure (see Fig. 1) extending an infinite distance into an elastic mass from its boundary surface, Professor Timoshenko has developed the relationship between the elastic properties of the two structures and the

---

[2] "Design of the Reinforced Concrete Road Slab," by L. E. Grinter, M. Am. Soc. C. E., *Bulletin No. 39*, Texas Eng. Experiment Station.

[3] "Design of Joints in Concrete Pavements," by R. D. Bradbury, Assoc. M. Am. Soc. C. E., *Proceedings*, Twelfth Annual Meeting, Highway Research Board, 1932.

[4] "Applied Elasticity," by S. Timoshenko and J. M. Lessels, Westinghouse Technical Night School Press, pp. 133–141.

deflection in the surrounding mass. Fig. 1 is a loading and deflection diagram that illustrates the sign convention adopted.

The relative stiffness of the structure and the mass is expressed by the following constant:

$$\beta = \sqrt[4]{\frac{K\,b}{4\,E_s\,I}} \quad \text{..........................(1)}$$

in which K = a "modulus of support" in an elastic mass, in pounds per inch[3]; b = the width of the structure; E_s = the modulus of elasticity of the structure (28 000 000 lb per sq in.); and I = the moment of inertia of the structure.

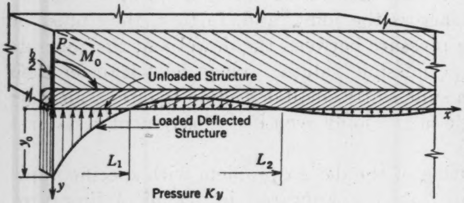


FIG. 1.—LOAD AND DEFLECTION DIAGRAM, SHOWING AN ELASTIC STRUCTURE EMBEDDED IN AN ELASTIC MASS

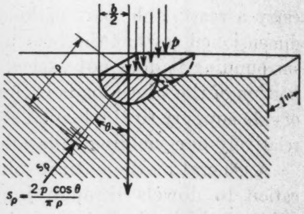


FIG. 2.—DISTRIBUTION OF STRESS IN AN ELASTIC MASS UNDER A DOWEL IF THE CONCRETE ABOVE THE DOWEL IS NEGLECTED

Modulus K is defined as the pressure intensity, in pounds per square inch, on the elastic mass, required to cause a 1-in. settlement. As developed by Professor Timoshenko, the general equation for the deflection, y, of the structure, in the mass, is:

$$y = \frac{e^{-\beta x}}{2\,\beta^3\,E_s\,I}\,\{P\cos\beta\,x - \beta\,M_0\,(\cos\beta\,x - \sin\beta\,x)\} \quad \text{.........(2)}$$

in which e = the base of Naperian logarithms; x = distances from the left end of the structure; P = the concentrated load, in pounds, acting downward at the face of the elastic mass; and M_0 = the bending moment in the structure at the left face of the elastic mass, in inch-pounds. Moment M_0 is positive clockwise. The slope of the structure may be established as the first derivative of Equation (2):

$$\frac{dy}{dx} = \frac{e^{-\beta x}}{2\,\beta^2\,E_s\,I}\,\{(2\,\beta\,M_0 - P)\cos\beta\,x - P\sin\beta\,x\} \quad \text{.........(3)}$$

For x equal to 0, the deflection, y_0, at the face of the mass is:

$$y_0 = \frac{P - \beta\,M_0}{2\,\beta^3\,E_s\,I} \quad \text{..........................(4)}$$

The pressure between the structure and the mass at any point is equal to $K\,y$ and, therefore, is derived directly from the deflection. The points where the pressure on the structure changes from positive to negative are obtained from Equation (2) for $y = 0$; thus:

$$\tan \beta L = 1 - \frac{P}{\beta M_0} \qquad (5)$$

in which L is the distance from the face of the mass to these points. The moment, M, at any point on the structure is obtained directly from Equations (2) and (3):

$$- E_s I \frac{d^2 y}{dx^2} = M = - \frac{e^{-\beta x}}{\beta} \{P \sin \beta x - \beta M_0 (\sin \beta x + \cos \beta x)\} \qquad (6)$$

From this value of M, the shear in the structure, V, at any point may be computed directly:

$$\frac{dM}{dx} = V = - e^{-\beta x} \{(2 \beta M_0 - P) \sin \beta x + P \cos \beta x\} \qquad (7)$$

Application of Design Formulas

The assumption is made that the dowels are not bonded to the concrete and that they fit snugly into the concrete mass. Under these assumptions it is evident that Equations (1) to (7) are directly applicable to long dowels. Load P is equivalent to the shear acting on the dowel, transferred from one road slab to the next. Moment M_0 is the moment in the dowel at the face of the joint incidental to the transfer of this shear. In practice, the dowels are usually not very stiff compared with the concrete pavement. It is an acceptable approximation, then, to presume that a point of contraflexure exists in the dowel at the center of the joint, in which case, for a joint width, a:

$$M_0 = - P \frac{a}{2} \qquad (8)$$

Formulas for the solution of long dowels may then be written in the following simple and usable forms: From Equation (4) the deflection of the dowel with respect to the concrete, at the face of the joint, is:

$$y_0 = \frac{P}{4 \beta^3 E_s I} (2 + \beta a) \qquad (9)$$

From Equation (3) the slope of the dowel at the face of the joint is,

$$\frac{dy_s}{dx} = - \frac{P}{2 \beta^2 E_s I} (1 + \beta a) \qquad (10)$$

From Equation (5) the points, L_n, where the pressure changes direction along the dowel, are:

$$\tan \beta L = \frac{2 + \beta a}{\beta a} \qquad (11)$$

Equation (7), for $\frac{dM}{dx} = 0$, determines the point, x_m, where maximum moment occurs in the dowel:

$$\tan \beta x_m = \frac{1}{1 + \beta a} \qquad (12)$$

By substituting this value in Equation (6), the maximum moment in the dowel is obtained:

$$M_{\max} = -\frac{P e^{-\beta x_m}}{2 \beta} \sqrt{1 + (1 + \beta a)^2} \qquad (13)$$

The stresses and deflections are proportional to P. The locations of the point of maximum moment and of the dowel action, as represented by L_n, are independent of the magnitude of P.

YIELDING OF CONCRETE IN WHICH DOWELS ARE EMBEDDED

Values of K in Equation (1) have not been determined experimentally. The only values known to the writer are those suggested[2, 5] by L. E. Grinter, M. Am. Soc. C. E., of 300 000 lb per in.^{3} and 1 500 000 lb per in.^{3}. K represents the elastic deformation of several inches of concrete above or below the dowel; and its numerical value, therefore, would be less than the modulus of elasticity of the concrete.

The dowels entering a concrete pavement slab are completely surrounded by concrete. The concrete above the dowel holds that immediately beneath the dowel by tension. The compressive stress in the concrete must then be less than the value existing at the dowel in reverse proportion to the depth beneath it. The stress condition illustrated in Fig. 2 is theoretically correct for loadings at a boundary surface. In actual road construction the vertical depth of the concrete beneath the dowel is usually from 3 in. to 6 in., and the vertical pressure in the concrete at the bottom of the slab cannot be greater than the bearing pressure exerted by the slab on the soil. It follows, therefore, that for the average grade of concrete now in use, the value of K probably is higher than 300 000 lb per in.^{3}. In this paper K has been assumed equal to 1 000 000 lb per in.^{3} for all sizes of dowels.

Since the value of K is dependent, to a major degree, upon the pressure immediately at the dowel, its value will probably increase with an increase in the size of dowels; and the experimental establishment of the value, $K b$, is highly desirable. However, since β varies as $K^{\frac{1}{4}}$ (see Equation (1)), a considerable change in K will not influence the stress distribution to any great extent.

DESIGN FORMULAS FOR THE ORDINARY CASE

All the design formulas may be solved directly. Table 1 affords a clear "picture" of the stresses in and around dowels. The maximum moment in the dowels is seen to occur some distance away from the joint, inside the concrete.

For shears other than 1 000 lb, the values of deflections and stresses change in direct proportion for values within the yield point of the steel or the ultimate bearing resistance of the concrete. Stresses above the yield point in the steel

---

[5] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1157.

(first reached at the point of maximum moment inside the concrete) have the effect of increasing the maximum bearing pressure on the concrete at the face of the joint. Under most construction conditions the bearing stress on the concrete is the critical one. Accordingly, the steel in the dowels should have a sufficiently high yield point so that it is not reached before the bearing stress on the concrete has exceeded any possible ultimate value.

TABLE 1.—STRESS CHARACTERISTICS OF DOWELS FOR A SHEAR OF 1 000 POUNDS; K = 1 000 000

Width of joint, a, in inches	Deflection, y_0, in inches, at the face of the joint	Bearing stress, $K y_0$, in pounds per square inch, imposed on the concrete at the face of the joint	DISTANCES, IN INCHES, FROM THE FACE OF THE JOINT TO THE POINT OF:			Maximum moment, in inch-pounds	Deflection, y_0, in inches, at the face of the joint	Bearing stress, $K y_0$, in pounds per square inch, imposed on the concrete at the face of the joint	DISTANCES, IN INCHES, FROM THE FACE OF THE JOINT TO THE POINT OF:			Maximum moment, in inch-pounds
			First stress reversal, L_1	Second stress reversal, L_2	Maximum moment, x_m				First stress reversal, L_1	Second stress reversal, L_2	Maximum moment, x_m	
	(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
	(a) ¾-INCH DOWELS; I = 0.016 INCH^4; β = 0.810 INCH^{-1}						(b) ⅞-INCH DOWELS; I = 0.029 INCH^4; β = 0.723 INCH^{-1}					
0	0.0022	2 160	1.94	5.81	0.97	400	0.0017	1 650	....	....	....	....
0.25	0.0024	2 380	1.82	5.70	0.84	490	0.0018	1 800	....	....	....	....
0.50	0.0026	2 590	1.73	5.60	0.76	575	0.0019	1 950	....	....	....	....
0.75	0.0028	2 810	1.65	5.53	0.69	670	0.0021	2 009	....	....	....	....
1.00	0.0030	3 030	1.59	5.46	0.62	770	0.0022	2 240	....	....	....	....
1.50	0.0035	3 460	1.49	5.36	0.53	980	0.0025	2 540	....	....	....	....
	(c) 1-INCH DOWELS; I = 0.049 INCH^4; β = 0.654 INCH^{-1}						(d) 1¼-INCH DOWELS; I = 0.120 INCH^4; β = 0.552 INCH^{-1}					
0	0.0013	1 300	2.40	7.20	1.20	495	0.0009	890	2.84	8.52	1.43	585
0.25	0.0014	1 410	2.28	7.08	1.07	575	0.00095	950	2.72	8.40	1.31	665
0.50	0.0015	1 520	2.19	7.00	0.99	665	0.0010	1 010	2.62	8.30	1.21	755
0.75	0.0016	1 620	2.10	6.90	0.90	760	0.0011	1 070	2.54	8.20	1.12	850
1.00	0.0017	1 730	2.03	6.84	0.83	860	0.0011	1 130	2.46	8.15	1.04	945
1.50	0.0019	1 940	1.91	6.71	0.72	1 060	0.0013	1 250	2.32	8.00	0.91	1 145

LENGTH OF DOWELS

The foregoing theoretical analysis of dowels, of course, is correct only for an infinitely long dowel. In Fig. 3, the typical diagrams for concrete pressure, dowel shear, and dowel moment have been plotted for an infinitely long 0.75-in. dowel and a joint opening of 1 in. The pressures between the dowel and the concrete decrease very rapidly with each succeeding stress change.

The effect of introducing short dowels can be estimated as a correction factor. The procedure is indicated for 0.75-in. dowels, cut at the second point of pressure change. This point (see Table 1) is 5.81 in. to 5.36 in. from the joint face. According to Equations (6) and (7) the shear and the moment in an infinitely long dowel at this point, for a 0.75-in. dowel and 1-in. joint opening, are: Shear, − 17.5 lb; and moment, + 21.5 in.-lb.

These values are less than 5% of the values at the face of the joint, the moment being of reverse sign. The correction in the actual stresses, made necessary by cutting the dowel at this point, is equivalent to considering the

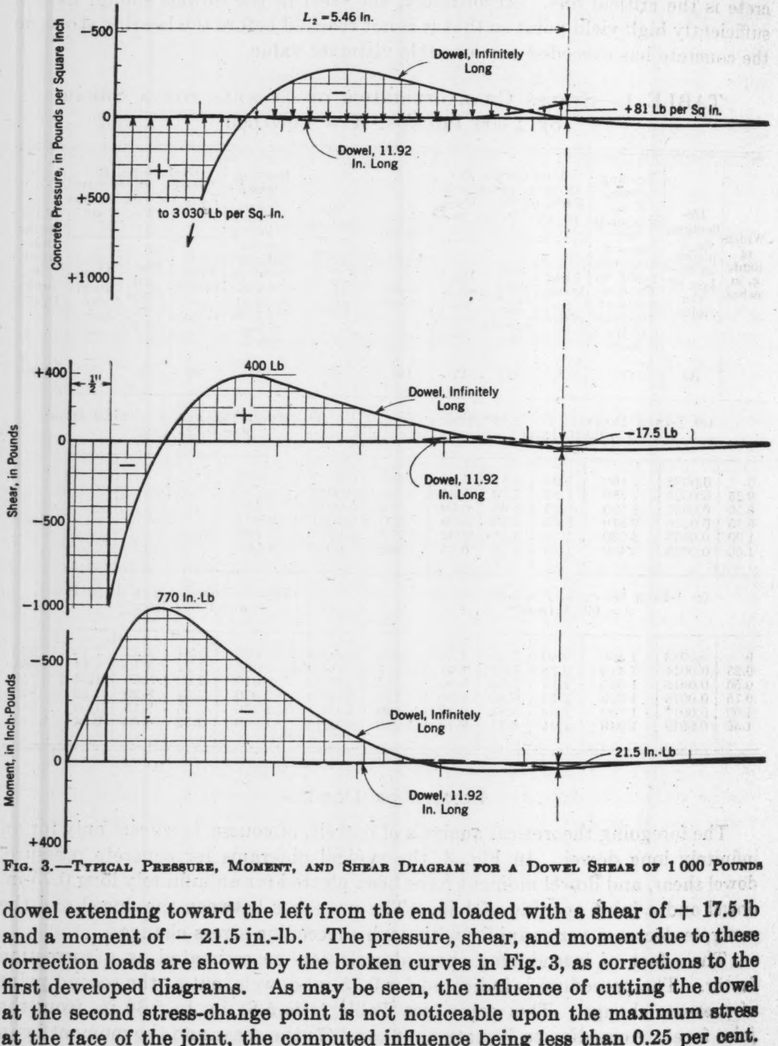


FIG. 3.—TYPICAL PRESSURE, MOMENT, AND SHEAR DIAGRAM FOR A DOWEL SHEAR OF 1 000 POUNDS

dowel extending toward the left from the end loaded with a shear of + 17.5 lb and a moment of − 21.5 in.-lb. The pressure, shear, and moment due to these correction loads are shown by the broken curves in Fig. 3, as corrections to the first developed diagrams. As may be seen, the influence of cutting the dowel at the second stress-change point is not noticeable upon the maximum stress at the face of the joint, the computed influence being less than 0.25 per cent. It is unwarranted to repeat the correction process back from the face of the joint.

The computation indicates that it is entirely unnecessary to extend the dowels beyond the second point of pressure change. Since values of shear and moment are quite low for some distance before the second stress-change point is reached, the dowels could be made as much as 25% shorter without appreciably affecting the maximum stresses in and around the dowels.

DOWEL DEFLECTION ACROSS A JOINT

Of primary importance in judging the dowel action in concrete pavements is the deflection across the joint. The less this deflection, the more nearly will the adjacent pavement slabs match, and thus the greater part of the wheel load will be transferred by the dowel from the loaded side to the unloaded side.

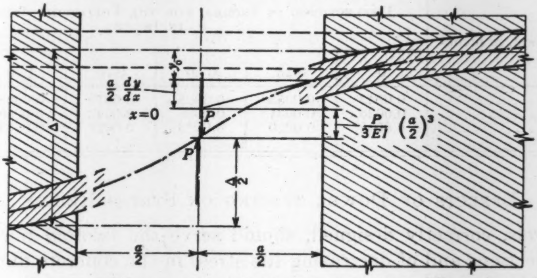


FIG. 4.—DEFLECTION OF A DOWEL ACROSS A PAVEMENT JOINT

For a given shear, the total deflection of a dowel is a function of: (1) The deflection at the faces of a joint; (2) the slope of the dowel, at the faces of the joint; and, (3) the deflection in the part of the dowel in the joint opening. The conditions are illustrated in Fig. 4, and may be defined by a simple formula under the assumption that the faces themselves remain parallel.

The deflection, Δ, across the joint is equal to twice the sum of the following quantities:

(a) The deflection at each joint face, y_0;

(b) The deflection due to the dowel slope over one-half the joint, $\frac{a\,dy_0}{2\,dx}$; and,

(c) The deflection of the dowel steel over one-half the joint, $\frac{P}{3\,E\,I}\left(\frac{a}{2}\right)^3$.

The deflection, Δ, is obtained directly from Equations (9) and (10) by adding Increment (c); that is:

$$\Delta = \frac{P}{2\,E\,I}\left(\frac{1 + (1 + \beta\,a)^2}{\beta^3} + \frac{a^3}{6}\right) \ldots\ldots\ldots\ldots\ldots (14)$$

The deflection is proportional to the dowel shear, P, as shown for 0.75-in., 1-in., and 1.25-in. dowels in Table 2.

In practice, the joint faces would not remain parallel. The slope of the pavement slab in the direction of the dowel, as experimentally observed in tests of the United States Bureau of Public Roads,[6] however, rarely exceeds 10% of the slope of the dowel at the face of the joint as determined by Equation (10). For maximum dowel shear, when the wheel load is near the joint, the two joint faces may slope in opposite directions, minimizing the influence. Therefore, the effect of pavement slope upon dowel deflection across joints may be neglected safely.

TABLE 2.—DEFLECTION ACROSS JOINT FOR 1 000-POUND DOWEL SHEAR AND PARALLEL JOINT FACES

Diameter of dowel, in inches	DEFLECTIONS, IN INCHES, FOR THE FOLLOWING JOINT WIDTHS, *a*, IN INCHES					
	0	0.25	0.50	0.75	1.00	1.50
0.75	0.0043	0.0053	0.0065	0.0079	0.0094	0.0134
1.00	0.0026	0.0031	0.0036	0.0042	0.0049	0.0062
1.25	0.0018	0.0020	0.0023	0.0027	0.0030	0.0039

EFFECTS OF DOWEL SPACING ON STRESS RELIEF

The dowels, correctly designed, should serve the twofold purpose of decreasing deflections and of decreasing the stress in the concrete due to concentrated wheel loads. The maximum stress is that which occurs parallel to the edge—for edge-line loading a tensile stress in the bottom of the slab immediately under the load; and, for corner loading, a tensile stress in the top of the slab some distance from the corner. In each case, this stress is proportional to the moment in the concrete and, in turn, to the load supported by the concrete.

Influence of Dowel Spacing Upon Moments.—A dowel placed immediately beneath the load may be assumed to decrease the stress in the pavement in the same proportion as it transfers part of the wheel load to the adjacent slab by the dowel shear. Dowels some distance from the point of loading cannot be considered as effective as those that are closer. They will be farther from the point of maximum moment, and their action will be influenced by shear conditions in the concrete itself. The ideal dowel condition would be that which would transfer shear along the line of the joint identical with the interior of the slab. When support across the joint is lacking, the shear in planes parallel to the joint will increase correspondingly. The combination of shear in the concrete and dowel shear, accordingly, influences the moment and stresses in the concrete.

Convenient diagrams for determining the moment under and near the wheel load have been presented by H. M. Westergaard, M. Am. Soc. C. E.[7] For loadings a considerable distance from any edge the maximum positive moment is beneath the load, the maximum negative moment is $1.8\,l$ from the point of load, l being the radius of relative stiffness as defined by Dean Westergaard. The moment diagrams for loading at an edge a considerable distance from any corner show a small maximum negative moment at approximately the same distance ($1.8\ l$) from the point of loading.

---

[6] "The Structural Design of Concrete Pavements," Pt. 4, *Public Roads*, September, 1936.

[7] "Computation of Stresses in Concrete Roads," by H. M. Westergaard, *Proceedings*, Fifth Annual Meeting, Highway Research Board, 1925.

At the points of maximum negative moment, the shear in the concrete in a plane perpendicular to the edge must equal zero. This shear is the sum of the forces on one side of the section: The subgrade reaction, the concrete shear in planes parallel to the joint, and the dowel shear. Beyond the point, $1.8\ l$, from the load, the moment changes very little. The influence of any one of the shear increments beyond that point, upon moment under the load, therefore, may safely be neglected. The effective dowel shear may reasonably be estimated in accordance with the following rule:

> The effective dowel shear decreases inversely as the distance of the dowel from the point of loading, to zero at a distance of $1.8\ l$. No dowels beyond that point influence the moment at the load point.

It is sufficiently accurate to consider the moment diagram of each dowel shear as identical to that produced by an equal wheel load, the dowel shear acting in an opposite direction to the wheel load. Table 3 gives the moment immediately under the wheel load of effective dowel shears for increasing distances from the point of loading to the dowel, the unit moments being taken directly from data presented by Dean Westergaard.[7]

TABLE 3.—PROPORTION OF EDGE MOMENT RELIEVED BY DOWEL SHEAR

Description	DISTANCE FROM THE LOAD POINT TO THE DOWEL AS A PROPORTION OF THE RADIUS OF RELATIVE STIFFNESS:							
	0	$0.2l$	$0.4l$	$0.6l$	$0.8l$	$1.0l$	$1.4l$	$1.8l$
Proportion of effective dowel shear	1.00	0.89	0.78	0.67	0.55	0.44	0.22	0
Moment at the Load Point, in Inch-Pounds:								
For a unit dowel shear	0.49	0.35	0.15	0.05	0.01	−0.02	−0.05	−0.06
For the effective dowel shear	0.49	0.31	0.12	0.03	0.005	−0.01	−0.01	0
Moment expressed as a proportion of the moment due to one dowel at the load point	1.00	0.64	0.24	0.07	0.01	−0.02	−0.02	0

From these data (Table 3), the relative effectiveness in decreasing slab stress may be developed numerically for any spacing, number, and arrangement of dowels relative to the load point. The sum of the effectiveness of the active dowels, expressed as percentages of the effectiveness of one dowel, immediately beneath the wheel load, for varying pavement depths, and for dowel spacings from 8 in. to 20 in., is illustrated in Fig. 5. The lowest efficiency occurs when the wheel load is between two dowels. With customary dowel spacing, wheel loads between dowels will occur most frequently (see solid lines in Fig. 5), and in rating the stress-relieving value of dowels relative to their spacing, they should be so compared. The average effectiveness for 5-in., 6-in., and 7-in. pavement thickness under these conditions is as follows:

Dowel spacing, in inches	Proportion of single dowel shear
8	1.55
12	1.00
16	0.60
20	0.35

If the dowels are to be effective in decreasing stresses in the concrete, they must be spaced close together.

These values, established directly from Table 3, are based on the assumption that the edge moment distribution is not changed. For very close dowel spacing and for very rigid dowels this assumption would be too approximate and the values must then be corrected for some moment distribution between that for edge loading and for interior loading. The values probably apply closely for dowel spacing of 12 in. with 0.75-in. and 1-in. dowels, and for a spacing of 12 in.; and also for 1.25-in. dowels at expansion joints.

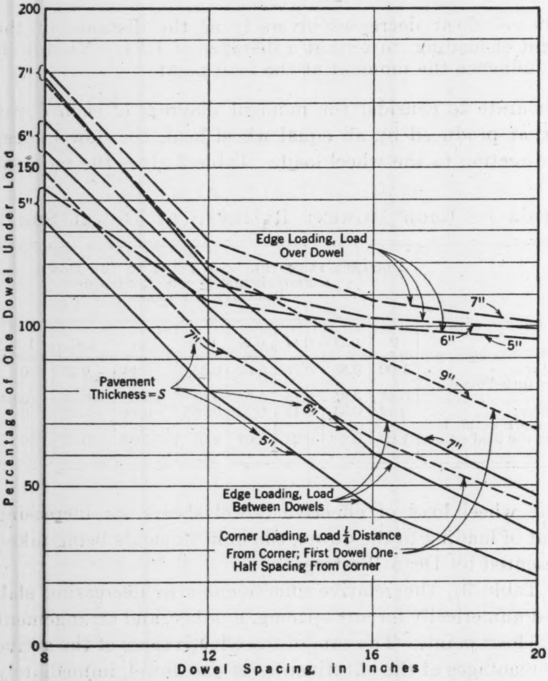


FIG. 5.—TOTAL EDGE STRESS RELIEF AS A PERCENTAGE OF THAT AFFORDED BY ONE DOWEL DIRECTLY UNDER A LOAD

ACTUAL STRESS RELIEF DUE TO DOWELS ACROSS TRANSVERSE JOINTS

Relation Between Pavement Stiffness and Dowel Stiffness.—For a complete evaluation of dowel effectiveness it is necessary to establish a basic value of one dowel at the loading point only. The rates of deflections of dowels have been established as shown in Table 2. The actual load transfer by a dowel directly under the load may then be determined if the deflection of the pavement edges or corners under a load is known.

The assumption that the deflection is directly proportional to the load carried by the concrete (see Fig. 6) approximates closely the results obtained in actual deflection measurements by the Bureau of Public Roads. Computed in accordance with methods established by Dean Westergaard the lines correspond to theoretical values of $k = 300$ for edge loading and $k = 200$ for corner loading.

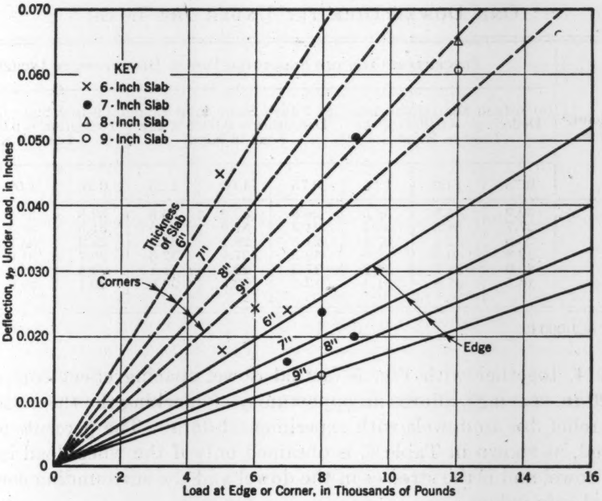


FIG. 6.—DEFLECTIONS OF PAVEMENT UNDER EDGE AND CORNER LOAD ON A SILT LOAM SUBGRADE

The deflection of the loaded pavement edge must equal the deflection of the dowel plus the deflection of the adjacent pavement edge, and the load on the adjacent pavement edge must be the same as the dowel shear. If W = the wheel load, P = the shear on a dowel directly under the load, and y_p and y_d = the deflection of the pavement and the dowel, respectively, for a unit load, the following values govern:

(*a*) The deflection of the loaded edge of a pavement, $y_p (W - P)$;
(*b*) The deflection of the dowel, $y_d P$; and
(*c*) The deflection of the adjacent edge, $y_p P$.

The sum of the deflections on either side must be identical, or:

$$y_p (W - P) = y_d P + y_p P \qquad (15)$$

from which equation the dowel load transfer is obtained, thus:

$$\frac{P}{W} = \frac{1}{2 + \frac{y_d}{y_p}} \qquad (16)$$

The values have been computed for common pavement dimensions in Table 4, with y_p obtained[8] from Fig. 6, and y_d from Table 2.

TABLE 4.—PERCENTAGE OF LOAD TRANSFER ACROSS TRANSVERSE JOINTS FOR PAVEMENTS OF STANDARD THICKNESS; (SILT LOAM SUBGRADE; ONE DOWEL DIRECTLY UNDER THE LOAD)

Width of joint, *a*, in inches	PERCENTAGES FOR THE FOLLOWING DOWEL DIAMETERS, IN INCHES:								
	(*a*) 6-Inch Slab; Joint Edge Deflection = 0.0033 × the Load in Kips*			(*b*) 7-Inch Slab; Joint Edge Deflection = 0.0025 × the Load in Kips*			(*c*) 9-Inch Slab; Joint Corner Deflection = 0.0045 × the Load in Kips*		
	0.75	1.00	1.25	0.75	1.00	1.25	0.75	1.00	1.25
0	30.2	35.8	39.5	26.8	32.9	36.8	33.8	38.8	41.8
0.25	27.7	34.2	38.2	24.2	31.0	35.6	31.5	37.3	40.8
0.50	25.3	32.4	37.1	21.8	29.1	34.3	29.2	35.7	39.8
0.75	22.8	30.5	35.6	19.5	27.1	32.6	26.7	34.0	38.6
1.00	20.6	28.7	34.2	17.3	25.2	31.1	24.4	32.3	37.4
1.50	16.5	25.8	31.4	13.6	22.3	28.0	20.1	29.6	34.9

* 1 kip = 1 000 lb.

Table 4, together with Fig. 5 (actual dowel spacing effect computed for 8-in. to 20-in. spacing), affords an opportunity of checking the theoretical value of stress relief due to dowels with experimental data. The percentage of load transferred, as shown in Table 4, is obtained only if the wheel load is directly over the dowel and if the stresses in the dowel and the surrounding concrete do not exceed safe values.

TABLE 5.—COMPARISON BETWEEN EXPERIMENTAL AND THEORETICAL VALUES OF DOWEL STRESS RELIEF (0.75-INCH DOWELS)

Position of load	6-INCH PAVEMENT; 0.75-INCH JOINT; 0.75-INCH DOWELS; 18 INCHES, CENTER TO CENTER						7-INCH PAVEMENT; BUTT-JOINT; 0.75-INCH DOWELS; 24 INCHES, CENTER TO CENTER					
	Observed Stress Relief				Computed Stress Relief One Dowel Under Load 22.8% (Table 4)		Observed Stress Relief				Computed Stress Relief One Dowel Under Load 16.8% (Table 4)	
	Stress Under Load, in Pounds per Square Inch, with Edge		Stress relief, in pounds per square inch	Stress relief (percentages)	Effectiveness for load position and spacing, Fig. 5 (percentages)	Adjusted stress relief computed (percentages)	Stress Under Load, in Pounds per Square Inch, with Edge		Stress relief, in pounds per square inch	Stress relief (percentages)	Effectiveness for load position and spacing, Fig. 5 (percentages)	Adjusted stress relief computed (percentages)
	Not doweled	Doweled					Not doweled	Doweled				
Between dowels...	297	260	37	12.5	48	11	275	215	60	22	30	8
At dowel.........	297	235	62	21.0	101	23	275	182	93	34	100	27

---

[8] *Public Roads*, October, 1936, p. 176 *et seq.*

Accurate measurements have been reported[8] by L. W. Teller, Assoc. M. Am. Soc. C. E., and Mr. E. C. Sutherland, from which comparative values have been computed in Table 5. The values in Table 5 for a 0.75-in. open joint check closely. For the 7-in. slab with butt-joint, the theoretical values are considerably lower than those developed experimentally, particularly for a wheel load between dowels. These experimental data indicate that there is considerable load transfer and joint restraint due to surface friction across the butt-joint, which will make the experimental values of stress relief greatly higher than what would be obtained with dowels alone.

For open contraction joints and expansion joints of conventional types, the foregoing theoretical method of computing stress relief due to load transfer apparently gives results, in substantial agreement with those observed experimentally. This is evidence that the assumptions in regard to concrete yield, K, and the influence of dowel spacing by the use of edge moment distribution, are fairly correct.

The method offers a possibility of determining the performance of dowels in regard to size and spacing, and to furnishing designs for any required degree of stress relief across a doweled edge. In no applicable case can the load transfer or stress relief exceed 50 per cent. For the close spacing of rigid dowels the method requires adjustment, because in that case it is not a sufficiently close approximation to assume that the moment distribution for an unsupported edge governs for a doweled edge.

STRESSES IN AND AROUND DOWELS

Table 1 gives the concrete bearing stresses around a 0.75-in. dowel for a shear of 1 000 lb. The moment in the dowel for bearing stresses from 2 160 lb per sq in. to 3 460 lb per sq in. varies from 400 in.-lb to 980 in.-lb, and the maximum fiber stress in the dowel varies from 9 700 per sq in. to 23 600 per sq in. Neither of these stresses exceeds that which can easily be taken by the materials; of the two, probably the concrete bearing stress is the critical stress. In joints from 0 in. to 0.75 in. wide and for normal subgrade conditions, 0.75-in. dowels may be expected to transfer not more than about 25% of the wheel load in common pavements as shown in Table 4. A dowel shear of 1 000 lb would then correspond to a single tire load of 4 000 lb. These values are in fair accord with those encountered in actual practice. If the dowels connect pavement slabs under which subgrade support is inadequate, they will be required to transfer more nearly one-half the wheel load, in which case stresses approximately double those for a 1 000-lb dowel shear may be expected to occur. The extreme stresses would then approach both the yield point in the steel and the ultimate compressive strength of the concrete as established by the usual tests, although, in the confined space surrounding the dowel, the ultimate concrete stresses may be considerably higher than those established in the usual tests.

A change from 0.75-in. to 1-in. dowels would improve the load transfer across joints from a 25% to a 33% maximum in accordance with Table 4. The concrete stress beneath the dowel for a 1 000-lb dowel shear would vary from 1 300 to 1 950 lb per sq in., the maximum moment in the dowel from about

495 in.-lb to 1 060 in.-lb, and the extreme fiber stress in the steel from 5 000 to 10 800 lb per sq in. The increased load-transfer capacity would noticeably strengthen the joint edges both against corner cracking and longitudinal cracking between the edge and the center joint. The decrease in stress both in the concrete and in the steel, with the use of 1-in. dowels, is such that their use in heavy-duty concrete pavements seems well warranted.

CONSTRUCTION CONDITIONS AND DOWEL PERFORMANCE

Effect of Inaccurate Dowel Alignment.—If all dowels across a transverse joint are parallel and the concrete slabs remain parallel, there is no resistance to movement in the joint except that exerted by frictional bond between the dowel steel and the concrete. If the dowels are not in alignment, however, their resistance to the movements of the joint will increase in proportion to the severity of the movements. Forces will arise to deflect both the dowels and the pavement edges sufficiently to make their deflection correspond to the relative movement. These are shear forces on the dowels, and their deflection may be computed according to established formulas, as illustrated in Fig. 4. Each one of the two deflections—that of the dowels and that of the pavement—is ascertainable independently. With common dowel spacings, the stiffness of the pavement edge will be so much greater than the stiffness of the dowels that the deflection of the pavement between adjacent dowels may be neglected.

The dowel deflection is given in Equation (14). In its application to a dowel on an angle with the direction of movement, the dowel deflection must equal the transverse component of the movement in a parallel displacement of the dowel; this is proportional to the movement and to the angular deviation of the dowel.

Under the assumption of a fixed pavement edge, the relationship between the deflection of the joint and the dowel misalignment then is:

$$\Delta = \frac{P}{2\,E\,I}\left(\frac{1 + (1 + \beta\, a_i)^2}{\beta^3} + \frac{a_i^3}{6}\right) = \alpha\, i \text{ . . . . } (17)$$

in which i = additional opening of the joint; a_i = new joint width; and α = misalignment of the dowel in the direction of movement, in radians. The relationship between the developed dowel shear, P_s, due to the misalignment, α, therefore, may be written:

$$P_s = \frac{2\,E\,I\,\alpha\, i}{\dfrac{1 + (1 + \beta\, a_i)^2}{\beta^3} + \dfrac{a_i^3}{6}} \text{ . . . . } (18)$$

Table 6 contains computed shear values for various joint widths, and joint movements. The data apply for dowel misalignment in the horizontal as well as in the vertical direction, provided the modulus of support in the concrete, K, is the same in both directions.

With the aid of Table 6, it is possible to find the loads on the dowels for any given construction condition. Shears and bearing stresses are shown for contraction joints and expansion joints in Fig. 7. If the misalignment should be greater than 1% (approximately ⅛ in. per ft of dowel), the dowel deflection necessary for readjustment will be proportionately greater. The resulting dowel loads and stresses will increase proportionately.

TABLE 6.—SHEAR, P_s, IN POUNDS, DUE TO A DOWEL MISALIGNMENT OF 1% (⅛ INCH PER FOOT OF DOWEL)

Final joint width, a_i, in inches	CHANGE IN JOINT WIDTH, i, IN INCHES						Final joint width, a_i, in inches	CHANGE IN JOINT WIDTH, i, IN INCHES					
	0.25	0.50	0.75	1.00	1.50	3.00		0.25	0.50	0.75	1.00	1.50	3.00
	Corresponding Vertical Dowel Deflection, in Thousandths of an Inch							Corresponding Vertical Dowel Deflection, in Thousandths of an Inch					
	2.5	5.0	7.5	10.0	15.0	30.0		2.5	5.0	7.5	10.0	15.0	30.0
(a) 0.75-INCH DOWELS							(b) 1.00-INCH DOWELS						
0	580	1 155	1 730	2 310	3 460	6 920	0	960	1 920	2 880	3 840	5 760	11 520
0.25	470	940	1 410	1 880	2 820	5 640	0.25	815	1 630	2 445	3 260	4 890	9 780
0.50	390	775	1 160	1 550	2 320	4 640	0.50	695	1 390	2 085	2 780	4 170	8 340
0.75	320	640	955	1 275	1 910	3 820	0.75	595	1 185	1 780	2 370	3 560	7 120
1.00	265	530	795	1 060	1 590	3 180	1.00	510	1 015	1 525	2 030	3 050	6 100
1.50	185	370	560	745	1 120	2 240	1.50	405	805	1 210	1 610	2 420	3 220
3.00	75	150	230	305	460	920	3.00	175	350	525	695	1 050	2 100
(c) 1.25-INCH DOWELS													
0	1 415	2 825	4 240	5 650	8 480	16 960	1.00	825	1 645	2 470	3 290	4 940	9 890
0.25	1 235	2 465	3 700	4 930	7 400	14 800	1.50	640	1 275	1 915	2 550	3 830	7 660
0.50	1 090	2 175	3 165	4 350	6 330	12 660	3.00	325	645	970	1 290	1 940	3 880
0.75	940	1 875	2 815	3 750	5 630	11 260	....	..	....	....	....	....	....

Since the bearing stress between the dowel and the concrete apparently is the critical stress, it has been developed from Table 1 and plotted in Fig. 7. As may be seen, exceedingly high concrete stresses accompany excessive dowel misalignment in transverse joints. Dowel loads from this source may exceed those caused by the normal load transfer function.

The actual forces will be less than those developed by assuming that the pavement is entirely rigid. To some extent, the direction of longitudinal movement follows the average alignment of all dowels. The force acting in the dowel, therefore, is dependent on the individual deviation from this average. The full dowel force is not developed if the deviation in alignment occurs gradually from dowel to dowel.

Strength of the Concrete Around the Dowel.—Some general observations of concrete failures around 0.75-in. dowels have been reported by Mr. A. R. Smith and S. W. Benham, Assoc. M. Am. Soc. C. E.[9] The data are classified for study in Table 7, in which the stresses to which the misalignments correspond, as read from Fig. 7, are given in Column (5).

The uniformity of the two sets (in Table 7, Items Nos. 1 and 4 against Items Nos. 2, 3, and 5) of computed stresses for the observed evidence points

---

[9] "Effect of Dowel-Bar Misalignment Across Concrete Pavement Joints," by Arthur R. Smith and Sanford W. Benham, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1133.

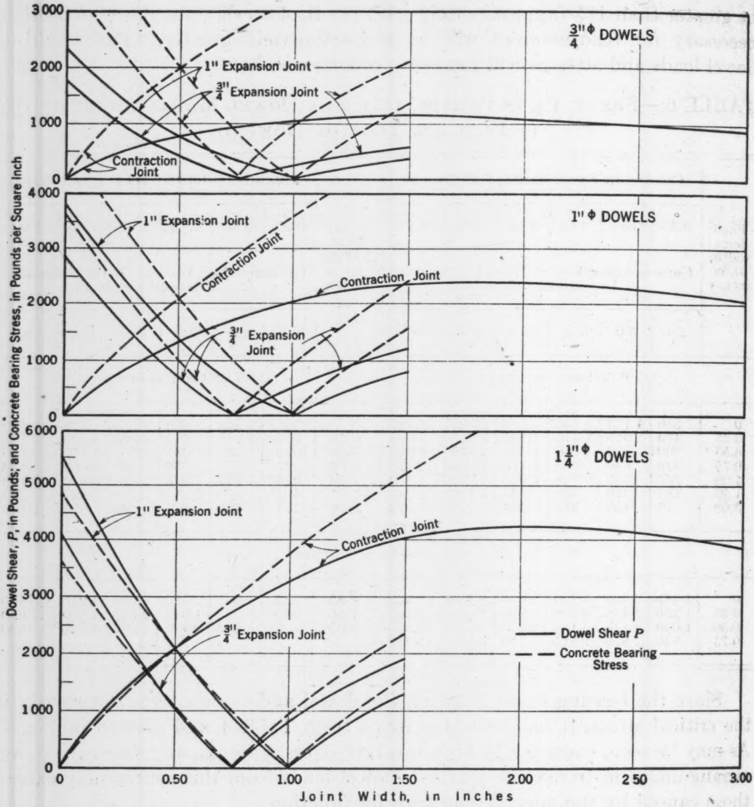


FIG. 7.—DOWEL SHEAR AND CONCRETE STRESS PRODUCED BY JOINT MOVEMENT DEFLECTING A DOWEL 1 PER CENT. ($\frac{1}{8}$ INCH PER FOOT OF DOWEL)

TABLE 7.—OBSERVED CONCRETE STRENGTH AROUND DOWELS

Item No.	Type of joint (1)	Degree of misalignment (2)	Additional opening,* i, in inches: Initial (3)	Additional opening,* i, in inches: Final (4)	Extent of spalling around dowels (5)	Concrete bearing stress, in pounds per square inch, from Fig. 7 (6)
1	Contraction	1%	0	0.75	Slight	2 700
2	Contraction	4%	0	0.75	Distinct	10 800‡
3	Contraction	6%	0	0.50	Failure	12 000
4	Expansion	1%	1.00	0.25*	Slight	3 300
5	Expansion	4%	1.00	0.25*	Complete†	13 000

* Additional closing at expansion joint. † Complete cracking and spalling. ‡ 4 × 2 700 = 10 800.

to distinctly separate stress conditions in the concrete. The slight spalling, with bearing stresses of about 3 000 lb per sq in., probably indicates failure of a secondary frictional nature, or direct tension immediately beside the dowel. The ultimate stress of more than 10 000 lb per sq in., undoubtedly indicates failure in compression, or for thin pavements failure in tension along the cone-shaped surface under or over the dowel. The behavior, including ten extensive cycles of movement, indicates that it may be safe, in design, to use a bearing stress of somewhat less than 3 000 lb per sq in. between the dowels and the concrete. The data indicate the presence of a high compression stress in the confined concrete surrounding the dowels, which had been expected. Higher stresses under exceptional conditions may be permissible because, although concrete failure would result around an exceptionally faulty dowel, such failure may not be noticeable in a pavement failure on good subgrade.

Permissible Limits of Dowel Misalignment

Theoretical analysis of stresses due to dowel misalignment establishes the fact that the dowels must be placed with considerably greater accuracy for wide joints than for narrow joints if both are to close to the same value of a_i. Fig. 7 also illustrates the fact that 1-in. dowels cause no greater bearing stresses in the concrete than 0.75-in. dowels for identical misalignment. If the limiting stress due to the average misalignment commonly encountered in practice is set at 2 000 lb per sq in. in the concrete, the misalignment of 0.75-in. expansion joints intended to maintain a minimum opening of 0.25 in. should not be more than 1% for either 0.75-in. or 1-in. dowels. The same permissible average dowel misalignment would apply to 0.75-in. or 1-in. dowels extending through a contraction joint with a maximum opening of 0.50 in.

It would seem impossible to tolerate alignment errors of 0.5 in. per ft of length, except under the most exceptional circumstances. The efficiency of the joint and of the dowels is seriously impaired by greater misalignment than $\frac{1}{8}$ in. per ft, although the edges of the joint may not fail.

Practical Problems in Alignment

Most pavement designs include spacer or distribution bars parallel to, and on each side of, transverse joints, to which bars the dowels are tied. The spacer bars are justified for reinforcement at the edge of the joint. However, wiring has not proved adequate to maintain the horizontal alignment of the dowels during the construction operations. Some pavement designs, accordingly, specify crimping or other mechanical processes, to locate the dowels exactly and also to extend the spacer bars exactly to the side forms so the dowels in both pavement lanes are parallel with the longitudinal axis of the pavement. The horizontal alignment can be made exact in that manner without increasing the material or work necessary in the construction of the pavement.

Satisfactory alignment in the vertical direction can be obtained with supports from the subgrade placed near the outer ends of each dowel. These supports must be substantial enough to retain their position during concreting. It should be possible to adjust the dowels with a spirit-level if inspection reveals them to be out of line more than the permissible tolerance. A great number of observations in the field have revealed that, with reasonable care and inspection, dowels supported near their ends may easily be placed with misalignments of less than $\frac{1}{8}$ in. per ft.

Dowels supported on the joints only, or very near the joints, cannot easily be maintained in satisfactory alignment, because they will be influenced by movements of the joint under the action of the fresh concrete. Particular care is required to maintain short dowels in correct alignment as the vertical deviation at their ends is not at once discernible in casual inspection.

CONCLUSION

Dowels across the transverse joints of pavements can be designed exactly by simple formulas. The value of K (1 000 000 lb per in.^{3}) used in this paper should be verified experimentally by actual load tests on dowels. It is believed to be sufficiently close, however, to permit its use for stress determination. Actual stress determinations show relatively high stresses both in and around 0.75-in. dowels under loads that may be encountered in practice. Increase in the size of dowels relieves the concrete stress quickly. The dowel length may be decreased to one-half the 24-in. length now common in actual construction.

Experimental and theoretical stress and moment determinations in pavements subjected to edge loading indicate that dowels which are farther away than 1.8 times the radius of relative stiffness are totally ineffective for the relief of traffic load stresses and that their effect decreases rapidly with increasing distance from the point of load, within the effective distance. Linear decrease in effective shear is proposed as a convenient and sufficient approximation.

Dowel deflection across the joints is primarily a function of dowel deflection and elastic slope at the face of the joint. Deflection of 1-in. dowels is approximately one-half the deflection of 0.75-in. dowels. The stress relief offered by single dowels is dependent on their deflection, and the rate of deflection of the pavement on the subgrade. By proper design and construction, stresses can be reduced 25% to 35% at the average joint opening. Dowels are effective for this purpose if they are spaced sufficiently close. The performance of a dowel can be predicted quickly by the method given in this paper, and the size of the dowel, as well as the proper spacing, can be determined for any desired stress relief. The results check closely the values that have been determined experimentally.

Stresses due to inaccurate dowel installation have been determined. With extreme joint movements, they may exceed the stresses due to traffic loads unless care is given to this construction detail. Observed concrete failures under known conditions of dowel misalignment indicate the presence of high ultimate concrete compression stresses in the confined space around the dowels. A design stress of 2 000 lb per sq in. between the dowel and the concrete appears justifiable. Dowels should be aligned with extreme care if they are to perform their function during the life of the pavement. Satisfactory dowel performance can be obtained with less weight of steel than is now used, but as the dowels are shortened, increasing care must be given the exactness of the supports and spacing of the dowels.

APPENDIX

NOTATION

The following notation conforms essentially with American Standard Symbols for Mechanics, Structural Engineering, and Testing Materials,[10] compiled by a committee of the American Standards Association with Society representation, and approved by the Association in 1932:

a = width of a joint between pavement slabs; a_i = new width after the joint has spread or closed;
b = breadth = the width of a structure embedded in an elastic mass;
E_s = modulus of elasticity of a structure; E_c = modulus of elasticity for concrete;
e = base of Naperian logarithms;
I = moment of inertia of a structure;
i = additional opening or closing of a joint from its original dimension;
K = modulus of support in an elastic mass, defined as the pressure intensity, in pounds per square inch, on a mass, sufficient to cause a settlement of 1 in.;
k = modulus of subgrade reaction;
L = length; $L_1, L_2, \cdots L_n$ = distances to Sections 1, 2, $\cdots$ n, from the face of an elastic mass, Sections 1, 2, $\cdots$ n being points where the pressure changes direction;
l = radius of relative stiffness of the pavement;
M = moment at any point of a structure and an elastic mass; M_0 = moment in a structure at the left face of an elastic mass;
P = load, or shear, on a structure at the face of the elastic mass; P_s = the load equivalent to the developed shear;
s = unit stress; s_ρ = unit stress at radial distance, ρ;
V = shear in the structure at any point;
W = total weight; wheel load;
x = distance parallel to the X-axis, or the horizontal axis of a structure; x_m = point in a dowel where maximum movement occurs;
y = deflection; distances parallel to the Y-axis; deflection of a structure relative to an elastic mass; y_0 = deflection at the face of an elastic mass; y_p = deflection of a pavement acted upon by a unit load; y_d = deflection of a dowel acted upon by a unit load;
α = misalignment of a dowel, in radians;
β = relative stiffness of a structure embedded in an elastic mass;
Δ = total deflection across a joint;
θ = angular distance corresponding to radius of stiffness, ρ;
ρ = radius of stiffness.

---

[10] A.S.A.—Z10A—1932.

DISCUSSION

L. E. GRINTER,[11] M. AM. SOC. C. E. (by letter).—The paper by Mr. Friberg presents the second restatement of the general principles of dowel design first presented briefly by the writer as one phase of the structural design of concrete pavements.[12] In 1932 R. D. Bradbury,[3] Assoc. M. Am. Soc. C. E., made use of these same principles to draw other conclusions as to the proper size and spacing of dowels. Since the writer cannot avoid the responsibility of having suggested these approximate methods of analysis to the profession, it seems necessary to point out some of the limitations that should be placed upon the use of such approximations before persons without a proper understanding of the problem reach unacceptable conclusions.

Change in Dowel Practice.—Mr. Friberg states that present practice is to place $\frac{3}{4}$-in. or $\frac{7}{8}$-in. dowels of 2-ft length at from 12-in. to 20-in. centers. When the writer made his study of this problem,[12] a common practice was to use $\frac{1}{2}$-in. or $\frac{5}{8}$-in. dowels at from 3 ft to 5 ft centers. Since that study showed the certain need for heavier dowels at much closer spacing, the trend in present practice is decidedly gratifying; but the trend toward the use of heavier dowels more closely spaced must be carried further and such dowels may need to be made longer than 2 ft for maximum efficiency, as will be demonstrated. This appears in conflict with Mr. Friberg's conclusion that dowels may be shortened to less than 2 ft. Naturally, this lack of agreement is produced by different points of view regarding K, the foundation modulus.

Dowel Deflection Curve.—Based upon mathematically derived formulas that assume perfect elasticity, originally published by Professor S. Timoshenko and Mr. J. M. Lessels,[13] the writer has made the computations of Table 8. In addition to Equation (1), the formulas needed are:

$$y_P = \frac{P\,e^{-\beta x}}{2\,\beta^3\,E\,I}\,(\cos\beta\,x) \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19a)$$

TABLE 8.—COMPUTATIONS OF RELATIVE DOWEL DEFLECTIONS

ANGLE βx		βx (times π)	$e^{-\beta x}$	$\cos\beta x$	$\sin\beta x$	$\cos\beta x - \sin\beta x$	DEFLECTIONS	
Degrees	Radians						y_P	y_{M_0}
0	0.000	0.00	1.00	1.0	0	+1.0	+1.0	+1.0
30	0.166	0.52	0.59	0.866	0.5	+0.366	+0.51	+0.216
45	....	....	....	0.707	0.707	0.0	....	0.0
60	0.333	1.05	0.35	0.5	0.866	−0.366	+0.175	−0.128
90	0.500	1.57	0.21	0.0	1.0	−1.0	0.0	−0.208
120	0.666	2.10	0.122	−0.5	0.866	−1.366	−0.061	−0.167
150	0.832	2.62	0.074	−0.866	0.5	−1.366	−0.063	−0.100
180	1.000	3.14	0.043	−1.0	0.0	−1.0	−0.043	−0.043
210	1.166	3.67	0.026	−0.866	−0.5	−0.366	−0.022	−0.009
240	1.333	4.20	0.015	−0.5	−0.866	+0.366	−0.007	+0.006
270	1.50	4.72	0.009	0.0	−1.0	+1.0	0.0	+0.009

---

[11] Vice-Pres. and Dean, Graduate Div., Armour Inst. of Technology, Chicago, Ill.

[12] "Design of the Reinforced Concrete Road Slab," by L. E. Grinter, M. Am. Soc. C. E., *Bulletin No. 39*, Texas Eng. Experiment Station, pp. 53–59, 75, and 79–83.

[13] "Applied Elasticity," by S. Timoshenko and J. M. Lessels, Westinghouse Technical Night School Press, pp. 133–153.

and,

$$y_M = \frac{M_0\, e^{-\beta x}}{2\,\beta^2\, E\, I} (\cos \beta\, x - \sin \beta\, x) \;\ldots\ldots\ldots\ldots\ldots (19b)$$

in which y_P is the deflection at any distance, x, along the dowel due to the load, P, applied at its end, and y_M is the corresponding deflection due to an applied end moment, M_0. The constants, E and I, belong to the dowel since the sub-grade factor, K, includes, in its complex make-up, the modulus of deflection of the sub-grade, the modulus of elasticity of the concrete slab, and the stiffness of the slab as measured by its thickness.

The curves of Fig. 8 make several points of importance clear: (1) If moment is neglected, the length of bar producing positive bearing pressure will be one-half the length producing negative bearing pressure; and (2) for resistance to end moment alone, the length of dowel producing positive pressure is decreased 50% but the length of bar to the second point of zero deflection, or of reversed bearing, is not changed greatly. Hence, that length of bar beyond the value of $\beta\, x = 270°$ cannot be very effective in reducing bearing pressure at the joint or under the load. Nevertheless, since dependence is being placed upon a theoretical analysis that actually assumes an infinite length to the bar, it would be the better part of wisdom to design the dowel to project a few inches beyond the point, $\beta\, x = 270°$, or where $x = \frac{1.5\,\pi}{\beta}$.

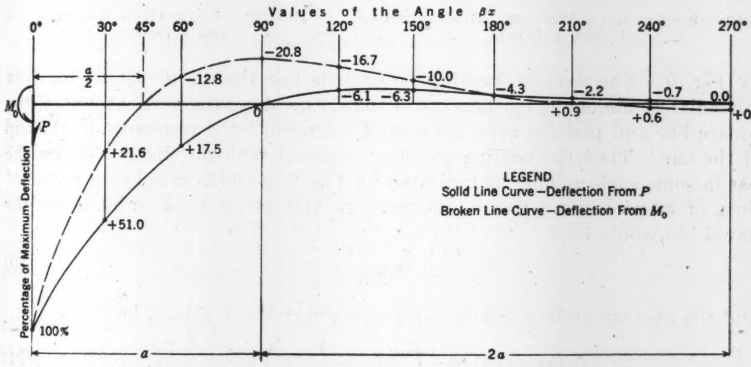


FIG. 8.—COMPARISON OF DOWEL DEFLECTIONS FROM LOAD AND MOMENT

Convenient Computation of Bearing Pressure.—Since both moment and load or shear are applied to the end of the dowel bar at the joint, the use of the length of bar up to the point, $\beta\, x = 0.5\,\pi$, for resistance to positive bearing pressure (with parabolic variation of bearing pressure) is not conservative. In fact, it is definitely liberal. However, it offers a logical procedure for computing dowel-bearing pressure that avoids the direct use of the factor, K (a factor that varies from joint to joint and even from dowel to dowel of the same joint). Fortunately, this factor is under a fourth root radical in the formula for β which means that the required length of the dowel bar is influenced far less directly by the sub-grade modulus than is the bearing pressure.

The Foundation Modulus, K.—Several persons have expressed an interest in the values of 300 000 lb per sq in. and 1 500 000 lb per sq in. that the writer has used on occasion for the factor, K, in Equation (1). This symbol denotes the settlement of the dowel bar, in inches, for a vertical load of 1 lb per sq in. applied to the top surface of the bar. If the dowel bar is supported on a massive foundation, the stress distribution under the bar may be somewhat as indicated

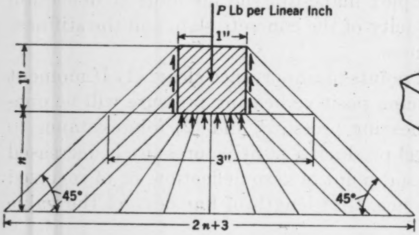


FIG. 9.—POSSIBLE PRESSURE DISTRIBUTION UNDER A DOWEL

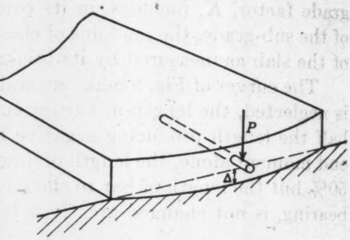


FIG. 10.—FOUNDATION MODULUS FOR A DOWEL

by Fig. 9. The average bearing pressure is less than 1 lb per sq in. even immediately under the bar because of the shears along the vertical sides of the square bar and perhaps even because of a tension bond resistance at the top of the bar. Then the bearing pressure decreases with the distance below the bar in some such manner as indicated by Fig. 9 in which it is based upon 45° lines of distribution. Hence, the average unit stress at n units below the dowel bar would be,

$$f_b = \frac{1}{2\,n + 3} \qquad \text{(20)}$$

and the average vertical compressive stress over the depth, n, becomes:

$$f_b\,(\text{average}) = \frac{1}{2}\left(\frac{1}{3} + \frac{1}{2\,n + 3}\right) = \frac{n + 3}{6\,n + 9} \qquad \text{(21)}$$

For large values of n this fraction becomes nearly $\frac{1}{6}$. Hence, one may write (approximately), $\delta_{1\ \text{lb}} = \frac{n}{6\,E}$, but,

$$K = \frac{1}{\delta_{1\ \text{lb}}} = \frac{6\,E}{n} \qquad \text{(22)}$$

This analysis affords no definite answer to the value of K for the case of a dowel bar projecting from a massive block of concrete; but it does indicate that it may be smaller than E. For instance, if the conditions indicated by Fig. 9 could be expected to hold for a depth of 5 ft below a 1-in. dowel bar set in a massive block of concrete, the value of K would be only 10% of E, or about 300 000 lb per sq in. The writer doubts whether there would be a reduction of this amount but there is another factor to be considered. The tendency, both in practice and in Mr. Friberg's recommendations, is to stress the concrete rather high in bearing under the bar. For stresses above common working stresses the modulus of elasticity of the concrete itself will decrease, and plastic deformation will take place to some degree. This may result in a considerable reduction of K, perhaps even to 300 000 lb per sq in., when the action illustrated by Fig. 9 is also considered. Such a low value, therefore, may be the proper one to use when dowel-bar misalignment in the horizontal plane is studied. Tests are needed to verify this possibility.

Finally, there is the fact that the actual condition to be studied for the case of vertical load transfer by a dowel bar is one for which the dowel is supported by a thin slab which, in turn, rests upon a sub-grade that is expected to re-act nearly elastically for low loads. As shown by Fig. 10, there is not only a compressive deformation in the concrete under the dowel, but the slab actually curves and sinks into the foundation mat. All these factors tend to increase the deflection by a large measure and, of course, they reduce the factor, K.

Assume, for example, that there are 3 in. of concrete below the dowel of Fig. 10. Then, at the soil level, the load might be spread over a width of $2n + 3 = 9$ in. A width of 10 in., however, would still give only a 10 to 1 ratio of the modulus K to the sub-grade modulus itself; for 100 in. the ratio would be 100 to 1. This is far from a satisfactory analysis but it may be somewhat indicative. For a sub-grade modulus of 300 lb per sq in. per in. of settlement (a rather firm material) the correspondingly indicated range of values of K would be from 3 000 to 30 000 lb per sq in. Perhaps the actual value of the foundation modulus is not this low, but nevertheless it calls attention sharply to the fact that present ideas regarding dowel design may have to be revised.

Incidentally, this was the weakness in the theory that the writer had in mind in 1931 when he specified 4-ft dowels.[12] His computations on bearing pressure were made for $K = 300\,000$ lb per sq in., which was regarded as the minimum value that the foundation modulus might approach for a very thick slab or block of concrete. For a thin slab on a soil foundation the modulus, K, was recognized to be subject to a probable serious reduction, and the minimum possible value of perhaps only 3 000 to 30 000 lb per sq in. dictated the specification for 4-ft dowels that was made. In a later publication, in which stresses from misalignment in the horizontal plane were being reported, a value of K of 1 500 000 lb per sq in. was chosen since the purpose was to set limits on the possible disastrous effect of misalignment rather than to study average conditions. Of course, this value was used only for the study of misalignment in the horizontal plane where the soil modulus is not a factor. All this should point out the very indefinite information available regarding the foundation modulus, K, and the need that it should be studied in a scientific manner. The writer believes that this modulus may show a maximum variation of a hundred-fold. Only tests can answer this question.

Longitudinal Warping Produces Severe Dowel Stresses.—A kind of dowel stress that has been given a minimum of consideration by the author, and by all other writers, is the bending produced by longitudinal warping of the pavement. The sub-grade modulus varies from section to section, and temperature variation through the slab is not uniform. The result is that no two undoweled slabs would ever warp in identically the same manner. Hence, there are differences of elevation between the two slabs at the joint that must be equalized by the dowels. The writer has no doubt but that this difference in elevation would amount to $\frac{1}{8}$ in. in a considerable percentage of joints and $\frac{1}{4}$ in., or more, for exceptional cases of non-uniformity. To produce a settlement of $\frac{1}{16}$ in. over an average slab width of only 1 ft near the joint where the sub-grade modulus is 200 (average material) would require a dowel force of $\frac{1}{16} \times 200 \times 144 = 1\,800$ lb per lin ft of joint. (It is assumed that each slab will be displaced $\frac{1}{16}$ in. for a $\frac{1}{8}$-in. differential settlement.) The value of $\frac{1}{16}$ in. chosen is about equal to the upward curling deflection of the slab for a temperature variation of 25° F. It is important to recall that W. K. Hatt,[14] M. Am. Soc. C. E., measured an upward corner deflection of a pavement slab of $\frac{1}{4}$ in. from moisture change only. Hence, it seems reasonable to expect at least $\frac{1}{8}$ in. of differential settlement between adjacent undoweled slabs where lateral curling and longitudinal warping produced by a combination of temperature and moisture changes may readily combine with variations of sub-grade modulus to produce non-uniformity.

The writer's crude estimate of the dowel shear per linear foot of joint as that force necessary to produce a settlement of $\frac{1}{16}$ in. over a slab area 1 ft square can be refined rather easily by taking into account the elastic curve of the slab. Thus, the slab of 20-ft width and 6-in. depth may be considered to be supported on an elastic foundation and loaded at the edge with a series of vertical loads which are the dowel shears from longitudinal warping. Let $K = 200$ lb per sq in. per in. of settlement; then, by Equation (1):

$$\beta = \sqrt[4]{\frac{200 \times 240}{4 \times 3\,000\,000 \times \frac{1}{12} \times 240 \times 6^3}} = 0.031.$$

The length of slab to the second point of zero deflection is defined by $\beta_x = 270° = 1.5\,\pi$; or, $x = \frac{1.5\,\pi}{0.031} = 157$ in. $= 13.1$ ft. For an end deflection of $\frac{1}{16}$ in., $\delta = 0.062 \text{ in.} = \frac{P - \beta\,M_0}{2\,\beta^3\,E\,I} = \frac{P}{2\,\beta^3\,E\,I}$ (nearly); and, $P = 0.062 \times 2 \times 0.031^3 \times 3\,000\,000 \times \frac{1}{12} \times 12 \times 6^3 = 2\,400$ lb, in which P is the dowel load per linear foot of joint. This value indicates that the dowel shear is approximately the load necessary to produce a settlement of $\frac{1}{16}$ in. over a slab area, 1 ft wide by 1 ft 4 in. long, for the particular case studied.

Severe Combinations of Dowel Stress.—In order to evaluate the author's conclusions, it is necessary to determine the possible severity of load combinations. This study will include: (1) A load transfer of 50% of the author's 4 000-lb rear wheel load (which is a smaller load than is permitted in many States); (2) longitudinal warping producing a differential settlement of $\frac{1}{8}$ in. between adjacent undoweled slabs; and (3) misalignment of $\frac{1}{8}$ in. per ft, or $\frac{1}{4}$ in. for a standard length dowel. Calculations will be made for three K-values: A value of 300 000 lb per sq in. as an upper limit with a soft sub-grade; an intermediate value of 30 000 lb per sq in.; and a minimum K-value of 3 000 lb per sq in. Dowels are $\frac{3}{4}$ in. at 12-in. spacing. For $K = 300\,000$, Equation (1) yields $\beta = 0.59$. The required length of dowel would be $2\,(1.5 \times \pi) \div 0.59 = 16$ in. For $K = 30\,000$, $\beta = 0.33$. The corresponding length of dowel would be $2(1.5 \times \pi) \div 0.33 = 29$ in. Finally, for $K = 3\,000$, $\beta = 0.187$. The dowel length then would be $2(1.5 \times \pi) \div 0.187 = 50$ in. Thus, there is the possibility of 4-ft dowels being required, but as a more probable case which is close to present-day practice a study will be made of a dowel of 30-in. length.

---

[14] "The Effect of Moisture on Concrete," by W. K. Hatt, *Public Roads*, March, 1925, p. 15.

The minimum possible bearing stress for a $\frac{3}{4}$-in. dowel, 30 in. long, will occur when positive pressure exists over one-third the length of 10 in. For parabolic variation of pressure and a load transfer of 2 000 lb, the bearing stress is $\frac{3 \times 2\,000}{0.75 \times 10} = 800$ lb per sq in. The corresponding bearing stress for longitudinal warping would be $\frac{3 \times 2\,400}{0.75 \times 10} = 960$ lb per sq in. To this total of 1 760 lb per sq in., which is already as high as any one is willing to recommend, must be added according to the author's estimate another 2 000 lb per sq in. for misalignment, a total of 3 760 lb per sq in. even with 30-in. dowels. Satisfactory functioning of much shorter dowels during the full life of the pavement is obviously questionable.

Conclusion.—The author has presented an interesting study of an old problem. The problem has received less consideration than it deserves because expansion joints, once placed, are not open to inspection until the road is torn up. Failure of road-slabs at other places than the expansion joints have been so frequent that dowel failures have received scant study and little comment. Nevertheless, the crudest possible calculations show rather conclusively that short dowels, widely spaced, are of little value for load transfer and that they subject the concrete to high bearing stresses which reduce, rather early in the life of the pavement, whatever initial effectiveness the dowels may have had.

The author's conclusions are reasonable except in regard to the lengths of dowels. His interpretation of the significance of the factor, K, which must combine in its make-up the sub-grade modulus, the modulus of elasticity of the concrete, and the slab stiffness as measured by its thickness led him to overvalue the significance of his calculations in regard to dowel length. No one can guarantee that the value of K for vertical loading may not be very much lower than the value of 300 000 used by the writer in his first studies of this problem, whereas the author's conclusions are based upon a value of 1 000 000 for K. The result of using a low foundation modulus in the author's calculations might be to revise his conclusions that 24-in. dowels may be shortened to 12 in. and lead to the conclusion that standard 24-in. dowels should be lengthened instead. No one can answer this criticism until adequate tests have been made to set limits upon the foundation modulus, K.

During the interim the only reasonable procedure in analysis is to study each design feature for the corresponding critical value of the sub-grade modulus. Thus, bearing pressure, which controls the diameter and spacing of dowels, must be studied for a relatively high value of the foundation modulus. The dowel length, on the other hand, will be controlled by a relatively low value of the foundation modulus. Even when tests have established the unknown relationships involved, it will still be proper to follow this procedure because the foundation modulus will never be constant for any reasonable length of roadway.

W. O. Fremont,[15] M. Am. Soc. C. E. (by letter).—Under "Present Construction Practice" the author states that the "exact mathematical solution" of the problem pertaining to a dowel structure of infinite length has been presented by Professor Timoshenko and Mr. Lessels. This "exact mathematical" theory is nothing more than a very approximate first tentative approach to the analysis of the actual behavior of a straight elastic bar of uniform rigidity on an elastic foundation. It is simulated roughly by the same bar, continuously connected to an absolutely rigid support by uniformly distributed elastic columns, of constant length and cross-section and constant elastic characteristics (see Fig. 11). It is assumed that each individual column must lengthen or shorten uniformly and independently, without affecting any of the other columns. This assumption implies that at the upper ends the columns are connected with the neutral plane of the bar by means of hinges. At the lower ends they are fixed to the absolutely rigid support. In this way, the bar is assumed to rest on an ideal brush with an absolutely rigid back, and with ideal bristles extending to, and connected with, the neutral plane of the bar by ideal hinges. It is implied: That no shear or friction acts between the bristles; that they do not bend; that they are of very small cross-section; and, that they are spaced close to one another. Let L = the constant length of each column; A_0 = the cross-section area; E = the modulus of elasticity; F_0 = the force in a column that causes lengthening or shortening; ΔL = the change in length of a column under an external load, F_0; N = number of columns per unit area; Σ_p = the sum of the forces in the uniformly stressed columns, referred to a unit area; and, k = modulus of the elastic foundation. Then,

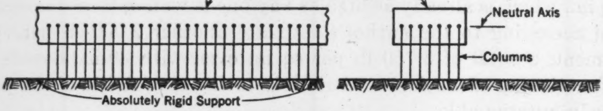


Fig. 11

$$F_0 = \frac{E\,A_0\,\Delta L}{L} \qquad (23a)$$

$$\Sigma_p = N\,F_0 = k\,\Delta L \qquad (23b)$$

$$k = \frac{N\,E\,A_0}{L} \qquad (23c)$$

and, if $N\,A_0 = 1$,

$$k = \frac{E}{L} \qquad (23d)$$

If $E = 4.5 \times 10^6$ lb per sq in. and $L = 4.5$ in., $k = 10^6$ lb-in.$^{-3}$.

The theory based on the foregoing assumptions is also referred to as the elementary theory of beams on elastic foundation. In adapting it to his problem, Mr. Friberg makes a number of assumptions, which the writer proposes to discuss.

---

[15] Bridge Designing Engr., State Highway Dept., Lansing, Mich.

Assumption 1.—The paper is predicated on the principle that the concrete surrounding the dowel behaves as is assumed in the elementary theory of beams on elastic foundations. Conditions similar to this ideal exist only in exceptional cases, if at all, in engineering practice. Nevertheless, the theory based on the foregoing assumption yields satisfactory approximations in some cases. Cases are known in which these approximations are not satisfactory. In dowel construction, the columns may be changed to thin plates of similar characteristics perpendicular to the dowel; and one can visualize easily what will happen when the concrete surrounding the dowel, especially near the ends, is suddenly released from all the shear forces interacting between the plates.

The elementary theory depends upon the assumption that Hooke's law holds. Shear tests on doweled concrete block specimens invariably leave permanent deformations. The first run of the test below the "critical point"[16] may produce a shear-deflection curve that is almost a straight line passing through the zero point for the total, elastic, and permanent deflections. The second run shows a considerable deviation from the straight line especially if the residual deformation of the first run is considered. There are plastic deformations that are not considered by the elementary theory. If the dowel is painted or greased the greater compressibility of the film of paint or grease affects the behavior of the system considerably, especially if the paint is destroyed with the passing of time or by friction between the dowel and the concrete.

It is evident that, by the adoption of the elementary theory for doweled pavement slabs, only a very rough approach to the actual conditions is made and one must be prepared for surprising results when derivations on the basis of this theory are compared with actual observations of construction work or test specimens.

Values of dowel shears in concrete blocks, at the point of failure, computed on the basis of the elementary theory, varied between 4 000 and 14 000 lb (350%) in one set of tests. In the case of dowels of circular cross-section there is a further complication as the distribution of pressures upon the concrete may be different from the one in the case of rectangular prismatic bars.

Assumption 2.—Under "Application of Design Formulas" Mr. Friberg states: "In practice, the dowels are usually not very stiff compared with the concrete pavement. It is an acceptable approximation, then, to presume that a point of contraflexure exists in the dowel at the center of the joint, * * *." By a method similar to the one proposed by Dean Westergaard[17] and, by the introduction of the elementary theory discussed herein under "Assumption 1," the writer has made the following observations:

Load at the joint, in pounds........................	9 000
Thickness of the slab, in inches....................	7
Modulus of elasticity of concrete, in pounds per square inch....................................	4.5×10^6

---

[16] "Joint Testing Experiments with a Theory of Load Transfer Distribution Along the Length of Joints," by J. W. Kushing and W. O. Fremont, *Proceedings*, Fifteenth Annual Meeting, Highway Research Board, December, 1935.

[17] "Spacing of Dowels," by H. M. Westergaard, *Proceedings*, Eighth Annual Meeting, Highway Research Board, 1928.

Joint opening, in inches	1
Diameter of dowels, in inches	$\frac{3}{4}$
Spacing of dowels, in inches, center to center	12
Subgrade modulus (bulk modulus of elasticity) in pounds per cubic inch	250
Shear force, T_0, in pounds	920
Moment (see Fig. 12), in inch-pounds, at the:	
Unloaded face, $M =$	− 369
Loaded face, $M_f =$	551
Center, $M_c =$	91
The relative stiffness, $\beta =$	0.793

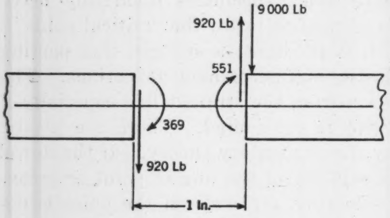


FIG. 12

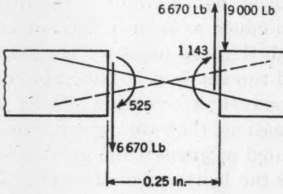


FIG. 13

From the foregoing, the maximum pressure on the concrete, as computed by the formula

$$s = 10^6 \frac{T + \beta M_f}{2 \beta^3 E I} \qquad (24)$$

is $\frac{920}{474} \times 10^3 + \frac{71.3}{474} \times 10^3 + \frac{364}{474} \times 10^3 = 2\,867$ lb per sq in.

Of this pressure, 150 lb per sq in. (that is, 5.25% of the total pressure) is due to the moment $M_c = 91$ in-lb. If the misalignment is uniformly 0.5 in. per foot of dowel, but alternating in opposite direction and symmetric with respect to the load, the condition shown in Fig. 13 was obtained for the center dowel when the opening was decreased to 0.25 in. by expansion. In this case,

$M_c = 6\,670 \times 0.125 - 525 = 308$ in-lb.; and, $s = \frac{6\,670}{474} \times 10^3 + \frac{244}{474} \times 10^3 + \frac{906}{474} \times 10^3 = 14\,100 + 515 + 1\,915 = 16\,530$ lb per sq in., in which 515 is due to M_c and is 3.1% of 16 530.

Assumption 3.—A third assumption involved in the paper is that the foundation modulus of the concrete is assumed to be $k_c = 10^6$ lb per cu in.

Assumption 4.—Under "Design Formulas for the Ordinary Case," the author states: "Under most construction conditions the bearing stress on the concrete is the critical one."

Tests by the writer indicate that the concrete beneath the dowel is subject to strains corresponding to very high compression stresses. According to the mathematical theory of elasticity these stresses must decrease in the direction away from the dowel, and therefore the destruction or plastic deformation of the concrete is limited to a region in the immediate vicinity of the dowel. If the dowel is near the neutral plane of the pavement slab this plastic deformation has no further effect on the slab itself.[18]

It has not been demonstrated that the plastic deformation around the dowel has ever impaired the strength of the pavement slabs directly, but such deformation impairs the load transfer and therefore increases the extreme fiber stresses in the slabs and decreases the strength of the structure indirectly. Therefore, as long as the fiber stresses are under control the pressure of the dowel on the concrete, with any resulting plastic deformation, should not be considered "critical."

Assumption 5.—In the sentence preceding "Dowel Deflection Across a Joint" Mr. Friberg states that, under the conditions cited, the dowels could be made as much as 25% shorter without appreciably affecting the maximum stresses in and around the dowels. If the plastic deformation is taken into consideration, as well as the effects of paint and grease, the points of stress reversal may be found to have moved deeper into the concrete. Furthermore, the use of caps reduces the effective length of the dowels. Therefore, it is necessary to consider, carefully, all the factors affecting the length of the dowels.

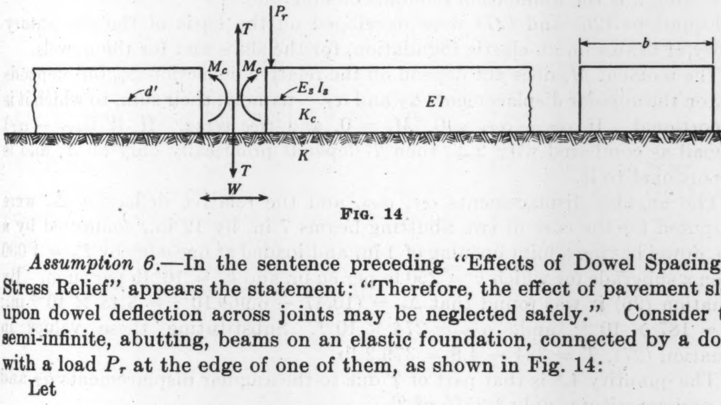


FIG. 14.

Assumption 6.—In the sentence preceding "Effects of Dowel Spacing on Stress Relief" appears the statement: "Therefore, the effect of pavement slope upon dowel deflection across joints may be neglected safely." Consider two semi-infinite, abutting, beams on an elastic foundation, connected by a dowel with a load P_r at the edge of one of them, as shown in Fig. 14:

Let

$$\Delta = 6\,n\,[1 + (1 + \beta\,W)^2] + 6\,m^3\,[1 + (1 + \epsilon\,W)^2] + \beta^3\,W^3 \quad \text{(25a)}$$

$$n = \frac{E_s\,I_s}{E\,I} \quad \text{(25b)}$$

$$\beta = \epsilon\,m = \sqrt[4]{\frac{k\,b}{4\,E\,I}} \quad \text{(25c)}$$

and,

$$\epsilon = \sqrt[4]{\frac{k_c\,d'}{4\,E_s\,I_s}} \quad \text{(25d)}$$

---

[18] "Plasticity," by A. Nadai, McGraw-Hill Book Co., Inc., 1931.

Then,

$$T = \frac{3\,n\,(2 + \beta\,W)}{\Delta}\,P_r \qquad \text{(26a)}$$

and,

$$M_c = \frac{n\,P_r}{2\,\beta\,[2\,(m + n) + \beta\,W]} \qquad \text{(26b)}$$

On the other hand, the static and elastic states of a dowel in a pavement may be defined by the angular displacements α_T and α_{T_r} of the abutting slabs at the faces of the joint and their relative vertical deflection Δ_v, independently of the vertical forces and moments acting on the slabs. Then the bending moment at the center of the dowel will be:

$$M_c = \frac{\epsilon\,E\,I\,(\alpha_T + \alpha_{T_r})}{2 + \beta\,W} \qquad \text{(27a)}$$

and, the shear forces T in the dowel will be:

$$T = \frac{6\,\epsilon^3\,E\,I\,[2\,\Delta_v + W\,(\alpha_{T_r} - \alpha_T)]}{6\,[1 + (1 + \beta\,W)^2] + \beta^3\,W^3} \qquad \text{(27b)}$$

in which d' is the diameter of the dowel; k_c is the foundation modulus of concrete; and, k is the foundation modulus of subgrade.

Equations (26) and (27) were developed on the basis of the elementary theory of beams on an elastic foundation, for the slabs and for the dowels.

The moment M_c does not depend on the relative deflection Δ_v, but depends only on the angular displacements α_T and α_{T_r}—namely, their sum, to which it is proportional. If $\alpha_T + \alpha_{T_r} = 0$, $M_c = 0$, and *vice versa*. If $W\,(\alpha_{T_r} - \alpha_T)$ is small as compared with $2\,\Delta_v$ then T depends practically only on Δ_v and is proportional to it.

The angular displacements α_T, α_{T_r}, and the relative deflection Δ_v were computed for the case of two abutting beams 7 in. by 12 in., connected by a $\frac{3}{4}$-in. dowel having a joint opening of 1 in. and loaded at one edge by $P_r = 1\,000$ lb, on a subgrade for which $k = 250$ lb per cu in. and $k_c = 10^6$ lb per cu in. By Equation (26) it was found that $\Delta_v = (10.47 - 6.96)\,10^{-3} = 3.48 \times 10^{-3}$ in.; $\alpha_T = 187 \times 10^{-6}$; and, $\alpha_{T_r} = 274 \times 10^{-6}$. Substituting these values in Equation (27), $T = 384 - 4.8 = 379.2$ lb.

The quantity 4.8 is that part of T due to the angular displacements α_T and α_{T_r} and constitutes only 1.25% of T.

Assumption 7.—From the Rule stated by the author under "Effects of Dowel Spacing on Stress Relief" it follows that the distribution of shear among the dowels is independent of the elastic characteristics of the joint construction and the amount of joint opening. The reasoning presented in the text preceding the Rule is not convincing and is incomplete. The shear considered near the points of maximum negative moments will be nearly zero only for points near the very edge. For the consideration of equilibrium, including the dowel shear forces, an element of finite dimensions must be considered. For such an element the shears referred to will not be zero on the basis of the reasoning presented in the paper.

For absolutely stiff, or rigid, joint units 50 % of the load will be transferred by one unit at the point of application of the load. For very elastic (say rubber units) the load transfer curve will approach the elastic curve of the loaded slab. In the latter case no load transfer will occur, and almost the entire load will be supported by the loaded slab alone.[19] In the case of dowels in concrete slabs the engineer will be unable even to approach a condition of absolute rigidity in joint construction. The characteristic condition, rather, would tend toward the other extreme.

Assumption 7 calls for a triangular load distribution. In the case of rigid joints there would not be any definable distribution. In the case of very flexible joints the distribution defined by the deflection curve will extend for a distance equal to more than $\pi \times l$ on both sides of the load; at $\pi \times l$ the deflections are still greater than 10% of those under the load.[20]

Field tests on doweled pavements (8-in. slabs, 11 ft wide, loaded on the edge at the center of the pavement, with a 9 000-lb load, 13 dowels of $\frac{3}{4}$-in. diameter, spaced 12 in.) produced edge-deflection curves for the loaded and unloaded edges extending the entire width of the pavement which did not intersect at any point. The relative deflections of the two slabs at the outer edges were still 11% and 15.6% of those under the load.

The radius of relative stiffness was computed as $l = \sqrt{\frac{4.5 \times 10^6 \times 8^3}{12 \times 161}}$ = 33 in. This shows that dowels beyond the point 1.8 l may still have quite a noticeable influence upon the bending moment under the load and the corresponding extreme fiber stresses. The load transfer distribution depends to a great extent on the rigidity of the joint construction.

Assumption 8.—Equation (16) is presented as the expression for defining the shear on a dowel directly under the load. When so defined, P is considered to be quite independent of the dowel spacing and the action of all the other dowels in the joint except the one directly under the load. Consider a pavement construction consisting of two abutting slabs, connected by joint units, on an elastic foundation, with one dowel directly under the load W in Fig. 15.

Equalizing the deflections (Fig. 15):

$$(W - T_0)\, y_p - 2\Delta_T + (M_{0r} - M_0)\, y_m + y_m' - y_m'' = T_0\, y_p + T_0\, y_d \ldots\ldots (28)$$

in which (neglecting the effect of M_0 on y_d):

$$y_m' = a_{11} M_1 + a_{12} M_2 + \cdots \ldots\ldots\ldots\ldots\ldots\ldots (29a)$$

and

$$y_m'' = y_m' + W\,(a_{11} T_1 + a_{12} T_2 + \cdots) \ldots\ldots\ldots\ldots (29b)$$

Substituting Equation (29) in Equation (28):

$$T_0 = \frac{W\, y_p - 2\,\Delta_T + [(M_{0r} - M_0)\, y_m - W\,(a_{11} T_1 + a_{12} T_2 + \cdots)]}{2\, y_p + y_d} \ldots (30)$$

---

[19] "Joint Testing Experiments with a Theory of Load Transfer Distribution Along the Length of Joints," by J. W. Kushing and W. O. Fremont, *Proceedings*, Fifteenth Annual Meeting, Highway Research Board, December, 1935, p. 154.

[20] "Computation of Stresses in Concrete Roads," by H. M. Westergaard, *Proceedings*, Fifth Annual Meeting, Highway Research Board, December 3–4, 1925, Fig. 9.

Ignoring the influence of the bending moments:

$$\frac{T_0}{W} = \frac{1 - 2\,\dfrac{\Delta_T}{W\,y_p}}{2 + \dfrac{y_d}{y_p}} \ldots \ldots \ldots \ldots \ldots \ldots \ldots (31)$$

in which,

$$\Delta_T = 2\,A_1\,T_1 + 2\,A_2\,T_2 + \cdots \ldots \ldots \ldots \ldots \ldots \ldots (32)$$

is the deflection at Points O and O' produced by the shears T_1, T_2, $\cdots$ alone; and, $W\,y_p$ is the deflection produced at Point O from Load W alone. It is clear

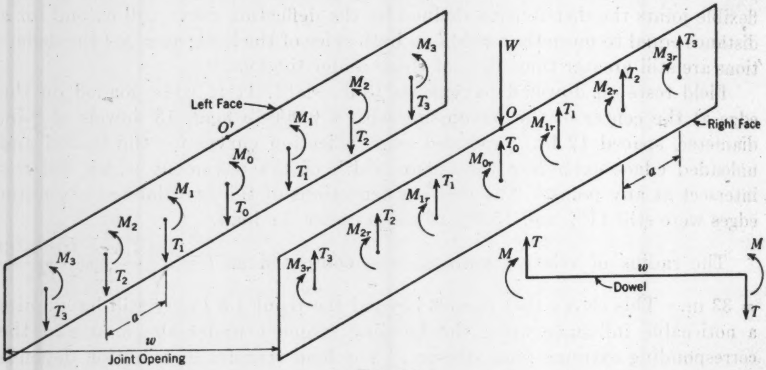


FIG. 15

that the doubled ratio $\dfrac{\Delta_T}{W\,y_p}$ cannot be neglected in Equation (31) without introducing considerable error.

Consider a 7-in. slab with $\frac{3}{4}$-in. dowels. The foundation is elastic, with a foundation modulus of 250 lb per cu in.; the joint is open 1 in.; five dowels are spaced 12 in. apart; and, a load of $W = 9\,000$ lb is concentrated over the center dowel. By a method similar to the one proposed by Dean Westergaard[17] and the introduction of the elementary theory described herein under Assumption 1, the writer has determined the following values: $T_0 = 920$ lb; $T_1 = 673$ lb; $T_2 = 480$ lb; $\Delta_T = 4.086 \times 10^{-6} \times 673 + 330 \times 10^{-6} \times 480 = 4.34 \times 10^{-3}$; $y_p\,W = 2.35 \times 10^{-6} \times 9\,000 = 21.18 \times 10^{-3}$; and (according to Table 2 of the paper), $y_d = 0.0094 \times 10^{-3}$. Consequently, by Equation (31):

$$T_0 = 9\,000\,\frac{1 - 2\,\dfrac{4.34}{21.18}}{2 + \dfrac{94}{23.5}} = 885 \text{ lb},$$

which is not seriously different from the value $T_0 = 920$ lb determined by the writer previously. Equation (16) of the paper yields:

$$T_0 = 9\,000\,\frac{1}{2 + \dfrac{94}{25}} = 1\,562 \text{ lb},$$

an increase of 70%. Mr. Friberg's distribution, as affected by Assumption 7, yields:

$$T_1 = \frac{1.8\,l - a}{1.8\,l}\,T_0 \dots\dots\dots\dots\dots\dots\dots (33a)$$

$$T_2 = \frac{1.8\,l - 2\,a}{1.8\,l}\,T_0 \dots\dots\dots\dots\dots\dots\dots (33b)$$

and

$$T_k = \frac{1.8\,l - k\,a}{1.8\,l}\,T_0 \dots\dots\dots\dots\dots\dots\dots (33c)$$

From Equation (32):

$$\Delta_T = 2\,T_0 \sum_{k=1}^{k=m} \left(1 - \frac{a\,k}{1.8\,l}\right) A_k \dots\dots\dots\dots\dots\dots (34)$$

in which $m + 1 > \frac{1.8\,l}{a} > m$; and, m is an integer. Substituting Equation (34) in Equation (31):

$$\frac{T_0}{W} = \frac{1}{2 + \frac{y_d}{y_p} + 4\,\frac{\sum_{k=1}^{k=m} A_k - \frac{a}{1.8\,l}\sum_{k=1}^{k=m} k\,A_k}{y_p}} \dots\dots\dots\dots (35)$$

in which values of A_k are read from curves presented by Dean Westergaard.[17] In the case of the present numerical example, the conditions stated for Equation (34) are satisfied when $l = 26.8$ in. and $m = 4$. Then: $A_0 = 2.35 \times 10^{-6} = y_p$; $A_1 = 2.043 \times 10^{-6}$; $A_2 = 1.650 \times 10^{-6}$; $A_3 = 1.300 \times 10^{-6}$; $A_4 = 0.935 \times 10^{-6}$; $\sum_{k=1}^{k=4} A_k = 5.928 \times 10^{-6}$; $\sum_{k=1}^{k=4} k\,A_k = 12.983 \times 10^{-6}$; and, $\sum_{k=1}^{k=4} A_k - \frac{12}{1.8\,l}\sum_{k=1}^{k=4} k\,A_k = 2.682 \times 10^{-6}$. Therefore, by Equation (35),

$$T_0 = \frac{9\,000}{2 + \frac{94}{23.5} + 4\,\frac{2.682}{2.35}} = 871 \text{ lb}$$

which checks, reasonably well, the results derived by other methods. Assumption 8, as expressed by Equation (16), should be revised to conform to Equation (35).

The maximum fiber stress at Point O, Fig. 15, may be expressed as

$$s = s_W - s_0 - 2\,s_T \dots\dots\dots\dots\dots\dots\dots (36)$$

in which s_W is the component of stress due to W; s_0 is the component due to T_0; s_T is the component due to T_1, T_2, etc., on one side of the load; and,

$$\gamma_0 = \frac{s_0 + 2\,s_T}{s_0} \dots\dots\dots\dots\dots\dots\dots (37)$$

represents the amount of total stress relief, expressed in the partial relief, produced by the dowel directly under the load as a unit.

Then, the total stress relief at Point O, expressed in terms of the stress s_W, corresponding to the state in which there is no relief (free ledge) as a unit, will be

$$\gamma = \frac{s_0 + 2\,s_T}{s_W} = \frac{s_0}{s_W}\gamma_0 \qquad (38a)$$

Since $s_W = u\,W$; and, $s_0 = u\,T_0$:

$$\gamma = \frac{T_0}{W}\gamma_0 \qquad (38b)$$

and—since $M_0 = m_{0,0}\,T_0$; $M_T = \sum_{k=1}^{k=m} m_{0,k}\,T_k$; $s_0 = \frac{M_0}{W_0}$; and, $s_T = \frac{M_T}{W_0}$—

$$\gamma_0 = 1 + 2\sum_{k=1}^{k=m} \frac{m_{0,k}}{m_{0,0}}\frac{T_k}{T_0} \qquad (38c)$$

in which $m_{0,k}$ are picked from the curves presented by Dean Westergaard.[17]

Formulas similar to Equations (31), (35), (37), (38*b*), and (38*c*) must be derived for the case in which Load W is placed between two dowels.

To compute γ one must know $\frac{T_0}{W}$, Equation (38*b*), in which T_0 is the shear in the dowel directly under the load, with all the other dowels acting according to Assumption 7 and not as assumed by Mr. Friberg in the derivation of Equation (16). For T_0 Mr. Friberg takes the shear in the dowel directly under the load, on the assumption that the other dowels are inactive. Therefore, T_0 in Equation (38*b*) must be determined according to Equation (35). Equation (38*c*) can be computed conveniently by means of graphs.

In introducing Table 3 the author states that the unit moments were taken directly from data presented by Dean Westergaard[7]; and, again, in discussing Fig. 6, he states that the curves were computed in accordance with methods established by Dean Westergaard. The values of y_p may also be obtained from Dean Westergaard's diagrams. Would it not be more logical to use these diagrams throughout the entire analysis, and then compare the theoretical results with experimental measurements, rather than mix theory with experiment?

The percentages of load transfer given in Table 4, the computed values of stress relief in Table 5, and all conclusions drawn on the basis of these tables, need to be adjusted to Equations (35), (38*b*), and (38*c*).

Assumption 9.—The effect of vertical dowel misalignment is determined by Equation (18) on the assumption of uniform alternate misalignment. Under such conditions the maximum fiber stresses in the slab are affected very little. Such ideal conditions of misalignment will occur seldom in actual pavements.

By the same method (with a 7-in. pavement and five dowels spaced at 12 in., the center dowel being misaligned 1 in. per ft of dowel; the original opening was 1 in. and the final was zero), the writer has found: $T_0 = 10\,740$ lb; $T_1 = -3\,380$ lb; and, $T_2 = -2\,025$ lb. The corresponding fiber stresses in the slab were found to be: $s = -10\,740 \times 0.0364 + 6\,760 \times 0.0123 - 4\,050 \times 0.00306$

= − 321.4 lb per sq in. For smaller misalignments this stress will be proportionately smaller, being zero for zero misalignment and − 80.4 lb per sq in. for $\frac{1}{4}$ in. misalignment. The actual misalignment stresses will be smaller owing to the relief afforded by plastic deformation and by the paint or grease effect, which must also be taken into consideration.

The pressure between dowel and concrete is not critical as long as the fiber stresses are under control.

The Nature of Assumptions.—In making assumptions for the simplification of problems in applied mechanics it is important to know the kind and amount of their effect upon the result. Another suggestion is to vary the assumed values so that, by definite variations, a successively closer and closer approach can be made to the exact solution.

Assumptions affecting the laws of mechanics, or conclusions derived from such laws, do not always indicate, directly and clearly, the effect they have upon the results without laborious investigation. Assumptions affecting the characteristics of the materials used are mostly of the same nature (as, for example, the assumption of the linear, or Hooke's, law instead of some curvilinear law). Assumptions affecting the general make-up of the system, without any sacrifice in the exactness of fundamental laws or the characteristics of the materials, often afford a much clearer and easier appraisal of the errors involved by using the approximate solution instead of the exact one. For instance, in the case of a doweled pavement slab construction on an elastic foundation the problem may be simplified by considering only one dowel under the load. It is clear, then, that by introducing three dowels instead of one, the designer approaches closer to the exact solution. With any one of such assumptions (1 or 3 or 5, dowels) he has the advantage of knowing "where he stands"; the road to closer solutions is clear and open and the preceding step helps to make the following one.

Conclusion.—The writer is of the opinion that the method of dowel action analysis introduced by Dean Westergaard,[17] supplemented and extended either by the introduction of the elementary theory of beams on elastic foundation (described herein as Assumption 1) and the consideration of plastic deformation and the effect of grease or paint, or by the use of experimental curves obtained in the laboratory with specimens of actual joint units, is the basic method. On the basis of the data so obtained convenient graphs can be prepared for the use of the practical and field engineer.

Any other methods may be used within definite limits, for definite joint constructions, if their agreement with the basic method has been established within these limits. This is especially important for the analysis of new types of joint construction.

Bengt F. Friberg,[21] **Assoc. M. Am. Soc. C. E.** (by letter).—The purpose of the paper was to present the rules governing the behavior of an elastic structure in a yielding mass in such form that their application to steel dowels, encased in concrete and crossing transverse joints in concrete pavements, would be convenient to the designing engineer. To cover the subject the paper included consideration of dowel spacing as well, although it must be recognized that any analytical evaluation of dowel spacing upon pavement stresses is subject to considerable approximation.

---

[21] Research Engr., Laclede Steel Co., St. Louis, Mo.

Credit for first having indicated the application of bent bars in an elastic foundation to the design of single dowels in concrete pavement joints may be claimed by Dean Grinter.[2] The general solution by Messrs. Timoshenko and Lessels[4] represents the actual conditions around a dowel closer than most other applications; it has been used advantageously in the far less exact structure of railroad tracks. Such approximations as occur are the results rather of non-uniformity in the elastic behavior of the surrounding concrete as evaluated by the modulus of support "K" than in approximations of the solution.

In developing the formulas for dowel design, a number of assumptions were made. Mr. Fremont has enumerated, checked, and substantiated several of these assumptions, and, based on his discussion, the following conditions presented in the paper may be agreed upon as approaching the actual conditions nearly enough for present-day designs:

(*a*) Hooke's law for bearing pressure as proportionate to deflection at any point along a dowel may be applied when contact between the dowel and the concrete exists;

(*b*) Flexible steel dowels of round bars may be assumed to act as hinges between the two slabs with a point of contra-flexure at the center of the joint;

(*c*) Critically high bearing stresses, although sufficient to cause plastic deformation or failure in the concrete around the dowel, need not be considered critical for the pavement;

(*d*) The dowel length in contact with the concrete need not be greater than to the second stress-change point on each side of the joint;

(*e*) Pavement slope at the joint can be neglected in computing the deflection of a dowel across the joint; and

(*f*) Misalignment stresses around the dowels, although critical for the concrete around the dowels, need not be considered critical for the pavement fiber stresses, especially as the largest tension load stresses in the bottom of the pavement edge cannot be added to the largest misalignment stresses. The largest misalignment stresses at the dowels occur when adjacent dowels are misaligned in opposite direction. The largest edge stresses may occur when two or more dowels deviate in the same direction.

The modulus of support of the dowel in the concrete, or, as it will be called, "the modulus of dowel reaction K," has been given a value of 1 000 000 lb per cu in., which is thought to be reasonable for concrete pavements. Dean Grinter has suggested possible K-values in the field as low as from 3 000 to 30 000 lb per cu in. Such a suggestion is based on the assumption that the pavement deflection, as such, is reflected in the factor K. Obviously, that is not the case. The rate at which the concrete around a dowel resists the pressure of the dowel in the concrete does not change with deflection of the body of the concrete. The sub-grade influences the modulus of dowel reaction only to the extent that it permits local deformation of the concrete directly below the dowel and introduces curvature of the pavement slab. The modulus of dowel reaction, therefore, is relatively independent of the material beneath the concrete. Considering the clear relationship between modulus of elasticity of the concrete and the modulus of dowel reaction and customary pavement dimensions, the value of K for reasonable pressures would rarely be less than 25% of the modulus of elasticity.

There is nothing logical in choosing an arbitrary length of dowel embedment and presuming that such a length equals the distance to the second stress-change point, only because such a choice avoids the direct use of the factor K. The value of K is of great influence not only upon the bearing stresses around the dowel, but upon its deflection as well. The relationship between K, deflection, and bearing stress is illustrated in Fig. 16 for a 1-in. open joint, and for dowel sizes from $\frac{3}{4}$ in. to $1\frac{1}{4}$ in. round, deformed. It is evident that the deflection of the dowel for K-values less than 500 000 lb per cu in. quickly becomes so great that the dowels would be considered useless for their basic function of load transfer and would not be used. Fortunately, K would scarcely assume the low values suggested.

Inordinately high values of K would likely result in plastic deformation in the concrete due to the increased bearing stresses. High K-values (which are probably far more common in occurrence) would then be balanced automatically by the increased deflection and decreased load transfer accompanying a dowel of resulting longer free length. The actual bearing stress at the surface of the dowel encased in concrete may be greater than the bearing stress distributed across the diameter of the dowel by as much as 25% (see the rule for radial distribution of bearing stress in Fig. 2).

The estimate of differential warping stresses, by Dean Grinter, indicating a resultant dowel shear of the order of 2 400 lb, does not take into account the fact that the dowel shear due to differential warping would act as a lifting force on one of the slabs, resisted only by the weight of the concrete. Dowel shears considerably less than 1 000 lb for the conditions cited would then be likely to occur. Warping due to moisture differentials would probably raise the two sides of the joint alike. Curling due to temperature differential through the slab would also be likely to cause equal deflection of the two sides of the joint. Warping, therefore, scarcely needs to be considered in the design of dowels. A serious difference in level between adjacent slabs is due, more likely, to the loss of sub-grade support under one of the slabs. High dowel shears of amount equal to, or exceeding, the dowel shears computed for one dowel only according to Equation (16) will then be produced by large wheel loads; but the load-bearing capacity will be limited quickly by the unsupported pavement's resistance to bending stresses.

The effect of dowel spacing upon load transfer and stress relief was presented in the paper with three assumptions to obtain a simple, fairly representative picture, not too tedious for use in actual design. The three assumptions were:

(1) The moment diagram under a dowel load is similar to that under the wheel load;

(2) The influence of dowel shear upon pavement stress decreases with increasing distance from the wheel load to zero at a distance of 1.8 l; and

(3) A dowel directly under the wheel load remains fully effective.

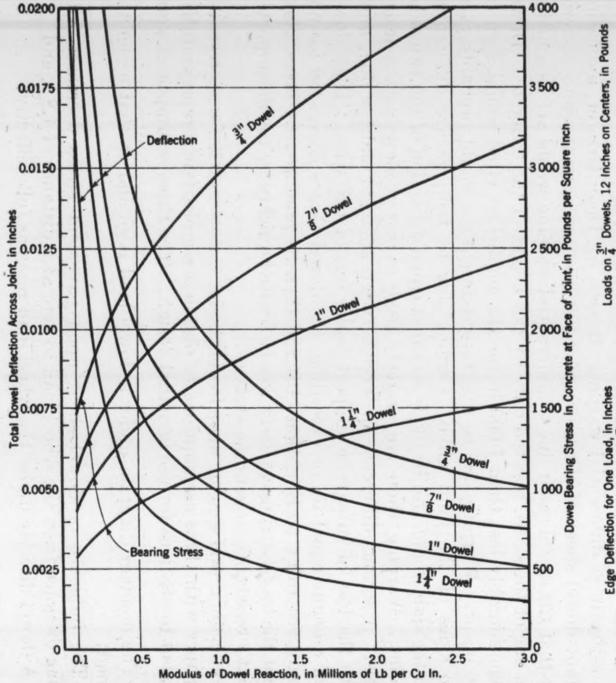


FIG. 16.—DEFLECTION AND BEARING STRESS FOR ROUND DOWELS CROSSING A ONE-INCH JOINT; 1000-POUND DOWEL SHEAR AT VARYING MODULI OF DOWEL REACTION

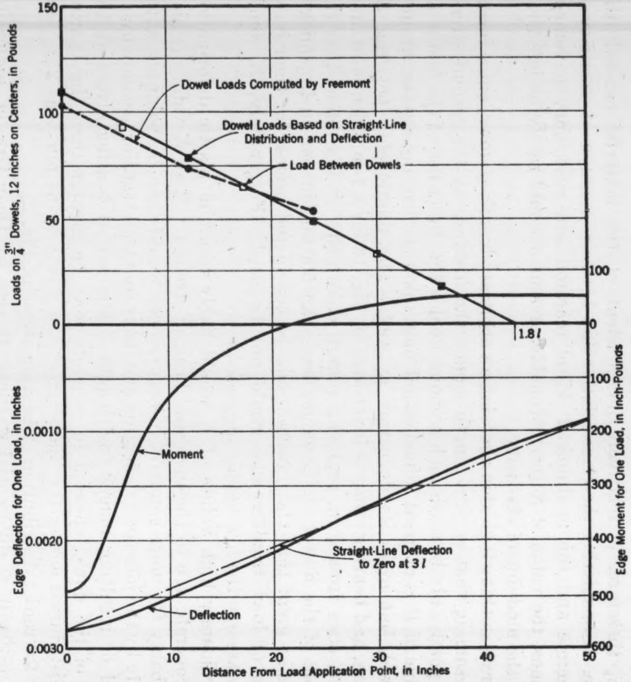


FIG. 17.—DEFLECTION, MOMENT, AND DOWEL LOADS ALONG THE EDGE FOR MODULUS OF SUB-GRADE REACTION 250 POUNDS PER CUBIC INCH; MODULUS OF DOWEL REACTION 1 000 000 POUNDS PER CUBIC INCH; AND RADIUS OF RELATIVE STIFFNESS "l" = 24 INCHES

Mr. Fremont has shown, quite correctly, that with a number of dowels crossing a joint the shear on the dowels nearest the wheel load is decreased. This should relieve high pressures around the dowels, but would also decrease the stress relief furnished by the dowels. Equation (16) is correct for one dowel only. However, even temporary lack of sub-grade support or voids around the dowels would immediately increase the load on the dowels nearest the wheel load. It seems wise to suggest that Equation (16) should be used for dimensioning of the individual dowel when spaced 12 in. or more apart under single tire loads. For the same reason the influence of dowels with sufficient clearance to be free to move, and some distance away from the wheel load where the pavement deflection is correspondingly smaller, would be too small to justify computation of their effect upon pavement stress.

The dowel shears computed by Mr. Fremont for a 7-in. pavement, a sub-grade modulus of 250 ft per cu in., and a 1-in. joint with five $\frac{3}{4}$-in. dowels spaced 12 in. center to center are shown in Fig. 17. Also shown are the moment diagram and the deflection diagram for a single load at the edge. The deflection diagram approximates closely a straight line with zero at a distance of 3.0 l from the load point, a condition typical for edge loadings in accordance with the diagrams developed by Dean Westergaard.[7] The moment quickly diminishes on either side of the load point and reaches a small negative maximum at a distance of about 1.8 l from the load point. Values of distributed dowel shear, based on the straight-line deflection diagram, and on the assumption of linear decrease in dowel loads to zero at a distance of 1.8 l, have been computed by the method described by Dean Westergaard,[17] adjusted to include dowel deflection. These values, plotted for the load over one, and between two, dowels, approach very near to those computed by Mr. Fremont without simplifying assumptions. The illustrated change in computed dowel shear between adjacent dowels is such that for all practical purposes dowels more than 1.8 l distant should be neglected even if the negative moment, maximum at that point, did not suggest that expedient. Therefore, computed values of dowel shear could be used for individual dowel dimensioning under conditions of reliable sub-grade support and accurate construction, especially for dowels on close spacing. The relative edge-stress relief for all dowels at variable dowel spacing, shown in Fig. 5, approximates very closely the actual conditions, expressing as it does the influence of spacing alone. The elastic characteristics of the joint and the joint opening are reflected in the basic value of shear established for a dowel directly under the load.

With the dowel shear distribution computed by Mr. Fremont, the edge-stress relief would be approximately 60% of that obtained under assumption of full effectivity of the dowel directly under the load. The actual edge-stress relief is believed to be considerably higher than that computed from distributed dowel shears. The pavement stress under the load is a function of sub-grade pressure which, in turn, is a function of pavement deflection. Dowels at considerable distance from the load point will be effective in decreasing the deflection, and therefore in affecting the distribution of sub-grade pressure near the load point, although their effect upon stress relief as gaged from the moment diagram would be to add to, rather than relieve, the pavement stress. Higher stress relief may be anticipated also, due to the true position of the dowel reaction at the edge, whereas the maximum edge stress under a wheel crossing a doweled joint occurs while the center of the contact area is some distance from the edge. The greater a concentration of effective dowel shears near the load point is assumed, the greater would be the computed shears on the dowels nearest the load point, and therefore, also, the computed stress relief. How closely the assumption of full effectivity of a dowel directly under the load will be approached with the dowel spacings that are now common remains to be established. For 18-in. dowel spacing the assumption agrees well with observed conditions. For critical comparison of various types of load transfer on different spacings, the relative, and not the absolute, value is sufficient.

The observed stress relief for the 6-in. pavement in Table 5 corresponded to a basic stress relief of 22.8% for one dowel directly under the load. To obtain this value with a computed dowel-shear distribution as presented in Fig. 17 would necessitate a dowel deflection across the joint of only 0.001 in. per 1 000 lb dowel shear, which is less than that obtainable with any possible modulus of dowel reaction for a $\frac{3}{4}$-in. dowel. The observed stress relief accordingly offers further evidence that the dowels nearest the load are more effective than the distributed dowel shears would indicate.

Thanks to the discussion, the value of the paper as an indication of conditions necessary to consider in the design of dowels across transverse joints in concrete pavements has been increased. Due to the uncertain factors upon which the mathematical analysis of a concrete pavement must still be based, it is scarcely recommended that tedious refinements be established for the detail of dowel design. The data give full recognition to the necessity of close dowel spacing for the relief of concrete pavement stresses. Future research may throw added light upon the actual deflection distribution and moments at joints so that the high effectivity of dowels near the load for pavement-stress relief may be substantiated theoretically.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2082

STRESS DISTRIBUTION AROUND A TUNNEL

By Raymond D. Mindlin,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. W. O. Richmond, Jacob Feld, J. H. A. Brahtz, and Raymond D. Mindlin.

Synopsis

The problem of the disturbance of stress around a tunnel is treated in this paper from the point of view of the theory of elasticity. The tunnel is considered to be a horizontal cylindrical hole of circular cross-section situated in a semi-infinite elastic solid under the action of gravity. Through the introduction of the bipolar co-ordinate system an exact solution of the classical elasticity equations is obtained which satisfies the boundary conditions at both the upper free surface of the solid and the free surface of the hole.

Formulas for the stresses are given in terms of infinite series. The series have been evaluated and the results are given in the form of tables and curves which show that the stresses are important when the hole is either very close to the upper surface of the solid or very far from the surface.

Introduction

In the study of the equilibrium of elastic solids, problems are often encountered in which the shape of the structure and the distribution of the loading are so irregular that the ordinary methods of mechanics and strength of materials are not adequate to cope with the situation. The analysis of the disturbance of stress around a tunnel presents just these difficulties. This problem is concerned with an elastic solid of large dimensions, pierced by a hole (the tunnel) and bounded by an unloaded upper surface (Fig. 1). Before this mass is pierced by the tunnel, it is in a condition of elastic equilibrium. The problem considered here is that of the disturbance of this original condition of stress created when the mass is pierced by the tunnel.

The methods of the theory of elasticity have been adapted previously to studies of the tunnel problem.[2] The most nearly complete discussions were

---

Note.—Published in April, 1939, *Proceedings*.

[1] Instr., Dept. of Civ. Eng., Columbia Univ., New York, N. Y.

[2] Since this paper was submitted Prof. Z. Anzo has called the author's attention to a paper "On the Stresses in a Gravitating Elastic Body Having a Circular Hole near Its Horizontal Surface," by Z. Anzo (in Japanese), Technology Reports of the Kyushu Imperial University, Vol. XII, No. 3, June, 1937.

given by S. Yamaguti,[3] and H. Schmidt,[4] who investigated the state of stress around a horizontal hole of circular cross-section, situated in a gravitating solid of indefinite extent. In his analysis, Yamaguti correctly considered the effect on the stresses of (1) the weight of the material removed from the hole, and (2) the stress concentration induced by an assumed type of initial stress in the surrounding medium. However, the influence of conditions on the upper surface of the solid was neglected. Accordingly, by St. Venant's Principle, the solution is applicable only when the distance of the hole from the upper surface is large in comparison with the radius of the hole. When such a restriction is made, however, a complicated solution of the type used by Yamaguti is no longer necessary because, in this case, the effect of the weight of the material removed from the hole is negligible in comparison with the stress concentration induced by the initial stress in the surrounding material. When the hole is far from any other boundary the latter effect can be obtained quickly by a method[5] well known in the theory of elasticity. The hole, with its center at point *A* (Fig. 1), is considered simply as situated in a uniform stress field composed of the principal stresses which were initially present at point *A*.

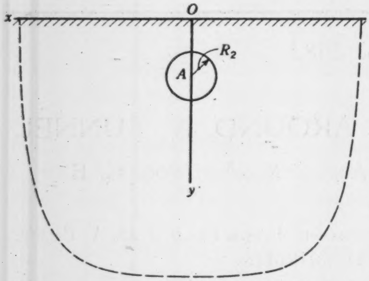


FIG. 1.—HORIZONTAL CYLINDRICAL HOLE OF CIRCULAR CROSS-SECTION IN A GRAVITATING, SEMI-INFINITE, ELASTIC SOLID

An exact solution of the elasticity equations is described here in which full consideration is given to the conditions at both the boundary of the hole and the free upper surface of the solid. The results are valid, therefore, regardless of the proximity of the two boundaries. In fact, the two boundaries may be taken so close together that the solution may be applied to the study of the dead load stresses in the thin-arched structure included between them. A simple method for treating this case is given in Appendix I. Finally, more detailed consideration is given to the assumed state of initial stress in the solid, which need not be the particular state assumed by Yamaguti.

NOTATION

The symbols used in this paper are defined where they first appear and are assembled alphabetically, for reference, in Appendix II.

STATEMENT OF THE PROBLEM

The tunnel is considered to be a horizontal cylindrical cavity of circular cross-section with its axis parallel to the *z*-axis of a rectangular co-ordinate system *x*, *y*, *z*. The surrounding material has, as its upper surface, the plane $y = 0$, but is otherwise unbounded. Technically such a body is known as a semi-infinite solid. The *y*-axis penetrates, in its positive direction, vertically downward and bisects the cross-section of the tunnel (see Fig. 1). The semi-infinite solid has a uniform weight per unit of volume, w, so that the problem involves a constant body force, w, per unit of volume, acting in the positive y direction.

---

[3] "On the Stresses Around a Horizontal Circular Hole in a Gravitating Elastic Solid," by N. Yamaguti, *Journal* of the Civil Engineering Society of Japan, Vol. 15 (1929), pp. 291–303.

[4] "Statische Probleme des Tunnel- und Druckstollenbaues," by H. Schmidt, 1926.

[5] "Theory of Elasticity," by S. Timoshenko, New York, 1934, p. 80.

The length of the tunnel is assumed to be large in comparison with its diameter. This factor and the uniformity of the body force permit a treatment of the problem as one of plane strain.[6]

One method of studying problems in plain strain involves expressing the components of stress in terms of a body-force potential, Ω, and derivatives of a stress function χ. In rectangular co-ordinates these expressions take the form[7]

$$\sigma_x = \frac{\partial^2 \chi}{\partial y^2} + \Omega \dots\dots\dots\dots (1a)$$

$$\sigma_y = \frac{\partial^2 \chi}{\partial x^2} + \Omega \dots\dots\dots\dots (1b)$$

and,

$$\tau_{xy} = - \frac{\partial^2 \chi}{\partial x \, \partial y} \dots\dots\dots\dots (1c)$$

The components (X, Y) of the body force per unit of volume are derived from the body-force potential by

$$X = - \frac{\partial \Omega}{\partial x} \dots\dots (2a); \qquad \text{and,} \qquad Y = - \frac{\partial \Omega}{\partial y} \dots\dots (2b)$$

The stress function χ is a function of x and y only and is governed by the fourth-order differential equation

$$\nabla^4 \chi = - \frac{1 - 2\nu}{1 - \nu} \nabla^2 \Omega \dots\dots\dots\dots (3)$$

in which ν is Poisson's ratio and $\nabla^4 = \nabla^2\nabla^2$ where ∇^2 is Laplace's operator $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$.

In the present case, $X = 0$ and $Y = w$ so that, from Equations (2), $\Omega = - w\,y$; whence $\nabla^2\Omega = 0$ and Equation (3) becomes

$$\nabla^4 \chi = 0 \dots\dots\dots\dots (4)$$

The problem, then, is that of finding a solution of Equation (4) that satisfies a particular set of boundary conditions. The straight boundary and the circular boundary, in this case, are unloaded, and, therefore, there will be four boundary conditions to the effect that the normal stress and the shearing stress on each of these boundaries is zero.

On a horizontal plane, at a great distance below the tunnel, there will be zero shearing stress and a uniform normal stress equal to $- w\,y$. On vertical planes normal to the x-axis and situated at a great distance from the tunnel, the distribution of stress will depend upon the state of stress assumed to have existed in the semi-infinite solid before the boring of the tunnel. Three initial states will be assumed.

---

[6] "A Treatise on Photo-Elasticity," by E. G. Coker and L. N. G. Filon, Cambridge, University Press, 1931, p. 125.

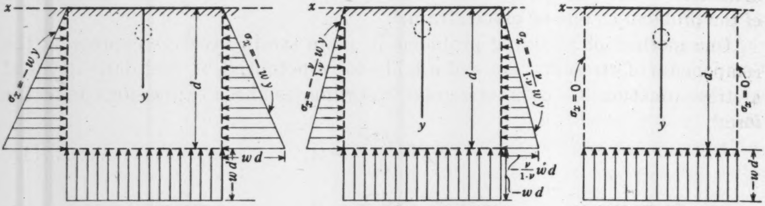
[7] *Loc. cit.*, p. 128.

Case I.—It may be assumed, for example, that each element of the solid is initially under a state of isotropic pressure. The stress in the earth at great depths is probably in this state, approximately. Under this assumption the lateral boundary condition will be a linear distribution of normal pressure $\sigma_x = - w\,y$ as shown in Fig. 2(*a*). Such a stress distribution in the semi-infinite solid, without the hole, may be expressed by means of the body-force potential

$$\Omega = - w\,y \quad (5a)$$

and the stress function

$$\chi_1 = 0 \quad (5b)$$



(*a*) CASE I, HYDROSTATIC PRESSURE

(*b*) CASE II, NO LATERAL DEFORMATION

(*c*) CASE III, NO LATERAL RESTRAINT

FIG. 2.—ASSUMED BOUNDARY CONDITIONS FOR THE SEMI-INFINITE SOLID

Case II.—It might be assumed, however, that initially (long before the boring of the tunnel) the material was restrained from lateral displacement during a supposed application of the gravitational field. In this case it may be shown that the lateral pressure is

$$\sigma_x = - \frac{\nu}{1 - \nu}\, w\,y$$

and that the initial state of stress may be expressed by

$$\Omega = - w\,y \quad (6a)$$

and

$$\chi_2 = \frac{1 - 2\,\nu}{6(1 - \nu)}\, w\,y^3 \quad (6b)$$

This is the state of initial stress (shown in Fig. 2(*b*)) which was assumed by Yamaguti.

Case III.—Another assumption which might be made is that there is no lateral stress at all (Fig. 2(*c*)) in which case the initial stress system may be represented by

$$\Omega = - w\,y \quad (7a)$$

and

$$\chi_3 = \tfrac{1}{6}\, w\,y^3 \quad (7b)$$

These three assumptions include a great range of possibilities for the initial state of stress.

CURVILINEAR CO-ORDINATES

Returning to a consideration of the four boundary conditions on the straight and circular boundaries, it is known that the mathematical form will be greatly simplified if the analysis is framed in a system of curvilinear co-ordinates which is particularly suited to the boundaries. The curvilinear co-ordinate system (α, β) which will be used is obtained through a conformal transformation of the type[8]

$$x + i\,y = f(\alpha + i\,\beta) \dots\dots\dots\dots (8)$$

in which $i = \sqrt{-1}$. Important parts will be taken in the solution by the stretch ratio, J, of the transformation and the angle ϕ which the tangent to the curve $\beta = constant$ makes with the x-axis. It is known[9] that these functions are given by

$$J\,e^{i\phi} = f'(\alpha + i\,\beta) \dots\dots\dots\dots (9)$$

in which e = base of Naperian logarithms. The expressions for the radii of curvature of the co-ordinate curves will be used also. If ρ_1 and ρ_2 are the radii of curvature of $\beta = constant$ and $\alpha = constant$, respectively (both positive for counterclockwise rotation of the tangent moving in the positive direction along the curve), then[9]

$$\frac{1}{\rho_1} = \frac{1}{J}\frac{\partial\phi}{\partial\alpha} = -\frac{1}{J^2}\frac{\partial J}{\partial\beta} \dots\dots\dots\dots (10a)$$

and

$$\frac{1}{\rho_2} = \frac{1}{J}\frac{\partial\phi}{\partial\beta} = \frac{1}{J^2}\frac{\partial J}{\partial\alpha} \dots\dots\dots\dots (10b)$$

Finally, the components of stress in curvilinear co-ordinates are given by[10]

$$\sigma_\alpha = \frac{1}{J^2}\frac{\partial^2\chi}{\partial\beta^2} + \frac{1}{J^3}\left(\frac{\partial\chi}{\partial\alpha}\frac{\partial J}{\partial\alpha} - \frac{\partial\chi}{\partial\beta}\frac{\partial J}{\partial\beta}\right) + \Omega \dots\dots\dots (11a)$$

$$\sigma_\beta = \frac{1}{J^2}\frac{\partial^2\chi}{\partial\alpha^2} + \frac{1}{J^3}\left(\frac{\partial\chi}{\partial\beta}\frac{\partial J}{\partial\beta} - \frac{\partial\chi}{\partial\alpha}\frac{\partial J}{\partial\alpha}\right) + \Omega \dots\dots\dots (11b)$$

and

$$\tau_{\alpha\beta} = -\frac{1}{J^2}\frac{\partial^2\chi}{\partial\alpha\,\partial\beta} + \frac{1}{J^3}\left(\frac{\partial\chi}{\partial\alpha}\frac{\partial J}{\partial\beta} + \frac{\partial\chi}{\partial\beta}\frac{\partial J}{\partial\alpha}\right) \dots\dots\dots (11c)$$

BIPOLAR CO-ORDINATES

A co-ordinate system that is ideally suited to the present requirements is the bipolar system. It was first applied to two-dimensional elasticity by G. B. Jeffery,[11] who gave a general solution of the elasticity equations in this co-ordinate system for the case of zero body force. His method will be adapted to the present requirements after account has been taken of the effect of the action of the gravitational force.

To obtain a system of bipolar co-ordinates, Equation (8) is particularized as

$$x + i\,y = i\,a \coth \tfrac{1}{2}(\alpha + i\,\beta) \dots\dots\dots\dots (12)$$

---

[8] "A Treatise on Photo-Elasticity," by E. G. Coker and L. N. G. Filon, Cambridge, University Press, 1931, p. 152.

[9] *Loc. cit.*, p. 153.

[10] *Loc. cit.*, p. 163.

[11] "Plane Stress and Plane Strain in Bipolar Coordinates," by G. B. Jeffery, *Transactions* of the Royal Society, London, England, Series A, Vol. 221 (1920), pp. 265–293.

in which a = distance from the origin to a pole, in bipolar co-ordinates. Equating real and imaginary parts in Equation (12) there results

$$x = \frac{a \sin \beta}{\cosh \alpha - \cos \beta} \qquad (13a)$$

and

$$y = \frac{a \sinh \alpha}{\cosh \alpha - \cos \beta} \qquad (13b)$$

From Equations (9) and (12)

$$J\, e^{i\phi} = -\frac{i\,a}{2} \operatorname{csch}^2 \tfrac{1}{2}(\alpha + i\,\beta) \qquad (14)$$

whence

$$J = \frac{a}{\cosh \alpha - \cos \beta} \qquad (15a)$$

and,

$$\tan \phi = \frac{\cosh \alpha \cos \beta - 1}{\sinh \alpha \sin \beta} \qquad (15b)$$

Equation (12) may also be written in the form

$$\alpha + i\,\beta = \log \frac{x + i(y + a)}{x + i(y - a)} \qquad (16)$$

Writing $x + i(y + a) = r_1\, e^{i\theta_1}$ and $x + i(y - a) = r_2\, e^{i\theta_2}$, and equating real and imaginary parts in Equation (16), there results

$$\alpha = \log \frac{r_1}{r_2} \qquad (17a)$$

and

$$\beta = \theta_1 - \theta_2 \qquad (17b)$$

in which r_1 and r_2 are the radii to a point in the x, y-plane from the points $(0, -a)$, $(0, +a)$, respectively, and θ_1, θ_2 are the angles which these radii make with the x-axis, as shown in Fig. 3.

The curves α = *constant* are coaxial circles of radius $a \operatorname{csch} \alpha$ having the poles $(0, -a)$ and $(0, +a)$ as limiting points. The centers of these circles lie on the y-axis at distances $a \coth \alpha$ from the origin. The x-axis is the curve $\alpha = 0$. This and a circle α = *constant* (positive) represent the boundaries of the problem.

The curves β = *constant* are circular arcs passing through the poles $(0, \pm a)$; β is the angle included between the radii r_1 and r_2; it has a discontinuity of 2π on passing between the poles.

In terms of θ_1 and θ_2, the angle ϕ which the tangent to β = *constant* makes with the x-axis is given by

$$\phi = \theta_1 + \theta_2 + \frac{\pi}{2} \qquad (18)$$

From Equations (11) and (15a) the components of stress in bipolar co-ordinates are found to be

$$a\sigma_\alpha = \left\{(\cosh\alpha - \cos\beta)\frac{\partial^2}{\partial\beta^2} - \sinh\alpha\frac{\partial}{\partial\alpha} - \sin\beta\frac{\partial}{\partial\beta} + \cosh\alpha\right\}\left(\frac{\chi}{J}\right) + a\,\Omega \quad (19a)$$

$$a\sigma_\beta = \left\{(\cosh\alpha - \cos\beta)\frac{\partial^2}{\partial\alpha^2} - \sinh\alpha\frac{\partial}{\partial\alpha} - \sin\beta\frac{\partial}{\partial\beta} + \cos\beta\right\}\left(\frac{\chi}{J}\right) + a\,\Omega \quad (19b)$$

and

$$a\tau_{\alpha\beta} = -(\cosh\alpha - \cos\beta)\frac{\partial^2}{\partial\alpha\,\partial\beta}\left(\frac{\chi}{J}\right) \quad (19c)$$

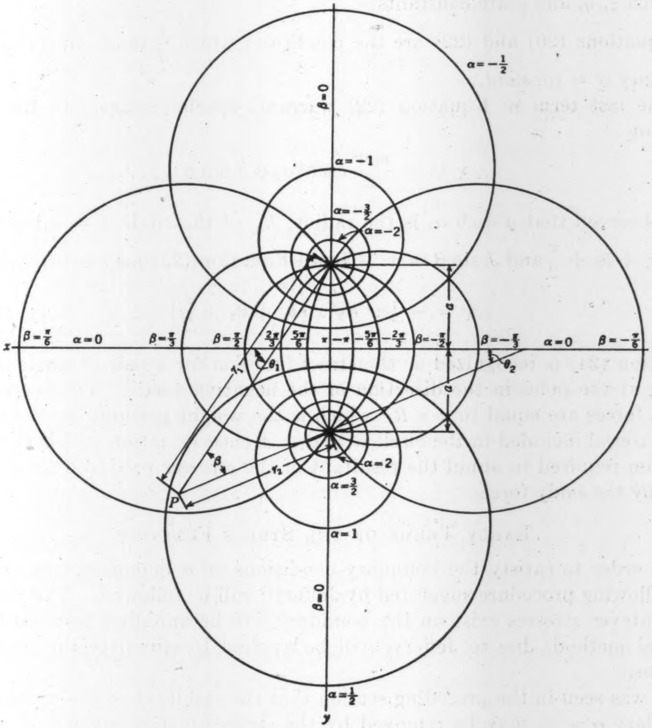

$\alpha=-\frac{1}{2}$
$\beta=0$
$\alpha=-1$
$\alpha=-\frac{3}{2}$
$\alpha=-2$
x
$\beta=\frac{\pi}{6}$
$\alpha=0$
$\beta=\frac{\pi}{3}$
$\beta=\frac{\pi}{2}$
$\frac{2\pi}{3}$
$\frac{5\pi}{6}$
π
$-\pi$
$-\frac{5\pi}{6}$
$-\frac{2\pi}{3}$
$\beta=-\frac{\pi}{2}$
$\beta=-\frac{\pi}{3}$
$\alpha=0$
$\beta=-\frac{\pi}{6}$
a
a
θ_1
θ_2
r_1
r_2
β
P
$\alpha=2$
$\alpha=\frac{3}{2}$
$\alpha=1$
$\beta=0$
$\alpha=\frac{1}{2}$
y

FIG. 3.—BIPOLAR CO-ORDINATES

BOUNDARY CONDITIONS

In each of the three cases, there are two curves, α = *constant*, which are free of stress. Since $\tau_{\alpha\beta} = 0$ on a free boundary α = *constant*, Equation (19c) requires that

$$\frac{\partial}{\partial\alpha}\left(\frac{\chi}{J}\right) = \xi \text{ (a constant)} \quad (20)$$

On the same boundary $\sigma_\alpha = 0$. Hence, noting that $-w\,y = -w\,J\sinh\alpha$: and substituting Equation (20) in Equation (19a) with $\sigma_\alpha = 0$, there results,

$$\left\{(\cosh\alpha - \cos\beta)\frac{\partial^2}{\partial\beta^2} - \sin\beta\frac{\partial}{\partial\beta} + \cosh\alpha\right\}\left(\frac{\chi}{J}\right) = w\,a\,J\sinh\alpha + \xi\sinh\alpha \quad\text{..(21)}$$

Equation (21) is an ordinary differential equation the solution of which is found by the usual methods[12] to be

$$\frac{\chi}{J} = \xi\tanh\alpha + \eta\,(\cosh\alpha\cos\beta - 1) + \zeta\sin\beta + w\,a^2\operatorname{csch}\alpha - \tfrac{1}{2}\,w\,a^2\operatorname{csch}^2\alpha\,\phi\sin\beta \quad\text{......(22)}$$

in which ξ, η, and ζ are constants.

Equations (20) and (22) are the conditions which $\frac{\chi}{J}$ must satisfy on each boundary $\alpha = constant$.

The last term in Equation (22) warrants special notice. In the stress function

$$\chi = -\frac{w\,a^2}{2}\operatorname{csch}^2\alpha_1\,\phi\,J\sin\beta \quad\text{......(23)}$$

it is observed that $a \operatorname{csch} \alpha_1$ is the radius, R_2, of the circle $\alpha = \alpha_1$; and that $\phi = \theta_1 + \theta_2 + \frac{\pi}{2}$ and $J\sin\beta = x$. Hence Equation (23) may be written as

$$\chi = -\tfrac{1}{2}\,w\,R_2{}^2\,x\left(\theta_1 + \theta_2 + \frac{\pi}{2}\right) \quad\text{......(24)}$$

Equation (24) is recognized as the stress function for a pair of single forces[13] acting at the poles in the direction of the negative y-axis. The magnitudes of the forces are equal to $w\,\pi\,R_2{}^2$ which is the weight per unit of thickness of the material included in the circle $\alpha = \alpha_1$. Hence Equation (23) is the stress function required to annul the resultant of the stresses produced on $\alpha = constant$ by the body force.

EARLY TERMS OF THE STRESS FUNCTION

In order to satisfy the boundary conditions on a boundary $\alpha = constant$, the following procedure suggested by Jeffery[11] will be followed. The resultant of whatever stresses exist on the boundary will be annulled first and then a general method, due to Jeffery, will be applied to eliminate the remaining stresses.

It was seen in the preceding section that the resultant of the stresses on a boundary $\alpha = \alpha_1$ may be removed by the stress function given in Equation (23). This function introduces many valued terms[14] in the displacements. In the present problem it is required that the displacements be single-valued in the region $\alpha > 0$. This may be accomplished by adding the stress function

$$\frac{\chi}{J} = \frac{1 - 2\nu}{4\,(1 - \nu)}\,a^2\operatorname{csch}^2\alpha_1\left[\alpha\sinh\alpha - \alpha\,(\cosh\alpha - \cos\beta)\right] \quad\text{....(25)}$$

---

[12] "Differential Equations," by H. T. H. Piaggio, London (1929), Chapter VII.

[13] "A Treatise on Photo-Elasticity," by E. G. Coker and L. N. G. Filon, Cambridge, University Press, 1931, p. 327.

[14] "A Treatise on Photo-Elasticity," by E. G. Coker and L. N. G. Filon, Cambridge, University Press, 1931, p. 326.

Equation (25) is perhaps more readily recognized when it is written in terms of r_1, r_2, θ_1, θ_2:

$$\chi = \frac{1 - 2\,\nu}{4\,(1 - \nu)}\, w\, R_2^2\, [(y - a) \log r_2 - (y - a) \log r_1] \ldots\ldots (26)$$

The term $(y - a) \log r_2$ in Equation (26) serves to eliminate the many-valued displacements in the region $\alpha > 0$ which were introduced by the term $x\,\theta_2$ in Equation (24).[13] Accordingly there should be included, in the stress function, the early terms

$$\frac{\chi_4}{J} = -\tfrac{1}{2}\, w\, a^2 \operatorname{csch}^2 \alpha_1$$

$$\times \left\{ \phi \sin \beta - \frac{1 - 2\,\nu}{2\,(1 - \nu)} [\alpha \sinh \alpha - \alpha (\cosh \alpha - \cos \beta)] \right\} \ldots . (27)$$

First Auxiliary Stress Function

The stress function χ_4 leaves only a self equilibrating system of stress on $\alpha = \alpha_1$; but, at the same time, it introduces stresses on the initially free boundary $\alpha = 0$. It is advantageous to remove the stresses introduced on $\alpha = 0$ by Equation (27) before proceeding with the application of the boundary conditions, Equations (20) and (22). In this way advantage may be taken of a great simplification of the boundary equations suggested by Jeffery.

The stress function required to annul the stresses produced on $\alpha = 0$ by Equation (27) is well known.[15] In terms of the bipolar notation it is:

$$\frac{\chi_5}{J} = \frac{w\, a^2}{8\,(1 - \nu)} \operatorname{csch}^2 \alpha_1 (\cosh 2\,\alpha - \sinh 2\,\alpha - 1) \cos \beta \ldots\ldots (28)$$

It should be observed that Equations (27) and (28), combined, represent the two-dimensional solution for a force of magnitude $w\,\pi\,R_2^2$ applied along the y-axis at Point $(0, a)$ in the interior of the semi-infinite solid $y > 0$.

Second Auxiliary Stress Function

It is now required to find a second auxiliary stress function, satisfying Equation (4), which will annul the stresses on $\alpha = \alpha_1$ introduced by $\chi_{1,2,3}$, χ_4, χ_5 and Ω, and at the same time produce no stresses on $\alpha = 0$. In curvilinear co-ordinates Equation (4) becomes[16]

$$\left(\frac{\partial^2}{\partial \alpha^2} + \frac{\partial^2}{\partial \beta^2}\right)\left\{\frac{1}{J^2}\left(\frac{\partial^2 \chi}{\partial \alpha^2} + \frac{\partial^2 \chi}{\partial \beta^2}\right)\right\} = 0 \ldots\ldots\ldots\ldots (29a)$$

Jeffery showed that, for the case of bipolar co-ordinates, Equation (29a) can be reduced to a linear equation with constant coefficients by considering $\frac{\chi}{J}$

---

[15] "Der Spannungszustand der durch eine Einzelkraft im Innern beanspruchten Halbscheibe," by E. Melan, *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 12 (1932), pp. 343–346; see also "Stress Systems in a Circular Disk under Radial Forces," by R. D. Mindlin, *Transactions*, A. S. M. E., Vol. 59 (1937), pp. A-115 to A-118; and "Force at a Point in the Interior of a Semi-Infinite Solid," by R. D. Mindlin, *Physics*, Vol. 7 (1936), pp. 195–202.

[16] "A Treatise on Photo-Elasticity," by E. G. Coker and L. N. G. Filon, Cambridge, University Press, 1931, p. 164.

instead of χ as the dependent variable. Equation (29*a*) then becomes

$$\left(\frac{\partial^4}{\partial\alpha^4} + 2\,\frac{\partial^4}{\partial\alpha^2\,\partial\beta^2} + \frac{\partial^4}{\partial\beta^4} - 2\,\frac{\partial^2}{\partial\alpha^2} + 2\,\frac{\partial^2}{\partial\beta^2} + 1\right)\left(\frac{\chi}{J}\right) = 0 \;\ldots\ldots (29b)$$

On account of the symmetry of the present problem, the solution must be even in β. Furthermore, since there is a discontinuity of $2\,\pi$ in β on passing between the poles, the solution must be periodic in β, of period $2\,\pi$. A solution of the form

$$\frac{\chi_6}{J} = f\,(\alpha)\cos n\,\beta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

will therefore be considered. Substituting Equation (30) in Equation (29*b*) there results:

$$\left(\frac{d^4}{d\alpha^4} - 2\,(n^2 - 1)\,\frac{d^2}{d\alpha^2} + n^4 - 2\,n^2 + 1\right) f\,(\alpha) = 0 \ldots\ldots (31)$$

Equation (31) is an ordinary differential equation whose solution is found to be, by the usual methods:[17a]

For $n \geqq 2$,

$$f_n\,(\alpha) = A_n \cosh\,(n+1)\,\alpha + B_n \cosh\,(n-1)\,\alpha$$
$$+ \; C_n \sinh\,(n+1)\,\alpha + D_n \sinh\,(n-1)\,\alpha \ldots\ldots\ldots\ldots (32a)$$

for $n = 1$,

$$f_1\,(\alpha) = A_1 \cosh 2\,\alpha + B_1 + C_1 \sinh 2\,\alpha + D_1\,\alpha \ldots\ldots\ldots (32b)$$

and, for $n = 0$,

$$f_0\,(\alpha) = A_0 \cosh\alpha + B_0\,\alpha\cosh\alpha + C_0 \sinh\alpha + D_0\,\alpha\sinh\alpha \ldots\ldots (32c)$$

Now, $A_0 \cosh\alpha$ and $B_1 \cos\beta$ give the same stresses; hence, A_0 may be taken equal to zero, which is equivalent to adding a constant term to the stress function. An inspection of Equations (20) and (22), with $w = 0$, reveals that χ_6 will produce no stresses on $\alpha = 0$, if

$$f\,(0) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33a)$$

and

$$f'\,(0) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33b)$$

Substituting Equations (32) in Equations (33), the following relations are obtained among the constants, which will assure the vanishing of σ_α and $\tau_{\alpha\beta}$ when $\alpha = 0$:

For $n \geqq 1$,

$$A_n + B_n = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34a)$$

for $n \geqq 2$,

$$(n+1)\,C_n + (n-1)\,D_n = 0 \ldots\ldots\ldots\ldots\ldots (34b)$$

Also

$$2\,C_1 + D_1 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34c)$$

and,

$$B_0 + C_0 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34d)$$

Jeffery has shown that the condition that the displacements be single valued requires that

$$D_0 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35a)$$

[17a] "Differential Equations," by H. T. H. Piaggio, London (1929), Chapter III.

and

$$D_1 + B_0 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35b)$$

Substituting Equations (34) and (35) in Equations (32) and writing E_n for $\frac{C_n}{(n-1)}$, the following form is found for the second auxiliary stress function:

$$\frac{\chi_6}{J} = B_0 \left[\alpha (\cosh \alpha - \cos \beta) + \sinh \alpha (\cosh \alpha \cos \beta - 1)\right]$$

$$+ A_1 (\cosh 2\alpha - 1) \cos \beta + \sum_{n=2}^{\infty} \{A_n \left[\cosh (n+1) \alpha - \cosh (n-1) \alpha\right]$$

$$+ E_n \left[(n-1) \sinh (n+1) \alpha - (n+1) \sinh (n-1) \alpha\right]\} \cos n \beta \ldots (36)$$

(The line of reasoning leading from Equations (33) to (36) is essentially that followed by Jeffery[11] in applying his general bipolar solution to the problem of a semi-infinite plate containing an unstressed circular hole, and under a uniform tension parallel to its straight edge. The process has been repeated here because Jeffery considered correctly only the first of the conditions given in Equations (34), with the result that the term $B_0 \sinh \alpha (\cosh \alpha \cos \beta - 1)$ is lacking in his Equation (66).[11] Consequently, in his application, the boundary conditions on $\alpha = 0$ are not satisfied.)

COMPLETE STRESS FUNCTION

The complete stress function (χ) is the sum of the initial stress function (χ_1, χ_2, or χ_3), the early terms (χ_4) and the two auxiliary stress functions (χ_5 and χ_6), as follows:

$$\frac{\chi}{J} = \frac{\chi_{1,2,3}}{J} - \frac{w a^2}{2} \operatorname{csch}^2 \alpha_1$$

$$\times \left\{ \phi \sin \beta - \frac{1 - 2\nu}{2(1-\nu)} \left[\alpha \sinh \alpha - \alpha (\cosh \alpha - \cos \beta)\right] \right\}$$

$$+ \frac{1}{8(1-\nu)} w a^2 \operatorname{csch}^2 \alpha_1 (\cosh 2\alpha - \sinh 2\alpha - 1) \cos \beta$$

$$+ B_0 \left[\alpha (\cosh \alpha - \cos \beta) + \sinh \alpha (\cosh \alpha \cos \beta - 1)\right]$$

$$+ A_1 (\cosh 2\alpha - 1) \cos \beta + \sum_{n=2}^{\infty} \left\{ A_n \left[\cosh (n+1) \alpha - \cosh (n-1) \alpha\right]\right.$$

$$\left. + E_n \left[(n-1) \sinh (n+1) \alpha - (n+1) \sinh (n-1) \alpha\right] \right\} \cos n \beta \ldots (37)$$

There remains only the determination of the arbitrary constants. This is accomplished by substituting Equation (37) in Equations (20) and (22) with the proper value for $\chi_{1,2,3}$ for each of Cases I, II, and III.

CASE I

In this case the first term on the right in Equation (37) is $\frac{\chi_1}{J} = 0$. The determination of the constants B_0, A_1, A_n, E_n by substitution of Equation (37) in the boundary conditions (Equations (20) and (22)) is perfectly straightforward except for one consideration. The difficulty arises from the appearance of a term $\frac{\sin^2 \beta}{(\cosh \alpha_1 - \cos \beta)}$ which results from the differentiation of the function $\phi \sin \beta$ with respect to α in Equation (20).

The values of the constants are determined by solving a set of simultaneous linear equations formed by equating coefficients of like terms of the type $\cos n\beta$. The term involving $(\cosh\alpha_1 - \cos\beta)$ in the denominator can only be adapted to this process by expanding it in infinite series as follows:

$$\frac{\sin^2\beta}{\cosh\alpha_1 - \cos\beta} = e^{-\alpha_1} + e^{-2\alpha_1} - 2\sinh\alpha_1 \sum_{n=2}^{\infty} e^{-n\alpha_1}\cos n\beta \ldots\ldots (38)$$

This is valid for $\alpha_1 > 0$. It is the appearance of this term in the first boundary condition which necessitates an infinite series solution. Were it not for this term the constants A_n and E_n would vanish and the final stress function would appear as the sum of a finite number of elementary functions.

Using Equation (38) after the substitution of Equation (37) in Equations (20) and (22), the following values for the constants result:

$$B_0 = -\tfrac{1}{2}\, w\, a^2 \operatorname{csch}^2\alpha_1 \left[2\coth\alpha_1 + \frac{1-2\nu}{2(1-\nu)}(\coth\alpha_1 - 1)\right] \ldots (39a)$$

$$A_1 = \tfrac{1}{2}\, w\, a^2 \operatorname{csch}^2\alpha_1 \ldots\ldots\ldots\ldots (39b)$$

$$A_n = -\tfrac{1}{2}\, w\, a^2 \operatorname{csch}^2\alpha_1$$

$$\times \frac{e^{-n\alpha_1}\sinh\alpha_1(\sinh n\alpha_1 \cosh\alpha_1 - n\cosh n\alpha_1 \sinh\alpha_1)}{\sinh^2 n\alpha_1 - n^2\sinh^2\alpha_1} \ldots\ldots (39c)$$

and

$$E_n = -\tfrac{1}{2}\, w\, a^2 \operatorname{csch}^2\alpha_1 \frac{e^{-n\alpha_1}\sinh^2\alpha_1 \sinh n\alpha_1}{\sinh^2 n\alpha_1 - n^2\sinh^2\alpha_1} \ldots\ldots\ldots (39d)$$

Substituting Equations (39) in Equation (37), the stress function for Case I is found to be:

$$\begin{aligned}\frac{\chi}{J} = -\tfrac{1}{2}\, w\, a^2 \operatorname{csch}^2\alpha_1 \Bigg\{ & \phi\sin\beta - \frac{1-2\nu}{2(1-\nu)}\alpha\sinh\alpha \\ & + \frac{5-6\nu}{2(1-\nu)}\coth\alpha_1(\cosh\alpha - \cos\beta)\alpha - \frac{3-4\nu}{4(1-\nu)}(\cosh 2\alpha - 1)\cos\beta \\ & + \frac{1}{2}\left(\frac{5-6\nu}{2(1-\nu)}\coth\alpha_1 - 1\right)\sinh 2\alpha\cos\beta + \sum_{n=2}^{\infty}\frac{2\sinh\alpha_1 e^{-n\alpha_1}}{\sinh^2 n\alpha_1 - n^2\sinh^2\alpha_1} \\ & \times [\sinh n\alpha_1 \sinh n\alpha \sinh(\alpha - \alpha_1) \\ & - n\sinh\alpha_1 \sinh\alpha \sinh n(\alpha - \alpha_1)]\Bigg\}\cos n\beta \ldots\ldots\ldots\ldots (40)\end{aligned}$$

The most important stresses are those in the circular boundary. Using Equations (19b) and (40):

$$\begin{aligned}[\sigma_\beta]_{\alpha=\alpha_1} = \frac{w\, a(\cosh\alpha_1 - \cos\beta)}{2\sinh^2\alpha_1}\Bigg\{ & \frac{2(1-\cosh\alpha_1\cos\beta)\sinh\alpha_1}{(\cosh\alpha_1 - \cos\beta)^2} \\ & - 4\cosh\alpha_1 - \left(\frac{5-6\nu}{1-\nu} + 2e^{-2\alpha_1}\right)\cos\beta \\ & - 4\sinh\alpha_1 \sum_{n=2}^{\infty} N_n \cos n\beta\Bigg\} \ldots\ldots\ldots\ldots (41)\end{aligned}$$

in which

$$N_n = \frac{n\, e^{-n\alpha_1} (\sinh n\, \alpha_1 \cosh n\, \alpha_1 - n \sinh \alpha_1 \cosh \alpha_1)}{\sinh^2 n\, \alpha_1 - n^2 \sinh^2 \alpha_1} \ldots\ldots (42)$$

In order to obtain numerical values for $[\sigma_\beta]_{\alpha=\alpha_1}$ it is necessary to tabulate values for the series in Equation (41); but it is found that the series converges very slowly, especially for small values of α_1. To facilitate the computations, the more slowly converging part may be separated by letting

$$N_n = n\, e^{-n\alpha_1} + R_n \ldots\ldots (43)$$

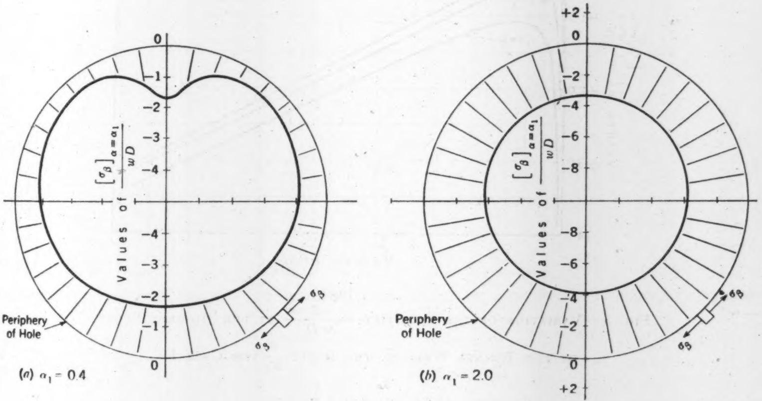
Noting that

$$2 \sum_{n=1}^{\infty} n\, e^{-n\alpha_1} \cos n\, \beta = \frac{\cosh \alpha_1 \cos \beta - 1}{(\cosh \alpha_1 - \cos \beta)^2} \ldots\ldots (44)$$

and substituting Equation (43) in Equation (41):

$$[\sigma_\beta]_{\alpha=\alpha_1} = \frac{2\, w\, a\, (\cosh \alpha_1 - \cos \beta)}{\sinh \alpha_1} \left\{ \frac{1 - \cosh \alpha_1 \cos \beta}{(\cosh \alpha_1 - \cos \beta)^2} - \coth \alpha_1 \right.$$

$$\left. - \frac{(7 - 8\, \nu)}{4\, (1 - \nu)} \frac{\cos \beta}{\sinh \alpha_1} + 2\, e^{-\alpha_1} \cos \beta - \sum_{n=2}^{\infty} R_n \cos n\, \beta \right\} \ldots\ldots (45)$$



(a) $\alpha_1 = 0.4$ (b) $\alpha_1 = 2.0$

FIG. 4.—DIAGRAM OF STRESSES $\frac{[\sigma_\beta]_{\alpha=\alpha_1}}{w D}$ AROUND THE TUNNEL

Values of R_n are given in Table 1. Values of $[\sigma_\beta]_{\alpha=\alpha_1}$ around the hole have been calculated for a wide range of values of α_1 and for three values (0, $\frac{1}{4}$, and $\frac{1}{2}$) of ν (see Table 2). In the typical curves in Fig. 4 the dimensionless ratio $\frac{[\sigma_\beta]_{\alpha=\alpha_1}}{w D}$ (in which D is the diameter of the hole) is plotted radially from the periphery of the hole. Fig. 4(*a*) gives the variation of $[\sigma_\beta]_{\alpha=\alpha_1}$ around the hole for a small value (0.4) of α_1—that is, when the hole is close to the surface; and Fig. 4(*b*) is for a larger value (2.0) of α_1. Both are for $\nu = \frac{1}{2}$.

TABLE 1.—VALUES OF R_n

R_n	α_1										
	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4
R_2	1.4663	0.5193	0.2043	0.0848	0.0364	0.0157	0.0069	0.0031	0.0014	0.0006	0.0003
R_3	0.7620	0.1852	0.0482	0.0127	0.0033	0.0034	0.0002	0.0001	....	....	....
R_4	0.3848	0.0615	0.0099	0.0016	0.0003	....	....	....	....	....	....
R_5	0.1884	0.0189	0.0018	0.002	....	....	....	....	....	....	....
R_6	0.0893	0.0054	0.0003	....	....	....	....	....	....	....	....
R_7	0.0409	0.0015	....	....	....	....	....	....	....	....	....
R_8	0.0181	0.0004	....	....	....	....	....	....	....	....	....
R_9	0.0078	0.0001	....	....	....	....	....	....	....	....	....
R_{10}	0.0033	....	....	....	....	....	....	....	....	....	....
R_{11}	0.0013	....	....	....	....	....	....	....	....	....	....
R_{12}	0.0005	....	....	....	....	....	....	....	....	....	....
R_{13}	0.0002	....	....	....	....	....	....	....	....	....	....
R_{14}	0.0001	....	....	....	....	....	....	....	....	....	....

Table 3 gives some idea of the relation between the radius (R_2) of the hole and the distance (c) of the center of the hole from the free surface for the range of α_1 for which calculations were made.

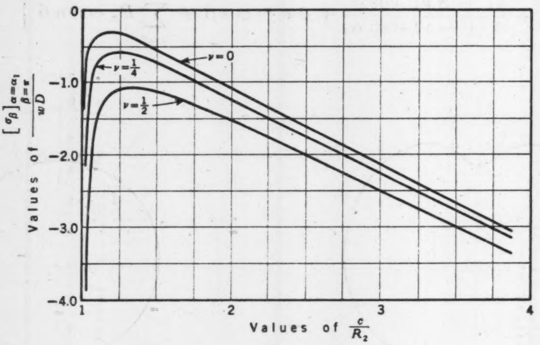


FIG. 5.—VARIATION OF THE QUANTITY $\frac{[\sigma_\beta]_{\alpha=\alpha_1, \beta=\pi}}{w D}$ AT THE HIGHEST POINT OF THE TUNNEL WALL *vs.* THE RATIO $\frac{c}{R_2}$ FOR CASE I

In Fig. 5 the quantity $\frac{[\sigma_\beta]_{\alpha=\alpha_1}}{w D}$ at the point of the hole nearest to the upper free surface is plotted against the ratio $\frac{c}{R_2}$ for three values of Poisson's ratio.

Figs. 4 and 5 and Table 2 illustrate the following significant facts for Case I:

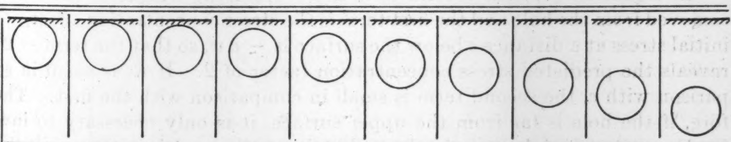
(1) The disturbing influence of the upper free boundary is not felt until $\frac{c}{R_2}$ is less than about 1.5;

(2) For $\frac{c}{R_2} > 1.5$, the stresses increase almost linearly with the depth;

TABLE 2.—VALUES OF $\frac{[\sigma_\beta]_{\alpha=\alpha_1}}{w\,D}$ (CASE I)

α_1	β									
	0	20	40	60	80	100	120	140	160	180
$\nu = 0$ (CASE I)										
0.2	−1.63	....	....	....	....	....	....	....	....	−1.35
0.4	−1.69	−1.07	−0.77	−0.71	−0.69	−0.65	−0.57	−0.49	−0.43	−0.40
0.6	−1.80	−1.42	−1.13	−0.93	−0.86	−0.76	−0.62	−0.46	−0.34	−0.29
0.8	−1.96	−1.72	−1.37	−1.20	−1.07	−0.93	−0.74	−0.59	−0.45	−0.40
1.0	−2.18	−2.01	−1.71	−1.49	−1.33	−1.17	−0.98	−0.80	−0.66	−0.61
1.2	−2.46	−2.33	−2.07	−1.84	−1.64	−1.46	−1.27	−1.08	−0.95	−0.90
1.4	−2.81	−2.71	−2.48	−2.24	−2.03	−1.82	−1.64	−1.45	−1.31	−1.26
1.6	−3.25	−3.17	−2.97	−2.73	−2.50	−2.28	−2.07	−1.89	−1.76	−1.71
1.8	−3.79	−3.72	−3.54	−3.31	−3.07	−2.84	−2.62	−2.44	−2.31	−2.27
2.0	−4.45	−4.42	−4.22	−4.00	−3.75	−3.51	−3.29	−3.11	−2.98	−2.94
$\nu = \frac{1}{4}$ (CASE I)										
0.2	−1.63	....	....	....	....	....	....	....	....	−2.18
0.4	−1.67	−1.05	−0.73	−0.66	−0.66	−0.69	−0.74	−0.78	−0.81	−0.85
0.6	−1.77	−1.39	−1.02	−0.89	−0.84	−0.79	−0.73	−0.66	−0.60	−0.58
0.8	−1.93	−1.68	−1.33	−1.16	−1.05	−0.96	−0.83	−0.74	−0.65	−0.62
1.0	−2.14	−1.97	−1.67	−1.45	−1.31	−1.19	−1.06	−0.93	−0.83	−0.79
1.2	−2.41	−2.29	−2.03	−1.80	−1.63	−1.48	−1.34	−1.19	−1.09	−1.06
1.4	−2.76	−2.66	−2.44	−2.21	−2.01	−1.84	−1.69	−1.55	−1.43	−1.39
1.6	−3.19	−3.11	−2.92	−2.69	−2.49	−2.30	−2.13	−1.98	−1.88	−1.84
1.8	−3.73	−3.66	−3.49	−3.27	−3.04	−2.85	−2.67	−2.52	−2.42	−2.38
2.0	−4.39	−4.36	−4.17	−3.96	−3.74	−3.53	−3.34	−3.19	−3.08	−3.05
$\nu = \frac{1}{3}$ (CASES I AND II)										
0.2	−1.61	....	....	....	....	....	....	....	....	−3.85
0.4	−1.64	−0.99	−0.63	−0.54	−0.59	−0.78	−1.06	−1.35	−1.58	−1.67
0.6	−1.73	−1.33	−0.93	−0.80	−0.79	−0.85	−0.95	−1.05	−1.12	−1.15
0.8	−1.86	−1.61	−1.25	−1.08	−1.01	−1.01	−1.01	−1.04	−1.05	−1.06
1.0	−2.06	−1.89	−1.58	−1.37	−1.28	−1.23	−1.20	−1.18	−1.16	−1.15
1.2	−2.32	−2.20	−1.94	−1.73	−1.60	−1.52	−1.46	−1.41	−1.38	−1.37
1.4	−2.66	−2.56	−2.35	−2.13	−1.98	−1.87	−1.81	−1.74	−1.69	−1.67
1.6	−3.08	−3.00	−2.82	−2.62	−2.46	−2.33	−2.24	−2.16	−2.11	−2.09
1.8	−3.61	−3.55	−3.39	−3.20	−3.02	−2.88	−2.77	−2.69	−2.63	−2.61
2.0	−4.26	−4.24	−4.06	−3.88	−3.71	−3.56	−3.44	−3.35	−3.29	−3.27

TABLE 3.—POSITION OF HOLE WITH RESPECT TO HORIZONTAL SURFACE FOR VALUES OF α_1 AND $\frac{c}{R_2}$



α_1	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
$\frac{c}{R_2}$	1.02	1.08	1.19	1.34	1.54	1.81	2.15	2.58	3.10	3.76

(3) For $\frac{c}{R_2} < 1.2$, the stresses increase with great rapidity as the hole approaches the upper surface of the solid;

(4) For materials of reasonable strength and density, the stresses are important only when the hole is very close to the upper surface of the solid or when it is very far from the surface;

(5) Variation of Poisson's ratio, ν, has little effect on the stresses except when the hole is very close to the upper surface; and,

(6) The stresses on the boundary of the tunnel are greatest at the lowest point of the tunnel cross-section except when α_1 is very small, in which case there are two equal maxima straddling the highest point.

In an analysis of this type, involving so much mathematical detail, it is important to determine whether or not the solution approaches, in a limiting case, a result that can be obtained directly by a simple process. When the hole is far from the upper surface, there are two factors that should be revealed in the solution: First, the stresses induced by the weight of the material removed from the hole should be very small in comparison with the stress concentration due to the initial stress in the surrounding material; and, second, by St. Venant's principle, the latter effect should be the same as for a hole in a uniform-stress field composed of the principal stresses present before the introduction of the hole. In Case I, the initial stress system is an isotropic pressure, and it is known[5] that in such a system the introduction of a hole doubles the stress.

To perform the limiting process in Equation (41), let $\alpha \rightarrow \infty$ while $a \rightarrow c$; $a \operatorname{csch} \alpha_1 \rightarrow R_2$; and $\beta \rightarrow \psi$ in which $\psi \left(= \frac{3\pi}{2} - \phi \right)$ is the angle between the radius from the center of the hole and the normal to the straight boundary, so that

$$\cos \psi = \frac{\cosh \alpha_1 \cos \beta - 1}{\cosh \alpha_1 - \cos \beta} \dots\dots\dots\dots\dots\dots (46a)$$

and

$$\sin \psi = \frac{\sinh \alpha_1 \sin \beta}{\cosh \alpha_1 - \cos \beta} \dots\dots\dots\dots\dots\dots (46b)$$

Making these substitutions in Equation (41):

$$[\sigma_\beta]_{\alpha=\alpha_1} = -2\,c\,w - R_2\,w\,\frac{3 - 4\,\nu}{2\,(1 - \nu)} \cos \psi \dots\dots\dots\dots (47)$$

In Equation (47) the second term arises from the weight of the material removed from the hole and the first term is the stress concentration effect. The initial stress at a distance c below the surface is $-\,c\,w$, so that the term $-\,2\,c\,w$ reveals the predicted stress concentration factor of 2. If R_2 is small in comparison with c, the second term is small in comparison with the first. Therefore, if the hole is far from the upper surface, it is only necessary to invoke St. Venant's principle to arrive immediately at the resulting stresses.

Cases II and III

The same general stress function, Equation (37), which was used for Case I is applicable to Cases II and III. The latter two differ from the former only in the first term of Equation (37). In Case I the first term was obtained from Equation (5*b*). In Case II, the first term is obtained from Equation (6*b*); and for Case III, Equation (7*b*) is used. Thus Cases II and III differ from Case I only by a term

$$\chi_{2,3} = G\,w\,y^3 \qquad (48)$$

in which, for Case II,

$$G = \frac{1 - 2\,\nu}{6\,(1 - \nu)} \qquad (49a)$$

and, for Case III,

$$G = \tfrac{1}{6} \qquad (49b)$$

To arrive at Cases II and III, therefore, it is only necessary to superpose on Case I the stress system arising from the initial stress represented by Equation (48). This stress system is obtained by combining the second auxiliary stress function (Equation (36)) with Equation (48) in such a manner that the stresses on the circle $\alpha = \alpha_1$ are annulled. Equation (48) can only be used with Equation (36) if it is expressed in the form $J\,f(\alpha)\cos n\,\beta$. To this end write

$$\begin{aligned}\chi_{2,3} = G\,w\,y^3 &= \frac{G\,w\,a^3\sinh^3\alpha}{(\cosh\alpha - \cos\beta)^3}\\ &= G\,w\,a^2\,J\left\{\cosh\alpha + 2\sum_{n=1}^{\infty}(\cosh\alpha + n\sinh\alpha)\,e^{-n\alpha}\cos n\,\beta\right\} \qquad (50)\end{aligned}$$

The complete stress function to be added to Case I to transform it to Cases II and III is, therefore,

$$\begin{aligned}\frac{\chi}{J} = G\,w\,a^2\Bigg\{&\cosh\alpha + 2\sum_{n=1}^{\infty}(\cosh\alpha + n\sinh\alpha)\,e^{-n\alpha}\cos n\,\beta\\ &+ B_0\,[\alpha\,(\cosh\alpha - \cos\beta) + \sinh\alpha\,(\cosh\alpha\cos\beta - 1)]\\ &+ A_1\,(\cosh 2\,\alpha - 1)\cos\beta + \sum_{n=2}^{\infty}\Big(A_n\,[\cosh\,(n+1)\,\alpha - \cosh\,(n-1)\,\alpha]\\ &+ E_n\,[(n-1)\sinh\,(n+1)\,\alpha - (n+1)\sinh\,(n-1)\,\alpha]\Big)\cos n\,\beta\Bigg\} \qquad (51)\end{aligned}$$

To determine the values of the constants, Equation (51) is substituted in Equations (20) and (22) with $w = 0$ in the latter; thus:

$$B_0 = 3\coth\alpha_1\,\mathrm{csch}^2\,\alpha_1 \qquad (52a)$$

$$A_1 = -\tfrac{3}{2}\,\mathrm{csch}^2\,\alpha_1 \qquad (52b)$$

$$A_n = -\frac{n\,(n^2 - 1)\sinh^2\alpha_1}{\sinh^2 n\,\alpha_1 - n^2\sinh^2\alpha_1} \qquad (52c)$$

and

$$E_n = \frac{n\sinh\alpha_1\,(n\sinh\alpha_1 + \cosh\alpha_1) + e^{-n\alpha_1}\sinh n\,\alpha_1}{\sinh^2 n\,\alpha_1 - n^2\sinh^2\alpha_1} \qquad (52d)$$

Again, the stresses $[\sigma_\beta]_{\alpha=\alpha_1}$ on the boundary of the hole are found by substituting Equations (52) and (51) in Equation (19*b*). The result is

$$\begin{aligned}[\sigma_\beta]_{\alpha=\alpha_1} = w\,G\,a\,(\cosh\alpha_1 - \cos\beta)\,\{&6\coth\alpha_1\,\mathrm{csch}\,\alpha_1\\ &+ 6\,\mathrm{csch}^2\,\alpha_1\cos\beta + 4\sinh\alpha_1\sum_{n=2}^{\infty}S_n\cos n\,\beta\} \qquad (53)\end{aligned}$$

in which

$$S_n = \frac{n\,(n^2 - 1)\sinh n\,\alpha_1}{\sinh^2 n\,\alpha_1 - n^2 \sinh^2 \alpha_1} \qquad (54)$$

The series in Equation (53) converges too slowly for calculation purposes. The slowly converging part may be separated out by letting

$$S_n = 2\,n\,(n^2 - 1)\,e^{-n\alpha_1} + T_n \qquad (55)$$

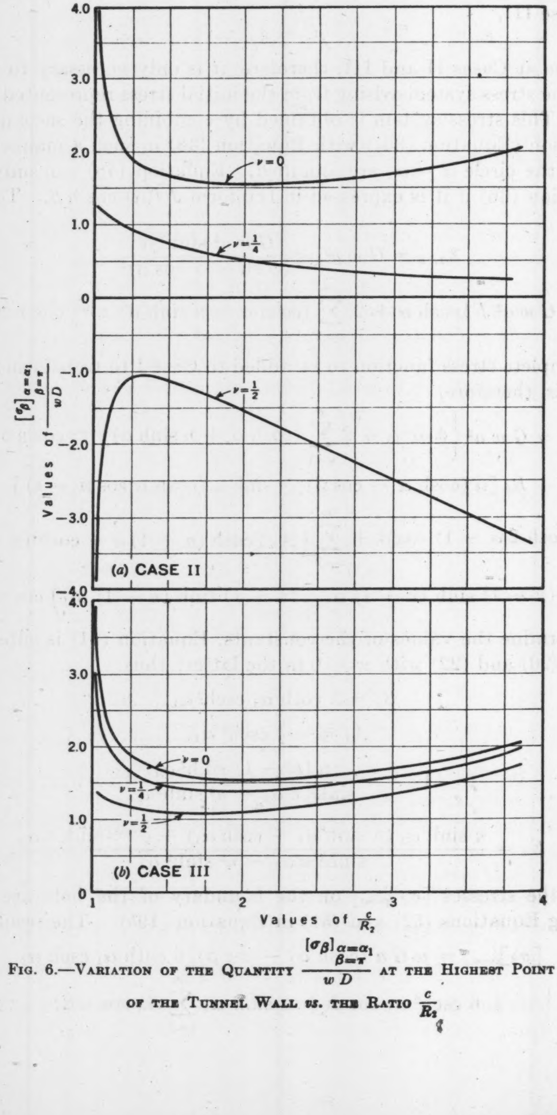


FIG. 6.—VARIATION OF THE QUANTITY $\frac{[\sigma\beta]_{\alpha=\alpha_1,\ \beta=\pi}}{w\,D}$ AT THE HIGHEST POINT OF THE TUNNEL WALL *vs.* THE RATIO $\frac{c}{R_2}$

Then, noting that

$$8\,(\cosh\alpha_1 - \cos\beta)\sum_{n=1}^{\infty} n\,(n^2 - 1)\,e^{-n\alpha_1}\cos n\,\beta = 6\,\mathrm{csch}^2\,\alpha_1$$
$$\times\,(2\cosh\alpha_1\cos 2\,\psi + \cos\psi + \cos 3\,\psi)\ldots\ldots\ldots\ldots(56)$$

and substituting Equation (55) in Equation (53):

$$[\sigma_\beta]_{\alpha=\alpha_1} = w\,G\,a\,(\cosh\alpha_1 - \cos\beta)\{6\coth\alpha_1\,\mathrm{csch}\,\alpha_1 + 6\,\mathrm{csch}^2\,\alpha_1\cos\beta$$
$$+\,4\sinh\alpha_1\sum_{n=2}^{\infty} T_n\cos n\,\beta\} + 6\,w\,G\,a\,\mathrm{csch}\,\alpha_1$$
$$\times(\cos\psi + 2\cosh\alpha_1\cos 2\,\psi + \cos 3\,\psi)\ldots\ldots\ldots\ldots(57)$$

Values of T_n are given in Table 4. Values of $[\sigma_\beta]_{\alpha=\alpha_1}$ for Case II and Case III are obtained by adding Equations (45) and (57) with G given by Equations (49*a*) and (49*b*), respectively. The results are given in Table 5.

The curves in Fig. 6 show the variation of the stress in terms of $\frac{[\sigma_\beta]_{\alpha=\alpha_1}}{w\,D}$, at the highest point of the tunnel as $\frac{c}{R_2}$ is varied. The conclusions that were drawn for Case I also hold for Cases II and III with the following additional comments:

(1) The wide variation of the stresses for the three separate cases shows the necessity of knowing the initial state of stress in the solid; and,

(2) The apparent great change in the stresses of Case II with variation of Poisson's ratio is to be attributed to variation of the state of initial stress rather than to any marked change with Poisson's ratio.

TABLE 4.—VALUES OF T_n

T_n	α_1										
	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4
T_2	41.4132	10.1670	3.3052	1.2281	0.4924	0.2047	0.0876	0.0382	0.0168	0.0074	0.0013
T_3	33.2080	6.1642	1.4073	0.3146	0.0867	0.0219	0.0055	0.0026	0.0003	0.0001	....
T_4	20.1894	3.1329	0.4583	0.0680	0.0099	0.0015	0.0001	....	....	....	....
T_5	16.2229	1.3750	0.1211	0.0103	0.0008	0.0001	....	....	....	....	....
T_6	10.1179	0.5326	0.0271	0.0012	0.0001	....	....	....	....	....	....
T_7	5.9144	0.1857	0.0053	0.0001	....	....	....	....	....	....	....
T_8	3.2620	0.0593	0.0005	....	....	....	....	....	....	....	....
T_9	1.7069	0.0176	0.0001	....	....	....	....	....	....	....	....
T_{10}	0.8523	0.0043	....	....	....	....	....	....	....	....	....
T_{11}	0.4100	0.0023	....	....	....	....	....	....	....	....	....
T_{12}	0.1882	0.0002	....	....	....	....	....	....	....	....	....
T_{13}	0.0828	....	....	....	....	....	....	....	....	....	....
T_{14}	0.0361	....	....	....	....	....	....	....	....	....	....
T_{15}	0.0140	....	....	....	....	....	....	....	....	....	....
T_{16}	0.0029	....	....	....	....	....	....	....	....	....	....
T_{17}	0.0005	....	....	....	....	....	....	....	....	....	....
T_{18}	0.0001	....	....	....	....	....	....	....	....	....	....

Once again it is desirable to check the results by applying a limiting process as in the preceding section. In this case there is available, for comparison, Yamaguti's approximate solution[3] for a hole far from the upper free surface with initial stresses corresponding to Case II.

TABLE 5.—VALUES OF $\frac{[\sigma_\beta]_{\alpha=\alpha_1}}{w\,D}$ FOR CASES II AND III

α	β									
	0	20	40	60	80	100	120	140	160	180
(*a*) CASE II AND CASE III, $\nu = 0$										
0.2	1.24	....	....	....	....	....	....	....	....	3.87
0.4	1.20	−1.62	−1.04	−1.03	−1.04	−0.53	0.73	1.89	2.16	2.05
0.6	1.25	−1.60	−1.69	−1.27	−1.00	−0.45	0.31	1.14	1.79	2.03
0.8	1.29	−2.43	−2.18	−1.68	−1.20	−0.63	0.15	0.93	1.54	1.77
1.0	1.36	−0.81	−2.53	−2.23	−1.47	−0.84	0.05	0.77	1.41	1.66
1.2	1.47	−0.48	−2.74	−2.89	−2.09	−1.07	−0.20	0.68	1.31	1.55
1.4	1.61	−0.22	−2.86	−3.53	−2.53	−1.44	−0.44	0.58	1.29	1.55
1.6	1.79	0.01	−2.96	−4.19	−3.59	−2.22	−0.75	0.48	1.31	1.60
1.8	2.04	0.24	−3.09	−4.89	−4.55	−3.00	−1.19	0.38	1.38	1.74
2.0	2.33	0.43	−3.24	−5.65	−5.65	−4.00	−1.79	0.19	1.52	1.98
(*b*) CASE II, $\nu = \frac{1}{4}$										
0.2	0.29	....	....	....	....	....	....	....	....	1.29
0.4	0.26	−1.41	−0.90	−0.87	−0.89	−0.61	0.14	0.81	0.91	0.79
0.6	0.26	−1.51	−1.39	−1.11	−0.93	−0.58	−0.11	0.41	0.82	0.97
0.8	0.24	−2.15	−1.87	−1.48	−1.14	−0.75	−0.23	0.27	0.68	0.83
1.0	0.22	−1.17	−2.22	−1.95	−1.41	−0.97	−0.44	0.12	0.56	0.72
1.2	0.21	−1.05	−2.47	−2.50	−1.93	−1.22	−0.62	−0.02	0.42	0.58
1.4	0.19	−1.00	−2.69	−3.07	−2.35	−1.59	−0.89	−0.20	0.30	0.48
1.6	0.17	−0.99	−2.92	−3.67	−3.22	−2.26	−1.25	−0.40	0.17	0.37
1.8	0.15	−1.02	−3.19	−4.33	−4.03	−2.96	−1.72	−0.64	0.04	0.29
2.0	0.13	−1.12	−3.51	−5.06	−5.00	−3.85	−2.34	−0.99	−0.08	0.23
(*c*) CASE III, $\nu = \frac{1}{4}$										
0.2	1.25	....	....	....	....	....	....	....	....	3.03
0.4	1.22	−1.60	−0.99	−0.97	−1.00	−0.57	0.57	1.61	1.78	1.61
0.6	1.27	−1.57	−1.57	−1.22	−0.97	−0.48	0.20	0.95	1.52	1.74
0.8	1.33	−2.39	−2.14	−1.64	−1.18	−0.65	0.07	0.78	1.34	1.55
1.0	1.40	−0.77	−2.49	−2.20	−1.45	−0.87	−0.13	0.64	1.25	1.48
1.2	1.52	−0.44	−2.70	−2.85	−2.08	−1.09	−0.27	0.57	1.17	1.40
1.4	1.66	−0.17	−2.81	−3.49	−2.52	−1.46	−0.50	0.48	1.16	1.41
1.6	1.85	0.07	−2.91	−4.15	−3.58	−2.23	−0.81	0.39	1.19	1.48
1.8	2.10	0.29	−3.04	−4.85	−4.53	−3.02	−1.24	0.30	1.27	1.62
2.0	2.39	0.49	−3.19	−5.61	−5.64	−4.01	−1.84	0.11	1.42	1.87
(*d*) CASE III, $\nu = \frac{1}{2}$										
0.2	1.27	....	....	....	....	....	....	....	....	1.36
0.4	1.25	−1.54	−0.90	−0.86	−0.94	−0.66	0.25	1.03	1.01	0.78
0.6	1.32	−1.51	−1.49	−1.13	−0.93	−0.54	−0.02	0.56	1.00	1.17
0.8	1.39	−2.32	−2.05	−1.56	−1.14	−0.70	−0.11	0.48	0.94	1.11
1.0	1.48	−0.69	−2.41	−2.12	−1.42	−0.91	−0.27	0.39	0.92	1.12
1.2	1.61	−0.35	−2.61	−2.78	−2.04	−1.13	−0.39	0.35	0.89	1.09
1.4	1.76	−0.07	−2.72	−3.42	−2.49	−1.50	−0.61	0.28	0.90	1.13
1.6	1.96	0.18	−2.82	−4.08	−3.55	−2.27	−0.92	0.21	0.96	1.23
1.8	2.22	0.41	−2.94	−4.78	−4.51	−3.05	−1.34	0.13	1.06	1.39
2.0	2.52	0.61	−3.08	−5.54	−5.61	−4.04	−1.94	−0.05	1.21	1.65

Note: Case II, $\nu = \frac{1}{2}$ and Case I, $\nu = \frac{1}{2}$ are identical.

Applying the limiting conditions:

$$[\sigma_\beta]_{\alpha=\alpha_1} = \frac{w}{1-\nu}\left\{-c - \tfrac{1}{2}R_2\cos\psi + 2(1-2\nu)\,c\cos 2\psi + (1-2\nu)\,R_2\cos 3\psi\right\} \quad \text{(58)}$$

In order to compare $[\sigma_\beta]_{\alpha=\alpha_1}$ in Equation (58) with Yamaguti's σ_θ (written $\overline{\theta\theta_1}$) note that his symbol θ is given by $\theta = \pi - \psi$.

When this substitution is made, the two formulas check except for an error in sign[17b] in Yamaguti's work.

APPENDIX I

Dead Load Moment and Thrust in an Arch

The results of the preceding sections can be applied to the arch-like structure formed between $\alpha = 0$ and $\alpha = \alpha_1$ for small values of α_1; but, when α_1 is small, the infinite series in the stress formulas converge very slowly. This is due to the fact that, for $\alpha_1 = 0$, the series actually diverge. Consequently, the computations are exceedingly laborious. The difficulty may be avoided if it is observed that the resultant force and couple on a section $\beta =$ *constant* connecting the two boundaries may be expressed in finite terms. In what follows, this statement will be proved and the results applied to Case I as an illustrative example.

Let s_α be an arc of the curve $\beta =$ *constant*, measured positively in the direction of increasing α. In traveling along the curve in the positive direction, the right-hand side is called the outside and the left-hand side the inside. Let $\bar{X}$ and $\bar{Y}$ be the forces per unit of thickness parallel to the x-axis and the y-axis which the material outside of s_α exerts on the material inside of s_α per unit of length of s_α. Then, if l and m are the direction cosines of the outward drawn normal to s_α referred to the x-axis and the y-axis:[18]

$$\bar{X} = l\,\sigma_x + m\,\tau_{xy} \qquad (59a)$$

and

$$\bar{Y} = l\,\tau_{xy} + m\,\sigma_y \qquad (59b)$$

Now $l = \dfrac{dy}{ds_\alpha}$; $m = -\dfrac{dx}{ds_\alpha}$ and σ_x, σ_y and τ_{xy} are given, in terms of χ and Ω, by Equations (1). Hence

$$\bar{X} = \frac{dy}{ds_\alpha}\left(\frac{\partial^2\chi}{\partial y^2} + \Omega\right) + \frac{dx}{ds_\alpha}\frac{\partial^2\chi}{\partial x\,\partial y} = \frac{d}{ds_\alpha}\left(\frac{\partial\chi}{\partial y}\right) + \Omega\frac{dy}{ds_\alpha} \qquad (60a)$$

and

$$\bar{Y} = -\frac{dy}{ds_\alpha}\frac{\partial^2\chi}{\partial x\,\partial y} - \frac{dx}{ds_\alpha}\left(\frac{\partial^2\chi}{\partial x^2} + \Omega\right) = -\frac{d}{ds_\alpha}\left(\frac{\partial\chi}{\partial x}\right) - \Omega\frac{dx}{ds_\alpha} \qquad (60b).$$

The x and y components of resultant force on the entire arc between the straight and circular boundaries are found by integrating Equations (60) along s_α, from the point of intersection of s_α with $\alpha = 0$ to the point of intersection of s_α with $\alpha = \alpha_1$. Calling these total x and y components H and V, respectively:

$$H = \int_0^{\alpha_1} \bar{X}\,ds_\alpha = \left[\frac{\partial\chi}{\partial y}\right]_0^{\alpha_1} + \int_0^{\alpha_1} \Omega\,dy \qquad (61a)$$

and,

$$V = \int_0^{\alpha_1} \bar{Y}\,ds_\alpha = -\left[\frac{\partial\chi}{\partial x}\right]_0^{\alpha_1} - \int_0^{\alpha_1} \Omega\,dx \qquad (61b)$$

[17b] The error in Yamaguti's paper first appears in his Equations (16). The second term in the second of his Equations (16) should be preceded by a plus sign instead of a minus sign. The error is carried through the remainder of the paper so that his formula for $\overline{\theta\theta}_1$ is incorrect.

[18] "A Treatise on Photo-Elasticity," by E. G. Coker and L. N. G. Filon, Cambridge University Press, 1931, p. 507.

Now $\dfrac{\partial \chi}{\partial y} = \dfrac{\partial \alpha}{\partial y}\dfrac{\partial \chi}{\partial \alpha} + \dfrac{\partial \beta}{\partial y}\dfrac{\partial \chi}{\partial \beta}$; or,

$$\frac{\partial \chi}{\partial y} = \frac{\partial \alpha}{\partial y}\left(J\frac{\partial}{\partial \alpha} + \frac{\partial J}{\partial \alpha}\right)\left(\frac{\chi}{J}\right) + \frac{\partial \beta}{\partial y}\left(J\frac{\partial}{\partial \beta} + \frac{\partial J}{\partial \beta}\right)\left(\frac{\chi}{J}\right) \ldots\ldots (62a)$$

and, $\dfrac{\partial \chi}{\partial x} = \dfrac{\partial \alpha}{\partial x}\dfrac{\partial \chi}{\partial \alpha} + \dfrac{\partial \beta}{\partial x}\dfrac{\partial \chi}{\partial \beta}$; or,

$$\frac{\partial \chi}{\partial y} = \frac{\partial \alpha}{\partial x}\left(J\frac{\partial}{\partial \alpha} + \frac{\partial J}{\partial \alpha}\right)\left(\frac{\chi}{J}\right) + \frac{\partial \beta}{\partial x}\left(J\frac{\partial}{\partial \beta} + \frac{\partial J}{\partial \beta}\right)\left(\frac{\chi}{J}\right) \ldots\ldots (62b)$$

Therefore

$$H = \left[\frac{\partial \alpha}{\partial y}\left(J\frac{\partial}{\partial \alpha} + \frac{\partial J}{\partial \alpha}\right)\left(\frac{\chi}{J}\right) + \frac{\partial \beta}{\partial y}\left(J\frac{\partial}{\partial \beta} + \frac{\partial J}{\partial \beta}\right)\left(\frac{\chi}{J}\right) + \int \Omega\, dy\right]_0^{\alpha_1} \ldots (63a)$$

and,

$$V = -\left[\frac{\partial \alpha}{\partial x}\left(J\frac{\partial}{\partial \alpha} + \frac{\partial J}{\partial \alpha}\right)\left(\frac{\chi}{J}\right) + \frac{\partial \beta}{\partial x}\left(J\frac{\partial}{\partial \beta} + \frac{\partial J}{\partial \beta}\right)\left(\frac{\chi}{J}\right) + \int \Omega\, dx\right]_0^{\alpha_1} \ldots (63b)$$

To find the total moment on the section, take moments of $\tau_{\alpha\beta}$ on β = *constant* about the center of the circle β = *constant* as the moment center. Then

$$M = \rho_1 \int_0^{\alpha_1} \tau_{\alpha\beta}\, ds_\alpha = \frac{J^2}{\left(\dfrac{\partial J}{\partial \beta}\right)_{\alpha 1}} \left[\frac{\partial}{\partial \beta}\left(\frac{\chi}{J}\right)\right]_0^{\alpha_1} \ldots\ldots\ldots\ldots (64)$$

In Equations (63) and (64) $\dfrac{\chi}{J}$ occurs only in the form of its boundary values and the boundary values of its first derivatives with respect to α and β; but, by Equations (20) and (22), these quantities are given in finite terms. Hence, by using Equations (20) and (22) in Equations (63) and (64), formulas for the force and couple on a section β = *constant*, connecting the two boundaries, may be found readily in finite terms.

Applying Equations (63) and (64) to Case I:

$$H = \frac{w\, a^2}{2}\left(\frac{1 - 2\,\nu}{2(1 - \nu)}\,\alpha_1 \operatorname{csch}^2 \alpha_1 - \coth \alpha_1\right) \ldots\ldots\ldots (65a)$$

$$V = -\frac{w\, a^2}{2}\left\{\left(\phi_1 - \frac{\pi}{2}\right)(\csc^2 \beta - \operatorname{csch}^2 \alpha_1) + \frac{2 \coth \alpha_1 \sin \beta}{\cosh \alpha_1 - \cos \beta}\right.$$
$$\left. - \frac{(1 - \cosh \alpha_1 \cos \beta)(\cosh \alpha_1 + \cos \beta)}{\sin \beta \sinh \alpha_1 (\cosh \alpha_1 - \cos \beta)}\right\} \ldots\ldots\ldots\ldots (65b)$$

and,

$$M = \frac{w\, a^3}{2} \operatorname{csch}^2 \alpha_1 \left\{\left(\phi_1 - \frac{\pi}{2}\right)\cot \beta - \frac{\sinh \alpha_1}{\cosh \alpha_1 - \cos \beta}\right.$$
$$\left. + \frac{5 - 6\,\nu}{2(1 - \nu)}(\alpha_1 \coth \alpha_1 - 1) + \sinh \alpha_1\, e^{-\alpha_1}\right\} \ldots\ldots\ldots (65c)$$

In terms of the dimensions shown in Fig. 7 these become

$$H = \frac{w}{2}\left\{\frac{1 - 2\,\nu}{2\,(1 - \nu)} R_2^2 \log \frac{c + a}{R_2} - a\, c\right\} \ldots\ldots\ldots\ldots (66a)$$

$$V = -\frac{w}{2}\{\Theta\,(R_1^2 - R_2^2) + y_1 f + x_1 c\} \ldots\ldots\ldots\ldots (66b)$$

and,

$$M = \frac{w}{2} R_2^2 \left\{ \Theta f - y_1 + \frac{5 - 6\nu}{2} \left(c \log \frac{c + a}{R_2} - a \right) + a^2 \frac{(c - a)}{R_2^2} \right\} \ldots (66c)$$

To determine the moment (M_0) at the crown of the arch it should be observed that

$$\lim_{\beta \to \pi} \Theta \cot \beta = - \frac{\sinh \alpha_1}{\cosh \alpha_1 + 1} = - \frac{y_0}{a} \ldots\ldots\ldots\ldots\ldots (67)$$

Hence

$$M_0 = \frac{w}{2} R_2^2 \left\{ - 2 y_0 + \frac{5 - 6\nu}{2} \left(c \log \frac{c + a}{R_2} - a \right) + \frac{a^2 (c - a)}{R_2^2} \right\} \ldots (68)$$

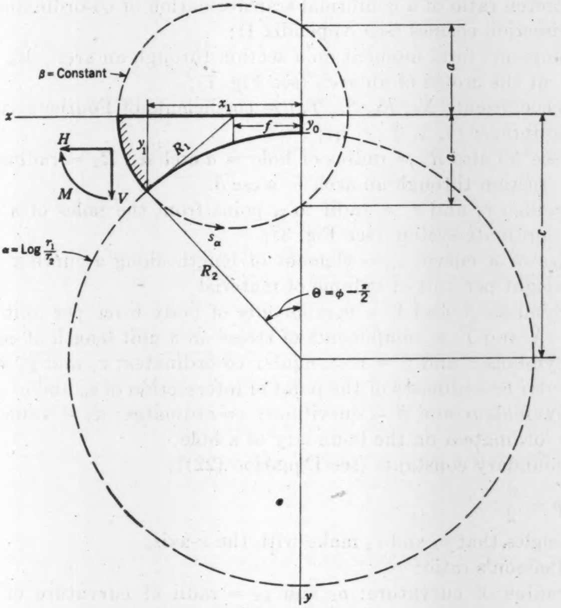


FIG. 7.—DIMENSIONS OF ARCH

APPENDIX II

NOTATION

The following notation conforms essentially with American Standard Symbols for Mechanics, Structural Engineering, and Testing Materials[19] compiled by a committee of the American Standards Association with Society representation, and approved by the Association in 1932. Where additional symbols are required, the notation used by Professor Timoshenko[20] is followed.

---

[19] ASA—Z10A—1932.

[20] "Theory of Elasticity," by S. Timoshenko, New York, 1934.

A = a constant ($A_0 \cdots A_n$; $B_0 \cdots B_n$; $C_0 \cdots C_n$; $D_0 \cdots D_n$);

a = a distance from the origin to a pole, in bipolar co-ordinates;

c = distance of the center of hole from the origin = $a \coth \alpha_1$

D = diameter of hole; (see also A)

d = depth below ground surface, to a horizontal plane (see Fig. 2);

e = Naperian base = $2.718 \cdots$;

f = $a \cot \beta$ (see Fig. 7);

G = a constant defined in Equation (49);

H, V (see Fig. 7); H and V = horizontal and vertical components of a resultant force, on a section through the arch;

$i = \sqrt{-1}$;

J = stretch ratio of a conformal transformation of co-ordinates;

l, m = direction cosines (see Appendix I);

M = moment; total moment on a section through an arch; M_0 = moment at the crown of an arch (see Fig. 7);

N = a coefficient (N_n, R_n, S_n, T_n) = coefficients of Fourier series;

n = an integer ($1, 2, 3 \cdots n$);

R (see N) also R_2 = radius of hole = $a \operatorname{csch} \alpha_1$; R_1 = radius of circular section through an arch = $a \csc \beta$.

r = radius; r_1 and r_2 = radii to a point from the poles of a bipolar coordinate system (see Fig. 3);

s = arc of a curve; s_α = element of length along a curve β = *constant;*

w = weight per unit of volume of material;

X, Y Symbols X and Y = components of body force per unit of volume; $\bar{X}$ and $\bar{Y}$ = components of stress on a unit length of curve;

x, y Symbols x and y = rectangular co-ordinates; x_1 and y_1 = rectangular co-ordinates of the point of intersection of s_α and α_1 (see Fig. 7);

α Symbols α and β = curvilinear co-ordinates; α_1 = value of the coordinate α on the boundary of a hole;

ζ, η, ξ = boundary constants (see Equation (22));

$\Theta = \phi - \frac{\pi}{2}$;

θ_1, θ_2 = angles that r_1 and r_2 make with the x-axis;

ν = Poisson's ratio;

ρ = radius of curvature; ρ_1 and ρ_2 = radii of curvature of the curves β = *constant* and α = *constant*, respectively;

σ = unit normal stress; σ_x, σ_y = components of unit normal stress in rectangular co-ordinates; σ_α, σ_β = components of unit normal stress in curvilinear co-ordinates;

τ = unit shear stress; τ_{xy} = unit shear stress in rectangular co-ordinates; $\tau_{\alpha\beta}$ = unit shear stress in curvilinear co-ordinates;

ϕ = angle between the x-axis and the tangent to the curve β = *constant;*

χ = stress function;

$\psi = \frac{3\pi}{2} - \phi$;

Ω = body-force function; and,

∇^2 = Laplace's two-dimensional operator ($\nabla^4 = \nabla^2\nabla^2$).

DISCUSSION

W. O. Richmond,[21] Esq. (by letter).—The stress distribution due to gravity in a semi-infinite elastic body pierced by a circular hole is an interesting problem and the author's treatment of it is a complete one. The application of the results in practise depend, of course, upon the accuracy with which the assumptions are fulfilled. If the shape of the tunnel is other than circular (as it often is), a mathematical solution is practically impossible and recourse must be had to experimental methods.

An interesting experimental approach to this problem was developed by Mr. S. Yamaguti,[22] whose method consisted of casting a flat plate of agar-agar in a horizontal position. A network of lines was placed on the plate, and a hole was cut, of a shape appropriately representing the tunnel profile. When the model was placed in a vertical position, the strain in the material due to its own weight could be measured from the distortion of the originally parallel lines, and hence the stresses could be determined. For a round tunnel profile Mr. Yamaguti obtained good agreement between his mathematical solution and the results of his model test.

Mr. Yamaguti's method of test reproduces the author's Case II with the appropriate value of Poisson's ratio for agar-agar. The author's Case I could also be reproduced by this method. It would only be necessary to cast the agar-agar plate in a vertical position, so that the stress distribution in the unpierced plate would approximate a hydrostatic pressure.

A second experimental approach to the problem is by the method of photo-elasticity. If the tunnel profile is some distance below the surface, the stress distribution may be obtained approximately by the method of superposition, as stated by the author. Assuming that the original stress distribution is as represented by Case II, for instance, the boundary stress around the profile is just determined for a uniform compression in a vertical direction of magnitude.

$$\sigma_y = - w\,c \qquad \text{(69)}$$

Then the boundary stress is determined, for a stress at right angles, of magnitude

$$\sigma_x = - \frac{\nu}{1-2\,\nu}\,w\,c \qquad \text{(70)}$$

in which the symbols used are those of the author in Fig. 2, and c is the distance of the hole from the free surface. The stress on the boundary is then obtained by adding the two boundary stress distributions. For a non-circular profile the boundary stresses are most easily determined by photo-elastic tests.

This procedure is sufficiently accurate if the profile is some distance below the surface. For the case shown in Fig. 4(*b*) (that is, for a value of Poisson's ratio of ½ and $\alpha_1 = 2.0$), this approximate method gives a uniform compressive stress around the boundary of the tunnel of $- 3.76\, w\, D$, in which w = density of the material, and D = diameter of the circular profile. The maximum stress by the author's computation is $-4.26\, w\, D$. The error in the approximate method is thus slightly more than 10%, which is perhaps accurate enough for practical purposes. Thus, the conclusion may be drawn that if the tunnel is more than four radii, or two diameters from the surface, the approximate solution obtained by superposition is sufficiently accurate.

---

[21] Asst. Prof. of Mech. Eng., Univ. of British Columbia, Vancouver, Canada.

[22] "On the Investigation of the Stress Distribution in a Tunnel with Agar-agar Model Experiments," by S. Yamaguti, *Proceedings*, Third International Cong. for Applied Mechanics, Vol. II, 1930, pp. 150–154.

A few years ago the writer (working under the direction of A. Nadai at the Westinghouse Research Laboratories) made some photo-elastic tests on holes of tunnel profile at the suggestion of F. C. Carstarphen, M. Am. Soc. C. E. The profiles tested were squares with various fillet radii at the corners; the ratio of fillet radius, r, to side of square, D, had the values $\frac{r}{D} = \frac{1}{3}, \frac{1}{6}$, and $\frac{1}{12}$. The boundary stress distribution was obtained photo-elastically when the bakelite plate which contained the holes was subjected to a pure tension parallel to an axis of symmetry perpendicular to the sides of the holes. The stress concentration factors (that is, the ratio of the maximum stress on the boundary of the hole to the stress in the plate a long way from the hole) from these tests are shown in Table 6. For the sake of completeness, the case of the circular hole in a tensile stress field, for which $\frac{r}{D} = \frac{1}{2}$, is included, the results for which can be obtained by computation.[23] It is interesting to note that a minimum stress concentration factor seems to result for a value of $\frac{r}{D}$ of about $\frac{1}{6}$. The stress concentration factor for a square hole with sharp corners would be theoretically infinite.

TABLE 6.—STRESS CONCENTRATION FACTORS

Item No.	Description	RATIO OF FILLET RADIUS, r, TO THE SIDE OF THE SQUARE, D			
		$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{12}$
1	Stress concentration factor for tension or compression........	3	2.6	2.5	2.6
2	Maximum compressive stress on boundary of tunnel profile (Case II, $\nu = \frac{1}{4}$)..	2.7	2.4	2.4	2.5

The stress distribution around the boundary of tunnels of similar cross-section was then computed from the boundary stresses determined by the photo-elastic tests. The case solved by the writer was the author's Case II, in which the principal stresses before the tunnel existed were given by Equations (69) and (70). The results obtained by this procedure for a value of Poisson's ratio of $\frac{1}{4}$ are shown in Item 2, Table 6. These results should be sufficiently accurate, provided that the tunnel is at least two diameters below the surface, that the material in which the tunnel is driven behaves elastically, and that the original principal stresses are as assumed.

---

[23] "Theory of Elasticity," by S. Timoshenko, 1934, p. 75.

Jacob Feld,[24] M. Am. Soc. C. E. (by letter).—For the condition explained in this paper—namely, a circular tunnel in a uniform elastic material—Mr. Mindlin's solution is the most complete available. For use in practical cases, however, the following items must be taken into account:

I.—The internal stress in the rock in its natural condition, which stress may be relieved in the local vicinity of the tunnel by the removal of the tunnel rock.

II.—The percentage of error resulting from the use of the method and formulas in this paper when rock conditions cannot be assumed as isotropic. It is seldom that a tunnel is driven through a uniform granite, which is probably the nearest approach to the material assumed in this paper. The more usual cases are tunnels driven through laminar rock, such as shales, schists, or limestones. It is extremely doubtful whether the stress determinations resulting from the formulas in this paper can be applied to a case where a blocky limestone is encountered, or, to take a different example, where schist in thin layers, lying edgewise to the face of the tunnel, is encountered.

III.—The effect on stresses around the tunnel if temporary bracing is wedged against the exposed rock during driving, or the effect on the rock stresses as the tunnel is lined with concrete and grouted under pressure; also the effect of various grouting pressures in balancing or over-balancing rock stresses.

In taking the foregoing suggestions into consideration, it might be pertinent to consider the assumption made under the heading "Statement of the Problem" that, at the circular boundary, the normal stress and the shearing stress are equal to zero. Such an assumption is not correct when rock is encountered which requires bracing to prevent movement. It seems to the writer that in the three cases mentioned as covering the initial states of the rock, although they cover a great range of possibilities, such range is chiefly in the variation of the Poisson ratio in Case II. Case I is that of hydrostatic pressure which may be applicable at tremendous distances below the earth's surface in rocks that are under such pressure that they are practically liquid, and Case III would only apply where a tunnel is driven through blocks of rock with open vertical seams, which case could not be considered isotropic. With reference to Case II and its dependence upon the Poisson ratio of the material, it might well be emphasized that such ratio may vary for rocks, depending upon the amount of the stress. However, as stated in Mr. Mindlin's paper, such variation has little effect on the stresses themselves, except when the tunnel is unusually close to the open surface. It is for such a condition (namely, shallow tunnels) that a careful stress analysis is most necessary.

J. H. A. Brahtz,[25] Esq. (by letter).—Congratulations are due the author for his excellent presentation of a very difficult theoretical subject. It is highly gratifying to have studies of this caliber brought to the attention of the engineering profession. In the mathematical theory of elasticity, relatively few exact solutions exist that may be applied directly to engineering problems. This paper is a definite contribution to this small group of exact solutions.

---

[24] Cons. Engr., New York, N. Y.

[25] Director, Photoelastic Laboratory, U. S. Dept. of the Interior, Bureau of Reclamation, Denver, Colo.

A characteristic behavior of additions to the exact-solution group seems to be that the underlying mathematics become more abstruse and complicated with each addition. This is particularly true in the cases treated by Mr. Mindlin, but in no way detracts from the value of the paper. However, it does limit its broad use and application to a relatively small group of engineers with a highly specialized mathematical background. Apparently, the author is well aware of this limitation and has minimized the attendant difficulties by presenting, in tabular and graphical form, the results of many computations. These simplify and reduce the labor greatly in applying the theoretical results to engineering problems.

In his study of the stresses due to body forces, Mr. Mindlin has considered three possible conditions that differ only in the ratio of horizontal stress to vertical stress along the vertical boundaries. To the mathematician the choice of this ratio represents no problem; it is merely called a boundary condition and is assumed to be known. To the practicing engineer the proper choice of this ratio is of utmost importance. For lack of definite information on the proper value of this ratio, the engineer is likely to employ a value that produces the most critical state of stress. This is on the side of safety but scarcely ever on the side of economy. Although the author has left this value "floating" (as he should, since a knowledge of it comes under the science of soil mechanics or geophysics, and not mathematics), it seems important to stress the need of experimental and statistical knowledge on this phase of foundation engineering.

It would have been interesting if the author had included one or more photoelastic checks and had compared the experimental maximum shear stresses with the computed values. It is realized, however, that body forces are difficult to apply to photoelastic models and that even when M. A. Biot's[26] method of employing boundary forces is used, formal and practical difficulties arise in the application of a continuously varying normal load on the closed boundary that corresponds to the tunnel surface.

Mr. Mindlin confined the application of the bipolar theory to body force or "dead load" stresses. For structures having shapes similar to those given in the paper, other common conditions of loading exist. These include the cases in which either the straight or circular boundaries are subjected to uniformly distributed loads. Although the general form for these cases has been given elsewhere,[27] it is felt that the particular cases in which one boundary is a straight line should be included in the paper, for completeness. Loadings of this type frequently arise in engineering practice. Usually they occur simultaneously with the case treated by the author. An example of this condition would be a circular tunnel in bed-rock below a river-bed or earth embankment. Sometimes they occur as the only loading; for example, a vertical shaft near the up-stream face of a masonry dam or a pressure pipe, or conduit, near a free surface. The latter is the case of uniform normal pressure on the inner boundary.

The writer is presenting the results obtained by auxiliary studies covering the conditions of uniform normal loads along the boundaries. The first part of

---

[26] *Journal of Applied Mechanics*, June, 1935.

[27] See, for example, "A Treatise on Photo-Elasticity," by E. G. Coker and L. N. G. Filon, Cambridge, University Press, 1931, p. 313, *et seq.*

the study is purely mathematical and is based upon Jeffery's general theory for stresses in a circular plate having an eccentric hole. A uniform normal pressure is applied to the outer boundary. If the radius of this boundary is assumed to increase without limit, the segment of the outer boundary in the vicinity of the hole approaches the limiting condition of a straight line. It should be noted that, in this limiting process, the outer boundary is still loaded on all sides, and the stress formulas obtained for the limiting case must be for the condition of the infinite boundaries, as well as the straight boundary, being loaded uniformly.

Although this is an important loading condition, a more common condition is one in which the straight boundary is loaded fully, and the vertical infinite boundaries are loaded only partly. The stresses for this condition can be obtained, however, by combining the results of a mathematical solution with those of photoelastic experiments, or by combining two mathematical solutions. The two mathematical solutions to be combined are the one given by the writer in this discussion and the solution by Mr. Jeffery referred to by the author.[11] However, Mr. Mindlin has called attention to an inconsistency in Mr. Jeffery's solution, and, rather than attempt to correct this fundamental error, the writer has used a combination of experiment and theory. The experimental problem treated by the writer is the problem treated by Mr. Jeffery. A table of numerical values for stresses at critical points has been developed by S. Timoshenko.[28] For the smallest value of $\frac{d}{r}$ given in the table, the boundary stress on the straight boundary opposite the hole is approximately twice the average stress in the plate, and of opposite sign. In his experimental study the writer was unable to note any change of sign in the stress along this boundary. An inspection of the isochromatic photographs included herewith bears out this point. Finally, the stresses are given for the cases in which a uniform pressure exists only on the internal boundary.

Another interesting application of bipolar co-ordinates arises in connection with the problem of percolation into a tunnel in pervious foundation or a shaft of circular cross-section situated near the up-stream face of a masonry dam.

Uniform Normal Loads on Boundaries.—Again making use of the bipolar transformation, shown in Fig. 1, the following boundary conditions are specified:

(I) The boundary $\alpha = \alpha_1$ is subjected to a normal load $\sigma_\alpha = 1$ and is free of shear—that is, $\tau_{\alpha\beta} = 0$.

(II) The boundary $\alpha = \alpha_2$ is free of load—that is, $\sigma_\alpha = \tau_{\alpha\beta} = 0$; and α_1 and α_2 are any of the positive α-circles shown in Fig. 1.

Now consider the function (see Equation (30))—

$$\frac{x_6}{J} = F = \sum_{n=0}^{\infty} f_n(\alpha) \cos n\beta \qquad (71)$$

The first two functions ($n = 0$ and $n = 1$) are

$$F_0 = A_0 \cosh \alpha + B_0 \alpha \cosh \alpha + C_0 \sinh \alpha + D_0 \alpha \sinh \alpha \qquad (72a)$$

---

[28] "Theory of Elasticity," by S. Timoshenko, New York. 1934, p. 79.

and

$$F_1 = (A_1 \cosh 2\alpha + B_1 + C_1 \sinh 2\alpha + D_1 \alpha) \cos \beta \ldots\ldots (72b)$$

The relations, $D_0 = 0$ and $-D_1 = B_0$, have been shown by the author to exist in order that displacements may be single valued; and he has shown that A_0 may be placed equal to zero since $B_1 \cos \beta$ gives the same type of stress. Hence $F_0 = -D_1 \alpha \cosh \alpha$ and

$$F = F_0 - F_1 = (A \cosh 2\alpha + B + C \sinh 2\alpha) \cos \beta + D\alpha (\cos \beta - \cosh \alpha) \ldots\ldots (73)$$

Substitution of Equation (73) in the general stress equations (Equations (19)) gives

$$a\,\sigma_\alpha = A (\cosh 2\alpha - 2 \sinh 2\alpha \sinh \alpha \cos \beta) + B + C (\sinh 2\alpha - 2 \cosh 2\alpha \sinh \alpha \cos \beta) + D \sinh \alpha (\cosh \alpha - \cos \beta) \ldots\ldots (74a)$$

$$a\,\sigma_\beta = A [2 \cosh \alpha \cos \beta - \cosh 2\alpha (4 \cos^2 \beta - 1 - 2 \cosh \alpha \cos \beta)] + B + C [2 \sinh \alpha \cos \beta - \sinh 2\alpha (4 \cos^2 \beta - 1 - 2 \cosh \alpha \cos \beta)] - D \sinh \alpha (\cosh \alpha - \cos \beta) \ldots\ldots (74b)$$

and

$$a\,\tau_{\alpha\beta} = (2 A \sinh 2\alpha + 2 C \cosh 2\alpha + D) (\cosh \alpha - \cos \beta) \sin \beta . (74c)$$

The constants A, B, C, and D are now determined by the boundary conditions (I) and (II). Omitting mathematical details, the constants are found to be:

$$A = \frac{a}{2} \frac{\sinh (\alpha_2 + \alpha_1)}{\sinh (\alpha_2 - \alpha_1) [1 - \cosh (\alpha_2 + \alpha_1) \cosh (\alpha_2 - \alpha_1)]} \ldots (75a)$$

$$B = A \frac{\sinh (\alpha_2 - \alpha_1) - \sinh 2\alpha_2 \cosh (\alpha_2 - \alpha_1)}{\sinh (\alpha_2 + \alpha_1)} \ldots\ldots (75b)$$

$$C = -A \coth (\alpha_2 + \alpha_1) \ldots\ldots (75c)$$

and

$$D = 2A \frac{\cosh (\alpha_2 - \alpha_1)}{\sinh (\alpha_2 + \alpha_1)} \ldots\ldots (75d)$$

It has been stated that Equations (75) are applicable to any two circular boundaries, $\alpha = \alpha_1$ and $\alpha = \alpha_2$, as shown in Fig. 1. It gives the stresses for unit normal tension on the boundaries of a circular cylinder with an eccentric circular hole.[27] If the outer boundary is free of force, and if a unit compression force acts on the boundary of the hole, it is only necessary to superimpose a unit hydrostatic state of compression, $\sigma_\alpha' = \sigma_\beta' = -1$, $\tau_{\alpha\beta}' = 0$, on to the stresses of the first solution.

Application to a Circular Tunnel.—If the ground surface is assumed plane (that is, a cylinder of infinite radius), α_1 is placed equal to zero in the formulas; hence, Equations (75) become

$$A = \frac{a}{2 (1 - \cosh^2 \alpha_2)} \ldots\ldots (76a)$$

$$B = A (1 - 2 \cosh^2 \alpha_2) \ldots\ldots (76b)$$

$$C = -A \coth \alpha_2 \ldots\ldots (76c)$$

and

$$D = -2C \ldots\ldots (76d)$$

The values of the constants for this case are plotted in Fig. 8 for various ratios of $\frac{d}{r}$, in which d is the depth of the center below the ground surface and r is the radius of the tunnel. The next step is to plot the stresses for unit com-

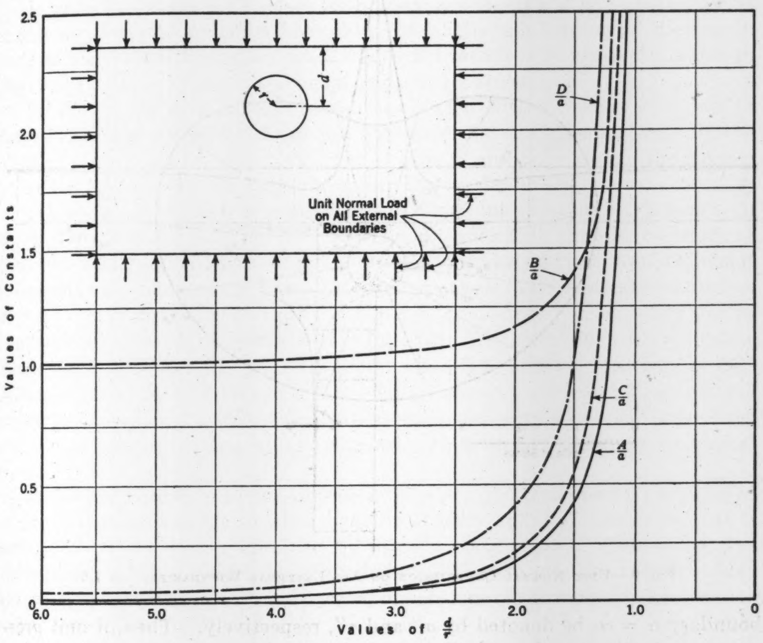


FIG. 8.—CONSTANTS A, B, C, AND D WHEN $\alpha_1 = 0$

pressive load, obtained by stress Equations (74), around the hole, along the ground surface, and along a vertical line through the center of the hole for $\frac{d}{r} = 6, 5, 4, 3, 2, 1, 1.5$ and 1.25, respectively.[29] All stresses due to this loading are compression and can be plotted in terms of the unit load. Fig. 9 illustrates the form of curves for $\frac{d}{r} = 1.5$.

As stated previously, the computations made for a unit normal load acting on the outer boundary may also be used to determine the stresses in the foundation due to a unit normal load acting on the boundary of the hole. It is obvious that, if a unit normal load is applied simultaneously on both the outer and inner boundaries, all points in the body will have unit normal stress in all directions. Such a body is said to be in a hydrostatic state of stress. Principal stresses are equal to the same quantity at all points, and all shear stresses are zero.

---

[29] *Technical Memorandum No. 597*, U. S. Bureau of Reclamation.

Let a normal stress at any point due to a unit normal load acting on the boundary $\alpha = 0$ be denoted σ' and the shear by τ'. Furthermore, let a normal stress and shear at the same point due to a unit normal load acting on the boundary $\alpha = \alpha_2$ be denoted by σ'' and τ'', respectively. Then, if unit pressures act simultaneously on both boundaries, the hydrostatic state gives:

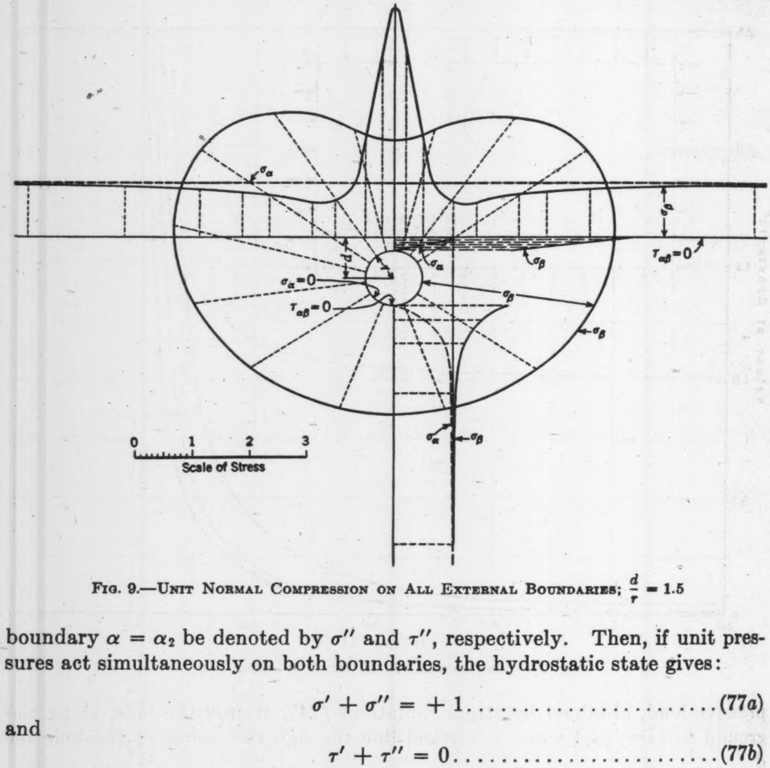

σ_α
σ_β
$\tau_{\alpha\beta}=0$
d
σ_α
σ_β
$\sigma_\alpha=0$
$\tau_{\alpha\beta}=0$
σ_β
σ_β
σ_α
σ_β
0 1 2 3
Scale of Stress

FIG. 9.—UNIT NORMAL COMPRESSION ON ALL EXTERNAL BOUNDARIES; $\frac{d}{r} = 1.5$

$$\sigma' + \sigma'' = +1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (77a)$$

and

$$\tau' + \tau'' = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (77b)$$

if compression stress is considered positive.

By solving Equations (77), the normal stress σ'' and the shears τ'' can be found in terms of σ' and τ', respectively, as follows:

$$\sigma'' = 1 - \sigma' \ldots\ldots\ldots\ldots\ldots\ldots\ldots (78a)$$

and

$$\tau'' = -\tau' \ldots\ldots\ldots\ldots\ldots\ldots\ldots (78b)$$

A negative result will then denote tension. Hence, if the normal stresses found for a unit normal load acting on $\alpha = 0$ are deducted from unity, the stresses due to a unit normal load acting on $\alpha = \alpha_2$ will be obtained. The shears found at any point of the body for a unit load acting on $\alpha = 0$ will be of equal magnitude but opposite direction for a unit load acting on $\alpha = \alpha_2$.

Equations (78) are easily solved graphically by curves such as those in Fig. 9. It will be found that the stresses σ_α'' are still compressive and attain their maximum value at the hole, diminishing asymptotically to zero away from the hole. The stresses σ_β'' become tension, maximum at the hole, and also approach zero asymptotically.

In order to obtain the stresses for a boundary loading of any magnitude, it is only necessary to multiply those obtained for the unit loading by the magnitude of the actual boundary load. In other words, the stress diagrams in Fig. 9 are influence lines in the usual engineering sense.

Finally, if the external boundaries are loaded with a normal compressive force P_0 and the perimeter of the hole with a normal compressive force, P_1, the solution is obtained by multiplying the stresses σ' and τ' by P_0 and the stresses σ'' and τ'' by P_1, and then superimposing the results.

It is to be noted that the stresses σ' and τ' are due to a unit compression on the entire external boundary $\alpha = 0$. This includes both the plane surface of the foundation and the infinite boundaries. In many practical applications it is desirable to remove all or part of the load on the infinite vertical boundaries. For this purpose the theoretical solution by Mr. Jeffery [11] (mentioned parenthetically by Mr. Mindlin in the text following Equation (36)), is applicable. However, the author has raised a doubt about the strict correctness of Mr. Jeffery's application of his general bipolar solution to this case; therefore the writer has conducted a series of photoelastic experiments which will serve both a practical purpose and as an experimental check on Mr. Jeffery's theoretical results.

Photoelastic Study.—In the photoelastic study of Mr. Jeffery's problem, of course, it was impossible to use a bakelite sheet of infinite dimensions. It is believed that the results obtained by using a plate of finite dimensions give valid results when the diameter of the hole and the distance from the free edge are small in comparison with the over-all dimensions of the model, in accordance with Saint-Venant's principle.

The model was constructed in rectangular form of bakelite, 0.5 in. thick, with vertical boundaries 2.75 in. wide by 4.62 in. long. A quarter-inch hole was drilled near one free edge of the model, such that $\frac{d}{r}$ was 6 (d is the distance from the center of the hole to the straight boundary and r is the radius of the hole). When the experiment was completed for the case $\frac{d}{r} = 6.0$, the model was reduced in width successively so that the ratio $\frac{d}{r}$ was equal to 5.0, 4.0, 3.0, 2.0, 1.0 and 1.5. The normal load on the lateral boundaries was maintained at a constant intensity of 1 000 lb per sq in.

The photoelastic interferometer [30] was used to measure the individual principal stresses along the shortest straight line connecting the straight edge of the model with the hole. The boundary stresses around the hole and along the free edge were obtained directly from the isochromatic photographs.[29]

---

[30] "Direct Optical Measurement of Individual Principal Stresses," by J. H. A. Brahtz and John Soehrens, Jun. Am. Soc. C. E., *Journal of Applied Physics*, Vol. 10, No. 4, April, 1939.

For example, the stresses along these lines for $\frac{d}{r} = 1.5$ are shown in Fig. 10, and the isochromatic photographs for $\frac{d}{r} = 1.5$ and 6.0 are shown in Fig. 11. In these the calibration of principal stresses along a free boundary is 0.173 p per fringe, in which p is the intensity of loading on the vertical boundaries and is assumed equal to unity. The probable error in the experimental stresses is ± 5 per cent. The calibration for maximum shears, of course, would be 0.0865 p. Note that the stresses along the straight boundary are compression throughout.

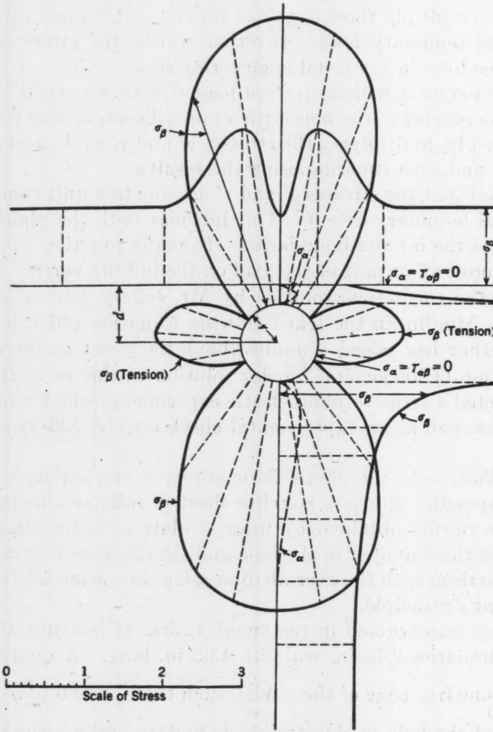
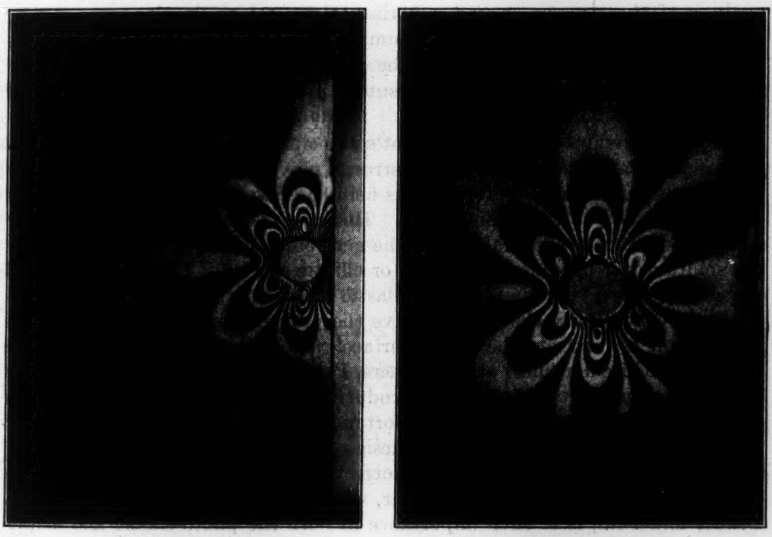


FIG. 10.—UNIT NORMAL COMPRESSION ON VERTICAL BOUNDARIES AT INFINITY; $\frac{d}{r} = 1.5$ (ALL STRESSES COMPRESSION EXCEPT AS NOTED)

The stresses plotted as in Fig. 10 for unit end loading of the vertical infinite boundaries, in connection with the stresses plotted as in Fig. 9 and combined with the author's results, permit of the complete stress determination for various boundary loading and body forces.

The theoretical results of the paper may be used with sufficient accuracy for civil engineering purposes to estimate the stress conditions around vertical shafts and inspection galleries near the faces of the dam, and pressure pipes running through the dam. These problems have been treated elsewhere[31, 32] under the assumption that the openings were far removed from the faces, whereas the present theory permits the opening to come as close to the faces as practical considerations will permit. In this application of the theory, it is customary first to compute the principal stresses at the center of the opening, as if the opening did not exist, and then apply these principal stresses as normal boundary loading far removed from the opening and compute the stresses in the vicinity of the opening by the present theory.



(a) $\frac{d}{r} = 1.5$ (b) $\frac{d}{r} = 6.0$

FIG. 11.—ISOCHROMATIC PHOTOGRAPHS; MAXIMUM SHEAR CALIBRATION 0.0865 p PER FRINGE; VERTICAL LOAD

It is hoped that the author, in his closure, will comment further on the apparent discrepancies in the theoretical and photoelastic results for a plate with an eccentric circular opening and under a uniform traction. This problem is important as it occurs in many forms in practical structures.

Finally, it should be noted that the author's bipolar transformation can be used also for the determination of the liquid pore pressures which may exist in the material surrounding the tunnel and the seepage into the tunnel. The

---

[31] "Introduction to the Stresses in the Wall Rock of Tunnels," by Frederick C. Carstarphen, M. Am. Soc. C. E., *The Mines Magazine* (Colorado School of Mines, Golden, Colo.), May, 1939.

[32] "Stresses Around Circular Holes in Dams and Buttresses," by I. K. Silverman, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 133.

theory and formulas for these effects may be found in the memorandum[29] as well as the details of the various theoretical and photoelastic studies mentioned and exemplified by the writer.

Acknowledgments.—The writer wishes to express his appreciation to the staff of the photoelastic laboratory and others of the Bureau of Reclamation, Denver, Colo., for their valuable assistance in the preparation of this discussion. Mr. John E. Soehrens, Associate Engineer, Mr. H. B. Phillips, Junior Engineer, and Mr. H. J. Kahm, Engineer Aid, performed the photoelastic experiments. Mr. C. N. Zangar, Assistant Engineer, and Mr. J. R. Bruggeman, were responsible for the mathematical analyses.

RAYMOND D. MINDLIN,[33] ASSOC. M. AM. SOC. C. E. (by letter).—An analysis of the stresses around a horizontal tunnel of circular cross-section, situated in a gravitating elastic medium, is presented in the paper. It shows how the stresses are affected by (1) the proximity of the free upper surface of the medium, (2) a certain range of assumed initial stresses in the medium, and (3) variation of Poisson's ratio.

A result predicted from St. Venant's principle was verified—that is, when the tunnel is far from the surface the stresses may be calculated by considering the tunnel situated in a uniform stress field composed of the principal stresses initially present at the tunnel level. However, even in this comparatively simple case, there are available from the mathematical theory of elasticity only the solutions for openings of circular or elliptic section. Professor Richmond has shown how a very simple photo-elastic experiment, combined with a judicious use of elasticity theory, will solve the problem for any shape as long as the opening is far from the upper surface. In his application to the square section with circular fillets at the corners, Professor Richmond finds that there is an optimum fillet radius which produces a minimum stress concentration factor. This is a result of some importance which might be applied usefully in other fields of stress analysis and design.

Professor Richmond has listed correctly the limitations inherent in his suggestion. One of these in particular, which was also pointed out by Messrs. Brahtz and Feld, is of such importance in both the photo-elastic and mathematical treatments of the tunnel problem that it warrants another repetition. It is that the stresses in the vicinity of the tunnel cannot be predicted, even approximately, without advance information as to the state of initial stress in the medium. This was stated in the original paper when comparing the widely different results obtained for Cases I, II, and III.

Mr. Feld mentions the question of the effect of Poisson's ratio on the stress distribution. As stated in the paper, the reason for the great change in the stresses of Case II when Poisson's ratio is altered is that, in this case, the boundary conditions are functions of Poisson's ratio.

The effect on the stresses of bracing or lining in the tunnel, mentioned by Mr. Feld, can be calculated by the methods of elasticity theory. The effect of grouting pressure can be estimated from Jeffery's solution for the hole under uniform pressure in the semi-infinite plate. The results obtained from this solution are indicated in the first part of the discussion by Mr. Brahtz and are treated in greater detail in the source to which Mr. Brahtz refers.[29]

---

[33] Instr., Dept. of Civ. Eng., Columbia Univ., New York, N. Y.

Mr. Brahtz reveals a very interesting point in his discussion of the problem of the semi-infinite plate with a circular hole near the straight boundary and under uniform stress parallel to the straight boundary. The point in question is whether or not there is a reversal of sign in the stress along the straight edge when the hole is close to this boundary. According to Jeffery's mathematical solution of the problem, there should be reversal of sign for $\frac{d}{r} < 2$. Mr. Brahtz made a photo-elastic experiment with $\frac{d}{r} = 1.5$ and found no reversal of sign, but he noted that the writer had called attention in his paper to an error in Jeffery's solution which prevents comparison with the experimental results.

The writer has corrected Jeffery's solution and finds that Mr. Brahtz's results are partly verified. The corrected solution indicates that there is no reversal of sign along the straight boundary. However, there are some discrepancies between the magnitudes of the stresses obtained in Mr. Brahtz's experiments and those found mathematically. These discrepancies result in percentage errors higher than the 5% estimated by Mr. Brahtz, especially in the regions of low stress. This is to be expected, since a small error in the measurement of phase retardation, while producing a negligible percentage error in regions of high stress, will produce a very large percentage error in regions of low stress.

The writer has calculated the corrected values for Jeffery's problem and has verified them photo-elastically for values of $\frac{d}{r}$ down to 1.1. The details of the results will be presented in another paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2083

THE UNIT HYDROGRAPH PRINCIPLE APPLIED TO SMALL WATER-SHEDS

By E. F. Brater,[1] Jun. Am. Soc. C. E.

With Discussion by Messrs. Franklin F. Snyder, LeRoy K. Sherman, Raphael G. Kazmann, Waldo E. Smith, and E. F. Brater.

Synopsis

The unit hydrograph principle was first presented in 1932 by LeRoy K. Sherman,[2] M. Am. Soc. C. E. Since then it has been developed into a usable method for surface run-off analysis. The most notable advance was the introduction of the distribution graph and pluviagraph in 1934 by Merrill M. Bernard,[3] M. Am. Soc. C. E.

Although the method has become quite generally accepted, the engineers and hydrologists who have made special studies in this field have suggested that further research be conducted to determine, more definitely, the limitations in the application of the unit hydrograph. One of the unsettled issues is the question of its usefulness on small streams.

It is hoped that the present investigation will provide a better understanding of the natural phenomena involved in the production of a unit hydrograph. The paper demonstrates that the unit hydrograph principle may be applied to small water-sheds, and that the distribution graphs and pluviagraphs are valuable tools for the analysis of surface run-off. The study further indicates that the unit hydrograph method is one of the best practical devices for predicting flood flows.

Introduction

The investigation described in this paper was conducted to determine whether the unit hydrograph principle is applicable to small streams and, if so, to determine its usefulness as a tool in the analysis of surface run-off.

Note.—Published in September, 1939, *Proceedings*.

[1] Instr., Civ. Eng. Dept., Univ. of Michigan, Ann Arbor, Mich. (The study was undertaken while the writer was employed by the Appalachian Forest Experiment Station.)

[2] "Stream Flow from Rainfall by the Unit Hydrograph Method," by L. K. Sherman, *Engineering News-Record*, Vol. 108, 1932, pp. 501–505.

[3] "An Approach to Determinate Stream Flow," by Merrill M. Bernard, *Proceedings*, Am. Soc. C. E., Vol. 60, January, 1934, pp. 3–18.

The study is based on continuous records of run-off and rainfall taken from twenty-two small water-sheds by the Appalachian Forest Experiment Station.[4] The water-sheds lie in the high rainfall belt of the Southern Appalachians. They vary in area from 4.24 acres to 1 876.7 acres, and in cover type, from old-growth forest to complete denudation.

The first part of the investigation was devoted to the selection of unit hydrographs and the preparation of distribution graphs on the twenty-two streams. The method of selecting the unit hydrographs from the discharge records of the water-sheds is presented, with a general consideration of the principles involved in the selection. The various factors to be considered in the separation of surface run-off from ground-water flow are discussed, and a method of procedure for making this separation on hydrographs from small streams is suggested. Distribution graphs were derived from each of the unit hydrographs, and the graphs from each stream were superimposed upon one another to permit the selection of a composite distribution graph for the stream. A detailed study was made of the rainfall that produced the unit hydrographs for the purpose of determining more definitely the conditions that govern the formation of such graphs.

The second part of the work consists of some applications of the distribution graphs for the purpose of further testing the adequacy of the theory, and to indicate the value of the unit hydrograph as a tool in the analysis of run-off phenomena. The composite distribution graphs were analyzed for the purpose of disclosing any correlations between the shapes of the distribution graphs, especially the peak percentages and the widths of bases, with the physical characteristics of the water-sheds. The distribution graphs of three of the streams were then utilized in the construction of pluviagraphs for comparison with actual hydrographs of run-off. The pluviagraph studies led logically to a consideration of surface run-off coefficients. Finally, a method is submitted for the use of the unit hydrograph principle as a practical method of predicting surface run-off.

BASIC DATA

The investigation is based on rainfall and discharge records collected continuously for periods varying from two to three years. Fifteen of the twenty-two water-sheds are forested, two are void of all vegetation, and the remainder have varying types of vegetative cover. They are situated in three separate experimental areas, the streams of each group being designated by the name of the experimental area together with their number within the area.

Locations of the Installations.—The first group of seven installations is in or near the Bent Creek Experimental Forest which is about 11 miles south of Asheville, N. C. Four of these are forested, and one is a mixture of forest and adjoining agricultural land, whereas the other two are over-grazed pasture and abandoned farm land. The location of Bent Creek and the other two experimental areas is shown in Fig. 1. Descriptive data concerning all individual water-sheds are given in Table 1, the vegetal cover in each case being as follows:

---

[4] Appalachian Forest Experiment Station, U. S. Forest Service, Department of Agriculture, Asheville, N. C. (see "Engineering Aspects of the Influence of Forests on Mountain Streams," by Richard A. Hertzler, Jun. Am. Soc. C. E., *Civil Engineering*, August, 1939, p. 487).

Stream No.	Description
Bent Creek 1	Forest; oak-chestnut type; a high percentage of old-growth timber; the forested areas were heavily settled and farmed prior to 1900; no large-scale lumbering operations have been undertaken on this area except for cutting and clearing on the individual farms. Scattered areas were burned from time to time, prior to 1926.
Bent Creek 2	Forest; two-thirds of the standing trees, including all non-commercial trees, were cut in the winter of 1930–1931; heavy skidding operations at that time greatly depleted the litter layer.
Bent Creek 3	Over-grazed pasture and abandoned old fields; sheet erosion well advanced.
Bent Creek 4	Approximately 50% forested, and 50% cultivated land and abandoned old fields.
Bent Creek 5 and 6	Forest; second growth of the oak-chestnut type.
Bent Creek 8	Abandoned old fields; eroded.
Copper Basin 1 and 2	Forest of the oak-chestnut type; this region has a relatively thin layer of soil, underlain by shales and sandstone.
Copper Basin 3 and 4	Grass-land; sheet erosion and some gullying in progress.
Copper Basin 5 and 6	No cover; top-soil completely removed and terrain deeply gullied.
Coweeta 1 to 9	The primary vegetation of all the Coweeta watersheds is second-growth forest of the oak-chestnut type, with scattered areas (less than 25% of the whole) of old-growth timber. The entire area was lumbered commercially as long ago as 1919.

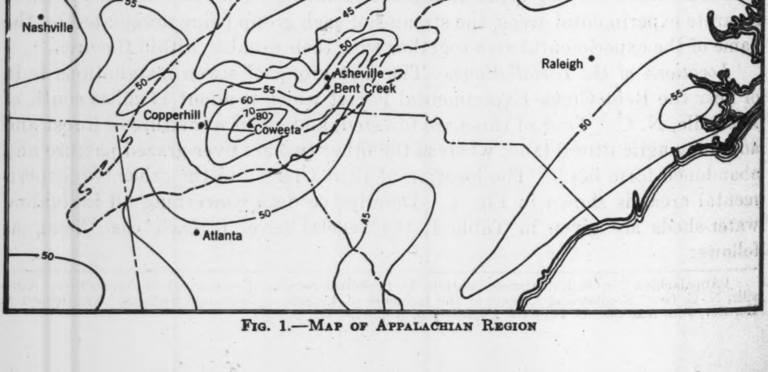

Charleston
45
40
Isohyetal Lines
Boundary of Southern Appalachian Mountains
50
50
55
Nashville
45
50
Asheville
Bent Creek
Raleigh
55
55
60
70
80
50
Copperhill
Coweeta
50
45
Atlanta
50

FIG. 1.—MAP OF APPALACHIAN REGION

TABLE 1.—DESCRIPTION OF WATER-SHEDS IN THE APPALACHIAN REGION (FIGURE 1)

Stream No.	Area, in acres	DISCHARGE, IN CUBIC FEET PER SECOND PER SQUARE MILE		ELEVATION ABOVE SEA LEVEL (ATLANTIC DATUM), IN FEET	
		Maximum	Minimum	Weir	Maximum of water-shed
	(1)	(2)	(3)	(4)	(5)
		(a) BENT CREEK WATER-SHEDS			
1	48.6	41.5	0.075	2 080*	2 450
2	17.3	84.3	0.055	2 020*	2 680
3	10.78	335.0	0.125	2 050*	2 150
4	773.95	38.2	0.189	....†	....
5	90.4	61.1	0.06	2 350*	3 100
6	71.5	68.3	0.23	2 400*	3 100
8	16.25	188.3	....	2 160*	2 220
		(b) COWEETA WATER-SHEDS			
1	38.77	41.60	0.35	2 300*	3 250
2	30.78	55.55	0.11	2 350*	3 314
4	10.18	83.03	0.29	2 750*	3 000
5	4.24	88.99	0.81	2 300*	2 500
		(b) COWEETA WATER-SHEDS (*continued*)			
6	21.88	66.28	0.25	2 300*	2 650
7	145.45	37.12	0.71	2 400*	3 491
8	1 876.7	57.76	0.75	2 300‡	5 097
9	1 787.9	67.58	0.72	2 250‡	4 887
10	212.3	36.57	0.87	2 450§	3 600
		(c) COPPER BASIN WATER-SHEDS			
1	19.0	144.4	0.005	1 632*	1 947
2	88.7	164.0	0.14	1 635*	2 492
3	6.0	832.0	0.00	1 528*	1 645
4	6.7	1 094.7	0.16	1 554*	1 698
5	15.6	1 263.2	0.04	1 695‖	1 821
6	4.7	1 433.9	0.00	1 719*	1 817

* Right-angle, V-notch. † 5-ft trapezoid. ‡ 12-ft trapezoid. § 120° V-notch. ‖ 3.5-ft trapezoid.

Coweeta Experimental Forest (see Table 1(*b*)) is about 15 miles south of Franklin, N. C., and 6 miles north of the Georgia State line. The two larger water-sheds of this group (Stream No. 8 and Stream No. 9) include the entire experimental area. All of the others lie within the boundaries of these two. The entire area is covered with second-growth forest.

The remaining six water-sheds are in the southeast corner of Tennessee, within or near the area known as the Copper Basin (see Table 1(*c*)). This region derives its name from the copper-mining and smelting activities in Copperhill and Ducktown, Tenn., about which the area centers. The Copper Basin consists of approximately 8 sq miles of completely denuded land surrounded by a zone of grass-land the area of which is 22 sq miles. The denudation was started by cutting the larger trees, but the principal damaging factor was the sulfur fumes resulting from the original open roast-heap method of smelting. The denuded area is deeply gullied and all top-soil is washed away. The grass-land has suffered from sheet erosion and is in early stages of gully formation. Two water-sheds were gaged in each of these zones. Two others are located just west of the Copper Basin in the Cherokee National Forest.

Methods of Measurement.—Stream flow was gaged by means of standard sharp-crested weirs, right-angle V-notch weirs being used on the smaller streams and trapezoidal weirs on the larger ones. Heads over the weirs were recorded continuously by float gages. Each weir was set accurately in a concrete dam, which was extended to rock bottom in all cases. Stilling basins behind all the weirs were more than adequate to satisfy requirements for proper contraction of the jet and for reducing the velocity of approach to a negligible quantity. The type of weir used in each case is indicated by footnotes in Table 1.

Rainfall was measured by means of standard Weather Bureau rain gages, with a smaller number of recording gages. There were twenty-seven standard rain gages and five recording gages on the Bent Creek water-sheds, twenty-three standard and six recording gages on the Copper Basin water-sheds, and sixty-three standard and six recording gages on the Coweeta water-sheds. Considering the total area of all the water-sheds, the average area per rain gage was thirty-seven acres. The gages were arranged to give the best possible sample, taking into account aerial distribution, elevation, and aspect. The average precipitation was computed by the Thiessen method.[5]

The Preparation of Unit Hydrographs and Distribution Graphs

The principle underlying the unit hydrograph method may be stated briefly as follows:[6] For any water-shed, the surface run-off resulting from a unit storm is distributed, according to time, in a characteristic manner, and this distribution is independent of the intensity of the rainfall.

A unit storm is a rain of an intensity sufficient to produce surface run-off which occurs within the period of rise of the hydrograph. The period of rise is the time from the beginning of surface run-off to the occurrence of peak discharge.

Definitions.—Since the terminology that is applied to this phase of hydrology is not entirely uniform, the following terms are defined according to their usage in this paper.

Surface Run-Off.—That portion of the precipitation which reaches the stream by means of overland flow.

Period of Surface Run-Off.—The time intervening between the appearance of surface run-off at the gaging station and the time when all surface run-off has passed the station. The period of surface run-off is equal to the period of overland flow plus the time required for the remaining channel storage, derived from overland flow, to pass the gaging station.

Period of Rise.—The time intervening between the beginning of surface run-off and the occurrence of the peak discharge.

Infiltration Capacity.—The maximum rate at which a given soil, when in a given condition, can absorb falling rain.[7]

Surface Run-Off Coefficient.—The percentage of the total rainfall on a water-shed that appears at the gaging station as surface run-off.

Unit Storm.—Isolated rainfall falling at an intensity greater than the infiltration capacity and having a duration equal to or less than the period of rise.

Unit Hydrograph.—A hydrograph of surface run-off resulting from a unit storm.

---

[5] *Journal*, New England Water Works Assoc., March, 1924, p. 26.

[6] Geological Survey Water-Supply Paper 772.

[7] "Surface Run-off Phenomena, Part I—Analysis of the Hydrograph," by Robert E. Horton, *Publication 101*, Horton Hydrological Lab., Voorheesville, N. Y.

Distribution Graph.—A graph having a time base equal to that of the unit hydrograph from which it was derived and having ordinates that represent the percentage of total surface run-off that occurs in certain selected equal time intervals. It provides a graphical representation of the characteristic distribution of surface run-off from a water-shed.

Selection of the Unit Hydrographs.—The first step in the preparation of the distribution graphs was the selection of unit hydrographs from the continuous run-off records. The method was to select isolated, clean-cut surface run-off graphs that occurred at a time of low flow (see Fig. 2). If insufficient graphs of this nature were available, those which were affected by small rains before or after the main one were used. In such a case the main graph was separated from its "parasite" after a careful study of the normal rising or receding sides of other surface run-off hydrographs from the same stream.

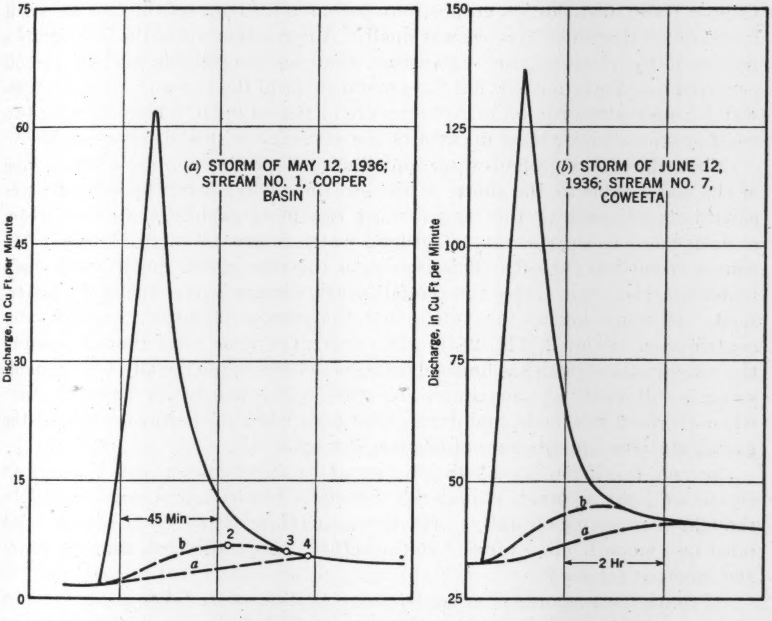


FIG. 2.—TYPICAL HYDROGRAPHS

This procedure was necessitated by a lack of knowledge of the type of rains that would produce unit hydrographs. In the light of the knowledge gained from the work, it is possible to select the unit storms from the rainfall records by selecting intense isolated rains, the duration of which was less than the period of rise indicated by the area and nature of the water-shed. The only advantage of selecting the unit storms from the rainfall records rather than from stream-flow records occurs when continuous stream-flow records in the form of hydrographs are not available.

The Separation of Surface Run-Off from Ground-Water.—The second step in the preparation of the unit hydrographs was the separation of surface run-off from ground-water contribution. Under these ideal conditions (isolated storms occurring at times of low flow) this procedure became relatively simple. The separation was made first as indicated by Base Line *a*, Fig. 2(*a*). A study of the ground-water build-up during rainfall later indicated that for most streams a base, such as Base Line *b*, was more nearly correct. In the case of Stream No. 1, Copper Basin, distribution graphs were prepared for both sets of base lines and it was found that the difference was small. An investigation of the hydrographs produced by rains of such intensities that no appreciable surface run-off occurred has also indicated that the general shape of the line separating ground-water from surface run-off is in the nature of Base Line *b*. The following two considerations lend weight in favor of the accuracy of this observation.

First, the rate of ground-water contribution depends upon the average slope of the water-table at the shores of the stream. Obviously, the ground-water elevations adjacent to the stream must rise more rapidly than the stream elevations to cause the rate of ground-water inflow to increase during the course of surface run-off. This is true in the case of the small water-sheds under consideration, where the rainfall nearly always covers the entire water-shed. It seems logical to assume that the maximum rate of ground-water contribution (Point 2, Fig. 2(*a*)) will occur at a time after the elevation of the water in the stream has finished its rapid decline, while the adjacent ground-water is still nearly at a maximum elevation. This point occurs near the time when overland flow ends, and during that time when the surface run-off at the gaging stations consists only of channel storage.

Second, it is inconceivable that the ground-water discharge could vary in an erratic or jerky manner. Similarly, the effect of channel storage cannot be thought of as ending abruptly. It follows, therefore, that the line of separation must be a smooth curve tangent to the actual hydrograph both where it leaves and where it rejoins it.

If continuous records of ground-water elevations were taken in conjunction with records of water elevation in the stream, it would be possible to determine the exact nature of this line of separation with considerable accuracy.

Although the preceding comment concerns especially the conditions on small water-sheds, the same general line of reasoning may be applied to larger water-sheds. However, in the latter case, unusual conditions may arise which require special consideration. For instance, it frequently happens that rain occurs only in the upper reaches of a water-shed and misses entirely the region where the gaging station is situated. When the stream rises at such a station, there must be a temporary reversal in the ground-water gradient and the ground-water contribution could conceivably be negative for a short interval. Even in this case, the volume of water required in most water-sheds to build up the underground water to the level of the stream would be quite small. Furthermore, a portion of the water flowing in the stream is ground-water contribution from the region in which the rain occurred, and it would be a very extreme case if this volume were not more than sufficient to supply the deficiency in the lower reaches. It can safely be assumed that, in all cases, the line of separation will have a minimum slope of zero and generally will slope upward during the period of overland flow.

On the basis of tests by the Appalachian Forest Experiment Station, thus far, it may be stated with considerable assurance that the line of separation for small water-sheds is a reverse curve of the nature of Base Line *b*, Fig. 2(*a*). The line is quite definitely fixed by the two points of tangency (Points 1 and 3) and by the maximum ordinate (Point 2).

Generally, even in the case of storms producing overlapping unit hydrographs, the point where surface run-off begins (such as Point 1, Fig. 2) is quite definite.

In the case of the hydrograph shown, the point where surface run-off ends (namely, Point 3) was selected because at this point the graph flattens out rather suddenly. Such a point is not always evident and its selection requires considerable judgment. Although this point cannot be chosen definitely, the maximum time that surface run-off continued after the rainfall ended usually may be selected with assurance, as for instance the time from the peak to Point 4, Fig. 2. This information becomes extremely important when applied to complicated hydrographs resulting from a series of rains, in which instance the selection of this point becomes much more difficult. In cases of this nature, studied by the writer, the lengths of base lines determined by different persons have varied by as much as 100 per cent. By utilizing the knowledge gained from the unit hydrograph, this difference would be more of the order of 5 per cent.

The high point (such as Point 2, Fig. 2(*a*)) probably occurs near the end of overland flow, or about half-way between the peak of the hydrograph and the end of surface run-off. Until more quantitative results are available, the height of this point must be a matter of judgment based on a study of the flow characteristics of the particular stream.

The Construction of the Distribution Graphs.—Having separated ground-water from surface run-off, the unit hydrographs are represented by the ordinates between the line of separation and the actual hydrograph. The unit hydrographs thus determined were then divided arbitrarily into time intervals, and the average rate of surface run-off was determined for each interval. These values were totaled and the percentage of the total occurring during each of the periods was computed. The resulting percentages are a numerical representation of the distribution graphs. Time intervals varying from 2.5 min to 30 min were used, the object being to represent the shape of the graph faithfully with a minimum of labor.

It should be noted that the magnitude of the percentages making up a distribution graph depends on the time interval selected. If 7.5-min intervals instead of 15-min intervals were chosen in a given case, it is evident that the percentages would be half as great. Therefore, for the purpose of comparison, all the distribution graphs were converted to a common time interval. For

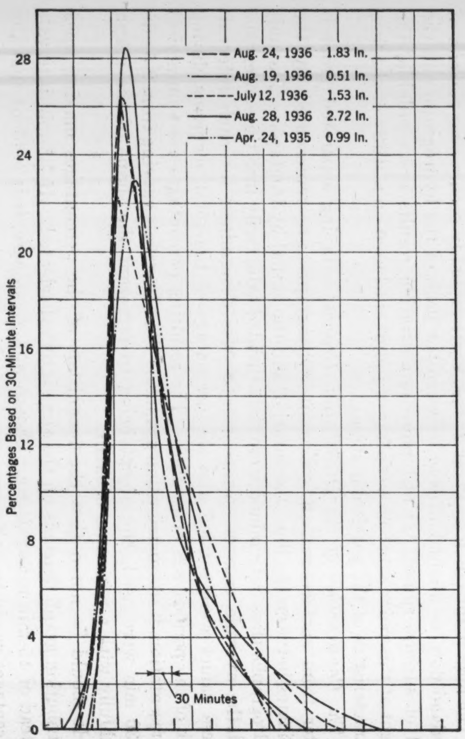


FIG. 3.—DISTRIBUTION GRAPH; STREAM NO. 9, COWEETA WATER-SHED; 1 788 ACRES; FORESTED

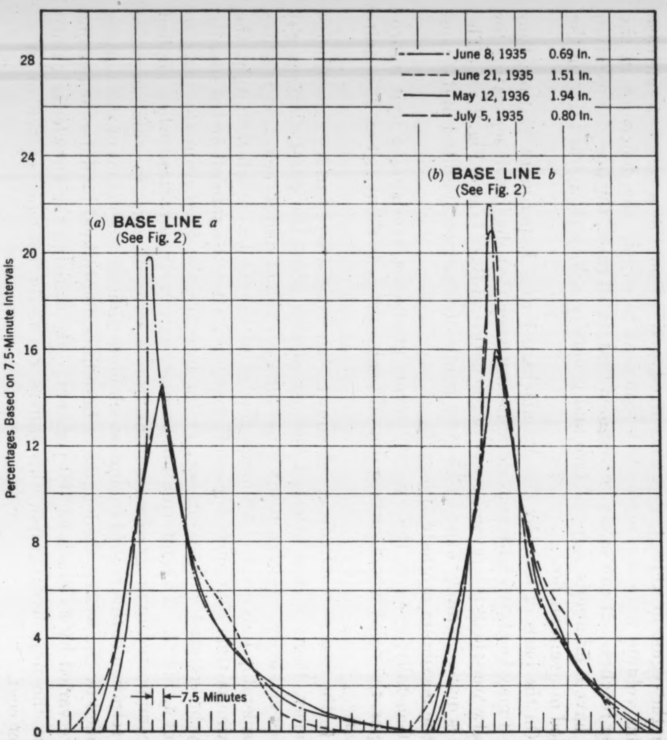


FIG. 4.—DISTRIBUTION GRAPH; STREAM NO. 1, COPPER BASIN WATER-SHED; 19.0 ACRES; FORESTED

example, percentages based on 15-min periods were divided by 6 to reduce the data to a 2.5-min basis. If these results are plotted, they must be spaced at 15-min intervals, since they represent the average 2.5-min interval for a period of 15 min.

The final step was to superimpose all the distribution graphs for each stream as nearly as possible on one another. Figs. 3 and 4 show the resulting clusters of characteristic graphs for two of the streams. A composite distri-

TABLE 2.—Data for Distribution Graphs

Interval No.	Stream Numbers:									
	1	2	3	4	5	6	7	8	9	10
(*a*) Bent Creek Water-Sheds										
T^* =	7.5	7.5	5.0	7.5	15.0	7.5	....	5.0	....	....
1	1.6	2.0	2.5	1.4	0.4	3.0	....	10.0	....	....
2	6.0	19.0	14.0	3.2	7.4	14.0	....	19.0	....	....
....	....	(52.0)†	....	....	....	(20.0)†	....	....	....	....
3	12.8	33.0	41.0	5.2	21.6	17.0	....	48.0	....	....
....	(20.4)†	....	(52.5)†	....	(34.0)†	....	....	(80.0)†	....	....
4	18.4	16.0	15.0	8.0	25.4	13.0	....	7.0	....	....
5	13.0	8.0	9.0	12.0	14.0	10.0	....	5.0	....	....
6	10.6	6.5	6.0	17.4	7.0	8.0	....	3.5	....	....
....	....	....	....	(18.7)†	....	....	....	....	....	....
7	9.6	4.5	4.5	15.2	4.6	6.0	....	2.5	....	....
8	6.8	3.5	3.0	9.0	3.6	5.0	....	2.0	....	....
9	4.4	2.5	2.0	6.2	2.8	4.0	....	1.5	....	....
10	3.6	2.0	1.5	5.4	2.8	3.6	....	1.0	....	....
11	2.8	1.5	1.0	4.5	2.4	3.1	....	0.5	....	....
12	2.4	1.0	0.5	3.7	2.0	2.7	....	....	....	....
13	2.0	0.5	....	2.8	1.6	2.4	....	....	....	....
14	1.6	....	....	2.0	1.6	2.0	....	....	....	....
15	1.3	....	....	1.4	1.2	1.6	....	....	....	....
16	1.1	....	....	1.0	0.8	1.3	....	....	....	....
17	0.8	....	....	0.7	0.4	1.0	....	....	....	....
18	0.6	....	....	0.5	0.4	0.8	....	....	....	....
19	0.4	....	....	0.3	....	0.6	....	....	....	....
20	0.2	....	....	0.1	....	0.5	....	....	....	....
21	....	....	....	....	....	0.3	....	....	....	....
22	....	....	....	....	....	0.1	....	....	....	....
(*b*) Coweeta Water-Sheds										
T^* =	15.0	15.0	....	15.0	7.5	15.0	15.0	30.0	30.0§	15.0
1	2.6	1.2	....	0.9	1.1	8.0	0.6	0.5	0.6	1.8
2	10.9	4.8	....	9.3	6.9	27.6	3.4	5.7	2.4	6.4
....	....	....	....	....	....	(42.5)†	....	....	....	....
3	21.7	12.2	....	31.6	18.5	28.7	6.4	13.4	9.0	12.2
....	(29.1)†	....	....	(46.8)†	(34.2)†	....	....	....	....	....
4	23.9	24.0	....	33.4	30.0	14.4	20.8	20.9	25.4	17.8
....	....	....	....	....	....	....	(21.5)†	(22.1)†	(28.6)†	....
5	16.1	30.4	....	10.5	16.5	9.0	19.6	17.7	23.3	20.0
....	....	(35.2)†	....	....	....	....	....	....	....	(21.0)†
6	9.6	15.7	....	6.2	8.9	5.9	14.2	13.4	14.5	15.2
7	6.4	6.8	....	4.1	6.0	3.7	10.0	9.7	9.7	11.3
8	4.3	3.1	....	2.3	4.3	2.0	6.8	7.2	5.9	7.0
9	2.7	1.4	....	1.2	2.9	0.7	4.6	4.7	3.9	3.8
10	1.3	0.4	....	0.5	1.7	....	3.5	3.2	2.6	2.5
11	0.5	....	....	....	1.1	....	2.5	2.0	1.6	1.5
12	....	....	....	....	0.9	....	2.0	1.1	0.8	0.5
13	....	....	....	....	0.6	....	1.6	0.6	0.3	....
14	....	....	....	....	0.5	....	1.2	....	....	....
15	....	....	....	....	0.2	....	1.0	....	....	....
16	....	....	....	....	....	....	0.7	....	....	....
17	....	....	....	....	....	....	0.5	....	....	....
18	....	....	....	....	....	....	0.3	....	....	....
19	....	....	....	....	....	....	0.2	....	....	....
20	....	....	....	....	....	....	0.1	....	....	....

TABLE 2.—(*Continued*)

Inter-val No.	Stream Numbers:									
	1	2	3	4	5	6	7	8	9	10
					(c) Copper Basin Water-Sheds					
*T** =	7.5‡	15.0	2.5	2.5	2.5	2.5	....	....	....	....
1	0.7	0.6	0.1	1.5	0.3	0.8	....	....	....	....
2	1.9	3.4	0.2	4.6	1.0	3.5	....	....	....	....
3	5.6	7.8	0.6	27.4	2.6	16.5	....	....	....	....
....	....	....	....	(37.5)†	....	....	....	....	....	....
4	9.4	14.0	8.0	12.9	4.1	40.0	....	....	....	....
....	....	....	....	....	....	(56.0)†	....	....	....	....
5	15.6	19.2	24.0	10.0	6.3	19.8	....	....	....	....
....	....	(20.2)†	(34.8)†	....	....	....	....	....	....	....
6	20.3	16.0	24.0	8.7	8.9	13.7	....	....	....	....
....	(21.0)†	....	....	....	....	....	....	....	....	....
7	14.0	11.6	10.1	7.4	11.75	3.8	....	....	....	....
8	8.7	8.2	7.4	6.1	15.25	1.1	....	....	....	....
9	5.7	6.2	6.2	4.8	18.6	0.5	....	....	....	....
....	....	....	....	....	(20.0)†	....	....	....	....	....
10	4.4	4.4	5.2	3.6	16.1	0.2	....	....	....	....
11	3.7	3.2	4.0	2.8	11.55	0.1	....	....	....	....
12	3.0	2.2	3.1	2.5	2.8	....	....	....	....	....
13	2.3	1.4	2.5	1.7	0.5	....	....	....	....	....
14	1.7	1.0	1.5	1.2	0.2	....	....	....	....	....
15	1.4	0.6	0.9	1.0	0.1	....	....	....	....	....
16	0.9	0.2	0.5	0.9	0.05	....	....	....	....	....
17	0.5	....	0.4	0.8	....	....	....	....	....	....
18	0.2	....	0.35	0.7	....	....	....	....	....	....
19	....	....	0.3	0.6	....	....	....	....	....	....
20	....	....	0.25	0.4	....	....	....	....	....	....
21	....	....	0.2	0.2	....	....	....	....	....	....
22	....	....	0.1	....	....	....	....	....	....	....
23	....	....	0.05	....	....	....	....	....	....	....

* *T* = time interval, in minutes. † Peak percentages are shown in parentheses.
‡ See Fig. 4. § See Fig. 3.

bution graph was then chosen for each case. This was done either by selecting one of the individual graphs as representing an average, or by drawing an average graph through the cluster and taking the percentages from it. The latter process involves a trial-and-error procedure in order to produce a total of 100 per cent. Numerical values of the composite distribution graphs are listed in Table 2.

Characteristics of the Distribution Graphs.—An examination of the twenty-two sets of distribution graphs similar to Figs. 3 and 4 showed that there was a close agreement among the graphs in each group. Furthermore, for any stream, the shapes of the graphs were singularly individualistic. The adherence to the characteristic shape for each of the streams appears to be much closer than in the case of streams draining larger water-sheds.

The rainfall varied considerably within the individual clusters of graphs. For example (see Table 3), unit hydrographs selected for Coweeta 9 were produced by rains varying from 0.51 in. to 2.72 in., those of Coweeta 5 by rains varying from 0.35 in. to 1.69 in., and those of Bent Creek 1 varied from 0.51 in. to 1.44 in.

The actual peaks of the unit hydrographs also varied widely within the individual cluster. For instance, the peak discharges varied from 9 to 95 cu ft per sec in the unit hydrographs used for Coweeta 9 and, in the case of Bent Creek 3, the peak discharge varied from 1.9 to 13.0 cu ft per sec.

The duration of each of the rains used in preparing the distribution graphs and the period of rise for each stream are given in Table 3. The period of rise corresponds approximately to the time of concentration in the so-called rational formula. The latter is usually defined as the time required after the beginning of rainfall for run-off to reach the point of concentration from the most distant point on the water-shed. These studies indicate that a somewhat different conception must be given to this time interval.

A careful study of Table 3 in conjunction with the corresponding distribution graphs will show that the peak discharge appears at approximately the same time after surface run-off starts, regardless of the duration or magnitude of the rainfall. Nearly every set of graphs provides illustrations of this. For

TABLE 3.—DESCRIPTION OF THE RAINS THAT PRODUCED DISTRIBUTION GRAPHS SIMILAR TO FIGURES 3 AND 4

Date of storm* (1)	Total rainfall, in inches (2)	Duration of rainfall, in minutes (3)	Date of storm* (1)	Total rainfall, in inches (2)	Duration of rainfall, in minutes (3)
(*a*) BENT CREEK WATER-SHED					
Stream No. 1; $P = 22$ min:			Stream No. 4; $P = 45$ min:		
August 22, 1935	1.22	....	August 29, 1936	0.71	25
July 1, 1936	0.75	20	September 20, 1936	0.44	15
July 2, 1936	1.44	30	September 29, 1936	0.69	30
July 20, 1936	0.51	20			
September 20, 1936	0.95	20	Stream No. 5; $P = 30$ min:		
			July 2, 1936	2.04	30
Stream No. 2; $P = 15$ min:			September 20, 1936	1.02	70
July 20, 1936	0.50	20			
August 6, 1936	0.58	20	Stream No. 6; $P = 12$ min:		
August 27, 1936	0.81	20	August 23, 1935	1.38	50
September 20, 1936	0.37	15	July 2, 1936	2.22	30
September 29, 1936	0.66	30	September 20, 1936	0.85	70
Stream No. 3; $P = 12$ min:			Stream No. 8; $P = 20$ min:		
July 2, 1936	1.03	20	June 7, 1936	0.74	20
July 6, 1936	0.64	15	July 1, 1936	0.53	20
July 23, 1936	0.41	30	July 6, 1936	0.64	15
August 10, 1936	1.13	20	July 20, 1936	0.63	20
(*b*) COPPER BASIN WATER-SHED					
Stream No. 1; $P = 38$ min:			Stream No. 4; $P = 8$ min:		
June 8, 1935	0.69	25	March 25, 1935	0.77	8
June 21, 1935	1.51	55	July 27, 1935	0.35	10
July 5, 1935	0.80	30	August 17, 1935	0.63	8
May 12, 1936	1.94	60	August 23, 1935	1.41	40
			October 28, 1935	1.10	12
Stream No. 2; $P = 60$ min:					
June 8, 1935	0.73	25	Stream No. 5; $P = 20$ min:		
June 21, 1936	1.48	55	April 2, 1935	0.29	8
July 20, 1936	0.67	25	April 3, 1935	0.73	15
August 10, 1936	0.72	35	July 29, 1935	0.67	8
			August 4, 1935	0.28	14
Stream No. 3; $P = 10$ min:					
April 2, 1935	0.35	8	Stream No. 6; $P = 10$ min:		
July 2, 1935	0.90	7	July 13, 1935	0.25	10
March 24, 1936	0.74	8	July 29, 1935	0.67	10
June 10, 1936	0.89	12	August 3, 1935	0.28	14
August 10, 1936	0.70	16	August 17, 1935	0.50	12

TABLE 3.—(*Continued*)

Date of storm* (1)	Total rainfall, in inches (2)	Duration of rainfall, in minutes (3)	Date of storm* (1)	Total rainfall, in inches (2)	Duration of rainfall, in minutes (3)
(*c*) Coweeta Water-Shed					
Stream No. 1; P = 45 min:			Stream No. 7; P = 60 min:		
August 22, 1935	1.43	30	May 11, 1936	1.51	60
August 23, 1935	0.33	35	June 10, 1936	0.79	30
July 12, 1936	1.74	40	June 12, 1936	0.92	22
July 17, 1936	0.68	20	July 22, 1936	1.18	65
August 19, 1936	0.97	20			
			Stream No. 8; P = 70 min:		
Stream No. 2; P = 60 min:			May 11, 1935	1.36	120
July 12, 1936	1.94	40	July 25, 1935	1.15	65
August 19, 1936	1.02	20	July 26, 1935	1.37	35
August 24, 1936	0.99	20	August 22, 1935	1.49	30
Stream No. 4; P = 25 min:			Stream No. 9; P = 75 min:		
August 11, 1934	1.40	....	April 24, 1935	0.99	42
May 11, 1935	1.38	60	July 12, 1936	1.53	35
August 19, 1936	0.72	20	August 19, 1936	0.51	25
August 24, 1936	1.91	20	August 24, 1936	1.83	20
Stream No. 5; P = 15 min:			August 28, 1936	2.72	75
July 12, 1936	1.69	35	Stream No. 10; P = 60 min:		
July 13, 1936	0.35	16	June 4, 1936	1.13	15
August 19, 1936	1.09	25	June 12, 1936	1.06	22
August 24, 1936	1.60	42	August 24, 1936	1.17	20
Stream No. 6; P = 30 min:					
August 11, 1934	....	....			
July 12, 1936	1.66	35			
August 19, 1936	1.02	25			
August 24, 1936	1.46	42			

* P = Period of rise.

instance, the period of rise for Stream 9, Coweeta (see Fig. 3), is 75 min. Four of the distribution graphs were produced by rains lasting only from 20 to 42 min. These graphs fall very well in line with the one produced by a much heavier rain which fell during the entire 75 min. On Coweeta 7, rains lasting 30 and 22 min, respectively, produced distribution graphs which coincide with those produced by rains of 60- and 65-min duration. In contrast to these shorter rains, about ten of the rains used were of a longer duration than the period of rise, although, in most cases, the major portion of the rain fell within this period. However, even in the case of the rain of August 23, 1935, on Copper Basin 4 (which lasted five times as long as the period of rise) a definite peak occurred at the same time as those of the other distribution graphs. This evidence may be summarized as follows:

(1) Any rain occurring in a time equal to or less than the period of rise will produce a unit hydrograph;

(2) A rain that lasts somewhat longer than the period of rise but with decreasing intensity also produces a unit hydrograph; and,

(3) There is a tendency for the period of rise to be unaffected even if the rain continues longer at a high intensity.

The foregoing considerations help to trace the sequence of events that constitute the process of surface run-off. This process is believed to be as follows: The first rainfall that occurs at a rate in excess of the infiltration capacity produces surface storage. As rainfall continues the surface water begins to move down the slopes in thin films and tiny streams. Surface tension and friction play an important part at this stage. As these minute rivulets increase in volume, their velocity becomes greater; but their path is tortuous and every small obstruction causes a delay until sufficient head is built up to overcome it. Upon its release, the stream is suddenly speeded on its way again. Each time that two or more of these little streams combine, the water is further accelerated in its down-hill path. It is the culmination of all these small contributions into a final combined accretion to the main stream which produces the ultimate hydrograph. It appears, therefore, that the period of rise has no particular relation to the time required for water to reach the gaging station from the most remote part of the water-shed, since the peak is merely the result of the simultaneous release of the accumulated rainfall from the major portion of the water-shed. The rain that falls near the end of the period begins its overland flow much more readily than that falling earlier, since it falls at a time when all the surface depressions have been filled and into water which is flowing with a relatively high velocity. An increase in the intensity of the rainfall over the minimum required to produce surface run-off increases the total surface run-off but does not materially change the time required to complete the foregoing process.

APPLICATIONS OF THE DISTRIBUTION GRAPH

It has been shown that unit hydrographs exist in, and may be isolated from, the discharge records of small streams. A study of the natural phenomena involved in the formation of a unit hydrograph has thrown light on some phases of the process of surface run-off. In the following studies, the distribution graphs are utilized to test the value of the unit hydrograph principle in surface run-off analysis.

Correlation of the Distribution Graph with Water-Shed Characteristics.—The variety of types and sizes of water-sheds upon which the investigation is based offers an opportunity to relate the characteristics of the distribution graphs with those of the water-sheds. The writer is aware that the exact dimensions of the distribution graphs are partly a matter of personal judgment, especially the widths of bases and the peak percentages. A change in the method of separating base flow from storm flow might change, considerably, the total length of base of the distribution graphs, although it would not materially affect their general shape. Furthermore, in some of the streams, the records may not have been long enough to produce a true unit hydrograph; however, as demonstrated later by the construction of pluviagraphs, the evidence is in favor of their adequacy. In view of the foregoing consideration, the correlations were made to include as much information as possible. None of the graphs contained "average curves." The only averages used were the composite distribution graphs for each stream.

For a comparison of peak discharge, the twenty-two composite distribution graphs were reduced to a 2.5-min basis. The peaks were then plotted in order of magnitude as shown in Fig. 5.

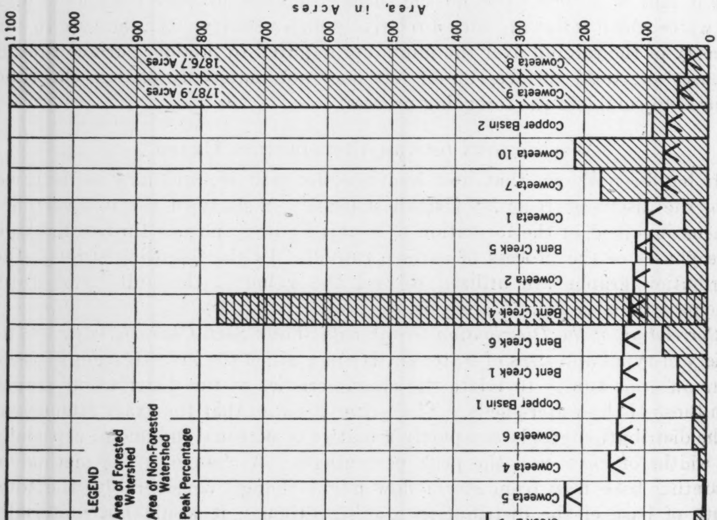


FIG. 5.—RELATION BETWEEN PEAKS OF DISTRIBUTION GRAPHS AND AREAS OF WATER-SHEDS

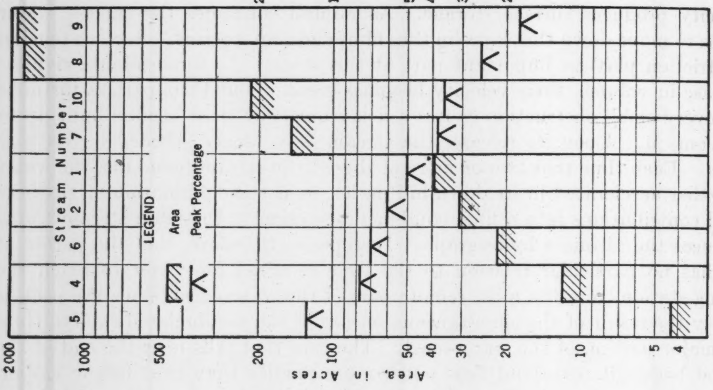


FIG. 6.—RELATION BETWEEN PEAKS OF DISTRIBUTION GRAPHS AND AREAS; COWEETA WATER-SHEDS

That the peak percentage varies according to the area of the water-shed is quite obvious. However, it is the manner in which the variation occurs that is especially interesting. For instance, if the forested water-sheds of the Bent Creek, Coweeta, and Copper Basin areas are considered separately, the agreement between peak percentage and area becomes consistent. That is, from left to right on the graph the areas are constantly increasing. This may be noted by a careful study of Fig. 5; but it is more clearly indicated in Fig. 6, in which only the Coweeta streams are plotted. The greatest peaks for a given area are found at Bent Creek—the smallest at Copper Basin and Coweeta. It is significant that the denuded and grassed areas have peaks much higher than would be indicated by the general trend of the forested areas.

It is interesting to consider, specifically, some of the individual variations from the general trend. For instance, Bent Creek 8 is considerably out of its place, considering merely area and peak. It has the largest area of any of the unforested water-sheds, but has the second highest peak. It is possible that this is due to the fact that this water-shed has nearly a semi-circular shape, with its outlet at the center of the circle, thus permitting all sectors of the water-shed to contribute their run-off simultaneously with practically no delay due to channel storage.

Water-shed 4 on Bent Creek has approximately the position in the graph that one would expect, being about 50% forested and 50% cultivated farmland, abandoned farmland, and cut-over land.

The peak percentage of Stream No. 2, Bent Creek, is considerably above the trend of the other forested water-sheds. Two-thirds of the standing trees were cut from this water-shed in 1930 and 1931, and skidding operations at that time were exceptionally heavy. Furthermore, drainage from approximately 100 yd of roadway discharged into the water-shed. Although the quantity of water entering from this source was not great, it entered in the form of a small stream and followed a fairly narrow and well-defined course to the main stream, discharging into it at a point about 100 ft from the weir. This latter difficulty was eliminated in subsequent records to determine whether the high peak was due to the scarcity of vegetative cover or to the unnatural drainage.

Fig. 7 was prepared to show the variation in the duration of run-off for a unit storm, as indicated by the lengths of bases of the distribution graphs. The bases are plotted using the area, in acres, as the zero ordinate for each graph. The types of water-sheds are indicated by the legend. As is to be expected, the length increases with the area. The sharp difference between the forested and unforested areas is apparent.

Although the individual variations from the general trend (with possible causes as stated previously) are interesting and perhaps significant, the important feature is that there appears to be a general relationship between the dimensions of the distribution graph and the area, and a logical reason for all the deviations. If this is true, the value of distribution graphs as a means of isolating the area factor in the analysis of surface run-off phenomena becomes apparent.

Instead of using a uniform interval of 2.5 min as a percentage basis in the foregoing analysis, it would have been possible, and might at first appear logical, to base the percentage upon a constant portion, say 10%, of the base of the distribution graphs. The effect of selecting the proper length of base lines becomes much more pronounced when this method is used. For instance, the peak percentages based on 2.5-min intervals are 4.0% for Base Line *a* and 4.4% for Base Line *b*, Fig. 2(*a*). Based on an interval equal to 10% of the length of bases, the peak percentages are, respectively, 43.6% and 31.9 per cent. The difference in the first case is 10% and in the latter case about 35% showing that personal judgment in selecting the base line would play a much more important part if the latter method were adopted.

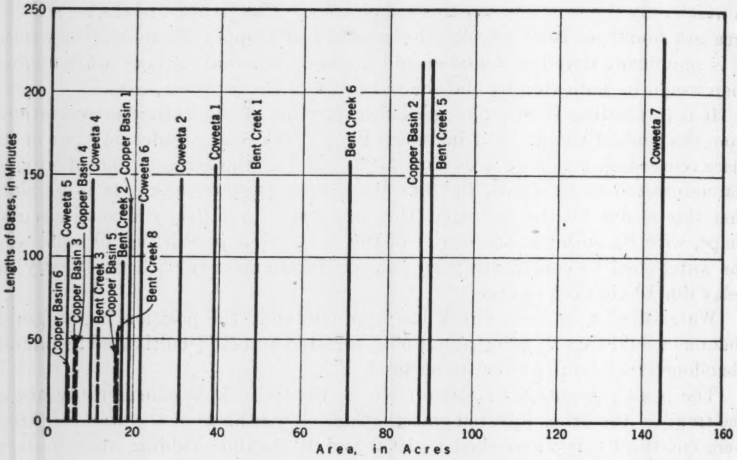


FIG. 7.—RELATION OF WATER-SHED AREAS TO BASES OF DISTRIBUTION GRAPHS

For this reason, and because the peak percentage based on a definite time interval may be translated into rate of surface run-off and therefore has a real practical value, the comparisons were all based on such intervals.

Application of the Pluviagraph.—The pluviagraph is a hydrograph of 100% surface run-off prepared by distributing a rain or a series of rains according to the ordinates of the distribution graphs. It is assumed that each unit storm produces a unit hydrograph, although other subsequent or succeeding rains may produce overlapping graphs. Then, since a distribution graph shows the successive periodic surface run-off contribution of a unit hydrograph, each unit storm must contribute to each time interval, after the beginning of rainfall, a portion determined by the successive percentages of the distribution graph. Only that rainfall which falls at a rate greater than the infiltration capacity becomes surface run-off. Therefore, a coefficient must be applied to the pluviagraph values to produce an actual hydrograph of surface run-off.

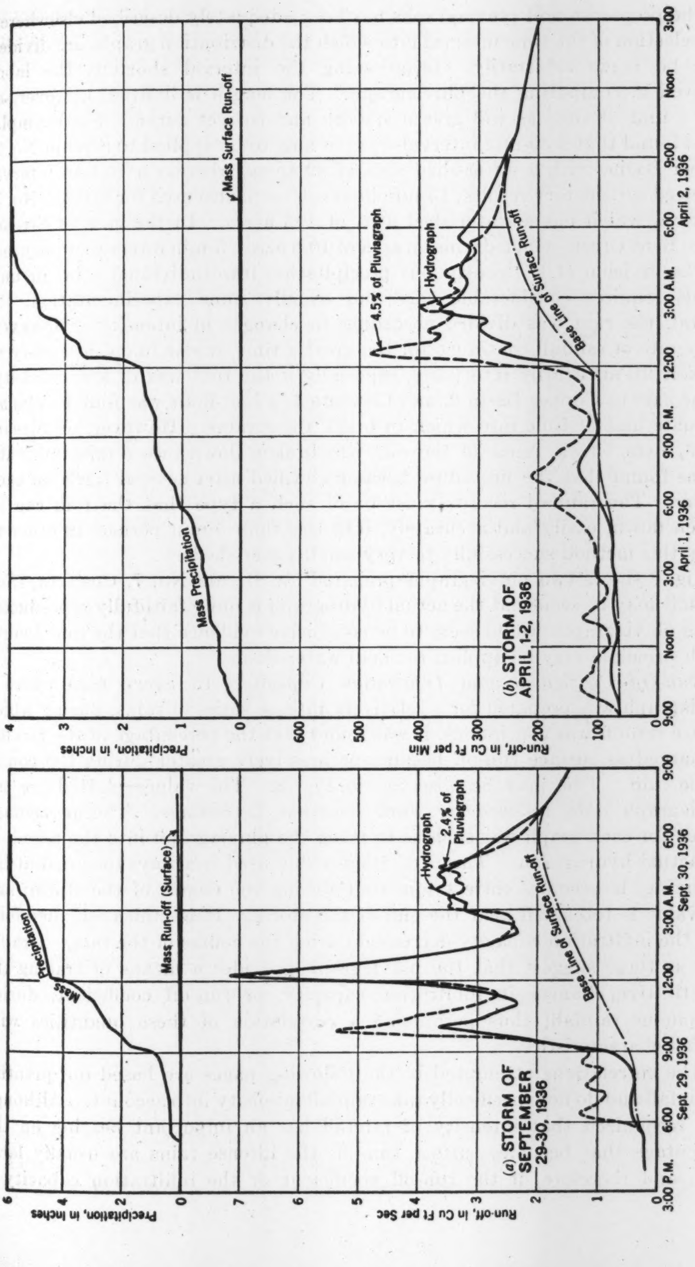


FIG. 8.—PLUVIAGRAPHS FOR TWO STORMS; STREAM NO. 7, COWEETA

The preparation of pluviagraphs has been adequately described elsewhere.[6] The selection of the time interval into which the distribution graphs are divided must be made arbitrarily. Lengthening the interval shortens the labor involved in computing the pluviagraph. The length is limited, however, to such a unit of time as will give a smooth and correct curve. For example, it was found that a 30-min interval was too long to be applied to Stream No. 2, Copper Basin, with a water-shed area of 89 acres, whereas a 15-min interval produced satisfactory results; 15-min intervals were also used for Stream No. 7, Coweeta, which has a water-shed area of 145 acres. In the case of Stream No. 3, Bent Creek, which drains an area of 10.8 acres, 5-min intervals were used.

The division of the continuous precipitation into individual rains or unit storms requires considerable care and usually some experimentation. In general, the rain was divided according to changes in intensity. However, if any rate of rainfall continued for too great a time, it was found necessary to break it up arbitrarily into parts, especially if the rain was of low intensity. In the case of Copper Basin 2, and Coweeta 7, a half-hour was found to be an adequate unit of time into which to break these rains. However, on Stream No. 3, Bent Creek, much of the rain was broken down into 5-min intervals. It was found that the procedure became clarified after several trials on each stream. The rainfall records must be of such a type that the rain can be broken down, easily and accurately, into less than 5-min periods in order to apply this method successfully to very small water-sheds.

Fig. 8 shows two pluviagraphs prepared for Stream No. 7, Coweeta, from which it may be seen that the actual hydrograph is quite faithfully reproduced. These pluviagraphs would seem to be conclusive evidence that the unit hydrograph principle may be applied to small water-sheds.

Run-Off Coefficients and Infiltration Capacity.—In every case where a pluviagraph was prepared for a relatively intense series of rains, during which surface run-off was continuous, it was found that the percentage of the rainfall discharged as surface run-off became progressively greater during the course of the rain. This may be observed in Fig. 8. The values plotted are the pluviagraph data reduced by some constant percentage. The percentage chosen for each graph was such as to bring the pluviagraph into the region of the actual hydrograph. The percentage value used is always too great at the beginning; it becomes correct sometime during the course of the storm; and the value is too small near the end of the storm. It is indicated, therefore, that the infiltration capacity decreased during the course of the rain. Graphs such as these suggest that the pluviagraph provides a means of tracing the quantitative changes in infiltration capacity, or run-off coefficient, during continuous rainfall, thus permitting a correlation of these quantities with rainfall characteristics.

The correlations attempted in the following pages are based on quantity of rainfall and do not specifically take rainfall intensity into account. Although it is recognized that intensity of rainfall has an important bearing on the percentage that becomes surface run-off, the intense rains are usually large ones, and therefore, if the run-off coefficient or the infiltration capacity is correlated with quantity of rainfall, the intensity will be taken into account somewhat. There are several reasons for leaving intensity "out of the picture." In the first place, the determination of mean rainfall intensity for a given series of rains often leads to a distorted result, dependent upon how much of the rain is taken into account. Furthermore, it is difficult to include antecedent rainfall in a correlation involving intensity rather than quantity of rainfall.

In order to determine, quantitatively, the variations in percentage run-off, the percentage which brought the pluviagraph into practical coincidence with the actual hydrograph was determined for each significant peak during the course of surface run-off. This was then related to the quantity of rainfall, which, according to the pluviagraph, had had an opportunity to be converted to run-off at the time when that coefficient was applicable. In Fig. 9, each group of connected points represents such a series of consecutive rains during which surface run-off was continuous.

Four of such series are plotted for Stream No. 2, Copper Basin, in Fig. 9(*a*), together with a number of points from individual rains. It will be seen that there is a definite relationship between accumulated rainfall and percentage of surface run-off, despite the fact that rain intensity and previous saturation are not considered. It will be noted that points plotted for the storm of April 5 and 6, 1936, have higher percentages than for the preceding storm of April 1 and 2, 1936. Undoubtedly, the first storm had an appreciable effect on the second one, and, if this could be taken into account, the two curves would probably come near to coinciding. As the study of soil moisture and its effect upon surface run-off is advanced, this may become possible, but for the present it is more important to know the range within which the coefficients will vary.

Fig. 9(*b*) is a similar graph for Stream No. 7, Coweeta. It will be seen that the trend is noticeably different in this case. The effect of previous saturation noted for the two April storms may be seen again for the same storms on this stream.

Three sets of coefficients in series, and a number of single ones, are plotted for Bent Creek 3, in Fig. 9(*c*). The series of storms occurring on January 7, 1935, is a good example of the rising percentage of surface run-off. At the time Rain No. 1 occurred, 1.72 in. of rainfall had fallen. Since surface run-off was intermittent to this point, the previous rainfall was not considered in plotting the points. The intensity of rainfall was relatively low throughout this rain. The fact that these points lie in a lower percentage range than the main trend would indicate that intensities of rainfall have an important effect in water-sheds of this nature. More evidence of this is found in the series of rains occurring on April 6, 1936. Rain No. 3 of this series was a much less intense shower than Nos. 1 and 2. It may be seen from the graphs that the percentage of surface run-off has dropped back for this rain.

Thus far, no mention has been made of the length of time in which a build-up in the coefficient is sustained after the end of surface run-off. In this connection it is interesting to consider Rain No. 4 of the series just mentioned. In Fig. 9(*c*), Point 4*b* is plotted according to accumulated rainfall, whereas Point 4*a* is plotted according to the rainfall that produced its run-off. A value lying between these two would fall in line with the general trend. A break in the continuity of surface run-off of 1 hr occurred between this rain and Rain No. 3. Although Rain No. 4 was nearly equal in amount and intensity to Rain No. 3, the percentage of surface run-off is much smaller. Surface detention may account for some of this difference, but the following considerations indicate that its effect could not have been great: (1) The slope is relatively great and erosion has advanced to such a stage on this water-shed that actual detention in the form of puddles on the ground must be very slight; and (2) in the several hours intervening between the rains, the relative humidity remained at nearly 100%, so that practically no evaporation occurred. Therefore, the quantity of rainfall previously intercepted and held by the vegetation had not been depleted, and detention from this source may be practically eliminated. The conclusion may be drawn, then, that the infiltration capacity increased considerably in the space of 1 hr despite the 1.60 in. of rainfall immediately preceding.

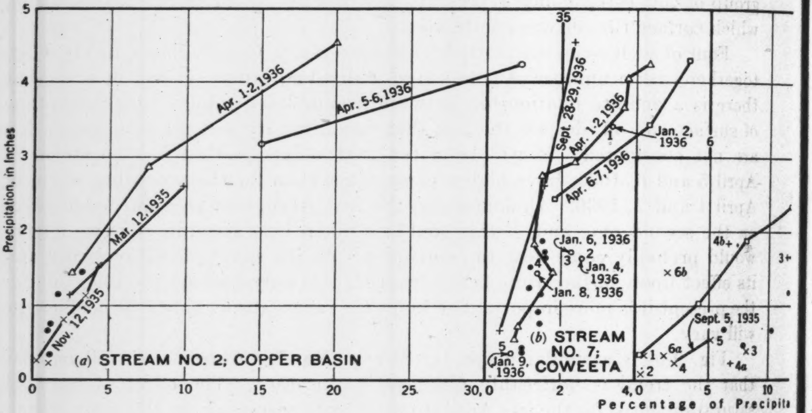
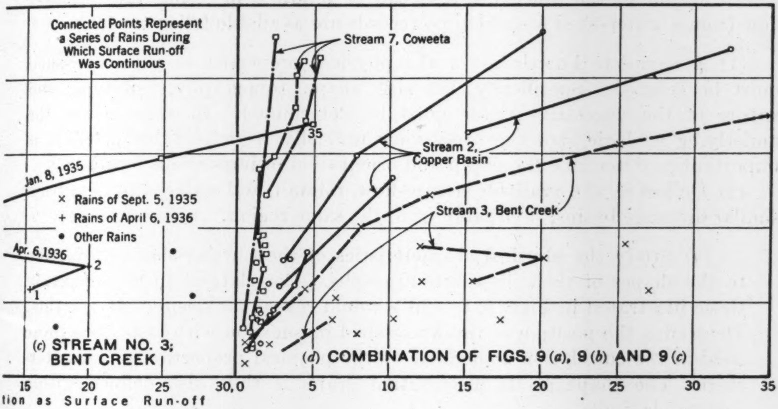


FIG. 9.—RELATION BETWEEN ACCUMULATED

Another illustration is furnished by the rains of September 5, 1936. Surface run-off resulting from these rains was interrupted for periods varying from a half to 1.5 hr between all of the rains except Nos. 4 and 5. For illustration, Point 6*b* was plotted according to accumulated rainfall whereas Point 6*a* was plotted according to the actual rainfall. Since Point 6*a* falls into line with the other points, it appears that the effect upon infiltration capacity of the previous 1.10 in. of rainfall is completely gone in 1.5 hr.

Five heavy rains occurring in the period from April 2 to April 9, 1936, on Coweeta 7 are plotted individually in Fig. 9(*b*). Breaks in the occurrence of surface run-off between the successive storms were 29, 28, 32, and 13 hr in duration, respectively. The amount of each rain is shown by the graph. It may be seen that the points plotted individually fall quite well in line with the general trend, which indicates that these rather heavy rains had little or no effect on the surface run-off coefficient of those following.

The points plotted for each stream in Figs. 9(*a*), 9(*b*), and 9(*c*) are all combined in Fig. 9(*d*). The definitely distinct trends of the points for the three streams are evident, despite the limitations in allowing for intensity of rainfall and previous saturation. If such a series were prepared for a sufficient number of streams, covering a wide enough range in area, cover type, slope, etc., the resulting information would be of value to the practicing engineer, as well as being an important step in the accumulation of a basic knowledge of surface run-off phenomena. It would give the engineer definite limits between which the coefficient could be expected to vary for a given type of water-shed. If he were then able to choose a representative distribution graph, he could predict, far more accurately than he could by the use of any empirical formula, the run-off to be expected from a given quantity of rainfall.



RAINFALL AND PERCENTAGE RUN-OFF

It will be noted that no attempt was made to separate winter and summer rains in the foregoing correlations. A similar study on northern streams would undoubtedly require such a separation. Furthermore, although the indications are that the effect of previous saturation on infiltration capacity is negligible on these water-sheds, it is likely that in other locations this effect may be extremely important.

The value of a series of investigations to determine run-off coefficients under varying conditions cannot be over-estimated. It is extremely desirable that data collected from different sections of the United States be gathered and analyzed to show the effect of soil structure, topography, vegetative cover, temperature, and other water-shed and rainfall factors on the surface run-off coefficient or infiltration capacity.

THE USE OF THE UNIT HYDROGRAPH IN ESTIMATING FLOOD FLOWS

The design of so many structures depends upon a knowledge of the maximum rate of run-off to be expected from a water-shed that any method that will give a more dependable determination of this quantity must be of inestimable value to the engineering profession. The unit hydrograph method possesses several advantages over other methods usually employed for this purpose. First, this method permits the engineer to estimate not merely the peak discharge, but the entire hydrograph of run-off. Second, the peak may be determined for any desired time interval. It thus provides a flexibility that is not provided by any empirical formula. Third, the quantities involved in computation by this method have physical conceptions which are familiar to the engineer. No empirical formulas with vaguely defined constants are involved; the physical characteristics of the water-shed have a straight-forward bearing on the computations. Finally, the method lends itself easily to the experimental determination of the quantities involved—namely, percentage of rainfall appearing as surface run-off and the shape of the distribution graph.

An outline of the procedure that can be followed in estimating the flood flow from a water-shed for which no records are available follows:

(1) A complete knowledge of the physical properties of the water-shed must be secured. Specifically, the size, shape, topography, soil type, and nature of the vegetative cover must be determined. In areas where the underlying geologic structure may vary radically from place to place, it is important to determine the depth and orientation of impervious strata.

(2) Collect all the available stream-flow, rainfall, and water-shed data from similar drainage basins, preferably from the same region.

(*a*) Study the physical characteristics of these water-sheds in relation to the shapes of their distribution graphs. Correlations in the nature of those illustrated in Figs. 5, 6, and 7 would reveal the desired information. Determine the position of the water-shed in question with respect to those used in the correlation as defined by the physical properties of the water-shed. The shape of its distribution graph is thus also defined within reasonable limits.

(*b*) Determine the range of variation of run-off coefficients. The accumulation of all the available data in graphical form as illustrated in Fig. 9(*d*) would show the variations of this coefficient with rainfall and would permit the selection of a maximum value.

(3) Determine the maximum storm that has occurred in the region which might reasonably be expected to occur on the water-shed being studied. Transpose this rainfall upon the water-shed and compute the volume of water applied to the water-shed.

(4) The portion of this volume that will appear as surface run-off may be computed by applying the coefficient determined in Item (2*b*). The hydrograph of surface run-off may now be constructed by apportioning this quantity according to the figures of the distribution graph selected in Item (2*a*).

Where flood estimates are to be made on a water-shed for which some records are available, the results may be obtained in a similar manner except that the records on the water-shed itself would be utilized to the fullest extent in the foregoing procedure.

It is evident that the determination of the maximum flood becomes more dependable as the number of available stream-flow and rainfall records increases. A great mass of such data has been collected in the United States, but its usefulness is limited until it is brought together and analyzed in a systematic manner. Furthermore, for very small water-sheds, average daily discharge records of a stream are of little value for any type of stream-flow analysis. It is necessary that records on such streams (and likewise the rainfall records) be continuous, and that they be available as such to permit their utilization in the advancement of the knowledge of stream behavior.

SUMMARY

The preparation of unit hydrographs and distribution graphs for twenty-two small streams and a study of their relation to the corresponding rainfall and water-shed characteristics have served to reveal some of the natural phenomena involved in the process of surface run-off as well as to point out the usefulness of the unit hydrograph principle in flood-flow prediction. The results are summarized as follows:

(1) Unit hydrographs are produced on small streams and may be readily selected from continuous run-off records;

(2) The ground-water contribution may be separated from the surface run-off with considerable assurance in the case of hydrographs resulting from isolated rains;

(3) The unit hydrograph is an important aid in the separation of ground-water contribution from surface run-off in the case of a complex hydrograph produced by a series of rains;

(4) Any rain that is sufficiently intense to produce surface run-off will produce a unit hydrograph if its duration is equal to, or less than, the period of rise;

(5) The period of rise is a function of the water-shed characteristics and is the time required for the major portion of the water-shed to release its accumulated rainfall load;

(6) There is a definite correlation between the shape of the distribution graphs and the water-shed characteristics;

(7) The reproduction of known flood hydrographs by the use of the pluviagraph provides a method of determining the variation of run-off coefficient or infiltration capacity on a water-shed throughout the period of surface run-off; and,

(8) The unit hydrograph principle provides a fundamentally sound method of predicting surface run-off.

The study has indicated certain lines of investigation that should be conducted to strengthen some features of the unit hydrograph principle and to provide a background of experimental data upon which to base its application in flood-flow prediction. The studies which it is believed would be most fruitful are:

(*a*) An intensive examination of the variations in stream levels and adjacent water-table levels during the course of surface run-off to provide a dependable basis for separating ground-water flow from surface run-off. An exhaustive study on only a few water-sheds would provide data upon which to base the separation in other cases.

(*b*) A study to determine whether a composite or average depletion curve may be used to estimate a hydrograph of run-off during intervals between rains.

(*c*) An investigation to determine the influence of rainfall and water-shed characteristics upon surface run-off coefficients. (Such a study should attempt to isolate and show the effect of various factors such as size, shape, and slope of the water-shed, vegetative cover, cultivation, air temperature, duration and intensity of rainfall, antecedent rainfall, etc. The results would provide a basis for estimating the coefficients in cases where no discharge or rainfall records are available, and would indicate how changes on the surface of the water-shed would influence run-off.)

(*d*) The preparation and systematic analysis of distribution graphs from a large number of streams showing variations in the shapes of the graphs with relation to water-shed characteristics. The knowledge provided by such an investigation in conjunction with a known surface run-off coefficient would permit the construction of flood hydrographs on streams where no discharge records are available.

ACKNOWLEDGMENTS

In its original form, this paper was a thesis presented to the University of Michigan, in 1937, in partial fulfillment of the requirements for the degree of Doctor of Philosophy. The work was first revised under the direction of members of the Department of Civil Engineering, with the special aim of making it a useful contribution to engineering literature. The study is based on unpublished records of rainfall and stream flow gathered by the Appalachian Forest Experiment Station, U. S. Forest Service, with which the writer was formerly associated.

DISCUSSION

Franklin F. Snyder,[8] Jun. Am. Soc. C. E. (by letter).—The usefulness of the unit hydrograph principle in analyzing rainfall and run-off records from small water-sheds is well illustrated in this paper. The author fills the gap, with respect to size of area to which the procedure is applicable, from the city block as demonstrated by W. W. Horner, M. Am. Soc. C. E., and F. L. Flynt, Assoc. M. Am. Soc. C. E.,[9] to drainage areas of several hundred square miles, and larger, as demonstrated by other investigators. Although the paper is quite complete, the writer would like to discuss several items with the hope of increasing the value of the paper as a guide for further studies.

The author has established a procedure for separating ground-water and surface run-off and apparently has been able to ignore the occurrence (if any) of sub-surface storm flow. On larger basins the unit hydrograph crests for large floods are frequently higher than the crest values obtained for small floods, and this is due in part to the difficulties encountered in handling true surface run-off, sub-surface storm flow, and ground-water flow.

From the statements on the separation of surface run-off from ground-water, it appears that the author's procedure includes, with the ground-water discharge, most of the sub-surface storm flow which has entered the surface channels within the confines of the basin. If this could be done for floods and basins of all sizes, it would eliminate the need to consider sub-surface storm flow in connection with unit hydrographs. Since sub-surface storm flow would become effective at the gaging stations shortly after the beginning of run-off, the inclusion of the former with run-off from the ground-water zone would explain the author's conclusion that the rate of ground-water contribution increases as soon as run-off begins.

The writer believes it possible that for a short period of time the ground-water discharge may actually decrease at a greater rate following the occurrence of run-off than before. At some time near the occurrence of crest discharge the ground-water discharge begins to increase and then probably follows a trend similar to that described in the paper. As shown by Mr. Brater, the actual transition would be smooth, regardless of its exact course. However, in practical applications, the separation can perhaps be made more consistent for application to different basins by the use of several straight lines rather than a curved line.

The author has concluded that the period of rise of a unit hydrograph is independent of the duration of rainfall so long as the duration of rainfall does not exceed the period of rise. This is contrary to results obtained from larger drainage basins, and there are several indications that it may be erroneous for the water-sheds under discussion.

There is usually a period at the beginning of rainfall during which no run-off occurs, and the rainfall in this interval is often referred to as initial loss. The duration of initial loss may vary considerably for different storms, especially in summer. The duration of initial loss should be subtracted from the duration of the storm to obtain the duration of the surface run-off producing rain. The latter period of time is the duration of rainfall that should be compared with the period of rise of the resulting discharge graph.

---

[8] Central Office, U. S. Weather Bureau, Washington, D. C.

[9] "Relation Between Rainfall and Run-Off from Small Urban Areas," by W. W. Horner and F. L Flynt, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 140.

Although the writer's experience has indicated that values of initial loss are usually smaller in the Southern Appalachians than for areas farther north, it is believed that such an adjustment to the values of rainfall duration in Table 3 would reveal a closer relation between duration of surface run-off producing rain and period of rise.

Table 4 gives the apparent period of rise, *P*, for the distribution graphs of Figs. 3 and 4 as scaled from the diagrams. There may be some errors and mistakes in these values due to difficulty in distinguishing between the individual graphs. The duration of rainfall, as listed in Table 3, and the value of *P* adopted by the author for each of the two basins is also given. The data show some relation between duration of rainfall and period of rise and it is believed the relation would be more pronounced if periods of initial loss were deducted from the values of duration of rainfall.

TABLE 4.—COMPARISON OF RAINFALL DURATION AND PERIOD OF RISE

Description	STREAM NO. 9; COWEETA; *P* = 75 MIN					STREAM NO. 1; COPPER BASIN; *P* = 38 MIN			
	April 24, 1935	July 12, 1936	August 19, 1936	August 24, 1936	August 28, 1936	June 8, 1935	June 21, 1935	July 5, 1935	May 12, 1936
Duration of rainfall, in minutes	42	35	25	20	75	25	55	30	60
Period of rise (from Figs. 3 and 4), in minutes	103	53	66	77	110	39	58	36	46

The best evidence that the unit of time used with unit hydrographs has an appreciable effect on the results and that the unit should be considerably less than the period of rise is given by the author under the heading "Applications of the Distribution Graph: Application of the Pluviagraph." It is stated that the interval or unit of time for the distribution graph is limited to such a length that the computations will give a smooth and correct curve. These same conditions undoubtedly controlled the break-down of the continuous precipitation into units of time. To accomplish this the author has used a unit of 30 min for Stream No. 2, Copper Basin; yet the period of rise is given as 60 min. The unit of time adopted for Stream No. 7, Coweeta Basin, is 30 min, and the period of rise is 60 min. The only other values given in the paper are for Stream No. 3, Bent Creek, for which the precipitation unit of time is 5 min, and the period of rise is 12 min.

The paper includes a very interesting discussion and illustration of variation in run-off coefficients. In considering the use of the pluviagraph to study the continuous variation in run-off coefficients (assuming the procedure is theoretically correct), it should not be overlooked that the factor obtained by dividing an observed rate of discharge by the corresponding pluviagraph rate is not the true run-off and rainfall ratio for any particular unit period of time except those for the first and last periods of a storm. The coefficient obtained by dividing the discharge rate by the pluviagraph rate is the weighted average of the actual ratios for the various unit periods, parts of the rainfall of which have been combined to obtain that particular pluviagraph rate. This fact does not necessarily impair the usefulness of the ratios between discharge and pluviagraph rates for comparative purposes, but it should not be disregarded when correlating the ratios with actual physical data.

The author has made an excellent analysis of what is undoubtedly a comprehensive collection of data. The availability of data such as those used in the analysis, and those being collected at numerous other experimental areas, suggests that it may not be long before the oft-heard statement decrying the lack of data must be replaced by one emphasizing the need of competent analysis.

LEROY K. SHERMAN,[10] M. AM. SOC. C. E. (by letter).—A careful analysis of unit hydrographs, derived from good rainfall and run-off data, has been made by the author. He has demonstrated conclusively that the unit graph method is applicable to small areas. In his earlier work with the unit graph, the writer dealt with 24-hr total rainfall records. Consistent graphs, which neglected rain intensity and distribution, naturally could not be found unless the area was large enough to furnish time of transit and channel storage sufficient to "iron out" the effects of changing rain patterns. When recording gages are used, as in the author's examples, consistent results will be found on both large and small areas. It has been found that:

(*a*) The unit of time must be less than, and preferably a fraction of, any concentration period; and

(*b*) The unit graph, or the percentage distribution graph, on the larger areas, reflects the pattern of effective rainfall. For example: A storm centered on the lower end of the basin gives a relatively high and early peak, and conversely, a storm centered at the upper part of the basin gives a lower peak rate of run-off for the unit graph. Both are correct. In applications for run-off estimates, the type of graph is selected which is best suited to the given rainfall pattern. This gives better results than the average of several derived unit graphs.

On the derivation of base flow, the author's reasoning, for the adoption of the upper line "*b*," seems logical for the particular basins involved. It is assumed that the author has located Point 4, on the tail of the hydrograph, by actual field observation. That is, Point 4 is the observed time when all material overland flow into the stream has ceased. The writer is of the opinion that many base flow lines are drawn for flood hydrographs which show too great an increase in base flow during the period of surface run-off. Many cases have been found in which the base flow was actually negative. This is due to bank storage of water by infiltration at overbank or high stages of the stream.

---

[10] Cons. Engr., Chicago, Ill.

Before ground-water outflow can increase under high river stages, the infiltration or bank storage must become sufficient to raise the ground-water table. The correct method of solving the problem on an experimental area such as this is, as the author suggests, to have observation wells placed along the bank of the stream for measurement of the ground-water outflow profile.

Mr. Brater has made a careful study of run-off coefficients. This study would have been more useful had he applied his work to infiltration capacity instead of run-off coefficients. Run-off has no definite relation to rainfall, since it is rainfall minus infiltration and other losses. Considerable study and investigation have been made on infiltration capacities and loss rates.[11] The time has come already when such loss rates can, and are, being used in engineering practice for estimating run-off from rainfall. The use of these infiltration capacities or loss rates gives far more accurate results than can be obtained by the most qualified hydrologists in estimating, or guessing, at the coefficient of run-off. There is no run-off unless the rainfall has an intensity greater than the infiltration capacity of the soil. The writer considers it a fundamental mistake to "leave intensity out of the picture." The author is adhering to an old procedure which is fundamentally erroneous and rapidly becoming obsolete.

As an illustration of the infiltration method, Table 5 has been compiled from data furnished by the author. The average infiltration capacity in this table was computed by the direct method, which is premised on the fact that the volume of rain excess is practically equal to the volume of observed surface run-off. Note the consistent decrease in infiltration capacity with duration of effective rainfall. The absorptive capacity of the initially dry soil is greater than the more saturated soil. The average infiltration capacity, following excess rain of more than one or two hours duration, will be only a little greater than the ultimate infiltration capacity.

TABLE 5.—AVERAGE INFILTRATION CAPACITIES

Basin	Date of storm	Average infiltration capacity, in inches per hour	Duration of effective rain, in minutes	Reference
Coweeta 7*a*	6/12/36	2.35 −	25	Fig. 2 (*b*)
Coweeta 7*b*	6/12/36	2.35 +	25	Fig. 2
Coweeta 7	4/ 1/36	2.76	10	Fig. 8 (*b*)
Coweeta 7	9/29/36	2.10	40	Fig. 8 (*a*)

When applying ultimate infiltration capacity to given rainfall, for the purpose of deriving the volume of run-off, it is frequently desirable to know the initial loss rates. The writer is of the opinion that the author's procedure of utilizing the pluviagraph (Fig. 8) to determine variation in run-off percentages can also be used to determine the changes in initial loss rates.

It is pertinent to note that the forested basins used by the author represent an extreme limit of low run-off and high infiltration capacity. This is illustrated by the following derived ultimate infiltration capacities:

---

[11] "Surface Runoff Phenomena," by R. E. Horton, M. Am. Soc. C. E., *Bulletin* 101, Voorheesville, N. Y.; also *Transactions*, Am. Geophysical Union, 1936, p. 302; 1937, p. 371; and 1938, pp. 430–436.

Place	Ultimate infiltration capacities, in inches per hour
Bethany, Mo.	0.15
Illinois	0.10–0.30
Industry, Ohio	0.16
Red River Basin, Okla.	0.21–0.25
North Concho Basin, Tex.	0.30
Lincoln, Nebr.	1.00
Colorado Springs, Colo.	2.00
Coweeta 7, N. C.	2.10
Copper Basin, No. 1, N. C.	3.00 ±

The flat lines of "mass surface run-off" in Fig. 8 indicate the high rate of infiltration on Coweeta No. 7. The writer has replotted the mass run-off for the storm of September 29, 1936, with an expanded vertical scale. This graph shows clearly the relation of run-off to the intensities of rain in excess of infiltration capacity.

RAPHAEL G. KAZMANN,[12] JUN. AM. SOC. C. E. (by letter).—A secondary conclusion advanced in this interesting paper is worthy of some attention. After describing two consecutive rains which followed one another closely, Mr. Brater states (heading, "Applications of the Distribution Graph: Run-Off Coefficients and Infiltration Capacity"), "The conclusion may be drawn, then, that the infiltration capacity increased considerably in the space of 1 hr despite the 1.60 in. of rainfall immediately preceding." This conclusion was based on the data from Bent Creek 3 which indicated that the run-off from Rain No. 4 was about one-half the run-off which resulted from Rain No. 3, although the two rains were "nearly equal in amount and intensity." The question may be raised logically: Can the infiltration capacity of a water-shed be increased by saturating it?

Admittedly, circumstances are conceivable under which such a phenomenon might occur: For example, the presence of a top layer of frozen earth that had been thawed out by the first rain might bring an apparent increase in the infiltration capacity of a water-shed. The same result would occur if the soil were very dry with all the interstices filled with air, if the first rain were a deluge and the second a slower, steadier precipitation. The circumstances would be analogous to those of an air-locked filter. In general, however, experience seems to indicate that the infiltration capacity of a water-shed decreases as the ground becomes more saturated, finally reaching a more or less well-defined minimum.

It is possible, of course, that during the period upon which the author based his conclusion the upper layer of soil was in a frozen or semi-frozen state. If, on the other hand, there was no frost in the ground (April is an in-between month, anyway), there is another possibility implicit in the data collecting methods used in the observations. It lies in the failure to correlate rainfall intensity with the other observations. Mr. Brater states (heading,

---

[12] Junior Hydr. Engr., Ground Water Div., U. S. Geological Survey, Washington, D. C.

"Applications of the Distribution Graph: Run-Off Coefficients and Infiltration Capacity"):

> "The correlations attempted in the following pages are based on quantity of rainfall and do not specifically take rainfall intensity into account. Although it is recognized that intensity of rainfall has an important bearing on the percentage that becomes surface run-off, the intense rains are usually large ones, and therefore, if the run-off coefficient or the infiltration capacity is correlated with quantity of rainfall, the intensity will be taken into account somewhat."

Considering this statement, it is not difficult to surmise how a questionable conclusion concerning increased infiltration capacity might have been deduced. From the rainfall records the quantity of rain and the total time of each of the periods of precipitation was known; but the distribution of the precipitation during these periods was unknown. This was due to the fact that Bent Creek 3 is the smallest of the Bent Creek water-sheds; consequently, since there were not enough recording rain gages to go around (five recording gages *versus* eight water-sheds), it probably did not receive a recording gage.

Probably the first rain was "spotty." Periods of light precipitation alternated irregularly with periods of very intense rainfall. The rain which followed (Rain No. 4) was probably steadier and had fewer periods of intense precipitation. Without recording gages, unless an observer was on the water-shed during the rains, this "fact" would be unknown. By simply dividing the total quantities of precipitation by the periods of time involved, it would appear that the intensities of precipitation were the same, whereas, although average intensities were nearly alike, the actual intensities differed widely. Assuming that the picture sketched is correct, the phenomenon was observed correctly but an incorrect, possibly too hasty, conclusion was drawn. The run-off from the second storm was less, chiefly because the rain, in general, was more evenly distributed and did not come down in spasmodic torrents. This more gradual rainfall gave the soil (eroded and wet though it was) a better chance to accommodate the rain—that is, the rainfall could infiltrate rather than run off on the surface.

WALDO E. SMITH,[13] M. AM. SOC. C. E. (by letter).—The author is to be commended for undertaking the task of determining distribution graphs for twenty-two small water-sheds, attempting to analyze the results, and to demonstrate applications. He has shown that the unit hydrograph principle is applicable to small water-sheds, as well as to large ones, with probably about the same general limitations. The distribution graph has justly become recognized as a handy tool in the kit of the hydrologist; but it should never be overlooked that the method is inherently an approximate one. At all times one should be aware of its shortcomings and inconsistencies, and should not expect too much refinement from it. For instance, it is self-evident that a tank of given size and outlet requires a longer time to become empty if it is full than if it is half full. It is likewise reasonable to expect that a channel full of surface run-off will require a longer time to be drawn down to ground-water flow than a channel only partly full; yet the width of base of the distribution graph is taken as constant for all cases, for a given water-shed. Similar deductions (some compensating to an unknown degree) can be made if ground-water flow is large or small, or if the average stream velocity is small because of overbank flow, or high because the stream is just at bankfull stage.

---

[13] New Philadelphia, Ohio.

The writer is unable to share the author's optimism with regard to the conclusiveness of all of the points stated in the "Summary," particularly the items referring to the relation, or lack of relation, of duration of rainfall to the period of rise. Possibly it is within the inherent accuracy of the method, on small water-sheds, to assume that within the period of rise the duration of rainfall has no effect upon the shape of the resulting hydrograph. For large water-sheds, it is known that this cannot be done. However, to the writer, it does not appear that this assumption can be made for water-sheds of the size under discussion, either in the instance of the author's data or in the case of other similar data that the writer has available. If rainfall of uniform intensity in excess of the infiltration rates continues for the duration of the distribution graph, the period of rise will be found to be about equal to the duration or base of the distribution graph. Perhaps the best test of the validity of the author's assumption on the basis of his data is a consideration of the period of rise. Mr. Brater does not discuss variations in this value, but if one may use values obtained as accurately as possible from the distribution graphs of Figs. 3 and 4(*b*), then Tables 3(*b*) and 3(*c*) may be extended as shown in Table 6 (rearranging in the order of duration of precipitation).

TABLE 6.—SUGGESTED EXTENSION OF TABLES 3(*b*) AND 3(*c*)

Date	Total rainfall	Duration of rainfall, in minutes	Period of rise, in minutes	Date	Total rainfall	Duration of rainfall, in minutes	Period of rise, in minutes
(*b*) COPPER BASIN WATER-SHED; STREAM NO. 1; $P = 38$ MIN				(*c*) COWEETA WATER-SHED; STREAM NO. 9; $P = 75$ MIN			
June 8, 1935	0.69	25	38	August 24, 1936	1.83	20	70
July 5, 1935	0.80	30	38	August 19, 1936	0.51	25	60
June 21, 1935	1.51	55	56	July 12, 1936	1.53	35	45
May 12, 1936	1.94	60	48	April 24, 1935	0.99	42	110
....	....	....	....	August 28, 1936	2.75	75	110
Average	....	....	45	Average	....	....	79

For the Copper Basin water-shed there seems to be, almost, a surprizing relation between duration of rainfall and period of rise; but if one examined only the first three of the Coweeta group, one might deduce that the period of rise decreases as the duration of rainfall increases. It would have been interesting if data on mass curves of rainfall and isohyetal maps for these storms and areas had been included in the paper. It is possible that in the case of the 35-min rain on the Coweeta water-shed, the major portion fell early in the period, or perhaps was heavier near the gaging station; but inasmuch as there is nearly a 150% variation from the minimum to the maximum period of rise, it is obviously affected by various factors. The statement concluding "Characteristics of the Distribution Graphs" is an example of the pitfalls into which one can be led by deductive reasoning. It is reasonable and rational, but it does not involve all factors.

In spite of the summary statements on lack of effect of duration of rainfall within the period of rise, under "Application of the Pluviagraph" the author has acknowledged that for the production of the pluviagraph in which "each unit storm produces a unit hydrograph," the precipitation must be divided into periods shorter than the period of rise, and "that a 30-min interval was too long * * * whereas a 15-min interval produced satisfactory results" for Copper Basin 2, with a given period of 60 min. If the author's statement that duration of rainfall within the period of rise has no effect is correct, it should be possible to put the 60-min rainfall into one of the four 15-min periods into which it was divided without effect upon the resulting pluviagraph.

The author has failed to give the lapse of time between the beginning of rainfall and the beginning of the rise. The period of the distribution graph should start with the time at which run-off producing rain starts. Is duration of rainfall, as given, the total time or only that period which presumably produced run-off?

The writer regrets that in engineering and hydrologic literature, distribution graphs are published so often, and that tables of distribution coefficients appear so seldom. The author has set forth coefficients in Table 2, for which he receives hearty approval; but Figs. 3 and 4 use the graph. The writer has found that he can determine a composite from several sets of distribution coefficients in tabular form much more readily than from their graph form, as Table 7 shows. Furthermore, a distribution graph as such has no direct use unless it is in block form as suggested by William T. Collins,[14] Jun. Am. Soc.

TABLE 7.—SUMMARY OF DISTRIBUTION COEFFICIENTS FOR 12-HOUR PERIODS; LICKING RIVER AT TOBOSO, OHIO; 672 SQUARE MILES

Date	Precipitation*	Surface run-off*	DISTRIBUTION COEFFICIENTS, BY PERIODS, IN PERCENTAGE OF SURFACE RUN-OFF													
			1	2	3	4	5	6	7	8	9	10	11	12	13	14
(1935)																
August 6	....	....	0.0	15.0	28.0	27.0	14.0	8.0	4.0	2.0	1.0	1.0	..	..	..	..
(1937)																
January 14	1.58	1.09	0.0	18.0	32.0	25.0	8.0	6.0	3.0	2.0	2.0	2.0	1.0	1.0	..	..
January 17	0.73	0.58	1.0	26.0	34.0	14.0	7.0	5.0	3.0	3.0	2.0	2.0	1.0	1.0	1.0	..
January 21	1.58	1.03	1.0	22.0	33.0	17.0	8.0	5.0	4.0	3.0	2.0	2.0	1.0	1.0	1.0	..
January 24	1.76	1.45	1.0	22.0	31.0	19.0	8.0	4.0	3.0	3.0	2.0	2.0	2.0	1.0	1.0	1.0
June 21	3.73†	2.11	6.4	25.6	30.0	17.8	8.2	5.0	3.7	2.5	0.8	..	..	..	..	..
December 17	2.65‡	0.59	4.9	25.8	30.1	19.2	10.2	5.0	2.4	1.3	0.7	0.4	..	..	..	..
(1938)																
March 13	1.64§	0.88	7.7	25.7	31.7	19.8	6.9	4.3	2.1	1.2	0.6	..	..	..	..	..
April 6	1.56‖	0.83	1.3	24.2	33.7	17.4	10.0	8.2	3.9	1.3	..	..	..	..	..	..
(1939)																
January 29	1.59‖	0.80	2.7	24.8	31.0	23.1	10.1	4.5	2.0	1.1	0.6	..	..	..	..	..
Average	....	....	2.6	22.9	31.5	19.9	9.1	5.5	3.1	2.0	1.2	0.9	0.5	0.4	0.3	0.1

* In inches. † Precipitation heaviest in the south-southeast. ‡ Precipitation heaviest in the southeast. § Precipitation heaviest in the central part. ‖ Precipitation somewhat heavier in the north.

---

[14] "Runoff Distribution Graphs from Precipitation Occurring in More Than One Time Unit," by William T. Collins, *Civil Engineering*, September, 1939, Vol. 9, p. 559.

C. E. Frequently it is plotted as a line connecting the points representing the coefficients, in which case its shape and consequent physical significance are lost.

Fig. 6 is particularly good in demonstrating, within a given region and general set of conditions, the effect of size upon the percentage of peak surface run-off. Fig. 7 is likewise very good; but as the author suggests, the values plotted involve a personal element in the selection of the terminal point of surface run-off and the resultant width of base of the distribution graph. The graph shows the probable trend of the effect of cover, but because so few data are given on factors such as slope and shape, it is not conclusive.

Mr. Brater would have done well to have extended his studies for Fig. 9 one step further and made determinations of infiltration rates. His statement under "Run-Off Coefficients and Infiltration Capacity"—"Graphs such as these suggest that the pluviagraph provides a means of tracing the quantitative changes in infiltration capacity [rate] * * * during continuous rainfall * * *" —is the most noteworthy in the paper. Had the durations of the successive rains (in excess of the infiltration rate) been given, with quantity of rainfall for the same periods, the reader would have been enabled to do this for himself. It is possible that rainfall intensities in some of the cases cited were just in excess of infiltration rates, resulting in the small percentages of run-off noted.

The reference to the shape of the curves of Fig. 9, as a key to the solution of water-shed problems in general, calls to mind a study[15] which showed that coefficients for a given small water-shed, plotted against a quantity of rainfall, scatter over a considerable area.

As for the use of the distribution graph for forecasting flood flows, this is already being done extensively for primary water-sheds and for sub-water-sheds in connection with the operation of the Muskingum water-shed flood-control reservoirs in Ohio, as well as in the Tennessee Valley, the State of Pennsylvania,[16] and elsewhere. Depletion curves of ground-water flow are in use in many offices[17] for determining ground-water flow during periods of run-off, as well as between rains.

On various water-sheds, there is about every conceivable variation of size, slope, soil type, soil profile, soil condition, drainage density, cover, and a real distribution and intensity of rainfall, many of which factors are variable with regard to time as well as with regard to location. These factors are further thrown together in an infinite number of combinations. As far as empirical means are concerned, in addition to the unit graph method and as adjuncts to it, the methods described by R. L. Gregory and C. E. Arnold,[18] Assoc. Members, Am. Soc. C. E., by W. W. Horner, M. Am. Soc. C. E., and F. L. Flynt,[19] Assoc. M. Am. Soc. C. E., and others, are helpful.

---

[15] "Modifications of the Index-Area Principle and the Anticipated Application of the Principle to Muskingum River Flood Control," by Waldo E. Smith, *Transactions*, Am. Geophysical Union, Part I, 1938, p. 455.

[16] "Synthetic Unit Graphs," by Franklin F. Snyder, Jun. Am. Soc. C. E., *loc. cit.*, 1938, p. 447.

[17] "The Structure of Discharge-Recession Curves," by B. S. Barnes, Assoc. M. Am. Soc. C. E., *loc. cit.*, Part IV, 1939, p. 721; also, "A Conception of Run-Off Phenomena," by Franklin F. Snyder, Jun. Am. Soc. C. E., *loc. cit.*, p. 725.

[18] "Run-Off—Rational Run-Off Formulas," by R. L. Gregory and C. E. Arnold, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1038.

[19] "Relation Between Rainfall and Run-Off from Small Urban Areas," by W. W. Horner and F. L. Flynt, *loc. cit.*, Vol. 101 (1936), p. 140.

Attempts have been made to determine, by analysis of the basin, the influence of each factor; but the great difficulty is that it is not possible to operate (as in a laboratory) to hold all variables constant, except one, and to study the effects of each one in turn. Possibly the best hope for an analytical solution is to plot run-off data with different slopes, soil types, cover, etc., each plot being a relatively homogeneous, small area; and then, after determining the extent to which each factor enters into the heterogeneous water-shed, to integrate, in effect, all of these values. This has been done with success, to the writer's knowledge, to the point that the correct volume or total run-off has been determined closely. The carrying of this flow through the tortuous ways of overland and subsurface flow, and channel and valley storage, to the gaging station is similarly under study with results, so far as the writer is apprised, as yet inconclusive. However, the number of variables involved and the evaluation of these variables even on a small water-shed suggest the difficulty in (and even the uncertainty of) the ultimate solution of the problems involved as suggested by the author. The distribution-graph method, as such, is probably not sufficiently refined to be of service in connection with such a solution if and when it is found. In the meantime, it serves as a very handy tool.

E. F. BRATER,[20] JUN. AM. SOC. C. E. (by letter).—The writer is gratified by the stimulating comments contributed by the various discussers. Some of the discussions provide additional valuable data. Several points of controversial nature were raised and discussed, and the writer wishes to add his views on these several points. In general, it should be said that the analysis was based on a study of twenty-two small southern Appalachian water-sheds. The results reflect the influence of the topography, soil, and vegetation of this region. It must be recognized, therefore, that some of the conclusions may have to be modified if they are to be applied to other localities.

The main purpose of the paper was to study the application of the unit hydrograph to small water-sheds. It seemed desirable to keep the discussion, as far as possible, in terms of well-known and accurately definable quantities. For this reason, run-off coefficients were used instead of the newer conception of infiltration capacity. The run-off coefficient is simply the percentage of a given rain which appears as flood flow in the stream. Its meaning is clear and straightforward and its value may be determined easily, within reasonable limits of accuracy.

Mr. Sherman suggests, however, that the paper would have been more valuable if infiltration capacities had been used. He has computed the infiltration capacity by the direct method for a number of the hydrographs given in the paper and has presented the results along with values from other parts of the United States.

The writer does not question that the infiltration capacity as defined by Robert E. Horton,[7] M. Am. Soc. C. E.—namely, "The maximum rate at which a soil, when in a given condition, can absorb moisture"—is a meaningful and important soil characteristic. It should be recognized, however, that the so-called infiltration capacity computed by the direct method is not the quantity defined by Mr. Horton and does not have any real physical significance. This may be rather forcefully illustrated by considering the three values which Mr. Sherman has obtained for Coweeta 7. It will be shown that there was no overland flow during these three rains, and therefore that the actual infiltration capacity was not reached.

---

[20] Instr., Civ. Eng. Dept., Univ. of Michigan, Ann Arbor, Mich.

The length of stream of Coweeta 7 is 11 100 ft. The width of the flowing stream during a heavy rain varies from about 20 ft to a mere trickle, but the gravelly bed of the stream plus the adjacent banks embraces a strip 20 to 30 ft wide over the length of the stream. Therefore, if all of the flood flow from any given rain can be accounted for by 100% run-off from a strip of land 11 100 ft long and not in excess of 25 to 30 ft wide, it must be concluded that no overland flow occurred. A rainfall on this water-shed of 0.92 in. on June 12, 1936, produced a total flood flow of 6 850 cu ft. This is equal to a volume of water 0.92 in. deep on an area 8.1 ft wide and 11 100 ft long. An 8.1-ft strip barely accounts for the actual area of the flowing stream, so there could not possibly have been any overland flow.

Similarly, rains of 3.98 in. on April 1, 1936, and 5.95 in. on September 29, 1936, produced volumes of flood flow of 50 000 cu ft and 64 500 cu ft, respectively. The widths of 11 100-ft strips from which 100% run-off would have to occur to produce these flood flows are 13.6 and 11.7 ft for these two storms. Here again there was no overland flow.

Since no overland flow occurred, the infiltration rates were simply equal to the rainfall rates, and the infiltration capacity, as defined by Mr. Horton, was not reached; yet Mr. Sherman has computed values of infiltration capacity of 2.35, 2.76, and 2.10 in. per hr, respectively, for these three rains. It is believed that the actual infiltration capacity of Coweeta 7 is more than 5 in. per hr. Overland flow has never been observed from any of the heavily forested areas in this region, except during severe cloudbursts.

It would be highly desirable to be able to compute the actual average infiltration capacity of a water-shed from the rainfall and run-off records. One of the requirements necessary to approach this ideal more closely would be to take into account the part of the water-shed from which 100% run-off occurs. This would include the areas of the lakes and streams as well as all impervious portions of the water-shed. The result of neglecting this factor is illustrated by the foregoing examples. Because of its variable nature, however, it would be difficult to estimate, accurately, the area of 100% run-off. For instance, during a prolonged, heavy rain, not only does the area of the permanent lakes and streams increase, but this area is frequently augmented by additional inundated areas. The error introduced by neglecting impervious areas causes computed values to be too small.

It should also be pointed out that even if the impervious areas and the areas of lakes and streams were carefully taken into account, the infiltration capacity computed by the direct method might still differ considerably from the actual infiltration capacity. This method of computation is based on the assumption that the rainfall excess is equal to the surface run-off. For this assumption to be true, it would be necessary for all the surface detention present at the end of each period of rainfall excess to be transported instantaneously into the flowing stream. This is not true, of course, since most of this surface detention has an additional opportunity of infiltering for periods varying from a few minutes to several hours. Therefore, an error is introduced, the magnitude of which depends on the variation in rainfall intensity. Another error which is inherent in this method of computing infiltration capacity results from the fact that depression storage is included with the total infiltration. All water left standing at the end of rainfall excess, whether it be in tiny puddles or good-sized lakes, is slowly dissipated as evaporation or infiltration. This process may continue for a number of days, and yet the direct method assumes all of this water to have been infiltered during the period of rainfall excess. This factor has little or no effect in the case of the water-sheds discussed in the writer's paper, but may be very important in flat or terraced water-sheds. The errors discussed in this paragraph are compensated for, to some extent, in the case of relatively impervious water-sheds, by residual, low-intensity rainfall which may follow the period of rainfall excess.

The values computed by this method may be very useful quantities for some purposes. It seems highly desirable, however, to avoid calling them infiltration capacities since that implies that they represent the maximum rates at which a given water-shed can absorb rainfall. For instance, they could be called infiltration indexes or rainfall excess indexes.

Mr. Snyder and Mr. Smith ask whether the values of duration of rainfall are corrected for initial lag. The initial lag is small on these streams, since the stream rise begins almost at the beginning of rainfall. The values given are corrected for any minor, low-intensity rainfall that may occur at the beginning and end of the storms. The period given is very nearly equal to the duration of run-off producing rain.

Mr. Sherman points out that the shape of the distribution graph is influenced by the rainfall pattern. Mr. Snyder and Mr. Smith note that the period of rise is not a constant quantity. These are pertinent observations because they emphasize the fact that the unit hydrograph is not a mathematically precise method for predicting the nature of flood flow but simply a workable one, sufficiently accurate for all practical purposes. The original study consisted of an analysis of seventy-eight storms on twenty-two water-sheds. The mean of the individual variations from the average period of rise on all of the water-sheds was about 25 per cent. Part of this variation is due to the presence or absence of a small fillet preceding the main portion of the rising hydrograph. The general shape of the hydrograph is not materially affected thereby.

Mr. Snyder concludes that this variation in the period of rise is due to the variation in the duration of rainfall. The writer indicated in the paper that the period of rise tends to remain constant despite variations in rainfall duration, provided the duration does not greatly exceed the period of rise. Another careful comparison, by the writer, of the period of rise with rainfall duration on all twenty-two of the water-sheds shows no evidence of a direct relationship between the two quantities. Mr. Snyder rightfully cites as evidence of the effect of rainfall duration on the period of rise the statements made by the writer concerning the method of breaking down continuous rainfall records into individual storms for use in reconstructing hydrographs from rainfall data. The paragraph to which Mr. Snyder refers reads in part as follows (see heading "Applications of the Distribution Graph: Application of the Pluviagraph"):

> "In general, the rain was divided according to changes in intensity. However, if any rate of rainfall continued for too great a time, it was found necessary to break it up arbitrarily into parts, especially if the rain was of low intensity. In the case of Copper Basin 2, and Coweeta 7, a half-hour was found to be an adequate unit of time into which to break these rains."

The writer did not, however, intend to imply that periods greater than 30 min would not produce adequate results because periods as long as 130 min were actually used.

Table 8 shows the manner in which the continuous rainfall records were broken down in order to reconstruct the hydrographs presented in Fig. 8. Although it may not be completely clear, due to the small scale of Fig. 8, the intervals chosen correspond to changes in rainfall intensity. The operation was performed without giving any attention to the period of rise. As a result, there is no regularity in the length of time intervals. It may be seen from Table 8 that the intervals vary from 15 min to 135 min. Since the actual hydrographs are quite faithfully reproduced by the reconstructed ones, despite this variation in the length of interval, it appears that the duration of rainfall has only a minor effect on the period of rise. There are other factors which cause the length of the period of rise to vary. The influence of the rainfall pattern was mentioned by Mr. Sherman. Other factors of importance in these small, steep water-sheds are the intensity at which rainfall begins, and the direction of approach of the rainstorm.

TABLE 8.—MANNER IN WHICH TWO CONTINUOUS RAINS WERE BROKEN DOWN FOR THE APPLICATION OF THE DISTRIBUTION GRAPH

(*a*) STORM OF APRIL 1 AND 2, 1936 (INTERVALS FROM 9:30 A.M. TO 5:00 A.M.)						(*b*) STORM OF SEPTEMBER 29 AND 30, 1936 (INTERVALS FROM 5:15 P.M. TO 4:15 A.M.)		
Time at beginning and end (1)	Duration, in minutes (2)	Precipitation, in inches (3)	Time at beginning and end (1)	Duration, in minutes (2)	Precipitation, in inches (3)	Time at beginning and end (1)	Duration, in minutes (2)	Precipitation, in inches (3)
9:30– 9:45	15	0.06	10:00–11:00	60	0.12	5:15– 6:45	90	0.32
9:45–10:15	30	0.14	11:00–11:45	45	0.04	6:45– 7:45	60	0.20
10:15–11:00	45	0.06	11:45–12:15	30	0.23	7:45– 9:00	75	0.20
11:00–12:45	105	0.06	12:15– 1:00	45	0.82	9:00–10:00	60	1.55
12:45– 2:30	105	0.18	1:00– 1:15	15	0.06	10:00–11:15	75	0.58
2:30– 3:00	30	0.05	1:15– 1:30	15	0.24	11:15–11:30	15	0.32
3:00– 4:00	60	0.06	1:30– 2:15	45	0.12	11:30–12:00	30	1.22
4:00– 4:45	45	0.08	2:15– 2:30	15	0.35	12:00– 1:00	60	0.14
4:45– 5:15	30	0.08	2:30– 3:15	45	0.06	1:00– 2:00	60	0.14
5:15– 5:45	30	0.08	3:15– 3:45	30	0.17	2:00– 2:45	45	0.47
5:45– 6:30	45	0.18	3:45– 4:30	45	0.17	2:45– 3:30	45	0.22
6:30– 7:00	30	0.09	4:30– 5:00	30	0.06	3:30– 4:15	45	0.25
7:00– 7:45	45	0.24	5:00–	....	0.08	4:15–	....	0.32
7:45–10:00	135	0.11	....	....	....	....	....	....

Mr. Smith isolates the following phrases as evidence of the relationship between duration of rainfall, within the period of rise, and the length of the period of rise: "that a 30-min interval was too long * * * whereas a 15-min interval produced satisfactory results." It is only necessary to read, in its entirety, the paragraph from which this quotation was taken to realize that the time interval referred to is simply the result of the number of divisions into which the unit hydrograph was separated, arbitrarily, in order to compute the distribution graph percentages. In reconstructing a hydrograph from rainfall data, these percentages are applied to the total rainfall to determine the portion contributed to stream flow in each corresponding interval. These portions are correctly plotted as blocks having a width equal to the time interval and height equal to the average rate of flow necessary to produce the given quantity of run-off. In order to construct a smooth curve through the tops of these blocks, they must be made reasonably numerous, especially when two or more of such unit hydrographs are superimposed upon one another. There can be no connection between this purely mechanical or drafting-board procedure and the period of rise of the unit hydrograph.

Mr. Smith makes the following statement in his discussion: "If rainfall of uniform intensity in excess of the infiltration rates continues for the duration of the distribution graph, the period of rise will be found to be about equal to the duration or base of the distribution graph." This statement is not consistent with the meaning of the period of rise as used by the writer. The hydrograph described by Mr. Smith is the combination of two or more unit hydrographs. Therefore, the rising part of the hydrograph consists of portions of two or more periods of rise, and should not be referred to as the period of rise.

Mr. Kazmann states that a decrease in run-off coefficient in the second of two consecutive rains on Bent Creek 3 might have been due to variations in rainfall intensity. The mass diagrams of rainfall for the two rains are quite similar; therefore, it is believed that the dry period of 1 hr which separated the rains permitted the soil to regain some of its capacity for the absorption of water.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2084

TENSION TESTS OF LARGE RIVETED JOINTS

By Raymond E. Davis,[1] and Glenn B. Woodruff,[2] Members, Am. Soc. C. E., and Harmer E. Davis,[3] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Charles F. Goodrich, Frederick P. Shearwood, Jonathan Jones, C. C. Winter, W. M. Wilson, J. M. Garrelts, A. E. Richard De Jonge, E. C. Hartmann and Marshall Holt, and Raymond E. Davis, Glenn B. Woodruff, and Harmer E. Davis.

Synopsis

An extensive series of tests on large riveted joints is described in this paper, including variations in length, type of splice, plate and rivet steel, pitch, and rivet pattern. The paper discloses the behavior of the various parts under load and provides information bearing on design practices. It was found that the usual assumption of equal distribution of load among rivets is satisfactory for design purposes, regardless of length of joint; and, that joints having as many rivets in the end row as in interior rows are as efficient as corresponding joints in which the net section is supposedly protected by using fewer rivets in the end row.

Introduction

The specifications commonly followed in the design of structural riveted connections are based largely on tests made at Cornell University, at Ithaca, N. Y., in 1904.[4] The joints tested at that time were relatively small, having a maximum of nine rivets in either end, and were made of softer plate, and rivet, steels than those now used even for ordinary bridges and buildings. Tests on joints fabricated with nickel-steel plates and rivets were conducted at the University of Illinois, at Urbana, Ill., in 1911;[5] but, again, there were no more than nine rivets in either end of the joint. Tests involving a larger

---

Note.—Published in May, 1939, *Proceedings*.

[1] Prof., Civ. Eng., Director, Eng. Materials Laboratory, Univ. of California, Berkeley, Calif.

[2] Cons. Civ. Engr., San Francisco, Calif.

[3] Associate Prof., Civ. Eng., Univ. of California, Berkeley, Calif.

[4] "Tests of Riveted Joints," a report of Committee on Iron and Steel Structures, *Proceedings*, A. R. E. A., Vol. 6, 1905, pp. 272–446.

[5] "Tests of Nickel-Steel Riveted Joints," by A. N. Talbot, Past-President and Hon. M. Am. Soc. C. E., and H. F. Moore, Eng. Experiment Station, *Bulletin No. 49*, University of Illinois, 1911.

number of rivets have been confined almost entirely to short joints of the type used in ship, and tank, construction.[6, 7] Experimental data on the behavior of long joints are rare, although theoretical analysis indicates that increasing the number of transverse rivet rows in each end above, say, five results in lower efficiency.

In connection with the design and construction of the San Francisco-Oakland Bay Bridge, it was found possible, in 1934, to begin two groups of tests with the object of securing information on the behavior of long riveted joints containing carbon and alloy steels and on related problems. One group, consisting of fatigue tests of relatively small joints, was conducted at the University of Illinois.[8] The other group, comprising static tension tests of large joints, was conducted at the University of California; these tests are described herein.

The program of static tension tests was planned to secure information on the following points: (1) The strength of large joints containing as many as twenty-eight transverse rows of rivets; (2) The relative behavior of joints having rivets of carbon or manganese steel; (3) The relative behavior of joints having plates of carbon, silicon, or nickel steel; (4) The relative merit of butt and shingle splices; (5) The partition of load among the transverse rows of rivets; (6) The effective net section of riveted plates; (7) Slip phenomena in large joints; and, (8) The proper design relation between tensile stress in plates and shearing stress in rivets. Throughout the tests, effort was made to secure data as to the action of the joints within the working range.

The abbreviation "kip" used throughout this paper denotes "kilo-pounds"; that is, 1 000 lb.

The Action of Riveted Joints

As an aid to the interpretation of test results, it is desirable to review certain general concepts of the action of riveted joints. These views are derived in part from theoretical considerations and in part from the results of these tests.

If perfect fabrication could be attained, the following conditions would exist: (*a*) Plates absolutely flat; (*b*) faying surfaces in contact at time of riveting; (*c*) holes matched perfectly; (*d*) at time of driving, holes completely filled by rivets; and (*e*) after cooling, rivets concentric with holes, with annular space around each rivet. During cooling, owing to the effort of the rivets to contract longitudinally against the restraint of the plates, tension is developed in the shank of the rivets. Lateral contraction of the rivet shank is caused in part by thermal contraction and in part by the Poisson-ratio effect due to rivet tension. The pressure between the plates caused by rivet tension results in a frictional resistance of the plates against slipping.

The behavior, under load, of a perfectly fabricated, two-rivet joint such as that shown in Fig. 1(*a*) may be considered in four stages: In the first stage static friction prevents slip; in the second stage the load is greater than the static friction, and the joint slips until the rivets come into bearing; in the

---

[6] "An Investigation of the Behavior and of the Ultimate Strength of Riveted Joints Under Load," by E. L. Gayhart, *Transactions*, Soc. of Naval Architects and Marine Engrs., Vol. 34, 1926, p. 55.

[7] "Tests of Joints in Wide Plates," by Wilbur M. Wilson, M. Am. Soc. C. E., James Mather, and Charles O. Harris, Eng. Experiment Station, *Bulletin No. 239*, Univ. of Illinois, 1931.

[8] "Fatigue Tests of Riveted Joints," by Wilbur M. Wilson and Frank P. Thomas, Eng. Experiment Station, *Bulletin No. 302*, Univ. of Illinois, 1938; see also "Fatigue Tests of Riveted Joints," by Wilbur M. Wilson, *Civil Engineering*, Vol. 8, August, 1938, pp. 513–516.

third stage, rivets and plates deform elastically so that the load-slip relation is linear; and, in the fourth stage, yielding of plates, rivets, or both, occurs until either plate fracture or complete shearing of the rivets results. This behavior is indicated by the diagrams of Fig. 1(*b*), which are reproduced from typical results of pilot tests on two-rivet joints tested at the University of Illinois[8] in connection with this investigation. The range of load over which Stage I extends is affected by the amount of friction between the plates. The friction depends on the tension in the rivets and the characteristics of the faying surfaces.

In Stage II, as slip progresses, the tilting of the rivets increases the pressure between the plates and hence increases the frictional resistance to movement. In lap joints, the bending due to eccentricity tends to cause the plates to peel apart at the ends and affects the amount of friction. These actions, together with the effect of non-uniform clearance around rivets, bending of the rivets, and distortion where contact is made between plates and rivets, extend Stage II over a range of load and cause the load-slip relation to take various forms.

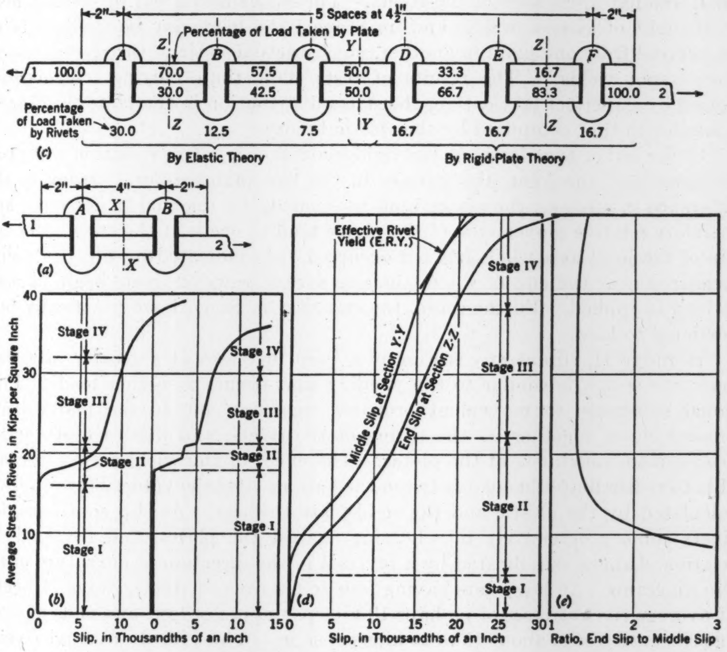


Fig. 1.—Action of Riveted Joints

Fig. 1(*c*) is a diagram of a joint with six rivets in line, showing the proportion of load transmitted by the rivets and the partition of load between plates, computed by the "elastic theory,"[9] which is based on the assumption that the rivets and plates deform elastically. Plate stresses in Fig. 1(*c*) are also computed on the assumption of equal load partition among rivets, which may be called the "rigid plate" theory. It is seen that by the elastic theory the end rivets take a relatively high percentage of load.

---

[9] "Work of Rivets in Riveted Joints," by A. Hrennikoff, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 99 (1934), pp. 437–489.

In Stage I, load is transferred from one plate to the other by friction. Stage I ends when the load is sufficient to cause movement of the joint as a whole—that is, when slip begins at the middle of the joint. This is shown by Fig. 1(*d*) in which the load-slip diagrams are generalized from the results of several tests in this investigation.

Although it is conceivable that the plates are in close contact over their entire faying surfaces, it seems likely that, in actual joints, owing to quilting action, the plates develop effective frictional resistance only over relatively small annular areas around the rivets. These localized areas of contact may be thought of as spot welds; and, in Stage I the load may be thought of as transferred from one plate to another, not by actual shear in the rivets, but by these "spot welds." The results of tests show that, barring irregularities caused by imperfect fabrication, the stress distribution in the plates in Stage I is similar to that computed by the elastic theory.

Under either the elastic or the rigid-plate theory, at any section except at the middle of the joint, the stresses in the two plates differ. Owing to the difference in stresses, the plates tend to elongate by unequal increments, and therefore relative movements of the plates tend to occur at the ends, although slip of the joint as a whole has not occurred. As indicated by Fig. 1(*d*), slips measured near the end of a joint having several rows of rivets begin as soon as load is applied. Furthermore, the end slips in Stage I are practically proportional to load.

In Stage II, slip at any section of a joint increases at a greater rate than load; the action is similar to the yielding that occurs at higher loads. Frictional resistance to movement probably increases due to the plates being pressed closer together by the tilting of the rivets, and due to tooth action arising from roughness of the plates. The effect of this yield action in Stage II is to redistribute the load between the various rivets to values between those calculated by the elastic and the rigid-plate theories. As slip continues, the rivets come progressively into bearing against the plates, and the load-slip relation changes as indicated by a reversal in the direction of curvature of the slip diagrams. In lap joints having four to six rows of rivets, Stage II begins at average rivet stresses of perhaps 10 kips per sq in., or less, and ends at average rivet stresses of about 15 to 25 kips per sq in. Thus, for the average riveted joint, the working range lies largely within Stage II.

In Stage III, where elastic action takes place with all rivets in full bearing, the ratio of end slip to middle slip is constant, as shown in Fig. 1(*e*), and remains so until plastic action begins. In this stage, the transfer of load at rivets again tends toward that called for by the elastic theory.

In Stage IV, yield occurs in the plates, rivets, or both; and slip again increases more rapidly than load. At the beginning, this increase in rate is caused in over-riveted joints by yielding of the plate material, and in under-riveted joints by yielding of the rivets; in either case there is no well-defined proportional limit. For other materials or structural members which may not have a well-defined yield, often a "useful limit point" (U.L.P.) is taken as that point on the stress-strain curve at which the rate of deformation is twice that below the proportional limit. Similarly, for the under-riveted joints of this investigation a load called the "effective rivet yield" (E.R.Y.) was taken as that where the rate of slip with respect to load was twice that in Stage III. In a balanced joint, there would be uncertainty as to whether the useful limit point or the effective rivet yield is reached first.

In the latter part of the yield range, rivets in the end rows of a number of the joints tested in this investigation sheared before the maximum load had been reached. The rivets in these joints were $\frac{7}{8}$ in. in diameter. Measurements of slip between plates at sections through the rivets indicated that premature rivet failure occurred when the detrusion was about 0.3 in.

It seems logical to suppose that in Stage IV, while the steels are deforming plastically, the partition of load among rivets tends to become equalized. The premature rivet failures caused by excessive strain cannot necessarily be interpreted as indicative of higher-than-average stresses in rivets near the ends of a joint.

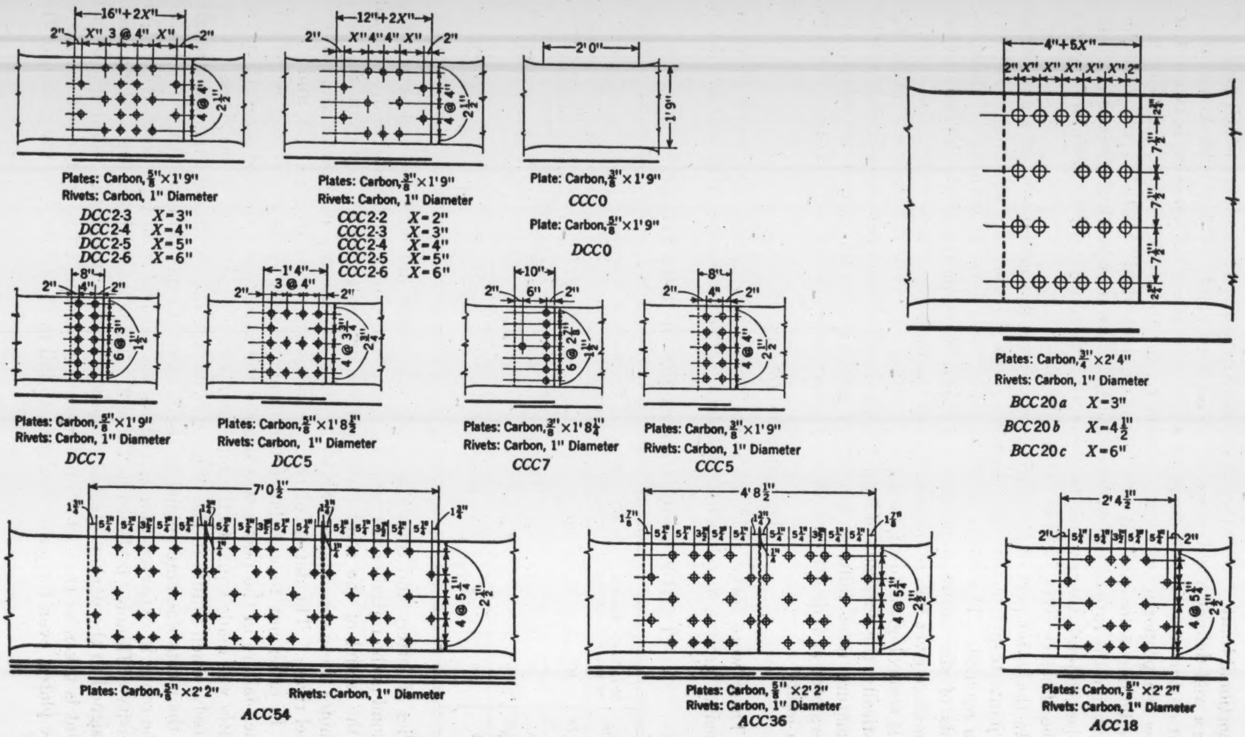
DESCRIPTION OF SPECIMENS

There were forty pairs of joints, which are herein considered in three principal groups, as shown in Table 1. The specimens are identified by a series of letters and numbers. The first letter indicates the test series; the second is the initial of the plate steel (carbon, silicon, or nickel); and, the third is the initial of the rivet steel (carbon or manganese). For example, ASC denotes those specimens of Series A which have silicon-steel plates and carbon-steel rivets. The details of the specimens are shown in Figs. 2, 3, and 4.

TABLE 1.—CLASSIFICATION OF TEST SPECIMENS

Series	Number of pairs	Figure number	Kind of splice	Number of plates	Designed to fail in:	Variables
C, D	16	2	Lap	1	Plate	Net width; end pitch; plate thickness.
A	15	2, 3, and 4	Lap	1, 2, and 3	Rivets	Size of joint; rivet steel; plate steel.
B	3	2	Lap	1	Rivets	Pitch.
F	6	4	Butt, shingle	1, 2, and 3	Plates	Size of joint; butt *vs.* shingle splices.

In order not to unbalance the design of the joints more than necessary, when failure in the plates was desired the maximum expected strength of the plates was made equal to the minimum expected strength of the rivets. Conversely, when failure in the rivets was desired the maximum expected strength of the rivets was made equal to the minimum expected strength of the plates. The range in expected unit strengths, established from pilot tests at the University of Illinois,[8] is shown in Table 2. Table 2 also shows the working stress assigned to the various steels. Unless otherwise stated, the working load of a joint is taken as that which produces the working stress at the net section of the plates, even for joints designed to fail in the rivets. In the design of the


16"+2X"
2"
X" 3 @ 4" X"
4 @ 4"
2½"
12"+2X"
X" 4" 4" X"
2'0"
1'9"
4"+5X"
2" X" X" X" X" X" 2"
7½"
2¾"
8"
4"
6 @ 3"
1½"
1'4"
3 @ 4"
4 @ 3¾"
2¾"
10"
6"
6 @ 2⅞"
4 @ 4"
7'0½"
1¾"
5¼"
3½"
1"
4 @ 5¼"
4'8½"
1⅞"
2'4½"

Plates: Carbon, ⅝"×1'9"
Rivets: Carbon, 1" Diameter
DCC2-3 X=3"
DCC2-4 X=4"
DCC2-5 X=5"
DCC2-6 X=6"

Plates: Carbon, ⅜"×1'9"
Rivets: Carbon, 1" Diameter
CCC2-2 X=2"
CCC2-3 X=3"
CCC2-4 X=4"
CCC2-5 X=5"
CCC2-6 X=6"

Plate: Carbon, ⅜"×1'9"
CCC0

Plate: Carbon, ⅝"×1'9"
DCC0

Plates: Carbon, ¾"×2'4"
Rivets: Carbon, 1" Diameter
BCC20a X=3"
BCC20b X=4½"
BCC20c X=6"

Plates: Carbon, ⅝"×1'9"
Rivets: Carbon, 1" Diameter
DCC7

Plates: Carbon, ⅝"×1'8½"
Rivets: Carbon, 1" Diameter
DCC5

Plates: Carbon, ⅜"×1'8¼"
Rivets: Carbon, 1" Diameter
CCC7

Plates: Carbon, ⅜"×1'9"
Rivets: Carbon, 1" Diameter
CCC5

Plates: Carbon, ⅝"×2'2" Rivets: Carbon, 1" Diameter
ACC54

Plates: Carbon, ⅝"×2'2"
Rivets: Carbon, 1" Diameter
ACC36

Plates: Carbon, ⅝"×2'2"
Rivets: Carbon, 1" Diameter
ACC18

FIG. 2.—DETAILS OF SPECIMENS; SERIES A, B, C, AND D

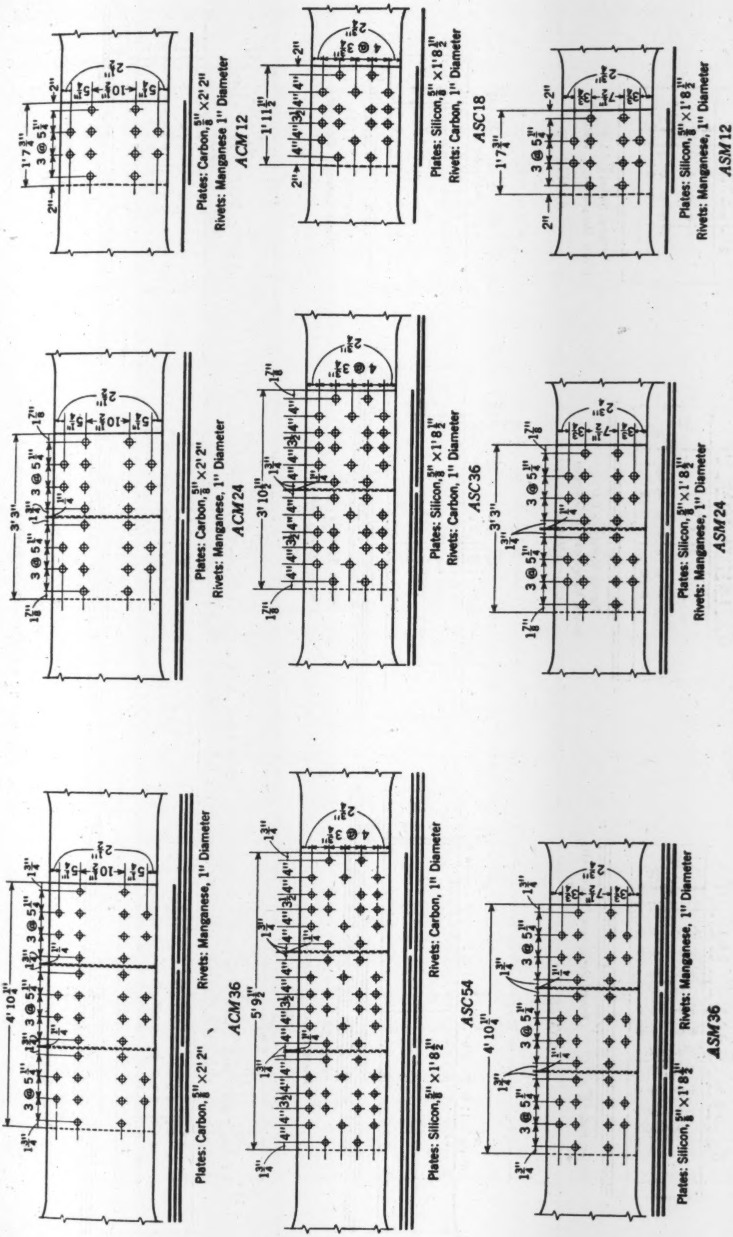
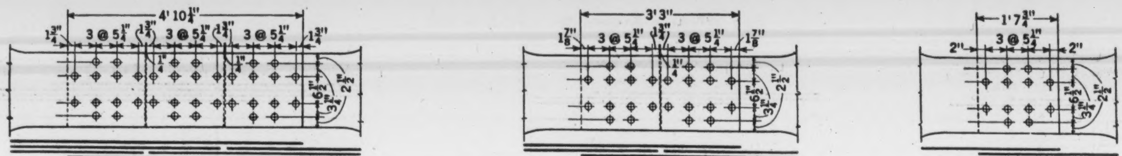

ACM12
Plates: Carbon, 5/8" × 2' 2"
Rivets: Manganese 1" Diameter
1' 7 3/4"
3 @ 5 1/4"
2"
2 1/4"
5 1/4"
10 1/2"
ASC18
Plates: Silicon, 5/8" × 1' 8 1/2"
Rivets: Carbon, 1" Diameter
1' 11 1/2"
4 @ 3 3/4"
ASM12
Plates: Silicon, 5/8" × 1' 8 1/2"
Rivets: Manganese, 1" Diameter
1' 7 3/4"
3 @ 5 1/4"
ACM24
Plates: Carbon, 5/8" × 2' 2"
Rivets: Manganese, 1" Diameter
3' 3"
1 7/8"
ASC36
Plates: Silicon, 5/8" × 1' 8 1/2"
Rivets: Carbon, 1" Diameter
3' 10 1/2"
ASM24
Plates: Silicon, 5/8" × 1' 8 1/2"
Rivets: Manganese, 1" Diameter
3' 3"
ACM36
Plates: Carbon, 5/8" × 2' 2"
Rivets: Manganese, 1" Diameter
4' 10 1/4"
1 3/4"
ASC54
Plates: Silicon, 5/8" × 1' 8 1/2"
Rivets: Carbon, 1" Diameter
5' 9 1/2"
ASM36
Plates: Silicon, 5/8" × 1' 8 1/2"
Rivets: Manganese, 1" Diameter
4' 10 1/4"

FIG. 3.—DETAILS OF SPECIMENS; SERIES A


4' 10¼"
1¾"
3 @ 5¼"
1¾"
3 @ 5¼"
1¾"
3 @ 5¼"
1¾"
¼"
6½"
3¼"
2½"
3' 3"
1⅞"
3 @ 5¼"
1¾"
3 @ 5¼"
1⅞"
1' 7¾"
2"
3 @ 5¼"
2"

Plates: Nickel, ⅝" × 1' 6" Rivets: Manganese, 1" Diameter

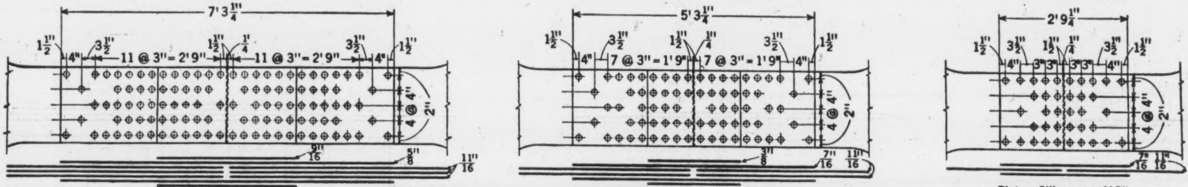
ANM36

Plates: Nickel, ⅝" × 1' 6"
Rivets: Manganese, 1" Diameter

ANM24

Plates: Nickel, ⅝" × 1' 6"
Rivets: Manganese, 1" Diameter

ANM12


7' 3¼"
1½"
4"
3½"
11 @ 3" = 2' 9"
1½"
¼"
11 @ 3" = 2' 9"
3½"
4"
1½"
4 @ 4"
2"
9/16"
⅝"
11/16"
5' 3¼"
7 @ 3" = 1' 9"
7 @ 3" = 1' 9"
⅜"
7/16"
11/16"
2' 9¼"
3"
3"

Plates: Silicon, t × 1' 8"
Rivets: Carbon, ⅞" Diameter

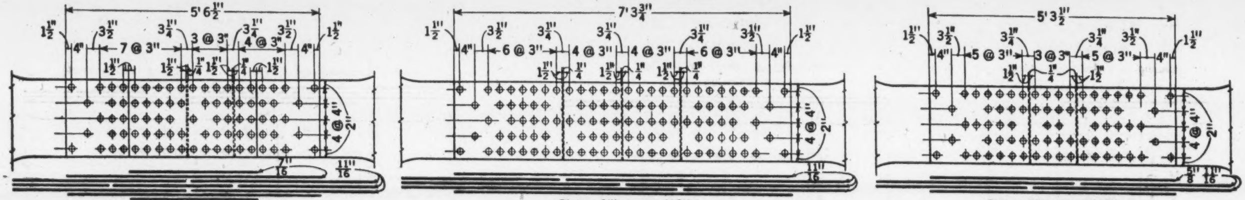
FSCC

Plates: Silicon, t × 1' 8"
Rivets: Carbon, ⅞" Diameter

FSCB

Plates: Silicon, t × 1' 8"
Rivets: Carbon, ⅞" Diameter

FSCA


5' 6½"
1½"
4"
3½"
7 @ 3"
3¼"
3 @ 3"
4 @ 3"
1½"
¼"
7/16"
11/16"
4 @ 4"
2"
7' 3¾"
6 @ 3"
4 @ 3"
4 @ 3"
6 @ 3"
5' 3½"
5 @ 3"
3 @ 3"
5 @ 3"
⅝"

Plates: Silicon, t × 1' 8"
Rivets: Carbon, ⅞" Diameter

FSCF

Plates: Silicon, t × 1' 8"
Rivets: Carbon, ⅞" Diameter

FSCE

Plates: Silicon, t × 1' 8"
Rivets: Carbon, ⅞" Diameter

FSCD

Fig. 4.—Details of Specimens; Series A and F

specimens, the net section was taken through the end row of rivets. In computing the area of net section, the diameter of the rivet hole was taken as the nominal diameter of the rivet, plus $\frac{1}{8}$ in. In computations involving rivet shear areas, the nominal diameter of the rivet was employed.

TABLE 2.—Range in Expected Unit Strength, and Working Stress

Steel	Unit Stress, in Kips per Square Inch					
	(a) Tension in Plate			(b) Shear in Rivets		
	Range of Unit Strength:		Working stress	Range of Unit Strength:		Working stress
	From	To		From	To	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Carbon	60	72	22	53	61	15
Silicon	77	93	28	....	....	....
Nickel	90	110	34	....	....	....
Manganese	....	....	....	71	92	20

Materials.—The steels were rolled at the Homestead plant of the Carnegie-Illinois Steel Corporation to the specifications of the San Francisco-Oakland Bay Bridge, which were similar to the corresponding specifications of the American Society for Testing Materials. All of the materials for the specimens met the specification requirements. The chemical properties of the steels were as listed in Table 3. The yield strength and ultimate strength of the mill coupons taken from plates are given subsequently in the tables showing the results of tests on joints (Tables 5, 9, and 17).

TABLE 3.—Chemical Properties of Steel
(Percentages of Total)

Chemical content	Plate Steel			Rivet Steel	
	Nickel	Silicon	Carbon	Carbon	Manganese
Carbon	0.29 to 0.30	0.30 to 0.37	0.17 to 0.22	0.13	0.22
Manganese	0.63 to 0.65	0.73 to 0.77	0.53 to 0.55	0.45	1.35
Silicon	0.19 to 0.24	0.23 to 0.31	....	....	....
Nickel	2.91 to 3.03	....	....	....	....
Phosphorus	0.027 to 0.035	0.020 to 0.028	0.017 to 0.031	0.020	0.026
Sulfur	0.028 to 0.033	0.031 to 0.047	0.031 to 0.041	0.033	0.021
Copper	0.22	0.25 to 0.27	0.24 to 0.30	0.26	0.30

The physical properties of the rivet steel are shown in Table 4. The specimens were fabricated at the Ambridge plant of the American Bridge Company in accordance with the best commercial practice. For the carbon-steel and the silicon-steel plates, the rivet holes were sub-punched with a die of diameter $\frac{1}{8}$ in. less than the nominal diameter of the rivet, and the rivet holes were reamed to a diameter $\frac{1}{16}$ in. larger than that of the rivet. For the nickel-steel plates, the solid plates were assembled and the rivet holes were drilled to a diameter $\frac{1}{16}$ in. larger than that of the rivet. Rivets were driven with a hydraulic machine.

The temperature of carbon-steel rivets at the start of driving ranged from 1 400° F to 1 800° F with an average of 1 570° F. For manganese-steel rivets the range was 1 400° F to 1 900° F and the average 1 680° F.

TABLE 4.—PHYSICAL PROPERTIES OF RIVET STEELS

Specimen	Property	Carbon steel		Manganese steel	Annealed manganese steel
Diameter of specimen, in inches		⅞	1	1	1
Rivet rod stock*	Percentage:				
	Elongation in 8 inches	31.0	33.0	24.1	27.6
	Reduction of area	58.0	58.8	55.5	62.7
	Strength, in Kips Per Square Inch:				
	Yield strength	39.9	39.6	54.6	49.2
	Tensile strength	58.0	57.2	81.0	74.7
Non-driven rivets†	Shearing strength in double shear in kips per square inch	....	42.1	54.5	....

* Tested at the mill. † Tested at the University of Illinois.

METHODS OF TESTING; INTERPRETATION OF DATA

Before each specimen was tested, gage lines for strain-gage observations were established at selected locations on the faces and edges of the plates; dials to determine the slip between adjacent plates were placed at one or more points along each edge; reference lines were scribed across each edge at intervals throughout the length; and, a coat of white brittle paint was applied to the faces of the plates.

The specimens were tested in the Southwark-Emery hydraulic testing machine at the University of California. Before the lower end of the specimen was gripped in the testing machine, initial strain-gage observations were taken. Up to a stress greater than the working stress but less than the yield stress, the load was applied in four or five increments, at a slow rate (approximately 500 lb per sq in. of gross section per minute). During the application of these load increments, observations of slip were made by means of the dial gages. After each increment of load had been applied, the load was maintained constant while strain-gage observations were made.

Above the highest load at which strains were measured, the load was applied at a faster rate, usually about 2 500 lb per sq in. of gross section per minute, up to the maximum load. During this final period of loading, the extent of plate scaling was recorded at intervals by means of photographs; written record was made of the progress of paint chipping from the rivet heads and of local failure of individual rivets or plate material; and, for some specimens the slip was observed by means of the dial gages up to about two-thirds of the estimated maximum load.

Determination of Yield Strength.—As load was applied, any marked yield or sudden slip in a specimen or its connections was reflected by a halt or lag in movement of the load-indicating gages of the hydraulic testing machine. For forty-two of the specimens, a semi-autographic record of the relation between load and time was obtained; for the other specimens, this relation was plotted from the log of the tests. On the load-time diagrams the successive loads at which various parts of the plates scaled were also noted. The yield load of the joint was taken as the first load at which the combined evidence of load lags and plate scaling was sufficiently marked so that it represented clearly the general behavior of the joint rather than minor sudden slips in the joint or in the pulling heads. Many of the joints did not exhibit a well-defined yield, and for seven of the joints no yield load could be assigned (Table 9, Specimens ASC54-2, ASM24-2, ASM36-2, ANM12-1, ANM24-2, ANM36-2, and BCC-20b-1). The yield loads determined as just described were probably close to those which might be determined from the load-elongation relations for the joint as a whole.

Attention is directed to the fact that herein the term "yield" does not indicate the same property as that reported for small joints by some investigators who considered yield as the load at which marked or sudden slip between plates first occurred.

Many of the joints with carbon-steel plates exhibited local scaling at loads appreciably below the yield load.

Usually, scaling of the plate began near rivets in the end row; and, for some specimens, the scaling through the yield range occurred largely outside the joint. It appears, therefore, that the yield strength, as determined, is a function of the stress conditions in the plate.

Determination of Effective Rivet Yield.—For joints failing in the rivets, it is believed that the load at which general rivet yielding occurs is of greater significance than the yield load based on plate action as just described. Herein a load called the "effective rivet yield" was determined, from the load-slip diagrams, as the point where the rate of slip with respect to load became twice the constant value it had attained while the joint was behaving elastically (Stage III, Fig. 1(*d*)). For the determination of this load, the slip measured at the middle of the joint was used; values obtained from slip measured at the ends of the joints were practically the same as those obtained from the middle slip. This load is analogous to the "useful limit point" used to define the practical proportional limit in materials or structural members which may not have a definite yield point. It was determined for all of the joints which had an effective rivet yield within the range of the slip measurements.

Use of Bending Frame.—The first three tests on lap joints disclosed that even at low loads the strains due to bending exceeded those due to direct stress, and that within the range of working loads the yield point of the steel was exceeded at critical locations. To reduce the bending within the length of the splice, the "bending frame" shown in Fig. 5 was used on most of the remaining lap joints (see Tables 5 and 9). After each increment of load had been applied to a specimen, the frame was adjusted until the bending within the length of the splice was reasonably small, as indicated by observations with a "curvemeter." The frame was removed after the last series of strain-gage observations had been completed.

Although the bending frame served its general purpose satisfactorily, the joints, as fabricated, were slightly warped and buckled both longitudinally and transversely, with the result that considerable local differences in bending stress were induced by loading and by application of the frame.

Determination of Curvature.—The curvometer used to determine curvature of the plate consisted of a rigid bar having at each end a leg which was applied to a gage point on the plate. At the center of the bar, a dial gage was mounted so that its plunger was in contact with the plate midway between the gage points. In order to convert the observed deflections into terms of bending strain at the surface of the plate, it was assumed that the elastic curve of the bent plate was a circular arc and that the neutral surface of the plate was midway between its faces.

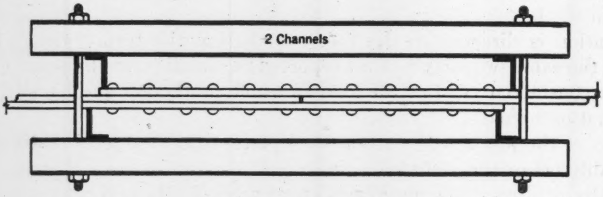


Fig. 5.—Bending Frame

Strain-Gage Measurements.—For most of the observations, 2.5-in., fulcrum-plate, strain gages were used; but for certain observations a gage length of 5, 10, or 20 in. was used. For a gage length of 2.5 in. the strain corresponding to the least reading by estimation was equivalent to a stress in the steel of 120 lb per sq in.

Many of the lap joints were curved while under load, even when the bending frame was used. The amount of bending within the length of the 2.5-in. strain-gage lines on the faces of the plates was determined, and curvature corrections were applied to the strain-gage observations.

With respect to the length of the joint, the longitudinal strain-gage lines were generally: (1) In the gross section of the plate outside of the splice; (2) in each space between rivet rows; and (3) spanning the rivet rows. Between rivet rows, for the single-plate and double-plate lap joints, in general, the gage lines were closely spaced on one-half of one face, and skeletonized on the remaining half of that face and on the opposite face. Corresponding gage lines were established on each edge of each plate. For the triple-plate lap joints and the butt joints, the observations were confined almost entirely to the edges of the plates. Transverse strains were measured only on the solid plates of Series C and D.

Slip Measurements.—The "slip," or relative movement of adjacent plates at a given section, was determined by means of dial gages, at the edges of the plates, by means of which movements of 0.0001 in. could be measured. Each gage on one plate was actuated by a lug attached to an adjacent plate. For very short, single-plate lap joints, the slip was measured at the middle of length; for the other single-plate lap joints, the slip at the ends was measured also. For multiple-plate joints, as a rule, the slip was measured at points where it was considered that minimum and maximum movement would occur in each single lap which composed a section of the joint as a whole. The slip gages were fastened to the plates at the middle of their thickness.

In many cases, sudden slips occurred during the progress of load application. In the majority of instances, after sudden slip occurred, the rate lagged temporarily, so that after an additional few thousand pounds had been applied the load-slip relation became coincident with what it would have been had the sudden slip not occurred. In these cases, the over-all characteristics were not altered by these erratic movements, and continuous curves are presented herein to show the general load-slip relation.

Although the middle slip is perhaps a characteristic of a joint acting as a whole, the end slip is considered significant with regard to the work done on the rivets in the end row. It is obvious that the distance of the end slip gage from the middle of the joint affects the ratio of end slip to middle slip. In making comparisons between joints, any differences in this distance should be taken into account.

Determination of Unit Elongation.—The elongation of a riveted joint under load includes both the strain in the plates and the slip between the plates. For use in determining the elongation, generally, a continuous chain of 2.5-in. and 10-in. strain-gage lines extended along each edge of the specimen. Furthermore, for some of the longer joints the over-all change in length was measured with an attached extensometer. For computing the unit elongation, the length of the joint was taken as the distance between ends of the plates forming the splice. The ratio of unit elongation of the joint as a whole to the unit elongation of the gross section of the plate, at the working load of the joint, is herein called the "elongation ratio." Although the unit elongation includes plastic deformation and slip as well as elastic deformation, it may be used as a basis for computing an effective modulus of elasticity of the joint.

The modulus of elasticity of the plates used in the joints was based upon strains measured on the plates outside the splice. The modulus generally ranged from 28 to 31 million lb per sq in.; the average was 29.7 million lb per sq in.

Computation of Joint Efficiency.—The efficiency was taken as the ratio of the stress in the gross section at ultimate load to the tensile strength of corresponding mill coupons. In this investigation, the efficiency was computed only for the specimens that failed in the plates.

Computation of Rivet Strength Ratio.—In so far as the performance of the rivet materials is concerned, a factor herein called the "rivet strength ratio" is of more significance than joint efficiency based on the plates which, for joints failing in the rivets, merely reflects the conditions of a particular design. The rivet strength ratio evaluates the performance of the rivets in a joint with respect to their potential shearing strength, as follows:

$$\text{Rivet strength ratio} = \frac{\text{Average shearing resistance per rivet in a joint}}{\text{Shearing strength of non-driven rivets in double shear}} \quad \cdots\cdots (1)$$

The ratio was computed only for the specimens that failed in the rivets.

TABLE 5.—TESTS ON LAP JOINTS DESIGNED

Specimen No. (for details, see Fig. 2)¶	Plate (Carbon Steel)				Rivets (One Inch: Carbon Steel)			Mill Coupons (Kips per Square Inch)		Specimens	
	Thickness, in inches	Width, in inches	Area, in square inches		Total number	Number in first row	End pitch, in inches	Yield strength	Tensile strength	End slip at rivet working load, in inches	Ratio of end slip to middle slip*
			Gross	Net (first row)							
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
CCC0-1	⅜	21	7.77	7.77	0	....	....	44.1	68.3	....	....
CCC0-2			7.77	7.77				....	....	....	....
Average			7.77	7.77				44.1	68.3	....	....
CCC2-2-1	⅜	21	7.80	6.96	12	2	2	42.9	69.3	0.0070	1.6†
CCC2-2-2			7.77	6.93				43.9	69.9	0.0080	1.2
Average			7.78	6.95				43.4	69.6	0.0075	....
CCC2-3-1	⅜	21	7.77	6.94	12	2	3	44.0	68.2	0.0107	1.2
CCC2-3-2			7.79	6.95				44.1	68.1	0.0105	1.3
Average			7.78	6.95				44.1	68.2	0.0106	1.3
CCC2-4-1	⅜	21	7.76	6.93	12	2	4	43.7	68.4	0.0105	1.5
CCC2-4-2			7.70	6.87				44.0	70.4	0.0091	1.4
Average			7.73	6.90				43.9	69.4	0.0098	1.5
CCC2-5-1	⅜	21	8.03	7.17	12	2	5	43.1	69.7	0.0122	1.5
CCC2-5-2			8.04	7.18				43.7	70.3	0.0047	2.0
Average			8.04	7.18				43.4	70.0	0.0085	1.8
CCC2-6-1	⅜	21	7.98	7.13	12	2	6	43.4	69.8	0.0128	1.7
CCC2-6-2			8.01	7.16				43.9	69.9	0.0086	1.5
Average			8.00	7.15				43.7	69.9	0.0107	1.6
CCC5-1	⅜	21	8.04	5.89	10	5	....	41.2	68.2	0.0080	....
CCC5-2			8.07	5.91				40.8	67.6	0.0098	....
Average			8.06	5.90				41.0	67.9	0.0089	....
CCC7-1	⅜	20½	7.80	4.77	8	7	....	42.3	67.4	0.0059	....
CCC7-2			7.84	4.79				42.1	68.4	0.0097	....
Average			7.82	4.78				42.2	67.9	0.0078	....
DCC0-1	⅝	21	13.12	13.12	0	....	....	38.1	68.1	....	....
DCC0-2			13.16	13.16				....	....	....	....
Average			13.14	13.14				38.1	68.1	....	....
DCC2-3-1	⅝	21	13.08	11.68	20	2	3	39.3	68.1	0.0072	2.0
DCC2-3-2			13.11	11.71				38.5	67.3	0.0034	2.1
Average			13.10	11.70				38.9	67.7	0.0053	2.1
DCC2-4-1	⅝	21	13.18	11.77	20	2	4	38.2	68.4	0.0078	2.7
DCC2-4-2			13.17	11.76				38.0	67.6	0.0077	3.0
DCC2-4-3			13.15	11.74				38.6	67.2	0.0080	2.5
DCC2-4-4			13.17	11.76				39.6	69.2	0.0070	1.8
Average			13.17	11.76				38.6	68.1	0.0076	2.5
DCC2-5-1	⅝	21	13.02	11.63	20	2	5	40.5	67.6	0.0082	2.3
DCC2-5-2			13.05	11.65				39.2	67.8	0.0058	2.2
Average			13.04	11.64				39.9	67.7	0.0070	2.3
DCC2-6-1	⅝	21	12.97	11.58	20	2	6	38.5	67.0	0.0070	2.6
DCC2-6-2			12.99	11.60				38.4	67.3	0.0075	2.1
Average			12.98	11.59				38.5	67.2	0.0073	2.4
DCC5-1	⅝	20½	12.79	9.28	16	5	....	38.5	67.9	0.0044	....
DCC5-2			12.80	9.29				37.5	67.3	0.0050	....
Average			12.80	9.29				38.0	67.6	0.0047	....
DCC7-1	⅝	21	13.09	8.18	14	7	....	38.3	67.7	0.0029	....
DCC7-2			13.09	8.18				37.5	67.8	0.0033	....
Average			13.09	8.18				37.9	67.8	0.0031	....

* At highest maintained load (25 kips per sq in. in plate, except as noted by reference (†)); at this load the ratios were practically constant. † At 20 kips per sq in. in plate. ¶ For all specimens CCC2 and DCC2, bending frame (Fig. 5) used up to stress of 25 kips per sq in. at net section of plate.

TO FAIL IN PLATES; SERIES C AND D

SPECIMENS (*Continued*)

Elongation ratio at plate working load‡	At Yield Load			At Ultimate Load					
	Stress in plate, in kips per square inch		Efficiency, percentages‖	Load, in kips	Stress in plate, in kips per square inch		Efficiency, percentages‖	Effective reduction in width due to rivets, in inches	
	Gross	Net			Gross	Net		By test (see Equation (2))	By formula (see Equation (3))
(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
....	37.4	37.4		491	63.2	63.2			
....	38.3	38.3		480	61.8	61.8			
....	37.9	37.9	86.0	485	62.5	62.5	91.5	....	....
1.33	38.2	42.8		399	51.2	57.3			
1.50	37.7	42.3		390	50.2	56.3			
1.42	38.0	42.6	87.5	394	50.7	56.8	72.8	5.71	4.87
1.63	36.7	41.1		388	49.9	55.9			
1.54	37.0	41.4		391	50.2	56.3			
1.59	36.9	41.3	83.7	389	50.1	56.1	73.4	5.59	3.94
1.38	38.7	43.3		400	51.5	57.7			
1.38	38.2	42.8		403	52.3	58.7			
1.38	38.5	43.0	87.7	401	52.0	58.2	75.0	5.25	2.62
1.41	37.5	42.0		428	53.3	59.7			
0.99	37.4	41.9		410	51.0	57.1			
1.20	37.5	42.0	86.4	419	52.2	58.4	74.4	5.38	2.25
1.31	36.8	41.2		420	52.6	58.9			
1.31	37.2	41.6		417	52.1	58.2			
1.31	37.0	41.4	84.7	418	52.4	58.6	74.9	5.27	2.25
2.72	35.5	48.4		423	52.6	71.8			
3.11	35.7	48.7		413	51.2	69.9			
2.92	35.6	48.6	86.8	418	51.9	70.9	76.5	4.94	5.63
2.46	35.9	58.7		356	45.6§	74.6§			
3.32	35.7	58.5		360	45.9§	75.2§			
2.89	35.8	58.6	84.8	358	45.8§	74.9§	67.5§	<6.60§	7.88
....	34.3	34.3		839	63.9	63.9			
....	35.3	35.3		841	63.9	63.9			
....	34.8	34.8	91.4	840	63.9	63.9	93.8	....	....
0.99	33.0	37.0		691	52.8	59.2			
0.87	33.6	37.6		703	53.6	60.0			
0.93	33.3	37.3	85.6	697	53.2	59.6	78.6	4.50	3.94
1.03	33.3	37.2		701	53.2	59.6			
0.93	33.0	36.9		698	53.0	59.4			
1.01	33.5	37.5		710	54.0	60.5			
1.05	33.7	37.8		717	54.4	61.0			
1.01	33.4	37.4	86.6	707	53.7	60.1	78.8	4.45	2.62
0.98	32.4	36.3		686	52.7	59.0			
1.00	33.3	37.3		695	53.3	59.7			
0.99	32.9	36.8	82.5	691	52.9	59.3	78.1	4.60	2.25
0.97	32.2	36.1		669	51.6	57.8			
0.93	32.8	36.7		684	52.7	59.0			
0.95	32.5	36.4	84.4	677	52.2	58.4	77.7	4.68	2.25
1.33	32.1	44.3		654	51.1	70.5			
1.36	32.1	44.2		671	52.4	72.2			
1.35	32.1	44.3	84.5	663	51.8	71.4	76.6	4.80	5.63
1.98	31.6	50.6		624	47.7§	76.3§			
2.18	31.7	50.7		622	47.5§	76.0§			
2.08	31.7	50.7	83.6	623	47.6§	76.2§	70.2§	<6.26§	7.88

‡ Ratio of unit elongation of joint to unit elongation of plate (gross section). § Failure in rivets.

‖ Ratio of stress in gross section of plate to corresponding strength of mill coupons.

TESTS OF LAP JOINTS DESIGNED TO FAIL IN PLATES; SERIES C AND D

The principal purpose of these tests on lap joints was to provide information regarding the efficiency of riveted tension members as influenced by: (1) The end pitch, or distance between the first and second transverse rows of rivets; (2) the effective net width of plate as controlled by the number of holes at a given section; and (3) the bearing stresses as controlled by thickness of plate.

The details of the specimens are shown in Fig. 2. All plates and rivets were of carbon steel. The rivets were 1 in. in diameter. The plates were nominally 21 in. wide, and were $\frac{3}{8}$ in. thick for Series C and $\frac{5}{8}$ in. thick for Series D. The $\frac{3}{8}$-in. material was thinner than required by standard specifications to develop the single-shear value of the rivets. Solid plates were included to provide data on the strength and efficiency (based on the strength of coupons) of wide plates without holes. The results of the tests are summarized in Table 5.

Tests of Solid Plates.—The efficiency at ultimate load, based on the strength of coupons tested at the mill, was 92% for the $\frac{3}{8}$-in. solid plates and 94% for the $\frac{5}{8}$-in. plates (Table 5). At the yield load, the efficiencies were 86% and 91%, respectively. These results are in agreement with those of other investigators.[7]

For loads up to 30 kips per sq in., longitudinal strains were the same at the middle of the plate as at the edge. For one $\frac{3}{8}$-in. specimen, the strains were measured at a stress of 35 kips per sq in., at which load the longitudinal strain was 11% greater at the middle of the plate than at the edge.

The specimens exhibited a marked yield at an average stress of 37.9 kips per sq in. for the $\frac{3}{8}$-in. plates and 34.8 kips per sq in. for the $\frac{5}{8}$-in. plates. The brittle paint on the specimens began to flake at stresses at, or slightly below, the yield point. This flaking exhibited the characteristic Lueders lines which started at the middle of the plate and progressed toward the edges as the loading was increased.

The plates failed by tearing apart at mid-length. For three specimens, a short transverse crack started at the middle of width of the plate; for the fourth specimen, the first crack started at one edge. The tendency for failure to start in the middle is attributed to the unequal distribution of longitudinal strain. The average elongation of the plates at failure was 29% in a 20-in. length. The reduction of area at failure was 28% for one $\frac{3}{8}$-in. plate and 35% for one $\frac{5}{8}$-in. plate.

Lateral and longitudinal strain-gage measurements were made on a grid of sixteen 5-in. squares which had been established on one $\frac{3}{8}$-in., and one $\frac{5}{8}$-in. solid plate. Within the working range, Poisson's ratio was found to be 0.28 for plates of both thicknesses. After failure, the ratio of lateral to longitudinal set was found to be 0.38 for the $\frac{3}{8}$-in. plate and 0.40 for the $\frac{5}{8}$-in. plate. These values are of interest in connection with the large transverse component of shear induced in widely spaced rivets in the end rows of long joints.

Effect of Number of Holes at a Given Section.—The effect of the number of rivets in the end row is shown in detail in Table 5 and summarized in Table 6. Of the riveted specimens, the highest total strength and efficiency was attained by the joints having five rivets in the end row at a gage distance of about 4 in. In the specimens having seven rivets in the end row at a gage distance of about 3 in., failure took place in the rivets rather than in the plates, although it is believed that plate failure was imminent at the time the joints ruptured.

For either of the sets of riveted joints, the calculated average stress in the net section at ultimate load was greater than that for the solid plates, as shown in Table 6. This difference is attributed in part to the difference between the assumed ($1\frac{1}{8}$ in.) and the actual ($1\frac{1}{16}$ in.) diameter of the rivet holes, and in part to what has been termed "the grooving effect" of notched plates.[10, 11] The joints having the lower bearing stresses ($\frac{5}{8}$-in. plates) exhibited the higher efficiencies.

TABLE 6.—EFFECT OF NUMBER OF RIVETS IN END ROW; SERIES C AND D

Plate thickness, in inches	NUMBER OF RIVETS		Symbol	Gage, in inches	Location of failure	Ratio of net to gross area	STRESS IN NET SECTION AT ULTIMATE LOAD		Efficiency percentages
	In first row	Total					Kips per square inch	Percentage of strength of solid plates	
3/8	0	0	CCC 0	....	Plate	1.00	62.5	100	91.5
	5	10	CCC 5	4	Plate	0.73	70.9	113	76.5
	7	8	CCC 7	$2\frac{7}{8}$	Rivets	0.61	74.9*	120*	67.5*
5/8	0	0	DCC 0	....	Plate	1.00	63.9	100	93.8
	5	16	DCC 5	$3\frac{1}{4}$	Plate	0.73	71.4	112	76.6
	7	14	DCC 7	3	Rivets	0.63	76.2*	119*	70.2*

* Strength of plates was higher than value shown, since failure was in rivets.

Effect of End Pitch.—The detailed test results for the group of joints to determine the effect of end pitch on strength and efficiency are shown in Table 5. All specimens failed in the plates along lines that passed through the rivet holes of the end row, as illustrated in Fig. 6. For the shorter end pitches, the fractures generally followed a zigzag course, passing through rivet holes in the first and second rows. The tendency for the fractures to extend through both end rows of rivets was less for the $\frac{3}{8}$-in. plates than for the $\frac{5}{8}$-in. plates, possibly because of higher bearing stresses in the $\frac{3}{8}$-in. plates. Furthermore, the $\frac{3}{8}$-in. plates exhibited a pronounced bulge at the end, opposite each end rivet, and the extent of this bulging was greater than that in any of the other specimens tested.

Fig. 7 shows a typical joint, with $\frac{5}{8}$-in. plates, after failure. The reduction in percentage of paint scaling from top to bottom of the splice reflects the transfer of stress from the near, to the far, plate. The characteristic Lueders lines where the mill scale and white paint have flaked off show the direction of maximum shear in the plate between rivets. The scaled areas at the front (upper) edges of the rivet heads indicate that the rivets have tilted forward as load was applied. At the bottom of the splice, it is seen that the slip between adjacent plates was about $\frac{3}{8}$ in. It is of interest to note also the extent to which the highly stressed section of the lower plate contracted laterally.

---

[10] "Johnson's Materials of Construction," by M. O. Withey and James Aston, John Wiley & Sons, Inc., New York, N. Y., Edition 7 (1930), pp. 663–666.

[11] "Strength of Materials," Part II, by S. Timoshenko, D. Van Nostrand Company, New York, N. Y., 1930, pp. 673–674.

The efficiencies of the joints at ultimate load, shown in Table 5, Column (20), are summarized in Table 7. The maximum difference in strength of joints with a given plate thickness was only about 5 per cent. Within this range, the $\frac{3}{8}$-in. specimens having staggered end rivets with an end pitch of 2 to 6 in. were less efficient than those with zero end pitch (five rivets in the end row). The $\frac{5}{8}$-in. specimens with staggered end rivets were somewhat more efficient than those with zero end pitch. If plates thicker than $\frac{5}{8}$ in. had been used, it is possible that the relative strength of the joints with staggered rivets would have been somewhat higher.

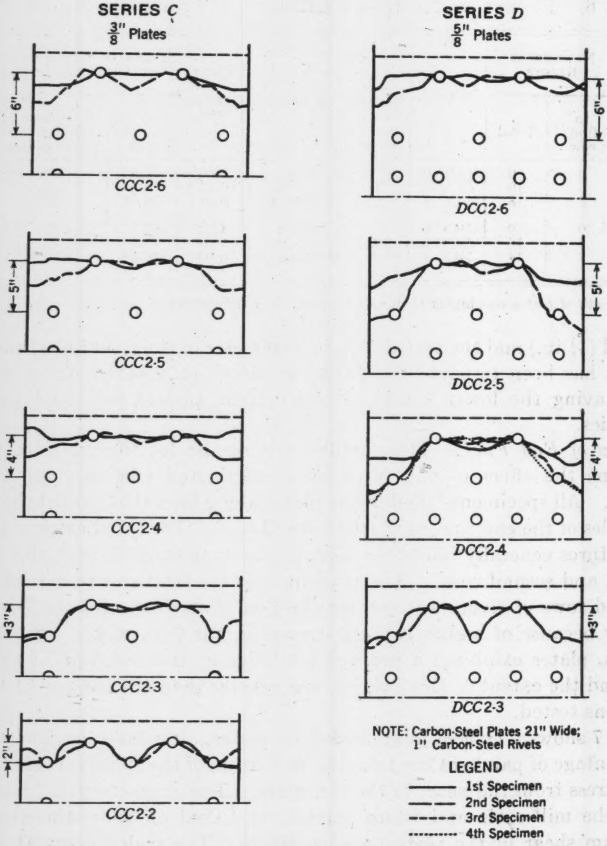

SERIES C
3/8" Plates
SERIES D
5/8" Plates
6"
CCC2-6
6"
DCC2-6
5"
CCC2-5
5"
DCC2-5
4"
CCC2-4
4"
DCC2-4
3"
CCC2-3
3"
DCC2-3
2"
CCC2-2
NOTE: Carbon-Steel Plates 21" Wide;
1" Carbon-Steel Rivets
LEGEND
1st Specimen
2nd Specimen
3rd Specimen
4th Specimen

FIG. 6.—LOCATION OF FRACTURE; SERIES C AND D; VARIABLE END PITCH

For both thicknesses of plate there was a decrease in the efficiency of the joints when the end pitch exceeded 4 in. A possible explanation is that the

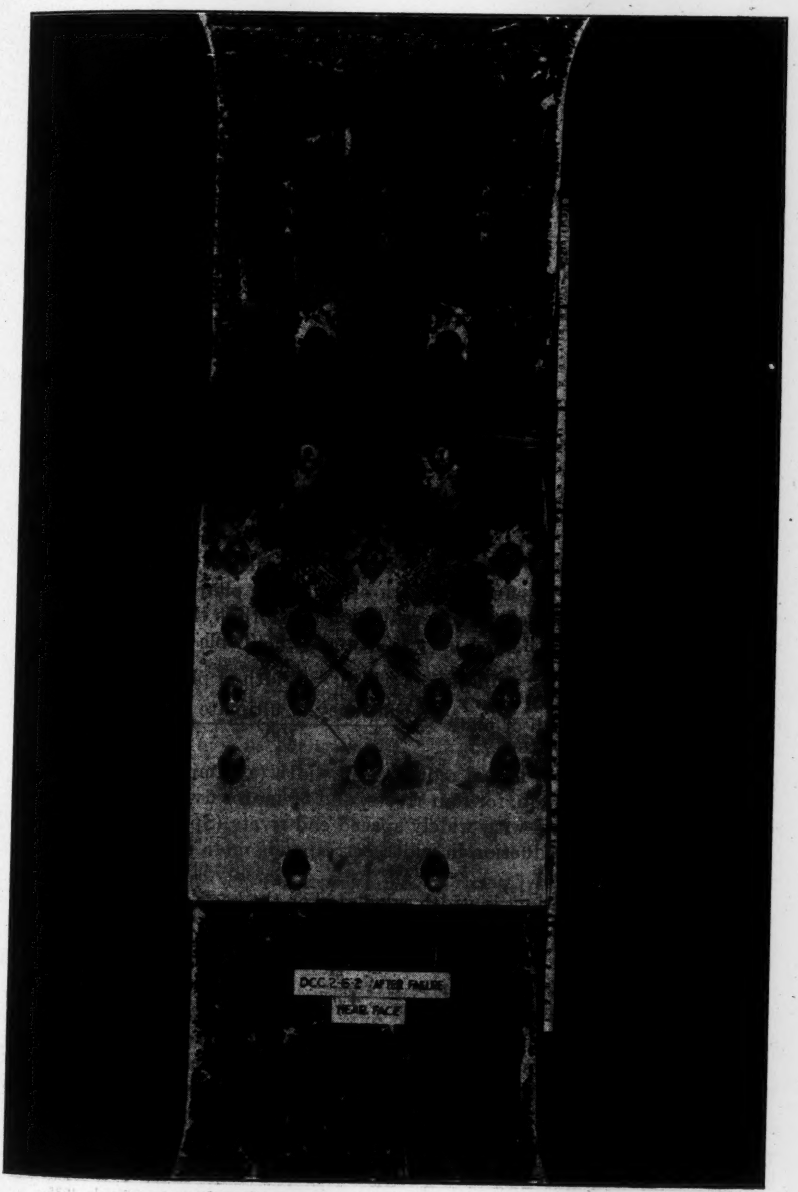

DCC 2-6-2 AFTER FAILURE
REAR FACE

FIG. 7.—TYPICAL PLATE FAILURE; SERIES D

high concentrations of stress at the extreme rivets of the longer joints precipitated plate failure. The efficiency of specimens in Series C ($\frac{3}{8}$-in. plates) was less than that for specimens in Series D ($\frac{5}{8}$-in. plates), possibly because of the higher bearing stresses in the $\frac{3}{8}$-in. specimens.

TABLE 7.—EFFECT OF END PITCH; SERIES C AND D

Series	Plate thickness, in inches	NUMBER OF RIVETS		PERCENTAGE EFFICIENCY OF JOINT, FOR THE FOLLOWING END PITCH					
		End pitch, 6 inches to 2 inches	End pitch, 0 inches*	End pitch, 6 inches	End pitch, 5 inches	End pitch, 4 inches	End pitch, 3 inches	End pitch, 2 inches	End pitch, 0 inches
CCC	3/8	12	10	74.9	74.4	75.0	73.4	72.8	76.5
DCC	5/8	20	16	77.7	78.1	78.8	78.6	....	76.6

* Five rivets in end row; other joints have two rivets in end row and three rivets in second row.

Effective Net Section.—The results of a study to determine the effective reduction in width of plates due to rivet holes are shown in Table 5, Column (21). The calculation is based on the assumption that the potential strength of a joint is the product of the strength of the mill coupons and the gross cross-sectional area of the plate; this assumption is not strictly true, as shown by the fact that the efficiency of the solid plates CCC0 and DCC0 was only 92% and 94% respectively. The effective width of a riveted plate is the gross width times the efficiency. The effective reduction in width is then the gross width minus the effective width; that is (see Column (21), Table 5)

$$\text{Effective reduction in width} = \text{Width of plate}$$
$$- \left(\text{Width of plate} \times \frac{\text{Efficiency at ultimate load}}{100} \right) \ldots\ldots\ldots (2)$$

Corresponding values of actual reduction in width (assuming $1\frac{1}{8}$-in. holes) are $2\frac{1}{4}$ in. for two rivets, $4\frac{1}{2}$ in. for four rivets, and $5\frac{5}{8}$ in. for five rivets. It is seen that for all of the joints with widely spaced end rivets (Specimens CCC2 and DCC2) the effective reduction in width corresponds more nearly to five rivets than to two rivets.

The effective reductions in width calculated on the basis of a typical formula used in current specifications[12] are shown in Column (22), Table 5. The formula is:

$$x = 1 - \frac{s^2}{4\,g\,h} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which x = the fraction of the hole to be deducted from b; s = the pitch, or the stagger; g = the gage of any two successive holes in a chain; and h = rivet diameter plus $\frac{1}{8}$ in. It is apparent that these requirements bear no relation to the results of the tests.

These results show that the use of fewer rivets in the end rows than in the interior rows does not make for the efficiencies that are calculated in accordance with common practice. The yield strengths (Table 5, Column (15)) show no large or consistent effect of end pitch or number of rivet holes at a given section on yield of joint.

---

[12] "Standard Specifications for Highway Bridges," Am. Assoc. of State Highway Officials, 1935.

TESTS OF LAP JOINTS FAILING IN RIVETS; SERIES A AND B

As shown in Figs. 2, 3, and 4, the specimens of Series A consisted of fifteen pairs, which included three different lengths of lap joint for five different combinations of rivet and plate material, as listed in Table 8. The joints were designed to fail in the rivets, and for a given length were designed to be of approximately equal strength. Lap, rather than butt, joints were used since within the capacity of the testing machine a greater variation in length of joint could be obtained by designing the specimens for single rather than double shear in the rivets. Lap joints also permitted the location of strain gages across the face of the main plates. The arrangement of the lap joints resulted in an eccentricity of $\frac{5}{8}$ in. for all joints. The uncertainties arising from, and the measures taken to correct for, the moments produced by this eccentricity have been mentioned previously herein (see headings, "Methods of Testing; Interpretation of Data: Use of Bending Frame," "Determination of Curvature" and "Strain-Gage Measurements").

TABLE 8.—CHARACTERISTICS OF SPECIMENS; SERIES A

Length of joint, in inches	PLATES (ALL $\frac{5}{8}$ INCH)			NUMBER OF RIVETS (ALL 1 INCH)	
	No.	Width in inches	Steel	Carbon steel	Manganese steel
20 to 29	1	26 20.5 18	Carbon Silicon Nickel	18 18	12 12 12
39 to 57	2	26 20.5 18	Carbon Silicon Nickel	36 36	24 24 24
58 to 85	3	26 20.5 18	Carbon Silicon Nickel	54 54	36 36 36

The specimens of Series A were designed to furnish the following information:

(1) Effect of length of joint upon average shearing resistance of rivets;
(2) Effect of plate material upon average shearing resistance of rivets;
(3) Comparative behavior of carbon-steel and manganese-steel rivets;
(4) Partition of rivet shear among the several transverse rows of rivets;
(5) Relation between the average shearing resistance of rivets and the shearing strength of non-driven rivets; and,
(6) Desirable ratio of tensile stress in plates to shearing stress in rivets, for use in design of large riveted connections.

The specimens of Series B consisted of three pairs of lap joints with $\frac{3}{4}$-in. by 28-in. carbon-steel plates and twenty-two 1-in. carbon-steel rivets arranged in full rows of four rivets (Fig. 2). The rivet pitch throughout the joint was

TABLE 9.—TESTS ON LAP JOINTS FAILING

Specimen No. (for details, see Figs. 2, 3, and 4)	Plates						Rivets (Diameter, One Inch)				Mill Coupons (Kips per Square Inch)		Specimens		
	Steel*	Width, in inches	Thickness, in inches	Number of plates	Area, in square inches		Steel*	Total number	Number in first row	Pitch, in inches	Yield strength	Tensile strength	Length of joint, in inches†	Bending frame (Fig. 5) used in working range	End slip at rivet working load, in inches
					Gross	Net (first row)									
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
ACC18-1	C	26	⅝	1	16.08	14.69	C	18	2	5¼	41.3	69.7	28½	No	0.0052
ACC18-2					16.13	14.73					39.5	68.2		Yes	0.0037
Average					16.11	14.71					40.4	69.0			0.0045
ACC36-1	C	26	⅝	2	32.32	29.52	C	36	2	5¼	38.8	68.8	56½	Yes	0.0038
ACC36-2					32.25	29.46					38.7	68.9		Yes	0.0057
Average					32.29	29.49					38.8	68.9			0.0048
ACC54-1	C	26	⅝	3	48.97	44.73	C	54	2	5¼	39.4	68.5	84½	No	0.0031
ACC54-2					48.99	44.75					39.4	67.9		No	0.0048
Average					48.98	44.74					39.4	68.2			0.0040
ASC18-1	S	20½	⅝	1	12.92	11.50	C	18	2	4	55.1	92.0	23½	Yes	0.0075
ASC18-2					12.93	11.51					54.5	90.8		Yes	0.0075
Average					12.93	11.51					54.8	91.4			0.0075
ASC36-1	S	20½	⅝	2	25.34	22.56	C	36	2	4	53.4	91.7	46½	Yes	0.0075
ASC36-2					25.70	22.88					51.3	90.8		Yes	0.0094
Average					25.52	22.72					52.4	91.3			0.0085
ASC54-1	S	20½	⅝	3	37.29	33.20	C	54	2	4	53.3	91.1	69½	No	0.0092
ASC54-2					38.69	34.45					51.7	91.2		No	0.0084
Average					37.99	33.83					52.5	91.2			0.0088
ACM12-1	C	26	⅝	1	16.18	14.78	M	12	2	5¼	40.5	68.6	19¾	No	0.0125
ACM12-2					16.20	14.80					38.6	68.7		Yes	0.0099
Average					16.19	14.79					39.6	68.7			0.0112
ACM24-1	C	26	⅝	2	32.44	29.64	M	24	2	5¼	38.6	69.4	39	Yes	0.0079
ACM24-2					32.44	29.63					38.8	69.3		Yes	0.0102
Average					32.44	29.64					38.7	69.4			0.0091
ACM36-1	C	26	⅝	3	48.67	44.46	M	36	2	5¼	38.6	68.6	58¼	No	0.0075
ACM36-2					48.98	44.74					39.0	67.0		No	0.0093
Average					48.83	44.60					38.8	67.8			0.0084
ASM12-1	S	20½	⅝	1	12.69	11.30	M	12	2	5¼	55.0	91.1	19¾	No	0.0125
ASM12-2					12.69	11.29					54.8	92.0		Yes	0.0123
Average					12.69	11.30					54.9	91.6			0.0124
ASM24-1	S	20½	⅝	2	25.11	22.35	M	24	2	5¼	51.1	92.0	39	Yes	0.0124
ASM24-2					25.26	22.49					53.3	92.0		Yes	0.0117
Average					25.19	22.42					52.3	92.0			0.0121
ASM36-1	S	20½	⅝	3	38.36	34.15	M	36	2	5¼	51.6	89.9	58¼	No	0.0109
ASM36-2					38.02	33.84					52.6	90.8		No	0.0132
Average					38.19	34.00					52.1	90.4			0.0121
ANM12-1	N	18	⅝	1	11.20	9.80	M	12	2	5¼	65.2	107.2	19¾	Yes	0.0112
ANM12-2					11.19	9.79					66.1	106.2		Yes	0.0115
Average					11.20	9.80					65.7	106.7			0.0114
ANM24-1	N	18	⅝	2	22.20	19.43	M	24	2	5¼	67.8	104.4	39	Yes	0.0092
ANM24-2					22.23	19.45					67.7	105.9		Yes	0.0123
Average					22.22	19.44					67.8	105.2			0.0108
ANM36-1	N	18	⅝	3	33.45	29.27	M	36	2	5¼	66.1	105.6	58¼	No	0.0101
ANM36-2					33.45	29.27					63.3	105.8		No	0.0143
Average					33.45	29.27					64.7	105.7			0.0122
BCC-20a-1	C	28	¾	1	20.66	17.34	C	22	4	3	35.6	68.8	19	Yes	0.0009
BCC-20a-2					20.68	17.36					35.2	68.5		Yes	0.0018
Average					20.67	17.35					35.4	68.7			0.0014
BCC-20b-1	C	28	¾	1	20.52	17.23	C	22	4	4½	37.0	66.0	26½	Yes	0.0021
BCC-20b-2					20.70	17.37					36.3	68.4		Yes	0.0018
Average					20.61	17.30					36.7	67.2			0.0020
BCC-20c-1	C	28	¾	1	20.49	17.19	C	22	4	6	37.5	65.4	34	Yes	0.0028
BCC-20c-2					20.49	17.20					36.9	65.8		Yes	0.0014
Average					20.49	17.20					37.2	65.6			0.0021
CCC7-1	C	20¼	⅜	1	7.80	4.77	C	8	7	6	42.3	67.4	10	No	0.0059
CCC7-2					7.84	4.79					42.1	68.4		No	0.0097
Average					7.82	4.78					42.2	67.9			0.0078
DCC7-1	C	21	⅝	1	13.09	8.18	C	14	7	4	38.3	67.7	8	No	0.0029
DCC7-2					13.09	8.18					37.5	67.8		No	0.0033
Average					13.09	8.18					37.9	67.8			0.0031

---

* C = carbon; S = silicon; M = manganese; and N = nickel.

† Measured from end to end of overlapping plates.

‡ In general, for the range in which this ratio was constant.

§ Ratio of unit elongation of joint to unit elongation of plates (gross section).

SPECIMENS (*Continued*)

Ratio of end slip to middle slip‡	Elongation ratio, at plate working load§	At effective rivet yield; shear in rivets		At Yield Load				At Ultimate Load				Number of rivets that failed before complete rupture
				Stress in plate, in kips per square inch		Shear in rivets			Shear in rivets			
		In kips per rivet	In kips per square inch	Gross	Net	In kips per rivet	In kips per square inch	Load, in kips	In kips per rivet	In kips per square inch	Ratio¶	
(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)
2.06	1.04			33.9	37.1	30.3	38.6	783	43.5	55.4		**
2.09	1.08			34.3	37.5	30.7	39.1	790	43.9	55.9		**
2.08	1.09	25.0	31.8	34.1	37.3	30.5	38.9	787	43.7	55.7	1.33	
....	1.13			31.4	34.4	28.2	35.9	1431	39.8	50.6		2+**
1.80	1.15			32.3	35.3	28.9	36.8	1460	40.6	51.6		5
1.80	1.14	25.4	32.3	31.9	34.9	28.6	36.4	1446	40.2	51.1	1.22	
1.40	1.03			33.3	36.4	30.2	33.4	2082	38.6	49.1		7
1.72	1.04			32.0	35.0	29.0	37.0	2157	39.9	50.9		13
1.56	1.04	27.8	35.4	32.7	35.7	29.6	37.7	2120	39.3	50.0	1.19	
1.44	1.42			44.0	49.4	31.5	40.2	780	43.3	55.2		0
1.48	1.28			43.6	48.9	31.3	39.8	769	42.7	54.4		0
1.46	1.35	....	....	43.8	49.2	31.4	40.0	775	43.0	54.8	1.30	
....	1.29			46.4	52.1	32.6	41.6	1529	42.5	54.1		**
1.82	1.18			46.0	51.7	32.8	41.8	1520	42.2	53.8		3
1.82	1.24	25.3	32.3	46.2	51.9	32.7	41.7	1525	42.4	54.0	1.29	
1.45	1.16			44.3	49.7	30.6	38.9	2168	40.1	51.1		1+**
1.35	1.19			‖	‖	‖	‖	2163	40.1	51.0		6
1.40	1.18	25.3	32.2	44.3	49.7	30.6	38.9	2166	40.1	51.1	1.21	
1.56	1.96			34.4	37.6	46.3	59.0	730	60.8	77.5		0
1.20	2.04			34.3	37.6	46.3	59.0	699	58.3	74.2		0
1.38	2.00	35.7	45.4	34.4	37.6	46.3	59.0	715	59.6	75.9	1.39	
....	1.51			32.0	35.0	43.3	55.0	1437	59.9	76.2		**
1.45	1.68			34.0	37.2	45.9	58.5	1483	61.8	78.6		2+**
1.45	1.60	38.8	49.3	33.0	36.1	44.6	56.8	1460	60.9	77.4	1.42	
1.11	1.33			33.7	36.9	45.6	58.1	1948	54.1	68.9		12
1.37	1.36			31.6	34.6	43.0	54.8	2085	57.9	73.8		2
1.24	1.35	37.5	47.8	32.7	35.8	44.3	56.5	2017	56.0	71.4	1.31	
1.51	1.63			44.9	50.4	47.4	60.4	680	56.7	72.2		0
1.24	1.73			44.0	49.4	46.5	59.2	683	56.9	72.5		0
1.38	1.68	35.6	45.3	44.5	49.9	47.0	59.8	682	56.8	72.4	1.33	
....	1.51			47.5	53.4	49.7	63.3	1417	59.0	75.2		0
1.55	1.37			‖	‖	‖	‖	1522	63.4	80.7		0
1.55	1.44	37.3	47.5	47.5	53.4	49.7	63.3	1470	61.2	78.0	1.43	
1.20	1.30			42.2	47.4	45.0	57.3	2075	57.6	73.4		2
1.45	1.36			‖	‖	‖	‖	2032	56.4	71.9		2
1.33	1.33	35.8	45.6	42.2	47.4	45.0	57.3	2054	57.0	72.7	1.34	
1.40	1.56			‖	‖	‖	‖	713	59.4	75.6		0
1.22	1.56			54.1	61.8	50.4	64.2	695	57.9	73.7		0
1.31	1.56	32.9	41.9	54.1	61.8	50.4	64.2	704	58.7	74.7	1.37	
....	1.39			57.7	65.9	53.4	67.9	1498	62.4	79.4		0
1.67	1.44			‖	‖	‖	‖	1430	59.6	75.9		0
1.67	1.42	36.7	46.7	57.7	65.9	53.4	67.9	1464	61.0	77.7	1.43	
1.14	1.23			55.6	63.5	51.7	65.8	2125	59.0	75.2		0
1.60	1.40			‖	‖	‖	‖	2155	59.9	76.2		0
1.37	1.32	38.8	49.3	55.6	63.5	51.7	65.8	2140	59.5	75.7	1.39	
1.44	1.15			32.8	39.0	30.8	39.2	975	44.3	56.4		0
1.35	1.27			33.4	39.8	31.4	40.0	975	44.3	56.4		0
1.40	1.21	....	....	33.1	39.4	31.1	39.6	975	44.3	56.4	1.34	
1.12	1.30			‖	‖	‖	‖	1003	45.6	58.0		1
1.19	1.23			33.2	39.6	31.3	39.8	1023	46.5	59.2		0
1.16	1.27	....	....	33.2	39.6	31.3	39.8	1013	46.1	58.6	1.40	
1.45	1.03			31.4	37.5	29.3	37.3	997	45.3	57.7		1
1.17	1.12			31.9	37.9	29.7	37.8	979	44.5	56.7		2
1.31	1.08	....	....	31.7	37.7	29.5	37.6	988	44.9	57.2	1.36	
....	2.46			35.9	58.8	35.0	44.6	356	44.5	56.7		2
....	3.32			35.7	58.4	35.0	44.6	360	45.0	57.2		3
....	2.89	....	....	35.8	58.6	35.0	44.6	358	44.8	57.0	1.36	
....	1.98			31.6	50.6	29.6	37.6	624	44.6	56.7		0
....	2.18			31.7	50.7	29.6	37.7	622	44.4	56.5		0
....	2.08	....	....	31.7	50.7	29.6	37.7	623	44.5	56.6	1.35	

‖ No yield load indicated by load rate data.

¶ Ratio of average rivet strength in the joint to strength of undriven rivets in double shear.

** One, two, or more rivets failed, as indicated by snaps in joint during loading; number could not be observed.

varied from 3 to 6 in., in order to determine the effect of pitch on strength of joint.

The results of tests on Series A and Series B are shown in Table 9. This table also gives the test results for two pairs of specimens of Series C and D (CCC7 and DCC7), which specimens failed in the rivets.

Series A

Strength.—The average strength of the rivets at ultimate load was obtained by dividing the total load by the total shearing area of rivets, based on the nominal diameter of the rivet. As between large and small joints of the same plate and rivet steel, the maximum difference in average rivet strength was only about 10%, as shown in Table 10, composed of selected data from Table 9.

TABLE 10.—RIVET STRENGTH OF LARGE AND SMALL JOINTS; SERIES A (Average Rivet Strength at Ultimate Load)

Plate steel	CARBON-STEEL RIVETS						MANGANESE-STEEL RIVETS					
	Kips per Square Inch			Percentages			Kips per Square Inch			Percentages		
	One plate	Two plates	Three plates	One plate	Two plates	Three plates	One plate	Two plates	Three plates	One plate	Two plates	Three plates
Carbon........	55.7	51.1	50.0	100	92	90	75.9	77.4	71.4	100	102	94
Silicon.........	54.8	54.0	51.1	100	99	93	72.4	78.0	72.7	100	108	100
Nickel.........	....	....	....	....	....	....	74.7	77.7	75.7	100	104	101

For the joints that had carbon-steel rivets in combination with a given plate material, the highest rivet strength was exhibited by the short (single-plate) joints and the lowest by the long (triple-plate) joints. However, for the joints that had manganese-steel rivets, the intermediate (double-plate) joints exhibited the highest rivet strength.

In order to investigate the reason for the somewhat higher strength of the manganese-steel rivets in the double-plate joints, two selected rivets from each of three joints were subjected to hardness tests. The rivets in the double-plate joint were found to be harder than those in the single-plate and triple-plate joints; furthermore, photomicrographs showed that the rivets in the double-plate joint had a finer grain structure. The difference may have been due to variations in cooling conditions. Another variable was the driving temperature, which averaged 100° F lower for the rivets of intermediate grip than for the other rivets.

The rivet strength ratio, or the ratio of the average shearing resistance per rivet in a joint to the shearing strength of non-driven rivets in double shear, is shown in Table 9, Column (28), and is summarized in Table 11.

The manganese-steel rivets exhibited higher rivet strength ratios than the carbon-steel rivets. As between the two kinds, the actual length of splice may have had some influence upon the rivet strength ratio; the joints with carbon-steel rivets contained two more rows of rivets per lap and were somewhat longer than the corresponding joints with manganese-steel rivets (see Figs. 2, 3, and 4; Series A). For these joints, all of which failed in the rivets, the kind of plate steel appeared to have no marked or consistent influence on the strength or the rivet strength ratio. As a group, the joints having manganese-steel rivets and nickel-steel plates exhibited the highest rivet strength ratios of any of the joints in the entire investigation.

TABLE 11.—Rivet Strength Ratios; Series A
(Percentages)

Plate Steel	Carbon-Steel Rivets			Manganese-Steel Rivets		
	One plate	Two plates	Three plates	One plate	Two plates	Three plates
Carbon	133	122	119	139	142	131
Silicon	130	129	121	133	143	134
Nickel	....	....	....	137	143	139

Effective Rivet Yield.—In an under-riveted joint the effective rivet yield has more significance, so far as rivet performance is concerned, than the yield strength of the joint. Average values of effective rivet yield, yield strength, and ultimate strength, taken from Table 9, are shown in Table 12. For a given rivet steel, the effective rivet yield did not vary greatly; and for both kinds of rivet steel, the effective rivet yield averaged 62% of the ultimate strength of rivets in the joints of this series (Series A). The yield strength of the joints varied with the type of plate steel as well as the type of rivet steel, so that the ratio of yield strength to ultimate strength was not constant.

TABLE 12.—Effective Rivet Yield, Yield Strength, and Ultimate Strength; Series A

Plate Steel	Carbon-Steel Rivets						Manganese-Steel Rivets					
	Rivet Stress, in Kips per Square Inch			Ratios of Rivet Shears			Rivet Stress, in Kips per Square Inch			Ratios of Rivet Shears		
	Effective rivet yield	Yield	Ultimate	$\frac{\text{Column (2)}}{\text{Column (3)}}$	$\frac{\text{Column (2)}}{\text{Column (4)}}$	$\frac{\text{Column (3)}}{\text{Column (4)}}$	Effective rivet yield	Yield	Ultimate	$\frac{\text{Column (8)}}{\text{Column (9)}}$	$\frac{\text{Column (8)}}{\text{Column (10)}}$	$\frac{\text{Column (9)}}{\text{Column (10)}}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Carbon	33.2	37.7	52.3	0.88	0.63	0.72	47.5	57.4	74.9	0.83	0.63	0.77
Silicon	32.3	40.2	53.3	0.80	0.61	0.76	46.1	60.1	74.4	0.77	0.62	0.81
Nickel	....	....	....	....	....	....	46.0	66.0	76.0	0.70	0.61	0.87
Average	32.8	39.0	52.8	0.84	0.62	0.74	46.5	61.2	75.1	0.76	0.62	0.82

The ratio of effective rivet yield to the yield strength and ultimate strength in tension of rivet-rod stock did not differ greatly as between the two kinds of

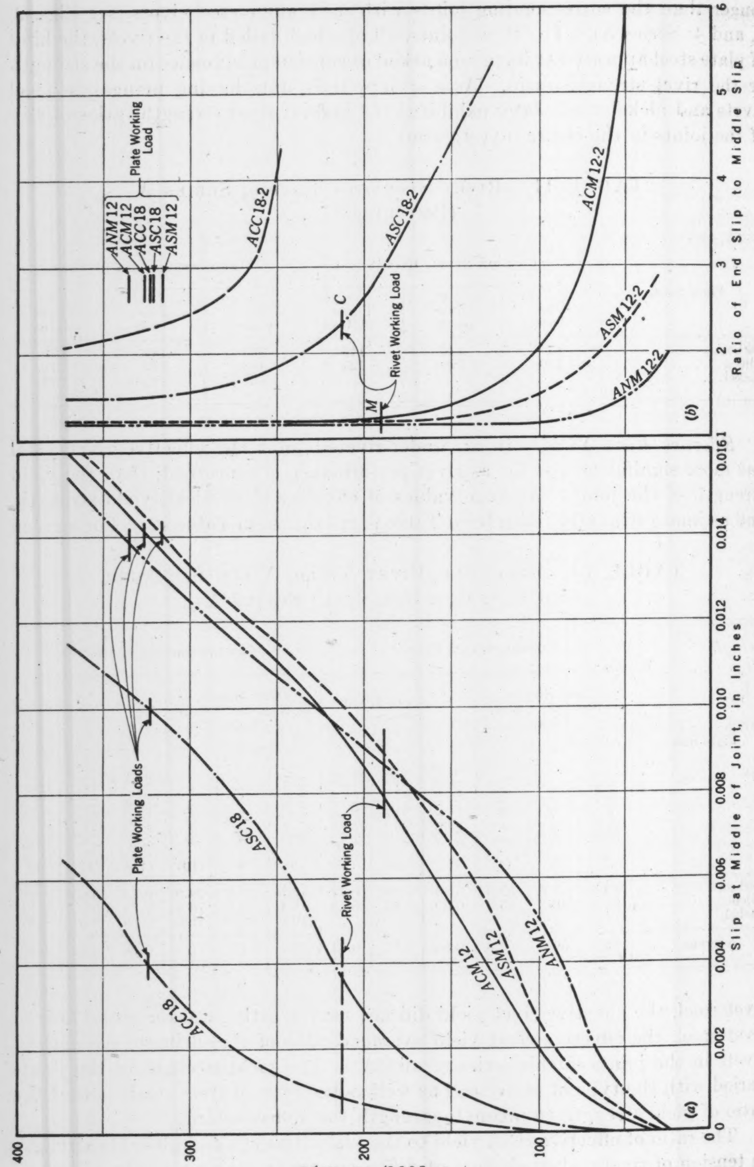


FIG. 8.—LOAD-SLIP RELATIONS FOR SINGLE-PLATE JOINTS OF SERIES A

rivet steel; for example:

Ratio	Carbon-steel rivets	Manganese-steel rivets
$\frac{\text{Effective rivet yield}}{\text{Yield strength in tension}}$	0.83	0.85
$\frac{\text{Effective rivet yield}}{\text{Tensile strength}}$	0.57	0.57

Slip.—The results of observations to determine slip for the joints of Series A are shown in Figs. 8 to 10. For the single-plate joints, Fig. 8(*a*) shows the

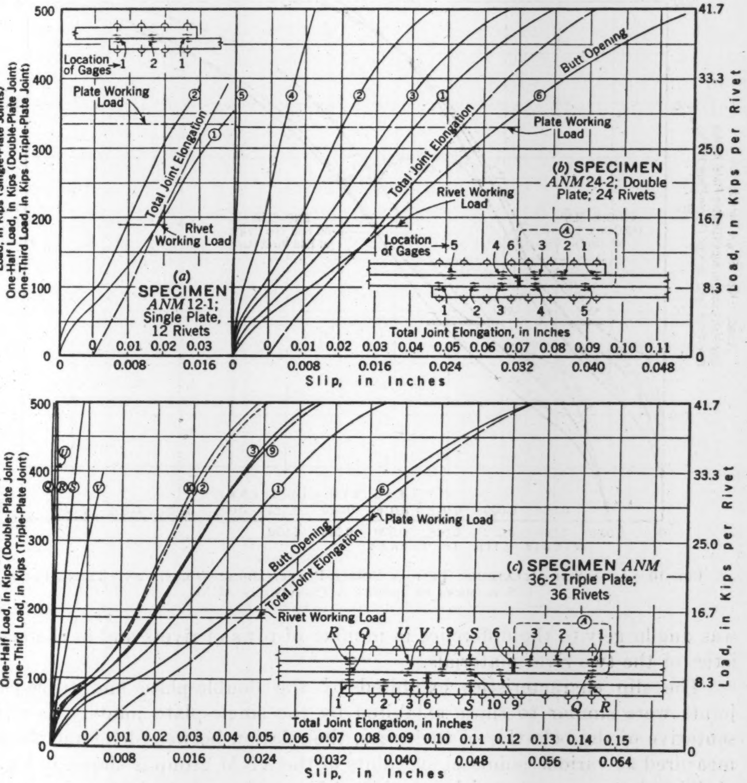


FIG. 9.—LOAD-SLIP RELATIONS AT VARIOUS PLACES ON TYPICAL SPECIMENS OF SERIES A

relation between load and middle slip. The joints with eighteen carbon-steel rivets (Specimens ACC18 and ASC18) slipped considerably less than those with twelve manganese-steel rivets, even if the slip were taken at the same load per rivet. Of the joints with carbon-steel rivets, those having carbon-steel plates slipped less than those having silicon-steel plates. All of the joints having manganese-steel rivets exhibited similar slip characteristics.

Fig. 8(*b*) shows the relation between load and the ratio of end slip to middle slip for one specimen of each of the pairs of single-plate joints. The end slip was measured midway between the first and second rows of rivets. Values in Fig. 8(*b*) are for one specimen of each pair. A constant ratio of end slip to middle slip—taken to be an indication that the rivets had come into full bearing—was attained at lower loads by the joints with twelve manganese-steel rivets than by those with eighteen carbon-steel rivets. Probably the difference was due largely to the difference in number of rows of rivets and in characteristics of the two rivet materials.

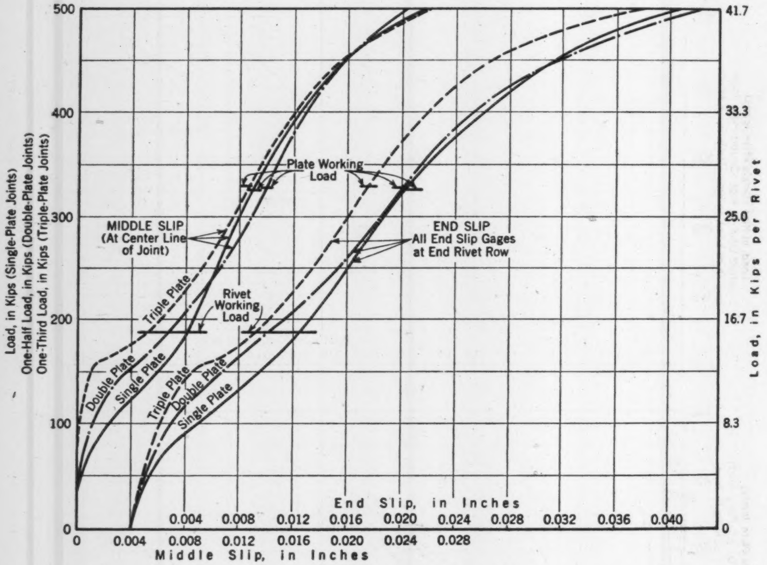


FIG. 10.—LOAD-SLIP RELATIONS; TYPICAL SINGLE-PLATE, DOUBLE-PLATE, AND TRIPLE-PLATE SPECIMENS OF SERIES A (SPECIMENS ACM)

The slip characteristics exhibited by the double-plate and triple-plate joints were similar to those exhibited by the single-plate joints. As representative of the behavior of the series, the relation between load and the slip measured at various points along joints of the ANM group is shown in Fig. 9. The plates were all of nickel steel, $\frac{5}{8}$ in. by 18 in.; and all rivets were of manganese steel, 1 in. in diameter. The relative movement of plates at the slip-gage locations is indicated by half-arrows in the inset diagrams. In the double-plate joint, the portion enclosed by the dotted line marked "A" (see inset, Fig. 9(*b*)) may be considered the equivalent of a single-plate lap joint, since at the end of the double-plate joint half of the total load is carried by each plate; and, at the middle of length of the joint, the load on the inner plate is zero while the outer plate still carries half the load. Similar characteristics hold for the triple-plate joint. Comparison of the end slips at Gage Locations 1 of the single-plate joint with those at Gages 1 and 3 of the double-plate joint, and at Gages 1, 3, and 9 of the triple-plate joint, shows that the slips at both ends of equivalent single laps in the multiple-plate joints did not differ greatly from the end slips for the single-plate joint at corresponding loads. Furthermore, in the multiple-plate joints, the "middle" slips (measured at all Gage Locations 2 and, for the triple-plate joint, at Gage 10 also) were similar to the middle slip of the single-plate joint. Slips at other points show the relative movements of the plates throughout the length of the joints.

A comparison of the relative magnitude of end and middle slip for a single, a double, and a triple-plate joint is afforded by the curves in Fig. 10. For the same unit stress in the plates, the middle slips and likewise the end slips were about the same as between the three sizes of joint. As in Fig. 8(*b*), the values are for one specimen of each pair.

The values of end slip at the rivet working load are summarized in Table 9, Column (16). With a few individual exceptions, the slips for single, double, or triple plates in any group having the same plate and rivet material were about the same. As noted in connection with the single-plate joints, the joints having carbon-steel rivets exhibited less slip than corresponding specimens having two-thirds as many manganese-steel rivets.

Four stages of action in riveted joints are indicated by the load-slip relations (see Fig. 1 and accompanying text). For the single-plate joints of Series A, Table 13 shows the approximate loads at the end of Stage I (no slip at middle of joint), at the beginning of Stage III (full bearing of all rivets, linear load-slip relation), and at the effective rivet yield which occurs when Stage IV is well started. For most of the joints, the rivet working load fell in Stage II; only for the specimens with manganese-steel rivets and nickel-steel plates was the working load definitely in Stage III.

TABLE 13.—AVERAGE STRESS IN RIVETS AT VARIOUS STAGES OF JOINT ACTION; SERIES A

Plate steel	EIGHTEEN CARBON-STEEL RIVETS					TWELVE MANGANESE-STEEL RIVETS				
	Kips per Square Inch					Kips per Square Inch				
	At working load	At end of Stage I	At beginning of Stage III	At effective rivet yield (in Stage IV)	Ratio, Column (4) / Column (2)	At working load	At end of Stage I	At beginning of Stage III	At effective rivet yield (in Stage IV)	Ratio, Column (9) / Column (7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Carbon....	15	6	27	33	1.8	20	5	23	47	1.2
Silicon.....	15	4	21	32	1.4	20	3	19	46	0.9
Nickel.....	....	....	....	....	....	20	2	13	46	0.7

Elongation.—The unit elongation of each joint relative to that of the main plate (gross section) varied from 1 to 2 at the plate working load (Table 9, Column (18)). The larger the joint, the less was the elongation ratio. It appears that the ratio was affected principally by the factors that influenced slip. For a single, a double, and a triple-plate specimen having nickel-steel plates and manganese-steel rivets, the variation in total elongation with increase in load is shown in Fig. 9.

Distribution of Strain.—The variation in longitudinal strain along a single, a double, and a triple-plate joint is illustrated in Fig. 11, for a stress at net section of 20 kips per sq inch. These curves (all edge strains) show that in the multiple-plate joints the stress in the outer plates was maintained at approximately the full value up to the beginning of the lap at the far end of the joint (marked "X"); between this point and the end of the plate the decrease in stress was similar to that in a single-plate lap joint. Irregularities in the strain diagrams may be due in part to localized stresses near the rivets and inequalities arising from fabrication.

Typical variations in longitudinal strain across a carbon-steel plate ($\frac{5}{8}$ in. by 26 in.) are shown in Fig. 12. The diagrams for Specimen ACC18-2 (eighteen, 1-in. carbon-steel rivets) show the observed strains, the corrections in strain determined by means of the curvometer, and the corrected strains, for a plate stress (at net section) of 20 kips per sq in. The diagrams for Specimen ACM12-2 (twelve, 1-in. manganese-steel rivets) show the corrected strains at plate stresses of 10 and 20 kips per sq in. The average strain across any section, employed in the load-partition studies discussed subsequently, was obtained by measuring, by planimeter, the areas under the corrected strain-distribution curves for each load.

Partition of Load Among Rivets.—The partition of load among rivets in the various transverse rows, determined from strain measurements along gage lines located between the rivet rows, is shown in Fig. 13. Data are shown for selected single-plate joints of each of the five types; for all of these specimens the bending frame (Fig. 5) was used to reduce the curvature of the plates, and curvemeter observations were made to provide a correction for change in curvature during loading. The curves show the percentage of the total load taken by a rivet in each row. Although there may be considerable error in the individual results, it is believed that the values are sufficiently accurate to show quantitatively the action of the joints tested. For this series, the specimen number defines the physical characteristics of each specimen; thus: ACM12-2 denotes Series A; Carbon-steel plates; Manganese-steel rivets; 12 rivets per joint; and the second specimen of the pair tested.

The load partition is shown at each of four loads, which differ as between the several types of joint. These loads varied from about one-half to about twice the rivet working loads. All the loads at which observations were taken lay above Stage I of joint action. Most of the observations were taken at loads within Stage II, which ended at rivet stresses of about 13 to 27 kips per sq in. For the joints with manganese-steel rivets, the third and fourth loads lay in Stage III; and for the joints in this group which had nickel-steel plates, the second load also lay in Stage III.

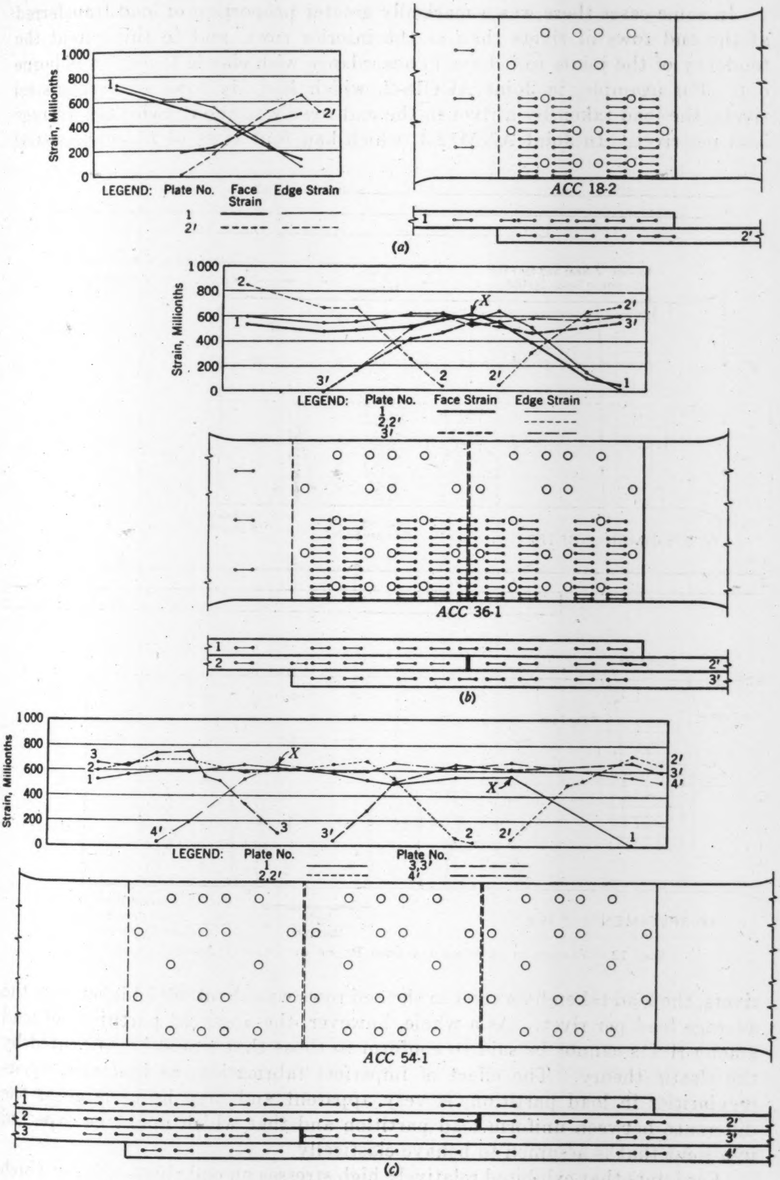
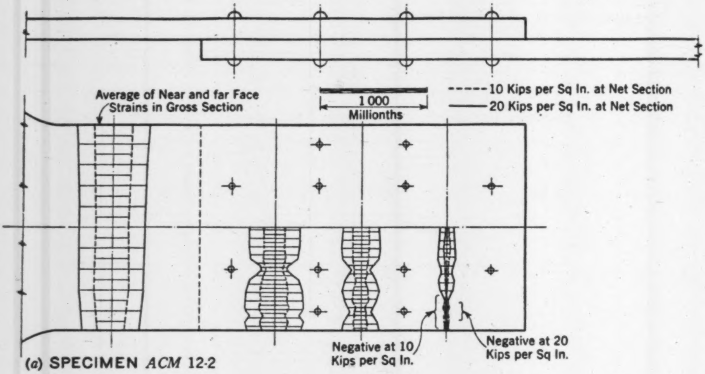
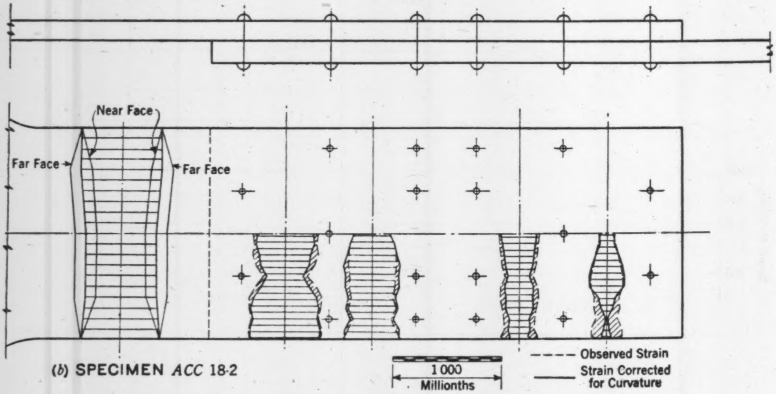


FIG. 11.—VARIATION IN STRAIN ALONG TYPICAL JOINTS OF SERIES A

In some cases there was a markedly greater proportion of load transferred at the end rows of rivets than at the interior rows, and to this extent the tendency of the joints to behave in accordance with elastic theory[9] was borne out. For example, in Joint ACC18-2, which had six rows of carbon-steel rivets, the load taken by a rivet in the end rows was about twice the average load per rivet. In Joint ANM12-1, which had four rows of manganese-steel rivets, the load taken by a rivet in the end rows was about 20% higher than the average load per rivet. As a whole, however, the observed partitions of load among rivets cannot be said to conform to those that would be computed by the elastic theory. The effect of imperfect fabrication, as indicated by irregularities in load partition, is very apparent and may be as large as the difference between uniform load partition and that which might be expected in a joint that is assumed to behave elastically.


Average of Near and far Face Strains in Gross Section
1000
Millionths
10 Kips per Sq In. at Net Section
20 Kips per Sq In. at Net Section
Negative at 10 Kips per Sq In.
Negative at 20 Kips per Sq In.

(*a*) SPECIMEN *ACM* 12-2

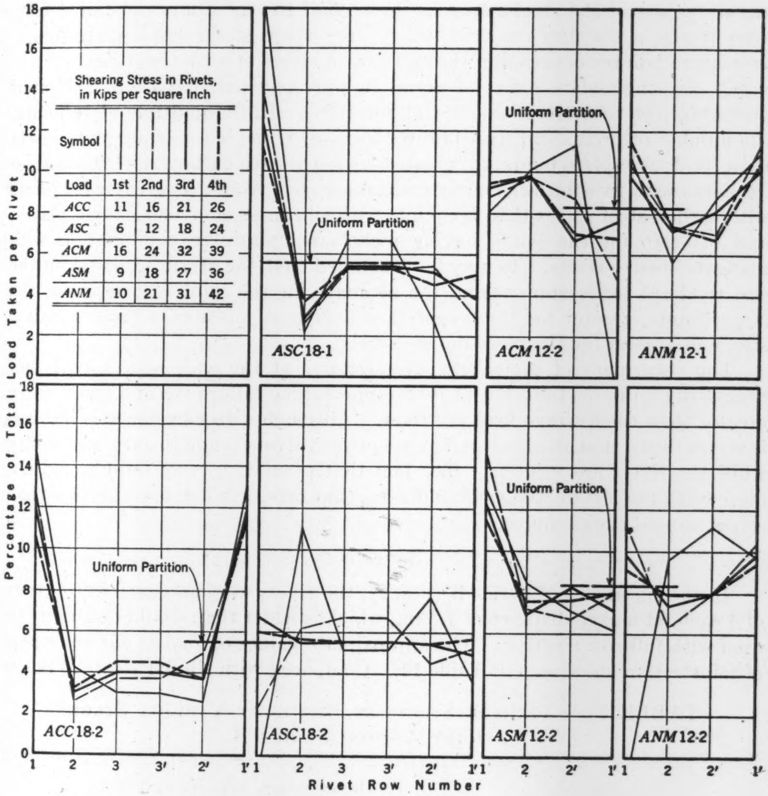

Near Face
Far Face
Far Face
1000
Millionths
Observed Strain
Strain Corrected for Curvature

(*b*) SPECIMEN *ACC* 18-2

FIG. 12.—VARIATION IN STRAIN ACROSS PLATE OF TYPICAL JOINTS; SERIES A

For joints that exhibited relatively high stresses on end rivets and for which two or more of the loads lay in Stage II (ACC18-2, for example) increasing the load reduced the share taken by the end rivets and tended toward more uniform partition of load. Upon this observation is based the preceding general statement that load partition is more nearly uniform in Stage II than in Stage I in which the joint may be behaving elastically (see text applying to Fig. 1).

For Specimen ANM12-1, in which the load partition tended to approach that required by elastic theory from the beginning, the relative stresses taken



Shearing Stress in Rivets, in Kips per Square Inch

Symbol	—	– –	—	– –
Load	1st	2nd	3rd	4th
ACC	11	16	21	26
ASC	6	12	18	24
ACM	16	24	32	39
ASM	9	18	27	36
ANM	10	21	31	42

Fig. 13.—Partition of Load among Rivets in the Various Rows; Series A

by the end rivets increased for the last two loads which were definitely in Stage III. This would indicate that after Stage II is passed, the load partition is again built up so as to approach, but not necessarily to conform to, that indicated by the elastic theory.

The elastic theory is not considered to be suitable for the prediction of load partition among rivets of a joint because: (1) The partition of load tends to vary through the successive stages of joint action, (2) ideal joints are not produced commercially, and (3) considerable slip occurs within the working range so that—except for short joints with high-strength plate and rivet materials—the rivets do not come into full bearing until the joint is stressed above the working load.

Premature Rivet Failures.—The number of individual rivets failing before complete rupture of the joint is shown in Table 9, Column (29). In nearly all cases of premature rivet failure, the rivets were in the end row of a lap; in the remaining cases, the rivets were in the second row of a lap and failed only after rivets in the first row had failed. For three of the triple-plate joints premature failures occurred in the end row of rivets of an interior lap.

Of the single-plate joints of Series A, only one pair (ACC-18) exhibited premature rivet failure (Table 9, Column (29)). In the multiple-plate joints, the number of premature rivet failures was: (*a*) Greater for triple-plate joints than for double-plate joints; (*b*) greater for joints with carbon-steel rivets than for joints with two-thirds as many manganese-steel rivets; (*c*) greater for joints with carbon-steel plates than for joints with silicon-steel or nickel-steel plates; and (*d*) zero for the joints having nickel-steel plates in combination with manganese-steel rivets. It may be concluded that the strain imposed on the end rivets at loads approaching the ultimate for the joint is greater for the larger joints, greater for the longer (less compact) joints of a given nominal size, and greater for the more ductile steels.

The occurrence of premature rivet failures at the ends of a joint is not necessarily an indication that at loads approaching failure the end rivets carry greater than the average load per rivet. Although direct evidence is lacking, it seems likely that the load partition between rivets tends toward uniformity while the rivets are yielding. The fact that premature rivet failures did not occur with the less ductile steels indicates that excessive detrusion, rather than stress, caused such failures.

Series B

In the specimens of Series B (Fig. 2), the rivets were arranged in full rows of 4 each, at a gage distance of 7.5 in., which is wider than would ordinarily be used with full-row riveting. The approximate values of load at various stages of joint action are shown in Table 14. Compared with similar joints in Series A, the joints of Series B exhibited relatively low slip. For example, as shown by the values in Table 9, Column (16), the slip of the joints of Series B at the rivet working load was less than half that of Joints ACC18.

TABLE 14.—AVERAGE STRESS IN RIVETS AT VARIOUS STAGES OF JOINT ACTION; SERIES B

Specimens	Pitch, in inches	RIVET STRESS, KIPS PER SQUARE INCH		
		Working Load	End Stage I	Begin Stage III
BCC20*a*	3	15	10	25
BCC20*b*	4.5	15	9	23
BCC20*c*	6	15	7	21

The differences in strength for all the specimens of Series B were small, although the joints with a pitch of 4.5 in. (0.6 of gage distance) were somewhat stronger than either the shorter or the longer joints. If the gage distance had been smaller (say, four or five rivet diameters) the optimum ratio of pitch to gage would not necessarily have been the same as in these specimens. Test results are summarized in Table 15. Specimens ACC18 of Series A (see Fig. 2) were not greatly different from the specimens of Series B except with regard to rivet pattern; in specimens ACC18 the net section was supposedly protected by having only two rivets in the first row and three rivets in the second row. Table 15 shows that these joints were not quite as efficient as the joints of Series B which had the full number of rivets in the end row. The last column of Table 15 confirms the discussion of Series A to the effect that end rows of rivets are subjected to larger strains in long splices than in short splices.

TABLE 15.—EFFECT OF PITCH ON STRENGTH; SERIES B

Series	Number of rivets in end row	Pitch, in inches	AVERAGE SHEAR IN RIVETS AT ULTIMATE LOAD		Number of premature rivet failures* (All in end rows)
			Kips per square inch	Percentages	
B	4	3	56.4	96	0
		4.50	58.6	100	1
		6	57.2	98	3
A	2	5.25	55.7	95	2 or more

* In two specimens.

TESTS OF BUTT AND SHINGLE JOINTS; SERIES F

For heavy tension members, the use of a "shingle" splice results in considerable economy of material and a shorter rivet grip than does the conventional butt splice. The specimens of Series F, shown in Fig. 4, were designed for comparison between butt and shingle splices. The plates were of silicon steel, 20 in. wide, and all main plates were $\frac{11}{16}$ in. thick. The rivets were of carbon steel and were $\frac{7}{8}$ in. in diameter. The distinguishing characteristics of the six pairs of specimens were as shown in Table 16. With the intention of insuring plate failure, the ratio of tensile stress in the plates to shearing stress in the rivets (T : S ratio) was made 1 : 0.54 (28 000 : 15 000) instead of the

TABLE 16.—TYPES OF SPECIMEN; SERIES F

Number of main plates	Symbol	Type of splice	Number of rivets
1	FSCA	Butt	38
2	FSCB	Butt	76
	FSCD	Shingle	76
3	FSCC	Butt	114
	FSCE	Shingle	114
	FSCF	Half-shingle	85 (116 shearing surfaces)

1 : 0.57 (93 000 : 53 000) established for rivet failure as a result of the pilot tests. In spite of this provision, in all specimens, except the single-plate butt joints FSCA, one or more rivets failed before plate failure occurred. The results of the tests are shown in Table 17 and Figs. 14 to 18 inclusive.

Behavior of Individual Joints

In order to secure a more general picture of the behavior of the parts of these joints than that afforded by the slip measurements alone, a series of 2.5-in. strain-gage lines was located on the edges of the plates, for one specimen of each pair. In calculating the share of the load taken by the various plates at any given section from the edge strains, it was necessarily assumed that the load transferred across a section was proportional to the observed strain. It is believed that the error involved in this assumption was reasonably small in the case of the multiple-plate joints since these joints had full-row riveting over a large proportion of the length.

TABLE 17.—TESTS ON BUTT

			PLATES (11/16 IN. BY 20 IN.; SILICON STEEL)			RIVETS		MILL COUPONS (KIPS PER SQUARE INCH)		SPECIMENS
				Area, in Square Inches						
Specimen No. (for details, see Fig. 4). (1)	Type of joint (2)	Length of joint, in inches* (3)	Number of main plates (4)	Gross (5)	Net (first row) (6)	Total number (7)	Number of shearing surfaces (8)	Yield strength (9)	Tensile strength (10)	End slip at rivet working load, in inches (11)
FSCA-1				13.74	12.37			51.7	92.1	0.0078
FSCA-2	Butt	33¼	1	13.84	12.46	38	38	51.3	90.8	0.0064
Average				13.79	12.42			51.5	91.5	0.0071
FSCB-1				27.58	24.82			51.2	90.5	0.0056
FSCB-2	Butt	63¼	2	27.62	24.86	76	76	52.8	93.3	0.0038
Average				27.60	24.84			52.0	91.9	0.0047
FSCD-1				27.51	24.76			50.9	90.1	0.0046
FSCD-2	Shingle	63½	2	27.66	24.89	76	76	51.7	90.6	0.0046
Average				27.59	24.83			51.3	90.4	0.0046
FSCC-1				41.19	37.07			54.0	92.5	0.0077
FSCC-2	Butt	87¼	3	41.00	36.90	114	114	53.7	90.8	0.0075
Average				41.10	36.99			53.9	91.8	0.0076
FSCE-1				40.16	36.15			52.5	91.9	0.0057
FSCE-2	Shingle	87¾	3	41.03	36.93	114	114	52.6	93.1	0.0061
Average				40.60	36.54			52.6	92.5	0.0059
FSCF-1	Half-			40.96	36.87			51.5	90.4	0.0052
FSCF-2	Shingle	66½	3	41.26	37.13	85	116	51.4	90.2	0.0056
Average				41.11	37.00			51.5	90.3	0.0054

* Length of inner (long) splice plates.
† Ratio of average unit elongation of joint to unit elongation of plates (gross section).
‡ Ratio of stress in gross section of plate to corresponding strength of mill coupons.

The behavior under load and the manner of failure, which are of particular interest in the case of the multiple-plate joints, are shown in Figs. 14 to 17, inclusive. In the figures, Diagrams *A* and *B* are the edge and face views of a joint and show rivet locations, gage-line locations, and the lines along which the parts of the joint separated at failure. In general, the location of final failure is shown for one of the two specimens at the left end of the diagrams and for the other specimen at the right end. In Diagrams *B*, a cross or a heavy dot drawn at a rivet location indicates that the rivet failed before complete rupture of the joint. Diagrams *C* show the percentage of load taken by each plate; the values were obtained from strain data by taking the summation of strains in all plates at any transverse section as 100 per cent. The points in Diagrams *C* correspond to the centers of the gage lines in Diagrams *A*. The stepped lines show the load that would be taken by each plate, calculated on the assumption of equal partition of load among rivets.

AND SHINGLE JOINTS; SERIES F

SPECIMENS (*Continued*)

Elongation ratio at plate working load†	At Yield Load				At Ultimate Load						
	Stress in Plate, in Kips per Square Inch		Efficiency, percentages‡	Shear in rivets, in kips per square inch	Location of failure§	Load, in kips	Stress in Plate, in Kips per Square Inch		Efficiency, percentages‡	Shear in Rivets, in Kips per Square Inch	
	Gross	Net					Gross	Net		All rivets‖	Remaining rivets¶
(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)
1.13	45.3	50.3		27.2	P	941	68.5	76.1		41.2	41.2
0.95	44.2	49.1		26.8	P	957	69.1	76.8		41.9	41.9
1.06	44.8	49.7	87.0	27.0		949	68.8	76.5	75.2	41.6	41.6
0.96	44.0	48.9		26.6	PR	1855	67.3	74.7		40.6	42.8
0.90	42.2	47.0		25.5	PR	1845	66.8	74.2		40.4	42.6
0.93	43.1	48.0	82.9	26.1		1850	67.1	74.5	73.0	40.5	42.7
0.80	42.4	47.2		25.5	PR	1782	64.8	72.0		39.0	40.6
0.85	43.7	48.6		26.5	PR	1816	65.7	73.0		39.7	41.9
0.83	43.1	47.9	84.0	26.0		1799	65.2	72.5	72.1	39.4	41.3
0.81	45.9	51.0		27.6	PR	2694	65.4	72.7		39.3	41.5
0.86	44.5	49.5		26.6	R	2626	64.0	71.2		38.3	46.5
0.84	45.2	50.3	83.8	27.1		2660	64.7	72.0	70.5	38.8	44.0
0.79	45.1	50.1		26.4	PR	2702	67.3	74.7		39.4	42.4
0.83	42.4	47.1		25.4	PR	2674	65.2	72.4		39.0	42.8
0.81	43.8	48.6	83.3	25.9		2688	66.2	73.6	71.6	39.2	42.6
0.90	43.6	48.4		25.6	PR	2595	63.4	70.4		37.2	37.9
0.93	42.7	47.4		25.2	PR	2708	65.6	72.9		38.8	39.9
0.92	43.2	47.9	83.9	25.4		2652	64.5	71.7	71.4	38.0	38.9

§ P = plate; and R = rivet.

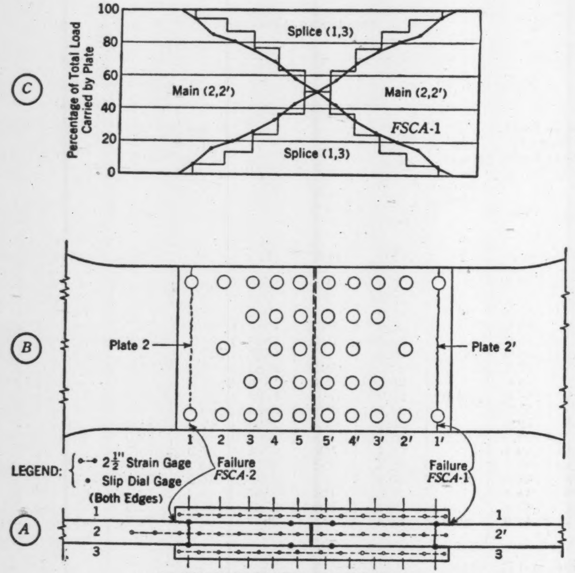
‖ Based on total number of rivets in joint.

¶ Excluding rivets which failed before maximum load was attained.

In all the specimens of this group of tests (Figs. 14 to 17) the main plates were $\frac{11}{16}$-in. by 20-in. silicon-steel plates, held by $\frac{7}{8}$-in. carbon-steel rivets (see Fig. 4). Slip gages were attached to both edges. In the single-plate and double-plate joints (Figs. 14 and 16(*a*)) the stress at the net section, 18.75 kips per sq in., was the average of 7.5, 15, 22.5 and 30 kips per sq in. In the triple-plate joints (Figs. 15, 16(*b*), and 17) the stress at the net section, 22.5 kips per sq in., was the average of 15 and 30 kips per sq in.

Single-Plate Butt Joints FSCA.—The strains in the main and splice plates were measured along chains of 2.5-in. gage lines, as shown in Fig. 14(*a*), Diagram *A*. The load partition computed from the strain is shown in Diagram *C*. Studies based on these data indicate that at the given loads the rivets in the end rows took more than the average share of the load, although the observed strains were undoubtedly affected by their location with respect to the various rivet rows which contained different numbers of rivets. The specimens failed entirely in the plates through an end row of rivets, as shown in Diagrams *A* and *B*, Fig. 14(*a*). First, the plate was torn from one end rivet to the edge, then the entire plate ruptured suddenly. The fracture was square and of a coarse, granular texture. Scaling of brittle paint from rivet heads occurred only in the end rows.

Double-Plate Butt Joints FSCB.—Strains were measured along chains of 2.5-in. gage lines on the edges of the plates, as shown in Fig. 14(*b*), Diagram *A*. Studies indicated that over the given range of loading the load per rivet was highest in the end row and in the two interior rows (9 and 10) next to the butt. In general, the progress of rivet scaling confirmed the load distribution determined from the strain measurements; marked scaling occurred in Rows 1, 10, 9, 8, and 7 in that order, whereas there was little scaling in Rows 2 and 3 and none in Rows 4, 5, and 6. At the end of the outer (short) splice plates, the inner (long) splice plates carried about 41% of the total load, as shown in Diagram *C*, Fig. 14(*b*). At the butt, the inner splice plates comprising 54% of the splice material carried about 58% of the total load, and the outer splice plates comprising 46% of the splice material carried about 42% of the load.


C
100
80
60
40
20
0
Percentage of Total Load Carried by Plate
Splice (1,3)
Main (2,2′)
Main (2,2′)
FSCA-1
Splice (1,3)
B
Plate 2
Plate 2′
1
2
3
4
5
5′
4′
3′
2′
1′
LEGEND:
2½″ Strain Gage
Slip Dial Gage
(Both Edges)
Failure
FSCA-2
Failure
FSCA-1
A
1
2
3
1
2′
3

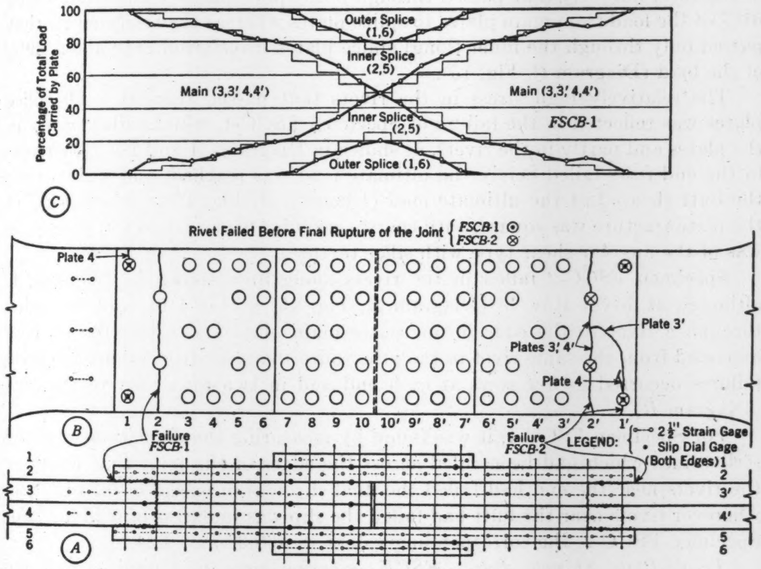
(*a*) SINGLE-PLATE JOINTS *FSCA*

FIG. 14.—PARTITION OF LOAD AMONG PLATES,

Before final rupture of the joint as a whole, all end-row rivets in both specimens failed. The plates fractured at the second row of rivets, except that one plate of Specimen FSCB-2 fractured half across the second row and half across the end row (Diagram *B*, Fig. 14(*b*)). About 80% of the fracture was square, and of coarse, granular texture; the remainder was of fine granular to silky texture, in part of the cup-cone and in part of the angular-shear type.

The relative displacements (slip) of the plates, observed after failure by measurements between lines scribed across the edges of the plates, disclosed that at sections where the rivets had failed, before complete failure of the joint, the slip was in excess of 0.33 in.

Double-Plate Shingle Joints FSCD.—Strains were measured along chains of 2.5-in. gage lines on the edges of the plates, as shown in Fig. 16(*a*), Diagram *A*. Over the stated range of loads, the load per rivet was found to be greatest in the two end rows and between the main plates at a butt. The load per rivet transferred directly from main plate to main plate at the middle of the joint was about the same as the average load per rivet. In general, the strain observations were confirmed by the progress of rivet scaling. Marked scaling occurred in Rows 1, 10, and 2 (in that order); little scaling occurred in Rows 4 to 7.



Percentage of Total Load Carried by Plate
100
80
60
40
20
0
Outer Splice (1,6)
Inner Splice (2,5)
Main (3,3,' 4,4')
Main (3,3,' 4,4')
Inner Splice (2,5)
Outer Splice (1,6)
FSCB-1
C
Rivet Failed Before Final Rupture of the Joint { FSCB-1 ⊙ FSCB-2 ⊗
Plate 4
Plate 3'
Plates 3,' 4'
Plate 4
B
1 2 3 4 5 6 7 8 9 10 10' 9' 8' 7' 6' 5' 4' 3' 2' 1'
Failure FSCB-1
Failure FSCB-2
LEGEND: { 2½" Strain Gage • Slip Dial Gage (Both Edges)
1 2 3 4 5 6
1 2 3' 4' 5 6
A

(*b*) DOUBLE-PLATE JOINTS *FSCB*

AND LOCATION OF FRACTURE; BUTT JOINTS

Before the joint failed as a whole, all but one of the end-row rivets failed. The plate fractures followed irregular lines which passed through rivet holes in one or more of the three rows next to an end (Diagram *B*, Fig. 16(*a*)). The fractures were in part square, with coarse, granular texture, and in part of the cup-cone and angular-shear type, ranging from fine granular to silky in texture.

Examination of the joints after fracture disclosed that wherever rivets had sheared, the relative displacement of the plates was in excess of 0.30 in.

Triple-Plate Butt Joints FSCC.—For this type of specimen, strains were measured only at selected locations along the joint, as shown in Fig. 15, Diagram *A*. The behavior of this joint was similar to that of the double-plate butt joint FSCB, in that the load per rivet was high at the ends and near the butt, and low just outside the ends of the outer (short) splice plates. In this pair of triple-plate specimens the outer splice plate was shorter than intended in the design. The progress of rivet scaling was in agreement with the computed distribution of load between rivet rows.

Along either half of the length of the joint, the twenty-seven rivets (47% of the total number) that passed through both splice plates transferred about 61% of the load from main plates to splice plates whereas the thirty rivets that passed only through the inner (long) splice plates transferred only about 39% of the load (Diagram *C*, Fig. 15).

The relatively high stress in the rivets that passed through both splice plates was reflected in the failure of Specimen FSCC-1, which failed partly in the plates and partly in the rivets, as shown in Diagrams *A* and *B*. The rivets in the end rows failed before the ultimate load was reached, and a rivet near the butt sheared at the ultimate load (Diagram *B*, Fig. 15). About 65% of the plate fracture was square, with coarse, granular texture, and the remainder was of the angular-shear type with silky texture.

Specimen FSCC-2 failed in the rivets along lines shown in Diagram *A*, although at Rivet Row 3′ (Diagram *B*, Fig. 15) a tear 4 in. long extended through a rivet to the edge in one outer main plate and a tear 0.5 in. long extended from the same rivet in the other outer main plate. Premature rivet failures occurred in five rows at each end and in two rows next to the butt (Diagram *B*).

For Specimen FSCC-1, it was found by measuring the displacement of the scribe lines which had been ruled across the edges of the joint that wherever the rivets near the end had failed the final slip was in excess of 0.33 in., and wherever rivets near the butt had failed the slip was in excess of 0.26 in. For Specimen FSCC-2, the corresponding values were 0.37 and 0.24 in.

Triple-Plate Shingle Joints FSCE.—Strains were measured at selected locations along the edges of the plates, as shown in Fig. 17, Diagram *A*. It was found that the rivets in the end rows took considerably more than the average share of the load, over the load range at which the measurements were made. The progress of rivet scaling was in agreement with the computed distribution of load between rivet rows. Between Rows 1 and 9 the scaling on the face of Plate 1 was much more severe than on the face of Plate 5, and between Rows 1′ and 9′ the scaling was much more severe on the face of Plate 5 than on the face of Plate 1.

The specimens failed partly in the plates and partly in the rivets, as shown in Diagrams *A* and *B*, Fig. 17. The rivets in the two end rows failed before the ultimate load was reached; and some rivets in the third and fourth rows sheared before final failure occurred (Diagram *B*). The plate fractures

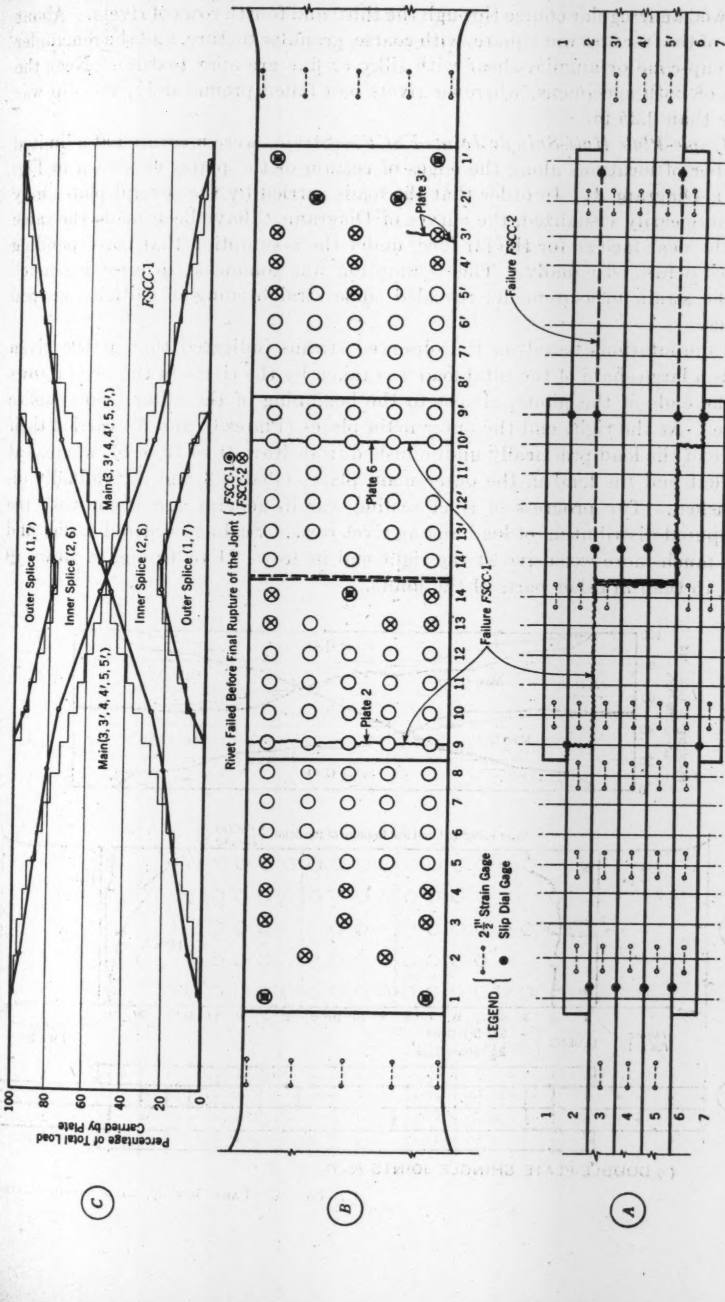
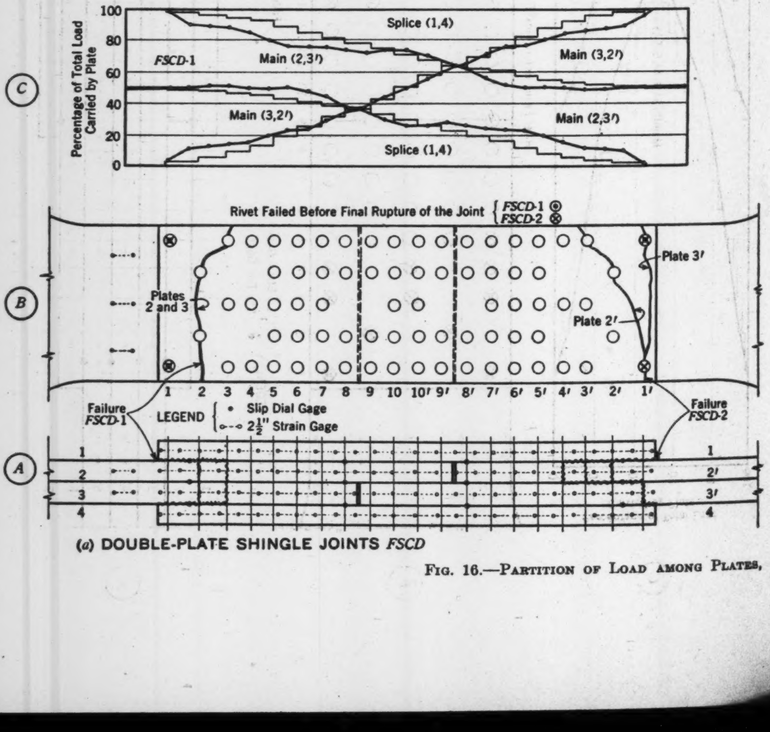

C
Percentage of Total Load Carried by Plate
100
80
60
40
20
0
Outer Splice (1, 7)
Inner Splice (2, 6)
Main (3, 3', 4, 4', 5, 5')
Main (3, 3', 4, 4', 5, 5')
Inner Splice (2, 6)
Outer Splice (1, 7)
FSCC-1
B
Rivet Failed Before Final Rupture of the Joint { FSCC-1 ⊗ FSCC-2 ⊠
Plate 3'
Plate 6
Plate 2
1 2 3 4 5 6 7 8 9 10 11 12 13 14
14' 13' 12' 11' 10' 9' 8' 7' 6' 5' 4' 3' 2' 1'
Failure FSCC-1
Failure FSCC-2
LEGEND
2½'' Strain Gage
Slip Dial Gage
A
1
2
3
4
5
6
7
3'
4'
5'

FIG. 15.—PARTITION OF LOAD AMONG PLATES, AND LOCATION OF FRACTURE; TRIPLE-PLATE BUTT JOINTS FSCC

followed an irregular course through the third and fourth rows of rivets. About 75% of the fracture was square, with coarse, granular texture, and the remainder was cup-cone or angular-shear with silky or fine granular texture. Near the ends of both specimens, wherever rivets had failed prematurely, the slip was more than 0.25 in.

Triple-Plate Half-Shingle Joints FSCF.—Strains were measured at a limited number of locations along the edges of certain of the plates as shown in Fig. 16(*b*), Diagram *A*. In order that the loads carried by the several plates may be more easily visualized, the curves of Diagrams *C* have been made the same for the near face as for the far face, under the assumption that corresponding plates deformed equally. The assumption was somewhat in error inasmuch as the strain measurements revealed some straightening of initially warped plates.

Computations based on the observed strains indicated that at the given loads a large share of the total load was taken by the rivets in the several rows at the ends of the joints, almost to the beginning of the outer (short) splice plate. At the right end the outer main plates (Plates 3′ and 5′) carried their share of the load practically undiminished from Row 21 to Row 14, whereas at the left end the load in the outer main plates (Plates 3 and 5) gradually diminished. The progress of rivet scaling was in general agreement with the computed distribution of load among rivet rows. Scaling occurred earlier and was much more extensive at the right end in Rows 21 to 19 and in Rows 16 and 15 than in other parts of the joints.


C
Percentage of Total Load Carried by Plate
100
80
60
40
20
0
Splice (1,4)
FSCD-1
Main (2,3′)
Main (3,2′)
Main (3,2′)
Main (2,3′)
Splice (1,4)
B
Rivet Failed Before Final Rupture of the Joint {FSCD-1 ⊙ FSCD-2 ⊗
Plates 2 and 3
Plate 3′
Plate 2′
1 2 3 4 5 6 7 8 9 10 10′ 9′ 8′ 7′ 6′ 5′ 4′ 3′ 2′ 1′
Failure FSCD-1
LEGEND { • Slip Dial Gage ⚬-⚬ 2½″ Strain Gage
Failure FSCD-2
A
1
2
3
4
1
2′
3′
4

(*a*) DOUBLE-PLATE SHINGLE JOINTS *FSCD*

Fig. 16.—Partition of Load among Plates.

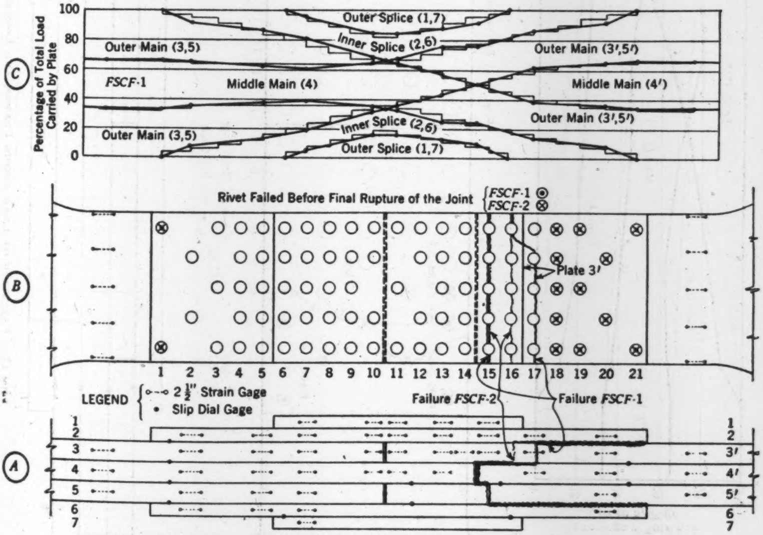
A consideration of all evidence available (strain-gage data, scaling, slip before and after failure, and premature rivet failures) would indicate that near the butts and near the ends of splices (abrupt changes in longitudinal section) the rivets carried relatively high loads.

The joints failed partly in the plates and partly in the rivets, at the end nearest to the butt of the middle main plate (Fig. 16(*b*), Diagrams *A* and *B*). As the rivets in the end rows failed progressively, more load was thrown on the rivets remaining between the end and the butt, and failure resulted. It was observed that, before final failure, rivets sheared up through the fourth row in both specimens. The greater part of the plate fracture was square, with coarse, granular texture, but in Specimen FSCF-1 about 5% of the fracture was cup-cone with silky texture. Wherever rivets failed in the left end, the slip was more than 0.31 in. for Specimen FSCF-1, and more than 0.37 in. for Specimen FSCF-2.

Comparison of Specimens

Efficiency.—The efficiency of the joints at ultimate load ranged from about 71% to 75%, as shown in Table 17, Column (21). The larger joints exhibited the lower efficiencies. There was little difference in efficiency as between butt and shingle joints of the same nominal size. The efficiencies at yield load, shown in Column (15), are 10% to 12% higher than those at ultimate load, but show the same general trend as between joints.

Elongation.—The unit elongation of the single-plate joints was about the same as that of the main plate outside of the joint (Table 17, Column (12)). The


C
Percentage of Total Load Carried by Plate
100
80
60
40
20
0
Outer Splice (1,7)
Inner Splice (2,6)
Outer Main (3,5)
Outer Main (3′,5′)
FSCF-1
Middle Main (4)
Middle Main (4′)
Outer Main (3′,5′)
Inner Splice (2,6)
Outer Main (3,5)
Outer Splice (1,7)
B
Rivet Failed Before Final Rupture of the Joint
FSCF-1 ⊙
FSCF-2 ⊗
Plate 3′
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
LEGEND
2½" Strain Gage
• Slip Dial Gage
Failure FSCF-2
Failure FSCF-1
A
1
2
3
4
5
6
7
1
2
3′
4′
5′
6
7

(*b*) TRIPLE-PLATE HALF-SHINGLE JOINTS *FSCF*

AND LOCATION OF FRACTURE; SHINGLE JOINTS

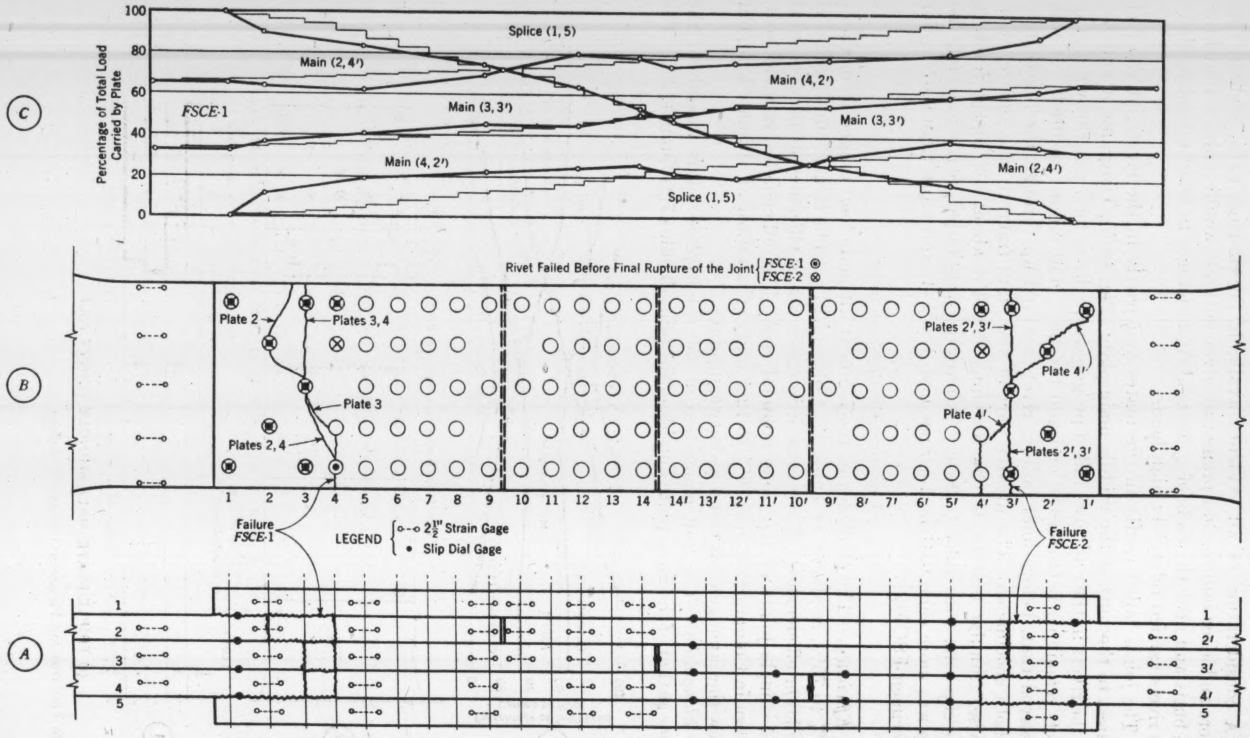


FIG. 17.—PARTITION OF LOAD AMONG PLATES, AND LOCATION OF FRACTURE; TRIPLE-PLATE SHINGLE JOINTS FSCE

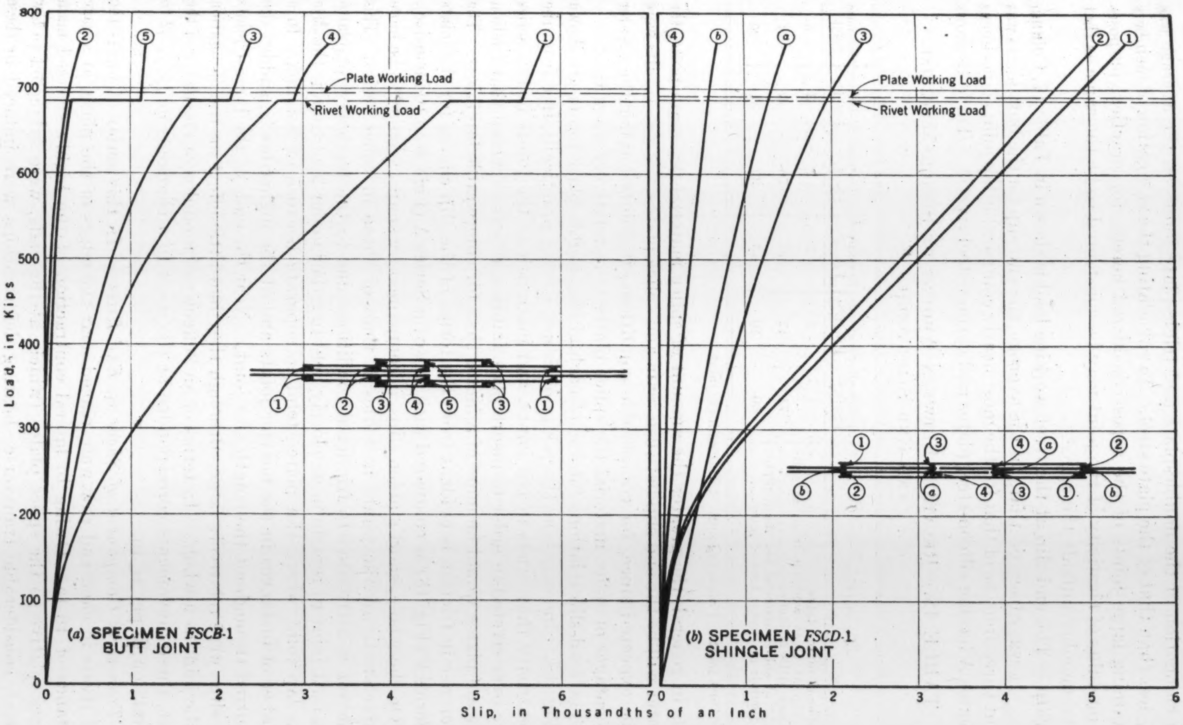


FIG. 18.—LOAD-SLIP RELATIONS; DOUBLE-PLATE JOINTS OF SERIES F

unit elongation of the double-plate and triple-plate joints on the average was 13% less than that of the plate steel. In calculating the elongation of members containing large joints of the types considered herein, for practical purposes the modulus of elasticity of the joint may be considered to be the same as that of the member outside the splice.

Slip.—The end slip at the rivet working load is shown in Table 17, Column (11). It was observed that, on the average, the end slip for the butt joints was about three-fourths of that for the most nearly comparable lap joints (specimens of Series A having silicon-steel plates and carbon-steel rivets). Table 18 shows that, in general, the greater the amount of splice material per main plate, the greater will be the end slip between main and splice plates. These results were to be expected since, for comparable conditions, in joints having the greater percentages of splice material the splice plates were relatively more rigid.

TABLE 18.—Relation Between Amount of Splice Material and End Slip; Series F

Description	Specimen FSCA*	Specimen FSCB*	Specimen FSCD†	Specimen FSCC*	Specimen FSCE†	Specimen FSCF‡
Number of main plates	1	2	2	3	3	3
End slip at rivet working load, in ten-thousandths of inches	71	47	46	76	59	54
Weight of splice material per main plate, in pounds per linear foot of joint	60	40	43	39	31	30

* Butt joint. † Shingle joint. ‡ Half-shingle joint.

The load-slip relations for one of each of the double-plate joints are shown in Fig. 18. Inasmuch as the joints of Series F were practically free from the eccentricity that exists in lap joints, and inasmuch as the joints in this series were over-riveted in order to insure plate failures, it was expected that friction between plates would be more of a factor than for comparable lap joints. The difference in friction is evident from the forms of the slip curves for the joints of Series F (Fig. 18) as compared with those for Series A (Figs. 8 to 10 inclusive).

On the whole, short sudden slips, accompanied by snaps, seemed to be more characteristic of the joints in Series F than of those in other series. This behavior is attributed to frictional conditions due to the tendency for plates in a butt joint to remain more closely held together under load than is the case in a lap joint where the plates tend to separate because of bending. In a number of instances in the heavier joints and at the higher loads, sudden slip occurred throughout the length of a joint. Near the ends of the largest joints the slips were relatively large, although they had little effect on the elongation of the joint as a whole. In terms of an "effective" modulus of elasticity of the joint, these movements corresponded at the most to a change from, say, 32 to 29 million lb per sq in.

Transverse Component of Shear in End Rivets.—In this series of joints the two rivets in each end row were placed near the edges of the plate, at a gage distance of 16 in. Owing to lateral contraction of the highly stressed main plates relative to the splice plates (which, at the ends, were subjected to low stress), considerable transverse shearing deformations were induced in these rivets. Although these rivets failed at relatively low computed stresses based on longitudinal shear, the strength was reasonably close to normal when the resultant of the longitudinal and transverse shear is considered.

Shear in Rivets at Ultimate Load.—In Table 17, Columns (22) and (23), is shown the average shearing stress in the rivets at ultimate load, based both on the total number of rivets and on the number which did not fail prematurely. Although most of the joints failed primarily in the plates rather than in the rivets, it is believed that the rivets were near failure at the ultimate load; in fact, one specimen (FSCC-2) failed entirely in the rivets. The values, in comparison with those of Series A (Table 9, Column (27)) indicate that the resistance per rivet shearing surface was materially less in double shear than in single shear. However, the comparison is not direct, particularly since in Series F the plates were stressed nearly or quite to their ultimate load and, therefore, had elongated greatly and had subjected the rivets near the ends to relatively large detrusions as compared with Series A. Because failure of the rivets was partly due to detrusion, and not entirely to the direct effect of loading, the rivet shearing values in Table 17 should not be used as a general basis for establishing working stresses in double shear.

CONCLUSIONS FROM THE TESTS

Since the specimens of this investigation were designed to conform reasonably well to standards of practice, the various problems and variables are, of necessity, interwoven. For example, in Series A and F the variations in nominal length of joint involve differences in actual length, thickness (number of plates), grip of rivets, and number of rivets; they are essentially variations in size rather than length. In no case is the width of joint a variable. The conclusions in this section apply only to the conditions and range of these tests.

Herein the term "rivet strength" denotes the average shearing resistance of the rivets in a given group or joint. The strength is computed by dividing the ultimate load by the total cross-sectional area of the rivets in shear, based on the nominal diameter of rivet.

Effect of Length of Joint.—In general, at the ultimate load the greater the length of the joints the lower was the rivet strength, regardless of the method by which the length was varied (that is, difference in pitch, rivet pattern, number of rivet rows, or over-all size). However, the differences were not large, and minor exceptions arose when the influence of other factors predominated. The general effect of length of connection is summarized in Table 19. For example, for joints with carbon-steel rivets and plates, the rivet strength was only about 5 kips per sq in., or about 10%, less in joints 7 ft long (eighteen rows of rivets) than in joints $2\frac{1}{3}$ ft long (six rows of rivets); for joints with carbon-steel rivets and silicon-steel plates the difference was even less. Furthermore, it appears that small variations in length are not important in comparison with variations arising from fabrication inequalities.

As between the butt joints of different length (FSCA, FSCB, and FSCC), the longer joints not only were lower in efficiency but also they failed essentially in the rivets rather than in the plates (Table 17).

Effect of Rivet Pitch.—From the tests of Series B, in which the pitch between transverse rivet rows was the only variable, it appears that over the range investigated (3 to 6 in.) the effect of pitch was small, although the 4.5-in. pitch (4.5 rivet diameters) gave slightly higher efficiency than either the 3-in. or the 6-in. pitch.

TABLE 19.—SHEARING STRENGTH OF RIVETS

Plate steel	CARBON-STEEL RIVETS							MANGANESE-STEEL RIVETS						
	Specimen number*	Number of:		Grip, in inches	Shear Stress in Rivets, in Kips per Square Inch			Specimen number*	Number of:		Grip, in inches	Shear Stress in Rivets, in Kips per Square Inch		
		Rivets	Rivet rows		At effective rivet yield	At yield load	At ultimate load		Rivets	Rivet rows		At effective rivet yield	At yield load	At ultimate load
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Carbon .	ACC2†	2	2	1¼	38.0	....	60.8	ACM2†	2	2	1¼	....	....	79.8
	GCC4†	4	2	1¼	38.0	....	62.1	GCM3†	3	2	1¼	....	....	93.6
	CCC7	8	2	¾	....	44.6	57.0	....	....	....	....	....	....	....
	DCC7	14	2	1¼	....	37.7	56.6	....	....	....	....	....	....	....
	BCC20*a*	22	6	1½	....	39.6	56.5	....	....	....	....	....	....	....
	BCC20*b*	22	6	1½	....	39.8	58.6	....	....	....	....	....	....	....
	BCC20*c*	22	6	1½	....	37.6	57.2	....	....	....	....	....	....	....
	ACC18	18	6	1¼	31.8	38.9	55.7	ACM12	12	4	1¼	45.4	59.0	75.9
	ACC36‡	36	12	1⅞	32.3	36.4	55.1	ACM24‡	24	8	1⅞	49.3	56.8	77.4
	ACC54§	54	18	2½	35.4	37.7	50.0	ACM36§	36	12	2½	47.8	56.5	71.4
Silicon . .	ASC2†	2	2	1¼	35.0	....	57.2	ASM2†	2	2	1¼	....	....	72.9
	GSC6†	6	2	1¼	35.0	....	55.7	GSM4†	4	2	1¼	....	....	70.3
	ASC18	18	6	1¼	....	40.0	54.8	ASM12	12	4	1¼	45.3	59.8	72.4
	ASC36‡	36	12	1⅞	32.3	41.7	54.0	ASM24‡	24	8	1⅞	47.5	63.3	78.0
	ASC54§	54	18	2½	32.2	38.9	51.1	ASM36§	36	12	2½	45.6	57.3	72.7
Nickel . .	....	....	....	....	....	....	....	ANM2†	2	2	1¼	....	....	86.0
	....	....	....	....	....	....	....	ANM12	12	4	1¼	41.9	64.2	74.7
	....	....	....	....	....	....	....	ANM24‡	24	8	1⅞	46.7	67.9	77.7
	....	....	....	....	....	....	....	ANM36§	36	12	2½	49.3	65.8	75.7

* One-plate specimens, with exceptions noted. † Pilot tests at the University of Illinois. ‡ Two-plate specimen. § Three-plate specimen.

Effect of Kind of Rivet Steel.—As compared with strength of rivets in small joints (pilot tests) or in a direct shear test of undriven rivets, in the large joints of Series A the manganese-steel rivets performed somewhat better as regards ultimate strength than the carbon-steel rivets. Although this difference is attributed in part to the difference in actual length of joint for a given nominal length, it seems fair to credit the manganese-steel rivets with this advantage since fewer rivets are needed. For the joints of Series A, all of which failed in the rivets, the average ratio of observed strength to the assigned rivet working stress (that is, the "factor of safety") was found to be 3.73 for the manganese-steel rivets as compared with 3.52 for the carbon-steel rivets.

Effect of Kind of Plate Steel.—In the joints that failed in the rivets, the effect of kind of plate steel was not large or consistent, although the decrease in shearing strength of rivets with length was somewhat more marked in the case of the carbon plate steel than with the less ductile plate steels. There was less tendency for premature failure of end rivets with the less ductile plate steels.

Single Shear vs. Double Shear.—Although the comparison is not direct, being based on the tests of joints of Series A and F [see heading: "Tests of

Butt and Shingle Joints; Series F (Comparison of Specimens): Shear in Rivets at Ultimate Load] the indications are that the strength of a rivet surface in double shear may be as low as 80% of the strength in single shear. These indications agree with the findings in the pilot tests[8] where non-driven rivets were tested in both single and double shear.

Net Section.—An outstanding finding of these tests is that the strength of the plates in a riveted joint is not directly proportional to the area of the net section, as commonly assumed in practice. The use of fewer rivets in the end rows than in interior rows is ineffective, as shown by the tests in Series C and D. Wide spacing of the end-row rivets (near the edges of the plate) is especially undesirable; in the tests of Series F it was found that differential lateral contraction of the opposing plates at each end of the joint subjected such rivets to lateral strain which, combined with the longitudinal strain, caused premature failures of these rivets.

In tests of joints having the full number of rivets in the end row (Series C and D), the efficiency was about 10% higher when the gage was about four rivet diameters than when a gage of about three rivet diameters was used.

The experimental results bore no consistent relation to the specification requirements for determining net section. In these tests, there was little difference in strength as between joints having an end pitch of four to six rivet diameters and joints having an end pitch of two to three rivet diameters, although the specifications would assign a smaller computed net section to the latter group. Regardless of the number of rivets in the end row, the net section found by deducting the projected sectional areas of rivet holes in the first two rows correlated better with the observed strengths than did the net section computed by standard specifications. It is apparent that there is nothing to be gained by using an elaborate formula to evaluate the effect of pitch on net section for joints of this type.

Butt vs. Shingle Splices.—There was little difference in efficiency as between the butt and shingle splices of Series F. The triple-plate shingle splices require less material for fabrication than the butt splice, as shown in Table 20; this advantage, however, may be offset by the somewhat greater difficulty of field erection for shingle splices. The partition of load between the splice plates agrees fairly well with the usual assumptions in designing these splices.

TABLE 20.—MATERIAL REQUIRED FOR VARIOUS TYPES OF SPLICE; SERIES F

Description	Butt splice FSCC	Shingle splice FSCE	Half-shingle splice FSCF
Weight of splice plates, in pounds.....	850	684	496
Number of rivets....................	114	114	85

Slip.—The relative movement, or slip, of adjacent plates is of interest in connection with the behavior of riveted joints. In fact, the rivets cannot develop resistance to shear until after slip occurs. Slip is believed to be of special importance with regard to structural members subjected to repetitions or reversals of load.

The load-slip relationship may be divided into these four stages: (I) Load is transmitted by friction between the plates, and general slip throughout the joint does not occur; (II) slip begins and the rivets are coming progressively into bearing; (III) the rivets are in full bearing and the load-slip relation is linear; and (IV) plastic action and yield occur in rivets, plates, or both (see heading "The Action of Riveted Joints").

These stages of joint action are not all developed to the same extent during the loading of joints of various types. In joints having four to six rows of rivets, the end of Stage I may occur at a rivet stress of 10 kips per sq in. or less. For the single-plate lap joints of Series A, the ratios of the loads at the beginning of Stage III (as nearly as could be determined) to the rivet working loads are shown in Table 13, Columns (6) and (11).

The ratio of slip measured near the ends of the joints to slip measured at the middle of length of a lap became a constant when the rivets came into full bearing (Stage III). The ratio was a function of the length of the joints.

In the tests of Series A, the joints containing carbon-steel rivets exhibited markedly less slip than those containing two-thirds as many manganese-steel rivets. As between the various series, the slip was less for joints with full-row riveting than for joints with any other rivet pattern.

Partition of Load Among Rivet Rows.—Under the assumption of perfectly elastic behavior of an ideal joint, in joints containing more than two rows of rivets, the rivets in the end rows are subjected to more than the average load per rivet.[9] In these tests, the results of strain measurements made over the working range showed that in actual joints the same tendency exists but that inequalities arising from fabrication conditions cause irregular and erratic load partition, and that the irregularities may be large as compared with the differences in load partition called for by the elastic theory.

The following general conclusions are drawn from various phases of the tests bearing on the distribution of stress in riveted joints:

(1) At low loads (Stage I), the load partition among rivets may be controlled largely by such conditions within a joint as warping of plates, variations in initial tension of rivets, and character of the faying surfaces;

(2) After slip becomes general throughout a joint (Stage II), there is a tendency for the shearing stresses to become equalized among the rivets in the various rows, although uniform partition of load is not probable;

(3) From limited data available, it appears that in Stage III the rivets in the end rows tend to take more than the average load per rivet;

(4) Although no direct evidence is available, it seems likely that in the early part of the yield range (Stage IV), the load partition among the rivets tends to become uniform; and,

(5) In the latter part of the yield range, the amount of plastic flow in the plates is so great that the resulting elongation is sufficient to cause detrusion of end rivets. For $\frac{7}{8}$-in. rivets, this rivet failure occurs when the end slip is about 0.3 of the rivet diameter, and occurs more frequently in the longer joints and in those made with the more ductile steels. It does not necessarily indicate that these end rivets are taking more than their proportionate share of the load.

In general, although it is probable that the partition of stress among rivets is not uniform at any stage of loading, the test results indicate that an assumption of uniform partition is as reasonable as any that has been made.

Recommendations Applicable to Design

The results of these tests, together with those of other investigators, appear to warrant the following statements relative to practice in the design of riveted joints for bridge construction:

(1) Nothing is gained by an attempt to detail a tension member with a critical net area greater than about 75% of the gross area;

(2) There is no justification for elaborate formulas to calculate the effect of rivet stagger on net section;

(3) Joints should be as compact as practicable (optimum results will probably be obtained by full-row riveting with a gage of about 4.5 rivet diameters and a pitch of 3.5 to 4 rivet diameters);

(4) Statements (1) to (3) lead to the suggestion that the allowable loads on riveted tension members be based on, and expressed in terms of, stress in the gross section; and, that the net section be not less than 75% of the gross section;

(5) The practice of assuming equal shear per rivet, regardless of length of joint, is satisfactory;

(6) Except in comparatively heavy structures where reduction in size or weight of splice is important, there is little reason for using manganese-steel rivets rather than carbon-steel rivets. Owing to the greater slip that occurs in joints with manganese-steel rivets, carbon-steel rivets are preferable in members subject to stress reversals; and,

(7) Working stresses recommended for use in the design of riveted joints in bridges, for steels commonly specified, are shown in Table 21. These recommended stresses were developed from a consideration of: (*a*) The strengths of the steels as indicated by standard mill acceptance tests; and (*b*) the relations between the strengths of mill coupons and the strengths of joints of structural proportions. The relationships between the strengths, in tension, of mill specimens and the properties of structural joints, based on a study of

TABLE 21.—Recommended Working Stresses for Riveted Bridge Joints

Property	Steel	Minimum Strength in Tension of Mill Coupons, in Kips per Square Inch		Recommended working stress, in kips per square inch	Ratio
		Yield	Ultimate		
Tension in gross section of plate	Carbon	33.0	60.0	16.0	1.00
	Silicon	45.0	80.0	20.0	1.25
	Nickel	55.0	90.0	22.5	1.40
Single shear in rivets	Carbon	28.0	52.0	15.0	1.00
	Manganese	42.0	75.0	20.0	1.33
Double shear in rivets	Carbon	28.0	52.0	13.5	0.90
	Manganese	42.0	75.0	18.0	1.20

data from various sources, are as follows:

Ratio	Tension in gross section of plate	Single shear in rivets (nominal diameter)
$\dfrac{\text{Useful limit point for a joint}}{\text{Yield strength of mill coupons}}$	0.72	....
$\dfrac{\text{Useful limit point for a joint}}{\text{Ultimate strength of mill coupons}}$	0.42	....
$\dfrac{\text{Ultimate strength for a joint}}{\text{Ultimate strength of mill coupons}}$	0.75	0.88
$\dfrac{\text{Effective rivet yield for a joint}}{\text{Yield strength of mill coupons}}$	....	0.83
$\dfrac{\text{Effective rivet yield for a joint}}{\text{Ultimate strength of mill coupons}}$	....	0.55

In computing these values, the useful limit point of plates in a riveted joint was assumed to be 85% of the yield strength of the joint.

The foregoing ratios are applied to the strengths of mill coupons to obtain the effective strengths of the materials acting in structural joints, as given in Table 22. The selection of strength values for mill coupons is based on an examination of mill reports on tests of steels of the given grades. It is believed that the foregoing ratios are conservative. Inasmuch as—to avoid rejections—the mills endeavor to supply material somewhat above the minimum specified, it should be safe to take values midway between the minimum and the average. Values taken in this manner are shown in Table 22 as "expected strengths."

TABLE 22.—EFFECTIVE STRENGTHS OF MATERIALS ACTING IN STRUCTURAL JOINTS

Steel	Expected Strength in Tension of Mill Coupons, in Kips per Square Inch		Useful Limit Point (for Plates) or Effective Rivet Yield (for Rivets)				Ultimate Strength	
	Yield	Ultimate	Based on yield, kips per square inch	Based on ultimate, kips per square inch	Average, kips per square inch	Ratio	Kips per square inch	Ratio
(*a*) Tension in gross section of plate								
Carbon..........	36.3	63.3	26.1	26.6	26.3	1.00	47.5	1.00
Silicon..........	47.6	83.7	34.3	35.2	34.7	1.32	62.9	1.32
Nickel..........	59.6	94.9	42.9	39.8	41.3	1.57	71.0	1.50
(*b*) Single shear in rivets (nominal diameter)								
Carbon..........	32.6	54.7	27.0	30.0	28.5	1.00	48.1	1.00
Manganese......	49.8	81.4	41.4	44.7	43.0	1.51	71.7	1.49

To obtain the recommended working stresses for carbon plate and rivet steels (Table 21), the strengths in Table 22 have been reduced by using a factor of about $1\frac{3}{4}$ in the case of the useful limit points and about 3 in the case of the ultimate strengths. When high-strength plate steels are used in place of ordinary carbon steel, some desirable property must be encroached upon to secure the added strength. The ductility and plastic flow of these high-strength steels are less than those of carbon steel, so that there is less opportunity for stress readjustment to take place and secondary stresses become relatively more important. For these reasons, working stresses should not be increased in proportion to strength. Likewise, the values of working stresses for manganese-steel rivets as compared with those for carbon-steel rivets should be lower than their relative strengths because of the greater slip that occurs in joints containing manganese-steel rivets. The rivet values in double shear per shearing surface have been taken as 0.90 times those for single shear.

ACKNOWLEDGMENTS

This series of tests was made possible by funds supplied by the San Francisco-Oakland Bay Bridge of which C. H. Purcell, Assoc. M. Am. Soc. C. E., is Chief Engineer and Charles E. Andrew, M. Am. Soc. C. E., is Bridge Engineer. The test specimens were designed by Ralph A. Tudor, Assoc. M. Am. Soc. C. E., Senior Bridge Engineer, under the direction of Glenn B. Woodruff, M. Am. Soc. C. E., Engineer of Design. Those acting in an advisory capacity in the development of the test program were C. F. Goodrich, Chief Engineer, American Bridge Company; Jonathan Jones, Chief Engineer of Fabricated Steel Construction, Bethlehem Steel Company; Leon Moisseiff, Consulting Engineer; and Wilbur M. Wilson, Research Professor of Structural Engineering, University of Illinois, all Members of the Society. The pilot tests were made under the direction of Professor Wilson.

The specimens were fabricated under contract at the Ambridge plant of the American Bridge Company from steel manufactured at the Homestead plant of the Carnegie-Illinois Steel Corporation. Of the coupons taken at the time the specimens were manufactured, about one-third were tested by the Carnegie-Illinois Steel Corporation under the supervision of J. M. Bailey, Principal Steel Inspector for the San Francisco-Oakland Bay Bridge; the remaining coupons were tested at the University of California.

The static tests on large joints were made in the Engineering Materials Laboratory of the University of California, during the period 1935 to 1937. The University of California was represented by Charles Derleth, Jr., M. Am. Soc. C. E., Dean of the College of Engineering, and Raymond E. Davis, M. Am. Soc. C. E., Professor of Civil Engineering, in charge of the Laboratory. The following members of the staff of the Engineering Materials Laboratory, University of California, participated in the conduct of the tests and the analysis of the results: J. L. Banville, E. H. Brown, Harmer E. Davis, Assoc. M. Am. Soc. C. E., H. D. Eberhart, Jun. Am. Soc. C. E., E. J. Garbarini, Jun. Am. Soc. C. E., Norman Haavik, J. W. Kelly, M. Am. Soc. C. E., Alexander Klein, Assoc. M. Am. Soc. C. E., J. F. Ryan, G. E. Troxell, M. Am. Soc. C. E., and E. L. Whittier. During the progress of the work, the San Francisco-Oakland Bay Bridge was represented by C. C. Winter, Assoc. M. Am. Soc. C. E., E. B. Payne, and R. L. Richardson, Juniors, Am. Soc. C. E.

Publication of the paper was made possible by financial assistance granted by the Engineering Foundation.

DISCUSSION

Charles F. Goodrich,[13] M. Am. Soc. C. E. (by letter).—It is the writer's opinion that the two groups of tests on structural riveted joints, made in connection with the design and construction of the San Francisco-Oakland Bay Bridge, are unequaled in their contribution to the knowledge of the behavior of such joints. Most of those who studied Professor Wilson's paper on "Fatigue Tests of Riveted Joints"[8] were considerably disturbed by the rather low fatigue values obtained, especially for the higher strength steels; but they were more especially disturbed by the low unit stresses at which slip occurred in the high-strength, or the so-called manganese, rivets. It seems to the writer that, in so far as static loads are concerned, the results of the present tests confirm, rather than disturb, one's confidence in riveted joints.

As the factor of fatigue is absent from the tests described in this paper, its authors quite properly have based their conclusions on the static tests of large, full-size specimens. This discusser would emphasize that there is considerable danger in projecting laboratory results on small specimens into the design of structures without a sufficient study of other mitigating factors, such as frequency of maximum loads, yielding of joints and other members and recovery between loads. Literature on projecting laboratory studies on fatigue into actual structures is sadly lacking. The profession needs such interpretation.

The authors have thrown considerable light on the action of riveted joints by considering four stages of joint action: Stage I, in which static friction prevents slip, the end of this stage being at the point at which slip at the middle of the joint takes place; Stage II, in which slip occurs until the rivets come into bearing; Stage III, in which rivets and plates deform elastically; and Stage IV, in which yielding of the plates, rivets, or both, occurs until either plate fracture or complete shearing of the rivets results.

Comparing joints of carbon rivets with those of manganese rivets, in which the number of rivets is in proportion to their shearing values, the authors, confirming Professor Wilson, find the average rivet stress at the end of Stage I much lower for manganese rivets than for carbon rivets (roughly, one-half) but, as the stages progress, the manganese rivets come into bearing and "catch up." At the beginning of Stage III they are roughly equal, and in Stage IV the manganese rivets have "come into their own" and their value, as compared to the value of carbon rivets, is about proportional to the strengths of the materials.

Regarding this matter of slip, it seems evident that engineers can no longer consider that the friction between the plates of a joint, due to tension developed by the rivets, is a measure of the joint value, even with carbon rivets in carbon plates—especially as working stresses are being continually increased as structural steel is improved in quality and strength.

With respect to high-strength steels and high-strength rivets, these tests have proved that friction between the plates disappears at stresses at or below the working stresses which are reasonable with such steels, and slip occurs in the connections of a member probably either before, or as soon as it receives its full-load stresses. As the rivets come into bearing at these low stresses, it would seem advisable to have the rivets fill the holes as nearly as possible. The writer believes that the optimum in this respect, with due regard to fabrication costs, has about been reached. Clearance between rivet and rivet hole cannot be reduced without causing expensive difficulty and delays in entering. Furthermore, the limit has about been reached in human strength to handle heavier, higher-pressure, rivet guns. The writer believes that the driving technique now being used on large structures by reputable contractors, under competent inspection, secures results that are reasonably satisfactory. Special endeavor to upset the rivets to fill the holes more completely was made in the case of the San Francisco-Oakland Bay Bridge, including investigation of sample drivings, but it is doubtful if it resulted in much better filling of holes in the actual structure than is accomplished in the usual best practice.

---

[13] Chf. Engr., Am. Bridge Co., Pittsburgh, Pa.

There is a balance in economy that must not be ignored. If structural engineers continue to increase unit stresses and then begin to worry about rivets not entirely filling the holes and several other points of workmanship, they may well question whether economy has really been effected. The rivets in the joints of the tension tests on large riveted joints, presented in this paper, were driven under usual shop practice. Some of the rivets were quite long and it is doubtful whether they filled the holes any better than usual; and yet, the joints all showed consistently excellent results. In fact, they were so consistently good as to inspire the writer with fresh confidence in driving technique and in riveted joints generally, especially large butt or shingle joints, data on the strength and behavior of which have been so sadly lacking until now. These tests confirm, in large joints (as have all similar previous tests on smaller specimens), the peculiar capacity of structural steel to transmit stress efficiently and safely from one part to another by means of rivets in shear and bearing.

The tests are a distinct warning that the connections of members subject to reversal of stress must be over-riveted, especially when using high-strength rivets. There are not many members subject to reversal, however, in which stresses are large enough to demand high-strength rivets. Continuous structures are likely to have several members subject to reversal, either at the design loads, or at a future over-load. Such members should be connected with an ample number of rivets to prevent slip—perhaps more than have heretofore been used.

The authors have arrived at some rather startling conclusions, one of which has been suspected by some engineers—namely, that designers have been wasting effort trying to maintain an optimum net section at the ends of connections or splices by the use of considerably fewer rivets in the end rows. The tests demonstrate that this is a source of weakness, in that "wide spacing of the end-row rivets (near the edges of the plate) is especially undesirable; * * * differential lateral contraction of the opposing plates at each end of the joint subjected such rivets to lateral strain which, combined with the longitudinal strain, caused premature failures of these rivets." Furthermore, the authors state: "The experimental results bore no consistent relation to the specification requirements for determining net section," and that "It is apparent that there is nothing to be gained by using an elaborate formula to evaluate the effect of pitch on net section for joints of this type." The authors have gone so far in this matter as to offer the proposal that "the allowable loads on riveted tension members be based on, and expressed in terms of, stress in the gross section"; then they propose that the members be so detailed "that the net section be not less than 75% of the gross section."

Based on their considerations of the tests, the authors have offered several recommendations, applicable to design, in addition to the one already mentioned regarding net *versus* gross section. It seems to the writer that these tests and recommendations should be given serious consideration by all structural engineers. There is room in this discussion for only a few points.

The authors have tabulated their recommended working stresses for use in the design of riveted joints in bridges, for steels commonly specified (see Table 21). The working stresses for carbon, silicon, and nickel steels are given respectively as 16.0, 20.0, and 22.5 kips per sq in. on the gross section. Assuming that the net section is the minimum recommended by the authors (namely 75% of the gross), these working stresses give unit stresses on the net section of 21.3, 26.7, and 30.0 kips per sq in., respectively. These unit stresses are quite high for general use in bridge design and are scarcely to be recommended for railroad bridges, light highway bridges, or most heavy highway bridges; and they are, perhaps, applicable only to bridges in which the dead load constitutes a large proportion of the total load—say 50 per cent. They are less, however, than was specified for the San Francisco-Oakland Bay Bridge.

The writer questions whether the authors have not based their recommendation of gross section unit stresses for tension members and 75% ratio on their large tension tests only, which are made up of plates alone. Have they also considered the effect of this recommendation on members built of rolled sections or a combination of rolled sections and plates? Consider, for instance, the simple case of a single angle. Any angle that has one hole for a $\frac{7}{8}$-in. rivet will have left a net section considerably greater than 75% of the gross. Therefore, a unit stress that is based on balancing the 75% ratio is likely to waste material in rolled shapes and even in plates. The gross-section-unit-stress method would be very easy to use, of course, but it would either call for too tight a ratio, gross to net, or it would make it all too easy to waste material and do away with all ingenuity on the part of designers and detailers directed toward maintaining as large a percentage, net to gross, as possible. It scarcely seems possible to find a unit stress based on gross section that will fit various conditions and not waste material.

The authors recommend a somewhat smaller ratio of increase in working stresses in the high-strength steels, as compared with those for carbon steels, than the ratios of minimum yields or tensile strengths of the mill coupons. Ratios of working stress of 1.25 and 1.40 are assigned for silicon and nickel steels, respectively, and 1.33 for manganese rivets, as compared with 1.00 for carbon steel. The authors' conservatism does not seem to be borne out by the results of the tests but is based rather on theoretical considerations as, when high-strength steels are used, some desirable property, such as ductility, plastic flow, and opportunity for stress distribution, must be sacrificed. The tests seem to confirm, rather than upset, the established custom of basing the unit-stress ratios on the yield ratios. Although fatigue may be a reason for a decreased ratio, such a reason must be kept separate from deductions resulting from the tests under discussion.

There is a factor (which may be called a safety factor) that engineers are prone to lose sight of—namely, that the steels, as produced, have actually better physical properties than the specified minima, because the mills, in order to avoid rejections, "play safe" and aim to stay well above the required minima. The average minima run, as the authors state, from 5% to 10% above the required. This tendency to "play safe" is greater in the production of high-strength steels than the ordinary carbon steels because of the greater monetary loss of rejections.

It seems to the writer that both Professor Wilson's tests[8] and those described in this paper have proved conclusively that rivets are not twice as strong in double shear as in single shear, and that specifications should be written to agree with what seems to be a well-established fact. The authors have recommended a ratio of 1.8.

The tests appear to justify Recommendation (5), which states that the practice of assuming equal shear per rivet, regardless of length of joint, is satisfactory. This is a very reassuring conclusion and inspires further confidence in large riveted joints.

The writer cannot agree with Recommendation (6), which states that, except in comparatively heavy structures where reduction in size or weight of splice is important, there is little reason for using manganese-steel rivets rather than carbon-steel rivets. The authors seem to connect this recommendation to the weight of splice material only, and to forget the economies to be obtained by using manganese rivets, not only in splices, but in the connections of members to gussets, where considerable economy is possible due to saving of material and number of rivets. Furthermore, the use of manganese rivets makes for shorter connections which are recommended by the authors. There are genuine difficulties involved in detailing connections for high-strength steels without a corresponding high-strength rivet. The writer believes that there is a need for high-strength rivets, although perhaps not exactly of the same chemical constituents as were used for the rivets for the San Francisco-Oakland Bay Bridge and in the tests presented in this paper. A somewhat more usable, perhaps softer, rivet will probably be developed for main members, such as will match silicon steel rather than nickel, as it seems likely that nickel-steel will be rarely used as compared with silicon, which bids fair to continue in wide use. A rivet that can be used with a steel of 45 to 50 kips per sq in. would seem to be the most desirable.

The writer wishes to congratulate the authors on the completion of an important series of tests, and on their careful and scientific analysis of them, resulting in material addition to the knowledge of the behavior of structural riveted joints, and in data which should (and undoubtedly will) be used widely by structural engineers in the preparation of specifications.

Frederick P. Shearwood,[14] M. Am. Soc. C. E. (by letter).—The distribution of the loads in the rivets and the stresses in the material of large riveted joints is one of the problems that, on account of its complexity and the many indeterminate factors which enter into its resistance, have been avoided by mathematical theorists; and these joints have been designed by simple empirical rules and the designer's common sense or judgment. The adequacy of riveted joints proportioned on the assumption that all the rivets in a joint are equally stressed is a confirmation of the "limit design theory" as presented by J. A. Van den Broek,[15] M. Am. Soc. C. E. There can be little doubt, and this paper confirms it, that it is the ductility of the material which enables the resistance to be safely distributed before the end rivets lose their resisting value.

---

[14] Cons. Engr., Dominion Bridge Co., Ltd., Montreal, Que., Canada.

Although careful designers have had to adopt the practice of disregarding the uneven distortion of material in a riveted joint, they have realized that this is not a really satisfactory method and these tests, and the able presentation of them, are most timely and useful contributions to structural engineering. Many of the findings should be applied to modify standard specifications and practice and so utilize more efficiently the resistance of these joints.

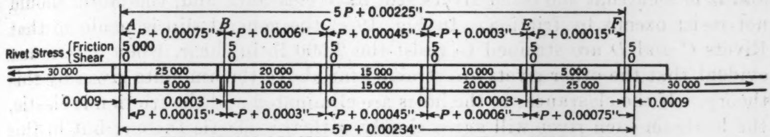
To compute, accurately, the stresses in each rivet of a joint it would be necessary tó know the exact clearance in the holes, the exact frictional resistance between all the surfaces in contact, and the relative deformations due to shear bearing and extension of the plates. These and some other factors are rather indeterminate and make the problem unsolvable; but, to visualize the approximate progressive action as a joint is loaded, is very desirable.

The action of riveted joints as described in the beginning of the paper does not appear to be possible if there is clearance in the holes and, unless the frictional resistance is elastic. Considering that there is clearance and that the friction is not elastic, all the stress must be resisted by the first row until it slips, because the strains in the connected plates must be alike if no movement occurs between them. For instance, in Fig. 19(*a*), if no movement occurs the strain in the two plates between Rivets *A* and *F* must be equal, and therefore Rivets *B C D* and *F* are redundant until the plates slip at *A* and *F* when *B* and *E* will resist the surplus stress; and the strain in the plates between *B* and *E* will be equal until they slip and *C* and *D* come into action. The condition when the load is just below the total frictional resistance of all the rivets will be as shown in Fig. 19(*a*) where Rivets *A* and *F* have slipped an amount equal to the difference in the strains in the upper and lower plates from *A* to *F*; whereas the slip at Rivets *B* and *E* is equal to the difference in strains from *B* to *E*. If the load is now increased so as to overcome the friction of all the rivets, the plates will slip until the end rivets come to a bearing and shear in them resists further movement. This condition is shown in Fig. 19(*b*), which shows that the interior rivets still have considerable clearance and so are only resisting by friction. If the load is doubled (see Fig. 19(*c*)), there will still remain a small clearance at Rivets *C* and *D*, and most of the extra load is taken by the first rows, *A* and *F*, and only a small amount by *B* and *E*.

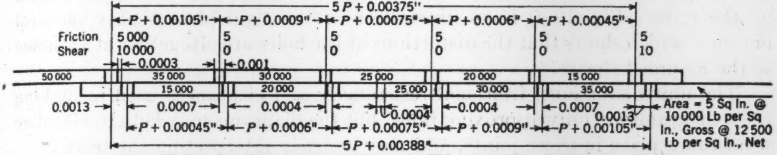
Figs. 19(*d*) and 19(*e*) show the relative position of the plates when the rivets are stressed according to Fig. 1. In Fig. 19(*d*) the general slip is made so that Rivets *A* and *F* are strained to the correct amount to resist 25 000 lb in shear,

---

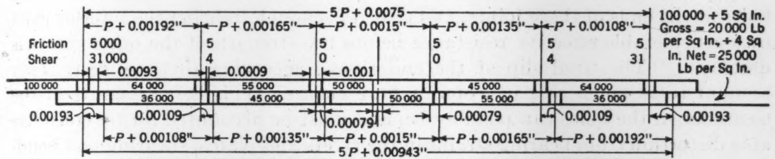
[15] "Theory of Limit Design," by J. A. Van den Broek, *Proceedings*, Am. Soc. C. E., February, 1939, p. 193.


5P + 0.00225"
A B C D E F
P + 0.00075" P + 0.0006" P + 0.00045" P + 0.0003" P + 0.00015"
Rivet Stress {Friction Shear
5 000 0
5 0
30 000
25 000 20 000 15 000 10 000 5 000
5 000 10 000 15 000 20 000 25 000
30 000
0.0009 0.0003 0 0 0.0003 0.0009
P + 0.00015" P + 0.0003" P + 0.00045" P + 0.0006" P + 0.00075"
5P + 0.00234"

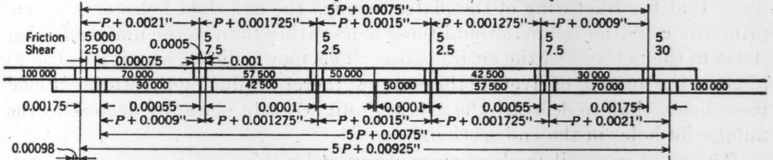
(*a*) TOTAL STRESS RESISTED BY FRICTION (6 RIVETS @ 5 000 LB EACH)


5P + 0.00375"
P + 0.00105" P + 0.0009" P + 0.00075" P + 0.0006" P + 0.00045"
Friction Shear
5 000 10 000
5 0 5 0 5 0 5 0 5 10
0.0003 0.001
50 000 35 000 30 000 25 000 20 000 15 000
15 000 20 000 25 000 30 000 35 000
0.0013 0.0007 0.0004 0.0004 0.0004 0.0007 0.0013
Area = 5 Sq In. @ 10 000 Lb per Sq In., Gross @ 12 500 Lb per Sq In., Net
P + 0.00045" P + 0.0006" P + 0.00075" P + 0.0009" P + 0.00105"
5P + 0.00388"

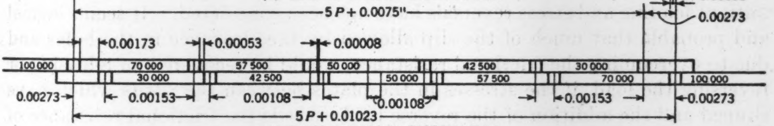
(*b*) TOTAL STRESS EXCEEDS FRICTION OF ALL RIVETS. (6 RIVETS = 8 330 LB, AVERAGE)


5P + 0.0075
P + 0.00192" P + 0.00165" P + 0.0015" P + 0.00135" P + 0.00108"
100 000 ÷ 5 Sq In. Gross = 20 000 Lb per Sq In., ÷ 4 Sq In. Net = 25 000 Lb per Sq In.
Friction Shear
5 000 31 000
5 4 5 0 5 0 5 4 5 31
0.0093 0.0009 0.001
100 000 64 000 55 000 50 000 45 000 64 000
36 000 45 000 50 000 55 000 36 000
0.00193 0.00109 0.00079 0.00079 0.00109 0.00193
P + 0.00108" P + 0.00135" P + 0.0015" P + 0.00165" P + 0.00192"
5P + 0.00943"

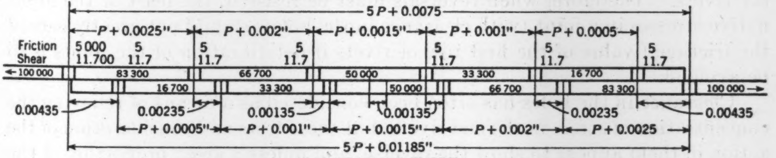
(*c*) 100 000-LB STRESS. 6 RIVETS = (16 666 LB PER RIVET, AVG) UNIT STRESS IN PLATES 20 000 LB PER SQ IN. GROSS, 25 000 LB PER SQ IN. NET


5P + 0.0075"
P + 0.0021" P + 0.001725" P + 0.0015" P + 0.001275" P + 0.0009"
Friction Shear
5 000 25 000
0.0005
5 7.5 5 2.5 5 2.5 5 7.5 5 30
0.00075 0.001
100 000 70 000 57 500 50 000 42 500 30 000
30 000 42 500 50 000 57 500 70 000 100 000
0.00175 0.00055 0.0001 0.0001 0.00055 0.00175
P + 0.0009" P + 0.001275" P + 0.0015" P + 0.001725" P + 0.0021"
5P + 0.0075"
0.00098
5P + 0.00925"

(*d*) ELASTIC THEORY DISTRIBUTION


0.00098
0.00273
5P + 0.0075"
0.00173 0.00053 0.00008
100 000 70 000 57 500 50 000 42 500 30 000
30 000 42 500 50 000 57 500 70 000 100 000
0.00273 0.00153 0.00108 0.00108 0.00153 0.00273
5P + 0.01023"

(*e*) ELASTIC THEORY DISTRIBUTION


5P + 0.0075
P + 0.0025" P + 0.002" P + 0.0015" P + 0.001" P + 0.0005
Friction Shear
5 000 11.700
5 11.7 5 11.7 5 11.7 5 11.7 5 11.7 5 11.7
100 000 83 300 66 700 50 000 33 300 16 700
16 700 33 300 50 000 66 700 83 300 100 000
0.00435 0.00235 0.00135 0.00135 0.00235 0.00435
P + 0.0005" P + 0.001" P + 0.0015" P + 0.002" P + 0.0025
5P + 0.01185"

(*f*) RIGID PLATE THEORY

Scale of Distortions =
1"/100
Clearance in Rivet Hole, (Inches)
0.001

Distortion in Plates per Pitch per 10 000 Lb = $\frac{10\,000 \times 4 \text{ In.}}{29\,000\,000 \times 5 \text{ Sq In.}}$ = 0.0003 In.

Distortion of Rivet and Hole per 10 000 Lb = 0.0003 In.

P (Pitch) = 4 In.; Area of Plate = 5 Sq In. Gross

FIG. 19

and it is seen that the other rivets still have clearance and, therefore, should not resist except by friction. In Fig. 19(*e*) the general slip is made so that Rivets *C* and *D* are strained to resist the 2 500 lb in shear, when it becomes evident that the other rivets are strained far above the amounts given by this theory. If the clearances in the holes are eliminated and the friction is elastic, the loads on each rivet will agree closely with this elastic theory; but in this case, as there is no clearance, there should not be any early sudden slip as shown by the tests. Fig. 19(*f*) is an illustration of the rigid plate theory (general practice) which shows that the distortions at the holes are altogether at variance to the assumed shears.

The unit distortions, frictional resistances, and clearances used in making these diagrams are only approximations; but the diagrams may help to visualize what takes place in these joints, and may assist in interpreting the tests.

The tests show that considerable deformation can occur in the rivets and holes at the ends of these joints, and that it is enough to bring the interior rows into considerable effective resistance before the strength of the outer rivets is destroyed. The total slip at the end rivets, according to these tests, may amount to as much as $\frac{3}{8}$ in. before failure occurs. This is far more than the clearance in the holes can possibly be, and must be accounted for by considerable distortion from bearing strains in the rivets and plates, shearing and bending in the rivets, and bending in the plates around the end rivets. It seems likely that the fracturing of the plates through the end rivet holes may be due primarily to initial failure around these holes rather than to the uniformly high stress in the net area of the entire plate. This may indicate why it is better to increase the number of rivets in the end row, thereby reducing the strains around these holes, than to decrease the average unit stress in plates by decreasing the number of holes in the end section.

The tests were all made with continuous loading and the effect of intermittent loading and stress reversals have not been considered. It seems logical and probable that much of the slip allowed by the clearance in the holes and due to overcoming the frictional resistance, would be recovered on releasing or reversing the load, if the stresses in the plates between the rivets which have slipped and the addition of the reverse load exceeds the frictional resistance of the rivets. Therefore, when reversals must be resisted, the least of the alternative stresses in a joint (with clearance in the holes) should not greatly exceed the frictional value of the first row of rivets if deterioration of the joints is to be avoided.

Clearance in the holes has often been considered as a means of relieving the concentration of stress in the end rivets; but these tests and consideration of the action in them appear to show the reverse, and unless a great proportion of the rivet resistance is due to friction, a better distribution is obtained without clearance. Cold-driven rivets may be the best means of obtaining this result as very little cooling occurs after the driving pressure is removed. Experience with cold riveting in tank work would seem to prove that such rivets do fill the holes thoroughly.

The authors' conclusions emphasize, indirectly, the importance of accurate workmanship because such faults as poor matching of holes must increase the amount of slip, and overstrain the vulnerable rivets. Unstraightened material, etc., must decrease the contacting surfaces and so decrease the friction between the plates. This feature suggests that drilling should give some advantage over punching, as the latter must deform the plates to some extent and so decrease the area of contact between surfaces.

Recommendations (1) and (2) of the paper are of great importance. For some time, there has been a fairly general feeling among designers that the spreading of rivets to save section area was more of a competitive effort to save metal and meet specifications than to gain genuine strength. These tests appear to show quite definitely that nothing is gained by doing it. Recommendation (5) seems rather too broad as these tests and theoretical considerations indicate that the strain in the end rivets must be increased as the length of the joint is increased. It also conflicts with reasons implied by Recommendation (3).

The magnitude and inequality of the distortions in riveted joints, as found by these tests, again emphasize that much of the designing cannot be based entirely on the elastic theories, and they also indicate how useful the ductility of steel can be in offsetting the errors in the assumptions used in designing and in safeguarding the mistakes and abuses of workmanship. The ductile and the excess elastic properties of steel must often function, but seldom are acknowledged.

JONATHAN JONES,[16] M. AM. SOC. C. E. (by letter).—The engineers of the San Francisco-Oakland Bay Bridge performed a notable service when they appropriated the sum required for a testing program on this large scale. Very seldom can a fraction of such a sum be found for structural research. One of the probable results of this report, however, is that it will justify reliance in future (as other phases of the riveted joint are investigated) upon smaller specimens and less expensive programs. That is, in future it may be adequate to reason from small to large, now that data have been presented on a project that covered both.

In accordance with the desires of those who formulated this program, considerable effort was made to acquire a knowledge of the details of the behavior of these joints prior to their failure. This behavior has been explained clearly in general terms, and for these specific joints certain intermediate loads, called "useful limit point" and "effective rivet yield," have been defined and recorded. This represents an inquiry as to whether such intermediate, rather than the ultimate, loads shall be the basis for determining design loads, by analogy with the customary use of tensile yield point, rather than ultimate strength, of main material.

The answer appears to be in the negative; the record suggests that the intermediate points in question are not so sharply defined, and not so important to structural behavior, but that the ultimate capacity of a joint, much more readily and certainly ascertainable, will suffice for designing.

Examining the lap joints (Table 5) which failed in the plates, one finds that the yield ratio (Column (14) divided by Column (18)) is 0.61 and 0.54 for $\frac{3}{4}$-in.

---

[16] Chf. Engr., Fabricated Steel Construction, Bethlehem Steel Co., Bethlehem, Pa.

and ⅝-in. plain carbon plates, and 0.75 to 0.62 for plates in joints, as against 0.64 and 0.56 for the mill coupons corresponding respectively thereto. This ratio is expected to be 0.55 to 0.60 in structural steel. Therefore, if the engineer designs for a certain factor of safety against failure of plates in a joint, he will automatically have an adequate factor against joint yield point.

Examining the lap joints (Table 9) which failed in the rivets, one finds that the rivet-yield ratio (Column (19) divided by Column (26)) is 0.57 to 0.71 for carbon rivets (1 in 5 being less than 0.60) and 0.56 to 0.67 for manganese rivets (1 in 9 being less than 0.60). Again, therefore, if the structural engineer designs with respect to breaking loads he will be in a usual relationship with the "effective rivet yield." The same comment holds true for the ratio between the yield load and the ultimate load for the more complex joints of Table 17, in which silicon plates were connected by carbon rivets.

On the basis of this comment, designers may base riveted joint design upon the breaking strengths determined in tests past, present, and future, with no fear of insufficient protection against some intermediate stage of yield. There is an important exception—the effect of pulsating or alternating loads upon joints which may slip readily; but this is a subject for direct testing in fatigue, rather than for reasoning from phenomena observed in static testing.

It is gratifying that these tests should have led to the authors' fifth conclusion that "the practice of assuming equal shear per rivet, regardless of length of joint, is satisfactory." Their record shows that this may be said of all of the rivets in any one joint. It is not so certain that it should be taken to mean that the same unit shear may be assumed for all of the rivets in a very long joint as for all of those in a very short one. Rivet strength in 7-ft joints was 10% less than in 2-ft joints (heading "Conclusions from the Tests: Effect of Length of Joint"). This is not a great difference, so long as specification values are not based on the results of the long joints. Rather than do that, the writer would urge that they be based on short joints and slightly reduced for larger ones, since there are possibly thousands of 2-ft joints designed and fabricated for every one of 7 ft.

Before any value can be assigned to a rivet, there must be knowledge of the relation between the ultimate shearing strength of a driven rivet, and the tensile strength of the bar from which it was manufactured. Neither this report nor the pilot test report[8] deals satisfactorily with this important ratio, because it would require that an individual heat be followed through from tension testing of the bar to shear testing of a joint, instead of using averages; and it would require starting with an annealed bar for the tensile test because the cooling conditions make "as-rolled" bars from the same heat so variable in tension test.

This has been done and reported elsewhere;[17] and the writer, in consequence thereof, approves the authors' proposed 15 kips per sq in. shear for carbon rivets (if tied to 52 kips per sq in. specified minimum tensile strength) and their 20 kips per sq in. shear for low-alloy rivets (if tied to 68 kips per sq in. specified minimum tensile strength, annealed). These should be safe values for static loads, as in buildings, and for bridge design if tied in to live loading somewhat in excess of that now operating.

---

[17] Report on Silico-Manganese Rivet A195-36T to Subcommittee II of Committee A-1, American Society for Testing Materials, by the Section on High-Tensile Rivet Steel, 1938.

Similar shearing tests (not published) have been made on eleven different compositions of low-alloy rivet steel, ranging from 64 to 83 kips per sq in. in annealed tensile strength. It was found that the ratio "shearing strength of driven rivet to annealed tensile strength of bar" was as low as 0.90 and as high as 1.065. The tests indicate that this ratio should be determined for each particular rivet steel proposed for use; with that information at hand, and with the data of the authors' experiments, the breaking strength of any structural joint in which the rivets are the weaker element will be predicted easily. The same is not true of the behavior of the plate material in joints that are adequately riveted or over-riveted.

Most, if not all, of the tension members in riveted bridge trusses, now standing, have been detailed on a certain assumption which these experiments tend decidedly to disprove. That assumption is that the safe strength at any right section is proportional to the net area existing on that section. Accordingly, economy of main material has been realized by so detailing as to start with a reasonable minimum number of rivet holes across any general section through the body of the member (tack rivets, attachments of diaphragms, etc.) and, on entering a splice or connection, to omit more and more holes as stress is passed through rivets into the splice or connection plate, until in the heart of the splice or connection as many holes are omitted from a section as the prescribed minimum spacing of rivets will allow.

In a typical chord of a bridge built in 1939, for instance, the computed net section at the entry into a connection is 85% of the gross, and in the heart of the connection it is found to be 73%, the difference being accounted for by stress unloaded into the connection plates. The evidence of the authors' tests is that the section of 85% net area is actually no stronger than that of 73% net area, the reason being found in stress paths and strain effects.

It is natural to inquire at once as to what extent existing bridges are deficient in capacity, as compared with the designers' intention, on the evidence of these experiments. To that end the writer has had computed the factors of safety of all of the splices tested in those experiments, dividing their "adjusted" yield loads and "adjusted" breaking loads by the working loads which would be permitted upon them under the present bridge design specifications of the American Railway Engineering Association (A. R. E. A.). By "adjusted" load is meant the actual experimental load, reduced in the ratio of the minimum yield point, or ultimate strength, of steel permitted under A. R. E. A. specifications, to the corresponding coupon value on the material used in these experiments. The resulting factors of safety, therefore, are presumed to represent the least that could occur in practice, and are less than any average values that might be assumed from any average strength of material as supplied.

These "adjusted" factors of safety are shown in Tables 23, 24, and 25, corresponding respectively to Tables 5, 9, and 17. Each line in these tables represents the average of two identical test specimens. The values in Column (5), Table 23, are obtained from Table 1 by multiplying Column (4), Table 1, by Column (14), Table 1. The adjusted yield load (Column (6), Table 23) equals $\frac{\text{Column (5)} \times 33}{\text{Column (4)}}$, to reduce to the value of the weakest permissible material. The adjusted ultimate load (Column (9), Table 23) equals $\frac{\text{Column (8)} \times 60}{\text{Column (7)}}$, to reduce to the value of the weakest permissible material.

TABLE 23.—Tests on Lap Joints that Failed in the Plates; Study Based on Table 5

Specimen No.	Net area, in square inches*	Allowable load, in kips*	Coupon yield point, in kips per square inch	Specimen Yield Load, in Kips		Coupon ultimate strength, in kips per square inch	Specimen Ultimate Load, in Kips		A.R.E.A. Safety Factor	
				By test	Adjusted		By test	Adjusted	Yield $\frac{\text{Column (6)}}{\text{Column (3)}}$	Ultimate $\frac{\text{Column (9)}}{\text{Column (3)}}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
CCC0	7.77	139.8	44.1	295	221	68.3	485	426	1.58	3.05†
CCC2–2	6.08	109.5	43.4	295	224	69.6	394	339	2.05	3.10
CCC2–3	6.54	117.7	44.1	287	215	68.2	389	343	1.83	2.93
CCC2–4	6.76	121.5	43.9	298	224	69.4	401	347	1.84	2.85
CCC2–5	6.92	124.3	43.4	302	229	70.0	419	359	1.84	2.88
CCC2–6	6.92	124.3	43.7	296	224	69.9	418	359	1.80	2.88
CCC5	5.90	106.2	41.0	287	231	67.9	418	370	2.17	3.49
DCC0	13.14	237.0	38.1	458	396	68.1	840	740	1.68	3.13†
DCC2–3	11.00	198.0	38.9	435	369	67.7	697	617	1.86	3.74
DCC2–4	11.76	212.0	38.1	437	379	68.0	700	618	1.79	2.92
DCC2–4	11.76	212.0	39.1	441	374	68.2	713	629	1.77	2.97
DCC2–5	11.64	210.0	39.9	429	355	67.7	691	612	1.69	2.92
DCC2–6	11.59	209.0	38.5	422	363	67.2	677	604	1.74	2.88
DCC5	9.29	168.0	38.0	411	357	67.6	663	590	2.12	3.51

* Column (2), net area by Specifications of American Railway Engineering Association, 1938, Par. 409; Column (3), the same, multiplied by 18 000 lb per sq in.

† No rivets.

Values for allowable load in Column (6), Table 24, are equal to Column (5) × 13.5 for carbon rivets and Column (5) × 18 for manganese rivets. The values in Columns (7) and (10), Table 24, copied from Table 4, are not very satisfactory because they are averages of more than one heat. These values were obtained from 95 "as-rolled" rods for carbon rivets and from annealed rods for manganese rivets. The adjusted yield load (Column (9), Table 24) equals $\frac{\text{Column (8)} \times 28}{39.6}$ for carbon rivets; and $\frac{\text{Column (8)} \times 38}{49.2}$ for manganese rivets; 28 kips per sq in. is the specified minimum yield point under specifications of American Society for Testing Materials (A. S. T. M.) A141–36 and 38 kips per sq in. (annealed) is the specified minimum yield point under A. S. T. M. Specification A195–39T. The adjusted ultimate load (Column (12), Table 24) equals $\frac{\text{Column (11)} \times 52}{57.2}$ for carbon rivets and $\frac{\text{Column (11)} \times 68}{74.7}$ for manganese rivets; 52 kips per sq in. is the specified minimum ultimate strength under A. S. T. M. Specification A141–36 and 68 kips per sq in. (annealed) is the specified minimum ultimate strength under A. S. T. M. Specification A195–39T.

In Table 25, the adjusted ultimate load for the plates (Column (9)) equals $\frac{\text{Column (8)} \times 60}{\text{Column (6)}}$, and for rivets (Column (10)) equals $\frac{\text{Column (8)} \times 52}{\text{Column (7)}}$, 60 kips and 52 kips being the minimum ultimate strengths for plates and rivet

bars specified by A. R. E. A. materials specification. The factor of safety at the yield point was not computed because the joint yield point was difficult to relate to the plate and the rivets, respectively.

TABLE 24.—TESTS ON LAP JOINTS THAT FAILED IN THE RIVETS; STUDY BASED ON TABLE 9
(C = carbon; S = silicon; Ni = nickel; and M = Manganese)

Specimen No.	STEEL		RIVETS		Allowable load, in kips	Coupon yield point, in pounds per square inch	SPECIMEN YIELD LOAD, IN KIPS		Coupon ultimate strength, in pounds per square inch	SPECIMEN ULTIMATE LOAD, IN KIPS		A. R. E. A. SAFETY FACTOR	
	Plates	Rivets	Number	Area, in square inches			By test	Adjusted		By test	Adjusted	Yield Column (9) / Column (6)	Ultimate Column (12) / Column (6)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
ACC18	C	C	18	14.14	191	39.6	549	388	57.2	787	716	2.03	3.75
ACC36	C	C	36	28.28	382	39.6	1 030	730	57.2	1 446	1 318	1.92	3.45
ACC54	C	C	54	42.42	573	39.6	1 640	1 160	57.2	2 120	1 930	2.02	3.36
ASC18	S	C	18	14.14	191	39.6	567	401	57.2	775	705	2.10	3.69
ASC36	S	C	36	28.28	382	39.6	1 178	831	57.2	1 525	1 388	2.18	3.65
ASC54	S	C	54	42.42	573	39.6	1 680	1 188	57.2	2 166	1 970	2.08	3.43
ACM12	C	M	12	9.43	170	49.2	557	430	74.7	715	650	2.53	3.82
ACM24	C	M	24	18.85	340	49.2	1 070	828	74.7	1 460	1 326	2.43	3.90
ACM36	C	M	36	28.28	510	49.2	1 595	1 232	74.7	2 017	1 830	2.41	3.59
ASM12	S	M	12	9.43	170	49.2	565	436	74.7	682	620	2.56	3.65
ASM24	S	M	24	18.85	340	49.2	1 195	924	74.7	1 470	1 340	2.72	3.95
ASM36	S	M	36	28.28	510	49.2	1 610	1 242	74.7	2 054	1 870	2.43	3.66
ANM12	Ni	M	12	9.43	170	49.2	606	468	74.7	704	640	2.75	3.76
ANM24	Ni	M	24	18.85	340	49.2	1 280	990	74.7	1 464	1 335	2.92	3.93
ANM36	Ni	M	36	28.28	510	49.2	1 860	1 440	74.7	2 140	1 940	2.82	3.80
BCC-20a	C	C	22	17.28	233	39.6	685	485	57.2	975	886	2.08	3.78
BCC-20b	C	C	22	17.28	233	39.6	685	485	57.2	1 013	922	2.08	3.95
BCC-20c	C	C	22	17.28	233	39.6	650	460	57.2	988	900	1.96	3.85
CCC7	C	C	8	6.28	85	39.6	280	198	57.2	358	326	2.33	3.84
DCC7	C	C	14	11.00	149	39.6	415	294	57.2	623	566	1.97	3.80

TABLE 25.—TESTS ON BUTT AND SHINGLE JOINTS; STUDY BASED ON TABLE 17.

Specimen No.	Net plate area, in square inches*	Rivet shear area, in square inches	ALLOWABLE LOAD IN KIPS		COUPON ULTIMATE STRENGTH, IN KIPS PER SQUARE INCH		ULTIMATE LOAD IN KIPS			A.R.E.A. SAFETY FACTOR	
			Plate, at 24 kips per square inch*	Rivets, at 13.5 kips per square inch*	Plates (silicon)	Rivets ($\frac{7}{8}$-inch carbon) Table 4	By test	Adjusted to plate	Adjusted to rivets	Referred to plate Column (9) / Column (4)	Referred to rivets Column (10) / Column (5)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
FSCA	12.42	22.8	298	308.5	91.5	58	949	830	850	2.78	2.75
FSCB	24.84	45.6	596	617	91.9	58	1 850	1 610	1 660	2.70	2.69
FSCD	24.83	45.6	596	617	90.4	58	1 799	1 595	1 610	2.68	2.61
FSCC	36.99	68.4	888	925	91.8	58	2 660	2 320	2 390	2.61	2.59
FSCE	36.54	68.4	877	925	92.5	58	2 688	2 325	2 410	2.65	2.61
FSCF	37.00	69.6	888	942	90.3	58	2 652	2 360	2 380	2.66	2.52

* Columns (2), (4), and (5), by Specifications of American Railway Engineering Association, 1938.

For lap specimens failing in plates (Tables 5 and 23), the safety factor for adjusted yield load varies from 1.69 to 2.17 and that for adjusted ultimate varies from 2.85 to 3.51. For lap specimens failing in the rivets (Tables 9 and 24), this safety factor for adjusted yield load varies from 1.92 to 2.92 and that for adjusted ultimate varies from 3.36 to 3.95. For butt and shingle joint specimens (Tables 17 and 25), this safety factor for adjusted ultimate varies from 2.61 to 2.79. As against these may be set the minimum factors of safety intended by the specification writers, $\frac{33}{18} = 1.83$ for yield load, and $\frac{60}{18} = 3.33$ for ultimate.

One interesting circumstance is that, as regards "adjusted yield load," the lap specimens CCCO and DCCO, both of them plain material without any holes or rivets, are more deficient in factor of safety than any of the riveted specimens. This is not true as regards ultimate strength. Deficiency in joint yield point, therefore, is not to be taken too seriously as a criticism of rivet arrangement.

The ratio $\frac{1.69}{1.83} = 92.3$ per cent. Therefore, existing margins against yield point, in carbon steel design, are apparently never below the intention by more than 8%, and seldom below at all; but $\frac{2.85}{3.33} = 85.5\%$ and $\frac{2.61}{3.33} = 78.3\%$; and therefore existing margins against ultimate, in carbon steel design, are occasionally as much as 15% below the intention, and in silicon steel they are habitually lower yet. Viewing this latter statement from another angle, it is as if carbon steel members had occasionally been designed for $\frac{18}{85.5} = 21$ kips per sq in. and silicon steel members for $\frac{24}{78.3} = 30.6$ kips per sq in., neither of which values would meet with general approval in a specification for a new bridge.

The authors wish to place tension-member design on a basis more in accord with their test results, and at the same time they apparently desire not to add too greatly to the current costs of bridgework. Accordingly they recommend: (1) That tension members be designed for gross section; (2) that 25% be taken out of the critical sections and no more be permitted to be taken out at other sections; and (3) that the allowable unit stress on gross section of carbon steel bridge members be 16 kips per sq in.

The writer believes that the first recommendation involves too many difficulties to be acceptable. For instance, many tension members are being built to-day with top and bottom cover plates, pierced with oblong holes, say, 12 in. wide, to permit access. These cover plates are reduced in net width by nearly 50 per cent. Other irregularities are encountered in detailing. Designers will have to treat such cases on the basis of the picture that exists, and the writer feels that a net-section basis, however modified, will necessarily be retained.

Since additions on the order of 10% to 15% to the gross section of riveted tension members are not acceptable economically, designers will begin a search for expedients whereby no such additions need be made if the findings of these tests are accepted. The authors realize this, and they have offered such an expedient, namely (see Recommendation (3)), that a gross unit stress of 16 000 lb per sq in., which is equivalent to 21 333 lb per sq in. on a 75% net section, be substituted for the 18 000 lb per sq in. of net section found in current specifications.

As one argument for such increase, the authors cite the excess of coupon strength they have found over the specified minimum, amounting to 10% at yield point and 5% at ultimate. To the writer this excess seems not available for design purposes. It is true that he has seen few yield-point reports that did not exceed the specified minimum, but he has seen many reports of ultimate tensile stress that were a few hundred pounds under or a few hundred pounds over. To depend, definitely, on all steel being, by some percentage, stronger than the specification requires is illusory. Designers should continue to assume that steel of the minimum permitted strength may be built into any critical part of their structures.

Another argument for increasing allowable unit stress in a joint is found by the authors in the fact that the ratio, $\frac{\text{Breaking load per square inch}}{\text{Coupon strength}}$, is greater than 100% (103.6 and 105.7) for Specimens CCC5 and DCC5 (all holes out), whereas it varies from only 83.3 to 96.7 for the nine Specimens CCC2 and DCC2 (two end holes). These findings are confirmed by E. L. Gayhart[6] in the following words:

> "A plate failure through the net section of full-row chain riveting may be expected to show a strength in excess (possibly 10 percent) of the theoretical value. The omission of alternate rivets in the outer row causes a loss of strength in the net section failure as compared with full-row riveting, and may even show less than theoretical strength."

Further confirmation is found in wide-plate tests by Wilson, Mather, and Harris[18] in which it appears that riveted lap joints with rivet spacing the same in all rows showed unit strengths on net section exceeding the coupon strength by 6.5% for two rows and by 7.5% for four rows. Other joints with certain rivets omitted from the outer rows, while their total capacity was thereby increased, showed less unit strength on net section than the coupons.

However, it will be noted that these reported increases of 3.6% to 10% over coupon strength, in specimens with all holes out, do not check with the authors' proposed increase from 18 000 to 21 333, or 18.5%, in working stress.

The increase over coupon unit strength was obtained by the authors (in Specimens CCC5 and DCC5), by Gayhart, and also by Wilson, Mather, and Harris on specimens with closely and uniformly spaced gage lines, creating a very straight and uniform flow of stress. Practically, such a condition cannot exist in built-up members, where flange angles, for instance, prevent this perfectly uniform spacing of gage lines. Before the results of Specimens CCC5 and DCC5 can be taken as a basis for design, they should be checked by similar specimens, except that the spacing of the gage lines should have variations up to at least an inch; and by still other specimens in which most of the 25% area deducted should be collected in one hand hole.

---

[18] "Tests of Joints in Wide Plates," by Wilbur M. Wilson, M. Am. Soc. C. E., James Mather, and Charles O. Harris, Eng. Experiment Station, *Bulletin No. 239*, Univ. of Illinois, 1931, Table 9, p. 44.

Furthermore, the even spacing of gage lines is not the only factor in favor of Specimens CCC5 and DCC5 in these tests. Comparing their designs (see Fig. 3), the distance from center of splice, where both plates are carrying the same unit stress, to the extreme rivet row where the stress in one plate is doubled and in the other reduced to zero, is seen to be 2 in. for Specimen CCC5 and 6 in. for Specimen CCC2–2. It is 6 in. for Specimen DCC5 and 9 in. for Specimen DCC2–3. Is it possible that the effect of differential plate stretch, elsewhere cited by the authors as a reason for failure in the end row, was here a little favorable to Specimens CCC5 and DCC5? It would seem that further tests are indicated, gradually increasing the pitch between the two rows of Specimen CCC5 and thus increasing this differential stretch.

Furthermore, some tests should be run in double shear. A factor which should thus be eliminated is the influence of bending on lap specimens. Whatever this factor may be, it may have been different on the short joint, CCC5, and the longer joints of the CCC2 series.

Therefore, the entire question of specification revision should await a more complete range of tests, simulating a number of practical arrangements of parts (plate with two flange angles, etc.) in which it is impossible to achieve an ideal stress path. It would seem that lighter and smaller specimens would serve this purpose.

Until further information is thus obtained, it would be premature to adopt a tensile stress of 16 kips per sq in. of gross section simply because of the behavior of the more or less ideal Specimens CCC5 and DCC5.

The more probable, although only implied, basis for the authors' recommendation as to unit stress is that the factors of safety corresponding thereto have been shown (as outlined in the first part of this discussion) to be all that exist in at least some parts of some existing bridges (and can at any time exist, under current specifications); and these bridges appear to be satisfactory. It seems doubtful that this will be a general attitude. It is possible that, in course of time, designers might adopt, and detailers would welcome, the principle of taking all holes out of all rows in splices and connections. It seems less likely that they will pass quickly from 18 to 21 kips per sq in. as a permissible unit stress on such sections.

It may be of interest to cite a test made at Lehigh University, in 1939, by Bruce G. Johnston, Assoc. M. Am. Soc. C. E.[19] A bar 17.5 in. wide was reinforced with a pair of pin plates attached by fillet and plug welding. In front of the pin, the pin plates tapered toward the center of the bar.

The welds at the forward end of the pin plates, where the differential elongation appeared in the authors' tests to be so severe on rivets, showed no distress. These fillets were equal to, but not in excess of, the requirements of the American Welding Society Specifications for Welding of Bridges. After failure (through bolt holes in the body of the bar), the whitewash had not scaled away from these welds.

---

[19] *The Welding Journal*, Am. Welding Soc. August, 1939, p. 253–5.

There is an interesting lead here toward an expedient for joint design to meet the situation revealed by the authors' tests—namely, before entering a riveted joint or splice, to weld on a reinforcing plate of such dimensions that full gross section can be used in the design, and full-row riveting can be used throughout the joint. This expedient must be regarded with caution because (*a*) its effect upon strength in fatigue remains to be proved by test, and (*b*) fabricators are not prepared to do welding on main members in riveted construction except at relatively high cost. Nevertheless, for long members and for fairly steady stresses, this expedient appears to the writer more valid than a considerable upward revision in tensile unit stress.

To summarize these comments, the experiments reported in the paper are important to structural designers and demand new thinking with respect to tension-member design. They point, however, toward many additional tests to be made before general changes in specifications are undertaken.

C. C. WINTER,[20] ASSOC. M. AM. SOC. C. E. (by letter).—The writer proposes a few problems and tests, and suggests a few changes in specifications or in the interpretation of the present ones.

Rivets.—The rivet is the fundamental element of a riveted joint. The two main variables of rivets are the grip and diameter. It is suggested that there is, or should be, some relation between small pin and rivet design. Fig. 20 shows the relation between bearing, shear, and bending based on a simple case. If the bending is only half as much as assumed in Fig. 20, it still appears to be a very important factor. The tests reported in the paper indicate that a decrease in strength can be expected as the grip increases, but the tests have a very limited variation of grip. Fatigue tests at the University of Illinois support this conclusion. German fatigue tests have developed breaks in bending.

The writer suggests that the specifications on long rivets be made more conservative; for example, "Rivets (long rivets) subjected to calculated stress and having a grip in excess of two diameters shall be increased in number at least 16% per inch of grip." Tests to isolate this problem should be relatively simple.

In Fig. 20 it appears that, for thin plates, bearing is important. These tests were made on relatively thick plates so that bearing for thin plates was not revealed. There appears considerable evidence that the allowable bearing stress can be quite high. It is probable that the rivet heads confine and assist the joint by distributing the load over a larger area.

End Distance.—The tests reported in the paper did not indicate much need for greater end distances, but these tests were for relatively thick plates. One proposed specification calls for the end distance, or edge distance, to be such as to give an area equal to the shearing area of the rivet. It appears reasonable to demand that this area be maintained between rivets to prevent tearing. Proposed: "The pitch should not be less than the end distance plus one-half the diameter of the hole."

---

[20] Res. Engr., Bridge Dept., State Div. of Highways, Sacramento, Calif.

Pitch.—In the "B" series, where full-row riveting was used, there is an indication that the maximum efficiency can be obtained when the pitch is two-thirds of the gage. This point can possibly be explained by the distribution of the Lueders lines. If the lines are at 45°, then with the pitch two-thirds of the gage, the maximum stress lines will fall between rivets and not through them. Photo-elastic studies should indicate the most efficient rivet pattern and the effect of pitch and gage on the stress in the plates.

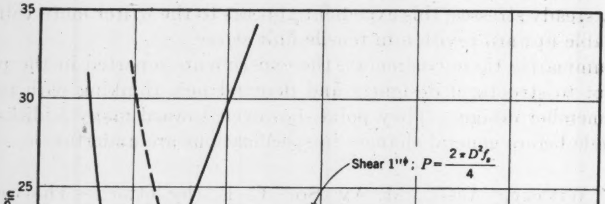


FIG. 20

Net Section.—These tests raised a large question about net section. Under the heading "Conclusions from Tests: Net Section" the following statement is made: "The experimental results bore no consistent relation to the specification requirements for determining net section."

The writer concurs that the tests do not agree with common practice and methods as shown in textbooks. However, net section, as used in the analysis of these tests and common practice, appears to be contrary to a strict interpretation of present specifications, which state that: "The net section shall be the least area which can be obtained by deducting from the gross sectional area, the area of holes cut by any straight or zigzag section across the member,

etc." If the foregoing specification permits the protection of the net section by omitting rivets in the first row, then the specifications need revision.

The grooving effect is important when full-row riveting is used. Why not take advantage of this property and use the nominal diameter of the rivet when the holes are subpunched and reamed?

TABLE 26.—COMPARISON OF PROTECTED NET AREA AND LEAST NET AREA

Specimen (see Figs. 2, 3, and 4)	Net Area, in Square Inches		Ultimate Strength, in Pounds per Square Inch		
	Protected area*	Least area†	Protected net area	Least net area	Coupon (mill)
CCC2-2-1	6.96	5.94	57.3	67.0	69.6
CCC2-2-2	6.93	5.92	56.3	65.9	69.6
CCC2-3-1	6.94	5.92	55.9	65.5	68.2
CCC2-3-2	6.95	5.94	56.3	65.8	68.2
CCC2-4-1	6.93	5.91	57.7	67.6	69.4
CCC2-4-2	6.87	5.87	58.7	68.7	69.4
CCC2-5-1	7.17	6.12	59.7	69.7	70.0
CCC2-5-2	7.18	6.13	57.1	66.8	70.0
CCC2-6-1	7.13	6.08	58.9	68.9	69.9
CCC2-6-2	7.16	6.10	58.2	68.3	69.9
CCC5-1	5.89	6.13	71.8	69.0	67.9
CCC5-2	5.91	6.14	69.9	67.2	67.9
CCC7-1	4.77	5.10	74.6	69.9	67.9
CCC7-2	4.79	5.13	75.2	70.1	67.9
DCC2-3-1	11.68	9.97	59.2	69.3	67.7
DCC2-3-2	11.71	9.99	60.0	70.3	67.7
DCC2-4-1	11.77	10.03	59.6	69.9	68.1
DCC2-4-2	11.76	10.03	59.4	69.6	68.1
DCC2-4-3	11.74	10.02	60.5	70.8	68.1
DCC2-4-4	11.76	10.03	61.0	71.5	68.1
DCC2-5-1	11.63	9.97	59.0	68.8	67.7
DCC2-5-2	11.65	9.95	59.7	69.8	67.7
DCC2-6-1	11.58	9.88	57.8	67.7	67.2
DCC2-6-2	11.60	9.90	59.0	69.2	67.2
DCC5-1	9.28	9.67	70.5	67.7	67.6
DCC5-2	9.29	9.67	72.2	69.4	67.6
DCC7-1	8.18	8.72	76.3	71.6	67.8
DCC7-2	8.18	8.72	76.0	71.3	67.8
FSCA-1	12.37	10.73	76.1	87.7	91.5
FSCA-2	12.46	10.81	76.8	88.5	91.5
FSCB-1	24.82	21.55	74.7	86.0	91.9
FSCB-2	24.86	21.58	74.2	85.6	91.9
FSCC-1	37.07	32.18	72.7	83.7	91.8
FSCC-2	36.90	32.02	71.2	82.1	91.8
FSCD-1	24.76	21.51	72.0	82.8	90.4
FSCD-2	24.89	21.60	73.0	84.0	90.4
FSCE-1	36.15	31.40	74.7	86.1	92.5
FSCE-2	36.93	32.05	72.4	83.4	92.5
FSCF-1	36.87	32.00	70.4	81.2	90.3
FSCF-2	37.13	32.22	72.9	84.0	90.3

* Protected net area is the first row net area as used in the tests.

† The least net area is based on the nominal diameter of the rivet, deducting one hole for each gage line.

Table 26 gives the ultimate strengths based on the nominal diameter of the rivet, one hole being deducted for each gage line, resulting in the least net section obtainable. It is found that the least net section is in fair agreement with the ultimate strengths based on mill coupons. A study of the data from

the tests by E. L. Gayhart[6] at the Bureau of Standards also gives more reasonable results when based on the least net section.

One very desirable quality of any specification is a basis upon which the ultimate strength can be predicted. The present tests do not have large riveted joints with full-row riveting and it seems desirable to perform these tests before the specifications limit the riveted joints to 75% efficiency. A set of comparable butt-welded joints might also be worthy of consideration.

Proposed: "In calculating the required net section of riveted tension members, net sections shall be used in all cases; and in deducting rivet holes, the holes shall be taken as: (*a*) $\frac{1}{16}$ in. larger than the nominal diameter of the rivet for punched holes; (*b*) the nominal diameter of the rivet for reamed or drilled holes. In determining the net width, one hole shall be deducted for each gage line."

Transverse Shear.—In regard to the findings on transverse shear on the rivets, the tests indicate many coarse square fractures with very little reduction of area. If there is transverse shear on the rivets, there must be transverse tension in the main plates and compression in the cover plates. These stresses probably contributed to the curling and buckling of the plates. The combined stress appears to have caused the square breaks. The question might be put: "What is the effect on yield, ultimate strengths and reduction of area if the lateral contraction is restricted?"

The tests seem to indicate the desirability of using full-row riveting, short compact joints, and ductile materials.

It appears that a co-ordinated program of tests should be undertaken and then a uniform specification written.

W. M. WILSON,[21] M. AM. SOC. C. E. (by letter).—To the writer, the two most important conclusions from the tests given by the authors are:

(1) The ultimate strength of a large group of driven rivets equals, approximately, the sum of the strengths of the individual rivets; and,

(2) Omitting alternate rivets from the outside row of a group of rivets, or increasing the transverse distance between the rivets, does not add greatly to the effective net section of a riveted tension member. Conclusion (1) is reassuring and highly gratifying. Conclusion (2) has been indicated by other tests, and will be the subject of the present discussion.

The specifications most used in the design of American steel bridges contain rules for determining the effective net section of riveted tension members. These rules assign considerable importance to the relation between pitch and gage if some rivets are omitted from the outside row and do not recognize the effect of the transverse spacing of the rivets upon the unit strength of the section through the rivet holes. (The outside row of rivets of a joint corresponds to the outside row of a gusset plate in the case of a riveted tension member connected to a gusset plate.) This is true although there has been a considerable quantity of experimental data available for several years, indicating that the ratio of pitch to gage for staggered rivets is a minor factor and that the transverse spacing of the rivets is a major factor affecting the strength of riveted tension members. The tests reported in this paper are of value in that they add to the accumulating evidence that present specifications are in error in both of these respects.

---

[21] Research Prof., Structural Eng., Civ. Eng. Dept., Univ. of Illinois, Urbana, Ill.

The writer has searched American literature to obtain experimental data to be used in a study of the effective net section of riveted tension members such as are used for truss members of steel bridges. With a few exceptions, the tests included in this study comply with the following requirements: (1) The width of the specimens varied from 13.5 in. to 22.5 in., inclusive; (2) the strength of the plate material as given by tests of standard control specimens was reported; (3) the specimens failed in the plate; (4) the specimens had either $\frac{7}{8}$-in. or 1-in. rivets; and, (5) the joints were generally in accord with what is accepted as good design practice. However, there were the following exceptions to the requirements: One specimen was only 10 in. wide; four specimens failed in the rivets but at a plate stress that made the tests of value. Forty-nine tests reported by four investigators complied with these requirements, except as noted, and have been used in the study (they included eight tests made at the Watertown Arsenal;[22] twenty-one tests made for the Navy;[6] four tests by Mr. C. O. Harris;[23] and, sixteen tests made at the University of California and reported in the paper being discussed). In addition to the foregoing, a number of tests of specimens 9 in. wide, made by Mr. Harris,[23] have been used in the study of the effect of the ratio of pitch to gage upon the effective net section.

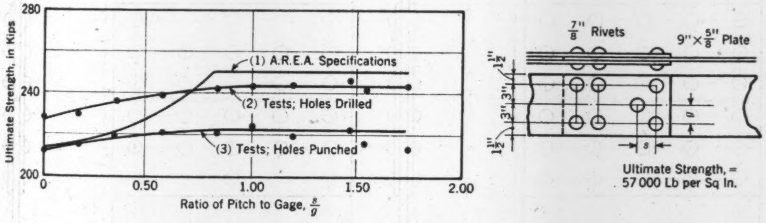


FIG. 21.—RELATION BETWEEN STAGGER OF RIVETS AND STRENGTH OF PLATE OF RIVETED JOINTS

Preliminary to the formulation of a rule for determining the effective net section of riveted tension members, the writer made one study to determine the effect of the ratio of pitch to gage on the strength of joints from which some rivets were omitted from the outer row, and he made a second study to determine the effect of the transverse distance between rivets upon the unit strength of the net section. These studies are presented herein.

The tests used in the study to determine the effect of the ratio of pitch to gage upon the strength of a joint from which rivets were omitted from the outer row include those made by Mr. Harris, some made for the U. S. Navy, and some made at the University of California. The tests made by Mr. Harris, and the details of the joint, are shown in Fig. 21. Two series of tests of riveted joints are reported: For one the holes in the plates were punched (Curve (3)), and for the other the holes were drilled (Curve (2)). A third series of tests was made by Mr. Harris for which the specimens were plates without joints, but with open holes of the same pattern as the rivets of the specimens with riveted joints. The results of these latter tests are shown by Curve (1) in Fig. 21. Tests were made in duplicate for each series and the small circles represent the average of the results of tests on duplicate specimens. The ordinates of the diagrams represent the ultimate load and the abscissas represent the ratio of the pitch to the gage. The strength of the plate material was determined by tests of standard control specimens for all of Mr. Harris' tests, and the ultimate strength for all specimens has been adjusted to a common plate strength of 57 000 lb per sq in. It is to be noted that the plates with open holes tested by Mr. Harris were stronger than the plates connected with rivets. The relation between the strength and the stagger, as computed by the American Railway Engineering Association specifications, is given in Curve (1), Fig. 21.

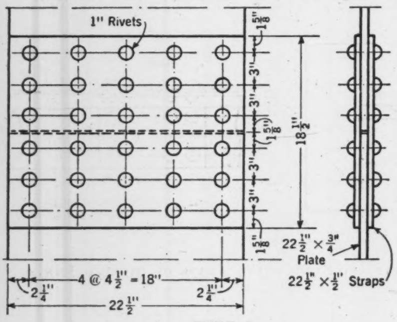
---

[22] "Tests of Metals," 1896, p. 247.

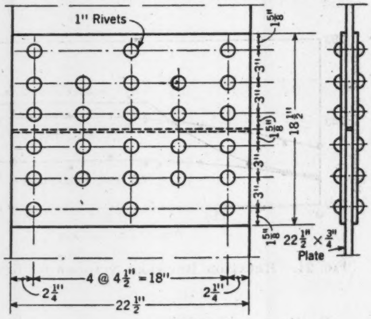
[23] "Effective Net Section of a Riveted Joint," by C. O. Harris, thesis presented to the University of Illinois in 1930 in partial fulfillment of the requirements for the degree of Master of Science.

Tests made for the U. S. Navy by E. L. Gayhart,[6] and used in this study, include six each with the rivet patterns shown in Table 27. Each group of

TABLE 27.—INFLUENCE OF RELATION OF PITCH TO GAGE UPON EFFECTIVE NET SECTION OF RIVETED TENSION MEMBER; TESTS FOR NAVY



TYPE *B*

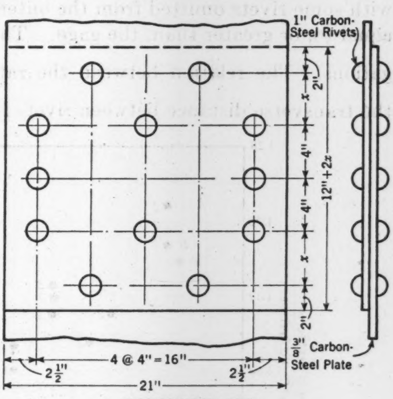


TYPE *D*

Item No.	Description	Type *B*		Type *D*	
		Medium steel	High-test steel	Medium steel	High-test steel
1	Maximum load, in pounds	824 000	1 089 000	853 000	1 053 000
2	Stress on gross section, in pounds per square inch	50 200	65 960	52 690	64 010
3	Strength of control specimen, in pounds per square inch	59 760	80 740	64 975	80 320
4	Effective net area, by test, in square inches	13.79	13.49	13.13	13.11
5	Net Section by American Railway Engineering Association specifications, in square inches	12.31	12.38	13.58	13.80

six specimens included three with carbon-steel plates and three with low-alloy plates. For the specimens with carbon-steel plates, the efficiency of the joints, the average of three tests in each instance, was 0.84 for Type B and 0.81 for Type D. For the specimens with low-alloy plates the corresponding efficiencies were 0.82 and 0.80. That is, changing the pitch from 0 to $\frac{2}{3}$ of the gage decreased the efficiency of the joint only about 3 per cent. (The term efficiency as used herein is the ratio of the ultimate strength as determined by test to the product of the gross area of the section and the unit strength of the control specimen.)

Two series of tests made at the University of California, and reported by the authors (Specimens No. *CCC*2 and *DCC*2, Fig. 2 and Table 5), had the rivet pattern shown in Fig. 22. The plate thickness was $\frac{3}{8}$ in. for Specimen *CCC*2 and $\frac{5}{8}$ in. for Specimen *DCC*2. The pitch, x, had values of 2 in., 3 in., 4 in., 5 in., and 6 in. for *CCC*2, and 3 in., 5 in., and 6 in. for *DCC*2. The values of the efficiency of the joints for the various values of x were as shown in Table 28. It is to be noted that varying x produced no large or consistent change in the efficiency of the joints.



SPECIMEN *CCC*2

FIG. 22

TABLE 28.—INFLUENCE OF RELATION OF PITCH TO GAGE UPON EFFECTIVE NET SECTION OF RIVETED TENSION MEMBERS. TESTS AT UNIVERSITY OF CALIFORNIA

Item No.	Description	SPECIMEN *CCC2*					SPECIMEN *DCC2*		
		$x = 2$ inches	$x = 3$ inches	$x = 4$ inches	$x = 5$ inches	$x = 6$ inches	$x = 3$ inches	$x = 5$ inches	$x = 6$ inches
1	Ultimate load, in thousands of pounds per square inch.	399.0	388.0	400.0	428.0	420.0	691.0	686.0	669.0
2	Stress on gross section, in pounds per square inch...	51.2	49.9	51.5	53.3	52.6	52.9	52.7	51.6
3	Strength of control specimen, in pounds per square inch.	67.7	67.0	66.7	67.5	68.5	65.8	65.6	65.1
4	Effective net section, by test, in square inches..........	5.89	5.79	6.00	6.34	6.13	10.50	10.46	10.28
5	Net section by American Railway Engineering Association specifications, in square inches...........	6.08	6.52	6.93	7.17	7.13	10.97	11.62	11.57
6	Efficiency..................	0.756	0.745	0.772	0.790	0.768	0.804	0.803	0.793

The results of the three independent investigations,[6, 23] agree in one important respect: The strength of riveted joints, with some rivets omitted from the outer row, is not greatly affected by the ratio of the pitch to the gage.

In the study to determine the effect of the transverse distance between rivets on the unit strength of the net section of the plates, only two types of joints were included: (1) Joints with no rivets omitted from the outer row; and (2), joints with some rivets omitted from the outer row but only those which had a pitch equal to, or greater than, the gage. They included tests from all four investigations. The relation between the ratio, $\frac{\text{Strength of net section}}{\text{Strength of control specimen}}$, and the transverse distance between rivets is shown by the small circles of Fig. 23.

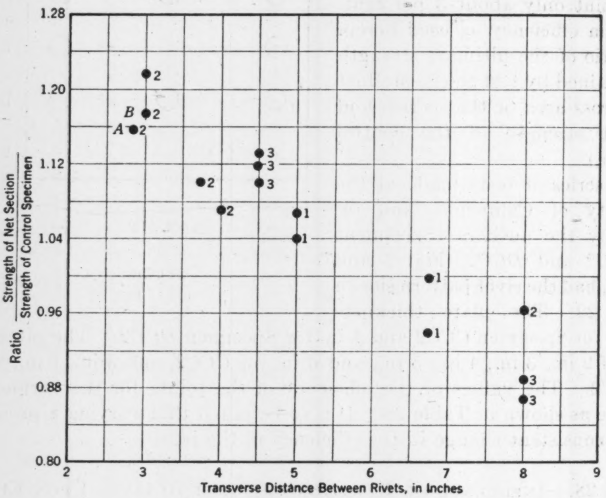


FIG. 23

The numeral adjacent to each circle indicates the number of tests of identical specimens averaged. The study includes twenty-eight tests and a variation in gage from $2\frac{7}{8}$ in. to 8 in. The points marked *A* and *B* are for specimens that failed in the rivets. It is apparent from Fig. 23 that the unit strength of the net section of a riveted tension member is greatly affected by the transverse distance between the rivets.

In view of the discussion contained in the previous paragraphs, it appears that any rule for determining the effective net section of a riveted tension member should take into consideration the net section through the outer row of rivets and also the transverse distance between rivets. Furthermore, it would appear unnecessary to consider the effect of minor variations in the ratio of the pitch to the gage for joints with rivets omitted from the outer row. With these ideas in mind the following empirical rule for determining the effective net section has been developed.

The effective net width (W_n) of a riveted tension member is given by the equation

$$W_n = (W_g - N\,D)\left(1 + \frac{D}{G}\right)0.85 \ldots\ldots\ldots\ldots\ldots (4)$$

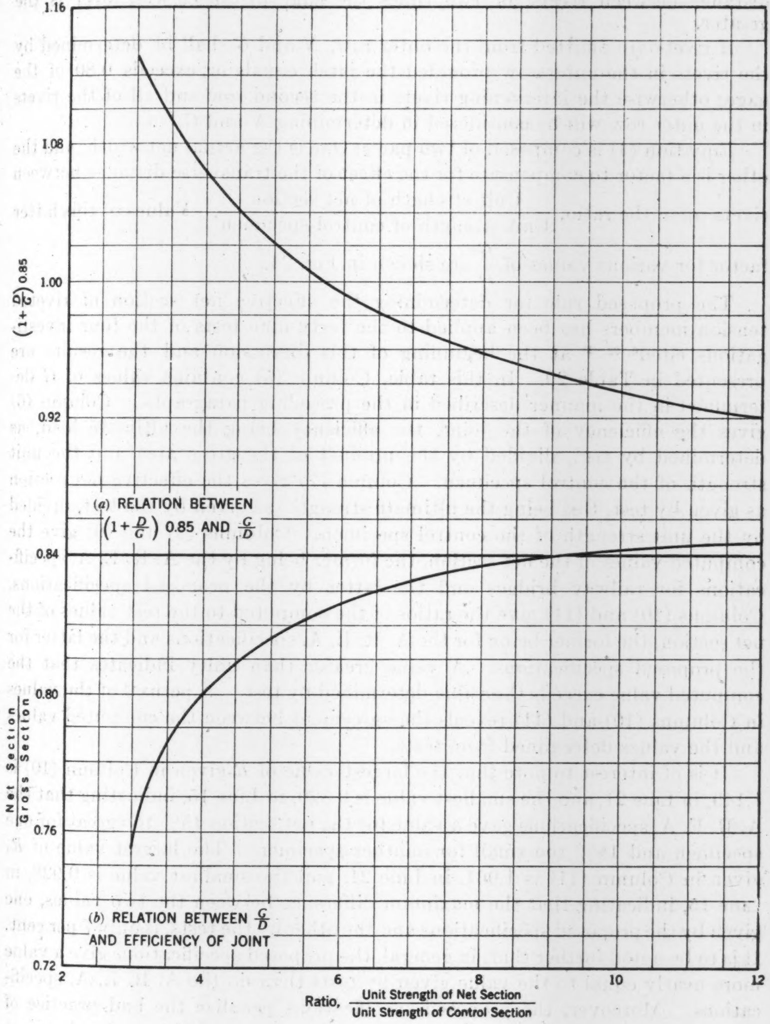

1.16
1.08
1.00
0.92
0.84
0.80
0.76
0.72
$\left(1+\frac{D}{C}\right)$ 0.85
$\frac{\text{Net Section}}{\text{Gross Section}}$
(*a*) RELATION BETWEEN $\left(1+\frac{D}{C}\right)$ 0.85 AND $\frac{C}{D}$
(*b*) RELATION BETWEEN $\frac{C}{D}$ AND EFFICIENCY OF JOINT
2
4
6
8
10
12
Ratio, $\frac{\text{Unit Strength of Net Section}}{\text{Unit Strength of Control Section}}$

FIG. 24

in which W_g = gross width of the member; D = nominal diameter of the rivet plus $\frac{1}{8}$ in.; N = number of rivet holes to be deducted; and, G = transverse distance between rivets, or two times the edge distance, whichever is the greater.

If rivets are omitted from the outer row, N and G shall be determined by the rivets in the outer row provided the pitch equals or exceeds 0.80 of the gage; otherwise the intervening rivets in the second row and all of the rivets in the outer row will be considered in determining N and G.

Equation (4) is composed of two parts: One is the actual net width, and the other is a factor to compensate for the effect of the transverse distance between rivets upon the ratio, $\frac{\text{Unit strength of net section}}{\text{Unit strength of control specimen}}$. Values of this latter factor for various values of $\frac{G}{D}$ are shown in Fig. 24.

The proposed rule for determining the effective net section of riveted tension members has been applied to the forty-nine tests of the four investigations cited[6, 22, 23] at the beginning of this discussion and the results are presented in Table 29. In this table, Column (5) contains values of G determined in the manner described in the preceding paragraphs. Column (6) gives the efficiency of the joint, the efficiency being the ultimate load, as determined by test, divided by the product of the gross area and the unit strength of the control specimen. Column (7) gives the effective net section as given by test, this being the ultimate strength, as given by the test, divided by the unit strength of the control specimen. Columns (8) and (9) give the computed values of the net section, the former being by the A. R. E. A. specifications for railway bridges and the latter by the proposed specifications. Columns (10) and (11) give the ratios of the computed to the test values of the net section, the former being for the A. R. E. A. specifications and the latter for the proposed specifications. A value greater than unity indicates that the computed value exceeds the value determined by test. A perusal of the values in Columns (10) and (11) reveals the agreement between the computed values and the values determined from tests.

It is of interest to note that the largest value of R given in Column (10) is 1.149, in Line 21, and the smallest value is 0.820, in Line 15, indicating that the A. R. E. A. specifications gave a value for the net section 15% too great for one specimen and 18% too small for another specimen. The largest value of R_1 given in Column (11) is 1.061, in Line 21, and the smallest value is 0.928, in Line 15, indicating that the maximum difference between the two values, one given by the proposed specifications and the other by the tests, is only 7 per cent. It is to be noted further that, in general, the proposed specifications give a value more nearly equal to the value given by tests than do the A. R. E. A. specifications. Moreover, the proposed specifications penalize the bad practice of using an extremely large transverse spacing of the rivets and large edge distances in the outside row of rivets, expedients which, according to A. R. E. A. specifications, result in a high efficiency but which, according to tests, resulted in little, if any, increase in efficiency.

In the proposed rule, *G* is defined as the transverse spacing, or two times the edge distance, whichever is the greater. If the transverse spacing varies, the widest space, or two times the edge distance, is to be used.

TABLE 29.—EFFECTIVE NET SECTION OF RIVETED TENSION MEMBERS; CARBON-STEEL RIVETS AND CARBON-STEEL PLATES

Line No.	No. of tests averaged	NOMINAL DIMENSIONS, IN INCHES		Value of *G*, in inches	Efficiency of joint	NET SECTION, IN SQUARE INCHES BY:			$R = (8) \div (7)$	$R_1 = (9) \div (7)$
		Thickness of plate	Diameter of rivet			Tests	American Railway Engineering Association specifications	Proposed specifications		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
					(*a*) WATERTOWN ARSENAL TESTS					
1	1	0.50	0.875	6.75	0.802	5.40	5.73	5.10	1.061	0.945
2	1	0.50	0.875	5.00	0.835	4.10	3.93	4.01	0.959	0.977
3	1	0.50	0.875	5.00	0.855	6.03	5.64	5.75	0.935	0.954
4	1	0.50	0.875	6.75	0.853	8.47	8.46	8.24	1.000	0.973
5	1	0.50	0.875	6.75	0.825	8.32	8.93	8.38	1.073	1.007
6	1	0.50	0.875	10.00	0.785	8.41	9.33	8.51	1.109	1.012
7	2	0.50	0.875	5.00	0.873	6.48	6.38	6.06	0.985	0.934
					(*b*) U. S. NAVY TESTS					
8	3	0.75	1.000	4.50	0.850	14.04	12.39	13.16	0.883	0.937
9	3	0.75	1.000	4.50	0.840	13.79	12.31	13.08	0.893	0.948
10	3	0.75	1.000	4.50	0.823	13.29	12.11	12.86	0.911	0.968
11	3	0.75	1.000	4.50	0.811	13.13	13.58	12.90	1.034	0.982
12	3	0.75	1.000	4.50	0.809	13.01	13.47	12.81	1.035	0.984
13	3	0.75	1.000	4.50	0.851	13.72	13.54	12.86	0.987	0.937
14	3	0.75	1.000	4.50	0.843	13.86	13.79	13.09	0.995	0.945
					(*c*) TESTS BY C. O. HARRIS					
15	2	0.625	0.875	3.00	0.812	12.20	10.00	11.32	0.820	0.928
16	2	0.625	0.875	8.00	0.813	9.64	10.00	9.55	1.037	0.990
					(*d*) UNIVERSITY OF CALIFORNIA TESTS					
17	2	0.375	1.000	3.00	0.708	5.52	4.78	5.58	0.866	1.010
18	2	0.625	1.000	3.00	0.735	10.22	8.70	10.16	0.851	0.994
19	2	0.375	1.000	5.00	0.783	6.31	5.90	6.14	0.935	0.973
20	2	0.625	1.000	5.50	0.797	10.20	9.28	9.49	0.910	0.931
21	3	0.375	1.000	13.00	0.777	6.16	7.08	6.54	1.149	1.061
22	1	0.375	1.000	5.00	0.756	5.89	6.08	5.93	1.032	1.007
23	1	0.375	1.000	5.00	0.745	5.79	6.53	5.92	1.128	1.023
24	2	0.625	1.000	13.00	0.798	10.37	11.60	10.71	1.119	1.035
25	1	0.625	1.000	5.00	0.804	10.50	10.96	9.94	1.044	0.947

If all of the transverse spaces are equal and if the edge distance is one-half of the transverse spacing, the net section can be expressed as a fraction of the gross area and the value of the fraction depends only upon the ratio of *G* to *D*. The relation between the ratio $\frac{\text{Effective net section}}{\text{Gross section}}$ and the ratio $\frac{G}{D}$ is given in Fig. 24(*b*). It is apparent from this diagram that the efficiency of a joint does not increase appreciably with an increase in the transverse spacing beyond a value of $G = 6\ D$.

The authors have drawn the conclusion that the efficiency of a riveted tension member does not greatly exceed 75%, irrespective of the transverse spacing of the rivets. If a different rivet pattern had been used for the joints with a large transverse spacing, it is believed that a different conclusion would have resulted. All of the specimens with a large transverse spacing that were tested by the authors had no rivets at the ends of the outside row, and the edge distance was abnormally large for the weakest section of the main plate. Thus, for Specimens *CCC*2 and *DCC*2, shown in Fig. 22, the edge distance for the outside row of rivets is 6.5 in. Geometrically, this is equivalent to a transverse rivet spacing of 13 in., whereas, for the interior rivets of the outside row, the spacing is only 8 in. Moreover, the authors report in another part of the paper that, beyond the yield point, the unit longitudinal strain in a wide plate is greater over the middle than it is at the edges. This being true, the harmful effect of placing the rivets on the weak section near the mid-width accentuates the harmful effects of a wide edge distance. This statement is supported by the tests reported in Table 29, which are discussed in the following paragraphs.

The only specimens for which the value of the effective net section determined by the proposed specification exceeds the value given by the tests is for those tests reported in Lines 5, 6, 17, 21, 22, 23, and 24, Table 29.

The details of the specimen used in the test reported in Line 5 are shown in Table 30 (Specimen 8510). The edge distance is 3⅞ in. The proposed specifications give a value for the effective net section 0.70% greater than the value given by the test.

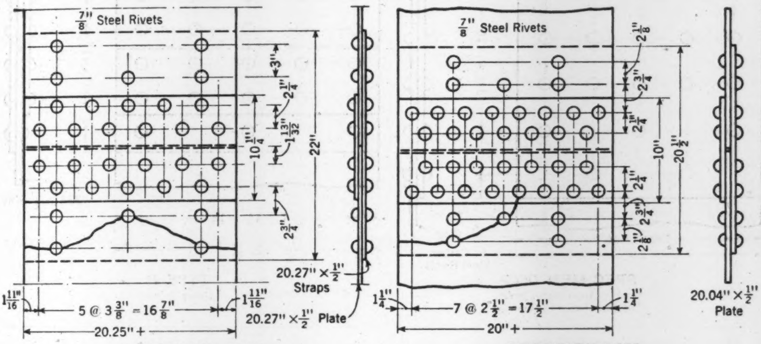
The details of the specimen used in the test reported in Line 6 are also given in Table 30 (Specimen 9375). The edge distance is 5 in. The proposed specifications give a value for the effective net section 1.2% greater than the value given by the tests.

The specimens used in the tests reported in Line 17, Table 29, had a close spacing and a small edge distance; but the fact that the computed value of the effective net section exceeded by 1.0% the value given by the tests is accounted for by the fact that the failure of the specimens was due to the shearing of the rivets and not by a failure of the plate. All other specimens for which the computed value of the effective net section exceeded the test value had the same rivet pattern as the specimens of Fig. 22. That is, the few instances in which the computed value of the effective net section exceeded the test value can be accounted for by the fact that the edge distance was large for the outer row of rivets.

The specimens shown in Table 31 have practically the same transverse spacing of rivets, but Specimens *DCC*2 have two rivets near the middle of the outer row, whereas Specimens of Type *F* have one rivet at the center and one at each end of the outer row.[6] The computed and test values of the efficiency are equal to each other and equal to 0.80 for Specimens *DCC*2, whereas, for Specimens of Type *F*, the computed efficiency is 0.80 and the value determined from the test is 0.85, the values being the averages from three tests in each instance. That is, although the computed value of the effective net section for Specimens *DCC*2 is reduced because of the large edge distance, the computed value is still greater, relative to the test value for these specimens, than it is for the specimens of Type *F*, which have a small edge distance.

The fact should be noted, however, that the specimens used for the tests reported in Lines 1, 2, 3, and 4, Table 31, had edge distances equal to one-half the transverse spacing and yet they developed an effective net width somewhat greater than the computed value.

TABLE 30.—Strength of Riveted Tension Members for which the Edge Distance of the Rivets in the Outside Row is Excessive



$\frac{7}{8}$" Steel Rivets
3"
$2\frac{1}{4}$"
$1\frac{13}{32}$"
$10\frac{1}{4}$"
22"
$2\frac{3}{4}$"
20.27" × $\frac{1}{2}$" Straps
$1\frac{11}{16}$"
5 @ $3\frac{3}{8}$" = $16\frac{7}{8}$"
$1\frac{11}{16}$"
20.27" × $\frac{1}{2}$" Plate
20.25" +
$\frac{7}{8}$" Steel Rivets
$2\frac{1}{8}$"
$2\frac{3}{4}$"
$2\frac{1}{4}$"
10"
$20\frac{1}{2}$"
$2\frac{1}{4}$"
$2\frac{3}{4}$"
$2\frac{1}{8}$"
$1\frac{1}{4}$"
7 @ $2\frac{1}{2}$" = $17\frac{1}{2}$"
$1\frac{1}{4}$"
20" +
20.04" × $\frac{1}{2}$" Plate

SPECIMEN 8510
Joint Failed in Plate, Holes Drilled

SPECIMEN 9375
Holes Drilled

Item No.	Description	Specimen 8510	Specimen 9375
1	Maximum load, in pounds	503 970	502 200
2	Stress on gross section, in pounds per square inch	49 960	46 900
3	Strength of control specimen, in pounds per square inch	60 550	59 700
4	Effective net section, by test, in square inches	8.32	8.41
5	Net section by American Railway Engineering Association specifications	8.93	9.33

As a result of the analysis of the data contained in Table 29, the following rules for the design of riveted tension members are submitted for the consideration of the profession:

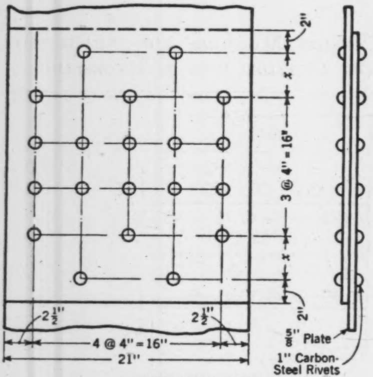
Rule 1. Effective net width = $(W_g - N\,D)\left(1 + \frac{D}{G}\right) 0.85.$

Rule 2. The edge distance shall not exceed 2.5 *D* for the outer row of rivets.

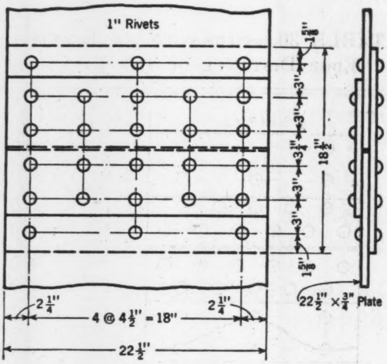
In the foregoing equation (Rule 1): W_g = gross width of the member; N = number of rivet holes to be deducted; D = nominal diameter of the rivets plus $\frac{1}{8}$ in.; and G = transverse distance between rivets or two times the edge distance, whichever is the greater.

For joints with rivets omitted from the outer row, only rivets in the outer row are to be considered in determining *N* and *G*, provided the pitch equals or exceeds 0.80 of the gage; otherwise all rivets in the outer row and the intervening rivets in the second row should be considered in determining *N* and *G*.

TABLE 31.—Influence of Edge Distance of Rivets in Outside Row Upon Strength of Riveted Tension Members



SPECIMEN *DCC2*
Carbon-Steel Plates Holes Sub-Punched and Reamed



TYPE *F*
Holes Drilled

Item No.	Description	Specimen *DCC2*			Specimen, Type *F*	
		x = 3 inches	*x* = 5 inches	*x* = 6 inches	Medium steel	High-test steel
1	Ultimate load, in kips	691.0	686.0	669.0	902.0	1 113.0
2	Stress on gross section, in kips per square inch	52.9	52.7	51.6	55.9	66.2
3	Strength of control specimen, in kips per square inch	65.8	65.6	65.1	65.7	81.6
4	Effective net section, by test, in square inches	10.50	10.46	10.28	13.72	13.64
5	Net section by American Railway Engineering Association specifications, in square inches	10.97	11.62	11.57	13.54	14.11

In the preceding discussion it has been assumed that the weakest section of a riveted tension member is through the outside row of rivets of the gusset plate at the ends of the member. The writer knows of no published reports of tests of riveted tension members with stitch rivets, but the tests of plates with holes by Mr. Harris, reported in Fig. 21, indicate that an open hole does not weaken a plate as much as a rivet transmitting stress. It is not reasonable to suppose that a stitch rivet weakens a plate more than an open hole. It would seem reasonable, therefore, that the foregoing rules of design, which are applicable where the rivets transmit stress, give conservative results where the rivets do not transmit stress but only serve as stitch rivets. The only probable exception to this statement is where stitch rivets are used to connect two parallel pieces of steel having widely differing stress-strain diagrams. Under this latter condition one piece, because of its lower ductility, might break before the other had been stressed to its ultimate strength. This condition is not likely to occur in a member fabricated of a carbon steel of structural grade because the part of the stress-strain diagram near the maximum stress is nearly horizontal for a considerable length. However, the behavior, under similar conditions, of a member fabricated from parallel pieces made of a steel which is not uniform in its physical properties is somewhat uncertain.

J. M. GARRELTS,[24] ASSOC. M. AM. SOC. C. E. (by letter).—The engineering profession is indebted to the engineers of the San Francisco-Oakland Bay Bridge who provided for this important series of tests, and to those who were responsible for making the tests and analyzing the results. As has been advocated by some consulting engineers, a series of well-planned tests, made in connection with the important structures which are built, would soon yield valuable information concerning the behavior of structures.

The fact that under different stresses "the plates in a riveted joint tend to elongate by unequal increments" is important and should be used in the analysis of many structural problems as well as in that of the riveted joint. As allowable unit stresses increase, structural design must be placed on a more scientific basis. The structural designer must be aware of the many indeterminate factors that affect the action of a structure and must have some knowledge of the influence of those factors.

In order to determine the effective net section and the effect of end pitch, additional data are needed concerning the behavior of plates with open holes under stress. The lines of fracture for the specimens shown in Figs. 6 and 7 all appear to originate in the regions of high stress concentration. When a condition of plastic yielding is reached, a redistribution of stress takes place, but initial fracture may occur in the region where yielding first occurs. If stress concentration at the edges of a hole has any bearing on the initial fracture, it is quite probable that the present ideas concerning net section need to be revised. The writer is of the opinion that, by additional study and tests, a rational analysis may result. In the meantime, the authors' recommendations appear to represent a reasonable procedure.

In view of the uncertain action of the riveted joint in Stage II, the authors' recommendations for assuming equal shear per rivet seem fully justified and no additional refinement is likely to result from further tests.

A. E. RICHARD DE JONGE,[25] M. AM. SOC. C. E. (by letter).—The tests described in this paper are perhaps the most elaborate tension tests conducted to date on large-size structural riveted joints. A variety of specimens was tested, in which several characteristics were changed systematically. The questions that were to be answered by these tests are clearly stated under the eight items at the end of the "Introduction", and the answers are given in "Conclusions from the Tests".

---

[24] Associate Prof., Civ. Eng., Columbia Univ., New York, N. Y.

[25] Adjunct Prof., Polytechnic Inst. of Brooklyn, Brooklyn, N. Y.

After reading the lengthy report, the writer who has made an extensive study of the world's literature on riveted joints[26], would like to take stock of what has really been accomplished by these very expensive tests. In so doing, he is not going to discuss the eight items previously mentioned, but will rather deal with those factors that have an influence on the strength of riveted joints, in so far as they have been treated by the authors.

(*1*) *Strength of a Riveted Joint.*—The meaning of the term "strength of a riveted joint" should be revised so as to be in accordance with actual facts. In the past, too much emphasis has been placed on the ultimate strength of a riveted joint. With the modern means of investigation, there is no reason why the method introduced more than one hundred years ago by Sir William Fairbairn[27] should still be adhered to to-day. In service, no actual joint is ever stressed anywhere near the breaking point. Therefore, it matters little whether the joint, if stressed to the breaking point, fails in the rivets or in the plates; in both cases the joint becomes useless. In fact, if the rivets would fail before the plates become distorted to any great extent, it would be possible to re-rivet the joint, thereby making it again serviceable. This is impossible, however, when the joint fails in the plates.

Consequently, it is of far greater importance to determine what happens in a riveted joint up to the maximum working load than to pull the joint apart—that is, subject it to conditions to which, ordinarily, it would never be subjected in practice. The only reason for the ultimate strength test is to provide data for a convenient, although wholly inadequate, method of calculation which, by the introduction of a suitable factor of safety, assures sufficient safety at the maximum working load that the joint is called upon to sustain. Substituting for this "factor of ignorance" a thorough knowledge of the phenomena that occur in the joint, from the moment a load is applied to it until this load has reached the maximum value allowable under ordinary working conditions, would lead to a rational method of calculation which, at present, is sadly lacking. The maximum working load should remain sufficiently far below the yield strength of the riveted joint rather than below the ultimate strength. The yield strength would then form the true strength limit for the joint, which can ordinarily be utilized. However, since the authors have followed a long-established precedent[27] in subjecting riveted joints to breaking loads, a discussion of the various factors that control the strength of a joint, up to the point of fracture, will be necessary.

(*2*) *Tests of Solid Plates.*—The authors have subjected two solid plates each of Series C and D to tension tests. They were of carbon steel, and had a width of 21 in. and a gage length of 24 in., two being $\frac{3}{8}$ in. thick and the other two, $\frac{5}{8}$ in. thick. Marked yielding occurred in the $\frac{3}{8}$-in. plates at 37.9 kips per sq in., and in the $\frac{5}{8}$-in. plates at 34.8 kips per sq in. The $\frac{3}{8}$-in. and $\frac{5}{8}$-in. plates failed, respectively, at strengths of 92% and 94% of that of the test coupons. The ratios of lateral set to longitudinal set, at failure, were 0.38 for the $\frac{3}{8}$-in. plates and 0.40 for the ⅝-in. plates. Poisson's ratio was found to be 0.28 for both plate thicknesses. The plates failed at mid-length. In three specimens, a short transverse crack started in the middle of the plate, whereas in the fourth the first crack developed at one edge. The authors measured the longitudinal strains at 30 kips per sq in. and found them to be uniform—that is, the strains were the same at the center as at the edges. In one ⅜-in. plate, the strains were measured at 35 kips per sq in. (that is, very close to the yield point) and were found to be 11% greater in the middle than at the edges.

---

[26] "Riveted Joints: A Critical Review of the Literature Covering Their Development, with Bibliography and Abstract from the Most Important Articles," by A. E. Richard de Jonge, *Research Publication*, A.S.M.E., 1940; see also *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 474.

[27] "Experimental Inquiry into the Strength of Iron with Respect to Its Application as a Substitute for Wood in Shipbuilding," by William Fairbairn, British Association for the Advancement of Science, Tenth Meeting, Glasgow, Scotland, Reports 1840, Vol. 1840, pp. 201–202; see also *Philosophical Transactions*, Royal Soc. of London, Vol. 140, 1850, pp. 677–725.

The authors attribute the tendency for the failure of the plates to start in the middle to the unequal distribution of longitudinal strains. To the writer, this seems to be correct, but it requires amplification to explain this behavior.

As long as the cross-contraction is approximately the same over the gage length, the strains are the same everywhere in the plate. A network of small squares scratched into the surface of the plate at the beginning would simply be changed into a network of rectangles. However, shortly before yielding becomes general, a slight necking occurs at the edges of the plate, usually at mid-length. This is an indication for a redistribution of the material. The thicker the plate, the easier this redistribution will proceed—that is, the more marked will the necking become. In the region of necking, the small squares are distorted into a kind of trapezoid with curved sides; but the orthogonal system of tension and compression lines that coincides with the sides of the original squares must remain orthogonal. The greatest lengthening of the tension sides of these squares, therefore, takes place at the center of the plate in the region where necking at the edges occurs. This appears to be the reason why the first crack must start in the middle of the plate instead of at the edges. As the redistribution of the material takes place easier in thick than in thin plates, it is to be expected that the final set must be greater in thick than in thin plates. This is precisely what was observed. Why failure started at the edge, in one case, cannot be determined without further knowledge of the fracture. It seems likely that, due to a slight notch effect at the edge, the crack started from this notch.

Because the redistribution of the material is more difficult in thin plates than in thick plates and a round bar, and because the elongation of the ⅜-in. plates is less than that of the ⅝-in. plates, as observed by the authors, the ultimate strength exhibited by the ⅜-in. plates did not reach that of the ⅝-in. plates and was lower than that of the test coupons (92%), whereas the ultimate strength shown by the ⅝-in. plates was somewhat higher, but still below that of the test coupons (94%). Thus, the foregoing explanation describes all the observed facts accurately. Furthermore, if a ⅜-in. plate is bent into the shape of a cylinder (that is, if it is formed into a tube of ⅜-in. wall thickness, with a lateral surface equal to the width of the original ⅜-in. plate), and if it is then pulled apart, it will most likely be found that the strength approaches much closer that of the test coupons, because the entire wall of the tube can now contract together with the latter (necking) at right angles to the surface. If the plate should become a bar, the thickness of which is equal to its width, and if it were possible to tear it apart, it would probably be found that its strength would approach that of the test coupon very closely. To the writer, these appear to be the reasons underlying the strange behavior of wide plates. As the tests by the authors have revealed these facts, it can be said that these experiments have served a useful purpose.

(*3*) *Effective Net Section and Grooving Effect.*—Of effective net section, grooving effect, rivet pattern, omission of rivets in the outer rows, pitch, and gage-line distances—all of which factors are intimately interconnected—let the first two be discussed at this point.

The authors claim that "an outstanding finding of these tests is that the strength of the plates in a riveted joint is not directly proportional to the area of the net section as commonly assumed in practice". Theoretical and experimental studies on the net section have previously been conducted by C. R. Young[28], M. Am. Soc. C. E., and by Professor Young and W. C. Dunbar[29], in which it was found that the net section, assumed to be stressed uniformly, is not thus stressed, and is not a simple function of the rivet-hole arrangement. T. R. Loudon[30] also found, by using photo-elastic analysis, that there are high stress concentrations near the rivet holes, and that the stress is, by no means, distributed uniformly over the section remaining between the rivet holes. Therefore, the authors can scarcely claim priority for their finding.

Probably the entire question of net section would long since have assumed a different aspect if the stress distribution, as found by Prof. E. G. Coker[31] and by Professor Loudon[30], had been considered more carefully; for a phenomenon, termed the "grooving effect," occurs in which the material in the vicinity of the rivet absorbs a greater part of the stresses than the material farther away so that the material between two closely spaced rivet holes exhibits an apparently greater strength. This phenomenon was first stated definitely by Daniel Adamson[32], in 1878, although, previously in 1872, Walter R. Browne[33] had shown that the stress distribution in the remaining cross-section is not uniform, the stress being greater near the hole. According to William Hovgaard[34], the material of mild steel plates, $\frac{3}{8}$ in. to $\frac{3}{4}$ in. thick, absorbs, between two rivets spaced at a gage of three times the diameter (3 *D*), a 10% higher stress than does the solid plate ahead of the rivets. This amount drops to 5% if the gage distance between rivets is four times the diameter (4 *D*). The grooving effect is of the utmost importance with regard to the rivet arrangement or rivet pattern, as will be shown subsequently.

(*4*) *Rivet Pattern, Omission of Rivets in the First Row, Pitch, and Gage-Line Distances.*—Only a few, and scarcely representative, rivet patterns were investigated by the authors. Long ago it has been shown that the omission of rivets in the first row, for the purpose of increasing the strength of riveted joints, is a fallacy[35] (Meldahl showed that, in the case of a saw-tooth like doubling plate, the maximum stress occurs slightly ahead of the ends of the saw-teeth, and that fracture actually started at those points); it is based on the misconception that the stress between the rivets of the first row is uniformly distributed (assumption by J. W. Schwedler[36]). Even as far back as 1860, Sir William Fairbairn[37] recommended, where greatest strength combined with lightest weight is desired in bridge girders, the use of chain riveting or placing the rivets in the various lines, one behind the other, instead of "in zigzag" as was customary at that time. (The writer, here, uses the term "rivets in line" for rivets placed longitudinally one after the other, and the term "row" for rivets spaced side by side transversely to the direction of the acting force.) The same recommendation was made again by Commander Gayhart, in 1926, on the basis of his very elaborate tests[6]. Gayhart showed conclusively that chain riveting (placing the rivets in line one after the other and using like lines, evenly spaced, side by side) is stronger than riveting in which, alternately, one or more rivets in the first row are omitted. Since this was well known at the time these tests were planned, the writer would have expected that the joints for these tests would have been laid out with due regard for this fact. The only specimens that approach fairly closely the chain riveting recommended by Fairbairn and Gayhart are those of Series B. However, due to the omission of two rivets in one of the inner rows of each specimen of this series, these specimens cannot be regarded as good examples of joints designed for greatest strength. Nevertheless, the tests proved that these joints had a better efficiency than other joints—for instance, those of Series A.

---

[28] (*a*) *Bulletin No. 2*, School of Eng. Research, Univ. of Toronto, 1921, pp. 233–244; (*b*) *Canadian Engineer*, Vol. 39, 1920, p. 427; (*c*) *ibid.*, pp. 512–513; and (*d*) *Engineering and Contracting*, Vol. 54, 1920, pp. 188–189.

[29] *Bulletin No. 6*, School of Eng. Research, Univ. of Toronto, 1926, pp. 51–65.

[30] Faculty for Applied Science and Eng., Univ. of Toronto, School of Eng. Research, *Bulletin No. 9*, 1932, pp. 231–239.

[31] *Transactions*, Inst. of Naval Architects, Vol. 53, Pt. 1, 1911, pp. 265–296, including discussion; see also "A Treatise on Photo-Elasticity," by E. G. Coker and L. N. G. Filon, Cambridge, 1931.

[32] "On the Mechanical and Other Properties of Iron and Mild Steel," by Daniel Adamson, *Journal*, Iron and Steel Inst., Vol. 1878, pp. 383–403.

[33] "Drilling Versus Punching," by Walter R. Browne, *Engineer*, Vol. 34, 1872, p. 362.

[34] "Structural Design of Warships," by William Hovgaard, E. and F. N. Spon, Ltd., Spon and Chamberlain, London, 1915, p. 121.

The reasons why full-row rivet spacing in the first row shows a greater strength are: (1) The grooving effect (the capacity of the material between narrowly spaced rivets to absorb greater stresses than the material between widely spaced rivets); and (2) the fact that the stress distribution in the case of narrow, even rivet spacing is far more uniform than that in the case of widely spaced rivets, as has been shown, for example, by Coker who used photo-elastic analysis[31]. Thus, by spacing the rivets in the outer rows evenly and closely, dangerous stress concentrations are avoided at these rivets.

Next, rivet pitch, particularly between the first two rows in the case where the rivets in these are spaced in zigzag, has been investigated by the authors, in Series C and D. In all cases, the fracture passed through the two outermost rivets and showed almost always a tendency to follow shear lines inclined at about 45 degrees. This tendency was greater in the ⅝-in. plates than in the ⅜-in. plates. Where the second row of rivets was near enough (Specimens CCC2–2 and DCC2–3), the fracture passed in zigzag through first and second row rivets[38]. An interesting feature is that, at yield load, Specimens CCC2–2 and CCC2–4 showed about the same efficiencies, whereas the specimens with the intermediate pitch of 3 in. (CCC2–3) exhibited considerably lower efficiencies. At ultimate loads, however, the efficiencies vary as shown in Table 7. In that case, the maximum efficiency for both ⅜-in. and ⅝-in. plates is reached for an end-row pitch of 4 in. For smaller, as well as for greater, pitches, the efficiency was lower.

---

[35] "Materialspannungen in ausgeschnittenen Platten," by K. G. Meldahl, *Schiffbautechnische Gesellschaft*, Berlin, Jahrbuch, Vol. 5, 1904, pp. 481–523.

[36] "Über Nietverbindungen," by J. W. Schwedler, *Wochenschrift des Architekten Vereins zu Berlin*, Vol. 1, 1868, pp. 451–452, 463–464, and 472–473.

[37] "Chain Riveting," by William Fairbairn, *Mechanics' Magazine*, Vol. 3, 1860, pp. 375–376; see also *Journal*, Franklin Inst., Vol. 70, 1860, pp. 237–239.

[38] See also "Weerstand van de doorsnede van een plaat tuschen nagels in een scheeve rij, berekend volgens de breukhypothese van Mohr," by P. P. Bijlaard, *De Ingenieur*, Vol. 46, 1931, pp. 47–57.

The rivet pitch throughout the joint was the only variable that was changed in the specimens of Series B. The gage distance was rather large (7.5 in.), and thus, in the light of what has been said about the stress distribution, the joints cannot be regarded as being representative of best or even ordinary practice. Nevertheless, the results are of interest in so far as they show that joints with 4.5-in. pitch throughout were stronger than either those with 3-in., or those with 6-in., pitch (see Table 15). It is to be regretted that no attempt was made to vary the gage-line distances as this would have revealed the optimum condition of pitch relative to gage distance. It is obvious that, when the gage-line distance is too great, stress concentrations at the outer row of rivets will appear so that the stress distribution will become uneven, resulting in reduced strength.

The effect of the change in gage distance, or of the number of holes in a given section, may be judged, to a certain extent, from Specimens CCC5 and DCC7. It should be kept in mind, however, that the former consisted of ⅜-in. plates, whereas the latter were composed of ⅝-in. plates. As may be expected from what has been said about the grooving effect, Specimens DCC7 with the narrow-gage distances of 3 in. sustained a greater load (more than 76.2 kips per sq in. of net section) than did Specimens CCC5 with the wider gage distances of 4 in. (70.9 kips per sq in.). The grooving effect of the narrow-gage distances proved sufficient to prevent failure in the plates in spite of the higher stress in the net section, the joints, in this case, failing in the rivets.

(*5*) *Bending in Lap Joints.*—In its essentials, the foregoing discussion, under Items (3) and (4), applies equally to lap joints and butt joints. The high bending stresses that usually occur in lap joints, particularly in short ones, were eliminated by the authors to a certain extent by the use of a "bending frame". While, thus, the values indicated in Table 5 are intended to represent data purely in tension without bending, it must be kept in mind that bending is always present in actual lap joints. The bending stresses that occur should be placed on the debit side in such joints. The values given by the authors, therefore, have only a theoretical interest and should be used with caution wherever bending stresses are present at the ends of the lap, particularly since the bending frame tended to increase the friction between the plates.

(*6*) *Efficiency of Lap Joints.*—In Table 5, the authors have given the efficiencies for lap joints that failed in the plates, as well as for those that failed in the rivets. The efficiencies of joints of Series C and D which failed in the plates vary from 72.8% to 78.8%, the mean being approximately 75 per cent. Considering the different types of rivet patterns, the different lengths of the laps, the different pitches and gage-line distances, and the different kinds of steel, it seems remarkable that the differences in the efficiencies were not very much greater. However, it should be emphasized again, that, for a joint used in actual construction, the ultimate efficiencies mean little; it is rather the efficiency up to, or at, the yield strength of the joint that is of importance and should so be considered by the designer.

(7) *Rivet Data.*—

(a) Driving Temperatures.—The strength of a riveted joint depends, to a large extent, on the clamping effect of the rivets, and this, in turn, on the temperatures at which the rivets were driven. Two temperatures are of importance—the one at which driving starts and the one at which it ends. The data given by the authors are extremely scanty. No definite temperatures are given. As to the temperature at which driving started, the authors give the range and average in a general way: For carbon-steel rivets from 1 400° F to 1 800° F, with an average of 1 570° F, and for manganese-steel rivets from 1 400° F to 1 900° F, with an average of 1 680° F. The far more important end temperatures at which the ram of the hydraulic riveter completes the forming of the snaphead, and that at which it leaves the rivet, are not stated; and yet, these have a deciding influence on the strength of the joint. Furthermore, no mention has been made as to how these temperatures were measured. It is hoped that the authors will give further information as to these important factors in their closing discussion.

An extensive investigation of the most suitable rivet temperatures at the beginning and end of driving was published by Yasuchi Taji[39], in 1914. Taji recommended the use of the lowest possible temperatures, with riveting done as quickly as possible. Although he used a wide range of temperatures at the start of driving the rivets, his general average, in hydraulic riveting, is 817° C (about 1 503° F). Although this average value is lower than that used by the authors, it is well within the range used by them. For the lower end temperature, Taji gives, as the best value, 650° C (about 1 200° F), and this is in remarkably good agreement with the optimum end temperatures given in 1886 by A. Considère[40], in France, who recommended an end temperature of from 600° to 700° C (about 1 100° to 1 300° F). If one considers that the average temperature used by Taji was almost 70° F lower than that used by the authors, it is questionable whether the end temperatures in the authors' case were close to the best end temperature recommended by Taji.

That better strength values may be obtained by using lower temperatures is borne out by some of the authors' specimens of Series A (manganese-steel rivets and double-plate joints). Without giving the actual temperature at the start of driving, the authors state that it was about 100° F less than in the other cases of this series; yet, the strength of these joints was found to be well above that of the other joints, and the rivets had a greater hardness and finer grain structure. Other investigators, too, have obtained better results at lower initial temperatures—for instance, Professors Young and Dunbar[41], whose grand average was 1 384° F. It is to be deplored that the authors did not follow up this clue by experimenting with somewhat lower temperatures, using joints of a definite type, but with rivets driven at different initial and final temperatures.

---

[39] "Some Recent Studies on Rivets, Riveting, and Riveted Joints," by Yasuchi Taji, Soc. of Naval Architects and Marine Engrs., Zosen Kyokwai, Vol. 13, September, 1914, pp. 127–181.

[40] "Mémoire sur l'Emploi du Fer et de l'Acier dans les Constructions," by A. Considère, *II. Annales des Ponts et Chaussées*, 6me série, Vol. 2, 1er semestre, 1886, pp. 5–149.

[41] "Rivets in Combined Tension, Shear, and Flexure," by C. R. Young and W. C. Dunbar, *Canadian Engineer*, Vol. 56, 1929, pp. 201–206.

(b) Rivet Strength.—As stated under Item (7), the strength of a rivet and, thus, also of the joint depends on the temperatures at the beginning and end of driving. The important question, however, is what the actual strength of a driven rivet is as compared with that of the undriven rivet and the stock from which the rivet was made. The authors have given yield strength and ultimate tensile strength for the rivet-rod stock, and the shear strength for undriven rivets, but no actual shear strengths of driven rivets except the mean shear strengths as computed from the tests of the joints. In Tables 9, 10, 12, 13, 14, 15, 17, and 19, average shear stresses are given which are computed for effective rivet yield, yield load, and ultimate load. For the latter, the authors have stated that it was found "by dividing the total load by the total shearing area of rivets, based on the nominal diameter of the rivet". However, nothing has been mentioned about how the other two values were obtained. At the "effective rivet yield" and yield load, the shear is certainly not distributed uniformly over all the rivets of the joint. Therefore, it is not clear how the shear values in these cases were obtained. At ultimate loads, however, it seems likely that the shear is more or less uniformly distributed.

Furthermore, it should be kept in mind that the rivet-rod stock and the rivet produced from it by a heating process, if subjected to shear, are stressed more or less biaxially, whereas the driven rivet, which, due to contraction by cooling, is under tension, is stressed triaxially if subjected to additional shear. This explains the differences in the shear values for rivet-rod stock, undriven rivets, and driven rivets.

For the foregoing reasons, the values tabulated by the authors cannot be regarded as absolute values, but only as relative values for the purpose of making some kind of a comparison possible. Caution is urged, therefore, in using these values.

(c) Single Shear vs. Double Shear.—Under this heading, the authors mention in their "Conclusions from the Tests" that a value as low as 0.80 is indicated for the ratio of double shear per shearing surface to that of single shear. The authors state that this value agrees with findings from their pilot tests. It is not quite clear how this value was derived, because if one compares, for example, the ultimate shear values per shearing surface for Specimens FSCA and ASC18 (namely 41.6 and 54.8 kips per sq in., respectively) their ratio is 0.76, which is a still lower value and much lower than that found by other investigators. E. S. Fitzsimmons[42], for example, obtained, for carbon-steel rivets in double shear (per two shearing surfaces), 84 kips per sq in., and for like rivets in single shear, 49 kips per sq in., from which the value of the ratio of double shear per single surface to single shear follows as 0.857. Other investigators have found this ratio to be approximately 0.9. As the aforementioned specimens vary considerably in rivet pattern, rivet diameter, and width and thickness of plates, this comparison is, probably, not a fair one. It would have been of great value if the authors had conducted tests with otherwise similar specimens, in single and in double shear, with one and more rivets.

---

[42] "The Shearing Strength of Rivets," by E. S. Fitzsimmons, *Iron Age*, Vol. 75, 1905, p. 2058.

In that case, reliance could be placed on an average value obtained thereby; but this, certainly, cannot be said about the value of 0.80 given by the authors. It should be mentioned, however, that, in their recommendations, the authors have used the value of 0.9, which they, too, probably consider as the most likely one.

(8) *Influence of Plate and Rivet Material.*—Two types of rivet steels were used in these tests—a carbon steel and a manganese steel. The tensile strength of the rivet-rod stock is given by the authors, for ⅞-in. and 1-in. carbon-steel rods, and for the 1-in manganese-steel rod, as 58, 57.2, and 81 kips per sq in., respectively. The great loss in tensile strength of the annealed manganese steel, to 74.7 kips per sq in., should be noted. It is to be regretted that the data on these steels are very fragmentary. In the first place, the shearing strengths in single and double shear of the rivet-rod stock, as stated, are not given; neither are the moduli of elasticity and shear, nor the relationship between elongation and stress. For "non-driven rivets", the shearing strength in double shear (probably per single shearing surface) is given for 1-in. carbon-steel and 1-in. manganese-steel rivets, but the shearing strength in single shear is not stated. If the maximum shearing strength (in single shear) is assumed to be 0.8 of the ultimate tensile strength of the rivet-rod stock (a factor which, usually, is not far from the actual), the shearing strength in single shear, for the carbon-steel and manganese-steel 1-in. rivets, would be obtained as about 45.8 and 64.8 kips per sq in., respectively. The ratios of double shear per single shearing surface to single shear would then follow as $\frac{42.1}{45.8} = 0.92$ and $\frac{54.5}{64.8} = 0.84$, respectively—values which are more in accord with those of other investigators as stated previously herein. In double shear, the undriven manganese-steel rivets were a little more than 29% stronger than the carbon-steel rivets. If the ultimate shearing strength of Specimens ACC–18 and ACM–12 is compared, it is found that the mean in the case of the former was 55.7 and in that of the latter 75.9 kips per sq in. Hence, in these joints, the manganese-steel rivets proved to be about 36% stronger than the carbon-steel rivets. However, one must not lose sight of the fact that the joint with carbon-steel rivets was longer than that with manganese-steel rivets, the plates, in both cases, being of carbon steel.

As regards the three kinds of steel used in the various joints, the authors have failed to state the exact ultimate strength of the plates used in each joint. Likewise, the moduli of elasticity and shear are not given and neither are the stress-strain relations. Thus, the checking of the elastic properties of the joints is made impossible. Unless the authors supply these values in their closing remarks, this entire set of data is useless from the point of view of developing a rational method of calculating riveted joints. The statement that the effect of the plate steel in joints which failed in the rivets was neither large nor consistent, although the decrease in shearing strength of the rivets with length was somewhat more marked in the case of the carbon-steel plates than with less ductile steels, should be questioned. One would have expected that the harder silicon steel would exert a stronger cutting action, a fact which is actually borne out by a comparison of Specimens ASC–18 and ACC–18. In the former, the average ultimate shearing value was 54.8 kips per sq in., whereas in the latter, it was 55.7 kips per sq in., thus showing that the shearing strength of the rivets in the case of silicon-steel plates is slightly less than that in the case of carbon-steel plates. This holds even for plates with manganese-steel rivets, for, in comparing the respective shearing values for Specimens ASM-12 and ACM-12 (namely, 72.4 and 75.9 kips per sq in.), the shearing strength of the rivets in these joints with silicon-steel plates is found to be somewhat lower than that of the joints with carbon plates. In general, the combination of nickel-steel plates and manganese-steel rivets seems to be better than that of silicon-steel plates and manganese-steel rivets. In order to arrive at a decision as to which combination is the best under a given set of conditions, further experiments appear to be necessary which should be made with the strongest joints that can be designed, keeping in mind the facts dealt with in this discussion.

(*9*) *Effect of Length of Joints and Premature Rivet Failure.*—In general, the authors have described, correctly, the effect of length of a joint. As is well known from former tests and theoretical considerations, the strength of a riveted joint decreases with its length[43]. It stands to reason that the longer the joint, the greater will be the end slip and the more likely will the rivets in the end rows shear off prematurely—that is, before ultimate failure of the joint. The more ductile the steel of the plates, the greater will be the elongation and slip, and the more readily will the end rivets be sheared off. Therefore, this tendency is greatest in carbon-steel plates, and less in silicon-steel and nickel-steel plates. For the latter, in combination with manganese-steel rivets, the authors did not observe any such tendency.

A fact not discussed by the authors is that, when the end rivets failed, the joint with the remaining rivets still proved strong enough to withstand the full load. As the outer rivets failed almost consistently, it is obvious that they carried a greater load than the others. On the other hand, the next row, or the over-next row when the rivets in the two or three outer rows failed, made the plates (on account of the closer rivet spacing in the subsequent rows) relatively stronger due to the grooving effect, and the stressing of the remaining rivets became more uniform due to the more uniform stressing of the plates. Thus, one cannot escape the conclusion that the rivets in the outer row (or rows) were unnecessary for developing the maximum strength exhibited by the joint. This is a further proof for the fact that, even at ultimate loads, joints with closer rivet spacing in the outer rows are stronger both in the plates and in the rivets. That the transverse contraction of the plates increases the relative displacement of the rivet holes in adjoining plates is obvious. However, the authors' statement that "excessive detrusion, rather than stress, caused such failures," cannot be accepted by the writer as the actual stress always increases until failure occurs. Unfortunately, engineers accept the usual stress-strain curves, in which the curve, after reaching a maximum, drops again, without being conscious of the fact that these curves are, for convenience, referred to the original cross-section of the test bar instead of (as they should be) to the smallest cross-section in the region of necking. If the curves were thus replotted, it would be found that they would rise continuously up to the point where fracture finally occurs. Therefore, the statement that these outer rivets failed by overstressing is fully justified, the so-called theory of limit design to the contrary notwithstanding[44].

---

[43] "Inanspruchnahme der Anschlussnieten elastischer Stäbe," by Ivan Arnovļević, *Zeitschrift für Architektur und Ingenieurwesen*, Vol. 14, 1909, pp. 89–106.

(*10*) *Butt and Shingle Joints.*—The efficiencies of the joints of Series F showed that the short butt joints had the highest efficiencies—75.2 per cent. Comparing the double-plate butt joints with the double-plate shingle joints (the respective efficiencies of which are 73.0% and 72.1%), it will be seen that the efficiency of the butt joints is slightly higher, as it should be according to theory[43, 45]. As is to be expected, the longer joints showed the lower efficiencies. The authors' statement that the advantage of the lighter shingle and half-shingle splices, compared with the heavier butt splices, is offset by greater difficulty in field erection should be noted. The triple-plate, half-shingle joint (Fig. 16(*b*)) does not seem to have been designed correctly, for it fractured, as one would expect, near the unsymmetrically arranged butt of the center plate. The authors state that the outer splice plates in the triple-plate butt joint (Fig. 15) were shorter than had been intended, and the twenty-seven rivets that passed through inner and outer splice plates transferred 61% of the load from the main plate to the splice plates, whereas the thirty rivets that passed through only the inner splice plates transferred but 39% of the load. The question should, here, be raised as to what this load was—whether it was the ultimate load or some other load—as the load distribution varies with the load. The authors have failed to state this clearly. In explanation of the somewhat unexpected load distribution in these joints, it may be said that the longer rivets were probably under a higher tension than the shorter ones, and that, for this reason, they exerted a greater clamping pressure on the plates. The friction, which, up to the time of fracture, undoubtedly played an important part, was probably the cause of this rather strange distribution of load. The same explanation holds also for the double-plate butt joints, although to a lesser extent.

The authors have undertaken extensive strain measurements on specimens of this Series F. It is to be regretted very much that the results of these measurements were not presented in tabulated form to enable the investigator to check the results by calculation. Apparently, what the authors have done (although they have not stated this clearly) is to compute the stresses in the net section of the various plates for different loads (7.5, 15, 22.5, and 30 kips per sq in., or 15 and 30 kips per sq in.) and then to average them and give them as percentages in the diagrams above the joints. If this understanding is correct, it must be stated that the curves given are of little value since the load distribution and, consequently, the strains and stresses vary with the load. In fact, it would have been better if the authors had given the curves for one definite load only, as they would then be available for checking eventual calculations that may be derived. At any rate, the authors' statement that the end rivets and those near the butt have taken a greater load than the remaining ones is significant.

---

[44] "Theory of Limit Design," by J. A. Van den Broek, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., February, 1939, pp. 193–216, particularly p. 195.

[45] "Zur Kraftverteilung in genieteten Stäben," by Ivan Arnovlević, *Oesterreichische Wochenschrift für den öffentlichen Baudienst*, Vol. 14, 1908, pp. 607–615.

(*11*) *Behavior of Riveted Joints.*—Last, but by no means least, elongation, slip, distribution of strains, and partition of load—in short, the behavior of riveted joints under load—will be discussed. All these factors are intimately interconnected, but are dealt with separately herein.

(a) Elongation.—Unfortunately, the authors have not given a definition of what they wish to be understood by elongation. In lap joints, it is either the change in length of the distance between the outer ends of the plates, or the change in length between the points on the main plates, coinciding at the beginning with the outer ends of the adjoining plates. Similarly, in butt joints, elongation is either the change in length of the splice plates, or the change in distance between the points on the main plates coinciding at the beginning with the ends of the splice plates. In other words, the authors should have stated if, by elongation, they mean the elastic and plastic change in length of the plates between their ends, without the end slip, or if the elongation is to include the end slip at either end. This makes quite a difference. A clear-cut definition would have helped greatly in gaining a correct understanding.

Furthermore, the authors use a ratio of unit elongation of the joint to unit elongation of the gross-section of the main plates, which, for lap joints, is shown (see Table 9) to vary from 1.03 to 3.32—that is, within wide limits. The actual law of elongation has not been discussed. However, as an "effective modulus of elasticity" for the joints of Series F, the authors give a value of about 29 000 000 lb per sq in. as against 32 000 000 lb per sq in. for the main plates. For this series, the elongation ratio varied from 0.79 to 1.13 which is about 43% on the basis of the lower, and about 30% on the basis of the higher value (see Table 17, Column (12)). The writer is unable to understand how this effective modulus of elasticity takes care of this range.

(b) Slip.—The authors also have failed to define clearly what they mean by "slip", or, rather, they have been using the term in two very different senses: (1) For the movement of an entire plate relative to another plate; and (2) for the movement of parts of one plate relative to parts of another while other parts of both plates do not move relative to one another. At places, the authors have called the former "middle slip" and the latter "end slip". The way in which the latter was measured is not beyond reproach, as the locations vary somewhat. In almost all cases, the measurements were taken in the joint somewhere between the first and second rows of rivets. Only in the short specimens, ANM–12, were they measured in the first row of rivets. However, slip occurs even ahead of the first row of rivets, and the proper position for measuring the end slip would have been directly at the end of the plates, or of the splices. It is unfortunate that this was not done as it becomes, thereby, almost impossible to compare, directly, the measured results with those derived from a rational method of calculation. The middle slip is used by the authors as an indication that all rivets have come into full bearing, and they have plotted curves of the ratio of end slip to middle slip. The latter, however, mean little since this ratio combines two different effects in it as has been shown previously herein. Unfortunately, the authors have given only typical load-slip curves and have not entered their test points in these diagrams, from which fact it may be inferred that these are idealized curves, or, at least, curves of averages. As the test points were omitted in the diagrams of the load-slip relationship, the investigator is not able to form his own judgment as to whether the curves shown are actually the best that can be passed through the test points, or if local anomalies exist, and, if so, at what loads. Apparently, the curves are faired (almost too much so), a fact that does not make them appear at all likely where sudden slips occur. The true test points, or better still, a tabulation of the test readings, would have been of far greater value for comparisons between tests and calculations.

The load-slip relationship of the joints (representing their load-elongation diagram, which is akin to the stress-strain curves) has been divided by the authors into four stages. With the exception of the first, these stages have been described more or less accurately. Regarding the exception, they state: "In the first stage, static friction prevents slip." This is incorrect if it refers to end slip, but is correct if it refers to middle slip, which shows clearly the ambiguity of the term "slip" as used in the paper.

Of interest are the authors' statements regarding sudden short slips and snaps which occurred in Series F. The writer believes that it would be much appreciated if one could see some of the actual load-slip curves (including all test points) which exhibit these short slips.

The authors are partly correct in stating that the slip has little influence on the elongation. This statement holds true for the later stages; but it does, or does not, hold true for the last part of the first stage, according to the definition of elongation chosen. That the more ductile steels exhibit less middle slip than do the harder steels is well known, particularly from shipbuilding. The unknown fact, however, is how much the slip actually is; and this depends on the friction factor, the state of the faying surfaces (which is not stated by the authors), the pressure exerted by the rivets, the extent to which, and intensity with which, this pressure is distributed over the faying surfaces, the clearances between rivet shank and hole, etc.

For evolving a rational method of computation, the friction factor and distribution of the rivet pressure over the faying surfaces are absolutely essential. The coefficient of friction is required for various kinds of steel plates, in single-sided friction (single shear) as well as in double-sided friction (double shear), and for a single rivet, a single rivet row, and double rows of rivets. Before the present tests were started, the writer had requested that, at least, some tests of single rivet lap and butt joints be included. It is unfortunate that this was not done. The distribution of the rivet pressure over the faying surfaces is of importance as the friction factor depends on it. This distribution varies considerably with the thickness of the plates.

Of interest, also, is the statement by the authors that, in the lap joints of Series A, the working load of the rivets, for most joints, fell in Stage II, but that for nickel-steel joints with manganese-steel rivets, it fell in Stage III. Hence, it seems as if such nickel-steel joints might be calculated according to the elastic theory only (Ivan Arnovljević, Cyril Batho, and Alexander Hrennikoff, Assoc. M. Am. Soc. C. E.). Another interesting fact may be mentioned—namely, that in butt joints the end slip proved to be about three-fourths of that of lap joints of similar proportions.

(c) Distribution of Strain.—The authors are to be commended for having attempted the arduous task of establishing the strains and, thereby, also the stresses over the full width of the main plates in the joint for, at least, a few joints. The picture revealed thereby is illuminating. It shows that, with the exception of the area near the rivet holes, the longitudinal strains are distributed fairly uniformly across the joint. There is only one drawback to the results obtained—namely, that they were derived by the use of a rather long gage length (2.5 in.). For this reason, the resulting strains are more or less average strains between rivet rows. Since the strains and stresses vary sharply from point to point near the rivet holes, the authors could not possibly have detected the actual strain distribution in the vicinity of the holes with the strain gages they were using. How the distribution of the strains and stresses around a rivet hole, filled by a rivet and transmitting forces from the rivet to the plate, or *vice versa*, actually looks may be seen from the data published by A. Hennig[46]. It would have been very interesting, in the light of what has been written previously herein about the grooving effect, if the authors had also determined the variation of the strains from point to point in a line along a rivet row—that is, transversely between rivets. However, extensometers of extremely short gage length would have had to be used in that case to obtain any accuracy at all.

From the diagrams of Fig. 11, however, it appears as if the strains have not been plotted correctly; or, rather, that the curves drawn through the plotted points do not represent the strains correctly. For example, in the diagram at the extreme upper left-hand corner of Fig. 11, the face strain labeled 1, starting from the left end, should remain constant in Plate 1 until, at least, the edge of Plate 2′. From then on, it should decrease to the first gage point between the first left-hand row of rivets and the second row, and, thereupon, follow the full curve shown by the authors, to its right end, from where the strain should drop to zero at the right-hand edge of Plate 1. The same holds true for the face strains in Plate 2′, only in reverse order. It appears that the authors have not used a sufficient number of gage points to obtain the true distribution. This also holds for the other two sets of curves in Fig. 11, but, for Specimen ACC54-1, not to quite the same extent.

If the strain, however, should actually have varied in the manner shown by the authors' curves, it would have been an indication of the fact that bending strains were present and, in that case, the strain on the opposite face of Plate 1, or of Plate 2′, respectively, should have been determined in order to eliminate the bending strains. Perhaps the authors can explain the course of the curves by giving further data.

In addition, the authors have measured the edge strains at the center of the plate edges. Their plots show that these strains do not differ too greatly from the average of the face strains, although, at places, the deviation is considerable, at least in the specimens of Series A to which Fig. 11 applies. Here, again, the long gage length used is an objectionable feature. As regards the course of the curves, remarks similar to the foregoing apply. The observations as to gage-line length apply also to the joints of Series F, for which the authors have, likewise, determined the strains, although at the edges only. More data would certainly be required for careful checks, and would be welcomed warmly.

---

[46] "Polarisationsoptische Spannungsuntersuchungen am gelochten Zugstab und am Nietloch," by A. Hennig, *Forschung auf dem Gebiete des Ingenieurwesens*, Vol. 4, 1933, pp. 53–63.

(d) Partition of Load Among Rivets.—From the measurements of the strains, the authors have determined the distribution of the load over the various rivet rows (Fig. 13). These do not seem to follow the elastic theory closely. On the other hand, if the strains plotted by the authors in Fig. 11(*a*) are used for determining the distribution of the load, as was done by the writer, one arrives at a curve (for 20 kips per sq in. in the net section of the plate) which differs from the curves of load partition plotted by the authors in the lower left-hand corner of Fig. 13, and which represents more closely the load distribution curves derived from the elastic theory. This is particularly true for the curve derived from the face strains. Unfortunately, this is the only check that can be made with the material given by the authors. It would be very desirable if further material could be published by them so as to make the tests useful to the profession.

For Series F, the authors have determined the load carried by each plate at the various points. These curves were derived from the edge strains. As has already been stated, the gage length was, apparently, too great to give accurate results. As regards the load for which the curves were plotted, this has already been discussed under Item (10) herein. Fig. 14(*b*) shows, in Diagram C, that the end rivets, apparently, take a larger share than the other rivets, but that the load in the main plate increases again at about the third outer rivet row, after it has dropped about 3% more between the second and third rivet rows. This seems, indeed, very unlikely, because it means that the load carried by the splice plates would, again, be transferred to the main plates, a fact which could only occur in the case where the splice plates are considerably more rigid than the main plates. An explanation by the authors of this behavior, which does not appear in any of the other joints (at least not at the first few outer rivet rows) would be welcomed. In the shingle joints, however, it is true that load from one main plate is first transferred to the splice plates and is then again transferred from the splice plates to the main plates. Without having more accurate data than the curves which are shown at a rather small scale, it is not possible to check the behavior of these types of joints.

(*12*) *General Remarks and Summary.*—In summing up the foregoing, the writer makes the following general observations:

The present procedure of designing riveted joints is antiquated. It dates back more than 100 years when Fairbairn, in 1838, on the basis of his first practical tests, suggested the use of the ultimate strength of a riveted joint, in conjunction with a suitable factor of safety, as the criterion for its strength at working loads. From this procedure, no progress has been made within the last century, and the enormous sums spent for testing specific joints have produced, as the only result, an enormous accumulation of incoherent data that have never been properly correlated. The fault of this procedure has been recognized by some far-sighted minds, but they have not been able to replace it by a better one. This is due to the problem never having been attacked properly, a fact that was revealed to the writer by the study of the world's literature on riveted joints[26]. The deeper reason for it is that every one dealing with this problem has attempted to generalize (extrapolate) from individual experiments instead of to follow the opposite "path" of investigating, first, the underlying general and basic laws, and of particularizing (interpolating), then, by applying them to the specific problem or joint under consideration. The numerous and costly tests conducted to date might then have served as valuable checks and would have quickly revealed either the correctness or incorrectness of the theory, or its weaknesses.

Although the authors, obviously, had the best intentions of making these tests a basis for obtaining useful information and a clear insight into the behavior of riveted joints, very little has been accomplished that was not known previously, as has been shown herein. Therefore, these tests must be regarded as having, primarily, added further statistical material, obtained at ultimate loads, to the already enormous amount of such statistical information available. The load-slip relationship has been cleared up somewhat, but the published data are insufficient to permit any rational theory of riveted joints, that may be devised, to be checked by them. The tests have failed to produce those basic data which are necessary for establishing a rational method of calculating riveted joints. No such method has been published as yet. Unless such a method is established by logical reasoning, which will give results that agree with practical measurements, it cannot be said that the problem of riveted joints has been solved completely.

The present method of calculation should be revised, at least so as to make the yield load the deciding factor and to give more weight to the behavior of riveted joints at working loads. In effect, this is a procedure similar to that recommended in the theory of limit design[44].

As regards the establishment of a rational theory of riveted joints, the following should be kept in mind:

Friction, which is frequently regarded as of doubtful value, provides the supporting force in most structures. Bearing of the rivets against the plates occurs only under exceptional conditions. In ships, for example, which, in heavy seas, are subjected to constant reversals of forces—that is, to alternating stressing—rivets, when cut out, seldom show any evidence of bearing; and yet, the joints of ships creak and squeak in heavy seas, an evidence of slip occurring under friction. No deterioration of the plate material on the faying surfaces has been observed, however. Riveted plate girders with splice plates in the web scarcely ever show evidence of the rivets bearing against the plates. They are held together rigidly by friction, as has been proved by tests. These are only a few examples showing the importance of friction as the supporting force. For this reason, friction in riveted joints should be studied extensively and far more than has been done in the past.

No information is available on how the rivet force used for clamping plates together spreads in these plates, nor how far its influence extends over the faying surfaces—that is, where the quilting action, or the lifting off of one plate from the other, starts. It is obvious that this influence depends on the rivet diameter, the diameter of the rivet head, and the thicknesses of the plates clamped together. To prevent over-stressing of the plates, the smallest possible pressure of the ram of the hydraulic riveter should be used, just sufficient to form the head properly, and the ram should be held on the finished rivet until it has cooled sufficiently in the shank to prevent plastic deformations in it during further cooling. Thus, the uncertainty of the rivet clamping force would be largely eliminated. The holes and the rivet shank in finished joints should be measured so as to obtain accurate information as to the clearance space between rivet and hole. Thus, basic data would be supplied for making a rational calculation of riveted joints possible.

As to specific recommendations, the writer offers the following, of which some coincide with similar ones made by the authors:

1. Riveted joint design should be based on the stress distribution along the joint, produced by the longitudinal forces, and particular emphasis should be placed on the stress peaks near the rivet holes;
2. Riveted joint design should be based, furthermore, on the distribution of the rivet clamping force (perpendicular to the plates), with particular emphasis on the extent to which, and the intensity with which, the rivet force spreads over the faying surfaces;
3. Chain riveting (full-row riveting) should be adopted generally;
4. Rivet pitch and gage-line distances depend on both the rivet diameter and the plate thickness;
5. Friction coefficients for various plate and rivet materials at rivet pressures should be determined;
6. Riveted joints should be laid out with reference to the yield strength on which working stresses should be based;
7. Riveted joints should be as short as possible (butt rather than shingle type); and
8. In general, carbon-steel rivets are more satisfactory than rivets of alloy steels except where it is necessary to save as much in weight as possible.

E. C. HARTMANN,[47] AND MARSHALL HOLT,[48] ASSOC. MEMBERS, AM. SOC. C. E. (by letter).—Referring to the heading, "Recommendations Applicable to Design," the writers were particularly interested in Statements (1) and (2). Regarding Statement (2) that "There is no justification for elaborate formulas to calculate the effect of rivet stagger on net section," the writers have assumed that "elaborate formulas" refer to Equation (3) taken from the specifications of the American Association of Highway Officials[12] (A. A. S. H. O.) and identical in effect with that in the A. R. E. A. Specifications. Having used this method of computing net section with satisfactory results, and having frequently tested riveted joints with efficiencies in excess of 75%, the writers decided to review the test results to check the authors' recommendations.

Figs. 25 and 26 show the types of specimens tested and Tables 32, 33, and 34 show the detailed test results. These specimens are from various investigations and do not represent a single planned series of tests. Some are too small to be considered structural joints in the ordinary sense, but these can be treated as scale models of structural joints.

---

[47] Research Engr., Aluminum Co. of America, Aluminum Research Laboratories, New Kensington, Pa.

[48] Research Engr., Aluminum Co. of America, Aluminum Research Laboratories, New Kensington, Pa.

In dealing with rivet sizes ranging from 3/16 in. to 7/8 in., it would be misleading to add the same fixed amount (1/8 in.) to the rivet diameters in arriving at the sizes of holes to be deducted. Therefore, all holes to be deducted were assumed to be 15% larger than the rivet diameter, a percentage which is consistent with current practice when applied to the range of sizes from 3/4 in. to 7/8 in.

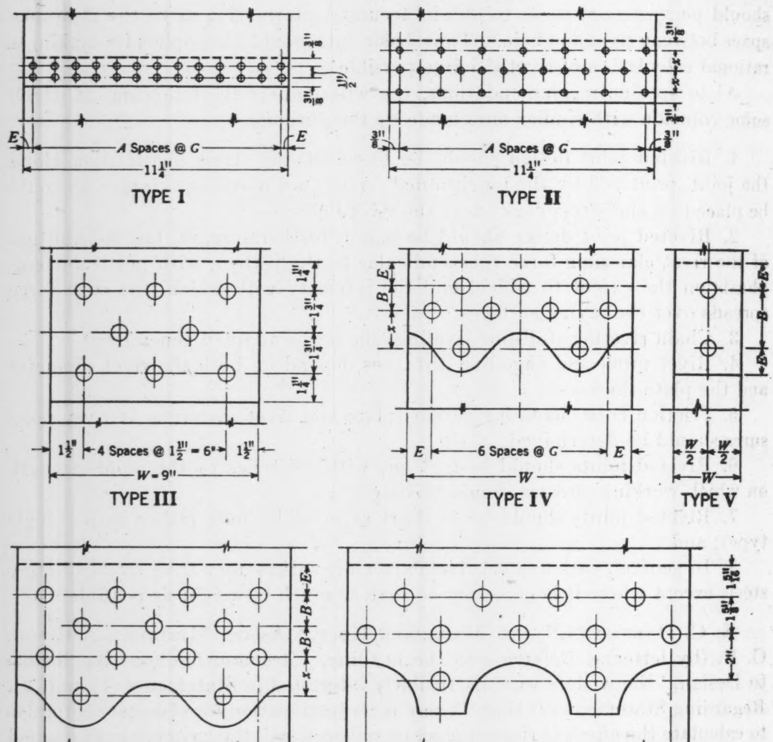

E
A Spaces @ G
E
11 1/4"
3/8"
1/2"
3/8"
TYPE I
3/8"
A Spaces @ G
3/8"
11 1/4"
3/8"
x
x
3/8"
TYPE II
1 1/2"
4 Spaces @ 1 1/2" = 6"
1 1/2"
W = 9"
1 1/2"
1 3/4"
1 3/4"
1 1/2"
TYPE III
B
E
x
E
6 Spaces @ G
E
W
TYPE IV
B
B
B
W/2
W/2
W
TYPE V
E
B
B
x
E
6 Spaces @ G
E
W
TYPE VI
13"/16
8 Spaces @ 1 5/8" = 13"
13"/16
14 5/8"
1 5/8"
1 3/4"
2"
TYPE VII

FIG. 25.—RIVETED JOINTS

Table 35 summarizes the test results and includes a parallel tabulation of the results of some of the tests made by the authors and some described in the authors' references. This table shows that each group of riveted joints had one or more joints with efficiencies considerably in excess of the 75% which the authors recommend as the useful upper limit for practical design. This indicates that possibly the authors' first recommendation is too conservative. Table 35 further shows that the commonly used A. R. E. A.-A. A. S. H. O. method of calculating net section gives predicted loads which average within 11% of the actual ultimate loads for a wide variety of joints in several different materials. This spread is not excessive when compared with the accuracy expected of other design formulas such as those for column strength, allowable compression in beam flanges, and spacing of web-plate stiffeners.

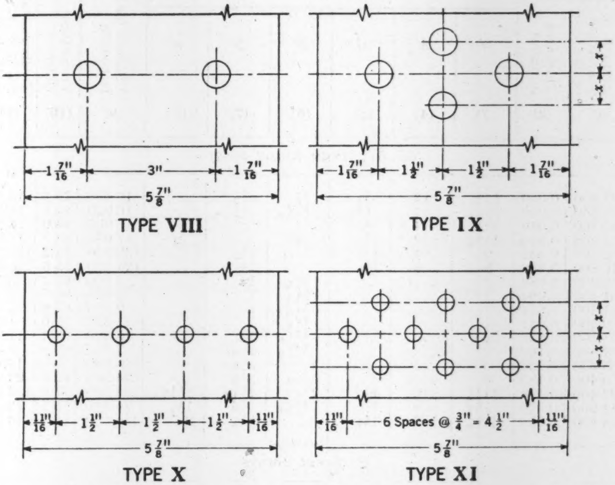


Fig. 26.—Plates with Open Holes or Idle Rivets

TABLE 32.—Characteristics of Test Specimens

Type*	Size of rivets, in inches	Material§	
		Rivets	Main Plates
(1)	(2)	(3)	(4)
	(a) Aluminum Alloy Joints		
I	3/16†	53S-W	52S-½H
II	3/16†	53S-W	52S-½H
III(a)	¾†	17S-T	17S-T
III(b)	¾†	17S-T	61S-T
V(a)	¼†	17S-T	17S-T
V(b)	½†	17S-T	17S-T
V(c)	⅝†	17S-T	17S-T
V(d)	¾†	17S-T	17S-T
VI(a)	¾‡	53S-W	53S-T
VI(b)	½‡	53S-W	53S-T
VII	⅞†	53S-W	52S
	(b) Steel Joints		
IV(a)	⅝†	Steel‖	Steel¶
IV(b)	½‡	Steel‖	Steel¶
IV(c)	7/16†	Steel‖	Steel¶
IV(d)	5/16‡	Steel‖	Steel¶
IV(e)	½‡	Steel‖	Steel‖
IV(f)	5/16‡	Steel‖	Steel‖

* See Fig. 25. † Single shear. ‡ Double shear. § For chemical composition and typical properties, see *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 1267. ‖ Carbon steel. ¶ Low-alloy steel containing approximately 1.45% manganese and 0.18% carbon.

The writers have yet to find any method of calculating net section of riveted joints and tension members which gives consistently better results

TABLE 33.—RESULTS OF STATIC TENSION TESTS ON RIVETED JOINTS THAT FAILED IN THE PLATES (SEE CHARACTERISTICS IN TABLE 32)

Specimen type*	MAIN PLATES		SPECIMEN DIMENSIONS, IN INCHES (SYMBOLS IN FIGS. 25 AND 26)						Ultimate load, in pounds	Percentage efficiency	Predicted load (A. R. E. A. method) in pounds	Ratio, predicted load to actual load
	Tensile strength, in pounds per square inch (1)	Thickness, in inches (2)	*W* (3)	*A* (4)	*G* (5)	*E* (6)	*B* (7)	*x* (8)	(9)	(10)	(11)	(12)
					(*a*) ALUMINUM ALLOY JOINTS							
I†	35 800	0.050	11 1/4	12	7/8	3/8	1/2	....	16 050	79.5	15 100	0.94
I†	35 800	0.050	11 1/4	11	15/16	15/32	1/2	....	16 920	84.0	15 500	0.92
II†	35 800	0.049	11 1/4	14	3/4	3/8	....	1/2	16 960	85.9	16 500	0.95
II†	35 800	0.050	11 1/4	12	7/8	3/8	....	3/4	17 080	84.7	17 400	1.02
III(*a*)†	62 200	0.332	9	4	1 1/2	....	1 3/4	....	138 250	74.2	142 700	1.03
III(*b*)‡	46 800	0.388	9	4	1 1/2	....	1 3/4	....	124 450	75.9	120 900	0.97
V(*a*)	60 900	0.082	1	....	....	7/16	1	....	3 060	61.3	3 560	1.16
V(*a*)	61 400	0.064	1	....	....	7/16	1	....	2 620	66.7	2 790	1.06
V(*a*)	61 400	0.051	1	....	....	7/16	1	....	2 060	65.8	2 230	1.08
V(*a*)	60 200	0.041	1	....	....	7/16	1	....	1 670	67.6	1 760	1.05
V(*b*)	63 300	0.125	2	....	....	13/16	1 3/4	....	10 800	68.4	11 300	1.04
V(*b*)	61 800	0.103	2	....	....	13/16	1 3/4	....	7 700	60.5	9 100	1.18
V(*c*)	63 300	0.125	3	....	....	1	2 1/8	....	16 650	70.0	18 100	1.09
V(*d*)	60 400	0.185	3	....	....	1 3/16	2 1/2	....	24 300	72.3	23 900	0.98
VI(*a*)†	38 400	0.479	11 1/2	6	1 1/2	1 1/4	1 5/16	1 11/16	164 200	77.6	151 400	0.92
VI(*b*)	41 900	0.330	7 5/8	6	1	13/16	7/8	1 1/8	79 450	75.2	72 500	0.91
VII	28 600	0.500	14 5/8	8	1 5/8	....	1 5/8	2	144 000	68.8	144 900	1.01
					(*b*) STEEL JOINTS							
IV(*a*)†	74 900	0.226	9 1/2	6	1 1/4	1	1 1/8	1 5/8	105 250	65.5	105 200	1.00
IV(*b*)†	74 500	0.228	7 5/8	6	1	13/16	7/8	13/16	97 250	75.1	91 600	0.94
IV(*c*)	83 800	0.143	6 3/4	6	7/8	3/4	13/16	1 1/8	67 100	82.9	55 400	0.82
IV(*d*)	85 400	0.149	4 3/4	6	5/8	1/2	9/16	3/4	47 900	79.3	43 100	0.90
IV(*e*)†	64 400	0.276	7 5/8	6	1	13/16	7/8	1 3/16	109 550	81.1	95 900	0.88
IV(*f*)	71 600	0.193	4 3/4	6	5/8	1/2	9/16	3/4	51 950	79.2	46 700	0.90

* See Figs. 25 and 26; one specimen tested of each type except as noted. † Two specimens tested. ‡ Three specimens tested.

TABLE 34.—RESULTS OF STATIC TENSION TESTS ON ALUMINUM ALLOY PLATES WITH OPEN HOLES OR IDLE RIVETS*

Type†	Diameter of hole, in inches‡ (1)	*x*, in inches† (2)	LOADS, IN POUNDS		Ratio, predicted to actual load (5)
			Ultimate (3)	Predicted** (4)	
VIII	25/32§	....	63 800	62 800	0.98
VIII	25/32	....	56 700	62 800	1.11
VIII	25/32‖	....	59 100	62 800	1.06
VIII	25/32¶	....	55 500	62 800	1.13
IX	25/32	1	51 300	54 800	1.07
IX	25/32	1 1/4	53 000	57 600	1.09
IX	25/32	1 1/2	51 400	61 100	1.19
IX	25/32	2 1/4	54 200	62 800	1.16
X	13/32	....	59 500	62 800	1.05
XI	13/32	1/2	50 700	50 800	1.00
XI	13/32	5/8	53 600	55 000	1.03
XI	13/32	3/4	54 600	60 100	1.10
XI	13/32	1 1/8	57 200	62 800	1.10

* All plates 1/4 in.; 17S-T alloy; tensile strength, 60 500 lb per sq in. † See Fig. 26. ‡ All holes punched and open except as noted. § Drilled holes. ‖ 3/4-in. rivets, normal driving pressure. ¶ 3/4-in. rivets, excessive driving pressure. ** Predicted by A. R. E. A. method.

than the relatively simple method outlined in the present A. R. E. A. Specifications. The only feature of this method which is open to serious criticism is its tendency to encourage inexperienced detailers to go too far in omitting rivets in the outside rows. It would seem that this weakness of the method could be easily overcome, without sacrificing any of the advantages of the method, by some simple restriction on the maximum spacing of rivets in the outside row. For example, a statement similar to the following, added to the article on net section in the A. R. E. A. Specifications, would serve the purpose:

TABLE 35.—COMPARISON OF MAXIMUM EFFICIENCIES AND RATIOS OF PREDICTED TO ACTUAL ULTIMATE LOADS*

Number of specimens	Alloy	Laboratory	Maximum efficiency (percentages)	RATIOS, LOAD PREDICTED BY A. R. E. A. METHOD TO ACTUAL ULTIMATE LOAD		
				Minimum	Maximum	Average
	(1)	(2)	(3)	(4)	(5)	(6)
25	Aluminum	Aluminum Research Laboratories	85.9	0.91	1.18	1.02
9	Steel	Aluminum Research Laboratories	82.9	0.82	1.00	0.91
14	Aluminum†	Aluminum Research Laboratories	....	0.98	1.19	1.08
39	Steel	National Bureau of Standards‡	85.0	0.88	1.06	0.98
24	Steel	University of Illinois§	87.1	0.93	1.05	0.99
26	Steel	University of California‖	78.8	0.92	1.22	1.11

* All specimens failed by tension in the plates. † Plates with open holes or idle rivets. ‡ Based on data from "An Investigation of the Behavior, and of the Ultimate Strength, of Riveted Joints under Load," by E. L. Gayhart, *Transactions*, Soc. of Naval Architects and Marine Engrs., Vol. 34, 1926. § "Tests of Joints in Wide Plates," by Wilbur M. Wilson, M. Am. Soc. C. E., James Mather, and C. O. Harris, *Bulletin No. 239*, Eng. Experiment Station, Univ. of Illinois, Tables 6 and 8. ‖ See Tables 5 and 17 of the present paper.

> The transverse distance between rivets in the outside row shall not exceed eight times the nominal diameter of the rivet, and the edge distance for the end rivet in the outside row shall not exceed four times the nominal diameter of the rivet.

RAYMOND E. DAVIS,[49] GLENN B. WOODRUFF,[50] MEMBERS, AM. SOC. C. E., AND HARMER E. DAVIS,[51] ASSOC. M. AM. SOC. C. E. (by letter).—Such value as these tests may have in developing a concept of the action of riveted joints under steady loads has been enhanced by the interpretation of the results of these and other tests by the several discussers. The writers express their appreciation to those who have given their time to analyze the results and present their conclusions.

Considerable discussion has centered about the matter of design provisions for the calculation of net area in a riveted tension member. Former experimenters have noted that the customary omission of rivets at the edge of a connection has little effect on its strength, and no priority was claimed by the writers in pointing out that the effective net section of a member, as determined by experiment, does not agree with results obtained from formulas in common use. However, since designers have not generally recognized this fact in practice, the writers emphasized it by the proposal that riveted tension members be designed on the basis of their gross area. In making this proposal, the writers were aware that the data did not prove conclusively that such a procedure would represent the best possible practice. Additional analyses of the data of these and other tests, made by the various discussers, have confirmed the writers' belief that adoption of the proposal would make for an improvement over present practice.

---

[49] Prof., Civ. Eng., Director, Eng. Materials Laboratory, Univ. of California, Berkeley, Calif.

[50] Cons. Civ. Engr., San Francisco, Calif.

[51] Associate Prof., Civ. Eng., Univ. of California, Berkeley, Calif.

It has been pointed out that the suggested unit stress of 16 000 lb per sq in. on the gross section for carbon steel is the equivalent of 21 300 lb per sq in. on the net section, and that such a unit stress is higher than permissible. Table 26 of Mr. Winter's discussion shows the close relation between the stress computed on the basis of the least net area and the strength of mill coupons. Such studies show that the unit stresses proposed are approximately those that obtain under the present practice of designing for 18 000 lb per sq in. on net section and taking the usual care to protect net section. Mr. Goodrich asks that designers be given a chance to use their ingenuity in protecting the net section. The writers believe that these attempts at ingenuity generally result in a design that has less efficiency than one in which the attempt to protect net section is disregarded. Mr. Goodrich mentions the case of a single hole out of an angle. Although no experimental data covering this case are known to the writers, they doubt that the ultimate strength, based on gross section, would materially exceed 75% of the coupon strength.

Professor Wilson's proposed formula corresponds very well with the available experimental data. It allows for the grooving effect with close pitches of rivets, and it properly penalizes wide pitches, although from the standpoint of practical design the formula would again tempt the ingenuity of the designer. The writers find no reason to doubt that full end riveting of the splice plates with a $\frac{G}{D}$-ratio of 4 represents as good practice as can be developed from existing information. Professor Wilson finds that this arrangement gives an efficiency of 80 per cent. To be on the conservative side the authors suggested a ratio of net to gross section of 75 per cent.

Mr. Jones' suggestion of a weld at the end of splice plates introduces a field for further experiments, and further knowledge as to the weldability of various steels may lead to economies in design. Naturally such a procedure would be limited to steels suitable for welding and might require an alloy other than silicon for tension members. Even with supplementary welding, however, the assumption should not be made (until after its truth is developed by experiment) that the strength of the splice will not depend upon the least net section.

It appears to be the consensus of opinion that friction between the plates cannot be depended upon to any material extent, and that the holes should be filled as completely as possible. Mr. Goodrich feels that the optimum with respect to filling of holes by rivets has been reached. Possibly specifications for steel structures should fix tolerances for the diameter of rivets and for the size of the holes. In the case of the San Francisco-Oakland Bay Bridge both the quality of fabrication and the standards of inspection were above the usual commercial standards. The measurement of rivets and holes, by calipers, developed the fact that the rivets were generally undersized and the holes generally oversized. Field tests of driving rivets developed that, with the field connections reamed, there was no trouble in entering the rivets when the diameter of the cold rivet was $\frac{1}{16}$ in. less than the diameter of the hole.

Mr. Goodrich disagrees with Recommendation (6) in regard to the use of manganese-steel rivets. The writers have little doubt that, in any actual design, Mr. Goodrich would agree with the writers as to joints, including gusset-plate connections, in which manganese-steel rivets should be used. The writers feel that, in general, present design practice tends toward the extreme in attempts to reduce weight of steel in structures, and that alloy structural steel or rivets should be used only when the economies gained by using the higher-strength materials are distinctly worth while.

The writers believe that Mr. Shearwood's criticism of the description of the action of riveted joints arose from his failure to realize that the action as shown in Fig. 1(*b*) was based on a simple specimen with only two rows of rivets. The action shown by Fig. 1(*d*) is representative of the cases illustrated by Mr. Shearwood. In this latter case, end slip occurs at very low average intensities of stress on the rivets; middle slip does not start until after considerable slip has occurred near the end of the joint.

Mr. Shearwood's suggestions as to cold-driven rivets open another field for further development. At present, the equipment for driving them is so heavy that they cannot be considered for field work.

Mr. Jones suggests that the ultimate capacity of a joint will suffice as a basis for selection of design stresses. This suggestion should be applied with caution to the results of tests on large joints. As an example of data which might lead to false conclusions, the behavior of the heavier specimens of the F series may be pointed out. These specimens were designed to fail in the plates. The first failures took place in the rivets, and occurred because of yield in the plates sufficient to shear the end rivets. It is possible that if the number of rivets in these specimens had been doubled, the rivets might have failed at the same total load, and the conclusion might have been drawn that the rivets had only half the unit strength actually developed.

Mr. de Jonge's discussion is valuable in suggesting fields for further research. He has also added an interesting explanation of the failure of wide plates without rivet holes. The writers believe that a careful reading of the paper will clear up many of the difficulties expressed by Mr. de Jonge. Space limitations called for extreme brevity in view of the amount of original data presented, so that neither speculative interpretation nor presentation of extensive detailed data was possible.

A few of his comments appear to require answer. The writers are unable to agree with his main thesis in regard to the development of a "rational theory of riveted joints." Five conditions of perfect fabrication were given in the paper. To these might be added: Uniform tension in all rivets, approximating the yield strength of the rivet material. With the most perfect practicable fabrication, these conditions cannot be attained. For example, tension in the rivets of a joint may vary from zero to the yield strength of the material. Furthermore, there can be no doubt that considerable plastic action takes place at comparatively low loads, thus invalidating the assumption of elastic rivet and plate materials, on which the proposed "rational theory" of necessity would be based. Any theorizing, therefore, must be based on assumptions which cannot be met in practice.

The discussion of rivet temperatures introduces the question of greatest possible tension in the rivets *versus* completeness of filling the holes. Possibly the decision should be on the basis of type of service required of the joint. For ship plating, or for members subjected to reversals of stress, high rivet tensions may be desirable. In this case it may be that the rivet temperatures suggested by Mr. de Jonge, and the "smallest possible pressure of the ram," should be used. However, for members subjected to comparatively steady stress in one direction, the evidence is clear that slip will occur within the range of working loads. In this case it appears desirable that the holes be filled as completely as possible. To achieve this result it appears necessary to heat the rivets to as high a temperature as possible without burning them, and to use such driving pressures as to upset the shank of the rivet completely. In either case, it is desirable that the bolting up prior to riveting bring the faying surfaces into contact, and that the action of the riveter be continued longer than is usual.

In view of the foregoing considerations, the writers are unable to agree with Mr. de Jonge's Recommendations 1, 2, and 5.

The writers agree with Mr. de Jonge as to the importance of understanding the action of riveted joints within the yield range, and it was with this idea in mind that so many strain measurements were made. However, as pointed out by Mr. Jones, the indications of these tests are that the ratio of rivet yield to plate yield does not differ greatly from the corresponding ratio at the ultimate. From these and other considerations, the writers cannot agree with Mr. de Jonge's Recommendation 6.

There appears to be no reason to rule out shingle-type joints, as Mr. de Jonge does in his Recommendation 7. This type of joint is often more economical of material than a corresponding straight butt joint, and is often necessary as a means of keeping the rivet grip within reasonable limits.

It is believed that Mr. de Jonge has misapplied the idea of the "true stress-strain diagram" calculated from the test of a steel bar in tension to the detrusion failure of rivets. In the failure of structural parts or members, one is concerned with the actual load (or total load *versus* total deformation) required to cause failure, and not with a unit stress calculated by dividing the load by reduced cross-sectional area.

On the matter of joint elongation, it was the writers' intention to show that for calculations involving elongation of members such as are necessary in computing deflections, in computing secondary stresses, or in analysis of statically indeterminate structure, it is unnecessary to take the joints into separate consideration. A joint occupies only a short proportion of the length of a member. The extreme ratio of 3.32 was for a joint which probably would not be used in practice.

Messrs. Hartman and Holt have contributed valuable additional data on the problem of the effective net section; the information having to do with aluminum alloys, small rivets, and very thin plates is of particular interest.

It is pointed out by Messrs. Hartman and Holt that the use of the A.R.E.A. formula for effective net section could be applied to the results of the University of California tests to compute the ultimate loads within an average error of about 10% (range, 30%). Although this may be true, the effective net widths computed by means of this formula bore no relation to the results of tests. Close agreement was found between values of effective net width, and width determined simply on the basis of full net section, without the use of any formula. In this connection, it may be well to emphasize that the tests reported in the paper were concerned with riveted connections having a number of rows of rivets such as are used in bridge construction, rather than with connections with few rows of rivets such as are used in ship and tank construction.

Mr. Jones' suggestion that further experimentation may well be done on small specimens is well taken. As expressed in the "Introduction" to the paper, the reason for the large specimens was that those responsible for the tests felt that current practice represented too extended an extrapolation of previously existing experimental data which had been obtained from tests of small specimens. As mentioned by most of the discussers, the confirmation of major design practices was a valuable result of the investigation.

It appears that all research programs develop the need of additional ones. From the standpoint of contribution to the art of structural design, the following appear to be most worthy of additional study at the present time:

1. Further confirmation of the conclusion that greatest actual efficiency is obtained by full-row riveting;
2. Determination of the ratio between gage and rivet diameter, for most economical results; and
3. Investigation of Mr. Jones' suggestion with regard to the use of a weld at the end of splice or gusset plates.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2085

COMBINING GEODETIC SURVEY METHODS WITH CADASTRAL SURVEYS

BY CARL M. BERRY,[1] ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. WILLIAM L. SAWYER, PHILIP KISSAM, GORDON MACLEISH, AND CARL M. BERRY.

SYNOPSIS

The purpose of this paper is to present the field and office technique developed on a large reservoir boundary survey, in which the economic consideration of the integral parts of such a survey has resulted in a more feasible and less costly treatment of land acquisition survey practise.

INTRODUCTION

The Grand Coulee Dam is situated on the Columbia River in northeastern Washington, 153 miles down stream from the point where the river enters the State. Four tributary rivers of importance—the San Poil, Spokane, Colville, and Kettle rivers—flow into the Columbia River at distances of 19, 47, 107, and 114 miles, respectively, up stream from the dam site. Numerous other streams of minor importance are tributary to the Columbia throughout the backwater area, some entering in narrow, precipitous canyons, and others across comparatively wide distributary delta plains. In all, a shore line of about 500 miles will be created by the lake behind this structure; the largest man-made structure in the history of the world. The deep Columbia Valley is characterized throughout as consisting of softly molded rolling hills and valleys, and lends itself most favorably to the development of a strong network of triangulation for co-ordination and control.

Summarized, the purpose of the boundary survey was: (*a*) To determine the extent and area of all real property situated and lying below a general taking line, or flow line, the elevation of which was fixed at 1 310 ft above mean sea level, and to fix and define the boundary lines of the area by executing a traverse of unbroken continuity lying along, or just above, the 1 310 contour; (*b*) to control the flow-line traverse properly with a network of triangulation of

---

NOTE.—Published in September, 1939, *Proceedings*.

[1] Civ. Engr., U. S. Bonneville Power Administration, Portland, Ore.

such accuracy as to hold the closing errors of the traverse within the prescribed limits; (*c*) to recover or properly re-establish the original survey corners and other cadastral points established by the General Land Office and reference them to the rectangular co-ordinate system; (*d*) to furnish the Legal Division of the United States Bureau of Reclamation with analyses of chains of title, in the cases of parcels having obscure or inaccurate descriptions in previous conveyances, and finally to furnish the Legal Division with properly closed descriptions of all parcels of real property involved; and (*e*) to monument the network of control triangulation permanently and connect this network to the adjacent existing first-order triangulation of the U. S. Coast and Geodetic Survey, thus referencing the entire system of co-ordinates to the Federal network on the 1927 North American Datum.

Methods of making control surveys, within themselves, are not new to the profession; nor are the many accoutrements of retracement and flow-line surveys. The combination of the two into a single, simultaneous operation encompassing the use of instruments ranging from the simple hand compass to the 12-in. direction theodolite, and of distance determination from the woodsman's pacing to invar taping, introduces a new procedure, and is distinctly a problem in surveying economics.

SPECIFICATIONS

Early in the work, it was decided that all project co-ordination should be placed on the horizontal and vertical data commonly accepted by the twenty-three members of the Federal Board of Surveys and Maps, of which organization the U. S. Bureau of Reclamation is a member. Thus all surveys would be referenced permanently to the National network and would provide not only a lasting usefulness of all Bureau surveys to Federal, State, and local agencies, but likewise would be of advantage to the Bureau in any future work performed by other agencies in this locality. Consequently, all horizontal control was to be referenced to the 1927 North American Datum, which has as its basis the Clarke Spheroid of 1866. Vertical control was to be referenced to the 1929 level net adjustment of the North American continent.

The requirement was established that all work be "check-position"; that is, that all position computations in the field would be made by at least two independent mathematical solutions, and that by a comparison of the two routes of computation a ratio check of the actual field error would result. This ratio check was set by specifications to be not more than one part in 5 000 for all reservoir co-ordination. Both the horizontal and vertical geodetic control were to be established by triangulation and precise leveling methods, adhering to the specifications and to office and field procedure of the U. S. Coast and Geodetic Survey.

CO-ORDINATES

In 1933–34 the Bureau of Reclamation established Triangulation Station Alpha at Latitude 47° 58′ 00″.844 and Longitude 118° 58′ 29″.827 as the origin of a rectangular co-ordinate system for the Columbia Basin Project. The geographic position was determined by first-order and second-order triangulation and adjusted by the direction method to the adjacent Umatilla-49th Parallel arc of the U. S. Coast and Geodetic Survey. Rectangular co-ordinates, raised to the datum plane 1 310 ft above sea level, were then computed.[3] This grid was designated the Grand Coulee Grid Datum (GCGD), and its use was extended to a point approximately 30 miles east, and at nearly the same latitude, as Station Alpha. The practise was, after adjustment, to convert the geographic positions of the main-scheme second-order stations to grid co-ordinates; then from grid distances and bearings derived from these co-ordinates, to compute grid co-ordinates of all intersection and resection third-order stations within the figures. The scale error, increasing with the departure from the origin, had enlarged to the point at which, if use of the Grand Coulee Grid Datum was to continue, it would be necessary to compute all third-order points first on true geodetic datum and then convert individually to the rectangular grid. The reason for this proposed conversion was to smooth out, proportionately, all grid discrepancies because, with departure from the origin, the grid scale was too long with respect to an east and west direction, too short in a north and south direction, and lines perpendicular on the spheroid were several seconds askew from normal on the grid. Thus, although the accuracy of the plane co-ordinates was well within one part in 25 000, or first-order, at the mouth of the Spokane River, the discrepancy would approximate one part in 3 500 at the point where the Columbia River enters the State.

Toward the close of 1935 the Lambert Conformal projection tables—one covering the north half, and another the south half of the State—were published for the State of Washington. The wide-spread adoption of similar systems by other States, particularly those in the East, quickly led to the decision to place all future Bureau co-ordination over the reservoir site on the North Washington System.

RETRACEMENT

Early in 1934 two retracement parties were organized; they began at the dam site and proceeded up stream in search of the General Land Office corners, some of which had been established by contract surveyors more than fifty years before. Each party, comprising seven men, was assigned one bank of the river, and pursued its search by the stadia-transit traverse method. Where corners or their accessories were lost a random hub was set, and the party subsequently returned to re-establish the corner by the proportionate measurement method.

The following year (1935) a part of the retracement work was assigned to the triangulation party, and the time-honored "timber-cruiser" method was inaugurated on the project. Two seasoned woodsmen, well trained and experienced in finding obscure bearing trees and bits of evidence, were employed. The woodsmen, starting from a known corner, and armed only with photostatic copies of the General Land Office original field notes, a foresters' staff compass or a hand compass, paced off the required distance in a cardinal direction, and recovered or marked for re-establishment the corner sought for.

Upon arrival at the scene of a corner the retracement engineer first made the usual diligent search for evidence of the corner monument or the corner accessories, as described in the original field notes. If found, or restored either from the accessories or upon reliable parol (word of mouth) evidence obtained from some local landowner, the full description was entered on a Cadastral Retracement Record form (see Fig. 1) and the corner designated as a Type "A" recovery. Often a diligent search of old records in Court houses or abstract offices would reward the efforts of the engineer by disclosing a reliable tie from some near-by identifiable object to the now lost corner. In a few cases Court adjudications locating the positions of disputed corners would form the basis of the recovery record.

---

[3] Computed from formulas in *Special Publication No. 71*, U. S. Coast and Geodetic Survey.

Failure to recover a Type A, or original corner, would lead to interviewing the adjoining landowners for support of established fence corners. If they were found to have been established beyond the statutory length of time required for the enjoyment of peaceful occupation by the abutting owners, these points were designated as Type "B" corners, or legal corners of occupancy. Here again, all available parol evidence was entered on the Cadastral Retracement Record form, and the attest of the parties interviewed was obtained.

After all possibilities of typing a corner as "A" or "B" were exhausted, the re-establishment method by proportionate measurement, or Type "C," was resorted to, and the methods prescribed by the General Land Office manual were followed rigidly.

SIGNALING

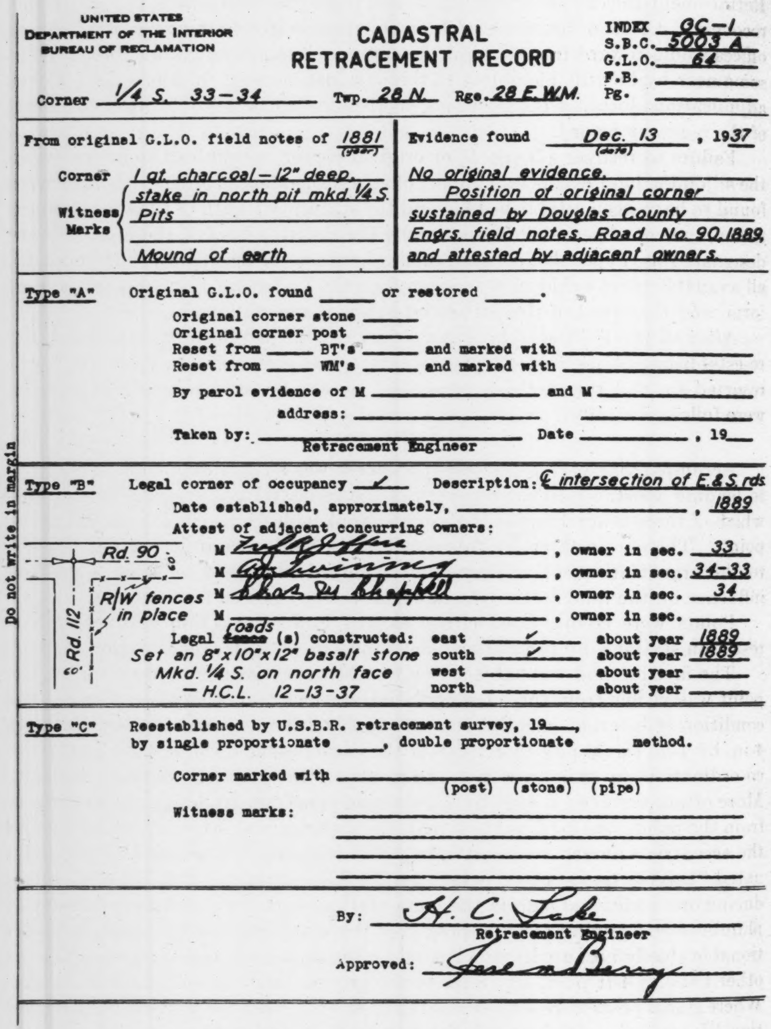
Accompanying each retracement engineer was a signalman who, in addition to lending assistance in the recovery of the cadastral points, would determine which of three general methods was most applicable for use in co-ordinating the point. The man performing this detail of the work was an important adjunct to the organization; his ability and developed judgment reflected a marked influence on the final cost of the newly developed procedure.

Points were co-ordinated either as: (*a*) Full intersection stations, (*b*) as resection stations, by the Pothenot method,[3] or (*c*) as traverse stations.

The first method was subject to a number of variations in procedure. If a point was visible from three triangulation stations and otherwise possessed the condition of location essential to a strong "fix," it was "signaled" by placing a 4-in. by 4-in. signal pole directly over the corner-stone itself and the method of co-ordination in this case was designated "Intersection station, central." More often, however, a 4-in. by 4-in. witness post, set from 6 in. to 20 ft away from the point, was used as the signal; the distance was measured carefully and the eccentric azimuth was read from a staff compass. This method was designated "Intersection station, point near." This latter method, although introducing one additional step in the computation work, obviated the necessity of plumbing and guying a signal pole over the stone monument, which is objectionable chiefly because livestock would often push over such a signal. On the other hand, a 4-ft post, set 18 in. in the ground, was secure from this danger. Where signal posts were set beyond the 20-ft limit from the corners, the designation "Intersection station, transit eccentric" was used, and the distance and direction were obtained by the signalman with a metric-foot tape and a light-weight transit. Often the densities of Indian allotment and cemetery corners, traverse control points, and adjacent section corners, would permit observation

---

[3] "Criteri e procedimenti per la formazione della mappe catastali con la fotogrammetria aerea," by Michele Tucci, R. Istituto Superiore di Ingegneria, Milano, 1935, pp. 16 and 18.



UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

CADASTRAL RETRACEMENT RECORD

INDEX GC-1
S.B.C. 5003 A
G.L.O. 64
F.B. ____
Pg. ____

Corner 1/4 S. 33-34 Twp. 28 N. Rge. 28 E.W.M.

From original G.L.O. field notes of 1881 (year)

Corner 1 qt. charcoal-12" deep.
stake in north pit mkd. 1/4 S.
Witness Marks Pits
Mound of earth

Evidence found Dec. 13 (date), 1937
No original evidence.
Position of original corner sustained by Douglas County Engrs. field notes, Road No. 90, 1889, and attested by adjacent owners.

Type "A" Original G.L.O. found ____ or restored ____.
Original corner stone ____
Original corner post ____
Reset from ____ BT's ____ and marked with ____
Reset from ____ WM's ____ and marked with ____
By parol evidence of M ____ and M ____
address: ____ ____
Taken by: ____ Retracement Engineer Date ____, 19__

Type "B" Legal corner of occupancy ✓ Description: ℄ intersection of E.&S. rds.
Date established, approximately, ____, 1889
Attest of adjacent concurring owners:
M ____, owner in sec. 33
M ____, owner in sec. 34-33
M ____, owner in sec. 34
M ____, owner in sec. ____
Legal ~~fence~~ roads (s) constructed: east ✓ about year 1889
south ✓ about year 1889
west ____ about year ____
north ____ about year ____

Set an 8"x10"x12" basalt stone
Mkd. 1/4 S. on north face
— H.C.L. 12-13-37

Type "C" Reestablished by U.S.B.R. retracement survey, 19__, by single proportionate ____, double proportionate ____ method.
Corner marked with ____ (post) (stone) (pipe)
Witness marks: ____

By: H. C. Lake
Retracement Engineer

Approved: ____

Do not write in margin

FIG. 1.—CADASTRAL RETRACEMENT RECORD FORM

upon from eight to ten "transit-eccentrics" from a single setup on an intersection station.

Necessity for the second method, that of the resection, or Pothenot method, arose chiefly from three situations: (*a*) That in which the second-order observing party failed to complete each of the prescribed "cuts" to a signal, either because the signal was disturbed or because of poor light and consequent poor conditions of visibility; (*b*) that in which, because of a corner poorly located for the requirements of a strong intersection, use of a three-point-fix observation from some advantageous point near the corner would enable the observer to select and make use of other non-occupied stations, as spires, cupolas, water-towers, etc., for a strong "fix"; or (*c*) that in which, subsequent to second-order observation over an area, it was decided to co-ordinate additional corners. In deciding upon the use of this method, the signalman left the choice of the "point near" and the objects to be observed upon, to the judgment of the three-point observer. Only a flag or banner was left at the corner to assist the observing party in finding the corner.

The third, or traverse method, was necessitated where dense stands of timber prohibited the use of triangulation methods. Three such traverses were used, two along the Columbia River near the town of Lincoln, Wash. (see Fig. 2), and one on the Spokane River near Little Falls, Wash., where "fixes" at each end of the traverses made it necessary to tape only in one direction. A fourth invar traverse was taped along an old railway road-bed in the Hawk Creek Canyon, and since a "fix" was impossible at the upper end, the traverse was double-taped to meet the requirement of check position. Corners and control points adjacent to these precise traverses were co-ordinated either by metric-foot taping, or by making them intermediate angle points in transit and steel tape loop traverses of third-order accuracy originating and ending at points along the precise traverses.

TRIANGULATION

First-Order.—First-order triangulation surveys comprised a chain of 25-mile quadrilateral figures extending from the Grand Coulee dam site east and north over the reservoir area to the International Boundary, and south from the dam site over the irrigation balancing reservoir area in the Grand Coulee to Coulee City, Wash., tying in to existing stations of the U. S. Coast and Geodetic Survey at each end, and to stations of the Geodetic Survey of Canada north of the Boundary. The network thus established extended to the high mountain peaks adjacent to the reservoir areas and established solitary control stations of first-order accuracy at about 25-mile intervals slightly above the reservoir flow lines. At the same time similar control stations were fixed in position at the heads of tributary reservoirs in the San Poil, Spokane, and Kettle rivers.

Besides the two 50-m invar tapes and stretching apparatus for measuring base lines, the Bureau obtained two instruments for first-order observations from the Coast and Geodetic Survey, a 12-in. direction theodolite, and a 7-in. repeating vertical circle.

Day-time observations of double zenith distances were made upon distant helioscope signal lights, using the vertical-circle instrument. Angles were

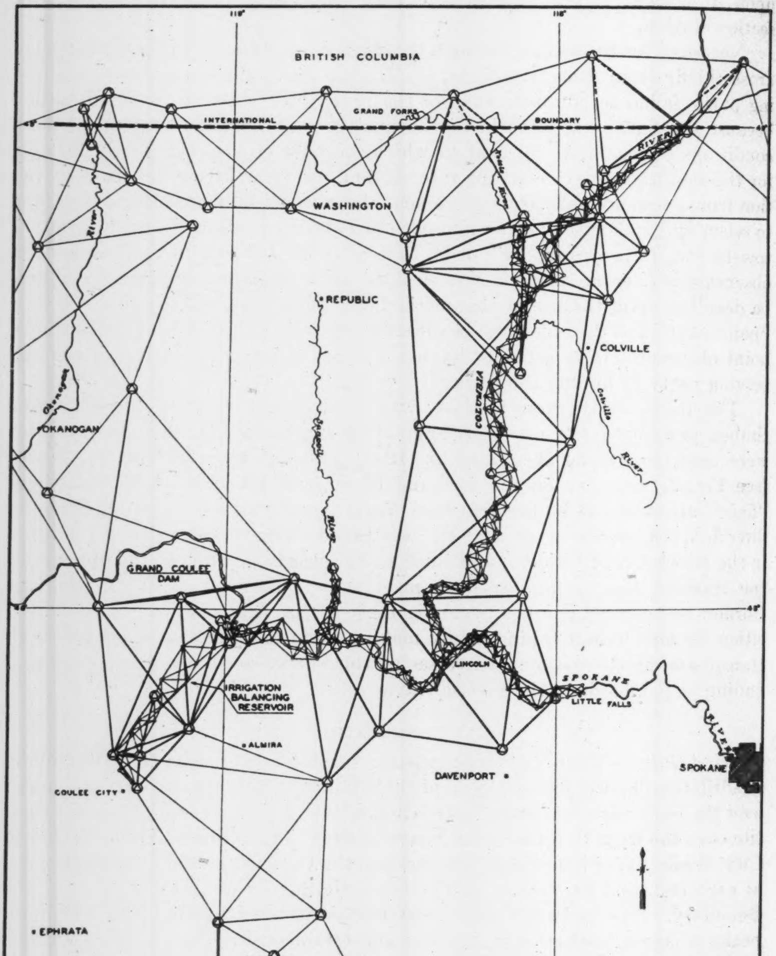

BRITISH COLUMBIA
INTERNATIONAL
GRAND FORKS
BOUNDARY
WASHINGTON
REPUBLIC
COLVILLE
OKANOGAN
Okanogan
River
Sanpoil
River
COLUMBIA
RIVER
Kettle River
Colville River
GRAND COULEE DAM
IRRIGATION BALANCING RESERVOIR
ALMIRA
LINCOLN
SPOKANE
RIVER
LITTLE FALLS
SPOKANE
DAVENPORT
COULEE CITY
EPHRATA
RITZVILLE
SCALE
10
0
10
20
49°
48°
119°
118°

FIG. 2.—GEODETIC CONTROL SURVEYS—COLUMBIA BASIN PROJECT

read on each of four verniers, direct to 10″. These latter observations formed the basis of the trigonometric leveling performed over the project.

In the Summer of 1934 a camp was established on the Columbia River 60 miles down stream from the International Border. Eight men were given a rigid course of training in the many duties and responsibilities of lightkeepers. Next, their adaptability for this rigorous and exacting work of back-packing, camping, and flashing code messages with helioscopes and electric signal lights, was tested by sending them out alone for two or three nights of work on a lone mountain peak. Those men found to be qualified were given regular assignments on reconnaissance work to locate all the stations of the first-order net. The actual observing work was next undertaken; 42 first-order stations were occupied, and the geographic positions of a total of 97 points determined. Among these were a number of fire-control towers, and their positions and azimuths between towers later were made available to the U. S. Forest Service, the U. S. Indian Field Service, and the State of Washington Division of Forestry. A number of aviation beacons were likewise "cut in," and their positions supplied to the U. S. Bureau of Air Commerce. Mapping operations of the U. S. Geological Survey were in progress near Kettle Falls during the seasons of 1935–1936, and positions of a number of their topographic control flags were supplied that Bureau. Still later, a part of the first-order net was used in controlling flights of aerial photographs taken by the Soil Conservation Service. First-order control for the project extended over an area of 3 038 sq miles.

Second-Order.—Between the control stations established by first-order work along the reservoir site, a secondary network, made up of quadrilateral figures averaging 1.75 miles in axial length, was monumented and observed. Lengths were expanded into the figures at the ends of each net from base lines measured with second-order accuracy, and, after making least-squares adjustment between the control stations, the latitudes and longitudes of the triangulation stations were converted to rectangular grid co-ordinates to simplify their use in conveying tracts. Spacing of first-order control stations along the flow lines permitted breaking up the second-order work into seven nets along the Columbia River. Three additional nets extended into the valleys of the San Poil, Spokane, and Kettle rivers. Division of the nets in this manner offered the advantage of having available, for the Right-of-way Division of the Bureau of Reclamation, the final "closed" plats and descriptions of any tract, within a few weeks after the second-order and third-order work in that net had been completed.

Measurements of angles between main stations, directions to cadastral signals and flow-line control points, and double-zenith-distance measurements for trigonometric leveling, were all observed simultaneously from the second-order stations; 7-in. theodolites reading to 10″, with single, opposite verniers, were used. Although this theodolite was of the "repeating" type, all pointings were made by the "direction" method, the high quality of the circle graduations permitting this variation. On the main network, 12 positions of the circle were read, with a residual of 5 sec from the mean specified as a maximum, whereas three positions were used in pointing the cadastral signals, and the same residual specified. Use of the direction method often permitted a station to be completed in much less time than it could have been by the method of repetition, because of poor light or fog obscuring some of the signals during a part of the observing period. Likewise, the direction method obviated the necessity of making the consequent local adjustment of sum angles in the lists of direction.

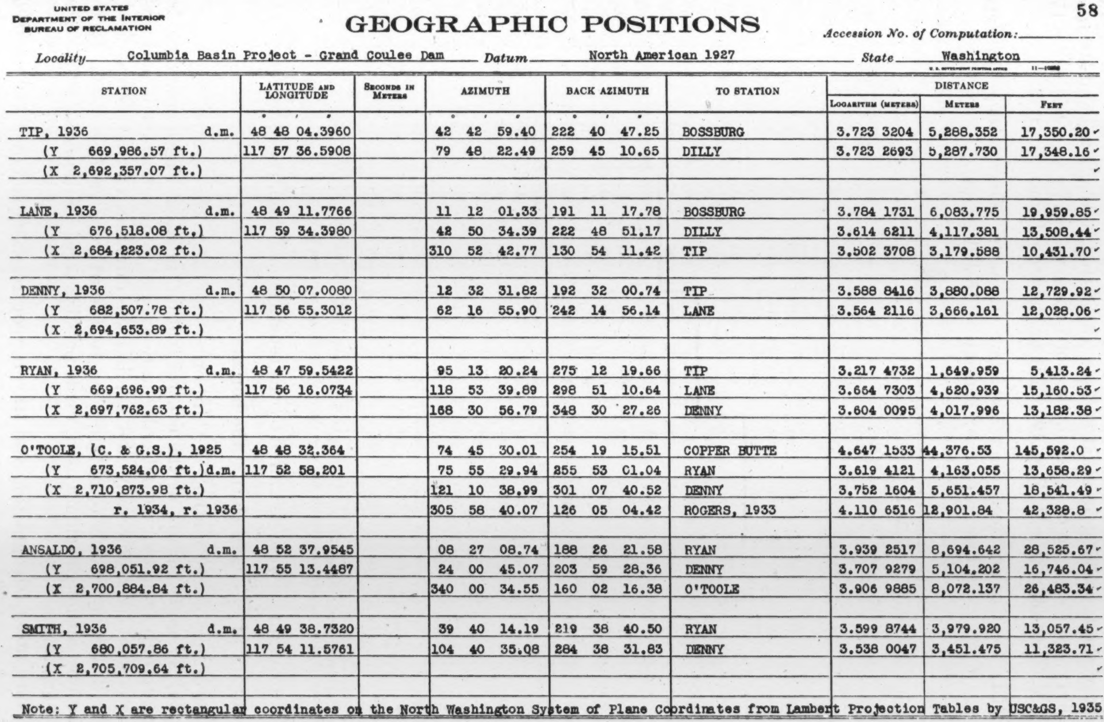
The parties occupied 342 second-order stations and determined the geographic positions of 2 020 points. An example of the notes for seven of these stations is presented in Fig. 3. Second-order control for the project extended over 437 sq miles.

Third-Order.—By triangulation or traverse of third-order accuracy, or by some combination of the two, the positions of all cadastral points within, and adjacent to, the reservoir area were co-ordinated and referenced to the existing monumented stations. In most cases this was accomplished by measuring the angular directions to signals set at or near the retraced cadastral monuments or flow-line angle points; and the procedure, because of its closely related performance to the second-order work, has already been described herein. The few remaining corners which lay concealed from second-order stations were co-ordinated by three-point-fix observations made from an open spot near the concealed monument. The practical aspects of this, the Pothenot method, or Italian "metodo di Pothenot," [3] are already rather well known to professional photogrammetrists, and use of this method represents the refinement necessary to its adaptation to cadastral surveying.

Instructions to the "three-point" observing party emphasized largely the necessity of selecting stations carefully for a strong resection, and further pointed out the dangers of a "fix" lying on or near the arc of the great circle, such a fix (commonly termed a "swinger") being indeterminate. Treatment of the resection problem may be found in any good manual on plane-table surveying, and the only further mention of the procedure in this paper is of the development of an arbitrary index to the strength of any given three-point-fix, which will be found with the description of the computing form in use on this project.

Rather frequently a landowner would appeal to the Right-of-way Division to complete appraisal and survey of his tract in advance of other work in that vicinity to forestall some pending litigation, and thus save both himself and the Government considerable delay and expense by the speedy consummation of the property transaction. In that case the three-point party would co-ordinate the adjacent corners and traverse control points quickly, and the necessary plat would be forthcoming within three or four days.

The use of a standardized metric-foot tape was found most applicable for co-ordinating points within 100 ft of the control station. For points lying beyond 100 ft and up to 500 ft, or for taping loop traverses, a standardized 300-ft or 500-ft steel tape was used. A standardization site was laid out with the invar tapes, and a calibration card was compiled for each of the two lengths of tapes. These cards were given to the chainmen for field use. Mounted on cardboard, they furnished a rapid means of determining the proper plus or minus correction for various conditions of temperature and catenary. For



UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

GEOGRAPHIC POSITIONS

58

Accession No. of Computation:

Locality Columbia Basin Project - Grand Coulee Dam *Datum* North American 1927 *State* Washington

STATION	LATITUDE AND LONGITUDE	SECONDS IN METERS	AZIMUTH	BACK AZIMUTH	TO STATION	DISTANCE		
						LOGARITHM (METERS)	METERS	FEET
	° ′ ″		° ′ ″	° ′ ″				
TIP, 1936 d.m.	48 48 04.3960		42 42 59.40	222 40 47.25	BOSSBURG	3.723 3204	5,288.352	17,350.20 ✓
(Y 669,986.57 ft.)	117 57 36.5908		79 48 22.49	259 45 10.65	DILLY	3.723 2693	5,287.730	17,348.16 ✓
(X 2,692,357.07 ft.)								✓
LANE, 1936 d.m.	48 49 11.7766		11 12 01.33	191 11 17.78	BOSSBURG	3.784 1731	6,083.775	19,959.85 ✓
(Y 676,518.08 ft.)	117 59 34.3980		48 50 34.39	222 48 51.17	DILLY	3.614 6211	4,117.381	13,508.44 ✓
(X 2,684,223.02 ft.)			310 52 42.77	130 54 11.42	TIP	3.502 3708	3,179.588	10,431.70 ✓
DENNY, 1936 d.m.	48 50 07.0080		12 32 31.82	192 32 00.74	TIP	3.588 8416	3,880.088	12,729.92 ✓
(Y 682,507.78 ft.)	117 56 55.3012		62 16 55.90	242 14 56.14	LANE	3.564 2116	3,666.161	12,028.06 ✓
(X 2,694,653.89 ft.)								✓
RYAN, 1936 d.m.	48 47 59.5422		95 13 20.24	275 12 19.66	TIP	3.217 4732	1,649.959	5,413.24 ✓
(Y 669,696.99 ft.)	117 56 16.0734		118 53 39.89	298 51 10.64	LANE	3.664 7303	4,620.939	15,160.53 ✓
(X 2,697,762.63 ft.)			168 30 56.79	348 30 27.26	DENNY	3.604 0095	4,017.996	13,182.38 ✓
O'TOOLE, (C. & G.S.), 1925	48 48 32.364		74 45 30.01	254 19 15.51	COPPER BUTTE	4.647 1533	44,376.53	145,592.0 ✓
(Y 673,524.06 ft.) d.m.	117 52 58.201		75 55 29.94	255 53 01.04	RYAN	3.619 4121	4,163.055	13,658.29 ✓
(X 2,710,873.98 ft.)			121 10 38.99	301 07 40.52	DENNY	3.752 1604	5,651.457	18,541.49 ✓
r. 1934, r. 1936			305 58 40.07	126 05 04.42	ROGERS, 1933	4.110 6516	12,901.84	42,328.8 ✓
ANSALDO, 1936 d.m.	48 52 37.9545		08 27 08.74	188 26 21.58	RYAN	3.939 2517	8,694.642	28,525.67 ✓
(Y 698,051.92 ft.)	117 55 13.4487		24 00 45.07	203 59 28.36	DENNY	3.707 9279	5,104.202	16,746.04 ✓
(X 2,700,884.84 ft.)			340 00 34.55	160 02 16.38	O'TOOLE	3.906 9885	8,072.137	26,483.34 ✓
SMITH, 1936 d.m.	48 49 38.7320		39 40 14.19	219 38 40.50	RYAN	3.599 8744	3,979.920	13,057.45 ✓
(Y 680,057.86 ft.)	117 54 11.5761		104 40 35.08	284 38 31.83	DENNY	3.538 0047	3,451.475	11,323.71 ✓
(X 2,705,709.64 ft.)								✓

Note: Y and X are rectangular coordinates on the North Washington System of Plane Coordinates from Lambert Projection Tables by USC&GS, 1935.

FIG. 3.—GEOGRAPHIC POSITION SHEET

example, referring to Table 1, for an observed 500-ft measurement at 32° F, with 200 ft of the tape lying on a flat surface (supported throughout) and 300 ft of catenary, under 15 kg of tension, the correction would be $\frac{2}{5}(-05) + (-15) = -0.17$, or $500.00 - 0.17 = 499.83$ ft as the true distance.

TABLE 1.—CORRECTIONS FOR 500-FOOT TAPE IN HUNDREDTHS OF FEET
(Two Supports; Tension = 15 Kilograms)

Length	CORRECTIONS FOR THE FOLLOWING TEMPERATURES, IN DEGREES CENTIGRADE:										
	−10	−05	00	05	10	15	20	25	30	35	40
Degrees Fahrenheit	14	23	32	41	50	59	68	77	86	95	104
25	01	01	00	00	00	00	00	00	00	00	01
50	01	01	00	00	00	00	01	01	01	02	02
75	02	01	00	00	00	01	01	01	02	02	03
100	02	02	01	01	00	01	01	01	02	03	03
125	02	02	02	01	00	00	01	02	02	03	04
150	04	03	03	02	01	00	01	02	03	04	05
175	06	05	04	03	02	01	00	01	02	03	04
200	08	06	05	04	03	02	01	00	02	03	04
225	10	08	07	06	04	03	02	01	01	02	03
250	12	10	09	07	06	04	03	02	00	01	03
275	15	14	12	10	08	07	05	04	02	00	01
300	19	17	15	13	11	09	08	06	04	02	00
325	21	20	18	16	14	12	10	08	06	04	02
350	26	24	22	20	18	15	13	11	09	07	05
375	31	28	26	24	22	20	18	15	13	11	09
400	36	33	31	29	27	24	22	20	17	15	13
425	43	39	38	36	33	30	28	26	23	20	18
450	49	46	44	41	38	35	33	30	28	25	22
475	57	54	51	48	45	42	40	37	34	32	29
500	65	62	59	56	53	50	47	44	41	38	35
500*	−11	−08	−05	−02	+01	+04	+07	+10	+13	+16	+19

* Tape supported throughout.
Note.—Values above the heavy line are plus; those below are minus.

Transit angles were measured by the repetition method, the observer recording one set of $\frac{3D}{R}$, and closing the horizon.

Variations in the method of third-order co-ordination included the following: (*a*) Triangle closure at the control station, in cases where cuts could be obtained only from two second-order stations; and (*b*) observation of one line of a three-point-fix reciprocally and holding the direction of that line as fixed, where it was impossible to obtain a cut to a fourth object for a check. Such variations, although seldom used, made possible consideration of the "fix" as a "check-position."

Third-order triangulation and traverse control extended over 406 sq miles, and a total of 1 934 points were co-ordinated by surveys of this accuracy. Including the positions of the 384 monumented stations occupied, a total of 4 051 control points and cadastral corners were co-ordinated over the project.

RELATED SURVEYS

The presentation, in this paper, of material pertaining to related surveys performed simultaneously with control and co-ordination will necessarily be limited to the discussion of the economies in time and effort resulting from the easy access to local control.

Topography.—Topographic mapping of parts of the draw-down area through town sites and across tracts of relatively high valuations made full use of adjacent control, both horizontal and vertical. Plane-table sheets were prepared in advance with the grid co-ordinate system, flow-line traverse, and such natural objects as might be used for resection cuts, plotted on them. Control proved of inestimable value in mapping these isolated areas.

Photogrammetry.—A stereotopographic map extending throughout the length of the reservoir site had been prepared jointly by the U. S. Army Engineers and the U. S. Geological Survey in 1930. A limited degree of control for the stereoplotting had been provided by the latter organization by plane-table triangulation, and although a faithful representation had been made of the terrain as to the proper shapes and sinuosities of the contours, the displacement toward or away from the river of the higher contours, due to lack of control away from the river banks, left in doubt the actual amount of ownership invasion of the 1 310 contour. The terrain below 1 100 was often the extent of topographic delineation on the maps, but in general the series provided a valuable aid to the Bureau's triangulation reconnaissance.

Early in 1934 a "flight" of single-lens aerial photographs was made for the project by a private contractor. This flight extended 75 miles up stream from the dam, embracing the area of the so-called low-dam backwater. A unit of the triangulation party was detailed to signal points on the ground to appear in the aerial photos, and to perform sufficient picture-point identification work and ground control to assemble the contact prints adequately into a controlled mosaic. Control points were selected on the prints and co-ordinated by substantially the same methods as were used in co-ordinating cadastral corners, and the section corners were identified wherever possible and pin pricked by the retracement engineers. In a few instances radial plots were made from the photos to enlarge upon some feature of a special right-of-way problem. After identification had been marked on the photographs, they were handed to the right-of-way appraisal crew, and greatly aided this feature of the acquisition work. Later they were utilized in the development of the new highway and rural road system which eventually replaced the existing road system of the area to be flooded.

Hydrography.—Control surveys, both horizontal and vertical, found widespread use in the field surveys to determine backwater effect of the reservoir pool. Sounding ranges were co-ordinated quickly throughout the backwater reaches, and profiles and soundings placed on the project datum from adjacent circuits of precise levels.

Miscellaneous.—Special field surveys called for by the Right-of-way Division embraced those made necessary by inaccurate or obscure descriptions found in previous conveyances. Often large discrepancies involving excesses or deficiencies, or perhaps overlapping tracts, would necessitate a retracement of State highway, rural road, and railroad rights of way, and would require that these tracts be related to the adjacent properties. More often, an obscure point of beginning, an unclosed description, or questionable orientation, would present the troublesome feature of a conveyance. Once carefully surveyed and related to the governing system of rectangular co-ordinates, a perpetuation of boundaries would result, and would permit, at any future period, the exact re-establishment of boundary markers subsequently lost or destroyed.

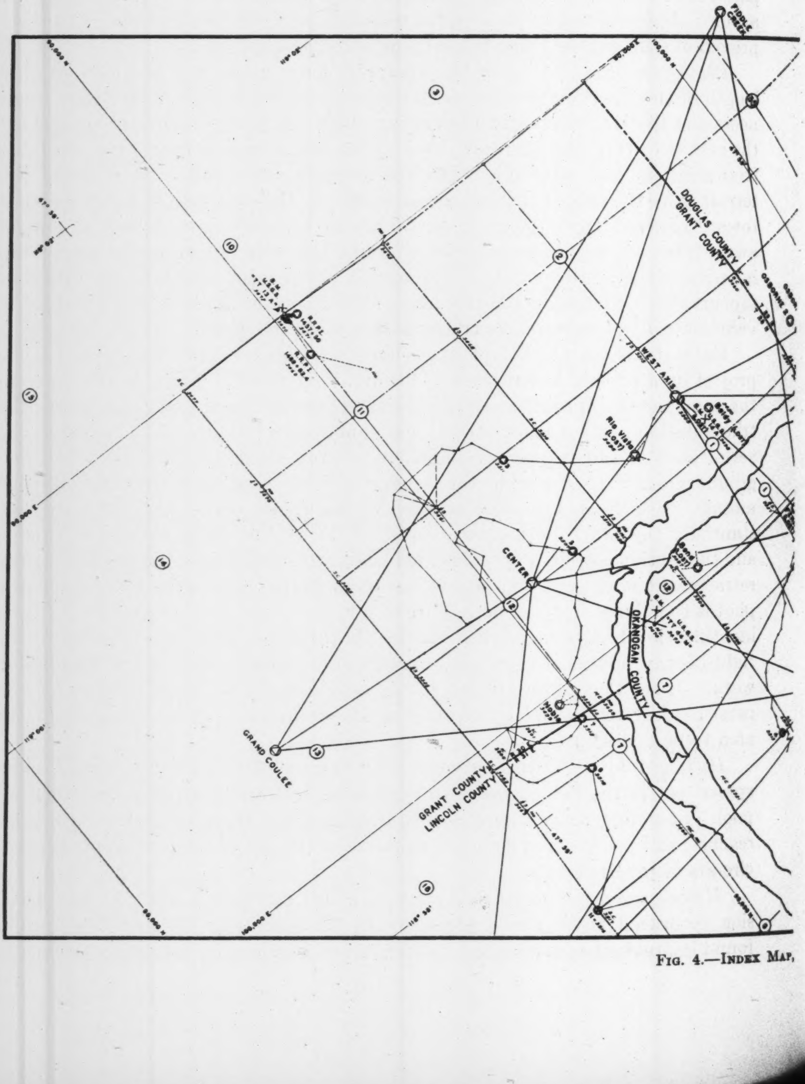
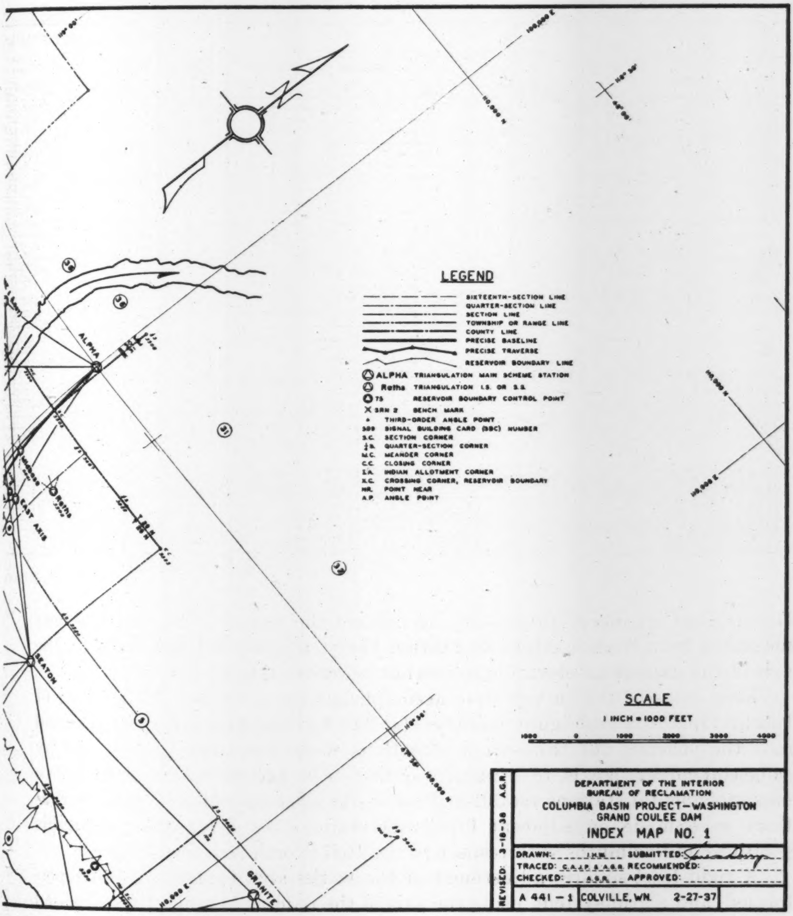

FIDDLE CREEK
90,000 N
100,000 N
DOUGLAS COUNTY
GRANT COUNTY
OSBORNE 2
WEST AXIS
Rio Vista (LOST)
Bailey (LOST)
Bond (LOST)
CENTER
OKANOGAN COUNTY
GRAND COULEE
GRANT COUNTY
LINCOLN COUNTY
90,000 E
100,000 E
119° 00'
119° 30'
47° 30'

FIG. 4.—INDEX MAP,

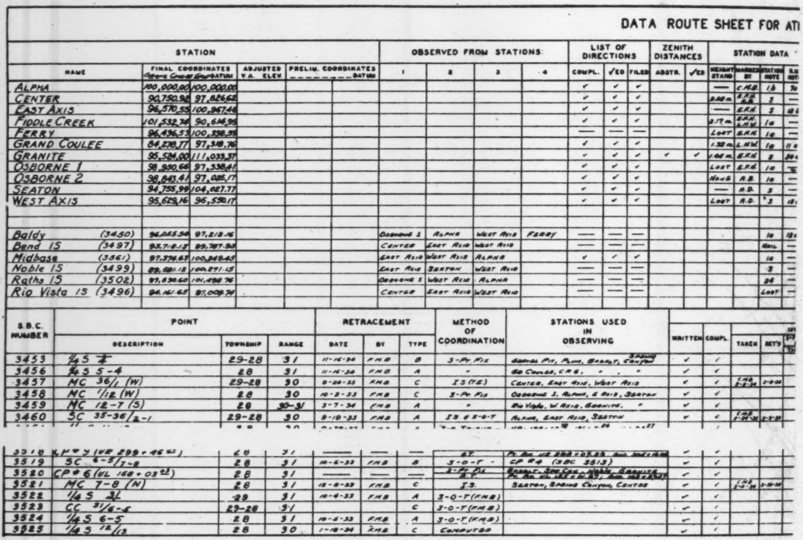
One interesting problem arose in the requirement for replacing long-span telephone crossings. The length of span and elevations of supporting towers were determined quickly by a three-point-fix theodolite observation, performing at the same time a double-zenith-distance set of observations. Elevations obtained by this method were reliable to within one foot of the true elevation.

Of no small degree of importance on the project was the exacting requirement, at the Grand Coulee dam site, for precise construction control. To provide adequate control for the building of the world's largest structure, the standard of accuracy was set at that of first-order triangulation and precise


LEGEND
SIXTEENTH-SECTION LINE
QUARTER-SECTION LINE
SECTION LINE
TOWNSHIP OR RANGE LINE
COUNTY LINE
PRECISE BASELINE
PRECISE TRAVERSE
RESERVOIR BOUNDARY LINE
ALPHA TRIANGULATION MAIN SCHEME STATION
Reths TRIANGULATION I.S. OR S.S.
75 RESERVOIR BOUNDARY CONTROL POINT
X SRN 2 BENCH MARK
THIRD-ORDER ANGLE POINT
389 SIGNAL BUILDING CARD (SBC) NUMBER
S.C. SECTION CORNER
¼S. QUARTER-SECTION CORNER
M.C. MEANDER CORNER
C.C. CLOSING CORNER
I.A. INDIAN ALLOTMENT CORNER
X.C. CROSSING CORNER, RESERVOIR BOUNDARY
NR. POINT NEAR
A.P. ANGLE POINT
SCALE
1 INCH = 1000 FEET
1000
0
1000
2000
3000
4000
ALPHA
EAST AXIS
Reths
SEATON
GRANITE
100,000 E
110,000 N
110,000 E
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION
COLUMBIA BASIN PROJECT—WASHINGTON
GRAND COULEE DAM
INDEX MAP NO. 1
DRAWN: I.H.M.
TRACED: C.A.J.P. & A.G.R.
CHECKED: A.G.R.
SUBMITTED:
RECOMMENDED:
APPROVED:
REVISED: 3-16-38 A.G.R.
A 441 — 1
COLVILLE, WN.
2-27-37

Vicinity of Grand Coulee Dam

leveling. Location of the axis of the dam, tentatively fixed by the U. S. Army Engineers some years before and marked by them with two iron pipes, was adopted subsequently by the Bureau of Reclamation as a result of its exploration studies. Projected outside the works area with the precise theodolite, the axis was marked permanently and referenced by grouting standard bronze disk markers in granite outcroppings, at points commanding a full view of the construction area. The stations thus marked were designated "east axis" and "west axis," and formed the south side of the first quadrilateral figure (see Fig. 4). Station Alpha, already set on the east bank near the center of the contractor's future town site, and Station Osborne, on the west bank in the Government engineers' future city, completed the figure. The base line was measured from Station Alpha to Station Ferry, this latter point lying on the axis of the dam at an elevation somewhat below Station East Axis.



DATA ROUTE SHEET FOR AT

STATION: NAME	FINAL COORDINATES Grand Coulee Grid DATUM		ADJUSTED V.A. ELEV.	PRELIM. COORDINATES ________ DATUM		OBSERVED FROM STATIONS: 1	2	3	4	LIST OF DIRECTIONS: COMPL.	✓ED	FILED	ZENITH DISTANCES: ABSTR.	✓ED	STATION DATA: HEIGHT STAND	MARKED BY	STATION NOTE	
ALPHA	100,000.00	100,000.00								✓	✓	✓			—	C.M.B.	16	7
CENTER	90,750.98	97,826.62								✓	✓	✓			2.00 m	E.R.H. A.B.	3	—
EAST AXIS	96,570.55	100,967.46								✓	✓	✓			—	E.R.H.	3	18
FIDDLE CREEK	101,532.74	90,614.95								✓	✓	✓			2.17 m	E.R.H. L.M.W.	10	—
FERRY	96,436.53	100,338.55								—	—	—			LOST	E.R.H.	10	—
GRAND COULEE	84,278.77	97,318.76								✓	✓	✓			1.32 m	L.M.W.	10	11
GRANITE	95,524.00	111,033.37								✓	✓	✓	✓	✓	1.45 m	E.R.H.	3	20
OSBORNE 1	98,950.66	97,338.41								✓	✓	✓			LOST	E.R.H.	10	—
OSBORNE 2	98,843.41	97,085.17								✓	✓	✓			NONE	A.B.	10	—
SEATON	94,755.99	104,027.77								✓	✓	✓			—	A.D.	3	—
WEST AXIS	95,629.16	96,550.17								✓	✓	✓			LOST	A.D.	3	18
Baldy (3450)	96,065.54	97,212.16				OSBORNE 1	ALPHA	WEST AXIS	FERRY	—	—	—					10	18
Bend IS (3497)	93,710.15	99,787.93				CENTER	EAST AXIS	WEST AXIS		—	—	—					NAIL	—
Midbase (3561)	97,374.63	100,949.45				EAST AXIS	WEST AXIS	ALPHA		✓	✓	✓					10	—
Noble IS (3499)	89,591.18	100,271.15				EAST AXIS	SEATON	WEST AXIS		—	—	—					3	—
Raths IS (3502)	97,630.60	101,498.76				OSBORNE 1	WEST AXIS	ALPHA		—	—	—					26	—
Rio Vista IS (3496)	94,161.65	97,009.74				CENTER	EAST AXIS	WEST AXIS		—	—	—					LOST	—

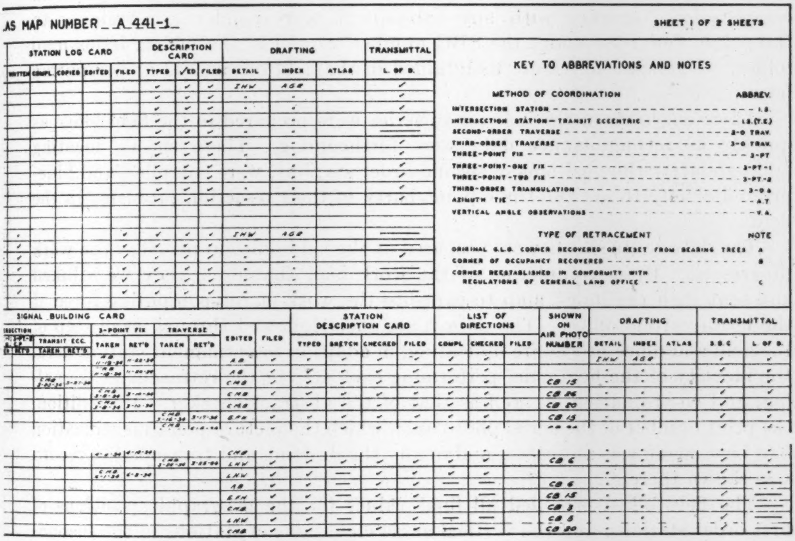
S.B.C. NUMBER	POINT: DESCRIPTION	TOWNSHIP	RANGE	RETRACEMENT: DATE	BY	TYPE	METHOD OF COORDINATION	STATIONS USED IN OBSERVING	WRITTEN	COMPL.	TAKEN	RET'D
3453	¼ S 7	29-28	31	11-16-34	FMB	B	3-PT. FIX	GRAVEL PIT, PLUME, BASALT, SPRING CANYON	✓	✓		
3456	¼ S 5-4	28	31	11-16-34	FMB	A	"	GD COULEE, CMG, " , "	✓	✓		
3457	MC 36/1 (W)	29-28	30	9-20-33	FMB	C	I.S. (T.E.)	CENTER, EAST AXIS, WEST AXIS	✓	✓	CMB 3-2-34	3-8-34
3458	MC 1/12 (W)	28	30	10-2-33	FMB	C	3-PT. FIX	OSBORNE 1, ALPHA, E AXIS, SEATON	✓	✓		
3459	MC 12-7 (S)	28	30-31	3-7-34	FMB	A	"	RIO VISTA, W AXIS, GRANITE, "	✓	✓		
3460	SC 35-36/2-1	29-28	30	9-18-33	FMB	A	I.S. & 3-0-T	ALPHA, EAST AXIS, SEATON	✓	✓	CMB 2-24-34	3-9-34
3518	[illegible]	28	31	—	—	—	S.T.	PT. ON UE 299+[illegible]	✓	✓		
3519	SC 6-5/7-8	28	31	10-6-33	FMB	B	3-O-T	CP #4 (SBC 3513)	✓	✓		
3520	CP #6 (UL 168+03)	28	31	—	—	—	3-PT. FIX S.T.	BASALT, SPR. CAN., NOBLE, GRANITE PT. ON UL [illegible]	✓	✓		
3521	MC 7-8 (N)	28	31	10-8-33	FMB	C	I.S.	SEATON, SPRING CANYON, CENTER	✓	✓	CMB 3-2-34	3-10-34
3522	¼ S 31	29	31	10-4-33	FMB	A	3-O-T (FMB)		✓	✓		
3523	CC 31/6-5	29-28	31			C	3-O-T (FMB)		✓	✓		
3524	¼ S 6-5	28	31	10-5-33	FMB	A	3-O-T (FMB)		✓	✓		
3525	¼ S 12/13	28	30	1-18-34	FMB	C	COMPUTED		✓	✓		

FIG. 5.—DATA

Four steps of the survey were accomplished in occupying the stations of this first quadrilateral figure: namely—(*a*) The 1 100-m base line was expanded into the scheme, (*b*) third-order directions were measured to the near-by cadastral corner signals, (*c*) directions of first-order accuracy were measured to 40 supplemental stations established for works area control, and (*d*) observations were made on adjacent first-order stations for fixing the geographic positions and azimuths with respect to the 1927 North American Datum.

A rigid least-squares adjustment of the works area control was next performed, and a grid system, using the axis of the dam as meridian, was devised for use over the area. Supplemental grid lines, parallel and normal to the axis, were established on the near-by hills at intervals varying from 400 to 600 ft, the taping being with invar tapes equipped with spring balances and thermometers. Resulting lengths were adjusted to values furnished for each tape by the U. S. Bureau of Standards. By "wiggling-in" in two directions, any of the intersecting lines might readily be reproduced as the dam construction progressed, and thus afford an easy means of holding form-setters' chaining errors to a minimum.

In the first-order leveling, two reciprocal river crossings were observed, and 16 permanent bench-marks were established throughout the works area for vertical control.



AS MAP NUMBER A 441-1

SHEET 1 OF 2 SHEETS

STATION LOG CARD					DESCRIPTION CARD			DRAFTING			TRANSMITTAL
WRITTEN	COMPL.	COPIED	EDITED	FILED	TYPED	✓ED	FILED	DETAIL	INDEX	ATLAS	L. OF D.
					✓	✓	✓	IHW	AGB		✓
					✓	✓	✓	•	•		✓
					✓	✓	✓	•	•		✓
					✓	✓	✓	•	•		✓
					✓	✓	✓	•	•		—
					✓	✓	✓	•	•		✓
					✓	✓	✓	•	•		✓
					✓	✓	✓	•	•		✓
					✓	✓	✓	•	•		✓
					✓	✓	✓	•	•		✓
					✓	✓	✓	•	•		✓
•				✓	✓	✓	✓	IHW	AGB		—
✓				✓	✓	✓	✓	•	•		—
✓				✓	✓	✓	✓	•	•		✓
✓				✓	✓	✓	✓	•	•		—
✓				✓	✓	✓	✓	•	•		—
✓				✓	✓	✓	✓	•	•		—

KEY TO ABBREVIATIONS AND NOTES

METHOD OF COORDINATION	ABBREV.
INTERSECTION STATION	I.S.
INTERSECTION STATION—TRANSIT ECCENTRIC	I.S.(T.E.)
SECOND-ORDER TRAVERSE	2-O TRAV.
THIRD-ORDER TRAVERSE	3-O TRAV.
THREE-POINT FIX	3-PT
THREE-POINT-ONE FIX	3-PT-1
THREE-POINT-TWO FIX	3-PT-2
THIRD-ORDER TRIANGULATION	3-O △
AZIMUTH TIE	A.T.
VERTICAL ANGLE OBSERVATIONS	V.A.

TYPE OF RETRACEMENT	NOTE
ORIGINAL G.L.O. CORNER RECOVERED OR RESET FROM BEARING TREES	A
CORNER OF OCCUPANCY RECOVERED	B
CORNER REESTABLISHED IN CONFORMITY WITH REGULATIONS OF GENERAL LAND OFFICE	C

SIGNAL BUILDING CARD										STATION DESCRIPTION CARD				LIST OF DIRECTIONS			SHOWN ON AIR PHOTO NUMBER	DRAFTING			TRANSMITTAL	
SECTION				3-POINT FIX		TRAVERSE		EDITED	FILED													
I.S. 3-PT-1 S.C.P		TRANSIT ECC.																				
	RET'D	TAKEN	RET'D	TAKEN	RET'D	TAKEN	RET'D			TYPED	SKETCH	CHECKED	FILED	COMPL	CHECKED	FILED		DETAIL	INDEX	ATLAS	S.B.C	L. OF D.
				AB 11-19-34	11-22-34			AB	✓	✓	✓	✓	✓	✓	✓			IHW	AGB		✓	✓
				AB 11-18-34	11-20-34			AB	✓	✓	✓	✓	✓	✓	✓			•	•		✓	✓
		CMB 3-23-34	3-27-34					CMB	✓	✓	✓	✓	✓	✓	✓		CB 15	•	•		✓	✓
				CMB 3-8-34	3-10-34			CMB	✓	✓	✓	✓	✓	✓	✓		CB 26	•	•		✓	✓
				CMB 3-8-34	3-10-34			CMB	✓	✓	✓	✓	✓	✓	✓		CB 20	•	•		✓	✓
						CMB 3-16-34	3-17-34	EFH	✓	✓	✓	✓	✓	✓	✓		CB 15	•	•		✓	✓
						CMB	6-6-34	[illegible]	✓	✓	✓	✓	✓	✓	✓		[illegible]	•	•		✓	✓
				4-6-34	4-10-34			CMB	•	•					•			•	•		•	•
						CMB 3-20-34	3-25-34	LHW	✓	✓	✓	✓	✓	✓	✓		CB 6	•	•		✓	✓
				CMB 6-1-34	6-8-34			LHW	✓	✓	—	✓	✓	✓	✓			•	•		✓	•
								AB	✓	✓	—	✓	✓	—	—		CB 6	•	•		✓	—
								EFH	✓	✓	—	✓	✓	—	—		CB 15	•	•		✓	—
								CMB	✓	✓	—	✓	✓	—	—		CB 3	•	•		✓	—
								LHW	✓	✓	—	✓	✓	—	—		CB 5	•	•		✓	—
								CMB	✓	✓	—	✓	✓	—	—		CB 20	•	•		✓	—

ROUTE SHEET

OFFICE

In 1934, during the first few months of control survey activity, headquarters for both the field crews and the computing detail were maintained at Almira, Wash., the town nearest the dam site. This period of intimate contact between the two units of the organization was utilized largely in initiating the procedure to be followed in the steps of converting field data to their final form of conveyancing plat. Later the two units were separated, the field office being moved successively to Davenport, Colville, and Coulee City, Wash.; and the computing detail was stationed permanently at the dam site (Coulee Dam, Wash.). To facilitate the transmission of field data an expeditious system of data route sheets and index maps was devised (see Figs. 4 and 5).

Index maps were oriented to accommodate a maximum of area, with a minimum of overlap, without regard to cardinal orientation; 37 such maps were required over the reservoir area, and were numbered successively up stream from the dam as A441-1, -2, etc. to -28 over the Columbia River area, A441-SP1 to -SP2 over the San Poil River, A441-SK1 to -SK5 over the Spokane River, and A441-KT1 to -KT2 over the Kettle River area.

Each cadastral corner or control point co-ordinated bore a serial number, first assigned to it by the signalman, and referred to as the Signal Building Card, or "SBC" number, of the point. For indexing, the "SBC" number was assigned to the lowest numbered index map and data route sheet of corresponding number, on which it first appeared. Thus, all data pertaining to each individually co-ordinated point, including its retracement record, signaling record, co-ordination list of directions, and its intersection or three-point-fix computation, together with any subsequent correspondence pertaining to the point, were filed under the SBC number of each in the proper index map folder. Duplicate files were maintained in the field offices and at computing headquarters.

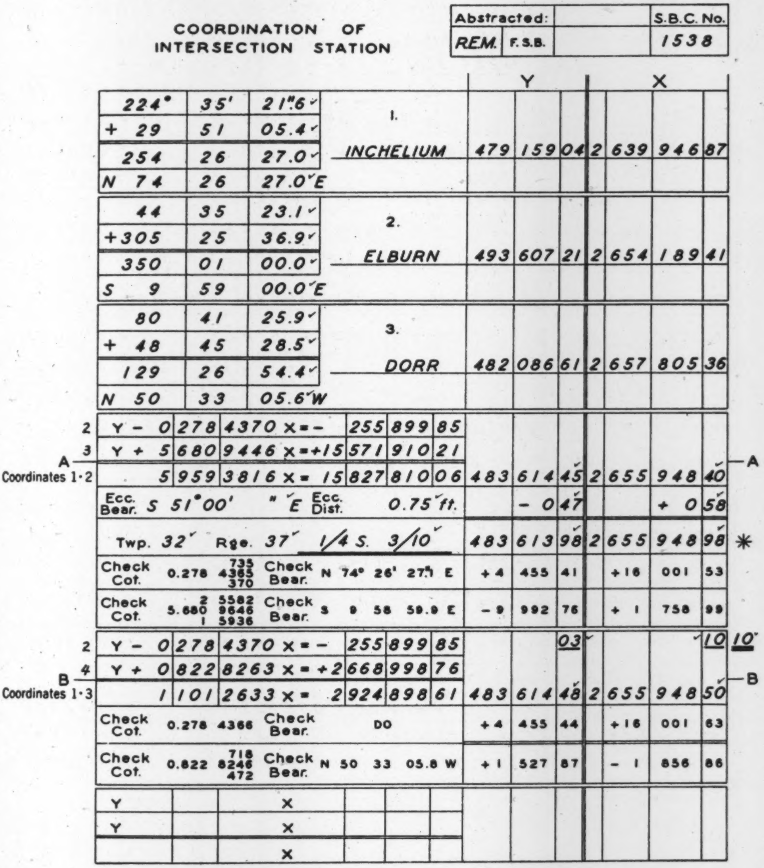
Advance sheets of the index map series were prepared in the field office as quickly as reconnaissance data were forthcoming. These sheets, locating approximately the positions of second-order stations, were compiled and blueprinted chiefly to aid the three-point party in their resection problems in the field.

The data route sheets were prepared at the field office in pencil as the work progressed. Upon completion of the work over that area, they were inked and sent with the index map to expedite the work of the computing force at the headquarters office. The data route sheet followed through each step of the field and office procedure indicating, in addition to permanent station data, the identity of the field chief performing each step of the retracement, signaling, and co-ordination, as well as that of the abstract checker. In addition, the print number of the aerial photo upon which the picture-point identification had been made was indicated, and finally the drafting and transmission record was shown thereon.

The field office provided all final values for the geographic positions of first-order stations, and the vertical angle elevations of stations in the second-order nets of triangulation. Least-squares adjustments for this work were made generally during inclement weather by members of the field parties skilled in higher mathematics. Station description cards were typed in the field office for all monumented stations and co-ordinated points, and photostatic copies of these cards were prepared for transmission to other Governmental agencies. At headquarters office, field data were resolved to final co-ordinate values, and the individual parcel right-of-way plats were prepared for the Legal Division. At this office all second-order nets were adjusted by the direction method of least squares.

Two forms were devised by Bureau computers which deserve attention, one for the intersection station solution (see Fig. 6), and the other for solution of the resection, or three-point-fix problem (see Fig. 7). Each makes provision for carrying the computation through traverse courses beyond the "fix" point to the corner sought. Instructions for the use of these forms are to be found in Appendices I and II. The mathematical formulas used in the forms are not new but the arrangement of terms is original and is believed to offer the utmost in speed where a calculating machine is available.

Intersection Computation Form.—At the headquarters office the data for coordinating most of the thousands of land monuments were received from the field offices in the form of "Lists of Directions," which were abstracted for each triangulation station of second-order accuracy and contained, in clockwise order, the names of from four to seven second-order triangulation stations and of as many as a hundred or more cadastral points (third-order stations). Opposite each name appeared the angle which the direction to that signal had been observed to make with the direction to the initial or first point on the list. The initial point was always one of the second-order stations.

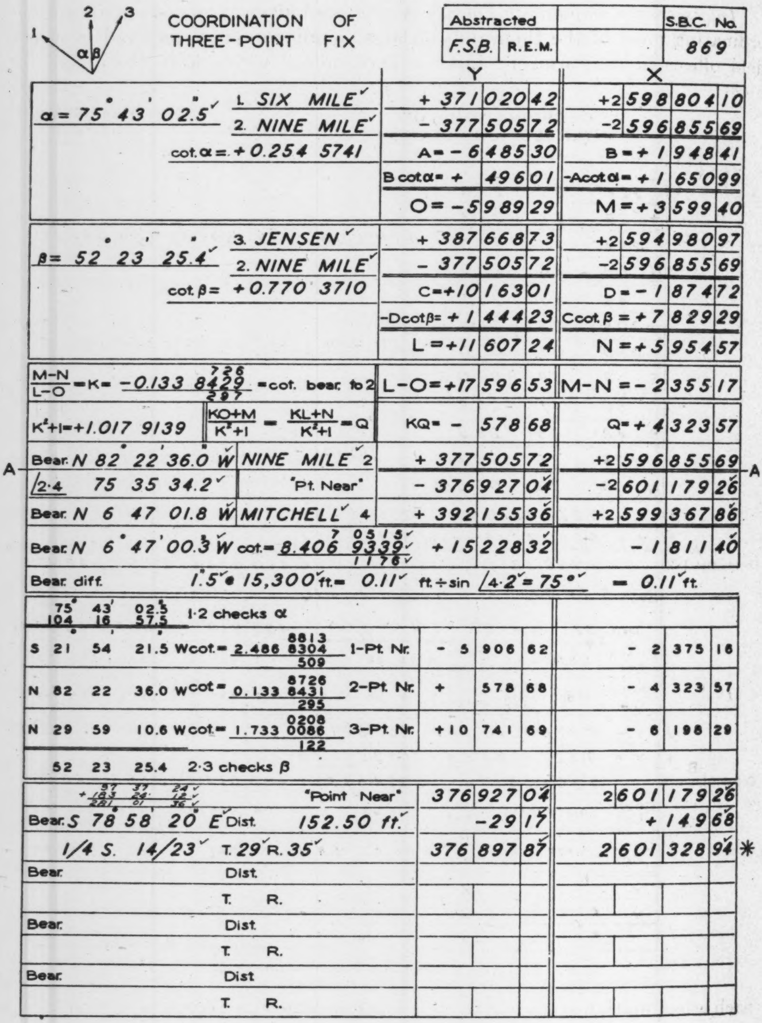


COORDINATION OF INTERSECTION STATION

Abstracted:			S.B.C. No.
REM.	F.S.B.		1538

							Y			X				
	224°	35'	21".6✓	1.										
	+ 29	51	05.4✓											
	254	26	27.0✓	INCHELIUM			479	159	04	2	639	946	87	
	N 74	26	27.0✓E											
	44	35	23.1✓	2.										
	+305	25	36.9✓											
	350	01	00.0✓	ELBURN			493	607	21	2	654	189	41	
	S 9	59	00.0✓E											
	80	41	25.9✓	3.										
	+ 48	45	28.5✓											
	129	26	54.4✓	DORR			482	086	61	2	657	805	36	
	N 50	33	05.6✓W											
2	Y − 0	278	4370 x = −	255	899	85								
3	Y + 5	680	9446 x = +15	571	910	21								
A Coordinates 1·2	5	959	3816 x = 15	827	810	06	483	614	45✓	2	655	948	40✓	A
	Ecc. Bear. S 51°00' "✓E		Ecc. Dist. 0.75✓ft.					− 0	47✓			+ 0	58✓	
	Twp. 32✓ Rge. 37✓ 1/4 S. 3/10✓						483	613	98✓	2	655	948	98✓	✳
	Check Cot.	0.278	735 4365 370	Check Bear.	N 74° 26' 27".1 E		+ 4	455	41		+ 16	001	53	
	Check Cot.	2 5.680 1	5582 9646 5936	Check Bear.	S 9 58 59.9 E		− 9	992	76		+ 1	758	99	
2	Y − 0	278	4370 x = −	255	899	85			03✓			✓	10	10"
4	Y + 0	822	8263 x = +2	668	998	76								
B Coordinates 1·3	1	101	2633 x = 2	924	898	61	483	614	48✓	2	655	948	50✓	B
	Check Cot.	0.278	4366	Check Bear.	DO		+ 4	455	44		+ 16	001	63	
	Check Cot.	0.822	718 8246 472	Check Bear.	N 50 33 05.8 W		+ 1	527	87		− 1	856	86	
	Y		X											
	Y		X											
			X											

FIG. 6.—INTERSECTION COMPUTATION FORM



COORDINATION OF THREE-POINT FIX

Abstracted			S.B.C. No.
F.S.B.	R.E.M.		869

		Y			X			
α = 75° 43′ 02.5″✓	1. SIX MILE✓	+ 371	020	42	+2	598	804	10
	2. NINE MILE✓	− 377	505	72	−2	596	855	69
	cot. α = + 0.254 5741	A = − 6	485	30	B = + 1		948	41
		B cot α = +	496	01	−A cot α = + 1		650	99
		O = − 5	989	29	M = + 3		599	40
β = 52° 23′ 25.4″✓	3. JENSEN✓	+ 387	668	73	+2	594	980	97
	2. NINE MILE✓	− 377	505	72	−2	596	855	69
	cot. β = + 0.770 3710	C = + 10	163	01	D = − 1		874	72
		−D cot β = + 1	444	23	C cot β = + 7		829	29
		L = + 11	607	24	N = + 5		954	57
$\frac{M-N}{L-O}$ = K = $-0.133\ \overset{726}{\underset{297}{8429}}$ = cot. bear. to 2		L − O = + 17	596	53	M − N = − 2		355	17
K² + 1 = + 1.017 9139	$\frac{KO+M}{K^2+1} = \frac{KL+N}{K^2+1} = Q$	KQ = −	578	68	Q = + 4		323	57
A— Bear. N 82° 22′ 36.0″ W✓	NINE MILE✓ 2	+ 377	505	72	+2	596	855	69 —A
∠2·4 75 35 34.2✓	"Pt. Near"	− 376	927	04✓	−2	601	179	26✓
Bear. N 6 47 01.8 W✓	MITCHELL✓ 4	+ 392	155	36✓	+2	599	367	86✓
Bear. N 6° 47′ 00.3″ W✓ cot. = $8.406\ \overset{7\ 0515}{\underset{1176}{9339}}$✓		+ 15	228	32✓	− 1		811	40✓
Bear. diff.	1″.5✓ • 15,300✓ ft. = 0.11✓ ft. ÷ sin ∠4·2✓ = 75°✓ = 0.11✓ ft.							

75° 43′ 02.5″ 104 16 57.5 1·2 checks α								
S 21° 54′ 21.5″ W cot. = $2.486\ \overset{8813}{\underset{509}{8304}}$	1-Pt. Nr.	− 5	906	62	− 2		375	16
N 82 22 36.0 W cot = $0.133\ \overset{8726}{\underset{295}{8431}}$	2-Pt. Nr.	+	578	68	− 4		323	57
N 29 59 10.6 W cot. = $1.733\ \overset{0208}{\underset{122}{0086}}$	3-Pt. Nr.	+ 10	741	69	− 6		198	29
52 23 25.4 2·3 checks β								

57 37 24✓ +183 24 12✓ 281 01 36✓	"Point Near"	376	927	04✓	2	601	179	26✓
Bear. S 78° 58′ 20″ E✓	Dist. 152.50 ft.✓		−29	17✓			+ 149	68✓
1/4 S. 14/23✓ T. 29✓ R. 35✓		376	897	87✓	2	601	328	94✓ *
Bear.	Dist.							
	T. R.							
Bear.	Dist.							
	T. R.							
Bear.	Dist							
	T. R.							

FIG. 7.—THREE-POINT-FIX COMPUTATION FORM

After the office had completed least-squares adjustment work on the second-order net, each list of directions received two items which served to fix its position and orientation—namely, the Y and X co-ordinates of the occupied station and the grid azimuth of the initial direction. The adjustments of the second-order observations resulted in slight changes in the observed directions to all the second-order stations on a list, including the direction to the initial station. Thus, the initial station would have an adjusted direction slightly above or below its observed direction of 0° 00′ 00″.00; but there remained, as the point to which all unadjusted third-order observations on the list were referred, a direction of 0° 00′ 00″.00 which was no longer the direction to any real object. However, there was no confusion in this, as the fixed grid azimuth of the residual 0° 00′ 00″.00 direction was very easily computed and recorded at the head of the list. The grid azimuth of the "cut" from the occupied station to any signal is thus the sum of the grid azimuth of the initial direction and the direction to the third-order signal.

On the North Washington System of Plane Coordinates, a Lambert conformal conic projection, every cut may be represented by an equation of the form:

$$y - m x = b \quad \text{(1)}$$

The plane co-ordinates of a third-order signal may be determined readily by a simultaneous solution of two equations which represent cuts from two second-order stations. Adopting this widely used principle of analytic geometry, the computation form devised for use on this project reduces the work of co-ordinating intersection stations to a routine requiring approximately 15 min of a computer's time for each point.

Each third-order signal was observed from three second-order stations. Only rarely were the three cuts exactly concurrent; usually they formed a small triangle of error. No attempt was made to determine the most probable position within this triangle. The co-ordinates of the point of intersection of the two cuts which intersected most nearly at right angles were adopted arbitrarily as final. Third-order observations were rarely in error by more than 3″, and the cut from second-order station to third-order signal seldom exceeded 3 miles in length. Under such circumstances it appeared that the foregoing intersection point was usually within 0.15 ft of the point that would have been obtained by a least-squares adjustment of the three observations. In the computing procedure the co-ordinates of the points of intersection of the strongest and second strongest pairs of the three cuts are determined. The displacement between these two pairs of co-ordinates is also the length of the shortest side of the triangle of error. It is a valued indicator of the accuracy of the adopted co-ordinates and as such is conservative because it is probably somewhat greater than the actual error. The co-ordinates of the third and weaker vertex of the triangle of error are not computed. The form (Fig. 6) is equally adapted to use in resection problems and variations where only one or two cuts from second-order stations are available but additional observations have been made from the third-order signal to the second-order stations.

UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

ADJUSTMENT STATISTICS OF TRIANGULATION NETS

COLUMBIA RIVER RESERVOIR --- COLUMBIA BASIN PROJECT

	FIRST-ORDER	SECOND-ORDER									
	Dam to Border	Breakdown figure C&GS to Dam	I	San Poil	II	Spokane	III	IV	V	Kettle	VI & VII
Total number of triangles	21	19	26	37	52	66	46	32	56	23	74
Number of triangles with plus closures	9	10	9	14	28	34	27	9	16	7	32
Number of triangles with minus closures	5	9	17	23	20	32	19	23	40	16	42
Number of concluded triangles	7	0	0	0	4	0	0	0	0	0	0
Average closure of triangles without regard to sign	1".32	2".435	1".572	2".11	2".42	1".64	2".38	2".05	2".10	3".38	1".97
Maximum closure of a triangle	3".08	7".02	7".58	6".65	6".61	4".45	7".40	5".27	6".09	6".95	7".64
Probable error of an observed direction	± 0".63	± 1".022	± 0".695	± 1".32	± 0".91	± 0".96	± 1".01	± 0".92	± 1".05	± 1".30	± 1".10
Mean error of an angle	± 0".70	± 1".113	± 1".250	± 1".14	± 1".09	± 1".147	± 1".727	± 1".448	± 1".503	± 2".202	± 1".443
Closure in length between bases after: (side and angle equations)	1/326,000	1/67,000	1/32,000		1/15,900				1/15,000		
or (side, angle, and azimuth equations)										1/97,800	
or (side, angle, azimuth, and latitude and longitude equations) have been satisfied.				1/59,400		1/65,000	1/26,000	1/22,000			1/800,000 (VI) 1/185,000 (VII)

FIG. 8.—TRIANGULATION STATISTICS

Three-Point-Fix Computation Form.—The form developed on this project for the solution of the resection, or three-point problem,[4] suggests several mechanical short cuts to the computer familiar with calculating machines. The example given in Fig. 7 required 20 min for the first computer and 15 min for the checker. Among the advantages of this form are: (1) One procedure takes care of all variations of the three-point problem; (2) necessity is eliminated for preliminary inverse computations between the fixed stations; (3) all computations are performed on one sheet; there is no need for scratch-paper computations; (4) warning is furnished if a fix close to the "great circle" is being computed; the quantities $L-O$ and $M-N$ both become very small and contain less significant figures than are required for K; (5) in its elimination of inverse computations, triangle computations, and all trigonometric functions except the cotangent, this form (Fig. 7) has many features in common with the Bureau of Reclamation form, Fig. 6 (as developed on the Columbia Basin Project) for computing intersection and resection stations, etc., in which an unknown point is quickly co-ordinated by simultaneously solving two or more equations of the form of Equation (1), every direction or "cut" on the plane co-ordinate system being exactly represented by such an equation. These similarities between the latter form and the three-point form contribute mutually to the development of speed in the work of an inexperienced computer who must become accustomed to all types of co-ordination problems. A discussion of the "fix strength index," as applied to this form (Fig. 7), is shown in Appendix II.

In one of the completed second-order nets, the third-order work disclosed a maximum side closure of 0.55 ft and an average of 0.21 ft. If one assumes the average length of sight to be 8 000 ft, the average closure of a three-point-fix or of a third-order intersection station is one part in 40 000, and the poorest closure one part in 15 000. There is no question that these accuracies were consistently attained. Adjustment statistics of the first-order and second-order nets are shown in Fig. 8.

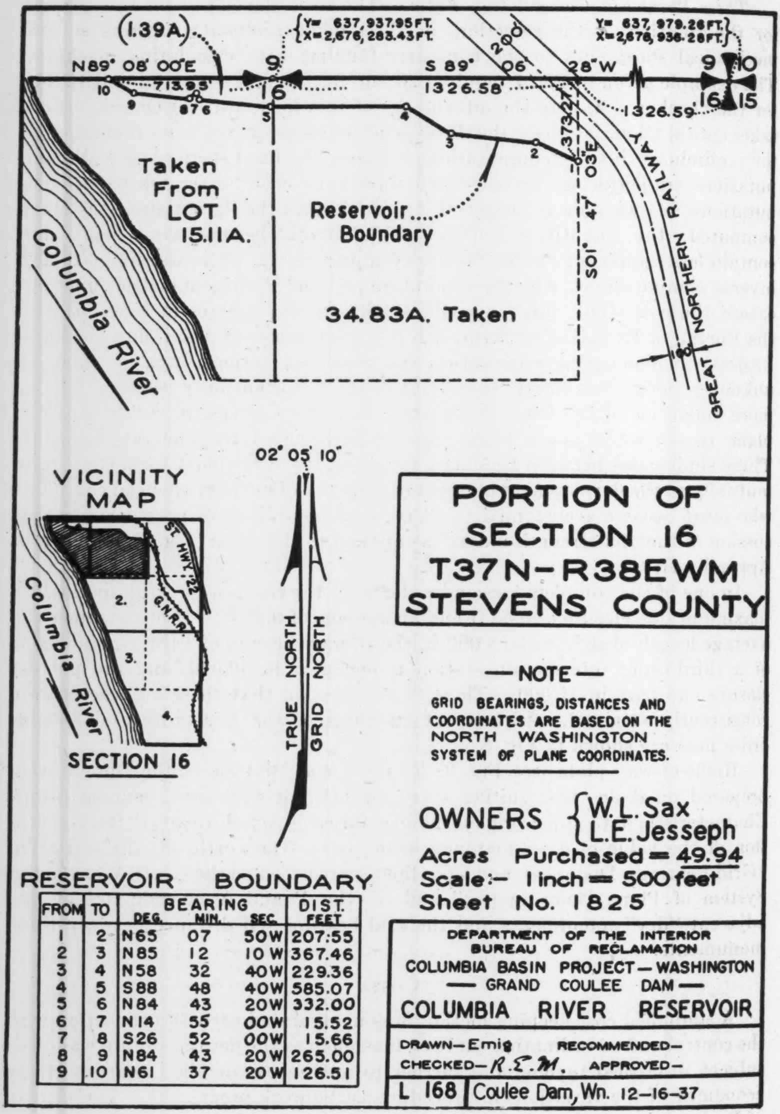
Right-of-way plats (see Fig. 9) for the Legal Division of the Bureau were prepared on scales best suiting them for binding with conveyancing forms. Characteristic data appearing on the plat for each parcel were: (*a*) Representation of the value of θ, or mapping angle, from true north, (*b*) the recitation "Grid Bearings, Distances, and Coordinates are based on the North Washington System of Plane Co-ordinates," and (*c*) the Y and X co-ordinates of two adjacent "dry" monuments and the grid bearing and distance between these monuments.

COSTS

A simplified cost keeping method was inaugurated early in connection with the control and co-ordination surveys, inasmuch as the new procedure was often subject to critical review, and further to facilitate a detailed analysis of the economies of the variations in procedure as the work progressed. The Bureau of Reclamation system of cost accounting, last revised in 1917 when the Bureau survey activities rarely, if ever, extended themselves beyond canal location and

---

[4] For formulas, see *Special Publication No. 145*, U. S. Coast and Geodetic Survey, pp. 98 and 100.


(1.39A.)
Y= 637,937.95FT.
X=2,676,283.43FT.
Y= 637,979.26FT.
X=2,676,936.28FT.
N89° 08' 00"E
713.95'
9
16
S 89° 06' 28"W
1326.58'
200'
9 10
16 15
1326.59'
10
9
8 7 6
5
4
3
2
1
373.27'
S01° 47' 03"E
GREAT NORTHERN RAILWAY
100'
Taken From LOT 1 15.11A.
Reservoir Boundary
34.83A. Taken
Columbia River
VICINITY MAP
ST. HWY. 22
G.N.R.R.
2.
3.
4.
Columbia River
SECTION 16
02° 05' 10"
TRUE NORTH
GRID NORTH

PORTION OF SECTION 16 T37N-R38EWM STEVENS COUNTY

—NOTE—

GRID BEARINGS, DISTANCES AND COORDINATES ARE BASED ON THE NORTH WASHINGTON SYSTEM OF PLANE COORDINATES.

OWNERS {W. L. Sax / L. E. Jesseph

Acres Purchased = 49.94

Scale:- 1 inch = 500 feet

Sheet No. 1825

RESERVOIR BOUNDARY

FROM	TO	BEARING DEG.	MIN.	SEC.	DIST. FEET
1	2	N65	07	50W	207:55
2	3	N85	12	10 W	367.46
3	4	N58	32	40W	229.36
4	5	S88	48	40W	585.07
5	6	N84	43	20W	332.00
6	7	N14	55	00W	15.52
7	8	S26	52	00W	15.66
8	9	N84	43	20W	265.00
9	.10	N61	37	20W	126.51

DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION
COLUMBIA BASIN PROJECT—WASHINGTON
—GRAND COULEE DAM—
COLUMBIA RIVER RESERVOIR

DRAWN—Emig RECOMMENDED—
CHECKED—R. E. M. APPROVED—

L 168 | Coulee Dam, Wn. 12-16-37

Fig. 9.—Right-of-Way Plat Used for Conveyancing

subdivisional surveys, was followed as the structure upon which to develop the new cost finding system:

Survey Cost Accounting: Grand Coulee Dam—Columbia Basin Project

Divisions

CR	Columbia River Reservoir
WA	Works Area (Precise Construction Control)
GC	Grand Coulee Balancing Reservoir
CB	Columbia Basin Economic Survey

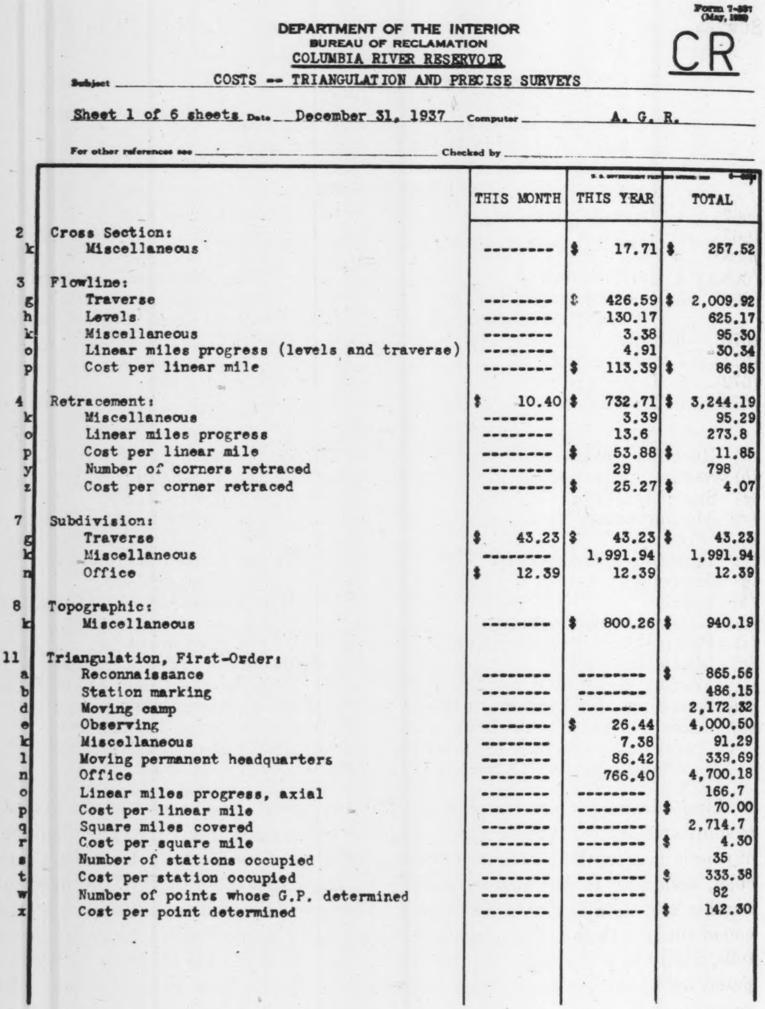
Class Number

(667)	1	Cross-Section
(668)	2	Location
(668)	3	Flow Line
(668*A*)	4	Retracement
(669)	5	Reconnaissance (other than triangulation)
(670)	6	Soil
(671)	7	Subdivision
(672)	8	Topographic
(673)	9	Trial Lines
(674)	10	Farm Units
(675)	11	Triangulation—First-Order
(675)	12	Triangulation—Second-Order
(675)	13	Triangulation—Third-Order
(675)	14	Triangulation—Fourth-Order
(675)	15	Precise Construction Control
(676)	16	Level Control, Precise
(676*A*)	17	Photogrammetric
(676*B*)	18	Hydrographic

Detail

(*a*)	Reconnaissance (675 only)
(*b*)	Station marking
(*c*)	Signal building
(*d*)	Moving camp
(*e*)	Observing
(*f*)	Base-line measurement
(*g*)	Traverse
(*h*)	Levels
(*i*)	Setting precise targets
(*j*)	Picture point identification
(*k*)	Miscellaneous
(*l*)	Moving permanent headquarters
(*m*)	Soundings, backwater studies
(*n*)	Office computing, drafting, general (not to include abstracting of field records)
(*o*)	Linear miles progress (axial)
(*p*)	Cost per linear mile
(*q*)	Square miles covered
(*r*)	Cost per square mile
(*s*)	Number of stations occupied
(*t*)	Cost per station occupied
(*u*)	Number of targets set Bench-marks set Stations set
(*v*)	Cost per station set
(*w*)	Number of points whose geographic positions or co-ordinates determined
(*x*)	Cost per point determined
(*y*)	Number of corners retraced
(*z*)	Cost per corner retraced

Limited time for cost keeping by the field organization necessitated a system of simple structure, yet one which might be relied upon to give, when called for, any unit cost with accuracy. Segregations were made daily by each party chief, assigning in the proper column of the daily labor report the account number and detail symbol pertaining to his crew's work for that day. At the end of the month the office clerk added to the slip the amounts for salaries, ferry tolls, fixed office charges, material costs, etc. Transportation costs were computed on a cost-per-mile basis furnished the field office by the project cost accountant for each car or truck, and assigned at the field office to the proper job number on a mileage basis. Recapitulating accounts required only 3 or 4 hr of the clerk's time at the end of each month. Unit costs were computed and tabulated twice annually, June 30 and December 31. A partial tabulation of the field and field office costs of control and co-ordination surveys over the Columbia River reservoir site is given in Fig. 10.



Form 7-[illegible]
(May, [illegible])

CR

DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION
COLUMBIA RIVER RESERVOIR

Subject COSTS -- TRIANGULATION AND PRECISE SURVEYS

Sheet 1 of 6 sheets Date December 31, 1937 Computer A. G. R.

For other references see ______ Checked by ______

			THIS MONTH	THIS YEAR	TOTAL
2		Cross Section:			
	k	Miscellaneous	--------	$ 17.71	$ 257.52
3		Flowline:			
	g	Traverse	--------	$ 426.59	$ 2,009.92
	h	Levels	--------	130.17	625.17
	k	Miscellaneous	--------	3.38	95.30
	o	Linear miles progress (levels and traverse)	--------	4.91	30.34
	p	Cost per linear mile	--------	$ 113.39	$ 86.85
4		Retracement:	$ 10.40	$ 732.71	$ 3,244.19
	k	Miscellaneous	--------	3.39	95.29
	o	Linear miles progress	--------	13.6	273.8
	p	Cost per linear mile	--------	$ 53.88	$ 11.85
	y	Number of corners retraced	--------	29	798
	z	Cost per corner retraced	--------	$ 25.27	$ 4.07
7		Subdivision:			
	g	Traverse	$ 43.23	$ 43.23	$ 43.23
	k	Miscellaneous	--------	1,991.94	1,991.94
	n	Office	$ 12.39	12.39	12.39
8		Topographic:			
	k	Miscellaneous	--------	$ 800.26	$ 940.19
11		Triangulation, First-Order:			
	a	Reconnaissance	--------	--------	$ 865.56
	b	Station marking	--------	--------	486.15
	d	Moving camp	--------	--------	2,172.32
	e	Observing	--------	$ 26.44	4,000.50
	k	Miscellaneous	--------	7.38	91.29
	l	Moving permanent headquarters	--------	86.42	339.69
	n	Office	--------	766.40	4,700.18
	o	Linear miles progress, axial	--------	--------	166.7
	p	Cost per linear mile	--------	--------	$ 70.00
	q	Square miles covered	--------	--------	2,714.7
	r	Cost per square mile	--------	--------	$ 4.30
	s	Number of stations occupied	--------	--------	35
	t	Cost per station occupied	--------	--------	$ 333.38
	w	Number of points whose G.P. determined	--------	--------	82
	x	Cost per point determined	--------	--------	$ 142.30

FIG. 10.—COST SHEET

Final costs per acre, based on the area over which retracement was required, varied from $0.305 to $0.345 for the different annual periods over which the work extended. Retracement costs, on a "cost-per-corner-retraced" basis, amounted to $4.07 for the 3-yr period over which this activity was pursued. Many of these corners represented widely-scattered points in country exceedingly difficult to reach, found to be required for subdivision purposes many months after the first stadia retracement parties had gone over the area. Often too, the necessary search through county records would mount the retracement cost of the individual corners to as high as $25. On the tributary streams where retracement work was totally completed by the triangulation party, 643 corners were recovered by the timber-cruiser method at a cost of $2.48 per corner, or one-sixth the estimated cost of retracement by the stadia method. Likewise, the data shown for flow-line levels and traverse costs per mile are to be understood with the modification that they represent reruns of isolated sections found to be in error between triangulation control points, and are necessarily higher than costs attained over sections of continuous run.

Conclusion

Beyond the point of retracing the original General Land Office corners and other cadastral points and boundaries, the method developed presents a complete reversal of the classical, or usual "ground methods," of co-ordination. Although it establishes a precedent "beyond the previous knowledge of the Court," the method is believed well grounded in the legal elements affecting boundaries and adjacent properties, in that the mathematical determination of azimuths and distances by the inverse method is supported by the closely controlled allowable limits of closure, as well as the requirement for the position check of each point. These elements, coupled with the permanency of such a survey in connecting with a definite mathematical relationship to the Federal network, alone justify the procedure, and in light of the cost element attained, further justification for the procedure is apparent.

The writer is fully aware of the many limitations placed on work of this nature. Generally, topography and forest cover are primary factors in the economic consideration of control and co-ordination surveys; but linked inseparably with characteristics of the terrain are the following considerations: (1) The project must be situated close to points in the Federal network, or it must be of sufficient magnitude to warrant carrying in the control from some distant network; (2) there must be no compromise with the high quality of instruments and equipment required for the control part of such a survey; and (3) the selection of personnel to prosecute such a project must sedulously guard against the inclusion of any but those best qualified in their respective fields of this highly specialized branch of engineering.

Acknowledgments

Acknowledgment is made to Mr. R. E. McGowan, Assistant Bureau Engineer in charge of headquarters computing office, for his aid in developing the two computing forms, Figs. 6 and 7, with their respective instructions and commentaries, in Appendices I and II.

All the work of providing geodetic control and co-ordination over the Grand Coulee Dam Reservoir and Balancing Reservoir sites was undertaken with a field and office personnel varying from a maximum of 28 men to an average force of 15 men. The work was done by the U. S. Bureau of Reclamation on the Columbia Basin Project, with headquarters at Coulee Dam, all field work being under the direction of Frank A. Banks, Assoc. M. Am. Soc. C. E., as Construction Engineer.

All engineering and construction work of the Bureau is under the direction of R. F. Walter, M. Am. Soc. C. E., Chief Engineer, with headquarters at Denver, Colo., and all activities of the Bureau are under the general charge of John C. Page, M. Am. Soc. C. E., Commissioner, with headquarters at Washington, D. C.

APPENDIX I

Computation of Third-Order Intersection Stations

The computer about to co-ordinate a third-order intersection station will first examine a field map and determine the strongest and second strongest pairs of the three cuts, the criterion being the closeness with which the intersecting cuts approach right angles. The cut common to both pairs is designated "No. 1," the remaining cut of the strongest pair "No. 2," and the remaining cut of the second strongest pair "No. 3." Referring to the sample computation in Fig. 6, the procedure begins by entering the names of the three second-order stations, Inchelium, Elburn, and Dorr, on Lines 1, 2, and 3 in the order described. From the list of directions for Station Inchelium the following items are transcribed to the right and left of the space on the form where the station name appears:

(*a*) The Y and X co-ordinates of Inchelium.

(*b*) The grid azimuth, 224° 35′ 21″.6 (the grid azimuth of the 0° 00′ 00″.00 direction of the list).

(*c*) The direction to quarter-section corner between Sections 3 and 10; *viz.*, 29° 51′ 05″.4.

Addition produces the grid azimuth of the cut from Inchelium; written as a bearing it is N 74° 26′ 27″.0 E. Below, in the central part of the form on Line 1, Fig. 6, the equation of the Inchelium cut may be formed. The factor m in Equation (1) is 0.2784370, which is the cotangent of the angle 74° 26′ 27″.0. The sign of the mx-term is minus for cuts in the northeast and southwest quadrants and plus for cuts in the northwest and southeast quadrants. Substituting the Y and X co-ordinates of Station Inchelium in the equation,

$$Y - 0.2784370\,X = b \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

the value of b is found to be − 255 899.85. It is advantageous to perform this substitution on a calculating machine in such a manner that the $-mx$-terms and Y-terms accumulate on the dials. Similar handling of the observations from Stations Elburn and Dorr completes the formulas on Lines 2, 3, 2, and 4 in the lower half of Fig. 6, as follows:

$$Y + 0.56809446\,X = b \quad \text{(3)}$$

and

$$Y + 0.8228263\,X = b \quad \text{(4)}$$

There is also space at the bottom of the form for a combination of Cuts 2 and 3, or of a possible fourth cut.

Proceeding with the use of Equations (2) and (3), it is evident that the Y-terms are readily eliminated by subtracting one equation from the other. The X-co-ordinate, obtained by machine division, is written on the right, in the X column. The corresponding value of Y results from a substitution of the X-value in either one of the original equations, preferably the one with the smaller cotangent. This substitution may be performed entirely on the computing machine. Following the same procedure with Equations (2) and (4), the co-ordinates of the 1·3 intersection appear in the lower part of the form.

The differences in Y and X between the 1·2 and 1·3 co-ordinates are 0.03 ft and 0.10 ft, respectively. The displacement between the two points of intersection is therefore 0.10 ft, usually written in the right-hand margin of the form. As this value lends assurance that the 1·2 co-ordinates are accurate within the specified limit of one part in 5 000, these are adopted as final.

Immediately below the 1·2 co-ordinate line, space is reserved for computing the co-ordinates of the monument proper if the signal is known to be eccentric. The eccentric distance from signal to monument, seldom exceeding 2 ft, along with carefully taken compass bearings observed from the signal to the monument and to the second-order stations, is taken directly from the Signal Building Card received from the field. On the sample computation form (Fig. 6), the bearing from the signal to the monument (S 51° 00′ E) has been referred to the Y-axis of the co-ordinate system by taking the angles between the compass bearings on the Signal Building Card and applying these to the reversed bearings in Sections 1, 2, and 3 in the upper part of the form.

Computation of the eccentric step should always be checked. In actual practice the work of the original computer appears in black ink, whereas that of the checker appears in red. For the purpose of differentiating in this paper the original computer's work is shown as tall, slope figures, whereas that of the checker's appears as small, vertical numerals. The checker does not examine the equations that his predecessor has used, but instead, computes the bearings to the 1·2 co-ordinates from Stations 1 and 2, and to the 1·3 co-ordinates from Stations 1 and 3. It is convenient to fold under the upper half of the form successively on the Lines *A–A* and *B–B*; the folded edge with the 1·2 or 1·3 co-ordinates uppermost may then be laid beneath the co-ordinates of Stations 1, 2, and 3 on these lists of directions and the sight differences in Y and X may easily be obtained by subtraction. Dividing the Y's by the X's gives the check cotangents which reproduce the foregoing bearings checked in Sections 1, 2, and 3, thus verifying all the computations leading to the adopted co-ordinates.

APPENDIX II

COMPUTATION OF THIRD-ORDER THREE-POINT-FIX STATIONS

The computer about to co-ordinate a three-point-fix station will first examine a field map and, noting thereon the relative positions of the fix and the four second-order stations observed, make the selection of the three stations which will constitute the strongest fix. The fourth object, in this case Station Mitchell, is reserved for checking purposes. Hence, Stations Sixmile, Ninemile, and Jensen become Objects 1, 2, and 3, respectively, on the computing form (see Fig. 7). (If the point sought happened to lie within the fixed triangle Sixmile-Ninemile-Jensen, any one of the three fixed stations could be designated Object No. 1; No. 2 and No. 3 would follow in clockwise order.) The names of the four stations are written on the lines labeled 1, 2, 3, 2, 2, and 4 in the upper half of Fig. 7. The known Y and X co-ordinates of these stations are entered in the spaces to the right of their names. From the List of Directions the three angles $1 \cdot 2$ (α), $3 \cdot 2$ (β), and $2 \cdot 4$ are obtained by subtracting mentally and entering in the spaces provided on the form. The cotangents of α and β, taken from tables, are copied on the lines labeled "cot α" and "cot β." (Interpolation between the interval values given in the tables is an operation of only a few seconds on the calculating machine. For an angle greater than 90°, the cotangent is negative, of course.) Fig. 7 is self-explanatory as the computer proceeds down the right-hand side of the form to Line L–O and Line M–N. When these two terms are divided as indicated on the left-hand side of the form, they produce the quantity K. Directly below this quantity, $K^2 + 1$ may be developed. To the right, substituting values in either of the two expressions $\frac{(K\,O + M)}{(K^2 + 1)}$ or $\frac{(K\,L + N)}{(K^2 + 1)}$ produces Q, which is written in the space provided on the extreme right. Immediately to the left of Q is a space for the term $K\,Q$. Adding algebraically produces the position co-ordinates of the "Point Near," the name given to the point sought or that point from which the three-point observation was made. The minus signs before the Y and X coordinates of "Point Near" should be ignored at this time but will be found to be somewhat advantageous in the checking operation involving Station Mitchell. In cases where the value of K is greater than unity, Q may be a small quantity; in such instances Q should be evaluated to several places of decimals, as these significant figures will be essential to accuracy in developing the value $K\,Q$.

At this stage of the computation, the problem has been solved to the extent of determining the co-ordinates of the three-point-fix. An experienced computer performs this much of the computation in 12 min. The remainder of the form is used for checking both the field and office work and for resolving the co-ordinates of the "Point Near" through the short traverse step to the coordinates of the concealed monument, in this case the quarter-section corner between Sections 14 and 23.

Returning to the use of the fourth object, Station Mitchell, the computer makes the algebraic subtraction indicated where the co-ordinates of Station

Mitchell have been written. Dividing + 15 228.32 by − 1 811.40 results in a value which is the cotangent of the bearing to Station Mitchell; evaluated on the left this bearing is N 6° 47′ 00″.3 W. On the third line above this may be written the bearing to Station Ninemile, derived from K, which is the cotangent of the bearing to Ninemile; the quadrant in which this bearing lies is determined by reversing the algebraic signs of the quantities $K\ Q$ and K. Adding (clockwise) the observed angle 75° 35′ 34″.2 again produces a bearing to Station Mitchell, N 6° 47′ 01″.8 W, which is 1″.5 divergent from the Mitchell bearing previously obtained. This quantity (1″.5) is a measure of the accuracy of the field observations given in the List of Directions, provided: (*a*) The office solution has been free from error, (*b*) the rectangular co-ordinates of the four second-order stations are entirely accurate, and (*c*) the fourth object is so situated that it offers a strong check against the other three.

The computer who has completed his work to, and including, the line "Bearing Difference" (Fig. 7), finding the field data checking satisfactorily, proceeds to the bottom part of the form and makes the simple traverse computation to develop the co-ordinates of the quarter-section corner-stone. His initials, F. S. B., at the top of the form, indicate he has abstracted these co-ordinates.

A second computer checks the first man's work but follows a partly independent procedure which begins with the assumption that the co-ordinates of the "Point Near" are correct. He first verifies the three angles α, β, and 2·4. Next, in the center part of the form, he makes the subtractions 1-Pt.Nr., 2-Pt.Nr., and 3-Pt.Nr. in the Y and X columns. (This subtraction is performed most simply by folding under the top half of Fig. 7 above Line A–A on the sample and then successively placing the folded edge directly beneath the co-ordinates of Stations Sixmile, Ninemile, and Jensen where they are listed on the abstract sheets; the subtraction is thus made without copying down any figures.) Dividing the Y-values by the X-values produces cotangents and consequently, on the extreme left in Fig. 7, the bearings from "Point Near" to Objects 1, 2, and 3, respectively. Bearings 1 and 2 should reproduce α, and similarly, Bearings 2 and 3 should reproduce β, thus checking the office work that produced these co-ordinates for "Point Near." Below this line the second computer checks all the work of his predecessor.

Use of the Fourth Object to Check the Observations

On the line "Bearing Difference," the value 1.5 sec is of little aid in deciding whether a specified accuracy of one part in 5 000 is being met. Therefore, the estimated length of the line between "Point Near" and Station Mitchell, 15 300 ft, is multiplied by the sine of 1″.5 to give 0.11 ft, which value, in turn, is divided by the sine of the angle 4·2 to produce the ultimate index of accuracy, the figure on the extreme right-hand end of the line which, in this particular example, remains 0.11 ft. The experienced computer can usually determine the terms on this line mentally and without resort to tables. The last angle is not always 4·2 but, rather, is the angle between the direction to 4 and whichever of the other three objects will produce the angle nearest to a right angle. This entire procedure that involves the fourth object is entirely arbitrary;

the reason for performing it in this manner may be seen in the following review of three-point problem computation where a fourth object is available:

(*a*) Mr. W. F. Reynolds has shown[5] the method of combining all four directions in a least-squares adjustment. This ideal method, of course, is not intended for general engineering use.

(*b*) The suggestion of making two different solutions is made by the U. S. Coast and Geodetic Survey,[6] which omits one of the four objects in the first solution and omits a different object in the second solution.

(*c*) A variation of Method (*b*) is popularly used because it is quicker and apparently furnishes as faithful a check. This method may be illustrated by a solution of the Bureau of Reclamation sample three-point problem on Coast and Geodetic Survey Forms Nos. 655 and 25. The fourth object, Station Mitchell, is introduced only in the fourth triangle on Form No. 25. In that triangle, the second angle is obtained by subtracting the known fixed angle, Jensen-Ninemile-Mitchell, from the second angle of the second triangle. The common side of the fourth triangle disagrees by 0.13 ft with the side in the first and second triangles. Values greater than 0.13 ft would appear if the fourth triangle were Point Near-Jensen-Mitchell or Point Near-Sixmile-Mitchell. The fourth triangle is always chosen so that its first angle is as near a right angle as possible. It will be noticed that this is a practice identical to that on the Bureau of Reclamation form (Fig. 7) in which the value 0.11 ft was divided by the sine of the angle 4·2, which was the angle between Station Mitchell and whichever of the other three objects produced the nearest to a right angle. If a number of examples were solved by both methods it would be noticed that the final index value (0.11 ft) on the line "Bearing Difference" in the Bureau of Reclamation form is practically identical with the check value (0.13 ft) appearing in the fourth triangle of the method of triangles mentioned. No matter which of the two methods is followed, the index or check value obtained by using the fourth object as shown in these examples is believed to be as trustworthy an indicator of the accuracy of the field observations as can be obtained without a least-squares adjustment. The average of these index values for a large number of three-point problems in a given district appears to represent the probable error in displacement for the points co-ordinated. In any one example, however, its index value does not necessarily represent its probable error in displacement because, as previously noted, a fourth object will sometimes be so situated that it is incapable of checking all of the other three directions, and *vice versa*.

---

[5] *Special Publication No. 138* (Manual of Triangulation Computation and Adjustment), U. S. Coast and Geodetic Survey.

[6] *Special Publication No. 145* (Manual of Second and Third Order Triangulation and Traverse), U. S. Coast and Geodetic Survey.

DISCUSSION

WILLIAM L. SAWYER,[7] ASSOC. M. AM. SOC. C. E. (by letter).—Mr. Berry's paper is an excellent description of an extended survey, including geodetic and cadastral methods, which, until recently, were rarely associated. From the manner in which these were correlated, it is evident that careful planning was a necessary preliminary to undertaking the work. This paper may well serve as a reference for any Federal, State, or political subdivision which is undertaking an extended survey, especially where cadastral surveys are involved.

The economy claimed by the author in the execution of the work is not to be disputed. In this connection it is gratifying to note that the cadastral points were defined in terms of plane rectangular co-ordinates on the Lambert conformal projection, thus tying the cadastral surveys to a system by which lost corners may be easily recovered or replaced with precision. Thus the economy to be realized by the many advantages of plane rectangular co-ordinates wil be extended into the future. Philip Kissam, Assoc. M. Am. Soc. C. E., has discussed these points in an excellent manner.[8]

The writer doubted the advisability of using 7-in., 0° 0′ 10″ repeating theodolites as direction instruments on second-order triangulation until noting the results of triangle closures in Fig. 8. The requirements of the U. S. Coast and Geodetic Survey[6] for second-order triangulation angles are, for average triangle closure, 0° 0′ 3″, except that an effort should be made to keep the average closure down to 0° 0′ 2″.5, and that the maximum closure should seldom exceed 0° 0′ 6″. With one exception (arc Kettle 3″.38) the triangle closure is less than 2″.5. The maximum triangle closure ranges from 4″.45 to 7″.64. In the opinion of the writer these results are worthy of note, considering the program of observing used. Doubtless, the accuracy obtained is due primarily to two points given in the conclusion and emphasized herein: The use of high-quality instruments and the selection of the best qualified personnel. The writer doubts the advisability of attempting second-order and third-order triangulations by the program outlined in the paper, except for those surveying organizations best equipped in personnel and instruments.

PHILIP KISSAM,[9] ASSOC. M. AM. SOC. C. E. (by letter).—The method of cadastral surveying described in this paper might well be used as a standard wherever cadastral surveys are made simultaneously over a large area. The paper is so inclusive and so well condensed that it might almost serve as a manual for this type of surveying.

There are a number of points that Mr. Berry describes which should be emphasized strongly and which are sometimes neglected in surveys of this kind. First, it is evident that all control survey points are monumented and marked with identifying numbers; descriptions of them are written and placed in a permanent record. This means, of course, that the survey is permanently located on the ground and it can be rerun for the purpose of locating any points that may be lost. Second, the work has been done according to the specifications of the U. S. Coast and Geodetic Survey, which means that the precision of the control is such that it can be made part of the fundamental triangulation net of the United States and, therefore, is available for use by any surveying agency. Third, the results have been computed in terms of the State plane co-ordinate systems, which means that there are available thousands of monumented positions within the area covered by the survey which have been determined in terms of these co-ordinates and therefore are useful as a basis for all types of surveying without the knowledge of geodetic reduction. Since the entire control system has been based on the accepted fundamental datum of the United States, the plane co-ordinates of these stations are the accepted plane co-ordinates recommended by the Federal Bureau of Surveys and Maps and contain an exceptionally low-scale correction.

---

[7] Associate Prof., Civ. Eng., Coll. of Eng., Univ. of Florida, Gainesville, Fla.

[8] "Proposed Improvements for Land Surveys and Title Transfers," by Philip Kissam, *Proceedings*, Am. Soc. C. E., April, 1939, p. 573.

[9] Associate Prof., Civ. Eng., Princeton Univ., Princeton, N. J.

With an arrangement of this kind the average land surveyor will be able to determine the State plane co-ordinates of all points on his surveys within reasonable distance of the points thus established. Thus, all surveys in that area can be easily referred to the same datum. The control points cannot be lost, and the positions of property lines can be so described that they are permanent in position. The advantages of a definite standard plane rectangular datum are so numerous that the decision to use such a system cannot be overemphasized.

The adoption of methods that tend toward the reduction in costs as well as the improvements in accuracy are excellent, and the careful cost accounting method used should be emulated. Of particular interest is the method of using long tapes—that is, tapes from 200 to 300 ft long for third-order traverse. Utilization of long tapes is a great timesaver, and it is especially important that the taped distances are corrected for slope rather than an effort made to hold the entire tape in a horizontal position. The simplicity of Table 1 makes possible the field reduction of all measurements and eliminates the necessity for too rigid standardization of methods of support which are necessary when computation is made entirely in the office. It is believed that further economies could have been obtained had a more modern, simultaneous-coincidence, micrometer transit been substituted for the 10-sec repeating instrument used for second-order triangulation. Accuracy, speed, and reduction in the first costs would probably have been obtained, if such a transit had been used, together with the advantages of using an instrument which would have been far easier to transport. The utilization of plane co-ordinates for practically all computation subsequent to the determination of geodetic position for first-order and second-order triangulation and control traverse is to be commended, and attention is drawn to the reduction in computation costs which this provides without, however, departing from a standard datum.

The right-of-way plan illustrated in Fig. 9 is exceptionally complete. However, there is no note or legend which indicates whether the bearings given are true bearings, or grid bearings, and it would seem proper that grid bearings only should be used and so noted.

The entire survey seems to have been well thought out and the administrative details nicely standardized. Mr. Berry is to be complimented upon his fine contribution.

GORDON MACLEISH,[10] ASSOC. M. AM. SOC. C. E. (by letter).—The engineer in charge of any extensive survey should find this paper of great interest. The methods described by the author for assuring accuracy in the work and for maintaining a complete historical record of all points established and co-ordinated are excellent; and the paper shows clearly the value of systematizing, carefully, the field and office procedures and arranging the observations and measurements so as to give the required information with the greatest economy of effort. It is to be hoped, also, that the paper will receive the attention it deserves from the legal profession and from the judiciary because, as the author well remarks, the indirect methods, although in many cases much more accurate than the orthodox "ground methods," are too often "beyond the previous knowledge of the Court."

The methods given by the author for the co-ordination of an intersection station and of a three-point fix (Figs. 6 and 7) are very concise and, if co-ordinates only are wanted, require a minimum of trigonometric functions to be looked up in tables and entered on the form. It often happens, however, that the bearings and distances are required as well as the co-ordinates; and in such cases it is necessary, of course, to look up the usual sine and cosine functions and to use additional lines for their computation; and still further work is required to check these quantities after they are computed.

It seems to the writer that the retention of tenths of seconds in angles and bearings in the computation of third-order intersections and three-point problems is a refinement that is scarcely warranted by the strength of such fixes. In Fig. 6 the location of the point from Inchelium and Dorr would be changed by only 0.01 ft, and in Fig. 7 the three-point fix would be changed by only 0.02 ft by using the nearest even seconds; and much time would be saved in the computations. When the use of tenths of seconds is justified it might be advisable to consider also the effect of the azimuth corrections in computing the longer fixes. A method of doing so will be given subsequently herein, whereby grid co-ordinates may be computed accurately from geodetic angles for lines of any length up to 25 miles or more, the length being limited only by the accuracy with which the azimuth correction terms can be computed.

The method given for the computation of an intersection station (Fig. 6) has the disadvantage of requiring the use of at least a ten-bank calculating machine unless the co-ordinates are all small; and even in that case the quantities m and b in Equation (1), or K in Fig. 7, may still be very large. This will be the case whenever the line in question is nearly north and south. It frequently happens, too, that an intersection or three-point fix is required in the course of a traverse, in which case it is very convenient to use a method of computation that utilizes the ordinary traverse form. The writer uses the following methods which he finds very quick and simple, and which he prefers for the reasons that:

---

[10] Cons. Engr., Santa Barbara, Calif.

1. A simple traverse form is used, whether for an intersection or a three-point, both of which are computed by the same process; and an eight-bank machine is large enough for all problems;

2. The calculations give, directly, the distances to, as well as the co-ordinates of, the unknown point, and are entirely self-checking for all quantities involved;

3. All trigonometric functions used are less than unity, thus requiring tables of tangents only up to 45°;

4. All digits on the left in either co-ordinate column that are common to all points may be disregarded without affecting any part of the work;

5. No unusual entries are required in the traverse form except the values of the tangents of the bearings; and

6. All three-point fixes are computed in the same manner whatever the relative positions of the points, the only effect of position being the addition of 180° to one or another of the bearings.

The time required by this method to calculate and check a three-point fix, including bearings and distances, is about the same as by the author's method; and for an intersection point it is probably a little less.

Consider first the solution for an intersection point, since this is the basis also of the three-point solution. Suppose Point P (Fig. 11) to be located by cuts from the known points A and B.

Let $\Delta x_{AB} = x_B - x_A$; $\Delta y_{AB} = y_B - y_A$; α and a = bearing and length of $A\,P$; and β and b = bearing and length of $P\,B$. It is evident from Fig. 11 that

$$\Delta x_{AB} - a \sin \alpha = (\Delta y_{AB} - a \cos \alpha) \tan \beta \ldots\ldots\ldots\ldots (5a)$$

and

$$\Delta x_{AB} - b \sin \beta = (\Delta y_{AB} - b \cos \beta) \tan \alpha \ldots\ldots\ldots\ldots (5b)$$

Whence,

$$a = \frac{\Delta x_{AB} - \Delta y_{AB} \tan \beta}{\sin \alpha - \cos \alpha \tan \beta} \ldots\ldots\ldots\ldots (6a)$$

or

$$a = \frac{\Delta y_{AB} - \Delta x_{AB} \cot \beta}{\cos \alpha - \sin \alpha \cot \beta} \ldots\ldots\ldots\ldots (6b)$$

$$b = \frac{\Delta x_{AB} - \Delta y_{AB} \tan \alpha}{\sin \beta - \cos \beta \tan \alpha} \ldots\ldots\ldots\ldots (7a)$$

or

$$b = \frac{\Delta y_{AB} - \Delta x_{AB} \cot \alpha}{\cos \beta - \sin \beta \cot \alpha} \ldots\ldots\ldots\ldots (7b)$$

As shown subsequently, the method of using these formulas is such that only their general form need be remembered.

The signs of the functions of the bearings are in strict accordance with the co-ordinate system used—that is, for the usual system as shown, a bearing in the northeast quadrant has all functions positive; a bearing in the southeast quadrant has the sine positive, the cosine negative, and hence the tangent

$\left(\frac{\text{sine}}{\text{cosine}}\right)$ negative, etc. The distance a may be found from either of Equations (6); but in order to keep all quantities as small as possible Equation (6*a*) is used if β is less than 45° and Equation (6*b*) if β is greater than 45°. Thus, tables of tangents to seconds of arc beyond 45° are not needed, and no function is greater than unity. Knowing a, the co-ordinates of Point P can be calculated, whence b becomes known as that distance necessary on the bearing β to reach the co-ordinate of Point B.

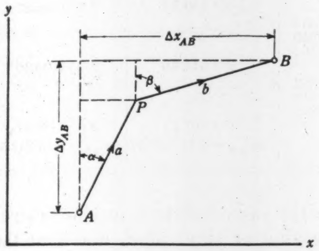


FIG. 11

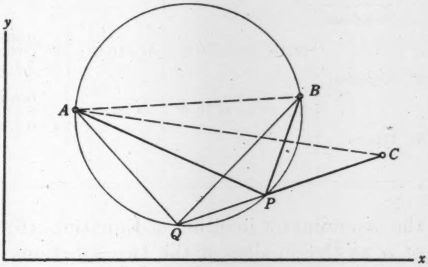


FIG. 12

By this method the solution of the author's intersection from Inchelium and Elburn in Fig. 6 (omitting the derivation of the bearings which can be inserted easily) would be made as shown in Table 2. The underlined digits in the co-ordinate columns need not be used in the computation.

Since β is less than 45°, use Equation (6*a*) and compute the denominator first, jotting it down. Then compute the numerator and without clearing it from the machine divide it by the denominator. This gives the value of a, thus,

$$a = \frac{(+\,14\,242.54) - (+\,14\,448.17)\,(-\,0.1760271)}{(+\,0.9633540) - (+\,0.2682333)\,(-\,0.1760271)} = 16\,610.23.$$

Having found the value of a, enter the y-co-ordinate of Point A in the machine and compute the y-co-ordinate of P. Then without clearing the machine enter $\cos\beta$ on the keyboard and crank up enough turns (positively or negatively) to arrive at the y-co-ordinate of Point B, the value of the distance b being given by the number of turns cranked. The x-co-ordinate column then serves as a check on the work and gives the x-co-ordinate of P in doing so. The distance b may be calculated by means of either co-ordinate column, the other serving as a check; but for greatest accuracy that column should be used which corresponds to the larger of the two functions (cosine or sine) of β. The only quantities not checked by the foregoing method are the entries of $\cos\alpha$ and $\sin\alpha$. Hence these should be verified or, as a final check, the value of b should be computed from Equation (7*a*) or (7*b*). If b is computed originally from Equation (7*a*) or (7*b*), the two co-ordinate columns furnish a double check on the entire work.

If the value of the denominator in Equation (6) or (7) is negative the numerator will be negative also, provided that the bearings are consistent with the given co-ordinates. In this case leave the numerator as a negative entry in the machine, enter the absolute value of the denominator on the keyboard and crank up positively until the result is zero. The value of a (or b) will then be given by the number of turns cranked. Note that the first term of the denominator in either of Equations (6) is the same function (cosine or sine) of α as the smaller of the two functions (cosine or sine) of β, and that the first term of the numerator is from the corresponding co-ordinate column; and this applies similarly for the use of Equations (7). If these two reminders are fixed firmly in mind Equations (6) and (7) need be remembered only as to general form, and the proper one will be chosen and used correctly almost without thought.

TABLE 2.—COMPUTATION OF INTERSECTION STATION

Point	Bearing	Distance, in feet	cos sin (tan, cot)	Y	X
A. Inchelium				$\underline{4}79\ 159.04$	$\underline{2\ 6}39\ 946.87$
	N. 74° 26′ 27″.0 E.	16 610.23	0.2682333 0.9633540 (+0.2784369)	+	+
P. X-station				$\underline{4}83\ 614.46$	$\underline{2\ 6}55\ 948.40$
	N. 9° 59′ 00″.0 W.	10 146.39 ✓	0.9848582 0.1733617 (−0.1760271)	+	−
B. Elburn				$\underline{4}93\ 607.21$	$\underline{2\ 6}54\ 189.41$ ✓
				$\Delta y_{AB} = +14\ 448.17$	$\Delta x_{AB} = +14\ 242.54$

Much time can be saved in the computation by carrying the co-ordinates (or their last seven or eight digits) continuously in the calculating machine and letting the multiplications accumulate, either positively or negatively as required. This saves writing down the individual values of Δx and Δy unless they are needed to compute a length or a bearing. They are never needed to compute areas, since cross-multiplication of co-ordinates (dropping the common digits on the left) gives the area much more quickly and with no additional columns of figures required. Thus the area of the triangle Inchelium-P-Elburn is:

$$\begin{aligned} &\tfrac{1}{2}\,[(39\ 946.87 \times 83\ 614.46) + (55\ 948.40 \times 93\ 607.21) \\ &+ (54\ 189.41 \times 79\ 159.04) - (79\ 159.04 \times 55\ 948.40) \\ &- (83\ 614.46 \times 54\ 189.41) - (93\ 607.21 \times 39\ 946.87)] = 83\ 868\ 164 \text{ sq ft.} \end{aligned}$$

This computation may be made without clearing or reading the machine until the double-area is reached. The sign of the result may be disregarded.

The following solution of the three-point problem is an analytical adaptation of a well-known graphical method, and consists almost entirely of two simple intersection computations exactly as in the foregoing.

Let Point P (Fig. 12) be located by measuring the angles subtended at P by the three known points A, B, and C. These angles may be given directly or as a list of directions from any initial. Choose, as A and B, those two of the three known points which subtend at P the angle nearest to 90°, and label them so that $A \rightarrow B$ is clockwise about P. Call the third known point C. The auxiliary point Q is the other intersection of the line $P\,C$ with the circle through A, B, and P. Denoting azimuths by α and directions by D, with subscripts for identification:

$$\alpha_{AQ} = \alpha_{AB} + (D_{PC} - D_{PB}) \dots\dots\dots\dots (8a)$$

($\pm$ 180° if P is within triangle $A\,B\,C$, or if C is counterclockwise $< 180°$ from A);

$$\alpha_{QB} = \alpha_{AB} + (D_{PC} - D_{PA}) \dots\dots\dots\dots (8b)$$

($\pm$ 180° if P is within triangle $A\,B\,C$, or if C is clockwise $< 180°$ from B). The azimuths in terms of the angles will be evident from Equations (8). Knowing these two azimuths Point Q may be co-ordinated as an intersection point by the foregoing method using Equations (6). Compute α_{QC} from differences of co-ordinates. Then,

$$\alpha_{CP} = \alpha_{QC} \pm 180° \dots\dots\dots\dots (9)$$

(or $= \alpha_{QC}$ if C is between P and Q);

$$\alpha_{PA} = \alpha_{CP} \pm 180° - (D_{PC} - D_{PA}) \dots\dots\dots\dots (10a)$$

and

$$\alpha_{PB} = \alpha_{CP} \pm 180° - (D_{PC} - D_{PB}) \dots\dots\dots\dots (10b)$$

Use Equation (10a) or (10b) whichever gives the best intersection at P with the line $C\,P$—that is, the angle at P nearest to 90°. Azimuths and angles may be carried forward easily on the calculating machine, adjusting degrees, minutes, and seconds for carry-over only at the end of the operation. By doing this repeated entries are avoided.

Point P is now co-ordinated as an intersection point as described. A final check is made by computing the third course from P by differences of co-ordinates and comparing the computed angle with the observed value. The agreement should be exact.

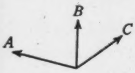
It is usually not known without some thought whether or not C is between P and Q, but this is of small consequence. The value of the length $C\,P$ as computed from Equation (6) should be positive; and if it comes out negative its absolute value is still correct, but the recorded directions of $C\,P$ and $P\,A$ (or of $C\,P$ and $P\,B$ if closure is made on Point B), together with the signs of their functions, should be reversed. These will be the only changes necessary, since the error in the assumed position of C will be apparent before any other value is computed.

Applying this method to the author's three-point fix given in Fig. 7, the solution would be as shown in Table 3. The underlined digits in the co-ordinate columns can be ignored in the computation, any changes in them being recorded by the machine.

The clockwise angle, Six Mile-P-Nine Mile ($= 75° 43' 02''.5$), is the nearest to 90°; so Six Mile is taken as Point A and Nine Mile as Point B, making Jensen Point C. It is evident immediately that P lies outside the triangle $A\,B\,C$, and that C is clockwise $< 180°$ from B. The azimuth of $A\,B$ is computed from Δx_{AB} and Δy_{AB}. The azimuths of $A\,Q$ and $Q\,B$ are found from Equations (8) by simple addition, using the machine to avoid duplication

of entries in the form; the azimuth of *Q C* is computed by co-ordinate differences; and the azimuths of *C P* and *P B* are found by addition using Equations (9) and (10*b*). The remainder of the work is identical with the foregoing for an intersection computation, both the points *Q* and *P* being co-ordinated in this manner using Equations (6) and (7).

TABLE 3.—COMPUTATION OF THREE-POINT FIX



	Directions	Angles
A. Six Mile	0° 00′ 00″.0	
		75° 43′ 02″.5
B. Nine Mile	75° 43′ 02″.5	
		52° 23′ 25″.4
C. Jensen	128° 06′ 27″.9	

Point	Azimuth	Length	cos sin (tan, cot)	*Y*	*X*
A. Six Mile				371 020.42	2 598 804.10
	215° 40′ 05″.8	5 498.24	0.8124065 0.5830915 (+0.7177337)	+	+
Q.				375 487.23	2 602 010.08
±180°	111° 23′ 08″.3	5 535.52 ✓	0.3646434 0.9311472 (−0.3916066)	+	−
B. Nine Mile				377 505.72 ✓	2 596 855.69
A–B	163° 16′ 40″.4		Δy_{AB}	= +6 485.30	Δx_{AB} = −1 948.41
Q–C	150° 00′ 49″.3		Δy_{QC}	= +12 181.50	Δx_{QC} = −7 029.11 ✓
C. Jensen				387 668.73	2 594 980.97
±180°	330° 00′ 49″.3	12 401.72	0.8661449 0.4997930 (−0.5770316)	−	+
P. Point Near				376 927.04	2 601 179.26
	97° 37′ 23″.9	4 362.13 ✓	0.1326596 0.9911617 (−0.1338425)	+	−
B. Nine Mile				377 505.72 ✓	2 596 855.69
C–B			Δy_{CB}	= −10 163.01	Δx_{CB} = +1 874.72
P–A	21° 54′ 21″.5	6 366.28	0.9277974 0.3730845 (+0.4021183)	−5 906.62	−2 375.16 ✓
P–B	97° 37′ 23″.9				
Angle: *APB*					
Computed	75° 43′ 02″.4 ✓				
Measured	02″.5				

If a fourth point *D* is observed as a check, compute the course *P D* by co-ordinate differences and compare the calculated angle from one of the other points with its observed value. It is assumed that *P* will be located by the three points that give the strongest fix.

For a given set of angles the strength of a fix is dependent on the relative length of the line *Q C* in comparison with *P A* and *P B*, and decreases with a decrease in the relative length of *Q C*, the problem being indeterminate when *Q C*, or *P C*, is zero. When *P* is on the circle through *A*, *B*, and *C* either Point *Q* or Point *P* is coincident with *C*. If *Q* and *C* are nearly coincident the fix is weak. On the other hand, if *P* is nearly coincident with *C* the fix will be strong if *Q* is sufficiently distant from *C* to give the required precision to the computed azimuth of *Q C* (the strength increasing also, of course, as the measured angles approach 90°); for in this case the computed position of *P* is comparatively in-

sensitive to changes in the observed direction of $P\,C$. A three-point fix with reasonable angles observed from a point close to C (the directions of $P\,A$ and $P\,B$ being accurately measured and the direction $P\,C$ being measured approximately) is a strong fix. Hence, the usual statement that P should never be close to the circle through A, B, and C needs qualification.

Effect of Azimuth Correction Terms.—Because the azimuth correction terms in either the Lambert or the transverse Mercator projection are different for each direction observed, a geodetic or measured angle is not exactly represented by the difference between the corresponding grid azimuths. These correction terms are small and in most cases can be neglected entirely; but with long lines and high precision work in certain districts they may be large enough to be worth considering.

Since the approximate grid co-ordinates of the points must be known before the azimuth correction terms can be computed, two methods of procedure to include their effects are available: (*a*) A preliminary graphical solution may be made on a good map and the corrections computed from scaled quantities (the measured angles are then corrected and a final analytical solution made directly); and (*b*) a preliminary analytical solution is made first, using the observed angles. In Method (*b*) the azimuth corrections are computed from the resulting approximate grid co-ordinates; and from these are computed corrections to the preliminary co-ordinates of the unknown point which will give its final co-ordinates.

The azimuth corrections in seconds of arc are given by the formulas:[11]

$$\epsilon\ (\text{Lambert}) = -\frac{x_2 - x_1}{2\,\rho_0^2 \sin 1''}\left(y_1 - y_0 + \frac{y_2 - y_1}{3}\right) \ldots\ldots (11a)$$

and

$$\epsilon\ (\text{transverse Mercator}) = +\frac{(y_2 - y_1)\,(2\,x_1' + x_2')}{(6\,\rho_0^2 \sin 1'')_a} \ldots\ldots\ldots (11b)$$

in which the sub-one subscripts refer to the station at which the azimuth originates and the sub-two subscripts refer to the station at which the azimuth terminates. These corrections are to be applied to the grid azimuths with the signs as given. For an intersection computation the azimuths, α, originate at the fixed points where the cuts are made, whereas for a three-point computation they originate at the unknown point. Corrections must be computed for both sides of each angle used—that is, a correction for each direction.

Correction of an Intersection Station.—Using literal subscripts to denote the origin and terminus, the letter R denoting any reference point (for example, ϵ_{AR} is the correction to the grid azimuth from A to R), the formula for an intersection is:

$$\alpha_{AP}\ (\text{grid}) = \alpha_{AR}\ (\text{grid}) + \text{Angle } R\,A\,P\ (\text{observed}) + (\epsilon_{AR} - \epsilon_{AP}) . (12a)$$

and

$$\text{Angle } R\,A\,P\ (\text{grid}) = \text{Angle } R\,A\,P\ (\text{observed}) + (\epsilon_{AR} - \epsilon_{AP}) \ldots (12b)$$

Then the change in the azimuth of $A\,P$ (which equals the change in the azimuth of $P\,A$) due to the azimuth corrections at A is

---

[11] *Special Publication No. 193* (Manual of Plane-Coordinate Computation), U. S. Coast and Geodetic Survey, pp. 13 and 171.

$$\Delta\alpha_{AP} = \Delta\alpha_{PA} = (\epsilon_{AR} - \epsilon_{AP})\ldots\ldots\ldots\ldots\ldots(13a)$$

and, similarly,

$$\Delta\alpha_{BP} = \Delta\alpha_{PB} = (\epsilon_{BR} - \epsilon_{BP})\ldots\ldots\ldots\ldots\ldots(13b)$$

Furthermore

$$\tan^{-1}\frac{x_A - x_P}{y_A - y_P} = \alpha_{PA}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(14)$$

and, taking the total differential with x_P and y_P variable, and substituting $x_A - x_P = s_{PA}\sin\alpha_{PA}$; $y_A - y_P = s_{PA}\cos\alpha_{PA}$; and $(x_A - x_P)^2 + (y_A - y_P)^2 = s_{PA}{}^2$:

$$\frac{\cos\alpha_{PA}}{s_{PA}}\,dx_P - \frac{\sin\alpha_{PA}}{s_{PA}}\,dy_P = -\,d\alpha_{PA}\ldots\ldots\ldots\ldots(15a)$$

Changing to increments and dividing by sin 1″ to change the value of $\Delta\alpha$ from radians to seconds of arc:

$$\frac{\cos\alpha_{PA}}{s_{PA}\sin 1''}\,\Delta x_P - \frac{\sin\alpha_{PA}}{s_{PA}\sin 1''}\,\Delta y_P = -\,\Delta\alpha_{PA}'' = -\,\epsilon_{AR} + \epsilon_{AP}\ldots(15b)$$

Similarly,

$$\frac{\cos\alpha_{PB}}{s_{PB}\sin 1''}\,\Delta x_P - \frac{\sin\alpha_{PB}}{s_{PB}\sin 1''}\,\Delta y_P = -\,\Delta\alpha_{PB''} = -\,\epsilon_{BR} + \epsilon_{BP}\ldots(15c)$$

As always, the signs of the functions are in accordance with the co-ordinate system. Equations (15*b*) and (15*c*) may be solved simultaneously for Δx_P and Δy_P, the coefficients being based on the results of the preliminary solution and the values of the ϵ's computed from Equation (11*a*) or (11*b*) according to the projection in use. A slide-rule may be used for the entire work.

Correction of a Three-Point Station.—At a three-point station the azimuth corrections are all based on the unknown Point P as origin; so if the directions are measured from an extraneous initial no correction need be computed for the initial since it will cancel out of the equations. Corrections will be required to the azimuths from P to each of the three fixed points A, B, and C, and to any additional point that is used for a check. The changes in the angles P_{AB} and P_{BC} due to the corrections are, in seconds of arc, $\Delta P_{AB} = \epsilon_{PA} - \epsilon_{PB}$; and $\Delta P_{BC} = \epsilon_{PB} - \epsilon_{PC}$. Angle $P_{AB} = \alpha_{PB} - \alpha_{PA}$, and applying Equation (14):

$$\tan^{-1}\frac{x_B - x_P}{y_B - y_P} - \tan^{-1}\frac{x_A - x_P}{y_A - y_P} = P_{AB}\ldots\ldots\ldots\ldots(16)$$

whence, by a process similar to that used in deriving Equations (15*b*) and (15*c*):

$$\left(\frac{\cos\alpha_{PA}}{s_{PA}\sin 1''} - \frac{\cos\alpha_{PB}}{s_{PB}\sin 1''}\right)\Delta x_P - \left(\frac{\sin\alpha_{PA}}{s_{PA}\sin 1''} - \frac{\sin\alpha_{PB}}{s_{PB}\sin 1''}\right)\Delta y_P$$
$$= -\,\Delta P_{AB} = -\,\epsilon_{PA} + \epsilon_{PB}\ldots\ldots\ldots\ldots\ldots\ldots(17a)$$

Similarly,

$$\left(\frac{\cos\alpha_{PB}}{s_{PB}\sin 1''} - \frac{\cos\alpha_{PC}}{s_{PC}\sin 1''}\right)\Delta x_P - \left(\frac{\sin\alpha_{PB}}{s_{PB}\sin 1''} - \frac{\sin\alpha_{PC}}{s_{PC}\sin 1''}\right)\Delta y_P$$
$$= -\,\Delta P_{BC} = -\,\epsilon_{PB} + \epsilon_{PC}\ldots\ldots\ldots\ldots\ldots\ldots(17b)$$

The coefficients are computed from the preliminary solution and the ϵ-values from Equation (11). Equations (17), solved simultaneously for Δx and Δy, give the corrections to the preliminary co-ordinates of P due to the azimuth corrections, a slide-rule being used throughout.

The writer is glad to observe a more extensive use of intersections and three-point fixes in geodetic and cadastral surveys that are based on accurate triangulation, provided that all such locations are checked independently as was required in the surveys described by the author. Used with discrimination and with a facile knowledge of the relations between form and strength of figure, they constitute a valuable means of locating the points whose positions are required, quickly, and with considerable, if not with the highest, precision. They are thus particularly suited to cadastral, topographic, photogrammetric, and non-urban property surveys.

The indirect methods of surveying, along with triangulation in general and conformal projection in particular, will acquire the legal and judicial sanction accorded to direct measurements and plane representation as they become more generally known, both as to detail processes and as to the results. Mr. Berry's paper is a valuable contribution toward the increase of this knowledge.

Carl M. Berry,[12] Assoc. M. Am. Soc. C. E. (by letter).—There is little question but that land-acquisition practise making use of the State-wide conformal systems, with these in turn dominated by geodetic control, is steadily establishing itself throughout the membership of the engineering fraternity. Indeed, the improvements to follow in title-transfer practise may well be stated as the logical corollaries of a controlled boundary survey, the adoption of which among surveyors, title examiners, and the inestimable host of the laity who are to some degree affected by land titles and boundaries of real property will manifest itself in the future as progress—a progress which will depend, as it has in the past, upon growth in understanding.

The comment of Professor Sawyer, wherein he "doubts the advisability of attempting second-order and third-order triangulations by the program outlined in the paper, except for those surveying organizations best equipped in personnel and instruments," is well taken. Although overemphasis of this point might, in effect, lend hesitancy to those organizations contemplating the use of control surveys, it is felt, nevertheless, that such retarded impetus would be preferable to the questionable results of ill-planned and executed projects which might otherwise result. The long-range objectives involved in the use of modern technique should not lose sight of the public-relations aspect of the problem.

Professor Kissam's discussion might well be augmented by the appeal in his excellent paper,[8] "Proposed Improvements for Land Surveys and Title Transfers," for surveyor and title examiner to combine in a joint effort to find ways and means for employing those excellent tools already at hand (modern survey equipment, skill, procedure, and established State systems of plane co-ordinates) toward eliminating boundary difficulties. His suggestion of the use of a more modern type of direction instrument is one to be considered seriously by any organization contemplating work of this nature. The writer fully agrees that the advantages pointed out by Professor Kissam would result. His further comment on the use of a note to indicate use of "grid" bearings on the right-of-way parcel plat is believed covered by the note, "Grid Bearings, Distances and Coordinates Are Based on the North Washington System of Plane Coordinates," which obviously applies to the tabulated values in the box "Reservoir Boundary," as well as to the values along section and subdivisional lines on the upper part of the sheet. Perhaps a clearer expression would result by substitution of the word "All" for "Grid" in the note, inasmuch as all values shown on the plat are in terms of "grid" values; the theta mapping angle is represented principally for the convenience of surveyors and title examiners in plotting previous deed calls by metes and bounds which would ordinarily follow "true" bearings. Plats of this type have met wide-spread favor among abstract and title companies, because of their scale representation of parcels and ownerships, and were generally used, in lieu of the usual lengthy and cumbersome typewritten metes and bounds descriptions, in negotiating the purchase contracts from landowners.

---

[12] Civ. Engr., U. S. Bonneville Power Administration, Portland, Ore.

The writer has been identified with the U. S. Bonneville Power Administration engaged in right-of-way acquisition for transmission lines in the States of Oregon and Washington, and has developed a plat embracing a square mile —one plat for each section of land invaded. Through use of a single master tracing for each section, the individual ownerships, often numbering twenty or more to the plat, are represented by coloring each parcel, a device saving much drafting room time and labor. At the same time much of the attendant possibility of errors in the preparation of the maps is removed. Although neither of these two States has yet adopted the Enabling Act, nevertheless it is felt that much groundwork has been laid toward its early introduction and passage.

The computing methods suggested by Mr. Macleish indeed look to be engagingly neat, concise, and easy to perform; certainly the forms require a minimum of written figures. As with all new designs, however, the writer would want to see an average group of computers utilize these forms for a month or so before concluding that the Macleish forms were definitely superior, the principal points of skepticism being (1) the recommendation to perform a maximum number of operations on the machine without jotting down intermediate figures, and (2) the necessity for remembering "rules-of-thumb" regarding signs of angles and choice of alternative "*a*" and "*b*" routes in the computations. Of course, most persons of ordinary ability and interest would have no trouble remembering what criteria governed in the different stages of the computations, but, as is frequently the case, not all members of the computing organization are of that caliber and for this reason it follows that whatever forms are used should be freely labeled to be as self-explanatory as possible. Forms should be designed so that the casual or occasional computer would not become lost in endeavoring to carry out intricate or involved computations on the keyboard of the machine—the steady or experienced computer will involuntarily take the short cuts anyway, whether the form calls for intermediate results or not. One of the most noticeable traits of computers is "going stale" on the computing forms after having been diverted to other drafting or computing work for occasional periods of a few weeks or so. In such cases the presence of ample labels and "mile-posts" in the computing form is most helpful.

From an administrative point of view, recognition must be given the fact that the majority of computers are recruited either from lists of applicants, often scantily qualified in both mathematics and calculating machine operation, or from chainman-rodman ranks during periods of inclement weather. Thus, computing forms should possess an essence of simplicity conducive to short training periods, and at the same time offer the necessary means of checking, mechanically, as the solution is being "cranked through." One of the worthy features of the Bureau of Reclamation three-point solution is its elimination of an inverse solution between the main scheme stations; often these values are not available from the geodetic reduction, particularly in such cases as water towers, spires, and other points originally located by second-order as intersection stations. Although the possible use of the check-position cut was mentioned in the discussion of the three-point solution, and further laudably mentioned in the discusser's conclusions, the Macleish forms have not provided for incorporating the "check-position" cut in each solution—a feature which the writer emphasizes as of utmost importance in any cadastral surveying technique which departs from the conventional ground methods.

Aside from the matter of computing forms, Mr. Macleish questions the justification for using tenths of seconds. The writer's decision to use tenths of seconds throughout was made, not as an attempt to imply a higher degree of accuracy than was actually attained, but rather to unify all computation work by removing from the computer's mind unnecessary mental "traffic signals" through the several steps of the computation. What actual justification there might be for this practise would doubtless be limited to the following:

On long lines of observations and weak fixes or intersections, the field-observed directions might actually be accurate within 0.5 sec. In such cases it seems quite worth while to compute the point with the use of tenths of seconds rather than round off the directions to the nearest whole second and risk distorting an angle from 0.5 to, say, 1.5 sec. If, then, it is worth while to compute an occasional point with tenths of seconds, it is logically economical to compute all third-order work with tenths of seconds, for unless one is using tables of functions listed for every whole second, the extra time required to interpolate the function of tenths of seconds on the machine is negligibly greater than the time required to interpolate to whole seconds. Moreover, computers work most efficiently if they handle the same number of decimal places in seconds, distances, etc., in all the operations throughout the day. For example, in occasionally carrying computations to tenths of a foot instead of to hundredths of a foot it was disclosed that in the long run there was more time lost than gained, because the computer had to stop to think that he must not put down the hundredths in his computations.

Mr. Macleish has commented regarding the requisite of a ten-bank computing machine for the Bureau of Reclamation intersection computation form.

Although this was never a matter of concern inasmuch as it seems to be the policy of the Bureau to purchase only ten-bank machines, the eight-bank machine could nevertheless have been used in all cases with but little inconvenience. To use an eight-bank machine the first two figures of the X co-ordinate would be omitted in forming Equation (1). After adding the two equations the X-value thus obtained by dividing would then be, say, 55 948.40 to which would be added the figures previously dropped to obtain the true X co-ordinate, 2 655 948.40. The large cotangents of lines bearing approximately north and south are no trouble because the rate of change of X is so small. On these very nearly north-south lines one can almost estimate the X co-ordinate before looking up the cotangent; to obtain the true X-value of the station the computer simply feeds the machine the customary number of significant figures, nine or ten if using a ten-bank machine, seven or eight if using an eight-bank machine, and the customary accuracy results in the answer. The only hazard of large cotangents is that an inexperienced computer may compute the Y-value of the intersection station by substituting the X-value in the equation having the large cotangent. However, if he immediately checks his Y (as he always should) by substituting X in the other of the pair of equations, he finds the Y-value does not check very closely and thereafter knows better than to expect north-south lines to offer accurate Y-values. Mr. Macleish has spent much careful thought and analysis on the subject, and is to be complimented for the masterly manner in which he investigated the subject and presented his discussion.

It is indeed gratifying that the discussers are all engineers who are well acquainted with this field of work, and the writer feels that an impressive assurance has been given a method which has as its objective an economical procedure for a controlled, permanently marked, and recorded land-acquisition survey.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2086

DEVELOPMENT OF THE COLORADO RIVER IN THE UPPER BASIN

By Thomas C. Adams,[1] M. Am. Soc. C. E.

With Discussion by Messrs. C. C. Elder, and Thomas C. Adams.

Synopsis

In a general way, this paper treats the past and prospective use of water of the Colorado River and its tributaries in the upper basin above Lee's Ferry, Ariz. Descriptions are given on the topographic, industrial, and agricultural features connected with the basin and the effect of the Colorado River Compact and the Boulder Canyon Project Act on the use of water. Water supply for the upper basin, after agreements between the States concerned are satisfied, is shown to be somewhat less than had been anticipated by some.

Conditions favoring and limiting future extension of irrigation in the upper basin are described. Additional irrigation projects of the normal type will be built; other projects, primarily to aid the livestock industry of the region by the irrigation of meadows and feed crops, are held to be desirable; and, the large diversion projects conveying water outside the water-shed of the Colorado River for irrigation, power, industrial, and municipal purposes are shown to be highly important in the general plan of upper-basin development. The hydro-power resources, the rôle to be taken by power generation in water development, and the relations between the use of water of the upper basin and the social, economic, and industrial growth of the basin and near-by regions, are discussed. The paper includes an appendix containing data on the irrigable areas of the upper basin and an analysis of the flow of the Colorado River at Lee's Ferry (where division is made between upper and lower basins) since the signing of the Compact, which is believed particularly significant as it handles flow during the period of lowest known river flow.

Topography and Industry

Lee's Ferry is situated on the Colorado River of the West, in Arizona, immediately below the State line between Utah and Arizona (see Fig. 1). The

---

Note.—Presented at the meeting of the Irrigation Div., Salt Lake City, Utah, July 21, 1938, and published in September, 1939, *Proceedings*.

[1] Associate Prof., Civ. Eng., Univ. of Utah, Salt Lake City, Utah.

area drained by this river and its tributaries up stream from the old ferry site is known as the upper basin. It is an extensive shallow basin "funnelling" to its river outlet, rimmed on the east by the crest of the Rocky Mountain Cordilleran and to the west by the somewhat lower crests of the Wasatch Mountains and those other mountains and high plateaus to the north and south. The basin comprises the highest area of so large proportions in the United States. It includes within its water-shed a varied topography of well-watered mountain heights, delightfully green mountain valleys and stream canyons, flat mesas, barren deserts, and profound box canyons. Adjacent to the basin, and just over the mountain crests on both the east and the west sides, are areas somewhat lower in elevation, having rich soil and more equable climate, and containing some of the most populous agricultural and industrial areas, of the western interior.

Localities within the upper basin and near-by are noted for their mineral wealth, and the region will continue for a long time to produce large quantities of non-ferrous and ferrous metals, coal, oil, natural gas, and a large variety of other non-metallic and rare minerals. The principal industries are stock raising, farming, mining and treating minerals, and the distribution of commodities produced elsewhere. The upper basin of the Colorado River is 104 000 sq miles in area and lies partly in each of the States of Utah, Colorado, Wyoming, and New Mexico. It is one twenty-ninth the area of continental United States and hence nearly twice the area of an average State. It is well supplied with railroads and highways but is only sparsely populated.

The mountain rims of the upper basin and other mountains lying within it receive a high rate of precipitation and from this small part of its area flows nearly 80% of the entire water supply of the Colorado River Basin, one thirteenth of the area of the United States. These mountains support no farming or agriculture other than open grazing because of the extremely short growing season. All other lands in the basin, with the exception of limited acreages of "dry-farm" wheat fields, are deficient in rainfall and must be irrigated to produce crops. Much of the area of the upper basin lies at the foot of the mountains and is itself high in elevation (more than 6 000 ft). It comprises broad plains, valleys, and mesas, the growing season of which is short. Agriculture is limited largely to the raising of livestock and the irrigation of feed crops. Lower parts of the upper basin, in southeastern Utah and adjacent parts of Colorado, have suffered deep erosion in recent geologic times and are a jumble of box canyons and desert mesas and plateaus which, in some places, are almost devoid of soil. Irrigated lands are limited to favorably disposed positions in stream valleys.

IRRIGATION DEVELOPMENT

Irrigation development of the upper basin began early (William H. Ashley[3] and his trappers entered the Uinta Basin in 1824) and has proceeded in the manner typical of western irrigation. At the beginning small ditches were constructed by individuals and associations of individuals. These were fol-

---

[3] "History of Utah," by Bancroft.

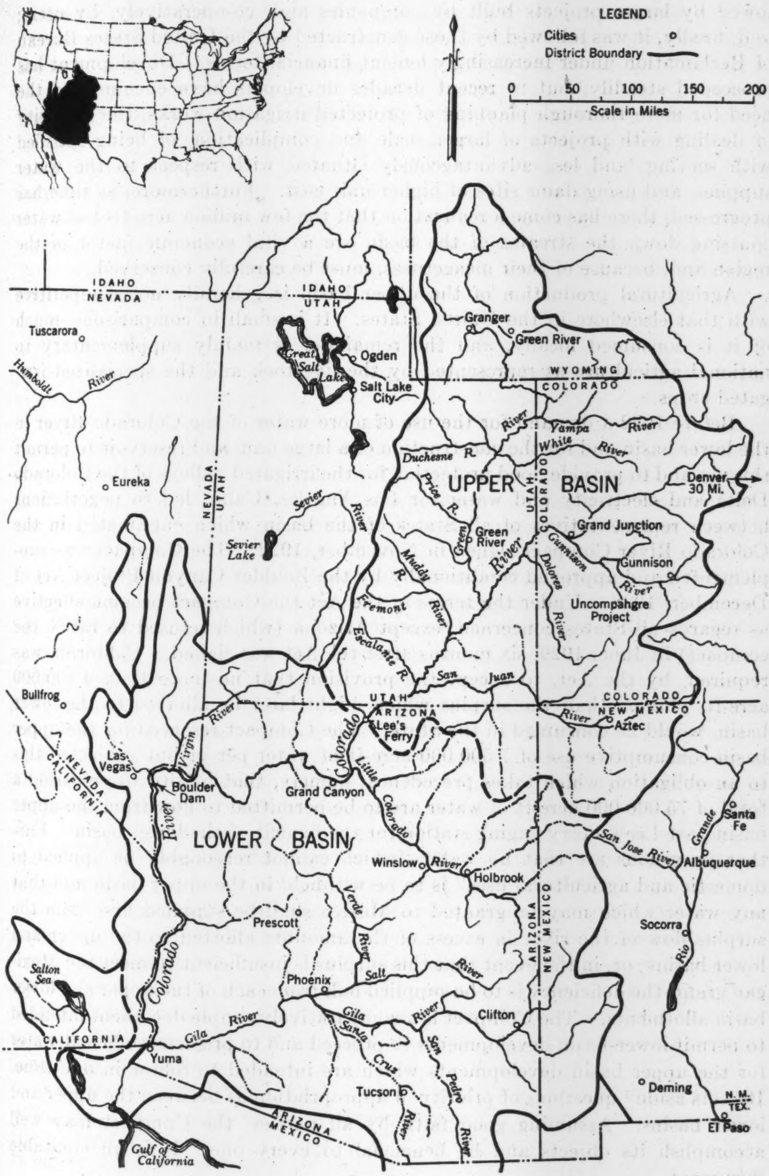

LEGEND
Cities
District Boundary
0
50
100
150
200
Scale in Miles
IDAHO
NEVADA
IDAHO
UTAH
Tuscarora
Humboldt
River
Great Salt Lake
Ogden
Salt Lake City
Granger
Green River
WYOMING
COLORADO
Yampa
River
White
River
Duchesne R.
Eureka
NEVADA
UTAH
UPPER
BASIN
UTAH
COLORADO
Denver 30 Mi.
Price R.
Sevier
River
Sevier Lake
Green
River
Green River
River
Gunnison
Grand Junction
Gunnison
Dolores
River
Muddy
R.
River
Fremont
Uncompahgre Project
Escalante R.
San
Juan
Bullfrog
UTAH
ARIZONA
COLORADO
NEW MEXICO
River
Aztec
Lee's Ferry
Colorado
Virgin
River
NEVADA
CALIFORNIA
Las Vegas
Boulder Dam
Grand Canyon
Little
Colorado
River
LOWER BASIN
Santa Fe
San Jose River
Grande
Albuquerque
Winslow
River
Holbrook
ARIZONA
NEW MEXICO
Prescott
Verde
River
Socorra
Rio
Salton Sea
Colorado
Phoenix
Salt
River
Clifton
Gila
Santa
Cruz
San
Pedro
River
River
CALIFORNIA
Gila
River
Yuma
Tucson
River
Deming
N.M.
TEX.
El Paso
ARIZONA
MEXICO
Gulf of California

FIG. 1

lowed by larger projects built by companies and, co-operatively, by users; and, finally, it was followed by those constructed by the United States Bureau of Reclamation under increasingly lenient financial terms. Development has proceeded steadily, but in recent decades developers have encountered the need for more thorough planning of projected irrigation works, the necessity of dealing with projects of larger scale and complication, of being satisfied with serving land less advantageously situated with respect to the water supplies, and using dams sites of higher unit cost. Furthermore, as time has progressed, there has come a realization that the few million acre-feet of water coursing down the streams of the basin are a vital economic factor of the region and, because of their meagerness, must be carefully conserved.

Agricultural production of the upper basin is generally not competitive with that elsewhere in the United States. It is small in comparison—much of it is consumed locally, and the remainder is mainly supplementary to national agriculture as represented by the livestock and the specialized irrigated crops.

Before 1922 a demand for the use of more water of the Colorado River in the lower basin and for the construction of a large dam and reservoir to permit this use and to provide flood protection for the irrigated valleys of the Colorado Delta and electricity and water for Los Angeles, Calif., led to negotiations between representatives of all States of the basin which culminated in the Colorado River Compact signed in November, 1922. The Compact was supplemented and approved conditionally by the Boulder Canyon Project Act of December, 1928. Under the terms of the Act the Compact became effective as regards all States concerned, except Arizona (which refused to ratify the compact) in June, 1929, six months after the Act was signed. California was required, by the Act, to accept the provision that no more than 4 400 000 acre-ft, plus one-half the surplus which might later be allotted to the lower basin, would be consumed in the State. The Compact reserved for the upper basin consumptive use of 7 500 000 acre-ft of water per yr but subjected this to an obligation which takes precedence; namely, that in any 10-yr period a total of 75 000 000 acre-ft of water are to be permitted to flow from the upper basin past Lee's Ferry gaging station for the benefit of the lower basin. Further provisions are that no water "which cannot reasonably be applied to domestic and agricultural uses" is to be withheld in the upper basin and that any water which may be granted to Mexico shall be supplied first from the surplus flow of the river in excess of the amounts allotted to the upper and lower basins; or, in the event that this surplus is insufficient to meet the Mexican grant, the deficiency is to be supplied half from each of the upper and lower basin allotments. The Compact is a comparatively simple document intended to permit lower-basin developments to proceed and to preserve sufficient water for the upper basin developments which are intended to follow in due time. It "sets aside" questions of priority of appropriation as between the upper and lower basins. Assuming good faith by all parties, the Compact may well accomplish its objects and be beneficial to every one. It is an equitable document.

Under the Boulder Canyon Project Act no work was undertaken; nor was any immediate benefit in the nature of construction intended, to the upper basin, with the exception of a moderate appropriation for studies of proposed developments throughout the entire basin. The Act set aside receipts from the use of works to be constructed under it after repayment to the Government of all costs of these is completed, in a fund which presumably would be devoted, at the direction of Congress, to further construction in the basin and partly in the upper basin. Through the construction of the storage reservoir at Hoover Dam (Boulder Dam in Fig. 1), the Act was intended to remove the possibility that litigation in support of prior appropriations made of Colorado River water in the lower basin would interfere with, or prevent, further developments in the upper basin.

APPRAISAL OF PAST EXPERIENCE

Sixteen years have now (1939) elapsed since the signing of the Compact and ten years since the passage of the Boulder Canyon Project Act; and it is of importance to upper-basin development to know what light and fruition intervening years have brought to the terms and intentions of these notable documents. Since 1922 many more discharge measurements of the Colorado River have been made and earlier discharge records corrected. Furthermore, the most severe drought period known has occurred, from which recovery is being made only during the present year. A review of the discharge of the Colorado River at Lee's Ferry since 1922 discloses an average measured discharge of 12 750 000 acre-ft per yr. To this may be added an average estimated upper-basin depletion of 2 180 000 acre-ft per yr, totaling 14 930 000 acre-ft per yr, which is an amount on the average for the seventeen years, 1922–1939, slightly less than sufficient to meet the Compact provision of 7 500 000 acre-ft per yr to each of the two divisions of the river basin. These values may be compared with an average, all-time, reconstructed flow of about 17 000 000 acre-ft per yr believed to be the yield of the river system above Lee's Ferry when the Compact was drawn. The low average discharge since 1922 would have the effect of reducing the all-time average to date by a small amount. For the last ten years of record (1929–1939), the values corresponding to those given are 11 720 000, 2 110 000, and 13 830 000 acre-ft per yr, respectively. To obtain the total quantity of water available for use from the Colorado River and its tributaries in both the upper and lower basins, after the terms of the Compact are applied, adjustments for gains and losses in the lower river must be made and a deduction taken for water granted to Mexico. No statement can be given of water that may be granted to Mexico. Only that title to water there established by treaty need be recognized and no treaty exists. The present use of water from the Colorado River in Mexico is about 1 000 000 acre-ft per yr.

A further study of the Lee's Ferry discharge shows that at the end of the water year, September 30, 1937, there would have been a deficiency in the 10-yr cumulative amounts (which the Compact specifies must be not less than 75 000 000 acre-ft for any 10-yr period) of 5.5 million acre-ft if use of water in the upper basin had reached the proportions requiring 7 500 000 acre-ft per yr in a year of normal stream discharge (which amount would be appropriately less in drought years) and in the absence of large, long-carry-over storage in the upper basin available to supplement stream discharge. Furthermore, even if the discharge of the river increases and remains above average during the ensuing years beginning with 1939, the deficiency in the 10-yr cumulative discharge will increase to perhaps the end of the water year of 1939 when it will possibly be 10 000 000 acre-ft. It is not likely that this 10-yr amount would again reach the 75 000 000 limit until well into the 1940's, barring continuously and phenomenally high discharge of the river.

Other studies of discharge show that in the ten years just passed an average of 5 800 000 acre-ft per yr would be available for upper-basin use from direct stream flow had the upper basin been using all available to it under the Compact, with adherence to the 10-yr cumulative provision included but without being obliged to share in an allowance of water to Mexico (see Column (12), Table 2 in the Appendix). Considerable storage would be required on upper tributaries of the river to equalize this average flow, throughout the 10-yr period, in a manner acceptable for upper-basin use. Water available to the upper basin under the conditions specified could be increased appreciably if large carry-over storage on the tributaries or the main river in the upper basin was available to assist in supplying 10-yr cumulative demands of the lower basin at Lee's Ferry.

These studies also show that an average of 6 700 000 acre-ft per yr during the ten years, 1929–1939, would be available from direct stream flow at the outlet of Hoover Dam under the foregoing conditions (see Column (14), Table 2). The equalization of this flow could take place in Lake Mead. This flow, plus allowable cumulative depletion of Lake Mead throughout the drought period, would constitute water available for lower-basin use on the main river plus one-half of any water granted to Mexico. A study has been made of the depletion of Lake Mead governed by the presumption that all requirements of the lower river will be met by a discharge of 7.5 million acre-ft per yr of water from Hoover Dam. The result is a maximum cumulative depletion of 20.0 million acre-ft in 1936 (see Column (16), Table 2).

Records are insufficient to show how frequently drought periods such as the one of the ten years, 1929–1939, will be repeated. Indications are that four droughts of considerable severity have been experienced in the past hundred years.

Construction of projects in the lower basin, provided for by the Boulder Canyon Project Act, has progressed rapidly and major items are (in 1939) substantially completed. Other, smaller, related projects have been authorized for the lower basin. The following has particular importance: Projects or divisions of projects in the lower basin which will draw their water supply from the main Colorado River are those (a) which are now (1939) fully planned; (b) for which money has been provided at least in part; and (c) upon which construction has begun or is about to begin. When 90% developed, they will require (together with projects at present (1939) operating) an annual quantity of water fully equal to the average available during the ten years, 1929–1939, to the lower basin under the terms of the Colorado River Compact from the flow of the river. The eventual needs of these projects are estimated herein at 6 200 000 acre-ft per yr, requiring the delivery at Hoover Dam of 7 000 000 acre-ft per yr with the average direct stream flow available at the outlet of the dam of 6 700 000 acre-ft per yr. If this demand is increased and particularly if it is burdened further with an additional grant of water for Mexican lands, Lake Mead will be seriously depleted in a series of drought years such as those through which the United States has just passed and under the conditions of river development which have been assumed.

However, a number of years will elapse before the lower-basin projects will need their full quantity of water. This time has been advanced due to conditions growing out of the years of depression through which the nation has been passing and by failure to realize earlier anticipations.

Appraisal of Future Trends

The consequences upon future use of water in the upper basin of the Compact, Boulder Dam Act, deficient stream flow, and accomplished and planned construction on the river may be briefly summarized as follows: (1) Early estimates of surplus, unallotted water are shown to be in error; (2) any increase in the use of water above that planned, and for which construction of works has begun in the lower basin, will require the appropriation of water which it has been presumed has been reserved for upper-basin use; (3) the small, future, financial aid that upper-basin development expected to receive from payments made to the Boulder Dam Fund by users of power and water is of doubtful maturity; (4) large storage volume upon the upper tributaries will be desirable for river regulation before final development is attained; (5) for a number of years large quantities of surplus water will flow into or through Mexico and will be available in part for use there with the possibility (according to the views of some) that use there will follow and finally that rights will be granted by treaty, with resulting ultimate detriment to the upper basin and perhaps to the lower basin. In pursuance of this condition an amicable agreement with Mexico should be made, if possible, protecting the water for United States use; or, failing this, a system for regulating the river should be adopted which will make further irrigation development in Mexico impossible through the resulting river regimen. The last may require the construction of additional reservoirs or, as a temporary expedient, the curtailment of some power development at Hoover Dam; but it is believed entirely feasible.

About 1 500 000 acres of land are now (1939) irrigated in the upper basin of the Colorado River. It is believed that this land consumes about 1.5 acre-ft per acre of water in years of plentiful water supply, or a total of 2 250 000 acre-ft. About 130 000 additional acre-ft have been diverted each normal year from the basin. Estimates of the additional area which, eventually, can be irrigated have been made and total about 2 500 000 acres; but these estimates are open to question, and studies are in progress, principally by the U. S. Bureau of Reclamation, to define, more accurately, the arable, and perhaps potentially irrigable, acreage. The results of recent surveys for the area drained by the Upper Colorado River in the State of Colorado (excluding the tributaries of the Gunnison and Dolores Rivers) have been made public and may be compared with an earlier estimate of potentially irrigable area in this basin of 290 000 acres. The recent survey gives the arable land in this same basin as 122 830 acres, less than half of the 290 000 acres (see comment on Table 3, in the Appendix). It is said, however, that the 122 830-acre estimate intentionally omits some land irrigable for meadow. Perhaps a similar reduction will be indicated for other subdivisions of the upper basin; but even these recent, more careful, surveys do not vouch for the general economy and feasibility of irrigating all the arable area which they disclose, nor for the adequacy of the water supply available to each area. Therefore, it remains uncertain how much of the 2 500 000 acres of upper-basin land, reported earlier as suitable for irrigation, will be irrigated.

By definition, the Colorado River Compact makes any area supplied with water from the river or its tributaries a part of the river basin even if it may lie without the water-shed (although this tacitly applies only to the signatory States and does not authorize nor encourage diversion of Colorado River basin water outside of States of the basin). The makers of the Compact fully intended that water of the river system should be used either within or without the basin as desirability later dictated. However, diversion of water from the upper basin for use on adjoining areas is now assuming an importance greater than was contemplated. The Colorado-Big Thompson Project is an object lesson to those whose engineering vision is restricted. A few years ago, this diversion (which will cost $44 000 000) was seriously considered only by a few of the most comprehending engineers. Others deemed the possibility of the undertaking as remote. The soundness of great diversions from the basin is due to the presence of highly developed agricultural, urban, and industrial areas just outside the basin which, although separated by mountain rims from the basin, are lower in elevation than much of the basin area. Hence, they have longer growing seasons and also possess abundant, highly productive soils, opportunities for the consumption of large quantities of electric power, and in general offer situations in which water supply has the best opportunity to stimulate an advanced economic and social development.

On the east side of the upper basin, in Colorado, twelve trans-mountain diversions have been operated for a number of years. The most important of these is the Grand River Ditch crossing the continental divide west of Ft. Collins, Colo. Only one is a tunnel, the Busk-Ivanhoe, west of Leadville, Colo., which makes opportune use of an abandoned railroad tunnel, shared with a highway. These diversions are said to have a normal aggregate capacity of 46 000 acre-ft per yr. The Moffat Water Tunnel Diversion was placed in operation in 1937. This structure is for the benefit of the City of Denver, Colo., and can divert 80 000 acre-ft per yr. The Jones Pass tunnel diversion, at the head of Clear Creek west of Idaho Springs, Colo., for the City of Denver has a capacity of 20 000 acre-ft per yr. This water is used to boost the low-water discharge of the South Platte River in the vicinity of Denver to avoid excessive pollution of this stream by the effluent from the Denver Sewage Treatment Works. After this use, it serves for irrigation and, when not required to increase the discharge of the river, may be turned into the Denver municipal supply. A notable tunnel trans-mountain diversion, constructed in 1938, is the Independence Pass (or Twin Lakes) which brings 48 000 acre-ft per yr into the head-waters of Arkansas River southwest of Leadville. The most pretentious diversion project that has reached the construction stage is the aforementioned Big Thompson project. It has many features and will divert 300 000 acre-ft of water beneath Rocky Mountain National Park for wide-spread benefit of the irrigated area of north-central Colorado. Power obtained from the falling water on the east side of the mountains is an important factor in the usefulness of the project.

Besides the trans-mountain diversions which have been noted, more than 17 others situated throughout the length of the crest of the Rocky Mountains in Colorado and dipping into New Mexico have been studied. Several studies have included field surveys and hydrography. Some of the projects are large.

Along the west rim of the upper basin, in Utah, nine diversions across the water-shed have been operated for several years. Much more important than the others is the Strawberry Tunnel which transports about 50 000 to 70 000 acre-ft per yr into the Utah Valley. In 1938, two, 1-mile diversion tunnels were built east of Spring City and Mount Pleasant, Utah, to transport 4 000 acre-ft each into the San Pitch Valley. These supersede two of the small earlier diversions. The Deer Creek project includes, as one of its features, a 5.5-mile tunnel to divert 32 000 acre-ft from the Duchesne River to the head of the Provo River, 35 miles east of Park City, Utah. More than eight other projects for diverting water from the upper basin to valleys lying to the west have been studied. In one of the most important of these it is planned to divert water from the Green River north of the Uinta Mountains to the Bear River, Wyoming. Investigations of two alternate plans including field surveys have been made by Mr. Ralf R. Woolley of the U. S. Geological Survey. A project of even greater magnitude has been studied by Mr. Kenneth Borg. This project would transport water from the Green River south of the Uinta Mountains and deliver it for use in the Salt Lake and adjacent valleys.

It is likely that development within the water-shed of the upper basin will progress in the form of a number of projects of moderate or small size. A considerable number of these have been given study. Localities exist within the upper basin where special crops and fruits grow luxuriantly. The produce is of such high quality that it is possible to overcome the handicap of remoteness and command a premium for it on national markets. Localities of this nature deserve expansion. Other localities, particularly the stream valleys at high elevations situated close to mountain and high plateau livestock ranges, support a productive livestock industry. It is said about half the feed for livestock in these areas comes from the ranges and half from cultivated fields. Western livestock raising is a highly specialized industry conducted mainly by men of great skill who have spent their lives in this occupation. When properly organized and managed, individual units may be very profitable; and extension of irrigation for the benefit of this industry, even at a cost that seems somewhat out of proportion to the low current prices for hay crops, is justified. Since 1936 the open range has been under close regulation; its use has been curtailed and made permissive in order to preserve its feeding capacity. Furthermore, use of the range has been made conditional upon the possession of commensurable property from which feed may be produced to supplement, to an extent more than required by natural conditions heretofore, the feed produced by the range. In this manner additional irrigated land for the production of livestock feed is assuming a new importance.

Developed water power in the upper basin of the Colorado River totals about 50 000 installed horsepower; and nearly all of it is along the Upper Colorado River in the State of Colorado where power-plants are within transmission distance of the Denver area. It is estimated that the upper basin can produce 2 000 000 primary horsepower with a regulated stream flow. Power sites on the Green River and main channel of the Colorado River, in their courses between Green River, Wyoming, and Lee's Ferry, are capable of producing the bulk of this output. Several gigantic dams would be required and the power would necessarily be made available in large blocks, with large initial investments being required. The power that can be developed will be lessened by future up-stream depletion due to irrigation development and the diversion of water from the basin. Proposals to place long reaches of the river and adjoining lands in national parks constitute a threat to the development for power and perhaps other purposes of the Green and Colorado rivers, in Utah. The necessity, or desirability, of making park reservations of these lands may be seriously questioned. Although they constitute scenic features of great importance and magnitude, and although money should be spent to provide means of access and travel, the natural scenery is not of a nature that it can be marred appreciably by such unrestricted human habitation and use as it is likely to undergo. Rather the few ranches, prospectors and nomad Indians who inhabit the country, add much to its picturesqueness. Dams and conduits that may be built will be only tiny spots in a vast wilderness of canyons and plateaus, largely of bare sandstone and shale. The few individual features requiring protection can be individually reserved without laying the "heavy hand" of park reservation over the entire area.

Conclusions

Conditions which will justify the development of the large power sites are unforeseen at present. Two possibilities are the construction of major dams for river control with the generation of power as a by-product, for sale at a price so low as to attract, to the vicinity, industries in which the cost of power bulks large; or an unforeseen technological development favorable to the development of power in this region.

This paper shows the development of the Colorado River in the upper basin to be varied and complex. Units in this development may be approximately grouped into: (1) Normal irrigation enterprises for assorted farming products; (2) irrigation of lands to produce livestock feed and meadow; and (3) the diversion projects. This classification is mainly an economic one.

Many of the undertakings for using water in recent years (and, most likely, many of those to be undertaken in the future) will furnish additional water to projects already operating. For some operating projects stored water is needed for late-season use when stream flow is normally low. Other operating projects exhibit the habits, not uncommonly displayed by western irrigation projects, of having a water supply that was initially over-estimated and hence unreliable, or having a canal system extended to serve lands for which no adequate supply of water is available; and under these conditions a supplementary supply of water is needed. In general, projects for supplying supplementary water where this is needed are regarded as particularly desirable because a small increment of investment commonly produces a relatively large increment of utility or return.

The proposed large diversions from the basin are especially complex and are the objects of particular engineering interest. Each includes a number of features such as: Collecting works; dams for diversion; storage reservoirs (for (*a*) water diverted, (*b*) water to supply deficiencies in lower reaches of the stream from which the diversion is made, and (*c*) to redistribute water near the points of use); pumping plants; electric generating stations; transmission lines; long tunnels; and, the more usual conduits and water control structures. The variety of purposes served by these projects includes irrigation supply, water for industrial and varied municipal purposes, improvement of recreational features, and the generation of power. With some projects should be included the functions of drainage, protection of land from torrential floods, and recovery of seepage, waste and return flow from low streams, drains or lakes. Some may serve the supplemental purpose of transporting power in the form of flowing water at a high elevation to a point where the generation of electricity can occur closer to markets and where greater power, by virtue of higher head, can be recovered.

It is an accepted fact that power generation is to play an important rôle in water development in the West. This is true in the upper basin of the Colorado River both for the projects within the basin and for the large diversions from the basin. Power generating stations, transmission lines, and pumps, free the energy gradient of the combined stream from many topographic limitations which would have to be met if water conduits alone were used. Increased efficiencies of electrical and hydraulic equipment now available and their lower cost and greater reliability—all coupled with the fact that a power system created as a part of water development and built primarily to serve this purpose, often does not need to meet separate development costs or the service requirements of systems built for other purposes—increase the usefulness of such power adjuncts to water projects. Then too some water development projects provide surplus power to sell at a profit to pay part of the cost of water development construction. However, if this is to continue to be an effective method of financing water projects, those interested in them must protect the power market and discourage abortive attempts to obtain cheap power through outside subsidy which, in turn, reduce or destroy the possibility of power development subsidizing water development.

The conviction is held by many that the water of the Colorado River System in the upper basin should be used in ways and places that will, most effectively, build communities with a broad industrial base, including not only agriculture but the winning and fabrication of mineral products and use of other industrial resources of the region. To the extent that this can be accomplished will come wide-spread advantages not only for residents of the basin and adjoining areas but for all areas within trading distance. This area may be indefinite in extent but it includes, more especially, the Pacific Coast and the Great Plains States. Communities dependent upon agriculture, supplemented by other industrial production, appear to have a more promising future in the inter-mountain west than communities dependent upon agriculture alone. This general thesis is further justification for the use of available water supplies to expand communities already established and experiencing healthy growth as a result of a combination of good agricultural conditions surrounding them, favorable position as trade centers, and possession of resources and facilities for other forms of industrial development. It is also a further justification for the great exportations of water from the basin which are being undertaken and proposed, and for the relatively high cost of these projects and the initial financial subsidization they need.

The general planning of the development of the Colorado River and its tributaries in the upper basin will continue. Such general plans are subject to drastic modification as new conditions develop and new information is available. Some past results are readily accessible in *Water Supply Papers* of the U. S. Geological Survey and various reports prepared by the U. S. Bureau of Reclamation and agencies of the States concerned. There remains, as distinct from general planning, a vast mass of engineering planning, of a detailed nature: (*a*) to unravel the hydrographic and other engineering complexities of individual projects; (*b*) to devise better and more economical combinations of their features; and (*c*) in short, by engineering ingenuity, to bring individual projects within the limits of an assured feasibility. The U. S. Bureau of Reclamation is undertaking a part of this heavy burden, successfully, and representatives of States and areas which are to participate in the benefits should assume another part of it both to bring more resources to the study and to insure realization of the opportunities which are within their grasp.

ACKNOWLEDGMENT

Fig. 1 has been reproduced from Drainage Basin Studies of the Water Resources Branch of the National Resources Committee.

APPENDIX

SUPPORTING DATA

FUTURE WATER SUPPLY

Table 1 contains an estimate of the future water supply to be drawn from the main channel of the Colorado River, below Lee's Ferry.

TABLE 1.—APPROXIMATE ESTIMATE OF FUTURE WATER SUPPLY TO BE DRAWN FROM THE MAIN COLORADO RIVER, DOWN STREAM FROM LEE'S FERRY

Item	Development (1)	Acres irrigated since 1937 (2)	Water diverted in recent years, in acre-feet (3)	Future irrigated area, in acres (4)	Future demand for water, in acre-feet per year (5)
1	Colorado Indian Reservation	7 500	....	100 000	270 000
2	Colorado River, Los Angeles Aqueduct	....	....	....	1 000 000
	Palo Verde Irrigation District:				
3	Developed by present canal system	23 000	225 000	75 000	....
4	By total anticipated development	....	....	115 000	200 000
	Gila Project:				
5	First unit, east of Yuma, Ariz.	....	....	150 000	405 000
6	Completed project	....	....	2 900 000	....
7	Yuma Project	60 000	450 000	80 000	325 000
8	Imperial and Coachella valleys	450 000	3 000 000	1 000 000	4 000 000
9	Total	....	....	....	6 200 000
10	Mexico	....	850 000	....	....
11	Hoover Dam losses	....	....	....	500 000
12	Net river losses below Hoover Dam	....	....	....	600 000
13	Sluicing and incidental loss at, and below, Imperial Dam	....	....	....	200 000
14	Power and irrigation losses, Lee's Ferry to Hoover Dam	....	....	....	300 000
	Gain, Lee's Ferry to Hoover Dam:				
15	Minimum	....	....	....	200 000
16	Maximum	....	....	....	700 000

Heading.—In the preparation of values of estimated future demand for water, attention has been given mainly, in the case of irrigation projects, to the indications afforded by physical features of the area served and of the construction completed, under way, or authorized. Thus constructed canal capacities and present experience with irrigation have been given more weight in arriving at values than general agreements or understandings regarding the distribution of water. But in all cases the figures adopted are intended to be conservative in not overstating the quantities.

Those familiar with Colorado River agreements may note some discrepancies between stated future demands for water and certain stipulations that have been made.

The following notes refer to corresponding items and column numbers in that table:

Item 8, Column (3).—Includes losses of the main canal through Mexico

Item 10, Column (3).—This quantity is water measured to irrigators in Mexico and does not include the transmission losses that will be incurred through the continuation of the Mexican Canal in operation after water for the Imperial Valley is drawn through the All-American Canal.

Items 3 and 4, Column (4).—The smaller acreage noted lies within the valley and is likely to be developed by the present canal system. The larger acreage includes good land on the mesa to the west of the Palo Verde Valley, which may be irrigated with a moderate pump lift.

Items 5 and 6, Column (4).—The smaller acreage is for the first unit of the Gila Project lying east of Yuma. It is reasonable to suppose that, when completed, this unit will be developed to a moderate degree within a few years. The entire Gila Project includes about 500 000 acres maximum; but other proposals have been made for the irrigation of the lands of the Gila River Basin of Arizona from the Colorado River and the large estimate given by Item 6 is perhaps the greatest aggregate acreage of these proposals.

Item 7, Column (3).—This is the quantity of water diverted for irrigation and excludes water diverted for power and other uses.

Discussion of Column (5), Table 1

Item 1, Column 5.—This estimate is based on 90 000 acres at 3.0 acre-ft per acre. Return flow from the Indian Reservation will re-enter the Colorado River for use below. A contract has recently been let for the diversion dam to serve this project.

Item 2.—The capacity of the Colorado River aqueduct is to be 1 640 cu ft per sec or 1 180 000 acre-ft per yr but it is intended to operate at 1 500 cu ft per sec or 1 090 000 acre-ft per yr. The date for the attainment of full operation was set by earlier forecasts at about 1970.

Item 4.—This estimate is based upon 90% of 75 000 acres at 3.0 acre-ft per acre. Return flow from the Palo Verde Irrigation District will re-enter the Colorado River for use lower.

Item 6.—This estimate is based upon 90% of 100 000 acres at 4.5 acre-ft per acre. It is to be noted that this acreage is less than two-thirds the acreage announced for the first unit of the Gila Project. Also return flow from this project will not be available, without pumping, to any United States irrigation undertaking.

Item 7.—This estimate is based upon 90% of 80 000 acres at 4.5 acre-ft per acre. Return flow from this project will be available only for use in Mexico. It is to be noted that the amount of water estimated for this project is less than the amount now being used on the project as indicated by the diversions made to the project less the large quantity of water returned more or less directly to the river after its use for power and maintaining optimum velocities and depths in a portion of the canal system. The present indicated use of water on this project is considerably greater per irrigated acre than the basis of estimates appearing in the foregoing table and this is true of the use of water on other irrigation projects diverting from the lower Colorado River.

Item 8.—This estimate is based upon 90% of 1 000 000 acres at 4.5 acre-ft per acre. The canal considered is the All-American Canal which will supply water to the present Imperial Valley, extensions to this within the Imperial

Irrigation District, to the East and West Mesas adjoining, the Coachella Valley, and possibly to San Diego, Calif., and vicinity, and other areas. The capacity of this canal (beyond Pilot Knob) is 10 000 cu ft per sec or 7 300 000 acre-ft per yr; 4 000 000 acre-ft are 55% of the canal capacity. Considering canal capacity to be 11% greater than the discharge (average) of the maximum month, the ratio of water diverted during the year to the annual canal discharging capacity is nearly 0.60 for the Imperial Canal and Yuma Main Canal (irrigation use only) within recent years.

Item 11.—This item is mainly evaporation. The value stated has been widely used as the evaporation from the reservoir some decades hence when the reservoir has been considerably silted and perhaps somewhat contracted in area. No doubt, the present evaporation is larger than this amount and there is some possibility that it is larger than is generally believed and will be larger in the future than the amount stated (for more information see Column (10), Table 2).

Item 12.—This is mainly evaporation from the river surface, adjacent flood plain, and Parker Reservoir to which is added stream inflow between the points considered. The latter is not great and the result is a rather large net loss as indicated. This net loss has been estimated as much as 200 000 acre-ft per yr greater than is given herein and may be considerably higher when other reservoirs are constructed on the lower river.[3]

Item 13.—It is expected that some loss of water from the Colorado River to the Gulf of California cannot be avoided. It is also considered by some that sluicing down the river of silt removed from All-American Canal water at Imperial Dam will require considerable water. Official estimates of water for incidental losses and sluicing are about twice the value stated in Table 1.

Item 14.—This amount includes allowance for evaporation from reservoirs which in the future are expected to be constructed on the Colorado River between the Hoover Dam and Lee's Ferry and for irrigation development on tributaries entering the river between these same two points. The two most important tributaries are the Little Colorado and the Virgin rivers. Official estimates are more than twice the amount given.

Item 15.—The gain from Lee's Ferry to Hoover Dam Outlet includes the discharge of the Little Colorado River, the Virgin River, and numerous smaller streams and springs. Evaporation from the river must be deducted and this may be from 50 000 to 100 000 acre-ft per yr. The net annual average gain has been variously estimated between the two values given. For reasons explained under Column (10), Table 2, the writer's estimate is near the low value.

DISTRIBUTION OF WATER BETWEEN UPPER AND LOWER BASINS

Table 2[3a] should not be interpreted as indicating how the water of the Colorado River will be distributed when development has reached a maximum, and during a dry period. Rather it is intended to show what restrictions the Colorado River Compact places upon the distribution of the water under these conditions. A modification in compact provisions may eventually appear desirable.

---

[3] The basic data for this estimate are from the Debler (U. S. Bureau of Reclamation) Report of 1934.

[3a] An addendum to Table 2 is included as Table 4 in the closing discussion.

The table was prepared for water years 1922 to 1937 which include the very dry period from 1931 to 1934 with sufficient years before and after to be indicative. There seemed to be no harm in setting forth presumptive data for later years to indicate what results would follow and the table has been extended to include these and derivations from them. It must be kept in mind, however, that all data for 1938 and following years are imaginary. Also it should be noted that the discharges assumed for 1938 and following years are high compared with the averages for periods passed. It is almost trite to state that if these discharges are as high as estimated, the conditions contemplated by the Colorado River Compact would be more quickly restored and less alarm would be warranted over shortages which would occur during such a dry period as centered about 1931 to 1934. If anything, Table 2 is slightly biased in favor of a larger usable water supply than will be realized from the flow of the river under conditions of the years considered. Explanations of the individual columns follow.

TABLE 2.—DISCHARGE OF THE COLORADO RIVER AT LEE'S FERRY, PRESENTING DISTRIBUTION OF WATER BETWEEN UPPER AND LOWER BASINS AS REQUIRED BY THE COLORADO RIVER COMPACT

(All Values in Millions of Acre-Feet per Year)

Year	Measured discharge, in millions of acre-feet per year	Ten-year cumulative discharges	ESTIMATED ANNUAL DEPLETION		Columns (2) and (4), minus Column (5)	Ten-year cumulative values from Column (6)	Measured discharge at Bright Angel, Ariz.	Ten-year cumulative values from Column (8)	Column (8) minus Column (2)	Annual estimated deficiencies	Water available for consumptive use	Annual discharge at Lee's Ferry based on conditions affecting Column (12)	Annual water supply, Lower Colorado River	Departures from cumulative sums of Column (14)	Cumulative depletion of Lake Mead
			Actual	Theoretical Future											
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
1922	16.4	....	2.4	7.9	10.9	....	....	....	....	....	....	10.9	....	....	....
1923	16.1	....	2.5	8.1	10.5	....	17.0	....	0.9	....	....	10.5	10.6	2.5+	....
1924	12.5	....	2.1	6.8	7.8	....	13.0	....	0.5	....	6.8	7.8	7.5	1.9+	....
1925	11.3	....	2.2	7.1	6.4	....	11.8	....	0.5	....	7.1	6.4	6.1	0.1−	....
1926	14.0	....	2.2	7.2	9.0	....	14.4	....	0.4	....	7.2	9.0	8.6	0.4+	....
1927	16.5	....	2.5	8.3	10.7	....	17.3	....	0.7	....	8.3	10.7	10.6	2.9+	....
1928	15.3	....	2.3	7.5	10.1	....	15.6	....	0.3	....	7.5	10.1	9.6	4.4+	....
1929	19.2	....	2.7	8.8	13.1	....	19.4	....	0.2	....	8.8	13.1	12.5	8.8+	....
1930	13.1	....	2.2	7.1	8.2	....	13.4	....	0.3	....	7.1	8.2	7.7	8.4+	3.8
1931	6.4	140.8	1.7	5.6	2.5	89.2	6.7	....	0.3	....	5.6	2.5	2.0	2.3+	9.3
1932	15.3	139.7	2.4	7.7	10.0	88.3	16.0	144.6	0.7	....	7.7	10.0	9.9	4.0+	6.9
1933	9.7	133.3	1.9	6.4	5.2	83.0	10.0	137.6	0.3	....	6.4	5.2	4.7	0.6+	9.7
1934	4.4	125.2	1.5	5.0	0.9	76.1	4.7	129.3	0.3	....	5.0	0.9	0.4	7.1−	16.8
1935	10.0	123.9	2.0	6.5	5.5	75.2	10.2	127.7	0.2	....	6.5	5.5	4.9	10.3−	19.4
1936	11.9	121.8	2.2	7.1	7.0	73.2	12.3	125.6	0.4	−1.8	7.1	7.0	6.6	11.8−	20.3
1937	11.9	117.2	2.2	7.1	7.0	69.5	12.4	120.7	0.5	−3.7	5.3	8.8	8.5	11.4−	19.3
1938*	14.0	115.9	2.2	7.2	9.0	68.4	14.5	119.6	0.5	−1.1	3.5	12.7	12.4	7.1−	....
1939*	15.0	111.7	2.3	7.5	9.8	65.1	15.5	115.7	0.5	−3.3	6.4	10.9	10.6	4.6−	....
1940*	15.0	....	..	..	10.0	66.9	15.5	....	....	....	4.2	13.3	13.0	0.3+	....
1941*	....	....	..	..	10.0	74.4	....	....	....	....	....	....	....	....	....
1942*	....	....	..	..	10.0	74.4	....	....	....	....	....	....	....	....	....

* All values below line (for 1938 and following years) imaginary.

Column (1).—The water year beginning October 1 of the previous year and ending September 30 of the year given. All data are for water years.

Column (2).—Measured discharge of the Colorado River at Lee's Ferry, in millions of acre-feet for the water year.

Column (3).—Ten-year cumulative discharges of the Colorado River at Lee's Ferry. The discharge for the year in question is added with the discharge for the nine preceding years.

Column (4).—Estimated annual depletion of the Colorado River above Lee's Ferry due to all present uses and diversions. To and including the year 1934, the depletions given were obtained from the Debler Report[3] on the water resources of the Lower Colorado River. This is the most complete and authoritative source of information. The writer has extended these estimates for 1935, 1936, and 1937 using the formula employed by the Bureau of Reclamation. The Debler Report gives values for calendar years but it is believed they are applicable without appreciable error to water years.

Column (5).—Estimated annual depletion in the upper basin of the Colorado River if development had proceeded to the limit imposed by an average use of 7 500 000 acre-ft per yr. Values for 1934 and earlier were obtained from the Debler Report.[3] Those for 1935, 1936, and 1937 were estimated by the writer following the Debler method. It is to be noted that values in this column vary from year to year with the dryness of the season.

Column (6).—Annual discharge remaining at Lee's Ferry after subtracting the values of Column (5) from the sums of the values of Columns (2) and (4). These latter sums are the total yield of the Colorado River and tributaries above Lee's Ferry. The items of Column (6) are amounts which would be available at Lee's Ferry to the lower basin had "full" upper-basin development existed during the time covered by the table, and without consideration for the provision of the Compact which required 75 000 000 acre-ft to flow past Lee's Ferry in any 10-yr period.

Column (7).—Ten-year cumulative sums of values of Column (6). It is these values that the Compact requires shall not be less than 75 000 000 acre-ft. Note that the values are less than the required 75 000 000 acre-ft for 1936 and 1937 and are likely to be less in succeeding years until 1943. The information in this column indicates that development in the upper basin cannot be based reliably upon an average consumption of 7 500 000 acre-ft of water per yr throughout a series of dry years but must be based upon some less amount; and that there is no surplus water available from the Colorado River over and above the allotments made by the Compact to the upper and lower basins.

Column (8).—Measured discharge of the Colorado River at Bright Angel, Ariz. Bright Angel is midway between Lee's Ferry and the Hoover Dam and below the mouth of the Little Colorado River, which supplies the principal inflow between the two points specified.

Column (9).—These are 10-yr cumulative values of the discharge at Bright Angel.

Column (10).—Differences between measured annual discharges at Lee's Ferry and Bright Angel. The net gain in the Colorado River between Lee's

Ferry and Hoover Dam outlet before deducting evaporation from Lake Mead has been taken numerically equal to these differences in later computations.

The smallness of the gain according to this assumption is somewhat surprising particularly as there is a large drainage area between the points considered. The Virgin River discharges from 100 000 to 400 000 acre-ft per yr into Lake Mead, and measurements by E. C. LaRue, M. Am. Soc. C. E., during August and September, 1923, disclose streams discharging an aggregate of 272.3 cu ft per sec into the Colorado River between Bright Angel and Black Canyon. Furthermore, a comparison of river discharge measurements made at Black Canyon and Bright Angel in 1932, 1933, and 1934 shows net river gains between these points of 200 000 to 400 000 acre-ft per yr. These years immediately precede the impounding of water in Lake Mead.

However, comparisons of the discharge of the Colorado River at Bright Angel and the Virgin River at Littlefield, Ariz., with the discharge from Hoover Dam, and accumulated visible storage in Lake Mead since the impounding of water began and during the period while the reservoir has been filling, show a remaining apparent loss of water after making a generous allowance for bank storage and a customary allowance for evaporation associated with Lake Mead, for most months and years. This may be the result of inaccuracies in discharge measurements, errors in the stated volume-stage relation for Lake Mead, too low an estimate of evaporation, temporary or permanent seepage from Lake Mead greater than was expected, or any combination of these factors.

A close study seems to indicate that: There is a very satisfactory consistency in the data which shows that discharge measurements are to be relied upon as being sufficiently accurate; there may be bank storage or seepage greater than estimated (10 ft of depth for the water area); there is a likelihood that the volume and area of the reservoir are greater for a given stage than has been stated heretofore; there is also a likelihood that evaporation is greater than estimated (6 ft of depth per yr) both because unit evaporation is greater and the area of the reservoir is greater; and, the apparent excess loss has ranged from 200 000 acre-ft in 1936 to 800 000 acre-ft in 1938. Inasmuch as part of this apparent loss is likely to prove to be an actual loss (evaporation and seepage other than bank storage)—perhaps of the order of the discharge of the Virgin River into Lake Mead (which is measured and was credited in the computations)—the net gain between Bright Angel and the outlet of Hoover Dam will continue to appear to be zero if the evaporation from Lake Mead is considered the same as before (at normal stages 500 000 acre-ft per yr when the reservoir area is somewhat decreased by silting).

This is an awkward manner of estimating the water resources of the river but appears to give correct results; and, until the volume-stage-area data for Lake Mead are confirmed or corrected, and until more data concerning the hydrology of the reservoir become available, no change in method is justified.

Column (*11*).—Annual quantities by which discharge passing Lee's Ferry would fail to provide a 10-yr cumulative discharge, at this station, of 75 000 000 acre-ft for any 10-yr period and under the conditions contemplated in the preparation of data for Columns (5), (6), and (7)—that is, an upper-basin development based on an average annual use of 7 500 000 acre-ft of water.

The values of Column (11) are obtained by noting the quantities in Column (7); and when these become less than 75 million, taking (in chronological order) differences between adjacent values. It would thus become apparent at the end of water year 1936 that the discharge of that year had been so low that the cumulative discharge for it and the nine preceding years had dropped to 73.2 million acre-ft; but, had the discharge at Lee's Ferry been 1.8 million acre-ft higher than it was assumed to be in Column (7), the cumulative discharge for 1936 and the nine preceding years would have been exactly 75 000 000 acre-ft. The 1.8 is the difference between 73.2 and 75.0.

Column (12).—Water available annually in the upper basin for consumptive use, based upon a development using 7 500 000 acre-ft per average yr except when this must be diminished to sustain the 10-yr cumulative discharge at Lee's Ferry at the 75 000 000 acre-ft specified by the Colorado River Compact. No shortage in the 75 000 000 acre-ft cumulative amount occurred until 1936; but this could not have been known until the end of 1936 and, therefore, the restitution of the shortage made in 1937. Similarly, it is considered that the restitution of the shortage of 1937 would be made in 1938, and so on. The values of Column (12) are obtained by subtracting from the values of Column (5) the values of Column (11) which appear on the lines next above. It is to be noted that the 10-yr-cumulative-amount provision of the Compact, if strictly adhered to, causes the greatest shortages to occur in the upper basin a matter of years after the end of the dry period. The only substantial effect of following this provision would be to hasten the restoration of the level of Lake Mead or other reservoir. The average of values of the column for the years 1931 to 1940, inclusive, is 5.8 million acre-ft.

Column (13).—Annual discharge at Lee's Ferry under the conditions upon which the data in Column (12) are based. Values are obtained by adding to the values of Column (6), the values of Column (11) appearing in the line next above (the sign is not considered). It is to be noted that under the conditions governing, very little water would be contributed to Lake Mead (or other reservoir on the lower river) in 1931 and 1934.

Column (14).—Annual water supply to the lower Colorado River reckoned at the outlet of Hoover Dam. Conditions of Columns (12) and (13) govern. To obtain the values of this column, the river gains of Column (10) are added to the discharges at Lee's Ferry given in Column (12); 0.8 million acre-ft are subtracted therefrom, representing a future depletion of 0.3 million acre-ft per yr between Lee's Ferry and Hoover Dam, and 0.5 million acre-ft, representing the annual loss from Lake Mead. The average of values of the column for 1928 to 1937, inclusive, is 6.7 million acre-ft. For further information on the probable evaporation from Lake Mead and considerations regarding the gain between Lee's Ferry and the outlet of Hoover Dam refer to the explanation of Column (10).

Column (15).—Departures of the cumulative sum of the values of Column (14) from the cumulation of the average for all years, applied successively each year. This is an indication of the storage requirements for equalizing flows of Column (14) for the entire period.

Column (16).—Likely cumulative depletion of Lake Mead during a drought period similar to the one of 1930 to 1937 after the Colorado River is developed in accordance with conditions stated for the data of Columns (12), (13), and (14). In the preparation of Column (16) the assumption is made that the requirements of the lower basin will necessitate the release of 7.5 million acre-ft per yr from Hoover Dam lake. This will include half of a possible concession to Mexican lands, the other half of which would move through the reservoir, and would be an additional release; but it is not considered in the present calculations because it would be from upper-basin allotments.

The available water from direct stream flow for lower-basin use as given in Column (14) is sufficient to supply the demand of 7.5 million acre-ft per yr without considerable depletion of Lake Mead until the end of the water year 1929; and the reservoir may be considered full at the termination of the spring flood of that year. A start therefore is made in 1930. This year is credited with flow available for the year and from this is deducted 4.0 million acre-ft representing the depletion of the reservoir for power and irrigation during the late summer and fall of 1929 and the spill required to obtain flood-storage volume before the expected spring flood of 1930; then 7.5 million acre-ft, representing the draft for 1930, is deducted. The item in Column (16) then is depletion of the reservoir at the end of the water year. For each subsequent year a credit is made for the flow available, and a deduction of 7.5 million acre-ft is made. This assumes that no spill, waste, or exclusively power use of water will be permitted under drought conditions.

The sum of the following quantities (each in million acre-ft) is 7.5 million acre-ft: Consumptive use by projects in the lower basin whose construction has begun, 6.2; channel losses between Hoover Dam and Yuma, 0.6; waste at Imperial Canal not recoverable in Mexico, 0.2; and, concession to Mexico, 0.5.

Such a drought coming several decades from now, when full development can be expected, would find Lake Mead partly silted. Perhaps its present capacity of 30 million acre-ft would be reduced to 25 million acre-ft. A depletion of 20 million acre-ft would leave 5 million acre-ft in the reservoir, with a head of 293 ft available at the power plant. This is a little more than half the normal head.

DISTRIBUTION OF IRRIGATED, IRRIGABLE AND ARABLE LAND

In Table 3, the data in Columns (2), (3) and (4) were presented by Daniel W. Mead, Past-President and Hon. M. Am. Soc. C. E., in 1929.[4] They are mainly census data and estimates that were available to the Colorado River Board. The data in Columns (5), (6), and (7) were compiled by the engineering staff of the U. S. Bureau of Reclamation under the direction of Frank E. Weymouth, Hon. M. Am. Soc. C. E. They are from a variety of sources including the U. S. Census Bureau.

The two sets of data are not strictly comparable for individual basins because divisions of the basins adopted are not the same; but they should be comparable for the entire upper basin (see Item 5, Table 3).

---

[4] "The Colorado River and Its Development," by D. W. Mead, *Journal*, Western Soc. of Engrs., July, 1929.

Data for arable lands in the basin of the Upper Colorado River have been cited in a report of the Bureau of Reclamation on the Colorado-Big Thompson Project. This report and the surveys of arable land within this basin were made under the immediate supervision of Porter J. Preston, M. Am. Soc. C. E.

TABLE 3.—ESTIMATES OF IRRIGATED, IRRIGABLE, OR ARABLE LANDS IN THE BASIN OF THE COLORADO RIVER ABOVE LEE'S FERRY, IN ACRES

Item	Basin of the:	D. W. MEAD			F. E. WEYMOUTH		
		Irrigated in 1920	Additional irrigable or arable	Total	Irrigated in 1922	Additional irrigable or arable	Total
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	Green River	643 000	1 212 000	1 855 000	532 000	1 067 000	1 599 000
2	Upper Colorado River	542 000	412 000	954 000	588 000	890 000	1 478 000
3	San Juan River	157 000	729 000	886 000	156 000	653 000	809 000
4	Remaining parts of the upper basin	128 000	102 000	230 000	171 000	130 000	301 000
5	Totals	1 470 000	2 455 000	3 925 000	1 447 000	2 740 000	4 187 000

For the basin of the Upper Colorado River in Colorado, excluding the Gunnison and Dolores river basins, comparisons with the Weymouth Report are as follows:

Report	Area, in acres
Porter J. Preston:	
Irrigated about 1935	256 300
Additional irrigable or arable	122 830
Total	379 130
Frank E. Weymouth:	
Irrigated in 1922	245 000
Additional irrigable or arable	290 000
Total	535 000

In the Preston Report the land is classified as arable but not necessarily irrigable from the standpoint of economy or adequacy of local water supply. Some areas which are said to be irrigable for meadow feed for livestock were omitted from the estimate intentionally. The Weymouth data for this basin are directly comparable in regard to the basin limits; but they include irrigated pasture land that was omitted from the Preston Survey.

DISCUSSION

C. C. Elder,[5] M. Am. Soc. C. E. (by letter).—As indicated by the title, the paper includes a comprehensive picture of Upper Colorado River Basin developments and a summary (see Appendix), in readily understood form, of the status of planning and investigations for future projects. The paper's chief value may well be (as was doubtless intended by the author) to broaden the viewpoint of engineers and others interested directly and too narrowly in special projects and localities, but who so often seem "too close to the forest" to see more than a few of "the trees."

For a considerable accomplishment toward this difficult goal, the author deserves much credit; and, as regards the portion of the paper covered by the title and relating specifically or chiefly to the Upper Basin, its data and conclusions appear to conform reasonably well with official and generally accepted standards, supported by an ample direct knowledge of the subject, being therefore fairly immune to criticism. So much can scarcely be granted for the numerous references, detailed quantitative data, and generally questionable conclusions concerning the water supply of the Lower Basin. Here, material departures from reality are noted, as this has been slowly achieved in the midst of the Colorado desert by generations of difficult pioneering; after long-continued and involved litigation; through conferences and negotiations without beginning or end, and therefore timeless as all eternity (or so they seemed); with agreements and contracts for water, valued and guarded like the last drink in the desert canteen; and, above all, by about one-half billion dollars of new water-supply construction within the decade 1930–1940, practically all of which must be repaid at full interest rates and without grants, subsidies, or other financial jugglery.

The author concludes (heading "Irrigation Development") that "The [Colorado River] Compact is a comparatively simple document." A mere reading of it might well seem to justify this appraisal, but not to any one who has listened to leading legal luminaries argue the pros and cons of rival interpretations of its innocent-looking phrases through six humid, but far from humorous, weeks of a Washington, D. C., summer. This argument has been interrupted occasionally during the last decade, as a result of the generated or prevailing heat, but as yet (1940) is far from being concluded.

The statement under the heading "Distribution of Water Between Upper and Lower Basins" that Table 2 "is intended to show what restrictions the Colorado River Compact places upon the distribution of the water under these conditions" (run-off as of 1922 to 1937) and the heading of the same table "* * * Distribution of Water Between Upper and Lower Basins as Required by the Colorado River Compact," are insufficiently qualified. No such restrictions or requirements as those of Table 2 are really inherent in, or definitely derivable from, the Compact; these are rather an expression very largely of "personal equation," being involved in more or less arbitrary assumptions and estimates, and in individual or regional interpretations of the Compact terms.

---

[5] Hydrographic Engr., Met. Water Dist. of Southern California, Los Angeles, Calif.

The Compact appears, in simple language, to allocate 8 500 000 acre-ft to the Lower Basin and 7 500 000 acre-ft to the Upper Basin. Quite improperly, in Table 2, the author assigns, for ultimate consumption, 8 800 000 acre-ft to the Upper Basin (for a year with run-off such as 1929, for example) while allotting the Lower Basin 6 200 000 acre-ft (in addition to Arizona's Gila River water use, which that State's officials insist must be called no more than 1 000-000 acre-ft annually). The Compact is certainly more subtle than has been suspected, if this result is "required" by its terms or can even be derived plausibly from them. The point is raised merely because it involves a general Compact interpretation which can be simply and briefly explained and understood, differing thus from some of the more involved hydrological problems.

The author has provided (see heading "Distribution of Water Between Upper and Lower Basins"; explanation of Column (5), Table 2) for Upper Basin development "to the limit imposed by an average use of 7 500 000 acre-ft per yr"; but it is literally true that nowhere in the Compact does the word "average" appear. It can be agreed that this usual interpretation of the Compact allocation, as adopted by the author, may finally become accepted by custom, or may prevail after long litigation and adjudication as an "equitable" interpretation. However, it is not consistent with present appropriative water-right law and practice in the Upper Basin States, where an upper diverter is not permitted to increase his claimed or decreed water right in a wet year at the expense of lower water users, merely in order to average the effects of past or future dry seasons.

If such an average allocation had been intended by the Compact allocations, presumably it would have been explicitly stated, as in Article III (*d*) with reference to the progressive cumulative 10-yr aggregate run-off as guaranteed at Lee's Ferry. It is not implied that the Upper Basin cannot, under the Compact, make use of surplus water in wet seasons, particularly to refill empty reservoirs (which holdover storage is not charged as a use until in later years it is actually diverted and consumed), as long as the Lee's Ferry flow is not reduced below the guaranteed minimum and the Lower Basin is not prevented from receiving its full allocation. In the long run, however, such excess use as listed in Table 2 must permit less holdover storage in Boulder Canyon reservoir and therefore, in some degree, reduce the supply available for the Lower Basin. Of course, the Upper Basin has the right to assert a claim to such excess water after 1963, and then secure an allocation of it if such claim is valid and justified, but apparently it has no such right of assertion of such a definite claim at this time. However, the present specific criticism is simply that the author has adopted the Compact interpretation most favorable to his own locality, without making clear that other interpretations and quantitative results are possible and may be considered by some to be more reasonable and equitable, and to conform more strictly to the terms of the Compact.

As regards the data of Table 1, the author might be presumed to have obtained the Arizona portion from California representatives, and *vice versa* for the California "discrepancies." The new Gila Project, for example, is listed as including 2 900 000 acres but allotted only 405 000 acre-ft for ultimate annual consumption (minimum requirements there being 4.0 to 4.5 acre-ft per acre). This is clearly an individual judgment as to the probable rate of development of this desert area, but if its settlement should proceed more rapidly than thus assumed by the author, and Arizona should continue outside the bloc of Compact States, the set-up of Table 2 may prove surprisingly in error. The carefully "staked-out" future water supply now depended upon by the Upper Basin would presumably be subject to appropriation in Arizona for use on this Federal project, largely defeating the Compact's attempted long-range protection. However unlikely, this is merely another of the numerous contingencies or variations that prevent the author's computations from being, with any degree of definiteness and finality, the one set-up to meet Compact requirements. As long as Arizona is not a party to the Compact, and until the terms of this treaty are interpreted with somewhat more judicial disinterestedness than now seems the custom, a wide range of such set-ups, rather than one, appears necessary to present an adequate picture of future water supply possibilities in the Colorado River Basin.

The author views with some alarm the possibility of severe water shortages in the Colorado River Basin during future drought cycles, this fate seeming to be assigned chiefly to the Lower Basin, even as happens to lower rather than upper diverters on an irrigation ditch. Without entirely discounting this pessimism, it is believed to be subject to various qualifications. The present series of years of low run-off is a reminder of a similar but rather less severe drought period that terminated in 1905. There is some basis for the opinion that droughts as severe as the present one may have a frequency of not more than once in a century; but considering only the period of record, 1897 to 1939, the average is materially reduced below the probable long-time mean by the inclusion in the reckoning of two major droughts and only one intermediate wet cycle. The period 1905 to date (1940) probably gives a more accurate approximation to the long-time mean, and on this basis the early estimate of average virgin flow referred to in the paper (heading "Appraisal of Past Experience") as 17 000 000 acre-ft seems not more than 2% to 3% too high.

Such average flows are of real value only when regulated by holdover storage for use during droughts. The effective active storage of Lake Mead will be increased greatly, long before needed by the Lower Basin for water supply, by the construction of additional reservoirs in the Upper Basin. The Bridge Canyon Dam will promptly cut off practically all silt inflow to Lake Mead for many years into the future, and every up-stream reservoir will have the same effect in some degree. Upper Basin and canyon reservoirs will aid in flood-control regulation, permitting the ultimate reduction of the present Lake Mead flood-control reserve of 9 500 000 acre-ft to a nominal amount. Below Lake Mead, Parker and Bullshead reservoirs will conserve winter power water. Instead of the author's 20 000 000 acre-ft of available holdover storage, there will finally be functioning (by the time needed) between 30 000 000 and 40 000 000 acre-ft of active regulatory capacity. The outlook thus appears much less gloomy—in fact, very satisfactory.

Similarly, on the credit side of the ledger, Item 13, Table 1, for silt sluicing at Imperial Dam has shrunk to a negligible amount with the rapid clearing of the Colorado River water that has already occurred there. Doubt has also been expressed that sufficient discount has been allowed for the high elevations at which proposed large transmountain diversions must be made, reducing materially the portion of the available run-off that can be so diverted. Other such proposed transmountain projects involving high pump lifts must, even in drought years, allot sufficient water (and, incidentally, additional water above this minimum) to the canyon power plants to generate the power required for pumping if commercial power and resulting revenues to help pay for such projects are to be produced. Such natural and economic restrictions on excess Upper Basin diversions may well appear more reassuring to Lower Basin water users than even the guarantees of the Colorado River Compact.

The author's theory (heading "Distribution of Water Between Upper and Lower Basins"; explanation of Column (11), Table 2[6]) of large errors in Boulder Canyon reservoir areas and capacities, stated to be of the order of 5% to 10%, or several thousand acres, seems absolutely untenable, even in that difficult canyon region, in view of the high engineering standards of the Bureau of Reclamation, with Boulder Canyon reservoir itself the best possible proof that these standards have been maintained. A series of air-survey maps, flown at varied water-surface elevations, should be able to answer this question of stage-volume relationship quickly and with conclusive accuracy. Meanwhile, it is considered that the large, unaccounted inflow to Boulder Canyon reservoir cited under "Distribution of Water Between Upper and Lower Basins" (discussion of Column (10), Table 2) can be explained more satisfactorily as bank storage, since the allowance of 10-ft depth over increments of wetted area can be called "generous" only in comparison with earlier estimates of 3 ft for bank storage. At Jackson Lake reservoir, with a capacity of 847 000 acre-ft, but with a water-level rise of only 8% of that at Boulder Canyon reservoir, a bank storage inflow from 10% to 15% of the reservoir capacity has been measured. In any case, whether Lake Mead is larger than assumed, or has unexpectedly large bank storage, the water supply on the Lower Colorado River will be aided materially, in time of drought, by such an increase of active or effective storage.

The writer feels much sympathy for the paper's concluding plea (see heading "Conclusions") "for the use of available water supplies to expand [Upper Basin] communities already established" and "in ways and places that will, most effectively, build communities with a broad industrial base" in addition to agriculture. This need is called a "further justification * * * for the relatively high cost of these [proposed transmountain diversion] projects and the initial financial subsidization they need," and with the latter statement little disagreement will be found, even in the Lower Colorado Basin and among engineers (those least radical or politically minded of mortals). Search the paper as one will, however, no reference to, or explanation of, any primary or initial justification of such "subsidization" can be found to which this appealing "further justification" must presumably be intended to be added in order to give to such proposed projects at least semblance or approximation to economic feasibility. The promotion of such large-scale "subsidization" of water exports is the only factor ever seriously viewed by Lower Colorado River Basin water users as constituting a genuine threat to the permanent safety of their future water supply. To engineers in general, it remains an unpleasant and distasteful idea, foreign to their mental processes and professional standards.

---

[6] Also personal letter from author dated February 3, 1939.

Thomas C. Adams,[7] M. Am. Soc. C. E. (by letter).—Mr. Elder's discussion is directed mainly at two points. One is the accuracy and adequacy of the writer's analysis of the division of water supply between the Upper and Lower Basins and the apparent intended use of water in the Lower Basin. The other is the justification of Government subsidy for water-supply development.

Consider first the matters of water supply. Table 2 is an application of the water-supply partition provisions in the Colorado River Compact to the distribution of the water of the river (and purports to be nothing else). The writer was led to prepare it because one of the problems vital to Upper Basin interests is the quantity of water available to the Upper Basin under the compact provisions and in conformity with the actual flow of the river. Table 1, giving estimates of water likely to be demanded by Lower Basin projects, was prepared because some believe that aggressive advancement of irrigation and other water-supply construction in the Lower Basin, with consequent large demand for water, is likely to result in an encroachment upon water supply generally supposed to be reserved by the Colorado River Compact for the use of the Upper Basin. In so far as the Upper Basin is concerned, the Colorado River Compact had an outstanding purpose—namely, to reserve an equitable portion of the undeveloped supply in the river system so that the Upper Basin might find it available for use at any future time. Furthermore, it is now coming to be more firmly believed that if projects of the Lower Basin make use of quantities of water so large as to encroach on the water supply reserved by the Compact to the Upper Basin, this water cannot be recaptured for Upper Basin use even though the compact provisions are explicit. One factor supporting this view is that it would immediately appear uneconomical to restrict the use of water by an operating project which involves a large investment, works of monumental proportions, many human settlements spread over a vast area of land, and far reaching, established, economic relations, in order to permit this water to be used on other, undeveloped lands where works are to be built, and all economic and other social advances are to be made.

Within the ten years 1930–1940, large developments in both the Upper and Lower Basins of the Colorado River have been, and in the future most likely will be, involved with Government financing dependent upon Congressional approval (and as some view it, also involved with a measure, greater or less, of Government subsidy). Therefore, the possibility of developments involving the withdrawal of water from use in one locality for use in another, even in conformity with previous agreements and with acknowledged equities, appears even more unlikely. An important factor in such a situation is that the wrong that might be done is not to an individual but is a more or less intangible wrong done to a community or section of the country, a situation which does not elicit so strong a personal interest from those who might defend equities nor so strong a public opinion upholding action with the same intention.

---

[7] Associate Prof., Civ. Eng., Univ. of Utah, Salt Lake City, Utah.

All should take special note that the question of the desirability of a legal formula which would withhold a natural resource for development in a specific locality at a late date is not now under discussion. In so far as this applies to the water of the Colorado River System, it was decided by contract when the Colorado River Compact was drawn and ratified.

It will bear re-statement that Table 2 was prepared to show the application of the partition terms of the Colorado River Compact to a period of river flow actually occurring. The partition terms apply to the reconstructed flow that was presumed to take place under pristine conditions. The terms are comparatively simple, and the provision is one of the most important in the Compact. Complications arise mainly from vagaries of river flow and the minds of men. Both of these are very complicated indeed; but very few have been bold enough to state that the Compact provisions will be set aside.

In the preparation of Table 2 it was necessary, of course, to use some judgment in the selection and ordering of data. In the few instances, where choice was not precluded by the definiteness of the data and strictly mathematical nature of operations, the writer chose to give preference to data and interpretations from authoritative sources. Such is the case with values in Columns (4) and (5) which record the Upper Basin annual demands for water under conditions of actual development in corresponding years, and future intended development for conditions of these years under terms of the Colorado River Compact, respectively. It is to be noted that later in Column (12), Table 2, it is found that deductions from the latter values—those of Column (5)—for years after 1936 are necessary because of reduced water supply.

The values of Column (5), Table 2, are not a constant 7.5 million acre-ft per yr because, in years of high runoff, water users will be able, and will find it convenient, to divert somewhat more than they normally divert. Where diversion is made by many thousands of small ditches (a condition that exists in the Upper Basin), it would be a great feat of water administration—one that has not been generally accomplished—to correct this condition. In years of low runoff, conversely, diversions will be below average for similar reasons; but the departures of values in Column (5) from 7.5 million acre-ft are not great. The largest of the values, 8.8 for 1929—a year of rather great stream flow—should compare with 12.5 million acre-ft from direct river flow available to the Lower Basin during this same year, not with 6.2 million acre-ft as inferred by Mr. Elder. However, if a uniform 7.5 million acre-ft per yr had been assumed in Column (5), the important conclusions drawn from Table 2 would not have been changed—namely, that (1) the water supply is critically limited; (2) the water partition provisions of the Colorado River Compact, although satisfactory when the river flow is high or moderate, have peculiar, undesirable consequences on the distribution of water in the presence of intended (future, ultimate) Upper Basin use of water and when prolonged, but to be expected, droughts occur.

Conclusions originally made from Table 2 are emphasized when this table is extended using discharges actually recorded in 1938 and 1939 in place of the imaginary values at first used. These discharges have become available since the table was prepared. The actual discharge for 1939 was considerably less than the imaginary value first used. An addendum for Table 2 is given in Table 4 using the actual values of discharge for 1938 and 1939 and an imaginary value for 1940. Of most interest in this table is the indication that in 1940, to meet conditions previously explained, including the compact conditions in the presence of ultimate Upper Basin use of water, it would be necessary to permit to flow past Lee's Ferry all water draining from the Upper Basin during the year, plus an additional amount of 0.6 million acre-ft. This conclusion, based upon a rather large imaginary river flow assumed for 1940, seems fully valid because the prospect of a river flow in 1940 of as great as 15 million acre-ft at Lee's Ferry, the assumed value, seems quite unlikely at the present writing (May, 1940). River flow, coupled with men's attempts to read the future and provide for it, does seem very subtle.

TABLE 4.—Discharge of the Colorado River at Lee's Ferry, Presenting Distribution of Water Between Upper and Lower Basins as Required by the Colorado River Compact

(Addendum to Table 2; All Values in Millions of Acre-Feet per Year)

Year	Measured discharge, in millions of acre-feet per year	Ten-year cumulative discharges	Estimated Annual Depletion		Columns (2) and (4), minus Column (5)	Ten-year cumulative values from Column (6)	Measured discharge at Bright Angel, Ariz.	Ten-year cumulative values from Column (8)	Column (8) minus Column (2)	Annual estimated deficiencies	Water available for consumptive use	Annual discharge at Lee's Ferry based on conditions affecting Column (12)	Annual water supply, Lower Colorado River	Departures from cumulative sums of Column (14)	Cumulative depletion of Lake Mead
			Actual	Theoretical Future											
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
1938	15.4	117.3	2.3	7.6	10.1	69.5	15.6	120.7	0.2	0.0	3.9	13.8	14.2	....	....
1939	9.4	107.5	1.8	6.2	5.0	61.4	9.6	110.9	0.2	−8.1	6.2	5.0	4.2	....	....
1940*	15.0	109.4	2.3	7.5	9.8	63.0	15.5	113.0	0.5	1.6	−0.6	17.9	18.4	....	....

* Values for 1940 are imaginary—believed based upon a discharge at Lee's Ferry, not likely to be reached.

The writer does not believe one can depart materially from the data of Tables 2 and 4 or the conclusions drawn therefrom while conforming to the terms of the Colorado River Compact and to the generally accepted interpretations.

Of course one must admit the possibility of other dispositions of Colorado River water. Large quantities of water may be absorbed in Mexico by appropriation sustained by international courts or by treaty grants. Arizona, not subscribing to the Compact, may legally appropriate a large quantity of water prior to its appropriation elsewhere and find ways of financing construction of necessary works. However, the vast expenditures required for water-supply development in Arizona must apparently be financed by the Federal Government and approved by Congress. Therefore it would appear that large-scale developments made contrary to the stipulations of the Colorado River Compact would involve flagrant violation of trust by the United States Government which is a party to the Compact. On the other hand, the Upper Basin may not desire to construct, or may be unable to finance, works in the Upper Basin and therefore water could be used elsewhere without protest. These and perhaps other possibilities exist, but they are not in accord with the preponderance of the opinions with which the writer is familiar.

The peculiarities of water distribution imposed by the Compact during drought periods appear to require an eventual modification of Compact provisions.

Referring to Table 1, which indicates the prospective water supply to be drawn reasonably soon from the main Colorado River below Lee's Ferry, the writer of necessity relied upon his own judgment to sort out acceptable items from the considerable quantity of discordant data. It again may be emphasized that an effort was made to be guided more by what is on the ground or posted on financial books and relatively less by statements of intentions.

Further comment may be made with respect to the Gila Project which was considered in Items 5 and 6 of Table 1. The construction of irrigation works for the first unit of this project, approximating 150,000 acres, is well advanced (May, 1940). So far as is known, there are not definite plans with respect to the time of construction and financing of additional works for other units of this project. The use of the term "Gila Project" by the Bureau of Reclamation is limited to an area of about 500,000 acres mentioned in the explanation accompanying Table 1. The value 2,900,000 acres was placed in Table 1 to indicate the extent of what some consider the possibilities of irrigation development in the Gila Valley. (Proposals, large and differing in detail, for the irrigation of areas approaching 2,900,000 acres in the Gila Valley of Arizona, are occasionally referred to as the Gila Project.) The writer did not go beyond an annual allotment of 405,000 acre-ft from the Colorado River to the Gila Project because this is ample water to supply the unit of the project for which there is assurance that works for its irrigation will be built; and also because each further appropriation of water for continuing use in the Lower Basin appears to add to a total which is already close to contravening the partition provisions of the Colorado River Compact. One would likely hold that, under the Compact, the irrigation of 2,900,000 acres of land in the Gila Valley of Arizona would require all water reserved for Lower Basin use.

A discussion of the fundamentals usually offered in justification of subsidies to irrigation and other water development in the United States was not the object of the paper. In the end, Government subsidization of water development or any other development is based on expediency, established policy, availability of money or credit, the adequacy of favorable sentiment, and other items. The final decision is made in political circles.

Unified judgment of the justifiableness of subsidization would be almost impossible to obtain. Even agreement of what constitutes subsidization seems impossible. Some will regard repayment of a Government investment in an irrigation project by 40 annual payments of 1/40 the first cost and without interest as complete repayment. (This is the present repayment plan of the Bureau of Reclamation—it governs repayment of advances for the construction of the All-American Canal which was built to convey Colorado River water to the Imperial Valley.) Others will compare this to a commercial investment which requires repayment in the same length of time, with interest at, say, 5%, and find the annual commercial repayment is 2.3 times the other. The latter, therefore, may conclude that the 40-yr repayment without interest is equivalent to approximately a 57% subsidy. If a Government rate of interest of 3% is taken, the subsidy which may be concluded is 42 per cent. The first WPA plan of financing (a portion of Parker Dam was financed in this manner) involved what is usually referred to as a 30% subsidy or "grant."

Referring generally to Government water-supply financing in the United States—after a general financing policy has been determined and applied, it has been true too often that subsequently "adjustments," "moratoria," and "commutation of payment" have intervened so that cost repayments have been considerably less than were originally anticipated. Members of the engineering profession of recognized high standing occasionally have supported these modifications as being entirely justified and sufficiently common so that they are regarded as a likely if not an inevitable consequence of Government financing and therefore that Government financing is tantamount to subsidization. Nearly all of the new water-supply construction involving approximately a half-billion dollars and forming part of the development of the Colorado River below Lee's Ferry has been financed by the Government, and concessions (for instance, reduction of the interest rate of the Reconstruction Finance Corporation to the Metropolitan Water District of Southern California from 5 to 4⅛%) have already been granted. In April, 1940, a determined effort was being made before Congress to obtain legislation permitting a reduction in the interest rate applying to repayments of the Government's investment in the Hoover Dam from 4 to 3 per cent. The bill embodying this change seems to have the benediction of all concerned. It also provides for deferring the beginning of repayment of that portion of the cost of the Hoover Dam which was allotted to flood control for about 50 years. Some will regard this as a substantial measure of subsidy and back their arguments with financial computations.

Throughout the American West, there is general agreement that Government subsidization of irrigation development to the extent of the current Bureau of Reclamation (Department of Interior) plan of financing is justified. Few advocate subsidization beyond this. It is the policy now in operation. Literature stating the pros and cons is voluminous. There is no need for the writer to re-state the case or particularly support it.

In Mr. Elder's discussion, mention is made of some technical points of hydrography. One is the apparent excessive loss from Lake Mead (not necessarily all real loss). Present data and their analyses seem to support fully the existence of excessive apparent loss and to indicate that a portion of the loss is genuine. It is probable that aeroplane surveys, which were referred to, and more data from the operation of the reservoir, will help to decide the nature of losses.

A second hydrographic point is the probable frequency of major drought periods. In this connection it is generally becoming apparent, even to optimists, that the region is having one drought that is truly severe. Data regarding recurrence of major droughts are indirect or only inferential. The First

Annual Report of the Metropolitan Water District of Southern California states:[8]

> "By analyses of old flood records, mission crop reports, miscellaneous notes, diaries of early settlers, and other indirect means, H. B. Lynch, consulting hydraulic engineer of Los Angeles, has extended these [meteorological] records with reasonable dependability back to the year 1769. The results show that there has been no change in mean climatic conditions in Southern California since that date at least, although drought periods are indicated that were both more severe (1822–32) and of longer duration (1842–83) than any that have occurred during the present century."

A third hydrographic point is the amelioration possibility of additional large reservoirs. This is a possibility, but the beneficial effects of reservoirs are strained by the evaporation at the water surface and the long hold-over storage required in connection with the Colorado River. The writer would emphasize in this connection, and in some others regarding the hydrography of the Colorado River (or other rivers), that statements of general effects of hydrographic components on the solution of hydrographic problems may be misleading. In place of dependence on such statements it is recommended that an analysis be made by suitable periods, and for each, to apply the conditions with respect to withdrawals and losses of water as they would occur and credit accretions of water provided by natural conditions and the conditions provided by judicious management of available water resources. All must be conducted with due regard to the fact that at any assumed time no extensive knowledge of future conditions is available other than general knowledge indicated by past experience. Pronouncements of how river discharge is to be regulated or distributed (based upon knowledge of average flows and unsupported statements of how reservoirs affect stream regimen) should be subjected to careful scrutiny.

A fourth hydrographic point is the prospective quantity of water that will be lost at or below Imperial Dam. In the preparation of Table 1 a rather low value for this loss was selected from among several authoritative statements of much larger values. Included in the losses in question, in addition to that required for silt sluicing, are: Incidental flood spill, return drainage water allowed to flow to waste because of high salt content, inevitable wastage from irrigation canals, and possible wastage of part of the power water on the Yuma Project or on the All-American Canal near the heading of the Imperial Canal.

---

[8] First Annual Report of the Metropolitan Water District of Southern California, 1939, p. 8.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2087

AN IMPROVED METHOD FOR ADJUSTING LEVEL AND TRAVERSE SURVEYS

BY CLARENCE NORRIS,[1] ESQ., AND JULIUS L. SPEERT,[2] ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. GEORGE H. DELL, HOWARD S. RAPPLEYE, R. M. WILSON, CLEVELAND B. COE, GEORGE D. WHITMORE AND C. C. MINER AND W. O. BYRD, ROBERT C. SHELDON, AND CLARENCE NORRIS AND JULIUS L. SPEERT.

SYNOPSIS

A simplified method for adjusting level and traverse surveys is presented in this paper. The results are identical with those obtained by least squares, but the method is so simple as to be understandable by any one possessing an elementary knowledge of algebra and surveying. The labor required for this method of adjustment is considerably less than that required by the usual least-squares methods, and is no greater than that of cruder methods.

The paper is divided into three parts: The main body gives detailed instructions for the adjustment, in such form that the computations may be performed by a clerk familiar with the use of a computing machine. For the benefit of those interested, the theory of this method is developed in Appendix I; and Appendix II explains a simple method of solving the adjustment equations.

INTRODUCTION

All surveys, whether for the determination of altitudes or of relative horizontal positions, are subject to two general types of error—systematic and accidental. Before any adjustment of a survey is attempted, all known systematic errors should be removed, leaving only those of the accidental type to be disposed of by the adjustment.

Frequent azimuth checks are usually made in a well-conducted traverse survey. A preliminary angle adjustment can be applied, therefore, before the computation of latitudes and departures is undertaken. The angle adjustment is so simple and well understood that it will not be discussed herein.

NOTE.—Published in October, 1939, *Proceedings*.

[1] Asst. Topographic Engr., U. S. Geological Survey, Washington, D. C.

[2] Asst. Topographic Engr., U. S. Geological Survey, Washington, D. C.

When latitudes and departures—or differences of latitude and longitude—have been computed for each course in a traverse net, some error will usually appear in the form of imperfect closures in the net. This error is due to two general causes—accidental errors in the straight-line measurements (including other errors treated as accidental), and errors of direction that still remain after the preliminary adjustment of the angles. Because of the complex interrelationship of these errors, and the practical difficulties involved in their separation, it is customary, except in the most precise surveys, to treat the traverse closure as a single accidental error with two components; namely, northing and easting (or latitude and longitude). In this paper it is assumed that all preliminary computations have been performed and that unadjusted north and east coordinates have been determined for both extremities of each line in the net. The resultant closure errors, in northing and in easting, will be treated as independent items, either of which may be adjusted without regard for the other.

Similarly, in a level net, the error will appear as imperfect closures, or differences of elevation, at the junction points. The method of adjustment to be described is applicable equally to a level or traverse net; but, because the latter presents a more general problem, having two errors to be adjusted, special attention will be given to the adjustment of a traverse net.

ADJUSTMENT OF A TRAVERSE NET

The method of adjustment to be described herein is based on the so-called "Junction Point method" for the adjustment of levels developed by Mr. D. H. Baldwin of the U. S. Geological Survey.[3] The theory of the new method is presented in Appendix I of this paper.

To illustrate the method and its application to traverse adjustment, a somewhat involved traverse net will be adjusted in detail. This net is shown diagrammatically in Fig. 1, with the junctions numbered for identification. The stations whose positions are to be held fixed as a result of earlier adjustment or of a more precise survey are indicated in Fig. 1 and are listed in Table 1 with their fixed positions. The results of the preliminary computations of north and east coordinates are listed in Table 2, the upper entry representing

TABLE 1.—GIVEN FIXED STATIONS AND POSITIONS

Station (see Fig. 1)	COORDINATES	
	North	East
A	248,367.22	126,478.29
B	328,946.18	129,817.26
C	315,406.20	154,926.18
D	319,029.42	240,792.22
E	250,612.16	232,163.26
F	293,936.85	187,391.63

---

[3] For a complete description of the Baldwin method see "Topographic Instructions of the United States Geological Survey," by C. H. Birdseye, M. Am. Soc. C. E., *Bulletin No. 788*, U. S. Geological Survey, pp. 156–160.

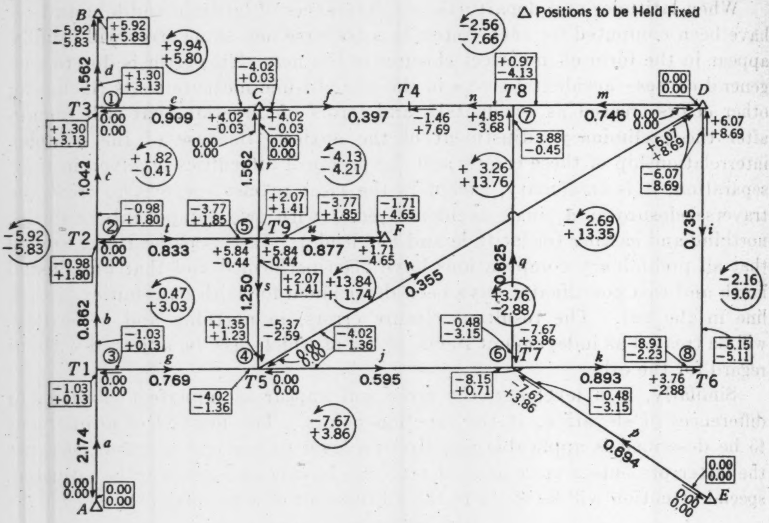

△ Positions to be Held Fixed
B
T3
T2
T1
A
C
T4
T8
D
T9
F
T5
T7
T6
E
a
b
c
d
e
f
g
h
i
j
k
m
n
p
q
r
s
t
u
2.174
0.862
1.042
1.562
0.909
0.397
0.746
0.833
0.877
0.769
0.595
0.893
0.355
0.625
0.735
1.562
1.250
0.694

Fig. 1.—Adjustment Diagram

TABLE 2.—Results of Preliminary Computations

Line	North	East	Computation Book		Number of stations	Station	Line	North	East	Computation Book		Number of stations	Station
			Book	Page						Book	Page		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(1)	(2)	(3)	(4)	(5)	(6)	(7)
a	248,367.22 259,848.50	126,478.29 127,206.14	G20 G20	1 3	23	*A* *T* 1	*k*	260,428.60 261,610.52	201,712.94 239,699.93	G22 G22	6 9	56	*T* 7 *T* 6
b	259,848.50 288,642.37	127,206.14 128,889.43	G20 G20	3 7	58	*T* 1 *T* 2	*m*	319,029.42 318,208.41	240,792.22 217,293.84	G22 G22	10 14	67	*D* *T* 8
c	288,642.37 312,167.15	128,889.43 127,416.52	G20 G20	7 10	48	*T* 2 *T* 3	*n*	318,208.41 316,984.82	217,293.84 182,300.06	G22 G22	14 18	70	*T* 8 *T* 4
d	312,167.15 328,940.26	127,416.52 129,811.43	G20 G20	10 12	32	*T* 3 *B*	*p*	250,612.16 260,420.93	232,163.26 201,716.80	G23 G23	1 5	72	*E* *T* 7
e	312,167.15 315,410.22	127,416.52 154,926.15	G20 G20	13 16	55	*T* 3 *C*	*q*	260,420.93 318,213.26	201,716.80 217,290.16	G23 G23	5 9	80	*T* 7 *T* 8
	315,410.22 316,986.28	154,926.15 182,292.37	G20 G20	16 20	56	*C* *T* 4	*r*	315,406.20 299,416.02	154,926.18 158,889.16	G24 G24	1 3	32	*C* *T* 9
g	259,848.50 255,027.73	127,206.14 159,716.78	G21 G21	1 5	65	*T* 1 *T* 5	*s*	299,416.02 255,022.36	158,889.16 159,714.19	G24 G24	3 6	40	*T* 9 *T* 5
h	255,027.73 319,035.49	159,716.78 240,800.91	G21 G21	5 15	141	*T* 5 *D*	*t*	288,642.37 299,421.86	128,889.43 158,888.72	G24 G24	7 10	60	*T* 2 *T* 9
i	319,035.49 261,606.76	240,800.91 239,702.81	G21 G21	15 19	68	*D* *T* 6	*u*	299,421.86 293,938.56	158,888.72 187,386.98	G24 G24	10 14	57	*T* 9 *F*
j	255,027.73 260,428.60	159,716.78 201,712.94	G22 G22	1 6	84	*T* 5 *T* 7	....	....	....	....	....	....	....

the beginning, and the lower entry the end, of the line indicated in Column (1), in accordance with the direction arrow on the line in Fig. 1. The items in Columns (4) and (5), Table 2, are for the computer's reference to the original computation books. Column (7) identifies the stations at the extremities of the various lines, as shown in Fig. 1. The positions shown in Tables 1 and 2 have been computed in plane coordinates. However, this method may be used equally well for adjusting a traverse in geodetic coordinates, differences of latitude and longitude being substituted for differences of northing and easting in the adjustment computations.

Special Conditions.—A few special and unusual conditions are included herein to show how such conditions may be treated in an adjustment. It will be noted that line *h* in Fig. 1 crosses line *q* without tying to it. The omission of a junction at this point merely eliminates one equation from the adjustment and presents no special problem. Although lines *f* and *n* were surveyed independently in the field, they are in reality a single line in spite of the closure between them, and they are treated as one line in the adjustment, the weight being that of the two lines combined. The same treatment could be applied to lines *i* and *k*; but, to illustrate the method, these are treated as separate lines, closing at junction 8 as indicated.

Junction Closures.—A junction closure is the distance by which the computed position of a point on one line differs from the computed position of the same point on another line. It is positive if the computed position on the closing line is the greater, and negative if it is the smaller. Stated otherwise, the junction closure is the distance that must be subtracted algebraically from the position on the closing line to obtain the corresponding position on the line or station closed upon. A junction closure is indicated by a short arrow parallel to the closing line, pointing toward the line closed upon, and by numbers representing the magnitude of the break or closure. The junction closures indicated in Fig. 1 are derived from inspection of Tables 1 and 2. For example, junction 1 is at the end of line *c* and the beginnings of lines *d* and *e* at station *T* 3. The position of station *T* 3 is the same on all three lines (see Table 2); therefore, zero closures are indicated between line *e* and lines *c* and *d*. The absence of a closure arrow at the extremity of any line implies a zero closure for that line on the junction involved. At junction 4, station *T* 5 has the same position on lines *g*, *h*, and *j*; but cn line *s* its position is 5.37 less in northing and 2.59 less in easting. Therefore, line *s* closes on junction 4 with closures of − 5.37 and − 2.59, as indicated. The position of station *C* on lines *e* and *f* is 4.02 greater in northing and 0.03 less in easting than the fixed position of station *C* (Tables 1 and 2), as indicated by the closures of these lines on the fixed station. Line *r*, Fig. 1, starts at station *C* with the correct final position, and its closure on *C*, accordingly, is zero. All closures, at junctions and at fixed stations, are determined in the same manner. When dealing with items of northing and easting, it is desirable always to write the northing quantity first, thereby avoiding confusion as well as the necessity of labeling each item.

Circuit Closures.—As the next preliminary step in the adjustment, it is desirable to investigate the circuit closures. Although these do not enter directly into the adjustment, they offer a key to the quality of the field work and preliminary computations, indicating places of apparent weakness and frequently aiding in the discovery of blunders.

The circuit closures may be computed by adding all junction closures counterclockwise around each circuit, changing the sign of a junction closure if its closure arrow is opposed to the counterclockwise route around the circuit. For this purpose, a group of lines connecting two fixed stations is treated the same as a closed circuit. In Fig. 1, the circuit closures are shown within the respective circuits, inside a curved arrow which indicates the direction of closure. Each circuit closure, and all the junction closures contained in it, may be checked from Tables 1 and 2 using Fig. 1 as a guide. Proceeding counterclockwise around the circuit as before, subtract the position of the beginning, and add the position of the end, of each line whose direction arrow agrees with the direction of progress around the circuit; reverse the operations, adding the beginning and subtracting the end, for each line whose direction arrow is against the direction of progress around the circuit. If a circuit connects two fixed stations, assume an imaginary line joining them, and treat the fixed positions as the extremities of this imaginary line. The circuit closures obtained by this method should check exactly those determined previously. Any discrepancy might be due to a mistake in the determination of the circuit closure by either method, or to a mistake in one or more junction closures, or to a combination of these mistakes.

The magnitude of any circuit closure in relation to the length of its circuit is a measure of the precision of the survey around that circuit. An excessively large closure might clearly indicate the existence of a gross blunder and lead to its detection. Large closures of opposite signs in two adjoining circuits indicate weakness, or possibly a blunder, in the line common to these circuits. In general, the algebraic difference between the circuit closures on opposite sides of any line may be used to form a rough preliminary estimate of the correction that the adjustment may be expected to force into that line. Thus, any line that appears to be below the standard of the remainder of the work may reasonably be omitted from the adjustment, in order that its weakness shall not detract from the strength of the remainder of the survey.

Weights.—In the adjustment of a level net, the lines are weighted inversely as their relative lengths. For a traverse, however, it is believed that the number of transit stations in a line offers a better index of the strength of that line. For convenience in solving the adjustment equations, to avoid coefficients either too large or too small, it is desirable that the magnitude of the diagonal terms of the equations be between 1 and 10. This condition can be produced most readily by using weights for the lines, such that their average value will be about 1. Therefore, the effective length of, or number of stations in, each line should be divided by the approximate average of all the lengths, or numbers of stations, before weighting. In this adjustment the lines will be weighted in inverse ratio to the number of stations in each, and the weight of each line in this example will be taken as 50 times the reciprocal of the number of stations in it. The number of stations in each line is listed in Table 2, and the weights are marked on the lines in Fig. 1.

As the preliminary positions and the closures have been computed to two decimal places, it is desirable that the weights be used to one place more, or three places, in order that there shall be no loss of accuracy due to uncertainty in the last decimal figure. If any line has been surveyed with greater or less care than the others, to entitle it to special consideration, its weight factor may be changed accordingly. In this example all lines are considered of equal strength.

TABLE 3.—ADJUSTMENT EQUATIONS

Equation	R	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	N	E	S
(1) *ab*		−3.513	+1.042							+ 5.593	+9.134	+12.256
c												
(2) *a*			−2.737	+0.862		+0.833				− 4.865	+0.367	− 4.498
b												
c												
(3) *a*				−3.805	+0.769					0.000	0.000	− 2.174
b												
c												
(4) *a*					−2.969	+1.250	+0.595			− 8.867	−6.322	−15.544
b												
c												
(5) *a*						−4.522				+15.199	+6.563	+19.323
b												
c												
(6) *a*							−2.807	+0.625	+ 0.893	−16.506	+9.963	− 7.237
b												
c												
(7) *a*								−1.768		+ 6.809	−7.754	− 2.088
b												
c												
(8) *a*									− 1.628	− 1.104	−8.959	−10.798
b												
c												
										+ 3.741	−2.992	
										(+0.749)		

Adjustment Form.—After the preliminary examination and preparation are completed, the adjustment may be begun. All junctions between lines of the survey should be numbered, except where lines meet at a fixed station. Thus, there should be a number for each junction for which a correction is to be determined. The form for the table of equations (Table 3) should be prepared, allowing one column for each numbered junction, two absolute columns, and an additional column on each side of the table for later use. Four lines should be allowed for each equation except the first, which requires only three. These groups of lines should be numbered in the same order as the column headings, and the first three lines in each group should be designated *a*, *b*, and *c*, respectively. The fourth line is left blank as a separation between the equations.

In the first equation lines *a* and *b* are combined. As a further guide, a horizontal line may be ruled above each *b*-line.

Equations.—The adjustment equations may now be formed by inspection

TABLE 4.—SOLUTION OF ADJUSTMENT EQUATIONS

Equation	R	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	N	E	S
(1) *ab*	0.284657	−3.513	+1.042							+ 5.593	+9.134	+12.256
c		(−1.0)	+0.2966							+ 1.5921	+2.6001	+ 3.4888
(2) *a*			−2.737	+0.862		+0.833				− 4.865	+0.367	− 4.498
b	0.411862		−2.428	+0.862		+0.833				− 3.206	+3.076	− 0.863
c			(−1.0)	+0.3550		+0.3431				− 1.3204	+1.2669	− 0.3554
(3) *a*				−3.805	+0.769					0.000	0.000	− 2.174
b	0.285796			−3.499	+0.769	+0.296				− 1.138	+1.092	− 2.480
c				(−1.0)	+0.2198	+0.0846				− 0.3252	+0.3121	− 0.7088
(4) *a*					−2.969	+1.250	+0.595			− 8.867	−6.322	−15.544
b	0.357143				−2.800	+1.315	+0.595			− 9.117	−6.082	−16.089
c					(−1.0)	+0.4696	+0.2125			− 3.2561	−2.1721	− 5.7461
(5) *a*						−4.522				+15.199	+6.563	+19.323
b	0.278242					−3.594	+0.279			+ 9.721	+4.855	+11.262
c						(−1.0)	+0.0776			+ 2.7048	+1.3509	+ 3.1336
(6) *a*							−2.807	+0.625	+ 0.893	−16.506	+9.963	− 7.237
b	0.376081						−2.659	+0.625	+ 0.893	−17.689	+9.047	− 9.782
c							(−1.0)	+0.2351	+ 0.3358	− 6.6525	+3.4024	− 3.6788
(7) *a*								−1.768		+ 6.809	−7.754	− 2.088
b	0.616903							−1.621	+0.210	+ 2.650	−5.627	− 4.388
c								(−1.0)	+0.1295	+ 1.6348	−3.4713	− 2.7070
(8) *a*									− 1.628	− 1.104	−8.959	−10.798
b	0.768640								− 1.301	− 6.701	−6.650	−14.651
c									(−1.0)	− 5.1507	−5.1115	−11.2613
					Back solution							
										+ 3.741 (+0.749)	−2.992	
CN		+1.3026	−0.9761	−1.0326	−4.0159	+2.0720	−8.1546	+0.9678	− 5.1507			
CE		+3.1328	+1.7963	+0.1322	−1.3599	+1.4063	+0.7142	−4.1332	− 5.1115			
CS		+3.4355	−0.1796	−1.9004	−6.3755	+2.4787	−8.4396	−4.1653	−11.2613			

from Fig. 1. For this purpose, as well as for the solution, a computing machine is almost indispensable. The original equations are written on the *a*-lines, the other lines being reserved for their solution. The diagonal term for each equation is entered in the line and column bearing the number of the corresponding junction, and is always negative. The coefficient of each of the remaining terms of an equation is entered to the right of the diagonal terms, on the line and in the column corresponding to the extremities of the corresponding line in the survey, as will be illustrated.

Diagonal Terms.—The diagonal term for any equation is the negative sum of the weights of all lines entering the correspondingly numbered junction. For example, at junction 1, Fig. 1, it is − (1.042 + 1.562 + 0.909) = − 3.513, as entered in Column (1), line 1*a*, Table 3. For junction 2, it is − (0.862

+ 1.042 + 0.833) = − 2.737, as entered in Column (2), line 2*a*. Similarly, all diagonal terms are determined from Fig. 1, and entered in Table 3, in the *a*-line and column corresponding to the number of the junction involved.

Coefficients.—The remaining coefficients in Equation (1), Table 3, are the weights of the individual lines entering junction 1 from the other numbered junctions. For example, line *c* enters junction 1 from junction 2, and + 1.042 is entered in Column (2), line 1*a*; lines *d* and *e* enter junction 1 from fixed stations and have no coefficients in the table. Similarly, line *b* enters junction 2 from junction 3, and + 0.862 is entered in Column (3), line 2*a*; line *t* enters junction 2 from junction 5, and + 0.833 is entered in Column (5), line 2*a*; line *c* enters junction 2 from junction 1, and + 1.042 is found to be already entered in Column (2), line 1*a*. It is not entered in Column (1), line 2*a*, because that space is to the left of the diagonal term, and only those spaces to the right of the diagonal terms are used. In like manner, the coefficients for all the equations are entered in Table 3.

Absolute Terms.—The absolute terms for northing and easting of each equation may now be computed and entered in Columns N and E, respectively. Each of these terms is the sum of the weighted closures of all lines entering its junction; it is the sum of the products of the weight of each line entering the junction by the sum of the closures in that line computed from the contiguous junction. It is essential that the sign and direction of each closure be observed carefully, the closure sign being held if its direction is toward the junction whose equation is being determined, and reversed if its direction is from the junction. For example, N for Equation (1) is 1.042 (0) + 1.562 (+ 5.92) + 0.909 (− 4.02) = + 5.593; E is 1.042 (0) + 1.562 (+ 5.83) + 0.909 (+ 0.03) = + 9.134. Similarly, for Equation (7), N = 0.625 (+ 7.67 + 4.85) + 0.397 (− 4.02 + 1.46) + 0.746 (0) = + 6.809; E = 0.625 (− 3.86 − 3.68) + 0.397 (+ 0.03 − 7.69) + 0.746 (0) = − 7.754. The absolute terms should be computed to the same number of decimal places as the coefficients.

When all the absolute terms have been computed and entered in Table 3, the adjustment equations are complete and may be checked. The absolute terms should be checked first, as follows: Compute the weighted closures of all lines into all the fixed stations in the same manner in which the absolute terms were computed; write the sum of all these northing closures (+ 3.741) at the bottom of the (N)-column and the sum of all these easting closures (− 2.992) at the bottom of the (E)-column. Including these items, the sum of all entries in each of these columns should be zero, or very nearly so. A small discrepancy in the last place may be due to the rounding off of residual decimals beyond the last significant figure. If preferred, the (N)- and (E)-columns may be checked simultaneously by recording only the sum of the northing and easting closures into the fixed stations (+ 0.749), and adding this to both columns to equal zero.

In computing all sums of products, the individual products need not be recorded, but should be accumulated in the computing machine. For this purpose the decimal points should be placed properly on all the dials, and should be observed carefully in all operations. Only the keyboard and multiplier dial are cleared after each multiplication, the product being left in the machine. Negative products are obtained with the reverse multiplier. The product dial is cleared only when the desired sum has been recorded.

(S)-Column.—The (S)-column at the right of Table 3 offers a further check on the formation of the equations, as well as a step-by-step check on the solution. The (S)-item for each equation is determined by adding to the negative sum of the weights of all lines entering a junction from fixed stations the entries in the (N)-column and (E)-column. For example, at junction 1, lines *d* and *e* enter from fixed stations; therefore S1 equals minus the sum of the weights of these lines plus N plus E, or $-(1.562 + 0.909) + 5.593 + 9.134 = +12.256$. At junction 3, only line *a* enters from a fixed station; hence $S3 = -2.174 + 0.000 + 0.000 = -2.174$. There are no fixed stations contiguous to junction 2; therefore, S2 equals the sum of N and E, or $-4.865 + 0.367 = -4.498$.

When the (S)-column is completed, the sum of all entries on line 1*a* through N and E should equal S1 exactly. The sum of all entries in Column (2) and on line 2*a* through N and E should equal S2. Similarly, the sum of Column (3) and line 3*a* should equal S3. Thus, each equation is checked by adding its column down to the diagonal term, then across through E, to equal its (S)-entry. A mistake in any coefficient will appear as a discrepancy in the check of the two equations containing that term. A discrepancy peculiar to a particular equation indicates a mistake in either its diagonal term or its (S)-term.

Solution of Equations.—The equations in Table 3 may now be solved by the Doolittle method[4] as shown in Table 4. The detailed solution especially adapted and simplified for use with a computing machine is explained with complete instructions in Appendix II. The (N)-, (E)-, and (S)-columns are treated identically in the forward solution, the check on the (S)-column being applied in the usual manner after each step. In the back solution, an independent computation is made for CN, CE, and CS for each equation, using only the appropriate items for each, and omitting the other two quantities from each computation. The back solution is checked, step by step, by adding CN, CE, and -1.0000 for each equation; the sum should equal CS for that equation, very nearly.

Corrections.—The (CN)- and (CE)-items of the back solution are the required corrections to be applied to the corresponding junctions of Fig. 1. These are rounded off to two decimal places and are shown in small rectangles with arrows pointing to the line at the extremity of which the correction is applied. It should be borne in mind that these corrections apply to the position closed upon at each junction. The correction to the extremity of the closing line may be determined by subtracting the junction closure from the computed correction. Thus, at junction 1, the computed correction applies equally to lines *c*, *d*, and *e*. At junction 7, the computed correction applies to lines *m* and *n*; the northing correction to the end of line *q* is + 0.97 − (+ 4.85) = − 3.88; its easting correction is − 4.13 − (− 3.68) = − 0.45. The corrections to lines at fixed stations are equal and opposite to the closures on these stations. The difference between the corrections at the ends of a line represents the correction to be distributed or prorated through the line.

---

[4] "Adjustment of Observations," by T. W. Wright and J. F. Hayford, Second Edition, pp. 114–121. Detailed instructions for the solution are given in several publications of the U. S. Coast and Geodetic Survey.

Final Test.—At this point the adjustment of the junctions is complete, corrections having been determined for each extremity of each line. One final check may be applied, however, to test the efficacy of the adjustment. According to the theory of least squares, the most probable values for the corrections should satisfy the condition that the sum of the weighted corrections applied to all lines around a given junction will be zero. This test may be applied around each numbered junction by multiplying the weight of each line entering it by the difference of the corrections applied to its extremities. For this purpose, the N- and E-corrections may be tested together, the corrections at the far end of each line being applied with their proper signs, and those at the near end with reversed signs. For example, at junction 1, 1.042 (− 0.98 + 1.80 − 1.30 − 3.13) + 1.562 (+ 5.92 + 5.83 − 1.30 − 3.13) + 0.909 (− 4.02 + 0.03 − 1.30 − 3.13) = + 0.02. In view of the number and magnitude of the items included in this test, the discrepancy of 0.02 is reasonable, and indicates a satisfactory adjustment. The same test may be applied to all numbered junctions in the net. In using lines *f* and *n* to check at junction 7, the break at station *T* 4 should be considered, thus: 0.397 (− 4.02 + 0.03 + 1.46 − 7.69 − 0.97 + 4.13) + 0.746 (0.00 + 0.00 − 0.97 + 4.13) + 0.625 (− 0.48 − 3.15 + 3.88 + 0.45) = − 0.01. In this example, the discrepancies at the eight numbered junctions vary from 0 to 0.02, with an average value of 0.009.

SUMMARY

A brief summary of the operations required for a complete adjustment may be of value in the formation of a mental picture of the entire process. For a traverse net, the preliminary computations should be completed and the N- and E-coordinates, or the latitude and longitude, should be determined for each junction. For a level net the orthometric elevation of each junction point should be determined. Then the operations of the adjustment should be performed in the following order:

1.—Prepare a diagram of the survey, showing junction closures, circuit closures, and weights of all lines. Number all junctions, except those at fixed stations.

2.—Prepare a form similar to Table 3 for the adjustment equations.

3.—Compute the diagonal terms as the negative sum of the weights of all lines entering each numbered junction.

4.—Enter the weight of each line in the space numbered according to the junction numbers at the ends of that line, using only those spaces to the right of the diagonal terms.

5.—Compute and enter in the (N)- and (E)-columns, respectively, the weighted closures in northing and easting of all lines entering each numbered

junction. In a level adjustment, of course, there would be only one absolute column.

6.—Check the (N)- and (E)-columns by summing them to zero with the weighted closures of all lines entering all fixed stations.

7.—Compute the (S)-entry for each equation as the negative sum of the weights of all lines entering its junction from fixed stations, plus its (N)- and (E)-entries. Add each equation down its column to its diagonal term, then across through its absolute terms to equal its (S)-term, exactly.

8.—Solve the equations by the Doolittle method, forward and back solutions.

9.—On the adjustment diagram, enter the results of the back solution as corrections to the lines closed upon at the numbered junctions.

10.—Subtract the junction closures from the computed corrections to determine the corrections at the ends of the closing lines.

11.—Test the adjustment by comparing with zero the sum of the weighted corrections in all the lines around each numbered junction.

12.—Distribute or prorate the difference between the end corrections of each line to the stations in that line, to complete the adjustment.

CONCLUSION

It is believed that the results of any survey, even a poor one, can be improved by appropriate adjustment, so that all parts of the survey shall fit together with mathematical nicety after the computations are completed. In other words, any survey worth making is worth improving at small additional cost by careful adjustment. The relatively small cost of adequately adjusting a survey undoubtedly improves the results to a greater degree than could be obtained by a similar expenditure for improved field technique. Obviously, poor work cannot be made good by adjustment, but the data resulting from any survey can be improved in value and usefulness if all mathematical inconsistencies are removed. When the funds available for a survey are limited, it is believed desirable to reserve a portion of the money for the purpose of adjustment.

In traverses, except for the most precise geodetic surveys, it is customary to combine all accidental errors in the closures of latitude and longitude, or latitudes and departures. In this respect, practically all methods of adjustment agree. Beyond this, variations exist, from adjustment by "estimation" or judgment to adjustment by least squares. For any adjustment, regardless of method, it is desirable that a diagram be prepared to show the various closures and the relative weights of the respective lines. The diagram, although not an essential part of the adjustment, is an invaluable aid in the determination and distribution of the adjustment corrections.

It is generally recognized that the method of least squares offers the best adjustment possible in accord with the theory of probability. Although the method described herein is not derived through a consideration of the squares of residuals or errors, it is based on the principle that the correction at any

junction shall be a weighted mean of the corrections brought in from the contiguous junctions (see Appendix I). This principle is in perfect harmony with the principle of least squares, and the two methods give identical results.

It may be emphasized, furthermore, that the adjustment equations that are set up by inspection from the adjustment diagram are identical in every respect with the normal equations that would be obtained by the use of the observation-equations method in least squares. This new method has a definite advantage over the latter in that the equations may be written by inspection, requiring no auxiliary tables. It has similar advantages over the condition-equations method, in that a table of correlate equations is not needed, and the solution of the adjustment equations gives the desired corrections directly, without the necessity of applying the correlates to the correlate equations and computing the line corrections before the junction corrections can be determined.

It is believed that the method of adjustment described herein is so simple as to be easily understood by any one familiar with the elements of surveying and of algebra. Although the Doolittle method is suggested as the shortest known method of solving the type of simultaneous equations obtained, these equations may be solved equally well by the usual algebraic methods. (For algebraic solution, the equations should be written in complete form; see Appendix I.) Although the method described is identical with least squares, it is not expected to have the psychological effect that the name "least squares" has upon most surveyors—the suggestion of something too complicated to be understood except by an expert mathematician. Some technical understanding is helpful, of course, in preparing and studying the closure diagram, but the mechanics of the adjustment are such that, if the instructions are observed carefully, a complete adjustment can be made by a clerk familiar with the use of the computing machine.

As compared with cruder methods of adjustment, this method requires little additional work for the theoretically most probable distribution of the errors. The only work peculiar to this method is the writing and solving of the simultaneous equations. In this respect there is a compensating saving of labor, particularly in a traverse adjustment, in that corrections for both north and east coordinates are obtained simultaneously, whereas in most methods these must be adjusted separately. As compared with the Baldwin method for a level adjustment, this method gives a more balanced adjustment, eliminates the need for exercise of judgment in the selection of junctions, and permits adjustment of an entire network at a single set-up. The time required for adjusting an ordinary level net by this method compares favorably with that required by the Baldwin method.

Because this method is suitable for the adjustment of any type of line survey, another possibility becomes apparent. In many surveys it happens that traverse and levels are run over the same routes for the purpose of determining horizontal and vertical control for all monuments. If the lines are so run that the weights to be used for adjusting the levels are in the same ratio as the weights to be used for adjusting the traverse, or can be made so, then both traverse and levels can be adjusted at the same time by using three absolute columns in the adjustment equations instead of one or two. With three absolute columns, the (S)-column should include the effect of all three, and the solution may be completed in exactly the same manner.

ACKNOWLEDGMENTS

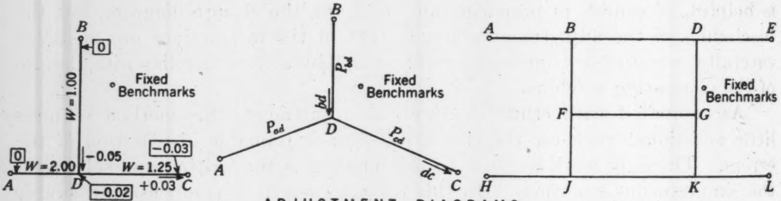
This method of adjustment was developed in the Section of Computing, of the U. S. Geological Survey, between 1936 and 1938. The short-cut approach to the theory through simple algebra was suggested by Mr. Norris; Mr. Speert arranged the mechanics of the adjustment in collaboration with other members of the Section. This paper is published by permission of the Director of the Geological Survey, United States Department of the Interior.

APPENDIX I

THEORY

As a necessary preliminary to the development of the new theory, the Baldwin method will be reviewed briefly.[3]

In the simple level net shown in Fig. 2, line *A D C* runs from bench mark *A* (the elevation of which is fixed) to another fixed bench mark, *C*, closing on bench mark *C* with an error of + 0.03 ft as indicated by the small arrow at point *C* and the number + 0.03 (this means that the elevation of bench mark *C*, as determined by line *A D C*, is 0.03 ft greater than the fixed elevation of bench mark *C*). Similarly, line *B D* begins at fixed bench mark *B* and closes on bench mark *D* with an error of − 0.05 ft (or the elevation of bench mark *D* as determined by line *B D* is 0.05 ft less than the unadjusted elevation determined by line *A D C*). Assume also that the lengths of *A D*, *B D*, and *D C* are 5, 10, and 8 miles, respectively.



ADJUSTMENT DIAGRAMS

FIG. 2 FIG. 3 FIG. 4

For the adjustment, the lines are to be weighted inversely as their lengths or in the ratio 2.00 : 1.00 : 1.25, as indicated in Fig. 2. This adjustment is based on the principle that the adjusted elevation of bench mark *D* is to be a weighted mean of the elevations brought in through the various lines tying to it. This principle is in strict accord with the principle of least squares and may be readily derived from it. The weighted mean is most conveniently obtained by considering corrections and closures instead of actual elevations. The correction to line $A\ D$ at point A (as shown in a small rectangle in Fig. 2) is zero; the correction to line $C\ D$ at point C (to correct the closure of + 0.03 ft) must be − 0.03; the correction to line $B\ D$ at point B is zero. As there are no breaks in continuity between bench mark D and the indicated corrections at points A and C, the corrections at these points may be used for weighting at point D; in order to refer to a common elevation at point D, the closure of − 0.05 must be added to the correction at point B before the latter may be weighted at bench mark D. Then the weighted correction at point D may be computed as the sum of the products of the various line corrections by the weights of their respective lines, divided by the sum of the weights; that is,

$$D = \frac{2.00(0) + 1.00(-0.05) + 1.25(-0.03)}{2.00 + 1.00 + 1.25} = -0.02 \text{ ft.}$$

To express this principle in a general equation (see Fig. 3), let A, B, C, and D represent the corrections at points A, B, C, and D, respectively; ad, bd, and cd represent the mathematical breaks or closures in the lines A to D, B to D, and C to D, respectively; and P_{ad}, P_{bd}, and P_{cd} represent the weights of the respective lines. Then $D = \dfrac{P_{ad}\,(A + ad) + P_{bd}\,(B + bd) + P_{cd}\,(C + cd)}{P_{ad} + P_{bd} + P_{cd}}$;

or

$$(P_{ad} + P_{bd} + P_{cd})\,D = P_{ad}\,(A) + P_{bd}\,(B) + P_{cd}\,(C) + P_{ad}\,(ad) + P_{bd}\,(bd) + P_{cd}\,(cd) \qquad (9)$$

Transposing all terms to the right side of the equation,

$$0 = -\,(P_{ad} + P_{bd} + P_{cd})\,D + P_{ad}\,(A) + P_{bd}\,(B) + P_{cd}\,(C) + P_{ad}\,(ad) + P_{bd}\,(bd) + P_{cd}\,(cd) \qquad (10)$$

Note that cd represents the closure or breaks in the line C to D, whereas dc represents the same closure or breaks in the reverse direction; thus,

$$cd = -\,dc \qquad (11)$$

Equation (10) represents a general equation for the correction at any junction in terms of the corrections at other points and the mathematical breaks in continuity of the lines connecting the junction with the other points. Ordinarily, the Baldwin method has been used to determine the correction at a particular junction when the corrections at the other points have been known.

Extension of Baldwin Method.—The same principle may be applied to a more complicated level net, as indicated in Fig. 4. Denote the correction to any junction by the letter used to designate that junction; the weight of a line by the letter P and the subscript letters of the ends of the line; and the mathematical breaks in continuity in any line by the lower-case letters of the ends of the line, thus: The break or closure in B to D is bd, that in D to B is $db = -bd$, etc.

As A, E, H, and L are fixed points, their corrections are known from the closures at these points, and need not be computed. Equations similar to Equation (10) may be written for each of the remaining junctions. Thus, at B,

$$\begin{aligned} 0 = -(P_{ab} + P_{bd} + P_{bf})\,B + P_{ab}\,(A) + P_{bd}\,(D) + P_{bf}\,(F) \\ + P_{ab}\,(ab) + P_{bd}\,(db) + P_{bf}\,(fb) \end{aligned} \qquad (12)$$

If $P_{ab} + P_{bd} + P_{bf}$ (the sum of the weights of all lines entering junction B) is designated P_b, and $P_{ab}\,(ab) + P_{bd}\,(db) + P_{bf}\,(fb)$ (the sum of the products of the breaks of the lines into the junction by the weights of these lines) is designated Q_b, Equation (12) becomes

$$0 = -P_b\,(B) + P_{ab}\,(A) + P_{bd}\,(D) + P_{bf}\,(F) + Q_b \qquad (13)$$

Similarly, equations for the other junctions to be adjusted may be written as follows: At D,

$$0 = -P_d\,(D) + P_{bd}\,(B) + P_{de}\,(E) + P_{dg}\,(G) + Q_d \qquad (14)$$

at F,

$$0 = -P_f\,(F) + P_{bf}\,(B) + P_{fg}\,(G) + P_{fj}\,(J) + Q_f \qquad (15)$$

at G,

$$0 = -P_g\,(G) + P_{gk}\,(K) + P_{fg}\,(F) + P_{dg}\,(D) + Q_g \qquad (16)$$

at J,

$$0 = -P_j\,(J) + P_{hj}\,(H) + P_{fj}\,(F) + P_{jk}\,(K) + Q_j \qquad (17)$$

and, at K,

$$0 = -P_k\,(K) + P_{jk}\,(J) + P_{gk}\,(G) + P_{kl}\,(L) + Q_k \qquad (18)$$

As there are six unknown junction corrections and six equations, a rigid solution is possible.

Equations (13) to (18) may be written most conveniently by inspection in the form of a table, the unknown corrections being used to designate the columns, in which only the numerical coefficients need be placed (see Table 5).

If the equations are written in the same sequence as the columns, it will be seen that the negative terms form a diagonal, about which the table is symmetrical, except for the Q-column; that is, all coefficients to the left (or to the right) of any negative term appear again above (or below) it in the same relative position. Thus, except for the Q-column, the table may be read across or down with the same results. This symmetry makes it possible to condense the table by omitting all coefficients to the left of the "diagonal" terms. Then each equation is read down to its diagonal term and across through its absolute term.

In this form the equations may be solved readily by the Doolittle method[4] to give the required junction corrections (see Appendix II).

TABLE 5.—ADJUSTMENT EQUATIONS

At point:	B	D	F	G	J	K	Q
B	$-P_b$	$+P_{bd}$	$+P_{bf}$				$+Q_b$ $+P_{ab}$ (A)
D	$+P_{bd}$	$-P_d$		$+P_{dg}$			$+Q_d$ $+P_{de}$ (E)
F	$+P_{bf}$		$-P_f$	$+P_{fg}$	$+P_{fj}$		$+Q_f$
G		$+P_{dg}$	$+P_{fg}$	$-P_g$		$+P_{gk}$	$+Q_g$
J			$+P_{fj}$		$-P_j$	$+P_{jk}$	$+Q_j$ $+P_{hj}$ (H)
K				$+P_{gk}$	$+P_{jk}$	$-P_k$	$+Q_k$ $+P_{kl}$ (L)

This method of adjustment is not only suitable for levels, but it is even more advantageous in its application to a traverse net. Instead of a single closure error at each junction, a traverse survey has two closures—in latitude and in longitude, or in northing and easting. These two closures are considered independently and, in other methods of adjustment, require individual treatment. However, by the method presented herein, the equations for the corrections at any particular junction are identical except for the absolute or Q-terms. Thus, both sets of equations may be combined in a single table, similar to Table 5, but having two absolute columns—one for northing and one for easting. The effort required for the solution of both sets is only slightly greater than that required for one alone.

To provide a check on the formation and solution of the equations, an extra column is added to the table. The items in this column are dimensionless numbers, each being the sum of the coefficients and absolute terms of its equation. This sum column is not an essential part of the adjustment, but makes it possible to check each step before proceeding to the next.

APPENDIX II

SIMPLIFIED SOLUTION OF ADJUSTMENT EQUATIONS BY THE DOOLITTLE METHOD

The Doolittle method is a semi-mechanical method for solving simultaneous equations of the symmetrical type formed in this method of adjustment. It is much quicker than the regular algebraic solution, especially when there are many equations to be solved simultaneously, and is particularly adaptable to the use of a computing machine. By this method, as by others, the equations are combined successively, one of the unknowns being eliminated at each step. When all the equations have been combined, the resulting equation contains only one unknown, which may be evaluated readily. In the back solution, the evaluated unknowns are substituted successively back into the combined equations for the evaluation of the preceding unknowns.

In Tables 3 and 4, the original equations have been written on the *a*-lines, and each equation is read down its column to the diagonal term, then across through the absolute terms. The *b*-lines, in Table 4, contain the combined equations, each of which includes the influence of those preceding it. Because of the elimination of one additional unknown, each combined equation contains one term less than the preceding one. The combined equations, on the *b*-lines, are read straight across. The equations on the *c*-lines are mathematically identical with the corresponding *b*-equations, but they express the principal unknown as an explicit function of the remaining unknowns. The items on the *c*-lines are the multipliers to be used in the formation of the combined equations on the succeeding *b*-lines. It is obvious that in the first equation the *a*- and *b*-lines are identical and need be written only once.

The procedure for combining and solving the equations follows a few definite rules; when once mastered, it becomes almost mechanical and requires practically no mental effort. The method is best illustrated by completing the solution of the equations in Table 3. The complete solution is shown in Table 4.

The first step in the solution is the formation of Equation (1*c*) from Equation (1*b*). Equation (1) represents the correction at junction 1 in terms of the corrections at all other junctions; therefore the entry in Column (1) (the diagonal term of Equation (1*a*)) will be called the principal term of Equation (1*b*). Similarly, the *b*-item under each diagonal term will be called the principal term of its equation. To express the correction at junction 1 as an explicit function of the other corrections, its coefficient should be made − 1.0. This is done by multiplying all items on line 1*b* by the negative reciprocal of the principal term. The negative reciprocal (0.284657) of − 3.513 is entered, for convenience, in Column R at the left of the table and is multiplied by the items on line 1*b* to give the items shown on line 1*c*. Line 1*c* may now be checked by adding it across through the absolute columns, including − 1.0 for the first term, to equal the (S)-item, very nearly. A small difference may be due to the dropped residual decimals beyond the last significant figure. In general, the *c*-lines should be recorded to one decimal place more than the *b*-lines, in order to retain the same number of significant figures.

The next step in the solution is the formation of Equation (2*b*) by combining Equation (1*b*) with Equation (2*a*). In this step, everything to the left of Column (2) is ignored and each item in Equation (1*b*) is multiplied by the (1*c*)-term in Column (2); then it is added to the (2*a*)-item in its own column. Thus, Equation (2*b*) is formed as follows:

1*b*-2(+ 1.042) × 1*c*-2(+ 0.2966) + 2*a*-2(− 2.737) = 2*b*-2(− 2.428)
1*b*-3(0) × 1*c*-2(+ 0.2966) + 2*a*-3(+ 0.862) = 2*b*-3(+ 0.862)
1*b*-5(0) × 1*c*-2(+ 0.2966) + 2*a*-5(+ 0.833) = 2*b*-5(+ 0.833)
1*b*-N(+ 5.593) × 1*c*-2(+ 0.2966) + 2*a*-N(− 4.865) = 2*b*-N(− 3.206)
1*b*-E(+ 9.134) × 1*c*-2(+ 0.2966) + 2*a*-E(+ 0.367) = 2*b*-E(+ 3.076)
1*b*-S(+12.256) × 1*c*-2(+ 0.2966) + 2*a*-S(− 4.498) = 2*b*-S(− 0.863)

Each item is designated by its line and column, thus: Item 2*a*-3 is on line 2*a*, Column (3). Equation (2*b*) may now be checked by adding it across to equal its (S)-term, very nearly. Equation (2*c*) is formed from Equation (2*b*) in exactly the same manner as Equation (1*c*) was formed from Equation (1*b*). It is checked by adding it across from − 1.0 through the absolute columns to equal its (S)-term, as before.

Equations (1*b*), (2*b*), and (3*a*) are now combined to form Equation (3*b*). In this step, everything to the left of Column (3) is ignored, and the *c*-items in Column (3) are used as multipliers for their respective *b* equations. As there is no item in 1*c*-3 (giving a zero multiplier), Equation (1*b*) drops out of this step. Each item in Equation (2*b*) is multiplied by item 2*c*-3 and then added to the (3*a*)-item in its own column to form Equation (3*b*).

Equation (3*b*) is added across to check on its (S)-term. Equation (3*c*) is formed from Equation (3*b*), and checked. This same routine is followed throughout the forward solution. The first four equations are all short ones and are combined easily, as explained.

Equation (5*b*) is formed by combining Equations (1*b*), (2*b*), (3*b*), (4*b*), and (5*a*). In this step, everything to the left of Column (5), Table 4, is ignored, and the *c*-items in Column (5) are used as multipliers for their respective *b*-equations. As there is no item in 1*c*-5, Equation (1*b*) drops out of this step. To compute an item in any column on line 5*b*, multiply each *b*-item in the column by the *c*-item of its equation in Column (5), and add the products to the (5*a*)-item above the term being computed, thus:

$$\begin{aligned}
2b\text{-}5 \times 2c\text{-}5 + 3b\text{-}5 \times 3c\text{-}5 + 4b\text{-}5 \times 4c\text{-}5 + 5a\text{-}5 &= 5b\text{-}5\\
4b\text{-}6 \times 4c\text{-}5 &= 5b\text{-}6\\
2b\text{-N} \times 2c\text{-}5 + 3b\text{-N} \times 3c\text{-}5 + 4b\text{-N} \times 4c\text{-}5 + 5a\text{-N} &= 5b\text{-N}\\
2b\text{-E} \times 2c\text{-}5 + 3b\text{-E} \times 3c\text{-}5 + 4b\text{-E} \times 4c\text{-}5 + 5a\text{-E} &= 5b\text{-E}\\
2b\text{-S} \times 2c\text{-}5 + 3b\text{-S} \times 3c\text{-}5 + 4b\text{-S} \times 4c\text{-}5 + 5a\text{-S} &= 5b\text{-S}
\end{aligned}$$

always using the *c*-items in Column (5) as the multipliers for the corresponding *b*-items in the other columns. Equation (5*b*) is now checked by adding across to 5*b*-S. Equation (5*c*) is formed by multiplying Equation (5*b*) by the negative reciprocal of its principal term, and is checked by adding across, including the − 1.0, to equal 5*c*-S, very nearly. Equation (6*b*) is formed by combining the previous *b*-equations with Equation (6*a*) as already explained, and is checked on 6*b*-S.

Back Solution.—The same routine is followed to the formation and check of Equation (8*c*), which expresses the correction at junction 8 as an explicit function of no other unknown. In other words, the corrections at junction 8 appear in the absolute columns on line 8*c*, and are − 5.1507 in northing and − 5.1115 in easting. As the first unknowns evaluated, these are entered on lines CN and CE, respectively, in Column (8) of the back solution. To provide a check, 8c-S is entered in CS-8. Then, in Column (8), CN + CE − 1.0000 should equal CS, very nearly. (This check was already applied on line 8c.)

The corrections at junction 7 may be evaluated next. Equation (7c) expresses each correction at junction 7 as an explicit function of the corresponding correction at junction 8, which is already known. The corrections at junction 8 (which will be called, collectively, C8) are therefore substituted into Equation (7c) to evaluate C7. As these corrections, in reality, are independent variables, it will be necessary, in the back solution, to substitute them separately, and to evaluate but one correction at a time. Thus:

$$\text{CN-8} \times 7c\text{-8} + 7c\text{-N} = \text{CN-7}$$
$$\text{CE-8} \times 7c\text{-8} + 7c\text{-E} = \text{CE-7}$$

and,

$$\text{CS-8} \times 7c\text{-8} + 7c\text{-S} = \text{CS-7}$$

The (CS)-item is not a correction but is used as a check on the others and is computed in the same manner. For the check, as before, CN-7 + CE-7 − 1.0000 = CS-7. Similarly, C6 is computed by substituting C7 and C8 into Equation (6c), and is checked by adding CN, CE, and − 1.0000 to equal CS, very nearly. Thus:

$$\text{CN-7} \times 6c\text{-7} + \text{CN-8} \times 6c\text{-8} + 6c\text{-N} = \text{CN-6}$$
$$\text{CE-7} \times 6c\text{-7} + \text{CE-8} \times 6c\text{-8} + 6c\text{-E} = \text{CE-6}$$

and,

$$\text{CS-7} \times 6c\text{-7} + \text{CS-8} \times 6c\text{-8} + 6c\text{-S} = \text{CS-6}$$

In like manner, all evaluated corrections are substituted successively into the next preceding *c*-equation for the evaluation of the corrections at the next preceding junction. The same check is applied to all; namely, CN + CE − 1.0000 = CS. Note that in Equation (5c) there are no items in Columns (7) and (8); C7 and C8, therefore, do not enter into the computation of C5, as their multipliers are zero. Similar conditions are found throughout the remainder of the back solution, materially reducing the labor required for this part of the problem. When all the corrections have been computed and checked, back to C1, they are entered on the diagram on Fig. 1, as already explained.

DISCUSSION

George H. Dell,[5] **Assoc. M. Am. Soc. C. E.** (by letter).—A clear and interesting description of a simplified adaptation of the junction point method to the adjustment of level and traverse nets is presented in this paper. It may be worth noting that the results given by this method are identical with those obtained by basing the adjustment upon appropriate circuit-closure and route-closure conditions. It should also be observed that in applying the junction point method it is necessary that at least one point in the net be treated as a fixed point, or otherwise an indeterminate system of equations will result.

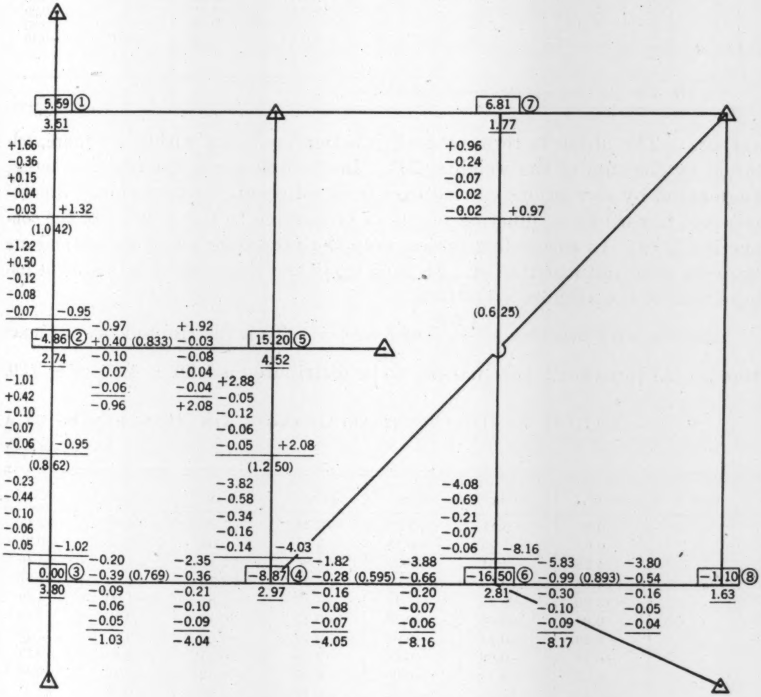

5.59 ①
3.51
+1.66
−0.36
+0.15
−0.04
−0.03
+1.32
(1.042)
−1.22
+0.50
−0.12
−0.08
−0.07
−0.95
−4.86 ②
2.74
−1.01
+0.42
−0.10
−0.07
−0.06
−0.95
(0.862)
−0.23
−0.44
−0.10
−0.06
−0.05
−1.02
0.00 ③
3.80
−0.97
+0.40 (0.833)
−0.10
−0.07
−0.06
−0.96
+1.92
−0.03
−0.08
−0.04
−0.04
+2.08
15.20 ⑤
4.52
+2.88
−0.05
−0.12
−0.06
−0.05
+2.08
(1.250)
−3.82
−0.58
−0.34
−0.16
−0.14
−4.03
−0.20
−0.39 (0.769)
−0.09
−0.06
−0.05
−1.03
−2.35
−0.36
−0.21
−0.10
−0.09
−4.04
−8.87 ④
2.97
−1.82
−0.28 (0.595)
−0.16
−0.08
−0.07
−4.05
−3.88
−0.66
−0.20
−0.07
−0.06
−8.16
6.81 ⑦
1.77
+0.96
−0.24
−0.07
−0.02
−0.02
+0.97
(0.625)
−4.08
−0.69
−0.21
−0.07
−0.06
−8.16
−16.50 ⑥
2.81
−5.83
−0.99 (0.893)
−0.30
−0.10
−0.09
−8.17
−3.80
−0.54
−0.16
−0.05
−0.04
−1.10 ⑧
1.63

Fig. 5.—Latitude Adjustments

The Doolittle method of solving the normal equations is unexcelled for use in offices where the execution of intricate adjustments is a major occupation. In fact, axisymmetrical systems of equations are so frequently encountered in various branches of engineering that it would be highly desirable to include the Doolittle method in the mathematics requirements of all engineering students.

---

[5] Associate in Civ. Eng., Univ. of Illinois, Urbana, Ill.

For organizations or individuals having to deal only occasionally with adjustments of the type under discussion, or for classroom work where an adequate supply of computing machines may not be available, the solution of the equations by successive convergence, with slide rule, offers a satisfactory substitute for the more precise Doolittle method. Such a solution is illustrated in Fig. 5, which deals with the latitude corrections in the example given by the authors. The absolute term at each junction is shown within a square, followed by the sum of the weights, ΣP. In the first cycle, the absolute terms, augmented by corrections carried over from adjacent junction points, are distributed toward these junction points in proportion to the weights of the connecting lines. In succeeding cycles, only the preceding set of corrections are brought over and distributed. In each cycle the sequence of adjustment follows that of the authors' equations.

TABLE 6.—LIST OF QUANTITIES DISTRIBUTED

Cycle	JUNCTIONS IN FIG. 5*							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	+5.59	−3.20	−1.01	−9.07	+10.41	−18.32	+2.73	−6.93
2	−1.22	+1.33	−1.93	−1.39	−0.18	−3.12	−0.69	−0.99
3	+0.50	−0.32	−0.46	−0.80	−0.44	−0.94	−0.21	−0.30
4	−0.12	−0.22	−0.28	−0.38	−0.23	−0.31	−0.07	−0.10
5	−0.08	....	....	....	....	....	....	....

* In Fig. 5 the junction numbers are encircled.

Starting with junction 1, $\frac{1.042}{3.51}$ of 5.59 (= 1.66) is distributed toward junction 2. At junction 2, the quantity to be distributed is 1.66 − 4.86 = − 3.20,

TABLE 7.—DIFFERENTIAL CHANGES (IN HUNDREDTHS OF A
INCREASE IN

Line	dC_{1N}	dC_{1E}	dC_{2N}	dC_{2E}	dC_{3N}	dC_{3E}	dC_{4N}
a	+0.86	−0.11	+2.93	−0.38	+$\underline{7.06}$	−0.91	+2.49
b	−0.05	−1.56	−0.18	−$\underline{5.26}$	+0.09	+2.64	+0.01
c	−4.28	−2.51	+$\underline{8.29}$	+$\underline{4.87}$	+2.18	+1.28	+1.46
d	+$\underline{23.66}$	+$\underline{13.80}$	+$\underline{10.62}$	+$\underline{6.22}$	+2.78	+1.62	+1.87
e	−$\underline{15.90}$	−$\underline{9.25}$	−$\underline{7.14}$	−4.17	−1.87	−1.09	−1.26
f	−0.04	−0.04	−0.12	−0.12	−0.16	−0.16	−0.63
g	−0.29	−0.14	−0.98	−0.49	−$\underline{4.68}$	−2.34	+$\underline{7.78}$
h	−0.19	−0.68	−0.64	−2.28	−0.81	−2.89	−3.29
i	−0.03	−0.10	−0.09	−0.36	−0.12	−0.46	−0.47
j	−0.46	+0.23	−1.55	+0.77	−1.96	+0.98	−$\underline{7.93}$
k	−0.02	−0.09	−0.08	−0.29	−0.10	−0.37	−0.39
m	−0.02	+0.08	−0.06	+0.27	−0.08	+0.34	−0.33
p	+0.02	+0.16	+0.08	+0.54	+0.11	+0.69	+0.43
q	−0.10	+0.08	−0.34	+0.27	−0.43	+0.34	−1.75
r	−1.11	−0.76	−3.75	−2.54	−1.77	−1.20	−$\underline{4.55}$
s	−0.08	−0.02	−0.25	−0.06	+0.50	+0.12	+2.78
t	−2.63	+0.05	−$\underline{8.88}$	+0.16	−1.77	+0.03	+1.23
u	+0.62	+0.84	+2.09	+2.85	+0.99	+1.34	+2.54
Maximum	50.36	30.50	48.07	31.90	27.46	18.80	41.19

the distribution toward junctions 1, 3, and 5 being − 1.22, − 1.01, and − 0.97, respectively. Similarly, at junction 3, the quantity to be distributed is − 1.01 + 0.00, or − 1.01, of which $\frac{0.862}{3.80}$ (= − 0.23) and $\frac{0.769}{3.80}$ (= − 0.20) are distributed toward junctions 2 and 4, respectively. A complete list of the quantities to be distributed is given in Table 6, and it is seen that after 4½ cycles all equations have begun to converge consistently. The fifth set of corrections in Fig. 5 is the result of series summations, obtained by multiplying the fourth set of corrections by the summation factor, which is found as follows: The absolute sum of the last set of eight quantities in Table 6 (= 1.67) was divided by the absolute sum of the preceding eight quantities (= 3.59), giving 0.465 as the average rate of convergence, r. The summation factor, $\frac{r}{1-r}$, therefore, is 0.87. The final values shown in Fig. 5 are the unknowns of the adjustment equations and are obtained by dividing the sums of the various columns by the weights of the corresponding lines.

With regard to the opening paragraph of the authors' "Conclusion," the writer agrees that mathematical inconsistencies should be eliminated by adjustment, and that careful surveys are thereby improved. The writer, however, is inclined to guard against placing undue reliance on such improvement, and believes that a high standard of precision can best be insured by rigorous field work. The results of a least-squares adjustment depend largely upon the weights, and, in a quantitative sense, surveys of high order are much less seriously affected by errors in judging the weights than are surveys of relatively low order.

The traverse used by the authors has been analyzed by the writer with a view to determining the effects of errors or variations in the weights. As a

FOOT) IN THE UNKNOWNS C_{1N}, C_{1E} $\cdots$ C_{8N}, C_{8E} DUE TO 10%
WEIGHTS OF LINES

dC_{4E}	dC_{5N}	dC_{5E}	dC_{6N}	dC_{6E}	dC_{7N}	dC_{7E}	dC_{8N}	dC_{8E}
−0.32	+1.23	−0.16	+0.70	−0.09	+0.25	−0.03	+0.38	−0.05
+0.33	−0.03	−0.88	+0.00	+0.09	+0.00	+0.03	+0.00	+0.05
+0.86	+1.93	+1.13	+0.41	+0.24	+0.14	+0.09	+0.23	+0.13
+1.09	+2.48	+1.45	+0.53	+0.31	+0.19	+0.11	+0.29	+0.17
−0.73	−1.67	−0.97	−0.36	−0.21	−0.13	−0.07	−0.19	−0.11
−0.63	−0.20	−0.20	−2.54	−2.54	−8.81	−8.81	−1.40	−1.40
+3.89	+1.97	+0.98	+2.21	+1.10	+0.78	+0.39	+1.21	+0.60
−11.72	−1.03	−3.66	−0.93	−3.33	−0.33	−1.17	−0.51	−1.82
−1.84	−0.15	−0.57	−1.90	−7.41	−0.67	−2.62	−5.19	−20.23
+3.98	−2.48	+1.24	+9.51	−4.76	+3.36	−1.69	+5.21	−2.62
−1.52	−0.12	−0.47	−1.56	−6.08	−0.55	−2.16	+3.28	+12.79
+1.39	−0.10	+0.43	−1.30	+5.59	−4.54	+19.39	−0.72	+3.07
+2.79	+0.13	+0.87	+1.73	+11.20	+0.61	+3.96	+0.95	+6.14
+1.39	−0.55	+0.43	−7.05	+5.58	+9.52	−7.54	−3.86	+3.06
−3.09	−9.10	−6.18	−1.29	−0.88	−0.46	−0.31	−0.71	−0.48
+0.68	−1.26	−0.31	+0.79	+0.19	+0.28	+0.07	+0.43	+0.11
−0.02	+3.85	−0.07	+0.35	−0.01	+0.13	−0.00	+0.19	−0.00
+3.46	+5.08	+6.91	+0.72	+0.98	+0.25	+0.35	+0.39	+0.54
39.73	33.36	26.91	33.88	50.59	31.00	48.79	25.14	53.37

basis for this study, use was made of expressions for the differentials of the unknowns of the normal equations with respect to the weights,[6] the calculations being based on hypothetical variations of 10% in the weights. The individual effects of these variations in weight are shown in Table 7, quantities of about 0.05 ft or greater being underlined, and the maximum combined effects being shown in the final line. For example, a 10% increase in the weights of lines *d* and *e* would produce corresponding changes of + 0.2366 ft and − 0.1590 ft, respectively, in the value of C_{1N}, and the maximum combined effect of such errors in all weights would be 0.5036 ft for this particular unknown.

Because of the complexity of the factors involved, it appears not unreasonable to suppose that, unless great care is taken, the inaccuracies in the assumed weights of some of the lines may be as great at 50%. The effects of variations greater than 10% in the weights would be approximately proportional to the quantities shown in Table 7. Therefore, it is seen that variations of 50% in the weights of certain individual lines would produce corresponding variations of from 0.25 ft to 1.00 ft or more instead of the underlined quantities, and the maximum combined effects of such errors or variations would range from 0.94 ft to 2.67 ft, approximately. However, if it were possible to limit the errors in the weights to the order of 10%, the corresponding errors in the final coordinates could not be materially in excess of 0.50 ft, and would probably be within 0.25 ft in most of the quantities.

It is thus seen that, in order to use the least-squares adjustment to best advantage as a means of improving the results of a low-order survey at a saving in field costs, it is desirable that great care should be exercised in determining the weights. Among the factors upon which the weight of a given line depends are: (*a*) The length of line, (*b*) the number of instrument stations or set-ups it contains, (*c*) the number of independent measurements (repetitions) entering into the data, (*d*) atmospheric conditions, (*e*) the character of the terrain, and (*f*) the quality of instruments and survey methods, including the personal equations or relative skilfulness of the various observers. It would appear that the development of a practical method of taking these factors into account in determining the weights would be a valuable step.

The authors are to be highly commended for presenting such a clear description of their improved junction point method and for their success in producing a really useful simplification of the method in question.

HOWARD S. RAPPLEYE,[7] M. AM. SOC. C. E. (by letter).—After all is said and done, what appears simplest to each individual is that with which he is most familiar. The method of adjustment proposed in the paper by Messrs. Norris and Speert undoubtedly appears to be simple to them because they are thoroughly familiar with it; yet the writer, who attempted to duplicate a previously completed adjustment of a simple level net by the new method, as a means of comparison, was not at all fully convinced that the statement (see "Conclusion")—"* * * if the instructions are observed carefully, a complete adjustment can be made by a clerk familiar with the use of the computing machine"—was not slightly overdrawn. Now that he has actually made an adjustment by the method, and has found that the results agreed within one unit in the last place with the results of the previous adjustment by another method, the writer can see that the proposed method has much to recommend it.

---

[6] *Bulletin No. 309*, Sections 7, 10, and 13, Univ. of Illinois Eng. Experiment Station, Urbana, Ill.

[7] Mathematician, U. S. Coast and Geodetic Survey, Washington, D. C.

Henry G. Avers, Assoc. M. Am. Soc. C. E., has presented[8] the complete adjustments of a level net, including six closed circuits and either one or two fixed elevations, solved by both the condition-equation and observation-equation methods. As a check on the method proposed by Messrs. Norris and Speert, this same net was adjusted by their method, and it was found that the resulting table of adjustment equations, except the "constant-term" and "check-sum" columns, was identical with that used in the observation-equation method[9] with all signs changed.

As to the relative speed of the various methods of adjustment, the writer is not yet sufficiently familiar with the Norris-Speert method to make an adequate comparison; but at present he fails to see any marked advantage of the proposed method over the older methods in this respect.

The fact that in the proposed method practically the same routine is used for the adjustment of both horizontal and vertical control surveys is an advantage. The routine is quite mechanical and, once one becomes familiar with the details of the procedure, little or no thought need be given to the underlying mathematics. A strict attention to algebraic signs, and care in the manipulation of the computing machine, should lead to excellent results.

However, the writer would like to caution any one who is attempting for the first time to adjust a traverse or level net by this method, and with no help other than the paper as printed, to select a small adjustment for practice purposes. Otherwise, failure to follow all the exacting detail may lead to erroneous results and eventual abandonment of a perfectly good method as being "too complicated."

In the writer's opinion a sample of a small level-net adjustment should have been included in the paper. He suggests that it be included in the closing discussion. It should be carried at least as far as the formation of the complete set of normal equations. An explanation of the application of the resulting corrections in the case of a level-net adjustment should also be included in order to make the paper more generally useful to those who may want to adjust levels as well as traverse.

R. M. WILSON,[10] M. AM. SOC. C. E. (by letter).—In 1927 a readjustment of the first-order triangulation net covering the United States was begun by the U. S. Coast and Geodetic Survey. The result changed, slightly, the geodetic coordinates for established triangulation stations throughout the country. Corrections to obtain proper agreement with the re-defined geodetic reference datum were thus imposed upon all dependent surveys which had used those stations before the readjustment.

---

[8] "Manual of First-Order Leveling," by Henry G. Avers, *Special Publication No. 140*, U. S. Coast and Geodetic Survey, pp. 53–74.

[9] *Loc. cit.*, p. 73.

[10] Chf., Section of Computing, U. S. Geological Survey, Washington, D. C.

The U. S. Geological Survey had in its files the accumulated results of its own control surveys, consisting of triangulation done since 1879 and transit traverse done since 1899, all referred to the old North American Datum. Because of the great volume of records involved, it was a real problem to find an economical way to revise the results of these surveys to make them consistent with the new North American Datum of 1927. The scheme adopted to refer the older triangulation to the new datum has been described by Mr. Speert in another paper.[11]

The older transit traverse by the Geological Survey had been extended into areas far from supporting triangulation. Now, however, arcs of new triangulation by the Coast and Geodetic Survey pass through many areas where support for the older traverse was needed urgently. In 1936, an allotment of funds by the U. S. Public Works Administration made it possible to send out field parties to make ties between the older traverse and the new triangulation. Thus the results of the traverse required revision not only because of changes in the coordinates of triangulation stations from which the traverse was originally extended, but also because of the strengthened ties to new arcs of triangulation.

Generally the traverse in a local project was given simple line-by-line adjustment as soon as it was computed, and much of it was subsequently readjusted one or more times due to the influence of newer traverse in later adjacent projects. In order that the effect of these old and sometimes doubtful local adjustments may be eliminated in the general revision to the new datum, the separate lines or elements of the traverse net are now (1940) being taken from the records as originally computed, freed from the corrections of all previous adjustments. Then they can be fitted together again, simultaneously and upon equal terms, under the present conditions of stronger basic control.

A suitable and conventional method of least squares was already available when these general revision adjustments were begun about 1934. Elements of the net were assembled into groups until a convenient number of circuits were formed and condition equations, one for each circuit, were solved in the usual manner. The volume of work thus accomplished is indicated by the following data. The number of circuits in one solution usually was about fifteen, although a few contained as many as twenty-five, and one contained thirty-six. About 180 such solutions were completed, all together involving some 40,000 linear miles of transit traverse surveys.

Out of that background the Norris-Speert method emerged. Now it is the number of unfixed junction points involved, rather than the number of circuits, that determines the size of an individual solution. The elements to be adjusted are assembled into groups as before, but with about twenty equations preferred instead of fifteen; in two instances, groups containing more than fifty equations have been solved. Since 1938 about seventy-five of these solutions have been done, so that nearly 18,000 additional miles of traverse lines have been adjusted by the new method. Approximately 43% of the traverse done before the results of the 1927 triangulation adjustment were available still needs readjustment, however. The Norris-Speert method has been used also to adjust a large part of the new traverse work of the Geological Survey, at the rate of about 7,000 miles per yr. It will be seen, therefore, that the method has been fully tested in practice.

---

[11] "Readjustment of Triangulation Datum," by Julius L. Speert, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1002.

The paper seems well written, and it is difficult to imagine how the ideas it contains could be more clearly expressed. Nevertheless, to one familiar with the computation there comes a feeling of regret that a really simple routine should be so masked, by the necessity of conveying it in print, as possibly to hide its simplicity from any one not having the perseverance to feel his way through it the first time.

The basic principle, which can be grasped readily by any one familiar with elementary algebra, is stated in the third paragraph of Appendix I thus: "* * * the adjusted elevation of bench mark *D* is to be a weighted mean of the elevations brought in through the various lines tying to it." This is easy to achieve if the "lines tying to it" all come from points whose elevations are fixed and known. The condition may be expressed as a simple algebraic equation. If a line comes from another point (called *G*, perhaps), where the elevation is not known, the equation can still be written for the elevation of *D* by including the elevation of *G* as an unknown. Another equation can then show the elevation of *G* with the elevation of *D* as an unknown. Then these two simple algebraic equations can be solved simultaneously for the unknown elevations at the two points *D* and *G*. Extending this principle, simultaneous equations, properly weighted, can be written for all unadjusted junction points of a complicated net. Solving them, the elevation determined at each junction will then be "* * * a weighted mean of the elevations brought in through the various lines tying to it." That principle is the important disclosure; the remainder of the paper is a description of the computing forms and routines as used by the Geological Survey to apply the principle efficiently in practice.

CLEVELAND B. COE,[12] ASSOC. M. AM. SOC. C. E. (by letter).—In presenting a method that may be used by any surveyor, to adjust the results of his work, the authors of this paper are to be commended greatly. Unfortunately, it is true that surveyors and designers alike tend to "shy away" from anything that suggests higher mathematics, and the name "least squares" is something to terrify the unwary. Consequently, the presentation of a simplified method

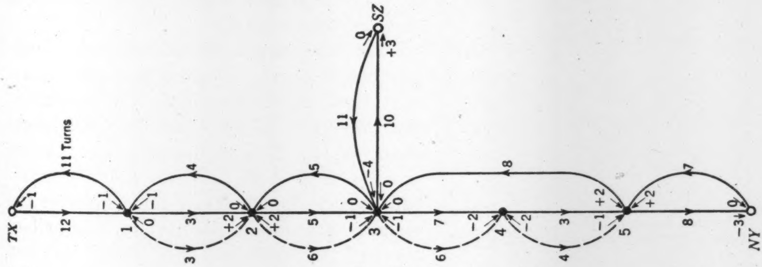


FIG. 6

---

[12] Associate Highway Engr. (TVA), Chattanooga, Tenn.

is a genuine service to the profession. The writer searched many texts without success for a simple method of adjustment for a series of interlocking traverses in connection with a fairly extensive contour survey of the town of Norris, Tenn., and the Norris-Speert presentation would have been extremely welcome.

TABLE 8.—Leveling Notes

Bench Mark	Bench Levels	
	Number of turns	Elevation
TX*(El. 72.22)		72.22
	12	
1		85.06
	3	
2		80.44
	5	
3		95.83
	7	
4		65.37
	3	
5		42.08
	8	
NY*(El. 25.01)		24.98
Check Levels		
NY		25.01
	7	
5		42.10
	8	
3		95.83
	5	
2		80.44
	4	
1		85.05
	11	
TX		72.21
Tie to Third Fixed Bench Mark		
3		95.83
	10	
SZ*(El. 76.16)		76.19
Check Levels		
SZ		76.16
	11	
3		95.79
Cross-Section Levels		
1		85.06
	3	
2		80.46
	6	
3		95.82
	6	
4		65.35
	4	
5		42.07

* Fixed bench mark.

Inasmuch as the application given in the paper is to traverse closures, the writer feels that the paper would be made more valuable to the average engineer by an example of adjustment in a very simple and typical case of levels run to establish bench marks along a surveyed route. For this purpose consider the notes in Table 8, reduced from a series of level lines. This is the common case of running from one fixed bench mark, closing on another, and establishing between them a number of new marks; then running back, turning on each mark, and closing on the original. A third set of readings is provided when the cross sections are taken and an additional tie to a third fixed bench mark also enters the adjustment. For clearness, the field procedure and closure results are shown in Fig. 6. For convenience, the closure results, originally in hundredths of a foot, are shown as integers. The elevations obtained by the first line run (shown in Fig. 6 as straight) are taken as the basic elevations, to which any computed correction is to be applied. Therefore, all closures on this line are zero except the final closures on fixed bench marks.

The lines are weighted inversely as the number of turns. This is a reasonable assumption, as the foresights and backsights are presumed to be equal and not longer than 300 ft. It is assumed further that the cross-section levels are not taken as carefully as the bench levels and therefore the weights are made, arbitrarily, one half of those of the bench and check levels, as given in Table 9. These weights are obtained by setting 3 on the CI-scale of the slide rule over 1 on the D-scale and reading directly under each of the other number of turns.

The table is then filled out in accordance with the Norris-Speert method, following these general rules (see Table 4);

1. Fill in all *a*-lines:
    - 1*a*-1 Negative sum of all weights entering 1,
    - 1*a*-2 Sum of weights entering 2 from 1; and similarly for 2*a*-2, 2*a*-3, etc.
2. Fill in N-column:
    - 1*a*-N Sum (weights × net closures) entering 1; and similarly for 2*a*-N, etc.
    - Check: Sum (weights × net closures) entering fixed bench marks. Place this sum at the bottom of N-column and add all items in the column, which gives zero.
3. Fill in S-column:
    - 1*a*-S Negative sum of weights entering 1 from a fixed bench mark, plus 1*a*-N; and similarly for 2*a*-S, etc.
    - Check: 1*a*-S = sum of all entries in line 1*a*;
      2*a*-S = sum of all entries in line 2*a*, plus all in Column (2); and similarly for 3*a*-S, etc.

TABLE 9.—WEIGHTS APPLIED TO LEVEL NOTES

Levels	Weights, for the Following Number of Turns:								
	3	4	5	6	7	8	10	11	12
Bench and check levels.....	1.00	0.75	0.60	0.50	0.43	0.38	0.30	0.27	0.25
Cross-section levels.........	0.50	0.38	....	0.25	....	....	....	....	....

TABLE 10.—ADJUSTMENT EQUATIONS

Equation	R	(1)	(2)	(3)	(4)	(5)	N	S
(1) *ab*		−2.77	+2.25				−1.75	−2.27
c								
(2) *a*			−3.70	+1.45			+2.50	+2.50
b								
c								
(3) *a*				−3.08	+0.68	+0.38	−3.24	−3.81
b								
c								
(4) *a*					−2.06	+1.38	−0.63	−0.63
b								
c								
(5) *a*						−2.57	+3.14	+2.33
b								
c								
							−0.02	

This completes the original entries in the table and the results are shown in Table 10.

The problem is then solved by the Doolittle method in accordance with the following general rules (see Table 11):

1. 1*b*-R = negative reciprocal of 1*b*-1;

2. Multiply all entries in line 1*a* by 1*b*-R and enter results in line 1*c*;

3. 2*b*-2 = 1*b*-2 × 1*c*-2 + 2*a*-2;

4. If there are any other entries in line 1*b*, multiply them by 1*c*-2 and add to the next *a*-entry below to get the next *b*-entry;

5. 2*b*-R = negative reciprocal of 2*b*-2;

6. Any final *b*-entry is the sum of the products of all *b*-entries and *c*-entries above it, plus the last *a*-entry;

7. Repeat until table is complete;

8. All *b*-lines in column N are filled by multiplying all *b*-entries above by the *c*-entries in the column in question and adding the *a*-entry;

9. Similarly for the *b*-lines in column S;

10. Compute the *c*-entries in columns N and S—1*c*-N = 1*b*-N × 1*b*-R and 1*c*-S = 1*b*-S × 1*b*-R—and similarly for 2*c*-N, etc.; and

11. Check the *b*-lines and *c*-lines by adding across. The S-column is the sum of the other columns.

TABLE 11.—SOLUTION OF ADJUSTMENT EQUATIONS

Equation	R	(1)	(2)	(3)	(4)	(5)	N	S
(1) *ab*	+0.361	−2.77	+2.25				−1.75	−2.27
c		−1.0	+0.812				−0.632	−0.820
(2) *a*			−3.70	+1.45			+2.50	+2.50
b	+0.510		−1.87	+1.45			+1.08	+0.66
c			−1.0	+0.775			+0.576	+0.353
(3) *a*				−3.08	+0.68	+0.38	−3.24	−3.81
b				−1.96	+0.68	+0.38	−2.40	−3.30
c				−1.0	+0.347	+0.194	−1.223	−1.682
(4) *a*					−2.06	+1.38	−0.63	−0.63
b	+0.549				−1.82	+1.51	−1.46	−1.77
c					−1.0	+0.830	−0.801	−0.971
(5) *a*						−2.57	+3.14	+2.33
b	+0.800					−1.25	+1.46	+0.21
c						−1.0	+1.168	+0.168
							−0.02	
CN (in 0.01 ft)		−0.753	−0.150	−0.938	+0.169	+1.168		
CS		−1.752	−1.149	−1.938	−0.831	+0.168		
CN in feet		−0.01	−0.00	−0.01	+0.00	+0.01		
Basic elevation		85.06	80.44	95.83	65.37	42.08		
Final elevation		85.05	80.44	95.82	65.37	42.09		

Solve backwards for each individual correction, shown at bottom of Table 11, as follows:

1. Correction at last bench mark is the same as the last *c*-entry in the N-column;

2. To obtain any correction, multiply all previously determined corrections by the *c*-entries in the line of the wanted correction and add to the N-value in that line.

Check: Fill out the CS-line by the same process, using the S-entries.

3. Then CN − CS = + 1.

Attention is called to the results immediately adjacent to the fixed bench marks; namely:

From	To	Difference, in feet by survey
TX	1	12.84
1	TX	12.84
SZ	3	19.63
3	SZ	19.64
NY	5	17.09
5	NY	17.10

A common inference is that the correct differentials between the foregoing bench marks are 12.84, 19.635, and 17.095, respectively, and many surveyors would start their adjustment of the net on this assumption. The computation of all corrections simultaneously shows the fallacy in this reasoning, as the most probably correct differentials prove to be 12.83, 19.66, and 17.08.

Inasmuch as the final elevations in this case are only to the nearest 0.01 ft, slide-rule computations were used throughout.

The writer agrees that the slight additional time required to adjust a circuit is well worth while and hopes that the average surveyor will be helped to grasp the mechanical process by the additional specific example set forth in this discussion. Acknowledgment is made to Mr. Speert for his suggestions as to the application of the new method to this typical example.

GEORGE D. WHITMORE,[13] M. AM. SOC. C. E., C. C. MINER,[14] ASSOC. M. AM. SOC. C. E., AND W. O. BYRD,[15] ESQ. (by letter).—The method of traverse and level adjustment described by the authors is correctly termed an "improved" and "simplified" method. Since April, 1939, these methods have been used consistently by the computing personnel of the Tennessee Valley Authority (TVA), to the exclusion of all other methods, for all traverse and level surveys of third-order or less accuracy. The authors infer, and the writers agree, that the described method may not always be ideally suited for adjustment of "most precise geodetic traverse" (first-order accuracy), since in traverse control of such precision, it is not always feasible to assume that all closure errors remaining after an azimuth adjustment are accidental errors of latitude and departure. In the cases of first-order and second-order level networks, the TVA computers continue to use the least-squares procedure recommended by the U. S. Coast and Geodetic Survey.

A point which always should be included in a paper on surveys adjustments is this: If the results of a traverse survey are to be published for use by other organizations and individuals, it is suggested that an inverse computation should always be made for each course of the traverse, from the finally adjusted coordinates of each pair of adjacent traverse stations, to determine an azimuth and a distance for each course which will agree exactly with the published adjusted coordinates. Otherwise, the preliminary or unadjusted azimuth and distance usually will disagree slightly with the values computed from adjusted coordinates, often to the confusion of the casual user of the traverse survey.

---

[13] Chf. of Surveys, Maps and Surveys Div., TVA, Chattanooga, Tenn.

[14] Chf. Computer, TVA, Chattanooga, Tenn.

[15] Asst. in Chg. of Computing, TVA, Chattanooga, Tenn.

The writers concur with the authors' statements that: (1) Results by the "improved" method are identical with results by the conventional least-squares method; (2) results by the "improved" method usually seem more logical and more consistent than results by the junction-point method; and (3) the labor involved in using the "improved" method is not greater and is often less than the labor required for any cruder method.

The writers are glad to note that the authors have stressed the use of a diagram as a desirable preliminary of this method. A detailed diagram showing the magnitude and signs of closures, direction of closures, length of line, or other data fixing weights, is almost an absolute prerequisite in any method of adjustment. As emphasized by the authors, such a diagram shows at once the lines having largest closures, and the locations of lines most likely to contain the large errors. Most really experienced survey computers appreciate this and make use of adjustment diagrams accordingly. There is a decided tendency among inexperienced computers, however, to try to work without a diagram on which all essential data are shown. Hence they often become hopelessly confused and end with results far different from those which the mere inspection of a diagram would reveal to be the most logical. If it does nothing else, the diagram gives the computer an over-all or "bird's-eye view" of the system which he is adjusting—certainly a desirable thing if the computer is to do any more than mere mechanical work.

The use of the number of set-ups in a traverse line as the basis for the weight of line, in contrast to using the length of line, appears to be a desirable improvement. The writer often has used a weight which would fall about midway between the weight determined by length and by the number of set-ups. In this connection, it would appear that no harm would result from rounding off each weight value to the nearest single decimal place, although, of course, in the solution itself the terms must be extended to the proper number of decimals. The authors suggest that where level control lines are parallel to the traverse lines, corrections can be determined through one solution for both the traverse coordinates and the elevations. In such a case, the use of the number of instrument set-ups in each traverse line as the basis for its weight may not always prove to give the most logical weight for the parallel level line. Computers who face the situation of paralleling traverse and level lines should consider, thoroughly, this matter of best weights for each type of survey before combining the level and traverse adjustments in one solution. For instance, in adjusting levels, the weight for each line could be based, conceivably, on any one or any combination of the following: (1) The number of level-instrument set-ups; (2) the total difference of elevation (including both uphill and downhill); (3) the length of the line; (4) the average length of sight; or (5) (as suggested in the paper) the number of transit stations in the parallel traverse line.

The unique feature of the method described appears to be in the use of the unadjusted position or elevation value at each junction point, instead of in the attempt to determine first an approximately adjusted value which will lie quite near the finally adjusted results. Through this procedure, a considerable portion of the "assumed-minus-observed" terms becomes zero, thus greatly reducing the number of operations and making the adjustment more completely adaptable to machine computation. It is found that by using the adjustment diagrams as recommended by the authors, the adjustment equations of the "improved" method can be tabulated from the diagram immediately, thus eliminating the necessity for preparing the usual auxiliary and correlate tables of the conventional least-squares procedure.

It is found, as stated by the authors, that the mechanics of this improved method are very simple and can be grasped quickly even by freshmen cooperative students who have had no previous computing experience. The writers have adjusted or caused to be adjusted many hundreds of miles of both traverse and levels by using this "improved" method, and recommend it in every way.

ROBERT C. SHELDON,[16] ASSOC. M. AM. SOC. C. E. (by letter).—As progress in engineering and surveying increases the need for, and use of, coordinate systems of surveying, in which all traverses are closed on points with fixed positions, the need for a satisfactory method of adjusting the closing errors of the traverse nets is correspondingly increased. The method presented in the paper fills this need admirably. It was the writer's first thought that any method that used the typical Doolittle method for the solution of the necessary simultaneous equations would be too difficult for the average engineering office to use. The authors, however, have so simplified the preliminary work of forming the equations as to make the method easy to use by any office with a computing machine.

The writer has applied the method on a traverse net that had previously been adjusted by another method. The net contained five fixed stations and eight interior junctions. The previous adjustment had been by what might be called "the single-line" method, or "path-of-least-resistance" method. That is, the field data between two fixed stations were computed into latitude and longitude increments in feet, after which the errors of closure at the second fixed station were distributed over the traverse in proportion to the lengths of the courses. After this distribution, the points on the line were considered as fixed stations and any lines starting or ending on them were adjusted to close.

The fixed points were triangulation stations that had been placed and adjusted so that, without question, their accuracy was of a higher degree than 1 : 10,000. The field methods used in running the traverses were such that accuracies of 1 : 10,000 could have been expected and 1 : 5,000 was required. The lines running between the fixed stations were well within the requirements, but several of the ties between the interior junction points were not.

The corrections to the traverses, to make them close on the fixed points, or junction points, by the single-line method were: Minimum 1 : 9,000, maximum 1 : 3,000, and the average over the net 1 : 6,000. After adjustment by the authors' method, the corrections to the traverses to make them close on the adjusted junction points were: Minimum 1 : 20,000 (except one line practically precise closure), maximum 1 : 5,000, and average over the entire net 1 : 11,000. The adjustment was made using the field values and not those found by the previous adjustment.

---

[16] Asst. Engr., Section of Surveys, The Panama Canal, Balboa, Canal Zone.

The time required in making the adjustment by the authors' method was a matter of hours. With experienced men, this adjustment on a net of this size would seldom require more than two days. The time lost in unsuccessful attempts to find errors in the field and office work on the traverses that did not close, using the single-line method, was a matter of several days.

The writer is in complete agreement with the conclusions of the authors and believes that the method presented will be a valuable aid to any engineering or surveying organization using a coordinate method of surveying. The writer's application of the method was made on the Panama Canal system using degrees, minutes, and feet of both latitude and longitude rather than the northing and eastings used by the authors.

CLARENCE NORRIS,[17] ESQ., AND JULIUS L. SPEERT,[18] ASSOC. M. AM. SOC. C. E. (by letter).—The comments of those who have discussed the paper have added vital information to it and have greatly increased its value to surveyors. A number of questions have been raised, also, which justify further discussion.

The writers cannot conceal their gratification at the unanimity with which most of those who have discussed the paper welcome the new method of adjustment. Although the method has withstood the test of use by the computers of the U. S. Geological Survey, as mentioned by Mr. Wilson, and its merit has been fully demonstrated, it was feared that its simplicity might be so hidden by the complex instructions required to present it that its real value might be lost.

There can be no question that, at first glance, the new method appears complex, and it was anticipated that the claims that it "is so simple as to be easily understood by any one familiar with the elements of surveying and of algebra" and that "a complete adjustment can be made by a clerk familiar with the use of the computing machine" would provoke disagreement. However, the experience of the Tennessee Valley Authority (TVA), as presented by Messrs. Whitmore, Miner, and Byrd, that the method was used satisfactorily "even by freshmen cooperative students who have had no previous computing experience," is a strong argument in support of these claims. The ease with which the method can be mastered is further attested by the experience of the Geological Survey. During the past few years, several computers who have come into the office have grasped the method apparently without difficulty. These men have included recent engineering graduates, a few college graduates with a knowledge of mathematics but no training in engineering or surveying, and a few older field men who have had relatively little mathematical background or have been out of school so long as to have lost all familiarity with it. In each instance, the mechanics of the adjustment were so readily acquired as to justify the writers' claim of simplicity.

---

[17] Asst. Topographic Engr., U. S. Geological Survey, Washington, D. C.

[18] Asst. Topographic Engr., U. S. Geological Survey, Washington, D. C.

In spite of all this, it was not the writers' intention to recommend that adjustments be left entirely in the hands of a clerk. However, it is believed that, under competent engineering supervision, the mechanics of the adjustment can be performed quite satisfactorily by untrained personnel.

The slide-rule solution of the equations suggested by Mr. Dell appears to be an excellent substitute for the Doolittle solution in those adjustments for which slide-rule precision is adequate, particularly for the adjustment of a level net. For a traverse adjustment, however, the entire process would have to be performed twice, and it appears that a single solution by the Doolittle method, even by slide rule, might be quicker.

Mr. Dell's recommendation that the Doolittle method of solving equations be incorporated in the mathematics requirements of engineering students suggests the desirability of including the theory of this method of adjustment in the surveying courses of such students. Because of its complexity and mathematical background, the method of least squares is not, so far as the writers are aware, taught to engineers except in advanced courses in geodesy or mathematics. The principle of weighted means, however, is so basic and simple that even a student of elementary surveying should be able to grasp it and appreciate its significance. Having once absorbed the theory of adjustment, even if the mechanics of the solution have been long forgotten, an engineer should have relatively little difficulty in making a simple adjustment if called on to do so in later years. Then the predicament described by Mr. Coe of searching "many texts without success for a simple method of adjustment for a series of interlocking traverses" could, to an extent, be avoided.

Messrs. Whitmore, Miner, and Byrd, like several of the others, recognize that results by the new method are identical with those by the conventional least-squares methods. Nevertheless, they seem to limit its use by the TVA to surveys of third-order limits or lower, and for first-order and second-order levels continue to use the longer methods described by the U. S. Coast and Geodetic Survey. The writers wonder why this should be so. Although the new method is not recommended for first-order traverse, it is fully adequate for adjusting the most precise levels. It must be emphasized that the new method is equally as "exact" as the conventional least-squares methods, the only difference being that the approach to the normal equations has been "streamlined."

They also suggest that an inverse computation be made to correct the azimuth and length of each course to agree with the adjusted coordinates of its ends. For this purpose, the writers propose the following modification of the formulas used in correcting triangulation lines after readjustment of datum:[11]

$$da = K\,(dE \cos a - dN \sin a) \dots\dots\dots\dots\dots\dots (19)$$

and

$$ds = dN \cos a + dE \sin a \dots\dots\dots\dots\dots\dots (20)$$

In these formulas, dN and dE are the adjustment corrections at the beginning of a course, A, minus those at the other end, B, in feet; K, in seconds, is $\frac{1}{s \sin 1''} = \frac{206{,}300}{s}$; K, in minutes, is $\frac{1}{s \sin 1'} = \frac{3{,}438}{s}$; a is the observed azimuth from A to B; s is the measured length of the course, in feet; and da and ds

are the corrections to be applied to a and s, respectively, to bring them into agreement with the adjusted coordinates of A and B. The quantities da and ds will always be so small that slide-rule computation will afford ample precision. If a number of lines are to be adjusted by these formulas, preparation of a simple computing form would help to standardize the work and save time.

Mr. Dell makes the interesting observation that, in this method, at least one station in the survey must be treated as a fixed point. In an isolated survey, where no "fixed point" is available, any one station may be assigned an arbitrary adjustment correction (preferably zero), and all other adjustment corrections may be referred to that assumed datum. If, later, a more satisfactory datum is established, the entire system may then be shifted by a uniform correction to fit the new datum.

A somewhat similar interesting problem recently came to the attention of the writers. A third-order traverse net was tied at several places to an old first-order traverse line and also to several triangulation stations. The triangulation was on the 1927 North American Datum, but the traverse line was on an old datum and, with the data available, could not be brought to the 1927 datum for the purpose of controlling the adjustment of the third-order net. It was desired to adjust the new traverse to the 1927 North American Datum, and still give the first-order line all the influence in the adjustment to which it was entitled. In other words, the first-order line could not be used to control the datum of the new traverse, but relative positions along its route were to be held as fixed by its original adjustment. Thus, the old line was free to be translated, but could not be distorted by the new adjustment. This was accomplished in the adjustment by treating all ties to the first-order line as if made to a single junction, this junction to receive an adjustment correction the same as any other in the third-order net. This had the effect of including the first-order line in the adjustment, and of giving its sections infinite weight, but at the same time allowing it perfect freedom in shifting its datum. By this procedure, all ties to the first-order line received the same datum correction and were brought to the 1927 North American Datum, and no distorting adjustment corrections were thrown into the parts of the first-order line.

Mr. Rappleye suggested that a sample of a level-net adjustment should have been included in the paper. It is hoped that the adjustment presented by Mr. Coe will fill that need. Although Mr. Coe's example may not be truly a "net," it has all the elements of a larger adjustment and fully illustrates the principles. In this connection, a few points in Mr. Coe's rules relating to Table 11 should be clarified. In the solution of the equations, the N- and S-columns must be carried down simultaneously with the body of the table in order to provide a check on each line before proceeding with the next line. Otherwise, a mistake in the solution may pass undetected and affect some of the subsequently computed values. In other words, rules 10 and 11 should follow immediately after rule 2. Similarly, rules 8 and 9 should be combined with rule 6, emphasizing that the c-factor used is always taken from the column of the principal term of the equation being formed.

Mr. Dell raises a vital point concerning the importance of careful determination of weight factors for the lines of a survey net, and Table 7 gives striking evidence of this importance. As brought out by Mr. Dell and several others, the proper weight of a line is a very complicated function of a large number of uncertain, and at present unmeasurable, factors. It might even appear, at a casual glance, that careful adjustment by any "precise" method is an unwarranted refinement so long as such uncertainty exists in the determination of weights. The problem of weights, however, enters into all methods of adjustment in about the same way, so that, strictly speaking, the matter of determining weights is not involved in the method used in making the adjustment. It appears that, whether weights are assigned in accordance with the simple rules suggested or whether a more complex procedure is used to include other factors involved, a reasonable adjustment may be expected if all the elements of the survey are properly balanced against one another. The importance of including all the elements of a survey in a general adjustment, rather than adjusting line by line, is illustrated by the example presented by Mr. Sheldon. Nevertheless, the writers agree that the determination of weights is perhaps the weakest factor that enters into an adjustment, and a practical means of evaluating all the uncertainties involved is much to be desired.

The wide applicability of this method of adjustment is illustrated by the variety of problems mentioned in the discussion. The traverse example in the paper was presented in terms of feet of northing and easting; the traverse of the Geological Survey, described by Mr. Wilson, is computed and adjusted in terms of degrees, minutes, and seconds of latitude and longitude; the surveys of the Panama Canal system, discussed by Mr. Sheldon, use degrees, minutes, and feet of latitude and longitude. Similarly, in levels, Mr. Coe's example is adjusted in foot units, and the adjustments mentioned by Mr. Rappleye are in metric units. Inasmuch as the units of measurement enter the adjustment only in the absolute columns, and as the absolute columns of an adjustment are not interdependent, it is not even essential that the same units be used in both columns. Thus, the fact that a second of latitude is different in length from a second of longitude does not interfere with a simultaneous adjustment of both. Similarly, under proper circumstances, seconds of latitude and longitude and feet or meters of elevation could be included in the same adjustment.

In conclusion, the writers wish to express their sincere gratitude to all those who have discussed the paper for the interest they have shown and for the valuable suggestions and comments offered.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2088

FLASH-BOARD PINS

By Chilton A. Wright,[1] and Clifford A. Betts,[2]
Members, Am. Soc. C. E.

With Discussion by Messrs. William P. Creager, Lincoln W. Ryder, E. T. Schuleen, John E. Field, Julian H. White, Harry H. Hatch, and Chilton A. Wright and Clifford A. Betts.

Synopsis

Formulas and stress values for the design of pipe supports on the automatic flash-board gates of dam spillways are developed and discussed in this paper. As a result of hydraulic laboratory tests of full-size flash-boards, supplemented by mechanical tests of the pipe supports and check tests of actual field installations outlined herein, data have been obtained which permit accurate predetermination of the lake level at which gates of this type will automatically fall and by-pass flood water.

In addition to the description of the tests and the analysis of the relations between water head, height of flash-boards, and size and spacing of pipe, the results have been consolidated to serve as a guide in the design of flash-board installations. On this basis the United States Forest Service has designed hinged flash-board gates, supported by steel pipes, and has installed them on a number of their dams. Field tests, made at these dams later, yielded results that closely check the laboratory tests.

Flash-Boards

Automatic flash-board gates are a logical development of the conventional flash-board for reservoir water-surface regulation. Being a structure that increases the height of a dam, usually on the spillway section (see Fig. 1), the flash-board is designed to fail under a predetermined head of water and fall, thereby increasing the flood-carrying capacity of the spillway. In the past many flash-board installations have been constructed, using light, inexpensive planks, supported by closely spaced pins, and were lost after failure during a

---

Note.—Published in May, 1939, *Proceedings*.

[1] Prof., Hydr. and San Eng., Polytechnic Inst. of Brooklyn, Brooklyn, N. Y.; formerly Hydr. Engr., National Bureau of Standards, Washington, D. C.

[2] Engr., U. S. Forest Service, Washington, D. C.

flood. Many other types of flash-boards have been used in which failure was induced mechanically by cutting anchor wires or posts or by lifting with traveling hoists.

Conservation of the storage at the top of a reservoir, where every foot of additional depth represents the cream of the storage, is of high economic value during dry periods, particularly on water-power and water-supply projects. On numerous streams where spring floods are followed by extended periods of low flow, flash-boards can be raised toward the end of the high-water stage and the storage can be augmented for carrying through the drought period. Control of water-surface fluctuations within narrow limits is advantageous and sometimes imperative for fish and game lakes, as well as on recreational developments. As a "safety valve" against back-water overflow or sustained inundation, flash-boards are distinctly effective. In arid regions conservation of water usually requires hand gates, but in humid regions where the flood menace, rather than the water loss, is the prime consideration, the cost of hand-operated flood gates is not justified when the same function can be performed by simple, automatic flash-board gates.

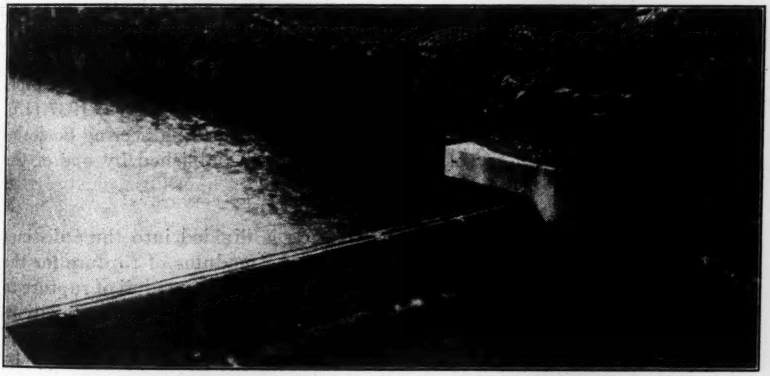


FIG. 1.—HINGED AUTOMATIC FLASH-BOARD GATE, 2.5 FEET HIGH BY 35 FEET LONG, AT LAKE SHERANDO DAM, GEORGE WASHINGTON NATIONAL FOREST

The cost of providing flash-boards is small compared with that of increasing the size of the spillway. In many cases it is almost prohibitive to provide spillway capacity, adequate to handle rare yet disastrous floods, unless flash-boards are used. For the control of large floods, a deep spillway gate can be constructed in multiple panels which will fall successively, thereby limiting the discharge in the channel below, except for exceptional floods, and using the pondage to equalize the flow.

Flash-boards of plank may be converted into flash-board gates by the addition of hinges and stiffening members of wood or steel. The entire structure can be made of wood, steel, or other structural material. After the flood the gates may simply be raised again and the broken supports replaced.

By providing dependable regulation at low cost, thereby protecting not only the dam but also the property below and the lake shores above, the automatic flash-board gate becomes an important factor in water conservation.

NECESSITY FOR TESTS

Such use of flash-boards presupposes failure of the supporting pipe within narrow limits of reservoir water level. Positive failure of the flash-boards is essential to successful operation, inasmuch as the entire dam is in danger if the flash-boards do not function as designed. Region Seven of the U. S. Forest Service had the problem of providing flash-boards for a number of earth dams similar to that shown in Fig. 1, where certain action of the flash-boards was essential.

A search of the published literature[3] for data to be used in the general design of supports for flash-board installations yielded only a few isolated tests of pins, made during construction, and gave no assurance that the pins or pipes would fail reasonably near the desired water head. In fact, discrepancies of 2 ft, or 50%, were obtained by using stresses suggested by different writers.

Thus, tests were required to furnish dependable design data. The Forest Service suggested and co-operated in a series of tests made in the National Hydraulic Laboratory, National Bureau of Standards, on various types and sizes of flash-boards, mounted on a low dam crest, and supported by pins consisting of various sizes of standard commercial steel and wrought iron pipe. A complete report of these tests was prepared under date of April, 1937 (15). A copy of this report has been filed for reference in the Engineering Societies Library.[4] A brief summary of the test results was published by one of the writers in 1936 (16).

SCOPE OF INVESTIGATION

Hydraulic Tests.—The general investigation is divided into three distinct parts: (a) The experimental determination of the modulus of rupture for the pipes used to support the flash-boards; (b) the use of these moduli of rupture to compute the head or water level at which various flash-board installations should fail, using various simplified moment formulas for this purpose; and (c) the comparison of these computed heads with the heads at which failure actually occurs in various flash-board installations in order to determine the accuracy with which the various moment formulas can be used.

The hydraulic tests were made of pipe ranging from $\frac{3}{4}$ in. to 3 in. in size (see Table 1). Spacing between the pipes was varied from 5 to 8 ft. The wider spacing of the larger pipes was found necessary in order that they might collapse under the limited water head available in the laboratory flume. The flash-boards ranged from 1 ft 10 in. to 5 ft 2 in. in height above the dam crest. In addition to measuring the water elevations above and below the dam, and deflections of the pipe under load, its behavior prior to, and after, the yield point was reached was observed. Panels about 3.5 ft by 5 ft having "V" joints and "butt" joints were tried before concentrating the tests on continuous flash-boards fastened together to operate as a single unit.

---

[3] See bibliography in Appendix I. Line numbers in parentheses are used to indicate references.

[4] 33 West Thirty-Ninth Street, New York, N. Y.

The moduli of rupture were computed from these tests and formulas developed for design. Then, as a ready means of checking the formulas, the laboratory set-up was assumed to be a field installation, the formulas were applied thereto, and their accuracy was judged by comparison with the test results.

Mechanical Tests.—In order to determine whether simple mechanical tests could be substituted for the more elaborate hydraulic tests and still yield the equivalent necessary values of modulus of rupture, samples were cut from the pipes used in the hydraulic tests. These specimens were tested as cantilevers in a testing machine, and the results were compared with those derived from the hydraulic tests.

TABLE 1.—DIMENSIONS FOR NESTING PIPE

Pipe size, in inches	Pipe Diameter,* in Inches					
	Standard		Extra Heavy		Sleeves	
	External	Internal	External	Internal	External	Internal
0.75	1.050	0.824	1.050	0.742	1.315†	1.062†
1.00	1.315	1.049	1.315	0.957	1.900	1.610‡
1.25	1.660	1.380	....	....	1.900	1.610
1.50	1.900	1.610	1.900	1.500	2.375	2.067
2.00	2.375	2.067	2.375	1.939	2.875	2.469
2.50	2.875	2.469	2.875	2.323	3.500	3.068
3.00	3.500	3.068	3.500	2.900	4.000	3.548
3.50	4.000	3.548	4.000	3.364	4.500	4.026
4.00	4.500	4.026	4.500	3.826	....	....

* Standard diameters from Federal Specification WW-P-403 for steel pipe.
† Federal Specification WW-P-351 for brass pipe = iron pipe size; this sleeve is the only brass pipe.
‡ Lined with tubing.

Pipe vs. Solid Pins.—Tests made on solid steel pins prior to these pipe tests had revealed certain inherent disadvantages of the solid type. A small variation in diameter, composition, or heat treatment made a relatively large change in load capacity—a striking contrast to the pipe behavior (from previous tests made for the Forest Service by the National Bureau of Standards). Furthermore, extensive tests on mild steel pins by the Pennsylvania Water and Power Company of Baltimore, Md., indicated that the solid pins continued to deflect and carry load until they had deflected nearly 90 degrees.

DESCRIPTION OF EXPERIMENTS

Hydraulic Tests

General Layout.—The hydraulic tests were made in the main concrete flume on the first floor of the National Hydraulic Laboratory (see Fig. 2). It is 12 ft by 12 ft in cross-section, and was supplied with water by three pumps having a combined capacity of 35 cu ft per sec. The pumps discharged into a concrete tank about 25 ft by 32 ft in section, at the upper end of the flume, as shown in Fig. 3. Two motor-operated gates, 6 ft square, were used to control the flow into the flume. During most of the tests, a bulkhead, built for use in another investigation, was present in the flume down stream from the experimental set-up, but finally it was removed to prevent the tail-water from rising too rapidly during the tests. A large drain was provided in the side of the flume down stream from the flash-boards to return the water to the supply basin below the floor.

About 52 ft down stream from the control gates, a concrete dam crest, 21 in. high and about 28 ft long, equivalent to the crest of an actual dam, was constructed across the flume. To obtain this length, it was necessary to place the crest diagonally as shown in Fig. 2. For the preliminary experiments, five flash-boards, each 5 ft long and 3 ft 6 in. high, were placed on the crest and were supported by pipes of the same diameter placed in sockets embedded in the crest of the dam at 5-ft intervals.

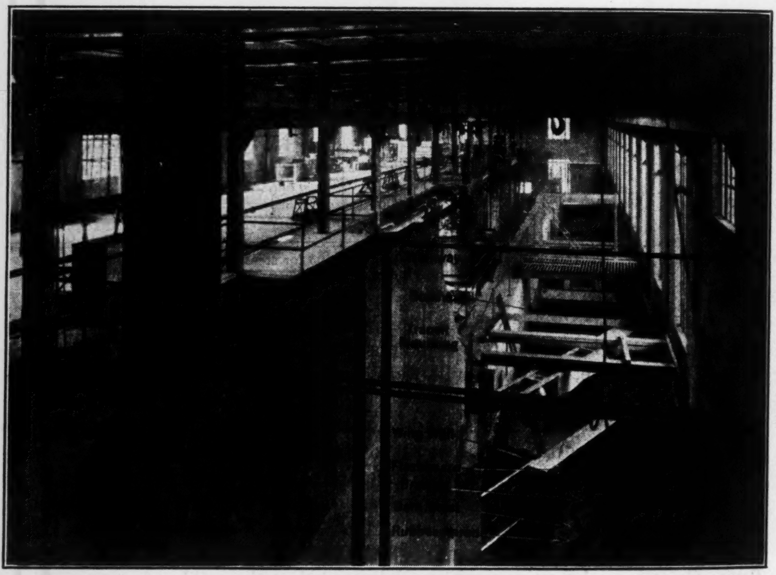


Fig. 2.—Hydraulic Laboratory Flume

In these preliminary tests, because the larger pipes were too strong to fail under a static water load produced by a head-water level even with the top of the flash-boards, a fixed wooden bulkhead with its crest 9 ft 4 in. above the flume floor was provided for increasing the head-water level (see Figs. 3, 4 and 5). Dressed lumber (Douglas fir) $2\frac{5}{8}$ in. thick was used for both the bulkhead and flash-boards, and the horizontal joints were made water-tight by 1-in. splines between the boards. The flash-boards tested in Preliminary Series No. 1 had wedge-shaped "V" joints at the ends (see Fig. 3). In Preliminary Series No. 2, the flash-boards were provided with "butt" joints, or square ends, bound with rubber to reduce leakage as shown in Fig. 4. A rubber apron on the up-stream face of the bulkhead prevented water from discharging over the top of the flash-boards as they deflected during the test.

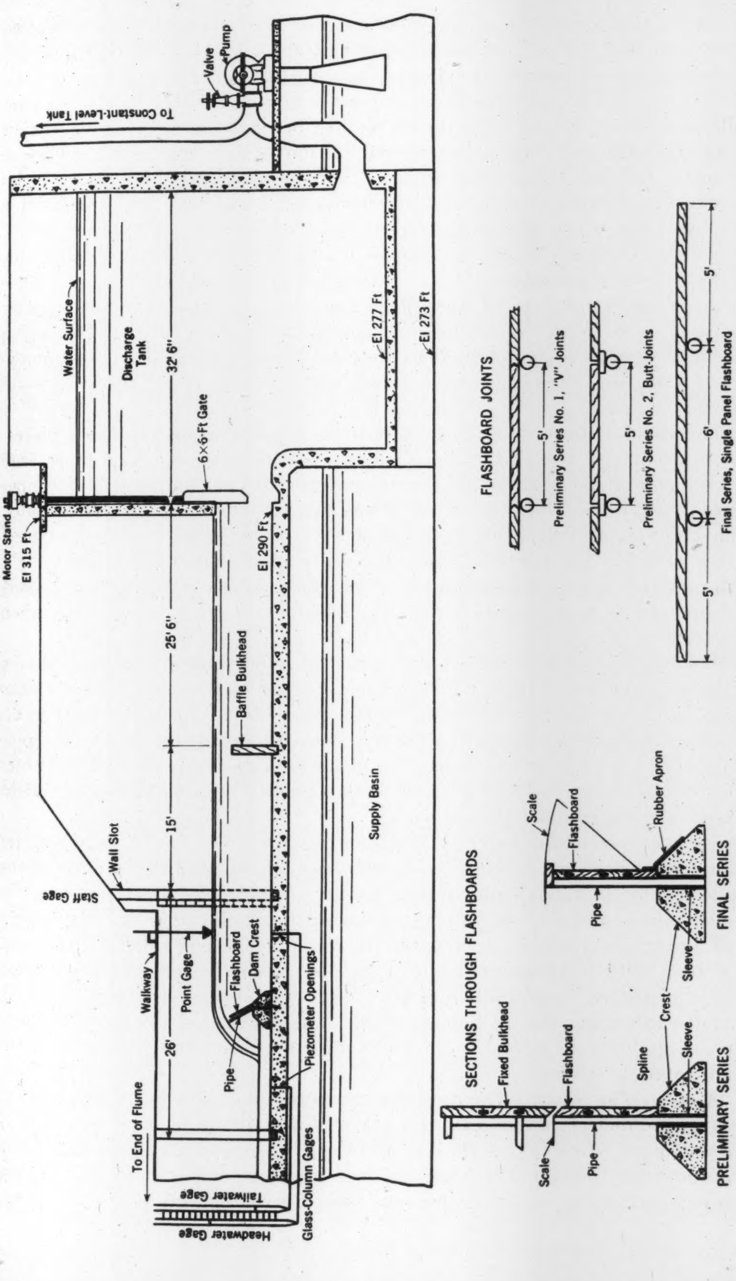

To Constant-Level Tank
Valve
Pump
Water Surface
Discharge Tank
32' 6"
El 277 Ft
El 273 Ft
6×6-Ft Gate
Motor Stand
El 315 Ft
El 290 Ft
25' 6"
Baffle Bulkhead
Supply Basin
15'
Wall Slot
Staff Gage
Walkway
Point Gage
Flashboard
Dam Crest
Piezometer Openings
26'
Pipe
To End of Flume
Glass-Column Gages
Tailwater Gage
Headwater Gage
FLASHBOARD JOINTS
5'
Preliminary Series No. 1, "V" Joints
5'
Preliminary Series No. 2, Butt-Joints
5'
6'
5'
Final Series, Single Panel Flashboard
SECTIONS THROUGH FLASHBOARDS
Fixed Bulkhead
Flashboard
Scale
Pipe
Spline
Crest
Sleeve
PRELIMINARY SERIES
Scale
Flashboard
Rubber Apron
Pipe
Sleeve
FINAL SERIES

FIG. 3.—GENERAL LAYOUT; FLUME, TANK, BAFFLES, AND FLASH-BOARDS (NOT TO SCALE)

For the final series of tests the fixed bulkhead was removed, and a one-piece flash-board 16 ft long and of heights varying from 1 ft 10 in. to 5 ft 2 in. above the dam crest was constructed. It was supported by only two pipes, symmetrically placed. The ends, bound with rubber, were closely fitted to wooden wing walls, as shown in Fig. 6. The rubber binding produced no appreciable friction on the end walls, and the flash-board could be moved very freely by hand before the test. One flash-board was balanced horizontally, and the distance of its center of gravity from the bottom was determined. This was used as a basis for computing the center of gravity of the other flash-boards. The lower part of the gate was braced, as can be seen in Fig. 2, to reduce the end deflections caused by water pressure.

The various pipes tested were fitted into the 4-in. sockets in the crest by means of pipe sleeves of various diameters, 20 in. long, the sleeves being machined when necessary to make them nest together properly. The dimensions for nesting pipes and sleeves are given in Table 1 and the sleeves are shown in Fig. 7.

The external diameters of the test pipes were measured in three places: The bottom, top, and at the top of the supporting sockets. The pipes were weighed so that the average internal diameters could be computed. For pipes larger than 2 in., the internal diameter was also measured directly. Computed from the weight, they showed an average variation from directly measured values of 0.3% for the mechanical-test specimens and 0.1% for the eight hydraulic-test specimens compared. The pipes were set up in the sockets and brought to a firm contact with the flash-boards by wooden shims when necessary.

Measuring Devices Used.—A scale, curved approximately to the arc of a circle, with the free length of the pipe as a radius, was fastened either on the top of the flash-board or to the pipe supporting the flash-boards (see Fig. 6). A transit mounted on top of the flume was used to read the scale as the pipe deflected. For later overflow tests, the pipe was extended up through the water to carry the scale (see Fig. 8). As a result of this there could be only a slight degree of vacuum under the nappe.

Up stream from the center of the dam, as shown in Fig. 3, a brass piezometer connection, with an opening $\frac{5}{8}$ in. in diameter, was placed in the floor of the flume and led to a glass-column gage having an internal diameter of 1 in. A similar provision was made for reading the tail-water elevation but this was not used, owing to excessive turbulence in the water below the dam. For the overflow tests with large quantities of water, in which the head-water was also greatly disturbed, a plumb-bob or point gage was used as indicated in Fig. 3. A staff gage, graduated in 3-in. intervals, was painted on the steel bulkhead slot in the flume up stream from the dam for approximate observation of the head-water elevations.

Non-Overflow Tests.—The head-water was raised gradually by means of the pumps, and deflection readings were taken at 3-in. intervals, as determined by the staff gage. Readings of the deflection, staff gage, and glass gage were taken simultaneously on signal. When the pipe began to deflect more rapidly, the readings were taken as the flash-board passed given deflection angles. When

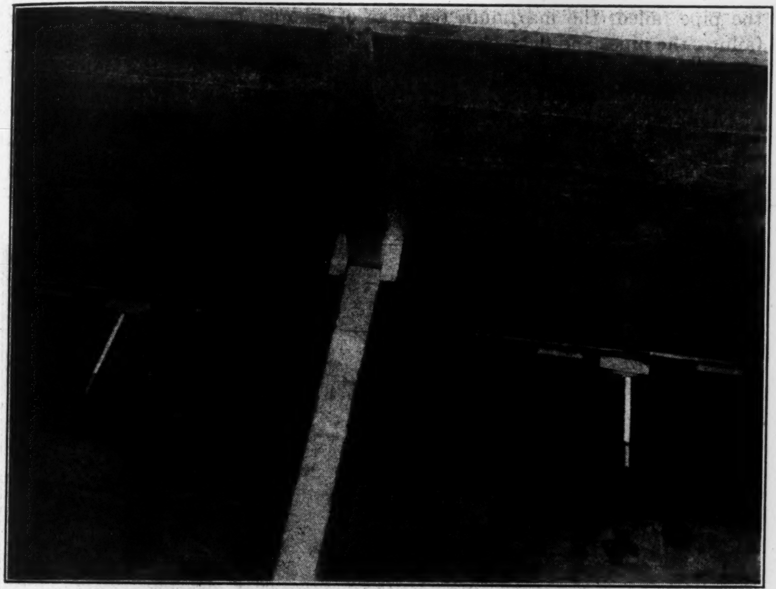


FIG. 4.—FLASH-BOARD PANELS WITH BUTT JOINTS

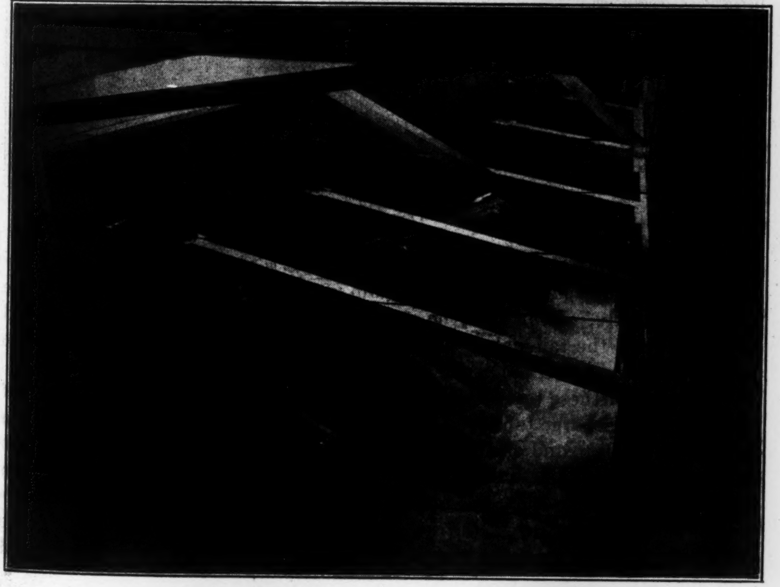


FIG. 5.—FLASH-BOARDS WITH "V" JOINTS AT FAILURE

the pipe failed, the maximum readings of all quantities were taken. After failure, the pipes and flash-boards were examined and photographed. In some of the preliminary tests the flash-boards did not fail because the water flowed over the top of the bulkhead before creating sufficient head for failure. A view of Non-overflow Test No. 4, Preliminary Series No. 1, is shown in Fig. 5.

Overflow Tests.—The same long, single flash-board that was used for the non-overflow tests was used for these tests, and the pipes were also set up in the same manner. However, it was necessary to use a quite different experimental procedure in the overflow tests, because the tail-water rose so rapidly that it backed up against the down stream face of the flash-boards unless failure occurred quickly. Even after the obstruction down stream from the dam had been removed, the tail-water occasionally rose until its surface was above the level of the dam crest, particularly in tests of the higher flash-boards.

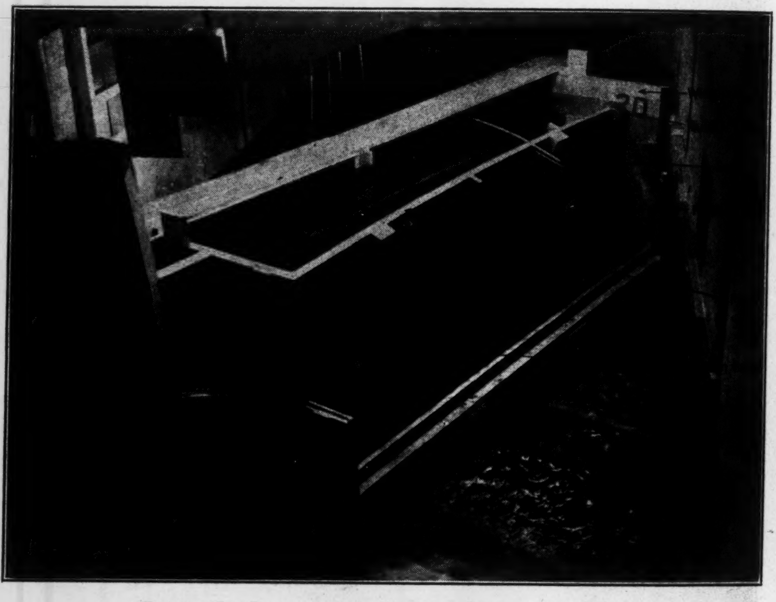


FIG. 6.—NON-OVERFLOW TEST NO. 20 AT FAILURE; 3-INCH PIPE

For some of these tests 35 cu ft per sec was not sufficient, and a method of utilizing the discharge tank was devised. The tank (see Fig. 3) was filled to a point near the top, and one 6-ft gate was opened slightly until the water began to flow over the baffle-bulkhead up stream from the flash-boards. As soon as the water appeared on the face of the flash-boards, simultaneous readings were made and were repeated at every additional 3-in. height. When the water reached the top of the boards, the gate in the discharge tank was opened rapidly, and the head of the water flowing over the boards was brought quickly to the maximum that could be obtained. In this manner a large discharge was produced over the flash-boards—at times probably as much as 150 cu ft per sec. The deflection was observed closely and, at the instant of failure, readings were taken of all quantities. The failure was fairly clear-cut, in that the movement of the pipe accelerated gradually until a sudden collapse occurred. A view of Final-Series Overflow Test No. 27 at the point of failure is shown in Fig. 8. If the tail-water rose above the dam crest level (base of flash-board) before failure of the flash-board pipes, the flume was drained and the test repeated.

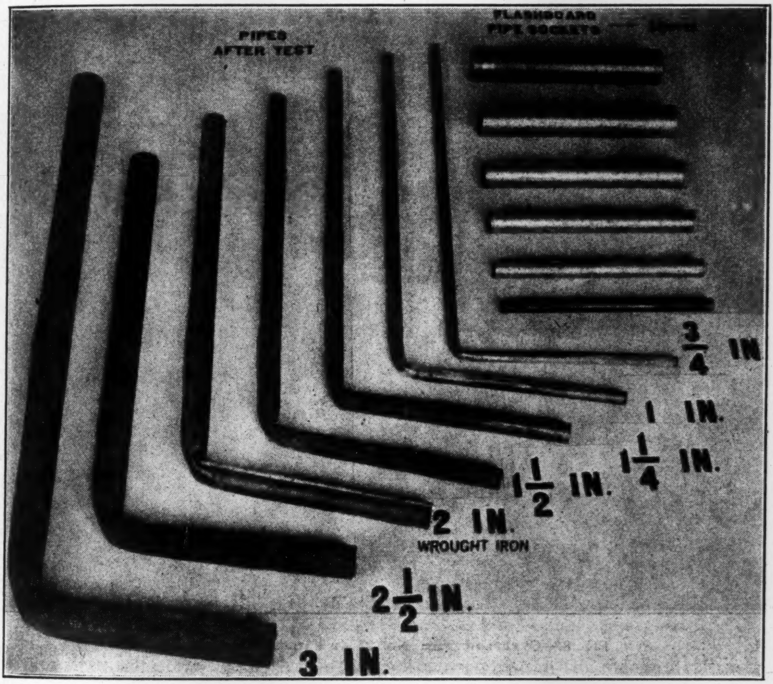


Fig. 7.—Pipes and Pipe Sleeves after Tests

Fig. 7 shows the typical appearance of the pipes after failure. The bending was concentrated at the section of the pipe in the plane at the top of the socket: All of the pipes collapsed at this section, most of them splitting along the seam, and some across the bend.

Accuracy of Measurement.—For most of the tests the head was determined by means of the glass-column gages, but in the later overflow tests the point gage was used. By comparative tests it was established that the head measurement was accurate to 0.01 ft for the non-overflow tests, 0.02 ft for most of the overflow tests, and 0.03 ft for tests with violent disturbance of the head-water.

Results of Hydraulic Tests.—The net result of the Preliminary Series No. 1 was to disclose that the indeterminate influence of the binding of the "V" joints was frequently as great as the supporting power of the pipe, and therefore such joints could not be tolerated. Preliminary Series No. 2 also showed sufficient binding in the 90° "butt" joints to interfere with the design of the flash-boards on a short spillway, although the effect would probably be insignificant on a long spillway. In both series the stresses in the pipes computed from the water loads were excessive.

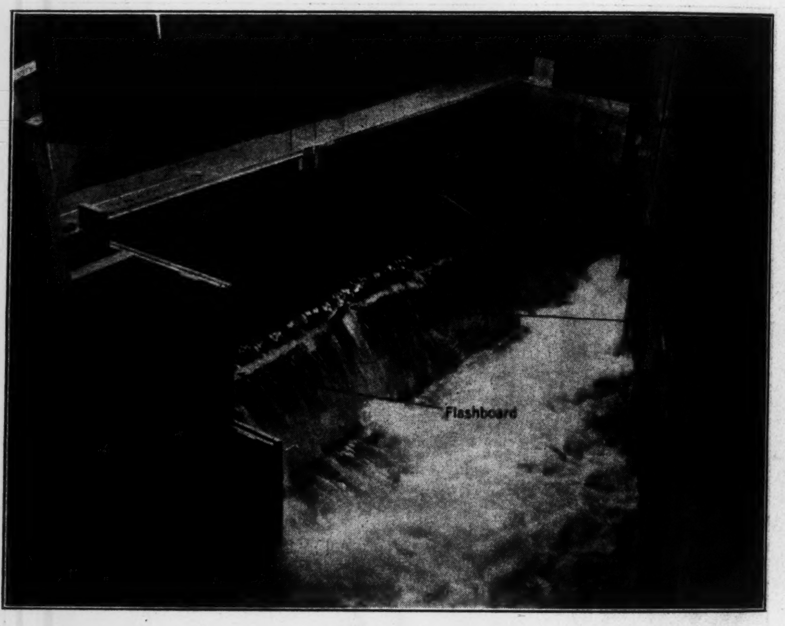


FIG. 8.—OVERFLOW TEST NO. 27 AT FAILURE; 1½-INCH PIPE

Accordingly, cognizance is given only to the results of the final series of tests, where end slots were eliminated, leaving free movement at the ends of the flash-boards along vertical walls at right angles to the center line of the gate. Notwithstanding turbulence, a possible slight degree of vacuum under the nappe, and varying rates of application of pressure, the steel pipe collapsed within sufficiently narrow limits to justify confidence that its behavior can be depended upon. The characteristic of uniform bending up to a critical yield angle from the vertical of from 20° to 30°, followed by a rapid collapse thereafter, is favorable for automatic flash-board control. Results based on this series are analyzed, together with those of the mechanical tests, under the heading "Results of Experiments."

Mechanical Tests

Duplicate specimens 36 in. long were cut from the pipes used for the hydraulic tests and were used for mechanical tests. The specimens varied from $\frac{3}{4}$ in. to 3 in. in size. One specimen of wrought-iron pipe was tested also. The internal diameters were computed in the same manner as for the hydraulic tests; that is, from the weight and external dimensions.

The tests were made in a hydraulic testing machine of 100 000-lb capacity. As shown in Fig. 9, each pipe was loaded as a cantilever beam. The assembly of pipe sleeves used in the hydraulic tests was used also in the mechanical tests to simulate the same conditions of support. The load was applied to the pipe 10 in. from the support through a steel plate corresponding to the actual flash-board loading. During each test the load was applied in increments of approximately 200 lb, and the deflection of the end of the pipe for each load was computed from the readings of two dial gages. After the yield point of the pipe was passed and the pipe deflected excessively, the dial gages were removed, and the vertical distance from the pipe to the platen of the machine was measured until the maximum load was reached.

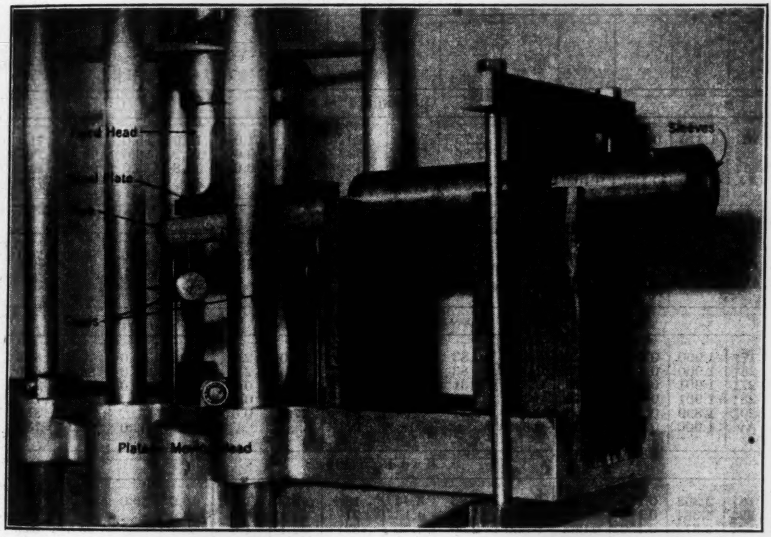


FIG. 9.—MECHANICAL TEST NO. 7, $1\frac{1}{2}$-INCH PIPE—PLATEN, GAGES, AND SLEEVES

RESULTS OF EXPERIMENTS

General.—In arriving at the principal objective—the modulus of rupture and design of flash-board supports—the data from the tests were analyzed in the

TABLE 2(*a*).—HYDRAULIC TESTS; FINAL SERIES

Test *	External diameter, D_e, in inches	WALL THICKNESS		Section modulus, S, in inches^3	FEET		Deflection, θ, at failure, in degrees§	MAXIMUM MOMENT, M, IN INCH-KIPS			STRESS, f, IN PIPE, IN KIPS PER SQUARE INCH		
		Thickness, t, in inches	Ratio, $\frac{t}{D_e}$		Height of flash-board, h	Head, H, on crest of dam at failure		Normal	Due to weight of flash-board	Total	Yield point	Modulus of Rupture	
												Normal moment¶	Simplified
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
THREE-QUARTER-INCH PIPE													
....	....	....	....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....	....	....	....
ONE-INCH PIPE													
26†	1.316	0.138	0.105	0.136	2.75	2.12	20	9.9	1.0	10.9	55.0	80.2	71.0
30‡	1.314	0.136	0.104	0.135	1.86	2.25	33	11.6	0.6	12.2	55.0	90.4	78.0
31‡	1.318	0.132	0.100	0.133	1.86	2.30	33	12.2	0.6	12.8	55.0	96.2	82.0
Av.	1.316	0.135	0.103	0.135	....	....	29	....	....	....	55.0	88.7	77.0
ONE-AND-ONE-QUARTER-INCH PIPE													
16†	1.663	0.136	0.082	0.232	3.83	2.54	20	17.8	2.0	19.8	50.0	85.4	70.0
....	....	....	....	....	....	....	....	....	....	....	....	....	....
Av.	....	....	....	....	....	....	....	....	....	....	....	....	....
ONE-AND-ONE-HALF-INCH PIPE													
17†	1.900	0.145	0.076	0.326	3.83	2.81	20	24.0	2.0	26.0	50.0	79.8	67.5
25†	1.900	0.141	0.074	0.318	3.83	2.83	20	24.4	2.0	26.4	55.0	84.0	69.0
27‡	1.901	0.150	0.079	0.336	2.31	3.00	21	24.2	0.6	24.8	50.0	73.8	71.5
28‡	1.901	0.150	0.079	0.334	2.31	3.15	22	26.9	0.6	27.5	....	82.4	78.5
29‡	1.899	0.156	0.082	0.342	2.31	3.20	22	27.6	0.6	28.2	....	82.4	81.0
Av.	1.900	0.148	0.078	0.332	....	....	21	....	....	....	52.0	81.3	73.5
TWO-INCH PIPE													
18†	2.368	0.154	0.065	0.554	3.83	3.44	18	43.7	1.9	45.6	50.0	82.3	73.0
32‡	2.384	0.170	0.071	0.616	2.50	4.15	21	48.1	0.5	48.6	....	78.9	83.5
33‡	2.372	0.150	0.063	0.550	2.50	4.15	(33) ||	52.1	1.0	53.1	....	96.6||	83.5
(24)†	(2.372)	....	....	(0.604)	(3.83)	(3.24)	(18)	(36.2)	(1.8)	(38.0)	(35.0)	(62.9)	....
Av.	2.374	0.158	0.066	0.573	....	....	20	....	....	....	50.0	80.0	80.0
TWO-AND-ONE-HALF-INCH PIPE													
19‡	2.863	0.206	0.072	1.068	3.83	4.16	17	74.0	1.7	76.0	40.0	71.2	66.5
22†	2.878	0.208	0.072	1.090	4.53	4.02	22	73.5	3.0	76.5	40.0	70.2	61.0
23†	2.866	0.214	0.075	1.103	4.53	4.20	18	79.9	2.5	82.4	....	74.6	70.0
Av.	2.870	0.210	0.073	1.087	....	....	19	....	....	....	40.0	72.0	65.8
THREE-INCH PIPE													
20†	3.508	0.226	0.064	1.800	5.17	4.93	14	126.0	2.3	128.3	45.0	71.2	69.0
21†	3.504	0.216	0.062	1.730	5.17	4.84	14	119.0	2.3	121.5	45.0	70.2	65.0
Av.	3.506	0.221	0.063	1.765	....	....	14	....	....	....	45.0	70.7	67.0
General Average, Steel Pipe				....	....	....	21	....	....	....	49.2	79.6	73.0

* All specimens are steel except Tests 24 and 10*a*, which were wrought iron.

† Non-overflow tests.

‡ Overflow tests.

|| Not used in average; deflection excessive.

§ At maximum moment (modulus of rupture).

¶ Column (11) divided by Column (5).

TABLE 2(*b*).—MECHANICAL TESTS

Test*	External diameter, D_e, in inches	WALL THICKNESS		Section modulus, S, in inches³	Deflection, θ, at failure, in degrees§	STRESS, f, IN PIPE, IN KIPS PER SQUARE INCH	
		Thickness, t, in inches	Ratio, $\frac{t}{D_e}$			Yield point	Modulus of rupture; normal moment
(1)	(2)	(3)	(4)	(5)	(8)	(12)	(13)
THREE-QUARTER-INCH PIPE							
1*a*	1.057	0.111	0.105	0.071	28	53.0	84.8
2*a*	1.047	0.118	0.113	0.071	32	53.0	86.8
Av.	1.052	0.115	0.109	0.071	30	53.0	85.8
ONE-INCH PIPE							
3*a*	1.314	0.138	0.105	0.135	23	52.0	79.8
4*a*	1.320	0.156	0.118	0.145	20	45.0	68.0
....	....	....	....	....	....	....	....
Av.	1.317	0.147	0.112	0.140	22	48.5	73.9
ONE-AND-ONE-QUARTER-INCH PIPE							
5*a*	1.659	0.137	0.083	0.229	23	45.0	68.6
6*a*	1.659	0.135	0.081	0.228	23	45.0	70.0
Av.	1.660	0.136	0.082	0.228	23	45.0	69.0
ONE-AND-ONE-HALF-INCH PIPE							
7*a*	1.894	0.149	0.079	0.329	23	45.0	70.7
8*a*	1.902	0.146	0.077	0.329	20	45.0	73.0
....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....
Av.	1.900	0.147	0.078	0.329	22	45.0	71.8
TWO-INCH PIPE							
(10*a*)	(2.373)	(0.177)	....	(0.624)	(18)	(30.0)	(60.6)
9*a*	2.372	0.158	0.067	0.561	20	45.0	80.4
11*a*	2.368	0.177	0.075	0.607	19	30.0	59.0
Av.	2.370	0.168	0.071	0.584	20	37.5	69.7
....	....	....	....	....	....	....	....
TWO-AND-ONE-HALF-INCH PIPE							
12*a*	2.867	0.216	0.075	1.102	21	43.0	74.5
13*a*	2.868	0.218	0.076	1.111	22	43.0	74.1
....	....	....	....	....	....	....	....
Av.	2.868	0.217	0.076	1.106	22	43.0	74.3
THREE-INCH PIPE							
14*a*	3.505	0.231	0.066	1.804	17	50.0	75.0
15*a*	3.506	0.231	0.066	1.817	17	50.0	72.3
Av.	3.506	0.231	0.066	1.810	17	50.0	73.6
....	....	....	....	....	22	46.0	74.0

following order:

(1) The actual bending moment on the pipe and the corresponding stress were computed as accurately as possible throughout the range of the test;

(2) Values of the modulus of rupture and the yield point for each pipe were determined for both the hydraulic and the mechanical tests;

(3) Values of the modulus of rupture determined for the hydraulic tests in Item (2) were used for the computation of the bending moment at which the pipe should fail; then, on this basis, formulas of various degrees of precision were set up for determining the corresponding water head;

(4) Simplified formulas for use in practical design were developed (moduli of rupture for use with these formulas were obtained by application of the formulas to the laboratory test heads);

(5) Using the values of modulus of rupture from Item (4) a simplified formula for standard sizes of pipe was set up for the prediction of the water head at failure;

(6) The laboratory set-up was assumed to represent a field installation. (The various design formulas, together with their appropriate value of modulus of rupture, were applied thereto and the resulting design head for failure was determined. These values were compared with the observed heads and the relative precision of the formulas determined);

(7) The simple method of design from Item (5) was applied to field structures and check tests were obtained; and,

(8) The original test data referring to Items (1), (2), and (4) are summarized in Table 2. Values in Column (14) are found by Equations (19*b*) or (20) (presented subsequently), divided by the section modulus for standard steel pipe given in Column (5), Table 3. The moment formulas in Items (3) and (4) are derived and discussed in the section on "Moment Formulas for Design." The heads computed for Item (6) are summarized in Table 3. The application of Items (5) and (7) is discussed in the section on "Design of Pipe Supports." Finally, suitable values of moduli of rupture and bending moment formulas are recommended in the "Conclusions." A summary of the bending moment formulas used in the computation of heads in Table 3 is presented in Table 4, the formulas being derived subsequently herein.

Notation.—The letter symbols in this paper are defined where they first appear and are assembled for convenience of reference in Appendix II. The term "kips," when used in this paper, denotes "kilo-pounds"; that is, thousands of pounds.

Computation of Stress in Pipe.—The cross-section of each pipe was assumed to be circular, and its section modulus was computed according to the standard formula:

$$S = \frac{\pi \, (D_e^4 - D_i^4)}{32 \, D_e} \qquad \text{.....................(1)}$$

in which: S = section modulus (Column (5), Table 2); D_e = external diameter of pipe; and D_i = internal diameter of pipe. The stress in the pipe was then computed from the usual formula

$$f = \frac{M}{S} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which M = bending moment in pipe; and, f = stress at the outer fiber.

Computation of Bending Moment.—Thick planks were used at the bottom of all of the flash-boards, and variations in height were obtained by changing the thin section at the top. Accordingly, the exact relation between the depth of the water and the bending moment produced by the water pressure on this irregularly-shaped flash-board could not be expressed readily in general terms;

TABLE 3.—COMPARISON OF PERFORMANCE; LABORATORY TESTS

Size of pipe, in inches	External diameter, D_e, in inches	WALL THICKNESS		Section modulus, S, in inches³	ADOPTED MODULUS OF RUPTURE, f, IN POUNDS PER SQUARE INCH	
		Nominal thickness, t, in inches	Ratio, $\frac{t}{D_e}$		Normal moment formula	Simplified horizontal moment formula
(1)	(2)	(3)	(4)	(5)	(6)	(7)
(a) STANDARD PIPE DATA FOR DESIGN						
0.75	1.050	0.113	0.108	0.070	88 000	80 000
1.00	1.315	0.133	0.101	0.135	88 000	77 000
1.25	1.660	0.140	0.084	0.235	86 000	73 000
1.50	1.900	0.145	0.076	0.330	82 000	72 000
2.00	2.375	0.154	0.065	0.560	82 000	70 000
2.50	2.875	0.203	0.070	1.070	73 000	70 000
3.00	3.500	0.216	0.062	1.750	73 000	70 000
Average	....	....	....	....	....	....

TABLE 3.—(*Continued*)

Size of pipe, in inches	Height of flash-board, h, in feet	DESIGNED HEAD FOR FAILURE, H, IN FEET				Test head, at failure, in feet	VARIATION OF DESIGN HEAD FROM TEST HEAD (PERCENTAGES)				Designed head, H, in feet*	Percentage variation of designed head (Column (18)) from test head
		Normal Moment (θ = 20°); Flash-board Moment		Horizontal moment	Simplified horizontal moment formula		Normal Moment (θ = 20°); Flash-board Moment		Horizontal moment	Simplified horizontal moment formula		
		Included	Neglected				Included	Neglected				
(1)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
(b) OVERFLOW TESTS												
0.75	....	no tests	....	....	....	....	....	....	....	....	....	....
1.00	1.86	2.33	2.30	2.38	2.24	2.28	+2	+1	+4	−2	1.83	−20
1.25	....	no tests	....	....	....	....	....	....	....	....	....	....
1.50	2.31	3.16	3.12	3.22	3.02	3.12	+1	0	+3	−3	2.48	−20
2.00	2.50	4.05	4.02	4.11	3.75	4.15	−2	−3	−1	−10	3.03	−27
2.50	3.83	4.19	4.17	4.32	4.24	4.16	+1	0	+4	+2	3.66	−12
3.00	....	no tests	....	....	....	....	....	....	....	....	....	....
Average	....	....	....	....	....	....	+1	−1	+3	−3	....	−20

TABLE 3.—(*Continued*)

Size of pipe, in inches	Height of flash-board, h, in feet	DESIGNED HEAD FOR FAILURE, H, IN FEET				Test head, at failure, in feet	VARIATION OF DESIGN HEAD FROM TEST HEAD (PERCENTAGES)				Designed head, H, in feet*	Percentage variation of designed head (Column (30)) from test head
		Normal Moment ($\theta = 20°$); Flash-board Moment		Horizontal moment	Simplified horizontal moment formula		Normal Moment ($\theta = 20°$); Flash-board Moment		Horizontal moment	Simplified horizontal moment formula		
		Included	Neglected				Included	Neglected				
(1)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)
(*c*) NON-OVERFLOW TESTS												
0.75	....	no tests	....	....	....	....	....	....	....	....	....	....
1.00	2.75	2.22	2.18	2.29	2.18	2.12	+5	+3	+8	+3	1.84	−13
1.25	3.83	2.61	2.61	2.72	2.58	2.54	+3	+3	+7	+2	2.21	−13
1.50	3.83	2.89	2.87	3.00	2.87	2.82	+3	+2	+6	+2	2.47	−12
2.00	3.83	3.46	3.43	3.58	3.39	3.44	−1	0	+4	−1	2.95	−14
2.50	4.53	4.12	4.10	4.27	4.21	4.11	0	0	+4	+2	3.66	−11
3.00	5.17	4.86	4.82	5.03	4.96	4.88	0	−1	+3	+2	4.31	−12
Average	....	....	....	....	....	....	+2	+1	+5	+2	....	−12

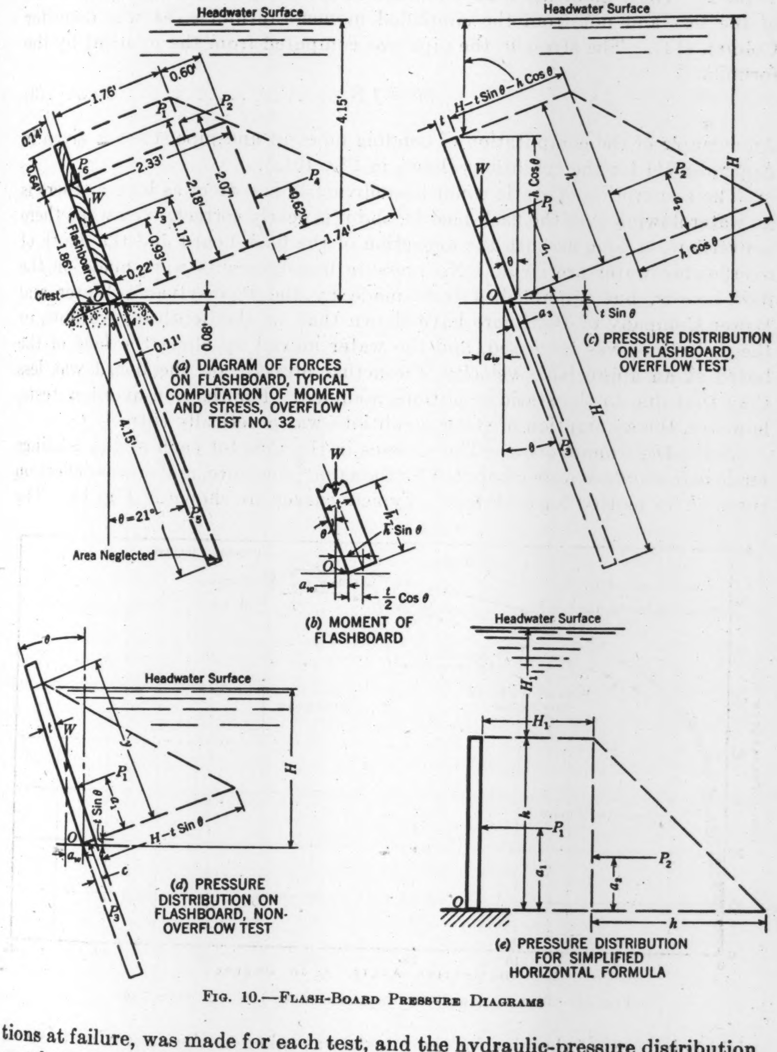
* For a (yield point) stress of 46 kips per sq in.

TABLE 4.—SUMMARY OF BENDING MOMENT FORMULAS

Case	Formula	Equation No.	Computed head given in Table 3 in Column No.:
(*a*) OVERFLOW CONDITIONS			
1	$M = M_{B^*} + M_N = M_B + w\,L\,h^2\left(\frac{H}{2} - \frac{h\cos\theta}{3} - \frac{t\sin\theta}{2}\right)$	(4*b*) and (8)	(9)
2	$M = M_{N'} = w\,L\,h^2\left(\frac{H}{2} - \frac{h\cos\theta}{3}\right)$	(9)	(10)
3	$M = M_H = w\,L\,h^2\left(\frac{H}{2} - \frac{h}{3}\right)$	(10)	(11)
3(*a*)	$M = 375\,L\,h^2\left(H_1 + \frac{h}{3}\right)$	(19*b*)	(12)
(*b*) NON-OVERFLOW CONDITIONS			
4	$M = M_B + M_N = M_{B^*} + w\,L\left(\frac{H^3 - 3\,H^2\,t\sin\theta}{6\cos^2\theta}\right)$	(4*b*) and (15)	(21)
5	$M = M_{N'} = w\,L\left(\frac{H^3}{6\cos^2\theta}\right)$	(16)	(22)
6	$M = H_H = \frac{w\,L\,H^3}{6}$	(17)	(23)
6(*a*)	$M = 125\,L\,H^3$	(20)	(24)

* Computed separately.

so it was computed individually for each case. Later, simplifications were made in the assumed loadings, and general expressions for the moment were derived on that basis. A diagram similar to Fig. 10, showing all of the conditions at failure, was made for each test, and the hydraulic-pressure distribution was determined for all parts of the flash-board, assuming static conditions. The effective bending moment, M, on the pipe was computed by taking the moments


Headwater Surface
1.76'
0.60'
0.14'
0.64'
P_1
P_2
P_6
2.33'
2.07'
2.18'
4.15'
P_4
0.62'
W
P_3
1.17'
0.93'
1.74'
1.86'
Flashboard
0.22'
Crest
O
0.08'
0.11'
4.15'
θ=21°
P_5
Area Neglected
Headwater Surface
$H-t\sin\theta-h\cos\theta$
t
W
$h\cos\theta$
h
P_1
P_2
a_1
a_2
$h\cos\theta$
θ
H
O
a_3
$t\sin\theta$
a_w
H
θ
P_3
W
t
h
$\frac{h}{2}\sin\theta$
θ
O
a_w
$\frac{t}{2}\cos\theta$
θ
Headwater Surface
t
W
y
P_1
$t\sin\theta$
a_1
H
$H-t\sin\theta$
O
a_w
c
P_3
Headwater Surface
H_1
H_1
h
P_1
P_2
a_1
a_2
O
h

(*a*) DIAGRAM OF FORCES ON FLASHBOARD, TYPICAL COMPUTATION OF MOMENT AND STRESS, OVERFLOW TEST NO. 32

(*b*) MOMENT OF FLASHBOARD

(*c*) PRESSURE DISTRIBUTION ON FLASHBOARD, OVERFLOW TEST

(*d*) PRESSURE DISTRIBUTION ON FLASHBOARD, NON-OVERFLOW TEST

(*e*) PRESSURE DISTRIBUTION FOR SIMPLIFIED HORIZONTAL FORMULA

FIG. 10.—FLASH-BOARD PRESSURE DIAGRAMS

of all the forces around the base of the board, O. The weight of the flash-board was appreciable in most of the tests, and hence its moment was added to that due to the water pressure. The moments are given in Columns (9) to (11), Table 2. These moments were computed individually for the exact conditions of the test and not from the simplified moment formulas, as was done for Column (14). The stress in the pipe was computed from the moment by the formula

$$M = f\,S \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

An example of the computation of bending moment and pipe stress is given in Appendix III for the conditions shown in Fig. 10(a).

The assumption of static conditions obviously is correct as long as there is no water flowing over the flash-boards, and it is nearly correct, even when there is overflow, as long as both the deflection of the flash-board and the depth of overflowing water are small. No pressure measurements were made on the flash-boards, but unpublished tests made by the Pennsylvania Water and Power Company of Baltimore have shown that, as the depth of overflow, or the deflection, was increased, and the water moved up along the face of the board at an appreciable velocity, the actual pressure on the board was less than that due to the static conditions assumed. For the non-overflow tests, however, the assumption of static conditions was practically correct.

Stress-Deflection Curves.—The stresses in the pipe for each of the readings made before failure were computed by the same procedure, and stress-deflection curves were plotted for each test. Typical curves are shown in Fig. 11. The yield point (18) of the pipe for these tests was taken as the approximate stress at which the stress-deflection curve breaks away from its initial straight section. It was desired to determine the stress at which the pipes first began to bend

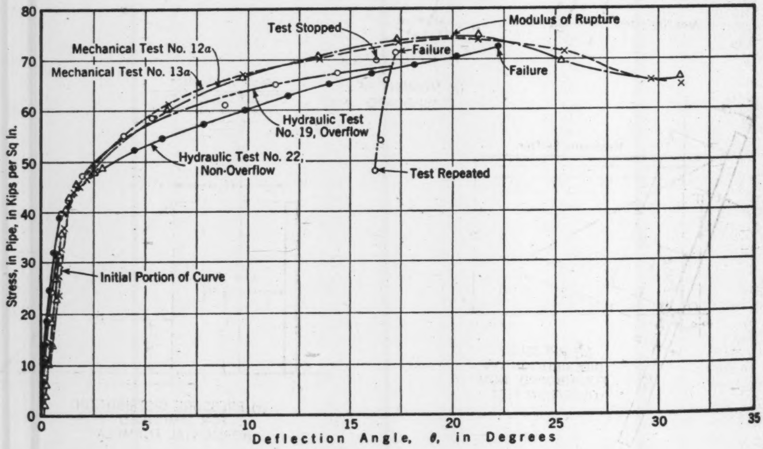


Fig. 11.—Typical Stress-Deflection Curves for a 2½-Inch Pipe

noticeably in these tests, since it was thought that they might possibly fail at the head causing this stress. When the deflection increases there is a considerable increase in the moment of the hydraulic load, even if the head remains constant. For this feature of the investigation, the term "yield point" seemed better adapted than other terms considered, although it does not accord with the standard definition. These values were determined from the curves and are given in Column (12), Table 2.

The stress computed from the maximum moment was called the "modulus of rupture," and these values are given in Columns (13) and (14), Table 2. (The term "modulus of rupture" has been used to indicate the maximum computed flexural strength, based on the original section of the pipe, in accordance with engineering practice.)

The stress-deflection curves for 2½-in. pipe, shown in Fig. 11, were selected as typical of both the hydraulic and mechanical tests. The curves are seen to be generally similar in shape throughout their range but to differ in the value of the modulus of rupture. It may be observed in Fig. 11 that, as the headwater was lowered and again raised for Test No. 19, the corresponding stress-deflection curve tends to be parallel to its initial direction. Furthermore, as would be expected, the curve comes back to its original position before going on to the point at which the pipe failed.

The beginning of the stress-deflection curve, within the limits of error of the test, was observed to be a straight line and served to determine the approximate yield point as already described at about 46 kips per sq in. The deflection of the pipe over the elastic range of the test was very small—that is, less than 2° as compared to that at its subsequent failure at 20° to 30° deflection. Extensive studies of the properties of the pipe within this range of loading failed to yield any results pertinent to the main objective (design of flash-board supports); consequently no further consideration was given to conditions within the elastic range.

The deflection at modulus of rupture was plotted against the corresponding size of pipe for both the hydraulic and mechanical tests, using the data given in Table 2. The average values for the hydraulic tests showed a fairly consistent agreement with the mechanical tests. In both types of tests the deflection at failure is less for the larger pipes than for the smaller ones. A probable explanation of the latter fact is given subsequently herein in connection with the relation of modulus of rupture to pipe size. For the hydraulic tests the deflection ranged from 15° to 30° and the average was taken as 20° for later use in connection with the design of flash-boards.

Yield Point Stress.—The stress at the yield point for all of the pipes tested was found to range from 40 to 55 kips per sq in. averaging 49 kips per sq in. Beyond this stress permanent bending occurred. (It was found later, in field tests, that a value of 46 kips per sq in., computed by the horizontal formula, was more nearly in accordance with the facts; some tests gave as low as 40 kips per sq in.) However, failure did not occur until the pipe had traversed the large deflection angle of 15° to 30°, the angle of failure being consistent for pipes of the same size. Thus, for an actual installation, if the pipes had been bent only slightly by a small flood, they still could be depended upon to fail at the desired head caused by a subsequent larger flood.

The wide spread between the yield point and the modulus of rupture of pipe supports is particularly significant. Instead of beginning decisively to fail at the yield point, the pipe did not actually fail until the stress value approached the modulus of rupture, although it was loaded beyond the yield point for a considerable period.

From these characteristics it is evident that in designing flash-board supports the diameter and spacing of the pipe should be such that the yield point, and resulting permanent bending of the pipe, will not be reached too frequently. The natural tendency is to build flash-boards too close to the maximum permissible water level in order to gain as much head and storage as possible. Then a small flood will bend the pipe, making the gate unsightly, or collapse it, requiring the pipe to be replaced. On the other hand, if the pins are designed for an initial stress of 30 to 50 kips per sq in., as well as for the ultimate modulus of rupture required, premature deformation due to debris, waves, or high water can be largely avoided.

The deviations of the heads at which the pipe should begin to bend, from the heads at which failure actually occurred, are shown in Columns (19) and (31), Table 3. These columns show, furthermore, that if the yield point is used in place of the modulus of rupture to compute the heads at which the pipes should fail, this will lead to values distinctly lower than the actual heads at failure. This comparison was made partly because of the belief (apparently held previously, but now shown not to be the case) that once the yield point of the pipes had been reached, the installation would go on to failure without any further increase in head.

Modulus of Rupture-Pipe Size.—In order to determine the proper modulus of rupture for the design of pipe supports, the relationship of modulus of rupture to pipe size was studied. The values of modulus of rupture given in Columns (13) and (14), Table 2, for both the hydraulic and mechanical tests, were plotted against the corresponding pipe size given in the sub-captions of the table (see Fig. 12). They range from 96 to 61 kips per sq in., and the values of the modulus of rupture, obtained from the hydraulic tests were approximately 20% greater for the smallest pipes than for the largest pipes. The values for the mechanical tests show a similar relationship between extreme sizes, although it is not so marked for the intermediate sizes. This relationship is due mainly to the greater relative wall thickness of the smaller pipes, which is indicated by comparing the values in Column (4), Table 2, with the standard pipe size. It is an accepted fact (19) that, when any structural member is stressed in bending beyond the elastic range, the stress distribution becomes non-linear, and the extreme-fiber stress computed on the assumption of a linear distribution exceeds the actual value. The greater the concentration of the material at the neutral axis, the greater this excess will be. Thus, although the actual stress occurring in the extreme fiber of two pipes may be the same, yet the computed value would be greater for the thicker-walled pipe. The larger pipes, with the relatively thinner walls, fail at smaller deflection angles and yield lower moduli of rupture.

Comparison of Hydraulic and Mechanical Tests.—For the mechanical tests the moments and resulting stresses were computed directly from the measured loads. The yield point for each pipe was determined from stress-deflection curves in the same manner as for the hydraulic tests and the results are given in Column (12), Table 2. Values of modulus of rupture determined from the maximum moment are given in Columns (13) and (14).

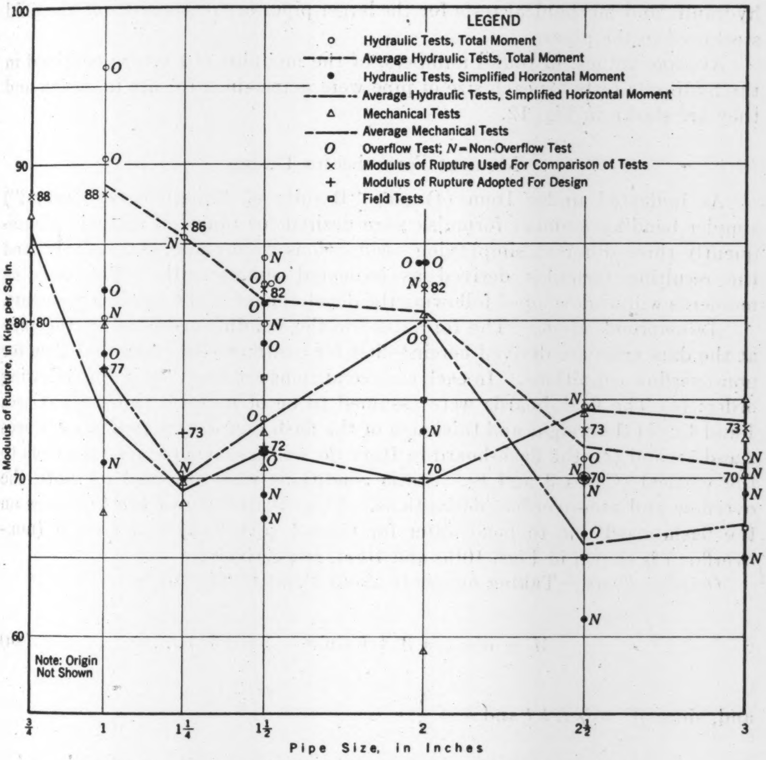


FIG. 12.—RELATION BETWEEN MODULUS OF RUPTURE AND PIPE SIZE

It is believed that the conditions were more nearly determinate, and more nearly in accordance with the assumptions made, in the tests of the 2½-in. and 3-in. pipes than for the smaller pipes. The results obtained in the mechanical tests agree quite well with those obtained in the hydraulic tests for the two largest sizes of pipe tested.

An exhaustive analysis of the discrepancies between the results of the mechanical and the hydraulic tests for pipe diameters of 2 in. and smaller was made (15*a*), but no obvious explanation of the lack of agreement was found. It was concluded that the discrepancy between the hydraulic and mechanical tests is a function of any or all of a number of minor factors, none of which can be evaluated with any certainty. These include: (*a*) Uncertainty as to the portion of the water load that was actually transmitted to the pipes; (*b*) possible uncertainties as to the exact values of the moments applied in the mechanical tests, due to the large deflection at failure; (*c*) uncertainties as to the exact shape and condition of the pipe at the support; and (*d*) insufficient number of pipes tested. The values of modulus of rupture determined by the hydraulic and mechanical tests for the larger pipes are reasonable for the mild steel used in the pipes.

Average values, in round numbers, of the modulus of rupture obtained in the hydraulic tests for each size of pipe were determined for use in design and they are shown in Fig. 12.

Moment Formulas for Design

As indicated under Item (4) (see "Results of Experiments: General") simpler bending moment formulas were desired for practical design. Consequently three different simplifying assumptions were made, successively, and the resulting formulas derived as indicated subsequently. The resisting moments will be developed following the development of the bending moments.

Assumptions Made.—The formulas for the bending moment on the pipe at the dam crest are derived herein—first for overflow conditions and then for non-overflow conditions. In each case conditions are assumed in the following order: (*a*) The flash-boards were assumed to be of uniform thickness (Cases 1 and 4); (*b*) the weight and thickness of the flash-board were neglected (Cases 2 and 5); and (*c*), the flash-board in Item (*b*) was assumed to be swung back to the vertical (Cases 3 and 6). Static conditions were assumed for both the overflow and non-overflow derivations. The distribution of the pressures on the flash-board due to head-water for Case 1 (overflow) and Case 4 (non-overflow) is shown in Figs. 10(*b*) and 10(*c*), respectively.

Overflow Tests.—Taking moments about Point O, Fig. 10(*b*):

$$M = W\,a_w = W\left(\bar{h}\sin\theta - \frac{t}{2}\cos\theta\right) \ldots\ldots\ldots\ldots (4a)$$

and, since $W = \gamma\,L\,h\,t$ and $\bar{h} = \frac{h}{2}$:

$$M_B = \gamma\,L\left(\frac{h^2\,t}{2}\sin\theta - \frac{h\,t^2}{2}\cos\theta\right) \ldots\ldots\ldots\ldots (4b)$$

in which: W is the weight of the flash-board; $\bar{h}$ is the distance from the base of the flash-board to its center of gravity; a_w is the lever arm of the weight W; γ is the specific weight of the timber used for the flash-board (36 lb per cu ft); L is the length of flash-board along the crest supported by one pipe (average spacing of the supporting pipes); and, M_B is the moment due to the weight of the flash-board. The flash-board moment was appreciable only at deflection angles greater than 5 degrees.

Case 1. Normal Moment.—The flash-board is assumed to be of constant thickness t. The trapezoidal pressure diagram is divided into a rectangle and a triangle for convenience, as shown in Fig. 10(c). Consequently, the moment of the water pressure normal to the flash-board is:

$$M_N = P_1\, a_1 + P_2\, a_2 + P_3\, a_3 \dotfill (5)$$

in which $a_1 = \frac{h}{2}$; $a_2 = \frac{h}{3}$; $a_3 = \frac{t}{2}$; and,

$$P_1 = w\, L\, h\, (H - t \sin\theta - h \cos\theta) \dotfill (6a)$$

$$P_2 = \frac{w\, L\, h}{2}\, (h \cos\theta) \dotfill (6b)$$

$$P_3 = w\, L\, t\, H \left(\text{neglecting } \frac{t}{2} \sin\theta\right) \dotfill (6c)$$

in which w is the specific weight of water (62.4 lb per cu ft). Hence:

$$M_N = w\, L\, h\, (H - t \sin\theta - h \cos\theta)\, \frac{h}{2} + \frac{w\, L\, h}{2}\, (h \cos\theta)\, \frac{h}{3} + w\, L\, t\, H\, \frac{t}{2} \dotfill (7a)$$

or

$$M_N = w\, L \left(\frac{H\, h^2}{2} - \frac{t\, h^2 \sin\theta}{2} - \frac{h^3}{2} \cos\theta + \frac{h^3}{6} \cos\theta + \frac{H\, t^2}{2}\right) \dotfill (7b)$$

The term containing t^2, in Equation (7b), can be neglected in comparison with the other terms; and thus, the normal-moment formula becomes

$$M_N = w\, L\, h^2 \left(\frac{H}{2} - \frac{h \cos\theta}{3} - \frac{t \sin\theta}{2}\right) \dotfill (8)$$

Case 2. Normal Moment Neglecting Weight and Thickness of Flash-Board.—Making $t = 0$ in Equation (8), the normal moment, neglecting the weight and thickness of the flash-board, is:

$$M_N' = w\, L\, h^2 \left(\frac{H}{2} - \frac{h \cos\theta}{3}\right) \dotfill (9)$$

Case 3. Normal Moment Considering Flash-Board Swung Back to the Vertical, and Neglecting Weight and Thickness of Flash-Board.—In Equation (9), make $\theta = 0$, and the normal moment (considering the flash-board to be swung back to the vertical, and neglecting the weight and thickness of the flash-board) becomes:

$$M_N'' = M_H = w\, L\, h^2 \left(\frac{H}{2} - \frac{h}{3}\right) \dotfill (10)$$

which is the horizontal moment formula.

Non-Overflow Tests.—The moment of the flash-board itself is the same as before (Equation (4b)).

Case 4. Normal Moment.—In Fig. 10(d) the moments are taken around Point O as in Equations (4):

$$M_N = P_1\, a_1 + P_3\, a_3 \dotfill (11)$$

in which $a_1 = \frac{y}{3}$; $a_3 = \frac{t}{2}$; y = slant height, wetted surface of flash-board; and,

$$P_1 = \frac{w\,L\,y\,(H - t\sin\theta)}{2} \qquad (12a)$$

$$P_3 = w\,L\,t\,H \qquad (12b)$$

$$M_N = w\,L\,y\,\frac{(H - t\sin\theta)}{2} \times \frac{y}{3} + w\,L\,t\,\frac{H\,t}{2} \qquad (12c)$$

Since

$$y = \frac{H - t\sin\theta}{\cos\theta} \qquad (13)$$

Therefore,

$$M_N = w\,L\left[\frac{(H - t\sin\theta)^3}{6\cos^2\theta} + \frac{t^2\,H}{2}\right] \qquad (14)$$

In Equation (14), expand $(H - t\sin\theta)^3$ and neglect terms containing t^2 and t^3; and the normal-moment formula is found to be:

$$M_N = w\,L\left(\frac{H^3 - 3\,H^2\,t\sin\theta}{6\cos^2\theta}\right) \qquad (15)$$

Case 5. Normal Moment Neglecting Weight and Thickness of Flash-Board.—If $t = 0$ in Equation (15), the normal moment, neglecting the weight and thickness of the flash-board, becomes:

$$M_N' = w\,L\left(\frac{H^3}{6\cos^2\theta}\right) \qquad (16)$$

Case 6. Normal Moment Considering Flash-Board Swung Back to the Vertical, Neglecting Weight and Thickness.—Considering the flash-board swung back to the vertical and neglecting weight and thickness, $\theta = 0$ and $t = 0$, from which, by Equation (15), the horizontal-moment formula becomes:

$$M_N'' = M_H = \frac{w\,L\,H^3}{6} \qquad (17)$$

Case 3(a) Simplified Horizontal Moment Formula.—Since it was found that compensating refinements, such as weight and thickness of flash-boards, angle of deflection, and increase of normal moment over horizontal moment, had no practical effect in the tests, the horizontal moment has been adopted as a working basis for design (see Item (7), heading "Results of Experiments: General"). The formulas were simplified for field and office use as follows (see Fig. 10(*e*)).

Overflow Tests.—In Equation (10), written in the form,

$$M_H = w\,L\left(\frac{H\,h^2}{2} - \frac{h^3}{3}\right) = M \qquad (18)$$

substitute w = 62.4 lb per cu ft and $H_1 + h = H$, in which H_1 is the head of water, in feet, above the top of the flash-board. Then, the moment, in foot-pounds, is

$$M = 62.4\,L\left(\frac{H_1\,h^2}{2} + \frac{h^3}{6}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(19a)$$

or, in inch-pounds,

$$M = 375\,L\,h^2\left(H_1 + \frac{h}{3}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(19b)$$

Non-Overflow Tests.—From Equation (17), the moment, in inch-pounds, is

$$M = 125\,L\,H^3\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(20)$$

Comparison of Computed with Measured Heads at Failure.—As indicated under Items (5), (6), and (7) (heading, "Results of Experiments: General"), the next step in the investigation was to compare the heads at which flash-board installations should fail, as computed by the various bending moment formulas (Table 4), with the heads at which failure actually had been observed to occur on installations. The laboratory installations used in this investigation to determine the modulus of rupture obviously afforded a convenient opportunity to make such a comparison. The flash-boards were assumed to be supported by standard steel pipe having the dimensions and section moduli given in Table 3. The moduli of rupture used were selected from Fig. 12 and are given in Column (6) of Table 3. For each size of pipe used, the two quantities, S and f, determined the resisting moment to be used in design. No allowance was made for any partial vacuum under the sheet of water flowing over the flash-boards, since they were ventilated at each end.

It should be noted carefully that this is really a comparison of the relative accuracy with which various approximate formulas, nominal pipe sizes, and moduli of rupture will determine the head at failure of a flash-board installation, as compared to the actual head at which failure occurred. The discrepancies found might be somewhat larger when comparisons are made with other installations.

The data for standard pipe, the dimensions of the flash-boards, and spacing of pipe were substituted in the bending moment formulas (Table 4) and the head at which the flash-board should fail was determined (Table 3).

The heads at which the pipes should begin to bend were computed for the horizontal formula using a stress of 46 kips per sq in. to determine the bending moment. The resulting heads are given in Columns (18) and (30) of Table 3. The heads at which pipe should fail are given in Columns (12) and (24); and, the heads at which failure actually occurred are given in Columns (13) and (25).

The differences between the computed heads and the measured heads are given in Columns (14) to (17) and (26) to (29). The values obtained from the normal-moment formula, including the moment due to the flash-board (Cases 1 and 4), and similar values neglecting the flash-board moment (Cases 2 and 3), are seen to be very nearly the same for both the overflow and the non-overflow tests. The average difference ranges from 2% high to 1% low, with a maximum variation from + 5 to − 3 per cent.

The more approximate and convenient simplified horizontal moment formulas, neglecting both the flash-board moment and the angular deflection

(Cases 3*a* and 6*a*), yielded results within an average of 3% lower than the test heads for the overflow and 2% higher for non-overflow tests. For this formula, values of modulus of rupture developed in the following section and given in Column (7), Table 3, were used. It is seen that the more accurate and complicated formula for normal moment, allowing for both the weight and thickness of the flash-board, probably does not improve the results sufficiently to justify its use. Therefore, Equation (20) for non-overflow tests, and Equation (19*b*) for overflow tests, are recommended for use in the selection of pipe for flash-board supports.

Modulus of Rupture.—In order to determine suitable values of modulus of rupture for design with Equations (19) and (20), values of the test data from Table 2 were first substituted in the formulas and the modulus of rupture computed. They are given in Column (14), Table 2. In Fig. 12, they are seen to be in very close agreement with those of the mechanical tests. Average values of the modulus of rupture in round numbers, to be used in Equations (19) and (20) for flash-board design, were selected for each pipe size.

DESIGN OF PIPE SUPPORTS

The use of the derived formulas Equations (19*b*) and (20), in combination with the relation $M = f\,S$ for the design of the flash-board pipes, can be reduced to a very simple procedure. Since the variables involved are the height of the water above the spillway, height of flash-board, and span between supports, it is customary to designate two of these and solve for a third. Usually the size of pipe is to be determined. If so, and the water level is fixed, it may be necessary to modify the pipe spacing by trial to compensate for the gap between standard pipe sizes. On the other hand, if the pipe size and spacing are fixed, a definite solution for the head results.

Variable Modulus of Rupture.—The maximum bending moment, being a function of the known section modulus of the pipe (computed from standard dimensions) and of the experimentally-determined modulus of rupture, is determined by the latter. This characteristic, as opposed to the expected constant modulus of rupture for a given material, such as steel, constitutes an outstanding feature of the tests. It means that for accuracy, different values of modulus of rupture must be used in determining the maximum bending moment for different sizes of pipe since the modulus of rupture ranges from approximately 70 kips per sq in. for 3-in. pipe to 90 kips per sq in. for ¾-in. pipe (see Fig. 12). It is here that the test results are most valuable, inasmuch as the range in modulus of rupture is by far the greatest variable in the design. Therefore, for reasonable accuracy in the control of head-water elevations, it is preferable to use the tabulated modulus of rupture that corresponds most nearly to the selected pipe size, rather than to assume an average value for all sizes of pipe. As already indicated, if pipe sizes larger than 3 in. are to be used, it is presumed that they will first be checked by additional tests.

Height of Flash-Board.—It is obvious from Equations (19) and (20) that the ratio of the height of the flash-board to the total head of water is important. For the same pipe and spacing, the higher flash-boards fail earlier or under less head of water than do lower boards. To express it another way, for a given height of water above a spillway crest, a high flash-board gate with only a few inches of water flowing over it will fall, whereas a lower flash-board (supported by the same pipe system), overtopped by several feet of water depth, will stand erect. This inherent characteristic must be considered in selecting the appropriate flash-board dimensions for a structure.

Hinges.—If a flash-board gate is hinged at the bottom, the hinges must be loosely fitted. Any binding at the hinge would prevent the flash-board either from swinging freely under water pressure or from transferring its load properly to the pipe support. In either case the load on the pipe would be reduced and it would fail at a higher head than designed.

Typical Design Procedure.—To illustrate design procedure assume a spillway channel 35 ft wide by 8 ft deep, the entire capacity (depth) of which is required in time of flood. It is desired to raise the normal lake level 3 ft (h) above the spillway-level (crest), limit the maximum flood height above this normal water level to 2 ft (H_1), and, at the same time, retain a 3-ft free-board between the maximum flood height and the top of the dam. The cost of widening this spillway channel, so that it would have the same flood capacity at a 5-ft depth, compared to this 8-ft depth, would be prohibitive. The procedure follows:

(1) Pipe spacing: The customary spacing for the supporting pipe for a gate of this height is from 3 to 5 ft since wider spacing requires a gate that is too ponderous to lift back into position after failure, and closer spacing multiplies pipe replacement units. Try eight pipe supports. Then the length supported by one pipe is $\frac{35}{8} = 4.4$. However, since the gate is sufficiently rigid to distribute the loading equally to all the pipe, they act as a whole and are best designed on a basis of total moment. To prevent "curling" at the ends, place a pipe 1 ft from each end of the gate and space the remaining six pipes symmetrically with respect to the center line of the channel.

(2) Determine pipe sizes: Use the overflow formula (Equation (19*b*)) for bending moment, with $L = 35$ ft. Then $M = 375 \times 35 \times 9\,(2 + \frac{3}{3}) = 354\,000$ in-lb. The section modulus $S = \dfrac{M}{f}$ with $f = 70\,000$ lb per sq in. (approximately) yields: $S = \dfrac{354\,000}{8 \times 70\,000} = 0.632$ which indicates pipe between 2-in. and $2\frac{1}{2}$-in. in diameter. By trial it is found that a $1\frac{1}{2}$-in. pipe at each end, with four 2-in. and two $2\frac{1}{2}$-in. pipes alternated symmetrically between, gives a resisting moment of 353 000 in-lb. This is within the working limits.

(3) Check of yield point: Substituting again the combined section moduli of these eight pipes in $M = Sf$ when $f = 46\,000$ lb per sq in. (yield point) yields: $M = 5\,040 \times 46\,000 = 232\,000$, which is equivalent to a head over the gate (Equation (19*b*)) of $H_1 = \dfrac{232\,000}{375 \times 35 \times 9} - \dfrac{3}{3} = 0.9$ ft. If it is probable that a head of 0.9 ft over the gate will occur frequently it would be better either to lower the gate height or use stronger pipe. This encroaches on the free-board during occasional flood peaks, but avoids frequent unsightly bending of the pipe.

INSTALLATION AT SHERANDO LAKE TESTED

An automatic flash-board gate 35 ft long by 3.2 ft high, designed in accordance with the findings of the Hydraulic Laboratory tests, was installed at Sherando Lake, 10 miles south of Waynesboro, Va., in the George Washington National Forest in the fall of 1935 (see Fig. 1). The wooden gate is hinged at eight points and is sufficiently rigid so that the hydraulic load is apparently equal on all of the eight supporting pipes. These pipes are spaced on about 5-ft centers with end pipes about 1 ft from the channel side-walls. The average length of gate supported by one pipe is 4.4 ft. The hinges are placed behind the pipes and are very loosely fitted to avoid binding or taking part of the pipe load.

Floods collapsed the gate three times during 1936. First, on March 20, when the lake level reached a head of 2.42 ft (determined by an automatic gage) over the top of the flash-board, or within 0.12 ft of that specified, the gate opened gently downward with an accelerating speed as it fell into the surging tail-water which cushioned its fall. At first the gate leaned more and more out of plumb until the angle was about 20°; then all pipes collapsed and fell simultaneously.

Following this successful field test of 1-in. pipe, the pipe size was increased progressively, using first $1\frac{1}{2}$-in. pipe and then 2 in. with $1\frac{1}{4}$-in. end pipes. In each case the gate fell when the water surface was 0.1 to 0.3 ft below the designed height—that is, on the safe side. This procedure of first testing a pipe size smaller than the design calls for is recommended where the pipe cost is negligible. This helps to ascertain the effect of local conditions—such as wind, wave and current action, ice thrust, presence of logs and debris, vibration, and partial vacuum under the gate.

Other Installations.—One gate costing $500 saved a spillway cost of $15 000. This particular gate was 30 ft long and consisted of two superimposed vertical sections, the upper 2 ft being designed to fail under a 1.5-ft head and the lower $4\frac{1}{2}$-ft section failing under 3-ft head. Vacuum under the nappe caused this gate to fall at about 7% lower head than would be the case with ventilated overflow.

Checks made on the operation of a number of installed flash-board gates indicate adherence to within about 5% of the design water heads, and in all cases the failure of the gate has been on the safe side.

It has been observed that, in installations where a considerable degree of vacuum is formed beneath the nappe, the gate fails under a lower head-water than otherwise. This offers the alternative of using heavier pipe supports or of breaking the vacuum by vent pipes or by other expedients. However, like all other field test results obtained by following this design method, this vacuum effect tends to lower the maximum flood stage behind the dam.

CONCLUSIONS

The following conclusions have been amplified to include some minor, as well as the major, results of the experiments and field tests because of the dearth of published information on flash-board installations.

(1) Flash-boards with standard, steel-pipe supports can be designed to fall consistently within narrow limits under specified conditions.

(2) In the design of pipe supports for flash-boards, the simplified horizontal moment formula (Equations (19*b*)) for ventilated overflow conditions, and Equation (20) for non-overflow conditions, will give results within practical limits (approximately 3 per cent). Stock commercial steel pipe can be used, the section modulus being computed from internal and external diameters taken from lists of pipe standards as shown in Table 2. The moduli of rupture and resulting bending moments recommended for use in these formulas are given in Table 5. The stress at the yield point may be taken as 46 000 lb per sq in.

TABLE 5.—RECOMMENDED VALUES, MODULI OF RUPTURE AND RESULTING BENDING MOMENTS

Description	Pipe Sizes (Nominal Diameter) in Inches						
	¾	1	1¼	1½	2	2½	3
Modulus of rupture, in pounds per square inch	80 000	77 000	73 000	72 000	70 000	70 000	70 000
Bending Moment, in inch-pounds	5 600	10 400	17 150	23 800	39 200	74 900	122 500

(3) If greater accuracy is desired, the appropriate normal-moment formulas (Equations (4), (8) or (15)), may be used with the moduli of rupture shown in Column (6), Table 3.

(4) The modulus of rupture to be used in design may be determined experimentally from mechanical as well as hydraulic tests for pipes larger than 1 in. in diameter.

(5) The ratio of height of flash-board to total head of water should be such that ordinary floods can be passed without permanent bending of the pipe supports. In other words, the yield point of the pipe must be taken into consideration in design.

(6) Any unusual local conditions must be considered in the application of the formulas.

(7) The load on flash-board pipes may be reduced and increased successively without producing failure, as long as the maximum load at which failure should occur is not reached. When the maximum load (head) is reached, however, the pipe will assuredly collapse and fall completely.

(8) Uniform bending up to a critical angle with the vertical, of from 20° to 30°, usually precedes failure.

(9) Vacuum under the nappe causes vibration and collapse of the flash-board pipes at lower heads than designed. This is generally on the safe side.

(10) Because of loading conditions and relative wall thickness, the modulus of rupture used in designing steel pipes for flash-board supports is considerably greater than the minimum specified ultimate tensile strength of the metal in the pipe. This variation is greater for the smaller pipes.

(11) The formulas recommended for design (see Table 4) do not allow for the indeterminate binding effect of wedge, or "V"-jointed, flash-boards, or the

interlocking and wedging of "butt"-jointed flash-boards, in preventing failure at the desired elevation of the head-water.

(12) The use of solid steel or iron pins for flash-board supports seems to be not as dependable as the use of standard steel pipe.

ACKNOWLEDGMENTS

The success of the work was due largely to co-operation between the U. S. Forest Service and the National Bureau of Standards.

All hydraulic tests were conducted in the Hydraulic Laboratory Section of the latter under the general supervision of H. N. Eaton, Chief of the Section. Mr. Eaton also carefully reviewed and edited the previous report as well as the manuscript of this paper. The apparatus was constructed by C. W. Elliot of the laboratory staff, and in making observations, other members of the staff, as well as men from the office of the Forest Service, participated. The mechanical tests were made in the Engineering Mechanics Section by A. H. Stang, and others, under the supervision of H. L. Whittemore, Chief of the Section. The work of these laboratories is supervised generally by H. L. Dryden under L. J. Briggs, Director of the National Bureau of Standards.

J. C. Dort, M. Am. Soc. C. E., proposed the laboratory experiments to check the design for the installation of flash-boards at Sherando Lake. F. L. Brown, Jun. Am. Soc. C. E., was assigned to the hydraulic laboratory during the tests and A. H. Ronka, Assoc. M. Am. Soc. C. E., assisted in the analysis of results.

Regional Forester, R. M. Evans, is Chief of Region 7, U. S. Forest Service. T. W. Norcross, M. Am. Soc. C. E., is Chief of the Division of Engineering, U. S. Forest Service, and F. A. Silcox is Chief of the Forest Service.

The Pennsylvania Water and Power Company of Baltimore loaned their voluminous test data freely and facilitated inspection of their flash-board installations.

APPENDIX I

BIBLIOGRAPHY

(1) **Thomas, B. F.** Movable Dams. *Transactions*, Am. Soc. C. E., Vol. 39, June, 1898, pp. 431–616.

(2) **Muller, Richard.** A Formula for Calculating Flashboards for Dams. *Engineering Record*, Vol. 58, No. 8, August 22, 1908, p. 208.

(3) ——. An Arched Dam at Las Vegas, N. M. *Engineering Record*, Vol. 62, No. 15, October 8, 1910, p. 404.

(4) **Scheidenhelm, F. W.** The Reconstruction of the Stony River Dam. *Transactions*, Am. Soc. C. E., Vol. 81 (1917), pp. 907–1110.

(5) **Kanekeberg, A.** Diversion Dam Consists Chiefly of Collapsible Units. *Engineering News-Record*, Vol. 99, No. 19, November 10, 1927, p. 769.

(6) **Jacobs, Joseph.** Improved Flashboard Design Increased Hydro Plant Capacity. *Power*, Vol. 68, No. 20, November 13, 1928, pp. 793–794.

(7) ——. Tilting Slab Flashboards Provide Emergency Spillway. *Engineering News-Record*, Vol. 105, No. 7, August 14, 1930, p. 263.

(8) **Bell, H. K.** Blocking Spillways of Dams is Dangerous. *Engineering News-Record*, Vol. 107, No. 2, July 9, 1931, p. 64.

(9) **Carter, C. E.** Hinged Flashboards for Headwater Control. *Electrical World*, Vol. 100, No. 3, July 16, 1932, p. 79.

(10) ——. Flashboard Resetting Platform Conserves Water. *Electrical World*, Vol. 102, No. 9, August 26, 1933, p. 273.

(11) ——. Power-Driven Hoists Handle McIndols Dam Flashboards. *Power*, Vol. 77, No. 11, November, 1933, pp. 570–571.

(12) **Barrows, H. K.** Water Power Engineering. McGraw-Hill Book Co., Inc., New York, N. Y., 1927, pp. 491–493.

(13) **Creager, W. P.** Engineering for Masonry Dams. John Wiley & Sons, Inc., New York, N. Y., 1929, pp. 248–254.

(14) ——. Relative Merits of Various Types of Spillway Gates on or Adjacent to Dams; Flashboards, Needles and Stop-Logs. *Electrical West*, Vol. 60, No. 6, May 15, 1928, p. 365.

(15) ——. Tests of Steel-Pipe Pins for Temporary Flashboards on Spillways. National Bureau of Standards Report, Washington, D. C., April 3, 1937; (a) Appendix III.

(16) **Betts, C. A.** Controlling Flashboard Drop by Collapsing Pipe Supports. *Engineering News-Record*, Vol. 116, No. 18, April 30, 1936, p. 627.

(17) ——. Unpublished Data on Flashboard Tests. Pennsylvania Water and Power Co., Baltimore, Md.

(18) Defined by Standard E6–36, A. S. T. M. Standards, 1936, Pt. I, Metals, p. 855, the yield point is: "The stress in a material at which there occurs a marked increase in strain without an increase in stress."

(19) **Fuller, Charles E. and Johnston, William A.** Applied Mechanics, Vol. II, 1919, p. 147.

APPENDIX II

NOTATION

The following notation conforms essentially with American Standard Symbols for Mechanics, Structural Engineering, and Testing Materials,[5] compiled by a committee of the American Standards Association with Society representation, and approved by the Association in 1932:

a = lever arm to a local force, P; a_1 = lever arm to P_1; a_2 = lever arm to P_2; a_3 = lever arm to P_3; and a_w = lever arm to the force W;

D = diameter of pipe; D_e = external diameter; and, D_i = interior diameter;

f = stress in the outer fiber of a pipe;

H = height of head-water above the crest of the dam at failure; H_1 = height of water surface above the top of the flash-board;

h = height of the flash-board above the base of the hinge; $\bar{h}$ = distance from the base of the flash-board to its center of gravity;

---

[5] A. S. A.—Z10a—1932.

L = length of flash-board, along the crest, that is supported by one pipe; average spacing of the supporting pipes;

M = maximum bending moment; M_B = the moment due to the weight of the flash-board; M_N = moment due to water pressure normal to the flash-board; the total moment acting on a supporting pipe is $M_N + M_B$; M_N' and M_N'' = the approximate moment due to water pressure on the flash-board; M_H = the moment due to water pressure on a flash-board assumed to be swung back to the vertical position—that is, the "horizontal moment";

P = concentrated loads; P_1, P_2, etc. are local forces;

S = section modulus;

t = average thickness of the flash-board;

W = weight of flash-board, in pounds per pipe;

w = specific weight of water (62.4 lb per cu ft);

y = slant height, wetted surface of flash-board;

γ = specific weight of the timber used for flash-boards;

θ = angular deflection of the flash-board from the vertical.

APPENDIX III

Computation of Moment and Stress

Overflow Test No. 32.—The pressure diagram for Test No. 32 is shown in Fig. 10 (a) and the pipe data are given in Table 2. The distribution of water pressure on the flash-boards is shown diagrammatically, and the force diagram thus obtained is divided into convenient areas for computing the moment. All dimensions are in feet. The summation of the moments of all the areas around the base at Point O represents the total water moment. The moment of W about Point O is the flash-board moment.

Referring to Fig. 10 (a): $H = 4.15$ ft; $W = 150$ lb; $\bar{h} = 1.17$ ft; $\theta = 21°$; $M = M_B + M_N$; and,

$$M_B = W\,(\bar{h} \sin \theta - 0.11 \cos \theta) \ldots\ldots (21)$$

Substituting appropriate values in Equation (21), $M_B = 540$ in-lb.

For normal moment take moments around Point O:

$$M_N = P_1\,(2.18) + P_2\,(2.07) + P_3\,(0.93) + P_4\,(0.62) + P_5\,(0.11) - P_6\,(0.18) \ldots (22)$$

Again, substituting appropriate values in Equation (22), $M_N = +\,7.96\,(12\,w\,L)$ in-lb; and, neglecting P_6, $M_N = 48\,130$ in-lb. Then, $M = M_B + M_N = 500 + 48\,100 = 48\,600$ in-lb.

The stress in the pipe (modulus of rupture) $f = \frac{M}{S} = \frac{48\,600}{0.616} = 78\,900$ lb per sq in. as shown in Column (13), Table 2.

DISCUSSION

William P. Creager,[6] **M. Am. Soc. C. E.** (by letter).—An excellent series of experiments, indicating the strength of flash-board pins, has been presented in this paper. It should be noted that all of the experiments were made with a completely aerated jet yet, in most cases in practice, the jet is not aerated and a vacuum load is added to the pins.

The designer aims to provide an installation which will surely fail before the water surface reaches an undesirable elevation. On the other hand, he does not want failure to occur too soon because that would result in a greater number of replacements. Therefore, he is anxious to design the pins with as great an accuracy as possible.

The authors recommend a modulus of rupture varying from 70 000 lb per sq in. for 3-in. and 2-in. pipe to 80 000 for ¾-in. pipe. The writer found, by tests[7] made prior to 1926, that a modulus of rupture between 42 000 and 58 000 lb per sq in. applied to a number of actual installations utilizing 2-in. to 3-in. pipes. The difference between these stresses and those found by the authors may be attributed to three factors:

(1) The tests made by the writer involved actual cases in which a vacuum occurred under the jet. The amount of the vacuum being unknown, the computed failure stresses of 42 000 to 58 000 lb per sq in. disregarded the vacuum. If this vacuum had not occurred, as in the case of the authors' experiments, the computed failure stress would have been higher.

(2) Present-day manufacture of such pipes requires a descaling operation which involves rolling the pipe in the finishing passes at lower temperatures, which has a tendency to raise the yield point.

(3) Present-day tolerances in pipe thickness are "tighter" than they were formerly. Hence, the average pipe thickness to-day is greater, resulting in greater bending strength.

If water higher than that anticipated would cause damage, the writer recommends the use of the authors' design stresses for the first installation. In all probability the effect of vacuum in the usual clear crest installation will result in the boards failing too soon and the use of somewhat stronger pins in succeeding installations.

Lincoln W. Ryder,[8] **Jun. Am. Soc. C. E.** (by letter).—In August, 1939, in connection with the design and construction of three small flash-board dams for gound-water level control on an open drainage channel, it was necessary to select pins that would fail within a small range in the height of water above the crest of the dam. It was required that the boards remain in place with 1 ft, and certainly fail with 2 ft, of water overtopping them. The purpose of the tests was to determine the maximum bending moments of the pipe specimens chosen tentatively by the designing engineer[9] in order to check and control the variation in overtopping to be expected in the field.

---

[6] Cons. Engr., Buffalo, N. Y.

[7] "Hydro-Electric Handbook," by William P. Creager and Joel D. Justin, Members, Am. Soc. C. E., Art. 132, p. 263.

[8] Asst. Engr., Metcalf & Eddy, Boston, Mass.

The apparatus, set up as shown in Fig. 13, differed from the straight cantilever principle utilized by the authors in having two specimens of the same size inserted into opposite ends of a close-fitting pipe sleeve. The load was applied at the middle of the sleeve through a stiff section of I-beam. The sleeve and stiff member simulated closely the action of the cantilever jig in the aforementioned test, there being no appreciable distortion in the sleeve. It is believed that this test rig is simpler than that used by the authors and will yield equally correct results.

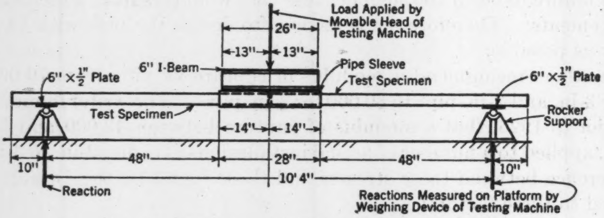


FIG. 13.—METHOD OF TESTING FLASH-BOARD PINS

The low height of the end supports would not allow the bending in the test of the pins to the greater than 20° amount possible in the authors' test; but modifications in this set-up, supporting the ends of the pins farther above the platform of the machine, would allow bends, limited only by the vertical travel of the screw or the ultimate breaking point of the pipe specimen. Such a modification is not necessary for the test as conducted, however, since the maximum bending moment was reached before a bend greater than 8° was made.

The specimens for the first test were 6-ft lengths of 2-in., nominal diameter, standard weight, seamless steel tubing marked "Pittsburgh-P-Seamless Tested 1000 lb. Grade A." Each specimen was inserted 14 in. into the 28-in. long, 2½-in. standard weight seamless tube sleeve, and was supported at points 48 in. from the ends of the sleeve on steel plates and rocker supports.

The specimens for the second test were 2½-in., nominal diameter, standard weight, seamless steel tubing marked and supported as for the first test. The 28-in. sleeve was of 3-in. standard steam pipe.

The results of the two tests, and comparisons with the values recommended by the authors are presented in Table 6. An examination of the results of the two tests shows that a departure from generally recommended design values is to be expected in individual cases. It is important that tests be made of the selected pipe when close control of the water level at failure of the boards is desired, with the thought in mind that a change in design may possibly be made, based on such test results.

The method of testing, using two specimens at one time, tends to give an average value for the modulus, although in the tests described herein the measured deflections showed that the 28-in. pipe sleeve moved downward always parallel to the bed of the machine, indicating the uniform strength of the two specimens.

---

[9] "Hydro-Electric Handbook," by William P. Creager and Joel D. Justin, Members, Am. Soc. C. E., p. 265.

TABLE 6.—COMPARISON OF TEST RESULTS

Item No.	Description	Test No. 1	Test No. 2
1	Nominal diameter of specimen, in inches	2.0	2.5
2	Maximum load recorded on testing machine, applied at the center of the test set-up, in pounds	1 500	2 600
3	Maximum bending moment,* in inch-pounds	36 000	62 400
4	Section modulus,† S, in inches^3	0.563	1.062
5	Modulus of rupture, $f = M/S$, in pounds per square inch	64 000	58 700
6	Recommended modulus of rupture for use in design (see Table 5), in pounds per square inch	70 000	70 000
7	Recommended bending moment for use in design (see Table 5), in inch-pounds	39 200	74 900
8	Comparison of results, in percentages	8	16

* $M = \frac{1}{2}$ (Item 2) times 48 inches.

† The section moduli were those computed from the standard dimensions for the weights of pipe tested.

Acknowledgment.—Irving Cowdrey, Associate Professor of Testing Materials, Massachusetts Institute of Technology, devised the method of testing and conducted the tests in the laboratories of the Institute; acknowledgment is made for permission to include a description of his rig in this discussion.

E. T. SCHULEEN,[10] ASSOC. M. AM. SOC. C. E. (by letter).—Considerable credit is due the National Hydraulic Laboratory for its tests on flash-boards and to the authors for their suggested methods of rational flash-board design. Apparently, the tests have been conducted in a thorough manner and the use of a full-size flash-board should increase the reliability of the data. It is to be regretted, however, that limitations of discharge in the laboratory precluded tests at higher overflows, particularly at the larger pipe sizes.

The Pennsylvania Water and Power Company conducted a large number of tests several years ago on flash-boards and flash-board pins at its hydroelectric development at Holtwood, Pa. These tests included mechanical bending tests on solid steel pins, wrought iron pipe, standard steel pipe, and extra strong steel pipe of various sizes. Tests were also made to determine the actual water pressure exerted on a flash-board deflected through various angles from the vertical under several depths of overflow.

The mechanical bending tests were conducted in the following manner: The specimen under test was inserted in a socket of 6-in., extra strong, pipe 12 in long. The socket was reinforced at the top with a pipe flange and grouted into a concrete floor. Suitable sleeves were used in the socket to give a snug fit with the various sizes of pipes and pins. The load or pull was exerted at a point 40 in. above the socket and was maintained normal to the specimen throughout the test by connecting the specimen, at the point of pull, through a spring-balance dynamometer to a 4-in. channel, which, in turn, was hinged to a support at the base of the specimen. A pull from a windlass was then exerted on the channel, causing it to rotate with the specimen. The pull normal

---

[10] Hydr. Test Engr., Pennsylvania Water & Power Co., Holtwood, Pa.

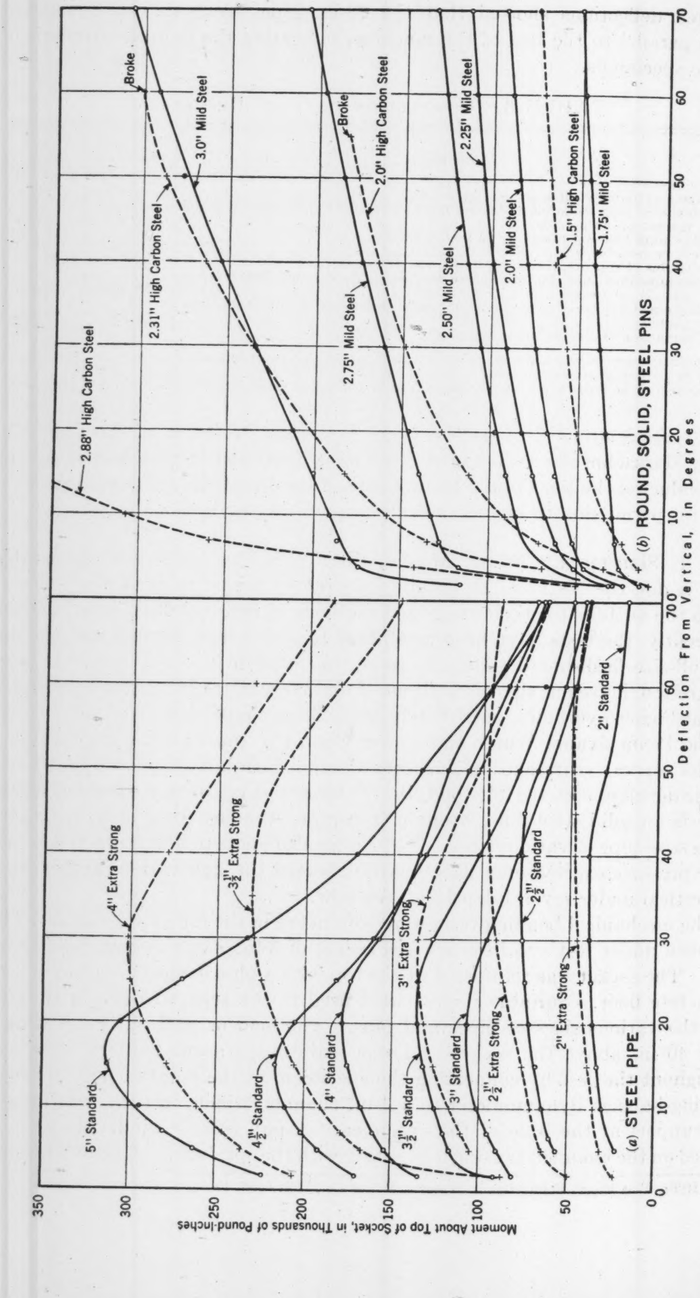


FIG. 14.—RESULTS OF MECHANICAL BENDING TESTS

to the specimen was read on the dynamometer, while the angle of deflection was read on a graduated scale. Since only the resisting moment at the top of the socket is required, the varying load distribution of the hydraulic pressure may be replaced by a single mechanical load such as was done in the test without introducing an appreciable error. Three specimens of each size of pipe or pin were tested and the results averaged.

The results of the tests on steel pipe are shown in Fig. 14(*a*) in which the resisting moment has been plotted against the angle of deflection. The writer feels that nothing is gained in reducing the resisting moment to unit stress by dividing through by the section modulus, which means dividing by a constant for a given pipe size. In addition, such procedure gives misleading stress values, as the actual section modulus is changing continually as deflection progresses. The curves in Fig. 14(*a*) show that pipe is well adapted for use as flash-board supports since the deflection increases with the moment until a definite angle, characteristic for each pipe size, is reached, beyond which the deflection increases while the moment decreases. This is a desirable characteristic which permits the pipe support, once the flash-board deflects to the angle of maximum moment, to bend over, completely and quickly, leaving little obstruction to the passage of water, ice, or debris. Table 7 summarizes the maximum moments and the angles of failure for the various sizes of pipe tests. The resisting moments of the individual test specimens of the same size varied within 5% of the averages given on the curves and in the table.

TABLE 7.—RESULTS OF BENDING TESTS ON PIPES

Size of pipe, in inches (1)	Maximum resisting moment, in pound-inches (2)	Deflection from the vertical, at maximum moment, in degrees (3)	Size of pipe, in inches (1)	Maximum resisting moment, in pound-inches (2)	Deflection from the vertical, at maximum moment, in degrees (3)	Size of pipe, in inches (1)	Maximum resisting moment, in pound-inches (2)	Deflection from the vertical, at maximum moment, in degrees (3)
(*a*) STANDARD STEEL			(*b*) EXTRA STRONG STEEL			(*c*) STANDARD WROUGHT IRON		
2.0	39 000	40	2.0	48 000	53	1.5	25 000	50
2.5	77 000	44	2.5	97 000	48	2.0	40 000	40
3.0	107 000	20	3.0	135 000	30	2.5	74 000	40
3.5	136 000	23	3.5	231 000	35	2.5†	98 000	50
4.0	178 000	20	4.0	300 000	30	..	....	..
4.5	216 000	15	2.5*	201 600	75	..	....	..
5.0	312 000	17	..	....	..	..	....	..

* Double extra strong. † Extra strong.

The Holtwood tests are comparable with the National Hydraulic Laboratory tests for pipe sizes of 2, 2.5, and 3 in. The angle of deflection at failure is greater in the Holtwood tests than the National Hydraulic Laboratory tests and the value of resisting moment is slightly less.

The results of some typical tests on solid steel pins are shown in Fig. 14(*b*). It is evident from these curves that the continued gradual increase in deflection with increasing moment makes solid pins unsuitable for flash-board supports, as very high forebay levels would occur before complete failure of the pins took place. Another disadvantage is the tendency for the pins to bend from 5 to 8 in. above the socket and thus continue to obstruct the passage of ice or debris after failure. It was found by test that these two difficulties could be overcome by cutting grooves on the front and back sides of the pin at the support, but this method is not practicable. The use of solid pins will usually be far more expensive than the use of pipe.

The computation of moment on a flash-board with overflow after deflection has started, with a normal moment formula, such as Equation (9), is obviously in error since static pressures are attributed to flowing water. An attempt to determine the magnitude of this error was made in a series of tests in Holtwood in which the actual moment of the board was measured at several conditions of overflow.

A test board 2.25 ft wide and 1.8 ft high was placed in a flume of equal width and was rigidly fastened at the bottom to a steel shaft free to rotate in ball bearings. On the outside of the flume, a cast-iron quadrant of 1-ft radius was keyed to the shaft and connected to a spring-balance dynamometer by a thin steel tape running over its edge. The dynamometer, in turn, was connected by a cable to a hand-operated winch. The quadrant and the board were counterbalanced so as to be in balance in any position. A seal of negligible friction at the edges of the board was provided by coating the walls of the flume with paraffin and scraping for close clearances. Four piezometers were placed in a vertical line on the board to indicate the pressure distribution. The board could be held at any angle under various heads and the moment about the bottom of the board, in pound-feet, could be read directly on the dyna-

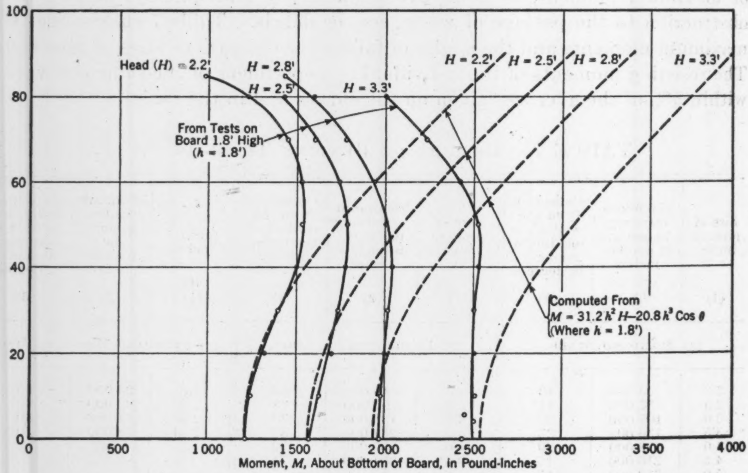


FIG. 15.—COMPARISON OF COMPUTED MOMENTS WITH ACTUAL MOMENTS AT VARIOUS ANGLES OF DEFLECTION

mometer. Tests on the 1.8-ft board were limited to four heads ranging from 2.2 ft to 3.3 ft.

The results of the tests are shown in Fig. 15, in which the measured moments, together with the moments computed from the normal moment formula, are plotted against the angle of deflection. At 2.2-ft head the test moment increases with the computed moment up to a deflection angle of 30° and then decreases, while the computed moment continues to increase. At the higher heads, the point of divergence between the test and computed moments occurs at lower angles of deflection, and at 3.3-ft head there is an appreciable difference at zero deflection. The piezometer readings indicated this reduction in pressure as the angle of deflection increased to be greatest near the top of the board.

An interesting corollary of the tests was the determination of the effect on moment of a vacuum on the down-stream side of the board. Since the test board was not aerated, a vacuum was found similar to that occurring on a long line of flash-boards. Breaking this vacuum by inserting a pipe through the nappe decreased the moment on the board approximately 7 per cent.

JOHN E. FIELD,[11] M. AM. SOC. C. E. (by letter).—In pointing out the uses and desirability of the pipe-supported, collapsible flash-board, this paper is so complete that little remains to be added. The authors are to be commended for the great care with which their research was conducted, and to be congratulated for the very practical conclusions made available to the engineering profession. The writer has used, and caused to be installed, such flash-boards in the spillways of canals and reservoirs, but until he was furnished with advance information and data by the authors, he has hesitated in recommending them unless installed under his personal supervision.

Spillway costs are no inconsiderable part of the entire cost of a dam, and with "fool-proof" collapsible flash-boards that cost is reduced materially and much of the valid objection to flash-boards in spillways will be answered satisfactorily.

The writer has seen many, and has installed some, collapsible flash-boards dependent more or less on mechanical operation, and his remembrance is that few were successful and reliable.

If the dam and spillway are close to a power-plant where experienced men are available at all hours, and especially where there are telephones to give warning of approaching floods, mechanically operated flash-boards in spillways can be tolerated.

The publication of the paper is opportune as records of the size of maximum floods have been considerably changed in the last few years; and ideas of what constitutes necessary spillway capacity have been greatly altered, with the result that the capacity and cost of spillways have been increased materially. Many existing spillways, heretofore considered to be of ample capacity, must be increased in size to conform to the probable maximum flood as determined by the later data.

The failures of more than 100 dams in New England, New York, and New Jersey, in March, 1936, were due to inadequate spillways and are a forceful argument for the re-examination and change in the design of existing spillways. An increased depth by lowering the floor or lip of the spillway with collapsible pipe-supported flash-boards, installed to maintain the necessary height of water in the reservoir, will often be the answer to the new problem.

---

[11] Cons. and Superv. Engr., Denver, Colo.; Engr. Examiner, PWA, Washington, D. C.

Two valid objections to an uncontrolled spillway are the probable loss of stored water, and the difficulty of stopping the outflow after the danger point has passed. These objections can be met in part by so selecting the pipe supports that those most accessible and easily replaced will fail before others. The authors have studied a flash-board so designed that the upper part collapses at a predetermined height of water and the lower part at a higher stage of water level. For a narrow spillway, pipe set horizontally in sockets, in both of the concrete abutments, could be made to support an upper portion of the flash-board independently of the lower portion, and would be free to swing down stream on hinges at the abutments. Local conditions will suggest many variations in the details of the design all based on the fundamentals shown in the paper. The writer has found this type of flash-board very satisfactory in canals, and the description of one actual problem may be of interest.

The cross drainage was taken into the canal primarily to secure the water for storage and also to obviate an expensive culvert or overshot. The cross drainage water from moderate rains could be carried by the canal; but storms were quite common during which the storm water would be in excess of the capacity of the canal to carry it. The proper location for the spillway was at a rock cut, up stream from the cross drainage channel; the lower bank of the canal was cut down to a level 2 ft below the high-water line and pipe supported flash-boards installed in the cut. They were designed to collapse when the water rose 1 ft above the high-water line so that when the pipe failed the wasteway would carry a depth of 3 ft of water equal to the maximum estimated inflow from the cross drainage. Under certain conditions the flood water will flow up the canal and be discharged through the spillway; this prevents an excessive flow below the point at which other cross drainage is probably entering the canal. The plans called for three such wasteways although the canal was provided with a regular wasteway with its floor somewhat lower than the bottom of the canal. This wasteway was provided with slide gates and hand-operated lifting devices.

In 1905, before the pipe-supported flash-boards were proposed, the writer provided wasteways in one large canal by cutting the lower bank, but in lieu of collapsible flash-boards a low earth dam was placed in the spillway. This dam had a crest level 1 ft above the high-water line and washed out whenever overtopped, thereby relieving the canal of its excess of cross drainage water. This design was considered "fool-proof," certain in operation, and easily replaced. The writer is now wondering why he did not think of pipe-supported flash-boards in this case; and he would have profited greatly had the contents of the paper been available to him at that time.

JULIAN H. WHITE,[12] ASSOC. M. AM. SOC. C. E. (by letter).—The extensive tests reported by the authors should help to restore confidence in the use of pipe flash-board pins. Although there is ample evidence that, in the past, their behavior under overflow conditions has been rather unsatisfactory, the results shown in Table 3 indicate that, by the use of the data obtained, failure can be predicted with remarkable accuracy. For practical purposes, a variation of only 2% of the head, as obtained by the authors, would meet every requirement. However, if such close results are expected with conditions different from those under which the tests were made, a better understanding of some factors not evaluated in the paper is necessary. The reduction of pressure at the top of flash-boards, due to overflow, and the rise of water against the flash-boards beneath the nappe can cause an error in head at failure in excess of 2 per cent.

---

[12] Engr., Howard M. Turner, Boston, Mass.

Through the courtesy of Charles M. Allen, M. Am. Soc. C. E., Director of the Alden Hydraulic Laboratory of the Worcester Polytechnic Institute, the writer made a few brief tests to obtain more information on these two factors, and the result may be of interest.

For the tests, a 5-ft glass-sided flume was narrowed to a width of 1 ft for a length of about 14 ft, and a wooden flash-board 1.51 ft high was set in the lower end. The back of the board was faced with a $\frac{1}{16}$-in. brass plate, set with the top above the wood to form a crest, and carefully drilled for 11 piezometers. The piezometers were set in the center of the board 0.010, 0.020, 0.040, 0.080, 0.161, 0.318, 0.450, 0.750, 0.950, 1.200, and 1.451 ft below the crest, and were connected to 0.5-in. gage glasses. Water, measured by venturi meter in a 12-in. pipe, passed through corrugated baffles before entering the flume. The head on the crest was measured by means of a piezometer set in the flume wall about 6 ft up stream from the flash-board, and in some tests was checked with a point gage. In certain runs, where a high velocity of approach was desired, the bottom of the channel was raised to a point 0.50 ft below the crest.

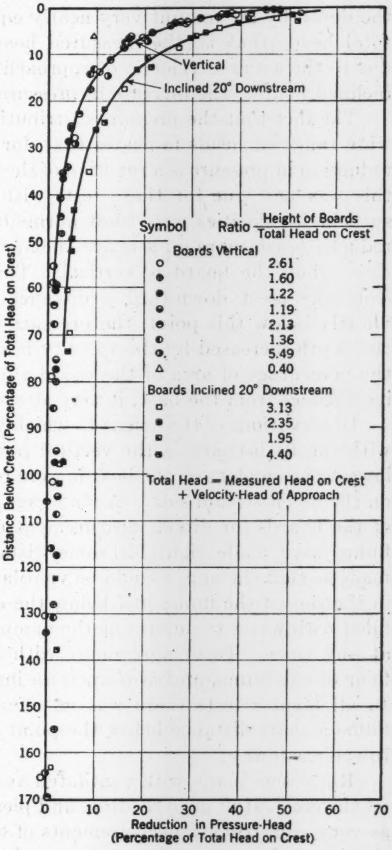


FIG. 16.—REDUCTION IN PRESSURE ON FACE OF FLASH-BOARDS WITH OVERFLOW

Measurements of pressure on the up-stream face of the flash-boards were made both with the boards vertical, and inclined down stream at an angle of 20° with the vertical. The heads ranged from 0.27 ft to about 1.20 ft, and the velocity head in the approach channel varied from 0.001 ft to about 0.13 ft. The results are shown in Fig. 16, in which the reduction in pressure for different points on the boards is expressed as a percentage of the total head on the crest. The tests showed that in the case with the boards vertical the pressure head at the bottom, and for some distance up, was greater than the depth by an amount very nearly equal to the velocity head. Therefore, the total head (that is, the measured head on the crest plus the velocity head due to the average velocity of approach) was used for the reference. With the inclined boards, the increase in pressure at the bottom was not so pronounced.

The fact that the pressure distribution is very nearly the same for a fairly wide range of heads indicates that, for the purpose of flash-board design, the reduction in pressure is a function of the total head on the crest only. Although this was not true for those tests with extremely high velocity of approach, such high velocities would not normally be met in practice. The pressure is reduced much more on a board inclined 20°, its approximate angle a failure, than when the board is vertical. The area affected in both cases extends from the crest downward a distance equal to about one half the head. Shortly below this point, the pressure is the same as for static loading, with the depth increased by the velocity head of approach. With heavy overflow, the percentage of area of the board affected becomes greater, and, because of its distance from the base, it may alter the bending moment considerably.

Observations of the height to which water behind the nappe rises were made with the flash-board in the vertical position only. For these tests, the three lowest piezometers on the boards were reversed so that they measured pressure on the down-stream side. A staff gage was mounted on the down-stream face of the boards for direct measurement of the water height. The sides of the flume were made tight for some distance down stream, and provision was made so that the nappe could be ventilated or sealed as desired. A piezometer in the side of the flume just below the crest was connected to a U-tube partly filled with water to determine the amount of vacuum existing under the nappe at any time. Tests were made with the water falling upon the horizontal floor of the flume, and also upon an inclined floor set at two different angles. In all regular tests there was no effect of back-water. The water left the flume a short distance below the point where it struck the floor, and dropped to the wasteway.

Runs were made with ventilated and unventilated nappe. The flow, head on the crest, staff gage reading, and piezometric heights were recorded, as well as very approximate measurements of the nappe and the angle of the jet with the floor. In the unventilated runs, the vacuum in feet of water existing under the nappe was also measured.

It was noted that when the underside of the nappe was fully ventilated, and no vacuum existed, the water in the gages connected to the piezometers was all at the same elevation. Although the piezometers were set at different heights in the flash-boards, and the water under the nappe was somewhat agitated, the piezometer gages agreed with the staff gage. Thus it was concluded that when ventilated the height of the standing water is a measure of the pressure exerted against the flash-boards, and the distribution is the same as for the static condition.

A. Schoklitsch[13] points out that the rise of water behind the nappe of a weir is not due entirely to the existence of a partial vacuum, but is partly caused by the reaction of the water being turned to its final direction of departure as it strikes the floor. Those tests with the ventilated nappe were analyzed on this reaction basis, and were found to agree very well. The equation given by Schoklitsch indicates heights of the standing water greater than those found, so that another expression was used.

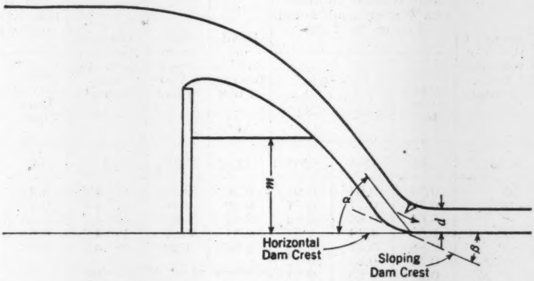


FIG. 17.—STANDING WATER BEHIND NAPPE

Referring to Fig. 17, the reaction of the water flowing at velocity V being turned through the angle α is balanced by the standing water behind the nappe, or

$$\frac{w\,q}{g}\,(V - V\cos\alpha) = w\left(\frac{m^2}{2} - \frac{d^2}{2}\right) \ldots\ldots\ldots\ldots\ldots (23)$$

and

$$m = \sqrt{\frac{2\,q\,V}{g}\,(1 - \cos\alpha) + d^2} \ldots\ldots\ldots\ldots\ldots (24a)$$

Where the sloping floor made an angle β with the horizontal, the equation

$$m = \sqrt{\frac{2\,q\,V}{g}\,(\cos\beta - \cos\alpha) + d^2\cos^2\beta} \ldots\ldots\ldots\ldots (24b)$$

was used. The value of the depth d is quite uncertain because of the curved transition, and with a sloping floor it becomes even more indefinite. The term $d^2\cos^2\beta$ is only an approximation used because greater refinement seemed unnecessary in view of its small value in comparison with the other term under the radical. Equations (24) do not consider losses and certain other phenomena and are therefore not exact. Probably the water behind the nappe above the height d accelerates the falling sheet in the horizontal direction, changing both the angle α and the velocity V. If the true values of these could be determined, other expressions would, perhaps, give better results. For the method used in estimating the reaction height for any particular flash-board installation, and for the few experiments with which it was checked, the formulas given appeared applicable.

---

[13] "Hydraulic Structures," by A. Schoklitsch, published by A. S. M. E., 1937.

The heights of the standing water as observed in the tests under various conditions are shown in Table 8, Column (4). The heights as calculated by Equations (24) from measurements of the nappe made in the tests are shown in Column (5). For these calculations, the measured angle α was used, and the thickness of the nappe at the foot of the fall gave a value for d which was also used to compute the velocity V from the known discharge. The agreement is very good in the tests where the floor was horizontal ($\beta = 0°$) and fair for the inclined floor.

TABLE 8.—RISE OF WATER BEHIND NAPPE

Head on crest of boards, in feet	Fall from crest to floor, in feet	Angle of the floor with the horizontal	HEIGHT OF THE STANDING WATER BEHIND THE NAPPE ABOVE THE FLOOR, IN FEET			Head on crest of boards, in feet	Fall from crest to floor, in feet	Angle of the floor with the horizontal	HEIGHT OF THE STANDING WATER BEHIND THE NAPPE ABOVE THE FLOOR, IN FEET		
			Observed from test	Computed from Equations (24)					Observed from test	Computed from Equations (24)	
(1)	(2)	(3)	(4)	(5)*	(6)†	(1)	(2)	(3)	(4)	(5)*	(6)†
0.78	1.5	0°	0.94	0.91	0.94	0.36	0.79	17° 45′	0.37	0.43	0.44
0.61	1.5	0°	0.79	0.77	0.81	0.50	0.86	17° 45′	0.47	0.53	0.57
0.45	1.5	0°	0.61	0.60	0.62	0.62	0.93	17° 45′	0.56	0.58	0.67
1.15	1.5	0°	1.25	1.14	1.27	0.77	1.00	17° 45′	0.65	0.74	0.80
0.72	1.5	0°	0.88	0.87	0.89	1.08	1.22	17° 45′	0.92	0.92	1.07
0.44	1.16	6° 24′	0.54	0.55	0.57						
0.58	1.17	6° 24′	0.64	0.65	0.71						
0.68	1.18	6° 24′	0.71	0.76	0.81						
0.77	1.20	6° 24′	0.80	0.86	0.90						
1.15	1.25	6° 24′	1.05	1.05	1.21						

* Using measured nappe dimensions from test.

† Using scaled nappe dimensions from plot of theoretical nappe shape.

In the case of a proposed flash-board installation, the thickness of the nappe at the foot of the fall, the angle of the jet at contact, and the distance of fall, if striking an inclined dam face, would not be known. To see what kind of approximation could be made in this circumstance, values of these unknown quantities for the conditions tested were scaled from a published plot of theoretic nappe shape. The resulting calculated heights of the water are shown in Table 8, Column (6). Although the agreement is not all that might be hoped for, it offers a method of estimating a factor which has been entirely neglected in many flash-board designs. As the moment varies approximately as the cube of the heads on either side of the boards, and as the water rises behind the nappe only a small part of the head on the up-stream side, the discrepancy of 20% shown in some of the computed figures would be much reduced.

The vacuum that may exist under an unventilated nappe can vary over a wide range. Apparently the water continually carries out small amounts of air, and if the fall is low the underside can become completely filled with water, provided there is no means of access for the outside air. The writer made too few experiments to learn just what the pressure conditions are under this circumstance.

When a partial vacuum exists under the nappe, the pressures which occur on the flash-boards are best illustrated by comparison with those obtaining when the nappe is fully ventilated. In the ventilated condition, a given flow caused the standing water to rise against the boards to a height of 0.79 ft, indicated both by the staff gage and the gages connected to the piezometers. With the same flow, but with the nappe sealed, a vacuum of 0.12 ft was observed. The jet was deflected inward approximately 6°, and the staff gage read 1.01 ft above the floor. Gages, connected to piezometers located 0.06 ft, 0.19 ft, and 0.56 ft from the floor, registered pressure heads of 0.95 ft, 0.91 ft, and 0.87 ft, respectively, above the floor. The gage to the upper piezometer differed from the staff gage by 0.14 ft, or about the height of vacuum in feet of water. The gages to the other two piezometers differed from the staff gage by amounts less than the vacuum, probably because of the effect of impact on the lower piezometers which are in a region where the water under the nappe is more disturbed. It is interesting to note that, although the partial vacuum decreases the pressure on the down-stream side of the flash-boards above the standing water, the total pressure due to the standing water itself is greater than it would be if the nappe were completely ventilated. There is still so much uncertainty regarding the question of vacuum that, in the case of a proposed installation, it would seem advisable to provide means to insure ventilation.

To apply the results of these experiments to a practical problem, the writer recalculated the bending moment at failure for Test No. 32, using the reduction in pressure given in Fig. 16. This happened to be the test with the greatest overflow, where the area affected by this reduction would be large. To be sure that the concrete dam used to support the pins had no effect on the pressure distribution, several runs were made using the same flash-board as previously, but with a wooden dam similar to that used in Test No. 32, all built to scale. No difference in pressure was noted.

The total bending moment, including the moment due to the weight of the flash-boards as given in Appendix III, was 44 200 in-lb, which is about 9% less than that computed by the normal moment equation. This gives a modulus of rupture for this test of about 71 700 lb per sq in., which is closer to the recommended value of 70 000 than the value in Table 2(*a*).

If the boards of Test No. 32 had been installed on a flat-crested dam, the standing water behind the nappe would have risen to about 1.9 ft, and this would cause a further reduction of the moment of about 6 500 in-lb. The combined effect of the reduction in pressure on the up-stream face and the back pressure would thus alter the bending moment by about 22 per cent.

Fortunately, the relationship between bending moment and head on the boards is such that the effect of any error in moment on the water-surface elevation is greatly reduced. In many flash-board installations the change in head due to the two factors discussed will be very small; however, where it is important that flash-boards fail within a narrow range, both the reduction in pressure on the face of the boards and the rise of water under the nappe must be considered.

HARRY H. HATCH,[14] M. AM. SOC. C. E. (by letter).—In practice, the authors' simplified horizontal moment formula is used in the design of flash-board pins. The equation is readily obtained from Fig. 10(e), in which $a_1 = \frac{h}{2}$ and $a_2 = \frac{h}{3}$, and it is usually given in the form:

$$M = \tfrac{1}{2}\, w\, h^2\, L \left(H_1 + \frac{h}{3} \right) \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

Expressed in inch-pounds, and with $w = 62.5$, Equation (25) becomes Equation (19b).

The reduction in the value of the modulus of rupture per unit area, with the increase of the pipe diameter, as demonstrated by the authors, is interesting and of vital importance. The authors report a 20% reduction in the unit rupture value between pipes of ¾-in. and 3-in. diameters. This raises the question regarding pipe sizes not within the range of the authors' tests. Will the same rate of reduction in unit value apply to larger diameters? In the absence of laboratory test results with which to answer the question, data from actual structures will be needed. For this reason, the following data may prove useful.

During the flood of September 21, 1938 (with a run-off of 600 cu ft per sec per sq mile), a section of the Cobble Mountain spillway flash-board, 44.92 ft long, functioned properly, in complete accord with its design. The pins consisted of 5-in., standard, wrought-iron pipes, and in this section they were spaced at 4.083-ft centers. The height of the flash-board was 7 ft and $H_1 = 2.25$ ft. Then: From Equation (19b), $M = 343\,840$ lb-in.; from standard pipe tables, $D_e = 5.563$ in. and $D_i = 5.045$ in.; from Equation (1), $S = 5.47$; and from Equation (2), $f = 62.9$ kips per sq in.

The average of the several measurements on some of the pipes resulted in $D_e = 5.60$ in. and $D_i = 5.07$ in., with the corresponding $S = 5.659$ and $f = 60.8$ kips per sq in. The difference in the ultimate unit stresses is around 3% in favor of the theoretical section modulus.

In this excellent paper, only Test No. 24 and Test No. 10(a) were with 2-in., wrought-iron pipes; but there is no value given for either of them under Column (14), Table 2, comparable with the writer's data for 5-in. pipe. It would be of interest if the authors could make this comparison possible.

CHILTON A. WRIGHT,[15] AND CLIFFORD A. BETTS,[16] MEMBERS, AM. SOC. C. E. (by letter).—The interest shown in this paper by the discussions, and by the interesting side-lights revealed, is appreciated.

The description, by Mr. Field, of his flash-board design for handling cross drainage in an irrigation canal was most interesting. Calling attention as it does to the practical applications of flash-boards, this discussion corroborates the experience of others who, since publication of this paper, have advised the writers of the dependable operation of installations based on the paper.

---

[14] Div. Engr., Springfield Water Works, Springfield, Mass.

[15] Prof., Hydr. and San. Eng., Polytechnic Inst. of Brooklyn, Brooklyn, N. Y.; formerly Hydr. Engr., National Bureau of Standards, Washington, D. C.

[16] Engr., U. S. Forest Service, Washington, D. C.

The formulas developed in the paper will not apply to the pipes set horizontally in the abutments.

The allusion to inadequate capacities of many existing spillways in the light of recent flood data is very important and is receiving consideration by engineers as shown by Mr. Creager in a lecture to the Metropolitan Section of the Society on March 20, 1940.

Hinged Flash-Boards.—The replacements mentioned by Mr. Creager under the heading "Flood of January 1, 1934," become unnecessary if hinged flash-boards are used. Since the publication of the paper (in May, 1939) a large number of such installations have been made in various parts of the United States. Instead of being satisfied with charging the loss of flash-boards that float away at high water to the added power head obtained from the use of these flash-boards, even for a period of a few days, some power companies are using hinges and are saving the boards as well as the power.

Other Tests.—The method of test described by Mr. Ryder is ingenious, but it had been tried by the writers and found wanting. When the use of pipes for pins was first considered by the U. S. Forest Service, two specimens each of galvanized-iron pipe, 3 in. and 4 in. in diameter, were tested in a 200 000-lb machine on a set-up similar to that described by Mr. Ryder. The 3-in. pipe was placed on rounded supports on a 14.4-ft span and loaded in the center by means of a 4-in. pipe 20 in. long, representing the socket support. The distance from the socket pipe to the support was made equal to the computed distance to the point of application of the water pressure (flash-board height 3 ft, overtopping head of water 4 ft). The machine loads were divided by 2 to obtain an average value for each pipe as a cantilever.

The maximum stresses determined by this test (58 700 lb per sq in. and 47 300 lb per sq in. for the 3-in. and 4-in. pipe, respectively) were considerably lower than the later mechanical test results shown in Table 2(*b*). The 3-in. pipe attained a maximum stress of 74 000 lb per sq in. in the mechanical test and 73 000 lb per sq in. in the hydraulic tests (Table 2(*a*)).

The maximum stresses were obtained at a measured deflection of 17° which was the same value as shown in Table 2(*b*) for the 3-in. pipe. However, the pipes were not broken, although they were bent to the limit of the machine, 45 degrees. It is believed that this method of testing cannot be depended upon to give stresses suitable for use in the design of flash-board pins. Actual hydraulic tests are much to be preferred.

The discussion by Mr. Schuleen was much appreciated by the writers, who had previously had access to the comprehensive test data of the Pennsylvania Water and Power Company. Although the methods of mechanical testing described by Mr. Schuleen differed from those of the writers, the principal results (maximum loads and deflections) are in general agreement.

The writers are in full agreement, that the characteristics of standard pipe make it structurally and economically adapted to use for flash-board support. Tests of steel pins made prior to the pipe tests disclosed how a small variation in the chemical make-up of steel pins can have a large and hazardous influence on failure head in the field. Commercial pipe, on the other hand, has a tolerance that requires little checking.

The characteristic definite deflection for maximum moment is shown for the results of the standard pipe tests. The value of 20° agrees closely with the writers' values in Table 2.

Although Equations (19*b*) and (20) presuppose the direct use of the resisting moment, it is thought that the computation of the stress from the maximum moment (using a section modulus based upon the original dimensions of the pipe) is of value in two ways. It determines the variation among individual pipes of a given size and also aids in determining a suitable average value to be used in design with tabular standard values of section modulus for a specified pipe size. The cantilever loading method of making mechanical tests seems to agree better with the hydraulic tests than the double cantilever method of testing.

The original data loaned to the authors from the Pennsylvania Water and Power Company included a large number of actual field tests of pipe-supported flash-boards on the Holtwood Dam. The recorded failures were at heads giving stress values in the pipes slightly lower than indicated in the mechanical tests of the company.

Mr. Hatch has reported an interesting field test on the Cobble Mountain spillway in which wrought-iron pipes were used. For purposes of comparison with the value of 62.9 kips per sq in. found by Mr. Hatch, the maximum stress computed by the simplified moment formula for Test 24 in Table 2 should be 56.3 kips per sq in. The value for Test 10*b*, 60.6 kips per sq in., was determined directly by mechanical test. Since steel pipe was much more readily available than genuine wrought iron, particularly in small sizes, it was adopted for test and was used by the Forest Service.

Pressure Reduction on Front of Flash-Boards.—In the data on actual dynamic pressure of the flowing water on the inclined flash-board (Fig. 15) it is to be noted that no noticeable variation from the assumed (static) pressure occurred up to a deflection angle of 20°. An interesting comparison is brought out by relating the maximum deflection attained before an appreciable departure from the theoretical value occurred, to the percentage of overflow head, $\frac{H - h}{h}$. The results in Table 9 show that, for all the overflow tests except possibly that of the 2-in. pipe, the error due to neglecting the dynamic effect of the water was negligible.

Mr. White's experiments were very interesting, showing an intelligent attack upon the perplexing question of the effect of pressure changes on flash-board action. Fig. 16, plotted in dimensionless terms, certainly emphasizes the hydraulic similarity of pressures on flash-boards made to different scales. For all ratios of length of boards to head on crest, the points seem to lie on the same curve (see Fig. 16). For any vertical board a pressure reduction of 10% can be expected at a distance below the top equal to 15% of the total head. This could be interpreted to mean that the pressure at a point 0.06 ft below the top of a 5-ft board with 0.4 ft of water going over the top would be reduced in the same amount as for the same point on a board 1 ft high with the same head. Although this seems difficult to believe, further inspection of Mr. White's tests might ascribe it to the added head of velocity of approach. The computation of the resulting total pressure would have to be made by integrating along the established pressure curve, such as, for example, Mr. White's determination of moment for Test 32, Table 2(*a*).

The reduction of pressure at the top of the flash-boards due to overflow, which is so well covered by Mr. White, is an interesting side-light but such a minor factor that it can be ignored except in unusual cases of very high approach velocities. For practical flash-board design the total head on the crest is sufficient.

TABLE 9.—Limiting Deflections for Static Pressures

Pipe size, in inches	Schuleen		Wright and Betts	
	Overflow head, $\frac{H-h}{h}$, in percentages	Limiting deflection angle before departure of static pressure from theoretical value (in degrees)	Overflow head, in percentages	Deflection at failure, in degrees
2.5	....	....	8	17
1	22	34	22	33
1.5	....	....	35	22
....	39	26	....	....
....	56	18	....	....
2	....	....	66	21+
....	84	....	....	....

Partial Vacuum Behind Flash-Boards.—Mr. Creager's able analysis of the lower stresses produced by flash-board pins tested under partial vacuum explains the variation of these from the writers' results. One of the reasons for running the tests described in the paper was the difficulty encountered when attempting to operate an installation based on Mr. Creager's table. The 42 000 lb per sq in. just did not apply to the standard pipe being used, which had a modulus of rupture of 70 000 lb per sq in.

The finding, indicated by Mr. Schuleen, that vacuum under the nappe decreased the moment approximately 7%, checks with findings at several installations that have been reported to the writers.

The paper did not attempt to analyze the effects of vacuum because in some cases on actual dams the pressure on the down-stream side of the boards is decreased, creating a "pull," whereas in others enough back-water is drawn up beneath the nappe to counteract this effect. Furthermore, the effect of vacuum can be readily corrected by the use of heavier pipe, the size of which can be computed from the observed failure head. Where it is impractical to ventilate the nappe, it is advisable to provide for larger sockets in case the heavier pipe becomes necessary.

As a practical matter, the degree of vacuum obtained would depend upon the height of the flash-boards, the clearance at end of the crest, the width of the flat channel immediately below the boards, and other factors, and should receive consideration in design of an installation. The effect of this vacuum would be to cause an earlier failure of the pins—that is, it would be on the safe side. Since an attempt to make an allowance for vacuum may very readily lead to the pipe failing at a higher head than desired, the recommendation in the last paragraph of Mr. Creager's discussion, which is also made in the original paper, should be emphasized.

In conclusion the writers wish to thank all those who have aided in shedding some light on the design of flash-board installations.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2089

EFFECTIVE MOMENT OF INERTIA OF A RIVETED PLATE GIRDER

BY SCOTT B. LILLY,[1] M. AM. SOC. C. E., AND SAMUEL T. CARPENTER,[2] ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. WILLIAM R. OSGOOD, CLYDE T. MORRIS, B. R. LEFFLER, E. NEIL W. LANE, LEWIS E. MOORE, W. E. BLACK, L. E. GRINTER, CHARLES M. SPOFFORD, E. MIRABELLI, EDWARD GODFREY, WALTER H. WEISKOPF, C. D. WILLIAMS, ALVIN B. AUERBACH, JONATHAN JONES, R. L. MOORE, J. M. GARRELTS AND I. E. MADSEN, FRANCIS P. WITMER, A. W. FISCHER, J. R. SHANK, HAROLD D. HUSSEY, CHARLES U. KRING, THOMAS C. SHEDD, AND SCOTT B. LILLY AND SAMUEL T. CARPENTER.

SYNOPSIS

The importance of the plate girder as a structural member has warranted many tests of its behavior under load. As an approach to the solution of this problem, a series of tests was planned to discover, if possible, how much the flange stresses in plate girders are affected by the rivet holes in the flanges. In other words, it is an attempt to find what, in this report, is called the effective moment of inertia, and, as far as the writers know, is the first attempt to secure such information. This report describes the methods used at Swarthmore College, at Swarthmore, Pa., for testing two plate girders and gives a resumé of the results. The intent was to obtain knowledge of the action of a plate girder under working conditions; therefore the girders were not loaded until they failed.

Both sets of tests show that the effective moment of inertia is very close to the gross moment of inertia, and that a good correlation of measured flange stresses with calculated stresses exists if the gross moment of inertia is used. Some data are also presented to justify the location of the neutral axis at the center of gravity of the gross cross-sectional area. Finally, the writers present a formula for effective moment of inertia.

NOTE.—Published in October, 1939, *Proceedings*.

[1] Prof., Civ. Eng., and Chairman, Div. of Eng., Swarthmore Coll., Swarthmore, Pa.

[2] Asst. Prof., Civ. Eng., Swarthmore Coll., Swarthmore, Pa.

INTRODUCTION

It is customary for structural designers to consider a net moment of inertia in the design of plate girders. For the purpose of this paper, "net moment of inertia" will be defined as the moment of inertia computed by deducting the effect of the material removed by holes in the tension flange. For clarity, "gross moment of inertia" is defined as the moment of inertia of the solid cross section of the plate girder, allowing no deductions for the effects of holes in the tension flange. The use of "net moment of inertia" in connection with the beam theory implies a displacement of the neutral axis that can be computed. The net moment of inertia assumes that holes on the gage lines of the tension flange are slotted for the full length of the girder. It is needless to say that the usual plate girder, having a rivet pitch in the flanges of from 2.5 in. to 6 in., does not represent a girder with slotted holes the full length of the tension flange. The practice of designers, who use the "gross moment of inertia" when computing the deflection of a plate girder, has been confirmed by some experimental data.

To secure an answer to the question of whether the net moment of inertia or the gross moment of inertia, or an intermediate value which may be called the effective moment of inertia, should be used in the determination of the flange stresses of a plate girder, the same section of the same plate girder was tested under pure moment with the flange in the following condition:

Series A, solid flanges (no holes in outstanding legs of flange angles or in cover plates);

Series B, open holes in outstanding legs of flange angles and in cover plates;

Series C, with machine bolts in the holes described in series B; and,

Series D, with rivets in the holes described in series B.

It is evident that if the same section of the same plate girder is tested under the conditions described as series A, B, C, and D, any displacement of the neutral axis will be disclosed by the strains in the web and in the flanges.

TEST GIRDERS

The choice of test girders was predicated upon maintaining a depth-to-span ratio consistent with practice, at the same time selecting a size that could be handled readily in the laboratory. The test girder chosen has a depth-to-span ratio of 1 : 10.6. Fig. 1 is a detail drawing of the test plate girder, PT-1, and represents the girder in the state in which it was received at the laboratory. The holes marked "A" were drilled in the shop after tests A1, A2, and A3 had been completed. Test girder, PT-2, was the same when received, except that the rivets connecting the web plate to flange angles in the pure moment section were spaced 2.5 in. apart. No holes were punched in the outstanding legs of the flange angles or in the cover plates in the center section of the girders. The make-up of the girders was the same throughout the full length, namely:

1. Web plate........	12 in. by $\frac{5}{16}$ in.
4. Flange angles......	2.5 in. by 2.5 in. by $\frac{1}{4}$ in.
2. Flange plates......	6 in. by $\frac{1}{4}$ in.
2. Flange plates......	8 in. by $\frac{1}{4}$ in.

The depth, back to back of angles, was 12.5 in.; and, 8-in. cover plates were used so that strains could be measured on the side of these plates next to the 6-in. cover plate. This procedure gave no useful information and was abandoned.

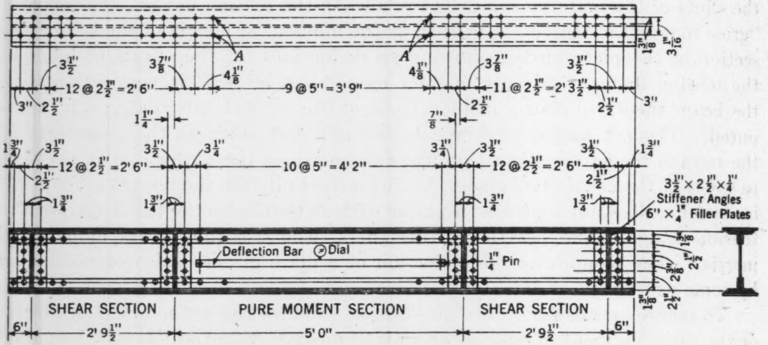

A
A
3½"
3⅞"
4⅛"
4⅛"
3⅞"
3½"
12 @ 2½" = 2' 6"
9 @ 5" = 3' 9"
11 @ 2½" = 2' 3½"
3"
2½"
2½"
2½"
3"
1⅜"
3 1/16"
1¼"
7/8"
1¾"
3½"
12 @ 2½" = 2' 6"
2½"
3½"
3¼"
10 @ 5" = 4' 2"
3¼"
3½"
12 @ 2½" = 2' 6"
3½"
1¾"
2½"
1¾"
1¾"
1¾"
1¾"
3½" × 2½" × ¼"
Stiffener Angles
6" × ¼" Filler Plates
Deflection Bar
Dial
¼" Pin
2½"
2⅜"
2½"
1⅜"
SHEAR SECTION
PURE MOMENT SECTION
SHEAR SECTION
6"
2' 9½"
5' 0"
2' 9½"
6"

FIG. 1.—PLATE GIRDER PT-1; LOCATION OF DEFLECTION BARS AND SECTIONS OF PURE MOMENT AND SHEAR

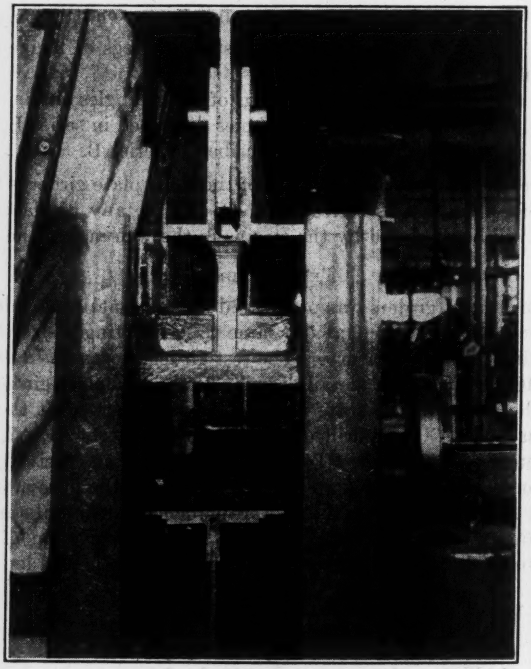


FIG. 2—VIEW OF TEST SET-UP, SHOWING THE FLANGE PLATE-CLAMPS IN PLACE

The "pure moment section" of the test beam was the only one in which the four comparative conditions of girder section could be secured. The cover plates were developed under the conditions of series A and B by bolts in the shear section only. Fig. 2 illustrates the typical installation of the flange clamps necessary to prevent the flange plates on the compression side from buckling during series A and B. The clamps were designed so that two $\frac{3}{8}$-in. set-screws came into point contact with the flange plates and two $\frac{3}{8}$-in. set-screws came into contact with the inner side of the outstanding legs of the flange angles along the gage lines. The plates were pulled into contact and held there by the clamps. Their construction was such that they added nothing to the flange area.

Description of Test Set-Up

The method of applying the loads was as follows: One end of the test girder rested upon a roller upon the platform of a 100,000-lb testing machine; the other upon a fixed cylindrical reaction point, supported by a steel trestle and adjusted to the same height as the platform of the testing machine. The load was applied to the test girder by an equalizer beam (see Fig. 3) through rollers situated at points 33.5 in. from the supports at each end of the test girder. (Test loads are noted in all cases, in this paper, as P and not 2 P. Therefore the bending moment in the pure moment section is always 33.5 P in-lb.) The load was brought to the middle of the equalizer beam by a loading beam. One end of this loading beam was connected to the movable head of the testing machine by a clevis and pin; the other end was attached to the corresponding end of the plate girder by two bolts, 1.75 in. in diameter, passing through steel slabs, top and bottom, contact being made through cylindrical reaction points. Contact between the loading beam and the equalizer beam was made by means of a roller. By this arrangement the concentrated load applied at each load point is exactly that which is weighed on the testing machine, making possible a total load of twice the rated capacity of the testing machine. In the presentation of data the testing machine load is the recorded value. In all cases, the total load on the girder is twice this value. The east end of the girder (see Fig. 3) rested on the testing machine.

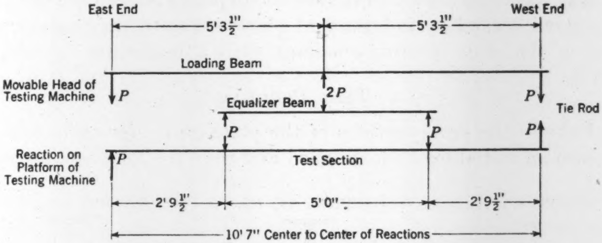

East End
West End
5' 3½"
5' 3½"
Loading Beam
Movable Head of Testing Machine
P
2P
P
Equalizer Beam
Tie Rod
P
P
P
Reaction on Platform of Testing Machine
P
Test Section
2' 9½"
5' 0"
2' 9½"
10' 7" Center to Center of Reactions

Fig. 3.—Load Analysis for Plate-Girder Test

Pure Moment Section.—The method of measuring the deflections in the pure moment section was as follows: Two pins (0.25 in. in diameter), spaced $25\frac{1}{8}$ in. each side of the mid-point of the girder, were passed through the web plate in such a manner that the tops of the pins were level with the center line of the web plate. These pins extended through the web plate sufficiently far on each side so that they could support steel side bars 1½ in. by ¼ in. in cross section. The holes in the ends of these bars were ¾ in. in diameter, and the distance between the supporting pins was 50.25 in. At the middle of the plate girder a third pin was placed through the web in a similar manner, and on each end of the pin was mounted an Ames dial, reading to $\frac{1}{10{,}000}$ in., with the contact points resting upon the side bars. (These dials are identified as the north and south dials.) Under this condition of loading, the middle of the test girder was subjected to a uniform moment, if the moment due to the weight of the girder itself is neglected. It is evident that the reading on the dials gave the deflection of the center line of the web below two points on that line which were spaced 50.25 in. apart (see Fig. 1).

To measure the stresses in the girder itself, gage points with 10-in. centers were drilled and readings taken with a 10-in. strain gage having a least reading of $\frac{1}{60{,}000}$ in. Assuming a modulus of elasticity of 30,000,000 lb per sq in., the least reading was equivalent to a stress of 50 lb per sq in. In the test procedure, readings were checked by independent observers until they agreed within six divisions on the dial, insuring readings with a maximum deviation of 300 lb per sq in.

TEST PROGRAM

Fig. 4 shows the cross sections of the plate girder for each series of tests. In each case an initial load was applied and then the load was increased until the increment of deflection in the pure moment section was 0.0300 in., or slightly greater. After each series had been run, with loads to give the aforementioned deflection, the load was increased to 63,000 lb for series C-3 and D-3 for specimen PT-1, and readings were taken in the pure moment section. A detailed statement of the test program follows:

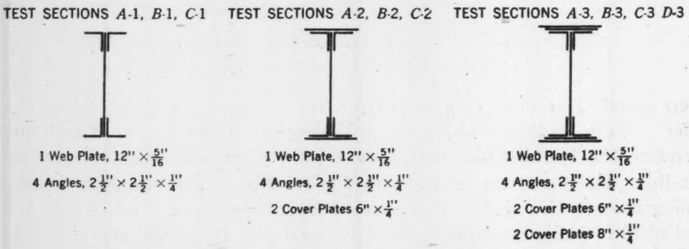


FIG. 4.—TYPICAL CROSS SECTIONS AND STRUCTURAL MAKE-UP OF THE TEST SPECIMENS

Test Section	Description (see Fig. 4)
A-1	Web plate with two pairs of flange angles riveted on, but with no holes in the outstanding leg.
A-2	6-in. cover plate added top and bottom. There were no holes in the plates in the

Test Section	Description (see Fig. 4)
	pure moment section. The plate was held in contact with the flange angles by the clamps shown in Fig. 2.
A-3	8-in. cover plate added to the 6-in. cover plate, top and bottom. In this case, also, the pure moment section had no holes in it and the plates were held in contact by the clamps.
B-1	Web plate and flange angles were tested as in test section A-1.
B-2	6-in. cover plates added, clamped, and tested as in test section A-2.
B-3	8-in. cover plate added to the assembly described in test section B-2, the two plates clamped and tested as in test section A-3.
C-1	$\frac{5}{8}$-in. machine bolts placed in the open holes ($\frac{11}{16}$ in. in diameter) of the outstanding legs, tightened with proper washers, and tested as in test specimen B-1.
C-2	6-in. cover plates added, bolted on, and tested as in test specimen B-2.
C-3	8-in. cover plate added to the assembly described under test specimen C-2 and tested as in test specimen B-3.
D-3	The two cover plates were riveted to the flange angles and tested as in test specimen C-3.

In series B (the section with open holes), the plate girder with the 6-in. and 8-in. plates attached was taken to a drill press and holes were drilled through the outstanding leg of the flange angles and the cover plates, in both compression and tension flanges. In specimen PT-1 the pitch was 5 in.; and in specimen PT-2 the pitch was 2.5 in. In series B, the holes were open in the pure moment section. (The pitch of the rivets in specimen PT-2 was first made 5 in. as in specimen PT-1, and tests were run which checked specimen PT-1 so closely that they are not reported herein. Then the girder was returned to the drill press and the additional holes drilled to make the pitch 2.5 in.)

Neutral Axis

Beginning with the assumptions of the common theory of flexure, it can be proved that the neutral axis must pass through the center of gravity of the cross section of a beam under flexure. This is substantially correct for homogeneous beams of the usual kind. However, in the design of riveted plate girders, the engineer must decide whether he will locate the neutral axis at the center of gravity of the gross cross section, or at the center of gravity of the net cross section, considering that the material that filled the holes has been removed from the tension flange, and that the holes in the compression flange are completely filled. If the net cross section existed at each section, as it would if it were obtained by slotted holes running the full length of the girder, and if the compression flange were solid, the center of gravity should undoubtedly be computed from the net cross section. The net section is not continuous, however, and it is questionable whether the rivets fill the holes of the compression flange sufficiently to consider the compression flange as solid.

That the neutral axis remains near the center line of the web plate is shown in Table 1, where data from tests on the two plate girders, PT-1 and PT-2, are presented. The observed results were obtained by measuring strains on 10-in. gage lines on both sides of the web plate. The gage lines were at distances of 1 in., 2 in., and 3 in. above and below the center line of the girder. A straight line was passed through the plotted data and the deviations obtained directly. The deviation of the neutral axis was also computed using the measured strains of the outside fibers of the girder, and these results are compared with those obtained from the web data. The experimental results all indicate that the neutral axis deviates only slightly from the center line of the web plate, and not as it has been considered to deviate by the designers using the net section. This means that the deformation of the compression flange is almost as great as that of the tension flange.

TABLE 1.—COMPARISON OF OBSERVED AND COMPUTED DISTANCES OF THE NEUTRAL AXIS ABOVE THE CENTER LINE, IN INCHES; PURE MOMENT SECTION

Description	(*a*) WEB PLATE AND FLANGE ANGLES			(*b*) WITH 6-IN. COVER PLATES ADDED			(*c*) WITH 6-IN. AND 8-IN. COVER PLATES ADDED			
	A-1	B-1	C-1	A-2	B-2	C-2	A-3	B-3	C-3	D-3
Observed, Web Data:										
Specimen PT-1	0.19	0.15	0.14	0.16	0.08	0.07	0.09	0.04	0.05	0.10
Specimen PT-2	0.09	0.05	0.01	0.04	0.09	0.05	0.17	0.07	0.07	0.10
Observed, Flange Data:										
Specimen PT-1	0.11	0.12	0.07	0.11	0.05	0.11	0.09	0.05	0.03	0.14
Specimen PT-2	0.09	0.05	0.04	0.12	0.15	0.13	0.23	0.16	0.07	0.08
Computed*	0.34	0.63	0.63	0.25	0.68	0.68	0.18	0.66	0.66	0.66

* Computed on the basis of net section.

TEST COUPONS

Tensile tests were made by the National Bureau of Standards on eight test coupons from specimen PT-1, obtained from the extra pieces of material sent by the fabricator (see Table 2). The coupons had a reduced width of 0.5 in. and a gage length of 2 in. The coupons for specimen PT-2 were tested at Swarthmore College. The results were so similar that the data are omitted for present purposes. The average value of the modulus of elasticity was 30,000,000 lb per sq in.

OBSERVED EFFECTIVE MOMENTS OF INERTIA

In the pure moment section, the deflection in a length of 50.25 in. was read to the nearest 0.0001 in. by Ames dials. To observe values of the effective

TABLE 2.—COUPON TESTS FOR TENSILE STRENGTH; SPECIMEN PT-1

Coupon No. (1)	Coupon cut from: (2)	Thickness, in inches (3)	UNIT STRESSES, IN KIPS PER SQUARE INCH				Percentage elongation in 2 in. (8)
			Young's modulus of elasticity (4)	Proportional limit (5)	Yield point (6)	Tensile strength (7)	
B1	Web plate	0.299	30,100	40	42.0	57.8	40
B1-3	Web plate	0.300	30,000	37	42.0	67.1	36
C	6-in. cover plate	0.253	29,600	40	53.0	72.5	26
C-3	6-in. cover plate	0.253	28,200	40	54.1	73.8	26
D	8-in. cover plate	0.252	29,800	37	44.7	60.8	39
D-3	8-in. cover plate	0.250	31,200	37	43.0	60.8	38
V	Flange angles	0.240	29,300	37	43.6	58.8	38
V-3	Flange angles	0.241	27,900	39	50.4	62.6	31
Average	....	....	29,500	..	...	...	..

moments of inertia, the moments of inertia of the gross section, I_G (Column (2), Table 3), were assumed to be correct for series A, and a load, P_G, was applied until the deflection in the pure moment section was 0.0300 in. In series B, C, and D, loads were applied to each test section of the correct value to cause a deflection of 0.0300 in. Then it was assumed that since the deflection, Δ, of the plate girder was the same in each of these tests, and since $\Delta \propto \frac{P}{E\,I}$,

$$\frac{P_G}{P_o} = \frac{I_G}{I_o} \qquad (1)$$

and the observed effective moment of inertia in each case is

$$I_o = \frac{P_o}{P_G} I_G \qquad (2)$$

in which P_o and I_o are the load and moment of inertia for any particular section. For example, the load that caused a deflection of 0.0300 in. for section A-2 was 26,600 lb and the gross moment of inertia was 315 in.^{4}; the load that caused the same deflection for section B-2 was 25,200 lb. Therefore, the observed effective moment of inertia for section B-2 is $I_o = \frac{25{,}200}{26{,}600} \times 315 = 298.5$ in.^{4}

In this manner, values of the observed effective moment of inertia for each test section were computed and tabulated in Column (1), Table 3. In comparing the values in Table 3(*a*) and Table 3(*b*), it should be recalled that the pitch in this pure moment section was 5 in. for specimen PT-1 and 2.5 in. for specimen PT-2. Instead of tabulating in Column (2), Table 3, the value of the gross moment of inertia opposite the test section of series A, the value of I_o was computed from the value of the deflection, 0.0300 in., as determined by the moment-area method, using the value of E as determined from the test coupons. These values check the values of the gross moment of inertia within the limits of the experimental error and justify the foregoing procedure.

When the values of the observed effective moment of inertia are plotted, the graphs show that they approach the values of the gross moment of inertia as the tests proceed from that giving the greatest deviation in series B (open holes), through series C (bolted sections), to series D (riveted sections). The deviation is a very small value in series D—namely, 2.1% of the gross moment of inertia for specimen PT-1 and 7% for specimen PT-2.

TABLE 3.—Effective Moments of Inertia

Test section	Moments of Inertia, in Inches^4				Flange Stresses, in Kips per Square Inch									
	Observed	Computed			Observed		Computed, Using the Method Indicated							
		Gross section	Net section				Observed moment of inertia		Gross section		Net Section			
			a*	b†	t‡	b§					Neutral axis eccentric		Neutral axis at center	
							t‡	b§	t‡	b§	t‡	b§	t‡	b§
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
(a) Specimen PT-1														
A-1	189.0	192.9	178.5	179.5	16.9	17.6	17.3	17.9	17.2	17.2	17.6	19.6	18.5	18.5
B-1	182.0	192.9	164.3	167.3	18.3	19.1	17.9	18.6	17.2	17.2	18.2	22.3	19.5	19.5
C-1	187.0	192.9	164.3	167.3	17.7	18.2	17.8	18.2	17.2	17.2	18.2	22.3	19.5	19.5
A-2	315.0	315.0	300.9	301.6	18.0	18.7	18.1	18.7	18.4	18.4	18.5	20.0	19.2	19.2
B-2	298.5	315.0	270.7	275.5	19.3	19.7	19.2	19.5	18.4	18.4	19.2	23.7	20.8	20.8
C-2	306.5	315.0	270.7	275.5	18.9	19.6	18.6	19.2	18.4	18.4	19.2	23.7	20.8	20.8
A-3	494.0	490.8	476.9	477.4	18.5	19.0	18.7	19.3	19.1	19.1	19.3	20.3	19.7	19.7
B-3	460.5	490.8	430.0	436.0	19.9	20.2	20.2	20.5	19.1	19.1	20.1	24.4	21.5	21.5
C-3	472.0	490.8	430.0	436.0	19.7	19.9	19.8	20.0	19.1	19.1	20.1	24.4	21.5	21.5
D-3	480.5	490.8	430.0	436.0	19.3	20.2	19.1	20.0	19.1	19.1	20.1	24.4	21.5	21.5
(b) Specimen PT-2														
A-1	189.0	192.9	178.5	179.5	17.7	18.1	17.0	17.5	17.2	17.2	17.6	19.6	18.5	18.5
B-1	177.0	192.9	164.3	167.3	19.4	19.8	18.7	19.0	17.2	17.2	18.2	22.3	19.5	19.5
C-1	179.0	192.9	164.3	167.3	19.0	19.2	18.5	18.6	17.2	17.2	18.2	22.3	19.5	19.5
A-2	315.0	315.0	300.9	301.6	17.9	18.6	18.1	18.7	18.4	18.4	18.5	20.0	19.2	19.2
B-2	282.0	315.0	270.7	275.5	20.0	20.9	20.2	21.0	18.4	18.4	19.2	23.7	20.8	20.8
C-2	295.0	315.0	270.7	275.5	19.4	20.3	19.3	20.0	18.4	18.4	19.2	23.7	20.8	20.8
A-3	500.0	494.0	480.1	480.6	18.5	19.8	18.1	19.4	19.1	19.1	19.3	20.3	19.6	19.6
B-3	442.0	494.0	433.2	439.0	20.6	21.0	20.8	21.8	19.1	19.1	20.1	24.4	21.4	21.4
C-3	453.0	494.0	433.2	439.0	20.0	20.9	20.5	21.0	19.1	19.1	20.1	24.4	21.4	21.4
D-3	460.0	494.0	433.2	439.0	20.3	20.3	20.2	20.6	19.1	19.1	20.1	24.4	21.4	21.4

* Neutral axis eccentric. † Neutral axis at center. ‡ Top flange. § Bottom flange.

FLANGE STRESS

In all of the test sections, flange stresses were obtained experimentally by measuring strains in the top and bottom flange. A 10-in. strain gage was used, with the gage lines placed symmetrically with regard to the longitudinal center line of the girder. Four gage lines were used for test sections A-1, B-1, and C-1, six gage lines for test sections A-2, B-2, and C-2, and eight gage lines for test sections A-3, B-3, C-3, and D-3. The layout of the gage lines on the 6-in. and 8-in. cover plates was such as to allow two gage lines to bisect the open holes in the flanges and later the bolts and rivets. The observed flange stress was computed from an average of the readings on all gage lines in every case, using the observed value of modulus of elasticity. The stresses were computed for a single load for each series, as follows:

Test sections	Load, P, in kips
A-1, B-1, and C-1	15.9
A-2, B-2, and C-2	26.6
A-3, B-3, C-3, and D-3	41.6

These particular values were selected because they caused an increment of deflection of 0.0300 in. in test sections A-1, A-2, and A-3 for specimen PT-1.

Using these loads the flange stress has been computed using the gross moment of inertia; using the net moment of inertia with the neutral axis at the center of the web plate; and finally using the net moment of inertia with the neutral axis at the center of gravity, considering the compression flange as solid and the tension flange as slotted.

The stresses computed by using the gross moment of inertia are compared with the observed stresses in Table 3, Columns (5), (6), (9), and (10). The computed flange stresses are somewhat lower than the observed stresses for all sections except for the initial solid flanges of series A. For those sections a good correlation exists. The stress in the compression flange is greatest for open holes, and gradually approaches the computed value as the method of holding the flange together is changed from clamps, to bolts, and to rivets. No such decrease is shown for tension flange. This probably indicates that bolts and rivets tend to relieve the stress in the compression flange.

In Table 3, Columns (5), (6), (13), and (14), the observed flange stresses are compared with the flange stresses computed using the net moment of inertia, with no shift allowed in the location of the neutral axis from the center line of the web plate. The computed flange stresses are about 2,000 lb per sq in. higher than the observed stresses throughout this comparison.

A comparison between the observed stresses and the flange stresses, computed by using the designer's net moment of inertia (see "Introduction") and assuming the neutral axis to be above the center line of the girder, in accordance with some design procedures, is shown by Table 3, Columns (5), (6), (11), and (12). The agreement between the compression flange stresses is very good. However, the computed stress for the tension flange is approximately 4,500 lb per sq in. higher than the observed. This would seem to indicate that this design procedure would give much larger flange areas than are needed to maintain safe stresses in the tension flange.

In addition to the foregoing information, Table 3 gives the flange stresses computed by the observed effective moment of inertia, using the observed deviation of the neutral axis as given in Table 1 from observed flange data. The excellent agreement between this column and the observed flange stress is an independent check on the observed results.

ACTION OF GIRDER UNDER HIGHER LOADS

After completing the tests where the loads were held to values giving a flange stress of about 20 kips per sq in., test sections C-3 and D-3 of specimen

PT-1 were subjected to loads causing flange stresses of 30 kips per sq in. The load was increased and decreased by increments, and strain readings were taken in the flange on both the up loading and the down loading.

The strain readings gave no indication of permanent set. A breaking of the scale around the rivet heads through the upstanding legs of the flange angles was noticed. A close observation showed a perfect set of orthogonal slip lines. No such breaking of the scale was noticed around the rivets through the flange plates, although this region was stressed higher. It is to be remembered that the holes in the cover plates were drilled, whereas the holes through the web and flange angles represented shop practice of punched holes. An investigation of one of these holes showed poor alinement of the plates and a very rough perimeter. Is it possible that the condition of the holes accounts for the observed strain lines?

STRESS CONCENTRATIONS

Stress concentrations, with their accompanying areas of plastic flow around the holes in the flanges of a plate girder, present some questions in regard to the validity and the interpretation of the strain gage readings explained herein. As treated in this paper strains have been reduced to terms of stress by applying Hooke's law. There may be objections to this procedure on two grounds: (1) That this does not represent the stress at any point; (2) that the average stress through the net section represents the true stress. In answer to these objections, the writers would point to Fig. 5, showing the way the stress varies across the net section. It is known that the maximum stress reaches the elastic limit, and further there is nothing that can be done about it; second, a unit stress arrived at by using a net area has no meaning.

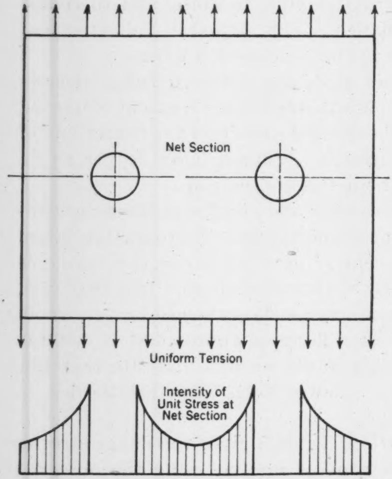


FIG. 5.—STRESS CONCENTRATIONS

It is reasonable to assume that any strain gage, operating on a 10-in. gage length over the holes, will register some plastic, as well as elastic, deformation. The writers do not know how to separate these components. A. Nadai states that the areas of plastic flow in the region of a hole are in the shape of a cross, the direction of the cross making an angle of approximately 45° with the principal stresses of tension or compression.[3] It is also well known that the intensity of stress is highest at the boundaries of the hole, which are on a diameter perpendicular to the applied principal stresses. Fig. 5 depicts this condition graphically. As the principal stress is increased, the stress at the boundary of the hole increases to the yield point value and plastic flow begins in the areas defined by the cross.

---

[3] "Plasticity," by A Nadai, McGraw-Hill Book Co., Inc., New York, N. Y., 1931, p. 103.

For any reasonable allowable design stress, the yield point stress will be reached at the hole, and plastic flow will occur. The designer's choice of 18 kips per sq in. instead of 20 kips per sq in. will not prevent this from happening, although it may limit, to some extent, the area over which plastic flow may extend.

Once a ductile material is strained beyond the yield point and the load is released, residual stresses are set up which will be beneficial in resisting the next applied load.

EFFECTIVE MOMENT OF INERTIA

It seems reasonable to believe that both the strength and the stiffness of a plate girder are affected by the size and spacing of the rivet holes in the flanges of a plate girder. If the pitch were small enough there would be a slot the full length of the girder, and if the diameter of the rivet holes were large enough there would be no material to carry the load. The writers have developed the following formula which incorporates the results of the tests described in this paper, and which contains terms that include the effect of the diameter and pitch of the rivet holes.

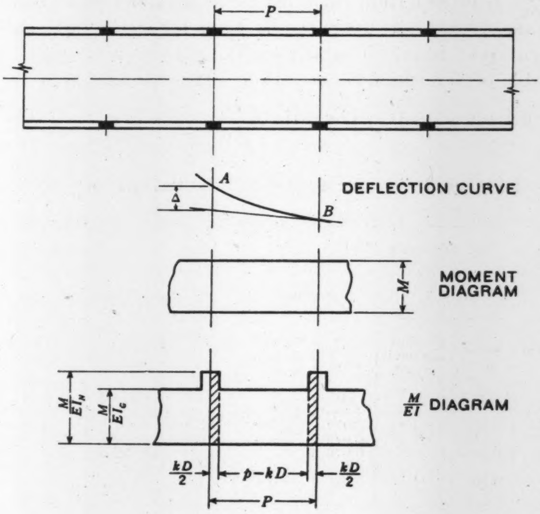


FIG. 6

Referring to Fig. 6, let L = length; M = moment; p = rivet pitch; D = diameter of rivet holes; $k\,D$ = the part of the rivet hole that is acting as a full net section; Δ = the deflection by point A with reference to a tangent to the elastic curve at point B; I_G = gross moment of inertia; I_C = clear moment of inertia (holes are deducted from the top and bottom flanges); and I_E = effective moment of inertia. The new term, I_C, the "clear" moment of inertia, has been introduced since these tests show that the neutral axis remains at the middle of the web plate. This fact is based on the practically equal stresses in the top and bottom flanges. By moment areas:

$$\Delta = \frac{0.5\,P\,M}{E}\left(\frac{p - k\,D}{I_G} + \frac{k\,D}{I_C}\right) = \frac{0.5\,P\,M}{E}\left[\frac{p + \left(\frac{I_G}{I_C} - 1\right)k\,D}{I_G}\right] \quad \text{..(3)}$$

In general, $\Delta = \frac{M\,L^2}{E\,I}$ times a constant; and, in the present case:

$$\frac{L}{I} = \frac{P}{I_E} = \frac{P + \left(\frac{I_G}{I_C} - 1\right)k\,D}{I_G} \quad \text{..................(4)}$$

That is,

$$I_E = \frac{I_G\,p}{p + \left(\frac{I_G}{I_C} - 1\right)k\,D} \quad \text{....................(5)}$$

The quantity $k\,D$ in Equation (5) must be determined experimentally. The value of k probably depends upon D, the diameter of the hole, and g, the distance between rivet lines. The writers do not have enough experimental results available to determine this relation. It is thought that the value of k must lie between 0.6 and 1.0. The ratio of $\frac{I_E}{I_G}$ is called "m." Table 4 shows the value of "m" obtained from experimental data compared with the computed "m," using k equal to 1 and 0.6, respectively. It will be noted that the percentages of deviation of the computed value from the observed value change

TABLE 4.—COMPARISON OF VALUES OF $m = \frac{I_E}{I_G}$

Section	(a) SPECIMEN PT-1					(b) SPECIMEN PT-2				
		k = 1		k = 0.6			k = 1		k = 0.6	
	Value of m by experiment (1)	Value of m (2)	Percentage deviation* (3)	Value of m (4)	Percentage deviation* (5)	Value of m by experiment (1)	Value of m (2)	Percentage deviation* (3)	Value of m (4)	Percentage deviation* (5)
C-1	0.970	0.952	−1.85	0.972	+0.2	0.928	0.907	−2.3	0.943	+1.6
C-2	0.970	0.955	−1.5	0.972	+0.2	0.936	0.914	−2.3	0.954	+1.9
C-3	0.962	0.962	0	0.976	+1.5	0.916	0.927	+1.2	0.956	+4.4
C-4	....	....	....	....	....	0.930	0.927	−0.3	0.956	+2.8
D-3	0.980	0.962	−1.8	0.976	−0.4	....	....	....	....	....

* All deviations are computed from the experimental value.

from positive to negative with these values of k; that is evidence that k has a value between 0.6 and 1.0.

CONCLUSIONS

1.—The measured flange stresses are smaller than the computed stresses using a "net" moment of inertia based upon a deduction of rivet holes from the tension flange. In the case of the girder with a 5-in. pitch, the difference was considerable; with the girder having a 2.5-in. pitch there was a closer correlation. This was anticipated and furnishes some evidence that an effective moment of inertia should depend upon the rivet pitch as well as the rivet diameter. The net moment of inertia, used in design with top and bottom flanges alike, should yield conservative stresses.

2.—The experimental values of the effective moments of inertia are less than the computed gross moment of inertia. The effect of rivet pitch is shown clearly. However, the test girder having the 5-in. rivet pitch and riveted flanges gave nearly an exact agreement between the experimental and computed values of the moment of inertia.

3.—The tests indicate that the neutral axis should be assumed at the center of gravity of the gross cross section.

4.—The effective moment of inertia can be computed by using the theoretical expression, Equation (5), which involves the rivet pitch. Further experimental evidence is needed to establish this conclusion, and the value of $k\,D$.

5.—Since stress concentrations reaching the yield point must exist as soon as the unit stress in the gross section approaches the stress ordinarily specified for structural work, and nothing can be done to prevent them, the writers believe that design methods should be made as simple as possible. Any method that may be adopted must give sections that have been proved in practice to be satisfactory.

It is believed that the data from these tests point to the acceptability of the gross moment of inertia for design purposes. For ordinary girders, under usual loadings, it requires no modification; for cantilever girders, or girders with small pitch, it should be used with a coefficient such as is indicated by Equation (5).

It is hoped that this paper will stimulate discussion of this subject and reveal additional data which may confirm or disprove these findings. Many engineers may have an opportunity to measure strains and deflections of full-size girders. If this could be done widely, and such data carefully studied, all uncertainties would soon be removed.

ACKNOWLEDGMENTS

The investigators wish to acknowledge the cooperation of The American Institute of Steel Construction in conducting this research project. F. H. Frankland, M. Am. Soc. C. E., and all members of the Research Committee have contributed many suggestions that have been incorporated in this study. The writers are particularly indebted to A. R. Wilson, M. Am. Soc. C. E., engineer of bridges and buildings, of the Pennsylvania Railroad Company, for his sympathetic and timely discussions during the preparation of the paper.

Mr. D. F. Windenburg, physicist at the Bureau of Construction and Repair of the U. S. Navy Yard in Washington, D. C., has cooperated throughout the project. His assistance in making several tension tests on riveted plate specimens is acknowledged. Robert DeMoyer, Assoc. M. Am. Soc. C. E., a graduate student at Swarthmore and an associate professor of civil engineering at Pennsylvania Military College, at Chester, was of great assistance while performing the tests. Mr. Paul Kutz and Mr. G. A. Bourdelais, of the Swarthmore Engineering Division, are responsible for many of the details developed during the set-up of the testing equipment. The Belmont Iron Works, Philadelphia, Pa., performed the final riveting of the test girders free of charge.

DISCUSSION

WILLIAM R. OSGOOD,[4] M. AM. SOC. C. E. (by letter).—In discussing the interesting and valuable paper by Professors Lilly and Carpenter, it may not be out of place to call attention to a paper by Friedrich Hartmann.[5, 5a] This paper treats the question theoretically and discusses a test made in 1891 in the light of the theory. Professor Hartmann comes to the conclusion that in the absence of considerations of fatigue, girders of symmetrical cross sections may be designed safely on the basis of gross moment of inertia. In the case of an unsymmetrical cross section with a heavier compression flange than tension flange, however, deductions should be made for the rivet holes in the tension flange.

The authors state that loads causing flange stresses of 30 kips per sq in. gave no indication in the strain readings of permanent set. Was any such indication noted in the readings of deflection?

It seems quite possible, as the authors suggest, that the condition of the holes in the web and flange angles might account for the observed strain lines around these holes. It is likely that severe initial strains due to punching existed around these holes, and also, if the holes were not completely filled by the rivets, higher strains under load would be expected than otherwise. The strain lines might appear in the upstanding legs of the flange angles and not in the outstanding legs because the latter were connected to plates with drilled holes. The drilled holes would probably tend to a less erratic variation of strain in the plates, and this tendency would affect the strain in the connected legs of the angles in the same way since strains in adjacent connected parts cannot be wholly independent of each other.

The authors state that it was their intent to obtain knowledge of the action of a plate girder under working conditions. Knowledge of the maximum load a girder can carry and the type of failure, particularly the behavior of the compression flange, is also so important that it would have been fortunate if this additional information could have been obtained for the girders tested.

CLYDE T. MORRIS,[6] M. AM. SOC. C. E. (by letter).—Professors Lilly and Carpenter are to be commended for the care with which their investigations were conducted and the completeness with which they have published the data secured. Experiments of this kind are important in extending the knowledge of strength of materials and confirming practical methods of design.

Presumably, the observed fiber stresses given in Column (6), Table 3, were computed from strain gage readings and, therefore, represent the average fiber stress over the 10-in. gage length. For girders PT-1 the holes in the outstanding legs and cover plates were spaced 5 in. apart, and therefore two holes would be included in the 10-in. gage length. Counting the holes as $\frac{11}{16}$ in. square (instead of round), which is on the safe side, the computed average fiber stress is $\frac{1}{10}\left[2 \times \frac{11}{16} \times \text{(Net I) stress} + 8\tfrac{5}{8} \times \text{(Gross I) stress}\right]$. For girders PT-2 the holes were spaced 2½ in. apart and the computed average fiber stress is $\frac{1}{10}\left[4 \times \frac{11}{16} \times \text{(Net I) stress} + 7\tfrac{1}{4} \times \text{(Gross I) stress}\right]$.

---

[4] Materials Engr., National Bureau of Standards, Washington, D. C.

[5] "Sollen genietete Träger mit Berücksichtigung der Nietschwächung berechnet werden?" von Friedrich Hartmann, *Der Stahlbau*, Vol. 12, February 17, 1939, p. 25.

[5a] A translation into English has been placed in Engineering Societies Library, 29 West 39th Street, New York, N. Y.

[6] Prof., Civ. Eng., Ohio State Univ., Columbus, Ohio.

TABLE 5.—STRESSES IN BOTTOM FLANGES

Girder	FROM TABLE 3				COMPUTED IN THIS DISCUSSION	
	Column (10) I_G	Column (14) I_N	Observed Column (6)		PT-1 5-in. pitch	PT-2 2½-in. pitch
			PT-1	PT-2		
B-1	17.2	19.5	19.1	19.8	17.52	17.83
B-2	18.4	20.8	19.7	20.9	18.73	19.06
B-3	19.1	21.5	20.2	21.0	19.43	19.76
C-1	17.2	19.5	18.2	19.2	17.52	17.83
C-2	18.4	20.8	19.6	20.3	18.73	19.06
C-3	19.1	21.5	19.9	20.9	19.43	19.76

Table 5 gives the stresses in the bottom flanges taken from Table 3 and as computed by the method just given. In every case the observed average stresses given in Column (6), Table 3, are greater than the computed stresses using the foregoing method, and are only slightly smaller than the stresses using the net moment of inertia.

From these tests the writer would draw the conclusion that the customary method of design, using the moment of inertia of the net section with the neutral axis at the center of gravity of the gross area (Column (14)), gives results only slightly on the side of safety, and that the use of the moment of inertia of the gross section (Column (10)), as recommended by the authors, is dangerous.

B. R. LEFFLER,[7] M. AM. SOC. C. E. (by letter).—Conclusion 5 of this paper—that for ordinary girders, under usual loadings, the tests indicate the acceptability of the gross moment of inertia for design purposes—is fallacious, and the writer will attempt to show where the false reasoning occurs.

For many years structural engineers have known that the calculated deflection of the ordinary plate girder, using the moment of inertia of the gross section, gives results which correspond closely to the results found by testing. The false reasoning lies in assuming that deflection is a true indication of stress in a riveted girder.

To understand the meaning of what follows, it should be remembered that the deflection of a plate girder is caused principally by the shortening of the compression flange, and the lengthening of the tension flange.

To show the fallacy more clearly, consider a bar, 100 ft long, 6 in. wide, and

---

[7] Engr. of Bridges, N. Y. C. R. R. Line West of Buffalo, Cleveland, Ohio.

0.5 in. thick, with a 1-in. hole at the middle and on the axis of the bar. If the bar were stressed to a given amount, a certain total stretch of the bar would be obtained. Now, if a similar bar of the same quality of metal, without a hole in it, were stressed to the same amount, the total stretch of this bar would not differ appreciably from that obtained for the bar with a hole in it; the longer the two bars, the closer would be the agreement in the total stretching. Consider the bars as reduced to a length of 1 ft, the hole still remaining in one of the bars: It is probable that the total stretch in the two short bars would differ considerably.

If the investigator confines himself to bars 100 ft long, he would be justified, apparently, in concluding that a hole in a tension bar does not weaken the bar. This reasoning is entirely as valid as the reasoning leading to Conclusion 5. In fact, the writer regards most static tests on riveted structural parts to be of little value as a guide to correct designing, particularly if most of the stress is caused by a live load.

The fact is that engineers have not yet learned how to design, correctly, a simple tension bar with one hole in it when such a bar is subject to frequent live loads. Even a tension bar without a hole cannot have a uniform distribution of stress, for the reason that any rolled material is subject to surface defects which act as stress raisers. About all that is known about a bar under tension is the amount of average stress across the bar.[8, 9]

Had the authors subjected the plate girder to an enormous number of repetitions of live load, they probably would not have reached Conclusion 5.

The ultimate average unit stress in any tension member can be determined only by the use of repeated loads, varying through a wide range of values. For a polished small specimen, the average unit stress will approximate the actual maximum unit stress; but such polished material is never used in actual structures.

The value of the average unit stress is a function of the range of stress and the number of applications. For a standard, Messrs. Wilson and Thomas[9] selected 2,000,000 applications. The average unit stress which produces failure after 2,000,000 applications is the ultimate strength of the material.

The authors have placed too much emphasis on the experimental location of the neutral axis. The location of the neutral axis can be varied over quite a wide range without materially affecting the results of calculation of stress in a plate girder, particularly in a deep girder. The internal resisting moment must equal the external moment. This internal resisting moment is a couple, the arm of the couple being the distance between the centers of gravity of the compressive stresses and the tensile stresses. The neutral axis is nothing but a dividing point between the compressive stresses and the tensile stresses; and this point is at the apexes of two triangles, granting the usual theory of flexure of a bent beam. Hence, a considerable variation in the location of the neutral axis can be made without affecting, to a great extent, the arm of the internal

---

[8] "Tension Tests of Large Riveted Joints," discussion by Wilbur M. Wilson, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., October, 1939, p. 1452.

[9] "Fatigue Tests of Riveted Joints," by Wilbur M. Wilson and Frank P. Thomas, Eng. Experiment Station, *Bulletin No. 302*, Univ. of Illinois, 1938.

couple. It is to be remembered, also, that the sum of the internal tensile stresses must equal the sum of the internal compressive stresses, and this fact alone is of much more importance than the exact location of the neutral axis.

The paper may lead to erroneous specifications. The desire to save metal and simplify the designing of a riveted plate girder is a strong inducement to consider Conclusion 5 as valid.

E. NEIL W. LANE,[10] ASSOC. M. AM. SOC. C. E. (by letter).—The importance of such basic experimental evidence as that supplied by the authors is seldom appreciated properly. Engineers, like men in other scientific groups in the search for truth, delve into detail to such an extent that sight is often lost of the major theme. Naturally this lays a weaker foundation in the form of an increased number of assumptions necessary to proceed with further detail. Messrs. Lilly and Carpenter have done a good job of attracting the attention of the profession to a major theme of structural design.

There has been considerable argument as to the relative merits of designing plate girders on the basis of the gross or net moment of inertia; and the younger, more inexperienced, and generally theoretical minded portion of the profession have tended to favor the more complicated and less accurate design procedure.

It is more or less common among structural engineers to use the gross moment of inertia in computing deflections because it will generally give a more reliable result than will the use of the net value. The authors have proved the experience that neither is exactly correct and that the users of the gross moment of inertia are nearly right. The experience of field engineers with full-sized girders under actual working conditions serves as further corroboration that the gross moment of inertia is nearly correct within the elastic or design range.

Use of the net moment of inertia has led to additional safety at the expense of appreciable economy, a false idea as to design accuracy, and a misrepresentation of the stress distribution.

The fabricating methods now in use are the best that can be obtained at a reasonable cost. For this reason, riveted plate girder flanges will always consist of many small areas (adjacent to the rivets) of high stress restrained and stiffened by surrounding areas of low stress. It should be emphasized that use of the girder causes a plastic flow of the metal at points of extreme stress that makes the girder more capable of withstanding repeated loading. Drilling of members in place and the use of turned bolts is a dream for any but unusual conditions. Welding, although a step in the right direction for continuity and distribution of stresses, has evils of its own that limit its use.

The authors are to be commended for including the effect of diameter and pitch of flange rivets in their proposed formula for the effective moment of inertia. However, the writer would like to see the authors' studies include the effect of various ratios of fabrication—such as, depth to span length, web thickness to depth, and flange area to web area.

---

[10] Junior Civ. Engr., National Advisory Committee for Aeronautics, Phoebus, Va.

Lewis E. Moore,[11] M. Am. Soc. C. E. (by letter).—The experimental determinations were made by measurements of deflection under varying conditions and certain conclusions are drawn, particularly with regard to design, that seem to call for discussion.

According to the laws of mechanics, deflections and flange stresses in a beam are interdependent, and, in general, therefore, the deflections should be in proportion to the average flange stresses. By average flange stress is meant the average, considering not only gross and net areas, but the proportion of length which each governs; in other words, it must be a weighted average.

Considering the ordinary plate girder, the net area of a flange governs only on a plane perpendicular to the flange and passing through the centers of the rivet holes. If this plane is pushed parallel to itself along the axis of the flange the net area will increase until, when the plane becomes tangent to the rivet holes, it becomes equal to the gross area. This gross area will govern until the plane becomes tangent to the next set of rivet holes. A simple method of obtaining the average flange area is to compute the volume of the flange between successive clusters of rivets, less the volume of the rivet holes. Using the girder of the experiments, and omitting the web, the 5-in., flange rivet spacing yields the following:

Description	Area, in square inches
2, 2½-in. by 2½-in. by ¼-in. angles	2.38
1, 6-in. by ¼-in. plate	1.50
1, 8-in. by ¼-in. plate	2.00
Gross area	5.88

The gross volume, center to center of rivet holes, is 5.88 in. by 5 in. = 29.40 cu in. The area of an $\frac{11}{16}$-in. rivet hole is 0.37 sq in., and the total length of holes is (¾ in. by 2) + (¼ in. by 2) = 2 in., making the volume of one set of rivet holes equal to 0.37 sq in. by 2 or 0.74 cu in. Subtracting this from 29.40 gives 28.66 cu in. net volume of steel between rivet holes.

The average flange area for the 5-in. length then is $\frac{28.66}{5} = 5.732$ sq in. The moment of inertia of the cross section will vary directly with the flange area. The deflection varies inversely as the moment of inertia. The ratio of the effective moment of inertia to the gross moment of inertia is then determinable from the average flange area, and theoretically, in the case in question, should be equal to $\frac{5.73}{5.88}$ or 0.975. This corresponds to the quantity "*m*" given by the author. For the particular experiment D-3 with dimensions of riveting as in the foregoing on the girder, "*m*" is given as 0.98.

This is a close agreement between theory and experiment and, incidentally, is a confirmation of the ideas of those who have believed it correct to compute deflections from gross moments of inertia.

The authors' conclusions regarding the position of the neutral axis agree with an opinion expressed by the writer in 1913.[12] The reasoning then was that the net section exists over only a short proportion of the beam and that between rivet holes the position of the neutral axis is certainly found by using the gross area. It seems entirely reasonable to assume that the neutral axis does not shift up and down at every rivet hole, but remains in substantially the same vertical position throughout the length of a properly designed beam. It seems reasonable, furthermore, to assume that this position will be nearer to the neutral axis of the preponderating section, which is the gross section.

---

[11] Cons. Engr., Boston, Mass.

[12] "Design of Plate Girders," by Lewis E. Moore, 1913, p. 42.

The question of the propriety of using the gross moment of inertia or the effective moment of inertia for designing is a quite different one.

The author states that conditions of stress on net areas through rivet holes are variable, and that the metal is stressed, in part, beyond the elastic limit at these locations, using customary working stresses.

As a simple illustration of what happens, consider the application to the experimental girder. Suppose the effective moment-of-inertia method is adopted, with the value determined by the experiment, which is 0.98 of the gross, and only the flange is used. With a design stress of 18,000 lb per sq in. one should use 18,000 × 0.98 = 17,640 lb per sq in. to calculate the gross moment of inertia, and to proportion the flange. The net area (with $\frac{11}{16}$-in. holes) of the flange is 5.88 − 1.02 = 4.86 sq in. The net area is then $\frac{4.86}{5.88} \times 100 = 83.0\%$ of the gross. This means that the average stress on the net area is $\frac{17,640}{0.830} = 21,200$ lb per sq in., or about 18.0% above the allowable of 18,000. In view of the fact that there is some unavoidable overstress at the rivet holes when designing in the usual way with usual stresses, it is doubtful if the effective moment of inertia method is proper to use when designing.

For quick designing by the section-modulus method the writer has found that, for ordinary plate girders, it is sufficient to determine the section by applying 85% of the allowable tensile stress to the moment and then using the tables for moments of inertia of gross sections. The designs so determined rarely need much revision of the main sections.

W. E. Black,[13] Jun. Am. Soc. C. E. (by letter).—While engaged in testing one-quarter size steel rigid frames at Lehigh University at Bethlehem, Pa., in 1938, the writer had occasion to investigate the effective moment of inertia that was being obtained in the tests. For one test specimen, a length of girder having constant cross section was tested as a simple beam loaded symmetrically with two equal concentrated loads. In the part of the beam having maximum moment, top and bottom flange strains were observed at various points, and the extreme fiber stresses were computed from them. The effective moment of inertia, as computed from these stresses, was found to be less than 3% smaller than the gross moment of inertia, and comparatively greater than the net moment of inertia. The cross section of the model girder at the location being tested was 6 in. deep, and consisted of a 6-in. by $\frac{5}{32}$-in. web plate and four 2-in. by 2-in. by $\frac{1}{4}$-in. angles, riveted together by 0.5-in. rivets having a pitch of 3 in. This section, although small, represents a girder of decidedly different proportions than that tested by the authors, and it is gratifying that results obtained in the two cases were similar.

---

[13] Instr., Dept. of Theoretical and Applied Mechanics, Univ. of Illinois, Urbana, Ill.

The method of computing the effective moment of inertia used in connection with the aforementioned rigid frame tests was based upon an empirical relation that has been used previously by some designers. In this method, the volume of metal removed by the rivet holes in a given rivet line is considered to be uniformly distributed along the length of the girder at the same distance from centroidal axis as the rivet line. Thus, the reduction in moment of inertia of a given section due to one line of rivets would be, approximately:

$$\Delta I = \frac{A_h\, g}{p}\, d^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which ΔI = reduction in moment of inertia due to the rivet line; A_h = cross-sectional area of rivet hole; g = grip of rivets; p = pitch of rivets; and d = distance from centroidal axis to center of rivet line.

In applying this relation, the neutral axis was assumed to remain at the centroidal axis of the gross section and the deductions were made from top and bottom flanges alike. This computed effective, or average, moment of inertia gave agreement within 1% of the experimental value of moment of inertia determined by the foregoing test.

To test the applicability of this simple method further, the writer prepared Table 6, which is based on the authors' tests. The experimental values are taken directly from the data presented in the paper. The computed values of $m = \frac{I_E}{I_G}$ (see Table 4 and accompanying explanation in the paper) and of the flange stresses were calculated using the effective, or average, moment of inertia just described. It will be noted that the comparisons are made for only the bolted and riveted sections, which may be expected to simulate an actual plate girder in behavior. The comparisons show that the computed average moment of inertia falls within approximately 1% of the experimental moment of inertia for test section PT-1, having a rivet pitch of 5 in., and within 4% for test section PT-2, having a rivet pitch of 2.5 in. The observed and com-

TABLE 6.—EFFECTIVE MOMENT OF INERTIA

Section	COMPARISON OF VALUES OF $m = \frac{I_E}{I_G}$				COMPARISON OF FLANGE STRESSES—STRESSES IN KIPS PER SQUARE INCH					
	Specimen PT-1 P = 5 In.		Specimen PT-2 P = 2½ In.		Specimen PT-1 P = 5 In.			Specimen PT-2 P = 2½ In.		
	By experiment	Computed, using average I	By experiment	Computed, using average I	By Experiment		Computed, using average I	By Experiment		Computed, using average I
					Top flange	Bottom flange		Top flange	Bottom flange	
C-1	0.970	0.972	0.928	0.943	17.7	18.2	17.7	19.0	19.2	18.2
C-2	0.970	0.974	0.936	0.945	18.9	19.6	18.9	19.4	20.3	19.4
C-3	0.962	0.975	0.916	0.953	19.7	19.9	19.6	20.0	20.9	20.0
C-4	....	....	0.930	0.953	....	....	....	....	....	....
D-3	0.980	0.975	....	....	19.3	20.2	19.6	20.3	20.3	20.0

puted values of flange stresses have a maximum divergence of 5% for any given section. It will be noted that the agreement obtained for the riveted sections is considerably closer than for some of the bolted sections.

The writer does not offer this method for determining the effective moment of inertia of a girder section in place of the authors' or any other theoretically sound formula, but rather to supplement such formulas as a simple working relation that will achieve approximately the same results. It is of interest to note that the values of $m = \frac{I_E}{I_G}$ computed by this method agree very closely with those computed by Equation (5), for $k = 0.6$. Regarding the application of any method for determining effective moments of inertia, the writer concurs with the authors that in the ordinary case, gross section may be used for design without fear of serious error. In those instances where the amount of material removed from a plate girder by rivet holes is unusually large, the test data indicate that an effective moment of inertia should be used. The method of determining the effective moment of inertia to be used in such cases requires, for complete justification, further tests on girders which have greater percentages of material removed by rivet holes, and from both the web and the flanges.

Many engineers, admitting the advisability of using an effective moment of inertia for the computation of girder deflections, still insist upon a net section when computing stresses. There is some basis for this stand in that the actual stresses existing adjacent to a rivet hole are probably much greater even than those computed by the use of net section. Were the structural failure of a plate girder to occur by rupture across a transverse row of rivets, the use of net section would be justified. However, since the structural failure of a plate girder, under static loads, will occur, not by rupture, but by the general yielding of a comparatively large portion of the girder near the location of maximum moment, the high local stresses near the rivet holes and the high average stress across a transverse row of rivets lose much of their significance. Many tests have borne out this predication. Therefore, the observed flange stresses obtained by the authors using a 10-in. gage length, although not the actual maximum stresses, are the significant stresses when the prospective failure of the girder is being considered. The agreement between the observed flange stresses and those computed using the observed effective moment of inertia (see Table 3) is evidence enough that the effective moment of inertia may be used for computing stresses.

L. E. GRINTER,[14] M. AM. SOC. C. E. (by letter).—The profession is indebted to the authors for prompt publication of these tests on plate girders. Recent changes in design specifications preceding the publication of these tests but possibly to be justified thereby, have upset standard procedures in plate-girder design and, naturally, have disturbed conservative designers. Net section, as well as its importance in design, has been taught to engineering students since the beginning of engineering education, and its sudden elimination from certain accepted specifications was too startling to be accepted without question. It is evident, therefore, that these tests, which are considered practically to justify the use of the gross moment of inertia, will be inspected with the greatest care.

---

[14] Vice-Pres. and Dean, Graduate Div., Armour Inst. of Technology, Chicago, Ill.

It is first noted that the two girders tested were only 12 in. deep. Hence, even the theoretical displacement of the neutral axis was a fraction of an inch, and the observed displacement was too small to be measured consistently by the available measuring devices. Another objection to a shallow girder as a built-up member for testing purposes is that the strains introduced by fabrication would produce a disproportionate influence upon the test results. The irregularities in Table I would undoubtedly have appeared far less serious if the girder had been several times as deep. For instance, rivet slip, if it had occurred, would represent a serious influence upon the test results for these shallow girders.

Neutral Axis.—Consistency is evident in the test results, reported in Table 1, in that all data indicate a small shift of the neutral axis in one direction. This shows the care with which the tests were made and confirms clearly the long-held belief that some deduction from the gross section is necessary to allow for the influence of holes whether open or filled. The fact that strain measurements were made on a 10-in. gage length which spanned from two to four rivet spacings indicates that the authors were not attempting to do more than determine an average eccentricity of the neutral axis. The second important observation from Table I is that the location of the neutral axis, as obtained from flange measurements, does not agree with the location as found from web strains. Since these strains were obtained from a section where the shear was zero, one can only conclude that there must have been some rivet slip, or the theory of "plane sections" failed to apply for some other unknown reason. This is rather disturbing if conclusions are to be drawn from these data.

If one notices that the average observed displacement of the neutral axis for all recorded data on specimen PT-1 is about one sixth or one seventh of the computed displacement, an important observation can be made. This specimen had $\frac{11}{16}$-in. holes at 5-in. spacing. Hence, less than one sixth of the length of the girder was influenced by the net section, and the observed "average" displacement of the neutral axis was of the magnitude that should have been expected. Whether the neutral axis actually shifts upward at each hole in the flange is questionable, perhaps, but the influence in added flange stress must be considered in design until it is proved non-existent. Such proof is still lacking. Fig. 7 shows, pictorially, the possible shift in the neutral axis above a flange hole. Proof of this action is also non-existent at the present time.

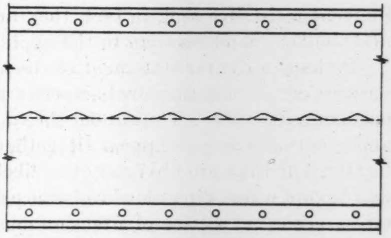


FIG. 7.—POSSIBLE UNDULATIONS OF NEUTRAL AXIS

Effective Moments of Inertia.—The method used to measure the effective moment of inertia was based upon the observation of deflection. Of course, this indicated, accurately, the proper moments of inertia for use in future deflection calculations, but it scarcely seems significant for maximum stress determination. A deflection is produced by the cumulative influence of the stress at all cross sections. Since a major part of the length of the girder functions as a solid cross section without holes, it is self-evident that the effective moment of inertia, when determined from measured deflections, will represent an "average" moment of inertia which will approach the gross moment of inertia closely. Surely there can be little connection between this "average" moment of inertia and the proper value for use in the flexure formula.

Flange Stress.—Again dependence is placed upon a 10-in. strain gage and recorded values are carefully determined averages of the readings taken from four to eight gage lines. Recorded values, therefore, do not represent even the greatest "average" strains for 10-in. gage lengths. What maximum strain would have been recorded on a shorter gage length is purely speculative. Naturally, it would have been higher than the "average" values recorded, which are from 2,000 to 4,500 lb per sq in. lower than computed values from net moment of inertia. It is clearly within the range of probability that strains could have been measured on ½-in. gage lengths quite as high as the theoretical values. In fact, considering possible torsion in the cross section, it would not be particularly surprising to find measured strains that exceeded the theoretical strains for gage lengths of 1 in. or less.

Stress Concentration.—There has been much confusion about the subject of stress concentrations, and discussions of rivet-hole stresses have not been free of such influence. There are high stresses around punched rivet holes, and there would be high stresses even if plastic distortion had not taken place during punching. However, residual stresses from temperature change, or from overstrain, and even stress concentrations from discontinuities, are unable to resist load. A tension member with a hole may have a high localized stress, but this stress does not increase the load capacity (in fact, it reduces it) because the average stress over the net area A_n is still $\frac{P}{A_n}$. In exactly the same manner the stress concentrations around the rivet holes of a plate girder do not produce moment resistance and, in fact, they are more related to fabrication methods and similar influences than to the applied moment.

Perhaps a clearer statement of the action of stress concentrations, residual stresses, etc., is that they are balanced stresses—that is, tension and compression occurring together. Thus under the smallest plastic flow they begin to reduce, and eventually they disappear altogether. Such stresses are not to be neglected lightly, but they are obviously less likely to produce failure than direct stress and flexure where direct load resistance is involved. Statics requires the average stress on the net section of a tension member to be equal to the load divided by the net area. The laws of statics permit no loose thinking in this matter, and it would be a very incautious investigator who questioned this fact or permitted test data to change his thinking in this regard. Perhaps bolts could be tightened through holes in a tension member with sufficient pressure to reduce the net effect of the holes, but engineers are not likely to be willing to let this possibility influence their belief that tension members must be designed for net section.

The lower flange of a girder is as much a tension member as is a chord eyebar of a truss. In fact, it serves the same purpose. Apparently any argument about stress concentration resisting load that can be applied to one can be applied to the other. Any thought that stress concentration over the entire section of the tension flange which is an increase of "average stress" can be neglected in the same way that residual stress and stress concentration around rivet holes are neglected is subject to serious criticism since it conflicts with the requirements of statics. Admittedly, this increase of stress over the section of the tension flange at a rivet hole may exist over a very short length of the flange and may scarcely be found at all when measurements are made over a 10-in. gage length. However, like a weak link in a chain, the effect upon the whole is serious, as statics demonstrates unerringly.

Further Tests.—It is not beyond the reach of modern testing methods to determine accurately the influence of rivet holes upon "total" or "maximum" flange stresses as distinguished from the authors' determination of "average" flange stresses over a 10-in. length of span. With sensitive gages, strains could be measured on $\frac{1}{2}$-in. gage lengths at a dozen or more positions across one section of the web and the tension flange in line with a rivet hole. It would be preferable to perform this test under light loading so that the problem of inelastic strain near rivet holes would not be involved. Then, by transferring unit strain into unit stress and by summing the total stress over the flange and web section, it would be possible to check either the so-called approximate or exact methods of plate-girder design. Such a test, to be most useful, should be made on a test girder of reasonable depth, as representative as possible of standard construction. Of course, there could be no particularly useful design information obtained from single deflection measurements because deflection is a measure of average resistance, whereas the interest in the present case is in the resistance of a particular section. The final check on the accuracy of such testing is obtained from statics. The total moment resistance of the section obtained as the summation of the moment resistances of its fibers must equal the applied bending moment.

Specifications.—Specifications should follow, and not lead, theory. Testing a procedure by introducing it into specifications before it has been checked by statics, well substantiated by tests, and even by use, is dangerous from more than one point of view. The questionable use of the gross moment of inertia in plate-girder design is a case in point. The influence upon the cross section is so minor that it is not likely to endanger a structure physically, but it seems to the writer that its introduction into specifications was far more serious, in that it tended to develop mental carelessness. The laws of statics are fixed matters that must not be changed by specification writers. When working stresses are to be increased, they should be straightforwardly raised, but "design moments" which are controlled by statics should not be reduced even if the influence upon the structure is the same. The profession suffered from the use of that artifice in flat-slab specifications, and there is no reason to reestablish a similar practice with reference to plate girders. The next step would be to omit the consideration of net section in tension members. Of course, this is no criticism of the authors' tests, which were undoubtedly well planned and carefully performed. The criticism is of premature use of anticipated results by specification writers.

CHARLES M. SPOFFORD,[15] M. AM. SOC. C. E. (by letter).—The writer has read this paper with much interest and wishes to commend the authors for their careful investigations and excellent report. That such investigations are needed is shown clearly by the fact that standard specifications for plate-girder design in the United States are not yet stabilized with respect to the allowance for rivet holes or the position of the center of gravity.

The authors' experimental method of determining the position of the center of gravity of the girder was such as to give the average moment of inertia over the central part of the girder only. In this part of the girder the bending moment is uniform, no web splice occurs, and there are no vertical lines of rivets except at the ends of the section. The results show clearly that under these conditions the centers of gravity of the girders tested lie nearer midheight than when determined by computations of the net section based upon deducting rivet holes in the tension part of the girder only. The calculated values given by the authors, however, give only the location of the center of gravity of a section through rivets, whereas the average center of gravity would be higher than this since rivet holes do not occur at every section.

For the girders tested, with rivets at 5-in. pitch in the vertical legs of the flange angles only, the position of the average center of gravity would be considerably greater than that computed by the authors and would conform quite closely to observed values. This would apply in lesser degree in cases where the rivet pitch is reduced to 2.5 in.

The writer feels that the results of the tests are scarcely sufficient to warrant definite conclusions for use in actual design. The girders tested were somewhat out of line with those generally employed in practice, since the section modulus of the web was much larger in proportion to the total value of the moment of inertia than is common in actual plate girders, particularly in bridge girders. This condition is especially true in the part of the girder in which the measurements were made, since the shear in this part of the girder was zero. Moreover, the weakest section of a large plate girder is generally at a web splice, and it would have been desirable if the authors could have included some tests for the value of I and the position of the center of gravity of the girder at such a splice.

The authors' conclusion that the data from these tests point to the acceptability of using the gross moment of inertia in design involves the conclusion that either the allowable unit stress should be reduced, or that rivet holes do not weaken a girder, which latter conclusion the writer believes designers will be slow to accept. It is to be hoped that Messrs. Lilly and Carpenter will find it possible to make additional tests along these lines.

---

[15] Hayward Prof. of Civ. Eng. Mass. Inst. Tech., Cambridge; Cons. Engr. (Fay, Spofford and Thorndike), Boston, Mass.

E. MIRABELLI,[16] M. AM. SOC. C. E. (by letter).—These tests show the effect of flange holes, bolts, and rivets on the deflection and on the average flange stress of a plate girder when subjected to pure bending by a static loading which induces average stresses within the elastic limit of the material. It is questionable if they show how the average flange stress at a critical section is affected by the presence of holes or rivets, particularly in a region subjected to the combined action of bending moment and shear. The tests are of definite value in contributing toward an understanding of plate-girder action, and there is no intention of disputing their importance. However, it is doubtful if this set of tests, by itself, provides enough information to form a sufficient basis for changing the current design practice of using net girder section. Following are some reasons why it appears to the writer that caution is warranted in abandoning the usual design procedure:

(1) The effective or observed moments of inertia given in Table 3 are based on observed deflections which are affected by all sections of the girder—those between rivet holes as well as those at the rivet holes. Such values of the effective moment of inertia are averages for the girder as a whole. They are proper ones to use in computing deflections, but are not correct for evaluating the stress at a critical section through rivet holes. The flange stress computed by the beam formula, using such a moment of inertia, is not the critical stress; it is some kind of an average stress. Similarly, the observed flange stresses that were measured with a 10-in. gage are the averages of stresses on all sections within the length of the gage. They are no indication of the average stress on a section passing through the rivet holes. It seems reasonable to assume that failure follows when the stress exceeds some critical amount on a single section without necessarily extending over an entire region. To prove this it is necessary only to test a perforated or riveted bar in tension and find that failure occurs on a section through the hole.

(2) The girders were tested in bending only, without simultaneous shear. Consequently, the flange rivets were not subjected to any stress due to flange-stress transfer which accompanies an increment in bending moment. Such rivets act simply as stitch rivets to hold the angles and cover plates together. Any stress that exists in the rivets is that due to initial tension in the shank caused by cooling, and whatever stress occurs in the rivet head through friction at the surface of contact with the plates or angles. The condition of the rivet is similar to that indicated in Fig. 8 in which a rivet is shown holding together four plates which are subjected to end forces, *F*. The only possible stress path across the gap, *G G*, is through the rivet heads. The strength of such a connection may be calculated if assumptions are made regarding the coefficient of friction under the rivet heads and the initial tension in the shank. The splicing action of the rivet heads, in some degree, makes up for the loss of section due to rivet holes in a plate girder and, to some extent, explains the increase in effective moment of inertia of the riveted specimen when compared with the specimen with open holes.

Now, if the condition in Fig. 8 is changed by the introduction of shearing forces, *S*, the total tension in plates *A* and *B* will be $T_1 = F + S$, and in plates

---

[16] Associate Prof., Civ. Eng., Mass. Inst. of Tech., Cambridge, Mass.

C and D will be $T_2 = F - S$. If $T_2 = 0$, there will be no stress to transfer from D to B or from C to A, although the total stress to be spliced is still $2F$. It is probable that some stress will pass from B to A by following a path which involves eccentricities; but it seems that the capacity of the rivet heads as a splice for direct stress is definitely reduced by the presence of the shearing forces. Now eliminate the gap, $G\,G$, by making C continuous with A, and B continuous with D, and the condition is similar to that occurring in the flanges of a girder. It seems reasonable to conclude that under this condition, also, the presence of shear will influence the amount of direct stress passing across the hole through the rivet head and thereby will weaken the section. In fact it has been found by test[17] that a stressed rivet has a greater effect in weakening a plate than has an open hole.

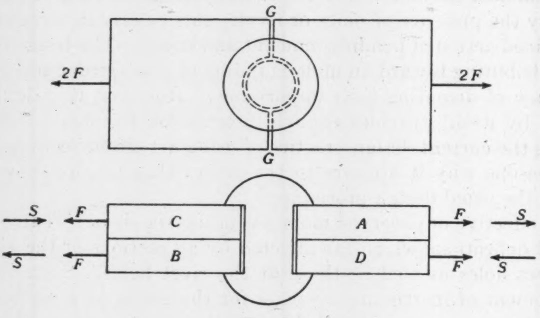


FIG. 8

Extension of the authors' conclusions to girder sections stressed simultaneously in bending and shear would not seem justified. The presence of stressed rivets in the flanges will reduce the effective moment of inertia. It is true that maximum bending moment and maximum shear do not occur at the same section except in cantilevers. However, maximum flange stress and maximum rivet shear may occur simultaneously at cross sections at the end of cover plates.

(3) The tests were made under static loading whereas, in service, girders are subjected to varying or pulsating loadings. The question of type of loading would not be involved if the rivet holes with the attending stress concentrations were not present. The effect of rivet holes differs with the two types of loading. This difference is shown by tests[18] which indicate that the tensile strength of a perforated or riveted bar under static loading is greater than that of a solid bar of the same net cross section (57,600, 59,700, and 62,300 lb per sq in. for solid, perforated, and riveted bars, respectively). However, the fatigue strength of the perforated and riveted bars is considerably less than that for the solid bar under a loading that pulsates from 2,820 lb per sq in. to the breaking stress (39,100, 29,900, and 24,300 lb per sq in. for solid,

---

[17] "Tension Tests of Large Riveted Joints," by Raymond E. Davis and Glenn B. Woodruff, Members, Am. Soc. C. E., and Harmer E. Davis, Assoc. M. Am. Soc. C. E.; discussion by W. M. Wilson, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., October, 1939, pp. 1454 and 1462.

[18] "Gemeinschaftsversuche zur Bestimmung der Schwellzugfestigkeit voller, gelochter und genieteter Stäbe," by K. Klöppel, *Der Stahlbau*, Berlin, June 19, 1936, pp. 97–112.

perforated, and riveted bars, respectively). In other words, the influence of stress concentrations may be disregarded under static loading but it cannot be disregarded under a vibrating loading. It is true that in a properly designed new structure the primary stress will not attain a value of 24,300 lb per sq in. and the condition is not nearly so bad as this value might indicate.

The stress concentrations adjacent to the rivet holes nullify the assumption of linear stress variation used in the ordinary beam theory and thereby lead to an observed moment of inertia which, under static loading, is larger than that computed for the net section. The difference between observed and computed moments of inertia for the "B" test sections (Table 3, Columns (1) and (4)) may be explained in part by this action. Under the action of a pulsating loading the aid given by the stress concentrations is diminished or lost entirely.

(4) These tests were not intended to reveal the useful carrying capacity of the girders—that is, the yield point of the girder as a whole, or the point at which the girder deflection increases rapidly with further application of loading. This is probably the proper point to which the margin of safety should be measured. If the girder deflection is in direct proportion to the loading until the useful carrying capacity is reached, the flange-stress intensity might be used as a measure of the factor of safety. Otherwise the factor of safety must be based on applied loading.

Due to the presence of stress concentrations at the rivet holes, it is quite possible that the linear relation between girder deflection and applied loading may cease to exist shortly after the stress in the vicinity of the rivet holes exceeds the yield point. This may occur while the average flange stress is still considerably within the elastic range. On the other hand, it is also possible that this linear relation may extend to a loading exceeding that at which the average flange stress is equal to the yield point stress. It would be interesting to know the relation between loading, strain gage reading, and deflection for the specimen PT-1 which was loaded to produce a flange stress of 30 kips per sq in.

It may be quite desirable and safe to design on the basis of gross moment of inertia as recommended by the authors. The writer feels that such a procedure would evaluate actual conditions correctly for only a small percentage of cases, and that in most cases the procedure would provide an actual margin of safety which would be less than the assumed amount. However, considering the fact that secondary stresses in plate girders are likely to be small, it may be permissible to reduce the margin of safety. From this point of view the weakening effect of rivet holes would be treated as a secondary effect which would be covered by the margin of safety. Until the influence of holes and rivets in girders is established more definitely, it is the writer's belief that the customary practice of using net section should not be abandoned.

Edward Godfrey,[19] M. Am. Soc. C. E. (by letter).—Conclusion 3 is correct. It agrees with what the writer has long held to be proper in design—namely, that the neutral axis of a girder should be considered in the center of gravity of the gross cross section. Furthermore, the sentence in Conclusion 5, "design methods should be made as simple as possible," expresses what the writer has stated many times.

---

[19] Structural Engr., Pittsburgh, Pa.

The last sentence in Conclusion 1 is not clear. It reads: "The net moment of inertia, used in design with top and bottom flanges alike, should yield conservative stresses." This does not seem to comport with the trend of the paper nor the other conclusions. The authors do not seem to be advocating the use of net moment of inertia in design but rather the reverse. The trend of the paper seems to be toward justification of the practice of using the gross moment of inertia. There is no question about the safety of design that uses the net moment of inertia.

Deflection measurements and extensometer measurements do not tell anything of value about the actual, or critical, stresses of perforated flanges of a girder or of perforated plates or angles in tension. These measurements show only the average stress in the length of piece where measurement is made. The extension beside the rivet hole is only a small fraction of that in the part not perforated, although the actual unit stress beside the hole may be very much greater than in the part not perforated.

Suppose, in a girder or I-beam, there were transverse saw slits instead of rivet holes. Deflection and extensometer measurements would be practically identical, in such members, with those of solid members of the same dimensions for stresses within the elastic limit. This is because the actual stretch beside the slit is only a very small fraction of the stretch between slits in the longitudinal direction. No one will contend that the members with the saw slits will be as safe as the solid ones.

The only difference between a plate perforated with saw slits and one perforated with rivet holes, so far as safety in tension is concerned, is that the former will probably break at, or a little above, the yield point, whereas the latter may hold until the usual ultimate stress is reached. The writer has made tests where ordinary steel specimens broke just above the usual yield point of the net area beside punched holes, whereas pieces with drilled or reamed holes broke at the usual ultimate stress.

The writer is quite out of agreement with Conclusion 5. It is a damaging admission to say that stresses as ordinarily specified for structural work result in unit stresses "reaching" the yield point of the steel as the unit stress in the gross section "approaches" that specified. This can mean nothing else but that, when the allowed unit stress on the gross section is reached, the actual unit stress beside rivet holes exceeds the yield point. The argument has been that rivets pinch the metal together, and the resulting friction under the head compensates for the loss of tension area in the perforated plates.

To say that nothing can be done to avert the condition is to forget that rational methods of design can be reverted to that treat a perforated plate as just what it is—namely, a plate that is weakened by the holes punched in it, good only for its net section in tension.

The method once universally used of finding the effective depth of a plate girder and proportioning the compression flange for its gross area and the tension flange for its net area is eminently rational and results in design to which exception cannot be taken from any standpoint.

The authors state, "Once a ductile material is strained beyond the yield point and the load is released, residual stresses are set up which will be beneficial in resisting the next applied load." This does not agree with abundant experience in the use of chains and axles. Chains will stand loads beyond their yield point for so long; then they are likely to break. Chain users know this, and they anneal their chains at times. Axles stand, repeatedly, the load that may finally break them. There is such a thing as fatigue, and frequently breaks occur with a comparatively small number of repetitions (not up in the hundreds of thousands or any such number).

Some years ago, in Pittsburgh, Pa., a wrought-iron plate girder broke apart in the tension flange and dropped the locomotive that it had doubtless carried many times. The flange plates broke sharply, very evidently from fatigue.

The rule that designs a plate girder for gross section in the tension flange may result (where thick metal and many cover plates are used) in 4 or 5 sq in. of metal punched away in one hole. This represents a stress of 70 to 90 kips being imposed on friction under the head of a single rivet, and that rivet might be loose.

Plate-girder design needs careful scrutinizing from a rational standpoint.

WALTER H. WEISKOPF,[20] M. AM. SOC. C. E. (by letter).—The profession is indebted to the authors for an investigation of an important feature of structural engineering. For many years holes have been deducted in computing the strength of plate girders as a matter of routine.

With the authors' measurements there is little room for dispute. When one considers that the net section extends over such a small length of girder as compared to the length between the holes, it is not surprising that the holes, whether open or filled by bolts or rivets, have small effect on deflections. Engineers have recognized this for many years and have generally used the gross moment of inertia when computing deflections or stiffness coefficients for use in designing statically indeterminate structures. Likewise, it is obvious that the holes have a negligible effect in shifting the neutral axis from the center of gravity of the section. For the same reason, also, a practically negligible increase in flange stress due to the holes was discovered by the method the authors employed in measuring such flange stresses. Since this stress was computed from the strain in a 10-in. length of flange, it was obviously determined by the average of the strains in many differential lengths of gross section and a much smaller number of differential lengths of net section. The method thus failed to detect the very high concentrations of stress around the holes well known to investigators of this subject and mentioned, but not quantitatively given, by the authors in Fig. 5.

With the authors' conclusions, however, there is room for very clear disagreement. The authors state that nothing can be done to prevent the high stress concentrations around holes. Actually, these can be eliminated very simply in all but girders with very large flanges by eliminating the flange rivets, either by using welded cover plates or by employing a flange composed of a split beam tee and two angles.[21]

---

[20] Cons. Engr. (Weiskopf & Pickworth), New York, N. Y.

The writer also emphatically disagrees with the authors' recommendation of using the gross moment of inertia with the usual fiber stress. Frequently, the area of the holes is very considerable as compared with the remaining flange section. In heavy cramped work a common flange consists of 8-in. by 8-in. angles and 17-in. or 18-in. cover plates, with four lines of rivets through the covers. Often the stagger is so small that the net section traverses all four rivets so that the cover plate area is as much as 22% rivet holes. This is far too important an item to treat lightly, and it is improper engineering to dismiss the high stresses around these holes with the casual remark "and nothing can be done to prevent them."

The purpose of strength computations is to provide a safe and reasonably efficient design. An efficient design is one in which all parts of the structure have about the same margin of safety or safety factor. It is obvious that a riveted girder with high stress concentrations at the holes is definitely inferior to a plain rolled beam or to a girder of another type which has no such holes; and a balanced design requires that this feature be given due consideration.

One way of compensating for the detrimental effect of the holes is the practice, extensively used, of designing by the net section. Another simpler, and perhaps more practical, method which has been suggested is to base the design on the gross moment of inertia but to reduce the fiber stress according to some function of the ratio of net to gross flange.

C. D. Williams,[22] Esq. (by letter).—Some interesting data are offered in this paper, although the conclusions may not be concurred in by all engineers. It should be noted that the stress intensities reported by the authors are based on average stresses as computed from the strains on 10-in. gage lengths. This set-up, of course, does not give the maximum stress values—a fact that is recognized by the authors.

It should be proper at this point to make some comparisons of their results with the results that would be expected when the stresses are computed according to the commonly accepted practice of deducting rivet holes in the tension flange.

The stretch of any finite length under stress is equal to the unit stress times the length, divided by the modulus of elasticity. When there are two or more unit stress intensities in the length considered, the stretch may be found by use of the average stress per unit length. If there were no reductions in area throughout the length the average stress and the maximum stress would ordinarily be equal. However, when there are local reductions in area, the average stress producing elongation will be increased thereby. The reduction in area and the length over which it occurs are both factors in the increase of stretch due to a given force since they influence the average stress over the finite length considered. In other words, the stretch of a finite length due to load is inversely proportional to the volume taking stress.

---

[21] "A Comparative Analysis of Plate Girders," by Walter H. Weiskopf and John W. Pickworth, Members, Am. Soc. C. E., *Civil Engineering*, November, 1934, p. 585.

[22] Associate Prof., Head of Dept. of Structural Eng., Fenn College, Cleveland, Ohio.

In considering the effect of a local reduction in area upon the deformation of a finite length longer than that which is reduced comparison may then be made between the reduced and unreduced volumes for the finite length.

TABLE 7.—COMPARISON OF OBSERVED AVERAGE STRESSES AND AVERAGE REDUCTION IN AREA

Section mark	TENSION FLANGE MAKE-UP		Gross area, in square inches	DEDUCTIONS			Percentage volume deduction*	Observed stress increase due to holes
	Angles	Plates		In square inches	In cubic inches	In cubic inches per inch of length		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(a) SPECIMEN PT-1: 5-IN. RIVET SPACING								
C-1	2	....	2.38	0.56	0.44	0.088	2.8	3.0
C-2	2	1	3.88	0.94	0.66	0.132	3.4	2.7
C-3	2	2	5.88	1.31	0.88	0.176	3.0	3.8
D-3	2	2	5.88	1.31	0.88	0.176	3.0	2.1
Average	....	....	....	....	....	....	3.05	2.90
(b) SPECIMEN PT-2; 2.5-IN. RIVET SPACING								
C-1	2	....	2.38	0.56	0.33	0.132	5.6	7.0
C-2	2	1	3.88	0.94	0.55	0.220	5.6	6.3
C-3	2	2	5.88	1.31	0.77	0.310	5.3	5.3
D-3	2	2	5.88	1.31	0.77	0.310	5.3	7.0
Average	....	....	....	....	....	....	5.45	7.15

* Or, the equivalent average area deduction.

Table 7 contains certain properties of the girders of series C and D taken from the details of the paper. These are the series that had holes in the tension flanges. Values in Column (5) are obtained by deducting two $\frac{3}{4}$-in. holes from the horizontal legs of angles and cover plates and one-half of a $\frac{3}{4}$-in. hole through the two vertical legs. The volume deduction shown in Column (6) is computed for sections in Table 7(*a*) as the volume of metal removed by two $\frac{3}{4}$-in. holes through the horizontal legs and cover plates and one $\frac{3}{4}$-in. hole through the two vertical legs; for sections in Table 7(*b*) the holes in the vertical legs are spaced 5 in.; therefore only one-half of the volume of these holes is included.

The results listed in Columns (7) and (8) are the deductions in volume and the percentage deductions in volume due to rivet holes. Column (9) is computed from Columns (1) and (2) of Table 3 and is the percentage by which the values of Column (1) are less than those of Column (2). Column (9), then, is the percentage of increase in the average stress which the authors found due to rivet holes. Column (8) is the percentage reduction of volume in the gage length due to rivet holes; or, inversely, it is the percentage increase in average unit stress.

The comparison of the values in Columns (8) and (9) is remarkable, especially in view of the authors' claim to an accuracy of only within about 1.5 per cent.

The writer would like to suggest the following conclusions on the basis of the data submitted by the authors:

1. In view of the fact that the average stress intensities as found by measuring strains on 10-in. gage lines check so closely the average values found by deducting rivet holes from the tension flanges, it would seem to follow that the maximum intensities of stress are more correctly given by use of the net moment of inertia than by the gross moment of inertia.
2. Since the measurement of strains for the purpose of locating the neutral axis was made on 10-in. gage lines and did not register maximum intensities of stress, local deviations of the neutral axis could be present without detection. Since the measurements all indicate that the average location of the neutral axis is moved up by drilling holes in the bottom flange, there seems ample justification of the usual procedure of considering the neutral axis to be at the gravity axis of the net section.
3. Since the average location of the neutral axis is only slightly above the gravity axis of the gross section this axis may be used satisfactorily for the computation of the deflection of the girder, as may the moment of inertia of the gross section; but deflection due to average stress and maximum stress intensity should not be confused.
4. Since the use of the gross section in computing stress intensity is obviously opposed to conservatism, and since there are likely to be stress concentrations in the region of rivets not affecting the longitudinal deformations, it seems proper to use the net moment of inertia in the design of girders until a more conservative and exact method is found.

Alvin B. Auerbach,[23] Jun. Am. Soc. C. E. (by letter).—To paraphrase an utterance of Hardy Cross, M. Am. Soc. C. E., a structure, not being cognizant of its designer's assumptions, always deforms in accordance to the laws of geometry and statics.[24] Both common design practice and experience agree that the deflections of a girder are a function of the gross section. Hence a suitable design method should:

1. Be based upon the deformation of the girder and therefore on the gross section;
2. Yield maximum computed stresses in consonance with the underlying assumptions of the specifications governing the design;
3. Be capable of being visualized by the user; and
4. Be adaptable to girders of unusual shape or section.

Under "Conclusions" the authors state: "It is believed that the data from these tests point to the acceptability of the gross moment of inertia for design purposes." The writer proposes to show that it is possible to analyze a structure so as to yield higher design stresses which are still in consonance with the average measured stress found by the authors.

For example, a method of design which meets Conditions 1 to 4 is the one utilizing the ratio of the equivalent net flange area to the gross. It was used in the U. S. Engineer Office, Rock Island (Ill.) District, for the design of the plate girders having a concave lower flange, and unsymmetrical sections. These girders were the service bridge spanning the movable gate openings on the Mississippi River dams. Using the relationship derived in standard texts on design of plate girders, the equivalent gross area is expressed by:

---

[23] 1st Lieut., Corps of Engrs., U. S. Army, Fort Du Pont, Del.

[24] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1373.

$$A_e = A_f + \frac{A_w}{6} \quad \text{.......................} \quad (7a)$$

and the equivalent net area by:

$$A_e' = A_f' + \frac{A_w}{\dfrac{6\,p}{p - r}} \quad \text{.......................} \quad (7b)$$

in which A_f and A_f' are the gross and net flange areas, respectively; A_w is the gross area of the web; p is the vertical pitch of rivets in the web (as in stiffeners); and r is the diameter of the rivet holes.

Accordingly, in compression,

$$s = \frac{M\,c}{I_{(\text{gross})}} \quad \text{.......................} \quad (8a)$$

and, in tension,

$$s = \frac{M\,c'}{I_{(\text{gross})}} \times \frac{A_e}{A_e'} \quad \text{.......................} \quad (8b)$$

TABLE 8.—EQUIVALENT FLANGE AREAS, IN SQUARE INCHES

Section	Dimensions, in inches	Equivalent Areas	
		Gross, A	Net, A'
Web	12 by 5/16	0.63	0.63
Angles	2½ by 2½ by ¼	2.38	2.04
Cover plate	6 by ¼	1.50	1.16
Cover plate	8 by ¼	2.00	1.66
Totals	....	6.51	5.49

Using the data supplied by the authors, for girder D-3, the areas are as shown in Table 8. From Column (9), Table 3, the computed stress for the gross section is 19.1 kips per sq in.; the maximum tensile stress would be $19.1 \times \frac{6.51}{5.49} = 22.7$ kips per sq in.; and thus the average tensile stress $\frac{22.7 + 19.1}{2} = 20.9$ kips per sq in. This is in fairly good agreement with the measured average tensile stress of 20.2 kips per sq in.

The results obtained by this method, although numerically slightly higher than other methods, better meet the desired conditions. The underlying reasoning is simple; it is easily adaptable to unusual girders. Also it does not encroach on the much abused factor of safety, which, already carrying allowances for defects of material, imperfections of workmanship, secondary stresses, local stress concentrations, and other intangibles, needs some protection lest it become the great unknown. It may be interesting to note that in military design of bridges in war time the factor of safety based on existing civil practice is to be flatly reduced by 20% for steel structures.

Most present-day specifications utilize design stresses based on double net areas, with results that are almost identical with the foregoing; but these specifications are not feasible for unsymmetrical girder sections. If stresses are to be raised, let the design stresses be increased openly rather than permit the increase to be hidden in illogical analysis.

JONATHAN JONES,[25] M. AM. SOC. C. E. (by letter).—When the Committee on Specifications of the American Institute of Steel Construction determined (1936) to adopt gross moment of inertia for the design of plate girders in building construction, it did so on the basis of a reasoning, satisfactory to its members, with respect to actual unit stresses on sections through rivet holes. That reasoning may be summarized somewhat as follows:

(1) A statically-loaded plate girder can scarcely be made to fail in the tension flange; the compression flange and the web usually determine the capacity.

(2) All manner of things have happened to overloaded crane girders in mill buildings, but not to the tension flanges.

(3) In normal, simple-span girders the flange area removed by rivet holes is less than 15%. The rivets are practically always machine-driven, and they induce a friction in the heads which carries part of the stress.

(4) If occasional rivets are loose, and the friction in Item 3 is nonexistent, the most severe stress that can occur is somewhat less than 23,000 lb per sq in., and the designer would rather have 23,000 in a plate girder flange than 20,000 in many other elements of a building.

(5) Flanges that are "chopped up" by closer spacing of rivets, on account of heavy shears to be developed in short distances, should not be required to carry still greater net stresses due to excessive reduction in area; and therefore a maximum exempted reduction of 15% resulting from a "chain" of holes was established.

(6) Holes filled with bolts, pins, or countersunk rivets have less hope of being assisted by friction, and should be deducted.

The specification clause[26] was drafted to authorize the use of gross sections, subject to the foregoing implied limitations. It was submitted to a number of engineers and met, not with unanimous, but with general, approval.

The desire for a research project arose after the specification had been thus drafted and adopted; its purpose was not to convince the Specification Committee, which had already acted, but to reconcile, if possible, some objectors to what the Specification Committee had done. No one expected to show, by a research project, that there are not peak points of high unit stress around rivet holes; or that, if a tension flange could be made to rupture, it would rupture elsewhere than through a hole. What could be shown, and has been shown, is that

(*a*) Exemption of rivet hole deductions to as much as 15% of the gross flange area is conservative (rivet hole deduction in test specimens 18.0% minimum and 23.4% maximum);

---

[25] Chf. Engr., Fabricated Steel Constr., Bethlehem Steel Co., Bethlehem, Pa.

[26] A.I.S.C. Specification for Design, Fabrication and Erection of Structural Steel for Buildings, Section 19*a*, 1936; also A.I.S.C. Manual on "Steel Construction," Third Edition, October, 1937, p. 272.

(*b*) Drilling holes in a solid flange increases the average unit stress over that in the solid material; but

(*c*) Bolts or rivets in these holes will go a long way toward reducing this average unit stress to what it was in the solid material;

(*d*) Bolts performed about as well as rivets in this respect, but these were tightened, of course, with considerable care.

The research project has been successful in demonstrating all that was hoped for. It has not disposed of the spots of overstress backed up by lower-stressed material, which were not the subject of this particular study. Engineers still may be divided into those who believe that the gross-section-modulus specification represents a sound attitude, and those who believe otherwise. For the former group, including the A.I.S.C. Specification Committee, it may be said that, as raw materials tend toward scarcity and high cost, it will be more and more necessary to dispense with inherited extravagances which a practical consideration of probable behavior may serve to eliminate.

R. L. Moore,[27] Assoc. M. Am. Soc. C. E. (by letter).—Certain phases of plate-girder behavior have always been open to question, namely: (1) What are the effects of rivet holes upon the behavior of a plate girder; and (2) what allowance should be made for their effects in design? As is generally recognized, specifications are not in agreement on the latter question. In some cases plate girders are proportioned on the basis of the gross section;[28] in others the net section of both tension and compression flanges is used in computing tensile stresses, whereas the corresponding compressive stresses are based on the gross section.[29]

Apparently, the logic behind the use of net sections in proportioning plate girders is that such a procedure provides some allowance for a reduction in strength resulting from rivet holes in the flanges. Little or no information, either from experience or tests, is available, however, to indicate how essential such a procedure may be. It is generally assumed, as stated in the paper, that bending deflections may be computed with reasonable accuracy, using moments of inertia based on gross sections. This view is supported by Table 3 in which the differences between observed effective moments of inertia and those computed on the basis of gross sections averaged only 3% in the case of specimen PT-1 and 6% for specimen PT-2, with the maximum individual differences amounting to only about double the foregoing percentages in each case. If moments of inertia for gross sections are satisfactory for computing deflections, it would seem to follow that they are also satisfactory for computing average flange stresses, since deflections are merely a reflection of the strains corresponding to such average stresses. The computation of stresses on the basis of net section may result in increases of from 10% to 15% over those obtained using gross sections; but such increases are of little significance in view of the stress concentrations of from two to three times the average actually produced in the flanges by the rivet holes. As long as questions concerning fatigue strength under repeated loadings are not involved, and average rather than maximum values of flange stress are used as a basis for design, the use of gross sections seems satisfactory.

---

[27] Research Structural Engr., Aluminum Research Laboratories, Aluminum Co. of America, New Kensington, Pa.

[28] See A.I.S.C. Specification for the Design, Fabrication and Erection of Structural Steel for Buildings, Section 19*a* (revised 1936).

[29] See A.R.E.A. Specifications for Steel Railway Bridges, Art. 426, 1938.

The authors state that their tests were planned to discover, if possible, how much the flange stresses in plate girders are affected by the rivet holes in the flanges. Unfortunately, they have presented no data pertaining to the actual stress distribution in the flanges; neither have they given any information to indicate what effect the stress concentrations at the rivet holes might have on the ultimate strength of the girders. The fact that some of the 10-in. gage lines used in their tests were located in line with the holes in the flanges means that the changes in length measured included hole distortions as well as strains in the material. Obviously, such deformations have no bearing upon the actual stress distribution, although they would be expected to be consistent with the strains corresponding to the measured deflections.

The observed stresses shown in Table 3 are in good agreement with the computed values based on either the observed effective moments of inertia or those based on gross sections, since there is little difference between these quantities. Although the stresses measured in the tension flanges were consistently higher than those found in the compression flanges, the agreement between measured and computed values based on net sections was not as satisfactory as that found for gross sections. The discrepancy would have been even greater if the authors had made reductions in area from both tension and compression flanges in computing net sections, which is the procedure suggested by the measurements made of the position of the neutral axis. In compression flanges, where the holes are filled tightly by rivets or by body-bound bolts, there may be some justification for assuming these flanges to be solid and making deductions from the tension side only. Where there are open holes in both tension and compression flanges, however (as in the authors' series B tests), or where the holes are filled with loose fitting bolts (as in the series C tests), there would seem to be no reason for not making deductions in area from both flanges if the question of net section is really a significant one.

In order to corroborate the authors' general conclusion that the gross-section method is satisfactory for computing plate-girder stresses in most cases, a few experimental data from a test on an aluminum-alloy plate girder have been included in this discussion. Fig. 9 shows the measured stress distribution in the extreme fibers of the web of a girder section subjected to pure bending. The stresses shown correspond to strains obtained by means of tensometers on ½-in. gage lengths midway between, and directly over, the flange rivets. The interesting fact about these data is that the stresses between rivets are higher than on the net section through the center of the rivets. Although this behavior is consistent with the elastic theory regarding the effect of stress concentrations produced by holes near the edge of a wide plate, it is contrary to the result expected when the net-section method is used to determine stresses in design. The stresses measured on the tension side of the web were consistently higher than those found on the compression side (as was also found in the authors' tests), but the maximum values in both cases are reasonably consistent with the stresses computed on the basis of the gross section. The use of the net section for the tension flange would increase the computed stresses in that flange by about 11%, but would give a greater difference between measured and computed stresses than that indicated in Fig. 9.

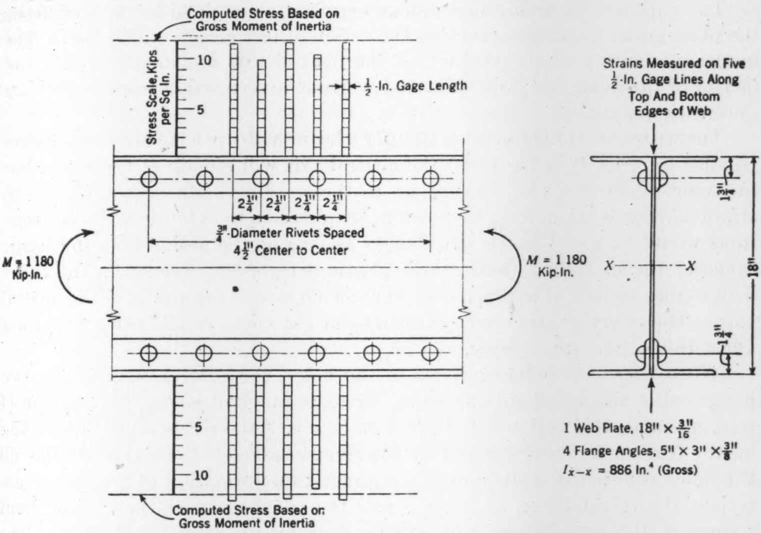

Computed Stress Based on Gross Moment of Inertia
10
5
Stress Scale, Kips per Sq In.
1/2-In. Gage Length
2 1/4"
2 1/4"
2 1/4"
2 1/4"
3/4"-Diameter Rivets Spaced 4 1/2" Center to Center
M = 1 180 Kip-In.
M = 1 180 Kip-In.
5
10
Computed Stress Based on Gross Moment of Inertia
Strains Measured on Five 1/2-In. Gage Lines Along Top And Bottom Edges of Web
1 3/4"
X
X
18"
1 3/4"
1 Web Plate, 18" × 3/16"
4 Flange Angles, 5" × 3" × 3/8"
I_{X-X} = 886 In.^4 (Gross)

FIG. 9.—MEASURED STRESS DISTRIBUTION IN WEB OF ALUMINUM ALLOY PLATE-GIRDER SECTION SUBJECTED TO PURE BENDING

The authors propose a formula for computing effective moments of inertia of plate girders which involves not only the rivet pitch and hole diameter, but an experimentally determined constant which they have evaluated within certain limits. Such a refinement in the computation of stresses and deflections scarcely seems justified from the data given in the paper. One objection that might be raised to such a procedure is that no consideration is given to the possible lack of integral action between the elements of a plate girder. For some proportions it appears that this factor may be of even more significance than the effect of rivet holes. The various parts of a plate girder function, of course, through their connections. Load can be transmitted from the web to the flange angles and cover plates only through the action of the rivets, which involves deformations and slip. It would have been of interest in this connection if the authors had included some of the strains measured in the web and flanges of the test girders. The two types of data did not indicate the same position of the neutral axis, as shown in Table 1. It is conceivable that they might also have thrown some light on the question of integral action.

J. M. GARRELTS,[30] ASSOC. M. AM. SOC. C. E., AND I. E. MADSEN,[31] JUN. AM. SOC. C. E. (by letter).—There is some difference of opinion as to the behavior of the elements of a plate-girder beam under load. With the adoption of increased unit design stresses, it becomes important for engineers to examine some of the fundamental assumptions that have been used for designing such beams.

---

[30] Associate Prof., Civ. Eng., Columbia Univ., New York, N. Y.

[31] Asst. Research Engr., Fritz Eng. Laboratory, Lehigh Univ., Bethlehem, Pa.

The authors' research represents an excellent attempt to learn more about the plate-girder beam by observing the action of test girders under load. The questions relating to the position of the neutral axis and to the moment of inertia of the beam are important and warrant investigation because they are uncertain quantities.

The writers are of the opinion that for a beam with open holes in both flanges (similar to series B in the tests) the neutral axis will remain at the center line for some range of loads, perhaps up to the working load. Since the stress-strain relation is taken to be the same in compression as in tension, the deformations would be equal in the two flanges as long as the action is in the elastic range of the material. Thus, until plastic deformation occurs on the entire flange cross section at an open hole, there is no reason for a shift of the neutral axis. The effect of stress concentrations at the edges of the holes would not affect this action appreciably.

When there are bolts or rivets in the holes, the deformations in the two flanges may also be essentially equal for some range of loads. In a region of pure moment, this action will depend on how well the rivets fill the holes, and on the friction forces developed by the rivet heads. If the rivets or bolts fill the holes sufficiently well to offer resistance to deformation of the flange materials, the transmission of forces across the net section will be of a different nature in the two flanges and will produce an unsymmetrical effect. The friction forces acting on the rivet heads will generally develop some tensile or compressive stresses in the rivet heads, thus providing some splicing action at that section.

The writers are in doubt as to what is meant by the statement (see heading "Conclusions"): "It is believed that the data from these tests point to the acceptability of the gross moment of inertia for design purposes." Do the authors refer to design computations for deflection, or for stress? If the latter is meant, the writers do not agree. A consideration of statics will show that the average stress on a cross section through a rivet hole must be larger than that on a gross cross section. In addition to the expected variation of stress due to stress concentration adjacent to the holes, strains measured on 0.5-in. gage lengths by the writers show a higher average unit strain at a cross section through a rivet hole than at a cross section taken between rivet holes. The 10-in. Whittemore strain gage, used in the authors' tests, measures the over-all average strain for all cross sections included in the 10-in. length, but does not give the average strain at the minimum cross section.

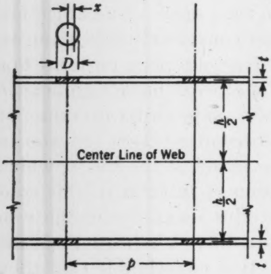


FIG. 10

It seems inconsistent to use the gross moment of inertia for proportioning the parts of a girder according to allowable stresses and an "effective" moment of inertia for computing deflections. The writers are of the opinion that an "effective" moment of inertia is an unnecessary refinement, because it represents the action that occurs only for the case of open holes, or for that in which the bolts or rivets do not fill the holes and therefore offer no resistance to deformation of the flange materials. If this procedure of using the weighted average of the moments of inertia for gross sections and sections through the holes is to be used, however, an average moment of inertia per unit of length of the girder can be determined by the following formula (see Fig. 10):

$$I_a = I_G - \frac{1}{p}\int_{-D/2}^{D/2}\left(2\sqrt{\frac{D^2}{4} - x^2}\right) t\, N \left(\frac{h}{2}\right)^2 dx$$

$$= I_G - (N\, t\, D)\left(\frac{h}{2}\right)^2 \frac{\pi}{4} \times \frac{D}{p} = I_G - I_H \times \frac{\pi}{4} \times \frac{D}{p} \ldots\ldots\ldots (9)$$

in which I_a = average moment of inertia per unit of length; N = number of rivet holes at cross section; D = diameter of rivet holes; p = rivet pitch; and I_H = the moment of inertia of the area of the holes, taken at the diameter of the holes.

Assuming

$$m = \frac{I_a}{I_G} = 1 - \frac{I_H}{I_G} \times \frac{D}{p} \times \frac{\pi}{4} \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

values of m are shown in Table 9.

TABLE 9.—COMPARISON OF VALUES OF $m = \frac{I_a}{I_G}$

Section	Value of m by experiment	Computed value of m	Percentage deviation	Value of m by experiment	Computed value of m	Percentage deviation
C1	0.970	0.971	+0.1	0.928	0.942	+1.5
C2	0.970	0.973	+0.3	0.936	0.945	+1.0
C3	0.962	0.975	+1.3	0.916	0.951	+3.8
C4	....	....	....	0.930	0.951	+2.3
D3	0.980	0.975	−0.5	....	....	....

The computed values of m check fairly well with the experimental values. However, the experimental values are not a true representation of $\frac{I_E}{I_G}$. This is readily seen when the method for determining m is considered. The value of m was computed as the ratio of the loads required to produce equal deflections for a given girder section and the corresponding section of series A. Since there were rivet holes in the vertical legs of the flange angles of the series A sections, the experimental values for m represent a comparison of the effective moment of inertia (I) for series B, C, and D with the effective I of series A.

The value of the factor k in the authors' equations is $\frac{\pi}{4}$, the same as the coefficient in Equations (9) and (10) for I_a. This quantity is a shape factor and depends only on the shape of the holes.

The exact behavior of the elements in a plate girder is unpredictable because of the uncertainty of the action of the rivets; but tests such as those that the authors report will aid in a correlation of the analysis with the possible structural action for which the girder should be designed.

FRANCIS P. WITMER,[32] M. AM. SOC. C. E. (by letter).—The interesting experiment reported in this paper has verified, satisfactorily, the general understanding among structural engineers that deflections of a plate girder are in practical accord with the gross, rather than with the net, moment of inertia. A similar relation, however, cannot be accepted so readily as regards maximum effective unit stress in the tensile flange. In the investigation of old plate girder structures whose sufficiency for modern loading was in question, the writer has found it convenient to use the method described below for the determination of these tensile unit stresses. Assume the girder to deflect in accordance with its gross moment of inertia. Since the section is symmetrical, this places the neutral axis at the center of the web plate, and also presupposes that the rivets hold the parts together with sufficient force to prevent slip and, therefore, to develop stresses which are in accord with bending theory.

If c is the distance from the neutral axis to the extreme fiber, the maximum unit stress on the gross section will be $s = \frac{M\,c}{I}$. If the distance from the neutral axis to any flange rivet line (either through the web or through cover plates) is y, the unit stress on the gross section at this line of rivets will be $s' = s\,\frac{y}{c}$.

This unit stress will occur in the part of the flange between rivets. A rivet hole will cut from the flange section an area equal to $2\,D\,t$, in which D is the diameter of the hole and t is the thickness of flange angle, or, for cover-plate rivets, the combined thickness of angle and cover plates for two lines of rivets.

The rivet hole may now be regarded as cutting from the section an area whose total stress equals $s'\,(2\,D\,t)$ and causing this stress to be detoured around the rivet and ultimately divided up over the entire net section of the flange. If A represents this net section, a unit stress of $\frac{s'\,(2\,D\,t)}{A}$ will be added to each unit stress in the gross flange section between rivets. Adding this increment to the maximum extreme fiber stress, s, in the gross section, will give the maximum tensile unit stress in the section.

This method of procedure is inconsistent in that it assumes the gripping action of the rivets to be sufficient to cause a distribution of fiber stresses between rivets which is proportional to the distance from the neutral axis; but it does not consider the frictional force under the rivet head as sufficient to prevent the increment of stress, due to the flange material cut out by the rivet hole, from being carried around the rivet. The results, however, are believed to be more nearly correct than those by either the method of net moment of inertia or that of net flange area. For a girder without cover plates, the result will usually be nearly the same as by the net flange area method; but if there are cover plates, it will generally be somewhat less than this value.

---

[32] Director, Civ. Eng., Univ. of Pennsylvania, Philadelphia, Pa.

A. W. FISCHER,[33] ESQ. (by letter).—The experiments and results reported in this paper were conducted with great care and refinement.

According to all the tests, the neutral axis remained fairly close in position to the center of gravity of the gross section; and this seems reasonable because the rivet holes in the tension flange do not remove much metal as compared to the entire volume of the tension flange. With a 5-in. pitch there is slightly less shift than at a 2.5-in. pitch, and that is as it should be; and, for average conditions, the writer will agree that, for the deflection of a riveted plate girder, it will be satisfactory to assume the neutral axis at the center of gravity of the gross section, and to use the gross moment of inertia. The computed deflection will then be a trifle less than it really is by experiment; but the variation will be small, and may never be more than 3% in error.

A riveted plate girder acts similarly to a truss. Every engineer will deduct the rivet holes in the tension flange of a steel truss, and for that reason it seems that rivet holes should be deducted from the tension flange of a riveted plate girder to ascertain the true stresses.

The authors failed to secure a true measure of the stress in the net section, for the tension flange, because of their testing procedure. The true stress at the section where the rivets occur cannot be computed from the readings of a 10-in. gage length. Only an average stress can be computed; and this average will be less than the actual at the rivet section. Furthermore, it is impossible to determine the true tensile stress at the section of a line of rivets if round holes are used. The only solution is to use slotted holes at the section where the stretch is to be measured, the slotted part being, say, 1 in. long, and the width being the diameter of the rivet hole. If the slotted part is actually 1 in. long, if a gage length of 1 in. is used, and if the average of several positions across the net area of the tension flange is used, then from such a test the average of the calculated results will represent the true stress at that section as closely as an experiment can be made to determine it. Tests should first be performed using slotted holes at the net section tested, and then the load should be removed. After the load is removed rivets shall be placed in the slotted holes and tested again to ascertain if the grip of rivet heads change the stresses.

The plate girder tested by the authors was rather shallow; a representative girder of reasonable depth should be tested, and several cover plates should be used so that the effect of rivet-head grip on the outer cover plate and the outstanding leg of the angles at the net section will be reduced to an average minimum.

One item that is not clear to the writer is just how the 10-in. gage length was converted to the true stretch of the bottom tension flange. A slight curvature in the beam seems to indicate that the actual gage length measured was the chord length and not the true arc length. It seems that the shorter the gage length the more definitely the true stretch can be ascertained.

The I_E-value as given in Equation (5) is too large. Before it is adopted, additional tests should be performed on representative girders, reasonably deep, and including several cover plates. The writer will agree that the pitch of the rivets in the tension flange should be used in the general equation so as to take care of the slight shift of the neutral axis, and the writer would not use a greater value for I_E than the following for the design of a riveted plate girder:

---

[33] Asst. Structural Engr., Construction Service, Veterans Administration, Washington, D. C.

$$I_E = \frac{p\, I_N}{p + \left(\frac{I_G}{I_N}\right) - 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

in which I_N is equal to I_G minus the moment of inertia of the rivet holes in the tension flange only, about the gravity axis of the gross section. The section modulus then equals I_E divided by the distance from the gravity axis of the gross section to the extreme tension fiber.

J. R. SHANK,[34] M. AM. SOC. C. E. (by letter).—Few engineers would think of using the net-section moment of inertia of a plate girder for computing the deflection for the reason that it is representative of so little of the length. The gross-section moment of inertia represents much more of it. A moment of inertia proportioned between these two is more likely than either of them to be the proper one to use. The deflection data of this paper afford excellent material for making a study of the proper basis for this proportioning. If the addition of holes in the tension flange causes an increase in the deflection, it is apparent that the angular movement due to the bending of that part at the holes is no longer the same as if no holes were present and the use of the gross moment of inertia is therefore only an approximation. The deflection may be computed in detail for gross section and net section progressively, as they come, or fairly accurately from gross and net sections in proportion to the parts of the lengths taken by the two. If the deflections given in this paper are equal to the deflections so computed, the conclusion must be drawn that the angular movement as computed from the net moment of inertia obtains at the regions containing holes and the fiber deformations and stresses are different from those at no holes.

Table 10 is an attempt to show the effect of the holes on the reduction of the load-carrying characteristics as illustrated by the moments of inertia. For a given deflection the moments of inertia are directly proportional to the loads carried. Columns (1) and (5) of Table 10 are taken directly from Columns (3) and (1) of Table 3. The percentage reductions are all computed from the A-value for each group. Column (3) is not found in the original paper. For PT-1 specimens having $\frac{11}{16}$-in. holes, spaced 5 in. apart, 13.7% of the entire length is at holes and 86.3% at no holes, or at gross section. Accordingly it is assumed that 86.3% of the girder has a moment of inertia of, say, 178.5 for specimens without cover plates, and 13.7% has a moment of inertia of 164.3. For convenience, the holes are considered to be $\frac{11}{16}$ in. square; thus:

$$178.5 \times 0.863 = 154.0$$
$$164.3 \times \frac{0.137}{1.000} + \frac{22.5}{176.5}$$

The effective moment of inertia for computing deflection or the proportional moment of inertia, as designated herein, is then 176.5. This procedure is substantially the same as that which would be used for computing the deflection of a girder having two different moments of inertia when the number of alternations of the two are allowed to increase toward infinity.

---

[34] Prof., Civ. Eng., and Asst. Director, Eng. Experiment Station, Ohio State Univ., Columbus, Ohio.

TABLE 10.—MOMENTS OF INERTIA AND PERCENTAGE REDUCTION DUE TO HOLES OPEN (B), WITH BOLTS (C), OR WITH RIVETS (D)

Test section	(a) SPECIMEN PT-1						(b) SPECIMEN PT-2					
	Net Section		Proportional		Observed		Net Section		Proportional		Observed	
	Moment of inertia, *I* (1)	Percentage of section A (2)	Moment of inertia, *I* (3)	Percentage of section A (4)	Moment of inertia, *I* (5)	Percentage of section A (6)	Moment of inertia, *I* (1)	Percentage of section A (2)	Moment of inertia, *I* (3)	Percentage of section A (4)	Moment of inertia, *I* (5)	Percentage of section A (6)
A-1	178.5	....	178.5	....	189.0	....	178.5	....	178.5	....	189.0	....
B-1	164.3	8.0	176.5	1.1	182.0	3.7	164.3	8.0	174.6	2.2	177.0	6.4
C-1	164.3	8.0	176.5	1.1	187.0	1.1	164.3	8.0	174.6	2.2	179.0	5.3
A-2	300.9	....	300.9	....	315.0	....	300.9	....	300.9	....	315.0	....
B-2	270.7	10.0	297	1.3	298.5	5.2	270.7	10.0	293	2.6	282.0	10.5
C-2	270.7	10.0	297	1.3	306.5	2.7	270.7	10.0	293	2.6	295.0	6.4
A-3	476.9	....	476.9	....	494.0	....	480.1	....	480.1	....	500.0	....
B-3	430.0	9.8	471	1.2	460.5	6.8	433.2	9.8	467	2.7	442.0	11.6
C-3	430.0	9.8	471	1.2	472.0	4.5	433.2	9.8	467	2.7	453.0	9.4
D-3	430.0	9.8	471	1.2	480.5	2.7	433.2	9.8	467	2.7	460.0	8.0

The percentage reductions shown in Column (2), Table 10, are those which would be expected if the holes ran in the form of slots for the full length of the girder. If the holes had no effect on the deflection, these percentages would all be zero. There are the two extremes. If the stresses at the holes are what would be expected from the net-section calculations, the reductions would be as shown in Column (4), Table 10. Column (6) shows the percentage reductions for the observed moments of inertia. The moments of inertia shown in Column (5) are larger than those in Column (1), and a reference to the paper shows them to be generally somewhat smaller than the gross section values of Column (2), Table 3. These differences are immaterial inasmuch as the comparisons are by percentages.

The material in the number 1 group, A-1, B-1, and C-1, is the most valuable since the A-values were not taken from girders where clamps were relied upon to develop the horizontal shear as was the case for the others. In all cases except one the effect of the holes was greater than would be expected from ordinary net-section calculations, taking into account the proportionate length occupied by the reduced section. The stiffening effects due to bolts and rivets are shown clearly. The discrepancies between the values of Columns (6) and (4), Table 10, are greater for the closely spaced holes. It appears that the holes have an effect over a length greater than their diameters and that the substitution of square holes for round holes is not nearly enough to cover this condition. In the cases of the close spacings these effects apparently met or overlapped.

The conclusion seems to be clear that the net-section method for the calculation of tensile fiber stresses indicates the actual situation much more accurately than the gross-section moment of inertia; and if designing plate girders by the gross-section moment of inertia is to be continued, it should be recognized clearly that considerably greater stresses occur at the rivet holes in the tension flange than the calculations indicate and that this fact must be taken into account when specifying allowable unit stresses.

HAROLD D. HUSSEY,[35] M. AM. SOC. C. E. (by letter).—The need of more tests of plate girders is emphasized in this paper. The tests reported were planned with care and were skilfully executed, and the writer believes that the authors' conclusions will be verified by further experimental work.

Under "Flange Stress" the authors state that the stress in the compression flange is greatest for open holes, and that it gradually approaches the computed value as the method of holding the flange together is changed from clamps, to bolts, and to rivets. The authors add that a corresponding decrease is not shown for tension flange, indicating that bolts and rivets tend to relieve the stress in the compression flange. The observed results in Column (6), Table 3, indicate that bolts and rivets tend to relieve the stress in the tension as well as the compression flange. This seems to offer evidence that the gripping action of rivets produces an appreciable reduction in the theoretical stress concentration at the sides of open holes in the flanges. Such stress concentrations may be as much as three times the gross unit stress, which indicates that they will reach the yield point of the material before the working stress is developed. It is significant that when the two girders were tested to 30 kips per sq in. there was no indication of permanent set. This is contrary to what one would expect. The fact that no set was produced is evidence of the relieving action of rivets.

The most important conclusion reached by the authors is that plate girders may be designed acceptably by the use of the gross moment of inertia. The most common consideration of this question has been on the basis of gross sections versus net sections of a tension member, when it should have been directed to the action of the plate girder.

A plate girder has one flange in tension and one flange in compression. The strength of the compression flange is determined by its resistance to buckling. It has long been known that, under static loads, the compression flange of a symmetrical girder will fail long before the tension flange has reached its ultimate strength. It has also long been known that, under static loads, it is practically impossible to make a girder fail in the tension flange, while the compression flange is still intact. This was well demonstrated in tests of plate girders which are summarized in Table 11.

These tests were made at the Ambridge, Pa., plant of the American Bridge Company in 1910. The girders were 30½ in. deep, back to back of flange angles; 28 ft long, center to center of bearings; and were made in the following general classes: Class A girders were symmetrical, without cover plates; class B girders were unsymmetrical, without cover plates, the compression flanges being heavier than the tension flanges; class C girders were symmetrical, with cover plates; class D girders were unsymmetrical, with cover plates, the compression flanges being heavier than the tension flanges; and the two class E girders were extremely unbalanced, with two angles only in the tension flanges and cover-plated compression flanges which were more than twice as heavy as the tension flanges.

---

[35] Designing Engr., Am. Bridge Co., New York, N. Y.

Load was applied to the girders at the third points of the span, through bearing blocks on the compression flanges. The two ends of the girders were held against lateral movement. The unsupported length of the compression flanges, therefore, was equal to the length of the girders except for such support as was supplied at the load points.

All of these girders failed in the same manner—that is, by buckling of the compression flanges at theoretical unit stresses which averaged 9% above the yield point. This was accompanied, in some cases, by buckling of the web plate. In only one case (Girder G11) was there failure of the tension flange. It is evident that the tension and compression unit stresses should be alike for symmetrical girders, when they are based on gross moment of inertia. The tension stress, on the basis of net moment of inertia, was somewhat higher (10% for the girders without cover plates and 20% for those with cover plates). The maximum tension stresses were still less than the ultimate—in the symmetrical girders these stresses varied from 73% to 81% of the ultimate.

Making the girders unsymmetrical resulted in an increase in the tension unit stress at failure without any appreciable change in the compression unit stress (in comparison with the yield point). These girders continued to fail by buckling of the compression flange or failure of the web plate until the dissymmetry reached the extreme values shown in girders G11 and G13, where the compression flanges were more than twice as heavy as the tension flanges. Girder G11 showed a maximum net tension stress of 105% of the ultimate tensile strength. As stated, this was the only girder in the series that was reported to have shown signs of failure in the tension flange. The report of this failure states, "Top (compression) flange and cover plates buckled and deflected downward. Complete failure of top and bottom flanges and web between loads."[36] Even in this girder the compression unit stress at failure was more than the yield point of the material.

The writer believes that it would be acceptable to design any of the girders listed in Table 11 by the gross moment of inertia method. The use of net moment of inertia seems to be an unnecessary refinement.

The plate girder is one of the most common types of structural members in use today, and its use as the principal member of large structures is increasing. The structural action of the plate girder is very complex and warrants much more study than has been given to it.

Considerable research has been conducted on many types of structural members, which has added to modern knowledge of their behavior. The column has proved a fruitful field of research since the Quebec Bridge disaster in 1907. Many column tests have been made in connection with the construction of some of the most important bridges. A very important series of tests of riveted tension members was conducted in connection with the building of the San Francisco-Oakland Bay Bridge. Whenever a structure is built which involves a quantity of eyebars, it is customary to specify full-size tests of several bars. The question is: Where can one find a report of tests of large plate girders made in connection with an important project?

---

[36] Unpublished report of tests at the Ambridge (Pa.) plant of the American Bridge Company, 1910.

TABLE 11.—TESTS OF PLATE GIRDERS, 30½ IN. DEEP AND 28 FT LONG

Girder mark	Thickness* of web plate, in inches	COMPRESSION FLANGE			TENSION FLANGE			Load at failure, in kips	UNIT STRESSES, IN KIPS PER SQUARE INCH				
		Thickness, in inches		Area, in square inches	Thickness, in inches		Area, in square inches		Theoretical Maximum			Physical Properties	
		Cover plates†	Angles‖		Cover plates†	Angles‖			Compression in flanges (I_g)¶	Tension in Flange		Compression flange (s_y)††	Tension flange (s_t)‡‡
										(I_g)¶	(I_n)**		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
CLASS A													
G5A	3/8	...	5/8	11.72	...	5/8	11.72	293.0	44.6	44.6	49.3	42.0	67.4
G5B	3/8	...	5/8	11.72	...	5/8	11.72	318.0	48.4	48.4	53.5	42.0	67.4
CLASS B													
G7	5/16	...	5/8	11.72	...	1/2	9.50	288.0	46.0	52.5	57.8	42.1	68.0
G8	3/8	...	5/8	11.72	...	1/2	9.50	290.5	45.2	51.3	56.7	42.1	68.0
G12	3/8	...	3/4	13.88	...	1/2	9.50	308.0	42.4	53.5	59.0	38.8	61.4
CLASS C													
G4A	3/8	1/2	1/2	16.50	1/2	1/2	16.50	396.8	43.2	43.2	52.1	37.3	64.6
G4B	3/8	1/2	1/2	16.50	1/2	1/2	16.50	393.0	42.8	42.8	51.6	37.3	64.6
G3A	1/2	3/4	5/8	22.22	3/4	5/8	22.22	488.0	39.7	39.7	47.7	34.3	62.0
G3B	1/2	3/4	5/8	22.22	3/4	5/8	22.22	463.0	37.7	37.7	45.3	34.3	62.0
CLASS D													
G10	5/16	1/2	7/16	15.36	3/8	7/16	13.61	344.3	41.1	44.8	53.9	35.1	66.2
G9	5/16	1/2	1/2	16.50	3/8	1/2	14.75	378.0	42.5	46.0	55.5	38.1	66.8
G6A	3/8	3/4	5/8	22.22	5/8	5/8	20.47	438.0	37.0	39.2	47.2	39.7	63.4
G6B	3/8	3/4	5/8	22.22	5/8	5/8	20.47	443.0	37.3	39.6	47.7	39.7	63.4
G14	1/2	1‡	3/4	27.88	1/2	1/2	16.50	558.0	39.6	55.5	66.8	36.9	68.3
CLASS E													
G11	3/8	3/8	1/2	14.75	...	3/8	7.22	288.0	38.4	58.5	67.0	33.4	63.9
G13	1/2	1⅛§	3/4	29.63	...	3/4	13.88	543.0	38.4	61.8	70.4	36.7	68.6

* Web plates all 30 in. deep. † Cover plates all 14 in. wide; one plate except as noted. ‡ Two 14-in. by 1/2-in. cover plates. § Two 14-in. cover plates, 5/8 in. and 1/2 in. thick. ‖ A pair of 6-in. by 4-in. angles in each case; outstanding leg, 6 in. ¶ I_g denotes "moment of inertia based on the gross area of both flanges." ** I_n denotes "moment of inertia based on the net area of both flanges." †† s_y denotes "yield-point stress in the compression flange." ‡‡ s_t denotes "tensile strength of the tension flange."

The plate girder has not been subjected to any concerted program of research—possibly because no major disaster has occurred from the failure of such a member. The writer believes, however, that there is need for such tests and that as a result engineers would be able to design more intelligently and with greater economy.

CHARLES U. KRING,[37] JUN. AM. SOC. C. E. (by letter).—Apparently, the authors presuppose that the theoretically correct method of designing a plate girder is by use of the moment of inertia, and that the crucial question is: "What moment of inertia should be used?" No mention is made of the widely used "flange-area method," probably because they regard it as only an approximate makeshift. For finding the actual stress in the extreme fibers, that is undoubtedly true, but, for that purpose, so is the moment-of-inertia method only a makeshift, since, as the authors point out, the actual stress in some points of the flanges will have reached the yield point when the maximum stress as found by the moment-of-inertia method is only 18,000 lb per sq in.

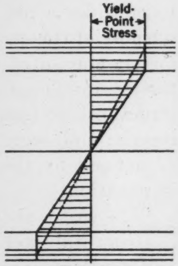


FIG. 11.

If it is agreed, then, that the ultimate aim of a method of design is not to produce a girder that will have a certain stress in the extreme fibers under the design load, but to produce a girder that will have a certain factor of safety under that load, then the writer believes it can be shown that, theoretically, the moment-of-inertia method is not as correct as the flange-area method.

A girder that is properly designed and properly braced against lateral buckling will fail by excessive deflection caused by plastic yielding of the flanges. The deflections will not be excessive, however, until the load is such that plastic yielding has occurred throughout the entire cross-sectional area of the flange.[38] Under that load, the ultimate load, the stress distribution will be as shown in Fig. 11. Under any less load, the stress in some fibers of the cross section of the flange will still be within the elastic range, and the deflections of the girder will still be of the magnitude of elastic deformations. Any additional load would cause the web to wrinkle and the girder to fail due to excessive deflection.

It is evident, then, that if a girder is designed by the flange-area method in which the allowable stress is assumed to be distributed uniformly over the entire flange area (net area in tension flange) the true factor of safety will be the yield-point stress of the material divided by the allowable stress. If 30,000 lb per sq in. were the yield-point stress and 18,000 lb per sq in. the allowable stress, the factor of safety would be 1.67. "Factor of safety" is defined herein as the ratio of the ultimate load to the design load. What the actual stress is at any point in the flange is not known, any more than it is when using the moment-of-inertia method, and, except where fatigue may be a factor, it is not important.

The flange-area method is slightly in error in regard to the addition of one-sixth the web area (one-eighth for tension flange) to the flange area, since under ultimate load it assumes the distribution of stress in the web to be as shown by the dotted line in Fig. 11. This error, however, is not only on the safe side (as an examination of Fig. 11 will show) but is also so small as to be negligible, except possibly in very shallow, heavy girders. Where the moment-of-inertia method is used, the factor of safety, even with the same allowable stress, will vary, being larger for shallow girders than for deep girders (as shown by the stress-distribution diagrams), and in no case will it be the yield-point stress divided by the allowable stress as is commonly assumed.

---

[37] Fellow, Civ. Eng., Univ. of Illinois, Urbana, Ill.

[38] "Theory of Limit Design," by J. A. Van den Broek, M. Am. Soc. C. E., *Proceedings*, February, 1939, p. 193.

From consideration of the foregoing, it is believed that the moment-of-inertia method is not only less theoretically sound than the flange-area method, but that the latter also satisfies the authors' requirement that "design methods should be as simple as possible."

THOMAS C. SHEDD,[39] M. AM. SOC. C. E. (by letter).—The first two sentences in the authors' introduction are: "It is customary for structural designers to consider a net moment of inertia in the design of plate girders. For the purpose of this paper, 'net moment of inertia' will be defined as the moment of inertia computed by deducting the effect of the material removed by holes in the tension flange." These sentences imply two things: First, that, in general, structural designers proportion plate girders on the basis of the moment of inertia; and, second, that in doing so they deal with the net moment of inertia as defined by the authors. There seems to be little basis in fact for either implication.

The writer always has been greatly interested in specifications for the design of structural steel bridges and buildings, and has studied minutely the changes and developments that have taken place since the first formal specification was issued by Clarke, Reeves and Company (later the Phoenix Bridge Company) on March 1, 1871. As a result the writer thinks it is the exception rather than the rule for structural designers to consider the moment of inertia of a girder, and even more exceptional for them to consider net moment of inertia as defined by the authors.

The fad for requiring the proportioning of plate girders by the "moment of inertia method" is a relatively recent and entirely unnecessary development. Quotations from some well-known specifications will give some clue as to when that development appeared.

The specifications issued by the American Railway Engineering Association in 1910 were widely used, and even more widely patterned after in the writing of other specifications. In that specification may be found:[40]

> "Plate girders shall be proportioned either by the moment of inertia of their net section; or by assuming that the flanges are concentrated at their centers of gravity; * * *."

These specifications represented general practice until they were superseded by the 1920 specifications, issued by the same organization. The 1920 specifications stated:[41]

---

[39] Prof., Structural Eng., Univ. of Illinois, Urbana, Ill.

[40] A.R.E.A. General Specifications for Steel Railway Bridges, Art. 29, 4th Ed., 1910.

[41] A.R.E.A. General Specifications for Steel Railway Bridges, Art. 116, 2nd Ed., May, 1923.

> "Plate girders shall be proportioned either by the moment of inertia of their net section including compression side; or by assuming that the flanges are concentrated at their centers of gravity."

A specification prepared and presented in 1923, by a committee of the Society, included the clause:[42]

> "Plate girders shall be proportioned by assuming that the flanges are concentrated at their centers of gravity, * * *."

A specification prepared by a joint committee of the Society and the American Railway Engineering Association, dated March, 1929, contained the statement:[43]

> "Plate girders shall be proportioned by the moment of inertia of their net sections, including the compression sides; or by assuming that the flanges are concentrated at their centers of gravity, * * *."

The original specification issued by the American Institute of Steel Construction contained the forerunner of present-day clauses in the form:[44]

> "Plate girders with webs fully spliced for tension and compression shall be so proportioned that the unit stress on the net section does not exceed the stresses specified in section five as determined by the moment of inertia of the net section."

It will be noted that this clause does not require that girders be proportioned by the so-called moment-of-inertia method—merely that they shall be so proportioned that fiber stresses computed from the net moment of inertia will not exceed a specified value. That requirement can be met easily and accurately without computing either the net or the gross moment of inertia.

The first widely accepted specification to adopt a definite requirement that girders be proportioned on the basis of the moment of inertia, as far as the writer knows, was the American Railway Engineering Association specification of 1935 which stated:[45]

> "Plate girders and other members subject to bending that produces net tension on one face shall be proportioned by the moment-of-inertia method, using the net section of the compression side as well as of the tension side."

This clause was revised in 1938 to read:[46]

> "Plate girders, I-beams, and other members subject to bending that produces tension on one face, shall be proportioned by the moment-of-inertia method. The neutral axis shall be taken along the center of gravity of the gross section. The tensile stress shall be computed from the moment of inertia of the entire net section and the compressive stress from the moment of inertia of the entire gross section."

---

[42] Final Report on Specifications for Design and Construction of Steel Railway Bridge Superstructure, *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), Art. 323, p. 482.

[43] General Specifications for Steel Railway Bridges, *Proceedings*, Am. Soc. C. E., December, 1929, Art. 427, p. 2658.

[44] A.I.S.C. Standard Specification for Structural Steel for Buildings, June 1, 1923, Art. 7.

[45] *Bulletin No. 374*, A.R.E.A. Specifications for Steel Railway Bridges, February, 1935, Art. 426, p. 21 (Vol. 36, p. 653).

[46] *Manual*, A.R.E.A. Specifications for Steel Railway Bridges, Art. 426, p. 15–15.

The writer can find little support, either in his experience or in an examination of specifications, for the authors' statements quoted at the beginning of this discussion. The definition of net moment of inertia given by the authors presumes a neutral axis resembling the edge of an old-fashioned bread knife. The most casual consideration of the flexed girder would seem to establish the plane of no stress as a continuous, smoothly curving surface which cannot depart more than an entirely negligible distance from the center of gravity of the gross section—it cannot conceivably "jitterbug" between the center of gravity of the gross section between holes and the center of gravity of the net section at holes.

To design plate girders by the moment-of-inertia method, as recommended by the authors, and required by many recent specifications, is an unnecessary waste of time and effort. It is an easy matter to proportion a girder by a simple modification of the commonly used "flange-area method," so that its extreme fiber stress, either on gross section or net section, will not exceed a specified value. One may proceed as follows:[47] Using the flange-area method, calculate the required net area of the tension flange in the ordinary way, multiply this area by the ratio of the estimated over-all depth to the estimated effective depth, proportion the girder, and check the estimated effective depth and the estimated over-all depth. If the estimated depths agree exactly with the actual values the girder as proportioned will have a calculated extreme fiber stress (using the moment-of-inertia method as defined in the specifications previously noted) not greater than the assumed value, and not less by more than 1% or 2% (usually less than 1%), except for very shallow and heavy girders, in which the discrepancy may be as much as 5%. If the differences between estimated depths and actual depths are considerable the designer will revise his calculations as he would in ordinary design. Obviously one may follow the same procedure if he prefers to design on the basis of gross area.

It should be clear that the flange-area method, when modified in the manner described,[47] would give exactly the same result as the moment-of-inertia method if effective depth were defined as the distance between the center of gravity of the compressive flange stresses and the center of gravity of the tensile flange stresses—the web area being translated into an equivalent flange area in substantially the usual manner.

In their conclusions the authors state that: "* * * the data from these tests point to the acceptability of the gross moment of inertia for design purposes." The writer is not sure he understands what is meant by the statement. If it means that one may compare girders he wishes to design with those which have been designed and built, with satisfactory results, on the basis of computations using gross moment of inertia, probably most engineers will agree, although they may not see any advantage in convenience or accuracy in so doing. If the statement is intended to convey the idea that one may apply the unit stress generally used on net section to gross section and secure a girder with the same margin of safety, the writer cannot agree—he finds no evidence in the data reported to support such a thesis.

---

[47] "Structural Design in Steel," by T. C. Shedd, John Wiley & Sons, Inc., 1934, p. 178 (first published in mimeograph form in 1928).

SCOTT B. LILLY,[48] M. AM. SOC. C. E., AND SAMUEL T. CARPENTER,[49] ASSOC. M. AM. SOC. C. E. (by letter).—The interest in this paper, as manifested by the many discussions, indicates that the structural design of plate girders is in a state of flux. The varied points of view that have been presented are of great value to all interested in this problem, and will be followed closely in planning future investigations.

As was to be expected, criticism of the writers' conclusions has been voiced by many discussers. On the other hand, all of the discussers who had experimental data to present have agreed that the gross moment of inertia is acceptable for design purposes. Although this agreement was indicated, several of the discussers based their conclusion on reasons which differ from those stated by the writers.

In general the discussions presented reflect the traditional thinking of the structural engineer in regard to net section as applied to tension members. The writers do not advocate the discarding of net-section rules for tension members, as so many discussers imply, but they wish to emphasize the fact that a plate girder is a structure in which tension and compression members are combined. They limited the original paper to the action of the girder under static loads within the working range, and were interested in over-all stress conditions rather than in the measurement of high localized stresses at, and around, the holes. The tests have shown that these strains are not greatly affected by these highly stressed areas, and that for all practical purposes the strains might be computed by the gross moment of inertia. Therefore, it is believed that the gross moment of inertia will represent working conditions of the girder more accurately than the net moment of inertia, since no matter what method of computation is used the real value of the high stress around the hole will be little affected. As several discussers have mentioned, these stress concentrations will be of importance if fatigue is a problem.

The tests also indicated that the over-all strains in the two flanges of the girder were practically the same, showing clearly that if the net-section advocates wish to be entirely right and consistent, they should deduct the holes from the compression flange as well as the tension flange. This is an elaborate statement of the fact that the stress concentrations exist in both flanges and that the rivets in the compression flange are no more effective in the so-called filling of the hole than those of the tension flange.

Although the writers restricted their paper to the action of a plate girder under static loads within the working range, the discussions indicate that the problem cannot be so limited. The discussions focus the attention on the compression flange rather than on the traditional tension flange. The writers often discussed failure by buckling while making the tests, but did not touch upon the subject in their paper. They regret this omission, but it has been ably discussed by several of the contributors, notably Mr. Hussey. Although the writers believe that their tests at working loads were sufficient to justify the use of the gross moment of inertia, a consideration of the stability of the compression flange makes more acceptable the use of the gross moment of inertia in design. This is true because in a failure by buckling a considerable portion of the girder is involved rather than a single cross section, and the gross moment of inertia tells more about the behavior of the entire structure.

---

[48] Prof., Civ. Eng., and Chairman, Div. of Eng., Swarthmore Coll., Swarthmore, Pa.

[49] Asst. Prof., Civ. Eng., Swarthmore Coll., Swarthmore, Pa.

The majority of the discussers seem to indicate that the use of the gross moment of inertia for determining deflections is common practice and such use is not questioned. The writers believe that this phase of the problem requires no more discussion, except to point out that in unusual cases Equation (5) is available to give a more accurate value of the effective moment of inertia.

The tests reported show that for all practical purposes the neutral axis may be considered at the centroid of the gross section, and that there is no indication of an undulating elastic line. It is true that a discontinuity exists in the stresses around the flange holes, and it is also apparent that conditions are the same in both flanges, giving a symmetrical cross section in this respect. By the principle of St. Venant, the boundary discontinuities will have no bearing on stresses normally expected in the interior of the web plate, and hence cannot affect the position of the neutral axis.

Neither the net moment of inertia, the gross moment of inertia, nor any other of the proposed methods of design gives the actual stresses in and around the holes of a plate girder. When the stress concentrations around such holes are given careful study, it becomes evident that no simple method can be used to evaluate them. They usually reach the yield point stress at the edge of the hole, and they cannot be modified by the methods of computation. The thought that a unit stress computed from a net moment of inertia gives a clearer idea of the safety of the plate girder is fallacious because plate girders do not fail by the rupture, or the tearing apart, of the tension flange.

If a high load is placed upon the girder, all parts of the tension flange will reach the yield point stress. Certainly then, if ever, the net section should govern because all stress concentrations have been wiped out. The compression and tension flanges will reach the yield point stress conditions at the same time. The tests made by the American Bridge Company, and reported by Mr. Hussey, show what happens when this condition is reached. The compression flange buckles and the plate girder fails—a functional failure—and the stresses and the stress concentrations in the tension flange are not vital factors. That is, these stresses did not measure the safety of the girder, so the traditional emphasis of the designer upon stresses computed in the tension flange by net-section methods has had no real bearing upon the safety of the girder. All of the tests reported by the discussers confirm these conclusions of the writers that safety does not lie in the computed stresses in the tension flange.

Mr. Osgood calls attention to an article by Friedrich Hartmann,[5] which should be read by every one interested in this subject.[5a] Professor Hartmann states:

> "The taking into account of the weakening effect due to rivets in those parts of a girder that are under tension was a matter of course so long as the static failure of these portions was considered as being the controlling factor in regard to safety. However, a girder becomes unfit for service much sooner, in general, when non-permissible deformations occur."

After a theoretical discussion calculated to give the basis for the foregoing statement, Mr. Hartmann describes the test of a pair of newly fabricated riveted girders made of open-hearth steel. These girders had a span of 10 m (32.81 ft), a depth-to-span ratio of approximately 0.1, and flanges of angles and cover plates. They were tested to destruction and failed by the buckling of the compression flange. The stress in the compression flange at failure slightly exceeded the compressive yield point of the material. Verification of this latter statement is noted in the tests reported by Mr. Hussey.

As stated, Mr. Hussey presented the data of a series of tests which bear directly on this problem. The plate girders were loaded to failure, and in every case except one the tension flange showed no distress and the compression flange failed by buckling. The one failure in the tension flange which Mr. Hussey reported occurred in a girder where the compression flange had twice the area of the tension flange. In general these tests supplied the data at failure requested by several of the discussers and bear out the statement of Mr. Hartmann that the safety of the girder is not controlled by the tension flange.

Professors Morris and Shank suggest methods of averaging the net sections with the gross sections to obtain flange stresses and a weighted moment of inertia, respectively. The writers believe that an average was obtained experimentally by using a 10-in. strain gage, since such measurements over the rivets (or holes) actually integrate the existing conditions.

Mr. Lane has expressed, very clearly, a thought that was uppermost in the minds of the writers, while they were planning the tests, when he stated: "*** riveted plate girder flanges will always consist of many small areas (adjacent to rivets) of high stress restrained and stiffened by surrounding areas of low stress." Highly stressed areas of this type have never given trouble.

Mr. Black computed the stresses in the riveted section of a rigid frame whose make-up was the same as that of a plate girder, and found that the measured stresses agreed with those obtained by using the gross moment of inertia. Mr. Moore reported tests made on an aluminum plate girder, in which he discovered that the stresses between rivets were higher than the stresses in the net section. He found that stresses computed from the gross moment of inertia correlated very well with those obtained by the use of a tensometer having a 0.5-in. gage length.

Mr. Jones has stated clearly the reasons for the adoption of the gross moment of inertia by the Committee on Specifications of the American Institute of Steel Construction. However, the writers want it clearly understood that the tests were not planned to confirm any action of any committee. They were designed to give essential data, and if the results seem to favor the gross moment of inertia it is simply because the confirmation is there.

The report of Professor Shedd on his careful research into the specifications of plate girders for the past fifty years is particularly valuable. Whether or not readers agree with Professor Shedd that girders should be proportioned by the "flange-area method," all will admit that as long as stresses govern the composition of flange areas, and the common theory of flexure is accepted, the moment of inertia, either gross or net, must be the final resort of the designer. The writers agree with Professor Shedd that the neutral axis does not resemble the old-fashioned bread knife.

The interest in the problems of the plate girder, which has been demonstrated by the large number of discussions, will insure many more investigations. The writers had only the results of two "pilot" test girders to guide them in writing their paper, but they are of the opinion that those tests, and the stimulating discussions, will go far in guiding future investigations, and hence will serve their major purpose. The writers are quite convinced that future investigations will be focused upon the compression flange rather than upon the tension flange. These future tests must include all types of cross sections of girders, varying degrees of restraint of the compression flange, and different depth-to-span ratios.

The writers close this discussion with the hope that each engineer will avail himself of any opportunity to test plate girders under working conditions and that he will also see to it that the results of his tests are given to the profession.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2090

FIELD TESTS OF A SHALE FOUNDATION

By August E. Niederhoff,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Harry H. Hatch, Jacob Feld, Jack R. Rountree, and August E. Niederhoff.

Synopsis

Field tests described in this paper were conducted on the shale foundation of Possum Kingdom Dam on the Brazos River near Mineral Wells, Tex. They included time-settlement tests on various sizes of square plates, a cyclic loading test, and a series of tests to determine the effect of bond and coefficient of friction of concrete monoliths on a prepared shale foundation. The tabulated and graphical field data presented are analyzed in an attempt to verify some of the current theories advanced by students of soil mechanics. Comparison is also made between somewhat similar tests performed in the laboratory on smaller specimens. Formulas and permissible load values are suggested for this particular foundation material.

Introduction

The necessity of placing dams on difficult foundations is becoming increasingly frequent as the better sites are put to use and as more and more control and conservation of water is demanded by the public. The engineer is thus faced with the problem of using a "second best" or a "third" or "fourth best" site, as the case may be.

Possum Kingdom Dam, near Mineral Wells, is one of several on the Brazos River designed to control floods and conserve water for useful purposes. Design requirements as set up contemplated a masonry dam for a head of 130 ft, founded in the blue shale underlying the sand and gravel in the river-bed at a depth of approximately 20 ft. The procedure adopted was the usual one of examining available literature on shale foundations, performing specific tests and experiments on selected samples in the laboratory, seeking advice from experts and consultants and then making field tests under natural conditions. In due course certain information was acquired and, more important, certain preconceived notions as to the extent of available information and the con-

---

Note.—Published in September, 1939, *Proceedings*.

[1] Civ. Engr., U. S. Army Engrs., U. S. Engr. Office, Portland, Ore.

sistency of test results acquired with identical technique were proved to be in error. Many of these curious anomalies in the data cannot be explained. This paper is not intended to be a profound dissertation on soils mechanics or foundation theories; it is confined to a description and record of field tests and such observations and deductions as seem to be warranted by the results.

The literature on shale foundations gives an excellent "bird's-eye view" of the problem and serves as a reassurance that such foundations have been utilized successfully in the past. All too frequently, however, reports of tests are not sufficiently comprehensive enough, are vague in description, or lack conclusions that could have been substantiated by later measurements. For maximum benefit to the profession the important boundary conditions of a problem should be stated, with a description of the material, a full record of the tests, the final working loads used in the design of the structure and, if available, the deformation of the foundation under actual loads. This last requisite cannot be furnished by the writer but it is hoped that it may be partly supplied by others during discussion.

Purpose of Field Tests

The general purpose of the field tests was to obtain data on the probable movement of the masonry dam on the shale foundation when subjected to sustained loads and a repetition of loads. This general purpose was divided into five component parts, as follows: (1) To study the sliding of concrete placed on a prepared shale foundation, under the influence of horizontal and vertical loads; (2) to observe the fatigue effect on the shale when the load varied for several cycles from a maximum to a minimum unit load; (3) to determine the time effect on an area of the shale under the weight of a constant load; (4) to establish the relationship between various sizes of loaded areas under equal unit loads and the settlement of those areas; and (5) to compare values from large-scale models loaded under natural conditions with similar values obtained in the laboratory on selected specimens, under assumed and controlled conditions of drainage and temperature.

At the time the tests were proposed, it was realized that conclusive data on each of the foregoing items might not be obtained and that supplementary points of interest would occur to the investigators during the tests. Both of these things happened and the result was a progressive accumulation of knowledge. Fundamental assumptions made prior to the tests included the observation that failure would probably not occur from compressive stresses in the shale but from the tendency of the dam to slide on the contact plane between the masonry and the rock. This item was given primary consideration. Another fundamental assumption was that the shale was less rigid than the concrete dam above it and that the deformations would be confined to the shale.

Geology

The rocks at the dam site are sedimentary in character, consisting of a top member of limestone and grading downward through various admixtures of sandstone, silty and sandy shale, conglomerate and finally a fine-grained blue shale. The blue shale bed is approximately 80 ft deep and is supported from below by a hard gray sandstone of unknown depth. This general stratification is apparent on the cliff on the left bank and in the trenches that were dug in the sloping ground on the right bank of the river. The Brazos River has cut through these rock barriers successfully and is now about 300 ft below the top of the limestone cliff. The cemented conglomerate and the sand and gravel in the river-bed have probably been brought down from the headwaters during flood periods and deposited at the site.

The top part of the Wolf Mountain shale member immediately below the base of the overlying limestone and conglomerate has been oxidized and altered by waters that have seeped through the relatively porous limestone and conglomerate and have followed along the top of the impervious shale. The color has been changed by oxidation of the iron content from gray to yellow; calcareous cement has been dissolved and the shaley content softened to clay. Except for this weathered zone at the top of the formation and a moderate general penetration of surface weathering, the sandstone, sandy and silty shales, and blue shale phases of the formation are satisfactorily dense, strong, and impervious. The various phases of the Wolf Mountain formation appear to have adequate bearing strength.

The shale, including the silty phases, slakes in water after being dried. The slaking propensities vary considerably depending on its composition and texture. Therefore, it was necessary to observe the usual precautions against surface deterioration in all foundation excavations where concrete was placed.

The foregoing considerations indicate immediately the point of greatest danger; that is, the tendency of the shale to slake and form mud seams. One method of retarding this tendency during construction was to spray the exposed shale surface with an asphalt solution. This protective covering keeps the shale from drying and preserves its rock-like character. Since concrete must be placed on top of this protective coating, the degree of bond between the dam and foundation was unknown.

Laboratory Tests of Shale

Prior to making field tests, an extensive series of laboratory experiments were made with the shale at the Agricultural and Mechanical College at College Station, Tex. Additional tests were run at the Vicksburg Soils Laboratory conducted by the U. S. Army Engineers at Vicksburg, Miss. In general, the specimens were cut from cores taken by shot-drill methods during the exploratory boring and were carefully preserved until tested. The greater part of these tests were made on cylinders, 6 in. in diameter, 12 in. high, or on parts trimmed from the 6-in. cores. Although they varied over a wide range, the results of these tests were of a predictable nature and for the lower stresses indicated that the shale behaved as an elastic, isotropic material. The average results of the first tests conducted are as follows:

Test	Average results
(*a*) Moisture content, expressed as a percentage of the dry weight	7.85
(*b*) Percentage of total volume occupied by voids	15.4
(*c*) Absorption of water, expressed as a percentage of the total weight	0.25
(*d*) Modulus of elasticity, in pounds per square inch, at a compression of 7.2 tons per sq ft	133 000

Test	Average results
(*e*) Crushing strength of 6-in. by 12-in. cylinders, in tons per square foot	30
(*f*) Apparent specific gravity	2.46
(*g*) Weight per cubic foot undisturbed, in pounds	154
(*h*) Shearing parallel to bedding plane, in tons per square foot when compressed at 7.2 tons per sq ft	13.2
(*i*) Shearing normal to bedding plane, in tons per square foot when compressed at 3.6 tons per sq ft	8.35
(*j*) Chemical constituents, expressed as percentages of the total weight:	
Silica	62
Alumina	26
Lime	0.1
Magnesia	2.0
Potash and soda	some
(*k*) Cohesion, in tons per square foot (Vicksburg tests)	4.5
(*l*) Angle of internal friction, in degrees (Vicksburg tests)	45

Item (*c*) is not a standard or recognized test. The method used was to weigh specimens containing all of their natural moisture and then immerse them in water at room temperatures for periods varying from 3 to 20 days. It was noted that the specimens remained hard during this time and continued to absorb water for about 10 days. After the period of immersion the specimens were towel dried and re-weighed. The percentage of absorption was taken as the difference in the two weights, divided by the initial weight.

It is also to be noted that Items (*h*) and (*i*) fail to conform with tests (*k*) and (*l*) made at the Vicksburg laboratory. Disparity between results obtained by different laboratories, and even differences between several results obtained by the same laboratory, are the usual problems that confront a foundation engineer. A second series of tests covering essentially the same points gave slightly higher results and will be discussed subsequently under the comparison of laboratory and field experimental data.

In conducting the laboratory tests an attempt was made to duplicate the technique used in testing other earth, shale, and rock samples. New methods and refinement of apparatus were given some consideration but available time made it expedient to follow such rules as were known to have given good results in other cases. A further discussion of the laboratory tests is beyond the scope of this paper and they are mentioned merely to afford an over-all view of the material on which the field tests were conducted.

SLIDING OF CONCRETE BLOCKS ON A SHALE FOUNDATION

On the right bank of the Brazos River the concave side of Possum Kingdom Bend has a gently sloping surface that formed an ideal place to dig a test trench down to shale. At the point chosen for the trench the elevation of shale was higher than low water in the Brazos River and all experiments were conducted without interference from the river or from inadequate drainage. Conglomerate and sandstone were excavated with the aid of a small power shovel and dump trucks, after drilling and blasting. When blue shale was reached, it was excavated to within 12 in. of final grade and an asphalt sealing solution was applied to the surface to keep the moisture within the shale from escaping. Immediately prior to casting the concrete blocks in place the last 12 in. of shale were removed by air tools and by hand, the surface was cleaned by an air jet and another sealing coat of asphalt solution was applied. The foundation for the test blocks had a 5° slope upward in the same direction as the direction of the horizontal force that was to be applied. This sloping contact plane enters into computations for the resultant load and the angle of friction between concrete and rock. The tangent of the angle of the resultant load on this 5° sloping plane from a line normal to it is the ratio of parallel to normal forces. The parallel and normal forces are the vectoral summation of proper components of the vertical and horizontal loads. No special effort was made to secure a sandpaper finish on the shale foundation and the small irregularities from the use of the air tools were believed to be comparable to the surface that would be obtained under the dam when it was constructed.

The concrete test blocks were made from local aggregates, river water and Portland cement. Frequent checks indicated an average weight of 153 lb per cu ft and concrete test cylinders made at the time the blocks were cast, as well as cores later drilled from the test blocks, showed an average crushing strength of a 6-in. (diameter) by 12-in. (high) cylinder to be appreciably more than 3 000 lb per sq in. Eight test blocks, measuring 5 ft by 5 ft by 5 ft were cast in place, but only three of these gave significant information about the shale foundation.

The apparatus for the tests consisted of vertical and horizontal hydraulic jacks for applying loads, and dial gages for measuring movements of the block. Two hydraulic jacks resting on the top of the block, with an overhead, ballasted steel platform for backing, transmitted a controlled and registered vertical force (see Fig. 1). The overhead platform spanned the test trench and rested on rollers placed on ledges or berms cut in the sandstone on either side of the trench about 10 ft above the foundation (see Fig. 2).

The rollers made the platform movable along the trench axis so that when one test was completed the entire platform was wedged over until it was directly over the next test block. In this manner considerable time was saved in erecting apparatus for the eight tests. The ballast on the platform was sand confined in bags.

Horizontal loads (see Fig. 3) were applied by five hydraulic jacks working against the base of the concrete test block and using the side of the trench for backing. Fig. 4 shows the apparatus in place. Shoring and blocking were necessary to distribute the loads properly to the shale backing. Jacks and hydraulic gages were calibrated after the tests were finished. Entire units were placed in a 300 000-lb testing machine, the load applied, and the gages were calibrated accurately.

Horizontal movement of the concrete blocks was measured by four dial gages at the lower four corners of the block. Vertical movement was measured by two dial gages at the front and rear on the center line of the block near the top. These dial gages were sensitive to movements of 0.001 in. It required some

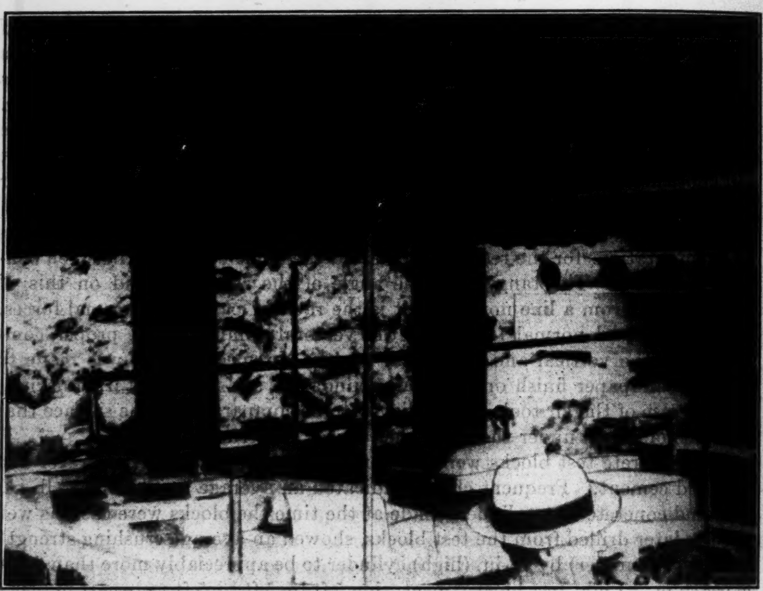


FIG. 1.—HYDRAULIC JACKS AND BLOCKING FOR VERTICAL LOAD APPLICATION

FIG. 2.—GENERAL VIEW OF THE TEST TRENCH AND APPARATUS

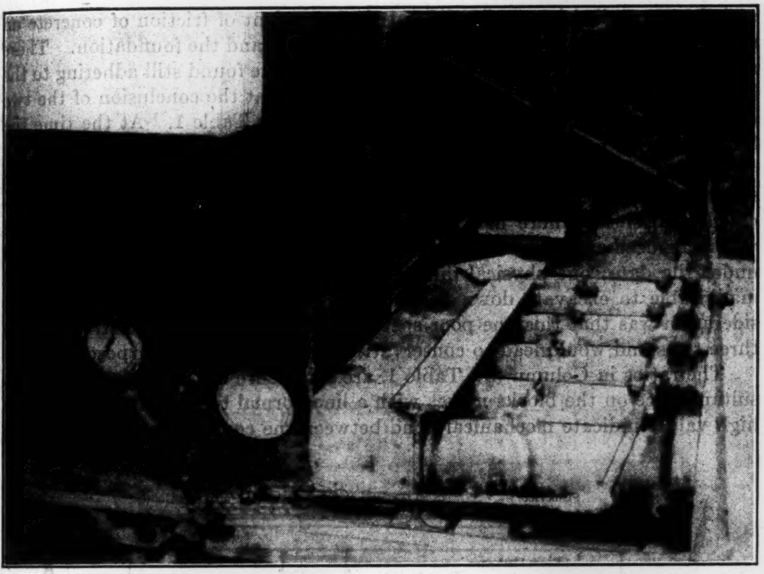


FIG. 3.—HYDRAULIC JACKS AND SHORING FOR HORIZONTAL LOAD APPLICATION

ingenuity to support these gages rigidly and independent from the test block. The framework finally used, as well as the relative positions of the six dial gages, is shown in Fig. 4.

The test procedure consisted of first applying the vertical load up to a predetermined value and then loading horizontally until the specimen failed. Movement of the block in a vertical and horizontal direction was recorded frequently during the loading. Time was kept from the beginning of the test until failure and a record was made at each reading interval to study the movement of the block.

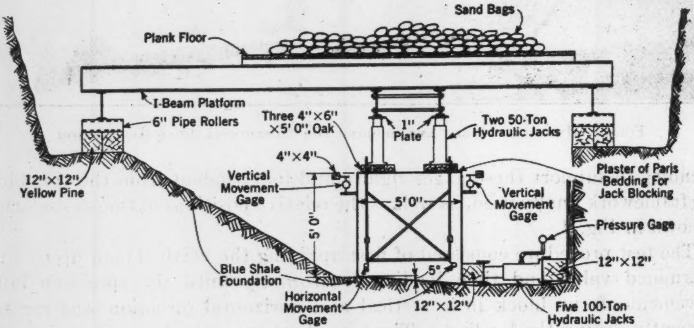
TABLE 1.—FIELD OBSERVATIONS ON TEST BLOCKS

Test Block No·	Time, in minutes (1)	Normal load, in tons per square foot (2)	Settlement, in inches (3)	Total horizontal load, in tons (4)	Shear, in tons per square foot (5)	Horizontal movement, in inches (6)	Ratio of horizontal to vertical load (7)	Ratio of parallel to normal load on foundation plane (8)	Coefficient of friction (9)	Percentage of base area in contact with shale* (10)
2	24	0.94	0.004	35.15	1.31	0.072	1.49	1.302	0.857	15
3	34	2.66	0.026	69.5	2.52	0.193	1.04	0.879	0.848	17
8	47	3.17	0.089	72.0	2.63	0.22	0.904	0.77	....	100

* Percentage of the base area of the concrete test block to which shale was adhering after the test.

Data on the three blocks giving most information about the shale have been tabulated in Table 1. After failure of the specimens both loads were released and then reapplied in an effort to get the coefficient of friction of concrete on shale, uninfluenced by bond between the monolith and the foundation. These results, with a record of the percentage area of shale found still adhering to the bottom of the blocks when they were tipped over at the conclusion of the two tests described, are given in Columns (9) and (10), Table 1. At the time the blocks failed (when no greater resistance to a horizontal load could be obtained) small cracks appeared in the shale immediately beneath the block. These small cracks, apparent at the bottom corners of the blocks in the shale, indicated local shear failure near the surface. The foundation for Block No. 8 was recognized as soft and inferior to that which ordinarily would be permitted under the dam; but physical difficulties in equipment and drainage made it inadvisable to excavate down to good shale. The fact that was given consideration was that this one poor specimen reduced the average stress for the three tests and would lead to conservative values for design purposes.

The ratios in Column (8), Table 1, are a measure of the angle that the resultant force on the blocks makes with a line normal to the foundation. The high values indicate mechanical bond between the concrete and shale prior to


Sand Bags
Plank Floor
I-Beam Platform
6" Pipe Rollers
Three 4"×6" ×5'0", Oak
1" Plate
Two 50-Ton Hydraulic Jacks
4"×4"
12"×12" Yellow Pine
Vertical Movement Gage
Plaster of Paris Bedding For Jack Blocking
Vertical Movement Gage
5'0"
5'0"
Pressure Gage
12"×12"
Blue Shale Foundation
5°
Horizontal Movement Gage
12"×12"
Five 100-Ton Hydraulic Jacks

TYPICAL SET-UP FOR ALL BLOCKS; BASE TYPICAL FOR BLOCKS 2, 3 AND 8

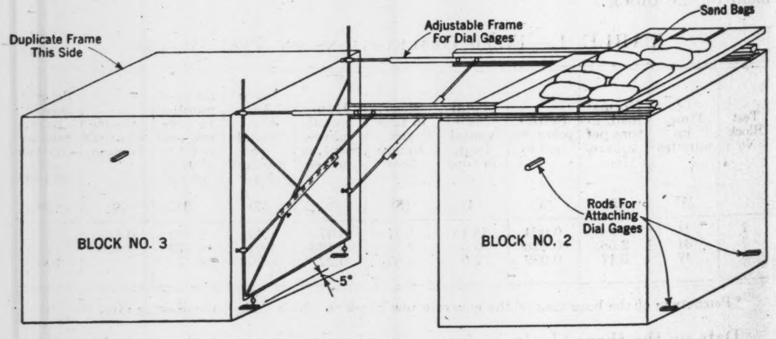

Sand Bags
Adjustable Frame For Dial Gages
Duplicate Frame This Side
BLOCK NO. 3
5°
BLOCK NO. 2
Rods For Attaching Dial Gages

FIG. 4.—GENERAL ARRANGEMENT OF APPARATUS FOR FOUNDATION TESTS

failure. The shear values in Column (5), Table 1, are taken along the plane of contact between the shale and concrete. They show an expected increase with the increase in normal load. The coefficients of friction of concrete on shale (Column (9), Table 1) include the effect of bottom irregularities in the foundation caused by scars and depressions from the use of hand tools in excavating. Subsequently these irregularities were filled with concrete when the block was cast in place. Visual inspection of the bottoms of the test blocks, when they were turned over upon completion of the tests, indicated that local shearing of the shale during the second test gave this rather high value for coefficient of friction. The time involved in the tests was a matter of minutes and corresponded to a "quick tests." This factor must be evaluated properly, particularly in any extrapolation of these data. A graphical stress-strain relationship is given in Fig. 5, the vertical load being kept constant for each of the tests, as follows:

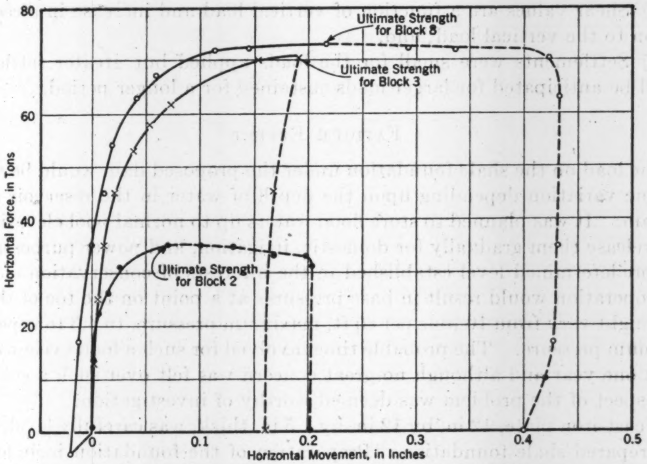


FIG. 5.—STRESS-STRAIN CURVES, BLOCKS 2, 3, AND 8

Test Block No.	Maximum vertical load, in tons
8	79.5
3	66.5
2	23.5

It should be noted in Fig. 5 that the specimens showed remarkable agreement with Hooke's law up to a fairly well defined point that might be termed the "yield point." This assumed yield point is about 63% of the ultimate strength and probably represents the first local failure in shear of a part of the shale immediately behind one of the protrusions of concrete that filled the irregularities of the surface of the foundation. The rate of increase in the ratio of horizontal load to vertical load was constant for all three specimens, to the point of ultimate strength.

The conclusions drawn from the foregoing data are:

(1) Bond existed between the concrete and shale in spite of the asphalt sealing solution that separated the two;

(2) The sealing solution preserved the rock-like qualities of the shale;

(3) If the dam is proportioned so that the ratio of the summation of horizontal forces to the summation of vertical forces does not exceed 0.50, a safe structure will result;

(4) Shear values are a function of vertical load and increase in direct proportion to the vertical load; and,

(5) Settlements were small for the loads applied but greater settlements should be anticipated for larger loads sustained for a longer period.

FATIGUE EFFECT

The load on the shale foundation under the proposed dam would be subject to some variation depending upon the depth of water in the reservoir behind the dam. It was planned to store flood waters up to normal pool elevation and then release them gradually for domestic, irrigation, and power purposes down to a predetermined level established in the interests of conservation of water. Such operation would result in base pressures at a point on the toe of the dam that might vary from 10 tons per sq ft, maximum pressure, to 4.3 tons per sq ft, minimum pressure. The probable time involved for such a load cycle would be about one year and although no great concern was felt over such a condition, this aspect of the problem was deemed worthy of investigation.

A cast-iron plate, 12 in. by 12 in. by 1.5 in. thick, was carefully leveled upon the prepared shale foundation. Preparation of the foundation included final hand excavation of shale and a sprayed application of an asphalt emulsion. This protective coating was renewed at 20-day intervals or as often as small patches showed wear from foot traffic. The cast-iron plate was seated upon a very thin layer of Portland cement mortar to obtain a level, horizontal surface and to fill the minute irregularities on the rock surface.

Controlled and registered vertical loads were applied through an hydraulic jack backed against a bin full of sand resting on an overhead platform. To assure axial application of the load a spherical bearing block was provided between the jack and the loading platform. Settlement readings were taken by means of small dial instruments measuring to 0.001 in. at the four corners of the test plate and at four points on the shale 6 in. removed from the edges of the plate.

A load cycle was considered complete when the pressure had been increased slowly to 10 tons and then decreased to 4.3 tons. In general, one cycle would require about six weeks during which time daily observations were taken of settlement, temperature, and weather. Some difficulty was experienced at first but corrections were soon made and the value of the test was not lost. Four cycles were completed in 246 days and the graphical record is indicated in Fig. 6.

In general, the shale behaved as an elastic material that would compress upon load application and rebound when the load was released during the first two cycles. The greatest average settlement of the four corners of the plate was 0.07 in., and 0.02 in. for the points on the shale 6 in. removed from the plate edge. The data are decidedly irregular and cannot logically be plotted as a stress-strain graph. In a gross manner it can be stated that there is a tendency toward elastic hysteresis since the "loading" line of a stress-strain graph does not coincide with the "unloading" line. This phenomenon was more apparent in laboratory tests of cyclic loading. A second observation is that the deformation at the end of the fourth cycle was more than that obtained in the first, second, and third cycles. There is a tendency for the settlement curve to flatten and become horizontal after repeated loadings, indicating some change in the structure of the rock. The permanent set of the area immediately under the test plate, and the slight upheaval of the points 6 in. removed from the plate, is typical of compaction and the probability of some plastic flow. However, the magnitudes of these movements of the foundation are small and it remains a matter of conjecture as to the number of loading cycles before definite fatigue effects could be noted. Certainly, for the loads involved in the test applied at the same rate for four cycles, the settlements are without structural significance. There is a recognized danger in assuming that a single test result on a specific, small area of rock can be used indiscriminately on the entire foundation. The difference between areas under test and under the prototype is too great to use the results of the experiment as any thing more than a trend or indication.

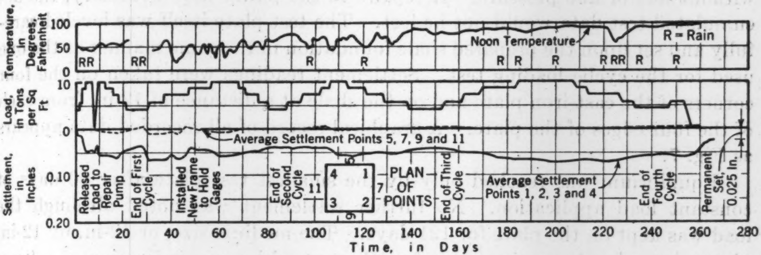


FIG. 6.—CYCLIC LOADING AND SETTLEMENTS, 12-INCH BY 12-INCH PLATE

TIME EFFECT ON SETTLEMENT

The time effect on settlement was investigated from the results of loading three cast-iron test plates and observing the settlement. Since the preparation and test procedure for obtaining the time effect is the same for all three plates, a description of only the 12-in. by 12-in. by 2-in. plate will be given. The final results will include data on all three plates. The dimensions of the other two plates were 6 in. by 6 in. by 2 in., and 24 in. by 24 in. by 2 in., respectively.

A registered and controlled vertical force was transmitted to the 12-in. by 12-in. plate through an hydraulic jack. The jack used the overhead loaded platform previously described, for backing, and axial application of the load was insured by a spherical bearing block between the jack and the platform. A refinement over the previous equipment consisted of a pressure regulator and check valve inserted in the pipe line between the pump and the jack. The pressure regulator gave constant pressure with a minimum of manual attention, and the check valve permitted withdrawal of the pump from the line without loss of line pressure. If repairs to the pump were necessary, the accumulated test data would not be lost. The test plate itself was leveled carefully and set upon the prepared shale foundation in the same manner as the one used for the cyclic loading test. Settlement readings were taken on the four corners of the cast-iron plate and on the shale at a distance of 12 in. from each of the four edges of the plate. A graphical record of all recorded data appears as Fig. 7.

Equilibrium was reached only by the smallest test plate after 60 days of constant load application. No further settlement was noted although the load was kept on the plate for 121 days. The medium size, or 12-in. by 12-in. plate, showed a decreasing rate of settlement with some indication of a "secondary" time effect after 70 days of constant load application. The large 24-in. by 24-in. plate continued to settle at a slow, uniform rate after the first 30 days of sustained load. The first 30 days was a period of decreasing rate of settlement.

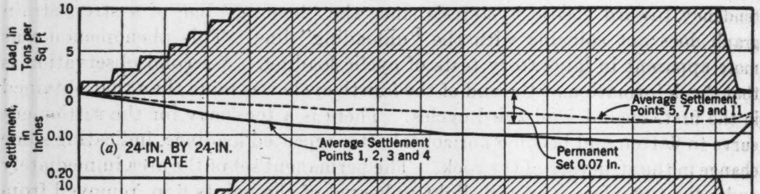


FIG. 7.—SETTLEMENT OBSERVATIONS UNDER LOADS OF TEN TONS PER SQUARE FOOT

Fig. 8 shows settlement plotted against time on a semi-logarithmic scale for the three plates. A smooth curve is drawn, based upon these points, and the difficulty of making simple, comprehensive statements from these test results is evident immediately. The three curves do not agree in shape. Although a general agreement of form can be said to exist for the time settlement curves for the medium and large plates, the data on the smallest plate do not give the same shape of curve. The conclusion drawn is that the rate of settlement is a function of the loaded area and the total load. Furthermore, the rate is slow enough (0.01 in. in 20 days for the largest plate) so that in the prototype the stability of the structure will not be affected. Continued settlement of the dam over a long period should be anticipated.

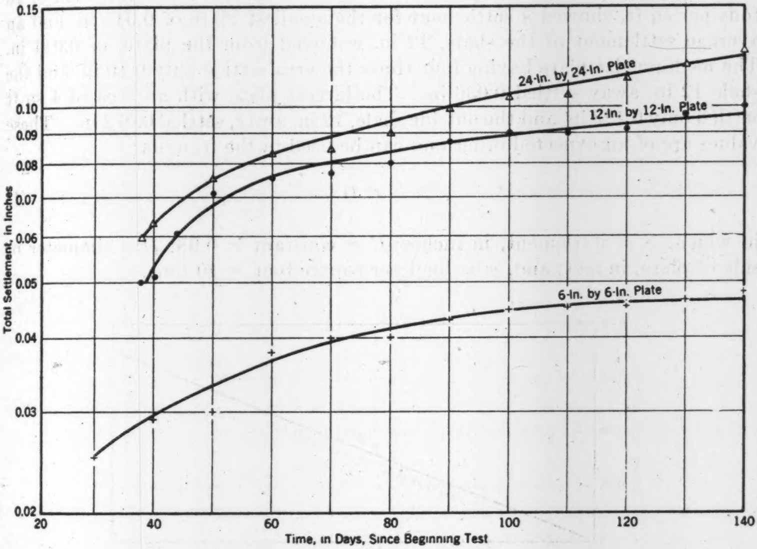


FIG. 8.—TIME-SETTLEMENT CURVES

The total settlement of the plates in all three cases was greater than the shale settlement 12 in. from the edge of the plates. No bulge in the surrounding shale was observed. This dish-shaped condition under load suggests that Boussinesq's formula, modified by a parameter, may be applicable. However, the Poisson's ratio of the shale is not known and a check on the foregoing supposition is precluded. No drift or plastic flow was discovered and settlement was a function of compaction only.

It has been stated[3] that the amount of settlement in soil is equal to the product of a constant depending upon the character of the soil, the diameter of the loaded area, and the unit load on the area. The tests on the three different sizes of square plates, each loaded with the same unit load, and founded on the same type of material, form the basis for establishing the relationship between the size of the loaded area and settlement. The equipment and tests are identical with that described under the heading "Time Effect on Settlement."

---

[3] "The Science of Foundations—Its Present and Future," by Charles Terzaghi, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 270.

The results of these tests, after 116 days under a constant unit load of 10 tons per sq ft, showed a settlement for the smallest plate of 0.048 in. and an average settlement of the shale, 12 in. removed from the plate, of 0.044 in. The medium size plate having four times the area settled only 0.10 in. and the shale 12 in. away settled 0.036 in. The largest plate with an area of 4 sq ft settled only 0.123 in. and the outside shale, 12 in. away, settled 0.072 in. These values are of an expected order and can be used in the formula

$$S = C\,D\,w \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which: S = settlement, in inches; C = constant = 0.08; D = diameter or side of plate, in feet; and, w = load per square foot = 10 tons.

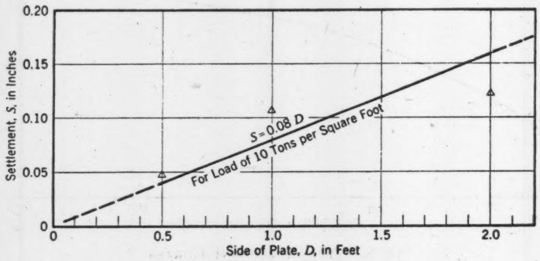


FIG. 9.—RELATION BETWEEN TEST RESULTS AND EQUATION (1)

Fig. 9 shows this relationship between experimental results and Equation (1) graphically; the straight-line variation may be assumed without too great a loss of accuracy. The deviations represent unexplainable phenomena that frequently crop up in foundation experiments in spite of painstaking efforts to insure comparable results.

Fig. 7 indicates that the permanent set under the 6-in. by 6-in. plate was 0.035 in. upon load release and 0.065 in. and 0.70 in., respectively, for the 12-in. by 12-in. plate and 24-in. by 24-in. plate when the load was released. This variation points to more compaction under the larger area and suggests that not only is the settlement under load a function of size but that the permanent set is likewise governed by the size of the loaded area.

The dish-shaped surface of the rock in the immediate vicinity of the three test plates indicates a cohesive medium offering resistance to shear. Since the shale around the smaller plate settled more than that around the 12-in. by 12-in. plate, it follows that there is some "edge effect" or disturbance. W. S. Housel,[8]

---

[8] "Notes on Soil Mechanics and Foundations," by Fred L. Plummer, M. Am. Soc. C. E., Edwards Brothers, Inc., Ann Arbor, Mich., 1936.

M. Am. Soc. C. E., has taken this factor into account in one of his older formulas for cohesive soils:

$$P = A\,m + l\,n \qquad (2)$$

in which: P = total allowable pressure in tons on footing; A = area of footing, in square feet; l = perimeter of footing in linear feet; and, m and n = two distinct constants. The constants can be established by considering the load causing equal settlement on two of the plates and checking against the third plate load for a similar settlement. This has been done for a settlement of 0.05 in. and the computed constants are $m = 6.2$ and $n = 0.525$. The agreement for other equal settlements is rather poor using these exact constants but there is no reasonable doubt that bearing capacity is a function of area and perimeter.

The conclusions that may be made from the experiments on loading and settlement of test plates on this shale foundation are:

(1) Cyclic loading of the foundation is not structurally important (the suspected plastic flow of the shale under repeated loads is small and at a slow rate);

(2) Continued settlement of the dam over a long interval at a slow rate can be expected logically;

(3) Careful proportioning of footings so that the larger footings carry smaller loads per square unit will prevent differential settlement; and,

(4) Edge effect of footings are important as well as the size of the loaded area.

CORRELATION OF LABORATORY AND FIELD TESTS

Rapid laboratory tests were made to determine shear and sliding values on small samples of shale 6 in. in diameter and about 2.5 in. high, on which was cast a neat sand-cement mortar cylinder of the same diameter and about the same height. The data in Table 2 can be compared with similar material for field tests on Blocks Nos. 2, 3, and 8, in Table 1. If the normal load is plotted against average shear (Item No. 2, Table 2), the resulting graph conforms to the equation

$$x = 3.7 + 0.9\,y \qquad (3)$$

TABLE 2.—LABORATORY OBSERVATIONS ON TEST BLOCKS

Item No.	Description	Normal Load, in Tons per Square Foot		
		10.8	7.2	3.6
1	Number of specimens	3	3	1
2	Average shear, in tons per square foot	12.46	10.8	6.9
3	Ratio of horizontal load to vertical load	1.15	1.50	1.91

One reason for the higher laboratory test values is that there was no asphalt coating separating the concrete and the shale. This is reflected by the lower bond or shear values for the field tests, amounting to an 81% reduction. Sliding friction was also lower by 28% for the field tests, for the same reason.

At the Vicksburg laboratory, three samples of shale were sandpapered smooth, coated with an asphalt sealing solution and a neat sand-cement mortar block was cast upon the top of the shale sample. The specimens were placed in a shearing apparatus with a known load normal to the plane of contact of the two materials, and then a load parallel to the contact plane was applied. After the slight bond was broken the coefficient of friction of the materials was 0.506. This value is 31% lower than that obtained by the field tests and gives a clear idea of the effect of irregularities in the contact plane in the field tests.

The shale was subjected to cyclic loading, at an accelerated rate, in the laboratory. A 6-in. by 12-in. cylinder of shale was placed in the center of a 16-gage corrugated iron culvert pipe 13.5 in. in diameter and 12 in. high. The annular space between the shale and the pipe was then filled with plaster of Paris. Load on the shale area 6 in. in diameter was applied through a disk measuring 4.91 in. in diameter. Compaction was measured by Ames gages. Two tests, for 16 and 24 cycles, respectively, varying from no load to 15.2 tons per sq ft gave a deformation of 0.006 in. and 0.0036 in. after four cycles. After sixteen cycles the permanent set was 0.0084 in. and 0.006 in. Elastic hysteresis and elastic after effects were both noted in these experiments. Likewise, it was apparent that the minimum deformation occurred under rapid application of load and an early release of that load. Since the conditions of the laboratory tests were totally different from the field tests no comparison is possible.

Several other unrelated facts were noted in the laboratory experiments, all of which defy comparison with the field tests. One of these was the 100% consolidation of a small shale specimen under a load of 3.576 tons per sq ft. Furthermore, by crushing many 6-in. cylinders of shale, 12 in. high, a general increase in crushing strength was noted for increase in depth at which the specimens were taken. This same relationship held for resistance to shear. Although this information "puts the final touches" to the pictured problem, greatest confidence is felt in the results of the field tests. Whatever else may be claimed about the field tests, it is undisputed that errors incident to sampling and to duplication of natural conditions are eliminated. It is believed that, in any comprehensive foundation investigation, the test results obtained in the laboratory should be supplemented by those taken in the field.

Loads Used in the Design of the Dam

Detailed plans were drawn for a gravity dam and a round-head buttress dam for the same site. The field tests indicated a safe bearing capacity of 10 tons per sq ft, assuming correct proportioning of footings. The maximum unit loads used in the design were 9.1 tons per sq ft for the gravity type of dam and 9.8 tons per sq ft for the round-head buttress type. The maximum average shear resistance on the plane of contact of these two types, with the foundation, was 2.3 and 3.6 tons per sq ft, respectively. This compares favorably with the field tests of 5.1 tons per sq ft under a much smaller vertical force. The ratio of the horizontal to the vertical forces used in design was 0.332 for the gravity dam and 0.486 for the round-head buttress dam. This is about six-tenths of the ultimate values from the field tests, 0.812.

DISCUSSION

Harry H. Hatch,[4] M. Am. Soc. C. E. (by letter).—Considering the individuality of every foundation material or its constituent soil particles, the difference in location and climatic conditions, the manner of, and personal equation in, investigation, and the variation in the experience, competence, and viewpoints of the investigators, one cannot be surprised that "consistency of test results," reported in this paper, is lacking.

The apparently negative result of a conscientious test is just as important as if its results agreed one hundred per cent with the preconceived notions. It is desirable to standardize tests and testing methods for the sake of economy of time and money; but on no job should an engineer be content with the "patent medicine" laboratory theories and notions. He should explore further, by independent research, to find the truth for the case in hand.

Only recently have the rule-of-thumb methods been discarded and scientific investigation applied to these jobs. Sufficient data are not available yet even to suggest a law of averages. It is essential that all such investigations should yield concise information in regard to the purpose and the method of conducting any test that is not well known or standardized. It is all the more important, therefore, to have before the profession the accurate description and results of every investigation of this nature in order to augment the store of information and knowledge on the subject; and the author should be commended for his purpose and efforts in presenting this paper.

Under the heading "Laboratory Tests of Shale," Items (*h*) and (*i*) do not entirely fail to conform with Items (*k*) and (*l*).

$$s = \frac{W}{A} \tan \phi + C \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which: s = unit shear; $\frac{W}{A}$ = normal unit load = external load per foot of area; ϕ = angle of internal friction; and C = unit cohesion.

The computed shear value for Item (*h*), according to Equation (4), is $7.2 \tan 45° + 4.5 = 11.7$ tons per sq ft, or 88.9% of the observed value, 13.2; and similarly for Item (*i*), it is $3.6 \tan 45° + 4.5 = 8.1$ tons per sq ft, or 97% of the actual value, 8.35. Items (*k*) and (*l*), therefore, give values about 11% and 3% less than the actual; and unless this consistency is purely by coincidence, the test results cannot be considered seriously in error.

The writer's experience has been that the coefficient of friction (and hence the angle of internal friction) is not constant for a given soil, and that it decreases with the increase of pressure. If C, the cohesion, is kept constant at 4.5 in order to have the actual and the computed shear values agree completely, the angle of internal friction should be 50° 23′ and 46° 55′ for Items (*h*) and (*i*), respectively, in this case. That is, the higher the pressure the larger the angle of internal friction will be, which does not agree with the writer's contention. Could the solid rock formation and its bedding planes account for this? On the contrary, with due respect to the difference in their normal loads, the actual shearing value parallel to bedding plane is higher than that normal to bedding plane, instead of vice versa as would be expected. Perhaps, after all, bedding planes in this solid rock may not be of any consequence during the shearing tests; or the value given to cohesion may be responsible for the inconsistency. More details as to the number of tests conducted for the two planes, with their individual results under different pressures and with a description of the method of averaging them, will be necessary to interpret the given averages intelligently.

---

[4] Div. Engr., Springfield Water Works, Springfield, Mass.

There are only two average shear values given. More would have been desirable, but these two will satisfy Equation (4) with the assumed constants $\phi = 53° 25'$ and $C = 3.5$ (see Fig. 10(*a*)). A description of the author's methods of testing cohesion and the angle of internal friction of a solid rock formation will be of interest and may throw some light on the subject.

Equations (4) and (3) are identical. In the latter $x = s$, the shearing force in tons per square foot; and $y = \frac{W}{A}$, the normal pressure in tons per square foot. Therefore, $C = 3.7$, and $\tan \phi = 0.9$, or $\phi = 42°$. As already stated, ϕ cannot remain constant as the pressure increases, even if shearing value increases with the increase of pressure. However, this relation does not follow the locus of a straight line, as given by Equations (3) and (4). Either equation is only approximate and, when obtained from the plottings of only two or three points, the equation becomes entirely inapplicable. In Fig. 10(*b*), Curve *A* gives the relation between Columns (2) and (5) of Table 1; Curve *B* is the plot of Item No. 2, Table 2; and *C* is the plot of Equation (3). According to Equation (3), or Curve *C* in Fig. 10(*b*), there are 3.7 tons per sq ft of cohesion between the concrete and the shale. Can there be any such cohesion between the concrete and the shale blocks? With no cohesion existing between the blocks, the extension of Curves *A* and *B* down to the origin can well be justified. From Fig. 10(*b*), it is perfectly evident that the relation between shear and normal pressure can be represented neither by a straight line, nor by a simple exponential equation, over the entire range.

From a practical point of view, what necessity is there for cohesion tests and angle of internal friction tests when actual shearing tests are likely to give a much better picture of the existing conditions in the structure?

It is preferable to have as many direct shear tests as possible under the circumstances, but a sufficient number of them at any rate, and to plot them all in order to obtain a more or less representative graph showing the relation between the shear and normal load, without paying any attention to the uncertain term of the angle of internal friction of a solid rock formation or its cohesion.

As stated by the author, the asphalt coating separating the concrete and the shale must have acted as a lubricating plane between the two and was mostly responsible for the lower shear and coefficient of friction values. Due to the asphalt alone, disregarding any other possible reason, it is questionable whether the two sets of test results can be considered comparable.

Although the two sets of tests may be unlike in more ways than one, the behavior of the coefficient of friction in both cases is according to the writer's experience; namely, the coefficient of friction is not constant, but decreases with the increase of vertical pressure. Insufficient data are available, however, to produce for this job a representative graph or equation expressing the relation between the coefficient of friction and the vertical load. Any such relation obtained from two or three tests would be practically of no value.

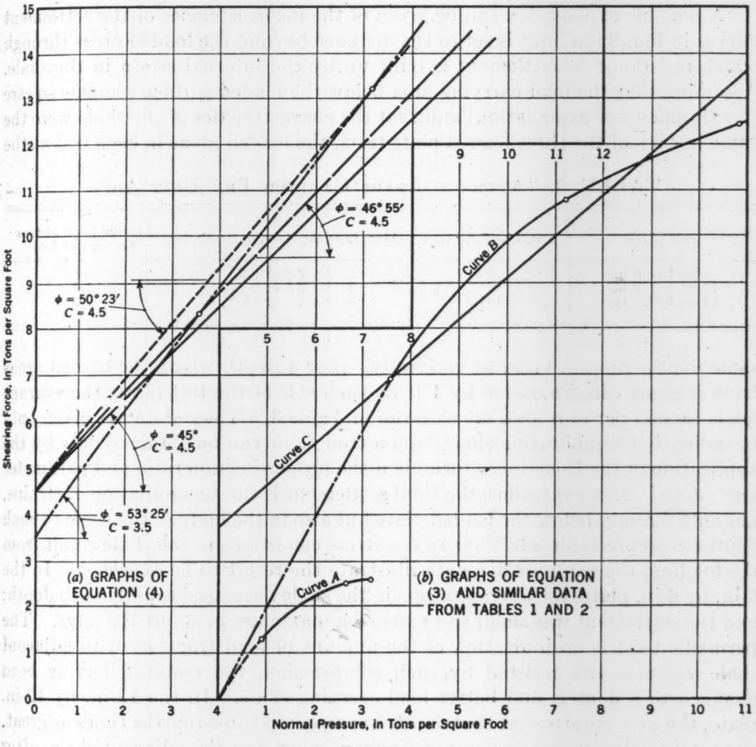


FIG. 10

JACOB FELD,[5] M. AM. SOC. C. E. (by letter).—A complete and unbiased description of field and laboratory tests, carefully planned and executed, is given in this paper. The value of the large-scale field tests is without question; the author's admitted lack of success in making the laboratory test results agree with the field test results should be considered a marked contribution to the study of foundations. No more valuable warning can be cited of the dangers from indiscriminate extrapolation of small-scale laboratory tests for use in design, unless such tests are tempered by actual experience with similar foundation materials or superseded by large-scale field tests.

---

[5] Cons. Engr., New York, N. Y.

As Mr. Niederhoff explains, the shale seemed to follow the laws of elasticity up to certain unit loadings, but with definite plastic flow characteristics.

A possible explanation can be given of the inconsistencies of the settlement curves in Fig. 8 if a limit is set to the distance beyond the loaded areas through which resistance to settlement is built up by the internal strain in the shale. Assuming that the load-carrying area below the loaded surface remains square (for simplicity of explanation) and that the characteristics of the shale were the same in each of the three loaded plate tests, the loaded areas in each test at the same depths are not stressed uniformly. For a depth where the loaded areas have increased in dimension by 1 ft on each side of the test plate, the average loads carried per unit area are as given in Table 3. These data are given only to indicate the qualitative effect; and a true value can be obtained only by the application of the Boussinesq formula if the proper Poisson ratio and parameter are known. However, since the total settlement is the accumulation of strains, not only directly below the loaded plate but also in the body of the shale to such depths as appreciable additions to the stress conditions in the shale result from the loading, the comparative data illustrate the result to be expected. In the 6-in. by 6-in. plate, the average stress in the shale decreased rapidly with depth; and the settlement was small and reached a maximum in about 100 days. The permanent set is an indication of the amount of load transferred to adjacent shale volumes and resisted by such compression, the material having been changed to a denser, and better load-carrying shale. In the 12-in. by 12-in. plate, the average stress at that depth was two and three fourths times as great, the total settlement was correspondingly greater, and the adjacent shale, after 140 days, had not yet taken up the load by being compressed. In the 24-in. by 24-in. plate, the same condition was also true to a larger degree.

TABLE 3.—AVERAGE LOADS CARRIED PER UNIT AREA

Test plate	Total load, in tons	Area, in square feet, at depth d	Avenue load, in tons per square foot
6 in. by 6 in.	2.5	2.5 by 2.5 or 6.25	0.4
12 in. by 12 in.	10	3 by 3 or 9	1.1
24 in. by 24 in.	40	4 by 4 or 16	2.5

An attempt to apply the results of a 6-in. by 6-in. test plate to the design of a dam foundation under the same unit loading can result only in unforeseen and apparently unexplainable settlements.

The description and results of the large-scale shear tests in shales are especially valuable as an addition to the compilation of data on foundation phenomena.

JACK R. ROUNTREE,[6] **JUN. AM. SOC. C. E.** (by letter).—The author is to be commended not only for presenting an interesting and valuable paper, but also for his purpose in placing before the profession a complete record of a carefully planned series of field tests. The results of only a few such tests have been published in engineering literature. In general, the physical properties of different shales are quite different; therefore it is seldom that the results of any one test may be correctly applied to a case at hand. A compilation of test results covering many types of shales, nevertheless, may be of considerable value in estimating the limits between which might fall the properties of a particular shale foundation.

---

[6] Asst. Structural Engr., TVA, Knoxville, Tenn.

This paper sets forth a very practicable field test for determining the resistance to sliding. The fissile character of some shales precludes the possibility of obtaining representative specimens large enough to be tested in the laboratory; in such instances, the engineer must necessarily resort to field tests in order to investigate the resistance to sliding. It is significant that the author recommends field tests to supplement laboratory tests, in "any comprehensive foundation investigation."

In many cases of dam construction, the contact surface between the foundation and the concrete is very irregular, and the problem of evaluating the resistance to sliding is reduced to a determination of the shearing strength of the foundation material. The sliding tests described by Mr. Niederhoff are well suited to the investigation of shearing strength of a shale foundation. At Test Block No. 8, where the underlying shale was described as "soft and inferior to that which ordinarily would be permitted under the dam," the failure evidently occurred within the shale, rather than at the contact surface; in this instance, the test results give a measure of the ultimate shearing strength of the "inferior" shale. In all cases where failure may occur within the shale it is believed advisable to remove lateral support by excavating a shallow trench around the boundary of the shear area. For a large shear area, such as that beneath a monolith of a gravity type dam, lateral support around the boundary of the area may have only a negligible effect upon the total shearing resistance. For small areas, however, this effect may be significant.

A series of tests conducted at the site of Watts Bar Dam on the Tennessee River in Tennessee represents an example in which the test methods and apparatus described by the author have been adopted in the testing of a different type of shale foundation. Due to distinct differences between the Watts Bar and Possum Kingdom shales, the results of the two series of tests are not similar. The Watts Bar tests were conducted by the Tennessee Valley Authority, in conjunction with the U. S. Army Engineers, for the purpose of determining the shearing strength of the shale in a direction parallel to the bedding planes.

The shale underlying the easterly portion of Watts Bar Dam is a part of the Rome formation and is classified as a cemented, clay and silty shale. It is fairly compact and, in general, unweathered. The shale is characterized as "fissile" and is comprised of very thin layers which are easily separated; the surfaces of these layers are smooth and glossy in appearance. The shale is interbedded with thin layers of hard sandstone which vary from ⅛ in. to 2 in. in thickness and which are normally from 1 in. to 24 in. apart. In general, the shale has a dip angle of from 20° to 30°, but occasionally, in limited areas, the bedding planes are horizontal. The apparent weakness of the material against movement parallel to the bedding prompted these tests.

Six concrete test blocks, 5 ft long, 3 ft wide, and about 4.5 ft high, were cast directly on the shale foundation. The contact surfaces were parallel to the bedding planes and were as nearly horizontal as the nature of the rock permitted. In contrast to the Possum Kingdom tests, where it was assumed that failure would occur by sliding of the concrete on the shale, the Watts Bar tests were predicated upon the assumption that failure would occur within the shale at a short distance below the contact surface. Immediately prior to testing, the shale around each block was trenched out to a depth of about 2 in. below the projection of the contact surface. Trenches were located about 6 in. from the edges of the blocks, and were carefully prepared by hand tools to insure that no shale under the contact surfaces was disturbed. At each block this trench served to remove lateral support from around a short prism of shale, thereby simulating the conditions existing for a large contact area, for which lateral support around the boundary of the area would afford very little, if any, additional shear resistance.

TABLE 4.—TEST DATA, WATTS BAR DAM, TENNESSEE

No.	Description	Test Blocks Nos.:					
		1	2	3	4	5	6
1	Preliminary period of sustained vertical load, in minutes	65	70	55	74	None	None
2	Increments of horizontal load, in pounds	5 000	5 000	5 000	5 000	5 000	10 000
3	Average rate of application of horizontal load, in pounds per minute	9 700	7 300	1 600	1 700	6 500	6 600
	Intensity of Load, in Tons per Square Foot, Parallel to Contact Surface:						
4	At failure	3.08	5.06	1.84	3.03	1.83	2.20
5	Estimated failure under sustained load	2.46	4.13	1.51	2.46	1.50	1.81
6	First sliding	2.30	3.31	1.51	2.53	1.34	2.20
7	Second sliding	1.46	2.06	2.86	1.32	....	....
	Intensity of Load, in Tons per Square Foot, Normal to Contact Surface:						
8	At failure	5.71	4.36	2.07	5.10	2.03	4.88
9	Estimated failure under sustained load	5.65	4.26	2.05	5.06	2.02	4.86
10	First sliding	5.63	4.18	2.05	5.06	2.00	4.88
11	Second sliding	3.10	2.11	5.10	2.52	....	....

The methods of loading the blocks and of measuring the deformations were similar to those described by Mr. Niederhoff. The general procedure consisted of bringing the vertical load up to a specified intensity, and then loading the block with increments of horizontal load until failure occurred. Deformations were observed after each increment of horizontal load had been applied. The failure point was taken to be that point at which a constant horizontal load produced a continued movement of the test block. After failure, all loads were relieved; the vertical load was then brought up to a specified intensity, and the horizontal load applied gradually until a slow, uniform sliding of the block was produced.

The speed of testing used at each block varied somewhat, as shown by Items Nos. 1, 2, and 3 of Table 4. After testing, each block was tipped over to allow inspection of the failure surface. At all blocks, the failure surfaces were from ¼ in. to 2 in. below, and essentially parallel to the contact surfaces; the shale was well bonded to the under side of the concrete test blocks. Similar failure surfaces were observed under Blocks Nos. 1, 3, 4, and 5; at Block No. 2, the failure occurred along a decidedly irregular surface; at Block No. 6, the failure surface was more nearly plane than at any other block, due to the exceptionally smooth bedding at that location.

At Blocks Nos. 1, 2, 3, and 4, the vertical load was sustained for a short period before application of the horizontal load. The effects of vertical deformation during the short period of sustained vertical load probably increased somewhat the shear strength of the material; under a long-time loading, such as during the construction period of a large gravity-type structure, a similar but more pronounced increase might logically be expected.

Fig. 11 shows the average horizontal deformation plotted against horizontal jack load, for Test Block No. 4. Similar curves were observed at all blocks except at Block No. 6. Fig. 12 shows the variation in the instantaneous value of the rate of change of deformation with respect to the horizontal jack load for Test Block No. 4; this curve suggests that progressive permanent yielding (shearing off of projections on the failure surface) occurred only in the vicinity of the failure load. Construction of curves similar to Fig. 12 for other test blocks led to the adoption of a value of 83% of the ultimate horizontal load as the average point of "break" in the deformation curves; a sustained horizontal load of this amount might finally produce failure; therefore this reduced amount is a more conservative measure of the strength of the material than the

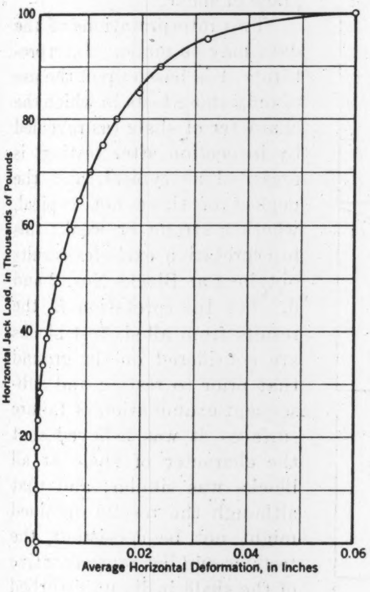


FIG. 11.—AVERAGE HORIZONTAL DEFORMATION, TEST BLOCK NO. 4

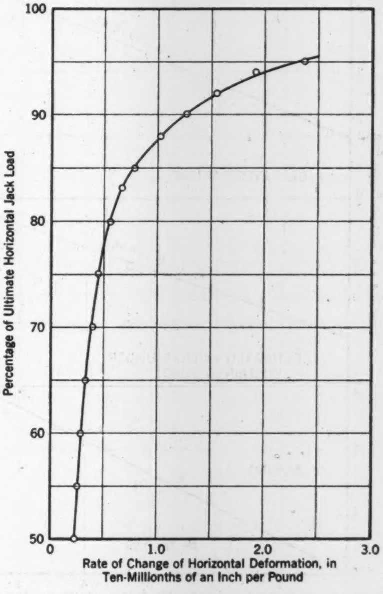


FIG. 12.—RATE OF CHANGE OF HORIZONTAL DEFORMATION WITH RESPECT TO HORIZONTAL JACK LOAD, TEST BLOCK NO. 4

ultimate failure strength indicated by the tests. Items Nos. 4 to 11 of Table 4 show the essential data obtained from the tests; the "estimated failure under sustained load" (Items Nos. 5 and 9) is based upon a horizontal load equal to 83% of the observed failure load.

In the interpretation of the test data, the internal friction theory as originated by Coulomb is used on the ground that it has been accepted by many engineers and that it affords a means of interpreting the data in a systematic manner. The resistance to movement is given by the expression:

$$s = s_1 + k\,q \quad \text{..........................(5)}$$

in which s is the total unit shearing strength, s_1 is the shearing strength under conditions of no load (pure shear strength), k is the factor of shearing-strength increase (assumed to be constant), and q is the intensity of load on the plane of shear.

Two interpretations of the data may be made. Interpretation *A* is based upon the use of only those tests in which the character of shale (as revealed by inspection after testing) is regarded as typical, and the neglect of those not typical, whether strong or weak; this interpretation excludes results obtained at Blocks Nos. 2 and 6. For Interpretation *B*, the results from all six test blocks are considered on the ground that prior to testing and subsequent examination of failure surfaces, it was believed that the character of shale at all blocks was similar, and that although the results obtained might not be consistent, the tests would be representative of the shale in its undisturbed form.

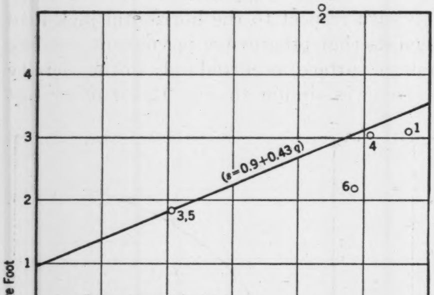


FIG. 13

The shearing strength of the shale is appraised at a value intermediate between those of the respective interpretations. Using the data for the observed failure loads, Equation (5) may be written (in units of tons per square foot):

$$s = 0.9 + 0.43\,q \quad \text{..........................(6)}$$

Fig. 13(*a*) shows the position of the straight line defined by Equation (6), together with the plotted points. The expression for the shearing strength based upon the "estimated failure under sustained load" data is:

$$s = 0.8 + 0.35\,q \qquad (7)$$

Fig. 13(*b*) shows the position of this straight line together with the plotted points from which Equation (7) was determined.

The sliding tests made after failure of the test blocks are the basis of the points plotted in Fig. 13(*c*). For the first four test blocks, sliding tests were made for two different intensities of vertical load. Assuming that s_1 in Equation (5) equals zero, a mean value of the coefficient of friction may be obtained for each block by constructing a straight line through the origin and through the average of the points obtained from the sliding tests for that block. Excluding the results at Block No. 2, an average value of the coefficient of friction of shale sliding on shale, parallel to the bedding planes, is 0.53, as shown by the slope of the straight line of Fig. 13(*c*).

The coefficient of friction so determined is probably too high; extension of the straight lines of Figs. 13(*c*) and 13(*a*) will indicate that, under higher normal loads, the resistance to sliding is greater than the total shearing strength. During the process of sliding, shearing off of projections on the failure surface continued; thus the construction of the straight line of Fig. 13(*c*) through the origin is probably not valid in this case.

The successful application of the Possum Kingdom test methods to the conditions existing for the Watts Bar tests affords additional proof of the value of Mr. Niederhoff's paper.

AUGUST E. NIEDERHOFF,[7] ASSOC. M. AM. SOC. C. E. (by letter).—One cannot but envy the delightfully simple routine of yesteryear's foundation engineer. His only field test for suitable rock was to hit it with a hammer and if the hammer rang, the rock was considered acceptable. Since that time loads have become heavier and more complex as superstructures have become larger. Engineers began to worry a little bit and took small samples of rock from the foundation to the laboratory where they could experiment on the "hand-picked" specimens. As long as the rock was hard and durable these laboratory tests were reassuring. No qualms of fear were felt that this quick analysis, wherein the specimen failed all at once like the poetic "one-hoss shay," did not give dependable results. Today, not only are the loads heavy but the foundation material is usually poor. There is also a dawning realization that there must be something to this fourth dimension concept sometimes called "time-effect" or "fatigue of materials." Also there is the growing suspicion that all foundations are not perfectly elastic media able to withstand the ravages of exposure; so the engineer again turns to field tests. The conduct of these field tests and the interpretation placed upon the results by the writer have elicited the views of several engineers. At least two cases in widely separated parts of the United States are known in which the field testing procedure, as described in this paper, has subsequently been used in whole or in part, yielding excellent results. The record is completed by the interesting and readable discussions of this paper that have added much to the store of knowledge on foundations.

---

[7] Civ. Engr., U. S. Army Engrs., U. S. Engr. Office, Portland, Ore.

Mr. Hatch contributes the observation that agreement between several laboratory results is reasonably good if they are properly evaluated. Using shear values derived from Equation (4) he finds the variation of consistency to be about 11 per cent. This is certainly within acceptable limits, but in the same paragraph he points out that laboratory shear values parallel to the bedding plane exceed those normal to the bedding plane. The expectation is naturally for results in the opposite direction and forms the basis for the writer's contention that laboratory tests should be taken with "a grain of salt." Too often they include numerous errors of sampling and duplication of natural conditions.

It is claimed that by experience Mr. Hatch has found that the coefficient of friction decreases with an increase in pressure. In Fig. 10 he has plotted shearing force against normal pressure, and the curved lines for both laboratory and field tests prove his point. The curves, of course, do not pass through the origin because there was evidence of definite adhesion between the concrete blocks and the shale. Another point that could be emphasized in Fig. 10 is the similarity of shape in the curves of the field and laboratory results. This unsuspected conformity between the two types of data is reassuring.

From a practical point of view it is debatable whether determining the angle of internal friction for a system composed of different materials such as concrete and shale, or for shale alone, is less important than knowing the actual shear value of the material. The writer believes both are important but that the answer in any particular case is necessarily influenced by the local conditions of contact between the concrete and the shale. For instance, a deep, narrow cut-off trench dug into the shale and filled with concrete to form a shear key presents one problem, whereas the same foundation using a pattern of small grooves cut into the shale to increase resistance to sliding is something quite different. Apparently the relative importance of shear values *versus* internal friction values must be decided on the merits of a particular case. In any event, determining the ratio of horizontal to vertical loads furnishes designers with a recognizable value. It is the kind of thing that gives them a comfortable feeling if they are sure their assumption is on the safe side.

For the foundation under consideration in the paper, the field data, and to a lesser extent the laboratory results, were enough to serve for the conclusions at the end of the section headed "Sliding of Concrete Blocks on a Shale Foundation." Criticism as to basing opinions upon a few tests can only be answered by the statement that even three field tests are preferable to having no results whatever. Limitations of money, time, and opportunity still harass the foundation engineer as well as other research professions.

The writer is pleased to note that Mr. Feld is of the same opinion regarding the dangers of extrapolating small laboratory tests to fit an entire foundation. The only time that this is excusable is when the failure values of small unconfined specimens are many times the actual loads to be placed on the foundation. Even then an engineer should run through preliminary calculations that tend to show that such a case actually exists. Table 3 demonstrates this procedure very nicely in addition to being an explanation of inconsistencies in the settlement curves in Fig. 8. Since writing the paper the author has noted that the presence of overburden loads in the proximity of the largest test plate may also have been responsible for erratic settlement. All of the test plates were placed in the bottom of a trench but the distance from the edge of the largest plate to the toe of the cut was a smaller proportion of plate diameter than was the case of the other two test plates. If the tests were to be done again the writer would see that no other loads were closer to the edges of the tested area than four diameters of the test plate. Mr. Feld is quite correct in his statement that tests and interpretations must be tempered by experience.

It is regrettable that discussion did not bring out actual measurements of settlement on Possum Kingdom Dam during construction. Data of this nature, if made available in the future, would be most conclusive as to the efficacy of the field and laboratory tests.

Mr. Rountree's discussion is clearly illustrated with results of tests conducted along similar lines and is one of the two aforementioned cases in which it is known that the test procedure has been used successfully. His observation that test blocks should not include the lateral support of abutting shale because of probability of erroneous interpretation of results is sound advice.

The fissile character of the foundation of Watts Bar Dam appears to make it a more difficult problem than that faced by the writer. The Tennessee Valley Authority has been known to adopt the best engineering methods in solving difficult design and construction problems and their decision to use field tests in this case illustrates the alertness of their engineering personnel. The writer was fortunate in having the opportunity of discussing these experiments with their engineers at Knoxville, Tenn., prior to the actual running of the tests. Agreement was reached that by no other means could a more lucid picture be given of the results of biaxial loading of a laminated shale foundation. The use of six test blocks and the several observations of various phases during and after the experiments guaranteed the maximum amount of information from these tests.

Equation (5) is the generalization of Equation (3) of the paper; and, as pointed out in one of the other discussions, the relation between shear and normal pressure is not a straight line over the entire range. In Fig. 13(*a*), it is possible to draw a smooth curve through all points except 6, indicating a somewhat smaller value of shearing strength under conditions of no load than that obtained by Mr. Rountree's analysis. This smooth but irregular curve does not pass through the origin, confirming the fact that shear occurred in the shale and not principally along the contact plane of the concrete and shale.

Acknowledgments.—Tests described in the paper were made as a part of the investigation and design of Possum Kingdom Dam by the U. S. Army Engineers. After the completion of investigation and detailed plans the dam was built under the direction of a State agency employing a firm of private engineers. For the Army Engineers, Captain H. A. Montgomery was District Engineer in charge. Acting Principal Engineer, W. L. Kuehnle, Assoc. M. Am. Soc. C. E., exercised supervision over the conduct of the tests and offered valuable suggestions and observations. The part of the paper under "Geology" was prepared from the report of Warren J. Mead,[8] Affiliate Am. Soc. C. E., submitted to the district engineer.

The writer is pleased to acknowledge the excellent assistance of Logan Woolley, Jun. Am. Soc. C. E., and Mr. Edwin Jensen in the compilation and recording of data. The preparation of this paper and the interpretation of test results were among the duties of the writer. Messrs. Jensen and Floyd Johnson, formerly of the U. S. Army Engineer Office at Mineral Wells, Tex., directed the field work.

---

[8] "Geological Conditions at Possum Kingdom Dam Site," by Warren J. Mead, dated May 14, 1936, pp. 2, 3, and 4 (not published).

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2091

RELATION OF THE STATISTICAL THEORY OF TURBULENCE TO HYDRAULICS

BY A. A. KALINSKE,[1] ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. HUNTER ROUSE, MARTIN A. MASON, J. C. STEVENS, BORIS A. BAKHMETEFF, CLYDE W. HUBBARD, JOHN S. MCNOWN, SAMUEL SHULITS, PAUL NEMENYI, BENNIE N. NETZER, AND A. A. KALINSKE.

SYNOPSIS

The basic principles of the statistical theory of fluid turbulence are outlined in this paper. Only those ideas and conceptions of importance in hydraulics problems are presented; the more complicated mathematical details are omitted. Turbulence can be characterized by intensity parameters which are the root mean square values of the fluctuating velocity components, and a length parameter proportional to the most probable eddy size. The specific parameters used in characterizing the energy dissipation and the diffusion power of turbulence are derived, and experimental techniques and data are presented to illustrate how these parameters are obtained. The two major problems in hydraulics in which knowledge of the turbulence mechanism is of importance are those involving the increase or decrease of energy dissipation, and the transportation of sedimentary materials. The practical application of the theories of turbulence to some specific problems is indicated. The experimental data were obtained by injecting color streams and suspended immiscible liquid droplets into turbulent water streams and by using motion-picture photography.

PURPOSE OF PAPER

Since about 1930, both in Europe and in the United States, considerable research on fluid flow has been concerned with the fundamental aspects of the mechanism of fluid turbulence. In aeronautics, wind-tunnel and atmospheric turbulence has been an important field of research since about 1925. Some of the findings of the aeronautics engineers and physicists have been permeating gradually into the hydraulics literature. Certain fundamental investigations

---

NOTE.—Published in October, 1939, *Proceedings*.

[1] Asst. Prof., Univ. of Iowa, Iowa Inst. of Hydraulic Research, Iowa City, Iowa.

on water turbulence have been conducted, and more and more are being started. However, all this talk and writing about turbulence generally leaves the practical hydraulic engineer quite uninterested. The practical usefulness, either immediate or future, of knowledge relating to turbulence does not seem very apparent.

It is the purpose of this paper to correlate the knowledge of fluid turbulence with the solution of some practical hydraulics problems, and to present the information on fluid turbulence, already in existence, in a more simplified form. Some experimental data obtained by the writer and others will be presented, and various experimental techniques for obtaining pertinent information on liquid turbulence will be discussed.

What Is Turbulence?

Turbulence, as it is present in wind-tunnel air streams, has been studied for some time and has gradually attracted the attention of hydraulics engineers, because there are many hydraulics problems which would be better understood if more were known of the mechanism of turbulence.

When observing turbulence by injecting a dye or some other material into water, or when observing smoke coming from a chimney, a most haphazard mixing process is seen, seemingly unpredictable in nature. Fundamentally, it may be said that a fluid at any point is in a state of turbulence if the direction and magnitude of its velocity vector vary irregularly with time. This variation is relatively rapid and in general cannot be predicted except in the probability sense. Turbulence may be caused as a result of high shearing stresses set up in a fluid in motion, by obstructions in the path of a fluid, by passing a fluid through a screen or honeycomb, or by some means such as mechanical stirring. Confusion has arisen in the minds of some regarding whether or not there is a definite periodic relationship between a velocity component at a single point with respect to time. True turbulence is of an entirely random nature without any kind of regular periodicity. A stream of discrete vortices which form directly behind a body immersed in a moving fluid, such as the von Kármán vortex street, is not true turbulence; it is referred to as vortex motion, which breaks down eventually into true turbulence.

Turbulence manifests itself in the form of large and small eddies, whirling about from one layer of fluid to another. These eddies can transfer momentum, mass, heat, etc., from layer to layer, and can also cause the high dissipation of energy that characterizes turbulent flow. It is apparent then that, to characterize various quantities regarding turbulence (such as, for instance, the size of the eddies, the velocity at a point, or the frequency of velocity fluctuation), average quantities must be dealt with and the principles of statistics employed in determining and in using these quantities. Additional fundamental ideas regarding turbulence can be found in the writings of H. L. Dryden,[2] Hunter Rouse,[3] Assoc. M. Am. Soc. C. E., and Boris A. Bakhmeteff,[4] M. Am. Soc. C. E.

---

[2] "Measurements of Intensity and Scale of Wind-Tunnel Turbulence," by H. L. Dryden, *Report No. 581*, National Advisory Comm. for Aeronautics, 1937.

[3] "Modern Conceptions of the Mechanics of Fluid Turbulence," by Hunter Rouse, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 463.

[4] "Mechanics of Turbulent Flow," by Boris A. Bakhmeteff, Princeton University Press, 1936.

BASIC PRINCIPLES OF STATISTICAL THEORY OF TURBULENCE

At the outset let it be noted that a complete and mathematically rigorous theory of turbulence has not as yet been developed. However, as far as the hydraulics engineer is concerned, such a theory, even if available, would be of little use. The mathematical difficulties are so great that the development of a rigorous analysis has taxed some of the best mathematical minds. The various ideas that have helped greatly in understanding the mechanism of turbulence are those developed by L. Prandtl,[5] G. I. Taylor,[6] and Theodor von Kármán,[7] M. Am. Soc. C. E. The basic statistical principles they have set forth have been checked experimentally, and are of fundamental importance in an understanding of the turbulence mechanism.

It is convenient to represent the varying velocity vector at any point in turbulent flow by means of the velocity components along three rectangular axes. The velocity in the direction of mean flow, which is taken along the x-axis, at any instant is represented as $(U \pm u)$, in which U is the mean velocity in that direction and u is the fluctuation in that direction. Obviously, the arithmetic mean of u is zero. The velocity along the other two axes, y and z, is represented as $(V \pm v)$ and $(W \pm w)$, respectively. If the mean flow is in the x-direction, V and W are zero. Since u, v, and w are quantities that vary with time, it is useful to represent them by certain statistical averages. Before this is done, the statistical distribution of these quantities should be investigated. That is, what is the form, for instance, of $f(v)$ which is a function such that $f(v_i)\, dv$ indicates the probability that the value of v lies between v_i and $v_i + dv$? Such a function, $f(v)$, is called a statistical distribution function. If the value of v could be determined at any point for a sufficiently long period of time the general form of $f(v)$ could be found.

The value of the velocity component v with respect to time is shown in Fig. 1. These data were obtained in a turbulent water stream at a point near the center of a 12-in. by 12-in. open channel where the mean velocity was 7.8 in. per sec (see heading "Experimental Technique"). A block diagram indicating the frequency of occurrence of various values of v is shown in Fig. 2. The smooth curve in Fig. 2 has the equation,

$$f(v) = y = \frac{1}{\sqrt{2\,\pi}\,\sqrt{\overline{v^2}}}\, e^{-a v^2} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which: $a = \dfrac{1}{2\,\overline{v^2}}$, to simplify typography, and $\sqrt{\overline{v^2}} = 0.725$ for the data in Fig. 1. The bar over a term or terms indicates a mean taken with respect to time.

This is the ordinary normal error law. Other investigators[8] have found the same type of distribution law, thus indicating the random nature of the velocity fluctuations. If the distribution function is of the same general form, different degrees of turbulence can be indicated by some average value of the quantities u, v, and w. The most significant average is that called the root mean square, or standard deviation, designated as $\sqrt{\overline{v^2}}$, for instance. The standard deviation indicates the spread of the distribution curve; also, its square, such as $\overline{v^2}$, is proportional to the energy of the turbulence for that velocity component.

---

[5] "Aerodynamic Theory," by W. F. Durand, Vol. III, J. Springer, Berlin, 1935, p. 162.

[6] "Statistical Theory of Turbulence," by G. I. Taylor, *Proceedings*, Royal Soc. of London, Vol. 151A, 1935, p. 421.

[7] "Turbulence," by T. von Kármán, *Journal*, Royal Aeronautical Soc., Vol. 41, No. 324, December 1937, p. 1109.

[8] "Investigation and Analysis of Velocity Variation in Turbulent Flow," by L. F. G. Simmons and C. Salter, *Proceedings*, Royal Soc. of London, Vol. 145, 1934, p. 242.

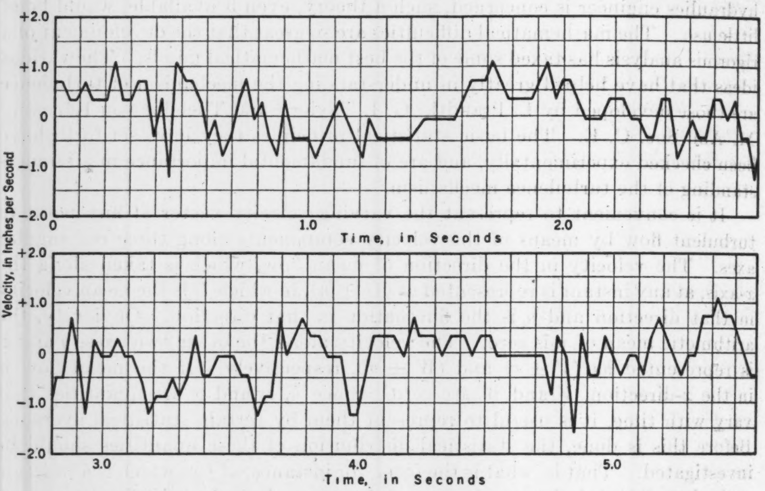


FIG. 1.—TRANSVERSE VELOCITY FLUCTUATION FOR TURBULENT FLOW IN A CHANNEL

Various types of turbulence can be designated depending on the relative magnitude and interrelationship of u, v, and w. The simplest type of turbulence, from the mathematical viewpoint, is that referred to as isotropic turbulence. In this type of turbulence $\overline{v^2} = \overline{u^2} = \overline{w^2}$, and also quantities of the type $\overline{u\,v}$, $\overline{u\,w}$, and $\overline{v\,w}$ are zero. These terms are referred to as mean products in statistics. If quantities such as $\overline{u\,v}$ are different from zero, a mean shearing stress and a mean velocity gradient exist in the fluid. In ordinary two-dimensional flow the mean shear stress, τ, is represented as,

$$\tau = \rho\, \overline{v\,u} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which ρ = unit density. A simple derivation of this is given by Messrs. Rouse[3] and Bakhmeteff.[4] Isotropic turbulence, therefore, cannot exist near a boundary. It is only approached in the center of large conduits and in artificially created turbulence, such as that beyond screens or grids. The turbulence beyond a hydraulic jump probably approaches isotropy.

Another fundamental concept that is useful in discussing turbulence is that of the correlation coefficient. In order to characterize a turbulent condition more completely, in addition to the quantities $\overline{u^2}$, $\overline{v^2}$, and $\overline{w^2}$, which characterize the intensity of the turbulence, a scale factor is necessary. Two different turbulent flows may have the same intensity, but the average size of the eddies may be entirely different. In order to identify the scale of the turbulence, Professor Taylor[6] has suggested that the instantaneous velocity components, either v or u, be measured simultaneously at two different points, along either the x-axis or the y-axis, over an appreciable period of time. Suppose v is measured simultaneously at two points a distance x apart. Let v_1 be the velocity at one point and v_2 that at the other point at the same instant, and define a number R_x such that,

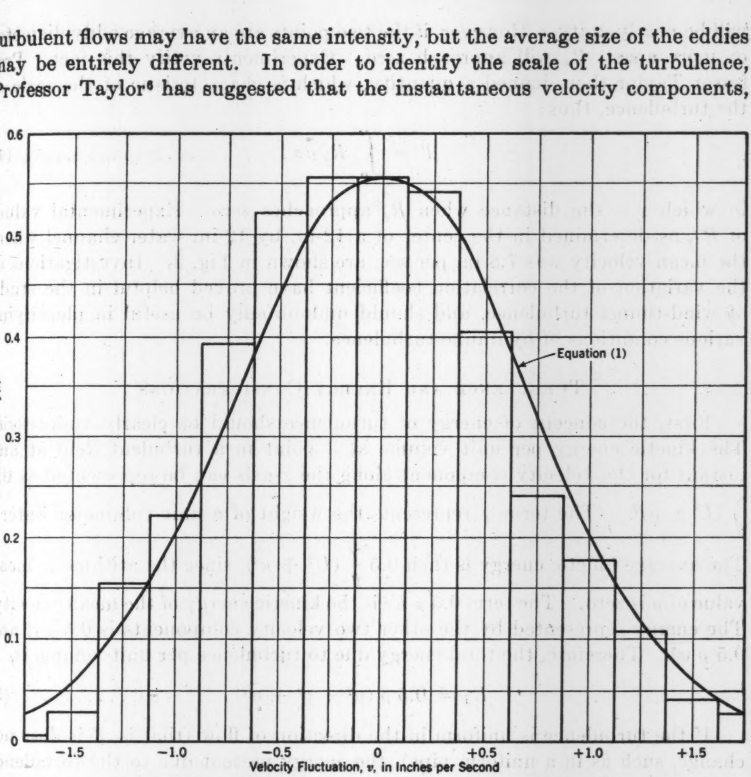


FIG. 2.—FREQUENCY DISTRIBUTION CURVE FOR TRANSVERSE VELOCITY FLUCTUATION

$$R_x = 1 - \frac{\overline{(v_1 - v_2)^2}}{2\,\overline{v^2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3a)$$

Since $\overline{v_1^2} = \overline{v_2^2} = \overline{v^2}$ Equation (3a) can be transformed into

$$R_x = \frac{\overline{v_1\,v_2}}{\overline{v^2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3b)$$

The term R_x is known in statistics as a correlation coefficient. Now, if the two points are close so that they are within the average size eddy, v_1 and v_2 will tend to be of approximately equal magnitude and of the same sign, and R_x

will be nearly unity. However, if the two points are an average eddy diameter or more apart, R_x will approach zero. Experiments verify this fact. Professor Taylor then defined a quantity, which is characteristic of the scale of the turbulence, thus:

$$l' = \int_0^x R_x \, dx \ldots\ldots\ldots\ldots (4)$$

in which x = the distance when R_x approaches zero. Experimental values of R_x, as determined in the center of a 12-in. by 12-in. water channel where the mean velocity was 7.8 in. per sec, are shown in Fig. 3. Investigations of the variation of the correlation coefficient have proved helpful in the study of wind-tunnel turbulence, and should undoubtedly be useful in identifying various conditions of hydraulic turbulence.

TURBULENCE AND ENERGY CONSIDERATIONS

First, the concept of energy of turbulence should be clearly understood. The kinetic energy per unit volume at a point in a turbulent fluid at any instant for the velocity component along the x-axis can be represented as $0.5 \frac{\gamma}{g} (U \pm u)^2$. (The term γ represents the weight of a unit volume of water.) The average kinetic energy is then $0.5 \frac{\gamma}{g} (U^2 + \overline{u^2})$, since the arithmetic mean value of u is zero. The term $0.5 \rho U^2$ is the kinetic energy of the mean velocity. The energy represented by the other two velocity components is $0.5 \rho \overline{v^2}$ and $0.5 \rho \overline{w^2}$. Therefore, the total energy due to turbulence per unit volume is,

$$E_T = 0.5 \rho (\overline{u^2} + \overline{v^2} + \overline{w^2}) \ldots\ldots\ldots\ldots (5)$$

If the turbulence is uniform in the direction of flow (that is, if it does not change, such as in a uniform pipe), the energy present due to the turbulence need not be taken into account in any energy equation. However, if the energy of turbulence varies from point to point, such as occurs in conduit expansions, beyond bends, through valves, etc., its omission, in any study of energy changes, results in an error. An important item regarding turbulent energy is that it is a form of degraded energy and will eventually be transformed into heat, and can never be recovered.

How does turbulence cause the dissipation of energy? All energy is dissipated into heat through the action of viscosity. If there were no viscosity, there would be practically no energy loss, and any turbulent energy created, such as in a sudden expansion, would tend to persist indefinitely. The high internal shear stresses set up by the turbulent eddies cause the large energy dissipation in turbulent flow.

In pipe flow, for instance, potential energy is being dissipated constantly into the form of heat. For turbulent flow, a simplified derivation for the rate of transference of the potential energy producing flow into energy of turbulence is as follows: Let the mean tangential shearing stress at a point in a circular pipe be τ. The mean difference in velocity between two layers of fluid is ΔU.

The work done per unit time by the force τ on an area $(\Delta x \, \Delta z)$ is then equal to $(\tau \, \Delta U \, \Delta x \, \Delta z)$. The work done per unit of time per unit volume of fluid is $\tau \frac{\Delta U}{\Delta y}$, or, in the limit, $\tau \frac{dU}{dy}$, in which $\frac{dU}{dy}$ is the mean velocity gradient. The mean unit shearing stress, τ, at a point distant y from the center of a pipe, of radius r is equal to:

$$\tau = \frac{h_f \, y \, \gamma}{2 \, L} \quad \text{.......................} (6)$$

in which: h_f = loss of pressure head in pipe length, L, and γ = unit fluid weight. This shear stress can also be represented as,

$$\tau = \mu \frac{dU}{dy} + \rho \, \epsilon \frac{dU}{dy} \quad \text{.......................} (7)$$

in which: μ = viscosity; ρ = unit density; and ϵ = mechanical viscosity due to turbulence (may vary from point to point). Except near the wall, in fully developed turbulent flow, the quantity $\mu \frac{dU}{dy}$ is very small in comparison to $\rho \, \epsilon \frac{dU}{dy}$. Thus in turbulent flow most of the potential energy is first transferred into turbulent energy at a rate represented by $\tau \frac{dU}{dy}$ per unit volume. The turbulent energy is then changed into heat by the action of viscosity. The rate at which potential energy is transferred into turbulent energy is not necessarily equal to the rate of energy dissipation into heat, at any one point. The turbulent energy can be diffused to other points by the action of the eddies, and then dissipated into heat.

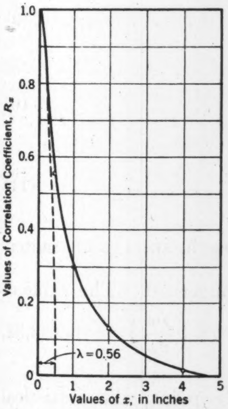


FIG. 3.—DISTANCE CORRELATION COEFFICIENT FOR TRANSVERSE VELOCITY, v

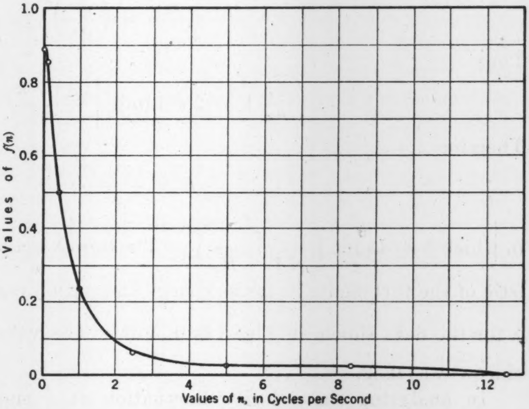


FIG. 4.—DISTRIBUTION OF TURBULENT ENERGY AMONG DIFFERENT FREQUENCIES

The mechanism by which turbulent energy is dissipated into heat is quite complicated, even in the simple case of isotropic turbulence. The rate of this energy dissipation is proportional to the viscosity times the mean square of the various instantaneous turbulent velocity gradients—that is, quantities such as $\mu \overline{\left(\frac{\partial v}{\partial x}\right)^2}$. The partial derivative sign is used because the component v also varies with y and z. The general formula for turbulent energy dissipation is given and discussed by Professor Taylor.[9] For isotropic turbulence Professor Taylor has shown that the mean rate of dissipation of energy per unit volume of fluid is:

$$K = 7.5\, \mu \overline{\left(\frac{\partial v}{\partial x}\right)^2} \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Various simple relations exist between the different mean turbulent velocity gradients in isotropic turbulence. For instance $\overline{\left(\frac{\partial v}{\partial x}\right)^2} = \overline{\left(\frac{\partial u}{\partial y}\right)^2}$, and $\overline{\left(\frac{\partial v}{\partial y}\right)^2} = \overline{\left(\frac{\partial u}{\partial x}\right)^2}$.

One method of determining the value of a quantity such as $\overline{\left(\frac{\partial v}{\partial x}\right)^2}$ is by use of the correlation coefficient concept. Recalling the definition of R_x from Equations (3), if the values $\overline{(v_1 - v_2)^2}$ are obtained for small values of x, then approximately,

$$\frac{\overline{(v_1 - v_2)^2}}{x^2} = \overline{\left(\frac{\partial v}{\partial x}\right)^2} \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Thus,

$$\overline{\left(\frac{\partial v}{\partial x}\right)^2} = 2\, \overline{v^2} \operatorname*{Limit}_{x \to 0} \left[\frac{(1 - R_x)}{x^2}\right] \ldots\ldots\ldots\ldots (10)$$

Therefore,

$$K = \frac{15\, \mu\, \overline{v^2}}{\lambda^2} \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

in which $\lambda = \operatorname*{Limit}_{x \to 0} \left(\frac{x}{\sqrt{1 - R_x}}\right)$. The term λ is a length, and is a characteristic of the turbulence as far as energy dissipation is concerned. The value of λ for the data shown in Fig. 3 is 0.56 in. The value of $\overline{\left(\frac{\partial v}{\partial x}\right)^2}$ is then 3.32, since $\overline{v^2}$ was 0.52.

In analyzing the velocity fluctuation at a single point, the statistical distribution function was found to indicate the frequency of occurrence of various values of v, as shown in Figs. 1 and 2. It is of interest to make a similar type of statistical analysis of the distribution of the energy of the turbulence, such as $0.5\, \rho\, \overline{v^2}$, among the various frequencies. This will be useful in showing what is the average magnitude of the frequency of the velocity fluctuation. A function $f(n)$ is to be determined such that $f(n)\,dn$ indicates the most probable proportion of the total energy, as represented by $0.5\,\rho\,\overline{v^2}$, lying between the frequencies n and $(n + dn)$. More exactly, $f(n)\,dn$ does not indicate the fractional part of the total energy lying between the frequencies n and $(n + dn)$, but rather, the probability of the total energy lying in that range of frequencies. The method of the determination of $f(n)$, from experimental velocity fluctuation data, has been indicated by the writer in a previous paper.[10] It can be shown that,

---

[9] "Statistical Theory of Turbulence," by G. I. Taylor, *Proceedings*, Royal Soc. of London, Vol. 151A, 1935, p. 430.

$$\overline{\left(\frac{\partial v}{\partial x}\right)^2} = \frac{4\,\pi^2\,\overline{v^2}\,\overline{n^2}}{U^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

in which $\overline{n^2} = \int_0^\infty n^2 f(n)\,dn$, since $\int_0^\infty f(n)\,dn = 1$. A plotting of $f(n)$ against n is shown in Fig. 4 for the velocity data shown partly in Fig. 1. The value of $\sqrt{\overline{n^2}} = 3.25$ and that of $\overline{\left(\frac{\partial v}{\partial x}\right)^2}$ is 3.56. This determination of $\overline{\left(\frac{\partial v}{\partial x}\right)^2}$ checks very well with the calculation by the correlation coefficient method as given by Equation (10). That the two methods are mathematically equivalent can be demonstrated. However, different data are used in each method to obtain the same final result.

DIFFUSION CHARACTERISTICS OF TURBULENCE

Turbulence eddies are capable of causing the exchange of momentum, heat, matter, etc., from one layer of fluid to another. Near solid boundaries the existence of a mean velocity gradient across a turbulent fluid indicates that momentum is being transferred from one layer of fluid to another. A temperature gradient indicates the transfer of heat; and a suspended material gradient in a turbulent stream indicates the transfer of matter. The characterization of a turbulent flow as to its diffusing or exchange power is an important item in any investigation of turbulence. The process of mixing in a turbulent fluid stream is often referred to as "eddy diffusion."

The laws of molecular diffusion have been applied to mass transfer processes in turbulent fluid streams.[11] However, the complete validity of this procedure is open to question, since the motion of fluid masses is not comparable to that of gas molecules. A common and convenient method of studying the diffusing power of turbulence is to observe the spread of heat, matter, or color from a point source of the heat, matter, or color injected in the turbulent fluid stream. Experiments were made by injecting a constant stream of color through a fine tube, and also by injecting colored droplets of a mixture of carbon tetrachloride and benzol, having the same specific gravity as water, and photographing the transverse spread of the color, or the droplets, with a motion-picture camera. The important quantity to determine from such experiments is the mean square cross-stream or transverse travel of the color at various distances downstream from the point of injection. Fig. 5 shows some sketches of typical motion-picture frames of the color and the droplets. The transverse travel at any distance x downstream is called Y and its mean square value $\overline{Y^2}$. Professor Taylor[12] has developed a theory relating x and $\overline{Y^2}$. His general premise is that the statistical correlation between the velocity of a fluid mass at one instant and at another later instant is high when the time interval is small, and this correlation approaches zero when the interval becomes large. This theory shows that $\overline{Y^2}$ varies as x^2 for small values of x, and gradually approaches a direct variation with x, as x increases. Professor Taylor[12] showed that

---

[10] "Application of Statistical Theory of Turbulence to Hydraulic Problems," by A. A. Kalinske and E. R. Van Driest, Jun. Am. Soc. C. E., *Proceedings*, Fifth International Cong. of Applied Mechanics, 1938.

[11] "Studies in Eddy Diffusion," by W. L. Towle, T. K. Sherwood, and L. A. Seder, *loc. cit.*, 1938.

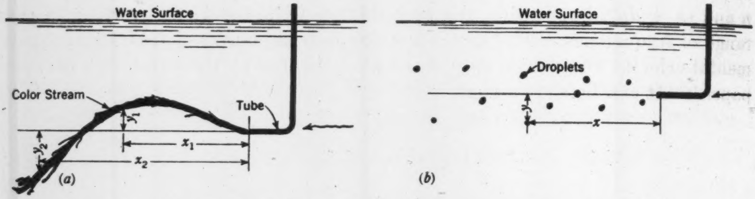


FIG. 5.—SKETCHES OF TYPICAL MOTION-PICTURE FRAMES, SHOWING COLOR STREAM, (a), AND COLORED IMMISCIBLE DROPLETS, (b)

$$0.5\,\frac{d\overline{Y^2}}{dt} = \overline{v^2}\int_0^t R_t\,dt \qquad (13)$$

in which: $\overline{v^2}$ = mean square transverse velocity fluctuation; R_t = correlation coefficient between velocity of a fluid mass at one instant and at an instant, t, later $= \frac{\overline{v\,v_t}}{\overline{v^2}}$; $t = \frac{x}{U}$; and U = the mean forward velocity. When R_t is near unity $\int_0^t R_t\,dt$ is equal to t and $\overline{Y^2}$ is given by:

$$\overline{Y^2} = \frac{\overline{v^2}\,x^2}{U^2} \qquad (14)$$

When R_t approaches zero, $\int_0^t R_t\,dt$ becomes a constant, k, and $\overline{Y^2}$ is given by:

$$\overline{Y^2} = \frac{2\,\overline{v^2}\,k\,x}{U} \qquad (15)$$

Fig. 6 shows experimental data obtained by the writer on the relation of $\overline{Y^2}$ and x in a turbulent water stream near the center of a 12-in. by 12-in. open channel at a point where the mean velocity was 7.8 in. per sec and $\sqrt{\overline{v^2}}$ was 0.78 in. per sec. Note the initial linear relationship of $\sqrt{\overline{Y^2}}$ and x; and at larger values of x, the linear relationship of $\overline{Y^2}$ and x. Note also that the initial slope of the $\sqrt{\overline{Y^2}}$-curve determines the value of $\sqrt{\overline{v^2}}$ according to Equation (14).

---

[12] "Diffusion by Continuous Movements," by G. I. Taylor, *Proceedings*, London Math. Soc., Vol. 20 1921, p. 196.

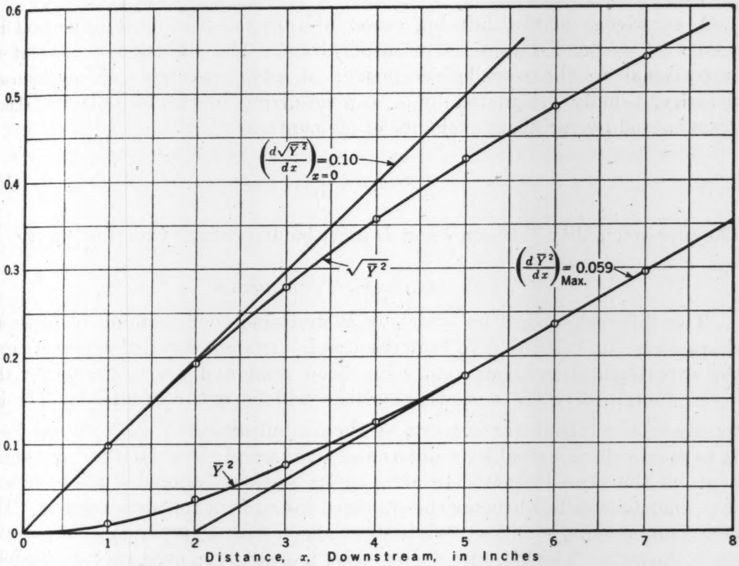


FIG. 6.—EXPERIMENTAL DATA ON DIFFUSION IN A TURBULENT WATER STREAM (Bars Over Squared Terms Indicate the Mean of the Squares and not the Square of the Mean)

A mathematical relationship between $\overline{Y^2}$ and x is not a very convenient means of characterizing the diffusing power of turbulence. A diffusion coefficient would be the logical thing to determine. Using the ideas of molecular diffusion theory, such a coefficient would be defined as:

$$D = \frac{U}{2}\left(\frac{d\overline{Y^2}}{dx}\right)_{\max} \dots\dots\dots\dots\dots\dots\dots (16)$$

The value of $\left(\frac{d\overline{Y^2}}{dx}\right)_{\max}$ for the data in Fig. 6 is 0.059. The dimensions of D are $\frac{L^2}{T}$, or a velocity times a length. The velocity is the root mean square transverse velocity, $\sqrt{\overline{v^2}}$, and the length term is the length associated with mixing processes. This length term is proportional to the size of the turbulent eddies, and to Prandtl's so-called "mixing length."

It should be added that the relative concentration of the color or droplets transverse to the turbulent stream varied according to the normal error curve; that is,

$$N = \frac{1}{\sqrt{2\,\pi}\,\sigma}\,e^{-b\,Y^2} \dots\dots\dots\dots\dots\dots\dots (17)$$

in which: $b = \frac{1}{2\,\sigma^2}$, to simplify typography; and $\sigma^2 = \overline{Y^2}$, so that $b = \frac{1}{2\,\overline{Y^2}}$. The probability that a particle would lie between Y and $Y + dY$ is given by $N\,dY$.

A knowledge of the diffusing power of turbulence is most important in studies of suspended material transportation. The diffusion coefficient is proportional to the so-called coefficient of eddy viscosity, or mechanical viscosity, usually designated by ϵ, and occurring in the formula for shear stress caused by turbulent exchange of momentum:

$$\tau = \rho\,\epsilon\,\frac{dU}{dy} \qquad (18)$$

The proportionality between ϵ and D must be determined experimentally.

Experimental Techniques

The different quantities which it is desirable to determine in order to characterize turbulence have been discussed. A more detailed explanation of the experimental methods that have been used and are available for the determination of these various quantities will be outlined briefly. The intensity of the turbulence is given by the magnitudes of $\sqrt{\overline{u^2}}$, $\sqrt{\overline{v^2}}$, and $\sqrt{\overline{w^2}}$. It is practically impossible to determine all three of these fluctuating components at the same instant. In aeronautics and wind-tunnel studies the hot-wire anemometer has become the standard instrument for measuring $\overline{u^2}$. The cross components, $\overline{v^2}$ and $\overline{w^2}$, can be determined with a special type of hot-wire anemometer.[13] The hot-wire anemometer has not been used very successfully in water, due primarily to difficulties such as collection of air bubbles and other foreign matter on the wire, the extreme magnification of the fluctuating current that is necessary in high-velocity water, and various other troubles. Nevertheless, it has been used in water at lower velocities and undoubtedly will find further use.

One of the most convenient methods of determining simultaneous values of u and v, or u and w, in water, is by studying the motions of particles suspended in the flowing water. If illuminated, such particles make streaks on motion-picture film, and from the length and direction of the streaks two components of the velocity can be determined. Particles made from a mixture of carbon tetrachloride and benzine, having a specific gravity equal to that of water, have been used. The particles are illuminated by a narrow plane of light, so that only those particles in the light are visible.

Another method used by the writer to obtain values of the cross velocity, v, for very short intervals of time, involves a thin color stream injected into the stream through a fine needle. A motion picture was taken of this stream close to the needle. The velocity v for very short intervals of time is obtained by measuring the transverse travel Y for short distances, x, downstream. Distance x is short enough so that the direction of the color stream is practically a straight line. The value of v is then $\frac{Y}{t}$, in which t is $\frac{x}{U}$. A large number of values of v enable the calculation of $\sqrt{\overline{v^2}}$. Such a color stream is shown in Fig. 5. The writer has found that good measurements of simultaneous values of u and v can be made by photographing, with a constant speed motion-picture camera, the travel of the immiscible droplets for a distance of about 1 in. from the point of injection. A magnifying type of close-focusing lens is used on the camera so that a picture is obtained of an area of about 2 sq in. Following the movement of the particles from frame to frame on the film permits the determination of u and v, and if sufficient data are obtained the quantities $\sqrt{\overline{v^2}}$, $\sqrt{\overline{u^2}}$, and $\overline{u\,v}$ can be calculated.

---

[13] National Advisory Comm. for Aeronautics, *Technical Report No. 581*.

Another method of determining $\sqrt{\overline{v^2}}$ is from diffusion experiments, by finding the initial slope of the $\sqrt{\overline{Y^2}}$-curve, as shown in Fig. 6, and using Equation (14). The energy dissipative characteristics are determined by finding a factor proportional to the mean square of the turbulent velocity gradients such as λ, which was defined by Equation (11), or the factor $\overline{n^2}$, as defined in Equation (12). The term λ can be determined by measuring the correlation coefficient R_x, and the term $\overline{n^2}$ is calculated by analyzing the velocity fluctuation at a single point. In air flow R_x is determined by use of two hot-wire anemometers placed a distance x apart. The writer was able to measure R_x by injecting color at two points, a distance x apart, simultaneously, and photographing the two streams together.

The determination of the diffusion coefficient by diffusion experiments has been discussed. Undoubtedly, this is the most direct means of finding the diffusion characteristic of turbulence. Evidence exists that the diffusion coefficient is related to the intensity of the turbulence as given by $\sqrt{\overline{v^2}}$, and the length factor λ. The term λ is proportional to the size of the eddies, and the greater it is, the greater is the diffusion for a given value of $\sqrt{\overline{v^2}}$.

Unquestionably, the photographic method is an accurate and ideal method for making fundamental studies of turbulence in the laboratory. It is time-consuming and tedious, particularly the analyzing of the data recorded on the motion-picture film. An instrument that would give accurate recordings of the instantaneous velocity in a direction either longitudinal or transverse to the flow would be useful. Of course, such an instrument would have to be compared with data obtained by the photographic method in order to insure that it is following the velocity fluctuations accurately and is not introducing turbulent effects of its own. A forward step in the development of such an instrument has been made by C. W. Hubbard,[14] who has developed a so-called "angularity" indicator which measures the mean angle of flow. This angle, of course, is proportional to the mean transverse velocity.

The cup type of current meter is very sensitive to turbulence. However, because of its inertia it does not follow all the velocity fluctuations exactly. It does give some information regarding the magnitude of the velocity fluctuations and the size of the eddies, if the speed of rotation of the vane is obtained, by some automatic recording arrangement, for short intervals of time over an extended period.

---

[14] "Investigation of Errors in Pitot Tubes," by C. W. Hubbard, *Transactions*, A. S. M. E., Vol. 61, 1939, p. 477.

RELATION OF TURBULENCE RESEARCH TO HYDRAULICS PROBLEMS

Energy Dissipation.—There are many problems in aeronautics, physics, chemical engineering, and meteorology in which knowledge of the turbulence mechanism is quite important. However, this paper will be confined entirely to outlining some specific problems in hydraulic engineering which could be solved more accurately and more completely with information on turbulence.

Considering the general problem of energy dissipation, there are various individual problems which require a knowledge of the manner in which energy is dissipated. There are some design problems in which it is desired to obtain the maximum energy dissipation possible, and some in which the energy dissipation must be a minimum.

In a conduit of expanding cross section, the additional losses of energy associated with such an expansion are directly attributable to the creation of turbulence energy beyond that ordinarily present in uniform flow. In such an expansion, whether gradual or sudden, there is first an increase in the energy of turbulence and then a decrease.

In order to study the energy transformations completely, the energy of turbulence as given by Equation (5) must be found, in addition to the ordinary kinetic energy, as computed from the average velocity distribution. Fig. 7 shows some typical velocity data as obtained by the method of streak motion pictures in a 3-in. circular pipe. Fig. 8 shows similar data for a section of a 3-in. by 5-in. expansion. The rate of flow in both cases was 0.0818 cu ft per sec. In computing the turbulent energy the transverse velocity component, $\overline{w^2}$, was assumed equal to $\overline{v^2}$; this is quite true for flow in a circular conduit, except very near to the wall. The total mean kinetic energy as obtained from the mean velocity distribution is given by,

$$E_m = \pi \rho \int_0^r y \, U^3 \, dy \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

The total energy of turbulence is

$$E_t = \pi \rho \int_0^r y \, U \, (\overline{u^2} + \overline{v^2} + \overline{w^2}) \, dy \ldots\ldots\ldots\ldots (20)$$

The value of E_m in the 3-in. pipe is 0.255 ft-lb, and E_t is 0.00793 ft-lb. In this case E_t is 3.1% of E_m. The value of E_m in the expansion at the 3.75 diameter section is 0.157 ft-lb, and E_t is 0.0135 ft-lb, or 8.6% of E_m. Note the relatively higher value of the turbulent energy in the expanding portion. Data such as that given in Fig. 8 enable the accurate study of the energy changes taking place from point to point.

Although turbulent energy can be created very quickly, for instance, in a sudden expansion, the dissipation of that energy does not occur immediately. The expression "shock loss" is used many times to describe the energy loss at

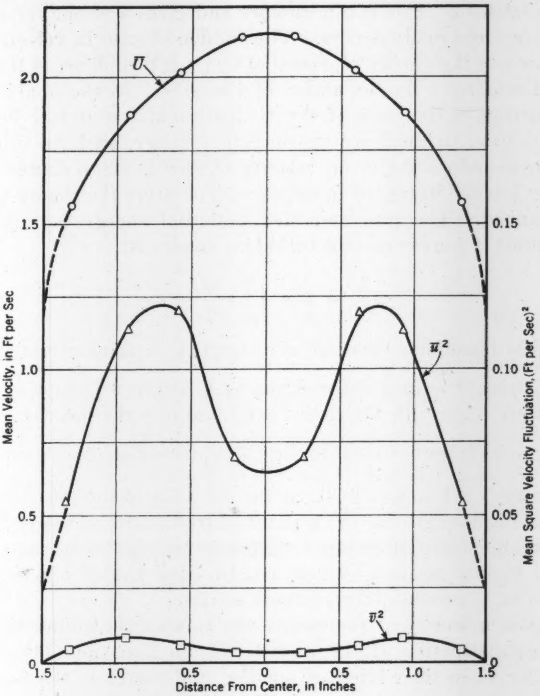


FIG. 7.—VELOCITY DATA FOR A 3-IN. UNIFORM CONDUIT

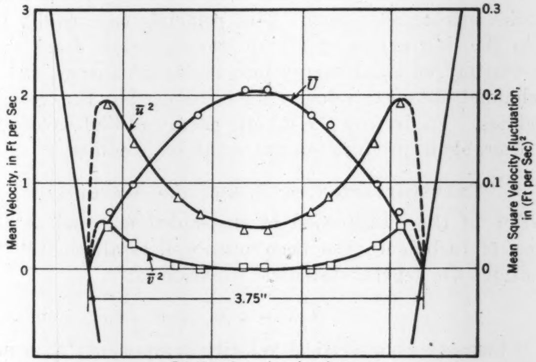


FIG. 8.—VELOCITY DATA FOR A SECTION IN A GRADUAL 3-IN. BY 5-IN. EXPANDING CONDUIT (Bars Over Squared Terms Indicate the Mean of the Squares and Not the Square of the Mean)

a sudden expansion. This is a misnomer and gives a wrong impression as to how the energy loss really occurs. The sudden change in velocity causes extreme turbulence; the energy represented by such turbulence is then gradually dissipated downstream by the action of viscosity. In the final analysis, viscosity, or friction, is the cause of the dissipation of energy into heat.

If it is desired to dissipate considerable energy, such as below dams or spillways, or to reduce the mean velocity of flow in steep chutes or conduits, much can be learned in regard to such problems from the theory of the turbulence mechanism. The rate at which potential energy per unit length of circular conduit is converted into turbulent energy is:

$$Q \frac{h_f}{L} = 2 \pi \int_0^r \tau \frac{dU}{dy} y \, dy \ldots\ldots\ldots\ldots (21)$$

in which: $\frac{h_f}{L}$ = loss of head per foot of conduit; Q = total weight of water flowing in unit time; τ = unit shear stress at y distance from the center; and r = pipe radius. For fully developed turbulent flow the shear stress is equal to $\rho \, \epsilon \frac{dU}{dy}$, in which ϵ is proportional to the diffusion coefficient, or more specifically, to the quantity $\sqrt{\overline{v^2}} \, l$, in which $\sqrt{\overline{v^2}}$ is the intensity of the transverse turbulent velocity, and l is a length factor determined by the size of the eddies. Therefore, the rate of potential energy transformation can be increased, assuming Q constant, by increasing ϵ, which can be done by introducing large-scale roughness so as to produce large, intense eddies.

For a given intensity of turbulence the smaller the eddies the higher the rate of energy dissipation will be, as is seen from Equation (11). The term λ will be smaller when the eddies are smaller, and therefore the average rate of energy dissipation will be greater. Thus, it appears that once the potential energy or mean kinetic energy of flow is in the form of turbulent energy, the dissipation of this turbulent energy can be hastened by the breaking down of the large eddies into smaller ones. This principle can readily be taken advantage of in the destruction of kinetic energy below dams and spillways. Thus, in converting potential energy into turbulent energy, such as for flow in a pipe or channel, the mean velocity can be reduced by the creation of intense large-scale eddies. To destroy the kinetic energy of high-velocity flow below a dam, it is desirable to produce intense, small-scale eddies.

TRANSPORTATION OF SUSPENDED MATERIAL

The relation of the distribution of suspended material to the exchange characteristics of turbulence has been quite well established.[15, 16] For equilibrium conditions the general statistical equation is:

$$\overline{v \, N} = c \, \overline{N} \ldots\ldots\ldots\ldots (22)$$

in which: v = instantaneous vertical velocity component; N = instantaneous

---

[15] "Review of Theory of Turbulent Flow and Sediment Transportation," by M. P. O'Brien, M. Am. Soc. C. E., *Transactions*, Am. Geophysical Union, 1933, p. 487.

[16] "Experiments on the Mechanics of Sediment Suspension," by Hunter Rouse, *Proceedings*, Fifth International Cong. of Applied Mechanics, 1938.

sediment concentration per unit volume; c = average velocity of fall of single sediment particles in still water; and $\overline{N}$ = mean sediment concentration at any point above the stream bed. The left side of Equation (22) indicates the net transport of material upward by the turbulence, and the right side is the quantity that settles due to gravity. Under equilibrium conditions they are equal. The value of N at any instant can be represented as $(\overline{N} \pm \Delta N)$. Then the value of $v\,N$ at any instant is $(v\,\overline{N} \pm v\,\Delta N)$. It is quite apparent that when v is upward, ΔN will be positive, and when v is down ΔN will be negative, since the concentration of sediment in general increases from top to bottom of a stream. The mean value of $v\,N$ is then $+\,\overline{v\,\Delta N}$, since the mean value of $v\,\overline{N}$ is zero, because the arithmetic mean value of v is zero. The question now arises as to how large ΔN is. This depends on the distance from which the suspended material is brought up or down or, in other words, on the scale of the turbulence or size of the eddies. Introducing a length factor, l, characterizing the diffusing scale of the turbulence: $\Delta N = l\,\frac{dN}{dy}$; and Equation (22) becomes

$$\overline{v\,l}\,\frac{dN}{dy} = c\,\overline{N} \quad \text{(23)}$$

The term $\overline{v\,l}$, which may be called β, is proportional to the diffusion coefficient defined in Equation (16). Whether β is identical to the momentum exchange coefficient, ϵ, occurring in the shear formula, must be investigated experimentally. To date they have been assumed equal, and preliminary investigations indicate that they are identical. Integration of Equation (23) leads to a relation between the unknown sediment concentration, $\overline{N}$, at any point y above the stream bed, and a known concentration, $\overline{N}_a$, at a point a distance above the bottom of a stream or canal:

$$\text{Log}_e\,\frac{N}{N_a} = -\,c\int_a^y \frac{dy}{\beta} \quad \text{(24)}$$

This expression for silt distribution has been checked both in artificially created turbulence and also in natural streams. The term β varies vertically in a natural stream, tending to increase away from the bottom, reach a maximum at about the mid-depth, and then decrease as the surface is approached. However, the exact variation has not yet been determined.

Another important problem regarding turbulence and suspended material is the predication of the actual concentration at some point (for instance, just above the bottom), knowing the composition of the bottom, the various hydraulic factors, such as mean velocity and depth of flow, and the intensity and diffusing power of the turbulence. Such a relation is necessary if the expression for sediment distribution (Equation (24)) is to be of great practical usefulness. The mechanism of the picking up of sediment from the stream bottom by the turbulent eddies merits intensive experimental investigation. However, such an investigation will be of little fundamental value if the intensity and the diffusing power of the turbulence is not determined at the same time.

In the problems of silt sedimentation, the diffusion characteristics of turbulence are also extremely important. To illustrate how the theory of turbulent diffusion can be applied to the sedimentation of silt, two general problems will be analyzed.

Problem 1.—Consider the problem in which sediment is settling in a turbulent stream, and it is desired to know what the silt distribution is at various distances downstream from the point at which settling starts, and at which point the silt distribution is known. The assumption will be made that the turbulence is of such magnitude that most of the silt that reaches the bottom is not picked up again. Referring to Fig. 9, if the concentration at section (1) at a point h_1 ft above the bottom is N_1 units per unit volume for a particular size of silt, what will be the distribution of these N_1 units at some distance downstream? If the particles were weightless in water they would be diffused according to the laws of turbulent diffusion, and their distribution would be according to the normal error curve as given in Equation (17) (with $\sigma^2 = \overline{Y^2}$). In Fig. 9, the point $y = 0$ would be at the height h_1. Then the average concentration at point h_2 at the section downstream will be:

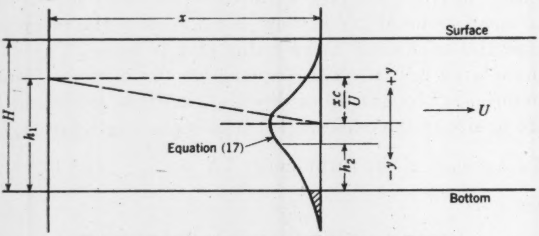


FIG. 9.—SEDIMENTATION IN A TURBULENT STREAM

$$\Delta N_2 = \frac{N_1}{\sqrt{2\,\pi}\,\sigma}\,e^{-b f}\,dy \qquad (25)$$

in which: $b = \frac{1}{2\,\sigma^2}$; $f = (h_2 - h_1)^2$; and $\sigma = \sqrt{\overline{Y^2}}$ = the root mean square diffusion distance as measured from the initial elevation, h_1. If the concentrations N_1 and ΔN_2 are per unit volume, dy can be taken as unity. In words, then, ΔN_2 is the area under the distribution curve between $(h_2 + 0.5)$ and $(h_2 - 0.5)$ times N_1, the concentration per unit volume at h_1. The term $\overline{Y^2}$ is a definite function of x (see heading "Diffusion Characteristics of Turbulence").

Since the silt settles in still water at a velocity of c, it is apparent that the highest concentration (the point on the distribution curve where $y = 0$) will be found a distance of $\frac{x\,c}{U}$ below h_1, or, in other words, the distribution curve will be shifted downward a distance equal to $\frac{x\,c}{U}$. For simplification the velocity U used is the average velocity in the entire stream. The concentration at point h_2 will then be

$$\Delta N_2 = \frac{N_1}{\sqrt{2\,\pi}\,\sigma}\,e^{-b\,i} \qquad (26)$$

in which: $i = \left[h_2 - \left(h_1 - \frac{x\,c}{U} \right) \right]^2$; and (as before) $b = \frac{1}{2\,\sigma^2}$, to simplify topography. To obtain the total average concentration per unit volume at h_2, values of ΔN_2 for all values of h_1, from 0 to H, must be summed:

$$N_2 = \int_0^H \frac{N_1}{\sqrt{2\,\pi}\,\sigma} e^{-b\,i}\,dh \qquad (27)$$

Note that N_1 is a function of h_1 and that h_2 and x are constants for each summation at any one point at section (2). Equation (27) gives the relation between the initial silt distribution and the distribution at another section downstream when settling is occurring.

Problem 2.—The other part of the sedimentation problem is that of finding how much silt settles in a distance x. Of course, the difference between the total concentrations at two points x distance apart is what has settled in the time required for the water to flow a distance x. However, this is a roundabout process of finding the quantity that has settled in the length x; therefore another method is used in finding this quantity. M. A. Velikanov[17] first presented the method outlined herein. The probable quantity of silt that will settle in a distance x from all the silt that passes a point h_1 in a given time is given by the shaded part of the area under the distribution curve, as shown in Fig. 9.

$$(\Delta M)_s = \frac{M_1}{\sqrt{2\,\pi}\,\sigma} \int_{-\infty}^{z} e^{-b\,y^2}\,dy \qquad (28)$$

in which: M_1 = total silt passing a unit area at point h_1 in a given period of time; and $z = \frac{x\,c}{U} - h_1$. The total silt that will settle in a unit time, in the reach x, is then given by

$$M_s = \frac{1}{\sqrt{2\,\pi}\,\sigma} \int_0^H M_1\,dh_1 \int_{-\infty}^{z} e^{-b\,y^2}\,dy \qquad (29)$$

Equation (29) can be simplified, especially if an approximate expression is sufficient. The probability integral can be evaluated readily by reference to books on probability and statistics. However, for the present it is deemed sufficient to present the basic equations only. The next important step is to conduct fundamental experiments in order to check these basic relations.

MISCELLANEOUS PROBLEMS

In addition to the general problems of energy dissipation and suspended material transportation, in which practical application is made of knowledge relating to turbulence, a few others may be stated, as follows:

(1) Many explanations have been proposed in the past as to the reason for the surface velocity in an open channel or stream being less than the maximum.

---

[17] "Theory of Probability Applied to Analysis of Sedimentation of Silt in Turbulent Streams," by M. A. Velikanov, *Transactions*, Scientific Research Inst. of Hydrotechnics, Leningrad, 1936, p. 50.

The fundamental explanation is related to the turbulence mechanism, and more specifically to the process of diffusion of fluid masses in the form of eddies which are formed at the channel boundaries and then diffuse into the main body of the stream.

(2) The relation between high surface-water velocity and the entrainment of air is dependent on the degree of turbulence in the water, which in turn depends on the depth of flow and the channel roughness.

(3) A problem that has recently been studied in detail is that of the relation of the turbulence in a conduit to the accuracy of the Allen salt-velocity method of measuring discharge.[18] In general, the conclusions seem to be that for a high degree of turbulence (that is, for relatively high velocity) the method is more accurate than for slight turbulence. Errors may also result if the measurements are made in a reach of pipe in which the turbulence is changing, such as beyond a bend, valve, pipe expansion, or other turbulence producing device.

(4) Instruments for measuring mean velocity and static pressure in a turbulent water stream are affected more or less by the fluctuating velocities. Current meters, especially those of the cup type, greatly over-register in turbulent water. The velocity head read by a pitot tube is greater than that due to the average velocity depending on the degree of turbulence. The static pressure read by a static-pressure tube placed in a turbulent fluid has a reading greater than the true static pressure by an amount proportional to the mean squares of the cross-velocity components. However, with intensities of turbulence ordinarily present in water conduits or channels, the error in the reading of a well-designed pitot tube or static-pressure tube should be less than 1%.

(5) In conclusion, it should be mentioned that in aeronautics the measurement of the intensity, and the scale of turbulence, is rapidly becoming a standard laboratory procedure in all fundamental experimental work. In the near future, undoubtedly, such measurements will be made part of many hydraulic laboratory investigations. For example, if model studies are made of problems involving energy dissipation or of problems dealing with suspended material transportation or bed-load movement, in order that such studies be fundamentally sound, dynamic similarity must be maintained as far as intensity and scale of the turbulence are concerned. Turbulence should be thought of in terms of definite parameters instead of just as a qualitative descriptive term relating to a general condition of flow.

ACKNOWLEDGMENT

Grateful acknowledgment is given to E. R. Van Driest and to J. M. Robertson, Juniors, Am. Soc. C. E., research assistants, for help in taking and analyzing data. Fig. 18 in the closing discussion was plotted from data supplied by Mr. Robertson in his thesis, "Diffusion Characteristics of Turbulence in an Open Channel," submitted to the University of Iowa in January, 1940, in partial fulfilment of the requirements for the degree of Master of Science.

---

[18] "Contribution à l' Étude de la Mesure des Débits d'Eau par la Méthode Allen," by Martin A. Mason, Assoc. M. Am. Soc. C. E., Doctorate Thesis, Univ. of Grenoble, France.

DISCUSSION

Hunter Rouse,[19] Assoc. M. Am. Soc. C. E. (by letter).—In welcoming the appearance of this latest interpretation of present-day theories of fluid turbulence, one cannot help being impressed by the rapidity with which the modern engineer profits by advances made in related fields of science. It was many decades before hydraulic engineers became fully aware of the value to them of the fundamental contributions of Boussinesq and Reynolds, whereas engineers of today already have before them in a clear, concise form an interpretive summary of the many significant developments made within the last few years. It is particularly fitting that this paper has been written by Professor Kalinske, for he is the first of the hydraulic engineers to have applied these statistical theories in the laboratory to problems primarily hydraulic in nature.

As is evident from even casual study, the readers for whom the paper is particularly intended are those whose efforts may immediately benefit by the adoption of the statistical method of approach—that is, research men seeking to further the science of hydraulics. (As if in illustration of the value of these methods to the research profession, the author, following the preparation of the present paper, collaborated in the application of the statistical approach to the analysis of the amount of material lifted from the bed of a sediment-bearing stream[20]—with results likely to prove of great worth in the ultimate solution of the suspended-load problem.) However, the writer feels that the practicing engineer may also find much of interest in this paper, particularly if he overcomes a natural tendency to let its mathematical aspects conceal the general information which it contains.

In past résumés of turbulence theories, it will be recalled that the subject matter was necessarily restricted to conditions of statistically steady, uniform flow, such as that in circular pipes, the growth of turbulence in an expanding boundary layer receiving only brief attention. The internal mechanism of the turbulence was investigated qualitatively, but only as a means of developing general expressions for velocity distribution and boundary resistance. Although the resulting equations were sufficiently exact for practical use, the qualitative analysis of the turbulence on which they were based was not accurate enough to establish the absolute intensity of the turbulence at any arbitrary point in the flow. That knowledge of this nature has become essential to further progress is shown by the attention now being given to quantitative analysis of turbulence in such widely different fields as meteorology, oceanography, and aeronautics—to which fields that of hydraulics is now added.

For the hydraulic engineer it is no longer sufficient merely to acknowledge the presence of turbulence in uniform circular pipes. He must realize, as Professor Kalinske points out, that such terms as "shock loss" and "impact loss" are definitely misnomers, that all hydraulic losses are ultimately attributable only to viscous dissipation into heat, and that by understanding the formation and subsequent decay of turbulence such losses can be controlled to a great extent. Such an understanding is probably instinctive to many an hydraulic engineer, although on every side one encounters the results of basic misconceptions. For instance, it has been suggested that one must devise a new word to distinguish the large-scale "turbulence" in rivers from the small-scale turbulence in pipes—although the large-scale "turbulence" in question really consisted of local eddies bearing the same relation to the actual turbulence of the river as the eddies at a change in pipe section bear to the normal turbulence of pipe flow. As noted by the author, eddies produced by local disturbances are by no means synonymous with true turbulence. Such eddies, however, are a primary source of turbulence, and the rate of subsequent energy dissipation is a matter of considerable importance. For example, many so-called "energy dissipators" could be made far more effective and less dangerous in their downstream influence if designers realized not only that the formation of large-scale eddies effectively reduces the mean velocity of flow, but also that the presence of such eddies is sometimes as harmful as that of a high mean velocity; as pointed out by Professor Kalinske, the production of intense small-scale eddies (approaching the state of isotropic turbulence) is an essential means of hastening energy dissipation.

---

[19] Prof. of Hydraulics, Univ. of Iowa; Cons. Engr., Iowa Inst. of Hydr. Research, Univ. of Iowa, Iowa City, Iowa.

[20] "The Relation of Suspended to Bed Material in Rivers," by E. W. Lane, M. Am. Soc. C. E., and A. A. Kalinske, *Transactions*, Am. Geophysical Union, Part IV, 1939, p. 637.

Of primary significance, evidently, is the fact that both the length scale and the velocity scale of turbulence govern the processes of diffusion and energy dissipation. The product of these scales is already familiar to the student of turbulence in the factor ϵ, the so-called "eddy conductivity," previously determinable only for uniform flow, and then only roughly through measurement of velocity distribution and resistance. Professor Kalinske has outlined two methods whereby the length and velocity scales for both uniform and non-uniform flow may be determined individually. He notes, however, that in either method the evaluation of the results is extremely tedious. Several hydraulic laboratories are already on the search for a simpler, and hence more practical, means of accomplishing such measurement in water, but their goal is by no means yet in sight. It is to be hoped that the present paper will also serve as a further incentive to this end.

MARTIN A. MASON,[21] ASSOC. M. AM. SOC. C. E. (by letter).—In an effort to clarify the present status of fluid turbulence research with respect to hydraulics, it seems pertinent to discuss briefly several concepts included in Professor Kalinske's paper, which may be sources of misunderstanding for the practical hydraulicians to whom the paper is addressed.

Unfortunately, much confusion of thought exists on the fundamentally important concept of the nature of turbulence. Among those thoroughly familiar with modern thought on the subject one finds turbulence as a generic term modified as isotropic, non-isotropic or anisotropic, initial, fully-developed, etc., each of which designations indicates to the experienced reader a characteristic phenomenon. To the practical man who is not familiar with the mathematical treatment of turbulence, however, these terms are usually meaningless. For these readers turbulence is a general designation for any or all types of irregular or agitated flow. Ordinarily flow in an open channel is "turbulent," as is the discharge over a dam spillway, the flow through a sluice gate, or the flow behind a bridge pier.

---

[21] Asst. Hydr. Engr., National Hydr. Laboratory, National Bureau of Standards, Washington, D. C.

Obviously, before the hydraulic engineer can benefit greatly from the work of the investigators in the field of turbulence theory consistency of thought must be established as to the nature of turbulence. In this connection, Dryden[22] makes a pertinent comment when he writes, "It cannot be stated too emphatically that the definition of turbulence is necessarily arbitrary and the most useful definition depends on the purpose of the study." Thus, for the purpose of studying the theory of turbulent flow it may be convenient to define turbulence (as Professor Kalinske has done) as being of an entirely random nature without definite periodicity with time, since the mathematical treatment is somewhat simpler for this case. In the case of flow along a boundary, which is the type most commonly encountered by hydraulicians, a more suitable definition of turbulence might be that it is motion such that correlation exists between the fluctuating velocity components at any point in the flow.

The translation of these statistical definitions into terms familiar to the average hydraulic engineer is beyond the scope of this discussion and the writer can emphasize only the importance of the suggestion that the reader not familiar with the concepts outlined consult certain works of Messrs. Dryden,[22, 23] Bakhmeteff, and Rouse.

If the foregoing definition of turbulence is admitted to be the most suitable for hydraulic engineering purposes, it is noted that the present statistical theory of turbulence does not offer immediate possibility of application to most hydraulic problems, inasmuch as it is chiefly concerned with isotropic turbulence. It must be admitted that, for the hydraulic engineer, at present, the statistical method of studying turbulence is only a useful tool which shows promise of being valuable under certain circumstances because of the simplifications it permits. Actually, at the moment, it is of little value for the solution of practical hydraulic problems, and hydraulic engineers should not expect it to supplant some of the older empirical methods of investigation. In this latter class one may cite the remarkable results achieved by L. Prandtl and his students, and by Theodor von Kármán.

Referring to Professor Kalinske's discussion of energy dissipation, it is suggested that, from the hydraulician's point of view, the determination of the time period necessary for the required dissipation of energy in the form of anisotropic turbulence is perhaps more important than the determination of the rate of dissipation. For example, consider the case of a dam spillway; the kinetic energy of the discharging water may be changed rapidly, and in substantial part, to anisotropic turbulent energy in the stilling basin; but, unless this turbulent energy is substantially dissipated in the basin, scour of the downstream bed will result. As shown by Professor Kalinske, the determination of the rate of dissipation of turbulent energy is rather complicated even for the relatively simple case of isotropic turbulence, a condition seldom, if ever, encountered in practice. It would seem certain, however, that (as indicated by the interpretation of the term λ as a measure of the diameters of the smallest eddies responsible for the dissipation of energy[6]) the smaller the size of the eddies the more rapid will be the dissipation of energy, and consequently the smaller the required basin length.

---

[22] "Turbulence and the Boundary Layer," by H. L. Dryden, *Journal of the Aeronautical Sciences*, Vol. 6, No. 3, January 1939, p. 99.

[23] "Turbulence, Companion of Reynolds Number," by H. L. Dryden, *Journal of the Aeronautical Sciences 1*, No. 2, 67, April, 1934.

As suggested by Professor Kalinske in "Miscellaneous Problems" (No. (5)), model studies involving energy dissipation, sediment transportation, or bed-load movement require that dynamic similarity exist between the intensity and scale of the turbulence in model and prototype. A somewhat related consideration is found in wind tunnel studies where a similarity of turbulence in the tunnel is necessary to obtain comparable results from different wind tunnels. This condition led Dryden to remark[22]

> "* * * the turbulence characteristics of the wind tunnel need to be known and taken into account in the interpretation of measurements. * * * We have to do essentially with a new independent variable whose effects are as important as those of Reynolds number and in many ways similar to those of Reynolds number * * *."

With this latter thought in mind, one may derive from the Navier-Stokes equations (which are applicable to turbulent flow since the velocity components used in the equations refer to instantaneous velocities) not only the usual parameters (Reynolds, Froude) governing the relation of model to prototype; but also, a third parameter having a form similar to Froude's number, which governs the viscous stresses in the two structures. The thought is suggested that this third parameter might be of the nature of a turbulence parameter, whose function is to relate the turbulence in model and prototype.

It should be remembered that similar Reynolds numbers for model and prototype do not alone guarantee similar conditions of turbulence unless there is complete geometric similarity. This may be shown as follows: The cause of the discrepancy in turbulence values in wind tunnels is the effect of the grids or vanes in the tunnel, the turbulence in the tunnel being a characteristic of the grid or vane installation. The corresponding source of discrepancy in hydraulic models would be the stilling racks or basin at the water supply end of the model. If such devices are not similar to those in the prototype, or do not establish flow conditions comparable to those in the prototype, there is no assurance that the intensity and scale of the turbulence in model and prototype are similar even if the same Reynolds number obtains for the two.

The possibility of the existence of a scale effect of turbulence between model and prototype would appear to be sufficiently real to justify the attention of model experimenters in those cases where the turbulence characteristics of the water are important.

Regarding experimental techniques, the writer had the opportunity to develop a method of observation of a stream injected into flowing water which uses fluorescent products illuminated by ultra-violet light.[18] The method shows some promise but has not yet been developed as fully as the excellent technique used by Professor Kalinske. Its principal advantage lies in the fact that the injected stream is seen as a lighted streak on a dark background, thus avoiding the interference of parasitic light reflected from the impalpable particles carried by the water.

In any photographic method employing foreign particles to reveal the motion of the water some uncertainty attaches to the assumption that the added particles truly follow the exact motion of the water particles. This would be true especially when the added particles are of the nature of droplets of an immiscible liquid, unless the droplets are exceedingly small. Similarly, the use of an injected stream of color may be questioned because of possible inertial and viscosity effects. However, Charles Camichel and his co-workers have shown[24] that aluminum particles approximately 0.1 mm in diameter, when washed in alcohol and introduced into a body of water, reproduce exactly the motions of the liquid particles they replace. It would be interesting to compare the magnitude of the droplets used by Professor Kalinske with the size of the particles recommended by M. Camichel.

A photographic method which has been used with considerable success in France for the study of the instantaneous distribution of velocities around a propeller would seem to offer great possibilities for the study of the turbulence problem. First suggested by Marey,[25] developed by numerous investigators, particularly M. Camichel,[26] and in 1937 completely described by M. Chartier,[27] the method is of particular interest because of its applicability to the study of any type of fluid motion in space by stereoscopic means.

In brief, motion pictures are taken of the movement of entrained aluminum particles, utilizing stroboscopic light. A comparison stereopticon of special design is then used in the study of each frame of the film obtained, or the motion may be studied by ordinary projection means. By the use of motion pictures combined with stroboscopic illumination the paths of the illuminated particles over very short time intervals are obtained as a series of dashed streaks, from which the particle velocities may be computed. The minimum interval of time recommended for the stroboscopic illumination is of the order of $\frac{1}{800}$ sec. An improvement of the method, resulting in a considerable saving of time, is the use of a stereo-projector so arranged as to locate the particle streaks on a coordinate system. This arrangement permits the determination of velocities within about 1% error, and the determination of position in space with about 5% error.

J. C. STEVENS,[28] M. AM. SOC. C. E. (by letter).—Professor Kalinske has been continuing the research project "Conversion of Kinetic to Potential Energy in Expanding Conduits" started by Frederic T. Mavis, M. Am. Soc.

---

[24] "Sur le régime permanent dans les chambres d'eau," by Charles Camichel, *Revue Générale d'Electricité*, Vol. 8, p. 331.

[25] "Le Mouvement des liquides étudié par la chronophotographie," by E. J. Marey, *Comptes Rendus de l'Académie des Sciences*, May 9, 1893, Vol. CXVI.

[26] "Notice sur les travaux de M. Camichel," November 15, 1929, p. 21.

[27] "Chronophotogrammétrie plane et stéréoscopique," by Charles Chartier, *Bulletin No. 114*, Publications Scientifique et Technique du Ministère de L'Air, 1937.

[28] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

C. E., for the Special Committee of the Society on Hydraulic Research.[29] The basic method of approach to this problem has been well set forth in this paper. The author has brought the practical aspects of fluid turbulence home to the hydraulic engineer.

Lately hydraulic engineers have recognized that the energy gradient in tortuous conduits is quite uniform and that energy losses are not concentrated at bends as was long believed. In the light of the author's treatment the explanation is simple: The so-called losses are merely heat dissipation through the viscous properties of water in turbulent motion that continue downstream far beyond the origin of the turbulence. In uniform flow the boundary contacts are a source of continuing turbulence, a balance having been reached between that and heat dissipation. Bends, expansions, and piers are sources of concentrated turbulence, and the heat dissipation from them persists for a long way downstream, thus giving a more or less uniform gradient of energy losses.

The author states (see paragraphs following Equation (21)): "* * * the smaller the eddies the higher the rate of energy dissipation will be." Perhaps this explains why the hydraulic jump is such a potent source of energy dissipation, most of the energy being lost in a comparatively short reach. This must be due to the violent internal eddies set up as a result of the sudden expansion, the visible, foamy, front roller being responsible for only a relatively insignificant portion of the total loss.

The data in Figs. 7 and 8 are of particular significance in explaining why kinetic energy cannot be converted to potential energy without inherent losses. As the author states (see heading: "Turbulence and Energy Considerations"), turbulent energy is a degraded form and is irrecoverable, thus being in broad conformity to the second law of thermodynamics. If, in Fig. 8, the water were caused to flow in the opposite direction, it is likely that the $\bar{u}^2$-curve and the $\bar{v}^2$-curve would become even less prominent than they appear in the uniform pipe of Fig. 7. This feature may be worthy of further investigation.

There is need of means and a method of measuring velocities at any point in a flowing prism without inserting something that itself causes turbulence. Professor Kalinske has solved the problem very cleverly by the use of a motion-picture camera and droplets of an immiscible liquid of the same specific gravity as water. These can be injected into the prism far enough upstream so that the slight turbulence caused by the needle becomes negligible at the camera.

The tedious labor involved in analyzing the data thus gathered, however, is a serious handicap to the method. Perhaps simpler, yet just as effective, means will be found in time.

BORIS A. BAKHMETEFF,[30] M. AM. SOC. C. E. (by letter).—There scarcely can be a subject of greater importance to the hydraulic engineer than the phenomenon of turbulence. Its outward apparent aspects have been known for years. Systematic detailed studies, however, and particularly quantitative appraisal of the inner mechanism, are a matter of recent development. So far, most of the investigations were connected with the aeronautical field. Mr.

---

[29] *Civil Engineering*, March, 1938, p. 195.

[30] Prof., Civ. Eng., Columbia Univ., New York, N. Y.

Kalinske is "breaking ground" in the domain of hydraulics. No praise is too high for this worthy initiative.

In the study of turbulence, as in other domains of physical science, the first requirement is the acquisition of facts—dependable records of actual observation. Observational technique in the domain of turbulence is exceptionally difficult. For air, the present-day needs are satisfied by the hot-wire anemometer, particularly as perfected by the National Bureau of Standards. For liquids, the hot-wire method has not asserted its usefulness; so recourse must be taken to photographing molar movements, rendered visible by that, or other, optical means. The outstanding achievement in the past is the work of A. Fage and H. C. H. Townend. Mr. Kalinske has come forward with new suggestions in this line. The successful development and application of the new technique constitutes, in the view of the writer, the principal contribution of Mr. Kalinske's paper. The experimental curves, as shown, justify the hope that the complex features, specifically characteristic of turbulence as it occurs in hydraulics, may some day really become revealed and elucidated. For the moment one may only regret that Mr. Kalinske did not describe his technique in greater detail.

A drawback of all cinematographic methods is their natural tediousness. For the moment that is an inescapable evil. No greater service could be rendered to the hydraulic profession, than if some ingenious inventor conceived a device that would permit observing and registering turbulence in liquids directly. The problem stands as a challenge, open to competitive imagination.

Mr. Kalinske purports to recapitulate briefly the principal results attained by the statistical method. The achievements of the English school are particularly emphasized. Perhaps the author has tried to encompass too much material into too narrow a space, because the nearly laconic brevity might be found somewhat trying. The paper, on the other hand, should rightly be considered as a pioneer attempt, and the engineer should not feel discouraged by the first encounter with a subject which, in the course of years, is certain to become a familiar every-day tool for whoever will be striving to penetrate and understand the baffling mysteries of the behavior of "water."

So far, the most tangible contributions of the statistical approach are not in the realm of a general all embracing "theory." With all the brilliance in recent advances, the "theory" has not reached the state of lucid certainty, which could make it a useful tool in the hands of the engineer. Statistical treatment, on the other hand, has matured into certain well-established basic definitions, which permit one to appraise and characterize turbulent phenomena in plausible numerical terms. The curves in Mr. Kalinske's paper pertaining to such definitions would seem to indicate the way through which these new concepts can be best conveyed to the "mind" of the profession. No tool is really available until it becomes physically tangible and until its dynamical essence can be explained in the simplest terms. The surest path is to lay open physical facts and make clear their practical meaning and significance. This is the next step to be achieved by the interpreters of the theory.

A question of especial interest is the spatial distribution of turbulent intensity in a cross section. The matter bears directly on efforts to unveil the mystery of the origin and spreading of turbulence. The writer is particularly interested in the form of the $\bar{u}$-curve as given in Fig. 7. As mentioned before, factual knowledge of turbulent phenomena is more than scarce and no opinions of too categorical a nature are warranted at this stage. Nevertheless it does seem that all observations heretofore concur,[31] that the axial component continually increases from the center toward the wall, at least to within the unstable border zone next to the laminar sublayer. The Fage and Townend[32] ultramicroscopic photos show the largest fluctuations as being in the closest proximity (distance less than 0.003 in.) to the wall. More recently such conceptions have been substantiated by observations at the Massachusetts Institute of Technology.[33] They stand, moreover, in close relation to the present-day views on the pendulatory character of the motion in the laminar zone, so admirably elucidated by Dryden.[34]

The writer would like to learn from Mr. Kalinske whether the particular outline of the $\bar{u}$-curve, shown in Fig. 7 (which reaches a maximum at about one half the distance and then declines) has been generally substantiated by repeated observations or may be ascribed to some accidental reason. The point is of vital interest in its possible bearing on the gradually crystallizing conception, that the eddies, which make for turbulence, are principally generated in a special active zone growing out of the instability of the laminar sublayer. Prandtl[35] most aptly characterized this zone as a "vortex mill" (Wirbelfabrik), assuming that the central portions of the fluid, into which the eddies are "cast off," play a more or less passive rôle.

The foregoing remarks tend to show how uncertain, as yet, is current knowledge of the inner mechanism of turbulence. They do not detract in any way from the value of Mr. Kalinske's contribution.

CLYDE W. HUBBARD,[36] Esq. (by letter).—Compliments are due the author on his clear and simple exposition of the theory of turbulence. More and more it is necessary for the hydraulic engineer to study the mechanism of turbulence in the hope that some of the unexplained phenomena of flowing water will become clear.

However, the author states in the conclusion of his paper that: "The static pressure read by a static-pressure tube placed in a turbulent fluid has a reading greater than the true static pressure by an amount proportional to the mean squares of the cross-velocity components." If this were true, the pitot differential would be less than the true velocity head and the coefficient of the pitot tube would be larger than unity. This is contrary to the experience of practically all investigators who have made tests with ordinary types of pitot tubes in pipes.

---

[31] "Statistical Measurements of Turbulence in the Flow of Air Through a Pipe," by H. C. H. Townend, *Proceedings*, Royal Soc. of London, Vol. 145A, 1934, p. 180.

[32] "An Examination of Turbulent Flow with an Ultramicroscope," by A. Fage and H. C. H. Townend, *loc. cit.*, Vol. 135A, 1932, p. 656.

[33] Discussion by J. C. Hunsaker of the Second Wright Brothers Lecture, *Journal of the Aeronautical Sciences*, Vol. 6, 1939, p. 105.

[34] "Turbulence and the Boundary Layer," by Hugh L. Dryden, *loc. cit.*, Vol. 6, 1939, p. 85.

[35] *Proceedings*, Fifth International Cong. for Applied Mechanics, 1938, p. 367.

[36] Asst. Prof., Hydr. Eng., Worcester Polytechnic Inst., Worcester, Mass.

The author also states that the error of a well-designed pitot tube or static pressure tube should be less than 1%. In other words, the customary form of pitot tube can be relied upon to read the true velocity within 1% in the turbulent flow normally found in pipes.

Tests described by the writer[14] and by E. Shaw Cole,[37] Jun. Am. Soc. C. E., in 1939, show that for the ordinary form of pitot tube the error caused by turbulence may be as much as 5% and ordinarily is about 2%. In connection with these tests the intensity of turbulence or "mean angularity" of flow was measured in pipes from 12 in. to 72 in. in diameter at velocities ranging from 3 to 8 ft per sec.

No measurable error in the reading of the impact orifice of a pitot tube was found to be caused by turbulence, which is in agreement with the author's statement. However, turbulence did appear to cause an error in the reading of the pressure head by either a wall piezometer or by a static orifice on the smooth tip of a pitot-static tube. This error in the pressure reading, although important in pitot-tube measurements, was not great enough to affect, seriously, the accuracy of pressure head measurements for pump and turbine efficiency tests.

Mr. Cole[37] describes tests with various simple pitot tubes using carefully constructed wall piezometers made in three 12-in. pipes having different flow patterns. The coefficients obtained for pitot tubes were different in each of the three pipes. Since the impact orifice was found to have no appreciable error, it can only be inferred that the change in coefficient was caused by errors in the wall piezometer readings.

The wall-piezometer error varied with the ratio of mean to center velocity, known as the pipe factor. The error increased proportionately as the pipe factor decreased below unity, so that a coefficient of 0.968 was obtained in the roughest pipe having a pipe factor of 0.77.

The writer described similar tests with a smooth pitot-static tube at the center of the same three pipes in the closure to his paper.[38] The same pitot tube was tested at the center of each of the pipes and the coefficients obtained were 0.992, 0.980, and 0.956. These coefficients, however, did not bear a simple relation with the pipe factor as did Mr. Cole's tests with wall piezometers; nor did the coefficient appear to depend on the intensity of turbulence alone.

It is possible that, when the scale (mean eddy size) as well as the intensity of turbulence are both measured successfully, some rational explanation of the error of the pitot-static orifice will be forthcoming. Near the wall of the pipe both the scale and intensity of turbulence are dependent on the condition of the pipe surface, and any turbulence caused by an upstream disturbance would be dissipated rapidly in the small intense eddies. It would be expected, therefore, that an error of the wall piezometer caused by turbulence would vary with the wall roughness and hence the pipe factor. On the other hand, the turbulence at the center of the pipe is largely of the isotropic type which does not produce shearing stress in the fluid. The scale and intensity of turbulence which exists may be dependent on upstream conditions, since the eddies are larger and the decay less rapid. Therefore, the error of a pitot-static pressure orifice at the center of a pipe, caused by turbulence, would not necessarily bear a consistent relation to the pipe factor or to the intensity of turbulence alone, but may be dependent partly on the scale of turbulence as yet not measured in these pipes.

---

[37] "Investigation of Errors in Pitot Tubes," by C. W. Hubbard, *Transactions*, A. S. M. E., Vol. 61, 1939, p. 495.

[38] *Loc. cit.*, p. 504.

One method of avoiding the effect of the existing turbulence in the center of a pipe on the pressure orifice of a pitot-static tube is to create a disturbance of the flow immediately upstream from it. A small ring was placed on the smooth tip just ahead of the pressure orifice and the tests in the three 12-in. pipes repeated. The coefficients obtained were 0.897, 0.885, and 0.889 as compared with 0.992, 0.980, and 0.956 with the smooth tip. The ring improved the consistency considerably, although the coefficient was decreased.

The test results described herein show that turbulence does appreciably affect the accuracy of pitot-tube measurements and causes the coefficient to be less than unity. However, the pitot tube remains an accurate instrument for flow measurement when properly used by experienced engineers. Errors caused by turbulence can be avoided by the use of a type of tube that is but slightly affected and by calibrating it under flow conditions similar to those under which it is to be used.

JOHN S. MCNOWN,[39] JUN. AM. SOC. C. E. (by letter).—Professor Kalinske has presented several excellent methods of approach to the difficult problem of turbulence, and has substantiated many of his conclusions by laboratory measurements. With so many phases of hydraulics depending on turbulence, the need for study and laboratory research on turbulence in hydraulics has become more and more pressing. This paper gives the fundamental principles, points out the fields of application, and describes the laboratory approach in detail. In addition, an important function fulfilled by this paper is the bridging of the gap between laboratory data and the mathematical theories of statistics and turbulence.

The photographic technique outlined by the author is an improvement over other existing methods for measuring instantaneous velocities in water. Any type of apparatus that must be placed in the water is indeed of doubtful accuracy until proved otherwise. One apparent difficulty with the photographic measurements, in addition to that of analyzing the pictures, is indicated by the angular character of the curve in Fig. 1 which gives the relationship between time and instantaneous velocity fluctuations. An increase in the number of pictures taken per second should smooth out the curve, but would increase the difficulty in analyzing the individual frames.

Several questions arise from the discussion of the quantity λ, defined, following Equation (11), by the formula

$$\lambda = \underset{x \to 0}{\text{Limit}} \left(\frac{x}{\sqrt{1 - R_x}} \right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

and its use in the determination of the energy loss resulting from turbulence.

---

[39] Instr. of Math., Mechanics and Hydraulics, Univ. of Minnesota, Minneapolis, Minn.

The author does not give the conditions under which the data used for this analysis were obtained or the degree to which the isotropy assumed in the development was attained. Furthermore, the method of determining the numerical value of λ is not fully understood by the writer. A knowledge of the functional relationship between x and R_x would be necessary to determine the value of the limit given in Equation (30) as the definition of λ. In a treatment of the same topic in a report by the Fluid Motion Panel of the Aeronautical Research Committee[40] a knowledge of the value of the quantity $\overline{\left(\frac{\partial v}{\partial x}\right)^2}$ is used in computing the value of λ, whereas the author computes λ independently and uses it to compute $\overline{\left(\frac{\partial v}{\partial x}\right)^2}$. The assumption of a parabolic distribution near the peak of the curve, as proposed by G. I. Taylor,[41] would make the reverse order of computation possible. The validity of such assumption depends upon the agreement between the curves at the peak.

The several lengths defined as helpful concepts in the study of turbulent flow must be interrelated to some extent. The concept of the correlation of instantaneous velocities at two points in a stream appears to be closely related to one of Prandtl's two definitions of a mixing length—namely, the diameter of the mass of fluid moving as a whole.[42] A comparison of Prandtl's mixing length, the length λ, the length represented by the x-intercept of the curve in Fig. 3 ($x = 4.6$ in., approximately), or the length criterion defined by Taylor would be interesting if possible. The aforementioned curve in Fig. 3 gives the relationship between x, the distance between two points, and R_x, the correlation between the instantaneous velocities at the two points. The x-intercept of this curve would be the least distance at which no correlation exists.

Among the hydraulic problems affected by turbulence is that of bed-load movement. One troublesome feature of bed-load studies is the lack of unity in interpreting the meaning of "beginning movement," and the application of this concept to other stages. Usually experimenters refer their results to this stage as a basis for computation in using such quantities as excess bottom velocity and effective tractive force. Conceivably part of the difficulty in reaching a common interpretation of this reference stage and the resulting relationships is caused by failing to consider the variable of degree of turbulence. A high degree of turbulence would cause an initial movement at comparatively low values of tractive force as the local velocities are of greater magnitude than for less turbulent streams. Furthermore, this difference becomes less with increasing velocity as the intermediate local velocities become more important. Thus, the effect of turbulence should be investigated in standardizing the results of comparable research problems. A difficulty in such an analysis is the probable absence of isotropy in the turbulence near the boundary where its effect is most important in this problem.

---

[40] "Modern Developments in Fluid Dynamics," by the Fluid Motion Panel of the Aeronautical Research Committee and edited by S. Goldstein, pp. 223–225.

[41] "Statistical Theory of Turbulence," by G. I. Taylor, *Proceedings*, Royal Soc. of London, Vol. 151A, 1935.

[42] "Hydrodynamics," by H. L. Dryden, F. G. Murnaghan, and H. Bateman, *Bulletin*, National Research Council, 1932, p. 396.

The use of the letter Q to represent weight of water flowing per unit of time is a departure from general hydraulic practice. Too much difficulty is encountered through unavoidable discrepancies in nomenclature without departing from the more or less standard usages. The letter W has been used for this quantity or, more commonly, $Q\,w$.

While questioning these few minor details the writer wishes to concur wholeheartedly with the basic principles presented in this paper. Very important in utilizing this work is a clear understanding of the underlying assumptions and restrictions applied in the derivation of the theory. Much appreciation is due the author for sifting out these concepts from the mathematical theory of turbulence and presenting those topics of most direct use to hydraulic theory and practice.

Samuel Shulits,[43] **Assoc. M. Am. Soc. C. E.** (by letter).—A need in present-day American hydraulics is to convince the so-called practical hydraulic engineer that fluid mechanics is a tool of much promise and not a scientist's toy. It is gratifying that Professor Kalinske devotes himself so capably to this task and that the Society is "blazing the trail."

Significance of the Statistical Fundamentals.—The frequency distribution curve in Fig. 2 is found to conform to Equation (1), the Gauss-Laplace law of frequency of errors, while other investigators are said to have verified this distribution law. Is it not a well-established mathematical law of statistics that deviations from the mean of any large number of measurements will array themselves in this fashion?

Isotropic turbulence is the name given to a flow situation such that $\overline{u^2} = \overline{v^2} = \overline{w^2}$ and $\overline{u\,v} = \overline{u\,w} = \overline{v\,w} = 0$. The physical significance of this mathematical definition may be of interest. Consider the so-called "precision index,"[44] $h = \frac{1}{\sqrt{2\,\overline{v^2}}}$. The larger it is, the more closely do the deviations cluster about the maximum ordinate (of the frequency distribution curve) at which $v = 0$; or the greater is the concentration of the observations about their mean. Since $\overline{v^2} = \frac{1}{2\,h^2}$, the first condition of isotropism merely states that, as a whole, each of the three rectangular velocity components departs from its mean value in the same general way.

If u and v are always of the same sign, $U + u$ and $V + v$ have like trends: $u\,v$ and the mean of the products $(\overline{u\,v})$ are always positive. A value of the instantaneous velocity along the x-axis greater than the mean will be found with an instantaneous velocity along the y-axis greater than the mean; the same applies to sub-average instantaneous velocities. A negative $u\,v$ indicates the coexistence of a sub-average and a super-average velocity along the x and y

---

[43] Associate Hydr. Engr., Flood Control Div., U. S. Engr. Office, Louisville, Ky.

[44] "Handbook of Engineering Fundamentals," edited by O. W. Eshbach, John Wiley & Sons, Inc., 1936, pp. 2–124.

axes—that is, an opposite trend of variation. A positive value of the algebraic sum of these products ($\overline{u\,v}$) stands for a like over-all trend of u and v whereas a negative quantity indicates an opposite course. If the grouping of u and v is quite accidental so that there is no relation between them, the sign of the product $u\,v$ will be negative and positive haphazardly and the mean product of a large number of observations will be zero. Consequently $\overline{u\,v} = 0$ denotes that there is no recognizable relation between u and v, as the plus and minus values of the products are collectively of equal weight. Therefore, the nature of u cannot be predicted from v and vice versa, and their relationship is entirely random, which is what one might envisage in the word "turbulence." Thus the second condition of isotropism is simply one mathematical statement of the disorderedness or "mixed-up-edness" of turbulence.

The transformation of the statistical mean product into a non-dimensional criterion results in the correlation coefficient, R_x. If v_1 always equals v_2, $R_x = 1$, whereas if v_1 is always the negative of v_2, $R_x = -1$; but $R_x = 0$ signifies no apparent interdependence between v_1 and v_2—their array (arrangement) is entirely accidental. When v_1 and v_2 are observed at points very close together (within the average size eddy), a close relationship (or correlation) should be expected, so that R_x should be nearly unity as Professor Kalinske states. The smaller x is, the more alike v_1 and v_2 should be, a situation that is easily visualized. On the other hand, the farther apart the points are, the more random should be the mathematical relationship, and the condition of seeming confusion (turbulence) prevails. Thus $R_x = 0$ is merely another parametric statement of turbulence. The physical connotation of Professor Taylor's characteristic scale of turbulence[6] (Equation (4)) follows easily: l' is the distance between two velocities when R_x approaches zero or the disconnectedness of the velocity concepts is absolute. The trend between R_x and x in Fig. 3 is also explainable in the light of the foregoing, although not the exact equation of the curve.

Professor Taylor's premise that "the statistical correlation between the velocity of a fluid mass at one instant and at another later instant is high when the time interval is small, and this correlation approaches zero when the interval becomes large," is an easily accepted physical dictum. The shorter the time interval between two measurements at one point, the more related they might be expected to be and therefore the closer R_t would be to unity. As the interval increases the relation should become more haphazard, and R_t approaches zero. Thus, the fundamentals of the statistical correlation method are used to express physical realities of turbulence.

The Diffusion Coefficient.—The coefficient of viscosity, μ, has the dimensions $\frac{F\,T}{L^2}$, or a unit force times time. In pure viscous flow the unit shearing stress or unit frictional force is completely given by $\tau = \mu \frac{du}{dy}$; but in turbulent flow, there is an additional friction or drag due to the transverse motion of the water particles. By analogy this frictional force has been expressed as $\tau' = \mu' \frac{du}{dy}$

in which μ' is an apparent viscosity. The total unit friction is then as expressed by Equation (7). Since $\mu' = \rho\,\epsilon$ and ρ has the dimensions $\frac{M}{L^3} = \frac{F\,T^2}{L^4}$, then ϵ has the dimensions $\dfrac{\frac{F\,T}{L^2}}{\frac{F\,T^2}{L^4}} = \frac{L^2}{T}$, a velocity times a length, or an area divided by time. Its proportionality to D, the diffusion coefficient of Equation (16), is readily seen. It is really a time rate of change of area and therefore its significance as a measure of transverse diffusion or particle transfer is apparent.

Whither Fluid Mechanics?—From about the middle of the nineteenth century to the present there may be said, without rigid accuracy, to be three phases in the development of hydraulics and fluid mechanics. The first period was a very prolific stage of empiricism (Bazin and Darcy, for example) to which the practical hydraulic engineer is indebted for much of what is still the most serviceable information. Hydrodynamics (Boussinesq and Lamb, for example) existed contemporaneously, but the hydraulician and the hydrodynamist do not seem to have collaborated. The former was prone to regard the latter as impractical and given to making premises not related to the facts. This first phase is still a powerful and indispensable overlapping one, from which modern fluid mechanics will derive much of its basic data.

Then followed a short second phase with Prandtl the leader, in which analogies and imaginative concepts were used, with the results of aerodynamic research as a basis. The main accomplishment was to coordinate and correlate old and new experimental data, with particular emphasis on Reynolds' number and dimensional analysis. Much basic data were gathered.

The present is the beginning of the third phase. As far as fluid mechanics is concerned, the other two periods had not been very fertile of broad theories, although productive of much data. The researchers suddenly realized that the experimental results must be regarded as arrays of fluid mechanics statistics if there was to be order in the research process and in the interpretation and synthesis of the information into basic concepts. The recent statistical theory of turbulence is more accurately the embryonic theories and facts of turbulence examined in the light of the laws of chance or statistics. It is not new theory—only the dawn of orderly research. The shame and wonder is that investigators were so late in their realization of this gross inadequacy in their methods. A debt of gratitude must be expressed to Professor Taylor for impressing the facts forcefully on modern hydraulics and to Professor Kalinske for his masterful applications.

Even the modern hydrologist does not seem to realize that hydrology is a collection of hydrologic facts requiring statistical analysis. There is very little theory to modern hydrology—merely an interpretation of data.

There is a great need for research of the kind Professor Kalinske has accomplished. With the present meagerness and paucity of basic laboratory and field data, statistical methods will avail very little. The statistical theory of turbulence does not obviate the observation and measurement of turbulent phenomena.

Paul Nemenyi,[45] **Esq.** (by letter).—The pioneer work done by Professor Kalinske in the experimental investigation of open channel turbulence and other turbulence problems pertinent to hydraulic engineering will stimulate many research workers to further inquiries. Eventually, it will also help practical hydraulic engineers to obtain a deeper insight into the fluid motion phenomena with which they deal in their daily work.

Of the many problems with which the paper deals, the writer proposes to discuss mainly one: The question of how a high rate of energy dissipation may be attained under various given conditions. The author appropriately distinguishes two main practical problems: (*a*) The disposition of energy dissipators all along a conduit or a channel in order to convert potential energy into turbulent energy, rather than into kinetic energy of the main longitudinal flow—or, in other words, to prevent the developing of large velocities; and (*b*) energy dissipation at the end of a chute to convert kinetic energy of the main flow into the more dispersed form of turbulent energy and into heat—that is, to destroy the large velocities present.

For several years, the writer has been interested in problem (*a*), which is important in the design of fishpasses, timber-floating channels, and sometimes pipes of surge tanks, and is related also to spillway design. He cannot agree with the author's opinion that quantitative turbulence research in its present stage can give suggestions or information relevant to the solution of this problem. The author bases his opinion primarily upon a formula for the rate per unit length at which potential energy is converted into turbulent energy in a circular pipe. The formula may be written in the following form:

$$\text{Rate of energy conversion varies as } \int_{y=0}^{r} \rho \sqrt{\overline{v^2}}\, l \left(\frac{dU}{dy}\right)^2 y\, dy$$

He concludes that in order to secure highly effective transformation of potential energy into turbulent energy, l, the size of eddies (being proportional to l) should be made as large as possible. Even apart from the fact that, as this discussion will show, the reliability of the foregoing energy conversion formula has very substantial limitations, the conclusion of the author is not convincing because in a turbulent flow v and $\frac{dU}{dy}$ are modified as soon as l is modified. All characteristic quantities in the integral being inseparably related, it cannot be decided without further study whether arrangements which produce large values of l would really tend to increase the rate of energy conversion. Indeed, there are good reasons for the assumption that the highest effectivity of energy dissipation can be obtained entirely without resorting to large turbulence eddies.

With little exception statistical inquiries into turbulent flow have been concerned with cases in which the average flow (for a sufficiently long period) is steady, straight, and uniform. The foregoing formula seems to have these limitations. Obviously no rough conduit or channel can strictly satisfy these conditions; and as the size and efficiency of the "roughness" increases, local deviations of average flow from the straight line will also increase in importance. This is one reason for the inadequacy of present-day turbulence statistics in producing a satisfactory treatment of energy dissipators. But even if turbulence research will extend to far more varied phenomena, however, energy dissipation studies will always need to go beyond turbulence study and go back to the more general concept of vorticity, supplemented by such concepts as boundary layers and discontinuity surfaces. The following remarks on the different possibilities of energy dissipation in long conduits will trace (aided by these concepts) the transformation of potential energy into the kinetic energy of the turbulence eddies, thus indicating the conditions under which this kind of energy can be generated at a high rate.

---

[45] Research Fellow, Iowa Inst. of Hydraulic Research, Univ. of Iowa, Iowa City, Iowa.

The first result of such a study is that, the shape of the "roughness" obstacles being given or chosen, the spacing must be recognized as a factor of equal consequence along with the size (height), which many times even today is believed to be the all-important factor. Figs. 10 and 11 illustrate this fact in its simplest and most marked instance. Fig. 10(*a*) shows that obstacles, placed so closely together that only very narrow slits remain between them, fail to disturb the boundary layer to any noticeable extent. In such slits the fluid is very nearly at rest, the pressure being the same as in the undisturbed boundary layer. The "obstacles" have scarcely any measurable energy dissipating influence upon the main flow. (Obviously, the known use of piezometric holes is based on this fact and in certain experiments piezometric holes have been actually replaced by slits in the cross-section direction.) In Fig. 10(*b*) the space between consecutive obstacles is approximately the same as the height of obstacles. The slowly moving material in the boundary layer cannot bridge the gap between the two obstacles; instead the faster moving material from the inner stream transfers some of its momentum to the neighboring fluid in the gap, or "bay." This in turn, by continuity, causes a backward stream near the bottom of the "bay." Thus a vortex, as indicated in Fig. 10(*b*), is generated. However, if, as assumed, the "bay" has a square or nearly square cross section, the vortex is locally fixed (trapped) and stable. As a result, it does not emit vorticity of any considerable amount into the inner stream and remains a poor energy dissipator although the height of the obstacle relative to the total width of the conduit may be very considerable. Only if the Reynolds number computed from the size of obstacle and the velocity of the main flow is extremely high may such a vortex of square cross section become unstable and thus become a more efficient energy dissipator; but no direct experimental evidence is available to support this view.

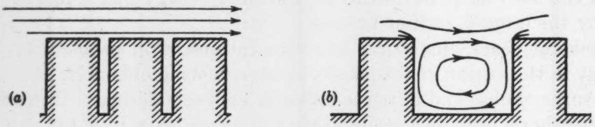


FIG. 10

For longer "bays" the situation is substantially different. Obviously, a very long oval-shaped vortex is unstable; it will give place to a more complex and less stable pattern of flow. Experiments of H. Schlichting[46] indicate, however, that the maximum energy dissipation efficiency of the type of roughness in question is reached only when the spacing of obstacles is approximately ten times their height. To understand this condition better one should first examine the case of the stream field in which there is only a single obstacle upon a smooth boundary. The writer has studied the stream fields experimentally for various shapes of individual obstacles.[47] Figs. 11 (α) and 11 (β) show the results for obstacles extending across the entire breadth of the channel, their cross sections being comparable to those under discussion. (In these figures, the arrows and corresponding numerals indicate velocities expressed as percentages of the undisturbed velocity.) It will be seen that the obstacle deflects the stream considerably, giving to it a momentum toward the inner part of the channel. Thus the region that is the seat of considerable energy dissipation extends far into the inner part of the flow. The lee of the obstacle, on closer investigation, showed velocity fluctuations approaching the same order of magnitude as the local average velocities given in Figs 11 (α) and 11(β). The unsteady part of the vorticity corresponding to these fluctuations is carried far into the inner channel and is the main source of energy dissipation there. If one now wishes to transform (at the highest possible rate) the potential energy all along the channel into such vorticity by the use of perpendicular obstacles, one must obviously place them equidistant, not too far from each other, but far

---

[46] "Experimentelle Untersuchungen zu dem Rauhigkeits-problem," by H. Schlichting, *Ingenieurarchiv*, Vol. 7, 1936, p. 1.

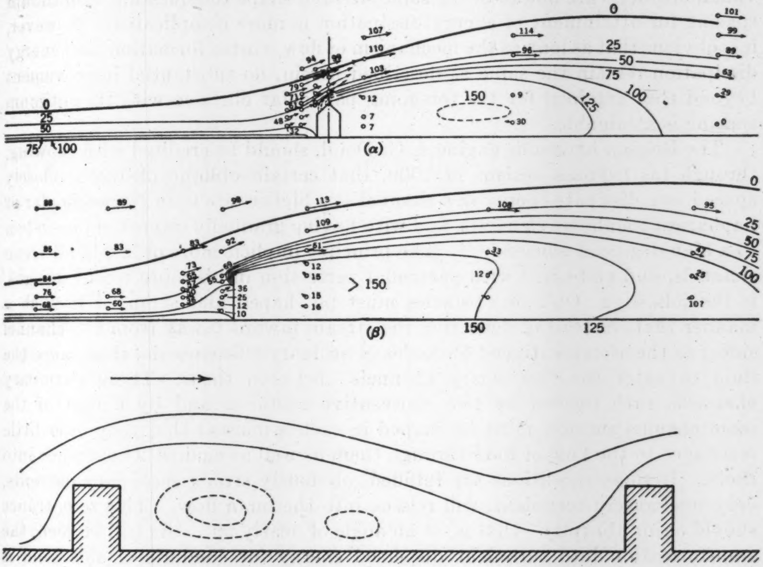


FIG. 11.—CURVES OF EQUAL TOTAL HEAD LOSS

enough to make the development of a considerable forward velocity on the windward side of each obstacle possible (Fig. 11(γ)). Without such a forward velocity the momentum necessary for the development of the large leeward vorticity would not be available. This reasoning explains the experimental finding by Schlichting concerning the optimum spacing. As his results indicate, the order of magnitude of optimum spacing remains roughly the same if, instead of obstacles extending across the breadth of the channel, separate roughness elements such as hemispherical shapes are used. Of course, if the individual obstacles are not all of the same size and shape the question of optimum spacing for attainment of energy dissipation is more complicated. However, it is obvious that as long as the mechanism of flow, vortex formation, and energy dissipation remain the same as described herein, no substantial improvement beyond that attained for the foregoing particular obstacle with its optimum spacing is attainable.

---

[47] "A New Device for Direct Stream Field Studies and Its Applications," by P. Nemenyi, *Ingeniørvidenskabelige Skrifter No. 39*, Copenhagen, 1935.

The Belgian hydraulic engineer, G. Denil, should be credited with showing, through his fishpass designs of 1909, that certain oblique obstacles, closely spaced, can dissipate energy in a channel at a higher rate than perpendicular or haphazard roughness elements, and with having gradually improved his system to a high degree of efficiency.[48] The principle implicit more or less in all Denil channels, and embodied with particular perfection in his more recent designs, is the following: Oblique obstacles must be shaped and arranged in such a manner that, instead of deflecting the stream inward (away from the channel sides) as the aforementioned obstacles of ordinary efficiency do, they cause the fluid to enter the "secondary channels" between them. These secondary channels, each formed by two consecutive obstacles and by a part of the main channel surface, must be shaped in such a manner that they offer little resistance to the flow of fluid through them as well as against its entrance into them. If these conditions are fulfilled, obviously strong secondary currents, only moderately turbulent, will reissue into the main flow. This re-entrance should occur abruptly—that is, at an angle of nearly 90°. As will be seen, the transformation into turbulent dispersed energy for conduits of this kind is concentrated for the greatest part in the vicinity of the areas where the secondary currents re-entering strike the main flow. In order that the gaps between these areas may be as small as possible, the obstacles should be as thin as practicable.

Since practically all of the designs of G. Denil made for various special purposes are too elaborate to be entirely economical, the writer has endeavored (first in a joint research with C. M. White,[49] and later during his work at the Iowa Institute of Hydraulic Research) to incorporate these principles of energy dissipation into new designs sacrificing as little as possible in efficiency while obtaining considerable simplification. Fig. 12 illustrates three of the designs developed—one of them a result of the London studies, the others of the Iowa studies. (It will be observed that the two latter designs (Figs. 12(*b*) and 12(*c*)) have almost exactly the same kind of baffles, but in one case are adapted to the conditions of a circular pipe and in the other to an open channel with a trough-like cross section.) These simple designs with the directions of flow traced in them well illustrate the foregoing principles of energy dissipation.

---

[48] "La Mecanique du poisson de rivière; qualités nautiques du poisson; ses methodes locomotrices; ses capacités; ses limites; resistances du fluide; effet de la vitesse; de la pente; resistance de seuil," by G. Denil, Bruxelles, 1938; see also "Fish Movements and Fishpasses," by P. Nemenyi (a review of G. Denil's book), *Nature*, September, 1939.

[49] "Report on Fishpass Research," by P. Nemenyi and C. M. White, mimeographed (printed report in *Journal* of the Institution of Civil Engineers is being prepared); see also: "Steep Channels Fitted with Energy Dissipators," by P. Nemenyi, *Transactions*, A. S. M. E., Vol. 60, 1938, Page A-118 (*Journal of Applied Mechanics*).

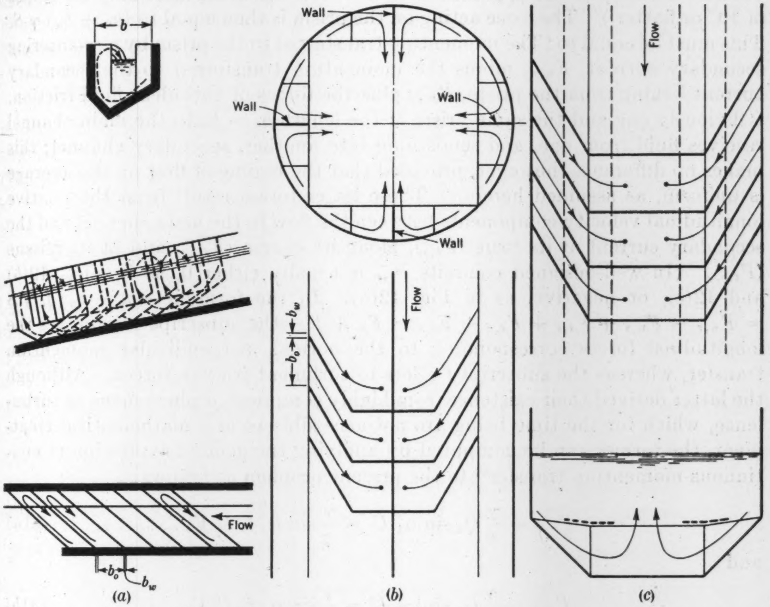


FIG. 12

While testing these and similar fishpasses the writer found the following quantitative explanation for the high effectiveness of this type of energy dissipating device. Suppose the free width of one secondary channel, measured in the direction of the axis of the main channel, is b_0; the wall thickness of secondary channels (measured in the same manner) is b_w; then $b_0 + b_w$ is the spacing of obstacles along the main channel. Applying the momentum theorem to a fluid prism (Fig. 13) of length $b_0 + b_w$ and of cross section f_0 corresponding to

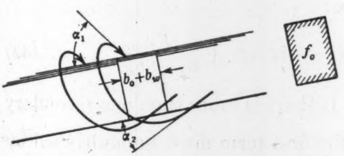


FIG. 13

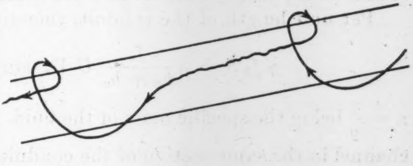


FIG. 14

the inner channel (free cross section), let S be the slope of the main channel, or the pressure loss per unit length if a closed conduit is being studied. (More exactly: $S = \sin \alpha$, in which α is the angle of the channel axis with the horizontal. The use of the approximation $S = \tan \alpha$ is acceptable only for slopes of 10° or flatter.) The force acting on the prism is then equal to $(b_0 + b_w) \gamma S$. This must be equal to: The momentum transferred to the prism by a re-entering secondary current, $F_{n,r}$; minus the momentum transferred to the secondary current issuing from the prism, $F_{n,i}$; plus the forces of turbulent fluid friction. (Obviously one and the same prism of the length $b_0 - b_w$ in the main channel receives fluid from one, and sends fluid into another, secondary channel; this makes no difference, however, provided that the regime of flow on the average is uniform, as assumed herein.) These latter forces result from the relative longitudinal velocity components between the flow in the main channel and the secondary current at its issue ($F_{t,i}$), along its course ($F_{t,c}$), and at its reissue ($F_{t,r}$). (In well-designed conduits $F_{t,r}$ is usually either 0, as in Figs. 12(*b*) and 12(*c*), or negative, as in Fig. 12(*a*). In the formula—$(b_0 + b_w) f_0 \gamma S = F_{n,r} - F_{n,i} + F_{t,i} + F_{t,c} + F_{t,r} = F_n + F_t$—the subscript n indicates the longitudinal forces corresponding to the normal, perpendicular momentum transfer, whereas the subscript t refers to turbulent friction forces. Although the latter derived their existence from highly complicated phenomena of turbulence, which for the time being are not accessible to any mathematical treatment, the former can be computed by applying the general expression of continuous momentum transfer[60] to the present problem as follows:

$$F_{n,r} = \frac{\gamma}{g} Q_s \sin \alpha_1 U = \frac{\gamma}{g} \sin \alpha_1 f_s U V_s \qquad (31a)$$

and

$$F_{n,i} = \frac{\gamma}{g} Q_s \sin \alpha_2 U = \frac{\gamma}{g} \sin \alpha_2 f_s U V_s \qquad (31b)$$

in which g = gravitational acceleration; Q_s, V_s, f_s = rate of flow, average velocity, and normal cross section of the secondary channel; U = average longitudinal velocity of the flow in the main (free) channel section; α_2 = angle of entrance from the main channel into the secondary current; and α_1 = angle of reissue of secondary current into the main channel.

Per unit length of the conduit, therefore:

$$\gamma f_0 S = \rho \frac{f_s}{b_0 + b_w} U V_s (\sin \alpha_1 - \sin \alpha_2) + \frac{F_t}{b_0 + b_w} \qquad (32)$$

$\rho = \frac{\gamma}{g}$ being the specific mass of the fluid. If there is more than one secondary channel in the same section of the conduit the first term must be multiplied by their number, n. (In Fig. 12(*b*), $n = 8$; in Fig. 12(*c*), $n = 2$.)

Theoretical considerations, as well as experimental facts, indicate that the first term is much larger than the second, for a well-designed channel. The structure of this first term contains much of what is essential for the understanding of this system of energy dissipation, particularly if it is compared with the corresponding, well-known formula for channels or conduits of ordinary type.

---

[60] For a graphic explanation of this simple, fundamental relation, see "Mechanics of Turbulent Flow," by Boris A. Bakhmeteff, Princeton University Press, 1936, pp. 38–39.

For a prismatic fluid body in the free cross section of the channel, with cross section f_0, and of unit length, it is generally accepted that in smooth channels as well as in rough channels of the usual types the following relation (a generalized form of Equation (2)) is valid:

$$\gamma f_0 S = \rho P_w \overline{u\,v} \qquad (33)$$

in which: P_w = "wetted perimeter" of the prism; u = fluctuation of longitudinal velocity; and v = velocity fluctuation perpendicular to the perimeter. (For each point of the wetted perimeter the average of $\overline{u\,v}$ for a long period of time must be computed; the quantity in Equation (33) is the average of these averages over the entire wetted perimeter.)

In the light of the remarks at the beginning of this discussion, it is known that Equation (33) cannot possibly be strictly correct for rough channels, because of the deviation of the average velocity vectors from straight parallel lines. For very large roughnesses this deviation may be quite considerable. It can scarcely be doubted, however, that the formula remains correct in so far as orders of magnitude are concerned. (Otherwise the numerous applications of this formula to rivers and other considerably rough conduits could not have led to favorable results.) Therefore, accepting the formula and comparing it with the formula of the highly effective energy dissipators discussed, one finds that in the latter the product of the velocity fluctuations is replaced by that of actual average velocities. On the other hand the entire wetted perimeter P_w of the prism is replaced by the expression $\frac{n\,f_s}{b_0 + b_w}(\sin\alpha_1 - \sin\alpha_2)$ which obviously corresponds only to a moderate part of the wetted perimeter, in accordance with the fact that the generation of turbulence eddies in such energy dissipators is restricted to part of the perimeter. However, this is a factor of lesser order of magnitude than the replacement of a fluctuation-product-average by a product of two average velocities. Thus, it is readily understood from the comparison of the two different mechanisms of momentum transfer that even the simplest of these energy dissipator designs (Fig. 12(*a*)) makes possible slopes, for a given limited average velocity, about 8 to 10 times larger than perpendicular obstacles of the same height with their optimum spacing would permit.

The results of intense momentum transfer produced by the energy dissipators discussed can well be compared to a shock or impact, in contrast to an abruptly expanding conduit, for which—as the author rightly points out—the expression "shock loss" is quite unsuited. Therefore, where the secondary current strikes the main flow, immediately intense but fairly fine-scale turbulence eddies are produced, without the intermediate stage of large-scale eddies. One could be tempted to speak of a large-scale regular vorticity with longitudinal axis; but it is doubtful whether this point of view would be either adequate or fruitful because the path of any individual particle is composed of two quite different kinds of paths alternating with each other in the manner indicated schematically in Fig. 14. It appears that no system of reference in uniform translation movement can be defined with respect to which the movement of the particles would have a closed orbit, and therefore the concept of vorticity is not strictly applicable to the phenomenon as a whole. Furthermore, the high rate of energy dissipation could not be explained by the presence of a vorticity with longitudinal axis, since it is known from other investigations that vorticity of this kind does not dissipate energy of any amount worth mentioning. For example, it is known that the secondary currents, assumedly connected with turbulent flow in a straight, smooth conduit of non-circular section, do not dissipate energy to any considerable extent.

Much more helpful in the understanding of the highly efficient roughness types discussed herein is the concept of "discontinuity surfaces." To be sure, discontinuity of velocity distribution in the strict sense of the word is impossible in any steady phenomenon. Suppose, for example, that a drainage channel meets a river at a certain angle, and that there is a sluice at the channel's mouth. In the instant when the sluice is lifted there is a genuine discontinuity between the velocity distribution in the channel and in the river, and in Fig. 15 line *A B* represents the surface of discontinuity. Quickly upon the generation of eddies, however, and their migration into the main stream, a kind of unity between the two velocity fields is established, no surface of discontinuity (in the strict sense of the word) being present; yet in a more or less narrow strip —tentatively indicated in Fig. 15 by shading—the transition between the velocity distribution of the channel and the river is particularly steep. Such regions of steep transition between two distinct velocity distributions can be represented schematically by a surface or, in the section, by a line of discontinuity; and it is in this sense that "discontinuity surfaces" are discussed in theoretical fluid mechanics.

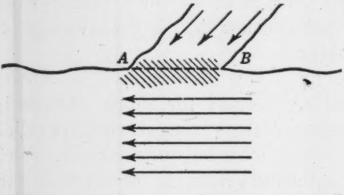


FIG. 15

There are obviously four extreme possibilities for the nature of such discontinuities, and the writer has represented them in Fig. 16 (the dots indicate streamlines perpendicular to the plane of the drawing). Actually any superposition of these cases can occur, but it is valuable to distinguish the distinct fundamental possibilities.

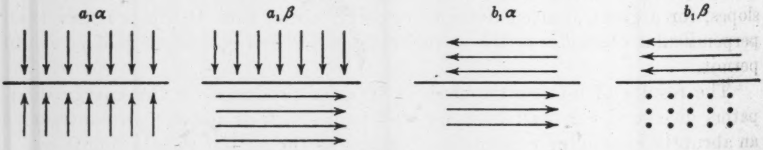


FIG. 16

Where the secondary current "strikes" the main flow, obviously, case $a_1\beta$ is realized with very good approximation. The foregoing momentum transfer studies explain the superior energy dissipating ability of this kind of discontinuity. The writer has made a little experiment with the channel in Fig.

12(*c*) to compare "discontinuity" $a_1\,\beta$ with $b_1\,\beta$. Obviously, in the middle section of each secondary current, its relation to the main current is very close to that in $b_1\,\beta$. Therefore, if this part of the baffles is covered by a longitudinal strip of thin steel sheet as indicated by a broken line in Fig. 12(*c*), the friction term $F_{t,c}$ is replaced by the known and extremely small friction of water upon a smooth steel surface. The comparison has shown that this does not, to any considerable extent, alter flow conditions in the main channel, thus indicating the complete lack of significance of the term $F_{t,c}$ compared with the total force, or with F_n which constitutes its main part. Similar relations can be expected to exist also between the energy conversions corresponding to these various forces. There are indirect reasons for believing also that "discontinuity" $b_1\,\alpha$ is a rather poor energy converter compared with $a_1\,\beta$. Indeed if one looks once more upon the ordinary roughness in its optimum spacing (Fig. 11(γ)), one will find that between the inner flow and the flow within the lee region of the obstacles a steep transition is occurring which can be represented schematically by a discontinuity surface of type $b_1\,\alpha$; and one can conclude from the comparatively moderate efficiency of this type of roughness that this type of "discontinuity surface" is not highly effective as a dissipator of energy.

The writer intends also to examine case $a_1\,\alpha$ by a comparative study of conduit design Fig. 12(*b*) with and without the four straight, longitudinal walls. Obviously these prevent occurrence of "discontinuity" of the kind $a_1\,\alpha$, thus securing secondary currents of greater intensity at the reissue into the main flow. These reissuing jets being of decisive importance for the energy dissipation in the conduit as a whole, the amount of increase in the dissipation caused by the walls will give an indication of the importance of the energy-dissipating effect of a discontinuity of type $a_1\,\alpha$.

It appears on the whole that the concept of "discontinuity surfaces," although an artificial abstraction, is, along with the general concept of momentum transfer, a fruitful aid in the analysis and understanding of the varied phenomena of energy conversion in conduits.

As to problem (*b*), that of destroying the large forward velocities present at the foot of an overfall dam or a chute, the writer fully agrees with Professor Kalinske's remark that, for security against scour or other destruction, transformation of forward flow into large-scale eddies or vortices is not sufficient and that disintegration into a finer-scale turbulence is indispensable. The writer believes, however, that the means to this end will have to be investigated experimentally and that it must be discussed in terms of general momentum transfer and "discontinuity" concepts. Although there are many valuable hydraulic research studies for special structures for scour prevention at the foot of dams, it appears that no general investigation of the problem from this fundamental point of view has been attempted thus far.

The difficulties of the problem are great and manifold. If compared with problem (*a*), discussed previously herein, two additional difficulties will be noticed immediately: First, that problem (*b*) is one of essentially non-uniform flow; and, second, that high-velocity shooting flow must be dealt with. Every experimenter knows the "evasive" character of such flow. Coming from a reservoir or river, the flow in problem (*a*) is relatively slow and can be easily deflected into the secondary channels; water shooting at high velocity is much more difficult to divert.

Nevertheless the writer believes that some of the considerations outlined in this discussion, concerning case (*a*), can well be utilized in case (*b*).

From an excellent study by Schoklitsch[51] it is known that the phenomenon of a "hydraulic roller" connected with the hydraulic jump should be considered as a backward flow of the sluggish water on top of the forward shooting flow, with a "discontinuity surface" between, rather than a single large vortex. From the foregoing comparative discussion it is known, however, that this particular kind of "discontinuity surface" is not one of very high energy dissipating efficiency, and this indicates that future studies may yield very substantial progress by revealing facts concerning "discontinuity" of the kind $a_1 \beta$ shown in Fig. 16. If the writer understands its function, the "jet breaker" ("stralkyvare") suggested by Erik Lindquist,[52] M. Am. Soc. C. E., appears to be an energy dissipator that produces a similar effect.

However, when considering the problem of destroying large forward velocities, it should be kept in mind that the task is not always best solved by dissipating energy at a particularly high rate—that is, by maximum velocity destruction per unit length of structure. It may be more advisable, in many cases, to dissipate the energy at a somewhat more moderate rate, if it can be located conveniently. Therefore it is not impossible that the structures generating an ordinary "hydraulic roller" will be used also in the future since they conveniently solve problem (*b*) by placing the dangerous vorticity, issuing from the discontinuity surface, well above the bed surface.

The writer believes that the most important field for the civil engineering applications of turbulence statistics lies in the problem of sediment transportation in canals and rivers. An entire series of Swiss investigations by aid of the bed-load trap[53] as well as the combined theoretical and experimental research of H. A. Einstein and G. Polya[54] seems to indicate that even the movement of the heavy bed load is essentially connected with the haphazard fluctuation-phenomena characteristic of turbulence. Nevertheless, it would be a fallacy to conclude that the problem of propelling granular materials by fluids can be treated, as a whole, as a problem of turbulence statistics, in the sense that Professor Kalinske makes use of this method. Paradoxically, although the bed-load movement obviously derives its energy from, and shares much of the properties of, the turbulence movement in the higher layers of the rivers, it also shows regular rhythms in many instances (such as shown not only by sand waves, but also, for long periods, by the generation of serpentines). This paradox will probably not find its complete clarification before turbulence will be understood in its relation to regular rhythmic vortex distributions on one hand, to momentum transfer by "grain bombardment" on the other. (The vortex theory of sand waves was first presented by G. Darwin and later developed by Vaughan Cornish[55] and the Swedish School of Geomorphologists.[56] The impact theory of sand waves has been presented by R. A. Bagnold in a long series of publications;[57] although the decisive importance of this factor is limited to Æolian phenomena [desert sand, etc.] it can scarcely be doubted that it must be included among the factors governing fluvial sand propulsion too.) With reference to the river problem the author's inquiries into open channel turbulence must be considered a great step forward to a far-off but very worth-while goal. The methodic aspect of the problem as a whole has been analyzed by the writer in a recent paper.[58]

---

[51] "Über die Energie-vernichtung durch Walzen," by A. Schoklitsch, *Die Wasserwirtschaft*, 1932.

[52] "Energyomvandling ved footen af overfall dammar," by Erik Lindquist, Stockholm.

[53] See "Untersuchangen in der Natur über Bettbildung, Geschiebe- und Schwebestofführung," Mitteilung Nr. 33, Eidgenössisches Amt für Wasserwirtschaft, Bern, 1939.

[54] "Der Geschiebetrieb als Wahrscheinlichkeitsproblem," by H. A. Einstein; and "Zur Kinematik der Geschiebebewegung," by G. Polya, Rascher, Zürich, 1937.

Finally, a few words in regard to the problems of gathering experimental data for the statistical study of turbulence: This question consists really of two parts—(1) how to make fluid particles visible, and (2) how to record the movement of particles made visible. To problem (1) the author has contributed a highly reliable technique. It should be stated that the ingenious ultramicroscopic method of A. Fage and H. C. H. Townend[32] does not require the addition of any strange material, the very fine suspensions present in drinking water being sufficient in many cases; for macroscopic visibility of air particles Mr. Townend[59] has suggested that it can be attained without strange admixture, but by localized heating through electric flashes combined with the "Schlieren technik"; the method is very interesting although it is rather unlikely to have any direct applicability for water.

The second half of the experimental problem consists of how to record the movement of particles made visible. The writer would like to call attention to L. Prandtl's suggestion[60] that a uniformly moving camera be used with speeds of systematically varied magnitudes. Mr. Prandtl showed how photographs taken with this method can lead to a very convenient system of velocity-fluctuation statistics. The writer should like to add the suggestion that such photographic records can be used for measuring vortex sizes. The camera moving at the speed corresponding to the average velocity of a certain longitudinal strip of the channel, the vortices in this particular strip will appear in their real size. The vortex size distribution in each strip found by this direct method should be compared with the corresponding average vortex sizes. These can be computed according to Mr. Taylor's suggestion[6] from the velocity data, as shown for the open channel by the author.

BENNIE N. NETZER,[61] JUN. AM. SOC. C. E. (by letter).—The practical hydraulic engineer's need has been for someone to put into actuality Mr. Kalinske's statement (see heading "Transportation of Suspended Material") that "Turbulence should be thought of in terms of definite parameters instead of just as a qualitative descriptive term relating to a general condition of flow." Many hydraulic model studies have been made of problems involving energy dissipation, for example, which have had as their net result only comparative indications of the intensity of turbulence, but no quantitative conclusions. The reason for this has been the lack of an instrument for measuring, accurately, the highly fluctuating velocities in turbulent flow.

---

[55] "Waves in Sand and Snow and the Eddies Which Make Them," by V. Cornish, London, 1914; also "Ocean Waves and Kindred Geophysical Phenomena," by V. Cornish, London, 1935.

[56] See, for example, "Beitrag zur Kenntnis der Transportmechanik des Geschiebes und der Laufentwicklung des Reifen Flusses," by H. W. Ahlmann. (Sveriges Geologiska Undersokning, Ser. C., No. 262, Stockholm, 1914.)

[57] See, for example, "The Transport of Sand by Wind," by A. Bagnold, *Geographical Journal* 99, No. 5, 1937.

[58] "The different approaches to the study of propulsion of granular materials and the value of their coordination," by P. Nemenyi, *Proceedings*, Round-Table Conference on the Rôle of Hydraulic Laboratories in Geophysical Research, Washington, D. C., 1939; publication pending.

[59] *Proceedings*, Royal Soc. of London, Vol. 145A, 1934, p. 80.

[60] "Über ausgeildete Turbulenz," by L. Prandtl, *Proceedings*, 2d International Cong. for Applied Mechanics, Zürich, 1927, p. 62.

[61] Asst. Engr., U. S. Engr. Office, Pittsburgh, Pa.

For a thorough understanding of the turbulence mechanism it is undoubtedly necessary to use some such method as the photographic method suggested by Mr. Kalinske. For quick, practical results, however, some other method that is less time-consuming and tedious and has more flexibility than the photographic method seems desirable. It is difficult to see how the photographic method can be used to measure velocities throughout an entire turbulent region with any ease and without consuming too much time. Model studies involving the analysis of turbulent flow in many hydraulic designs require a quick, practical way for measuring velocities.

Mr. Kalinske states (see heading, "Experimental Techniques") that "An instrument that would give accurate recordings of the instantaneous velocity in a direction either longitudinal or transverse to the flow would be useful." For the determination of the intensity of turbulence in a practical manner such an instrument is necessary.

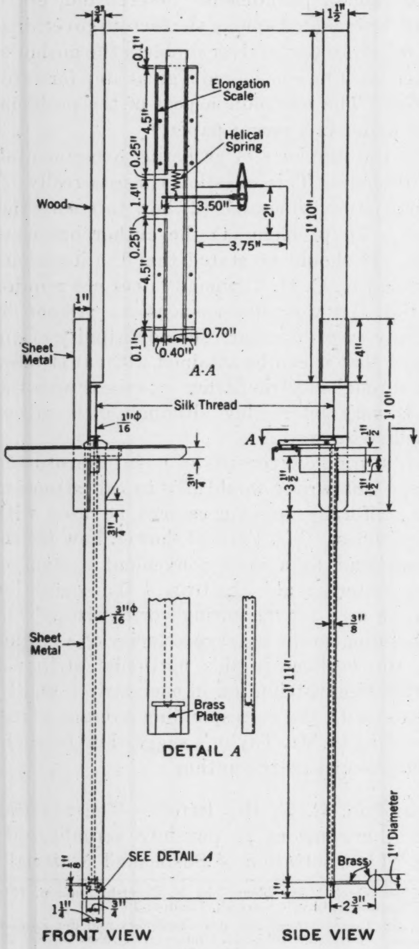


Fig. 17

Fig. 17 shows an instrument that may fulfil Professor Kalinske's requirements. The instrument was designed by H. A. Thomas, M. Am. Soc. C. E., and was used by the writer in experimental work on erosion below dams.[62] The instrument is a helical spring dynamometer that is very sensitive to turbulence and will measure the instantaneous maximum, mean, or minimum velocities at any point in a region of turbulent flow. The meter is essentially a vertical rod, encased in a streamlined tube, with a slightly cupped disk attached to its lower end by a horizontal arm. The torque caused by a current of water impinging on the cupped disk is transmitted by the rod to a horizontal pointer which is restrained in its rotation by two pins. The torque caused by the water current is balanced by a helical spring attached to the pointer and to a slide which moves at right angles to the pointer. When the torque is balanced, the pointer is free of both pins (similar to a scale beam), thus indicating the mean velocity. To ascertain the instantaneous maximum or minimum velocities the spring is stretched until the pointer is just moved from either pin. By suspending the vertical rod by a light thread its weight is removed from the bearings and friction is reduced to an absolute minimum. The instrument is calibrated by measuring the velocity of a jet of water moving with uniform velocity, and then determining the elongation of the helical spring in the same jet.

A. A. KALINSKE,[63] ASSOC. M. AM. SOC. C. E. (by letter).—The writer appreciates the many encouraging comments made by the various discussers, and especially the suggestions and advice submitted by experienced hydraulic engineers. These suggestions will serve as guides for further investigations in this field. Professor Rouse, who has given much thought to various theoretical and experimental phases of the turbulence problem, summarizes in a concise manner the items in the paper which relate to hydraulics problems. His effective analysis of the main features of the paper should prove helpful in dispelling fears that prospective readers may have regarding the presence of the more or less complicated mathematical formulations.

Verbal and privately written comments, other than the published discussions, seemed to indicate to the writer that many hydraulic engineers had the idea that the various theories and analyses presented in this paper had for their purpose the displacement of past ideas on water flow and the demonstration that various practical rules now in common use were all wrong. Nothing could be further from the truth. A study of the inner mechanics of fluid flow and attempts to understand the phenomena of turbulence have for their purpose the providing of basic, generalized information. It is hoped that such fundamental information will permit the bringing together of many apparently different hydraulic problems and will show that there is much that is common among them. Of course, some of the things that will be learned about turbulence will perhaps have no direct practical application. Much of such information will merely provide a better physical picture of the mechanics of turbulent flow. However, there are hydraulic problems for which an increased knowledge of turbulence is vital to their ultimate, complete solution. The recognition of the practical importance of the various conceptions regarding turbulence that were presented is evident in the remarks of the discussers.

---

[62] "Erosion Below Dams," by Bennie Netzer. Thesis presented to Carnegie Inst. of Tech., in 1938, in partial fulfilment of the requirements for the degree of Master of Science in Civil Engineering.

[63] Asst. Prof., Univ. of Iowa, Iowa Inst. of Hydraulic Research, Iowa City, Iowa.

Mr. Stevens states that: "In uniform flow the boundary contacts are a source of continuing turbulence, a balance having been reached between that and heat dissipation." This is a significant and fundamental fact regarding the origin and dissipation of turbulence. Professor Bakhmeteff submits some very pertinent remarks relating to this phenomenon. There is no question but that the boundary regions in a straight conduit are the major source of the turbulence. The writer ventures the opinion that the physical law controlling the creation and shedding of the eddies from the viscous boundary layer in smooth conduits, and from the roughness projections in rough conduits, may some day be formulated. Professor Bakhmeteff raises the question relating to whether the intensity of turbulence is generally distributed across a conduit as shown by the writer's data. Repeated observations in smooth circular conduits by the writer and others have indicated definitely that the longitudinal component of the turbulent velocity fluctuation tends to reach a maximum in the region between the wall and about two thirds the distance from the center.

The writer appreciates the importance of having the proper fundamental concepts regarding the type of turbulence one is talking about, as was shown by Mr. Mason. True turbulence is probably best defined as being motion in which there is no periodicity in the fluctuation in magnitude of the three velocity components, v, u, and w. From a practical standpoint turbulence is nonisotropic if there is a mean velocity gradient normal to the direction of flow. Mr. Mason says that the present statistical theory of turbulence does not offer an immediate possibility of application to hydraulics problems since it is concerned chiefly with isotropic turbulence. This is not entirely true. One of the most important practical hydraulic problems depending for its complete solution on a better knowledge of turbulence is that of sediment transportation. Application of the concepts arising from the statistical theory has enabled the writer to study the mechanics of the placement of sediment into suspension in the bed region of rivers. Certainly the turbulence in this region is far from being isotropic. From the experimental standpoint, application of Professor Taylor's statistical theory of turbulent diffusion[41] has permitted the direct measure of the coefficient of diffusion in an open channel (see Fig. 18). A study of this coefficient is all-important in problems involving the transportation of suspended material. In general, so far as hydraulics is concerned, the statistical theory of turbulence has not provided and probably will not provide for a long time a direct and exact solution to any practical problems. However, it has provided and will provide tools with which hydraulic engineers will be able to get a better insight into the physical phenomena, and also will provide a basic framework from which orderly research can proceed.

Mr. Mason notes various other photographic techniques that have been used and suggested for studying turbulence. The size of the suspended immiscible liquid particles used by the writer averaged about 1 to 2 mm. It appeared that particles of this size were small enough for studying the diffusion and intensity characteristics of the turbulence in the main body of flow in pipes or channels. Studies near the boundary region require the use of much smaller particles, and a magnifying lens system is necessary for taking pictures in this region.

Mr. McNown mentions that the turbulence must be considered in the study of sediment transportation as bed load. This is also pointed out by Mr. Nemenyi in his discussion. There can be no doubt that this is true, and fundamental studies of bed-load movement will have to give consideration to parameters which describe the turbulence quantitatively. Probably one of the most urgently needed fundamental experimental investigations in regard to sediment transport is a study of the zone near the bed where bed material is picked up and transported in suspension. Increased knowledge regarding this transitional zone in the bed region will be the key to the ultimate solution of the problems of prediction of the absolute concentration of suspended sediment. Professor Rouse has stated that:[64] "* * * analysis of the sediment problem as a whole will first become possible when bed-load and suspended load can be expressed as functions of the same flow parameters." In other words, the flow characteristics that control the movement of material along the bed also control the development of the turbulence that is responsible for the placement of material in suspension from the bed region.

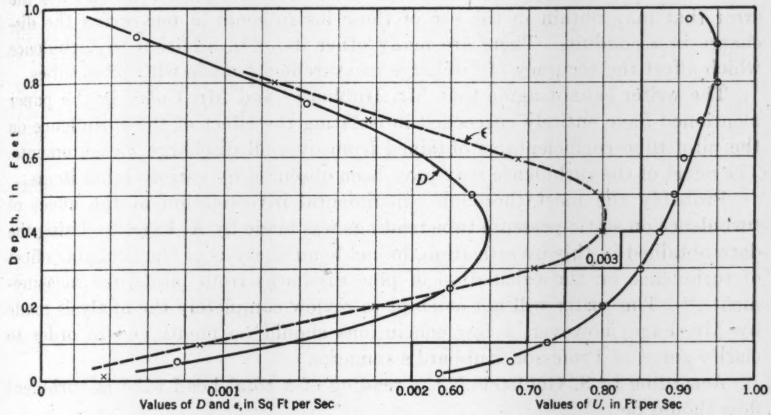


FIG. 18.—VARIATION OF D, ϵ, AND $\overline{U}$, IN THE CENTER OF A SMOOTH CHANNEL

The calculation of the length term λ, which is defined by Equation (30) in Mr. McNown's discussion, was made by substituting in this equation values of R_x for small values of x. This is in accordance with the procedure followed by Professor Taylor.[65] A knowledge of the functional relationship between x and R_x is not necessary in order to calculate λ to a degree of accuracy commensurate with the accuracy of the experimental data.

In the paper the writer makes a brief statement regarding the effect of turbulence on the measurement of static pressure in turbulent flow and the measurement of mean velocity by means of a pitot tube. From some measurements made by himself and various studies by others he concluded that for the ordinary turbulence present in pipe or channel flow the error in the "reading" of a static pressure tube or pitot tube due to the turbulence should not be greater than about 1%. Professor Hubbard, in his discussion, disagrees with this view. First of all it should be emphasized that the writer was talking about the error in the "reading" of a static-pressure tube or a pitot tube and not the error that may obtain in the use of these instruments in measuring the discharge in a conduit. There are many other items in addition to turbulence which affect the accuracy of discharge measurements made with pitot tubes.

---

[64] "An Analysis of Sediment Transportation in the Light of Fluid Turbulence," by Hunter Rouse, U. S. Dept. of Agriculture, Soil Conservation Service, *Technical Publication No. 25*, July, 1939.

[65] "Statistical Theory of Turbulence," by G. I. Taylor, *Proceedings*, Royal Soc. of London, Vol. 151A, 1935, p. 456.

The writer cannot agree that Mr. Hubbard[14] and Mr. Cole[37] in the paper mentioned have entirely succeeded in isolating the effect of the turbulence on the pitot-tube coefficients as obtained from over-all discharge measurements. The effect of the turbulence may have been obscured by various other items.

Probably the most thorough experimental investigation of the effect of turbulence on static-pressure tube readings was made by A. Fage.[66] Using the data obtained in this investigation, he made an analysis of the probable effect of turbulence on the calculation of pipe discharge from pitot-tube measurements.[67] The writer will not attempt to review completely the analysis made by Mr. Fage; however, a few conclusions should be mentioned in order to clarify some of Professor Hubbard's remarks.

According to S. Goldstein,[68] the reading of a total head tube in turbulent flow should be

$$H = \bar{h} + \frac{\bar{U}^2}{2\,g} + \frac{\overline{q^2}}{2\,g} \qquad \text{(34)}$$

in which $\overline{q^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$, the sum of the mean squares of the turbulent velocity components along the three coordinate axes; $\bar{h}$ is the local mean static pressure head; and U the local mean velocity. Mr. Fage points out that according to the vorticity transport theory the value of $\left(\bar{h} + \frac{\overline{q^2}}{2\,g}\right)$ is constant across the turbulent wake behind a body placed in a fluid stream. At the edge of the wake where the stream is undisturbed the static pressure is equal to $\left(\bar{h} + \frac{\overline{q^2}}{2\,g}\right)$. This indicates that the true local static pressure varies across a turbulent wake. These conceptions are extended by Mr. Fage to the turbulence in a pipe, the main body of the turbulent stream corresponding to the turbulent wake. The reading of a static-pressure tube placed in a turbulent fluid, according to Mr. Fage's investigation, is equal to $\bar{h} + K(\overline{v^2} + \overline{w^2})$, in which K is an experimental constant which will depend on the design of the tube used. However, in any case the reading will tend to approach the value of the pressure at a smooth wall, h_w.

If the pitot tube reads a pressure equal to that given in Equation (34), and the other side of the differential gage is connected to a wall piezometer, Mr.

---

[66] "On the Static Pressure in Turbulent Flow," by A. Fage, *Proceedings*, Royal Soc. of London, Vol. 155A, 1936, p. 576.

[67] "The Estimation of Pipe Delivery from Pitot-Tube Measurements," by A. Fage, *Engineering*, Vol. 145, June 3, 1938, p. 616.

[68] "Measurement of Total Head and Static Pressure in Turbulent Streams," by S. Goldstein, *Proceedings* Royal Soc. of London, Vol. 155A, p. 570.

Fage arrives at the following expression for the fractional overestimation of pipe delivery:

$$\int_0^1 \frac{\overline{q^2}}{\overline{U}_m^2}\left(\frac{\overline{U}_m}{\overline{U}}\right) d\left(\frac{y}{r}\right) + \int_0^1 \frac{(\overline{h} - h_w)}{\overline{U}_m^2/2\,g}\left(\frac{\overline{U}_m\,y}{\overline{U}\,r}\right) d\left(\frac{y}{r}\right) \ldots\ldots (35)$$

in which: $\overline{U}_m$ = mean velocity in cross section; y = distance from center; and r = pipe radius. Professor Hubbard's conclusion that if, as the writer states, a static pressure tube has a reading greater than the true static pressure by an amount proportional to the cross-velocity components, the coefficient of a pitot tube will be greater than unity, is erroneous. The effects of turbulence tend to make the coefficient of a pitot tube less than unity. By various transformations Mr. Fage reduces Equation (35) and gives as an approximate expression for the percentage of overestimation: $\frac{190\,\tau_0}{\rho\,\overline{U}_m^2}$, in which τ_0 is the wall shear and ρ is the fluid density. The maximum value of the expression occurs at the lower Reynold's numbers and is about 1.0%.

In addition to the effect of turbulence, Mr. Fage shows that an overestimation of discharge results from pitot-tube readings due to the fact:

> "* * * that the total head registered by a pitot tube in a region of total pressure gradient is not associated in general with the geometric center of the mouth of the tube, but with an effective center which is displaced from the geometric center towards the region of higher total head."

However, this correction should ordinarily not exceed 0.5% except when a large tube is used in a small pipe. Another item that may influence the reading of a static pressure tube is the deflection of the stream lines of the average flow by the presence of the tube in a region of high velocity gradient—that is, near conduit walls.

Everything considered, Mr. Fage's studies should not be considered as the final word in regard to the effect of turbulence on the readings of pitot tubes and static-pressure tubes; however, his analyses appear to be fundamentally sound and indicate the factors that must be studied in any experimental work on this particular problem.

In his discussion Mr. Shulits wonders whether it is a law of statistics that deviations from the mean of any large number of measurements will be distributed according to the normal error law. This is not true since, if (for instance) the fluctuations were periodic, the distribution of the deviations would certainly not be according to the error law. The fact that velocity fluctuations in fully developed turbulent flow are distributed according to the normal error law is a definite characteristic of turbulence, and not just a mere coincidence or the general behavior of any fluctuating quantity

Mr. Shulits states that the eddy viscosity, ϵ, which is proportional to the coefficient, D, as defined in Equation (6), is a time rate of change of area. Professor Taylor[6] definitely shows that the diffusion coefficient is proportional to the product of the root-mean-square of the transverse velocity and the distance a mass of fluid travels before the correlation coefficient between the velocity at the beginning and at the end of the distance is zero. To state that this coefficient is a time rate of change of area has no physical significance and is merely a "play" on the dimensions of this quantity. The kinematic viscosity, ν, of fluids has the same dimensions as ϵ or D—that is, $\frac{L^2}{T}$. A mere knowledge of the dimensions does not permit a physical interpretation of ν. However, analysis of the physical processes by which, for instance, a gas exhibits the property of viscosity, definitely shows that ν is proportional to the product of the mean velocity of the gas molecules and their mean free path. Professor Taylor's analysis gives a similar physical significance to the quantity ϵ or D.

Experimental data on the variation of D and ϵ in the center of a smooth open channel, 1 ft deep and 2.5 ft wide, are shown in Fig. 18. The mean velocity in a vertical section is 0.865 per ft sec. Values of D were obtained from data similar to those shown in Fig. 6 and ϵ was computed from the relation:

$$\tau = \rho\, \epsilon \frac{d\bar{U}}{dy} \qquad (36)$$

The computation of ϵ was not very accurate since some assumptions had to be made regarding the value of the shear. Nevertheless, the proportionality and perhaps even the equality between D and ϵ are evident.

The writer agrees with Mr. Shulits that the statistical theory of turbulence does not in any way obviate the necessity of observations and measurement; however, it does give direction to such research work.

Mr. Nemenyi makes a very worthy contribution in regard to the retardation of flow in conduits having various types of boundary obstacles. He remarks that energy dissipation studies will have to go beyond turbulence study and into the concepts of vorticity, boundary layers, and discontinuity surfaces. This is true, but it should be added that turbulence studies in general must also go back to these concepts. Since turbulence tends to originate at boundaries and surfaces of discontinuity, the laws controlling the creation of vortices will have to be studied if hydraulic engineers are to learn more about the physical processes in regard to the origin of turbulence. Mr. Nemenyi's remarks relating to the various types of surfaces of discontinuity that may be produced are very enlightening.

Present knowledge seems to point to the fact that ordinary turbulent flow in conduits is the result of the dispersion of eddies from the boundaries into the main body of the fluid stream. Their origin, whether from a surface of discontinuity at the viscous boundary layer in smooth conduits or from roughness projections in rough conduits, is probably governed by strict physical laws. The behavior of an individual eddy in the main fluid stream, of course, is unpredictable.

In order to retard the mean forward velocity of flow in a uniform conduit having a given hydraulic gradient and cross section of flow, the total rate of pressure or potential energy conversion must be decreased. This total rate of energy conversion per unit length of conduit is equal to $Q \gamma S$, in which Q is the discharge, γ the specific weight, and S the hydraulic slope. According to Equation (21), this rate of conversion for ordinary turbulent flow in a circular conduit can be represented by $2\pi \int_0^r \rho\,\epsilon \left(\frac{d\overline{U}}{dy}\right)^2 y\,dy$. The relation between ϵ and $\frac{d\overline{U}}{dy}$ is expressed by Equation (36). The average shear will remain constant if the cross section of flow and S remain the same. Any change in ϵ should result in a proportionally equal and opposite change in $\frac{d\overline{U}}{dy}$. From the foregoing expression for pressure energy conversion (see Equation (21)) it is seen that an increase in ϵ will result in a decrease in Q, or in the mean velocity for a given cross section. In other words, to reduce the mean forward velocity it is necessary to secure intense mixing between the slowly moving water at the boundary and the water in the center portion. This can be done by an increase in the diffusion coefficient, ϵ.

Ordinary roughness projections, as pointed out by Mr. Nemenyi, are not effective for producing a large retardation of the forward velocity. Specially designed obstacles that forcibly throw the water at the boundaries into the main portion of the stream are much more effective in retarding the mean forward velocity of the stream. Though the flow in such special conduits is obviously not directly comparable to ordinary turbulent flow, nevertheless the general concept that intense transverse mixing must be obtained, if a large reduction in forward velocity is to be secured, is still applicable.

In his discussion of the application of the theories of turbulence to problems of sediment transport, Mr. Nemenyi states that it is paradoxical that various regular phenomena manifest themselves in turbulent streams in which the velocity fluctuations are supposedly irregular. Difficult as it may be to imagine how irregular fluctuations can create regular phenomena, nevertheless it does appear that turbulence may even have a direct influence in the production of such effects as for instance the formation of sand ripples on a river bottom. In a paper[69] published in 1939 M. A. Velikanov gives a mathematical analysis which seems to indicate that a plane sand bottom in a uniform stream is transformed into a rippled one by the direct action of the turbulence of the stream.

In regard to Mr. Netzer's comments on the measurement of fluctuating velocities in model studies it should be kept in mind that the photographic methods, as used by the writer, have many advantages even if they are time-consuming. Since no apparatus is introduced into the stream, there is definite assurance that the results obtained are not influenced by, or are peculiar to, the measuring apparatus. Furthermore, the investigators obtain a definite mental picture of the turbulence, which is often as important as the actual quantitative measurements. The time required in analyzing the data obtained on motion-picture film can be shortened by utilizing the principles of random sampling theory. Nevertheless, any instrument, capable of measuring even approximately the turbulent velocity fluctuations directly, would be useful. A checking of the measurements obtained with an instrument such as that shown in

---

[69] "Formation of Sand Ripples on the Stream Bottom," by M. A. Velikanov, *Question 3, Report 13*, published for meeting of International Geophysical Assoc., September, 1939, Washington, D. C.

Fig. 17, against data obtained by photographic studies, would indicate how the instrument responds to the velocity fluctuations in turbulent flow.

It should be mentioned that excellent data on velocity fluctuations in large streams can be obtained with current meters, especially with those of the smaller type. Although, to a certain extent, the data are qualitative, they nevertheless permit the study of the relative magnitude of the velocity fluctuations under different conditions. One of the problems in which a knowledge of the magnitude and frequency of the velocity fluctuations in natural streams is of importance is in suspended material sampling. Before suspended material can be determined accurately and economically, it is important to know how the concentration fluctuates; this fluctuation in sediment concentration is closely allied with the velocity fluctuations of a turbulent stream.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2092

SEWAGE DISPOSAL PROJECT OF BUFFALO, NEW YORK

BY SAMUEL A. GREELEY, M. AM. SOC. C. E.[1]

WITH DISCUSSION BY MESSRS. C. A. HOLMQUIST, ARTHUR J. BULGER, C. R. VELZY, THOMAS M. NILES, AND SAMUEL A. GREELEY.

SYNOPSIS

The purpose of this paper is to make of record, with proper explanation, some of the engineering data accumulated during the design and construction of the sewage disposal project of Buffalo, N. Y. During the three years 1936 to 1939, the Buffalo Sewer Authority completed the construction of intercepting sewers, sewage treatment works, and storm-water relief sewers, at a total project cost of $15,000,000. The sewage disposal works were placed in operation toward the end of June, 1938.

GENERAL STATEMENT

The Buffalo Sewer Authority was created by a special act of the Senate and Assembly of the State of New York and became a law April 8, 1935. The five members of the Authority were first appointed in June, 1935, and later in the year, on December 7, the Authority appointed the organization which was to serve as designing and supervising engineers. Meanwhile, on June 18, the Authority made application to the Federal Emergency Administration of Public Works for a loan and grant, in the amount of $15,000,000, to aid

> "* * * in financing the construction of a system of trunk, intercepting and connecting, lateral and outlet sewers, storm water drains, pumping and ventilating stations, disposal or treatment plants or works, and other appliances and structures to provide an effectual and advantageous means for relieving Niagara River, Buffalo River and Lake Erie from pollution by the sewage and waste of the City of Buffalo and relieving the City of Buffalo from inadequate sanitary and storm water drainage and for the sanitary disposal or treatment of the sewage of the City of Buffalo."

The estimates of cost in this application included construction work in the sum of $13,200,000, and a sum of $1,800,000 to cover preliminary expenses, land and right of way, engineering, interest during construction, legal, and

---

NOTE.—Published in October, 1939, *Proceedings*.

[1] Greeley & Hansen, Chicago, Ill.

administrative costs. The proposed storm-water relief sewers were for the Bird Avenue Sewer District only.

The first investigation of the sewage disposal problem of Buffalo was made between 1909 and 1918 by the International Joint Commission in connection with an investigation of the pollution of boundary waters. The conclusions stated in their final report are briefly summarized as follows:

> "It is advisable to consider the Niagara and Detroit Rivers as a class by themselves, so far as remedial and preventive systems are concerned. As has been stated, the most serious condition existing is the bacterial pollution of these streams. To remedy this evil, sewage treatment should be applied in connection with dilution so far as is necessary to bring their waters to the standard mentioned—a mean annual cross-sectional average of B. coli not exceeding 500 per 100 c.c. This necessary sewage purification can be effected by fine screening or sedimentation, and when necessary by chemical disinfection, at a cost which will impose no unreasonable burden upon the urban community responsible for the present pollution."

Furthermore, it has been clear for a number of years that the waterways within the city limits of Buffalo have been grossly polluted, including Buffalo River, Buffalo Harbor, and Black Rock Harbor. The function of the sewage disposal part of the project, therefore, is threefold, as follows:

(*a*) To relieve Buffalo River and its immediate tributaries from undue pollution;

(*b*) To relieve Buffalo and Black Rock Harbors from undue pollution; and,

(*c*) To remove contamination from Niagara River and to reduce the bacterial content of its waters in order to avoid placing an undue pollutional load on water purification works of communities below Buffalo which derive their water supply from the Niagara River.

A number of the existing sewers had also been overloaded and much in need of relief. As this was especially true of the existing sewers in the Bird Avenue District, it was decided to include storm-water relief sewers for this sewer district in the project.

Action for undertaking the project was stimulated by the orders of the New York State Department of Health, issued in March, 1934, which required that during 1935 the City of Buffalo must engage engineers, take steps to acquire land on which to construct sewage treatment works, submit plans for the Bird Avenue storm-relief drains, and make a satisfactorily comprehensive general plan for interception and treatment of the entire sanitary sewage of the city. Detailed plans and specifications for a substantial part of the work were required to be completed by April 1, 1936, and construction work started on or before July 1, 1936. The order required that the treatment works be completed and placed in operation on or before October 1, 1937.

Actual construction work was begun on a part of the Bird Avenue storm-water relief drains in December, 1935, and a contract for borings, which ultimately required the expenditure of about $38,000, was started in January, 1936. The first major construction contract went into actual construction in June, 1936, under bids which had been advertised on April 23, 1936. An average monthly rate of construction was maintained during twenty-two consecutive months (from August, 1936, through May, 1938) of $547,000.

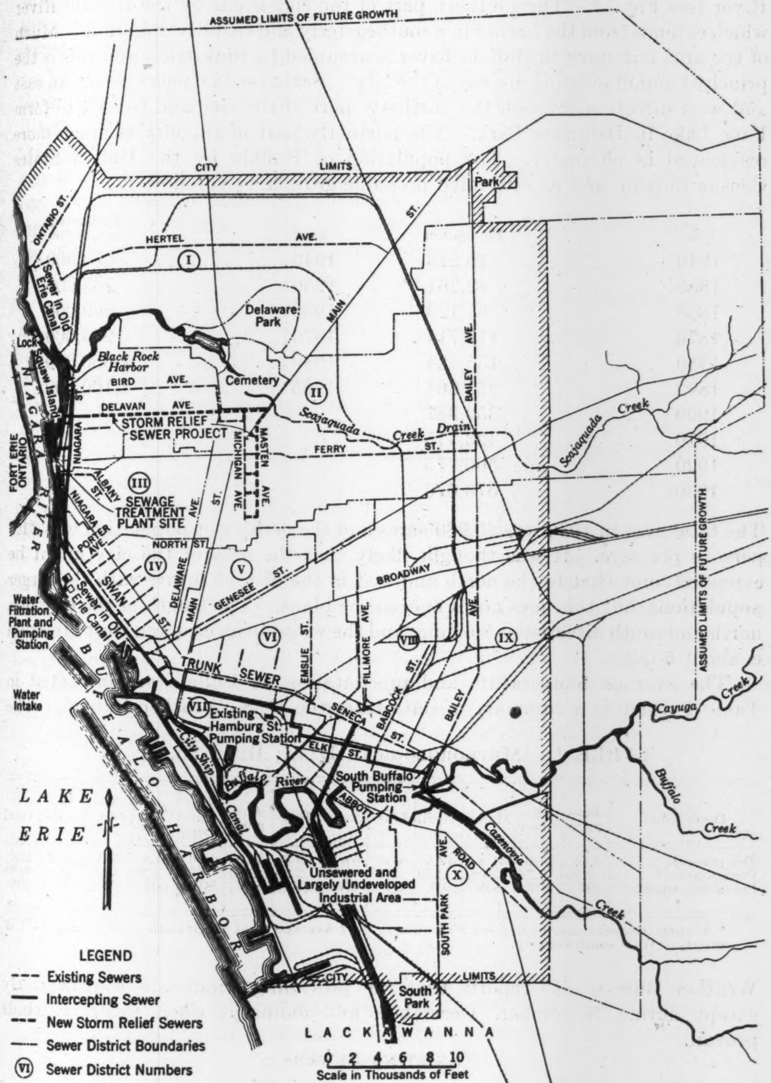

ASSUMED LIMITS OF FUTURE GROWTH
CITY
LIMITS
Park
ONTARIO ST.
HERTEL
AVE.
I
MAIN
ST.
Sewer in Old Erie Canal
Delaware Park
BAILEY
AVE.
Lock
Squaw Island
Black Rock Harbor
BIRD
AVE.
Cemetery
II
NIAGARA
DELAVAN
AVE.
Scajaquada
Creek
Drain
STORM RELIEF SEWER PROJECT
MASTEN
MICHIGAN
AVE.
FERRY
ST.
Scajaquada
Creek
FORT ERIE ONTARIO
RIVER
III
ALBANY ST.
SEWAGE TREATMENT PLANT SITE
DELAWARE
AVE.
NIAGARA
PORTER AVE.
NORTH ST.
IV
V
GENESEE
ST.
BROADWAY
SWAN
MAIN
EMSLIE
ST.
FILLMORE
AVE.
VIII
IX
ASSUMED LIMITS OF FUTURE GROWTH
Water Filtration Plant and Pumping Station
Sewer in Old Erie Canal
VI
BUFFALO HARBOR
TRUNK
SEWER
BABCOCK
ST.
BAILEY
Water Intake
VII
Existing Hamburg St. Pumping Station
SENECA
ST.
ELK
ST.
Cayuga
Creek
City Ship Canal
Buffalo River
South Buffalo Pumping Station
ABBOTT
Buffalo
Creek
LAKE ERIE
N
Cazenovia
ROAD
AVE.
X
Unsewered and Largely Undeveloped Industrial Area
SOUTH PARK
Creek
LEGEND
Existing Sewers
Intercepting Sewer
New Storm Relief Sewers
Sewer District Boundaries
VI Sewer District Numbers
CITY
LIMITS
South Park
LACKAWANNA
0 2 4 6 8 10
Scale in Thousands of Feet

FIG. 1.—GENERAL PLAN OF SEWAGE DISPOSAL PROJECT

The City of Buffalo, New York

Buffalo is situated along the eastern shores of Lake Erie and the Niagara River (see Fig. 1). The southern part of the city is cut by the Buffalo River which extends from the harbor in a southeasterly and easterly direction. Much of the area tributary to Buffalo River is occupied by industries, and this is the principal manufacturing district of the city. Scajaquada Creek flows in an east and west direction through the northerly part of the city and is used to form Park Lake in Delaware Park. The northerly part of the city is much more residential in character. The population of Buffalo by the United States Census Bureau, and its estimated probable growth, is as follows:

Year	Population	Year	Estimated population
1840	18,213	1940	670,000
1850	42,261	1950	765,000
1860	81,129	1960	860,000
1870	117,714	1970	955,000
1880	155,134	1980	1,050,000
1890	255,664	1985	1,100,000
1900	352,387		
1910	423,715		
1920	506,775		
1930	573,076		

The total area in 1930 was 26,980 acres and the density of population was 21.3 persons per acre. It was thought likely that the area of the city might be extended somewhat to the north and east in the distant future with the larger populations, but this does not influence the plans. The length of the city in a north and south direction is 8.6 miles and the width in an east and west direction is about 5 miles.

The average temperature and precipitation at Buffalo are indicated in Table 1 which is a summary of data from the U. S. Weather Bureau. The Weather Bureau also reports that the prevailing winds are southwesterly except during November, December, and January, when westerly winds prevail.

TABLE 1.—Meteorological Data; Buffalo, N. Y.

Description	January	February	March	April	May	June	July	August	September	October	November	December	Total
Temperature*	25.4	24.0	31.8	42.5	54.0	64.5	69.9	68.7	62.6	51.4	39.4	29.4	47.0
Precipitation†	3.23	2.74	2.68	2.53	2.85	2.88	2.99	2.81	2.88	3.20	2.98	3.21	34.98
Prevailing wind‡	W	SW	SW	SW	SW	SW	SW	SW	SW	SW	W	W	SW

* Average temperature, in degrees Fahrenheit. † Average total precipitation, in inches. ‡ W = westerly; SW = southwesterly.

Existing Sewers

Practically all the sewers of Buffalo are on the combined plan, their total length being about 710 miles. The first sewers were built prior to 1850;

during the 5-yr period prior to 1855, 23 miles were constructed; and in the 5-yr period prior to 1895, 139 miles were built. Thus, the existing sewers of Buffalo have been in service, in general, for many years. At present (1939) there are about 62 outlets, all of which have now been connected to the new intercepting sewers.

The city is divided topographically into two parts, one draining naturally to the Buffalo River and the other directly to Black Rock Harbor and the Niagara River. With a few exceptions, the main drainage system has developed along these natural lines. In the area north of the lower part of the Buffalo River, a system of canals formerly connected with the old Erie Canal. Some of the city sewers discharged into these canals, which have been covered over and serve as storm overflow sewers. Except in the case of District VII (see Fig. 1), the sewers of Buffalo are on the so-called combined system, with sewage and storm water carried in the same sewers. In District VII the dry-weather flow is confined in part to a separate system of sewers and conducted to the Hamburg Street Pumping Station. This station contains a steam power plant with three horizontal, centrifugal pumps, driven by steam engines. It was formerly operated in conjunction with a refuse incinerator, which has not been used in regular operation since before 1929.

Conditions in the harbor, the canals, and the Buffalo River became objectionable at an early date, and to alleviate this pollution, the so-called Swan Trunk Sewer was built in 1883. This sewer, with a later extension, runs roughly parallel to the Buffalo River and the harbor from Bailey Avenue to Albany Street, where it passes under Black Rock Harbor to the Niagara River. It is 8 ft in diameter for most of its length, and was designed as an intercepting sewer to receive the dry-weather flow from all the area north of Buffalo River and south of the Bird Avenue sewer district. It also receives the discharge of the Hamburg Street Pumping Station. In later years it has been much overloaded and, especially near the upper end, overflows have been frequent or sometimes continuous during dry weather. The capacity of this sewer is utilized in the new intercepting sewer system.

The population and industries adjacent to Buffalo River have continued to discharge their sewage into the sluggish waters of the river. Nearly all the large industries along Buffalo River have private sewers directly to the river.

There are several small areas, chiefly in small separate municipalities, outside the present city limits which discharge sewage into the Buffalo sewers, amounting in the total to about 1,000 acres, with a population of about 13,000.

The characteristics of the major sewer districts of Buffalo are as shown in Table 2.

WATERWAYS

The waterways of Buffalo pertinent to the sewage disposal project are (see Fig. 1): Niagara River, Buffalo Harbor, Black Rock Harbor, Buffalo River, Cazenovia Creek, and Scajaquada Creek.

Niagara River.—The drainage area of the Niagara River, at the outlet of Lake Erie, is about 175,000 sq miles. Based on records of the U. S. Lake Survey, the average discharge is 203,000 cu ft per sec. A discharge of 176,000 cu ft per sec is exceeded 90% of the time. The minimum monthly average discharge of record (142,000 cu ft per sec) occurred in February, 1934; and the maximum discharge (251,000 cu ft per sec) occurred in June, 1876. Thus the discharge is relatively large and uniform.

The elevation of the water surface of the Niagara River opposite the sewage treatment plant on Bird Island covers quite a range. It is estimated that an elevation of − 9.7 (Buffalo City Datum) is exceeded on the average for 99% of the time and an elevation of − 5.7 is exceeded for only 1% of the time. Extraordinary fluctuations in the level of Lake Erie cause momentary peaks in the water level in the river at Bird Island much higher than the foregoing. Thus an elevation of + 2.0 is reached, on the average, once in three years, and an elevation of + 5.0 has an estimated frequency of occurrence of once in fifty years.

TABLE 2.—Characteristics and Estimated Average Sewer Quantities; Main Sewer Districts

District No. (see Fig. 1)	District	Estimated Population, in Thousands		Area, in Acres		Estimated Average Sewage Flow			
		1930	1985	1930	1985	1930		1985	
						Mgd*	Gcd†	Mgd*	Gcd†
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
I	Ontario-Hertel	91.2	232.5	4,470	6,920	17.1	187	37.4	161
II	Bird Avenue	159.7	304.5	5,770	8,760	24.4	154	40.4	133
III	Albany Street	32.9	50.0	860	860	3.4	103	5.4	108
IV	Lake Shore	31.9	46.0	730	730	5.1	160	9.0	196
V	Genesee Street	33.0	40.5	800	800	9.4	285	10.2	252
VI	Michigan-Emslie	55.3	68.7	1,060	1,060	10.3	186	13.8	201
VII	Hamburg Street Pumping Station	20.3	14.0	760	760	7.8	384	8.1	578
VIII	Fillmore-Babcock	65.3	77.2	2,365	2,365	20.8	318	25.0	324
IX	Bailey Avenue	40.2	150.8	2,145	4,830	10.5	261	28.6	190
X	South Buffalo	43.2	115.8	2,410	4,140	13.3	308	27.3	236
	Parks, etc.	....	....	1,220	1,220	....	....	....	....
	Water area	....	....	1,840	1,840	....	....	....	....
	Other non-sewered areas	....	....	2,550	2,550	....	....	....	....
	Totals	573.0	1,100.0	26,980	35,835	122.1	213	205.2	186

* Mgd = million gallons daily. † Gcd = gallons per capita per day.

Below Buffalo, the Niagara River is used as a water supply by Tonawanda, North Tonawanda, Lockport, Niagara Falls, Youngstown, and Fort Niagara, in New York, with a total population of about 150,000, and by the cities of Fort Erie and Niagara Falls in Canada. There is an increasing use of the water for recreation, including boating, fishing, summer homes, and some bathing, mainly limited, at present, to the Canadian shore and to the westerly shore of Grand Island. The average number of B. coli per cubic centimeter at the Niagara Falls water-works intake was 36.7 in 1936 and 27.5 in 1937, with individual samples as high as 240. The analyses of the Sewer Authority laboratory show as great a concentration as 2,400 B. coli per cu cm in the most contaminated sections of the river downstream from Buffalo. The lowest content of dissolved oxygen found during the river surveys of the Buffalo Sewer Authority in July, 1936, was 6.0 ppm.

Buffalo Harbor.—Buffalo Harbor is made artificially by breakwaters in Lake Erie. Little sewage is discharged directly into Buffalo Harbor and it has not been an important factor in the design of the project.

Black Rock Harbor.—On the other hand, Black Rock Harbor has received a considerable quantity of sewage and was seriously polluted. In fact, it is a waterway connecting the New York State Barge Canal with Buffalo Harbor and is relatively long and narrow, having the shape more of a canal than of a harbor. Its length is 16,500 ft, the area of an average cross section is 8,800 sq ft, its surface area is 8,100,000 sq ft, and it has a liquid volume at mean depth of 140,000,000 cu ft. Some circulation (about 100 cu ft per sec) is maintained by the operation of a small outlet at the lower or northerly end of the harbor and by operation of the lock. Variations in the level of Lake Erie also have a marked effect in mixing relatively unpolluted lake water with the harbor water, especially in the upper or southerly end.

The principal source of pollution of Black Rock Harbor was the Bird Avenue outlet which discharged about 25.0 mgd of sewage into the harbor about 3,000 ft south of its lower end. This is about 20% of the sewage of the entire city. An indication of the effect of this pollution is given by the data in Table 3.

TABLE 3.—ANALYSES OF BLACK ROCK HARBOR WATER, FROM INDIVIDUAL GRAB SAMPLES

Date	AT FERRY STREET		AT NORTH END	
	Dissolved oxygen, percentage of saturation	5-day B.O.D., in parts per million	Dissolved oxygen, percentage of saturation	5-day B.O.D., in parts per million
1936				
June 25	78	1.7	0	28
July 29	79	15.0	7	16
August 25	69	4.3	0	22
September 30	51	2.8	0	31
December 3	59	8.1	4	28
1937				
January 5	32	4.3	12	30
February 2	44	11.0	19	19
March 9	60	4.6	13	3
April 6	32	5.2	0	30
May 5	10	5.9	0	39
June 2	16	2.3	0	22
July 7	66	1.3	0	6
August 3	62	....	0	5.4
September 8	40	2.6	0	25
October 5	61	1.0	0	25
November 9	28	....	0	....
December 7	47	4.6	0	39

Buffalo River and Cazenovia Creek.—Buffalo River and its main tributary, Cazenovia Creek, have a drainage area of 436 sq miles. There is no continuous record of gagings of the quantity of water flowing in Buffalo River. However, the U. S. Geological Survey has a record of routine gagings on Cattaraugus Creek, the drainage area of which is adjacent to that of Buffalo River and includes 488 sq miles. Using the Cattaraugus Creek gagings as a basis, the discharge of Buffalo River has been computed as follows:

Period	Discharge, in cubic feet per second
Annual average	650
Average from May 1 to October 31	355
Annual median	340
Median from May 1 to October 31	180

A discharge of 100 cu ft per sec is exceeded 90% of the time for the year as a whole, and for the period from May 1 to October 31, a discharge of 80 cu ft per sec is exceeded 90% of the time.

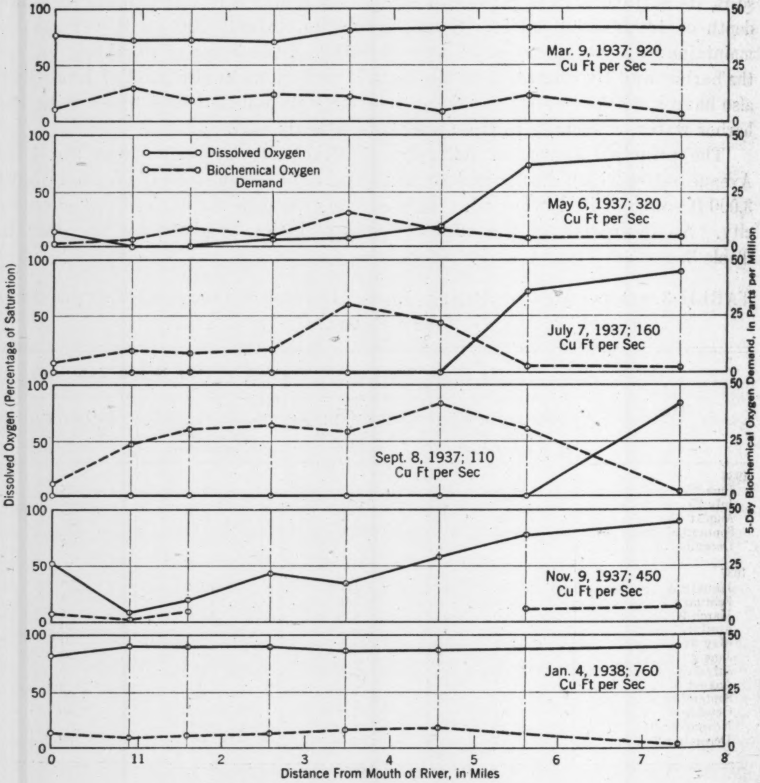


FIG. 2.—CONDITIONS IN BUFFALO RIVER

Fig. 2 shows the dissolved oxygen and the 5-day B.O.D. (bio-chemical oxygen demand) for five typical river surveys during 1937. The river discharges indicated were determined by actual gagings on the dates of the surveys. During the surveys of May, July, and September the dissolved oxygen was practically exhausted.

Scajaquada Creek.—Flowing through the northerly section of the city into Black Rock Harbor, Scajaquada Creek has a drainage area of 24 sq miles. Where it flows through Delaware Park, it has been dammed to form a small lake. Due to the pollution of waters of the creek, sludge deposits in this lake have caused objectionable odor conditions in the past. The section of the creek between the east city limits and Main Street has been enclosed in a large reinforced concrete drain.

ENGINEERING AND LABORATORY WORK

Much field engineering work was required to be done promptly, including surveys for design and for the acquisition of land and rights of way, investigations of the best location for intercepting sewers, the supervision of borings, examination and gaging of existing sewers, and planning and organizing a laboratory. Much useful work was accomplished through the laboratory, in determining sewage quantities and characteristics and the condition of the water in many of the waterways. Studies and reports were made on most of the larger industries. Investigation of the industrial sewages of Buffalo was a major item of the engineering and laboratory work.

During two periods in 1936 (May 14 to 27 and August 5 to 22), and one in 1937 (July 13 to August 28), several of the large sewers were gaged and sampled to determine the dry-weather characteristics of the sewage. The 1937 study was the most extensive and all the following data refer to it. Each sewer was sampled for a period of 8 to 10 days. Chlorine demand was determined on separate samples collected at bi-hourly intervals. Bacterial analyses were made on two samples daily. All other determinations were made on 24-hr composites, built up from samples taken at 15-min intervals and proportioned according to sewage flow. Table 4 gives the results of this work. The sewage

TABLE 4.—SUMMARY OF 1937 STUDIES OF SEWAGE CHARACTERISTICS BASED ON GAGINGS AND ANALYSES

Sewer	Districts (see Fig. 1)	Tributary area, in acres	Tributary population	Average sewage flow, in million gallons daily	ANALYTICAL RESULTS, IN PARTS PER MILLION						Potential of hydrogen, pH
					Total solids	Suspended solids	Volume of suspended solids	5-day B.O.D.; 20° C	Alkalinity	Chlorine demand	
Bailey Avenue......	IX	2,100	40,000	9.95	594	98	68	132	219	3.6	7.4
Babcock Street.....	VIII‡	400	2,000	4.70	1 930	472	375	498	296	28.4	8.8
Fillmore Avenue....	VIII‡	1,350	53,000	9.30	661	183	146	225	207	10.5	7.1
Emslie Street......	VI‡	550	32,000	6.10	639	186	153	238	180	13.4	7.0
Swan Trunk*.......	IV to IX	7,200	235,000	49.41	596	171	134	168	190	8.1	7.1
Albany Street......	III	860	33,000	3.75	....	220	....	205	....	5.7	....
Bird Avenue.......	II	5,340	143,000	24.53	533	145	101	162	177	4.2	7.1
Totals†........		13,400	411,000	77.69	....	....	....	....	....	....	....
Estimate for the entire city.......		21,370	573,000	110	....	153	....	148	....	7.3	....

* This sewer includes the flow from all the first four. † The total includes only the last three items—see preceding footnote. ‡ In part.

quantities were determined from measuring the head on weirs built in the sewers.

During 1936, investigations were made as to the effect of storms on the quantity of suspended solids in the sewage, with special attention to the characteristics of the first flushings, which would be taken to the treatment works. In all, some 150 samples were collected, in twelve studies.

The effect of storm runoff on suspended solids is indicated by the study of the Bird Avenue Sewer, results of which are given in Table 5.

TABLE 5.—EFFECT OF STORM RUNOFF ON SUSPENDED SOLIDS; BIRD AVENUE SEWER

(a) AUGUST 19, 1936; TOTAL RAINFALL, 0.55 IN.			(b) SEPTEMBER 23, 1936; LOCAL SHOWER					
Time (p.m.) (1)	Sewage flow, in million gallons daily (2)	Suspended solids, in parts per million (3)	Time (p.m.) (1)	Sewage flow, in million gallons daily (2)	Suspended solids, in parts per million (3)	Time (p.m.) (1)	Sewage flow, in million gallons daily (2)	Suspended solids, in parts per million (3)
5:30	25.3	208	7:30	28.5	....	9:00	38.0	532
6:30	120.6	964	7:50	32.8	204	9:15	34.7	600
7:30	104.0	1,220	8:05	41.9	132	9:30	33.0	300
8:30	56.7	696	8:15	46.2	508	9:45	32.8	252
9:30	37.6	364	8:30	46.6	704	10:15	31.0	208
10:30	28.6	172	8:45	43.1	844	....	....	....

Monthly surveys of Niagara River have been made since June, 1936. During the summer months, when the use of a boat was possible, samples were taken at six different cross sections of the river between the site of the treatment plant and the lower end of Grand Island, immediately upstream from the intake of the Niagara Falls water works. About forty stations were sampled at these times. During winter months, samples were obtained only from the two Grand Island bridges, a total of thirteen stations. Analyses were made of each sample to determine the numbers of B. coli and of total bacteria on agar-agar at 37° C. On a few occasions, determinations were also made of dissolved oxygen (D.O.), bio-chemical oxygen demand (B.O.D.), solids, nitrogen, oxygen consumed, alkalinity, and chlorides.

Daily bacterial analyses of the raw water at the Niagara Falls filter plant have been made for the Buffalo Sewer Authority since May, 1936. Two studies were also made to determine the hourly variations in bacterial content of the river water at this point.

Monthly samples from about ten points along the Buffalo River and Cazenovia Creek have been collected and analyzed since April, 1936. The stream discharge and temperature are determined and the following analyses made: Potential of hydrogen, dissolved oxygen, bio-chemical oxygen demand, total solids, suspended solids, oxygen consumed, total nitrogen, chlorides, alkalinity, 37° bacteria, and B. coli. Samples at special locations and for special purposes are collected at various times.

On each of the Buffalo River surveys samples were also collected and analyzed from the following points in the harbors: City Ship Canal at Michigan Avenue, mouth of Buffalo River, Black Rock Harbor at Ferry Street and at Bridge Street.

Occasional studies have been undertaken to determine conditions in the Scajaquada Drain and in Park Lake, with particular attention to sludge deposits and the rate of digestion. Many other more or less extensive investigations have been made covering the chlorine demand of the sewage, hourly variations in sewage characteristics, analyses of materials used in construction, etc.

INDUSTRIES

Buffalo has a relatively large number of industries which produce sewage. Many of these are situated in the southerly part of the city which drains naturally to Buffalo River. Some of the larger plants discharge directly into Buffalo River and do not use the sewers. The chemical industries form an important group, including seven major plants, among which are those of the General Chemical Company and the National Analine and Chemical Company. There are two relatively large coal coking plants, one of which has been closed. There are twenty-four meat packing plants, eleven breweries, fifteen dairies, eight oil handling and refining plants, five rubber reclaiming plants, and five tanneries.

Many of these industrial establishments have been surveyed and the quantities and characteristics of the sewage determined by gagings and analyses. It has been considered that the sewage from some of the industries is not likely to be readily amenable to treatment. It has thus been desirable to undertake a cooperative program with the industries for the removal of pollution from the Buffalo River and from the sewers of the city. This program has proceeded on a satisfactory basis. Properly, it has not been developed until the later steps of construction and the early steps of operation of the treatment plant.

STORM-WATER RELIEF SEWERS

Storm-water relief sewers were planned to serve the Bird Avenue sewer district which has an area of 5,490 acres. The main sewer for this district was inadequate throughout its length, especially the section above Main Street, which has capacities, in its several sections, of only 166 to 665 (gcd), based on estimated 1985 populations. Overflows had been built from this sewer into the Scajaquada Drain, a large sewer which, more or less, is parallel to the Ferry Street sewer to the north. This caused the pollution of Park Lake in Delaware Park. Several lateral sewers in this area were also of insufficient capacity, especially that in Michigan Avenue.

Storm-water relief sewers were planned for a rate of rainfall estimated to occur once in ten years. A runoff coefficient of 0.60 was used for the area within the city limits and of 0.22 for rural areas outside of Buffalo. The size of the necessary relief sewers ranged from 3.5 to 12.0 ft in diameter. The total cost of the storm-water relief sewers was $2,330,000. Plans and specifications for this part of the project were prepared by the City Sewer Department.

INTERCEPTING SEWERS

Intercepting sewers were designed to serve a future population of 1,100,000. The average quantity of sewage of each of the ten major sewer districts (see Fig. 1) was estimated as shown in Table 2. The unit quantities used in computing these sewage flows are as follows:

	Unit rates of average sewage flow
Gallons per capita per 24 hr; domestic	70
Gallons per Acre per 24 Hr:	
Industrial	
North Buffalo	10,000
East Buffalo	12,000
South Buffalo	10,000
Commercial	20,000
Infiltration	
North Buffalo	1,000
East Buffalo	2,000
South Buffalo	4,000

A summary of the estimated average sewage quantities, in million gallons daily, is as follows:

Description	1930	1985
Domestic	40.1	77.0
Industrial	31.6	52.0
Commercial	16.0	24.0
Infiltration	34.4	52.2
Totals	122.1	205.2
Totals, in gallons per capita per day	213	186

For the design of sewers and structures serving only part of the city, the estimated average sewage quantities were increased by factors to allow for a different distribution of the future growth than that estimated. These factors varied with the size of the area served and with the estimated density of population, having a maximum value of 1.5. The factors were applied to the domestic, industrial, and commercial quantities, but not to the infiltration.

For area that will overflow to the Buffalo River the intercepting sewer capacities were based on 3.0 times the estimated 1985 average flow, and for all other areas on 2.5 times the estimated 1985 average flow.

Based on rainfall records and a runoff coefficient of 60%, a computation was made to indicate approximately the number and duration of overflows of combined sewage into the waterways. No very exact computation was attempted, but the approximate results are given in Table 6. A proportion of the total sewage flow varying from about one third of one per cent, in the near future, up to almost one and one third per cent in 1985 will be overflowed during rain storms. Kutter's formula, with $n = 0.013$, was used in determining the capacity of all the intercepting sewers.

Careful studies were made to determine the economical balance between the size and slope of the intercepting sewer and pumping costs. In order to utilize all the head available and at the same time keep the sewer below existing and future utilities, much of the main intercepting sewer was built at a flatter grade than the slope of the hydraulic gradient at maximum flow, so that at times of storm the 11-ft and 9-ft portions of the sewer will operate with the hydraulic gradient as much as 2.5 ft above the crown of the sewer.

TABLE 6.—ESTIMATED FREQUENCY AND DURATION OF OVERFLOW FROM INTERCEPTING SEWERS FOR THE SIX SUMMER MONTHS, MAY TO OCTOBER

Intercepting sewer capacity, in gallons per capita per day	Population density per acre	AVERAGE DRY-WEATHER FLOW; 150 GALLONS PER CAPITA PER DAY		AVERAGE DRY-WEATHER FLOW; 200 GALLONS PER CAPITA PER DAY		AVERAGE DRY-WEATHER FLOW; 250 GALLONS PER CAPITA PER DAY	
		Number of overflows	Hours of overflow	Number of overflows	Hours of overflow	Number of overflows	Hours of overflow
400	20	95	230	104	257	....	....
	30	83	187	90	211	98	245
	40	76	161	82	180	90	211
	50	69	135	76	161	83	187
	60	63	117	70	140	79	168
600	20	79	168	82	181	86	195
	30	66	127	70	140	74	153
	40	56	101	61	112	65	124
	50	49	83	53	93	57	103
	60	42	71	47	79	51	89
800	20	68	132	70	140	73	147
	30	54	95	56	101	59	107
	40	44	75	47	79	49	85
	50	37	58	40	64	42	69
	60	32	47	34	52	37	57
1,000	20	59	106	61	111	63	116
	30	45	74	47	59	49	83
	40	36	56	37	60	39	63
	50	29	43	31	47	33	50
	60	24	32	26	35	28	39

A condensed profile of the main intercepting sewer is shown in Fig. 3, and typical sections of the intercepting sewer are shown in Fig. 4. As shown on the profile, the capacity of the sewer varies from 367 to 1,047 gcd, with the higher capacities along the Buffalo River.

For sewers between 3.5 ft and 7 ft in diameter, bidders were allowed to choose any one of four sections: Monolithic concrete, reinforced concrete pipe, brick, and segmental tile. Reinforced concrete pipe (types A and H, Fig. 4) was the choice of bidders in every case, except for monolithic concrete (type B) for earth tunnel in division 0, Fig. 3.

For rock tunnels the horseshoe shape (type C) was used, with a plain concrete lining having a minimum thickness of 8 in. For the larger sewers in open cut, the semi-elliptical section was chosen after extensive investigations of the relative economy of several sections. Drawings of types D, E, and G in

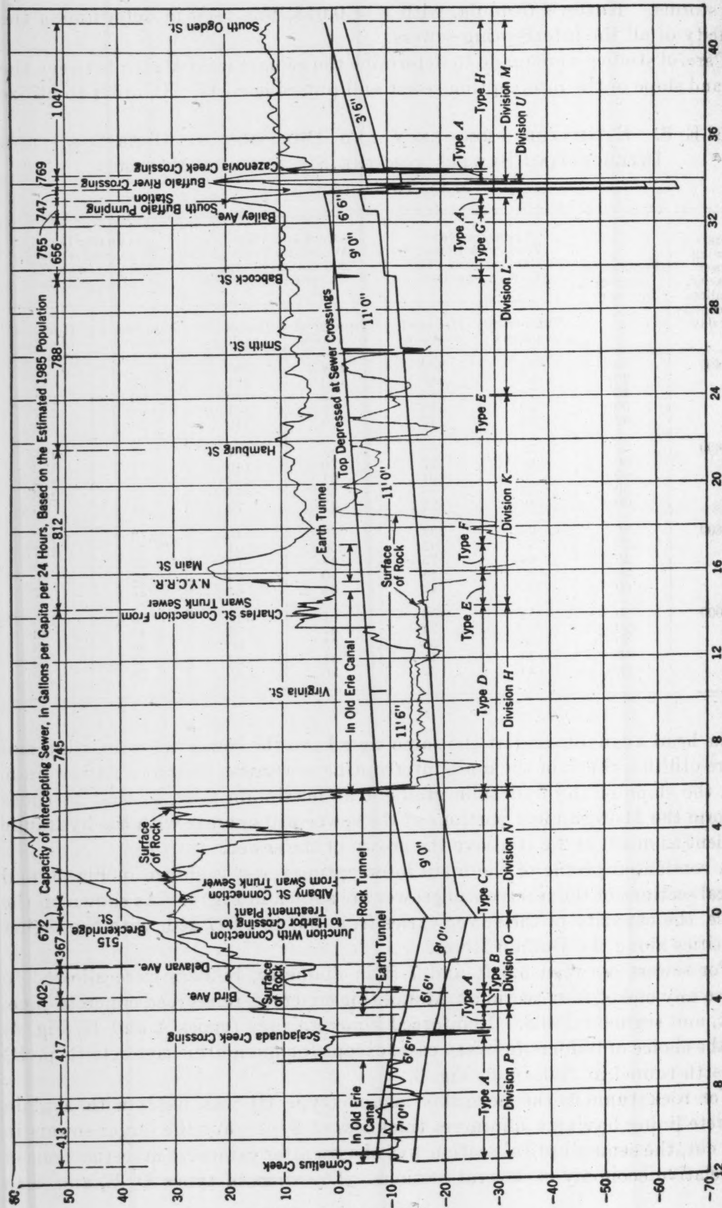


FIG. 8.—GENERAL PROFILE OF INTERCEPTING SEWERS

Fig. 4 show the sizes and dimensions used. "Standard" sections were used for shallow cuts where no extraordinary loading was anticipated. "Heavy" sections were used for deep cuts or where other heavy loads were anticipated, such as from railroad tracks. The entire length of the 11.5-ft, type D section in the abandoned Erie Canal was designed for heavy loads to provide for a proposed relocation of main-line tracks of the New York Central Railroad.

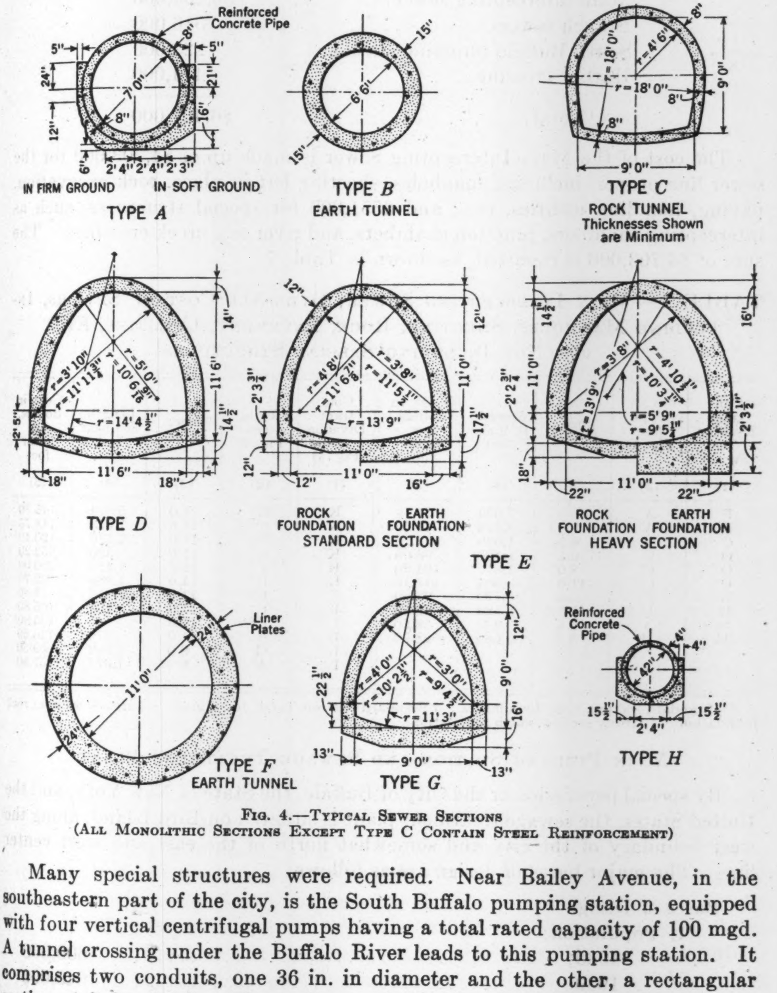

Reinforced Concrete Pipe
5"
5"
8"
7' 0"
24"
21"
12"
16"
8"
6"
2' 4" 2' 4" 2' 3"
IN FIRM GROUND
IN SOFT GROUND
TYPE A
15"
6' 6"
15"
TYPE B
EARTH TUNNEL
8"
r = 18' 0"
r = 4' 6"
r = 18' 0"
8"
9' 0"
8"
9' 0"
TYPE C
ROCK TUNNEL
Thicknesses Shown are Minimum
r = 3' 10"
r = 11' 11 3/4"
r = 10' 6 9/16"
r = 5' 0"
14"
11' 6"
14 1/2"
2' 5"
r = 14' 4 1/2"
18"
11' 6"
18"
TYPE D
12"
r = 4' 8"
r = 11' 6 7/8"
r = 3' 8"
r = 11' 5 1/2"
12"
11' 0"
2' 3 3/4"
r = 13' 9"
17 1/2"
12"
12"
11' 0"
16"
ROCK FOUNDATION
EARTH FOUNDATION
STANDARD SECTION
14 1/2"
16"
11' 0"
r = 3' 8"
r = 4' 10 1/2"
r = 10' 3 1/2"
r = 13' 9"
2' 2 3/4"
r = 5' 9"
r = 9' 5 1/4"
2' 3 1/2"
18"
22"
11' 0"
22"
ROCK FOUNDATION
EARTH FOUNDATION
HEAVY SECTION
TYPE E
Liner Plates
24"
11' 0"
24"
TYPE F
EARTH TUNNEL
12"
r = 4' 0"
r = 10' 2 1/2"
r = 3' 0"
r = 9' 4 1/2"
9' 0"
22 1/2"
r = 11' 3"
16"
13"
9' 0"
13"
TYPE G
Reinforced Concrete Pipe
4"
4"
42"
15 1/2"
2' 4"
15 1/2"
TYPE H

Fig. 4.—Typical Sewer Sections
(All Monolithic Sections Except Type C Contain Steel Reinforcement)

Many special structures were required. Near Bailey Avenue, in the southeastern part of the city, is the South Buffalo pumping station, equipped with four vertical centrifugal pumps having a total rated capacity of 100 mgd. A tunnel crossing under the Buffalo River leads to this pumping station. It comprises two conduits, one 36 in. in diameter and the other, a rectangular section, 4 ft by 6.5 ft. The main crossing under Black Rock Harbor to the

Sewage Treatment Plant comprises two conduits, each 8 ft in diameter. Some 55 intercepting and regulating chambers were built, as well as a large number of special structures including creek and sewer crossings, junction chambers, and several lateral separate sewers.

The estimated final construction cost of the intercepting sewers (exclusive of unsettled claims) may be summarized as follows:

Main intercepting sewer	$5,254,000
Branch sewers	516,000
South Buffalo pumping station	223,000
Harbor crossing	414,000
Total	$6,407,000

The cost of the Main Intercepting Sewer is made up of $4,703,000 for the sewer line proper, including manholes, sheeting left in place, rock excavation, paving, handling utilities, etc.; and $551,000 for special structures, such as intercepting chambers, junction chambers, and river and creek crossings. The sum of $4,703,000 is itemized, as shown in Table 7.

TABLE 7.—MAIN INTERCEPTING SEWER; OVER-ALL COST OF SEWERS, INCLUDING MANHOLES, SHEETING, ROCK, PAVEMENT, UTILITIES, ETC., BUT NOT INCLUDING SPECIAL STRUCTURES

Contract Division (see Fig. 3) (1)	Type (see Fig. 4) (2)	Size, in feet (3)	Length, in linear feet (4)	Cost, in dollars, per linear foot (5)	Contract Division (see Fig. 3) (1)	Type (see Fig. 4) (2)	Size, in feet (3)	Length, in linear feet (4)	Cost, in dollars, per linear foot (5)
P	A	6.5	2,059	51.70	K	E*	11.0	5,074	145.30
P	A	7.0	4,578	41.60	K	E†	11.0	1,273	148.75
O	B	6.5	1,049	132.10	K	E‡	11.0	1,176	120.40
O	C	6.5	1,407	93.00	K	E§	11.0	490	252.20
O	C	8.0	1,877	104.20	K	F	11.0	1,410	260.00
O	C	11.0	504	147.10	L	E*	11.0	1,989	92.70
N	C	9.0	5,720	97.90	L	E†	11.0	2,864	73.60
H	D	11.5	8,582	80.50	L	E‡	11.0	105	105.80
M	A	6.5	495	58.60	L	E§	11.0	85	135.90
M	H	3.5	9,680	28.90	L	F	11.0	370	136.60
....	....	....	....	....	L	G	9.0	2,909	69.50
....	....	....	....	....	L	A	6.5	1,375	57.90

* Standard section, rock foundation. † Standard section, earth foundation. ‡ Heavy section, rock foundation. § Heavy section, earth foundation.

MAIN PUMPING STATION AND SEWAGE TREATMENT PLANT

By special permission of the City of Buffalo, the State of New York, and the United States, the sewage treatment plant is situated on Bird Island, along the west boundary of the city and somewhat north of the east and west center line. The major bases of design are as follows:

Bases of Design

1. Population:

1935	600,000
1939	640,000
1950	750,000

2. Sewage Quantities (All in Million Gallons Daily):

Capacity of intercepting sewer (1985)	563
Capacity of main pumping station	570
1950 basis (population 750,000)	
Average	150
Maximum:	
Clarification and disinfection units	570
Conduits	570
4-hr maximum	570
24-hr maximum	450
2-day maximum	375
3-day maximum	300
Maximum month	200
Minimum (for less than one hour)	80
Near future basis (population 640 000):	
Average	128
Minimum (for less than one hour)	65

3. Sewage Characteristics:

Suspended Solids in Raw Sewage Average (1950):

Parts per million	180
Pounds per capita per 24 hr	0.3

Suspended Solids Removed and To Be Disposed of:

Period	Percentage removal	Pounds per 24 hr
Average	48.0	108,000
Maximum		
24 hr	38.4	448,000
2 days	38.4	351,000
3 days	38.4	278,000
Month	40.0	135,000

(Assume that, as a yearly average, the volatile matter is 70% of the total suspended solids and that at times of maximum flows it is 50%.)

Quantities of Grit:

Period	Cubic yards per million gallons	Cubic yards per 24 hr
Average	0.15	22.5
Maximum		
24 hr	1.0	200.0
Month	0.25	50.0

4. Grit Chambers:

Basis of design, in million gallons daily	570
Number of units or channels	12
Average depth of sewage, in feet	8.0
Length, in feet	65
Width, in feet	10
Velocity, in feet per second:	
Maximum	1.05
Minimum	0.75

5. Screens:
    - Coarse trash racks with bar-spacing of 5.5 in.
    - Bar screens mechanically cleaned with clear spacing of bars at 0.75 in.
6. Sedimentation Tanks:
    - Displacement period at average flow of:

Hours	1.33
Minutes	80.00

7. Raw Sludge Quantities:
    - Percentage of moisture in sludge 96
    - Other quantities:

Period	Gallons of wet sludge per 24 hr	Pounds of dry solids per 24 hr
Average	324,000	108,000
Maximum		
24 hr	1,345,000	448,000
2 days	1,053,000	351,000
3 days	833,000	278,000
Maximum month	405,000	135,000

(Assumed weight of wet sludge, 62.5 lb per cu ft)

Chemicals for conditioning raw sludge, in pounds per 100 lb of dry solids:

	Average	Maximum
Ferric chloride (100%)	2.0	3.0
Lime (CaO)	8.0	12.0

A general plan of the sewage treatment plant is shown in Fig. 5 and a profile showing the high-water and low-water hydraulic grade lines is shown in Fig. 6. The elevations in Fig. 6 are referred to Buffalo City Datum, which is 575.89 ft above mean tide at New York, N. Y. (1903 level adjustment, U. S. Army Engineers). The gradients show that,

(*a*) With river level at − 5.7 and with a sewage flow of 570 mgd, 330 mgd will be discharged through the submerged outlet and 240 mgd through the overflow weir;

(*b*) With river level at − 9.7 and with a sewage flow of 570 mgd, 500 mgd will be discharged through the submerged outlet and 70 mgd through the overflow weir;

(*c*) With river level at − 5.7, sewage flows of as much as 225 mgd will be discharged through the submerged outlet, without overflow; and,

(*d*) With river level at − 9.7, sewage flows of as much as 470 mgd will be discharged through the submerged outlet, without overflow.

The treatment plant occupies a somewhat triangular shaped site, having an area of 23.0 acres. At the southerly end is the main pumping station which lifts the sewage about 23 ft at average sewage flow. The sewage then flows by gravity through a screen and grit chamber, venturi meters, aerated channels, four sedimentation tanks, and a submerged outfall to the Niagara River.

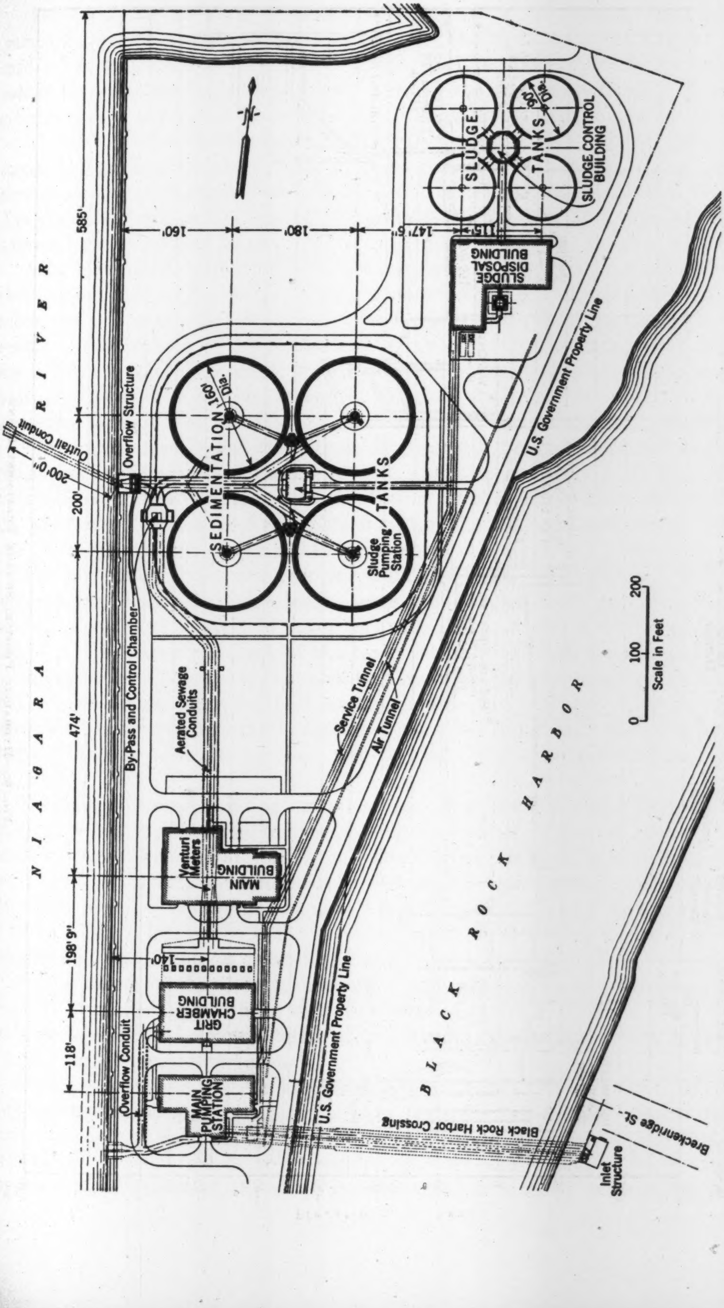

N I A G A R A R I V E R
585'
160'
180'
147'5"
115'
Overflow Structure
Outfall Conduit
200'0"
200'
474'
198'9"
118'
By-Pass and Control Chamber
Aerated Sewage Conduits
Overflow Conduit
SEDIMENTATION TANKS
160' Dia.
Sludge Pumping Station
SLUDGE TANKS
90' Dia.
SLUDGE CONTROL BUILDING
SLUDGE DISPOSAL BUILDING
U.S. Government Property Line
Service Tunnel
Air Tunnel
MAIN BUILDING
Venturi Meters
140'
GRIT CHAMBER BUILDING
MAIN PUMPING STATION
B L A C K R O C K H A R B O R
Black Rock Harbor Crossing
Breckenridge St.
Inlet Structure
0
100
200
Scale in Feet

Fig. 5.—General Plan of Sewage Treatment Plant

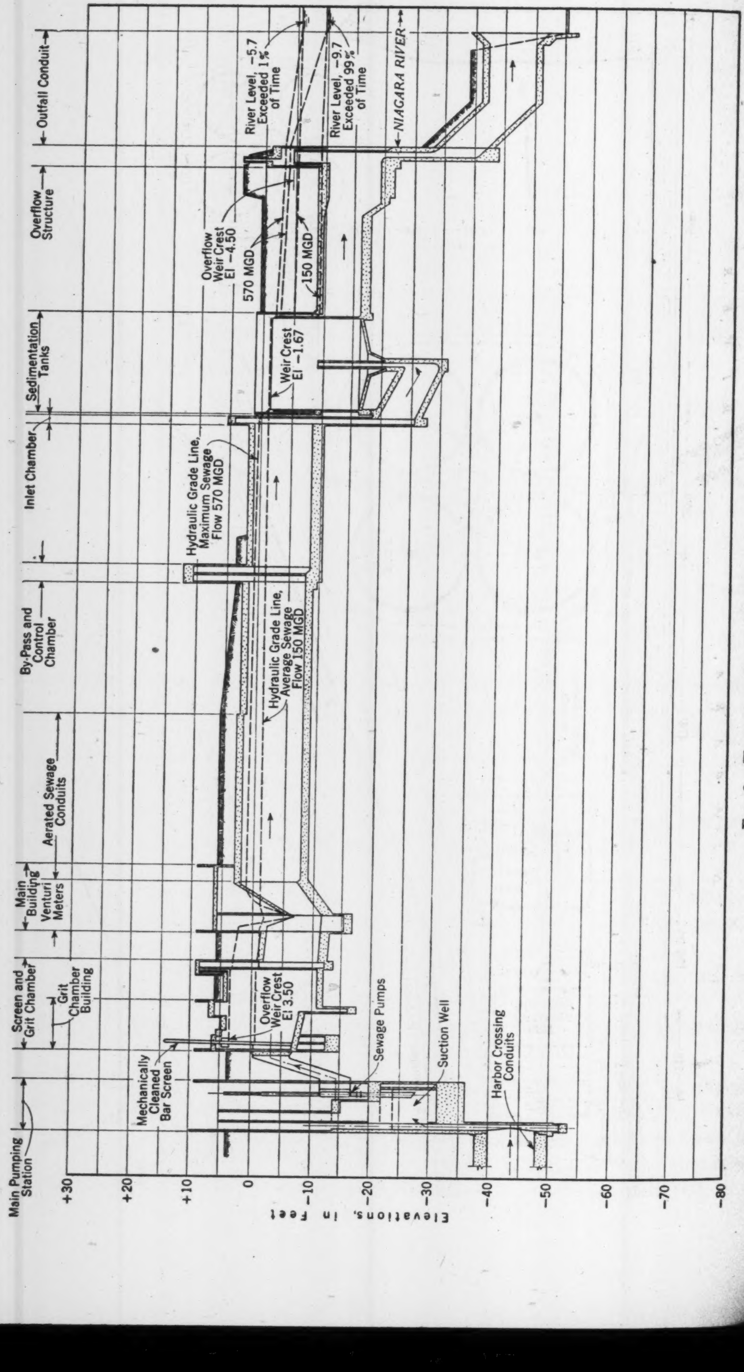

Main Pumping Station
Mechanically Cleaned Bar Screen
Screen and Grit Chamber
Grit Chamber Building
Venturi Meters
Main Building
Aerated Sewage Conduits
By-Pass and Control Chamber
Inlet Chamber
Sedimentation Tanks
Overflow Structure
Outfall Conduit
Sewage Pumps
Suction Well
Harbor Crossing Conduits
Overflow Weir Crest El 3.50
Hydraulic Grade Line, Average Sewage Flow 150 MGD
Hydraulic Grade Line, Maximum Sewage Flow 570 MGD
Weir Crest El -1.67
Overflow Weir Crest El -4.50
570 MGD
150 MGD
River Level, -5.7 Exceeded 1% of Time
River Level, -9.7 Exceeded 99% of Time
NIAGARA RIVER
Elevations, in Feet
+30
+20
+10
0
-10
-20
-30
-40
-50
-60
-70
-80

FIG. 6.—HYDRAULIC PROFILE, SEWAGE TREATMENT PLANT

The pumping building contains six vertical, close-coupled, centrifugal pumps, each with a rated capacity of 100 mgd and a low-head capacity of 120 mgd. The total capacity of the pumping station is thus somewhat more than that of the intercepting sewer system. Two of the pumps are fitted with two-speed motors to assist in uniform regulation of flow into the sedimentation tanks.

There are twelve grit chambers. Each pair operates as a unit with one bar screen and pump, and there are no cross connections between these units. Screenings are discharged into grinders and returned thence to the sewage. The grit chambers proper are 60 ft long and 10 ft wide; and they have an average liquid depth of 10 ft.

After leaving the grit chambers, sewage flows through two conduits under the main building, each conduit having a venturi tube. Following the venturi tubes, the two conduits are equipped with plates and piping for aerating the sewage. This section of the conduits is 285 ft long and traverses a part of the site left open, for the present, to provide for future structures. Chlorine is added to the sewage at the far end of these aerated conduits near the sedimentation tanks. The chlorination equipment is housed in the main building.

The sewage then flows through a by-pass and control chamber to four circular sedimentation tanks which provide a displacement period of 80 min at rated capacity. Each of these sedimentation tanks is 160 ft in diameter, and the side-water depth is 14.5 ft. Effluent from the sedimentation tanks flows into the Niagara River about 200 ft from shore, at a depth of about 40 ft.

The suspended solids are pumped from the sedimentation tanks to four sludge storage tanks, having a total capacity of 750,000 cu ft, or 1.0 cu ft per capita. Each tank is 90 ft in diameter, with side and center depths of 27 ft and 38 ft, respectively. Each of the tanks is equipped with a floating cover.

In the center of the space occupied by the sludge storage tanks is a building that houses sludge pumping and control equipment with provision for the delivery of sludge at given rates into one or all of the four tanks, the transfer of sludge from one tank to another, and to the sludge-disposal building (see Fig. 5).

The sludge disposal building contains three de-watering filters of the vacuum type, three flash-type sludge incinerators, and the boiler plant for heating the sludge storage tanks and all buildings. Each drum filter is 11.5 ft in diameter and 14 ft long, with a filtering area of 500 sq ft. It is estimated that these filters will de-water the sludge to a moisture content of 70%, at the average rate of 5.0 lb of dry solids per sq ft per hr when conditioned with about 2.0% ferric chloride and 8.0% lime (expressed as CaO).

Sludge cake from the filters is discharged on to a belt conveyer extending the length of the building which, in turn, discharges to a cross conveyer where the sludge is weighed and elevated to another longitudinal conveyer which delivers it to the sludge incinerator units.

Sludge cake, mixed with previously dried sludge, is delivered to flash dryers in the sludge incinerators; thence it is blown into the furnaces for combustion. Any deficiency in heat units is made up by burning oil or gas. The sludge incinerators include ash-removal equipment, fly ash cyclones, and other appurtenances. Each of the three incinerators has a rated capacity of 100 tons of sludge cake (containing 70% moisture) per 24 hr. Two of the incineration units will probably be sufficient to dispose of the average quantity of sludge.

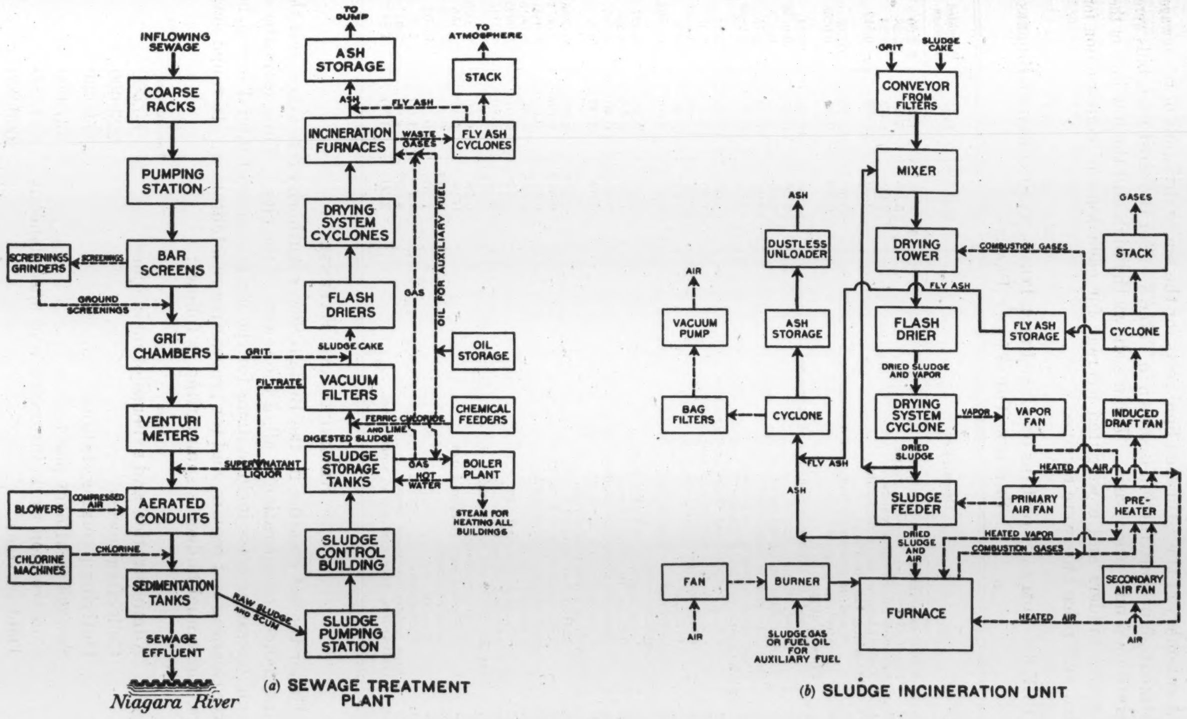
The various features of the sewage treatment plant are connected by a service tunnel along one side of which the main ventilating duct is constructed. The several parts and buildings of the plant where sewage is exposed are provided with large ventilating fans which withdraw air and discharge it into this main ventilating duct. The chimney of the treatment plant, rising 175 ft above the ground, is actually two concentric ducts. The inner of these is for the hot gases from the sludge incinerators and the boiler plant, and the outer, or annular one, is for ventilation and receives the air blown through the main ventilating duct.

The main building houses the office and laboratory, garage, machine shop, blowers for the aerated conduits, chlorine equipment, and the plant water supply, including pumps and elevated tank. A flow diagram of the sewage treatment plant is shown in Fig. 7(*a*), and of a sludge incineration unit in Fig. 7(*b*).

The approximate total cost of the main sewage pumping station and sewage treatment plant was $4,378,891, distributed as follows:

Contract division	Total cost
Preparation of site	$ 166,867
Pumping station and grit chamber	914,429
Sedimentation tanks	772,921
Pumping equipment	324,000
Electrical work	115,537
Sluice gates	49,375
Sludge de-watering equipment	74,965
Incineration equipment	257,500
Sludge disposal building	298,670
Sludge storage tanks	414,807
Brick stack	28,881
Electrical work	35,978
Boiler plant equipment	69,314
Outfall structure	102,897
Main building, service tunnels, sewage conduits, and meters	600,546
Trucks	13,266
Yard water piping equipment	22,327
Roads and walks	54,941
Wall around sedimentation tanks	16,670
Landscaping	20,000
Miscellaneous informal bids	25,000
Total	$4,378,891

The part of this total sum required for sewage treatment is about $3,500,000, which is equivalent to $4.67 per capita or $23,300 per mgd of design capacity.


INFLOWING SEWAGE
COARSE RACKS
PUMPING STATION
BAR SCREENS
SCREENINGS
SCREENINGS GRINDERS
GROUND SCREENINGS
GRIT CHAMBERS
GRIT
VENTURI METERS
AERATED CONDUITS
BLOWERS
COMPRESSED AIR
CHLORINE MACHINES
CHLORINE
SEDIMENTATION TANKS
SEWAGE EFFLUENT
Niagara River
RAW SLUDGE AND SCUM
SLUDGE PUMPING STATION
SLUDGE CONTROL BUILDING
SLUDGE STORAGE TANKS
SUPERNATANT LIQUOR
DIGESTED SLUDGE
FERRIC CHLORIDE AND LIME
VACUUM FILTERS
FILTRATE
SLUDGE CAKE
FLASH DRIERS
DRYING SYSTEM CYCLONES
INCINERATION FURNACES
ASH
ASH STORAGE
TO DUMP
FLY ASH
WASTE GASES
FLY ASH CYCLONES
STACK
TO ATMOSPHERE
GAS
OIL FOR AUXILIARY FUEL
OIL STORAGE
CHEMICAL FEEDERS
BOILER PLANT
GAS
HOT WATER
STEAM FOR HEATING ALL BUILDINGS
GRIT
SLUDGE CAKE
CONVEYOR FROM FILTERS
MIXER
DRYING TOWER
COMBUSTION GASES
FLY ASH
FLASH DRIER
DRIED SLUDGE AND VAPOR
DRYING SYSTEM CYCLONE
VAPOR
VAPOR FAN
DRIED SLUDGE
SLUDGE FEEDER
PRIMARY AIR FAN
DRIED SLUDGE AND AIR
FURNACE
HEATED AIR
HEATED VAPOR
COMBUSTION GASES
HEATED AIR
PRE-HEATER
SECONDARY AIR FAN
AIR
INDUCED DRAFT FAN
CYCLONE
STACK
GASES
FLY ASH STORAGE
FLY ASH
ASH
CYCLONE
ASH STORAGE
DUSTLESS UNLOADER
ASH
BAG FILTERS
VACUUM PUMP
AIR
FAN
AIR
BURNER
SLUDGE GAS OR FUEL OIL FOR AUXILIARY FUEL

(*a*) SEWAGE TREATMENT PLANT

(*b*) SLUDGE INCINERATION UNIT

FIG. 7.—FLOW DIAGRAMS

A summary of the estimated final costs for the major divisions of the sewage treatment plant construction is shown in Table 8. For the most part, bids were received on a unit price basis. Table 8 shows the high and the low bids of the successful bidders and a weighted average for each kind of construction for which unit price bids were received.

TABLE 8.—SUMMARY OF BIDS ON UNIT PRICE ITEMS, BY SUCCESSFUL BIDDERS; SEWAGE TREATMENT PLANT

Item No.	Description	High	Low	Weighted average
1	Earth excavation, in cubic yards	$ 4.00	$ 1.00	$ 2.90
2	Rock excavation, in cubic yards	15.00	5.00	13.28
3	Selected fill, in cubic yards	3.00	1.00	2.22
4	Embankment, in cubic yards	0.70	0.40	0.512
	Sheeting Left in Place:			
5	Wood, in thousand feet, board measure	60.00	25.00	52.67
6	Steel, in pounds	0.03	0.02	0.0258
	Sheet Piling:			
7	Wood, in thousand feet, board measure	125.00	40.00	93.34
8	Steel, in pounds	0.06	0.04	0.0556
9	Timber piles, in linear feet	1.00	0.35	0.553
10	Steel H-piles, in linear feet	10.00	1.50	2.75
11	Steel reinforcement, in pounds	0.05	0.0425	0.0488
	Concrete, in Cubic Yards:			
12	Class A-1, formed floors and footings	25.00	19.50	20.71
13	Class A-2, formed floors and footings	17.00	13.00	15.48
14	Class B, fill	12.10	10.00	11.38
15	Pre-cast	50.00	35.00	43.95
16	Rubbed finish, in square feet	0.10	0.09	0.0963
	Expansion Joints:			
17	Copper plate, in linear feet	1.10	0.75	0.871
18	Mastic, in square feet	1.00	0.25	0.296
19	Asphalt, in square yards	1.02	0.40	0.785
20	Common brick masonry, in cubic yards	50.00	25.00	34.33
	Cast-Iron Pipe, in Pounds:			
21	Bell-and-spigot, 12 ft	0.057	0.045	0.0486
22	Bell-and-spigot, 3 ft to 12 ft	0.079	0.056	0.0672
23	Bell-and-spigot, specials	0.09	0.076	0.0831
24	Flanged, 12 ft	0.068	0.06	0.0615
25	Flanged, 3 ft to 12 ft	0.10	0.07	0.0925
26	Flanged, specials	0.12	0.10	0.1143
	Structural Metal, in Pounds:			
27	Structural steel	0.08	0.05	0.0570
28	Miscellaneous steel and iron	0.18	0.117	0.156
29	Iron castings	0.163	0.12	0.123
30	Bronze, etc.	1.20	0.52	0.903
31	Aluminum	1.70	1.00	1.36
32	Pipe railing, in linear feet	4.00	2.00	2.03
33	Galvanized floor grating, in square feet	2.50	1.80	1.84

REVENUES AND EXPENSES

Under the act creating it, the Buffalo Sewer Authority has taken over the maintenance and operation of all of the sewers of the city. Thus, estimates of annual operating costs include items for the maintenance of lateral sewers, for sewer cleaning and repairs, and for progressive rehabilitation. In round numbers, the annual budget is summarized as follows:

Item	Annual sum
Administration and expense of main office	$ 80,000
Collecting sewers	223,000
Outlying pumping stations	59,000
Sewage treatment plant	397,000
Construction work, insurance, and miscellaneous	141,000
Debt service	600,000
Total	$1,500,000

The Authority has adopted a two-part rate schedule for raising the annual revenue. This rate schedule provides that 55% of the total sum (or $825,000) shall be derived from a charge based on water consumption; and that the remainder (or $675,000 per yr) shall be derived from a charge to be paid by all taxable property in proportion to the assessed valuation. The portion of the rate applicable to water use is 40% of the water rates, and the portion applicable to assessed valuation is about $0.73 per $1,000.

ACKNOWLEDGMENT

Many individuals have contributed to the progress and outcome of the Buffalo Sewage Disposal Project. Probably the only satisfactory acknowledgment would include a roster of all of the engineers and administrative officers connected with the work. Among those who have devoted considerable effort are the following:

Members of the Buffalo Sewer Authority.—Daniel H. McCarriagher, Chairman, Henry W. Keitzel, Kevin Killeen, William M. Coon, Myron G. Lehman, George W. Lennon, and Christy J. Buscaglia.

Members of the Firm of Greeley and Hansen.—C. R. Velzy (later superintendent of operation for the Sewer Authority), and J. R. Rumsey, Members, Am. Soc. C. E.; F. C. Sellnow and L. G. Williams, Associate Members, Am. Soc. C. E.; and P. E. Langdon, S. M. Clarke, J. K. Geisey, and T. M. Niles. The firm of Greeley and Hansen served as designing and supervising engineers.

The City of Buffalo, N. Y.—Carl Howell, engineer in charge of sewers for the City, Edward P. Lupfer, M. Am. Soc. C. E., directing engineer for the Buffalo Sewer Authority, and G. E. Symons, chief chemist.

U. S. Public Works Administration.—Harold Epstein, project engineer (later general manager in charge of operation and maintenance of the entire sewerage system of the Buffalo Sewer Authority), and E. L. Peterson, supervising engineer.

DISCUSSION

C. A. Holmquist,[2] Esq. (by letter).—The pollution of Niagara River by sewage and wastes from Buffalo and other municipalities has been a matter of grave and growing concern to the Sanitary Engineering Division of the State Department of Health ever since its organization in 1906. For many years, the polluted Niagara River has been the source of water supply for the cities of Lockport, Tonawanda, North Tonawanda, and Niagara Falls, and the villages of Lewiston, N. Y., Youngstown, and Fort Niagara, having a combined population of approximately 140,000. Years ago, and before water-filtration plants had come into common use, these communities suffered repeated outbreaks and an extremely high incidence of typhoid fever. Niagara Falls, for example, prior to construction of its water-filtration plant in 1912, had the highest typhoid-fever death rate of any city in the state, reaching a maximum of 185 per hundred thousand in 1905 and averaging 131.2 for the 12-yr period ending in 1912.

The installation of water-filtration and chlorination plants at Niagara Falls in 1912 produced the immediate effect of lowering the typhoid death rate greatly. There has not been a single typhoid outbreak in the city traceable to the public water supply since 1913, and there have been only two deaths from typhoid since 1925 from any cause. This naturally speaks well for the efficient operation of the city's water-filtration plants. The improvements made to the water supplies of other communities along the Niagara River produced similar effects on the typhoid-fever death rates.

A number of investigations of the pollution of the Niagara River by the Department of Health have shown that the sewage and wastes discharged into the river from Buffalo have a tendency to become stratified laterally and to "hug" the easterly shore of the easterly channel of the river. This condition naturally has afforded considerable protection to the water intakes which are located near the westerly shore. This so-called "natural barrier" and the greatly reduced typhoid-fever death rates, as a result of water-supply improvements, have been advanced as arguments against the construction of sewage treatment facilities by the city of Buffalo.

Limitations in the Public Health Law made it difficult to act to require installation of sewage treatment facilities by Buffalo in the face of reductions of the typhoid-fever death rate which had occurred. Furthermore, industrial wastes were a large and important factor in the pollution of the Niagara River, and the Public Health Law exempts industrial wastes from application of the law unless they are mixed with sewage. Over the course of many years the State Department of Health had tried repeatedly, but without success, to have the laws amended to give it more authority to control pollution and to simplify the somewhat cumbersome procedures established for its enforcement. These were some of the difficulties which the Department of Health had always encountered in its repeated efforts to have Buffalo provide adequate treatment facilities.

---

[2] Director, Div. of Sanitation, New York State Dept. of Health, Albany, N. Y.

The several investigations made by the Department of Health over a period of fifteen years showed clearly that pollution of the Niagara River was increasing, and that at times the lateral stratification of pollution in the river was not so effective a barrier as it was thought to be at one time. The Department of Health always recognized, notwithstanding the reduction in typhoid fever which had been achieved, that the situation was one which could easily lead to an explosive outbreak under the failure of water-treatment processes. It was for this reason that every effort was made to secure action by the City of Buffalo.

The entire matter of abatement of pollution along the Niagara frontier "came to a head" in 1931 when the governor was convinced by the state commissioner of health that the situation held great promise for a calamity in the near future. Accordingly, the governor called into conference representatives of all the Niagara frontier municipalities and requested them to show cause why orders should not be issued requiring the abatement of the dangerous pollution. Buffalo, represented by its mayor, commissioner of public works, and corporation counsel, admitted its responsibility for the pollution but claimed that it could not finance the cost of treatment works without greatly exceeding the limit of its borrowing power. The other municipalities along the river agreed in effect to install treatment facilities as soon as Buffalo proceeded with a solution of its problem (which was, of course, the major problem).

No definite action resulted immediately from this conference, although it served the good purpose of impressing upon Buffalo and these other communities that pollution of the river could not be tolerated much longer. The Depression occurred and it was impossible, for a time, to induce municipalities to make appropriations for public improvements.

Then in March, 1933, came a "wave of pollution" which swept down Niagara River and through Lake Ontario, affecting practically every water plant with intakes extending into either the Niagara River or Lake Ontario, and causing an outbreak of about 10,000 cases of gastro-enteritis in the City of Niagara Falls. This experience and the threats to the lives and health of thousands of persons served to strengthen, greatly, the Department of Health's case against the City of Buffalo.

Funds were becoming available from the U. S. Public Works Administration (PWA) about that time, and persistent efforts were made to induce the city to take advantage of the opportunity of federal aid for construction of the necessary facilities. Still the city did not move toward solution of this problem except through the construction of some storm-water relief sewers.

Early in 1935 another wave of pollution swept down Niagara River, which again threatened the safety of many public water supplies. The state commissioner of health decided that positive action against the city must be taken, and accordingly he instituted proceedings against Buffalo early in 1935, which resulted in the prompt issuance of an order, approved both by the governor and the attorney general, and which contained the following conditions:

(1) That a consulting engineer be engaged by May 1, 1935;

(2) That the purchase of the land for the sewage treatment works be authorized by June 15, 1935;

(3) That detailed plans and specifications for the Bird Avenue storm drain be submitted to the State Department of Health for approval by September 1, 1935;

(4) That comprehensive plans for the interception and treatment of the entire city's sanitary sewage, and necessary storm-water relief sewers, be submitted to the State Department of Health for approval by October 1, 1935;

(5) That detailed plans and specifications for sewage treatment works and a sanitary sewer system be submitted to the State Department of Health for approval by April 1, 1936, and that work be started by July 1, 1936, and the treatment works completed and placed in operation by October 1, 1937; and

(6) That by October 1, 1935, the City of Buffalo submit to the State Department of Health, for approval, a satisfactory program for the construction of the remaining interceptors, pumping stations, force mains, and storm-water relief drains to provide for the interception and treatment of the sanitary sewage of the city, together with the percentage of storm-water flow to be treated by the sewage treatment works.

However, its existing indebtedness made it impossible for Buffalo to finance the works by ordinary methods. Accordingly, the city had legislation enacted providing for the creation of a sewer authority with power to take over the sewer system, issue bonds, and establish sewer rentals for the financing of the cost of the city's share of the project and pay for the operation of the works. The city received a 45% grant from the PWA, amounting to approximately $7,000,000.

Almost simultaneously the City of Niagara Falls undertook the construction of intercepting sewers and sewage treatment works, which were completed in 1939. It was necessary, however, to take steps to order the cities of Tonawanda and North Tonawanda to install similar works. With the installation of treatment works for these two remaining cities, the chapter of pollution of the Niagara River will be closed, and one of the most serious cases of pollution in New York State will have been abated.

ARTHUR J. BULGER,[3] ESQ. (by letter).—The author's purpose to record some of the engineering data accumulated during the design and construction of the project has been ably effected. The information is a valuable addition to the fund of such data.

A statement of the factors leading to the selection of the percentage of annual costs to be raised by a charge based on water consumption, and its relation to that portion of the annual costs to be derived from an ad valorem tax, would have been both interesting and timely. Payment of annual costs for sewerage works by a rental charge for services is not new, but its use has been limited until the past few years to a relatively small number of plants.

In principle the two-part rate schedule adopted by the Buffalo Sewer Authority appears to offer a more equitable distribution of cost to benefits than an all-revenue schedule or payment entirely by ad valorem tax. The issuance of part revenue bonds and part general obligation bonds as security for construction funds has resulted, in some cases, in a similar allocation of costs. However, in this procedure, operating expenses are usually paid entirely from revenues.

---

[3] Director, Eng. Div., Federal Works Agency, Public Works Administration, Washington, D. C.

Digressing somewhat from the stated purpose of the author, the project at Buffalo is of peculiar interest because of its international aspect. Although a problem of the local community, the completion of the project should be gratifying to the nation since it has removed a condition displeasing not only to the citizens of Buffalo but also to the nation's good neighbors in the north as well. Furthermore, this project is an example of the influence of the Public Works Administration in materially accelerating the job of removing undue pollution from surface waters.

The Buffalo Authority and its engineers are to be congratulated on a job well conceived and executed.

C. R. Velzy,[4] M. Am. Soc. C. E. (by letter).—Some of the operating results of the sewage treatment works at Buffalo are reported herein, with comments on the accomplishments thus far attained by the sewage disposal project.

The first sewage, amounting to about 20 mgd, was turned into the interceptor on June 18, 1938. Following that date, other connections from various parts of the Buffalo sewer system were opened into the interceptor until, by July 1, 1938, the entire flow of the city was being received, pumped, and treated at the treatment works. Although final construction was not completed for more than a year thereafter, the structures essential to the starting of treatment were ready for use on this date, and treatment has continued without interruption since that time.

Records of operating results have been kept from the beginning, some of the more pertinent being included in Table 9. Where available, corresponding data from the bases of design are shown. It should be borne in mind that, due to construction and testing activities during the first eighteen months of operation, the results do not, in all cases, represent normal conditions. Partial interruption of settling tank operation and return of heavy supernatant to the settling tanks during interruptions in sludge disposal processes are illustrative of interferences with normal operation.

Table 9(*a*) contains data on population and flows. It is worthy of note that the population has not increased as rapidly as was expected in 1935. The average flow has been very close to the rate estimated, but, as might be expected, maximum rates have varied somewhat from the estimate. The maximum rate of 560 mgd occurred on September 14, 1938, and lasted about one hour.

The data on grit and suspended solids shown in Table 9(*b*) indicate raw sewage characteristics in accordance with the bases of design. The removal of suspended solids was handicapped by necessary irregularity in plant operation. Results obtained in some of the more normal months indicate that the removal will range from 43% to 50%, with an effluent containing somewhat less than 100 ppm of suspended solids.

---

[4] Works Supt., Buffalo Sewer Authority, Buffalo, N. Y.

Although the chlorine demand has been consistent, there are wide variations in short-period demand due to return of heavy supernatant and also apparently to the occasional discharge of industrial wastes of high demand. By hourly sampling and testing, the application of chlorine is made to follow the requirements closely, and a satisfactory kill of coliform bacteria is maintained at all times as shown in Table 9(*c*).

TABLE 9.—OPERATING DATA, SEWAGE DISPOSAL PROJECT OF BUFFALO, N. Y.

Item	Description	Bases of design	July, 1938, to June, 1939	July, 1939, to January, 1940
	(*a*) POPULATION AND FLOW CHARACTERISTICS			
1	Population	640,000	600,000	600,000
	Flow:			
2	Average, in gallons per capita per 24 hr	200	217	205
3	Average, in million gallons per 24 hr	128	130	123
4	Maximum monthly average, in million gallons daily	200	170	133
5	Maximum daily flow, in million gallons	450	334	240
6	Maximum rate, in million gallons daily	570	560	397
	(*b*) SOLIDS			
	Suspended Solids:			
7	In raw sewage, in parts per million	180	200	178
8	In pounds per capita per 24 hr	0.30	0.36	0.31
9	Percentage removal	48.0	41.0	37.1
10	Removal, in pounds per 24 hr	92,000	88,000	69,000
11	Grit, in cubic yards per million gallons	0.15	0.16	0.15
	(*c*) CHLORINATION			
12	Demand, in parts per million	....	5.82	6.27
13	Average use, in pounds per 24 hr	....	4,880	6,797
	Percentage Kill of Coliform Bacteria:			
14	Maximum monthly average	....	99.9	99.9
15	Minimum monthly average	....	94.2	97.2
16	Average	....	98.8	98.9
	(*d*) SLUDGE CHARACTERISTICS AND GAS			
	Percentage of:			
17	Moisture in raw sludge	96	96.6	92.6
18	Volatile matter in raw sludge	70	57.6	57.7
19	Moisture in digested sludge	....	87.8	89.5
20	Volatile matter in digested sludge	....	41.5	46.3
21	Gas production, in cubic feet per 24 hr	....	320,000	531,000
	(*e*) SLUDGE DISPOSAL			
	Percentage of:			
22	Ferric chloride in dry solids	2 to 3	2.31	2.95
23	Lime (CaO) in dry solids	8 to 12	9.25	10.25
24	Moisture in sludge cake	70	61.8	62.3
25	Volatile matter in sludge cake	....	38.8	37.2
26	Filter rate, in pounds of dry solids per square foot per hour	5.0	....	8.3
27	Incineration: Dry solids in pounds per furnace hour	2,500	....	2,610

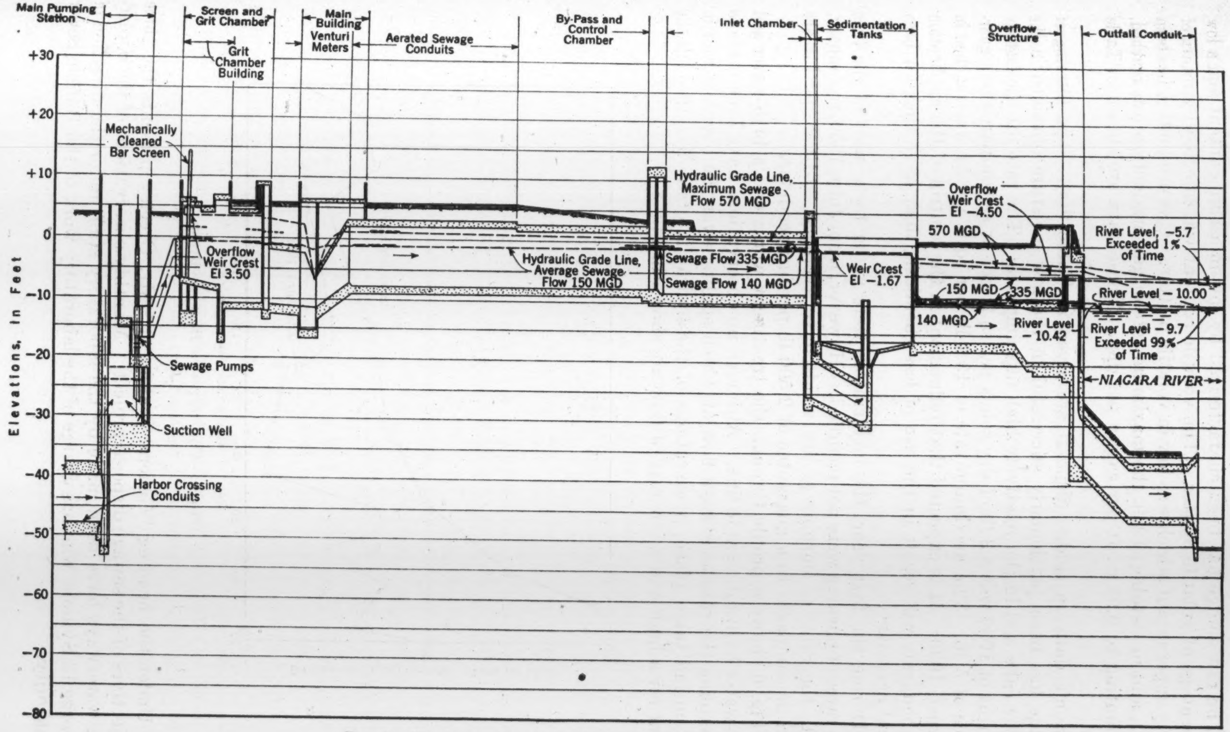

Main Pumping Station
Screen and Grit Chamber
Grit Chamber Building
Main Building
Venturi Meters
Aerated Sewage Conduits
By-Pass and Control Chamber
Inlet Chamber
Sedimentation Tanks
Overflow Structure
Outfall Conduit
Elevations, In Feet
+30
+20
+10
0
−10
−20
−30
−40
−50
−60
−70
−80
Mechanically Cleaned Bar Screen
Overflow Weir Crest El 3.50
Sewage Pumps
Suction Well
Harbor Crossing Conduits
Hydraulic Grade Line, Average Sewage Flow 150 MGD
Hydraulic Grade Line, Maximum Sewage Flow 570 MGD
Sewage Flow 335 MGD
Sewage Flow 140 MGD
Weir Crest El −1.67
Overflow Weir Crest El −4.50
570 MGD
150 MGD
335 MGD
140 MGD
River Level −10.42
River Level, −5.7 Exceeded 1% of Time
River Level − 10.00
River Level − 9.7 Exceeded 99% of Time
NIAGARA RIVER

FIG. 8.—HYDRAULIC PROFILE; MARCH, 1938; SEWAGE TREATMENT PLANT

The raw sludge has been characterized by low moisture content and a low percentage of volatile matter. The volatile content was affected by return of heavy supernatant which was low in volatile matter. Gas production has been good, and in October, 1939, the maximum rate of gas production was reached, amounting to 0.93 cu ft per capita per day. These data are shown in Table 9(*d*).

Some data on sludge disposal are given in Table 9(*e*). The filtering of sludge has been particularly successful in the low moisture obtained in the sludge cake and in the relatively high filtering rate. The rate for the last six months of 1939 was 8.3 lb of dry solids per sq ft per hr as compared to a guarantee of 5 lb. The maximum rate of 13.4 lb per sq ft per hr was reached in August, 1939. It is expected that refinement in the control of the application of chemicals will result in the use of less chemicals than have thus far been found necessary.

During the first year, the incinerators were under tests most of the time and rates of burning were not recorded. However, from July, 1939, to January, 1940, the rates of burning of dry solids have averaged 2,610 lb per furnace hour as compared to a guarantee of 2,500 lb per furnace hour.

Fig. 6 shows a profile through the treatment works and a high-water and low-water hydraulic grade line. Although no observations have been made to determine the rate of sewage flow at which spilling over the overflow weir in the outfall takes place, observations of the hydraulic grade line through the plant for a high and a low rate of flow are shown in Fig. 8.

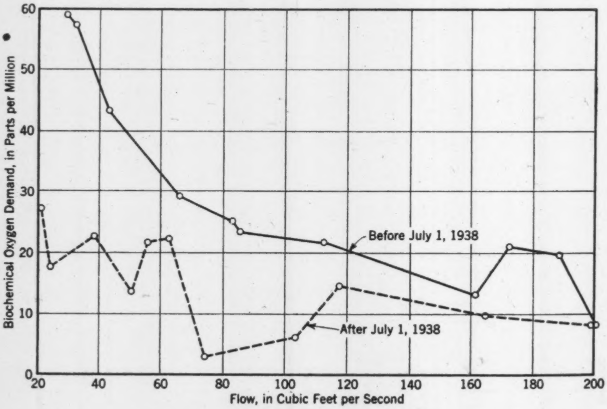


FIG. 9.—CONDITIONS IN BUFFALO RIVER

Under the heading "General Statement" Mr. Greeley presents the three objectives of the sewage disposal project. Monthly observations of the bordering waterways have been made by the laboratory staff under the direction of George E. Symons who has charge of the laboratory work of the Buffalo Sewer Authority.

Results of observations on the conditions in the Buffalo River before the treatment works went into operation and after they were in operation are shown in Fig. 9. These data are from averages of analyses of samples taken at four sampling stations in the middle stretches of the river somewhat above the outlet. The data and results herein are based on flows as great as 200 cu ft per sec. It will be noted that the data taken after July 1, 1938, even for a lower average flow, show a definite decrease in 5-day B.O.D. Observations showed a reduction in coliform bacteria of 43%. A definite increase in dissolved oxygen is evident in that, before July, 1938, it was found only in flows greater than about 160 cu ft per sec, whereas, after that date, it was found in flows as low as 100 cu ft per sec.

Some parts of Buffalo Harbor are directly affected by the Buffalo River. These parts enjoyed an improvement corresponding to the improvement in the river. Before July, 1938, Black Rock Harbor became quite septic in the summer months, with a high bio-chemical oxygen demand and coliform content. This was due to the discharge of one of the main sewer outlets into the harbor. After the treatment works went into operation, the waters of this harbor became fresh, with a dissolved oxygen content of about 6 to 7 ppm in the summer months. The reduction in a 5-day B.O.D. is shown in Table 10.

TABLE 10.—COMPARISON OF BIO-CHEMICAL OXYGEN DEMAND, IN PARTS PER MILLION

Date (1938)	January	February	March	April	May	June	July	August	September	October	November	December
Before July 1	28	15	3	30	35	29	17.	11	24	28	..	33
After July 1	3	..	..	5	0	4	0.2	1	2	2	2	2

Monthly samples have been taken and analyses made on the Niagara River, and it has been observed that the quantities of coliform bacteria have been reduced about 97% from the average for some two years previous to the beginning of operation. Water works below Buffalo report longer filter runs and reduction in the quantity of wash water and chlorine required. Reduction in sludge deposits and the return of game fish immediately below Buffalo are reported.

By intercepting Scajaquada Creek the Main Intercepting Sewer has removed a flow of sewage from Park Lake, an attractive body of water in Delaware Park, and has thus eliminated a serious odor nuisance which previously prevailed during the summer months.

With the completion of construction and testing, normal operating results as well as refinements and economies are anticipated. Further work on industrial waste disposal, with the continuation of excellent cooperation from the industries involved, will result in further improvement in the condition of adjacent waterways.

The operating results at the treatment works and the effects upon the waterways of Buffalo obtained during these first months of operation indicate the soundness of the bases of design and show accomplishment of the objectives which are described in Mr. Greeley's paper.

THOMAS M. NILES,[5] M. AM. SOC. C. E. (by letter).—The tunnel crossing under the Buffalo River and the main crossing under Black Rock Harbor to the sewage treatment plant are described briefly in this paper. Two additional crossings, not mentioned by name in Mr. Greeley's paper, were built under Cazenovia Creek and Scajaquada Creek. Structures of this kind are frequently required in connection with sewer work.

Black Rock Harbor is a relatively short navigable canal along the east bank of the Niagara River which carries traffic past the rapids at the head of the river. At the point of crossing it has a total width of about 400 ft, a channel width of about 250 ft, and a channel depth of about 20 ft. To allow for a future navigable depth of 27 ft, the War Department requested a clear depth of not less than 32 ft over the crossing structure, measured from low water datum. This was the only one of the four stream crossings at which active navigation had to be maintained during construction.

Although the Buffalo River is considered navigable as far as Bailey Avenue, the point of the sewer crossing, it is not now (1940) used this far up. The stream at this point is some 200 ft wide and 5 to 7 ft deep below low water. To provide for future navigation, the required depth over the crossing structure was 30 ft. Actually, this crossing was constructed in rock tunnel at a greater depth.

Cazenovia Creek and Scajaquada Creek are about 120 ft wide and 50 ft wide, respectively, and are both about 10 ft deep. The clear depth necessary was 14 ft for the former and 16 ft for the latter. Although both may be considered navigable, no development for deep draft vessels is contemplated.

The crossings were designed to provide velocities sufficient to avoid deposition of organic solids at minimum flows and sufficient to scour sand and grit at maximum flows, with reasonably low loss of head. Other controlling factors were simplicity of construction, simplicity of operation, ease of inspection and maintenance, and the provision of facilities for cleaning if found necessary.

Two-pipe conduits were used for all of the crossings. Those for Cazenovia Creek and Scajaquada Creek are circular reinforced concrete pipes encased in concrete. In the Buffalo River Crossing, the smaller pipe is circular and the larger a modified rectangular section with the longer dimension vertical with a semicircular crown. The harbor crossing consisted of two pipes of equal diameter. For all except the harbor crossing, where flow is controlled manually by sluice gates, dry weather flow is confined to the smaller pipes by means of weirs.

Table 11 presents data on sewage quantities, pipe sizes, velocities, and estimated loss of head for the four crossings. In the two creek crossings the pipes are of the same cross sections throughout their lengths and the inlet and outlet portions are sloped down from inlet chambers and up to outlet chambers which meet the inverts of the connecting sewers. The river and harbor crossings have vertical inlet and outlet shafts, and in the case of the river crossing the uptake pipes in the outlet shaft are reduced in size to 30-in. diameter and 48-in. by 54-in. rectangular to provide slightly higher velocities than in the horizontal part of the crossing.

---

[5] Engr., Greeley & Hansen, Chicago, Ill.

Inlet and outlet chambers for the creek and river crossings are provided with removable slabs to permit introduction of dewatering pumps and other equipment which may be useful in connection with inspection and cleaning. During dry-weather flow, the larger pipes of the creek crossings may be pumped out and inspected without disturbing the flow in the smaller pipes. Inspection of the smaller pipes involves construction of temporary dams to divert the flow through the larger pipes. Sluice gates are provided at both the inlet and outlet ends of the river and harbor crossings to permit diversion of flow to either pipe while the other is being inspected.

TABLE 11.—DESIGN DATA FOR STREAM CROSSINGS, BUFFALO SEWAGE DISPOSAL PROJECT

Crossing	1940 Sewage Quantities, in Million Gallons Daily			Pipe Diameters, in Inches		1940 Velocities, in Feet per Second					Estimated loss of head, in feet, at maximum flow
						Smaller Pipe				Maximum velocities, larger pipe	
	Minimum	Average	Maximum	Smaller	Larger	Minimum	Average	Maximum	At maximum flow		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Cazenovia Creek......	5.5	11	62	27	54	2.1	4.3	7.5†	3.9	5.0	1.0
Scajaquada Creek.....	9	19	97	36	60	2.0	4.2	7.7†	5.9	5.6	1.0
Buffalo River........	7	14	87	36	*	1.5	3.0	5.5†	4.2	4.3	2.3
Black Rock Harbor...	66	132	566	96	96	1.0	2.0	8.7	8.7	8.7	5.5

* Special shape (48 in. by 77 in.), area = 24 sq ft.

† Maximum velocities occur in the smaller pipes just before the larger pipes come into service. Advantage is taken of an available head somewhat higher than the head available at maximum total flow.

The outlet structure of the Buffalo River crossing includes a separate access shaft between the two riser pipes sufficiently large to accommodate a dewatering pump and other equipment that may be needed. Manholes at the bottom of this shaft permit access into the horizontal crossing pipes, and gate valves are provided for draining the pipes into the shaft.

The harbor crossing riser pipes are within the inlet chamber of the main pumping station at the treatment plant. They are vertical tubes, 8 ft in diameter, opening into surge chambers with an overflow connection to the river. These tubes extend 2 ft below the inverts of the horizontal crossing pipes to form pockets for the collection of large stones which may not be carried upward, and from which material may be periodically cleaned by means of a clamshell bucket. A portable dewatering pump is provided to facilitate inspection and cleaning.

Construction of the two creek crossings was by open cut with cofferdams in two sections, confining the stream flow to the inactive half of the work. Aside from occasional flooding from high water, no unusual difficulties were experienced.

Alternate methods of construction, open cut or rock tunnel, were available to bidders on the Buffalo River crossing. The low bid was for rock tunnel and the contractor exercised his option to construct the tunnel at a lower elevation than shown on the plans in order to obtain more favorable tunneling conditions. Even so, it was found necessary to install timbering for roof support. Occasional grouting of seams successfully controlled the flow of water into the work.

Borings had shown the material at the site of the Black Rock Harbor crossing to be unsuitable for tunneling. Accordingly, the construction method available was open cut, either by cofferdams or by subaqueous methods. Although proposals were received on both methods, the low bidder's choice of the subaqueous method involved less difficulty in maintaining navigation during the progress of the work. Considerable additional dredging would have been required to provide sufficient widths and depths of channels at either side had cofferdams been used throughout.

Approximate average construction costs for the four stream crossings, including inlet and outlet structures, were as follows:

Crossing	Total	Per foot of length
Scajaquada Creek	$ 15,000	$150
Cazenovia Creek	27,000	150
Buffalo River	75,000	300
Black Rock Harbor	425,000	800

The crossings were placed in operation in June, 1938, and have given satisfactory service. No operating difficulties have been encountered and there has been no need for cleaning.

The foregoing discussion of one feature of the work on the Buffalo project is perhaps in more detail than the relative importance of this feature should warrant. Nevertheless, it has seemed that the material presented may be of value in acquainting the reader with some of the detailed considerations which entered into the design. The paper itself is an admirably complete and concise review of the essential features of the project and contains sufficient detailed information to present a clear understanding of the problem at Buffalo and the manner in which it was solved.

SAMUEL A. GREELEY,[6] M. Am. Soc. C. E. (by letter).—The writer was very much interested in the discussions submitted. Those by Messrs. Velzy and Niles add much of interest to the record of the engineering data of the Buffalo Sewage Disposal Project.

Mr. Bulger comments on the two-part rate schedule adopted by the Buffalo Sewer Authority to provide the necessary annual revenue. The collection of revenues under this two-part rate schedule was done for two complete fiscal years on June 30, 1940, and the results were gratifying. The following is quoted from a publication of the Manufacturers and Traders Trust Company of Buffalo, in 1940, with reference to the sale by public bidding of $500,000 of bonds of the Authority:

> "Receipts for the fiscal year [1938–1939] have been uniformly good, being 94+% of that part of the Sewer Rent based upon assessed valuation of property and 93+% for that part of Sewer Rent based upon water consumption."

---

[6] Greeley & Hansen, Chicago, Ill.

The purpose of the two-part rate schedule is to provide equity in the contribution of persons to the necessary annual revenue. The two-part rate has been commented on in a publication of general interest by the Buffalo Sewer Authority (dated 1938) as follows:

> "The Buffalo Sewer Authority in trying to fix upon the most equitable method of raising the money to pay for building and operating the new sewer system, decided that it would be unfair to put the entire burden on real estate owners; also, that it would be unfair to put the entire burden on water users. After a very thorough and painstaking study of all of the facts, it was decided to divide the expense.
>
> "Fifty-five percent of the annual revenue, or $825,000, is collected on the basis of water use, and forty-five percent, or $675,000, is collected as a charge for sewage disposal service computed on the basis of assessed valuation."

It is the writer's opinion that the two-part rate permits equitable distribution of the cost of sewerage and sewage disposal service, particularly in cities having combined sewers, such as Buffalo.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2093

FUNCTIONAL DESIGN OF FLOOD CONTROL RESERVOIRS

By C. J. Posey,[1] Jun. Am. Soc. C. E., and Fu-Te I,[2] Esq.

With Discussion by Messrs. Edward Soucek, J. C. Stevens, Edward J. Bednarski, Roland A. Kampmeier, Edgar E. Foster, Sherman M. Woodward, M. Kindinger, P. Wilhelm Werner, Waldo E. Smith, J. O. Jones, W. C. Hammatt, and C. J. Posey and Fu-Te I.

Synopsis

The functional design of a flood control reservoir has as its objective the determination of the relation between the storage space that must be provided and the corresponding reduction of the flood peak. The method of incorporating this relation into the study of the economic balance of a flood control system has been explained in detail by Sherman M. Woodward,[3] M. Am. Soc. C. E. Mr. Woodward's "five-sixths rule," which heretofore has provided the only direct method for the functional design computations, is limited in its application to reservoirs of such proportions that the maximum outflow is small compared with the average inflow during the flood, and to reservoirs with orifice-type outlets and a certain type of depth-capacity relationship. In this paper the writers have generalized Mr. Woodward's method and have extended it to apply to reservoirs with either orifice- or weir-type outlets, in valleys of a wide range of morphological configuration. Although the relationships derived can be used in the design of multiple-purpose reservoirs, the present discussion is restricted to reservoirs for flood control only.

Introduction

A necessary preliminary step is the estimation of the maximum flood run-off that may occur at the given site. This is usually based upon a statistical study of meteorological and hydrological data, and includes a determination of the shape of the flood hydrograph, as well as its volume. Methods of making

---

Note.—Published in October, 1939, *Proceedings*.

[1] Associate Engr., Iowa Inst. of Hydr. Research; Associate Prof. of Hydraulics and Structural Eng., State Univ. of Iowa, Iowa City, Iowa. Mr. Posey transferred to Associate Member on April 10, 1939.

[2] Prof., Coll. of Eng., National Southwest United Univ., Kuenming, Yunnan, China; formerly Graduate Student, Dept. of Mechanics and Hydraulics, State Univ. of Iowa, Iowa City, Iowa.

[3] "Hydraulics of the Miami Flood Control Project," by Sherman M. Woodward, Technical Reports of the Miami Conservancy District, State of Ohio (1920), Part VII, Chapter XII, p. 286.

these studies have been the subject of extended discussion in recent years,[4] and further treatment is not necessary in this paper. Suffice to it say that the flood hydrograph used for the design is made somewhat greater than the maximum probable flood to provide a margin of safety. No matter how carefully this work is done, the design flood will have the kind of uncertainty that is attached to any prediction of future events. This sets a limit of accuracy beyond which it is useless to carry subsequent computations.

In order to determine the storage necessary to accomplish a desired reduction of peak flood flow, it is necessary to route the design flood through the reservoir—that is, to compute the moderating effect of the reservoir storage upon the flood hydrograph.

In general, flood-routing methods are of two types:

(1) Step methods, in which the flood inflow hydrograph is divided into steps or time intervals of such short duration that during each one the inflow and outflow rates may be assumed to be constant. The actual depth-capacity data for the reservoir site are used and the discharge-depth relation must be known. An irregular inflow hydrograph may be approximated, as closely as desired, by using a large number of steps.

(2) Integration methods, in which the given flood hydrograph is replaced by an equivalent flood of uniform intensity. The depth-capacity data are represented by an empirical formula. It is necessary to know only the exponent and approximate zero-depth of the discharge-depth relation.

The first step methods to be used required a trial-and-error solution of each step. Later, graphical methods were developed which eliminated this cut-and-try process.[5] Results of the step computations include the outflow and stage at the end of each step, so that step methods are well adapted for operation studies. In application to functional design, however, step methods are indirect. It is necessary first to assume the size of the outlet. The corresponding maximum storage and maximum outflow are determined by means of routing computations. Another size of outlet is then assumed, and the process is repeated until the desired part of the curve representing the relation between storage and percentage reduction in flood peak has been obtained. On the other hand, integration methods give the storage required for a given maximum outflow directly, with considerable saving of time and labor.[6]

Depth-Capacity Exponent.—The relation between depths and capacities in the reservoir may be represented by an empirical expression of the form

$$s = C\,h^m \qquad (1)$$

in which: s is the storage capacity of the reservoir at a depth h; h is the depth of water above outlet; C is a constant; and m is the "depth-capacity" exponent. For the determination of m, logarithmic plotting is used. The depths above outlet location (zero depth of the orifice type of outlet, or the crest of a weir type of outlet) are plotted as ordinates and the corresponding capacities as abscissas. A straight line best fitting the plotted points is drawn. The exponent m is the reciprocal of the slope of the line, the slope being measured directly with a scale having uniformly spaced divisions, disregarding the logarithmic scale of the plot. The constant C is not needed in the analysis.

---

[4] Technical Reports of the Miami Conservancy District, Part V; also "Analysis of Run-Off Characteristics," by Otto H. Meyer, Assoc. M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., November, 1938, p. 1769.

[5] "Slide Rule for Routing Floods through Storage Reservoirs or Lakes," by C. J. Posey, *Engineering News-Record*, April 25, 1935, Vol. 114, p. 580.

[6] "Hydraulics of the Miami Conservancy District," by Sherman M. Woodward, Technical Reports of the Miami Conservancy District, Part VII, Chapter VII.

A similar procedure may be followed with the depth-area data, and the depth-capacity exponent found by integration of the empirical relation so determined. It is better, however, to determine the depth-capacity exponent directly, because integration of the empirical relation derived by logarithmic plotting is more likely to introduce error than numerical summation of the original survey data.

In studying the depth-capacity curves for more than 250 reservoir sites, m was found to vary from 1.03 to 4.74. The following classification, given by R. A. Sutherland, Assoc. M. Am. Soc. C. E., was found to be justified.[7]

Classification	Values of m
Lake type	1.0 to 1.5
Flood-plain and foothill type	1.5 to 2.5
Hill type	2.5 to 3.5
Gorge type	3.5 to 4.5

In almost all cases, the points lay very nearly along a straight line on the logarithmic plot. The use of the exponential formula to obtain capacities at higher levels than those for which survey data are available is inadvisable. For some sites, the logarithmic plot will follow a straight line over a considerable depth, and then deviate sharply to one side. For others, the points will lie on a curve. Since volumes near the bottom of the reservoir are relatively insignificant, it is important that the best straight line should fit the data at the higher levels most closely. If the straight line is a close approximation to the data at higher levels, discrepancies at the lower levels can be ignored, with inappreciable error in the results. Some judgment is required in deciding whether the degree of approximation is satisfactory. No rigid rule can be formulated. In doubtful cases a check should be made by the step method.

Equivalent Uniform Flood.—The equivalent uniform flood corresponding to a given irregular flood hydrograph is a flood of uniform intensity, having the same total inflow as the given flood, which will produce the same total amount of storage with the same maximum rate of outflow from the reservoir. It is possible to determine a uniform flood exactly equivalent to a given irregular flood by detailed routing computations. It has been found, however, that even for hydrographs differing considerably from rectangular, it is not difficult to select, arbitrarily, an equivalent uniform flood that will give results sufficiently accurate for design purposes. For example, consider the triangular flood shown in Fig. 1. This flood was routed by the step method through a reservoir (m = 2.5) with the orifice type of outlet. The maximum outflow reached was 5,160 acre-ft per hr and the maximum storage, 117,100 acre-ft.

---

[7] "Some Aspects of Water Conservation," by R. A. Sutherland, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 164.

If the duration T of an equivalent uniform flood is assumed to be 40 hr, the intensity I should be equal to the total volume of the flood divided by 40, or 7,320 acre-ft per hr. Routing this uniform flood through the reservoir by the step method the maximum rate of outflow reached was found to be 5,140 acre-ft per hr, which is very close to the value obtained for the triangular flood.

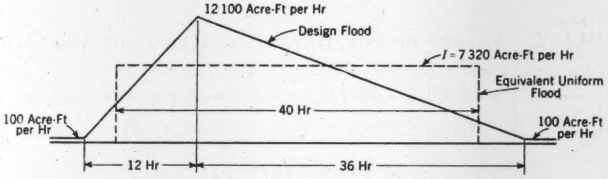


FIG. 1.—TRIANGULAR FLOOD AND EQUIVALENT UNIFORM FLOOD

The error in the maximum outflow rate and the maximum storage, corresponding to different assumptions for the duration of the uniform flood, is given in Table 1. It is seen that any estimate for the duration of the equivalent uniform flood between the values of 38 hr and 42 hr gives satisfactory accuracy, in this case. Similar results from other comparisons made by the writers confirm the belief that arbitrary selection of equivalent uniform floods is not difficult. It should be remembered that there is considerable uncertainty in the shape and volume of the "design" hydrograph.

TABLE 1.—ERRORS IN THE MAXIMUM OUTFLOW RATE AND MAXIMUM STORAGE

Duration assumption, T, in hours (1)	Inflow, I, in acre-feet per hour (2)	Maximum outflow rate, O, in acre-feet per hour (3)	Percentage error in maximum outflow rate (4)	Maximum storage, S, in acre-feet (5)	Percentage error in maximum storage (6)
38	7,700	5,195	+0.68	121,000	+3.3
40	7,320	5,140	−0.39	115,000	−1.8
42	6,970	5,080	−1.55	108,000	−7.8

Outflow Ratio.—The outflow ratio, x, is the ratio of the maximum outflow from the reservoir to the uniform inflow of the rectangular flood. It is the same as the "protection ratio" defined by Mr. Woodward.[6] The writers recommend that the name "protection ratio" be abandoned as misleading. The higher the value of this ratio, the greater the maximum outflow and the less the protection.

THE DETENTION RATIO

The detention ratio is the ratio of the maximum reservoir storage reached during the flood to the total volume of the uniform flood. By definition, then,

$$d = \frac{S}{I\,T} \qquad (2)$$

in which d is the detention ratio, S is the maximum storage, and $I\ T$ is the total volume of the uniform flood. The detention ratio is a function of the depth-capacity exponent, m, the outflow ratio, x, and the exponent governing the discharge of the outlets. The writers have computed values of d over the practical range of the outflow ratio and depth-capacity exponent, for reservoirs with orifice-type outlets (Table 2(*a*)) and for reservoirs with weir-type outlets (Table 2(*b*)). The values of d were computed with the aid of the infinite series developed in the Appendix. They can be checked (indirectly) by any of the standard step methods.

TABLE 2.—VALUES OF THE DETENTION RATIO FOR RESERVOIRS

Values of m (1)	$x = 0.1$ (2)	$x = 0.2$ (3)	$x = 0.3$ (4)	$x = 0.4$ (5)	$x = 0.5$ (6)	$x = 0.6$ (7)	$x = 0.7$ (8)	$x = 0.8$ (9)
(*a*) ORIFICE-TYPE OUTLETS								
1.0	0.933	0.864	0.794	0.722	0.647	0.570	0.489	0.399
1.5	0.925	0.848	0.771	0.692	0.611	0.528	0.441	0.348
2.0	0.920	0.839	0.757	0.674	0.590	0.504	0.415	0.322
2.5	0.916	0.832	0.748	0.662	0.575	0.488	0.397	0.303
3.0	0.914	0.828	0.741	0.654	0.565	0.476	0.384	0.290
3.5	0.912	0.824	0.736	0.647	0.557	0.467	0.374	0.279
4.0	0.911	0.822	0.732	0.642	0.551	0.460	0.366	0.271
5.0	0.909	0.818	0.726	0.635	0.542	0.450	0.355	0.258
6.0	0.908	0.815	0.722	0.630	0.536	0.442	0.347	0.250
(*b*) WEIR-TYPE OUTLETS								
1.0	0.959	0.916	0.870	0.822	0.769	0.710	0.645	0.567
1.2	0.955	0.907	0.858	0.805	0.748	0.686	0.618	0.536
1.4	0.951	0.900	0.846	0.790	0.730	0.665	0.593	0.509
1.6	0.948	0.893	0.836	0.777	0.714	0.647	0.573	0.486
1.8	0.945	0.887	0.828	0.766	0.700	0.632	0.555	0.467
2.0	0.942	0.882	0.821	0.756	0.689	0.617	0.539	0.451
2.2	0.940	0.878	0.814	0.748	0.679	0.605	0.525	0.437
2.4	0.938	0.874	0.808	0.740	0.669	0.594	0.514	0.425
2.6	0.936	0.870	0.803	0.734	0.661	0.585	0.504	0.414
2.8	0.934	0.867	0.798	0.728	0.654	0.577	0.495	0.404
3.0	0.933	0.864	0.794	0.722	0.647	0.569	0.486	0.395
3.5	0.930	0.858	0.785	0.710	0.633	0.553	0.468	0.377
4.0	0.927	0.853	0.777	0.700	0.621	0.539	0.453	0.362
4.5	0.925	0.848	0.771	0.692	0.611	0.528	0.441	0.349
5.0	0.923	0.844	0.766	0.685	0.603	0.519	0.431	0.338
5.5	0.921	0.841	0.761	0.679	0.596	0.511	0.422	0.330
6.0	0.920	0.839	0.757	0.674	0.590	0.504	0.415	0.322

It is worth noting that for manually controlled outlets operated at 100% efficiency,

$$d = 1 - x \quad \text{(3)}$$

ILLUSTRATIVE EXAMPLES

The significance and utility of the detention ratio in the functional design of flood control reservoirs will be illustrated by two examples.

Example 1.—Find the relation between the maximum outflow and the storage required for controlling a flood of 50,000 acre-ft in 24 hr by an orifice-outlet reservoir at reservoir site A, Fig. 2. The depth-capacity exponent for this site is 2.25. The average inflow is $I = \dfrac{50{,}000}{24} = 2{,}080$ acre-ft per hr. The computations are shown in Table 3, the column headings of which are

TABLE 3.—COMPUTATIONS, EXAMPLE 1

Outflow ratio, x (1)	Maximum outflow, $O = x\,I$, in acre-feet per hour (2)	Detention ratio, d, from Table 2(a) (3)	Storage required, in acre-feet ($S = 50{,}000\, d$) (4)
0.4	835	0.668	33,400
0.5	1,040	0.582	29,100
0.6	1,250	0.496	24,800
0.7	1,460	0.406	20,300
0.8	1,670	0.312	15,600

self-explanatory. The curve showing the relation between the maximum outflow and the storage required can be plotted from the data in Columns (2) and (4).

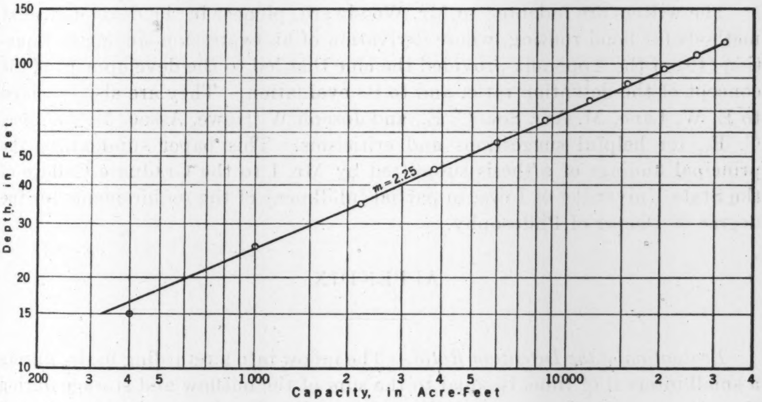


FIG. 2.—DEPTH-CAPACITY CURVE FOR RESERVOIR SITE "A"

Example 2.—Assume that, for the same example, political or other non-engineering reasons dictate that the maximum storage available shall be 25,000 acre-ft. What is the maximum flood reduction which may be obtained by the construction of a reservoir with an orifice type of outlet? By Equation (2), $d = \dfrac{25{,}000}{50{,}000} = 0.50$. From Table 2($a$)—for $m = 2.25$, and $d = 0.50$—$x = 0.595$. Hence, the maximum outflow $= x\,I = 0.595 \times 2.080 = 1{,}240$ acre-ft per hr. If the weir type of outlet is used instead of the orifice type, from Table 2(b)—for $m = 2.25$, and $d = 0.50$—$x = 0.725$. Hence, the maximum outflow $= 0.725 \times 2{,}080 = 1{,}510$ acre-ft per hr.

Manually-Controlled Outlets.—There has been much argument as to the relative merits of manually-controlled and automatic (fixed) outlet structures for flood control reservoirs. Comparisons based upon items as difficult to evaluate as fallibility should be postponed until the relative economy has been investigated. Tabulated values of the detention ratio afford an easy approach to such study. For example, assume that reservoir site *A* of Example 1 is to be used to reduce the maximum outflow from a flood of 50,000 acre-ft in 24 hr to 1,000 acre-ft per hr, and that the average cost of storage near the top of the reservoir is $30 per acre-ft. By interpolation in Table 3 the storage required, if orifice-type outlets are used, is found to be 29,900 acre-ft. With manually-controlled outlets, an efficiency of 100% could presumably be reached, in which case 25,900 acre-ft of storage would be required. The saving in storage is 4,000 acre-ft, representing $120,000. If manually-controlled outlets are to be economical, this $120,000 must exceed the capitalized cost of the maintenance and operation of the mechanical gates, plus or minus any difference in the original cost of the outlets.

ACKNOWLEDGMENTS

The writers are indebted to Mr. Woodward, pioneer in the development of methods for flood routing, whose derivation of an expression similar to Equation (19) of the Appendix provided the clue that led to the development of the concept of the detention ratio, and to its evaluation. They are also indebted to E. W. Lane, M. Am. Soc. C. E., and Joseph W. Howe, Assoc. M. Am. Soc. C. E., for helpful suggestions and criticisms. This paper summarizes the principal findings of a thesis submitted by Mr. I to the Graduate College of the State University of Iowa, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

APPENDIX

Evaluation of the Detention Ratio.—The inflow into a retarding basin, during a small interval of time, is equal to the sum of the outflow and storage during that interval of time, evaporation and percolation losses being neglected. Thus,

$$i\,dt = o\,dt + ds \qquad (4)$$

in which: i = average inflow rate during the time dt; o = average outflow rate during the time dt; dt = a small increment of time; and ds = increment of storage capacity during the time dt. The outflow from a reservoir can be expressed in terms of the depth h:

$$o = B\,h^n \qquad (5)$$

in which n is 0.5 for the orifice type of outlet, and 1.5 for the weir type. The storage capacity can be approximated by a monomial exponential function of the depth:

$$s = C\,h^m \quad \text{(1)}$$

so that

$$ds = C\,m\,h^{m-1}\,dh \quad \text{(6)}$$

Substituting these relations into Equation (4),

$$i\,dt = B\,h^n\,dt + C\,m\,h^{m-1}\,dh \quad \text{(7)}$$

For a constant inflow, I, during the entire flood period, Equation (7) becomes

$$I\,dt = B\,h^n\,dt + C\,m\,h^{m-1}\,dh \quad \text{(8)}$$

Transposing and collecting like terms,

$$dt = \frac{C\,m}{I}\left(\frac{h^{m-1}}{1 - \frac{B}{I}\,h^n}\right)dh \quad \text{(9)}$$

Equation (9) can be transformed into an infinite series by direct division,

$$dt = \frac{C\,m}{I}\left[h^{m-1} + \left(\frac{B}{I}\right)h^{m+n-1} + \left(\frac{B}{I}\right)^2 h^{m+2n-1} + \left(\frac{B}{I}\right)^3 h^{m+3n-1} + \cdots\right]dh \quad \text{(10)}$$

Integrating, term by term,

$$t = \frac{C\,m}{I}\left[\frac{h^m}{m} + \left(\frac{B}{I}\right)\frac{h^{m+n}}{m+n} + \left(\frac{B}{I}\right)^2 \frac{h^{m+2n}}{m+2\,n} + \left(\frac{B}{I}\right)^3 \frac{h^{m+3n}}{m+3\,n} + \cdots\right] \quad \text{(11)}$$

Substituting $o = B\,h^n$ and $s = C\,h^m$

$$t = \frac{s\,m}{I}\left[\frac{1}{m} + \left(\frac{o}{I}\right)\frac{1}{m+n} + \left(\frac{o}{I}\right)^2 \frac{1}{m+2\,n} + \left(\frac{o}{I}\right)^3 \frac{1}{m+3\,n} + \cdots\right] \quad \text{(12)}$$

At the end of the uniform flood, when the elapsed time has reached the value T, the storage of water in the reservoir will have attained its maximum value S, and the outflow rate its maximum value O. At this instant, Equation (12) becomes

$$T = \frac{S\,m}{I}\left[\frac{1}{m} + \left(\frac{O}{I}\right)\frac{1}{m+n} + \left(\frac{O}{I}\right)^2 \frac{1}{m+2\,n} + \left(\frac{O}{I}\right)^3 \frac{1}{m+3\,n} + \cdots\right] \quad \text{(13)}$$

If the ratio of the maximum outflow to the uniform inflow is denoted by x, the "outflow ratio," the total inflow during the flood period will be

$$I\,T = S\,m\left(\frac{1}{m} + \frac{x}{m+n} + \frac{x^2}{m+2\,n} + \frac{x^3}{m+3\,n} + \cdots\right) \qquad (14)$$

The total outflow during the reservoir-filling period, T, is equal to the total inflow minus the maximum storage,

$$\int o\,dt = I\,T - S = S\,m\left(\frac{x}{m+n} + \frac{x^2}{m+2\,n} + \frac{x^3}{m+3\,n} + \cdots\right) \qquad (15)$$

The average outflow during the period T is

$$\frac{\int o\,dt}{T} = \frac{I\left(\dfrac{x}{m+n} + \dfrac{x^2}{m+2\,n} + \dfrac{x^3}{m+3\,n} + \cdots\right)}{\left(\dfrac{1}{m} + \dfrac{x}{m+n} + \dfrac{x^2}{m+2\,n} + \dfrac{x^3}{m+3\,n} + \cdots\right)} \qquad (16)$$

The ratio, r, of the average outflow to the maximum outflow is given by

$$r = \frac{\dfrac{\int o\,dt}{T}}{O} = \frac{\dfrac{\int o\,dt}{T}}{x\,I} \qquad (17)$$

so that

$$r = \frac{\left(\dfrac{1}{m+n} + \dfrac{x}{m+2\,n} + \dfrac{x^2}{m+3\,n} + \cdots\right)}{\dfrac{1}{m} + x\left(\dfrac{1}{m+n} + \dfrac{x}{m+2\,n} + \dfrac{x^2}{m+3\,n} + \cdots\right)} \qquad (18)$$

By factoring and direct division the following expression is obtained, which gives the limiting value of r as x approaches zero:

$$r = \frac{m}{m+n}\left[1 + \frac{n^2\,x}{(m+n)(m+2\,n)} + \frac{(m^2 + n\,m + 2\,n^2)\,n^2\,x^2}{(m+n)^2\,(m+2\,n)\,(m+3\,n)} + \cdots\right] \qquad (19)$$

The detention ratio can be evaluated by substituting values of r into the equation

$$d = 1 - r\,x \qquad (20)$$

which follows from the definitions of d, r, and x. Equations (18) and (20) were used in computing the values of the detention ratio given in Table 2.

Time of Emptying.—The time required to empty a reservoir after the cessation of the uniform flood is found by letting the inflow equal zero in Equation

(8), which then becomes

$$\text{Zero} = B\, h^n\, dt + C\, m\, h^{m-1}\, dh \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

Transposing,

$$dt = -\frac{C\, m}{B}\, h^{m-1-n}\, dh \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

Integrating,

$$\int_T^t dt = -\frac{C\, m}{B} \int_H^h h^{m-1-n}\, dh \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

and,

$$t - T = \frac{1}{m-n}\, \frac{C\, m}{B}\, (H^{m-n} - h^{m-n}) \ldots\ldots\ldots\ldots (24)$$

except when $m - n =$ zero. From Equation (24),

$$t - T = \left(\frac{m}{m-n} \right) \frac{S}{O} \left[1 - \left(\frac{o}{O} \right)^{(m-n)/n} \right] \ldots\ldots\ldots (25)$$

The total time T' required to empty the basin after the uniform flood ceases is found by letting o equal zero in Equation (25) and therefore

$$T' = \left(\frac{m}{m-n} \right) \frac{S}{O} \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

DISCUSSION

Edward Soucek,[8] Jun. Am. Soc. C. E. (by letter).—A very convenient method for analysis of the action of flood control reservoirs with fixed outlets has been given by the authors. The selection of an equivalent uniform flood to represent a given non-rectangular inflow hydrograph may prove troublesome, despite the assurance of the authors that this operation is not difficult. A 40-hr uniform flood of equal total inflow is proved to be "equivalent" to the triangular flood shown in Fig. 1 for a particular outflow rate but no basis for the pre-selection of this duration is given. It seems that errors in estimates of the duration of the rectangular flood considerably exceeding those of 5% which are used in Table 1 are easily possible.

It is true that the paper deals primarily with the use of the rectangular flood—not its determination—so if it were not for the example given, this question would not be raised. The writer believes that the latitude which is possible in the establishment of the equivalent uniform flood is reduced if its total inflow is made equal to that of the hydrograph which is to be replaced only up to the time of maximum outflow. The inflow rate after this instant may remain constant indefinitely or may drop immediately to zero without effect upon either the maximum outflow rate or the maximum storage volume. It appears that the exclusion of an extraneous part of the inflow hydrograph makes the selection of a rectangular flood easier.

Considering the triangular hydrograph shown in Fig. 1, if a maximum outflow rate of 5,160 acre-ft per hr is contemplated, the maximum storage and outflow rate occur 32.8 hr after the beginning of the flood. The total inflow up to this time is 253,000 acre-ft. The remaining 40,000 acre-ft of inflow and the corresponding 15.2 hr of time are neglected. By plotting a mass curve of the given flood, the writer found that the least deviation of accumulated inflows between a rectangular hydrograph and the given inflow hydrograph resulted when the duration of the uniform flood was 31 hr and the intensity was 8,160 acre-ft per hr. By the application of Table 2, the maximum storage for this rectangular hydrograph was found to be slightly more than 116,000 acre-ft, a close check on the step-method solution. This elementary procedure may merely prove the contention of the authors that the selection of the equivalent uniform flood is easy but is intended to substantiate a preference regarding the total inflow of the uniform flood.

In Table 2, the small effect on the detention ratio of considerable variation in the depth-capacity exponent is very significant. Since the point of zero depth is fixed by the elevation of the outlets, a monomial exponential function cannot be fitted to the data as accurately as would be the case if the point of zero depth could be adjusted at will. This restriction is apparently unimportant except for high outflow ratios.

For orifice-type outlets, the depth-discharge exponent may be expected to remain sensibly constant at the value of 0.5. For weir-type outlets, however, the use of 1.5 for the depth-discharge exponent is somewhat in error, owing to the tendency for the constant B in Equation (5) to increase with the head. When only instantaneous discharges are being considered, the use of a constant exponent and a varying coefficient is justified by the resulting convenience of calculation. When a monomial exponential function with both the coefficient and exponent held constant is necessary, it might be better to modify the exponent. Assuming what appeared to be a reasonable variation of the coefficient B with the head, the writer plotted logarithmic rating curves for ogee spillways designed for 5-ft, 10-ft, and 20-ft heads and found that the exponent 1.6 represented the depth-discharge relation better than the exponent 1.5. The effect is smaller than might be anticipated from the variation in B which was used—a linear variation from 3.1 at zero head to 4.0 at the design head. As in the case of the depth-capacity exponent, the difference seems unimportant for low outflow ratios.

---

[8] Asst. Hydr. Engr., The Panama Canal, Diablo Heights, Canal Zone.

J. C. STEVENS,[9] M. AM. SOC. C. E. (by letter).—By substituting a uniform inflow for the variable inflow of the actual flood, the authors of this paper propose to simplify the problem of routing floods through reservoirs. This uniform inflow is selected by trial to have such a duration T that it will leave the same volume stored in the reservoir, and produce the same maximum rate of outflow as the actual flood event. It cannot determine any rate of outflow other than the maximum; yet other values of the outflow during the flood are frequently of considerable importance. Neither can it determine the time of occurrence of the actual outflow, which is generally of paramount importance.

The question arises as to whether the simplification resulting from this method of approach is not more apparent than real, and whether one might not solve the problem for the actual flood event in less time than that taken to find the "equivalent uniform flood," particularly if it is important to know the actual pattern of outflow.

For example, consider the basic relationship given in Equation (4). Since both the outflow o and the storage capacity s are single-valued functions of the depth above the outlet h, the latter may be eliminated from the equations and, therefore,[10] $s = f(o)$ from which

$$\frac{ds}{do} = f'(o) \qquad (27)$$

but $f'(o)$ represents the slope of the curve $f(o)$ at any given point. If a curve is plotted, or if a table of storage increments corresponding to outflow increments at various values of outflow is compiled, it is possible to obtain the function $f'(o)$ and, once for all, have the essential data to route any flood through that reservoir.

Let t = any finite time interval; i = average inflow during that interval; o_1 = outflow at the beginning of that interval; o_2 = outflow at the end of that interval; and θ = slope of the outflow-storage curve as previously explained for the average value of outflow during that interval; that is, $\frac{ds}{do} = f'(o) = \theta$.

---

[9] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[10] "Computing Reservoir Outflow and Height from Inflow and Capacity," by J. C. Stevens, *Engineering News-Record*, December 22, 1921, p. 1031, Equations 2 and 3.

It is well to fix the significance of θ well in mind. It merely means that for each cfs-increment (cfs = cubic feet per second) in outflow the storage increment is θ sfd (second-foot-day; increment may be either + or −).

TABLE 4.—VALUES OF θ FOR USE IN ROUTING A FLOOD

Description	LAKE LEVELS (FROM ASSUMED DATUM):										
	28	29	30	31	32	33	34	35	36	37	38
Outflow,* o	15.5	17.7	20.1	22.7	25.6	29.1	33.0	37.0	41.2	46.0	51.0
Difference, Δo		2.2	2.4	2.6	2.9	3.5	3.9	4.0	4.2	4.8	5.0
Storage,† S	185	210	235	262	289	316	343	370	398	426	454
Difference, ΔS		25	25	27	27	27	27	27	28	28	28
Ratio, $\frac{\Delta S}{\Delta o} = \theta$		11.3	10.4	10.4	9.3	7.7	6.9	6.7	6.7	5.8	5.6

* In thousands of cubic feet per second. † In thousands of second-foot-days.

TABLE 5.—COMPUTATIONS FOR ROUTING THE FLOOD OF NOVEMBER-DECEMBER, 1917, THROUGH COEUR D'ALENE LAKE, IDAHO

Date (1917)	HOUR*		Period, t, in days	In-flow,† i	$\frac{\Delta s}{\Delta o} = \theta$	$\frac{\theta}{t} + \frac{1}{2}$	$\frac{\theta}{t} - \frac{1}{2}$	$\frac{i}{\frac{\theta}{t} + \frac{1}{2}}$ (cfs)†	o_1 times $\frac{\text{Column (7)}}{\text{Column (6)}}$ (cfs)†	Out-flow,† o_2	Lake level, in feet
	From	To									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(November)											
25	0	24	1	....	....	....	....	....	....	21.2	30.4
26	0	24	1	13.0	10.0	10.5	9.5	1.2	19.2	20.4	30.1
27	0	24	1	20.0	10.0	10.5	9.5	1.9	18.4	20.3	30.1
28	0	12	0.5	31.0	10.0	20.5	19.5	1.5	19.3	20.8	....
28	12	24	0.5	41.0	10.0	20.5	19.5	2.0	19.8	21.8	30.7
29	0	6	0.25	47.0	9.5	38.5	37.5	1.2	21.2	22.4	....
29	6	12	0.25	51.0	9.5	38.5	37.5	1.3	21.8	23.1	31.2
29	12	18	0.25	56.0	9.5	38.5	37.5	1.4	22.5	23.9	....
29	18	24	0.25	60.5	9.0	36.5	35.5	1.7	23.2	24.9	31.7
30	0	6	0.25	68.0	9.0	36.5	35.5	1.9	24.2	26.1	....
30	6	12	0.25	72.5	8.5	34.5	33.5	2.1	25.3	27.4	32.5
30	12	18	0.25	76.0	8.5	34.5	33.5	2.3	26.4	28.7	....
30	18	24	0.25	78.0	8.0	32.5	31.5	2.4	27.8	30.2	33.3
(December)											
1	0	6	0.25	79.0	8.0	32.5	31.5	2.4	29.2	31.6	....
1	6	12	0.25	78.7	8.0	32.5	31.5	2.4	30.6	33.0	34.0
1	12	18	0.25	78.0	7.5	30.5	29.5	2.6	31.9	34.5	....
1	18	24	0.25	75.0	7.0	28.5	27.5	2.6	34.3	36.9	35.0
2	0	12	0.5	68.0	6.5	13.5	12.5	5.0	34.2	39.2	....
2	12	24	0.5	62.0	6.0	12.5	11.5	5.0	36.0	41.0	35.9
3	0	12	0.5	57.5	6.0	12.5	11.5	4.6	37.7	42.3	36.2
3	12	24	0.5	53.0	6.0	12.5	11.5	4.1	38.8	42.9	36.3
4	0	12	0.5	48.0	5.5	11.5	10.5	4.2	39.2	43.4	36.4
4	12	24	0.5	41.5	5.5	11.5	10.5	3.6	39.6	43.2	36.4
5	0	24	1	35.0	5.5	6.0	5.0	5.9	36.0	41.9	36.1
6	0	24	1	30.0	6.0	6.5	5.5	4.6	35.4	40.0	35.7
7	0	24	1	28.0	6.0	6.5	5.5	4.3	33.4	37.4	35.1
8	0	24	1	26.5	6.5	7.0	6.0	3.8	32.0	35.0	34.5
9	0	24	1	24.0	7.0	7.5	6.5	3.2	30.4	33.6	34.2

* Midnight = 0. † In thousands of cubic feet per second.

Substituting Equation (27) in Equation (4)

$$i\,dt = o\,dt + f'(o)\,do \quad \text{(28)}$$

Equation (28) may be translated into the finite form

$$o_2 - o_1 + \frac{(o_2 + o_1)t}{2\,\theta} = i\,\frac{t}{\theta} \quad \text{(29)}$$

Solving for the outflow at the end of the time interval t:

$$o_2 = \frac{i}{\frac{\theta}{t} + \frac{1}{2}} + o_1 \frac{\frac{\theta}{t} - \frac{1}{2}}{\frac{\theta}{t} + \frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

Starting with an initial outflow o_1, the outflow o_2 is found directly, which then becomes the o_1 for the next interval.

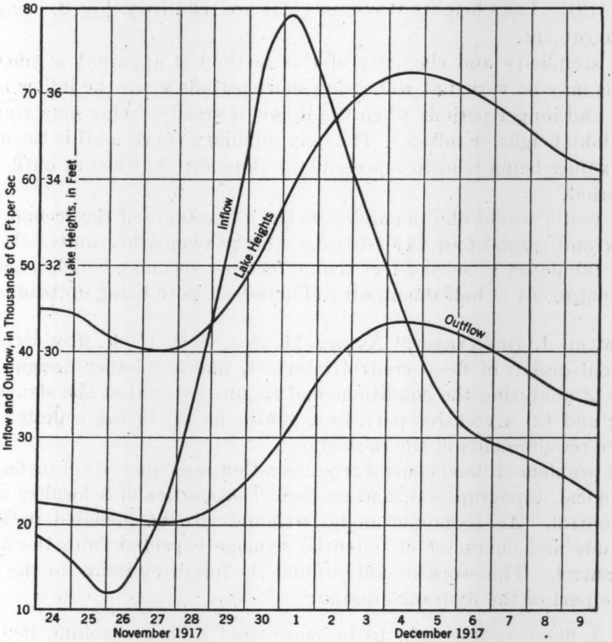


FIG. 3

The ease and rapidity with which this formula is applied in a step-by-step process, and the accuracy of the outflow pattern obtained thereby, rather obviate the necessity of trying to find an equivalent uniform inflow. This point so deserves emphasis that the writer believes an example of its application is warranted.

To illustrate, the flood of November-December, 1917, is routed through Coeur d'Alene Lake, Idaho. This lake forms the head of the Spokane River. At the outlet, a rating curve has been developed. The control is the 8 miles of river between the lake and Post Falls, Idaho. The storage capacity of the lake has been determined by surveys. The principal tributaries are the Coeur d'Alene and St. Joe rivers on which inflow was measured and a small percentage added for other tributaries. Table 4 gives the data necessary to route the flood. These data, with any given inflow, are all that is required to route any flood through this lake.

Table 5 shows the computations for routing the flood of November-December, 1917. The average inflow for each time interval in Table 5 was read from the inflow curve of Fig. 3. Values of θ were also read from Fig. 3, taken to the nearest 0.5 only. The outflow o_2 in Column (10), Table 5, is the sum of columns (8) and (9) and applies to the end of the periods indicated, the preceding value being the outflow o_1 at the beginning of the period as used in Column (9). Lake heights (Column (11)) are read from Fig. 3, using outflow as the argument.

The simplicity and elasticity of this method is apparent at once. Time intervals may be varied at will, using short periods when the inflow is varying rapidly and longer periods when the inflow is steady. One may start at any known lake height or inflow. The only auxiliary curve used is the θ-curve of Fig. 3, values being read to represent the slope for the average outflow during each period.

The writer would like to emphasize the advantages of the second-foot (cfs) for flow and second-foot-day (sfd) for storage over other units. Inflows are usually tabulated in second-feet (cubic feet per second), which avoids a conversion; also, sfd = half the number of acre-feet, both being units of volume.

EDWARD J. BEDNARSKI,[11] ASSOC. M. AM. SOC. C. E. (by letter).—The functional design of flood control reservoir, like any other design, is: (1) A process of analyzing the conditions and requirements that the structure must satisfy; and (2) a creative part, or a synthesis, producing a design that will fulfil the requirements of the problem.

The problem of flood control requires a thorough analysis of meteorological, hydrological, topographical, and geological properties of a locality in need of flood control. An economic analysis should also be included regarding the magnitude and character of potential damage expected from the absence of flood control. This work should produce the following items for the use in the creative part of the hydraulic design:

(1) A flood hydrograph, to be considered as a controlling factor of the design;

(2) A maximum rate of runoff within the watershed of the locality which could be tolerated under existing, or properly improved, conditions; and

(3) The topographic and economic data pertaining to every possible reservoir site in the form of elevation-capacity curves and the highest allowable water elevation.

The problem confronting the designer is the selection of a site for a reservoir which satisfies the foregoing stipulations and, if it does, the determination of the size and the type of the outflow facilities.

The authors present a method based on a mathematical treatment of a differential formula (Equation (4)). In order to do so they disposed of the equation

---

[11] Structural Engr., Los Angeles, Calif.

$$i = f(t) \qquad \text{(31)}$$

assuming that it can be substituted by

$$I = \frac{\int_0^T i\,dt}{T} \qquad \text{(32)}$$

an average rate of inflow. The outflow, however, is considered variable, starting from zero; then it reaches a maximum Q which is not supposed to be larger than I. Contrary to the notation of the paper the writer uses q and Q instead of o and O, respectively.

Unfortunately the writer has met with a number of cases in which $\frac{Q}{I}$ was greater than 1.

In the light of experience it appears quite futile to attempt a mathematical solution of the problem. A better approach seems to be a study of Equation (31) as it is represented by a hydrograph, in the form of Equation (32), or a mass curve that can be obtained from the hydrograph by numerical or graphical summation.

Quite contrary to the assumptions of the authors are the conditions of the problem to determine the least possible constant rate of outflow for a given amount of storage. Here q is constant but i is variable; and in Equation (5) $n = 0$ and $q = Q$, ($q = B\,h^n$).

This problem is solved easily, as follows: Consider the mass curve (a) shown in Fig. 4. Draw the same mass curve (upper part of it) downward from the first by a distance numerically equal to the maximum available storage (curve (b)). A straight line between these two curves tangent to each (see curve (c)) solves the problem. The tangent of the angle that this line makes with the horizontal axis, expressed in terms of acre-feet-per-hour yields the answer. The diagram is self-explanatory.

The foregoing solution is applicable when the outlet conduits are so located as to have sufficient head to pass the required discharge when storage in the reservoir is assumed to be equal to zero. However, if a certain initial storage is required to build up the necessary head, the second mass curve (b) should be drawn below the first by a distance equal to the difference in storage between the required initial and the maximum available storage. A straight line (c) tangent to these two curves will again produce the solution of the problem.

The problem of spillway length and the elevation of the spillway crest cannot be solved so simply. The outflow is not constant any more and all that is known about the outflow mass curve is that it is tangent to the mass curve or the horizontal axis at the time t_0, and parallel with the tangent to the mass curve at t_e, the time when the outflow is expected to be equal to the inflow. Between these two points the curve has no point of inflection and the angle of the tangent to this curve at any point is smaller than a tangent to the inflow mass curve at the same time. With some practice, it is possible to predict whether there is any possibility of designing an ordinary spillway with a given

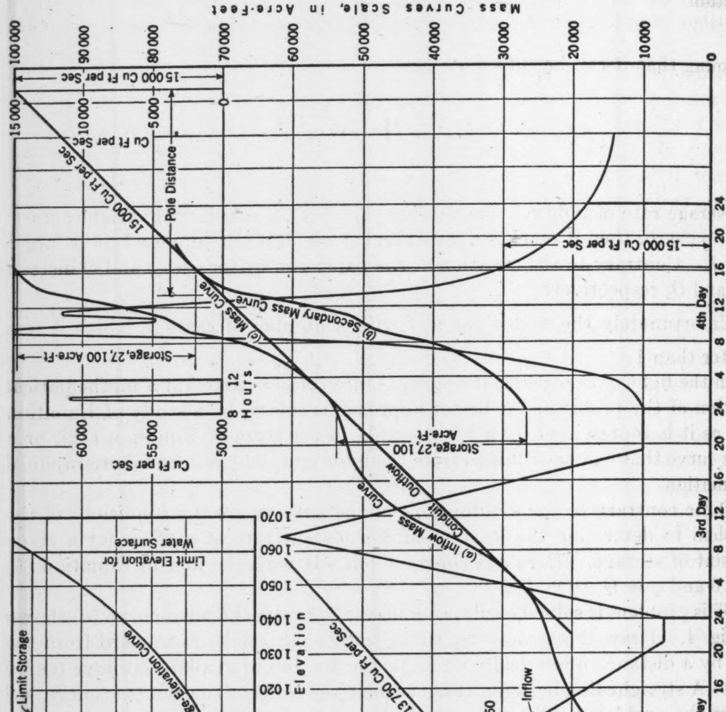


FIG. 4.—TO DETERMINE THE LEAST POSSIBLE CONSTANT RATE OF OUTFLOW FOR A GIVEN QUANTITY OF STORAGE

highest permissible elevation of the water surface and the elevation of the spillway crest.

Using Equations (1) and (5) it is possible to determine the necessary length of the spillway crest embodied in the constant B, as follows:

$$B = \left(\frac{C}{S}\right)^p Q \qquad (33)$$

in which (to simplify typography) $p = \frac{n}{m}$. This may or may not be the solution.

Whether the spillway length thus determined is satisfactory could be found by means of mass and storage-elevation curves by arithmetical computation or graphically.[12] Evidently there is no general solution. Each case must be solved by trial and error, which is not unusual and many, if not most, engineering problems are solved in this manner.

The limits of this discussion do not permit a detailed description as to the use of mass and storage-elevation curves in order to save some of the drudgery connected with the hydraulic design, and the writer regrets to state that an ingenious idea of using an average value of the inflow rate (as suggested by the authors) does not relieve the designer from a mass of tedious and painstaking labor, unavoidable in the process of the solution by trial and error.

Roland A. Kampmeier,[13] **Jun. Am. Soc. C. E.** (by letter).—This interesting and worth-while paper extends the principle of Mr. Woodward's "five-sixths rule"[3] to a wide variety of possible reservoir conditions.

The "five-sixths rule" states that the ratio, r, of average outflow to maximum outflow, will be approximately five-sixths under certain conditions—namely, when the reservoir has an orifice-type outlet; when the exponent, m, in the depth-capacity relationship is approximately 2.5; and when the ratio, x, of maximum outflow to assumed constant inflow is not too large. From Table 2 and Equation (20) it may be determined that their cases cover a range of values of r from 0.67 to 0.94 for the orifice-type outlet and from 0.41 to 0.85 for the weir-type outlet, the lowest values of r corresponding to the lowest values of m and of x. The effect of m is much more pronounced than the effect of x.

It may be noted that the use of weir-type outlets, to the exclusion of orifice-type outlets, would be unusual among reservoirs for flood control only (to the discussion of which the paper is limited), and that the paper indicates that the "five-sixths rule" is fairly satisfactory for orifice-type outlets when m is in the usual range of 2.0 to 3.5, and when x is less than about 0.6 (which would probably be the usual case in reservoirs designed exclusively for flood control).

Since the effect of the exponent m on the ratio r is so pronounced, it may be useful to emphasize that for a given reservoir the same value of m would not be likely to apply in comparing orifice-type and weir-type outlets. A weir-type outlet, or spillway, would normally be placed at a considerably higher level than orifice-type outlets, or sluices. If Equation (1) represented the depth-capacity relationship above the level of the sluices, the depth-capacity relationship above a higher spillway level could be represented approximately by a similar equation only if a smaller value of m were used. It would be interesting to know whether the values of m between 1.03 and 4.74, found by the authors for more than 250 reservoirs, were based on the depth-capacity relationships above sluice or spillway levels, as the case may be (as might be assumed from the applications in this paper), or above the bottom of the reservoirs, applying the equation as used by Mr. Sutherland.[7]

---

[12] See, for instance, "Hydraulik," by Ph. Forchheimer, Verlag und Druck von B. G. Teubner, Leipzig und Berlin; August 3, 1930, pp. 416, 417; or "Graphical Analysis of Spillway Capacity," by E. J. Bednarski, *Western Construction News*, February 25, 1932, p. 113 (see also erratum, *loc. cit.*, March 25, 1932, p. 168).

[13] Associate Prof., Hydr. Eng., Univ. of Tennessee, Knoxville, Tenn.

The authors mention (but do not discuss in detail) their method of selecting an equivalent uniform flood to replace the assumed design hydrograph, a method which differs from that used by Mr. Woodward,[6] who considered only that part of the hydrograph preceding the instant when the outflow equals the inflow and the maximum storage capacity has been reached. The authors' example (see Fig. 1) does not indicate whether or not a shorter equivalent flood of greater intensity would have been used if less storage capacity had been assumed available. The impression is left with the writer that the authors would use the same equivalent flood for all assumed conditions of outlet size and storage capacity. Suppose that the maximum outflow in the case indicated in Fig. 1 were 8,050 acre-ft per hr; using the equivalent flood shown would mean assuming an apparently unsatisfactory value of x of 1.1.

The writer questions whether the authors have used properly Mr. Woodward's term, the "protection ratio," which they state to be the same as their "outflow ratio," x, the ratio of the maximum outflow to the uniform inflow of the equivalent flood (see "Introduction: Outflow Ratio"). The writer understood the original use of the term "protection ratio" to mean the ratio of the maximum flood inflow to the maximum outflow,[14] whereas the term "outflow ratio" was originally used[15] with a different meaning than that assigned to it by the authors.

Appreciation of the data presented is obscured by the fact that the term x, in Table 2 and in Equation (3), does not reflect the actual reduction in maximum discharge. It might have been desirable to use values of the "protection ratio," as the writer understands its meaning, as headings of the columns in Table 2, instead of the values of the authors' "outflow ratio." The writer realizes that the use of the protection ratio would require assumptions as to the shape of the hydrograph. The isosceles-triangle form of hydrograph would seem to be a reasonable assumption somewhat more similar to the true hydrograph in most instances than the equivalent rectangular flood.

EDGAR E. FOSTER,[16] ASSOC. M. AM. SOC. C. E. (by letter).—The authors have undertaken to furnish a much-needed means of analyzing problems arising in regard to the functional design of reservoirs. There are three general problems to solve in functional design—namely,

---

[14] "Hydraulics of the Miami Conservancy District," by Sherman M. Woodward, Technical Reports of the Miami Conservancy District, Part VII, Chapter VII, Fig. 73, p. 197.

[15] *Loc. cit.*, p. 200.

[16] Associate Engr., U. S. Engr. Office, Omaha, Nebr.

(*a*) Design flood; that is, the magnitude and hydrograph of a flood upon which the operation of the reservoir is to be based;
(*b*) Outflow attainable; and
(*c*) Capacity needed to obtain the desired measure of control.

The reservoir design flood is built upon the quantity of rainfall in a suitable storm, by means of distribution factors; or it is derived from a selected flood (perhaps the maximum) experienced on the watershed. In any case, it is based upon conditions not touched upon by the authors' paper and hence need not be discussed further herein. The reduction of outflow (Problem (*b*)) and storage required (Problem (*c*)) are the questions with which the authors deal. These two factors are reciprocal in that when one is given the other may be obtained.

The problem of the reduction of outflow may be decided from the channel-carrying capacity below the reservoir, or some other critical point downstream. Under these circumstances there remains the problem of determining how much capacity is needed to reduce the design flood to the desired maximum outflow. On the first glance it seems that the authors had provided a method to yield the answer. Unfortunately, however, it is not possible to find the "equivalent rectangular" flood as readily as is indicated in the paper. The only sure method of determining the equivalent rectangular flood is that of routing the design flood through the reservoir; and, when that is done, the required capacity is known, and it becomes unnecessary to apply the method, since all elements are then known.

The writer made an attempt to determine the capacity of a reservoir under investigation by the authors' method. The pertinent data for the reservoir are as follows: Reservoir design flood, peak discharge, 32,400 cu ft per sec; volume, of flood, 311,000 acre-ft; total duration, of flood, 12 days; outlet discharge fixed at 10,000 cu ft per sec; total capacity of reservoir, 545,000 acre-ft; and m, derived from the capacity curve, 3.15. The problem is to fix the capacity to be set aside for flood control in order to reduce the design flood from 32,400 to 10,000 cu ft per sec.

On the basis of a 12-day period, the equivalent rectangular flood is 13,000 cu ft per sec, from which $x = 0.77$. Then from Table 2, by interpolation, $d = 0.316$. The capacity needed, therefore, should be $0.316 \times 311{,}000$ or 98,000 acre-ft. However, by routing this flood through the reservoir, it was found that approximately 150,000 acre-ft were needed. An attempt was made also to utilize the time (6.2 days) during which the design flood was above the maximum outlet discharge. This duration of the rectangular flood required a storage capacity of 202,000 acre-ft.

In the foregoing example (which is typical of reservoir design floods), there is no period other than the full duration of the design flood that can be taken logically as the length of the equivalent rectangular flood. That period is too long, however, and its use does not provide sufficient storage. The time may be diminished continuously with increasing requirements for storage, but there is no way of determining the correct volume without routing through the reservoir. There seems to be no logical point on the hydrograph, therefore, at which the length of the equivalent rectangular flood can be fixed.

In certain cases it may be desirable to reverse the procedure of the foregoing example. The volume of storage available may be fixed, instead of the permissible outlet discharge, and the problem is then to determine the maximum reduction. This problem also requires the calculation of the quantity x; hence, the difficulty of the equivalent rectangular flood is met again.

It is highly desirable to start from the known factors of a problem and to work progressively toward a solution without resorting to trial methods; but it is apparent that the difficulty of the equivalent design flood prevents such a course in analyzing flood storage capacity with the authors' method.

Sherman M. Woodward,[17] M. Am. Soc. C. E. (by letter).—An enlightening, interesting, and useful extension of the methods used about 1915 in the design of the reservoirs of the Miami Conservancy District at Dayton, Ohio, is presented in this paper. At that time the so-called "five-sixths rule" was finally developed by the engineering staff of the District after a prolonged period of studying and puzzling over the various complex relationships involved in the determination of the required capacity of the several reservoirs and the dimensions of their respective outlets. It was recognized that no great virtue or precision should be attached to the exact numerical value, five-sixths, and that this coefficient should be expected to change with circumstances and should be checked in each important case by further detailed calculations.

The practical value of the "five-sixths rule" is that for the purpose of approximate estimates and comparisons in preliminary design, before dimensions are finally fixed, it gives results in a few minutes which would otherwise be obtained only after days of laborious computation. The rule was evolved as a result of a determined effort to discover a general, broad-visioned, practical method of approach to a complicated problem, an approach which should give due consideration to the main, essential factors and at the same time minimize the unimportant, non-essential factors.

The authors have performed a valuable service in throwing new light upon some of the relationships. The writer believes that the possibilities of the subject have not yet been exhausted and hopes that one effect of the paper will be to reveal new directions for further profitable exploration.

One item of particular importance in the paper is the direct comparison that can now be made between the relative advantages of the use of automatic versus manual-controlled reservoir outlets.

M. Kindinger,[18] Assoc. M. Am. Soc. C. E. (by letter).—Table 2 can be condensed to a single set of data by tabulating d against $\frac{n}{m}$ instead of m. Its use will then be extended to a wide range of outlet types besides those of the orifice and weir.

---

[17] Chf. Water Control Planning Engr., TVA, Knoxville, Tenn.

[18] Res. Engr., Umgeni Water Scheme, Durban, Union of South Africa.

In Equation (5), substituting the value of h determined from Equation (1), and considering o and s to be maxima,

$$O = \frac{B\,s^u}{C^u} \qquad \text{(34)}$$

in which (to simplify typography), $u = \frac{n}{m}$. An inspection of Table 2 will reveal that for the same values of u and x the detention ratio is identical in both sets, as demonstrated for a few values of u in Table 6.

TABLE 6.—VALUES OF m AND d COMPARED WITH TABLE 2

$u = \frac{n}{m}$	m		d			
	$n = 0.5$	$n = 1.5$	$x = 0.2$	$x = 0.4$	$x = 0.6$	$x = 0.8$
0	...	...	0.800	0.600	0.400	0.200
0.1	5	15	0.818	0.635	0.450	0.258
0.2	2.5	7.5	0.832	0.662	0.488	0.303
0.5	1	3	0.864	0.722	0.570	0.399
1.0	0.5	1.5	0.896	0.789	0.656	0.498
1.5	0.33	1.0	0.916	0.822	0.710	0.567
2.0	0.25	0.75	0.939	0.849	0.749	0.619

The formulas given in the Appendix can be converted by substituting u for $\frac{n}{m}$. For example, Equation (19) then becomes:

$$r = \frac{1}{1+u}\left[1 + \frac{u^2\,x}{(1+u)\,(1+2\,u)} + \frac{(1+u+2\,u^2)\,u^2\,x^2}{(1+u)^2\,(1+2\,u)\,(1+3\,u)} + \cdots\right] \qquad \text{(35)}$$

The statement that the equivalent uniform flood can be selected readily on an arbitrary basis is open to question. The close agreement shown in Table 1 is not a convincing demonstration because it is always possible to compute the exact equivalent uniform flood which, for any flood volume one wishes to choose, will give the same storage and peak outflow as the original: $x = \frac{O}{I}$; and (if V is the flood volume) $I = \frac{V}{T}$. Therefore,

$$T = \frac{x\,V}{O} \qquad \text{(36)}$$

In the authors' example $d = \frac{117{,}100}{292{,}800} = 0.40$; but interpolating from Table 2, $x = 0.697$ and, therefore, $T = \frac{0.697}{5{,}160} \times 292{,}800 = 39.4$ hr. The problem most common in practice, however, is the reverse one. The engineer must guess, or otherwise determine, T first, and then use its value to compute the value of d required to reduce the peak outflow to a particular magnitude.

It is clear that with outflow peaks near to, or greater than, 7,320 acre-ft per hr the 40-hr equivalent will give unreliable results. Evidently, therefore, T must be so chosen as to give an equivalent flood intensity I greater than the peak outflow; and hence, T will vary from case to case.

Furthermore, the peak outflow is not influenced in any way by that part of the flood curve that lies beyond the point of equalization (the point where outflow equals flood inflow and where, also, the outflow is at its peak). Hence, no apparent purpose is served by the condition that the equivalent uniform flood shall have a volume equal to that of the entire flood. Rather, it would appear to be advantageous if the volume of the equivalent were made equal to that of the inflow or flood at the point of equalization.

In studying flood routing problems the writer has found that the mass curve is far more serviceable than the intensity curve.

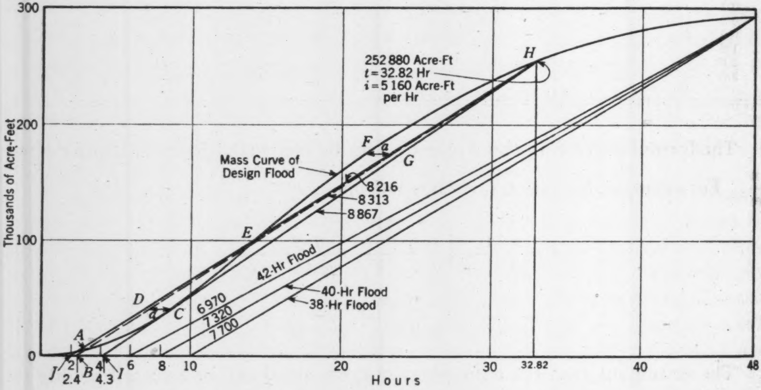


FIG. 5.—MASS CURVES OF VARIOUS SUBSTITUTE UNIFORM FLOODS COMPARED WITH MASS CURVE OF DESIGN FLOOD

In Fig. 5, which is a plot of the authors' example, H is the point of equalization. The storage volume will depend upon the shape and dimensions of the part of the curve $O\ C\ E\ F\ H$. Obviously, any equivalent uniform flood should deviate from this curve as little as possible. Starting from the known value of S and computing T, omitting the volume of inflow after the point of equalization, the period of the exact equivalent is found to be 30.8 hr. It is shown in Fig. 5 by the full line $J\ H$.

The following rule enables a substitute uniform flood curve to be drawn which will fit the original closely and yet give a value of the storage S which errs on the side of safety:

> Draw the chord $B\ H$, shown dotted, in such a manner that both the length of intercept, $H\ E$, and the horizontal divergence, $F\ G$, of the convex

part are respectively equal to, or larger than, the corresponding intercept, *E A*, and divergence, *D C*, of the concave part.

It can be proved, or verified by means of any of the step methods, that:

(*a*) A part of a mass curve concave upward gives a lesser flood-reducing effect than the chord joining its extremities and that the reverse is the case for a convex curve (it will be realized that the greater the increment in storage required to deal with the increment of the flood volume represented by the part of the flood curve in question the smaller will be the flood-reducing effect); and

(*b*) This difference in flood-reducing effect between chord and curve increases with length of chord, amount of divergence, and rate of outflow.

Even if the concave part, *A C E* (except for its reversed position), were identical in shape and dimension with the convex part, *E F H*, the larger outflow prevailing during the latter (convex) period would result in a greater difference in flood-reducing effect between chord and curve than would obtain during the earlier concave period. In addition, the flatter slope of the part, *O A*, in the beginning is superior in flood reduction to the chord *B A*. Hence, taking the substituted chord as a whole, it will have a lower flood-reducing effect than the original. In other words, it requires a higher storage *S* to reduce the peak to *O*. Another substitute that is easier to draw but has the disadvantage of giving a still more conservative result is the chord *I H* (Fig. 5), which is tangent to the concave part. For the purpose of comparison, computations have been made for $\frac{n}{m} = w = 2.0$, which gives a value of *u* at the opposite end of the field to that covered by the authors' $\frac{n}{m} = \frac{0.5}{2.5} = 0.2$. Storage *S* was determined graphically for the original flood and, for each of the uniform flood substitutes, by computation utilizing Tables 2 and 6. The results are summarized in Table 7.

TABLE 7.—COMPARISON OF MAXIMUM STORAGE FOR VARIOUS SUBSTITUTE UNIFORM FLOODS

Line	Type (see Fig. 5) (1)	*I* (2)	*O* (3)	*V* (4)	*x* (5)	*u* = 0.2		*u* = 2.0	
						d (6)	*S* (7)	*d* (8)	*S* (9)
1	Original	12,100	5,160	252,880	0.426	0.463	117,100	0.708	179,000
2	*J H*	8,216	5,160	252,880	0.628	0.463	117,100	0.733	185,300
3	*B H*	8,313	5,160	252,880	0.621	0.469	118,600	0.737	186,400
4	*I H*	8,867	5,160	252,880	0.582	0.504	127,440	0.759	191,900
5	44-hr	6,688	5,160	292,800	0.772	0.329	96,400	0.639	187,100
6	40-hr	7.320	5,160	292,800	0.705	0.392	114,800	0.688	201,500
7	36-hr	8,150	5,160	292,800	0.635	0.456	133,500	0.729	213,500

Arbitrary selection of the equivalent flood can lead to great uncertainty. Thus, in line 5, Column (7), Table 7, a 10% variation in *T* results in a negative error of nearly 19% in *S*, whereas line 7, Column (9), shows that a similar variation in the opposite direction involves a result 19% too high. The substitute uniform flood drawn according to the foregoing rule shows positive errors of less than 5%.

P. **Wilhelm Werner**,[19] Assoc. M. Am. Soc. C. E. (by letter).—This interesting study is valuable for the investigation of an important type of storage problem. It must be borne in mind, however, that it is strictly applicable only under certain conditions. For instance, if the reservoir outlet consists of a tunnel of some length, the discharge rating curve cannot be expressed, even approximately, by a monomial exponential function such as Equation (5). In the case of a pure orifice outlet the tailwater may sometimes influence the discharge so as to cause a considerable deviation of the rating curve from the monomial exponential function.

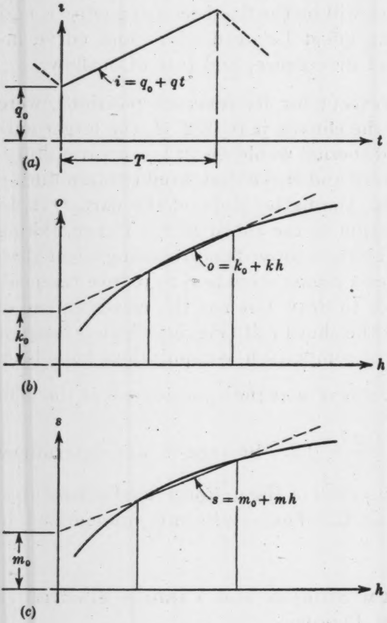


FIG. 6

As stated by the authors, flood routing studies are commonly executed by the trial-and-error step method or the integration method. In many cases, however, a combination of the two methods is useful. One such combination (which might be called the "straight-lines integration method") may be of interest in this connection. In its application this method is of the functional type, the integration, however, being executed in steps over a number of straight lines covering, as nearly as practicable, the given characteristic curves. Using the same nomenclature as the author, as far as possible, the procedure may be demonstrated as follows.

The inflow may be written

$$i = q_o + q\,t \qquad (37)$$

in which q_o is the intercept on the i-axis and q the slope of the straight line representing the inflow intensity during the interval T (see Fig. 6(*a*)). The outflow may be written

$$o = k_o + k\,h \qquad (38)$$

in which k_o is the o-intercept and k the slope of the straight line, substituting the outflow intensity curve in the part under consideration (see Fig. 6(*b*)).

---

[19] Civ. Engr., A. B. Vattenbyggnadsbyran, Stockholm, Sweden.

The depth-capacity may be written

$$s = m_o + m\,h \qquad (39)$$

in which m_o is the s-intercept and m the slope of the straight line to be substituted for the considered part of the depth-capacity curve (see Fig. 6(c)).

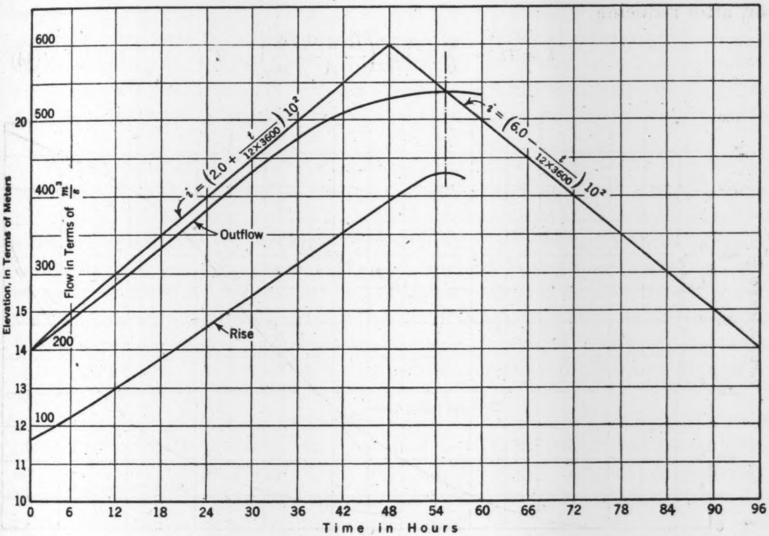


FIG. 7.—INFLOW HYDROGRAPH

Substituting Equations (37), (38), and (39) in the general differential equation (Equation (4)), and reducing:

$$m\frac{dh}{dt} + k\,h = q\,t - (k_o - q_o) \qquad (40)$$

Of the constants, q_o, m, and k are always positive, whereas q and k_o may be either positive or negative. If $t = o$ and $h = h_o$ are corresponding values, after integrating Equation (40), and reducing:

$$h - h_o = -\,(B + h_o)(1 - e^{-\alpha}) + A\,t \qquad (41)$$

in which (to simplify typography) $\alpha = \frac{k}{m}\,t$; and

$$A = \frac{q}{k} \qquad (42a)$$

and

$$B = \frac{1}{k}(k_o - q_o + m\,A) \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42b)$$

If a theoretical maximum occurs in some interval, it is determined thus:

$$\frac{dh}{dt} = -\,(B + h_o)\,\frac{k}{m}\,e^{-\alpha} + A = 0 \;\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

or, after reducing,

$$t = t_m = \frac{m}{k} \times \log\left(\frac{B + h_o}{A}\,\frac{k}{m}\right) < T \;\ldots\ldots\ldots\ldots\ldots (44)$$

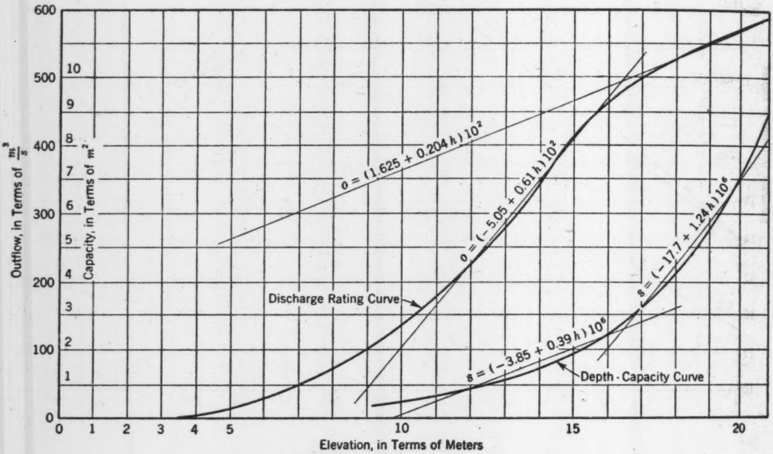


FIG. 8.—DISCHARGE RATING CURVE AND DEPTH-CAPACITY CURVE

The application of the foregoing deductions may be illustrated by the following example: The rise of the water level in a reservoir is to be determined for the flood given by the inflow hydrograph in Fig. 7. The water discharges through a by-pass tunnel, the discharge rating curve of which is shown in Fig. 8. The depth-capacity curve is also given in Fig. 8.

TABLE 8.—COMPUTATION OF RISE IN A RESERVOIR

h_o	q_o	q	k_o	k	m	A	B	$B + h_o$	$h - h_o$	$\frac{t}{3,600}$	h
11.60	200	$\frac{1}{432}$	−505.0	61.0	390,000	$\frac{1}{26,300}$	−11.3	+ 0.3	4.80	37.3	16.40
16.40	511	$\frac{1}{432}$	+162.5	20.4	1,240,000	$\frac{1}{8,820}$	−10.15	+ 6.25	1.45	10.7	17.85
17.85	600	$-\frac{1}{432}$	+162.5	20.4	1,240,000	$-\frac{1}{8,820}$	−28.35	−10.5	0.70	7.1*	18.55

* This value computed by means of Equation (44).

The computations, in three main steps, are summarized in Table 8, and the result is shown graphically in Fig. 7. A check by means of the usual trial-and-error step method shows a close agreement, the discrepancy in the total rise amounting to about 2%.

In comparison with the functional method developed by the authors, the foregoing suggestions possess the advantage of being applicable with sufficient accuracy to any storage problem of the nature in question, including all types of reservoirs and outlets. It also avoids the uncertainty in estimating the equivalent uniform flood. Compared with the trial-and-error step method it effects a saving in time and labor, inasmuch as the tedious cut-and-try operations have been eliminated. For office use a graphical evaluation of Equation (41), in some form or other, is often very helpful.

WALDO E. SMITH,[20] M. AM. SOC. C. E. (by letter).—An interesting contribution to the study of the functional design of reservoirs for flood control is found in this paper. Often the time-saving features of this method of establishing tentative basic values for a reservoir for preliminary estimates and as a guide in design have been overlooked, and it is hoped that this reminder of earlier studies will serve to call attention to this method of approach. However, the construction of gate controls on outlet works is the current tendency for flood control dams, and with this type of outlet, functional analysis loses much of its significance. It may still serve, none-the-less, on preliminary studies to give the value of the minimum allowable size of outlet.

The authors have failed to recognize the most common justification for the use of gate or mechanically controlled outlets. They assume that such outlets are for the purpose of maintaining 100% "efficiency," meaning, presumably, that outflow is uniform from the beginning to the end of the flood. The writer believes that this is incorrect usage of the term "efficiency" unless the only area one is attempting to protect from floods is that immediately below the dam. Ordinarily, one must relieve a prospective flood situation at some point or points considerably downstream, and must vary the outflow to accommodate the uncontrolled inflow into the channel at points between the control works and the points being protected. It is to provide for this needed flexibility that gate controls are ordinarily used. To quote from the Official Plan for the Muskingum Watershed Conservancy District:[21]

> "*Reasons for Selection of Gate Controlled Reservoirs.*—Careful study of the tributary arrangement of the Muskingum River and the time of travel of past flood crests on the tributaries show that the use of retarding basins will not provide either as efficient or as economical flood control as will gate controlled reservoirs. The principal objection to the retarding basin scheme as applied to the tributaries of the Muskingum River is based on the design principle of the system, that is, that the maximum outflow from any reservoir during the design flood must not, when combined with the outflow from other reservoirs and/or flows from uncontrolled areas, cause flows in excess of those for which provision has been made at any point downstream. * * * With gate control, the outflow from the reser-

---

[20] New Philadelphia, Ohio.

[21] "Official Plan for the Muskingum Watershed Conservancy District," Vol. I, pp. 23 and 24.

voirs can be reduced during high discharge from uncontrolled areas and increased after the rate of runoff below the reservoirs has receded."

In discussing the depth-capacity exponent, m, the authors state that depths above the outlet location are used, whether the outlet is an orifice or a weir. It has been the writer's procedure to base this value, and the resulting exponential equation, on the height above approximate zero reservoir capacity, as explained by Professor Woodward for the case of variable depth.[22] In all instances that the writer can recall, it has been possible to make nearly all of the points fall very close to a straight line on logarithmic paper. The resulting equation in the form:

$$s = C\,h^m \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

has the advantage of defining the storage accurately throughout the depth of the reservoir. For general purposes, the method suggested by the authors has two undesirable results: First, it has the tendency to change the natural valley characteristics as indicated by m and C; and second, it frequently results in a plotting of points that does not approach a straight line, with the result that no satisfactory equation can be written.

Pleasant Hill Reservoir in Ohio is a case in point. There a weir-orifice outlet will maintain the water surface at an elevation of about 1,020 ft, nearly 50 ft above the channel bottom. Fig. 9 shows the depth-capacity curves based on the value of approximate natural zero capacity and based on Elevation 1,020. The uppermost plotting was the first attempt, based on an estimated zero capacity at Elevation 968. A second attempt, not shown, was made before arriving at the second line from the top. This straight line passes very close to all points, with zero capacity at Elevation 973. The formula for this curve is expressed by Equation (1), with $C = 0.29$ and $m = 2.80$. It serves capacity needs throughout the entire depth of the reservoir as well as of any part in which one might be especially interested, and m and C serve as correct indications of morphological conditions within the valley, as mentioned by the authors.

Using only that portion of the reservoir capacity above Elevation 1,020 and measuring depth from that elevation. a third plotting of points is shown. These do not follow a straight line. If one uses the equation of the tangent at the upper end of the curve defined by the points, $C = 180$ and $m = 1.58$, values of capacity determined in the vicinity of the highest elevation will be quite satisfactory; but nearer to the weir-orifice level, the values will serve no purpose. If one derives the equation of the secant, drawn through the uppermost and lowermost points shown—$C = 590$ and $m = 1.29$—in the vicinities of these two points values of total storage may be approximated, but intermediate values and differentials will have little significance. As demonstrated subsequently, the value of m for the entire reservoir gives a better value of maximum outflow, in this case, than either of the other two values determined. It should be noted that the characteristics of the valley apparently change

---

[22] "Hydraulics of the Miami Conservancy District," by Sherman M. Woodward, Technical Reports of the Miami Conservancy District, Part VII, pp. 118–121.

markedly, *m* changing from 2.80 (hill type) to 1.58 (flood-plain foothill type) to 1.29 (lake type). Possibly the latter value has some use in expressing the nature of the valley between two given horizontal planes; but by taking other than the natural zero of storage capacity, the measure of natural valley characteristics expressed in *m* and *C* is destroyed. That this acts likewise for other reservoirs is shown by reference to Table 9 for Senecaville Reservoir in Ohio.

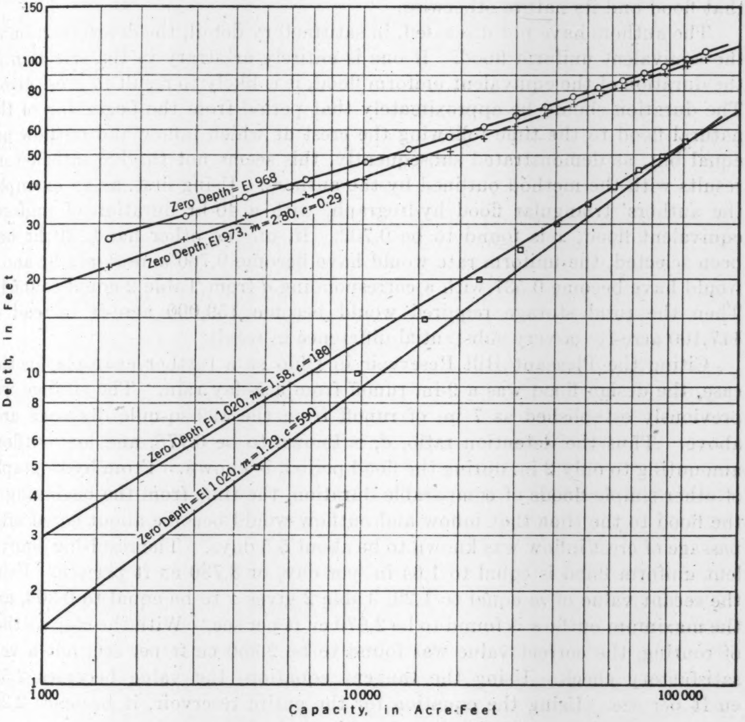


FIG. 9.—DEPTH CAPACITY CURVES; PLEASANT HILL RESERVOIR, MUSKINGUM VALLEY, OHIO

TABLE 9.—CONSTANTS IN EQUATION (1); SENECAVILLE RESERVOIR, OHIO

Elevation of zero depth	Description	CONSTANTS:	
		C	*m*
805	Natural	30	2.20
812	Sill of outlet gates	188	1.79
831	Crest of conservation weir	2,430	1.24

In Table 2, it should be noted that d varies relatively slightly within the entire range of m, as compared with the variation within the range of x. Thus factors entering into the variation of x are likely to be of greater importance that those entering into m. One of these is the value of the inflow rate of the uniform equivalent flood, which, in turn, is dependent on the duration of that flood and its nature otherwise.

The authors have not discussed, in satisfactory detail, the determination of the equivalent uniform flood. If one is entirely arbitrary in the selection of the duration of the equivalent uniform flood, it is likely to result in gross error. The duration should be approximately that period from the beginning of the natural flood to the time following the crest at which inflow and outflow are equal but, as demonstrated subsequently, this seems not to give satisfactory results with the method outlined by the authors. Using first, as an example, the authors' triangular flood hydrograph, with a 40-hr duration of uniform equivalent flood, x is found to be 0.702. If, on the other hand, 30 hr had been selected, the uniform rate would have become 9,760 acre-ft per hr and x would have become 0.537 with a corresponding d from Table 2 equal to 0.543. Then the total storage required would become 159,000 acre-ft instead of 117,100 acre-ft—a very substantial difference in result.

Citing the Pleasant Hill Reservoir in Ohio as a further example, in this case, the design flood was a 9-in. runoff from a 5-day rain. The storage was previously established as 7 in. of runoff from the 199-sq-mile drainage area above. Thus the detention ratio, d, is known to be 0.778, and low outflow, amounting to only 2 in. during the flood period, is known. From hydrographs of other simple floods of comparable duration, the time from the beginning of the flood to the time that inflow and outflow would become about equal after passage of crest inflow was known to be about 5.5 days. The resulting equivalent uniform flood is equal to 1.64 in. per day, or 8,780 cu ft per sec. Using the secant value of m equal to 1.29, Table 2 gives x to be equal to 0.303, and the maximum outflow is found to be 2,670 cu ft per sec. With the step method of routing, the correct value was found to be 2,350 cu ft per sec, not a very satisfactory check. Using the tangent equation, the value becomes 2,530 cu ft per sec. Using the equation for the entire reservoir, it becomes 2,280 cu ft per sec, the best of the three by this method.

Assuming the five-sixths rule to hold, however, one can establish the approximate maximum rate of outflow directly, knowing its volume to be 2 in. from the watershed above, over a period of 5.5 days. The average, which is five sixths of the maximum, is 1,950 cu ft per sec. The maximum, in turn, is found to be 2,340 cu ft per sec, a very good check. It should be noted that this does not require the use of any uncertain values of m or x.

The authors suggest the term "outflow ratio" for the ratio between maximum outflow rate and the inflow rate for the equivalent uniform flood, which is also designated as x. This is an excellent suggestion, but the writer would suggest that it be used also with respect to the ratio between the maximum outflow rate and the maximum inflow rate of the natural flood from which the equivalent uniform flood is derived. They are in error, however, in stating that this term designates the same value as Professor Woodward's "protection ratio." At no point has the writer been able to find that Professor Woodward gave any definition for x other than $\frac{O}{I}$.[23] "Protection ratio," as used by him without definition,[24] is a satisfactory expression, and is the peak inflow divided by the peak outflow. It is a ratio always equal to, or greater than, 1, and the reciprocal of outflow ratio if the authors will accept the writer's modification of their term "outflow ratio."

J. O. JONES,[25] ASSOC. M. AM. SOC. C. E. (by letter).—This paper is a distinct contribution to the literature on flood routing. It recognizes the fact that the storage capacity required is dependent, chiefly, upon the total volume of the flood, and only in a minor degree upon the flood distribution.

When the authors' method is applied to the problem of estimating the storage capacity required for a given maximum flood, with a given maximum outflow, a fairly large range of values of volume is obtained, depending upon the assumed duration of the equivalent uniform flood. This may be seen by applying Equation (2) and Table 2 to a concrete example.

The two floods shown in Fig. 10 have the same volume (140,000 acre-ft) but are markedly different in distribution. The exponent, m, in Equation (1) is 2.18. Hence, the values shown in Table 10 are obtained. The variation in the results may be inconsequential, in view of the more or less speculative nature of the design flood. The percentage variation in the required storage, however, is considerable. The step method as applied herein by the writer indicates that an equivalent uniform flood of 28,000 acre-ft per day with a duration of $T = 5$ days will agree exactly with design flood No. 1 but will underestimate, slightly, the required capacity for flood No. 2. Hence, it appears that a criterion is needed, which will indicate, at least approximately, the value of T in Equation (2).

TABLE 10.—DESIGN FLOOD, MARAIS DES CYGNES

Item	Description	DURATION T, IN DAYS			
		4.0	4.8	5.6	6.4
1	Equivalent uniform flood, in thousands of acre-feet	35.0	29.2	25.0	21.9
2	Outflow ratio, x	0.5	0.6	0.7	0.8
3	Detention ratio, d	0.585	0.498	0.409	0.315
4	Storage (S) required, in thousands of acre-feet	81.9	69.7	57.2	44.1
5	Maximum depth, h, in feet	43.5	40.4	36.8	32.8

It may be that a criterion can be obtained graphically. As a matter of fact, the volume of storage required is represented by the area between the flood hydrograph and the outflow hydrograph. Although the outflow curve cannot be obtained, except by the step method or its equivalent, a fair approximation may be had by considering the outflow curve to be a straight line drawn from zero to the point of maximum outflow (line *E E*, Fig. 10). The area between the straight line and the flood hydrograph overestimates the required capacity in every case. In the present example (Fig. 10) the excess is 19% for flood No. 1 and 14% for flood No. 2. An improvement may be had by noting that the outflow hydrograph is quite flat at the maximum point. By sketching a curve, which is required to pass through the points *E*, and which retains the characteristic shape of the outflow hydrograph, one may obtain a result quite close to that obtained by approved step methods.

---

[23] "Hydraulics of the Miami Conservancy District," by Sherman M. Woodward, Technical Report of the Miami Conservancy District, Part VII, p. 186.

[24] *Loc. cit.*, p. 197.

[25] Prof. of Hydraulics, Univ. of Kansas, Lawrence, Kans.

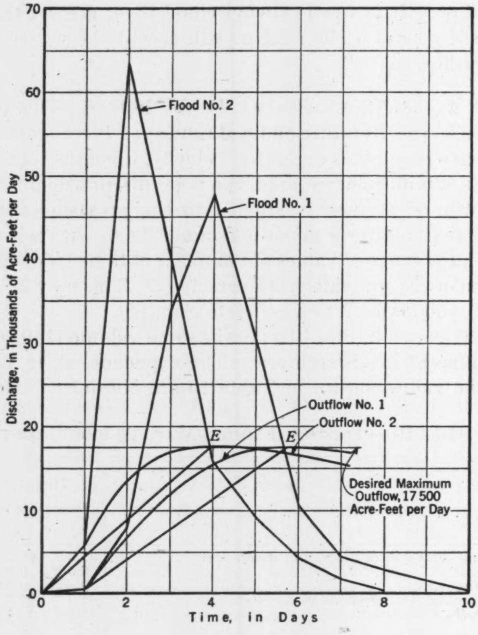


FIG. 10

W. C. HAMMATT,[26] M. AM. SOC. C. E. (by letter).—There seems to be a tendency in recent years to go into intricate and exact mathematical analyses of problems which are fundamentally of a practical nature. This tendency causes one to lose sight of, or at least to have one's vision obscured toward, the fundamental principles involved.

In the matter of the design of storage for protection against flood damage, certain factors are taken into account, any one of which may be the ruling one, depending on the facts in the particular case. These factors are as follows: (1) The expected flood flow of the stream, in intensity, duration, and frequency;

---

[26] Civ. and Cons. Engr., Los Angeles, Calif.

(2) possible storage capacity of the proposed reservoir; (3) the capacity of the stream bed below the reservoir outlet to handle flows without flood damage; and (4) the economics of the situation—that is, the comparative study of possible property damage from flood peaks, and the cost of protective storage. In spite of the methods developed in recent years, factor (1) is still an uncertain one, based on historical analysis of the watershed itself and of others with similar characteristics; and its uncertainty has been demonstrated frequently by the occurrence of floods far in excess of the predictions developed by customary methods of analysis. Factors (2) and (3) are capable of practically exact computation. Factor (4) is capable of exact analysis under existing conditions, but is subject to the uncertainty of development and values at a future date.

Beginning with factor (4), the economic study, and calculating the maximum cost that can be assumed for flood protection, considering that unlimited storage capacity can be developed, the computer works back through factors (3), a constant, and (1), an estimate, to obtain factor (2), the desired solution. It is a fundamental principle that the solution of a problem, wherein one or more of the factors are inexact, cannot itself be exact. It follows that, since factor (1) is an estimate only, the engineer can only design to meet this assumed condition, and that simplified methods of computation may be used with as much likelihood of success as more exact mathematical ones.

If the possible storage capacity is limited to less than that necessary for complete control, the problem resolves itself into a mere determination of the rectangular outflow to utilize the entire storage capacity available, always assuming that the benefits derived by this regulation are in excess of its cost. Since the total quantity, rate, and duration of the inflow are indeterminate, a graphical solution, preferably by an assumed mass curve based on historical data, is sufficiently accurate. There is nothing new in this method of treatment, the only recent development being the improved methods of flood prediction, and the writer believes that further refinement is not consistent with the uncertainty of the basic data.

C. J. Posey,[27] Assoc. M. Am. Soc. C. E., and Fu-Te I,[28] Esq. (by letter).—Some of the discussers contend that the writers' method of functional design is more complicated than the step-routing methods which give, as a result, the complete outflow hydrograph. For the problem at hand, in which only the maximum outflow and maximum storage need be found, this is decidedly not true. Given data on a reservoir site and an inflow hydrograph, a number of corresponding values of maximum outflow and maximum storage can be computed by the writers' method in less than a tenth of the time needed to determine the same number of values by the fastest step-routing methods available. The routing methods not only involve more computation, but are indirect as applied to this problem.

The accuracy of the writers' method hinges upon the validity of three steps in the solution: (1) Representation of the capacity-depth data for the reservoir site by the exponent m; (2) representation of the inflow flood hydrograph by an equivalent uniform flood hydrograph; and (3), use of the writer's values of the detention ratio in Equation (2). Each step has been questioned or criticized by one or more of the discussers. The points raised pertinent to these steps will be discussed in reverse order.

---

[27] Associate Engr., Iowa Inst. of Hydr. Research; Associate Prof. of Hydraulics and Structural Eng., State Univ. of Iowa, Iowa City, Iowa.

[28] Prof., Coll. of Eng., National Southwest United Univ., Kuenming, Yunnan, China; formerly Graduate Student, Dept. of Mechanics and Hydraulics, State Univ. of Iowa, Iowa City, Iowa.

The correctness of the tabulated values of the detention ratio, for use in Equation (2), can be verified by checking the mathematical derivation given in the Appendix. This is admittedly a somewhat lengthy procedure, but it is not at all necessary for an understanding of the method, nor even for verifying the values in Table 2. They can be verified by step-routing computations, a more lengthy and indirect method, but one that would be preferred by many. A partial check on the correctness of the tabulated values is afforded by the agreement shown, in the appropriate area of the table, with Mr. Woodward's "five-sixths rule." Professor Kampmeier points out that the agreement holds over a considerable range of values of m and x. None of the discussers who express doubt as to the correctness of the writers' method presents any evidence that the values in Table 2 are faulty.

In discussing the determination of the equivalent uniform flood, it should be remembered that the design flood itself is very uncertain. Even after the expenditure of considerable time and energy on the hydrological studies, the size and shape of the design flood cannot be guaranteed to within 20% or 30%. Precise quantitative prediction of the magnitude and frequency of floods is an art that the hydrologist has not yet mastered. Mr. Hammatt states of the expected flood flow that "its uncertainty has been demonstrated frequently by the occurrence of floods far in excess of the predictions developed by customary methods of analysis."

On most large engineering projects, the work must proceed simultaneously on several fronts. Functional design studies for the reservoir cannot be postponed until the final results of the hydrological studies are available. In the early stages, when the design flood has not been definitely determined, the writers' method of arbitrary selection of the equivalent uniform flood should give results that are sufficiently accurate for preliminary studies.

Use of the word "arbitrary" was probably ill advised. The writers assumed that a certain amount of judgment would be used in selecting the uniform flood. By use of the rules suggested by Messrs. Soucek and Kindinger for selecting the uniform flood, however, the amount of skill necessary to obtain good results is greatly reduced—so much that the method of the paper (with the following revisions) can be used for the final design, giving greater accuracy, in most cases, than the most optimistic hydrologist would claim for his design flood.

For preliminary studies, a mass curve of the inflow is plotted. A straight line approximating the mass curve for the entire flood is then drawn. The uniform flood corresponding to this straight line may be used instead of the "arbitrarily" selected uniform flood of the paper.

For final design, the mass curve of the inflow hydrograph is plotted, but the straight line approximating the mass curve is made to fit not the entire flood, but only that portion up to the time that the inflow rate equals the desired maximum outflow rate. This gives a different equivalent uniform flood for each different outflow, but the gain in accuracy is considerable, especially for the higher values of outflow. Subsequent computations with the several uniform floods are the same as for the "arbitrarily" selected flood of the paper. In contrast to the method outlined for preliminary design, this method ignores the tail of the inflow hydrograph, a procedure to increase the accuracy originally recommended by Mr. Woodward.

Messrs. Soucek and Kindinger both give rules for drawing the straight line best approximating a portion of the mass curve of inflow. The writers find little difference in the results. This phase of the problem seems worthy of further study.

Isolated cases may be encountered to which the writers' method of functional design will not apply, even if use is made of the refinements suggested by Messrs. Soucek and Kindinger. These cases can be recognized in advance. An uncertain value of m, an unusual inflow hydrograph, or a high value of x may warn of the necessity for a check by the step method. In any case, the outlet size selected as a result of the functional design studies should be checked by step-routing methods.

Professor Kampmeier and Mr. Smith both note that the value of m for a given site varies as the elevation of the zero depth changes. This does not indicate that any lack of definiteness attends the use of this exponent, but rather that the storage-depth relation for a given site changes as the zero depth is changed to correspond with the elevation of the outlet. A satisfactory straight line can be drawn through the upper portions of any of the rows of plotted points in Fig. 9. Slight differences in m are inconsequential, as Mr. Smith himself points out.

Professor Kampmeier inquires as to the zero depth used in determining the values of m for the 250 reservoir sites. Practically all of these were based on depths measured from the bottom of the reservoir. Professor Kampmeier is correct in stating that the value of m based upon a higher spillway level would be smaller. Use of the writers' values of the detention ratio for reservoirs with weir-type outlets presupposes that the reservoir is full to spillway level at the beginning of the flood, and that the outflow at the beginning of the flood is insignificant.

Professor Jones suggests an alternate method for the functional design that represents the ultimate in simplicity. The outflow hydrograph is sketched by eye upon the plot of the inflow hydrograph. Measuring the area between the two by planimeter gives the maximum storage directly. The method would be indirect in determining the maximum outflow corresponding to a given maximum storage. Compared with the writers' method, Professor Jones' method allows for greater flexibility in taking into account the actual shape of the inflow hydrograph, but it would require considerable skill to sketch the correct curves for different values of m and n—that is, for different characteristics of the reservoir site and outlet. With sufficient practice, always followed by a check by the step method, one might possibly develop the skill necessary to obtain accurate results by Professor Jones' method.

The writers are glad that the discussion brought out the special routing methods given by Messrs. Stevens, Bednarski, and Werner. Although they do not apply advantageously to the problem of the paper, they possess advantages in certain situations, and are not, to the writers' knowledge, available elsewhere. The method described by Mr. Stevens, especially, should be studied by every student of flood-routing methods.

Professor Kampmeier and Mr. Smith call attention to the writers' error in referring to Mr. Woodward's "protection ratio." Mr. Woodward defined the protection ratio to be "the proportion which the maximum flood inflow bears to the maximum outflow through the conduits," so that it is not identical with the writers' outflow ratio, and is not misleading, as the writers stated. Both terms are necessary and their use should be continued. The construction of tables similar to Table 2, embodying the protection ratio, as suggested by Professor Kampmeier, would be a difficult but possibly profitable task.

Mr. Smith questions the writers' assumption of 100% efficiency for the operation of gate-controlled outlets, apparently believing that, through modern data-gathering and forecasting methods, efficiency greater than the "100%" assumed by the writers could be reached. Although the writers will admit the theoretical possibility of greater efficiency in operation, they believe that a study of the records of the operation of reservoirs with gate-controlled outlets would show that the assumption of 100% efficiency is too high.

The writers are especially grateful to Mr. Kindinger for his important contribution in pointing out that the detention-ratio tables may be condensed and generalized by introducing $u = \frac{n}{m}$. Although it seems advisable to retain the present form of the tables for use in most cases, in which n is either $\frac{1}{2}$ or $\frac{3}{2}$, Mr. Kindinger's suggestion opens up wide fields of usefulness for the writers' method. The ogee spillways mentioned by Mr. Soucek, problems in the filling and emptying of tanks, the design of highway culverts with pondage, and many other cases, where the discharge relationship varies as some power of the head different from $\frac{1}{2}$ or $\frac{3}{2}$, can be solved advantageously by use of the writers' method and tables, with the modification suggested by Mr. Kindinger. The details of these various applications need to be developed carefully.

The writers wish to thank all of the discussers for their interest. If this paper results in an improvement in the functional design of flood control reservoirs, much will have to be credited to the discussers' efforts.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2094

RECONSTRUCTION OF THE WALPOLE-BELLOWS FALLS ARCH BRIDGE

By H. E. Langley,[1] Esq., and Edward J. Ducey,[2] Esq.

With Discussion by Messrs. J. B. French, Lewis D. Rights, Conde B. McCullough, H. M. Nelson, and H. E. Langley and Edward J. Ducey.

Synopsis

The three-hinged, open-web, steel, arched highway bridge between the Towns of Walpole, N. H., and Bellows Falls, Vt., was one of the several crossings over the Connecticut River which were seriously damaged during the spring flood of 1936. For example, Fig. 1(*a*) shows the New Hampshire end of the span, Fig. 1(*b*), the Vermont end; and Fig. 1(*c*) is a detail view of the damage to the bottom chord. It is apparent that the lower chord sections and the adjacent web members of the up-stream truss below high-water level were so badly crippled that it was necessary to close the bridge to all vehicular traffic; and the responsible authorities were compelled to decide between major repairs to the structure and its replacement with a new one.

This bridge had been in service only since 1908, and the steelwork in general had been carefully maintained so that a considerable expenditure for repairing the damage was warranted. The consulting engineers who designed the structure[3] were called upon to submit a design for repairs. They formulated a general plan involving supports near each end, with four along the interior of the arch, all supported on timber pile groups. Detailed plans were prepared, and bids were taken for the work on this basis. The successful bid ($120 000) included all operating hazards and was somewhat in excess of the anticipated costs. The danger element involved in this undertaking was relatively high because a further accident to the main span might not only result in the loss of the structure but would probably involve difficulties at the Bellows Falls power plant, situated immediately below the bridge, which is a main link in the electric power system that serves this region. In studying the situation the Bridge Division of the New Hampshire State Highway Department decided that it was more feasible for the State to assume the risks incident to this extra-

---

Note.—Published in April, 1939, *Proceedings*.

[1] Designing Engr., Bridge Div., New Hampshire Highway Dept., Concord, N. H.

[2] Designing Engr., Am. Bridge Co., Pittsburgh, Pa.

[3] *Transactions*, Am. Soc. C. E., December, 1908, Vol. LXI, p. 253.

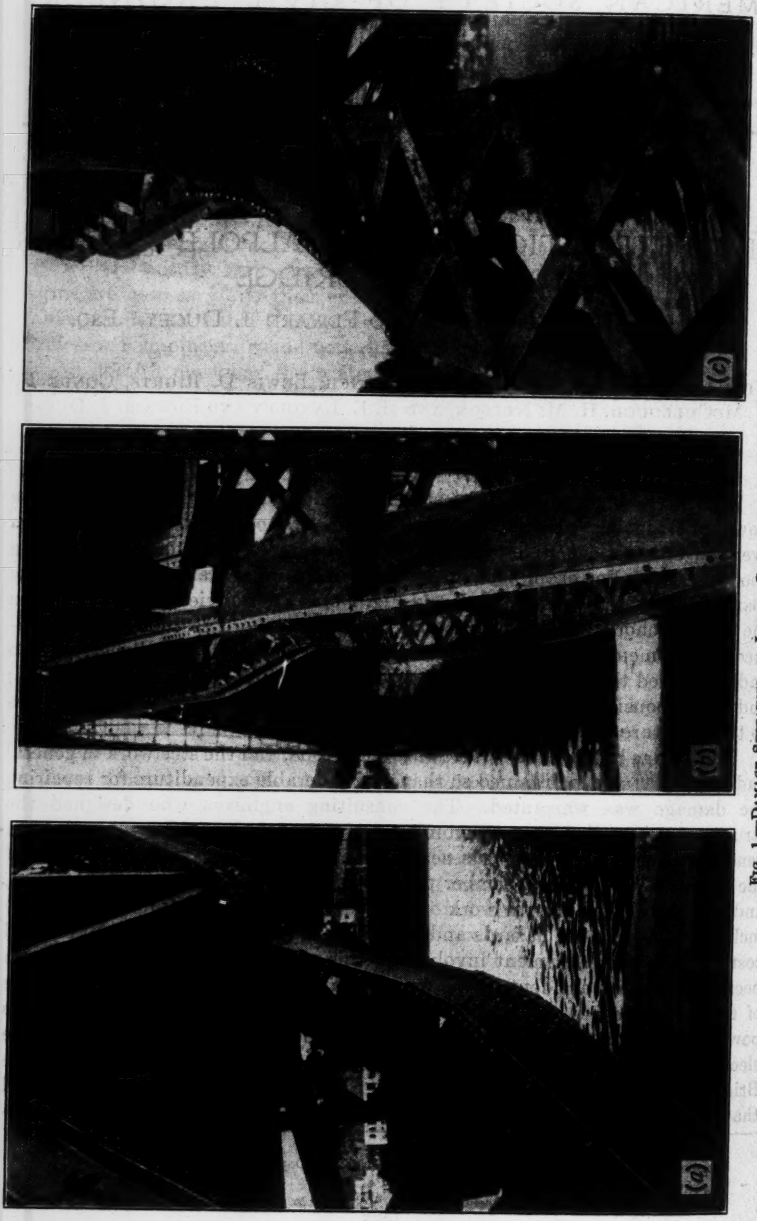


FIG. 1.—DAMAGE SUSTAINED BY LOWER CHORD AND ADJACENT MEMBERS

hazardous procedure than the contractor. To do this it was necessary to let the fabrication and erection contract on a "cost-plus" basis to a competent structural steel contractor who would co-operate with the State forces in driving the piles and placing the concrete.

In consultation with the contractor's engineers a study was made to eliminate some of the more hazardous features of the undertaking by keeping the river as clear of falsework as the condition of the structure would admit. Accordingly, a plan was devised incorporating the use of temporary cable ties (see Fig. 3) to take the horizontal thrust of the arch, thus converting the span temporarily into a tied three-hinged arch to be carried on falsework towers near the original supports. With the falsework at the ends of the span it could not only be protected, if necessary, from floating ice by means of ice breakers or fenders extending out from the river banks, but it could be constructed from the shores as well.

The use of temporary cable ties and other features introduced in this work seem somewhat unusual, and it is believed that a description of the procedure followed will be of interest to engineers and contractors generally.

INTRODUCTION

The original structure consisted of two ribs in each truss—a top chord and a bottom chord—each being a parabola of the same general dimensions, spaced one above the other 14 ft apart vertically. The top chord consisted of two 15-in. channels with a cover plate 24 in. wide and laced at the bottom flanges. The bottom chord consisted of two similar channels laced top and bottom, with additional side plates making the gross area approximately equal to that of the top chord. The diagonal members were made up of two 5 in. by $3\frac{1}{2}$ in. by $\frac{3}{8}$ in. angles, generally, single laced. This construction is shown quite clearly in Fig. 2. Since the two ribs are parabolas of the same dimensions, the thrust line under uniform load would be midway between them because the spring-line pins and the pin at the crown were placed midway between the two. The ribs in each truss were drawn together in the last two panels at either end of the span to meet at the spring-line pins so that the thrust from the two main arch trusses was carried by the skew-back shoes to the concrete abutments which were supported principally on wooden piles. The floor system, which was hung from the trusses with loop-rod hangers, had a system of lateral or wind bracing, which had for the chords of the system two members in the planes of the two main trusses. These extended throughout the length of the span, and were connected to the bottom chords of the main trusses by slotted connections to provide expansion due to temperature changes. Although these members were comparatively light, being made up of two 6-in. by 4-in. by $\frac{1}{2}$-in. angles, together with the diagonal bracing in the top-chord lateral system, they played an important part in saving the structure from collapse when the up-stream truss was damaged.

A very interesting and complete description of the construction of this bridge, together with discussions covering the erection of this and similar structures, was presented by Lewis D. Rights, M. Am. Soc. C. E., on April, 1, 1908.[3]

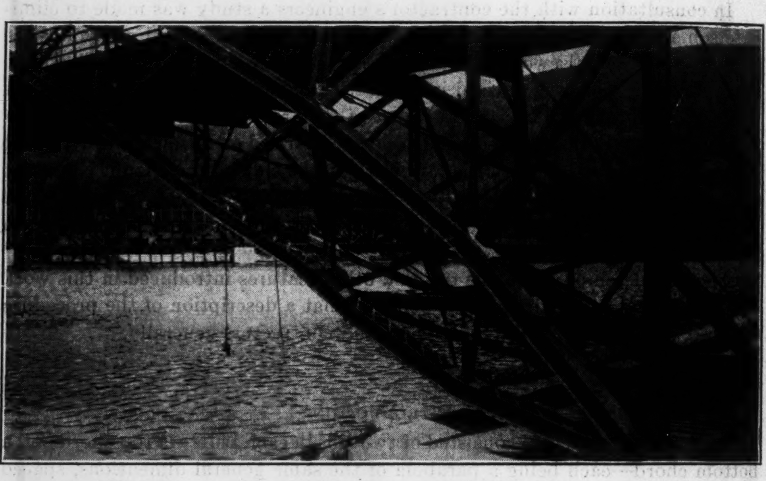


FIG. 2.—VIEW OF DAMAGED MEMBERS AT THE VERMONT END OF THE SPAN

THE PROBLEM

At the time the structure was built the low-water level was several feet below the present pool elevation, but improvements in the power development facilities on the Connecticut River have raised the normal pool to its present level (Elevation 115.5) a foot or so above the original spring-line pins. The depth of water now varies from about 19 ft at the abutments to about 50 ft at mid-channel. Borings taken at the abutments and at the temporary tower sites indicated about 20 ft of mud, sand, and boulders which gradually consolidated into a series of alternate strata of hard sand and gravel at a depth of about 35 ft, underlaid with solid rock at about 60 ft to 70 ft below the riverbed. The high water during the flood of 1936, which caused the damage to the main span, reached Elevation 125.5 (see Fig. 3) and covered the roadway at the New Hampshire end of the span.

The direct damage was caused by the ice jam which collected back of the main arch and extended entirely across the river. The impact and the sustained pressure from this ice so disabled the up-stream truss that the lower chord ceased to function as originally designed, and the greater part of its share of the load was distributed to the top chord. The diagonals $U_2 L_3$ and $U_3 L_4$ and the vertical $U_3 L_3$ (Fig. 3) assumed part of the thrust from the lower chord and distributed it to the top chord, while the ties which formed the chords of the lateral bracing in the floor took up the greater part of the remainder. Although the crippled chord sections undoubtedly carried some stress, it was

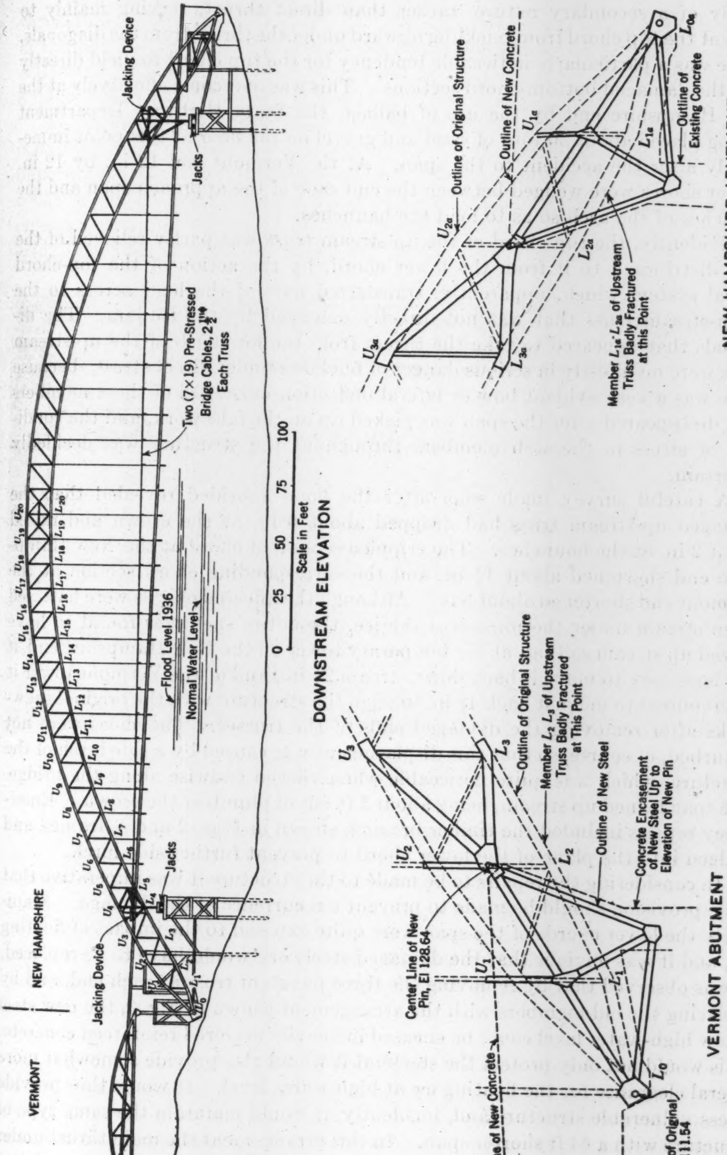

VERMONT
NEW HAMPSHIRE
Jacking Device
Jacks
Jacking Device
Jacks
Two (7×19) Pre-Stressed Bridge Cables, 2¼" Each Truss
Flood Level 1936
Normal Water Level
0 25 50 75 100
Scale in Feet
DOWNSTREAM ELEVATION
U_0 U_1 U_2 U_3 U_4 U_5 U_6 U_7 U_8 U_9 U_{10} U_{11} U_{12} U_{13} U_{14} U_{15} U_{16} U_{17} U_{18} U_{19} U_{20}
L_0 L_1 L_2 L_3 L_4 L_5 L_6 L_7 L_8 L_9 L_{10} L_{11} L_{12} L_{13} L_{14} L_{15} L_{16} L_{17} L_{18} L_{19} L_{20}
Center Line of Original Pin, El 111.54
Outline of New Concrete
Center Line of New Pin, El 128.64
Concrete Encasement of New Steel Up to Elevation of New Pin
Outline of New Steel
Member L_2L_3 of Upstream Truss Badly Fractured at This Point
Outline of Original Structure
VERMONT ABUTMENT
Outline of Original Structure
Outline of New Concrete
Member L_{1a}-L_{2a} of Upstream Truss Badly Fractured at this Point
Outline of Existing Concrete
NEW HAMPSHIRE ABUTMENT
L_0 L_1 L_2 L_3 U_1 U_2 U_3
L_{0a} L_{1a} L_{2a} L_{3a} U_{1a} U_{2a} U_{3a}

FIG. 3.—ADOPTED PLAN OF FALSEWORK FOR ALTERATIONS AND REPAIRS

largely of a secondary nature, rather than direct thrust, serving mainly to prevent the top chord from buckling upward under the thrust from the diagonals. There was a particularly noticeable tendency for the top chord to yield directly over the damaged bottom-chord sections. This was overcome effectively at the New Hampshire end by the use of ballast, the State Highway Department having placed about 50 tons of sand and gravel on the floor at this point immediately after the accident to the span. At the Vermont end 12 in. by 12 in. timber shores were wedged between the end shoe of the approach span and the haunches of the arch so as to load the haunches.

Evidently, the top chord of the up-stream truss was partly relieved of the load distributed to it from the lower chord, by the action of the top-chord lateral system which, apparently, transferred part of the load across to the down-stream truss that was not directly damaged by the ice jam. The diagonals that appeared to take the thrust from the lower rib of the up-stream truss were manifestly in serious danger of buckling under direct stress, because there was a very evident bow or lateral deflection in several of these members that disappeared after the span was picked up on the falsework, and the condition of stress in the web members throughout the structure was decidedly uncertain.

A careful survey made soon after the flood subsided revealed that the damaged up-stream truss had dropped about 6 in. at the crown and raised about 2 in. at the haunches. The crippled section of chord at the New Hampshire end shortened about 1¾ in. and the corresponding chord section at the Vermont end shortened about 3 in. Although the affected chords were battered down stream under the impact of the ice, the entire span was found to have moved up stream so that, at the temporary tower on the New Hampshire end, it was necessary to move it back down stream 2½ in., and at the Vermont tower it was required to move it back 4¾ in. to align the structure with the original skewbacks after removing the damaged ends of the trusses. The shoes were not disturbed, of course, so that this displacement was caused by a side lurch of the structure, which was quite noticeable when viewed endwise along the bridge. The span leaned up stream, being about 1 ft out of plumb at the crown. Emergency repairs included the timber bracing, shown in Figs. 2 and 4, framed and wedged into the plane of the lower chord to prevent further side lurch.

In considering the repairs to be made to the structure it was imperative that some provision should be made to prevent a recurrence of the damage. Manifestly the lower chords of the span were quite exposed to the impact of floating ice, and it was obvious that the damaged steelwork would have to be replaced. It was observed that by removing the three panels of truss at each end, and by replacing the old members with the arrangement shown in Fig. 3, the new steel below high-water level could be encased in heavily armored reinforced concrete. This would not only protect the steel but it would also provide somewhat more lateral clearance for the floating ice at high-water level. It would thus provide a less vulnerable structure and, incidently, it would maintain the same type of structure with a 54 ft shorter span. In this arrangement the main thrust under uniform load is carried, by a single member, from the new pin set on the original thrust line to the original skew-back. The effect of partial loading is taken care of through the cantilever action of the new steel below the pin, which steel is supported by a group of four steel piles under each truss, at a new shoe set approximately 20 ft forward of the skew-back shoes.

The obvious uncertainty of stress throughout the structure, and the general instability of the span, injected a hazard into the problem which became an important factor in the procedure to be followed in making the repairs. Little could be done toward making preliminary repairs, other than the emergency repairs already described, to secure the structure sufficiently to carry erection loads. It was impossible, therefore, to construct temporary falsework by working from the span, and it was impractical to reach the site with floating equipment because of the location and the proximity of the power company's dam situated in the river immediately below the span. Therefore, in addition to keeping the river as free from falsework as possible, the plan of procedure which seemed most feasible was the one involving the least number of temporary towers and so devised that they could be erected by working out from the shores on temporary bents, keeping entirely free of the damaged span so as to disturb it as little as possible.

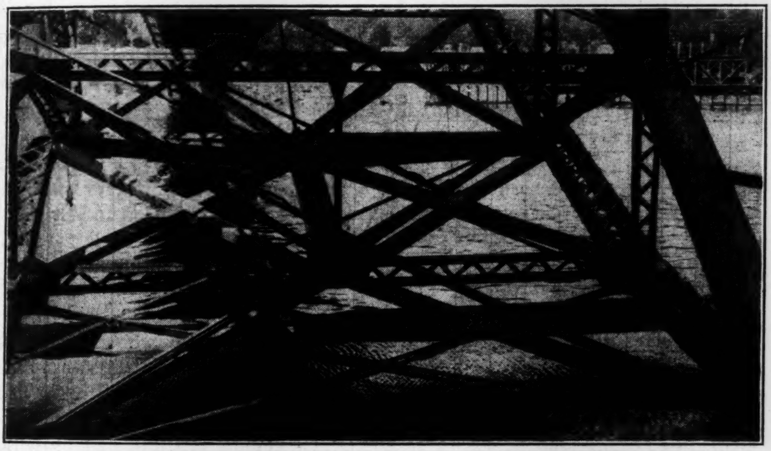


FIG. 4.—EMERGENCY BRACING IN THE PLANE OF THE BOTTOM CHORD; VIEW FACING ALONG THE BRIDGE TOWARD NEW HAMPSHIRE

THE PLAN ADOPTED

The plan decided upon was to pick up the span on temporary wooden pile towers placed as close to the shores as the steel to be removed would permit, and to take the thrust by means of temporary ties, and then remove the damaged ends and replace the old steel with new material after the structure had been leveled up and re-aligned on the temporary towers.

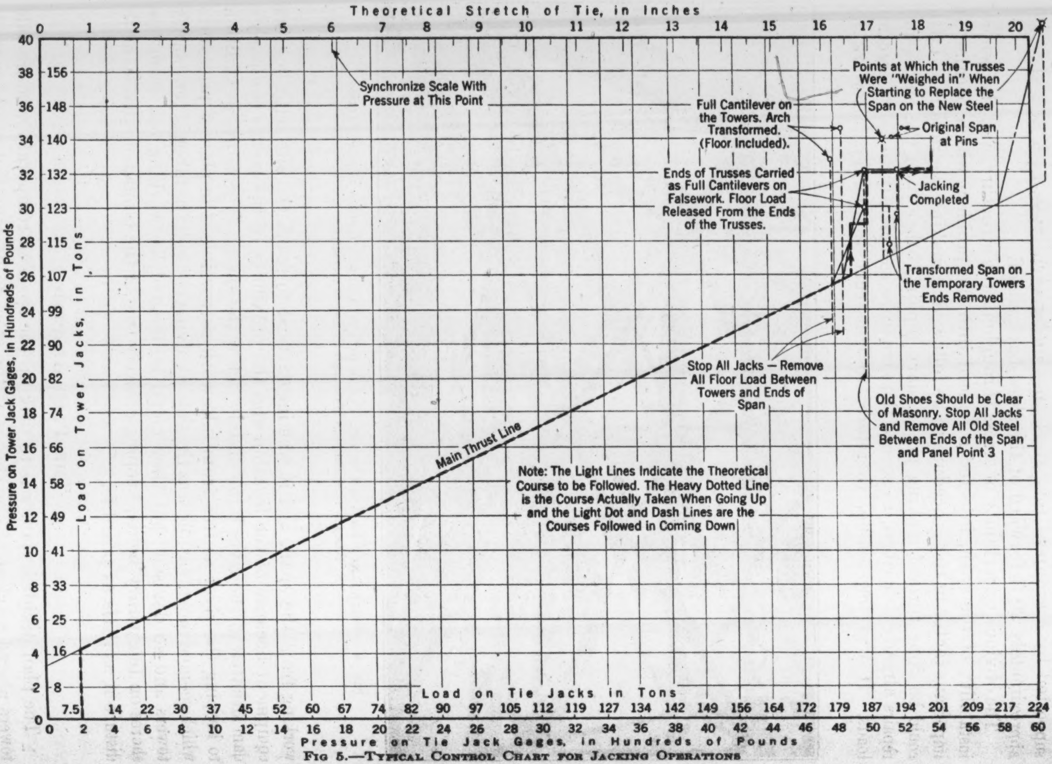


FIG 5.—TYPICAL CONTROL CHART FOR JACKING OPERATIONS

Handling the structure in this manner required that the ties be designed to take a thrust of 195 tons for each truss, with an allowance for an over-load estimated at 20%, and that each tower be designed to carry a vertical load of 143 tons at each truss. The total weight of the span was 572 tons before removing the old steel. These loads varied somewhat during the process of reconstruction as the removal of old steel and the addition of new members, removal of the floor system, etc., altered the weight of the span with a corresponding variation of the stress in the ties. This is shown in Fig. 5, with an over-load of 18% due to snow and ice, and the shifting of the old pavement (which was torn up and piled along the up-stream truss during the progress of the work).

The difficulty of taking these reactions on the trusses at points where such severe concentrations were not provided for in the original design presented somewhat of a problem, especially because provision had to be made for jacking devices to take up the stretch in the cable ties, and any temporary members used would have to be inserted without disturbing the structure, or without impairing the strength of the members carrying stress. This was accomplished by inserting a system of temporary members in the trusses at each tower, as shown in Fig. 6. A tower member, *A O C D*, was used to carry the vertical reactions and to intersect with the thrust member *O B*, on the thrust line, at Point *O*. The tie stress was then introduced at Point *O* by placing the jacking devices back of this point so that the line of action of the tie would pass through Point *O*.

During the jacking operations the stresses were introduced into the tie and the tower member by picking the proper increment of stress in each so as to keep the resultant stress coincident with Line *O B* as nearly as possible and tangent to the thrust line at this point. The stress from the thrust member was taken up at Point *B* by two distributing members, *B A* and *B C*, and delivered to the upper and lower chords at Points *A* and *C*. As shown in Fig. 3 a brace was used to steady the base of the tower member at the New Hampshire end but this was so arranged that it served no other purpose.

All of the new steel was completely fabricated in the shop. The temporary members were designed so that they could be erected by parts without impairing the strength of the members in the structure to which they were to be connected, except for the removal of a pair of lacing bars in each plane of lacing on the top and bottom chords. Field connections were drilled in the old members, either to a template or to match the new steel blocked in place on the falsework. Main connections were fully riveted and secondary connections and braces were bolted.

For each tie a pair of $2\frac{1}{2}$-in. galvanized bridge cables (6 by 19 on a 7 by 7 core, pre-stretched) were used to straddle each truss and were supported throughout the span on 2 in. by 6 in. wooden cross arms bolted to alternate hangers. These cables had an ultimate strength of 305 tons each and were pre-stretched to 120 tons, approximately 20% greater than the calculated working load. The modulus of elasticity of these cables after pre-stretching was 19 600 000 lb per sq in., for loads of less than 130 tons. Because of their relatively light weight in proportion to their strength, their flexibility, the ease with

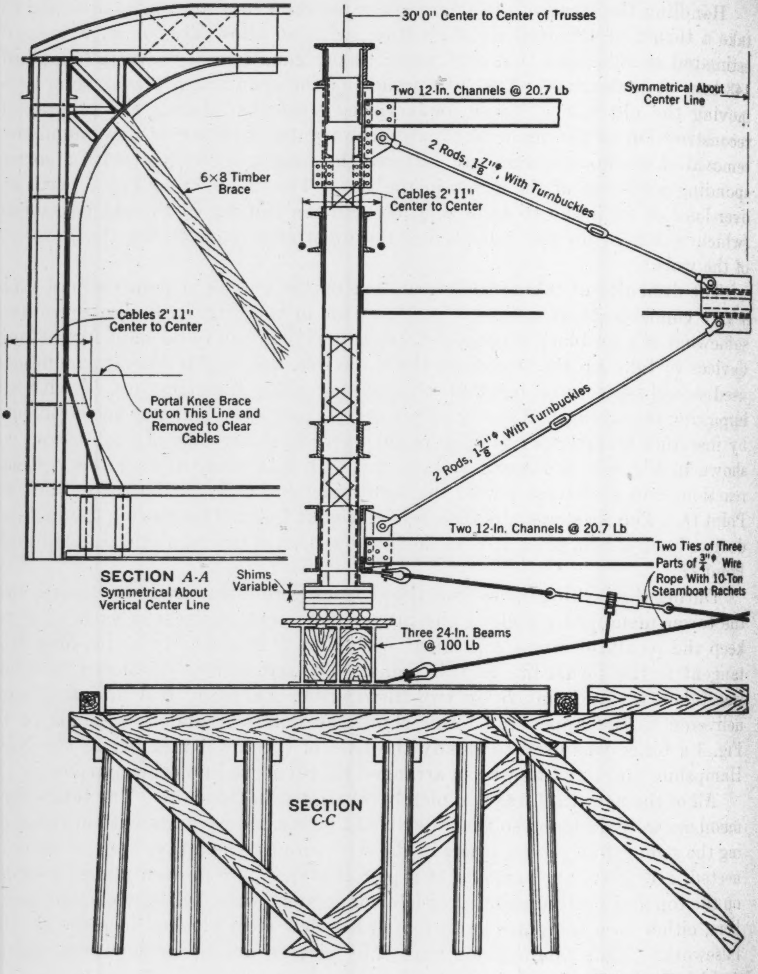
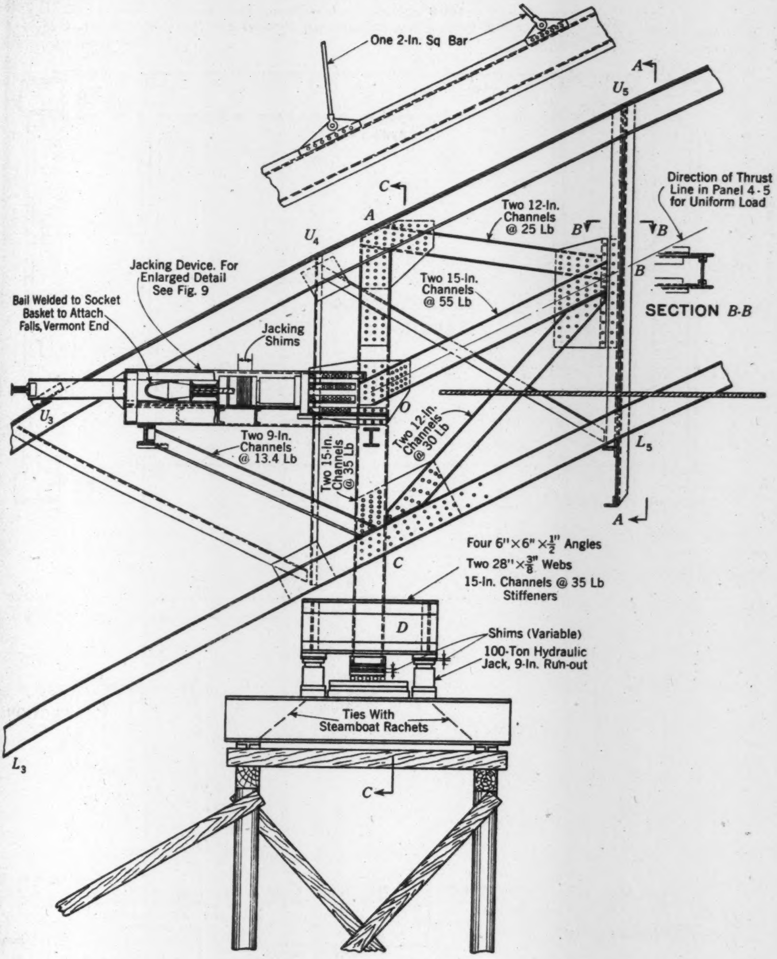

30' 0" Center to Center of Trusses
Two 12-In. Channels @ 20.7 Lb
Symmetrical About Center Line
2 Rods, 1 7/8"φ, With Turnbuckles
6×8 Timber Brace
Cables 2' 11" Center to Center
Cables 2' 11" Center to Center
Portal Knee Brace Cut on This Line and Removed to Clear Cables
2 Rods, 1 7/8"φ, With Turnbuckles
Two 12-In. Channels @ 20.7 Lb
Two Ties of Three Parts of 3/4"φ Wire Rope With 10-Ton Steamboat Rachets
SECTION A-A
Symmetrical About Vertical Center Line
Shims Variable
Three 24-In. Beams @ 100 Lb
SECTION C-C

FIG. 6.—TOWER AND

which they could be erected, and their adaptability for connecting to the jacking beams, these cables were particularly well suited for the purpose. They were cut to a carefully measured length and the sockets were added at the mill so that all cables would have the same length when subjected to the same stress. They were pulled into position in the jacking beams to an initial stress of about 5 tons each and their length under this stress was checked at


One 2-In. Sq Bar
A
U5
Direction of Thrust Line in Panel 4-5 for Uniform Load
C
A
Two 12-In. Channels @ 25 Lb
B
B
B
U4
SECTION B-B
Jacking Device. For Enlarged Detail See Fig. 9
Two 15-In. Channels @ 55 Lb
Bail Welded to Socket Basket to Attach Falls, Vermont End
Jacking Shims
O
U3
Two 9-In. Channels @ 13.4 Lb
Two 15-In. Channels @ 35 Lb
Two 12-In. Channels @ 30 Lb
L5
A
Four 6"×6"×1/2" Angles
Two 28"×3/8" Webs
15-In. Channels @ 35 Lb Stiffeners
C
D
Shims (Variable)
100-Ton Hydraulic Jack, 9-In. Run-out
Ties With Steamboat Rachets
L3
C

Jacking Layout

the mill to make certain that adequate clearance would be provided in the jacking devices to accommodate both the stretch and sag.

Jacking Operations

The general plan of the cable and jacking device layout is shown in Fig. 7, the cable lengths being as follows:

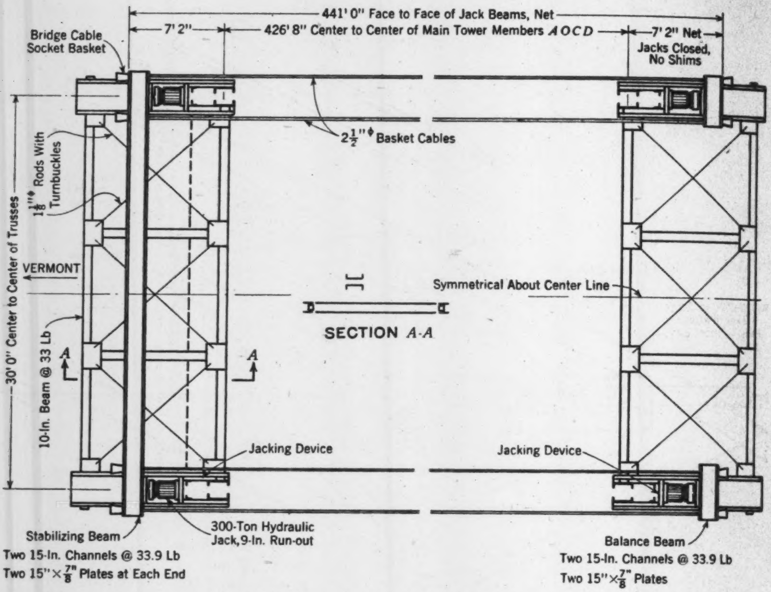

441' 0" Face to Face of Jack Beams, Net
7' 2"
426' 8" Center to Center of Main Tower Members AOCD
7' 2" Net
Jacks Closed, No Shims
Bridge Cable Socket Basket
$2\frac{1}{2}$"ϕ Basket Cables
$1\frac{1}{8}$"ϕ Rods With Turnbuckles
30' 0" Center to Center of Trusses
VERMONT
10-In. Beam @ 33 Lb
A
A
SECTION A-A
Symmetrical About Center Line
Jacking Device
Jacking Device
Stabilizing Beam
Two 15-In. Channels @ 33.9 Lb
Two 15" × $\frac{7}{8}$" Plates at Each End
300-Ton Hydraulic Jack, 9-In. Run-out
Balance Beam
Two 15-In. Channels @ 33.9 Lb
Two 15" × $\frac{7}{8}$" Plates

FIG. 7.—LAYOUT OF JACKING DEVICE, SHOWING ARRANGEMENT OF JACK BEAMS AND STABILIZING BRACES

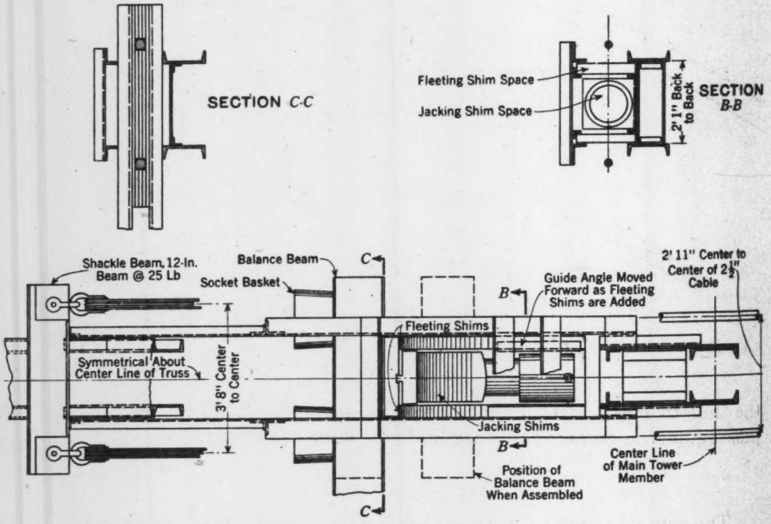

SECTION C-C
Fleeting Shim Space
Jacking Shim Space
2' 1" Back to Back
SECTION B-B
Shackle Beam, 12-In. Beam @ 25 Lb
Balance Beam
Socket Basket
C
C
B
B
Guide Angle Moved Forward as Fleeting Shims are Added
2' 11" Center to Center of $2\frac{1}{2}$" Cable
Fleeting Shims
Symmetrical About Center Line of Truss
3' 8" Center to Center
Jacking Shims
Position of Balance Beam When Assembled
Center Line of Main Tower Member

FIG. 8.—PLAN OF JACKING DEVICE, SHOWING PART PLAN OF FALLS USED TO STRETCH THE CABLES INTO PLACE

Distance from center to center of main towers...		426 ft 8 in.
Length of each jacking device from the face of jacking beam to the center line of the main tower (7 ft 2 in. times 2)................		14 ft 4 in.
Net distance, face to face of beams............		441 ft 0 in.
Allowance for sag, cable supported 27 ft on centers, or at cross arms....................	6½ in.	6½ in.
Allowance for distortion of truss due to buckling of bottom chord..........................	3 in.	3 in.
Provision for normal shims..................	6 in.	6 in.
Gross length of cable at 10 000-lb stress........	1 ft 3½ in.	442 ft 3½ in.
Stretch of cable (calculated); load, 10 000 to 200 000 pounds............................	1 ft 4½ in.	
Marginal jacking capacity 9 in. at each end.....	1 ft 6 in.	
Total jacking capacity, two jacking devices..	4 ft 2 in.	

The tension in each tie was equalized between the two cables by means of a short balance beam to which the cables were attached at one end of each truss, and the system was stabilized by means of a beam extending across the roadway from one truss to the other. The other end of all cables was attached at this beam. These beams (also referred to as jacking beams) were all free to rotate about a vertical axis, on the center line of each truss, by resting on seats in the jacking devices, provided with knife-edge bearings. This system permitted the independent movement of the trusses so that one could be moved longitudinally, or one tie length could be varied without affecting the other when supporting the temporary towers. The jacking devices being in compression, it was necessary that they be stabilized by means of some kind of bracing system. This was provided as shown in Fig. 7, using diagonal braces composed of rods fitted with turnbuckles and clevises so that the rods could be let out or taken up as might be required to rack the bracing.

Because of the limited clearance for the jacking beams used to equalize and stabilize the cables, it was necessary to provide two jacking devices, one at either end of each tie so that the tie could be taken up or let out at either end. However, both jacking devices were not to be used simultaneously. One 300-ton jack with a 9-in. run-out was used in each jacking device, and shims were provided for fleeting the jacks as well as for carrying the load independently when the jacks were not actually in operation. Fig. 8 shows the detail of the jacking device and, with Fig. 9, shows the assembly of the jack beams and the method of attaching the cables to the beams. Both cables were pulled into place at once, with the falls attached to socket baskets, a bail being welded on them. The falls were set 3 ft 8 in. apart so that the socket baskets would clear the jacking device. When the cables were pulled into

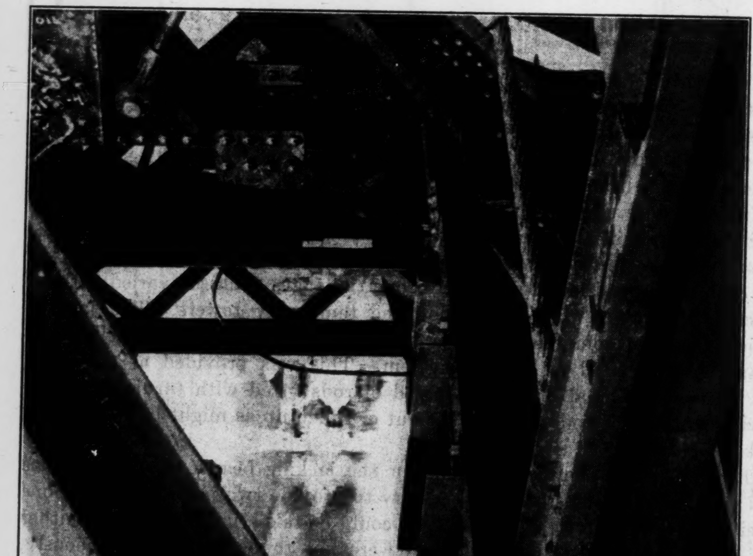


FIG. 9.—DETAIL VIEW OF THE JACKING DEVICE AND THE ANCHORAGE OF THE CABLES TO THE STABILIZING BEAM

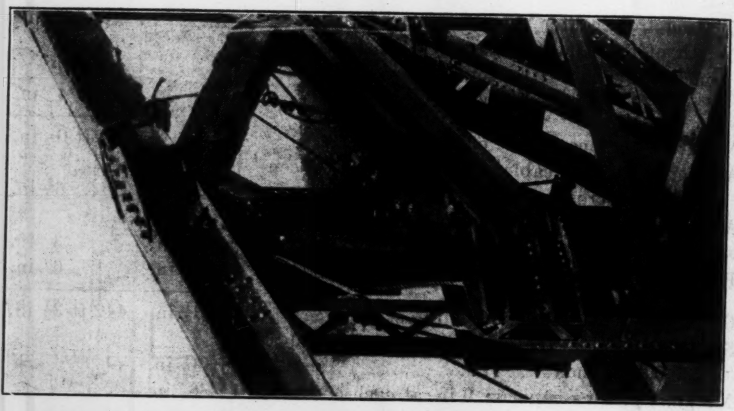


FIG. 10.—VIEW OF DISTRIBUTING MEMBERS, WITH THE JACKING DEVICE REMOVED TO SHOW THE JACK SEAT STIFFENERS

position, they were drawn together to a spacing of 2 ft 11 in. and the remaining beams were assembled in place. Fig. 10 shows the jacking device removed to give a view of the stiffeners and stiffener plates on which the base slab supporting the jacks was seated.

The arrangement of the jacks and shims on the towers is shown in Fig. 6. Four 100-ton hydraulic jacks, also having a 9-in. run-out, were used on each tower, two under each truss. A built-up box girder was provided to carry the load from the jacks to the tower member (see *A O C D* in Fig. 6). The tower member was framed through the girder and was fitted with a base slab to rest on a set of shims when the jacks were not in operation. To provide for movement of the span on the towers these shims were carried on a double nest of rollers so that movement could be obtained both longitudinally and laterally. Longitudinal movement was governed by the jacks in the main ties and lateral movement was controlled by a system of "steamboat" ratchets, shown as a part of the bracing in the plane of the tower in Fig. 6. All diagonal bracing in the towers, except that in which the ratchets were used, was composed of rods with turnbuckles and clevises for adjustment when aligning the span.

Each of the temporary wooden towers was composed of two bents of twelve piles each. The position of all piles, particularly those in the bents forming the approach for the traveler from the shore, had to be located very carefully to avoid interference with the old steel and the emergency bracing, or with the new members. The main towers were braced rigidly and the services of a diver were required to place the bracing below water level. All piles, including the steel piles used in the permanent construction, were driven with a double-acting steam hammer (No. 9-B-2), in swinging leads handled from the boom of the traveler, the operator working out from the shore on the temporary bents. These bents also served to support the old floor system at times during the progress of the work. The old floor system over the new work was not removed completely until the span was almost fully transformed into a tied arch and carried on the temporary towers. The purpose of this procedure was to avoid disturbance to the structure, or any changes in the condition of loading, until the span was well under control.

Jacking operations on major bridge structures are quite common but they are normally conducted under circumstances such that the conditions of stress in the structure are known, and they may be kept under control throughout the procedure. In the case of the jacking operations, in transforming this arch span, the engineers and erectors were confronted with the problem of beginning to operate jacks on a structure in which the stresses were mostly unknown and with many of the members manifestly over-stressed. A jacking load applied in the wrong position or in the wrong direction, or the successive applications of the loads in the wrong order, might easily have caused a failure in some over-stressed member.

The general layout of the jacking equipment is shown in Fig. 11. All pipe, tubing and fittings were $\frac{1}{2}$-in. double, extra-strong stock. All pumps, gages, and central control valves were placed in a group on the floor of the bridge and connected to the jacks at different elevations, by copper tubing.

Emergency stop-and-release valves were placed in the lines, at the jacks. The tie jacks and the tower jacks in pairs, were placed on separate pipe lines and each line was connected so as to operate normally from a separate hand pump. However, the hook-up was so arranged that both pumps might be "cut in" on either line by means of the stop-and-release valves; but, as the two lines worked under different pressures, they could not operate together.

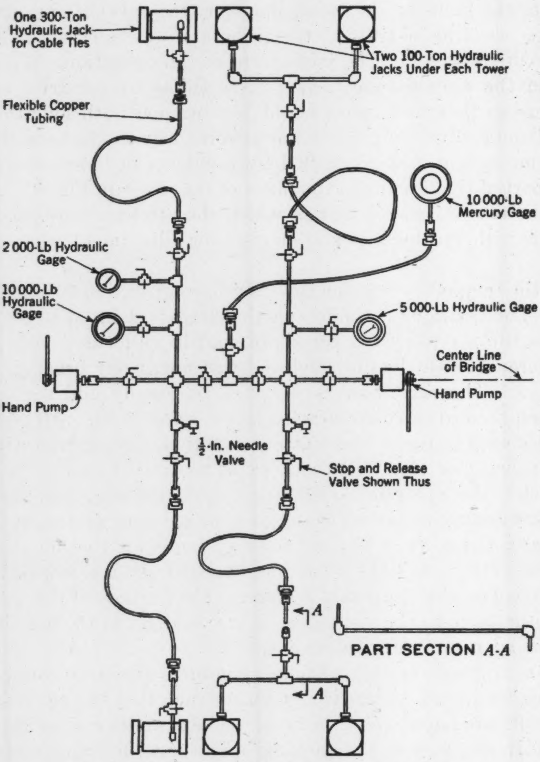

One 300-Ton Hydraulic Jack for Cable Ties
Two 100-Ton Hydraulic Jacks Under Each Tower
Flexible Copper Tubing
10 000-Lb Mercury Gage
2 000-Lb Hydraulic Gage
10 000-Lb Hydraulic Gage
5 000-Lb Hydraulic Gage
Center Line of Bridge
Hand Pump
Hand Pump
½-In. Needle Valve
Stop and Release Valve Shown Thus
A
PART SECTION A-A
A

Fig. 11.—Layout of Hydraulic Jacking System

Separate hydraulic gages were used on each line to control the pressure in the jacks; 5 000-lb gages were used with the tower jacks, and 10 000-lb gages in combination with 2 000-lb gages (for low pressures) were used with the tie jacks. Mercury gages (10 000-lb pressure capacity) were connected into the system at either end of the span for use as master gages in the event any discrepancies appeared in the readings of the hydraulic gages. However, it was not necessary to use the master gages because the others had been adjusted and calibrated at the tool house before they were shipped to the field, and they worked perfectly in every way. Their sensitiveness was very apparent on the gages governing the stress in the ties where the slightest variation in pressure on the jacks, as indicated by the gages, was instantly discernible on the yardsticks clamped in the jacking devices to measure the stretch in the ties. All pumps and gages at either end of the span were placed in a group on the floor of the bridge. There was no hydraulic connection between the jacking system at one end of the span and that at the other; but a telephone system fitted with headsets was provided as a means of co-ordinating the work. A foreman was placed in charge of the jacking equipment at each end; and, with the gages arranged so that he could read the jack pressure at a glance, he applied the pressure as directed by the engineer in charge, or the superintendent, who directed the operations by working from jacking charts prepared in advance (see Fig. 5).

Two jacking charts were required, one for each end of the span. Differences in the weight localized over the towers—such as the temporary steel, removal of part of the floor system, sand ballast, etc.—necessitated somewhat greater working pressure in the tower jacks under the New Hampshire end. Fig. 5 is the chart for the Vermont end only; the other, although different, was essentially the same so far as this description is concerned.

In starting to transfer the weight of the span to the falsework, the temporary steel (which weighed approximately 26 tons and which had been carried on the falsework by shimming up tight under the foot of the temporary tower members shown as $A\ O\ C\ D$ in Fig. 6) was first to be picked up on the tower jacks at 350 lb pressure. The initial stress on the ties (5 tons on each cable) corresponded to a gage pressure of 260 lb on the tie jacks. Actually the initial tie stress was less than 5 tons per cable and the load on the tie jacks was picked up at 200 lb pressure. According to the jacking chart this initial pressure on the tie jacks called for a total pressure on the tower jacks of 410 lb. The first step then, having taken the tie load on the jacks at one end, was to raise the pressure in the tower jacks to 410 lb at both ends of the span, working simultaneously. Both trusses were jacked in unison from the start, with the corresponding jacks in each truss "cut in" on the same line. This first step placed the entire jacking system "on the chart." Thence using the tie stress to govern the tower reactions in accordance with the chart, the tie stress was stepped up by 200-lb increments of pressure alternating with the tower jacks and stepping up the latter simultaneously at all points. Thus, with the tie jacks at 200 lb and the tower jacks at 410 lb the ties were stepped up to 400 lb while the tower jacks "stood by." The towers were then stepped up to 505 lb while the tie jacks "stood by." This process was repeated until a pressure of 2 600 lb was obtained on the tie jacks, corresponding to a tension of 97 tons in each tie with a total stretch of $8\frac{3}{4}$ in. on the cables. To keep the stretch in the ties about equally divided between the two ends of the span the jacking operations on the ties were then switched to the other end of the structure and the process was continued as before until a pressure of 4 780 lb on the tie jacks was obtained. This pressure corresponded to a stress of 178 tons on each tie, or 89 tons on each cable. This was the limit of the jacking that should be done, according to the calculated reactions, by following the direction of the thrust line on the chart, because it was the maximum stress obtainable in the tie with the structure resting on the towers and with the ends cantilevered under full floor load. During the jacking to this stage the sand and gravel ballast that was placed over the haunches of the up-stream truss, at the New Hampshire end, was gradually removed and the nuts were removed from the old anchor bolts in the skew-backs.

In this stage the increment of pressure on the towers jacks was approximately 95 lb, which corresponded to an increase of load of 4 tons on the tower. In effect this amounted to a concentration of 4 tons on each truss at each of the towers. It was thought that the most seriously over-stressed members in the structure, before any jacking was done, were the diagonals, U_2-L_3 and U_3-L_4, and the verticals U_2-L_3 (see Fig. 3). These diagonals were in compression and the verticals were in tension. To jack upward on the tower member (that is, to apply the increment of the tower reaction before the increment of tie stress was applied) would operate to increase the stress in these members; but, applying the tie-stress increment first, and then applying the tower-load increment, would serve to reduce these stresses. Therefore, this was the procedure adopted.

When the stress of 180 tons on the tie was obtained, all jacking was stopped and the floor deck removed; part of the steel work in the floor system was left supported on the temporary pile bents, but all floor load was removed from the first three panels at each end of the span. The approach span at the Vermont end was also supported on falsework at the east end to permit the removal of the old shoes a short time later.

Referring to the jacking chart in Fig. 5, it will be observed that there are two positions plotted for the positions of the jack pressures after departing from the main thrust line: One is for the maximum, and the other is for the minimum, calculated dead weight of the span. When jacking was resumed after the floor loads were relieved, the tie jacks were "stepped up" by increments of 100 lb of pressure, each being followed by a "step-up" of the tower jacks by an increment of 300 lb of pressure. After one "step-up" on the tie and one "step-up" on the towers the tie stress was determined as at 4 980 lb per sq in. jack pressure, a total of 185 tons. At this tie stress the chart indicates that the old shoes should be clear of the masonry skew-backs. An inspection of the shoes failed to indicate that this was so, and the tie jacks were once more stepped slowly to a jack pressure of 5 000 lb per sq in. The tower jacks were then "stepped up" slowly and when the pressure in these jacks reached 3 200 lb per sq in., an observer stationed at the down-stream shoe, at the New Hampshire end, reported that there was a very definite crack in the ice and grout around that shoe but no change could be observed at the other shoes. The ties were then stepped up to 5 200-lb jack pressure and it was observed that the joint U_3 on the up-stream truss (New Hampshire end) had opened about $\frac{1}{16}$-in. This joint, as well as all others at which the new steel was to be connected to the old, had been prepared previously for dismantling by replacing the rivets with bolts. The down-stream jacks all around were then cut off the lines and the up-stream truss was stepped up to 5 400 lb per sq in. (201 tons), on the tie, and to 3 300 lb per sq in. (136 tons), on the tower, and then returned to the previous position corresponding to the down-stream truss. This was simply a test movement. Movements at the joints that were bolted indicated that the members were carrying no stress of any importance, and consequently the stress in the trusses between the jacking devices and the ends of the span was released. The total stretch in the ties at this point was observed to be 17$\frac{5}{8}$ in.; and the total rise in the span directly over each tower, as determined by taking elevations on the floor beams at these points, was found to be $\frac{7}{8}$ in. The span was landed on shims in this position and the jacks released. The nuts were placed back on the anchor bolts at the Vermont end so as to stabilize the structure on the falsework and the old steel was completely removed from the New Hampshire end.

The total jacking time required for this work was about 12 hr including the time required for testing the jacking system for leaks, etc., and for checking the gages, one against the others. During the first stage of the jacking, which increased the tie stress to 178 tons, the weather conditions were very inclement. This stage of the work was begun at 8:00 A.M. on November 10, 1936. The day was quite cold, the temperature being approximately 32° F, and soon after the work was started it began to rain, and continued to rain and sleet until noon. After noon a high wind rose and continued for the remainder of the day. This made progress with the jacking very slow so that by evening the first stage of it was only about half completed. Work was continued the following morning; the weather was clear and the temperature was approximately 20° F, but it was much more favorable than the day before. The remainder of the jacking necessary to bring the tie stress up to 180 tons was completed in about 2$\frac{1}{2}$ hr. After about a day required to remove the floor load from the ends of the span the jacking was completed in approximately 1.5 hr. Under favorable weather conditions the entire jacking operations would have been completed in about 7 hr of actual jacking time.

After the removal of the old steel at the New Hampshire end, the jacks on the temporary tower at the same end were raised about 15 in., rotating the span vertically about the old pins at the Vermont end and, of course, raising the jacks at the Vermont tower proportionately. The tie jacks at the New Hampshire end of the span were also "taken up" (that is, the ties were shortened) about 5 in. on the up-stream truss and about 3 in. on the down-stream truss. The tower jacks on the up-stream truss, New Hampshire end, were then lowered about 2 in. and the bracing rods in the tower were adjusted at the same time by means of the turnbuckles. The difference of 2 in. in the distance that the ties were shortened relative to each other removed most of the sag in the up-stream truss and almost straightened the span vertically, bringing it back almost plumb. The actual pressures on the jacks during this set of operations naturally did not change except that, in the case of the tie jacks, the tie stresses increased about 10 tons to overcome the roller friction. The New Hampshire end was free and it was simply moved to make clearance for the erection of the new steel. At this time also the span was moved down stream 2.5 in. on the New Hampshire tower and almost 4.5 in. on the Vermont tower by means of the steamboat ratchets shown in the lower stage of bracing in Fig. 6. This jacking, together with the adjustment of various turnbuckles in the temporary rod bracing, was done simultaneously with the jacking to "take up" on the ties. Otherwise it is doubtful whether the span could have been moved so far laterally on the Vermont tower.

STEEL ERECTION

Before removing the old steel the spring-line pins were referenced carefully and the center line of the structure was established on the towers to correspond to the skew-backs. There was a rumor locally that, during high water in 1927, a wash-out had occurred back of the New Hampshire abutment, causing it to settle. Careful measurements were made to determine the relative positions of the pins all around, and levels were taken to check their elevations. It was found that the old pins were within $\frac{1}{16}$ in. of the same elevation. The nominal span was 540 ft, center to center of the old pins, but the measurements taken indicated the up-stream truss pins to be 540.05 ft and the down-stream pins to be 539.91 ft, center to center—a difference of $1\frac{3}{4}$ in. The measurements also showed that this difference was divided between the two abutments—$\frac{1}{2}$ in. to the New Hampshire abutment and $1\frac{1}{4}$ in. to the Vermont abutment. This finding seemed to indicate that each abutment had rotated about a vertical axis causing a change of span for both trusses. The up-stream truss was lengthened $\frac{5}{8}$ in. in span and the down-stream truss was shortened $1\frac{1}{8}$ in. This seemed rather improbable since there were no cracks in the wing walls nor any other visible indications of settlement; and, since the New Hampshire abutment (which was supposed to have been the one affected) had the least skew, no further attention was paid to the settlement rumor. However, the new steel was set to "square up" with the center line of the structure by working from the referenced position of the old pins in the down-stream truss to establish the length of the reconstructed span according to it. The up-stream truss span was shortened $1\frac{3}{4}$ in. in this manner.

Before placing the new main members the light-welded steel trusses shown in Fig. 12 were shored and blocked in place to serve as guides to hold the steel piles in exact position while driving. These trusses were designed for this purpose as well as to serve as reinforcement for the concrete piers. The weight of the piers was assumed to be carried by the piles entirely, without any other support, although the rip-rap, on which the piers were placed and through which the piles were driven, was washed off and cleaned of mud, etc., by a diver using a fire hose. The timber crib shown in Section *A–A*, Fig. 12, had been filled with rock and capped with concrete some time after the original span was completed. The rip-rap apron around the old crib was cleared away where necessary and 12 in. by 12 in. timbers were cut away to make room for the new steel piles. The piles were driven to a penetration of about 65 ft to rock and were capped with steel grillages to support the forward shoes of the new steel work. The shoes were set in position carefully on the grillages, to measurements from the referenced position of the old pins. The wood sheet-piling was then driven to provide forms, and the concrete placed by means of a tremie. Anchor bolts 3.5 in. in diameter, two for each shoe, were placed in the shoes and the concrete was poured to the bottom of the grillages. The posisions of the shoes were checked, the nuts on the anchor bolts locked down tight,

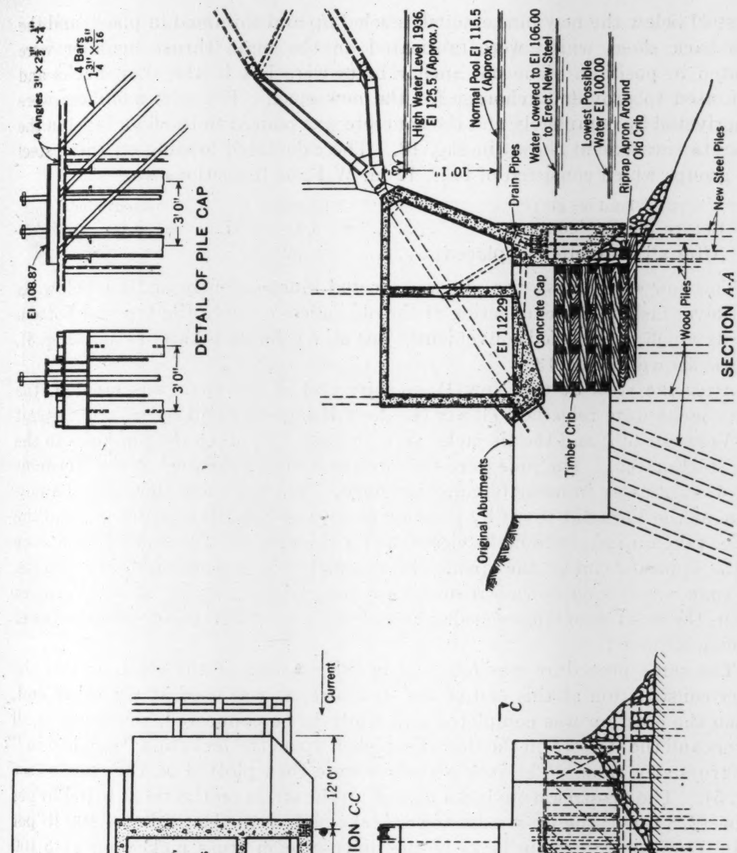

DETAIL OF PILE CAP
El 108.87
4 Angles 3"×2½"×¼"
4 Bars 1¾"×5/16"
3'0"
3'0"
SECTION A-A
High Water Level 1936, El 125.5 (Approx.)
Normal Pool El 115.5 (Approx.)
Water Lowered to El 106.00 to Erect New Steel
Lowest Possible Water El 100.00
Riprap Apron Around Old Crib
New Steel Piles
10' 1"
Drain Pipes
El 112.29
Concrete Cap
Wood Piles
Timber Crib
Original Abutments
SECTION B-B
Timber Crib, Rock Filled, Shown With Concrete Cap Removed
5' 4½"
3'0"
30'0" Center to Center of Trusses
Original Abutments
Top Laterals of Welded Trusses, 1 Bar 1¼"×3/16" Bottom Laterals Same
SECTION C-C
12'0"
Current
A
A
B
B
C
C
EAST ELEVATION
Top of First Pouring of Concrete El 107.75
Welded Truss See Detail of Pile Cap

FIG. 12.—CONSTRUCTION LAYOUT OF THE VERMONT PIER

the steel below the new pins erected, leveled up and shimmed in place, and the skew-back shoes which were fabricated on the main thrust member were grouted in position. The old anchor bolts were left in the skew-backs and were used to provide anchorage for the new steel. The new members were then riveted up completely and the concrete was poured to the floor level in the concrete encasement shown in Fig. 12. The calculated loading on these steel pile groups which consisted of four, 12 in. W F, 55 lb sections was:

Load per pile:	With wind	Without wind
Before placing concrete	+ 4 to − 47	− 5 to − 38
After all concrete was placed	− 67	− 58

The new steel above the pin was erected independently of that below, in cantilever fashion, by connecting at the old splices on both the top and bottom chords which were placed conveniently just above Joints U3 and L3 (see Fig. 3). This is shown in Fig. 13.

After the steel at the New Hampshire end of the span was erected, the tower jacks were released to lower the New Hampshire end by rotating about the Vermont pin; and the tie jacks were "let out," to make the pin holes in the new steel match. The pins were then driven and the old steel at the Vermont end was cut away immediately and removed. The span was then raised about 15 in. at the Vermont tower by rotating about the New Hampshire pin and the cables taken up again to make clearance for the new erection as had been done at the opposite end of the span. To complete the straightening-up process, the span was moved down-stream approximately $\frac{1}{2}$ in. farther at the Vermont tower, the up-stream truss was dropped about 2 in. at this point, and the tower bracing adjusted.

The same procedure was followed in the erection of the steel, and in the other construction at this end of the structure, as was used at the other end. When the erection was completed and ready to be connected, the loads at all towers and the stresses on the ties were picked up on the jacks and "weighed in," one truss at a time. The jack pressures were then plotted on the charts (see Fig. 5). The up-stream tie jacks picked up the stress on the tie at 6 100 lb per sq in. (228 tons); the up-stream tower jacks picked up the load at 4 100 lb per sq in. (168 tons); the tie jacks in the down-stream truss picked up at 5 100 lb per sq in. (190 tons); and, the tower jacks under this truss picked up at 3 400 lb per sq in. (140 tons). Due to the fact that several inches of snow and ice covered the deck of the span at this time and during the previous month, the old asphalt plank flooring had been piled along the up-stream truss, permitting the old plank sub-floor to become thoroughly soaked with water. The pressures at which the jacks picked up the loads were considerably higher than those required to raise the span at the beginning.

After the loads were weighed in on the jacks, the span was lowered, rotating about the New Hampshire pin, and the ties were released slowly so as to bring the pin holes into position for driving. The jack pressures did not change during this operation so that as the pins were driven the pressures in the jacks were already charted.

As soon as the pins were driven the structure was lowered immediately and landed on the abutments. Since the up-stream truss was the heavier of the two, it was started down ahead of the other and the pressures in the jacks controlling it were lowered to meet those of the jacks controlling the down-stream truss. The corresponding jacks on the two trusses were then thrown in on the same lines and the two trusses were thence brought down in unison. Thus, the down-stream jacks were "cut off" the lines but were held under pressure, and the tower jacks at both ends of the up-stream truss were lowered simultaneously by opening up a release valve in the line while the tie-jack line was kept closed and the pressure was permitted to "drift" as the tower jacks released the pressure. These jacks were not turned loose but were held under control by "stepping down" at 200-lb increments.

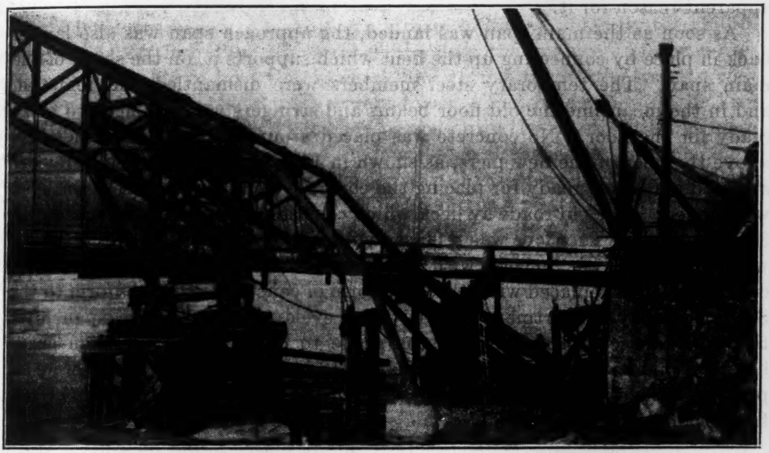


Fig. 13.—New Steel Erected at the New Hampshire End of the Span Ready for Driving the Pins

As the jacks were stepped down the ties gave up the load very leisurely, accumulating sag between the supporting cross arms until the original initial stress in the tie was reached. The shims were placed under Tower Members *A O C D* (Fig. 6) to take the local concentration of temporary members away from the trusses, and the release valves in all jacking lines were thrown wide open. The total time required to release the jacks and land the span was a little less than 2 hr.

Throughout the jacking operations in raising the span on to the towers, and again when landing it on the abutments, a peculiar difference between the action of the tower jacks at the two ends was manifest. As the stress in the tie was "stepped up" along the thrust line, the tower jack pressure at the end of the span, where the tie jacks were in operation, had to be pumped to the full increment of pressure, whereas the tower jack pressure at the opposite end drifted automatically in exact accordance with the charts, with little or no pumping after the falsework in the towers had taken up the initial settlement.

At first this was thought to be the result of excessive settlement in the tower where the jacking was required; but after switching the jacking procedure to the opposite end of the ties this same peculiar action continued as before: The jacks on the tower, which had been drifting as stress was jacked into the tie, now had to be jacked the full increment, and the others, which had previously required the full increment of jacking, now drifted with the tie stress. The same perplexing phenomenon continued when landing the span, with no very apparent reason for it.

As soon as the main span was landed, the approach span was also landed back in place by connecting up the bent which supports it, on the shoes of the main span. The temporary steel members were dismantled and removed, and in the meantime the old floor beams and stringers were replaced to make ready for the floor. No concrete was placed around the new steel members above the floor of the new piers, as shown in Fig. 12, but the forms were constructed and made ready for placing the concrete encasing as soon as the span was landed. The old roadway deck which consisted of asphalt plank laid on a 3-in. wood plank sub-floor was replaced with a 3-in. concrete floor and the old wooden sidewalk was replaced with 2-in. floor. The old railing, which consisted of gas pipe, was replaced with a modern form of construction to conform to the State Highway Department standards. Levels taken over the span after the work was completed showed the grade to conform to a uniform curve having a middle ordinate of approximately 18 in. and the floor beams were all practically level, the maximum difference in the elevation of the ends of any floor beam being only $\frac{3}{8}$ in.

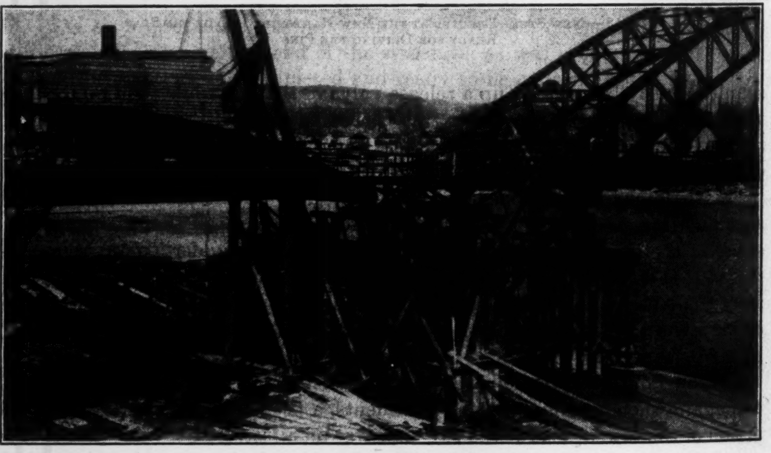


Fig. 14.—Falsework at the Vermont End

Working Conditions

The weather conditions at the time this work was in progress were very erratic and generally unfavorable. The late fall and winter seasons began quite cold and before the first of December the river was frozen over with ice approximately 10 in. thick. About this time it became necessary to lower the pool level to reach the old steel around the skew-backs. This broke the ice along the shores but it caused no serious trouble and the ice quickly sealed up as the cold weather continued. About December 9, with several inches of snow in the Connecticut River basin, the temperature rose suddenly and in three days the precipitation was reported variously as 3 in. to 4 in. Under the effects of this rise in temperature and the rain, the ice broke loose on the river above the pool level and began to move down stream quite rapidly. By the skillful manipulation of the rolling lift gates in the dam the engineers of the power company were able to hold this in check and permit the State forces to construct an ice fender to shield the Vermont tower. Fig. 14 is a view of the Vermont end immediately after the ice had passed down the river. It broke loose about 300 ft above the bridge, and the lower edge of the ice can be seen below the bottom chord of the truss. The ice is still intact around the falsework due to the protection of the fender. Since there was no fender to protect the New Hampshire tower, the gates were kept closed at that end of the dam, to hold the ice and to throw the current to the main channel on the opposite side, discharging the ice past the tower which was protected by the fender. This was so successful that it was decided that it would not be necessary to build a fender to protect the New Hampshire tower.

CONCLUSION

This project is somewhat novel in the history of arch-bridge construction, as was the design and erection of the original span.[3] The magnitude of this undertaking was by no means measured by the quantity of new material involved; nor was it measured by the cost of the project. The cost of a new structure is scarcely a criterion of the amount of the work incident to the planning and careful procedure which was necessary, and although the final cost was approximately three times the original cost of the superstructure, a new span would have cost several times the amount of the repairs and, in addition, the cost and hazard of removing the old structure would have to be taken into consideration. Because of the depth of the pool at this point and possible damage to adjacent property, the removal of the old span as it stood at the beginning of repair operations would have been quite a problem and undoubtedly would have cost at least half as much as the proposed repairs.

The load capacity of the span is adequate for the location. The arch trusses were found to have capacity for the H-15 loading and the floor system, including the slightly greater weight of the new concrete-steel deck, has a capacity 20% greater than the H-10 loading—that is, the H-12 loading, applied in accordance with the American Association of State Highway Officials Specifications. Therefore, no special alterations were necessary to raise the structure to the traffic requirements. However, when designing the new deck and hand-rail, the roadway was widened from 20 ft to 21.25 ft, and the width of the sidewalk was established at 4 ft 10 in. The new steel and foundations were designed for the H-15 loading, and since the steel in the main trusses is in such excellent condition that it will probably last indefinitely if properly maintained, it is possible, if future traffic demands it, to revise the entire structure to the H-15 loading at relatively little cost.

On the basis of replacement costs the State Highway Department, by repairing the damaged span, provided an adequate structure to meet the traffic requirements at approximately one-third the cost of removing the old structure and replacing it with a new one of the same capacity. By the use of the specific method adopted to do the work enough was saved to provide for the new deck and hand-rail.

As compared to the original structure, having a span of 540 ft with the spring-line pins 14 ft below present high water, the reconstructed arch has a span of 486 ft with the spring-line pins 3 ft above high water, assuring ample protection against a repetition of the damage at the same high-water level.

This work was done by the engineering and erection forces of the American Bridge Company, who furnished and fabricated the steel, and who furnished the erection equipment, in co-operation with the engineers of the Bridge Division of the New Hampshire Highway Department and their construction forces.

Although all operations were made difficult throughout the work by weather conditions, ice, high water, and extreme cold weather the progress and the final results were very satisfactory to the State Authorities and the bridge is adequate in every way for modern traffic.

DISCUSSION

J. B. French,[4] M. Am. Soc. C. E. (by letter).—The work described in this paper shows boldness and resourcefulness in planning and much skill and constant alertness in the supervision of its execution. It does not appear that any traffic, either vehicular or pedestrian, had to be maintained at any stage of the work, and therefore the risk of loss of life was confined to the workmen engaged. Nevertheless, it is obvious that any mistake, either in planning or in execution, might easily have wrecked the damaged structure, with the loss of the lives of the workmen and serious property damage. While the work was in progress, it undoubtedly appeared spectacular to those who stopped to give it intelligent attention, but after it was finished the spectacular quality necessarily disappeared and left no monumental features to attract attention. In fact, the best evidence that the work described has been completed effectively will be the failure of present observers to discover that any repairs or "reconstruction" had ever taken place.

Work of this kind, like foundations and structural frames hidden from view by subsequent construction, and all falsework and temporary structures for purely erection purposes constitute a most important part of engineering construction; detailed descriptions of such work, such as are given in this paper, are always valuable contributions to the publications of the Society.

It seems to the writer, however, that the value of the paper would be enhanced if a more detailed statement of the actual cost of the various parts of the work, as executed, could be included. In the "Synopsis," the authors state that "the successful bid" on the "design for repairs" submitted by the "consulting engineers who designed the structure" was $120 000; and under the heading "Conclusion" several general statements are made as to costs, but no clear statement is made of the cost in detail of doing the work as described.

Lewis D. Rights,[5] M. Am. Soc. C. E. (by letter).—The original bridge between Bellows Falls, Vt., and Walpole, N. H.—a 540-ft arch span across the Connecticut River, built in 1904—was an ambitious project years ahead of its time. When it was planned, the Advisory Engineer, the late J. P. Snow, M. Am. Soc. C. E., and the Consulting Engineer, J. R. Worcester, Hon. M. Am. Soc. C. E., both recognized the advantage of keeping the structure above possible ice damage; but the Bridge Committee, composed of local business men from both sides of the river, would not hear to the extra expense.

In 1906, a year after the bridge was finished, a flood damaged the bottom lateral bracing at one end and the consulting engineer recommended that a reinforced concrete slab be installed to brace the lower chord and protect the lower laterals, but there was no authority at hand to see that this was done.

In 1928, the dam was raised, making the pool 10 or 12 ft higher; the canal was enlarged and the power-house expanded. It was inevitable that sooner or later this elevation of the dam would cause a destructive flood.

---

[4] Cons. Engr., Jamaica, N. Y.

[5] Roselle, N. J.

At this time some strong State agency could have bargained with the power company to provide the necessary protection around the abutments as a partial offset for the benefit of increased power production. As is usually the case, the local authorities were incapable of handling these matters against a large power corporation, especially as the bridge was an interstate project. Had these precautions been taken, the heavy expense indicated in the paper might have been avoided. Fortunately, the entire bridge was not lost and the State of New Hampshire was able to take charge and repair it.

Offhand it would seem that the cheapest method of supporting the bridge during repairs would have been to use falsework towers as was done originally. So far as the writer knows the conditions were the same as when the bridge was first erected, except that the water was deeper in the pool. This would mean longer, and possibly a few more, piles but it would have been balanced by shorter timbers in the framing of the towers.

If the work had been done after the spring rise, when the ice was gone and before freezing weather, there would have been very little risk, even if a summer flood had occurred. Assuming that the erection cost of the bridge originally was one-half the cost of the superstructure, or $20 000, and taking into account the new repair material, it is difficult to understand the lump-sum bid of $120 000. On the "cost plus" plan the State assumed the risk from operating hazards, and if this had been done for the lump-sum plan this bid would probably have been much cheaper. Allowing for the changes in the abutments it would still leave a difference of $50 000 to $75 000 to be explained.

The writer hopes that the authors will give the costs of the repairs in detail; that is, the abutments, new repair steel, erection including cables and the new floor as well as information as to what was covered in the lump-sum bid.

All this does not in the least detract from the high praise due the engineers for the skill whereby the span was shortened from 540 ft to 486 ft and the spring-line pins raised 17 ft. The success of their scheme is the result of bold design, careful planning, and constant care.

The original bridge was erected in upper New England, over a capricious river, in the depth of winter because of the delays of the Committee in awarding the contract. The bridge was repaired, also, in the winter months, probably due to the delays of the "powers above" in reaching decisions. It is to the glory of engineers that they are willing to accept difficult and hazardous conditions forced by circumstances beyond their control, without wasting time quibbling about them, and that they then proceed to overcome these hazards successfully.

CONDE B. McCULLOUGH,[6] M. AM. SOC. C. E. (by letter).—Congratulations are due the authors, not only on a most coherent presentation, but also on an exacting piece of reconstruction, efficiently and safely performed. As aptly emphasized in their report, jacking operations on major structures are normally conducted under conditions of rigid stress control, whereas in this case the distribution of stress at the beginning of jacking operations was highly problematical. The care which they exercised in controlling both the magnitude and the position of the applied thrust leaves little to be desired.

---

[6] Asst. State Highway Engr., State Highway Dept., Salem, Ore.

It is gratifying indeed to see a "repair job" of this character accorded the distinction of a detailed technical write-up. In general, repair work has earned few plaudits, notwithstanding the fact that its technique often calls for design and erection procedures more complicated and involved than in new construction. Moreover, marked economies are generally effected; and in the present instance, the expenditure of approximately $120,000 operated to save the cost of a new structure which the authors point out would have amounted to several times this sum. Economies such as this are all too frequently lost sight of in the enthusiasm for new construction.

An instance which occurred many years ago involving the Morrison Street Bridge in the City of Portland, Ore., illustrates the economies which are sometimes possible. This structure consisted of a 384-ft draw-span flanked on one end by a 205-ft fixed truss span and on the other by two spans at 267 ft. In 1919 this structure had apparently been outgrown by its traffic, and was in a rather unfortunate condition of disrepair. The floor system was inadequate, the eye-bar chains over the draw-span towers were seriously overstressed, as were most of the compression members in the bottom chord with the span open, and popular opinion had it that the bridge was outmoded, inadequate, and unsafe. After a rather complete investigation, the writer recommended a system of repairs estimated to cost $208,000, based upon the following economic comparisons:

(*a*)	Estimated cost of repairs	$ 208,000
(*b*)	Estimated total maintenance expense for ensuing 12-yr period	111,000
(*c*)	Total cost during ensuing 12-yr period for repairs, renewals, replacements, and ordinary maintenance (Item (*a*) + Item (*b*))	$ 319,000
(*d*)	Annual interest charge on capital to reconstruct ($2,500,000 at 5%)	125,000
(*e*)	Total interest expense for 12-yr period	1,500,000
(*f*)	Financial saving realized by repairing old structure (Item (*e*) − Item (*c*))	$1,181,000

This report was finally adopted, the repairs made at a cost of slightly more than $200,000 and the structure is still carrying unlimited and unrestricted traffic after an extended service period of nearly twenty years. This instance is typical of many wherein distinct economies accrue from the maintenance of old construction even if the repair work involved is complicated and somewhat hazardous.

In connection with the adopted jacking procedure, the authors mention the extreme sensitiveness of the system, the slightest variation in pressure on the jacks being instantly discernible in the device for measuring cable stretch. They also mention a rather peculiar difference between the action of the tower jacks at the two ends, involving an automatic drifting of the far jacks with little or no pumping. These two qualities—extreme sensitiveness and an inclination toward erratic behavior—seem to be somewhat characteristic of all hydraulic jacking setups.

In 1931 and 1932, the writer had the opportunity of experimenting with an hydraulic jacking system in connection with the decentering of the highway bridge over the Rogue River in Oregon. This structure consists of a group of seven 230-ft reinforced concrete arch spans, and the decentering was performed in accordance with the Freyssinet method, by means of an hydraulic system, consisting of a battery of four 275-ton jacks installed at the crown of each rib. During decentering operations, observations were made of: (1) Total jack thrusts and moments; (2) crown openings at extrados and intrados; (3) vertical movements at crown, spring line, and intermediate points; (4) internal strains at extrados and intrados at five sections in each span; (5) rotation of rib axis at crown, quarter points, skewbacks, and piers; (6) horizontal movements at spring line; and (7) temperature and shrinkage.

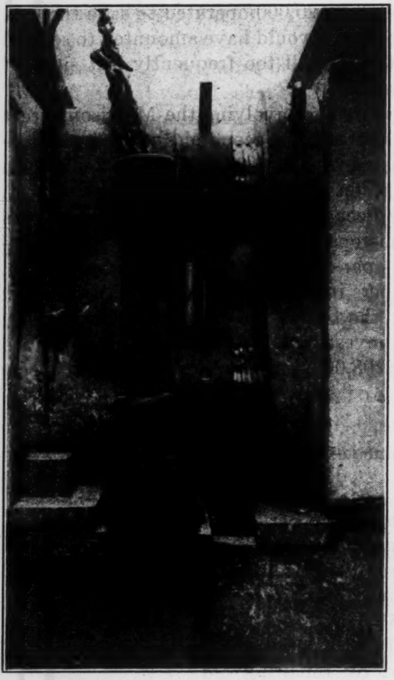


FIG. 15

Jack pressures were measured by calibrated gages, crown openings with a sliding scale and vernier, vertical movements by means of a system of water levels, horizontal movements by weighted invar wires, and internal strains by an installation of electric telemeters. The hydraulic jacks which were manufactured especially for this job by the manufacturer, who had previously designed and constructed similar equipment for M. Freyssinet, are shown in Fig. 15.

A graph (Fig. 16) shows the relation between crown thrusts as measured by the jacks, and the horizontal crown spread. It will be noted that as the jack thrust was increased the rate of crown spread increased gradually to Position 4. This increase in rate was due to the fact that, as more and more of the load was taken by the arch, the reaction of the centering against the ribs became correspondingly less, with the result that the frictional resistance to rib shortening was decreased. In Position 4, the safety rings were tightened, and the arch was allowed to stand overnight on the jacks, during which period the crown gap closed 0.03 in. due to a tightening up of the safety rings. When the decentering thrust was again applied, an additional crown spread of 0.1 in. was observed without increase in thrust. A shortening of 0.02 in. can be accounted for by the intervening drop in temperature; and the residue represented restrained shrinkage in the concrete taken up as the arch was lifted clear of its centers.

This graph, typical of a large number of similar ones for the various spans, indicates the marked sensitiveness of the system, very slight temperature and shrinkage effects being readily detectable on the pressure gages.

Another interesting phenomenon is indicated in Fig. 17 which contains graphs of crown thrust *versus* crown spread for Spans 6 and 7. It will be noted that whenever the direction of motion was reversed, a change in thrust of about 70 000 lb was necessary to reverse the motion. This rather puzzling condition persisted throughout the entire range of these experiments, and was closely checked by the telemeter readings as well. It was thought at first that jack friction might account for it, at least partly; however, a careful calibration of the jacks indicated no frictional resistance within the limits of accuracy of the gages. The only other logical explanation offered was that the joints and articulations in the spandrel structure, although theoretically free to move, possessed a distinct and fairly constant resistance to reversing motion.

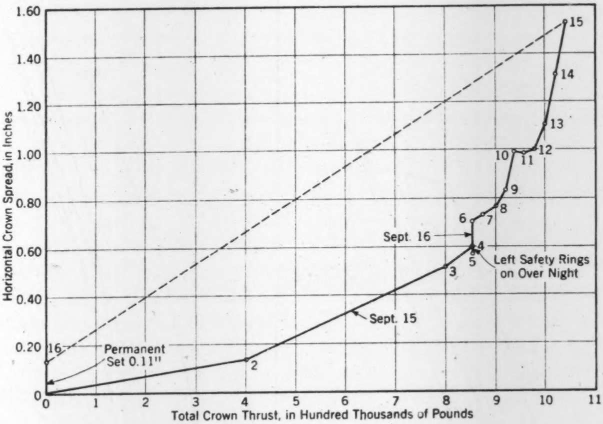


FIG. 16.—SPAN NO. 1; FIRST RUN, SEPTEMBER 15 AND 16 (CENTERS STILL IN PLACE)

The writer was reminded of this rather perplexing action when he noted the authors' description of the drifting of the distant tower jacks during the adjustment of the Walpole-Bellows Falls structure. Contradictory and confusing phenomena are not infrequent in hydraulic adjustment setups of this character.

All in all, this paper constitutes a timely and valuable contribution to the literature. More attention should be paid to the scientific design and execution of repair work. Furthermore, it would appear that there is a greater field of utility for hydraulic jacking adjustments than is evidenced by present usage. The "weighing in" of reactions on draw-spans and on continuous trusses and girders not only has the advantage of greater accuracy but also operates to eliminate any stresses that may be thrown into the superstructure through elastic support displacements or a plastic "take-up" in the foundations. For extensive installations of this kind it would appear that there are four independent methods for stress determination which may be used as a mutual check. These are: (1) Strain measurements by means of a system of extensometers, telemeters, or other suitable device; (2) deflection measurements, including angular rotation measurements at joints and at critical sections of the structure under adjustment; (3) measurement of reaction displacements at the jacks; and (4) gage readings on the jack-pressure lines.

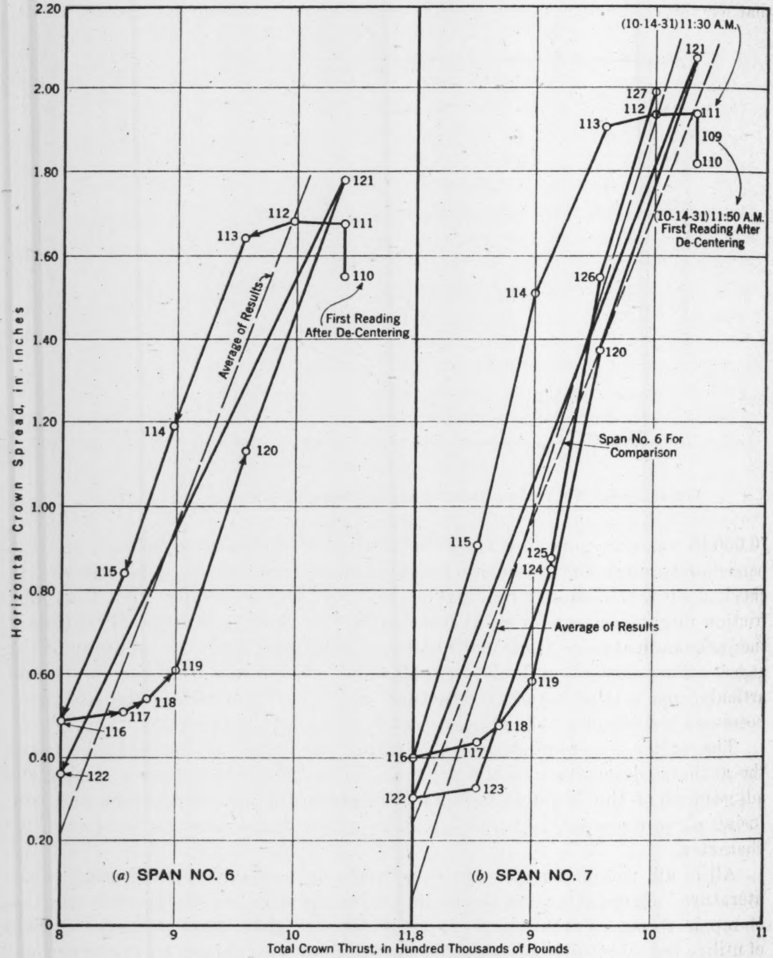


FIG. 17.—SECOND RUN, OCTOBER 14 (CENTERS REMOVED)

The paper will undoubtedly serve a most useful purpose in stimulating interest in this somewhat complicated phase of construction endeavor.

H. M. NELSON,[7] ASSOC. M. AM. SOC. C. E. (by letter).—An ice jam in the Connecticut River, composed of very large, thick cakes, which had been stalled for about four days at a point 6 miles above the Walpole-Bellows Falls steel arch bridge, passed through the bridge in the afternoon and evening of March 17, 1936. As the surface of the water and ice at that time was only slightly above the concrete protection which was placed in front of the skewbacks and extended beyond the first panel points, when the dam just below the bridge was raised some years ago, the bridge steel suffered no damage during the passage of the heavy ice jam. After the passage of the ice jam the river continued to rise, and, at about 1:00 a.m. on the morning of March 19, a crash at the bridge was heard. It is probable that the damage to the New Hampshire end of the bridge occurred at that time and that it was caused by a large ice floe. Because it was dark there were no eyewitnesses. The water was then at about Elevation 121.6.

During the early forenoon of March 19, when the Connecticut River was at Elevation 124.6, and near the peak of its flood, a floe, composed of ice that had been thrown out on a meadow during the run of the ice jam (or that had floated out of some bay that had not broken up when the ice jam came through), was seen approaching the bridge rapidly. The river bends sharply to the left in the vicinity of the bridge, and the floe was carried to the Vermont side in such a manner that the blunt front end of it struck the lower chord of the upstream arch near the second panel point. This floe appeared to have an area of a quarter of an acre or more. Just previous to the breakup, the river ice had been found to be 18 to 24 in. thick. Assuming it to be an average of 21 in. thick, the floe must have had a weight in excess of 500 tons. The velocity of the river above the bridge was about 4 miles per hr. The impact caused the entire structure to vibrate so violently that the writer, who was leaning over the bridge sidewalk railing to observe, was thrown to the floor. When the ice floe swung around toward the center of the river and passed along, it could be seen that the lower chord had a sharp offset about equal to its width. The actual damage to the New Hampshire end of the bridge was not discovered until the next day when the flood waters had receded somewhat.

The decision to reject the first plan of repair which contemplated placing a large number of pile bent supports in the river channel was a wise one, as was proved later when, in January, the ice in the river broke up over a distance of nearly 12 miles and came to rest only about 2 miles above the bridge. At this time the bridge was supported on the temporary pile bents driven at the edges of the river and protected by up-stream braced pile bents and sheeting to form a shear structure, according to the revised plan.

The authors have written a very clear and complete description of an ingeniously designed bridge-repair project completed under difficult river and weather conditions.

---

[7] Hydr. Engr., New England Power Service Co., Boston, Mass.

H. E. LANGLEY,[8] ESQ., AND EDWARD J. DUCEY,[9] ESQ. (by letter).—The writers are gratified to find that their paper was of some interest to the engineering profession, and they wish to thank the several members for their discussions. They have gained considerable satisfaction from these discussions, particularly the one submitted by Mr. McCullough, which are indicative of the careful reading that the paper received. Mr. Nelson's authoritative description of the flood and the ice floes, together with the effects on the structure, is invaluable. The January thaw to which he refers was of short duration. Had it continued for a somewhat longer period, the temporary protective works, the temporary ice fender, etc., undoubtedly would have been subjected to a severe test. Therefore, as he points out, their cost and construction were quite justified.

As revealed by Mr. French's discussion, the writers did not state that pedestrian traffic was maintained over this bridge throughout the entire progress of the work, and they did not stress sufficiently the seriousness of the property damage that would have resulted from the loss of the structure. The damage to the Power Company's plant would have been so serious if this span were dropped in the river that the safety of the structure during repair operations was the supreme consideration in planning the procedure.

The writers propose to close the discussion of their paper by considering, in connection with several features referred to in previous discussions, the costs of the various parts of the work, together with some of the more important reasons for adopting the scheme used in making the repairs.

It is interesting to note that the lateral bracing of the original structure was damaged by flood conditions the year after the bridge was completed. At that time steps were taken to provide a reinforced-concrete slab in the planes of the lower chord members from L_0 to L_2 to reinforce these members; and the necessary forms were constructed but the concrete was never placed.[10] As stated by Mr. Rights, the raising of the pool level in 1928 some 10 to 12 ft should have been contingent upon the proper protection of the abutments. The writers are not in a position to state why neither was done. The State Highway engineers recognized the hazard to which the structure was subjected and had studied several schemes for protecting it. Lack of funds, however, prevented the consummation of any of these schemes prior to the flood of 1936.

Mr. Rights states that had it been possible to proceed with the reconstruction during the summer months, falsework in the river would not have been subjected to unusual risks. Granting that this might be true, it was impossible, unfortunately, to do so for several reasons. The flood occurred in the spring, and there were several other important bridges over the Connecticut River, and other rivers as well, which were destroyed. It is obvious that some time was required to survey the damage on all of these bridges, estimate the probable cost of replacements, prepare designs, and determine on the proper sequence of construction. Except for pedestrian traffic, the reconstruction of this crossing was not so urgent as was most of the others, since there was a modern concrete arch bridge over the river a short distance below. It seemed advisable, therefore, to make only such temporary repairs as would insure the stability of the structure for pedestrians only until proper repairs could be made, and in the meantime, replace, as rapidly as possible, the structures that had been destroyed at other crossings.

---

[8] Designing Engr., Bridge Div., New Hampshire Highway Dept., Concord, N. H.

[9] Designing Engr., Am. Bridge Co., Pittsburgh, Pa.

[10] *Transactions*, Am. Soc. C. E., Vol. XLI, December, 1908, p. 264.

In some cases the State Highway engineers found it necessary to construct temporary bridges in order to maintain traffic with as little interruption as possible. To prevent loss of material in these temporary structures it was desirable to complete the construction of the new bridges at these locations prior to the high waters of the following spring. Although every effort was made to complete the program along these lines, two temporary bridges over the Connecticut River were lost a few weeks before the new bridges were ready for traffic.

The survey of the damage to this arch span and the plans for the reconstruction were made by J. R. Worcester, Hon. M. Am. Soc. C. E., and his associates, who were consulting engineers. Although these plans were furnished to the State within a reasonable time, and bids were called for shortly after the plans were received, it was not until the last of June, 1936, that bids were obtained. Obviously, then, unless the work could have been completed in about five months, which did not seem altogether probable, it would have been extended into the winter months, or otherwise delayed entirely until the following spring. The condition of the bridge was such that it seemed very unwise to let it stand through another winter, subject to floods and ice action, and the plan proposed by the consulting engineers seemed too vulnerable to the same elements to be undertaken at the time of year when the evident uncertainties involved in the work were so likely to cause delays. It was because of these considerations that the scheme finally adopted was proposed rather than because of economy alone. Even if it had been possible to proceed with the work during summer months, however, the writers are of the opinion that the scheme adopted would have been somewhat more economical.

The scheme originally proposed for doing the work called for the erection of six temporary wooden towers to support the arch trusses during the removal of the damaged ends and the reconstruction of the abutments. These towers were arranged in a general way very similar to the original falsework used to erect the span in 1904.[11] They varied in height from about 25 ft above normal water level at the ends of the span to about 80 ft near the middle. They were to be carried on piles, forming the lower stages below normal water level, some of which were to be driven on a batter to provide lateral rigidity.

For the construction of these towers 85 000 fbm of framed timber, about 15 tons of miscellaneous steel beams, and 144 wooden piles, 45 to 60 ft long, would have been needed. Long timber piles in this section of the country are difficult to obtain locally and are relatively expensive if imported from other sections. The fact that it was not considered safe to work from the bridge deck to construct this falsework (because of the damaged condition of the structure) would have necessitated some means, such as auxiliary falsework or the construction of floating equipment, to move the erection rig, pile driving outfit, etc., from tower site to tower site. The river at this location is not navigable, so that floating equipment, if used, would have to be brought in by overland transportation and assembled at the site. Furthermore, any such equipment would necessarily have been quite elaborate to meet the requirements on the work. Auxiliary falsework not only would have been expensive but would have filled the channel completely with this kind of timber construction. The original falsework used in the erection of the bridge was constructed by the use of "Chicago Booms," each tower being erected from the previous one, and the steel was erected by the same booms in a similar manner. Obviously, this method would not be practicable for erecting falsework with the structure already in place because of the interference with the operation of the booms, especially where long piles were to be driven on a batter. Altogether, this scheme seemed to the contractor's engineers to be a more expensive and hazardous procedure than the scheme adopted, which included in the material required for its construction 18 000 fbm of framed timber; 27 tons of steel beams; 63 tons of fabricated steel temporary members; 4 bridge cables; and the driving of 56 wooden piles, 45 ft long—all of which could be erected by working off the shore with comparatively simple erection equipment.

---

[11] *Transactions*, Am. Soc. C. E., Vol. XLI, December, 1908, p. 253.

In this procedure the amount of field work was considerably reduced by increasing the amount of material to be fabricated in the shops while the field work was progressing. This method, furthermore, provided an open channel with a minimum of falsework so placed that it could be protected by the construction of ice fenders extending out from the river banks.

TABLE 1.—COST DISTRIBUTION

Item No.	Description	False-work (1)	Removing and replacing steel (2)	Abut-ments (3)	Railing and deck (4)	Total (5)
1	Engineering and engineering superintendence	$ 900.00	$ 1 209.26	$ 1 100.00	$ 128.03	$ 3 327.39
2	Designing and drawings	808.00	305.92	832.82	....	1 946.74
3	Structural steel (shopwork only)	5 001.62	2 241.98	200.00	....	7 443.60
	Materials and Freight:					
4	Structural steel	3 762.38	2 695.87	2 323.00	....	14 651.25
5	Cables	3 770.00	....	....	....	
6	Falsework timber	2 100.00	....	....	....	
	Erection:					
7	Labor	3 930.00	13 726.48	4 984.90	5 122.80	27 764.18
8	Plant and equipment	1 179.00	4 117.94	1 495.50	1 182.18	7 974.62
9	Insurance (prorated)	3 084.19	3 443.30	1 276.76	818.57	8 622.82
10	Overhead	778.94	1 214.25	387.51	232.29	2 612.99
11	Profit	2 215.85	3 504.77	1 201.05	512.28	7 433.95
12	Subtotal, project on cost-plus basis	27 529.98	32 459.77	13 801.54	7 996.15	81 787.44
	Work Not Covered by Cost-Plus Agreement:					
13	Abutments	....	....	26 293.07	....	26 293.07
14	New I-beam deck and railing	....	....	....	15 930.12	15 930.12
15	Temporary ice fender	7 364.63	....	....	....	7 364.63
16	Subtotal	7 364.63	....	26 293.07	15 930.12	49 587.82
17	Total	$34 894.61	$32 459.77	$40 094.61	$23 926.27	$131 375.26

The actual cost of the project and the distribution of the costs to the different subdivisions of the work are shown in Table 1. From this table it will be noted that the cost of the falsework amounted to $27 529.98. This included all labor and materials required to lift the span from the abutments and make it ready for the removal of the parts to be replaced with new steel members, as well as the labor necessary to remove the same falsework material at the end of the job.

The contractor's estimate of the cost of the equivalent falsework required to accomplish the same purpose, working in accordance with the original plans for repairs, was $72 000, which did not include any allowance for contingencies. The difference between this estimated cost and $27 529.98, the actual cost of falsework as shown by Table 1, represents a saving in favor of the scheme adopted because the bid submitted by the contractor was tendered on a competitive basis and happened to be the only bid received, although plans and specifications were taken out by several contractors.

Adding the amount of the contractor's estimate for the falsework to the total of Column (2), Table 1, work common to both plans of procedure, makes a total of $104 459.77, which would represent the cost for the work on the basis of the original plan, exclusive of any allowance for contingencies or hazards. This amount deducted from $120 000, the amount of the original bid, leaves $15 540.23, approximately 15% to cover these items. The actual direct expenditure for contingencies is represented by the item of $7 364.63 for the cost of the ice fender. Had a second ice fender been found necessary for the protection of the New Hampshire end of the project, the total expenditure for contingencies would have been very close to the contractor's estimate allowance.

The original bid covered only the items listed in Columns (1) and (2); the State was to construct the necessary reinforced-concrete additions to the existing abutments for the protection of the steel. The decision to renew the railing and deck was made at a later date and the assignment of part of the work of reconstructing the abutments to the contractor was incorporated in the cost-plus agreement at the time it was decided to handle the project on this basis.

The reconstruction of the abutments proved to be the most difficult and time-consuming part of the work due largely to the difficulties encountered in driving the piles. Although the original plan was altered by the substitution of 8 steel H-piles for 42 wooden piles at each abutment (which facilitated the driving through the rip-rap), interference with old timber cribbing necessitated the employment of a diver to assist with the work. Had the original plan been followed the writers are convinced by the experience with the foundations and abutments that the falsework would have been destroyed and the structure would have been lost because the work, according to the revised plan, was progressing when the January flood occurred and had been delayed only about six weeks because of the revision of the plans.

In Table 1, Items 4, 5, and 6 include the cost at the bridge site of the steel bearing piles, reinforcing cages, temporary and also permanent fabricated steel, plain beams, and cables. Items 7 and 8 include the driving of the steel bearing piles, erecting all steel in the foundation and superstructure, all falsework labor, removing old steel and falsework, removal of old timber deck and railing, and painting new steel. Item 13 includes the placing and removal of the cofferdams, the placing of concrete and all reinforcing steel not included in Items 4 through 8, and the furnishing and placing of stone rip-rap around the abutments.

Reference to the tabulated costs indicates that the work covered by the original bid of $120 000 was accomplished for approximately $67 350, due to changes in the method of procedure. This in no way reflects on the original plans prepared by the consulting engineers. They were well conceived and prepared; they served excellently as a basis for the invitation for bids and for the designation of the work to be accomplished. The job was essentially a steel-bridge erection project, and on such work the choice of methods is always best left to the erectors. Governed by available equipment, personnel, and other factors, no two contractors are likely to choose the same procedure, and the success of the project often depends upon ability to recognize opportunities and the skilful change in plans to take advantage of them.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2095

WIND BRACING IN STEEL BUILDINGS

FINAL REPORT OF SUB-COMMITTEE NO. 31 COMMITTEE ON STEEL OF THE STRUCTURAL DIVISION

Introduction

After a period of activity extending over ten years, during which it has issued six Progress Reports, some of them of considerable length, the Sub-Committee submits its Final Report. It is impracticable and unnecessary, of course, to restate in detail all the facts elicited during the investigation. The more important aspects of these and the conclusions and recommendations made, however, are summarized in what follows, under the general headings of (*A*) Prescribed Wind Force; (*B*) Provisions for the Resistance of Wind Force; (*C*) Allocation of Wind Force to Braced Bents; (*D*) Types of Wind Bracing; (*E*) Methods of Analysis of Shallow Wind Bracing Systems; (*F*) Methods of Design for Shallow Wind Bracing Systems; (*G*) Methods of Design for Deep and Mixed Wind Bracing Systems; (*H*) Secondary Stresses; (*I*) Torsional Effects; (*J*) Deflection and Vibration; (*K*) Permissible Stresses; (*L*) Stability; and (*M*) Conclusions and Recommendations.

(*A*) Prescribed Wind Force

1. External Forces on Buildings with Plane Surfaces Normal to the Wind.—In its First Progress Report,[1] the Sub-Committee reviewed at some length the available information respecting the magnitude and character of the wind force that might be exerted on a building and made the tentative recommendation that, including gust effects, the force be considered as a pressure of 20 lb per sq ft on buildings with vertical plane faces for the first 500 ft of height, increased by 2 lb per sq ft for each additional 100 ft.

In consideration of the very serious effects of certain high winds that have occurred in continental United States and in Canada, and the great increase of hazard with increasing height, the Sub-Committee, in its Sixth Progress Report,[2] modified the original wind-force suggestion by recommending that the 20 lb per sq ft applicable for the first 500 ft of height be increased, above this 500-ft level, by 3.5 lb per sq ft for each additional 100 ft of height.

---

[1] *Civil Engineering*, March, 1931, p. 478.

[2] *Proceedings*, Am. Soc. C. E., June, 1939, p. 996.

Still further consideration of the matter led the Sub-Committee to accept the view of Robins Fleming[3] that the increase of the wind force with height should begin at the 300-ft level, rather than at 500 ft. Above 300 ft the force, as finally recommended, increases by 2.5 lb per sq ft for each additional 100 ft of height, thus giving 37.5 lb per sq ft at the 1,000-ft level and 50 lb per sq ft at the 1,500-ft level. The relation of this final recommendation to the previous progress recommendations and to certain other proposals is shown in Fig. 1.

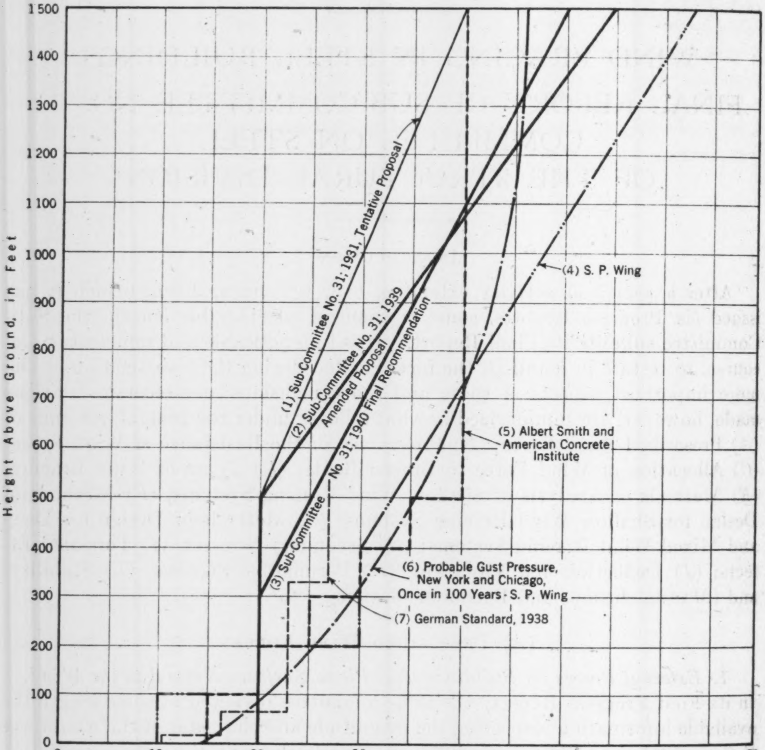


Fig. 1.—Revised Wind Force Proposed for Tall Buildings and Its Relation to Other Proposals

It is to be understood that the wind force recommended herein is in the nature of a minimum standard wind load for the United States and Canada. Special wind-force specifications should be formulated locally for areas that are definitely known to be subject to hurricanes or tornadoes. Sufficient allowance has been made in the prescribed wind force for the effect of air currents striking vertical faces obliquely, either in a lateral or vertical sense.

---

[3] *Proceedings*, Am. Soc. C. E., September, 1939, p. 1279.

From Curve (3), Fig. 1, the adopted wind force of 20 lb per sq ft for the first 300 ft of height corresponds to a wind velocity of only 77.8 miles per hr. Consequently, a substantial amount of shielding is tacitly assumed to exist. Having regard for the uncertainty respecting the permanence of the shelter for any building, and the desirability of having the same basic wind force for the lower portions of tall buildings as for low buildings, the Sub-Committee finally recommends that no additional shielding be assumed and that the omission of wind force for any portion of the height by reason of shielding should not be permitted.

2. Pressure and Suction.—As was pointed out in the First Progress Report,[1] it is not necessary to divide the wind force into pressure and suction effects when dealing with the bracing of tall buildings. However, this may need to be done for structures with rounded roofs, such as armories, hangars, and drill sheds, and for mill or other buildings with large, open interiors and walls in which large openings may occur. For any building, possibly high local suction effects, due to bad aerodynamic properties of the building profile, should be investigated in relation to secondary members and the attachments of roofing or siding.

A study made by the Sub-Committee of the wind forces observed for windward and leeward vertical walls of a large number of models and reported in its Fifth Progress Report,[4] indicated that, where q is the velocity pressure of the wind, an external pressure of 0.8 q may be exerted on the windward wall of buildings of average ratio of height to width and an external suction of 0.5 q on the leeward wall. For an air density of 0.07651 lb per cu ft, corresponding to 15° C, at 760 mm of mercury, and where the velocity V is expressed in miles per hour, the velocity pressure in pounds per square foot is given by

$$q = 0.002558\ V^2 \qquad (1)$$

From Eq. 1, it is seen that the total combined force on the outside of the windward and leeward vertical faces of an average building may be 1.3 q, or approximately,

$$p \doteq 0.0033\ V^2 \qquad (2)$$

On the basis of Eq. 2, the wind velocity corresponding to a wind force of 20 lb per sq ft is 77.8 miles per hr. The velocity pressure for this wind velocity, according to Eq. 1, is 15.5 lb per sq ft.

3. External Forces on Plane Surfaces Inclined to the Wind.—Following a study of all the available experimental data, the Sub-Committee, in its Fifth Progress Report, proposed certain external wind forces on the sloping surfaces of symmetrical gable roofs. These have been slightly revised and have been restated so as to give the force in pounds per square foot, rather than in terms of the velocity pressure. For surfaces not more than 300 ft above the ground and for both symmetrical and unsymmetrical gable roofs, where α is the slope of the roof to the horizontal, in degrees, the recommended wind forces are as follows:

---

[4] *Proceedings*, Am. Soc. C. E., March, 1936, pp. 398–407.

(1) Windward Slope—

(*a*) For α not greater than 20°, a suction of 12 lb per sq ft.

(*b*) For α between 20° and 30°, a force of

$$p = 1.20\,\alpha - 36 \qquad (3)$$

the negative value indicating suction.

(*c*) For α between 30° and 60°, a force of

$$p = 0.30\,\alpha - 9 \qquad (4)$$

the positive value indicating pressure.

(*d*) For α greater than 60°, a pressure of 9 lb per sq ft.

(2) Leeward Slope—

(*a*) For all values of α in excess of zero, a suction of 9 lb per sq ft.

The external wind forces applied to the windward slope are indicated graphically in Fig. 2.

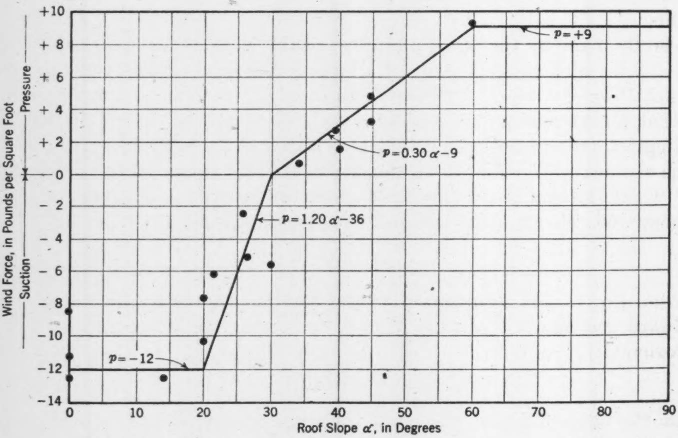


FIG. 2.—RELATION OF EXTERNAL WIND FORCE ON WINDWARD SLOPE OF GABLE ROOF TO ROOF SLOPE

4. External Forces on Rounded Roof Surfaces.—A corresponding study of the observed external wind forces on rounded, or approximately rounded, roof surfaces, reported in the Fifth Progress Report,[4] led to recommendations which in this Final Report are revised and stated in pounds per square foot of surface. Considering the rounded roof as represented roughly by a circular arc passing through the two eaves and the ridge, or, where the roof slope starts from ground level, through the springings and the ridge, and letting the ratio of rise of the arc to its span be r, the recommended wind forces, for surfaces not more than 300 ft above ground level, are as follows:

(1) Windward Quarter of Roof Arc—

(*a*) For roofs resting on elevated vertical supports:

When r is less than 0.20, a suction of 12 lb per sq ft.

When r is greater than 0.20, a pressure of

$$p = 30\,r - 6 \qquad (5)$$

Alternatively, when r is between 0.20 and 0.35, a force of

$$p = 80\,r - 28 \qquad (6)$$

the negative sign indicating suction.

(*b*) For roofs starting from ground level:

For all values of r, a pressure of

$$p = 19\,r \qquad (7)$$

(2) Central Half of Roof Arc—

(*a*) For roofs resting on elevated vertical supports, a suction of

$$p = 15\,r + 11 \qquad (8)$$

(*b*) For roofs starting from ground level, a suction of 11 lb per sq ft.

(3) Leeward Quarter of Roof Arc—

For all values of r greater than zero, a suction of 9 lb per sq ft.

The external wind forces applied to the windward quarter and to the central half of the roof are indicated graphically in Fig. 3.

5. *External Forces on a Flat Roof.*—For a flat roof a normal suction of not less than 12 lb per sq ft should be considered as applied to the entire roof surface.

6. *External Forces on Walls Parallel to the Wind.*—On walls parallel to the wind an external suction of 9 lb per sq ft should be considered.

7. *Internal Wind Forces.*—As was pointed out in the Fifth Progress Report,[4] the wind force to be assumed in design is to be taken as the applicable external wind force (given in paragraphs 3–6) only in case the building is airtight. Such a condition will rarely arise. Normally, air leakage due to the usual small openings around windows, doors, skylights, and eaves will give rise to an internal pressure or suction of from 0.25 q to 0.35 q, depending on whether the openings are chiefly in the windward or in the leeward surfaces.[5] In the rapid building up of gust velocities, the air transfer may be so slow that the internal force is less than this. In standard N–790 of The Netherlands,[5] the external wind forces have all been adjusted to take account of an internal suction of 0.2 q.

Where openings are of substantial size, the internal wind force may be of considerable magnitude. It may be pressure or suction, depending on whether the openings are in windward surfaces, in leeward surfaces, or in surfaces parallel to the wind. Very large internal pressures have been, and may be, built up because of the breaking of windows on the windward side of buildings by reason of flying gravel from the roof or other objects carried by the wind. The Sub-

---

[5] *Proceedings*, Am. Soc. C. E., September, 1936, pp. 1116–1119.

Committee is of the opinion that the only buildings for which the internal wind force should be restricted to the amounts mentioned in the preceding paragraph are those for which the construction is such that the doors and windows cannot be broken in, or with only very small areas taken up by windows and doors.

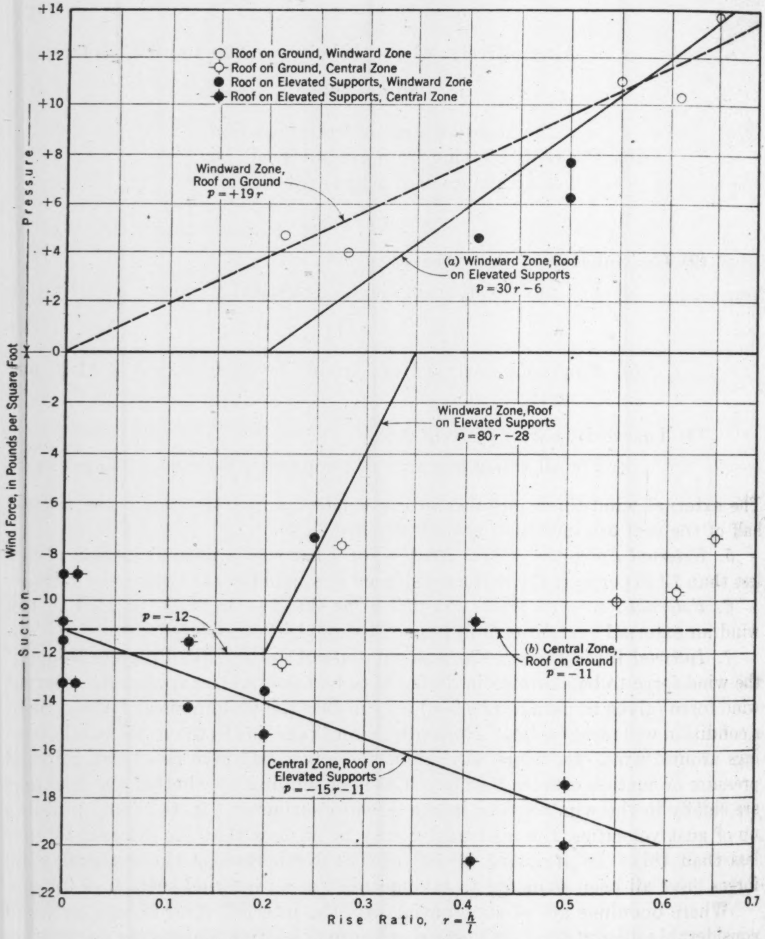


FIG. 3.—RELATION OF EXTERNAL WIND FORCE ON ROUNDED ROOF TO RISE RATIO

Particularly large internal forces may arise when the windward or the leeward side of a building is completely open. The German regulations, effective since June 18, 1938,[6] provide that, for structures which are entirely open on one or more sides, or which can be opened, or which by reason of one or more openings in one or more sides are at least one third open, or can be opened to this extent, a normal internal force of 1.2 q acting outwardly is to be assumed as applied to the underside of the roof. This is in addition to the external suction of 0.4 q on the leeward side of plane roofs, and is specified with a view to insuring proper anchorage of the roof as well as of the attachment of roofing sheets and purlins.

---

[6] Regulations for wind loads contained in DIN 1055, Blatt 4, *Stahlbau-Kalender*, 1939, pp. 182–185.

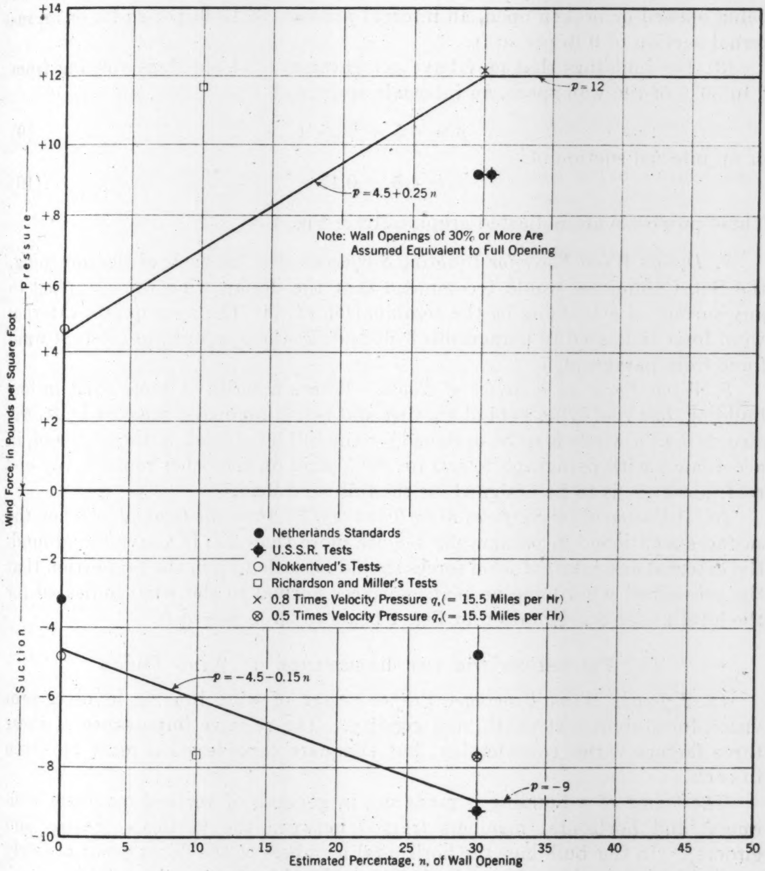


FIG. 4.—INTENSITY OF INTERNAL WIND FORCE

Based upon the investigations reported in the Fifth Progress Report,[4] and on subsequent study and observation, it appears that reasonable allowances for internal wind force would be as follows:

(1) For buildings that, although nominally airtight, with closed doors and windows and unbroken glass, are nevertheless more or less "air leaky," by reason of numerous distributed small openings, an internal pressure or suction of 4.5 lb per sq ft normal to walls and roof.

(2) For buildings with 30% or more of the wall surfaces open, or subject to being opened or broken open, an internal pressure of 12 lb per sq ft, or an internal suction of 9 lb per sq ft.

(3) For buildings that may have percentages, n, of openings varying from 0 to 30% of the wall space, an internal pressure of

$$p = 4.5 + 0.25\,n \qquad (9)$$

or an internal suction of

$$p = 4.5 + 0.15\,n \qquad (10)$$

These proposals are indicated graphically in Fig. 4.

8. Design Wind Force for Building Surfaces.—On the basis of the foregoing, the Sub-Committee would recommend that the design wind force applied to any surface of a building be the combination of: (1) The appropriate external wind force indicated in paragraphs 3–6; and (2) the appropriate internal wind force from paragraph 7.

9. Wind Force on a Series of Roofs.—Where a series of roofs exist in one building, one roof lying behind another and being nominally masked by it, the structure as a whole is to be designed for the full wind load on the first roof, in accordance with paragraph 8, and for 80% wind on the other roofs. Any one roof, however, is to be designed for the full wind load.[7]

10. Adjustment for Surfaces More Than 300 Ft Above the Ground.—When the surfaces mentioned in paragraphs 3–8 are more than 300 ft above the ground, the external and internal wind forces should be scaled up in the proportion that the prescribed wind force on plane surfaces normal to the wind indicated for the level under consideration in Fig. 1 bears to 20 lb per sq ft.

(*B*) Provisions for the Resistance of Wind Force

11. Effective Wind Bracing.—Proper design of wind bracing involves provision for stability, strength, and rigidity. The relative importance of these three factors varies considerably, but adequate consideration must be given to each.

The frame of a building is made up, in general, of vertical members (columns) and horizontal members framed between the verticals (beams and girders). In tier buildings the horizontal members of the floors occur at fairly regular intervals throughout the height and, with the floor slabs, form horizontal diaphragms uniting the columns at each floor level. The purpose of the wind bracing is to preserve this frame from undue distortion when subjected to horizontal forces.

In a building frame as described, the tendency of the panels formed by the columns and their connecting beams or girders to distort, due to horizontal thrust, is resisted by the connections at the columns and by the members in bending and direct stress. Where several columns occur in a line, they form a bent which (to a degree depending on the rigidity of the beams, their connections, and the nature of the bracing utilized) tends to resist the horizontal thrust in the manner of a vertical truss with the columns forming multiple chords, and the columns, beams, and girders forming the web. The columns on the windward side of the axis of the bent become tension chords and those on the leeward side, compression chords.

---

[7] DIN 1055, Blatt 4, p. 2.

In a successful design the building must be assumed to act as a unit, and not distort appreciably when subjected to the applied lateral loads.

12. Floors as Horizontal Diaphragms.—The floors of a building must act as horizontal diaphragms for the delivery of wind-load increments that arise in each story to the braced bents that are provided to receive them. Bracing planes usually run in more than one direction, and the floors must possess the necessary strength and rigidity under loads in their own planes to deliver these wind loads. If the floor is incapable of serving as a horizontal distribution unit, horizontal bracing should be provided between the braced bents. Most of the customary systems of floor construction used in fireproof buildings possess the requisite strength for this purpose.

13. Rôle of Walls and Partitions.—As ordinarily constructed, the walls of a modern tall building cannot be relied upon to absorb any appreciable fraction of the applied wind force. Apart from the uncertainty as to whether the walls act integrally with the frame at the outset, it is obvious that if stressed heavily under horizontal loading they would crack and cease to be a dependable element of strength.[8] In setback buildings, the walls in the planes of the tower sides, of course, are discontinued below the roof of the widened portion of the building. Nor can wind resistance be counted upon for partitions, which, in general, are removable. For these reasons, whatever may be their rôle in lessening deflection and vibration, walls and partitions in buildings having a high ratio of height to width should be ignored in strength calculations, and provision should be made in the structure frame for the entire recommended wind load.

It is recognized, of course, that buildings of low height-width ratio having heavy continuous walls derive much bracing advantage from the walls. Whether added bracing is necessary or not must be left to the analysis and judgment of the designer in each particular case.

(*C*) ALLOCATION OF WIND FORCE TO BRACED BENTS

14. Placement of Wind Bracing.—Whereas in buildings of moderate height-width ratio it is generally possible to adapt sufficient bracing to almost any proposed frame, in tower-like buildings placement of adequate bracing should be given attention in the early architectural studies.

If the floors meet the requirements already given (or in the event that they do not, if they are adequately supplemented by bracing in horizontal planes), there need be no maximum limit of spacing of braced bents, nor any restriction imposed on carrying all the wind force to the end or side walls, if the bracing therein is sufficient to provide for the entire wind force.[9]

---

[8] *Proceedings*, Am. Soc. C. E., June, 1939, p. 1024.

[9] *Civil Engineering*, March, 1931, p. 482.

In order to avoid obstruction of desired clear spaces, bracing often may be placed advantageously in outside walls, or between elevators or around permanent shaft or service areas. Provision can be made in the floor construction for horizontal transfer of shears at offset levels.

15. Wind Resistance in Accordance with Bent Stiffness.—Granted that the floors are capable of serving as effective distribution diaphragms for the applied wind loads, and that the columns at a given floor level remain at fixed distances from one another, the various braced bents passing through the floor can be considered as taking loads in proportion to their rigidities and inversely as the deflections that would be produced in them under equal loads. For symmetrical towers under symmetrical loads the deflection of the braced bents at any floor will be equal and the loads will be distributed in proportion to the bent rigidities.

The method of determining the relative indexes of rigidity was explained carefully in the exposition of the Spurr method in the Appendix of the Third Progress Report.[10]

(*D*) TYPES OF WIND BRACING

16. General.—The type and arrangement of wind bracing required will vary greatly with the size and shape of the building, and the judgment and experience of the designing engineer will have an important influence on its form, placement, and details.

Although it is not practicable to standardize the types of bracing suitable for special buildings, such as armories, hangars, sheds, and mill buildings, certain types are repeatedly used for tier, or multiple-story buildings.

For buildings of moderate height and proportions, shallow bracing involving some form of "knuckle" connections is most frequently used throughout, as was pointed out in the Second Progress Report.[11] This method of providing against wind force entails considerable deflection of the building, and consequently for high or narrow frames the inability of such bracing to hold deflection within reasonable bounds without greatly increasing the column and beam or girder sections makes it very desirable to adopt diagonal bracing of some kind.

Bracing consisting of full diagonals, knee braces, or *K*-frames, with various arrangements in respect of openings in the panel, have been shown in the Second Progress Report. The most effective and economical bracing consists in introducing either single or double diagonals crossing from one intersection of a girder with a column to the diagonally opposite intersection.

17. Combination of Bracing Types.—When gussets, knee braces, or frames are introduced, they add materially to the strength and rigidity of the beams and columns, so that their use tends to concentrate the wind resistance in the panels where they are employed. Although the use of one type of bracing in one panel of a bent and another type in adjoining panels presents no great difficulty, analysis must be based on equal web deflections in such panels, so that the bent may work as a unit.

In the design of wind bracing details the paramount consideration is the lessening of distortion and the prevention of large bent deflections arising therefrom. Desirable practice relating to the design of details was presented in the Second Progress Report.[11]

---

[10] *Proceedings*, Am. Soc. C. E., December, 1933, p. 1621.

[11] *Loc. cit.*, February, 1932, pp. 220–222.

(*E*) METHODS OF ANALYSIS OF SHALLOW WIND BRACING SYSTEMS

18. General.—For investigating the adequacy of an existing or proposed system of wind bracing, whether properly proportioned or not, a convenient and sufficiently accurate method of analysis must be available. To this end the Sub-Committee has reviewed a number of existing methods applicable to bents with shallow bracing and has developed some others of novel character.

In the Third Progress Report[12] the merits and limitations of the portal and the cantilever methods were discussed and a detailed illustration of the application of the Spurr method to a typical situation was given. The general status of these methods may be stated briefly as follows:

19. The Portal Method.—The portal method, which is generally satisfactory for buildings up to 25 stories in height and of moderate height-width ratio, does not commonly concern itself with elastic relationships. Its virtue lies in simplicity. The assumed distribution of horizontal wind shear is purely arbitrary. With equal beams on equal spans, excess stiffness in the exterior columns tends to increase the shear taken by the outside panels, particularly in low buildings with large story heights, where the columns are relatively flexible and the chord effects are negligible. However, in a high building, the chord action overbalances the web effect of the extra stiff outside columns, and the inside panels actually carry more shear than the outside panels. With irregular column spacing, errors may also occur due to the relation of beam stiffnesses being inconsistent with portal behavior.

20. The Cantilever Method.—The simple cantilever method, which rests on the assumption that the axial wind stresses in the columns, regardless of column areas, are proportional to the distance from the neutral axis of the bent, is generally suitable for buildings of moderate height-width ratio, not more than 25 to 35 stories high. Although it is less frequently used than the portal method, it is more suitable than the latter for high, narrow buildings.

The horizontal wind shears in the various panels are arbitrarily distributed irrespective of the relative stiffness of girders and columns, and of the ability of the girders to transfer increments of column stress as required in cantilever behavior. The degree of accuracy, therefore, will be largely incidental. With equal beams on equal spans, inaccuracy results from the unequal drifts from column and beam bending produced by the assumed stress condition. Unequal beams on unequal spans may cause similar inaccuracies. The resulting design may, or may not, be an improvement from the standpoint of accuracy in any particular case, as compared with the design which would have been obtained by the use of the portal method.

21. The Spurr Method.—For frames having a height-width ratio of 4 or more, increased accuracy of treatment becomes more and more necessary as the slenderness ratio increases. The Spurr method is fundamentally sound for all buildings, since it is based on true elastic behavior and concerns itself with the geometry of distortion. It rests upon the assurance that all members have been so proportioned that the frame will act as a true cantilever and that the floors remain plane during and after deflection. It may be used in whole or in part for buildings of moderate height.

---

[12] *Proceedings*, Am. Soc. C. E., December, 1933, p. 1618.

In an elastic structure of multi-story type which is subjected to horizontal wind forces, the true moments and shears in the various members are dependent upon the relative rigidities of all the members, if they are properly connected. Inelastic action or slip in the connections will affect the distribution of stresses throughout the structure, and such effects are to be guarded against as far as possible by care in the design of the connections. Elastic action in the connections, whether deep or shallow, or in knee braces or frames, is unavoidable, and quite properly such effects should be ascertained and evaluated in the design.

In high frames the change in length of columns under axial wind stresses is an important element. In real wind problems, such as are presented by high, slender frames, the aim should be the creation of a design which is "prudently" or "correctly" proportioned. In a frame so proportioned, the change in length of columns will have no material effect upon the web analysis. On the other hand, a badly proportioned frame will produce irregular changes in the lengths of columns under axial wind stresses, which condition will make it practically impossible to obtain an accurate analysis.

From a practical standpoint, therefore, it is essential that the elastic proportions of the web system be controlled in such a manner as to simplify the calculations. This is readily done by an assumed distribution of the horizontal wind shears throughout the structure so as to maintain a plane after bending in the various wind bents. If the building is to act as a unit, all bents must deflect alike.

The primary objective of the Spurr method, which is in reality more a method of design than one of analysis, is to insure that each wind bent acts as a true cantilever. Once this condition is attained, the investigation of stresses and deformations is simplified very greatly.

22. The Cross, Goldberg, and Grinter Methods.—In the Second Progress Report, a comparison was made of the stresses originally calculated by slope deflection in the classic 20-story Wilson and Maney bent with those found by the Cross method of distributing end moments.[13] This showed the entire sufficiency of the Cross method, provided that secondary bending stresses due to column axial distortion from wind are ignored. The greatly increased speed of calculation as compared with the slope-deflection method was particularly apparent. A concise statement of the successive steps to be taken in the application of the Cross process was presented in the report.

The Sub-Committee also examined other methods based upon theoretical considerations. In the Fifth Progress Report[14] the applicability of the Goldberg[15] and Grinter[16] methods was discussed. The method introduced by Henry V. Spurr, M. Am. Soc. C. E., in 1930,[17] which takes account of the axial distortions of columns due to wind, was outlined in some detail in the Third Progress Report.[18]

---

[13] *Proceedings*, Am. Soc. C. E., February, 1932, p. 213.

[14] *Loc. cit.*, March, 1936, p. 407.

[15] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 962.

[16] *Loc. cit.*, p. 610.

[17] "Wind Bracing, the Importance of Rigidity in High Towers," by Henry V. Spurr, McGraw-Hill Book Company, Inc., New York, N. Y., 1930.

23. Witmer Method of K-Percentages.—In the Sixth Progress Report,[19] consideration was given to the relation between the vertical wind reactions developed at the various columns of a symmetrical 4-column bent under the action of wind forces. These relations were determined in connection with varying dimensions of bent and varying sizes of girders and columns, the relations being derived by theoretical considerations, as well as from experimental tests of model bents made by Francis P. Witmer, M. Am. Soc. C. E., and Harry H. Bonner.[20] Diagrams were presented to facilitate computation of wind reactions for such bents.

Careful consideration was also given to the relations between vertical wind reactions of columns and actual dimensions for bents, both symmetrical and unsymmetrical, and with varying numbers of columns. Conclusions were based upon theoretical grounds by a consideration of the behavior of the bents as determined by the Cross method of analysis. A routine procedure called "a method of K-percentages" was described, based upon the actual sizes and dimensions of the bent, by consideration of which the vertical wind reactions at all columns might be obtained with reasonable accuracy. From these reactions, by a statically determinate process, the wind moments may be readily computed. For bents having a wide variety of proportions, these moments were found to be in sufficiently close agreement with results obtained by rigid theory to be considered generally permissible, and by reason of their rational basis to be preferable to those found by the older, approximate portal and cantilever methods, which used reactions developed according to some arbitrary assumption of behavior, regardless of the actual sizes in the bent.

It should be pointed out, with reference to a comment of Samuel T. Carpenter,[21] Assoc. M. Am. Soc. C. E., that, although the method of K-percentages was not derived from any concrete mathematical formula, it had its origin in phenomena discovered to be inherent in the application of Cross analyses to a great number of differently proportioned bents. As stated in the Sixth Progress Report,[19] it was observed that in every case the ratios of vertical wind shears in the different bays changed in value very slightly from cycle to cycle, and that their value as found from a simple consideration of the stiffness factors of the girders and columns was practically the same as after a complete Cross analysis. The sufficiency of a 3-story compromise bent for determining these shear ratios, and therefore the vertical wind-reaction ratios, was inferred after a comprehensive experimental study of Cross-analyses phenomena in such bents.

The engineer is not always concerned with the preparation of a new design. His duties frequently call for an investigation of an existing structure or the check of another's design whose method of computation is not known to him. He must then compute wind stresses with nothing before him except a set of actual or designed sizes. In such a case, unless the sizes happen to be in such relation as to develop the cantilever type of reactions, the Spurr method will be inapplicable. Then the only recourse generally would be to one of the more laborious rigid theoretical methods. In this contingency it is believed that the proposed method of *K*-percentages may prove of practical value. The preliminary design required before this method can be applied should not constitute an insuperable objection. Such a condition is inherent in practically all bridge designing problems which involve structures of a statically indeterminate character, and the Spurr method itself is not free from this requirement, as the sectional areas of columns must be determined before wind reactions in accord with the cantilever principle can be found.

---

[18] *Proceedings*, Am. Soc. C. E., December, 1933, p. 1611.

[19] *Loc. cit.*, June, 1939, p. 969.

[20] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 416.

[21] *Proceedings*, Am. Soc. C. E., November, 1939, p. 1635.

Discussions of the Sixth Progress Report have indicated that the method of *K*-percentages might be considered to possess practical utility. In this matter, however, the intention of the Sub-Committee has not been to advocate a new method of computation as a substitute for those already in use, but rather to indicate the desirability and practicability of determining at the outset, with reasonable correctness, all external reactions resulting from any assumed set of wind forces acting upon a bent of any assumed proportions. Wind moments computed from these reactions are considered to have a more dependable basis than those found by the older approximate methods, while at the same time the great expenditure of time and labor required by methods which are in more or less precise accord with rigid theory is avoided.

24. Applicability of the Various Methods to the Problem of Analysis.—Methods of computation are so largely a question of individual judgment among engineers that the Sub-Committee hesitates to recommend one method in preference to another. In view of the studies which it has made with regard to bents having small connections in pure bending, however, it feels justified in offering the following suggestions which bear upon the appropriateness of specific methods under certain specific conditions:

(1) If a preliminary design of the bent is made to accord with vertical floor loads alone, with an assumption of discontinuous girders, and with certain other assumptions as recommended by Professor Witmer in a discussion of the Fifth Progress Report,[22] and if, furthermore, secondary moments due to axial distortion of the columns under direct wind stress are neglected, direct wind stresses and vertical wind reactions in the interior columns may be considered equal to zero and the bent may be properly analyzed for primary wind stresses by the portal method.[22, 23]

(2) If the bent has been proportioned by the Spurr method,[17] true cantilever behavior will exist, floors will remain practically plane, and the secondary bending stresses from column axial distortion will be negligible and properly may be ignored.

(3) If any other method of designing is adopted than is described under paragraphs (1) or (2), and if wind stresses are then computed by assuming arbitrarily a particular behavior of the bent under wind, without due consideration of the relative sizes of girders and columns, there is no assurance whatever that the assumed behavior will be realized. Consequently, wind reactions and resulting stresses from wind will be without theoretical justification.

---

[22] *Proceedings*, Am. Soc. C. E., November, 1936, pp. 1496–1498.

[23] *Loc. cit.*, March, 1936, p. 409.

(4) If a preliminary design is first made upon any selected basis, primary wind stresses always may be computed by any of the rigid theoretical methods or their modifications—for example, by the slope-deflection method or by the methods of Cross,[13] Goldberg,[15] or Grinter.[23] Of these methods, slope deflection is the least practicable from a standpoint of time and labor required. The Cross method is far more advantageous in these respects, but is still very laborious. The Goldberg and Grinter methods offer abbreviations in the amount of work with some compensating loss in exactness, but the Sub-Committee's study of them indicated that they possess practical advantages and produce substantially correct results.

(5) Having a preliminary design made by any assumed method, or an existing structure whose method of design is unknown, primary wind stresses may be computed very readily and with sufficient theoretical correctness by the method of K-percentages.[19]

(*F*) METHODS OF DESIGN FOR SHALLOW WIND BRACING SYSTEMS

25. Non-Elastic Methods.—Having assumed the proper applicability of one of the methods of analysis outlined in part (*E*) of this Report, the determination of wind moments, shears, and thrusts and the proportioning of members and details naturally follows, subject, of course, to whatever inexactness may be involved in the theory adopted. Methods that ignore the elastic properties of the bent, such as the portal and the simple cantilever methods, have distinct limitations, as was stated in paragraphs 19, 20, and 24.

26. The Spurr Method.—Assurance of bent behavior in accordance with theoretical assumptions can be had with all necessary accuracy by proportioning according to the Spurr method, which is in accordance with the principles of elasticity. A preliminary design is required only for columns in order to obtain their sectional areas, from which vertical wind reactions can be determined that will be in keeping with the cantilever theory. The complete design may then be made in accordance with the elastic principles upon which the method is based and, in consequence, the relation of reactions will be the same in the finished structure as in the preliminary design.

To produce the reactions that are associated with true cantilever action, a definite relation must exist between the moments of inertia of girders in the several bays of any floor, and this always will require relatively larger girders in some bays than would be required for vertical floor loads alone.[20] The net effect of this condition upon the ultimate economy of the design cannot be ascertained without further investigation. There would appear to be justification for an inference that this method might lead to some loss in economy as compared with a bent designed in accordance with paragraph 24 (1). The Spurr method, however, has a further advantage of controlling the rigidity of the bent against horizontal deflection. If design upon the principles of the Spurr method is desired, an *a priori* determination of the approximate relation between moments of inertia of girders in the several bays may be obtained readily by use of the formula ΣI varies as $V_g L_g^2$, having first found values of V_g by the cantilever theory.[24]

---

[24] *Proceedings*, Am. Soc. C. E., June, 1939, p. 977, and Fig. 5(*d*), p. 983.

In following the Spurr method,[25] the designer's first step should be the preparation of preliminary plans and a column schedule based on dead-load and live-load requirements. The next step is the selection of vertical bents in the frame in each direction which can reasonably be expected to act in resisting the wind. The relative rigidities of the several bents should then be ascertained, based on the analysis of the bents for chord deflection. The proper percentage of the total wind load on the structure should be distributed to each bent in such a manner as to make the chord deflections equal, based on the assumption that horizontal planes through the bents remain plane after bending. Usually, the web deflections of the several bents (that is, the deflection due to the distortion of the beams and their connections and the bending of the columns) can be made alike by "tuning," so that the bents will give equal deflections under the allocated loadings. This principle must hold whether deep or shallow bracing is used. To this end it may be necessary to stiffen some panels and loosen others.

The foregoing conditions determine the amount of wind shear in the various panels of each bent throughout the structure. The web system should then be designed to produce the same drift in all panels in any one story. With the wind shears in the panels of each bent fixed, the web deflections in all panels of the bent must then be the same. In extremely high and slender buildings the designer should consider the necessity of placing a limitation upon the static deflection permitted under the design load.

(*G*) METHODS OF DESIGN FOR DEEP AND MIXED WIND BRACING SYSTEMS

27. General.—Since for equal deflections deep bracing is much more economical than shallow bracing, it should be used whenever practicable. The wind force and the possible effect of gravity loads may be readily cared for in the design in accordance with statical principles.[17]

Complications of building layout often render uniformity of type in wind bracing for different parts of a frame impossible of attainment. Several different types may be used in the various bays of a braced bent in one story but, so, the design should be in accordance with the principle that the estimated web deflections in the various bays should be the same.

(*H*) SECONDARY STRESSES

28. General.—In recent years, considerable thought has been given to more exact methods of design and analysis, especially the latter. Columns and beams subject to bending moments bend, and members subject to direct axial stress change in length. Such actions in the frame have a direct bearing on the distribution of wind shears, and, consequently, on the bending moments in the members.

In attempting to arrive at a more accurate analysis of frame action, the slope-deflection and the Cross methods were developed, both aiming to reach true shear distribution. Both methods are somewhat cumbersome in application for many-storied frames, and, to keep them at all workable, the assumption is usually made that the joints remain fixed in elevation and that there is no change in length of any of the members.

---

[25] *Proceedings*, Am. Soc. C. E., December, 1933, pp. 1621–1631.

In a high building which has been proportioned without due attention to the need for establishing such elastic relationships as will lead to regularity in the vertical movements of the successive floor joints, these movements will be irregular and not readily calculable. The existing complexity is evident when it is remembered that not only do the vertical reactions of the web system upon the columns affect the movement of the joints, but these movements affect the reactions. It is true, unfortunately, that an accurate analysis of an injudiciously proportioned frame of many bents and stories is a difficult and impracticable task.

In actual wind problems the aim should be the creation of a design that is so proportioned that the floor joints will move in a predetermined and regular manner in order that the predetermined stresses may be realized. An elastic relationship yielding this desirable result is attained when the floor joints are maintained in a plane after bending. Such action is possible only when the web system is so proportioned and connected to the columns as to transfer to them the vertical increments of load consistent with true cantilever action. Consequently, the beams in the interior panels must be more rigid than those in the exterior panels. As stated in paragraph 21, the Spurr method of design[25] produces a bent which will maintain planarity of floors and in which the lengthening and shortening of the columns will not affect the accuracy of the calculations.

Unless the bent acts as a cantilever under wind, as assumed in the Spurr method, secondary wind moments can be computed only by a second operation after primary direct wind stresses in columns have first been found by some other process, as the *K*-percentage method. This can be accomplished by applying the Cross method to the axial wind distortions in columns, thus determining separately the secondary moments by which the primary moments should be corrected.[13] This is a laborious process. It is believed that a satisfactory abbreviated procedure may be devised for the purpose but, so far, this has not been achieved.

29. Relative Magnitude of Secondary Effects.—According to the portal method of design, which allocates all the column direct stresses to the exterior columns, and consequently produces the same shear in all panels of the bent, the axial deformations of the exterior columns due to wind stress result in nonplanarity of floors and secondary moments in the members. George E. Large,[26] M. Am. Soc. C. E., has investigated the effect of direct deformation of the columns on the wind stresses in portal-designed bents, and states:

> "The results of two investigations conducted upon portal-designed bents show that correction for the direct deformation of columns is hardly necessary in bents of height to base ratios less than 5, while for bents twice as slender the corrections should be made to avoid large errors in girder shears, on the side of danger."

The investigations of Professor Large show that there is a decided tendency for a portal-designed bent of three equal panels to swing over to cantilever action. This results in errors in the assumed girder shears of from 10% to

---

[26] "Settlement Stresses in Continuous Frames," by George E. Large, *Bulletin No. 108*, Eng. Experiment Station, Ohio State Univ.; also, *Proceedings*, Am. Soc. C. E., June, 1939, p. 984.

15% in a 26-story bent with a height-width ratio of 5, and errors as high as 41% in a 55-story frame with a height-width ratio of 10.

(*I*) Torsional Effects

30. General.—Buildings which are symmetrical about two axes, both as to exterior and as to arrangement of bracing, are unusual, except for monumental towers and free-standing tower buildings. Even on a symmetrical building the distribution of the wind pressure is not uniform, due to surrounding structures and to eddies caused by the building itself. Although the wind pressure may act symmetrically over the face of a building, if the building is unsymmetrical in plan about an axis parallel to the direction of the wind, the eccentricity of the center of rotation of the building with respect to the resultant wind force will cause torsion and unsymmetrical stress distribution. If the building is unsymmetrical about both axes, a wind from any direction will cause torsion.[27]

31. Method of Analysis.—An analysis of torsional stress distribution, based upon the work of Albert Smith,[28, 29] M. Am. Soc. C. E., was prepared by Professor Large and published in the Sixth Progress Report of the Sub-Committee,[30] to which attention is directed.

As a result of the analysis mentioned, it appears that, although a rapid and often sufficiently accurate method may be based upon the neglect of all deformations except girder bending, it is desirable to include the effect of axial length changes of the columns. Professor Large found that over-all economy does not lie in the elimination of torsional moment by the stiffening of certain bents so as to move the torsional center to the line of action of the resultant wind force.

(*J*) Deflection and Vibration

32. General.—Quite apart from the obvious necessity for providing adequate strength in structural members and details and for the proper distribution of wind forces among the bents, care must be taken to insure that deflections and vibrations are kept within such limits as render tall buildings comfortably habitable.

Only experience will reveal the relation between computed maximum deflection and comfortable occupancy. By reason of the present impossibility of appraising in advance the restraining or damping effect of the non-skeleton parts of a building upon deflection, it is convenient to use, as a measure of the stiffness, the relation between the maximum deflection of the top of the frame and its height, assuming the frame to carry the entire lateral force. This might perhaps be called the "deflection index." Because of the damping effect of walls, partitions, and floors, it does not represent the actual deflection, but some multiple of that quantity. Nevertheless, it is a convenient standard by which to judge the probable stiffness of a building.

It has been found that tall buildings with a deflection index of 0.002 under assumed triangular loadings somewhat lighter than those recommended by the

---

[27] "Final Report of the Committee of the Structural Division on Florida Hurricane," *Proceedings*, Am. Soc. C. E., August, 1928, p. 1757.

[28] "Wind Bracing," by Albert Smith, *Journal*, Western Soc. of Engrs., February, 1933, p. 1.

[29] "Basis of Design for Hurricane Exposure," by Albert Smith, *Proceedings*, Am. Concrete Inst., Vol. 27 (1931).

[30] *Proceedings*, Am. Soc. C. E., June, 1939, pp. 988–996.

Committee have a satisfactory behavior in the matter of deflection and vibration. In very high buildings, this value represents not only a satisfactory occupancy basis, but about the upper practicable limit of attainable stiffness without large additions to the girder sizes for wind effect only. It is possible that higher theoretical frame deflections would be tolerable, but relaxation in this regard should come only as a result of accumulated experience.

Instances are known of well-designed buildings of 20 to 25 stories in height, having deflection indexes upward of 0.004 or 0.005 that have behaved satisfactorily. In addition to securing comfortable occupancy, a proper limitation of deflection guards against injury to internal and external masonry.[8]

33. Computation of Deflections.—Although, as previously stated, the actual deflection of a building is only a fraction of that computed, nevertheless the computed deflections of the bents under a unit wind load provide the only rational basis for apportioning the total wind force among the bents, and for judging flexibility.

The theoretical deflection of a wind bent is due to (1) chord action and (2) web action. The former arises from direct lengthening and shortening of the columns, whereas the latter arises from (*a*) reverse bending of girders, (*b*) reverse bending of columns, and (*c*) deformation of connections and bracing.

34. Chord Deflection.—In designing a tall building, the chord deflection of each bent due to a unit loading is computed as soon as the column schedules for dead load plus live load are available. Then the wind is divided among the bents inversely as the chord deflections, thus producing the same chord deflection for all bents. Ultimately, the girder and connections are so selected that the deflection of all bents due to web action is also a uniform amount. Thus, the total computed deflection of the structure is the same for all the bents, which satisfies elastic requirements.

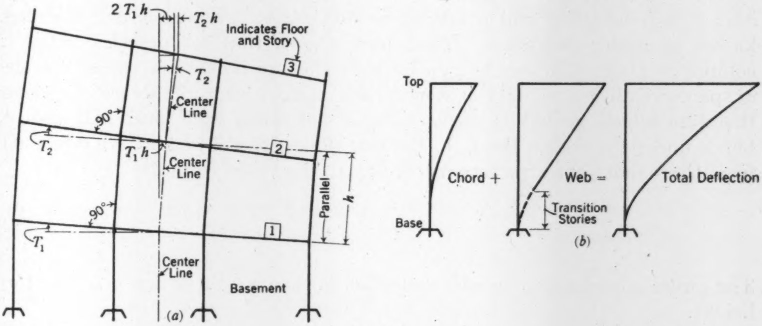


FIG. 5

A useful method of computing chord deflection is that developed by Professor Large.[31] Proceeding in accordance with it, and remembering that chord deflection results from the accumulation of column direct deformation throughout a number of stories, consider first the basement story, in which the generation of the tilt angle T_1, Fig. 5(*a*), causes a lateral deflection at the ceiling of story 1 of $T_1 h$. Then, after the next story deforms, the deflection at the ceiling of story 2 equals $2 T_1 h + T_2 h$, etc., if h is constant. The lower stories make the largest contribution to total chord deflection, measured at the top of the bent, (*a*) since their high column unit stresses produce comparatively large tilt angles, T, and (*b*) because their "leverages" to the top of the bent are longest. Fig. 5(*b*) shows the shape of the deflection curves. Contrary to popular conception, the upper portion is practically straight.

---

[31] "Tests and Design of Steel Wind Bents for Tall Buildings," by George E. Large, Samuel T. Carpenter, and Clyde T. Morris, *Bulletin No. 93*, Eng. Experiment Station, Ohio State Univ., p. 24.

35. Chord Deflection Formula.—For regular bents subjected to a uniformly distributed wind load, the overturning moment is proportional to the square of the distance from the top of the bent. Column areas will be very nearly directly proportional to the same distance. Consequently, the unit chord stress in a column increases directly with the distance measured down from the top. At the top of such a bent the chord deflection may be shown to be:[31]

$$\Delta_c = \frac{2\,s\,h^2}{3\,E\,L} \qquad \text{(11)}$$

In this expression s is the wind unit chord stress in the outside column at the base of the bent, h is the height of the bent, L is the base width of the bent, and E the modulus of elasticity.

An examination of Eq. 11 will show that, of two bents differing only in base width and number of panels, if one has half the width of the other, it can be assigned only one fourth as much of the wind load. The expression may also be used in an investigational manner to find approximate chord deflections for injudiciously proportioned bents. When so used, it should be applied only to the outside columns.

36. Web Deflection.—Unless a wind bent is very slender, the largest single item of web deflection will ordinarily be that due to reverse bending of girders, known as girder deflection. Tests have shown that for practical ratios of column to girder stiffness, the girder contraflexure points are near the center of span, even in the vicinity of structural irregularities.[32] The careful designer therefore selects girders of such stiffness that when each clear half span, b, bends under the design shear, V, the end slopes, θ, will be approximately uniform throughout the structure, according to the Spurr formula:

$$\theta = \frac{V\,b^2}{3\,E\,I} \qquad \text{(12)}$$

The girder contribution to web deflection is then $\theta\,h$, in which h is the story height.

Because of the appreciable distance between the end of a girder and the center of joint rotation on the geometric axis of a column, the girder end is displaced vertically as the angle θ develops. The net effect of this phenomenon is to diminish the girder contribution to web deflection. Mr. Spurr recognizes this by conveniently deducting 6 in. from the clear half span to obtain the bending length, b, to use in Eq. 12. This practice has been demonstrated to be satisfactory from a theoretical standpoint.[33]

---

[32] "Tests and Design of Steel Wind Bents for Tall Buildings," by George E. Large, Samuel T. Carpenter, and Clyde T. Morris, *Bulletin No. 93*, Eng. Experiment Station, Ohio State Univ., pp. 26–33.

The columns also bend in reverse curvature under wind, although their contribution to web deflection is seldom greater than 10% of that of the girder system. Their principal contraflexure points have been found by test to lie quite close to the midstory level, except in the vicinity of structural irregularities, such as fixed bases[34] and setbacks. Columns bend over their full lengths, and have minor contraflexure points near the midheight of the split-I, shallow connection to the girder, or at about half the depth of a knee brace.[35] Eq. 12 is also correct for computing column end slopes, if clear half lengths are used.[33] The resulting angle, θ, is measured from a straight line connecting the two ends. The column contribution to web deflection is then $\theta\, h$.

Little is known about the elastic deformation of wind connections, particularly split-I, shallow connections. The tests of J. Charles Rathbun,[36] M. Am. Soc. C. E., showed angular deformations as great as 1 : 1,000 under the design moment specified by the American Institute of Steel Construction. This is considerably more than the 1 : 6,000 design value sometimes used by Mr. Spurr[17] and suggests that such connections may contribute as much to web deflection as the girders. So long as this uncertainty exists, an ideal practice of arranging column spacing, which will produce the same wind moment and, therefore, identical wind connections entirely across a floor, should be encouraged, particularly for slender towers.[28]

37. Deflection of Lower Stories.—Fig. 5(b) is intended to show that a uniform web deflection of, say, 0.0015 h will not exist in lower stories just above a fixed base. Depending upon conditions, the designer needs from 3 to 6 stories to make the transition[32] to the established slope; and in these stories the location of the contraflexure points of the columns must grade gradually from a very high location in the basement story to the midstory level in the uppermost transition story. In these situations, a scale layout drawn with angles and deflections exaggerated about 400 times is very helpful.

38. Investigation to Find Web Deflection.—The foregoing discussion of web deflection has been addressed to the designer, who presumably is free to make the computed deflections whatever he prefers. The investigation of an existing slender bent for stress and deflection is more difficult; and in the case of bents originally proportioned with utter disregard for elastic requirements a satisfactory solution may be practically impossible, due to unmanageable direct deformation of columns.

(*K*) PERMISSIBLE STRESSES

39. Permissible Stresses for Combined Loading.—Careful consideration has been given to the discussion of the permissible stresses proposed by the Sub-Committee, and, having regard to all the facts, the recommendations contained in the Second and Third Progress Reports[13, 37] are here reiterated. They are as follows:

---

[33] "Tests and Design of Steel Wind Bents for Tall Buildings," by George E. Large, Samuel T. Carpenter, and Clyde T. Morris, *Bulletin No. 93*, Eng. Experiment Station, Ohio State Univ., p. 46.

[34] *Loc. cit.*, pp. 26–27.

[35] "Settlement Stresses in Continuous Frames," by George E. Large, *Bulletin No. 102*, Eng. Experiment Station, Ohio State Univ.

[36] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 542.

For members or details subject to wind stress only, except rivets and bolts, the permissible stress should be the same as that allowed for dead load or for dead load and live load.

For members subjected to stresses arising from the combined similar action of wind and other loads, and for rivets and bolts subject to wind stress, the wind stress up to $33\frac{1}{3}\%$ of the other stresses may be neglected, the excess wind stress being considered as equivalent to an added live-load stress, provision being made for it at the basic working stress for dead load and live load only.

A somewhat higher fraction of the wind stress may be neglected in the case of the columns of very high buildings, the exact amount being best left for the determination of the designing engineer in the light of all the circumstances.

Members subject to a combination of wind, dead load, and live load, of course, should have a section not less than the section required for dead load and live load alone at the normal working stress.

40. Members Subject to Combined Axial and Bending Stress.—Columns will be required to resist axial loads that arise from both gravity and wind effects and at the same time to resist wind moments, while girders will be called upon to resist moments due to both gravity loads and wind loads and, in addition, a wind thrust. It is necessary that the design of these members for the combined effects of the loads mentioned should be consistent with the proportioning of members for axial stress only, at the foregoing permissible stresses. This will be realized if

(*a*) All columns are proportioned, at the normal working compressive stress for dead load plus live load, for a hypothetical axial load equal to

$$P = \frac{2}{3} P_g + P_w + \frac{M_w d}{2 r^2} \dots\dots\dots\dots\dots\dots (13)$$

(*b*) All girders are proportioned, at the normal working flexural stress for dead load plus live load, for a hypothetical moment equal to

$$M = \frac{2}{3} M_g + M_w + \frac{2 P_w r^2}{d} \dots\dots\dots\dots\dots\dots (14)$$

in which: P_g = axial gravity load; P_w = axial wind load; M_g = gravity load moment; M_w = wind moment; d = depth of girder, or width of column; and r = radius of gyration in the direction of bending. In no case should the section selected be less than that required for gravity load effects only.

(*L*) STABILITY

41. Factor of Safety.—In order that a building may be secure against overturning, the Sub-Committee recommends that the factor of safety against uplift be not less than 1.5. In other words, the axial tension due to wind load in the critical column should not be more than two thirds of the dead-load compression

---

[37] *Proceedings*, Am. Soc. C. E., December, 1933, p. 1603.

in the bottom story of the column, unless the columns are securely anchored to the foundation by anchors proportioned at the permissible stress applicable to members carrying wind load only. Moreover, the foundation itself should be adequate for 1½ times any uplift that may be imposed on it.[38]

(*M*) CONCLUSIONS AND RECOMMENDATIONS

The principal conclusions and recommendations of the Sub-Committee are as follows:

(1) As a standard wind load for the United States and Canada the Sub-Committee recommends a uniformly distributed force of 20 lb per sq ft for the first 300 ft above ground level, increased above this level by 2.5 lb per sq ft for each additional 100 ft of height, no omission of wind force being permitted for the lower parts of the building by reason of alleged shelter. Special wind-force specifications should be formulated locally for areas that are definitely known to be subject to hurricanes or tornadoes.

(2) For proportioning the wind bracing of tall buildings it is not necessary to divide the wind force into pressure and suction effects, although this should generally be done for structures with rounded roofs, for mill or other buildings with large open interiors, and for walls in which large openings may occur. The effects of possible high local suction should be investigated in relation to secondary members and the attachment of roofing or siding.

(3) For plane surfaces inclined to the wind and not more than 300 ft above the ground, the external wind force may be pressure or suction, depending on the exposure and the slope. For a windward slope inclined at not more than 20° to the horizontal, a suction of 12 lb per sq ft is recommended; for slopes between 20° and 30° a suction uniformly diminishing from 12 lb per sq ft to zero; and for slopes between 30° and 60° a pressure increasing uniformly from zero to 9 lb per sq ft. For the leeward slope, for all inclinations in excess of zero, a suction of 9 lb per sq ft is recommended.

(4) For roofs that are rounded, or may be represented roughly by a circular arc passing through the two springings and the eaves, the wind force will depend not only upon the exposure and the ratio of rise to span of the equivalent circular arc, but also upon whether the springings are elevated above the ground or are on the ground. Where the surfaces considered are not more than 300 ft above ground level the recommended external wind force is as follows:

(*a*) On windward quarter of the roof arc, when the roof rests on elevated vertical supports and where the rise ratio is less than 0.20, a suction of 12 lb per sq ft is recommended; and for a rise ratio varying from 0.20 to 0.60, a pressure increasing uniformly from zero to 12 lb per sq ft, or, alternatively, for rise ratios between 0.20 and 0.35, a suction varying uniformly between these limits from 12 lb per sq ft to zero is recommended. For roofs springing from the ground level a pressure, for rise ratios varying from zero to 0.60, uniformly increasing from zero to 11.4 lb per sq ft, is recommended.

---

[38] *Proceedings*, Am. Soc. C. E., February, 1932, p. 215, and *Loc. cit.*, December, 1933, p. 1606.

(*b*) For the central half of the roof arc, where the roof rests on elevated vertical supports, with rise ratios varying from zero to 0.60, a suction uniformly varying from 11 lb per sq ft to 20 lb per sq ft is recommended; for roofs starting from ground level, a suction of 11 lb per sq ft, regardless of the rise ratio, is recommended.

(*c*) For the leeward quarter of the roof arc, for all values of the rise ratio greater than zero, a suction of 9 lb per sq ft is recommended.

(5) It is recommended that for a flat roof a normal external suction of not less than 12 lb per sq ft should be considered as applied to the entire roof surface.

(6) On walls parallel to the wind it is recommended that an external suction of 9 lb per sq ft should be considered.

(7) Even for buildings that are nominally airtight, internal wind forces of either pressure or suction may exist, varying from 3 to 6 lb per sq ft, and depending on whether the openings are generally in the windward or in the leeward surfaces. Large internal pressure may arise due to the breaking of windows in the windward side of buildings by reason of flying gravel from the roof or other objects carried by the wind. Still larger internal forces of pressure or suction may arise when the windward or leeward side of a building is completely open. The Sub-Committee recommends that for buildings that are nominally airtight an internal pressure or suction of 4.5 lb per sq ft should be considered as acting normal to the walls and the roof. For buildings with 30% or more of the wall surfaces open, or subject to being open, an internal pressure of 12 lb per sq ft, or an internal suction of 9 lb per sq ft, is recommended; for buildings that have percentages of wall openings varying from zero to 30% of the wall space, the recommendation is an internal pressure varying uniformly from 4.5 to 12 lb per sq ft, or an internal suction varying uniformly from 4.5 to 9 lb per sq ft.

(8) The Sub-Committee recommends that the design wind force applied to any surface of a building be a combination of (*a*) the aforementioned appropriate external wind force, and (*b*) the appropriate indicated internal wind force.

(9) Where a series of roofs exists in one building, one roof being nominally masked by another, the structure as a whole should be designed for the full wind load on the first roof and for 80% of the wind load on the other roofs. Any one roof should be designed for the full wind load.

(10) When wind surfaces are more than 300 ft above the ground, the external and internal wind forces should be scaled up in the proportion that the prescribed wind force on plane surfaces normal to the wind fixed by recommendation (1) at the level under consideration bears to 20 lb per sq ft.

(11) The capability of the floors to serve as horizontal diaphragms for the delivery of wind-load increments in each story of the braced bents should be investigated. If they appear to be inadequate, an efficient system of horizontal bracing should be provided between the braced bents to distribute the load.

(12) Although walls and partitions may serve to lessen vibration and deflection, they should be ignored in strength calculations for buildings having a

high ratio of height to width and provision should be made in the structural frame for the entire recommended wind load.

(13) If floors, with or without added bracing, are sufficient to serve as effective horizontal diaphragms, there need be no maximum limit of spacing of braced bents, nor any restriction imposed on carrying the wind force to the end or side walls, if the bracing therein is sufficient to provide for the entire wind force.

(14) If the floors are capable of serving as effective distribution diaphragms, the various braced bents may be considered as taking loads in proportion to their respective rigidities. The allocation of wind load to braced bents should take into consideration the eccentricity of the total lateral load to the lines of resistance.

(15) Although shallow bracing may be of sufficient strength, it may entail excessive deflection of the building. Consequently, for high or narrow frames, it is very desirable to adopt diagonal bracing of some kind. Full diagonals, knee braces, or K-frames, with various arrangements in respect of openings in the panel, may be used advantageously. Although the use of one type of bracing in one panel and another type in adjoining panels is permissible, the calculated web deflections in all the panels in any story of the bent should be the same, so that the bent may work as a unit.

(16) Although adequate over-all strength may be provided by wind bents designed in accordance with either the portal or the simple cantilever method, the assumptions upon which these methods rest do not ordinarily make possible a properly balanced design. These methods, however, are useful for making a rapid strength analysis.

(17) Investigation of frames having a height-width ratio of 4 or more by some method recognizing the elastic properties of the bent is necessary. If a bent has been so proportioned that it will act as a true cantilever and the floors remain plane during and after deflection, the problem of investigation is greatly simplified, in that secondary stress calculations are obviated.

(18) For purposes of analysis the Cross method of moment distribution has been found to be highly valuable, although for tall frames the large number of cycles necessary to effect a sufficiently accurate solution renders it impracticable. Both the Goldberg and the Grinter methods make it possible to effect a rapid and sufficiently accurate analysis.

(19) As a means of conducting a rapid investigation of a bent, the method of K-percentages is recommended by the Sub-Committee as particularly valuable. This routine procedure is based upon a consideration of the relative stiffnesses of the members of the frame from which the vertical wind reactions at all columns may be obtained with reasonable accuracy.

(20) Assurance of bent behavior in accordance with theoretical assumptions and deflections within desired limits can be had only when the bent is proportioned in accordance with the principles of elasticity. When the height-width ratio of a building exceeds about 5, a method of design, such as that of Spurr, which maintains planarity of floors under wind loads, should be used. Otherwise, proper provision for secondary stresses caused by the axial deformations of the columns should be made.

(21) Where high buildings have been proportioned without insuring regularity in the vertical movements of the successive floor joints, great complexity is introduced and an accurate analysis becomes a difficult and impracticable task. Unless a bent is so designed that the floor joints move in a predetermined manner, secondary wind moments will need to be computed after the primary direct wind stresses have been found in the columns. This may be accomplished by an application of the Cross method, but it involves a laborious procedure. On investigation, bents designed in accordance with the portal method may show large errors in girder shears in the upper portion of frames having a large height-width ratio. In buildings with setbacks provision must be made for unequal axial deformations of the columns.

(22) The effects of torsion should be investigated, particularly if the building is unsymmetrical in plan about an axis parallel to the direction of the wind. If the building is unsymmetrical about both axes, a wind from any direction may cause torsion. Although a solution sufficiently accurate for many purposes may be based upon the neglect of all deformations except girder bending, it is desirable to include the effect of axial length changes of the columns. Over-all economy is not served in any attempt to eliminate torsional moment by the stiffening of certain bents.

(23) Vibrational characteristics of buildings and the relation of sensation thereto must be determined experimentally before they can be designed with certainty to yield a desired degree of comfort to the occupants. Until such vibrational characteristics are more fully understood, it is recommended that the maximum computed deflection of the building at the top, when the structural frame is assumed to carry the entire lateral force, should not exceed 0.002 of the height.

(24) The Sub-Committee recommends that for members or details subject to wind stress only, except rivets and bolts, the permissible stress should be the same as that allowed for dead load or for dead load and live load.

For members subjected to stresses arising from the combined similar action of wind and other loads, and for rivets and bolts subject to wind stress, the wind stress up to 33⅓% of the other stresses may be neglected, the excess wind stress being considered as equivalent to an added live-load stress, provision being made for it at the basic working stress for dead load and live load only. Higher combined stresses may be acceptable in the columns of very tall buildings.

Members subject to a combination of wind, dead load, and live load, of course, should have a section not less than the section required for dead load and live load alone at the normal working stress.

(25) Members subject to both axial stress and bending should be proportioned for the combined effects in such a manner as to be consistent with the proportioning of members for axial stress only at the stresses permitted in recommendation (24). Formulas are submitted to insure this result. In no case should the section selected be less than that required for gravity load effects only.

(26) The Sub-Committee recommends that the axial tension due to wind load in the critical column should not be more than two thirds of the dead-load compression in the bottom story of the column, unless the columns are securely

anchored to the foundation by anchors proportioned at the permissible stress applicable to members carrying wind load only. The foundation should be adequate for 1½ times any uplift that may be imposed on it.

ACKNOWLEDGMENTS

The Sub-Committee wishes to express its thanks to many persons, and in particular to Mr. Spurr and Professor Large for assistance received in the course of the preparation of this Final Report.

Respectfully submitted

CLYDE T. MORRIS
N. A. RICHARDS
FRANCIS P. WITMER
C. R. YOUNG, *Chairman.*

January 12, 1940.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2096

IMPORTANT EVENTS, DEVELOPMENTS, AND TRENDS IN WATER SUPPLY ENGINEERING DURING THE DECADE ENDING WITH THE YEAR 1939

REPORT OF THE COMMITTEE ON WATER SUPPLY ENGINEERING OF THE SANITARY ENGINEERING DIVISION

At the request of the management of the Sanitary Engineering Division of the Society, the Committee on Water Supply Engineering of the Division undertook to make its current annual report on events, progress, and trends in water supply engineering cover the past decade instead of only the year. The Committee has done as much with this ambitious undertaking as the time of its members has permitted but recognizes that much interesting matter is not included.

The reports of previous years have included notes of important water supply projects currently in process of construction. Such reports were published in *Civil Engineering* for March, 1934, and April, 1935; and in *Proceedings* for December, 1934, April, 1935, March, 1936, April, 1937, March, 1938, and March, 1939. The present report lays particular emphasis on the progress and trends in water supply engineering.

The past decade has been peculiar in that it began with the great depression and lies entirely in the depression period. This depression is still continuing into the new decade although with some present abatement. The decade has witnessed the greatest government spending of all time. A full share of this spending has gone into water supply construction. The water supply engineering profession had an important part in advocating and encouraging this spending policy and has taken advantage of it. To some extent, however, private water utilities have offset the government spending on water supplies because their expenditures have had to be greatly decreased. The federal government's total expenditure in the decade has been more than $70,000,000,000 as compared with about $30,000,000,000 for the previous decade of great prosperity. Expenditures by private interests for capital goods decreased to $45,000,000,000 in the past decade as compared with $71,000,000,000 in the previous decade of great prosperity. The Committee has no exact statistics on the total expenditure for water supplies in the decade just passed as compared with the previous one, but is of the opinion that the total is greater than in the previous decade, although the engineering control of these expenditures has not always been so orthodox.

DROUGHTS, DRY WEATHER YIELDS, FLOODS, AND SPILLWAYS

At the beginning of the decade there was a very severe drought in the eastern and midwestern part of the United States. Again, from the spring of 1939 to the spring of 1940, drought conditions were severe.

As to floods, the past 10-yr period has brought floods of greater severity than any of those of the past century, and in some localities the floods have been of an intensity that probably occurs only once in several centuries. These floods have radically affected the design of spillways for dams, flood channels, flood protections for water works, and levee levels.

In New England, one of the areas in which long-time rainfall and runoff records are available is in the vicinity of Hartford, Conn. The rainfall record extends back to 1868, although the stream-flow records for the Nepaug and East Branch rivers start only with the year 1912.

In 1937, Caleb M. Saville, M. Am. Soc. C. E., stated[1] that from the Hartford record the maximum calendar-year rainfall occurred in 1920, amounting to 56.95 in. and that the minimum occurred in 1935, which totaled 28.90 in.

He stated that in this area the "great" flood came in March, 1935, on both the Farmington and Connecticut rivers. In the local watersheds, the runoff from the East Branch watershed of the Farmington River, with 62 sq miles and with 8.14 in. of rainfall from March 11 to noon, March 18, was equivalent to 11.7 in. with a peak runoff of 135 cu ft per sec per sq mile. In the Nepaug watershed of 32 sq miles, the corresponding figures for this time period were: Rainfall 6.95 in., runoff 8.8 in., and peak flow 98 cu ft per sec per sq mile. On November 2 to 4, 1927, with a rainfall of 6.52 in., a peak flow of 141.8 cu ft per sec per sq mile was recorded on the East Branch.

As a result of these and other lesser storms and studies of their flows, the new Barkhamsted (Conn.) Reservoir of the Hartford system, which has an earth dam, is designed for a peak flow of 500 cu ft per sec per sq mile. Mr. Saville explains that one reason for this large spillway capacity was that the additional cost for the large capacity was small. This reservoir is on the East Branch of the Farmington River at Barkhamsted; it has a drainage area of 52.7 sq miles and a capacity of 30,000,000,000 gal.

The Farmington River joins the Connecticut River 6 miles above Hartford with about 600 sq miles of watershed. The watershed of the Connecticut River at Hartford is estimated at 10,560 sq miles. The maximum flood-gage height in a period of 93 years previous to 1936 was 29.8 in 1854, while in March, 1936, it was 37.6 with a runoff of 28.4 cu ft per sec per sq mile as compared with 19.5 cu ft per sec per sq mile in 1854. Previous to 1936, the flood heights reached had a frequency estimated as once in 25 years, and in 1936 the flood height was estimated to be one that would occur once in 300 years.

---

[1] "New England Droughts and Floods in Their Relationship to Water Supply," by Caleb M. Saville, *Journal*, New England Water Works Assoc., Vol. 51 (1937), No. 3, pp. 326–366.

Comparison may be made with the heavy flood that occurred in November, 1927. This was much more severely felt in Vermont than in any of the other New England states. W. F. Uhl, M. Am. Soc. C. E., has stated[2] that the November, 1927, flood did $40,000,000 worth of damage, with two thirds in Vermont, and the 1936 flood did $70,000,000 worth of damage, with $55,000,000 in the Merrimack and Connecticut river valleys. In the 1927 flood, the runoff of areas of 20 and 40 sq miles reached 300 cu ft per sec per sq mile; for 200 sq miles, 200 cu ft per sec per sq mile; and for 1,000 sq miles, 114 cu ft per sec per sq mile. For the 1936 flood, the high runoffs for the small watersheds did not reach the maximum shown for 1927, but for 170 sq miles, 180 cu ft per sec per sq mile were shown. It was stated that the 1936 flood exceeded previous records for the greater part of New England. It was further stated that at Hartford the 1936 flood exceeded any for at least 150 years. In the Merrimack River, the flow of 1936 exceeded by 60% any floods in the previous 200 years.

It has been reported[3] that the summer flood of July, 1935, on small streams in New York State reached 2,000 cu ft per sec per sq mile for watersheds of 5 sq miles and less. For 100 sq miles, the flow was recorded up to 200 cu ft per sec per sq mile. The floods of March, 1936, showed lower flows, generally, than the 1935 flood.

On September 21, 1938, a hurricane swept over New England, and the rainfall exceeded previous records for large areas. The 16-in. fall covered 250 sq miles, a 14-in. fall covered 1,800 sq miles, and 6,600 sq miles had a 12-in. rain. With such precipitation, a flow of 175 cu ft per sec per sq mile came from the West River, Mass., compared with 127 cu ft per sec per sq mile in the November, 1927 storm. From a 178-sq-mile area on the Deerfield River, the recorded flow was 316 cu ft per sec per sq mile. On the small tributaries of this same river, flows of 700 cu ft per sec per sq mile were found for areas of 25 sq miles.

Special attention has been directed to the New England floods of November, 1927, March, 1936, and September, 1938, as in the short space of 12 years three major floods have visited this area, although the three floods did not set up maximum flows at exactly the same points in the New England area.

Pittsburgh, Pa., experienced its worst flood on record in March, 1936, when a crest of 46 ft was reached, compared with the previous crest of 35.5 ft, which represented the highest level for 86 floods that had occurred since 1856. The drainage areas of the Allegheny River and the Monongahela River at Pittsburgh total 18,900 sq miles.

The Ohio Valley flood of January, 1937, surpassed all previous floods that had occurred during the 175 years of the occupancy of the valley by white men. At Louisville, Ky., the runoff of the flood was estimated at 8 in. of rainfall, as against 6 in. for the 1884 flood which was the most severe flood that had been recorded previously; the flood level was 12 ft higher in January, 1937, than previously. It was estimated that the flood level was probably about 4 ft higher than it had been in the 1773 flood, which may have been of about equal volume to that of 1937.

---

[2] "Flood Conditions in New England," by W. F. Uhl, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), pp. 578–612.

[3] "The New York Floods of 1935 and 1936," by Arthur W. Harrington, M. Am. Soc. C. E., and Hollister Johnson, Assoc. M. Am. Soc. C. E., *loc. cit.*, pp. 613–621.

In California, a very heavy flood which caused much damage occurred on December 9 to 12, 1937, and a 12-in. rainfall was reported for the Yosemite Valley.

In the Southern California area, a flood on March 2 to 5, 1938, caused much damage in the Los Angeles section. It is reported that the rainfall was as great as 30 in. in some of the watershed areas.

The drought in 1930–1931 in the Croton River watershed of the New York water supply system was not quite as severe as the drought of 1879–1883. The Croton River rainfall and runoff records have been continuous since 1868. The Croton system has a tributary watershed of 375 sq miles and a storage capacity of 103,000,000,000 gal. From 1911 to 1931, the estimated dependable supply from the Croton watershed was considered to be 335 mgd. As a result of the drought of 1930–1931, which really extended as a dry period from 1929 to 1932, engineers of the New York City (N.Y.) Water Department determined that 25% of the total storage should be considered as a special reserve, and that the dependable yield should be estimated only on the basis of utilizing 75% of the storage. This reduced the estimated dependable yield of the Croton watershed from 335 to 300 mgd.

In the Schoharie and Ashokan watersheds, which comprise the Catskill system and have a combined area of 571 sq miles, the record of rainfall and runoff is available from 1907. The dryest period on record in these watersheds occurred from May 1, 1930, to March 23, 1931. During this period, a draft of 601 mgd would have practically drained the main reservoir, which is the Ashokan Reservoir, holding 131,000,000,000 gal. Prior to this drought period, the Catskill system was operated on the basis of 585 mgd for dependable supply, utilizing the total storage available. After this drought of 1930–1931, 75% of the available storage was assumed to be available for maintaining draft, and the dependable yield was reduced from 585 to 500 mgd.

In the Hartford district, Mr. Saville reports that the maximum drought period was between 1879 and 1883, when, for 5 years, the average rainfall was 35.72 in. and the lowest year in that period was 33.64 in., or 77.1% of the average. For the 5 years beginning with 1931, the average was 39.66 in. which was the next lowest 5-yr average, with the 1935 rainfall of 28.90 in. only 66.2% of the average. Four very low 5-yr periods have been noted since 1836. Two have been mentioned already; the other two started respectively with 1845, averaging 39.70 in., and 1910, averaging 41.01 in. The 100-yr period was secured by using for 1836 to 1868 the Amherst, Mass., and Springfield, Mass., records, which are longer than the Hartford record, and these were interpolated for Hartford. This interpolation was made by developing the relationship between these three records for the years for which records are available at all three stations.

From 1874 to 1886, or thirteen consecutive years, the Hartford rainfall was below the average and averaged 40.40 in., or 7.4% below the long-time average. However, for a 2-yr period, the minimum was 1930–1931, averaging 34.94 in., or 80% of the average.

The lowest runoff, for twelve consecutive months recorded since 1912, into the Nepaug Reservoir of the Hartford system, with a tributary watershed of

32 sq miles, occurred from March, 1930, to February, 1931, the runoff being 9.43 in., or 41.9% of the average annual runoff. The lowest runoff for sixty consecutive months was from January, 1928, to December, 1932, when the runoff averaged 18.69 in. or 82.9% of the long-time average. For forty-eight of the sixty months the figures were, respectively, 15.41 in. and 68.5%.

As a result of these runoff records, with storage available of 9,540,000,000 gal and 32 sq miles of watershed, the working safe yield of Nepaug Reservoir was taken as 21 mgd.

Although the drought of 1930–1931 was approximately of the same severity as that of 50 years earlier, the effects of this drought in influencing the responsible engineers to decrease the estimated dependable yield of watersheds having rainfall and runoff records extending back some 70 years is the item of special interest to water supply and sanitary engineers.

During the past 10 years it is obvious that the trend has been toward making greater allowance for maximum floods and spillway capacity, and reducing the estimated dependable yield of watersheds, such action being brought about by the floods and droughts of this 10-yr period, viewed in the light of earlier records of rainfall and runoff.

EARTH DAMS

The earth dam has gained in importance and in engineering interest during the past decade. In height, and particularly in volume, the magnitudes of recently completed and proposed structures have set precedents. At present, a rolled-fill dam approximately 400 ft high is under construction on the White River in the State of Washington. The Fort Peck Dam on the Missouri River in Montana has an embankment volume of approximately 122,000,000 cu yd. In large measure, this expansion has been due to the federal program of public works construction.

Both the hydraulic and rolled-fill methods of construction continue to be used, the choice depending on the character of material available, the adaptability of equipment to local conditions, and the comparative resulting costs. In general, hydraulic fill construction is limited to projects of large volume because of the cost of plant installation; but the increased size, power, and speed of available transportation and compaction equipment have so reduced the comparative cost of rolled-fill construction that more than two thirds of the dams 100 ft or more in height, built during the past 10 years, have been constructed by this method.

The increased capacity of construction equipment is notable: 25-cu-yd trucks, 30-cu-yd scrapers, 8-cu-yd draglines, and dredges rated at 1,500 cu yd per hr are now obtainable. For compaction of rolled fills, the sheepsfoot roller, because of its greater concentration of deeper pressure and its ability to leave the surface loose and ready for the next layer, has become standard equipment. In hydraulic fill practice, belt conveyers have been used to lift and transport material from the borrow pit to the hog box, and rubber-lined pipe has been successfully employed in the delivery lines from pump to equipment. Booster pumps in the delivery lines have made it possible to carry material a distance of 5.0 miles in some cases.

As a result of these developments in equipment, the cost of earth fill in place has been reduced greatly, being now (1940) about two thirds of that prevailing 10 years ago. Present prices per cubic yard range between 30¢ and 50¢, and contracts, both in the rolled-fill and hydraulic methods, have been let at less than 20¢ per cu yd.

The rapidly developing science of soil mechanics has been responsible, in considerable measure, for the confidence necessary to the undertaking of the unprecedented magnitudes of recent structures. Beginning in the United States about 1925 with the work of Karl Terzaghi, M. Am. Soc. C. E., the intensive study of the physical properties of soils became quickly popular. As a result, a more rationalized design and better control in the construction of dams has been made possible. This has not prevented some partial failures, however, such as those at the Tappan Dam on the Muskingum River in Ohio, the Fort Peck Dam on the Missouri River, and the Marshall Creek Dam on Marshall Creek, Kansas.

As evidence of the increasing interest in the design of dams and soil mechanics, several notable conferences have been held, with published results, as follows:

1. *Proceedings*, First International Congress on Large Dams, Stockholm, 1933.
2. *Proceedings*, Second International Congress on Large Dams, Washington, D. C., 1936 (Supt. of Documents, 5 Volumes).
3. *Proceedings*, International Conference of Soil Mechanics, Harvard University, 1936, 3 Volumes.
4. *Proceedings*, Soils and Foundations Conference of the U. S. Eng. Dept., Boston, Mass., June 17–21, 1938—printed January, 1939.

The Soil Mechanics and Foundations Division of the Society was created in 1936, and this Division has prepared a manual[4] containing a list of some 300 publications on soil mechanics, many of which deal with the design of earth dams.

Soil testing laboratories are now generally provided for the continuous testing of borrow-pit material for size gradation, compaction under varying moisture contents, and for settlement and shearing strengths under load. Shear and permeability tests are made of "undisturbed" samples of the foundation material. As the fill progresses, other tests of undisturbed samples of the fill for gradation, weight, moisture content, compaction, and shear are made.

Lines of seepage have been studied in model dams between glass slides to indicate the probable flow net, and model tests of spillways and outlets are becoming customary. In some cases, a gelatin model loaded with shot has been used to forecast the shear zones.

On the basis of the information thus collected, the most dangerous line of failure in shear is derived, and the section of the embankment necessary to provide the desired factor of safety is determined. Settlement under load is forecast and the permissible rate of construction or application of load is controlled accordingly.

---

[4] "Selected Bibliography on Soil Mechanics," *Manual of Engineering Practice No. 18*. American Society of Civil Engineers.

A refinement of the embankment design has thus been made possible and such partial failures as have occurred point to the foundation material (through which the line of maximum shear may pass and the evaluation of which is dependent on the number and validity of so-called undisturbed samples) as the element of greatest risk. In any case, the factor of safety to be applied remains a matter of judgment.

The fundamental requirments of an earth dam are reasonable watertightness and permanent stability against sloughing when the material is saturated and subject to the effect of percolating water. Present-day design, therefore, generally includes a central impervious section and more or less pervious shells with the shearing strength necessary for stability. Masonry core walls are seldom used and even key walls to connect the earth fill with the rock are questioned, reliance against seepage through the embankment being placed on a section of selected soil. Provision is made for safe discharge of such seepage as may result by adequate drainage in the form of gravel or rock fills, with reversed filters at the toe or inside the lower downstream pervious section.

The most interesting and cost saving development in method of providing a subsurface cutoff is that of sinking concrete caissons after a preliminary lowering of the ground water. This method was used in the construction of the Quabbin Reservoir dike, and dam of the Metropolitan District Water Supply Commission of Boston, Mass., and is to be used in the Merriman Dam of the New York City supply. An increased confidence in the value of steel piling as a subsurface cutoff has been indicated.

A marked increase in the provision of spillway capacity has been a notable development of the past decade.

In summary, although there has been little change in the theory of design, laboratory and field tests of the physical properties of soils have made possible a more definitely quantitative design and have supplied a basis for better evaluation of future success or failure. It is not to be understood, however, that the laboratory has eliminated the need for engineering judgment founded on experience. The choice of a factor of safety for shearing of the fill and foundation, for example, is the crux of design and is always an act of purest judgment.

Trends in Design and Construction of Masonry Dams

During the past decade no substantially new form of dam has been constructed. (The Coolidge Dam, in Arizona, of the multiple-dome type, was practically completed in 1928.) Rather, the outstanding development has been the design and construction of the earlier-type dams to greater heights than ever before. Hoover Dam, in Nevada and Arizona, constructed from 1931 to 1935 and having a maximum over-all height of about 727 ft, is designed to raise the water level of the Colorado River about 578 ft, whereas Shasta Dam of the Central California Project has a maximum over-all height of about 550 ft, and is intended to raise the waters of the Sacramento River about 490 ft. The former is massive, involving both gravity and arch action; the latter is a curved gravity-type. The Shasta spillway, located in the center of the dam, will be the highest ever built. There is no demonstrated limitation against still greater heights of dams, given appropriate physical conditions of site.

Outstanding in another respect is the Parker arch dam of the Metropolitan Water District of Southern California, which, although it raises the water of the Colorado River, in California and Arizona, only about 105 ft, has a maximum over-all height of about 320 ft—that is, about two thirds of its maximum height is under the original river level.

In the design and construction of smaller dams, as well as of the larger dams such as the foregoing, changes and tendencies have evolved, the more important of which are mentioned, although by no means exhaustively, in the following statements:

Foundations.—Clearly in the decade 1930–1939 there has been a continuation of the previous tendency to explore, more extensively and carefully, Nature's materials in foundations and abutments. The normal tendency in this respect has been augmented by the fact that, many of the better dam sites already having been utilized, it is becoming more and more necessary to construct dams where foundation conditions are relatively inferior. Geophysical methods are used somewhat, particularly for preliminary exploration, but for final exploration, borings—especially core borings—are used. The desire for still more intimate knowledge of foundation and abutment conditions finds expression in shafts and tunnels and in core borings of sufficient diameter (30 in. and larger) to allow direct inspection.

Because of the fact that, in the main, failures of masonry dams have been by sliding, there is growing recognition of the need for caution where the foundation material is unconsolidated or involves approximately horizontal stratification, with shales receiving special attention. Low resistance against horizontal movement is remedied by tying the dam deeply into the foundation material—for example, by anchoring walls of concrete, an expedient that is hardly efficacious for dams as high as 200 ft. Fortunately, higher dams of masonry are rarely attempted upon foundations having such characteristics.

Consolidation by grouting of yielding or fissured rock forming the foundation or abutments of dams (practiced, for instance, at Norris and Claytor straight gravity dams, in Tennessee and Virginia, respectively, and at Shasta curved gravity dam) is becoming standard procedure. Consolidation generally is under the entire base of the dam, but particularly at the downstream portions. Abutments naturally receive intensified attention in this respect in the cases of arched dams.

Solution channels in limestone and dolomitic foundations, especially where there is much jointing or faulting, have resulted in extensive, deep pressure-grouting operations in the rock at the upstream faces of the dams in order to minimize underflow and uplift. Such grouting is being done to substantially greater depths and with substantially greater pressures than previously was common practice. Among the newer dams receiving such treatment are especially those in the carbonate-rock regions of the Appalachian Mountains, including Norris and Chickamauga dams of the Tennessee Valley Authority (TVA) on the Clinch and Tennessee rivers, respectively, and Claytor Dam on New River in Virginia, the latter being the largest dam built in the United States with private funds since the beginning of "the depression."

Earth foundations still limit substantially the heights of any dams founded upon them. However, it is recognized that within reasonable limits concrete dams, especially of the buttress type, may be constructed safely upon such foundations if adequate protection is provided against seepage under and around the dams and against scour at the foot of any spillway.

No longer is it taken for granted that a hard rock containing fissures or joints, or even only bedding planes, is immune from scour. There is greater use of concrete aprons and baffles to insure dissipation of the energy of the spillway discharge before it comes in contact with the stream bed. Scarcely any important spillway is constructed now without such special provision for prevention of scour. Inasmuch as the conditions of height of dam, depth of overflow, material in the river bed, and depth of tailwater are never alike at any two dams, the opportunity for empirical deductions from actual dams has been limited and there has been much recourse to model testing.

Temporary retention, in place, of unconsolidated material by means of freezing has been practiced, as, for example, at the Gilbertsville dam site of the TVA, around an exploration shaft, and at Grand Coulee Dam, in Washington, for preserving steeper-than-natural slopes around the foundation excavation.

Design.—Especially for the higher dams, analyses are directed to determination of the maximum (principal) stresses and not merely their horizontal and vertical components. As to considering a masonry dam as an elastic structure, there is practical unanimity, but not so as to assumptions, stated or tacit, that the elastic properties of a given dam or its foundation are uniform or fully known, or that the behavior of a high gravity dam having keyed and grouted longitudinal joints is in such respect actually that of a structure to which, as a monolithic whole, the theory of elasticity is applicable.

Likewise, there has been an increase in the extent and refinement of mathematical analyses. In the latter respect there are two schools, the one inclined to rely almost solely upon such analyses, the other tempering the use of the purely theoretical considerations by means of the empirical. Within the latter category there is included the use of tests of models, albeit with recognition of a lack of complete homogeneity in the prototypes and differences in behavior of the latter from that of small specimens of the actual materials of which a dam is to be built or upon which it is to be founded.

The special complication of stress determination in arch dams, involving as it does stresses in three directions, has been not merely the subject of various new theoretical approaches as to solution but has quite logically been the subject of extensive and refined model tests, as in the case of Hoover Dam.

It is recognized that foundation deformation affects stresses in the dam. For this reason, especially in the cases of some of the higher dams, such as Norris and Shasta, the angles between the faces of the dam and foundation are eased.

Although the analysis of resistance to sliding by considering shearing resistance or a combination of shearing and frictional resistances is substantially older than the last decade, yet that method, in contrast with the former reliance upon friction alone, has been used increasingly. Especially in cases where pressure-grouting has been done to establish at the upstream face practically complete cutoff against underseepage, and where relief drainage has been provided downstream from the cutoff, such physical provision and the results of such of the highly desirable tests of actual uplift pressure at dams as are available, have led to some degree of reliance upon the assumption that the uplift forces will be substantially less than under the common, older assumption of straight-line decrease from full headwater to tailwater pressure. In the cases of certain larger gravity dams there has been introduced, primarily by the designing force of the Bureau of Reclamation, analysis of the effect of "twist" as caused by transfer of load between adjoining monoliths and between the dam and the abutment rock. Indeed, in Grand Coulee Dam vertical slots are left to allow for adjustment in respect of twist; after adequate adjustment is believed to have taken place the slots are to be filled with concrete and grouted

More and more, in regions subject to earth movements, analyses are taking into account the increased stresses due to earthquakes. Where it is found necessary to locate masonry dams across faults, and where there are specific possibilities of movement at known faults, special expedients in design have been adopted to ameliorate the results of any such movement. Outstanding in this respect are the Morris gravity dam, in California, with its "open joint" and the buttress-type Rodriguez Dam, in Lower California, Mexico, with its footing arch over the fault zone and its tying together of pairs of buttresses in a "tower" construction to resist earthquake action.

There is increasing recognition that, in general, the maximum floods of record constitute only woefully inadequate measures of the maxima which may occur; the result is increased provision for flood discharge. On the other hand, with the recognition that such special earthquake stresses and maximum flood loadings will occur only rarely, if ever, it is not unusual to permit higher unit stresses therefor than for the normal maximum loads.

It has become general practice to provide gates on spillway crests in order to permit continuous utilization of the head (and the corresponding storage capacity) represented by the height of the gates. In consequence spillway gates have been constructed to greater heights than heretofore, and the desire to insure against clogging by debris has resulted in greater lengths of clear openings. Such surmounting of the aforementioned Parker arch dam with crest gates 50 ft high necessitated corresponding thickening of the arch ring at the spillway crest so as to allow the gates and their spillway piers to function by gravity action with the arch dam as foundation. Where topographic and stream flow conditions permit, however, the practice is to separate flood discharging facilities from the arch dam proper; this has been done at both the Owyhee (in Oregon) and the Hoover dams.

Marshall Ford gravity dam on the Colorado River of Texas, designed by the Bureau of Reclamation and completed as to its first stage in 1939, is unique in its provision for increase in height (from 190 ft to 268 ft) by additions only on the upstream side.

Construction of Dam Proper.—Although the use of portland-cement concrete, instead of stone masonry, is predominant, the infirmities of concrete receive great attention. Indeed, the Board of Water Supply of New York City, in general, builds exposed masonry surfaces of stone, not concrete.

Outstanding among the aims in present-day construction of masonry dams is improvement of the quality of the concrete—not its strength, which generally is ample, but its durability and water tightness. Wet concrete is outmoded; complete, but not excessive, vibration, as a necessity as well as a desideratum for the placing of the relatively dry concrete, is a fully accepted process. The primary effort has been to prevent cracking. To this end cements of lower heating characteristics have been developed, the most important early applications being at Morris and Hoover dams. Temperature control has been practiced also by embedding pipes in the concrete for circulating cooling water, as in the cases of the larger dams constructed under Bureau of Reclamation specifications, or by the precooling of mixing water or the aggregates themselves, as at Hiwassee Dam, in North Carolina. Decrease in height of lifts to about 4 ft, or even less at the foundations, and longer time intervals between placing of superimposed lifts have likewise been found helpful; indeed, the emphasis upon quality has resulted in definite restraint upon speed of construction.

For the purpose of preventing disintegration of concrete exposed to alternate freezing and thawing, and of obtaining greater resistance to seepage through the upstream concrete below reservoir level, there is continuance (for example, in the TVA construction) of the practice of placing richer concrete near the faces. As to this practice, however, there is not complete agreement, for there are those, competent to have opinions, who attach greater weight to the resultant lack of uniformity and to such disadvantage as there is in the richer concrete because of greater, thermally caused volume changes; consequently they advocate relatively lean concrete even for dam faces.

There is more nearly unanimity as to the feasibility of using lean mixtures (1 bbl or less per cu yd) for the interior masses, particularly where they are quite dry and thoroughly consolidated by vibration. Differences of view in this respect involve mainly the extent to which leanness may be carried. However, there are those, including the late Thaddeus Merriman, M. Am. Soc. C. E., who have subscribed rather to the view that, granting certain weaknesses of portland cement, including its thermal activity in early life, nevertheless there is no adequate substitute for plenty of good cement. There is a strong tendency to specify cement content in terms of water-cement ratio in addition to, or even without, minimum quantities of cement per cubic yard.

For any masonry dam regardless of height, the art recognizes that prevention of transverse (to the axis) cracking, not to mention the practical needs of construction, requires division of the structure into blocks by means of transverse joints. Distances between joints are commonly, and generally successfully, 40 ft to 50 ft, but, where crest-gate lengths dictate, the foregoing consideration may be overruled, as in the case of Claytor Dam, with its spacing, in the spillway section only, as high as 60 ft.

The need to avoid cracking had led to the practice, initiated on a large scale at Hoover Dam, of dividing the structure not only by transverse but also by longitudinal joints, likewise 40 to 50 ft apart. Such practice results in constructing a dam by blocks, with resultant need for uniting the blocks into monoliths. Engineers have sought to accomplish this by somewhat elaborate keying and later grouting of all such joints, as at Hoover, Grand Coulee, and

Shasta dams, where the intent has been to make a huge monolith of the entire dam, once the concrete has cooled to practically its ultimate temperature. Metal seals at such joints are provided to divide the grouting into predetermined sections.

As to arch dams, there is general agreement that transverse joints, properly grouted, insure the intended arch length and hence immediate arch action despite previous shrinkage of concrete. So far as is known, circumferential joints in arch dams (as used at Hoover Dam, for example) have not been criticized. However, as to huge dams dependent solely on gravity, it has been suggested that keying and grouting of joints parallel to the axis have not yet been demonstrated to result, in that respect, in elastic action and margin of safety equivalent to those of a monolith.

An alternative to such use of transverse contraction joints involves, in effect, expanding the joint so that it becomes a slot several feet wide, and, after normal temperature has been attained in the mass, filling the slot with concrete, without provision for grouting. Except for the cooling slot in Hoover Dam, this alternative appears to have found application only in Europe.

There is increasing recognition of the importance of the treatment of horizontal construction joints with the aims of providing adequate bond, or at least shearing resistance between successive lifts; of obviating disintegration at the exposed faces (in particular immediately below such joints); and of minimizing seepage along the joints, with its detrimental effect upon durability. The extreme instance of precaution in this respect is probably under the specifications for Claytor Dam, but great care is exercised under other specifications, including those of the Bureau of Reclamation. Sandblasting of the surfaces of horizontal joints is now (1940) practiced at Grand Coulee Dam and is commanding increasing attention elsewhere.

Buttress-Type Dams.—Much of the foregoing is pertinent also to so-called hollow dams, consisting of flat or arched decks supported by piers. The use of these structures has continued where need for economy in masonry has dictated, and also where there are anticipated differences, in foundation deformation or even fault movement, between different sections of a dam. In dams of this kind, as in the cases of gravity and single-arch dams, the past decade has witnessed the overtopping of previous heights. Thus, the flat-slab, Rodriguez Dam, with an over-all height of 250 ft, and the multiple-arch, Bartlett Dam, in Arizona, with an over-all height of about 286 ft, successively assumed first place.

The trend is toward wide spacing of buttresses. In this respect the Don Martin, roundhead, buttress-type dam, in Mexico, involved a spacing of 29.5 ft between centers of buttresses, being at least one third greater than previous buttress spacings for a flat-slab dam. The massive buttress of the Possum Kingdom flat-slab dam, in Texas, completed in 1940, has a spacing of 40 ft; but even this is a "far cry" from the 180 ft of the Coolidge multiple-dome dam, in Arizona, completed in 1928.

Records and Reports.—There is a distinct increase in effort to determine and record accurately the forces actually effective upon and within completed dams and the behaviors of structures, including the constituent concrete, under external and internal stresses. However, in this respect it is only the trend and not the total of accomplishment which is laudable.

The "Technical Report on Construction of Morris Dam," in 1935, sponsored by the Water Department of the City of Pasadena, Calif., set a fine example of recording for the use of the profession the history and much detail as to design and construction of that structure.

ROCK-FILL DAMS

Rock-fill dams, as a distinctive type of structure, had their origin in the mining regions of the Sierra Nevada of California. There they constituted a very logical solution of the local problem—provision of storage at remote locations, where transportation of material and equipment would have been costly, where sound rock in unlimited quantities constituted the most easily available material for construction, and where a very large portion of the local population was accustomed to the use of explosives in mining activities. The same type of structure has been used since elsewhere, where conditions were generally similar; it has been used also where yielding or movement of foundations was to be anticipated. In 1939, very complete information concerning rock-fill dams, from the earliest examples in the mining regions of California to the largest recent structures, was made available.[5]

In Mexico the low cost of labor and the high cost of imported equipment has led to the adoption of the rock-fill type for five important dams: Charcas Dam, State of Guanajuato, 1933, 49 ft high; Taxhimay Dam, State of Hidalgo, 1934, 137 ft high; Madero Dam, State of Hidalgo, 1938, 153 ft high; San Ildefonso Dam, State of Querétaro, 1938, 189 ft high; and Tepuxtepec Dam, State of Michoacán, 1934, 126.5 ft high. In regions of Sweden, within the Arctic Circle, where the cost of concrete would be very high on account of the remote location, rock-fills have been used. In Chile, the possibility of earthquake damage to other types of dams has led to the choice of the rock-fill type. In Algeria, difficult foundation conditions have led to the use of rock-fills rather than masonry or concrete dams.

Extent of Use.—However, rock-fill dams have not been used as widely as a number of other types of structure. For example, in California, which may be considered the home of the rock-fill type, only 6.6% of the 588 dams under state supervision on January 1, 1936, were rock-fill structures; 64.5% were earth-fill; and the remainder, 28.9%, were of concrete or masonry; about the same percentages were reported to apply to the 950, more or less, dams examined. In a list of sixty major dams in California, thirty-four were concrete or masonry, and twenty-six were embankment dams, including ten important rock-fill dams of heights varying from 122 to 320 ft. In a similar list[6] of 600 important dams in the world, there were 38 rock-fill dams of which 17 were built in the period from 1930 to 1938.

Rock-fill dams are conspicuously lacking in the extensive construction program of the Bureau of Reclamation, although earth dams with heavy rock blankets on the downstream faces are numerous. Published reports of the Bureau show that rock-fill designs (as well as others) were considered, but discarded for the types shown, for the following structures: Stony Gorge Dam, Ambursen type, 142 ft high, 1928; Bartlett Dam, multiple arch, 270 ft high, 1939; and Deadwood Dam (in Idaho), concrete arch, 165 ft high, 1931. Although these are the only published cases of rock-fill dams having been discarded by the Bureau of Reclamation after comparison with other designs, it is possible that there are others.

---

[5] "The Design of Rock-Fill Dams," by J. D. Galloway, M. Am. Soc. C. E. (and the accompanying discussions), *Transactions*, Am. Soc. C. E., Vol. 104 (1939), pp. 1–92.

[6] "Dam Building Reaches a Climax," by Robert A. Sutherland, Assoc. M. Am. Soc. C. E., *Engineering News-Record*, December 10, 1936, p. 807.

Salt Springs Dam, in California, completed in 1930, still remains the highest purely rock-fill structure in existence. However, its height (313 ft above stream bed) has been exceeded by San Gabriel No. 1, in California, a rock-fill with an impervious earth-fill section.

Trends in Design.—Rock-fill dams vary widely in design. As a definite starting point in a discussion of these variations, the typical rock-fill structure may be taken as "A loose rock-fill forming the mass of the dam; an impervious face next to the water; and a rubble cushion between the two." Adequate design usually centers in two problems:

(1) Constructing the fill in such a manner as will cause the major portion of the total settlement to take place during construction and before placing of the watertight facing; and

(2) The selection of a type of facing that is sufficiently strong to withstand such loads as may come upon it and at the same time is flexible enough to follow without extensive cracking or fracture such residual settlement as may later occur in the fill.

The difficulties that have arisen in the solution of these problems under the differing conditions at various sites have led to modifications of the primitive or pure type of rock-fill, and these modifications appear to fall into two general groups:

(1) The substitution of dry rubble in place of the loose rock-fill, or (as the latter is often termed in California) the "drop-fill" section, forming the main portion of the dam; this substitution varies in extent, in some cases being only a moderate thickening of the dry-laid rubble cushion. In extreme cases a complete dry-rubble fill is substituted for the "drop-fill."

(2) Abandonment of the thin impervious facing of the dam (usually of reinforced concrete) and the adoption of a modified section in which the body of the dam is of rock-fill of various classes, and watertightness is secured either by an impervious earth blanket on the upstream side of the rock-fill, or an impervious earth core either in the upstream half of the dam, or as a central core of varying width.

The most complete change under group (1) is to be found in certain of the Algerian rock-fill dams, in the Shing Mun Dam (China), and in the Tepuxtepec Dam (Mexico); the most complete change under group (2) is to be found in the revised design of San Gabriel No. 1, and certain later dams.

In the United States, a large part of the knowledge of the behavior of rock-fill dams and the methods necessary for proper construction of the fill and design of facing has been derived from several very large structures, among them the Dix River, Salt Springs, Bucks Creek, and the two San Gabriel dams, in California. The latter are of particular importance in their apparent influence on subsequent designs.

Both of the San Gabriel dams were originally designed as typical rock-fills. Construction of Dam. No. 2 was started in July, 1932, and proceeded steadily until early in 1934 when it was interrupted on account of settlement that resulted from the heavy rains in December, 1933, and January, 1934 (and that reached as much as 12 ft vertically), wrecking much of the laminated concrete facing. This serious trouble served to focus the attention of all designers on the inherent difficulties in providing a type of facing satisfactory for very large structures, and on the care needed to reduce to a minimum the shrinkage in the fill.

At the same time, by coincidence, troubles of another type developed at San Gabriel Dam No. 1; there, prohibitive costs were resulting from the very great excess of quarry waste. This led to a change in the design of San Gabriel Dam No. 1 to a rock-fill with impervious earth section, which served not only to utilize quarry waste, but also eliminated concrete facing that would have been required with the rock-fill design, and with it went such troubles as had been encountered at San Gabriel Dam No. 2.

The precedent (of eliminating facing and substituting a section of impervious fill) set in the redesign of San Gabriel Dam No. 1 appears to have "carried weight," for since that time several dams consisting principally of rock-fill, but with impervious earth sections instead of facing, have been designed or constructed, and a continuous transition can be traced from typical rock-fills to earth-fills, with little difference between any two succeeding designs.

Progress in Knowledge of Ground Water

The past decade has been notable for a considerable increase in knowledge of ground water and in the inauguration of systematic surveys for the further increase of knowledge. Much of the information has concerned the limitations and exhaustion of ground-water sources. Extensive investigations were made of the ground-water levels and salt content on Long Island, N. Y., by private utilities and by the City of New York, followed by more extensive and continuing investigations by the state, national, and county governments. These emphasized the fact, previously known only to a few, that the western end of Long Island was already overpumped, with resulting increase in salinity. Since this fact became known, the state authorities have disapproved practically all applications for wells on western Long Island—that is, in Kings County and parts of Queens County of New York City, except for use in tight, noncorrosive cooling systems, in which cases wells for returning the water to the ground are required. Heating of the ground water has occurred in some of these cases where wells were too close together.

Immediate influence in ground-water levels between wells more than 10 miles apart in the water-bearing stratum called Lloyds gravel next above the rock on Long Island has been noted by systematic observations of the water levels. Pumping at one well results in decreased levels at distant wells.

Joint state and national investigations in New Jersey have disclosed salt intrusion caused by overpumping of wells in areas along the Raritan and South rivers.

Texas, particularly regions supplying the City of Houston, has been studied intensively, overpumping has been found, and increase in salinity noted. The region of Miami, Fla., has suffered salt intrusion caused by drought and reduction of water storage in the Everglades by too much drainage. A joint investigation by several agencies has been inaugurated.

In California, where ground water is still the lifeblood of the fruit, nut, and vegetable culture which forms the principal business of the state, the closest attention continues to be paid to ground-water levels and to the legal rights of persons using existing wells or projecting new wells. The irrigation or spreading of water on sandy and gravelly stream beds by the aid of a series of low retaining dams, to increase the yield of wells, has been practiced successfully, especially in the San Gabriel Valley south of Los Angeles. Two large dams, San Gabriel Nos. 1 and 2, for conserving flood flows for subsequent irrigation, have been constructed there in recent years.

In many parts of the United States ground-water investigations have been made because of the increased importance of the ground-water reservoirs in periods of drought which have occurred with unusual intensity in many parts of the country during the past decade.

The great advance in the scientific approach to the subject of ground water will be evident from the following enumeration of items of progress, description of which cannot be encompassed in the space available for this summary:

1. Establishment of a section on hydrology, of the American Geophysical Union. Valuable contributions to ground-water information are contained in reports of the Committee on Underground Waters for each of the years 1932 to 1939 inclusive, published in the *Transactions* of the Union for those years.

2. Establishment of a number of hydrological laboratories for the specific purpose of conducting research on the water-bearing characteristics of soil and rock.

3. Expansion in establishment of state well-drillers associations with consequent improvement in logs of wells and in preserving other valuable data.

4. Beginning in 1936, of annual publications by the U. S. Geological Survey of reports entitled "Water Levels and Artesian Pressure in Observation Wells in the United States."

5. Study of ground water by oil companies in connection with disposal of oil-field waters.

6. Development of a method of estimating the quantity of ground water available in an area, based on discharge by plants and evaporation from the soil. This was in the ground-water study in Escolante Valley, Utah (see U. S. Geological Survey, *Water Supply Paper No. 659a*).

7. Recognition of the existence of pollution by salt water and wastage of ground water by corrosion of well casings, and ways to correct the conditions in the Hawaiian Islands, New Mexico, Florida, and Texas (U. S. Geological Survey, *Water Supply Paper No. 796a*). Similar work has been done in California.

8. Discovery that yields from certain aquifers have come from stored water and not from recharge to the aquifer in the outcrop area. Such withdrawal of water has been accompanied by the compression of the aquifer.

9. General use of automatic water-stage recorders at selected inactive wells in areas under investigation.

10. Collateral-ground water studies made in connection with soil erosion studies and the attempt to reduce runoff (U. S. Geological Survey, *Water Supply Paper No. 679a*).

11. Better understanding of the "skimming process" for safely obtaining fresh ground water in regions subject to sea-water encroachment.

12. Use of geophysical method originally designed for use in prospecting for oil but also used for locating buried pre-glacial valleys containing ground water. The so-called resistivity method has been used in Texas and the Hawaiian Islands for determining the depth at which salt water will be encountered.

Among the recent valuable publications on ground water are a treatise by C. F. Tolman entitled "Ground Water" and a volume by Morris Muskat entitled "The Flow of Homogeneous Fluids Through Porous Media."

A device called a water mine, consisting essentially of a caisson from which horizontal wells in star formation are driven out sometimes to a distance of more than 200 ft, has been used successfully in Portugal, and also near the Thames River in England and in Ohio.

Graveled wells are being used increasingly, although wells adapted to the peculiar conditions, such as the California stovepipe well in which the casing is perforated after the well has been placed, continue to be used. There is an increasing use of brass and other copper alloys instead of iron or steel for screens both of the perforated-plate and the wire-wound types. Deep-well pumps instead of suction pumps are increasingly used, partly from prudence owing to uncertainty as to when the ground water will recede below the depth possible for suction; increased efficiency of turbine pumps have hastened this tendency. There is a steady increase in the use of turbine well-pump units with submersible electric motors directly connected below the pump.

AQUEDUCTS, TUNNELS, AND LARGE TRANSMISSION MAINS

The past decade has witnessed the construction of some of the most important aqueduct, tunnel, and large transmission pipe-line systems thus far constructed in the United States, including such mammoth undertakings as the 250-mile tunnel and conduit construction of the Metropolitan Water District of Southern California bringing water from the Colorado River to the Los Angeles metropolitan area and the 85-mile tunnel of the New York City water supply to make available 540 mgd from the Delaware River and Rondout Creek.

The work of the Metropolitan Water District of Southern California, in addition to the 250 miles of main aqueduct, includes about 140 miles of large branches. Of this total of nearly 400 miles, 92 miles were in tunnel. The conduit included some of the largest reinforced-concrete pipe thus far built, much of it being in sizes of from 9 ft to 12 ft 8 in. in diameter and designed for heads up to 285 ft. The making, transporting, and placing of pipe of this size necessitated the adoption of construction methods developed especially for the job and the terrain through which it passed.

The Delaware River aqueduct being constructed to supply New York City involves an 85-mile tunnel varying in diameter from 13.5 ft to 19.5 ft. This tunnel is of the deep-pressure type throughout its length, being located from 300 to 1,000 ft below the ground surface. Its construction involves 31 shafts, the length of the shafts alone exceeding 3.5 miles if placed end to end. Increased speed of construction has been notable in all this work.

Materials Used in Large Conduit Construction.—In general there has been a tendency toward the adoption of reinforced concrete or welded steel in large pipe construction. The use of cast iron has been less common than in former decades. However, there are a few notable exceptions, particularly since the availability of the new cast-iron-pipe specifications developed by Sectional Committee A21, American Standards Association (A.S.A.) (Sectional Committee on Specifications for Cast Iron Pipe and Fittings).

A review of recent literature covering approximately 600 miles of large conduit construction (36 in. and larger) indicated that approximately 40% were constructed of steel; 40% of reinforced concrete, with or without a steel cylinder embedded; and 20% of cast iron.

Reinforced-Concrete Pipe.—The first shop-built reinforced-concrete pipe of large diameter used in the United States was that at Baltimore, Md., about 1914. A recent examination showed this to be in an excellent state of preservation with its interior surface practically unimpaired in smoothness. From 1914 until 1930 approximately 225 miles of reinforced-concrete pipe in sizes 24 in. and more have been laid. In the decade between 1930 and 1940 an additional 380 miles have been laid. In other words, there has been 160% as much large concrete pipe laid since 1930 as in the 15 years prior to that year. This decade has also witnessed the use of reinforced-concrete pipe in larger sizes, up to 12 ft 8 in. in diameter, and under heavier heads than previously. The 54 in. crosstown Denver (Colo.) line operates under a 400-ft static head.

The earlier reinforced-concrete, pipe-line joints were of the jute-filled lead gasket type. In recent lines this is being superseded to a considerable extent by a rubber gasket type of joint. In this joint the rubber gasket is placed in a recess of the steel joint ring on the spigot end of the pipe. The shape of the steel joint ring on the bell end of the pipe is such that as the pipe is pulled home the rubber is compressed. The space inside and outside between the spigot and bell ends of the pipe is then filled with cement mortar in the same way as with the jute-filled lead gasket type of joint previously used.

One of the principal advantages of the rubber ring joint is that all calking is eliminated. Surrounded by cement-protected metal and thus protected from its chief enemies, light and air, the rubber is expected to have an indefinite life. Considerable evidence exists as to its durability under similar conditions.

Flow tests of reinforced-concrete pipe indicate little reduction of carrying capacity with age. An 18-yr-old reinforced-concrete pipe in Denver has still its original Williams-Hazen coefficient of 147.

Steel Pipe.—Practically all steel pipe lines in recent years have been of welded construction, thus eliminating the rough interior caused by laps and rivets and the longitudinal ridges of lock-bar construction and, in conjunction with improved linings, materially increasing the carrying capacity. Large steel pipe lines are now commonly made up in the shop in sections of from 30 ft to 40 ft—generally the latter and occasionally longer—the upper limit being determined by the necessities of railroad transportation and ability to follow the curvature of the railroad. Field joints are usually either welded, bell-and-spigot, slip joint (in which the bell is made by expanding one end of the pipe or by welding on a collar, the joint being made up by welding or by riveting and calking), or Dresser coupled. Probably a combination of the first and last named types of joint is used most frequently.

Standard specifications for lock-bar, riveted, and welded steel pipe 30 in. and larger have been developed by a committee of the American Water Works Association (A.W.W.A.) and are now (1940) under discussion. Specifications for smaller steel pipe, and for linings and external coverings of tar enamel and cement mortar, are well advanced.

Cast-Iron Pipe.—The use of cast-iron pipe in large sizes is receiving renewed attention since the standard specifications by Sectional Committee A21 have been made available. These specifications, which have resulted from 13 years of testing, research, and study by a Sectional Committee (A21) organized under the American Standards Association and sponsored by the American Gas Association, American Society for Testing Materials, American Water Works Association, and New England Water Works Association, have devised methods of design which take account of earth and truck loads as well as internal pressures. The adoption of spinning or centrifugal production methods for pipes of large size has made rapid strides and at the present time sizes up to 48 in. in diameter are being produced thus. Centrifugally cast pipe designed according to the procedure developed by Sectional Committee A21 was accepted for 81,000 ft of 30-in. pipe line at Corpus Christi, Tex., 121,000 ft of 30-in. pipe line at Greenville, S. C., and approximately 25,000 tons at Wichita, Kans., largely 42 in. and 48 in. in diameter.

Specifications for Purchase.—It has been quite common practice in recent years, on large conduits, to issue specifications designed to secure competition from two or all three of the aforementioned materials. Usually these specifications include bases of evaluation by which carrying capacity, probable life, and leakage, as well as fixed charges on the cost, are given consideration. There has been considerable variation in the bases of evaluation, depending upon the background of experience of the designer, the character of the soil as affecting exterior corrosion and of the water as affecting the interior, the maintenance of carrying capacity, and life. Officials of the Public Works Administration (PWA) have taken a hand in fixing these bases of evaluation on projects in which the federal government was providing a grant in aid. In a relatively few projects the only consideration has been that of price. "Protection from corrosion" is treated subsequently, covering both transmission and distribution mains.

TRENDS IN THE DESIGN AND CONSTRUCTION OF DISTRIBUTION SYSTEMS

Pipe Materials.—Cast iron continues to be the predominant material for the construction of distribution systems. The processes for producing cast-iron pipe have been greatly improved, resulting in greater uniformity in thickness and in metal strength. The process for centrifugal casting in metal molds without cores was introduced into the United States about 1922. The method of centrifugal casting in sand molds without cores dates from 1926, and pipe horizontally cast in green sand, with multiple gates, from 1922. Previous to this all cast-iron pipe was cast vertically in pits. At present only a small percentage of pipe is cast in pits and that is mostly in large sizes. Within the past 3 years (1937–1939) the centrifugal methods have encroached rapidly on the large pipe field.

Cement asbestos pipe was introduced from Italy in 1930. It has been produced there since 1916. It is now available from a number of factories distributed over the United States in sizes from 4 in. to 20 in., inclusive, and its use is increasing.

Steel pipe, with the various protective coatings mentioned elsewhere, has regained some of its former popularity. The cycle through which steel pipe for distribution systems has passed is an interesting one. In the middle and latter part of the last century it was used extensively in the eastern part of the United States in riveted form with thin plates and with a thick inner lining of natural cement mortar and often an outer mortar coating also. During the first quarter of the present century it was common practice for the more promotional type of developer in California, and perhaps some other states, to use thin riveted steel pipe in all sizes, using metal of 12, 14, and even 16, gage. No mortar protection was used but the surface protection was usually an ordinary hot asphalt dip, and lengths were joined by driving a tapered end into the preceding length after heating the coating so that it would seal the joint. Several miles of 9-in. and 11-in. riveted pipe, 16-gage thickness, laid in 1873 and 1876, have just been replaced in one California city because of a change in system operation involving increased pressures.

Then, for a time, perhaps due to an easier market in cast iron, little was done to advance the use of steel pipe in the distribution network although larger steel pipe, generally of substantial thickness, was used extensively for transmission lines. Recently, welded steel pipe with comparatively thin steel and tar enamel coating has been used to a considerable extent in competition with cast-iron and cement-asbestos pipe.

Now has come, particularly on the Pacific Coast, a further development of steel pipe, somewhat after the fashion of the cement-lined and covered pipe of the past century but applying the improvements afforded by the development of spun cement lining on the inside and reinforced gunite protection outside. The steel is much thinner than would be considered now for an all-steel pipe of the same diameter, being designed for the bursting stress with a minimum thickness dictated by experience to produce a pipe stiff enough to hold its shape during the spinning process. After spinning and guniting, the pipe can be handled safely, hauled by placing in cradles on truck or car, and laid with a stiffleg and sling. The normal thickness of lining on the Pacific Coast is about

½ in. and of gunite exterior about ¾ in. with mesh reinforcing. The pipe is welded and may be rolled from rectangular plates or spirally, from a strip. The joint between lengths may be of the bell-and-spigot design, calked with cement, and finished with mortar both inside and out. The steel is completely protected. For California, pipe of this design becomes cheaper than cast iron at approximately 16-in. diameter. For eastern United States the division point is at a somewhat larger size.

Methods of Laying.—There is undoubtedly a trend toward better laying methods designed to give the pipe the best chance, and the work of A.S.A. Sectional Committee A21 is undoubtedly responsible for some of this improvement. Doubtless many engineers have been astonished to learn to what extent pipe laid on blocks is more likely to fail and the strengthening which may be accomplished by tamping beneath and on the haunches of the pipe. More attention will be paid to these things in the future. For given conditions and factor of safety the additional cost of tamping is more than saved in the price of the pipe.

The Committee's elaborate work which has extended over 13 years has resulted in agreement among its membership, composed of manufacturers and engineers, on a revised method of getting at the design of cast-iron pipe barrels which takes into account not only the internal pressure but the external loading and manner of laying as well. This method is backed by a tremendous amount of data, either the result of experiments directed by the Committee or the work of earlier investigators. As a result of this work pipe can be designed for various pressures, various depths of cover, and different soils or laying conditions with a uniform factor of safety.

Joints.—The most notable trend in joints is the continued effort to depart from the use of lead.

Several lead substitutes have been used to a considerable extent and joints of neat portland cement have been found economical and very satisfactory. Lead joints are sometimes placed at intervals to provide a certain amount of ductility to the line. With improvement in the manufacture of rubber, rings of this material are being used with a follower for making up bell-and-spigot joints. Past history as to durability of rubber gaskets in enclosed locations is reassuring. Rubber also has the advantage of being easily sterilized.

Braided hemp for packing has shown economy over the old-fashioned yarn because of the saving in time. Proper lengths for a given size of pipe can be cut quickly and accurately, resulting in uniform joints and saving of material. It has been found to give less difficulty in sterilization of the line than loose jute which sometimes protrudes into the pipe and apparently acts as a culture for bacteria, probably harmless but giving undesirable laboratory reactions and proving extremely virile in resistance to chlorin. Main laying specifications show a tendency to be more stringent as to leakage than formerly.

Services.—During the past decade there has been a great advance in the design of services. The lead service is very much less used than formerly. Galvanized steel or wrought iron is still used, but a large number of communities have abandoned this material in favor of copper tubing which is now available at a reasonable price. Copper offers the advantages of resistance to corrosion;

flexibility, requiring no fittings to change direction for the purpose of changing elevation or passing an obstacle; and, since it comes in 60-ft lengths, requiring no coupling for any reasonable length. Up to this time no important drawbacks to its use have appeared, although there have been a very few reports of brittleness and there is an occasional report of green stains of origin as yet undetermined—possibly stray currents, possibly electrolytic action between dissimilar metals. Copper fittings are available for use with the tubing, either requiring expanding the end of the tube or the so-called streamline type which are soldered in place. With all copper from main to meter, the service may be considered as of equal permanence to any other part of the system.

Cement-lined steel pipe and fittings are also available in greatly improved form and are preferred over copper by some engineers. At the present time the cost is somewhat less. Cement-lined cast iron is available in sizes of 1½ in. and more, and is standard practice in many localities for larger services.

Valves and Hydrants.—No great advances or new ideas are to be noted in the design of valves and hydrants.

The A.W.W.A. has issued a new set of specifications which is intended to produce a more standardized valve.

Fittings.—Sectional Committee A21, in addition to its work on cast-iron pipe, also inaugurated some work on the design of fittings. Tests for strength and also tests for friction losses have been made. For fittings 12 in. in diameter, and smaller, a short body standard has been devised and is being used to a considerable extent as are other short body designs made by various manufacturers as standards of their own. For larger fittings no changes have as yet been decided upon, but the strength tests have indicated need for strengthening certain of the old standard designs in tees, crosses, and wyes having outlets the same or nearly the same as the main pipe, thus producing large flat surfaces at junctions of branch and main passages. Such fittings, though listed with certain pressure ratings, have seldom been used at those ratings and are not properly so rated.

PROTECTION OF PIPE LINES

There has been an increasing interest during the past decade in the phenomenon of decreased capacity of pipe lines caused by corrosion and tuberculation. Several outstanding reports have been contributed by committees of technical societies listing a very large number of flow-coefficient determinations that show, almost invariably, large reductions in carrying capacity. Tar dip had begun to yield to improved linings as much as 30 years ago, and a strong impetus was given to this movement in the decade previous to that included in this report by the introduction of spun cement linings in cast-iron and subsequently in steel pipe. The progress was slow, however, and water supply superintendents and engineers were seemingly less enterprising than their predecessors of 50 to 70 years ago, who laid many miles of thin steel pipe lined with natural cement mortar of very substantial thickness. None of this pipe has been reported as having deteriorated materially as regards its lining, although most of it has been retired on account of structural weakness.

As late as the report of this committee[7] for the year 1938, the cast-iron pipe lined with cement or other improved protection did not exceed about 10% or 12%, as reported to the committee, and as late as 1936 the value[8] was only about 7%. A recent canvass made by the manufacturers, for the Committee, indicates that the percentage has increased to about 29. This increase is caused largely by the development of a thin, spun cement lining, which is at first thoroughly rubbed on to the iron with the pipe spinning at low speed and then smoothed centrifugally by a higher speed. The thickness varies from $\frac{1}{16}$ in. for smaller sizes to $\frac{1}{8}$ in. for larger sizes. It is frequently protected from rapid leaching by a spray coat of asphalt.

For steel pipe in the East, spun tar enamel has continued to be the rule except for spun cement lining in stock-welded steel pipe in diameters of 25 in. and smaller. Tar enamel has been used extensively on the Pacific Coast, but the East Bay Municipal District and the City of Los Angeles appear to have standardized during the past few years on spun cement lining for steel pipe with reinforced gunite exterior coating. The City of San Francisco, Calif., has installed steel pipe coated with tar enamel without the usual priming coat. It is becoming the almost invariable custom to sandblast or steel shot blast steel pipe before coating; and also to whitewash the exterior coating where tar enamel is used to protect both interior and exterior enamel from the heat of the sun. Such whitewash has been found to improve the resistance of the coating to soil stresses. Tar enamel of improved quality, "plasticized," has come into use during the past decade. The laboratory methods of testing the primer and the enamel have become highly developed as have also methods of testing the enamel for pin-holes both in shop and field by high potential electric current.

Lining of Large Pipe in Place.—During the decade, a machine for applying cement mortar to large pipe lines in place by a centrifugal thrower followed by revolving trowels, has been developed and used for the lining with cement mortar of about 63,000 lin ft of steel main from 48 in. to 54 in. in diameter. It is understood that a greater amount is on order, indicating increased future use of this method. Most of the installations have been in the reconditioning of old pipes, but new work will include the lining of pipes laid without lining and then to be lined in place by this process. Linings are generally $\frac{3}{16}$ in. or $\frac{1}{4}$ in. in thickness. The process consists of dashing cement mortar on the interior of the pipe by a traveling machine containing essentially a centrifuge for throwing the mortar and a set of trowels attached to revolving arms which smooth the lining.

One who is acquainted with the development and practice during the past 15 years in the lining of cast-iron pipe with cement mortar, centrifugally, and with the developments of cement mortar lining of steel pipe on the Pacific Coast during the past 5 years, cannot but be struck by the extreme differences in the proportions and thicknesses of the linings and in the details of workmanship. The cast-iron pipe linings began with a mortar having three parts cement to one part sand, and in only a few cases, as far as the Committee is aware, has any mortar less rich than 1 : 1 been used. The early spun linings for cast-iron pipe were, in some cases, as thin as $\frac{1}{16}$ in., and the recently adopted A.S.A. standard provides for not less than $\frac{1}{8}$ in. thickness for the small pipe, $\frac{3}{16}$ in. for the intermediate sizes, and $\frac{1}{4}$ in. for the larger sizes. A few consumers have specified thicker linings but have been content with an increase of $\frac{1}{16}$ in. or so because of the manufacturers' fear of cracking or separation.

---

[7] *Proceedings*, Am. Soc. C. E., March, 1939, p. 429.

[8] *Loc. cit.*, April, 1937, p. 693.

The cement lining that has been responsible during 1939 for the great increase from about 12% to nearly 30% in the percentage of cast-iron pipe which is cement lined was described previously herein as having a thickness of $\frac{1}{16}$ in. to $\frac{1}{8}$ in., according to the size of the pipe. This lining is supplied without extra charge as compared with tar dip and, therefore, has been accepted by many who do not feel moved to pay for a cement lining of standard thickness.

In steel pipe on the Pacific Coast, linings of about $\frac{1}{2}$ in. average thickness are specified with limits of $\frac{3}{8}$ in. to $\frac{5}{8}$ in., or $\frac{3}{8}$ in. to $\frac{3}{4}$ in., and with mortar having proportions of 1 cement to $2\frac{1}{2}$ sand with even 1 part to 3 parts permitted in some cases. This latter proportion is, roughly, nine times as lean as the mortar that was commonly used in cast-iron pipe during the greater part of the past decade. In some cases, also, the steel pipe specifications provide for removal of laitance from the interior of the lining after spinning and its replacement with a surface finish of troweled mortar.

Some have recognized the importance of the choice of a proper cement for the lining of pipe, and the laboratories of several producers have investigated this matter; some of them are still investigating. The engineering profession has been very backward in attacking this question as a research matter. Obviously, a thin lining can be successful over a considerable period of time only if the cement is of very low solubility, and even a thick lining will lose its smoothness, with the resulting decreased carrying capacity, if the surface mortar becomes roughened by loss of cement. The favorable long-time experience with cement-lined steel pipe of the past century is unfortunately not directly applicable because it was done with natural, not portland cement.

Eastern manufacturers of steel pipe appear to be uncertain as to the safety of spinning a cement lining and subjecting it to the distortions caused by backfilling. As far as the Committee is aware, the western steel pipe lined with cement has been all covered with reinforced gunite and there is speculation (as yet unanswered by the test of construction and use) as to whether mortar lining in a large steel pipe not covered with gunite would be damaged by backfill even if the pipe, while being backfilled, were subjected to water pressure. The attachment of stiffening angles, or the use of special plates rolled with stiffening ribs, has been canvassed, but the expense involved renders this type of construction uneconomical as compared with other available types.

Lining of Small Pipe in Place.—During the past decade the Australian method of lining small pipe in place after cleaning it has been developed and is in extensive use in Australia, New Zealand, South Africa, and England. A recent beginning has been made in South America and the United States. The Committee is informed that in this country the method has been used in Pennsylvania, Massachusetts, and South Carolina and is about to be used in Florida and elsewhere.

Exterior Coatings for Pipe.—For the exterior protection of cast-iron water pipe, coal tar dip and, for cement-lined cast-iron pipe, various kinds of bituminous varnish as well as tar dip, are the prevailing methods of protection. For salt water and other corrosive conditions, tar enamel put on either over tar dip or over a special primer is frequently used, following the successful experience inaugurated about 25 years ago. In recent months cement mortar about ¾ in. or more thick, wrapped on with an open-mesh cotton fabric which is subsequently brushed over with mortar, has been used in some cases on pipe exposed to severe corrosion, as in salt muck.

On the Pacific Coast a process has been developed recently by which a mixture of bituminous filler with graded gravel and asbestos is fed hot, under pressure, to the exterior of a pipe which is mounted in a machine, so that the pipe comes out with a protective material having a thickness of ⅜ in. to ½ in. This development is made for oil pipes but is being applied under license.

During the last decade, the exterior protection of steel pipe used in water supply has been either of tar enamel or tar enamel with asbestos felt or kraft paper wrapping, or of gunite. One large job had a wrapped mortar covering. Reference has already been made to the East Bay Municipal District and the Los Angeles standard construction of mortar lining and reinforced gunite exterior coating for steel pipe. In very corrosive ground this gunite is placed over tar enamel on the exterior of steel pipe.

Cathodic Protection for Pipe Lines.—For some years well-informed water supply engineers have been watching the rapid development for the protection of gas and oil lines by the cathodic protection method. One long steel water main on the west coast (the Mokelumne pipe line of the East Bay Municipal District), following a partial failure of the bituminous and felt wrapping coating originally provided, has been equipped with cathodic protection. A recent specification for a long water main provided for a tar enamel coating and for the removal of stones and muck from backfilling to be used within 6 in. of the pipe. It was the intention that the engineering records should strongly recommend making periodic inspections of this pipe line and applying cathodic protection if and when required. A pipe line must have continuous electric bonding either through welded joints or by very generous bonds across joints in order that the cathodic method of protection may be applicable. A well-known coupling contains a metallic spiral spring in the rubber gaskets designed to furnish such a bond by the compression of the spring against the metal of the pipe and coupling.

This cathodic protection appears of such interest and potential importance to water supply engineers and owners that a general description and appraisal of the process which has been kindly furnished by one of the contributors outside the Committee membership has been included in this report. (See heading "Increasing Use of Cathodic Protection").

Concrete Incasement of Old Steel Pipe.—More than 22,000 ft of 10-in. and 14-in. pipe in the Vista, Calif., irrigation district have been saved from destruction and given a new lease of useful life in very corrosive ground by incasement in concrete reinforced by hoops and longitudinal rods and placed inside a metal form by pneumatic pressure.

TANKS

Within the past decade the large elevated tank has come into its own. The earliest steel elevated tanks (the first of which was built at Laredo, Tex., in 1893) were practically all built with hemispherical bottoms. This practically limited the size to small capacities unless a relatively large variation in head could be permitted. A notable exception to the ordinary range of sizes was the tank built for the Louisville (Ky.) Water Company in 1905. This tank stored 1,200,000 gal at a total height of 220 ft, a diameter of 45 ft, and a water-level variation of 87.8 ft. Such a range of head would ordinarily be undesirable. With the hemispherical type of bottom design, if the variation in head is kept within a reasonable range for large capacities, the diameter would become so great that the relation of the vertical side-wall height to the height of the hemispherical bottom would be all out of proportion. For all practical purposes, therefore, the hemispherical bottom tank was limited to about 250,000-gal capacity.

This inherent difficulty with hemispherical-bottom tanks led to the design of the ellipsoidal type of bottom, the vertical depth of which is about half that of the hemisphere. With this type of design it was practicable to build tanks of larger capacity with relatively lower head variation. This design is well adapted for capacities up to about 600,000 gal.

Beginning about 1930, elevated tank builders developed a design having a substantially flat bottom supported upon beams, girders, and columns which made it practicable to store water in any quantity desired, with substantially any range in head desired. Further variations in design have included oblate spheroids with bottoms flatter than tops and with water filling against the curving top as well as sides and bottom. These progressive improvements in design have been reflected in the growing use of large elevated storage tanks. The past fifteen years have seen the installation of more than 250 elevated tanks larger than 500,000-gal capacity, of which 77 were 1,000,000 gal or over.

Some of the contributing factors, other than the developments in design, which have brought about this great increase in use of large elevated storage capacities, are discussed separately herein.

Appearance.—Coincident with the developments in design, there has been a studied effort to make the elevated tank an attractive structure. This phase of the problem received a real impetus by a nation-wide competition of elevated tank designs conducted by one of the leading tank manufacturers a few years ago. Several hundred designs were submitted and substantial prizes were awarded for the best designs. Out of this competition grew an appreciation of the fact that an elevated tank need not be a blot on the landscape. Tanks now have a more streamlined appearance, the old conical roof has largely been supplanted by the graceful curved lines of the upper part of the tank, in which the roof serves as a water holder, and in many of the larger tanks tubular columns rather than structural built-up members have been used.

A notable illustration of this type of construction is the Blue Ridge tank recently completed at Indianapolis, Ind. In this tank the central riser is replaced by four centrally grouped tubular columns, thus better distributing the weight and permitting the center control compartment, or vault, to be placed in the center of the four-column group. One of these four central columns is utilized as the riser pipe connecting the tank to the distribution system. Another serves as an access tube to the tank interior, and within it is placed the ladder, overflow pipe, pressure pipe for flushing, and conduits for electric wiring. In addition to the four centrally grouped tubular columns, the tank is supported by twelve outer columns. All columns are 54 in. in diameter. The bottom of the tank is supported on twelve radial girders constructed as cantilevers, the outer wall of the tank being 9 ft outside of the row of columns.

Construction Methods.—Within the past decade elevated tank construction has changed from almost 100% riveted, to almost 100% welded, construction, particularly for the larger sizes. This change took place almost over night. Specifications for welded elevated tank construction are now in progress of preparation through a joint committee of the American Water Works Association and the American Welding Society. It is believed that they will be presented to the respective societies for approval during 1940.

Economics of Elevated Tanks.—The use of elevated tanks is no longer confined to small water works systems, as illustrated by installation of four tanks at Detroit (Mich.) of 1,500,000 gal capacity each, and one of 2,000,000 gal; Buffalo, N. Y., with three tanks of 2,000,000 gal each; Columbus, Ohio, with two of 1,000,000 gal, one of 1,500,000 gal, and one of 2,000,000 gal; Milwaukee, Wis., two of 1,000,000 gal each, Sacramento, Calif., two of 3,000,000 gal each, etc.

Unusual Designs.—Like all other engineering matters, the selection of type and dimensions of tank is largely dependent upon the local requirements. Illustrative of this fact are the installations at Kenosha and Racine, Wis. In each of these cities elevated storage was desirable in the amount of about 1,500,000 gal. It was found that, with the topographical conditions, distribution system layout, and the clear-water storage capacities at the filtration plants, it would be practicable to build a 3,000,000-gal stand-pipe for substantially the same cost as a 1,500,000-gal elevated tank. The top half would be available and used as high service storage and the bottom half would be made available by pumps, if desired, when the water level was drawn below mid-height. The lower level, in effect, is reserve clear-water storage capacity.

Sacramento is illustrative of a unique design in which a steel shell designed to resist full internal pressure is bonded with and encased in a concrete shell. The shell was poured while the steel tank was full of water so that the concrete is under no stress when the tank is full and in compression at all stages of lower water level. The design provides resistance to possible surge of the contained water that might be caused by an earthquake.

In the municipal water system of Syracuse, N. Y., a group of three large steel stand-pipes having a combined capacity of about 8,000,000 gal have been covered inside and out with reinforced gunite about 2 in. thick. The bottoms are entirely of reinforced gunite. Two existing tanks were coated with gunite several years ago and the gunite has remained thus far (1940) in good condition.

Protection of Tanks.—Paint continues to be used except in a few cases where tanks have been protected by the use of gunite, and recently by cathodic methods which will be mentioned subsequently herein. Standard red lead, with sometimes an admixture of litharge, continues to be used by many conservative engineers, but there has been a great increase in the use of synthetic vehicles wholly or partly replacing linseed oil and in the use of aluminum paint or mixtures of aluminum and other pigments, notably graphite. Although water works superintendents and engineers are ever optimistic, and listen with patience to descriptions of new paint products which are thought by their manufacturers to be superior to the old paints, there appears to be no conclusive evidence that modern paint has a longer life than the paint used in past decades.

In recent months the cathodic method of protecting ferrous metals, previously exploited and used to some extent for the protection of boilers, condensers, and heaters, and the kindred adaption of the cathodic process to the exteriors of buried pipe, have been adapted to the protection of steel tanks. Anodes of stainless steel or graphite are hung in the tanks and a direct electric current is passed in such a way as to make the tank cathodic, which tends to transfer the corrosive effects from the tank to the renewable anode rods. There are many of these installations recently made, beginning about 1936, but the Committee has been unable to secure a genuine appraisal of the effect as yet, although favorable opinions are expressed guardedly.

In one case noted, the throwing off of rust following the application of the cathodic method caused an old, very rusty tank to leak at certain seams and rivets. With new tanks, or tanks in good condition, there is promise, based upon the brief experience with tanks and on the longer experience with boilers, condensers, and the exteriors of buried pipe, that the method will be helpful in preventing corrosion.

Increasing Use of Cathodic Protection.—Cathodic protection as a means of protecting the exterior of pipe lines has been known for a number of years. In March, 1908, Herman Geppert obtained a German patent and in this patent mentions placing the anodes close to the pipe and connected to the positive pole of a generator while the pipes were connected to the negative pole of the generator. In 1910 Herbert W. Harker and Charles C. McNamara, Juniors, Am. Soc. C. E., gave the results of the use of cathodic protection in the prevention of corrosion of boilers. In 1915 a 60-in. steel gas main in New York was protected by cathodic protection by connecting the negative side of the generator to the pipe line and the positive side to an electrode placed in the East River.

These facts are mentioned, not as the total amount of work done in these years on cathodic protection, but to show that cathodic protection was recognized as early as probably 1905 and 1906. However, it was not until within the past ten or twelve years that the use of cathodic protection on pipe lines received very much attention.

During 1935 engineers throughout the United States were reporting results of cathodic protection installations to protect pipe lines. In those days the success of the installations was generally measured by the reduction in the number of leaks which occurred on lines that had been protected. For example, on one 5-mile section of a pipe line in Louisiana in the beginning of 1931 five leaks were reported. At the end of 1931, forty-two leaks were reported, and at the middle of 1932 fifty-five leaks were reported. At that time cathodic protection was installed and development of leaks continued for about six months following the installation, after which time the cathodic protection definitely prevented further development of leaks.

S. G. Rhodes was probably one of the first engineers to protect a large uncounted pipe line cathodically. One of his most recent installations is on a bare 24-in. steel water line[9] which is successfully protecting the line.

The extremely cold weather of 1936–1937 damaged numerous fruit trees in California due to freezing temperatures. As a result of this the California citrus growers installed a piping system near the surface of the ground conveying fuel oil to pressure burners distributed throughout the groves, which are burned when temperatures are low enough to harm the fruit trees. These pipe lines, being near the surface of the ground, were subject to rather severe corrosion and cathodic protection has been used successfully to protect these networks.

During the 1939 Golden Gate Exposition on Treasure Island in California, the networks of five utility systems were subject to the severe corrosion of the non-homogeneous, highly corrosive filled soil. Failures were so rapid that action had to be taken immediately to reduce the number of leaks. By the use of a combination of insulating joints properly installed and cathodic protection, pipe and cable failures have been practically eliminated. The status of this work is reported by W. E. Schneider.[10]

In the past few years practically all the oil and gas lines in the Southwest and Middle West are being protected in the "hot spots," or regions of corrosive soil, by cathodic protection. Many different types of installations are made. The most common source of power for cathodic protection of pipe lines is the copper oxide rectifier of a type developed for the purpose. This source of power seems to be the most satisfactory where power is available. Where power has not been available, such as on the oil and natural gas lines running through open country on right of ways, other sources of power have been used, such as gasoline-driven motor generator sets, natural gas-driven motor generator sets, storage batteries, and—one of the most recent developments—windmill generators. These windmill generators are mounted on a tower and have a large blade, which is driven by wind having a velocity of 4 miles per hr or higher.

Another method of generating current for cathodic protection has been the use of zinc bars placed in the ground adjacent to the pipe line. Probably the first use of zinc for cathodic protection was in boilers, where a nugget of zinc was placed inside the boiler and connected to the boiler sides. The bottoms of ships were also protected by this method years ago. Because of the potential difference generated by the dissimilarity between the metals, zinc and iron, current is driven through the soil from the zinc bars to the pipe line. It is reported that a 1-in. square zinc bar of pure electrolytic zinc would generate approximately 0.025 ampere when connected with the pipe line. These bars are placed adjacent to the pipe and spaced approximately 4 ft apart for 22-in. pipe, and approximately 10 ft apart for 8-in. pipe.

---

[9] "Cathodic Protection of a Bare 24-in. Steel Line," by T. C. Marshall, *Gas Age*, April 13, 1939.

[10] "Cathodic Protection on Treasure Island," by W. E. Schneider, *Gas*, August, 1939.

A cost analysis of one of the large installations of zinc anode cathodic protection shows that the actual cost for protecting 6-in. welded steel lines amounts to approximately 23.6 cents per ft of bare line, and for an 8-in. welded steel line approximately 25.2 cents per ft of bare line.

The cost of cathodic protection is controlled in the most part by the anode system used. Theoretically, the farther away from the pipe line that the anode can be placed, the larger the area of pipe that can be protected for approximately the same power cost. However, it is not always practical to place the anode at a sufficient distance from the line and in these cases the distributed anode system has been used. If any other structures are in the vicinity of the pipe line to be protected, such a structure might "shield" the pipe line from the anodic current with the result that corrosion of the structure might be quite serious. Therefore, it is impossible to apply cathodic protection to any one structure in a city network without giving due consideration to all the other structures present. Cathodic protection of mains joined with material that is not a good conductor should not be considered unless all such joints are bonded with a copper wire of sufficient capacity.

The use of cathodic protection for the protection of structures has increased by leaps and bounds within the past few years. Theoretically, it can be used as protection for practically any structure in any location, but there are some situations where the cost of installing cathodic protection might be prohibitive, such as in a city network where one structure is to be protected without the inclusion of other structures and where the anode would have to be placed immediately adjacent to the structure and extended over the full length to be protected. Another difficult and costly installation would be to attempt to protect the inside of a small water pipe line or to prevent corrosion produced by brass valves in small water lines. However, it can be used to protect pipe lines, water tanks, metallic building structures footed in corrosive soils, underground cable systems, inside of boilers, condensers, etc.

Practically all the companies owning underground piping or cable systems are successfully using cathodic protection in one form or another. Hundreds of miles of water, gas, and oil lines are now protected by this method. One company alone reports that the expenditure to date on cathodic protection amounts to several hundred thousand dollars.

WATER METERS AND GAGES

The types of water meter used to measure water delivered to domestic and other consumers are generally as follows: (a) Positive displacement meter, (b) current meter, (c) compound meter, and (d) fire-line, compound, proportional meter. There have been no material changes in the fundamental design of these meters, but improvements have been made in the choice of non-corrosive metals for the moving parts and casing, and in some cases the enclosure of the gear trains in oil.

Public-service commissions are prescribing narrower limits of accuracy than formerly, and flow tests resulting from this movement seem to prove that there is a much greater quantity of water consumed at low rates than was heretofore suspected. The net result has been more care in manufacture and the choice of material, resulting in greater accuracy.

The meters generally used for measuring the output of pumping stations and reservoirs are, in essence, rate-of-flow meters to which, in general, integrating devices are added to totalize the quantity of water passing. Improvements in meters of this category have continued during the past decade as indicated below. The propeller type has been improved by the use of chrome steel alloys for the bearings and the use of mineral base plastics for propellers, and these have been found not to crack or change in hot or cold water. Special meters of this type have been developed for use in open channels. One is installed in the new Croton Aqueduct at Dunwoodie Station, New York. Others will be installed in the intake channels of the filtration plant by the Metropolitan Water District of Southern California. Vertical installations have been used in the intake cribs in Chicago, Ill. A special meter for measuring the output of individual suction wells from 4 in. to 8 in. in diameter has been developed.

In place of a Venturi tube or an orifice, there have been a number of installations in which the losses in head in the tapered cones leading to pumps or the pump casings themselves or elbows have been used in connection with the indicating and recording devices. An open-flow nozzle for use with indicating and recording devices has been developed and used. Further fundamental research work for establishing the law of Venturi-tube coefficients with changes of size and ratio of throat to main has been done to a considerable extent during the decade.

In 1936 the Joint Committee of the A.G.A. and American Society of Mechanical Engineers (A.S.M.E.) published the results of its work on the determination of coefficients for the thin-plate orifice. Reynold's numbers have been used in the mathematical classification of the results. This work is being used by instrument companies.

Devices have been invented and used by which the head at the throat or orifice may be negative without injury to the accuracy. The primary and the secondary elements of these meters are connected by tubes containing air instead of water. Summation indicators and recorders are also in use which totalize the flow of several Venturi meters as well as indicate the difference between the flows of two or more Venturis. Meter tubes and indicators which will measure flow and indicate and record flow in either direction are in use. Indication and control of the ratio of rates of flow through two Venturi meters can be had; so also can be had meters to act as controls for chemical feed equipment, automatic water softening equipment, etc. A new principle for totalizing the flow has been developed which utilizes the force developed from a gyroscopic couple.

Telemeters and Telegages.—There have been many developments during the decade in the art of indicating totalizing and recording rates of flow and indicating and recording pressures at a distance. Some of these make use of rented telephone wires and others are operated through special wires. One of these systems, which may be defined as the time-impulse system, is a distinct product of the past decade and is in extended use in important water supplies.

Parallel with the development of telemetering and telegaging has come the distant control of valves and pumps. The operator in this case is able to see exactly what he is doing by the indications of telemetering and telegaging devices. There are important installations of this kind both in Baltimore and in Pittsburgh.

TEN YEARS' PROGRESS IN CENTRIFUGAL PUMPS

During the past 10 years the changes in centrifugal pump design have not been of a radical nature but have had mostly to do with refinements to produce higher efficiency and longer life.

Standard Pumps.—In the standard type of pump, of moderate size, there has been a distinct shift from sleeve to ball bearing, although there are many large operators who still prefer the sleeve type. The use of ball bearings, particularly on relatively small pumps, permits of closer running clearances, thereby reducing the leakage through the rings, and resulting in improved efficiency. In the larger pumps, however, it is still common practice to use sleeve bearings with the outboard equipped with a ball thrust bearing as well to take any unbalanced thrust that might occur due to changes in hydraulic conditions. Efficiencies are becoming higher year by year, 88 or 89% not being at all unusual for a pump operating under favorable conditions.

Large Centrifugal Pumps of High Efficiency.—The peculiar needs for boosting a large quantity of water over the high divide traversed by the new aqueduct which will carry water from the Colorado to the Metropolitan Water District of the Los Angeles region caused the development of vertical centrifugal pumps of very high efficiency—that is, approximately 92 or 93%. A difference in operating efficiency of 1% on all the pumps in various booster stations along this aqueduct meant a gain or loss of about $40,000 per yr in power costs. These pumps have lifts of 440, 300, and 150 ft, respectively, and capacities of about 130 mgd each. Pumps of about this efficiency have been obtained in other large installations.

Mixed Flow and Propeller Pumps.—In the large capacity and low-head field, the mixed-flow and propeller-type pumps have been progressing rapidly—the mixed flow particularly in sewage work and the propeller type in the drainage field. Inasmuch as the characteristics of these pumps, particularly as to horsepower, vary from the straight centrifugal unit, it is quite important to give these features close study, especially where synchronous motors or rigid starting current limits are involved. Pumps of these two types have the great advantages of being somewhat cheaper and are suitable for operation at higher speeds without sacrifice of efficiency, thus causing some saving in space and additional saving in the price of the driving motor. Most of these pumps are of the vertical type.

High-Pressure Pumps.—High-pressure, multi-stage pumps continue to be used and have been developed for very high-pressure boiler feed and hot oil pumps. Most of these installations have proved to be very satisfactory.

Well Pumps.—Three developments in well pumps are noteworthy. First is the increasing trend toward the use of water lubrication. The development of the rubber cutless bearing combined with the use of extremely hard metal for shaft journals has greatly increased the use of this type of lubrication.

Second is the development of the so-called submersible pump in which the motor is placed below the pump to which it is directly connected and the entire unit is submerged in the water of the well. The unit is thus close-coupled without a long shaft and has the advantage that it is noiseless, can be placed in a crooked well without bending the shaft, and requires no chamber to cover the motor. A considerable number of these units are in use. The type most commonly used employs a mercury seal to keep the water from the motor, which runs in oil.

The third development is that of a small, efficient, helical pump to operate at speeds not greater than 1,750 rpm with capacities of 10 to 75 gal per min in wells of 4-in. and 5-in. diameter. This has been a field dominated in the past almost entirely by the piston type of pump.

Methods of Drive.—In pumping stations, the electric motor drive is "holding its own" rather well, but increasing thought is being given to the use of Diesel engines with step-up gears driving the pump at, say, 1,800 rpm, thereby obtaining a light and efficient unit. The pump and gear are less expensive than the larger pump of low speed which would be suitable to be direct-connected to the engine. The Diesel drive, of course, is very well adapted to pumping into a direct system as it permits easy speed variation to meet demands without appreciable loss in over-all efficiency or the necessity of costly speed control, as frequently occurs in the case of the motor units. In many cases a generator also can be driven by the same engine, thereby producing power for well and small booster pumps, station auxiliaries, etc. In stations where the total horsepower is from 100 to 400, and where little or no reservoir capacity exists, it is quite likely that this type of unit will come into more general use.

Motor Mounted Pumps.—In the past few years the small, separate pump and motor have been largely replaced by the block-type unit, where the pump case is mounted on an extension of the motor frame, doing away with coupling or pump bearings. These units save space and are cheaper than the four bearing unit.

WATER HAMMER

Water hammer has received an increasing amount of attention during the past decade. In the case of penstocks for hydroelectric power, mathematical predictions as to the intensity of water hammer and provisions for limiting the intensity have been the rule for many years; in recent years supply mains and force mains are receiving similar attention from well-informed engineers.

Whenever the rate of flow in a pipe changes there is a pressure rise or fall which may be large if the alteration in discharge is great. The connection between the change in discharge and pressure rise is somewhat complicated, but may be explained briefly as follows:

When, for instance, water is flowing through a valve at the end of a long pipe connected to a reservoir and the valve is quickly partly closed, a decrease in the pipe velocity results and this is accompanied by an increase in pressure. This pressure rise starts at the valve and travels up the pipe at nearly constant pressure, until it reaches the open reservoir, and from there is reflected back as an equal decrease in pressure which finally reaches the valve; from the latter it begins its cycle over again. These pressure "waves" move at from 2,500 to 4,000 ft per sec, and are finally extinguished by friction. Should the valve be closed before the return wave reaches it (that is, within 2 sec in a pipe 2,500 ft long for the lower wave velocity), there is a pressure rise amounting, in feet, to approximately one hundred times the original velocity of the water; but the rise in pressure is very much less if the time of closure is at least twice the time taken for the wave to make its return trip. The rise in pressure is independent of the pipe length so long as the closure is quick enough.

For the ordinary gate valve on a long pipe, perhaps 70% of the gate closure may be made before the discharge is much affected, and this part of the movement may be made very quickly, but the last 30% of the movement, which is really the effective part, should be made with caution. The matter has been discussed in a paper entitled "The Action of Valves in Pipes," by R. W. Angus.[11]

For short pipes, gate-valve closure rarely can be made before the pressure wave returns to it because of the short time elapsed, and therefore short, ordinary pipes with gate or globe valves are in little danger from water hammer. A serious source of trouble, however, and one only recently fully appreciated, may arise in pipe lines supplied by electrically driven centrifugal pumps when the power supply fails suddenly. Most of these pumping units have very little inertia and consequently begin to slow down almost instantaneously after the driving power ceases; and being unable to maintain the head on the system, they cease pumping. Only the momentum of the water in the pipe maintains the flow, which continues upward through the pump, the pressure at the latter being practically atmospheric. As soon as the energy in the water is spent, the column begins to reverse, flowing downward through the pump, which frequently runs backward, and the water acquires a very high velocity unless hindered in some way; it then often happens that the check valve, always used on such systems, suddenly closes and there is a corresponding pressure rise and shock, which are often surprizingly great. The ordinary check valve is capable of closing very rapidly once the flap starts to move downward, and short, low-pressure pipes are usually in more danger than larger ones because the momentum of the water is small and reversal of flow may occur before the check valve has time to move. One case was observed on a sewage pump with ordinary check valve working against a static head of 56 ft and delivering 3,930 U. S. gals per min through 338 ft of 20-in. pipe, in which the pressure rise, due to sudden shutdown, was about 60 lb per sq in., and the blow actually shook the pump every time the power failed. In this case the reversal of flow was quick, but in a pipe several miles long, reversal may require more than 25 sec, giving the check valve ample time to close safely.

Various types of check valves have been devised to avoid danger, these being usually of a type in which the first part of the closure is rapid and the final part is slow; for a long pipe it is stated that an ordinary check valve, with a dashpot to prevent rapid final closure, is effective, particularly if a small, open by-pass is made around the valve to prevent the sudden extinction of the final part of the flow. Such a valve has been developed in Canada.

---

[11] *Journal*, A. W. W. A., Vol. 30, November, 1938, p. 1858.

Another cause of trouble from power failure is due to the sudden change in slope in hydraulic gradient, because after pumping stops the gradient will slope downward from the reservoir surface to the pump and may easily cut the pipe axis. Should it fall 34 ft or more below this axis, there will be a separation of the water column, the two parts traveling at different velocities; and when reversal of flow occurs the columns will re-unite and a dangerous pressure may result. Admission of air at the high point, and its later release, and the use of a proper type of check valve, should prevent trouble.

Distribution networks of cities and towns are not usually dangerously affected by water hammer, except through some accidental cause, such as sudden closure of a locomotive filling valve or fire stream, or the shutting down of a pump. In these systems the wave, which starts at the trouble making valve, travels into all distribution branches where it changes its rate of travel and its pressure rise according to the size of pipe in which it moves. Dead ends may be a source of danger, but usually in networks there are enough open branches to damp out the pressure wave before damage is done; theoretical study of these systems offers only as much difficulty as is experienced in the design of them for proper distribution, and quite good approximation may be arrived at.

It is now recognized that care should be taken in the type of service sold from a system. Many locomotives draw directly from a fairly long isolated pipe with a quick-closing valve at its outlet, and there is every possibility of the full flow being stopped before the return wave has reached the valve; such lines should have valves that cannot be closed rapidly, and, if necessary, should be equipped with dashpots. The fact that the pressure wave travels through the system shows that a pipe may break a mile away from the locomotive, if the valve is closed quickly enough.

The profession is in need of help from operating engineers who can furnish much data from which the present theories can be corrected or checked. The following types of information would be most helpful: (*a*) In cases of pipe bursts due to suspected water hammer, what are the sizes and lengths of pipes and what was the water velocity? (*b*) Was there a power interruption at the pump house? (*c*) What type of check valve was used? (*d*) Did the pump reverse, and what was the maximum pressure recorded at the station? (*e*) Are any data available as to the parting of the water column at pump cutout in a pipe line which has a sudden change of gradient? (*f*) What has been the experience with cone or other types of special check valves in reducing water hammer? (*g*) Has any trouble been experienced with low-head installations, such as those in which water is being lifted to filter beds; if so, what has been its nature? (*h*) What information is available about broken check valves?

Reliable pressure records of water hammer may be obtained by means of a steam-engine indicator recording on a drum, 4 in. to 6 in. in diameter, driven by a synchronous clock motor. Recording gages are usually too slow in action, but may sometimes be adapted to the work if only slightly altered.

When engineers cooperate by sending, to some central body, details of their water hammer experiences, the present theories will be very much improved and simplified.

PROGRESS IN WATER TREATMENT AND FILTRATION IN THE DECADE 1930–1939

Great progress in the treatment and filtration of water has been made within the past decade. New treatments have been introduced, and, although there has been no material change in the design of water purification plants, there has been improvement in the operation and maintenance of the plants. The use of activated carbon for taste and odor removal, ammonia in connection with chlorin for better sterilization, acid-treated sodium silicate for improving coagulation, and sodium hexametaphosphate for preventing precipitation of calcium carbonate and for corrosion control are the main developments in the treatment of water. Effort to maintain filter beds in good condition and improved laboratory technique have been the main development in plant operation. The addition of surface washes to filters and the flocculator type of mixing basin have been the main improvements in design.

TREATMENT OF WATER

Activated Carbon for Taste and Odor Removal.—The most important development in the treatment of water has been the use of activated carbon for removing objectionable tastes and odors. Experiments in 1928 and 1929 indicated that activated carbon was a practical means of removing tastes and odors from water, but it was not until 1930 that this material began to be used in the treatment of public water supplies. The use of carbon in the powdered form has extended rapidly since that date, and by the end of 1939 it was being used at least part of the time in nearly one thousand water filtration plants. The material was first used sparingly, and, although it produced some improvement in the quality of the water, the taste and odor were not eliminated completely.

Within the past several years increasing quantities of carbon have been used, and efforts to reduce the taste and odor compounds below objectionable concentrations are rapidly becoming the practice in all well-operated purification plants. There are instances where, for periods of short duration, the amount of carbon required to give satisfactory elimination of taste and odor has exceeded 1,000 lb per million gal. This represents a cost between $30 and $40 per million gal for taste and odor elimination alone. The use of any material in amounts that require such a high expenditure shows the importance placed upon the palatableness of water.

There are several places where beds of granular, activated carbon have been installed for passing the filtered water through the beds, but decidedly the greatest use of activated carbon is in the powdered form. The powder is added to the water in advance of filtration, and is removed by the filters. Not only was the use of activated carbon in water treatment developed in the decade just ended, but there has been great improvement by the manufacturer in the absorptiveness of the carbon. The better grades of carbon are now more than twice as absorptive as they were when carbon first was used. In addition, the price of the material has been reduced. The cost of removing taste and odor from water by the use of activated carbon is now less than one third the cost when carbon first began to be used.

Ammonia.—Another outstanding development in the treatment of water within the past 10 years is the use of ammonia for lessening chlorinous taste and for producing better sterilization of water. Like activated carbon, the effectiveness of ammonia in lessening chlorinous taste was established 2 or 3 years prior to 1930, although its extensive use did not start until then. Now (1940) a large number of water purification plants use ammonia.

Ammonia lessens the rate at which residual chlorin disappears in the water, and it is possible to maintain residual chlorin throughout practically the entire distribution system. Furthermore, it is possible to maintain a higher residual chlorin in the water than heretofore without producing objectionable chlorinous taste.

Fluorin Affects Teeth.—That certain waters cause defective tooth enamel seemed well established in 1926, although the substance causing the effect was not then known. In 1931 fluorin was found to be present in considerable quantities in several waters where mottled tooth enamel was of frequent occurrence and was suspected of being the cause. Definite proof that fluorin was responsible soon was established. Means of removing fluorin from water have been developed although the cost is still too high to be extended to public supplies if other sources of water are available at a reasonable cost.

Silicate Treatment.—The presence of silica in water was found to have considerable influence upon coagulation of the water with aluminum sulfate. This discovery led to the development of a silicate prepared by acid treatment of sodium silicate which will aid the coagulation of water with aluminum sulfate and ferrous sulfate. Use of the silicate to aid coagulation is likely to be extended to a number of water supplies where the water does not coagulate easily.

Softening.—Continued progress has been made in water softening, although there have been no major developments in the treatment. A number of plants using the lime-soda process have been constructed. The use of zeolite for softening municipal supplies has been extended also. The addition of carbon dioxide to lime-soda treated water for preventing incrustation of the sand grains in the filter beds and deposit in the piping distribution system is becoming common practice. Recently, sodium hexametaphosphate has been used to prevent lime incrustation, and there are indications that this material may be used to considerable extent in water softening plants using the lime-soda process. Also it has the possibility of aiding in pipe-corrosion control.

Flow through thickly concentrated and agitated sludge to hasten the softening action, followed by upward flow through a sludge blanket, has been developed as a substitute for the usual type of lime-soda softening plant. These devices are known as "accelators" or "precipitators."

BETTER OPERATION

There has been considerable improvement in the operation of water purification plants within the past decade. Chief interest has been directed to maintaining the filter beds in better condition, to good conditioning of the water before filtration, and also to improved sterilization of the water.

Turbidity of Filtered Water.—Accurate measurement of the turbidity of the filtered water has become common practice. This is resulting in clearer water being produced by the plants. Instruments have been developed for making low turbidity measurements and, although some of these instruments were developed prior to 1930, very few filtration plants made accurate measurements of the turbidity of the filtered water prior to this date. In many plants the turbidity of the filtered water is maintained below 0.2.

Odor-Threshold Test.—Use of the odor-threshold test as a guide to taste and odor removal treatment has become common practice. The water is diluted with odor-free water to the extent that the odor is just detectable. Operators now attempt to reduce the odor to some definite odor threshold.

Bacterial Quality of Water.—There has been improvement in the bacterial quality of the water in many filtration plants within the past few years. Several outbreaks of gastro-enteritis have caused present requirements for the bacterial quality of water to be questioned by a few. As a result, higher chlorination is being used in many plants. Prechlorination, used to a limited extent prior to 1930, is now (1940) the practice in a number of filtration plants. The superchlorination of water has made little progress, although it received renewed attention in 1939. High chlorination is used for two purposes—one to produce better sterilization of the water, and the other to aid in reducing taste and odor. The present indications are that there will be extension of high prechlorination treatments for these two purposes.

Sterilization of New Water Mains.—In some cities it has been the practice for years to sterilize new water mains before placing them in service. Within the past decade it became common practice in nearly all cities. The difficulty in obtaining good bacterial samples from new mains, even after sterilization, has been so great in some places that consideration is being given to the use of material other than jute for packing the joints. Sterilizing of jute by steam seems to be successful in permitting sterilization of mains.

DEVELOPMENTS IN FILTRATION PLANTS

Mixing Basins.—The mechanical type of mixing basin is being installed in nearly all of the new filtration plants. There has been a drift toward the use of mechanical mixers of the flocculator type. In some plants there are two stages of mixing. The first stage is rapid initial mixing, and the second stage is gentle agitation for forming the coagulation into large flocculated particles.

Settling Basins.—There has been little change in the general design of settling basins within the past decade. Most of the softening plants have settling basins equipped with continuous sludge-removing equipment. Two-story settling basins have been constructed in two or three filtration plants, and such basins are included in the design of the new filtration plant for Chicago. Local conditions, which made deep basins desirable, was the reason. There was no belief that a two-story basin is superior to the usual type of settling basin.

Filters.—The design of filters has remained largely the same within the past decade, but there has been considerable improvement in some of the parts of the filter. Surface washing systems have been installed in the filters of a number of filtration plants. The high rate of wash—that is, a rate to expand the bed about 50%—is being used in many places. There has been a drift toward the use of coarser filtering materials and higher rates of filtration. The use of crushed anthracite coal in the place of sand for the filter beds has been adopted in a few plants. The design of the filter underdrainage systems has not undergone any material change, although porous-plate filter bottoms are being tried in a few filtration plants. These plates are intended to eliminate the necessity for using gravel in the filters. Filter-rate controllers have been simplified and made more positive in their action.

New Filtration Plants

The construction of new filtration plants was almost at a standstill during the first 3 or 4 years of the depression (1930 to 1933, inclusive). However, since then a large number of plants have been constructed, and others are under construction. Many of the older plants have been extended and the treatment facilities improved.

Trends in Valuation of Public Utilities

The decisions of the United States Supreme Court are the milestones which indicate directional trends in valuation practice. Such decisions arise from cases brought before the court by the utilities on claims of confiscation under the due process clauses of constitutional amendments IV and XIV.

These decisions, although frequently expressing definite opinions on important phases of valuation philosophy, are more frequently only indicative of the court's position by refusal to disapprove the action of some lower court or regulatory commission.

Interpretation—involving the personal equation and perhaps requiring more legal knowledge than is vouchsafed to the run-of-the-mine engineer—is necessary, therefore, and any attempt by an engineer to deduce directional trends is more or less foolhardy. The following is offered with the realization of the limitations of engineers acting alone without benefit of counsel.

The United States Supreme Court first announced its general theory of valuation for rate-making purposes in the Smyth-Ames case (169 U. S. 466)—namely, that a utility is entitled to a reasonable return on the fair value of its property devoted to public service, and that, in determining such fair value, actual cost, reproduction cost, earning capacity, and various other factors are "all matters for consideration and each to be given such weight as may be just and right in each case."

The court has held to this general principle for more than 40 years, but without determination of the weight to be given to the various factors and not without repeated differences of opinion in the court as to their relative import. As an example of these differences, Justice Felix Frankfurter, in his minority opinion in Driscoll versus Edison Light and Power Company (28 P. U. R. (N. S.) 66 April, 1939), wrote: "The decree below was clearly wrong. But in reversing it, the court's opinion appears to give new vitality needlessly to the mischievous formula in fixing utility rates in Smyth-Ames."

As to the determination of cost, the U. S. Supreme Court has not established the relative weights to be given to actual and to reproduction cost. Leading cases following the World War of 1914–1918 gave increased importance to reproduction cost, and in McCardle versus Indianapolis Water Company (272

U. S. 400, 1926) predominant consideration was given this factor. Since 1930, the tendency appears to be more toward actual cost. Thus, in Los Angeles Gas and Electric versus California Railroad Commission (289 U. S. 287, decided May, 1933), actual cost is emphasized, and a similar broadening of historic cost in rate-base determination is indicated in Clark's Ferry Bridge Company versus Public Service Commission of Pennsylvania (2 P. U. R. (N. S.) 225, decided February, 1934), and in Dayton Power and Light Company versus Ohio Public Utilities Commission (3 P. U. R. (N. S.) 279, decided April, 1934). In his minority opinion on Driscoll versus Edison Light and Power Company (supra), Justice Frankfurter commends the practice of the Massachusetts Commission of using prudent investment as a yardstick of value. Likewise in McCardle versus Indianapolis Water Company (38 P. U. R. (N. S.) 468), Justice Hugo L. Black, in dissenting, refers to "imaginary reproduction value." These references are from minority opinions but may be significant in view of the rapidly changing personnel of the U. S. Supreme Court.

The recent practice of public service commissions has indicated a tendency to establish, as of some date, a rate base—whether founded on actual or reproduction cost—and to go forward therefrom on the basis of actual cost.

In reference to going value, recent decisions of the U. S. Supreme Court indicate a definite tendency on the part of the court to require that going value, if allowed, be adequately proved with respect to the particular property being valued. As stated in the Dayton Power and Light Company versus Ohio Public Utilities Commission case, decided by the U. S. Supreme Court April 30, 1934, "Going value is not something to be read into every balance sheet as a perfunctory addition." As stated in the Los Angeles Gas Corporation versus Railroad Commission of California, "It calls for consideration of the history and circumstances of the particular enterprise."

The reasonable rate of return has been related, in general, to the prevailing interest rates. Previous to the World War of 1914–1918, 6.00% was generally allowed. Under postwar conditions between 7.5 and 8.0% was considered necessary to avoid confiscation. In the Indianapolis Water Company case (1926) it was held that "the evidence is more than sufficient to sustain a rate of 7.0% * * * and recent decisions support a higher return." In Driscoll versus Edison Light and Power Company (1939) (supra) a 6.0% return was held not to be confiscatory.

In brief, decisions of the U. S. Supreme Court during the past decade seem to point to an increasing recognition of actual cost in the determination of fair value, a minimizing of going value as a separate element (except in so far as it is proved in each case), a downward trend in the rate of return, and, in general, a tendency to grant a wider discretion to the state regulatory commissions.

ACKNOWLEDGMENTS

The Committee has sought and received the aid of a number of engineers and specialists not enrolled in its membership and wishes to acknowledge with gratitude its indebtedness to the following persons: Prof. Robert W. Angus, for the technical portions of the section on Water Hammer; Frederick W. Scheidenhelm, M. Am. Soc. C. E., for the section on Masonry Dams; W. P.

Creager, and J. D. Justin, Members, Am. Soc. C. E., for much of the matter used in the section on earth dams; Frederick H. Fowler, M. Am. Soc. C. E., for a very complete article on rock-fill dams which was necessarily condensed to accommodate the available space; Henry Horandt, for much of the data used in the section on meters; and A. V. Smith, for the section on the increasing use of cathodic protection. Various producers of pipe and pipe protection at the Committee's request supplied statistics as to use of improved linings. Pump manufacturers supplied information relative to progress in the manufacture of their products.

F. A. Barbour L. R. Howson
J. R. Baylis J. A. Wade
W. W. Brush Thomas H. Wiggin, *Chairman*

Committee of the Sanitary Engineering Division on Water Supply Engineering

January 17, 1940.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2097

THE RELATION OF THE SOCIETY TO NATIONAL DEFENSE

ADDRESS AT THE ANNUAL CONVENTION, DENVER, COLORADO, JULY 24, 1940

BY JOHN P. HOGAN,[1] PRESIDENT, AM. SOC. C. E.

It has been customary for the President of the Society to prepare an Annual Address early in the summer, and, after careful editing by the staff, to have it published well in advance of the Annual Convention and distributed in pamphlet form. The march of events has been so rapid in the past six months that it has seemed to me that such an address would totally disregard the realities that confront us at the moment and those that may confront us in the near future. I therefore propose to devote a considerable part of my address to the subject in which the engineers of the country are most vitally involved—the subject of National Defense.

Before proceeding to this vital topic I wish to report on the state of the Society. This can be done in part only by the President, since, like a butterfly, he flits in and out yearly, whereas the management is carried on year in and year out by the members, acting through the Local Sections, the Board of Direction, and our hard-working and efficient staff. During the ten years of depression, or partial depression, through which we have passed, the activities of the Society have been greatly increased and decentralized.

We have 64 Local Sections, to which all of the members have been assigned, and to whose support the Society contributes nearly $22,000 a year. The Board of Direction has delegated to these Local Sections an increasing amount of its authorities and responsibilities as rapidly as the Local Sections have been organized to assume them, reserving only the right of review and of formulation of general policies. As a measure of this decentralization I can state that when I first came on the Board of Direction in 1922 the meetings of the Board consumed three days, whereas at the Spring Meeting in 1940 one day was sufficient.

The Technical Divisions have grown in number, size, and importance. With *Civil Engineering* as a vehicle, these Divisions have afforded an opportunity for the discussion of many technical topics of current interest, and have provided more serious discussions and theories for technical proceedings. The latter, as published in *Proceedings* and *Transactions*, have maintained their former high standard of excellence.

---

[1] Parsons, Klapp, Brinckerhoff & Douglas, New York, N. Y.

The Society has reached one of its peaks in membership, totaling today (July, 1940) approximately 16,500 in all grades, of whom about 4,000 are Juniors. In addition there are 6,000 members of Student Chapters in 123 technical universities and colleges throughout the country who are affiliated with the Society. Since there were only about 500 Juniors in 1919, and no Student Chapters, the accession to interest or membership of more than 9,000 youths has brought additional problems.

The first has been a demand for earlier and more intensive teaching of ethics, professional relations, and behaviorism. We are working to satisfy this demand.

The second has been the desire of these younger members and affiliates to educate themselves so as to be able to present and discuss their problems publicly. This is being handled by the Local Sections through the organization of Junior and Student Chapter forums, the presentation of prizes for papers, and special provisions for the presence of both groups at meetings.

The third has been the increased demand for more welfare service; this demand has not been confined to the youth. It is the sense of your Board of Direction that this demand should and must be satisfied, insomuch as it does not interfere with the legal status of the Society.

The management of the Society is to be congratulated that we have come through the depression with our assets unimpaired, our budget balanced, and our activities and usefulness increased, but there are certain red lights ahead, of which the principal one is the uncertainty of the times and what they may bring forth. Furthermore, Past-President Sawyer pointed out to you last year that the Corporate Membership of the Society has been steadily declining, and that our membership has been maintained only by the greatly increased number of Juniors. Largely through his efforts, membership drives were quietly started in a number of localities. These should be continued and extended to other localities.

The Society takes great pride in the technical achievements of six great permanent branches of the government services: The Coast and Geodetic, the Geological Survey, the Bureau of Public Roads, the Bureau of Reclamation the Corps of Engineers, and the Tennessee Valley Authority. In general, it is my belief that the new or emergency engineering organizations have not been particularly competent or efficient. I am opposed to the creation of more such organizations and in favor of "winding up" those already existing as rapidly as possible.

In speaking of National Defense, a subject much in the public mind today, the commonest question of the isolationist, or pacifist, is: "Why should any one attack us?" Why did they attack Poland, Norway, Holland, Belgium, and France? In fact there are many reasons why they should and will unless we are able to defend ourselves. One answer alone is enough. For the past two years all the movable wealth of Europe in the form of gold, securities, diamonds, etc., has been moved into the United States. Ships slip into New York Harbor with 50 to 60 million dollars worth of gold on board, without comment, although two years ago it would have created a sensation. One shipment from a small non-belligerent country included 125 million dollars in gold and a billion and a quarter in securities.

As a people we are prone to sign a check and forget about it. In this liberal and careless spirit we have just appropriated about 10 billion dollars for national defense. What have we now, and what will the new money do for us?

We have an efficient navy but most of the capital ships are old and slow. Some replacements may be expected within the next two years, but the capital ships provided by the present appropriations will not come into a service for four years.

If our navy is unable to protect us, what have we on shore? We do not have enough modern armament, ammunition, or high explosives for our present small army and National Guard, and we will not begin to produce enough for about eight months. Proper arms and ammunition for the minimum defensive army of 2,000,000 men will probably not be available until the fall of 1942.

We still have last hopes in aviation and luck, but neither of these will protect us from an economic strangulation through the loss of our foreign trade; we have already lost the trade of Europe and China. Notwithstanding the fact that we are at least a year late, we should, nevertheless, go ahead vigorously to make up for our neglect; and the Society, together with the other Founder Societies, the architects, and contractors, has cooperated since last February with the Army and Navy Munitions Board to this end.

I would like to emphasize the fact that the Construction Advisory Committee to the Army and Navy Munitions Board, which has been working for the past four months and has actually been commissioned since May 6, is composed entirely of the heads of the various organizations in the industry or their designees, and is therefore truly representative of the construction industry. I am proud of the fact that five out of the six members of the Committee are also members of the Society.

I should also like to make it very clear that the Committee has confined itself solely to questions of policy and has not attempted any selection. It did, however, initiate the formation of the Committee on Preparedness of all national engineering societies, and the architects, for the purpose of census and classification of personnel under the chairmanship of George T. Seabury, M. Am. Soc. C. E., Secretary of the Society.

The general purpose and aims of the Advisory Committee to the Army and Navy Munitions Board have been:

(1) To secure proper legislation permitting the use of management contracts on fixed fees. This has been accomplished.

(2) To prevent, as far as possible, the letting of agency contracts to manufacturing concerns to design, construct, and operate, because of our firm belief that this procedure would only result in confusion and, probably, scandal.

(3) To have satisfactory forms of construction contracts approved for management construction. This has been accomplished.

(4) To segregate engineering and architectural contracts from construction contracts. This has been established in principle in the army and representations have been made to the navy to correct some present difficulties.

(5) To establish the principle that there shall be no competition in fees. This seems to be generally accepted but the situation needs watching.

(6) To establish suitable regulations and forms for sub-contracts of all forms. This has been accomplished in the army but still remains unsatisfactory in the navy where the engineer, or architect, may be in the position of a sub-contractor.

(7) To establish the principle of utilizing the services of existing engineering and architectural firms for design instead of expanding existing organizations. This has been established in principle in both services but will be more utilized by the army on account of its greater needs.

(8) To urge as widespread geographical distribution as possible of technical and construction contracts to avoid overloading and favoritism. In this connection attention is invited to the practice of the architects in the New York City Working Development of taking in local associates to assist and share in the work of the principal architects.

(9) To prevent undue migration of technical and construction personnel either between different government jobs or between these jobs and normal construction work already in progress or anticipated.

(10) To insure adequate compensation for engineers and architects under the various classifications.

(11) To insure adequate fees for consulting services.

Due to the appointment of the Advisory Board to the National Defense Council by President Roosevelt, and the doubt in regard to the relation between the two committees, it has been necessary for this Committee to work in confidence. However, in June of this year W. S. Knudsen appointed William Harrison, chief engineer of the American Telephone and Telegraph, as his construction adviser, and the latter selected S. F. Voorhees, M. Am. Soc. C. E., and past-president of the American Institute of Architects, as a member of our Committee and as his assistant. It is the present intention that our Committee will remain as advisory to the Army and Navy Munitions Board, recommending on technical matters through it to the Services, and on questions of policy, to the National Advisory Board.

The question of engineering personnel for our armed forces may later become of moment should there be an immediate increase in the army. During the last war engineering troops formed about 7% of the army. An immediate increase to 10% is contemplated in our existing forces. It is reported that in the German army engineering troops form about 14%.

I have dwelt on this subject at length because it is something we have neglected too long. This is a familiar spot for the engineer, however, for he is often called upon to do immediately something that should have been done a year before. I am sure the professions will respond to the limit of their abilities.

In closing I can only quote to you the words uttered by John Curran 150 years ago, and published in the Saturday Evening Post.[3]

> "It is the common fate of the indolent to see their rights become a prey to the active. The condition upon which God hath given liberty to man is eternal vigilance; which condition if he break, servitude is at once the consequence of his crime and the punishment of his guilt."

---

[3] *Saturday Evening Post*, July 13, 1940.

MEMOIRS OF DECEASED MEMBERS

CHARLES DAVID MARX, Past-President and Hon. M. Am. Soc. C. E.[1]

DIED DECEMBER 31, 1939

Charles David Marx, emeritus professor of civil engineering, after a lingering illness died in his home in Palo Alto, Calif., on December 31, 1939. On this last day of the decade passed a famed member of Stanford's "Old Guard," a teacher who throughout a long life served well his community, his state, and his nation.

"Daddy" Marx—as he was affectionately called even when as a young man he established the Civil Engineering Department at the opening of Stanford University, at Stanford University, Calif., in 1891—was born in Toledo, Ohio, on October 10, 1857, the son of Joseph Eugene and Johanna (Pulster) Marx.

His father, who was born in Germany, came to the United States soon after the troubled year 1848 and was a journalist most of his life, remembered as a founder of the Ohio *Staats-Zeitung* (later The Toledo *Daily Express*). By appointment of President Lincoln during the Civil War, Joseph Marx served as American consul in Amsterdam, the Netherlands, from 1863 to 1866.

In this Dutch city Charles D. Marx spent several boyhood years. To supplement his private education in Amsterdam, he attended public schools in Toledo for a few years after 1866. Returning to his father's former homeplace in Baden, Germany, the youth studied at the Karlsruhe Realgymnasium, from which he was graduated in 1876. One of his teammates on the rugby squad at this school was F. Marion Crawford, the writer.

In the fall of 1876 Mr. Marx entered Cornell University, at Ithaca, N. Y., and was awarded the degree of Bachelor of Civil Engineering, with the class of 1878. Abroad once more, as a graduate student at Karlsruhe Polytechnicum, at Karlsruhe, Germany, he was named an assistant instructor in 1880–1881, and passed the Diploma examination as civil engineer in the same year.

Back in the United States, the young engineer was employed first by the Boston, Hoosac Tunnel and Western Railroad Company, successively as levelman, transitman, and assistant engineer on their proposed western expansion. He then became an engineer for the federal government on both the Mississippi and the Missouri River improvement projects, until Cornell University called him to serve as assistant professor from 1884 to 1890.

During the summers of his Cornell years he acted as inspector and assistant engineer, and at one time was principal assistant to the late Emil Kuichling,[2] M. Am. Soc. C. E., then city engineer at Rochester, N. Y.

An incident that was to influence his life came when, traveling on a train to a Cornell meeting in Ithaca, he became acquainted with David Starr Jordan. The first president of Stanford was so impressed by the young man that, when the University was opened in 1891, "Daddy" Marx was offered the first chair in engineering.

---

[1] Memoir prepared by a Committee appointed by the Board of Direction of the Society, consisting of John D. Galloway, F. H. Fowler, S. B. Morris, Leon B. Reynolds, and C. B. Wing, Members, Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXIX (1915), p. 1438.

Professor Marx was serving at that time as head of the civil engineering department at the University of Wisconsin, at Madison, Wis., having transferred there the previous year. His name was among the first ten appointments to the Stanford faculty, announced April 8, 1891; and that autumn he began a period of service as professor of civil engineering which was to last without interruption until his retirement as emeritus professor in 1923.

One of his earlier contributions to Stanford welfare was his professional advice in connection with the rebuilding of the University after the earthquake in 1906, when he was appointed on an engineering commission whose other members were William F. Durand and Prof. Charles B. Wing, M. Am. Soc. C. E. From 1917 until his retirement, Professor Marx served as acting president during the absences of President Ray Lyman Wilbur. He was executive head of the Civil Engineering Department, and a member of the advisory board and other university committees.

As early as 1896 Professor Marx was elected to the board of trustees of the City of Palo Alto. Thereafter, except for a break from 1904 to 1908, he held civic office until 1933, when he resigned from the chairmanship of the Board of Public Works, on which board he had held a position for twenty-four years. In 1898 when it was discovered that the city sewerage system installed under his supervision would not cost the full amount of the bond issue voted for the purpose, Professor Marx proposed that the remainder be used toward the inauguration of municipally-owned water and electric plants. It was due largely to his efforts that Palo Alto was also a pioneer among California cities in the operation of a municipal gas system; and through his guidance of the Board of Public Works, the Stanford engineer was responsible in a high degree for the economical development of the city's public works and utilities. Professor Marx was also prominent for many years in Palo Alto financial circles.

Professor Marx acted as consultant on a number of industrial and governmental projects. A list of the important engagements with approximate dates is as follows:

1893—consultant on the foundations of the San Francisco Ferry Building, in which he was assisted by Professor Wing;

1901—author of the report on Irrigation Problems in the Salinas Valley, California, published by the United States Department of Agriculture, Office of Experiment Station, Washington, D. C., as *Bulletin No. 100*;

1903—author of a report on the project of the Colorado Power and Irrigation Company at Denver, Colo.;

1904—author of a report on the St. Vrain Power Project, Colorado;

1906–1907—member, with Wynne Meredith and John D. Galloway, Members, Am. Soc. C. E., of the Board of Advisory Engineers on the Design of a Hydraulic Power Plant and Dams of the Stanislaus Power Company, later a part of the Pacific Gas and Electric Company;

1907—author of a study of the Buena Vista Pumping Plant of the San José Water Company;

1909—consultant to the Board of Directors of the Monterey County Water Works;

1912—chairman of the first State Water Commission, created on March 23, 1912, to administer the waters of California, and in this position Professor Marx assisted in framing the Water Laws of the state;

1912—associated with the late John R. Freeman,[3] Past-President and Hon. M. Am. Soc. C. E., in making early studies of water supply for the City of San Francisco, Calif.;

1913—consulting engineer to the City of Salinas, Calif., on the valuation of water and gas works owned by the Coast Valleys Gas and Electric Company;

1921–1923—consultant to the State of California in the preparation of *Bulletin No. 5,* "Flow in California Streams," and, in 1929–1931, of *Bulletin No. 28,* "Aspects of a Salt Water Barrier on Sacramento River";

1923—author of a report to the Board of Directors of Montecito Water District, Santa Barbara, Calif.;

1925–1926—member and chairman of the Board of Engineers, with G. B. Hegardt and Charles T. Leeds, Members, Am. Soc. C. E., of the Port of Oakland Harbor Development;

1926—member of the Peninsula Water Conference Committee;

1926—author of a study and report on the sewerage system of Reno, Nev.;

1927—member of the Board of Consulting Engineers, with F. H. Fowler and Charles H. Paul, Members, Am. Soc. C. E., of the San Gabriel Flood Control Board for the Los Angeles County Flood Control District;

1927—consultant to the San Diego Water Company;

1928–1929—member, with Mr. Paul, the late A. J. Wiley,[4] M. Am. Soc. C. E., and F. L. Ransome, geologist, of the Consulting Board to the East Bay Municipal District to investigate the causes of the partial failure of Lafayette Dam;

1929—member of the Engineering Commission on Sanitation and Drainage for the City of Oakland, Calif.;

1929—member of the Hoover-Young Commission that finally determined the location of the San Francisco-Oakland Bay Bridge across San Francisco Bay;

1930—adviser to the Public Works Engineering Corporation on the Chenery Reservoir, California;

1930—member of the San Francisco-Oakland Bay Bridge Advisory Committee of the San Francisco Real Estate Board;

1931—with C. F. Tolman, geologist, author of a geological and engineering report on the proposed dam at El Capitan Damsite No. 2 on the San Diego River to the City of San Diego;

1931—member, with J. B. Lippincott, Hon. M. Am. Soc. C. E., and John Buwalda, geologist, of the consulting board on Alterations and Improvements for Enlargement of the Chatsworth (Calif.) Reservoir;

---

[3] For memoir, see *Transactions,* Am. Soc. C. E., Vol. 98 (1933), p. 1471.

[4] For memoir, *loc. cit.,* Vol. 96 (1932), p. 1577.

1932, 1933–1935—member, with W. L. Huber, and F. C. Herrmann, Members, Am. Soc. C. E., as consultants to the State of California on the Bouquet Canyon Dam and the large earth and rockfill San Gabriel Dam No. 1. In 1932 Professor Marx was appointed Chairman of the Engineers' Advisory Board for the Reconstruction Finance Corporation, Washington, D. C. (in this capacity, he passed on such projects as the San Francisco-Oakland Bay Bridge, the bridge across the Mississippi River at New Orleans, La., the Colorado River Aqueduct of the Metropolitan Water District of Southern California, and numerous other projects throughout the United States);

1935—consulting engineer to the Metropolitan Water District of Southern California on the Design and Construction of the Cajalco Dam.

Professor Marx contributed several papers to the *Transactions* of the Society, including: "Experiments on the Flow of Water in the Six-Foot Steel and Wood Pipe Line of the Pioneer Electric Power Company at Ogden, Utah"[5] (with C. B. Wing, and the late Prof. L. M. Hoskins), and "Idealism and Art in Engineering."[6]

He was recipient of the honorary degree of Doctor of Laws (LL.D.) from the University of California in 1918 and that of Doctor of Engineering from the Karlsruhe Technische Hochschule in 1926. Professional affiliations and honors include: presidency of the American Society of Civil Engineers (1915) and Honorary Membership (1927); presidency of the San Francisco Section of the American Society of Civil Engineers; presidency of the Pacific Association of Consulting Engineers; presidency of the Joint Council of Engineering Societies in San Francisco; membership in the Western Society of Engineers, the Wisconsin Academy of Science; charter membership in the Society for the Promotion of Engineering Education; and chairmanship of the Arch Dam Committee of the Engineering Foundation. From 1926 to 1932 he was a member of the Executive Board of the American Civic Association. In 1929 he was a member of the American Committee to the World's Engineering Congress in Tokyo, Japan. In 1917 and 1918 he was a member of the Executive Committee on World Camp Community Service at Camp Frémont. In 1920, he was State Chairman of the National Public Works Department Association.

Professor Marx belonged to the Theta Delta Chi fraternity. His clubs included the Commonwealth Club of California (of which he was an honorary member and an early member of its committees on Water Resources and on Highways), the University Club, Menlo Country Club, and Monterey Peninsula Country Club.

In presenting Professor Marx to President John F. Stevens, Hon. M. Am. Soc. C. E., on the occasion when he was made an Honorary Member of the Society, Walter L. Huber said:

> "During these many years (at Stanford University) his influence has been felt not only by the hundreds of graduates of his own department,

---

[5] *Transactions*, Am. Soc. C. E., Vol. XL (1898), p. 471.

[6] *Loc. cit.*, Vol. LXXIX (1915), p. 1329.

> but by literally thousands of other young men, including even those who have graduated from rival institutions—it is a graduate of a rival institution who today presents him. With this record it is little wonder that throughout our country, and particularly through all of our western states, he has come to be affectionately known as 'Daddy Marx.'"

In 1888, Professor Marx was married to Harriet Elizabeth Grotecloss of Suffern, N. Y. Three children, Roland Grotecloss, Dorothy Grotecloss (Mrs. Edward B. Sherwood), and Alberta (Mrs. Harlan B. Granham), survive him; another daughter, Stephanie, died in her girlhood. Also surviving him are a brother, Guido Marx, and two sisters, Mrs. Frederick Will and Mrs. C. W. Curtis.

Public recognition of his contributions to the life of the University and the nation was afforded Professor Marx in his eightieth year when, in March, 1937, a campus luncheon in his honor was attended by active and emeritus members of the School of Engineering faculty. President Wilbur presided and stated that Professor Marx had built a place for himself in the affections of thousands of Stanford students, and had contributed to the solution of national engineering problems at Washington and on the Pacific Coast. This occasion was marked by the unveiling of a portrait of "Daddy" Marx painted by Mrs. Winifred Rieber of New York, N. Y., which had been commissioned by a group of Stanford graduates and which hangs in the Stanford Art Gallery.

"Daddy" Marx was best known through a long life as a great teacher who aroused in many generations of students a hunger for useful knowledge. He was sought out by many people from all walks of American life who needed the advice that he could give; and he gave freely and cheerfully of his store.

He had a keen sense of the dignity of the human race. Fellow engineers who accompanied him to China after an international meeting at Tokyo tell how he refused to ride in a jinrikisha because it exemplified inhuman toil.

"Daddy" left behind him a Stanford legend concerning his remarkable memory for facts and faces. It is said that after the first day of class work he could address any of his students by name, and recall personal incidents when meeting one of his acquaintance after many years. This habit of mind not only endeared him to students, but formed a personal bond of interest that made more effective his task as a teacher. His rugged geniality was a familiar part of campus life, and his spontaneous humanity inspired a ready response in everyone who knew him.

With the death of Charles David Marx, one of the last of the "Old Guard," passes a phase of Stanford history. The span of his service as a teacher comprised the origin and growth of Stanford University to its present place in the modern world; and not the least among those who were responsible for that growth was the beloved and dynamic man whom President Wilbur has termed "one of the real pillars of Stanford University."

Professor Marx was elected a Member of the American Society of Civil Engineers on October 7, 1896, and an Honorary Member on October 10, 1927.

GUSTAV LINDENTHAL, Hon. M. Am. Soc. C. E.[1]

DIED JULY 31, 1935

One of the gifted few among engineers who by technical genius combined with force of personality place a lasting impress on their art, Gustav Lindenthal was an acknowledged leader in the field of bridge construction for half a century. When he died at the age of 85, on July 31, 1935, he long had been the last remaining link between the present generation of civil engineers and the outstanding men of the nineteenth century.

His professional career extended from early engineering work in construction of the Centennial Exposition at Philadelphia, Pa., in 1876 to a commanding share in the development of modern bridge design during the 1880's and 1890's; later to construction of the Manhattan and Queensboro bridges as bridge commissioner of the City of New York, N. Y.; and still later to construction and execution of the world-famous Hell Gate arch and the Sciotoville continuous bridge. Perhaps the most dramatic element of his career was his 45 years of continuous effort to build a great railway suspension bridge across the Hudson River at New York.

Gustav Lindenthal was born in Brunn, Austria, on May 21, 1850, the son of Dominick and Franciska Schmutz Lindenthal. He was educated at Politechnicum (Dresden, Germany) College and received practical training from 1866 to 1870. He came to the United States in 1874, at a time of depression, and was forced to turn to handicraft for a livelihood, working for some months as a stone-mason on the Memorial Hall at the Centennial Exposition, then being built. In the course of this work, however, the engineers in charge of Exposition construction recognized his ability and transferred him to designing. Soon he was in charge of the two permanent buildings, Memorial Hall and Horticultural Hall, both still standing in Fairmount Park. When Exposition construction came to an end he obtained a position with the Keystone Bridge Company of Pittsburgh, Pa., as computer and designer.

In 1878 he was appointed bridge engineer of the Atlantic-Great Western Railroad at Cleveland, Ohio. Rebuilding many of the early structures of that railway further developed his technical skill, and his association with outstanding railway engineers, the late Charles Latimer[2] and Arthur Mellen Wellington,[3] Members, Am. Soc. C. E., chief among them, increased the breadth of view and determination that characterized him throughout his life.

In 1881 Mr. Lindenthal took up private engineering practice in Pittsburgh, and soon found himself in charge of the design and construction of important bridges for railroads and bridge companies. His earliest major structure was the Smithfield Street bridge over the Monongahela River[4] built in 1882, replacing a suspension bridge built by the late John A. Roebling, M. Am. Soc. C. E. It was distinguished both by the novelty of its double-elliptical trusses of 350-ft span and by the use in parts of the structure of steel, then new in bridge construction. He also built the Allegheny River bridge at Seventh Street, Pittsburgh, a four-span continuous suspension bridge with double-braced eyebar chains. Other important works were continuous bridges at McKeesport and at Herr's Island. The originality and boldness of conception of all these structures, their excellent adaptation to the site and service, and their striking appearance ranked him at once among the leading bridge engineers of his day.

---

[1] Memoir prepared by F. H. Frankland and F. E. Schmitt, Members, Am. Soc. C. E.

[2] For memoir, see *Proceedings*, Am. Soc. C. E., Vol. XV–XVI (1889–90), p. 137.

[3] For memoir, *loc. cit.*, Vol. XXI (1895), p. 199.

[4] For description, see *Transactions*, Am. Soc. C. E., Vol. XII (1883), p. 353.

In the late eighties the desire of the Pennsylvania Railroad to obtain direct rail entrance into New York City brought Mr. Lindenthal into close contact with the late Samuel Rea, Hon. M. Am. Soc. C. E., and with the problems of bridging the Hudson River and the waters between Long Island and the Bronx mainland. Pres. George B. Roberts instructed Mr. Rea to "find the best bridge engineer you can hire," and Mr. Rea, who had known of Mr. Lindenthal's unusual bridge operations in Pittsburgh, called on him to advise the company.

The plan for a New York entrance first centered around a proposed terminal at Washington Square, but the cost of such an entrance for the Pennsylvania alone appeared excessive. Mr. Lindenthal was commissioned to learn the attitude of the other railroads on participating in the bridge; the outcome was the formation in 1890 of the North River Bridge Company, whose incorporators included Thomas F. Ryan, financier, and Garrett V. Hobart, later Vice-President of the United States. A federal charter was obtained, and the company proceeded energetically to develop plans for the bridge. However, litigation delayed the beginning of field operations, and just as this was successfully terminated the business depression of 1893 forced indefinite postponement of the project.

Although brought to a halt, the enterprise gave rise to a memorable engineering investigation. In preparing to pass on the North River Bridge Company's application for a permit to build the bridge, the War Department, in 1894, appointed a board of engineers to advise concerning the feasibility of this and two other bridge projects also involving spans of unprecedented length—Mr. Lindenthal's proposed bridge having the unheard-of span length of 3,100 ft. This board made two reports, the first as to the maximum span practicable for suspension bridges and the other on the length of span most suitable for a bridge between Fifty-Ninth and Sixty-Ninth Streets, New York City. The first report determined 4,335 ft as the longest practicable suspension span, and the second report stated that a minimum span of 2,000 ft was required at the designated locations and that it was practicable to build a span of 3,100 ft as proposed by Mr. Lindenthal.

For a decade the Hudson River crossing remained dormant. Finally in 1904 the Pennsylvania Railroad, on finding that the other roads were reluctant to join, decided on a tunnel entrance to New York. Mr. Lindenthal was named a member of the board of engineers to direct the tunnel and terminal undertaking, and the bridge was again relegated to a distant future.

Mr. Lindenthal did not give up the Hudson River bridge project but revised his plans to suit the rapidly progressing northward movement of population and trade in the city. He had, in the early nineties, first chosen a new site at 23d Street, and after the World War, another still farther north, at 57th Street, with associated plans for a crosstown highway connection and its extension to the Long Island bank by the Queensboro Bridge, afterward modified to suit a proposed two-span arch bridge crossing at Fifty-Seventh Street.

Meantime, he was enlisted in the solution of other large bridge problems as Bridge Commissioner of New York City in 1902–1903. Growth of traffic between Long Island and Manhattan demanded two new East River bridges, one adjacent to the Brooklyn Bridge, the other some miles north at Blackwells (Welfare Island), later named Manhattan and Queensboro bridges. They offered prime opportunity for bold conception. Mr. Lindenthal first planned an eyebar chain suspension bridge with rocker towers for the 1,470-ft span of the Manhattan Bridge; in the following administration this design was transformed into a wire-cable design with fixed-base cellular plate columns as towers, a model for most large suspension bridges of later years. For the shorter spans of the Queensboro Bridge, crossing two separate channels over an island, he designed a double cantilever without suspended spans, and this design was adopted.

For probably the first time in American bridge history an architect was called in to collaborate on the design of these two structures—because of Mr. Lindenthal's conviction that a major bridge should be distinguished by good appearance as well as by strength and traffic capacity.

Soon after Mr. Lindenthal's service as bridge commissioner he was entrusted with the exceptionally important and difficult task of connecting the Pennsylvania-Long Island trackage with the New York, New Haven and Hartford Railroad in the northern part of the city by a bridge and viaduct crossing Hell Gate. In general location this connection had been projected years before by Mr. Rea and Mr. Lindenthal, when studying the New York entrance problem. Mr. Lindenthal was appointed chief engineer of the New York Connecting Railroad, and it engaged nearly all his energies for ten years, from 1907 to 1917.

Hell Gate arch was the most notable element of this enterprise. Its span of nearly a thousand feet, its four-track deck with ballasted concrete trough floor, the unprecedented size and weight of its members, and above all its striking outline, justly made it the best-known bridge of its time.

Before Hell Gate was completed, the Norfolk and Western Railroad required a bridge across the Ohio River to connect its main line with routes to Cincinnati, Ohio, Columbus, Ohio, and Chicago, Ill. The selected site, a few miles east of Sciotoville, Ohio, presented a river width of more than 2,000 ft and the requirement of two navigation channels. From 1914 to 1917, Mr. Lindenthal built a two-span structure 1,550 ft long, continuous over a central pier; it remains perhaps the boldest continuous bridge in existence.

After the World War, the desirability of bringing the New Jersey railroads, other than the Pennsylvania, into New York City again came to the fore. The lines joined in forming a committee of chief engineers under the chairmanship of the late John F. Wallace,[5] Past-President, Am. Soc. C. E., to develop a comprehensive scheme for efficient freight and passenger handling by way of Mr. Lindenthal's projected Fifty-Seventh Street. Mr. Wallace died in 1921, and the late Francis Lee Stuart,[6] Past-President, Am. Soc. C. E., joined in the work as vice-president and consulting engineer of the North River Bridge Company, Mr. Lindenthal being president and chief engineer. Agreement was reached on all major elements of the project, and in 1922, when working plans were well advanced, application for a permit was made to the War Department by the Bridge Company. It was not until 1928 that the department acted, however, and then it rejected the application on the ground of insufficient clearance height. Revised plans were developed to meet the clearance limits stipulated, but in 1931 these plans were again rejected, this time on the ground that no bridge should be built at the site proposed. Mr. Lindenthal was active in attempting to overcome the objections of the War Department to the Fifty-Seventh Street site until his work was taken over, to a great extent, by others a few years before his death.

In 1928 he had completed his last major bridge enterprise, which was the design and construction of a group of large bridges in Portland to carry the city's principal thoroughfares across the Willamette River. These structures are among the noteworthy engineering monuments of that city.

It was often said of Mr. Lindenthal during his lifetime, and with truth, that he never built two bridges alike. In this fact lies the best expression of his habit of looking on each bridge problem as new and unique, a problem whose proper solution could hardly be the same as that of any prior bridge problem. Moreover, he took up each bridge project broadly, seeking first a conception of general form that would offer the best solution and going on to stresses and details only as the last step. A gift of creative originality was obviously essential for such procedure, and this gift he possessed in high degree. Often his designs required the invention of new details, and he was never at a loss for an answer to such demands.

An innate love of beauty in engineering works went hand in hand with this seeking for the structurally best form. In part it seemed to spring from a conviction that a form satisfying the eye will also satisfy the demands of strength. In part, however, it reflected an esthetic sense that impelled him to seek the aid of men specially qualified in esthetic design whenever he had to deal with an important bridge project.

Throughout much of his career Mr. Lindenthal was a strong advocate of eyebar chains in place of wire cables as main carrying members of suspension bridges. His early success with the Seventh Street Bridge at Pittsburgh gave him assurance of the practicability and economy of such construction, and his design for the Manhattan Bridge contemplated the use of eyebar chains as has been mentioned. The precedent of the Brooklyn, Niagara, and other major wire-cable bridges was sufficiently strong to prevent the wide use of chains; nevertheless Mr. Lindenthal's views were a contributing influence in the successful development of high-strength, heat-treated eyebars by the American Bridge Company some 20 years ago.

---

[5] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1635.

[6] For memoir, *loc. cit.*, Vol. 101 (1936), p. 1493.

Mr. Lindenthal's bridge work was at all times his major preoccupation. He found time, however, to deal with other important problems, some of them later made the subject matter of papers which he presented before engineering societies, while others were embodied only in private reports to his clients. His study of the curve-resistance of railway cars and the design of rocker side bearings to reduce this resistance, and a study of an inverted U-rail to replace the standard T-rail in order to assure better metallurgy and structure of the steel, were among his more important investigations. His active mind also carried him outside the field of engineering problems, and he interested himself in many scientific and public matters.

Gustav Lindenthal contributed a number of important papers to the American Society of Civil Engineers and other engineering organizations, but throughout his life was too active to find the leisure necessary for writing books. His abilities and achievements were recognized by many distinctions conferred on him in the course of his career, including honorary degrees given him by a number of universities.

Nature had endowed Mr. Lindenthal with a powerful frame as impressive as his forceful and magnetic personality; but although he enjoyed robust health during most of his life, he had to undergo a series of severe operations after he had passed the age of 80. He never recovered fully from their effects, and passed away at his Metuchen home on July 31, 1935, in his 86th year. A more complete memoir is filed, for reference, in Engineering Societies Library.[7]

In 1902 he was married to Gertrude Weil of New York, who died in 1905. In 1910 he was married to Carrie Herndon of Durham, N. C., who survives him together with a daughter, Franciska Lindenthal.

Mr. Lindenthal was elected a Member of the American Society of Civil Engineers on May 3, 1882, and an Honorary Member on October 7, 1929.

HENRY BISSELL ALVORD, M. Am. Soc. C. E.[1]

DIED APRIL 20, 1939

The life of Henry B. Alvord was a remarkably full and fruitful one, notwithstanding the fact that it fell some sixteen years short of the allotted three score years and ten. In this relatively short span of time, he was able to encompass not only substantial achievements in his professional field of civil engineering but also a great variety of other accomplishments which gave to his personality a symmetry and charm far from common. Father of a very happy family of four children—two sons and two daughters—Henry Alvord drew a rich and satisfying enjoyment from his family life and responsibilities, which reflected itself in his personal philosophy and manner.

---

[7] 29 West 39th Street, New York, N. Y.

[1] Memoir prepared by E. A. Gramstorff, Assoc. M. Am. Soc. C. E.

From a long line of sturdy forbears reaching back three hundred years to Alexander Alford (name later changed to Alvord), of County Dorset, England (who settled in Northampton, Mass., in 1640), Henry Alvord inherited the vigorous traditions of New England pioneers. His father, Henry Clay Alvord, of Bolton, Conn., was educated for the Congregational ministry at New York University in New York, N. Y. His mother—née Alice Bissell—of Charlemont, Mass., came of a family originally located in Wilmington, Vt.

During the first pastorate of the Rev. Henry Clay Alvord, at Montague, Mass., a son was born on March 24, 1885, and christened Henry Bissell Alvord. When the boy was but a year old, his father accepted a call to the Old South Congregational Church in South Weymouth, Mass., where he served continuously until his death thirty-three years later.

Young Henry B. Alvord was educated in the public schools of Weymouth, Mass., from which he entered the Massachusetts Institute of Technology, at Cambridge, Mass., to study civil engineering. He was graduated from the Institute in 1907 but remained for three subsequent years as an assistant in the Department of Civil Engineering. During the summer months of the latter period, he was employed by the Commonwealth of Massachusetts for the purpose of establishing town boundaries and the geodetic position of their corners. In this work he showed a very high regard for precision and accuracy in his measurements. It was probably through this experience that he developed the keen interest in geodetic matters which remained with him throughout his life.

In the course of these summer assignments, he met, and later married at Bethlehem, N. H., Margaret Graham, daughter of John Graham from Langholm, Scotland, a Congregational minister who held various pastorates throughout New England.

Following his marriage, Mr. Alvord accepted a position as instructor in civil engineering at Bowdoin College, in Brunswick, Me., was made assistant professor the following year, and served continuously in this position until June, 1914, when he left the field of teaching to gain further professional experience. He served first as office assistant and librarian for the Aberthaw Construction Company, leaving their employ in March, 1918, to accept an appointment as chief of party with the county engineer in Braintree, Mass.

In 1919 he returned to teaching at Wentworth Institute in Boston, Mass., but after one year resigned to become assistant professor of civil engineering at Northeastern University, in Boston, in September, 1920. A year later he was made professor of civil engineering and chairman of the department, a position which he filled continuously thereafter until his death.

During nearly two decades in which he led the Department of Civil Engineering, Professor Alvord left the impress of his personality upon many hundreds of engineering students. He helped the College of Engineering grow from a very small, struggling institution to one that was strong, sound, and adequately housed. Throughout his service on the faculty, Professor Alvord was always eager to cooperate with other members of the staff in improving the work of the college. He had a genuine and unflagging interest in student activities, as well as in the professional aspects of instruction. Always a lover of the outdoors, and a particularly capable tennis player, he was able to meet students on their own ground outside of the classroom. One of his special hobbies was the planning of inspection trips and expeditions upon which faculty and students could mingle in friendly social intercourse.

In the course of his long teaching experience, Professor Alvord taught most of the subjects that are given in an undergraduate curriculum in civil engineering, but for the last decade of his life he concentrated his effort largely in the field of engineering structures.

During summer vacation periods he worked on various engineering jobs. At one time he served the Great Northern Railway Company in full charge of the triangulation survey for the Cascade Tunnel, and, at another time, he completed a triangulation of the City of North Adams, Mass. In the summer of 1923–1924 he served as instructor at the Harvard Summer Camp at Squam Lake, N. H.

He was a member of the American Concrete Institute and served as its secretary from 1917 to 1920. Other technical societies with which he was associated are the Boston Society of Civil Engineers (of which, at the time of his death, he was a director and faculty advisor of the student chapter at Northeastern University); the Society for the Promotion of Engineering Education; and the Massachusetts Federation of Planning Boards, of which he was secretary for two years and treasurer at the time of his death. He also served as a member of the Planning Board of the City of Melrose, Mass., from 1931 to 1934.

As a young man he studied the piano at the New England Conservatory of Music under Charles Dennee, and, together with his wife, thoroughly enjoyed a great interest in and love for music throughout his life. Both Professor and Mrs. Alvord were active members of the Beethoven Club of Melrose, of which he was a past-president. He developed (particularly in his later years) a whole-hearted interest in social, political, and economic problems, and was active in municipal and church affairs.

Professor Alvord is survived by his wife and four children, Margaret (Mrs. R. L. Briggs), John R., Graham G., and Jean Ruth.

Professor Alvord was elected an Associate Member of the American Society of Civil Engineers on June 6, 1927, and a Member on August 15, 1938.

JAMES MACFARLANE ANGLE, M. Am. Soc. C. E.[1]

DIED OCTOBER 30, 1939

James Macfarlane Angle was the eldest of four children, two boys and two girls, born to E. J. and Mary Clymer (Macfarlane) Angle. He was born in

---

[1] Memoir prepared by a Committee of the Alabama Section composed of J. L. Land, M. Am. Soc. C. E., and H. C. Wells, Assoc. M. Am. Soc. C. E.

Towanda, Pa., on September 15, 1884. As a youth, Mr. Angle attended the public and high schools of Towanda, later going to Bingham Military School at Asheville, N. C., for a year. He attended the Virginia Military Institute at Lexington, Va., for two years, and later completed his professional education by a 4-yr course at The Pennsylvania State College, at State College, Pa., from which he received the degrees of Bachelor of Science in Engineering in 1910 and Civil Engineer in 1923.

Almost immediately after graduation he began work in the engineering field. He served the Pennsylvania Steel Company, in Steelton, Pa., as draftsman until 1911, the U. S. Reclamation Service on the Flathead Project in Montana as rodman and instrumentman during 1911, the Great Northern Railway Company in the State of Washington as chairman and inspector in 1912, and the Northern Pacific Railway Company in the State of Washington as inspector on timber and concrete construction during 1913.

In April, 1914, Mr. Angle was employed by the Allegheny County (Pennsylvania) Engineering Office of Pittsburgh, and served as draftsman and inspector on the construction of bridges. He also assisted in making road and bridge locations and plans until he entered the U. S. Army in October, 1917.

Mr. Angle's services in the U. S. Army date from October, 1917, until April, 1919. He was commissioned a First Lieutenant and experienced various phases of army engineering, such as directing, mapping, and railway construction with the 45th Engineers; and Company Commander and Personnel Adjutant, 545th Engineers. He went overseas with the 545th Engineers in September, 1918, contracted influenza and other ailments immediately after landing, and was soon sent back to the United States for treatment. He was honorably discharged from the Army in April, 1919.

He then returned to the Allegheny County Engineering Office and served there until November, 1919, at which time he went with Morris Knowles, Incorporated, of Pittsburgh and served as computer on sewer system investigation until September, 1920.

In November, 1922, he entered the service of the U. S. Bureau of Public Roads and was connected with this office until his death in October, 1939. While with the Bureau of Public Roads, Mr. Angle climbed steadily from an associate highway bridge engineer to highway bridge engineer and senior highway bridge engineer. His responsibilities increased from checking and designing to district bridge engineer, which covered both designs and construction for the 8th District of the Bureau of Public Roads.

Mr. Angle's wide range of experience in diversified work gave him a well-rounded knowledge of the engineering field, and he was considered an authority on movable bridge spans. He was also interested in the profession as a whole, and built up quite a collection of engineering books, which he donated to the Montgomery (Ala.) Engineers' Club before leaving Montgomery.

His retiring disposition won him many close and confidential friends, but did not reveal the intellect of the individual to the extent that he deserved. In addition to his career as an engineer, he was a Mason, a member of the Presbyterian Church, a gentleman of the highest type, and an honor to the engineering profession and the community in which he lived.

Mr. Angle died at the residence of a sister in Pittsburgh on October 30, 1939. He was buried in the family cemetery at Towanda, and is survived by his aged mother; a brother, Philip E. Angle; and two sisters, Mrs. Guy F. Batchelor and Mrs. Thomas W. Oberhauser.

Mr. Angle was elected an Associate Member of the American Society of Civil Engineers on January 17, 1921, and a Member on July 12, 1937.

WILLIAM JOHN ASH, M. Am. Soc. C. E.[1]

DIED JULY 10, 1939

William John Ash was born in New York, N. Y., on April 27, 1884, the son of William Robert and Ernestine J. Ash. He received his early education in the public schools, and was graduated from Cooper Union, in New York, in 1909, with the degree of Bachelor of Science. In 1913, he received the degree of Civil Engineer from the same institution.

From November, 1905, to February, 1907, he acted as rodman and inspector of construction for the New York Central Railroad Company at Yonkers, N. Y. In February, 1907, he secured a position with the Bureau of Engineers at New Brighton, N. Y., first as draftsman and designer, and then as assistant chief draftsman of the construction division, on design and supervision of various municipal improvements.

From August, 1914, to November, 1917, Mr. Ash was with the Public Service Commission as assistant engineer. During the latter part of this time he was in charge of field and office work, as assistant to the section engineer.

In November, 1917, he became associated with the Air Nitrates Corporation in New York, first as squad leader on U. S. Nitrate Plant No. 2, at Muscle Shoals, Ala., and then, after July, 1918, as assistant engineer of designs on U. S. Nitrate Plants Nos. 3 and 4. In December, 1918, he left the Air Nitrates Corporation to become civil and mechanical engineer for the American Cyanamid Company. Among other things, he directed the design of buildings.

From December, 1919, to May, 1920, Mr. Ash was with the Central Dyestuff and Chemical Company, in Newark, N. J., as designing engineer in charge of building alterations, installation of chemical plant equipment, etc. He returned to the American Cyanamid Company in May, 1920, and was in charge of designs of chemical plant equipment and construction.

From August, 1921, to October, 1922, he was with M. Shapiro and Sons, in New York, as civil and structural engineer, designing steel buildings and foundations, and was also in charge as superintendent on reservoir construction at Liberty, N. Y., for the Workmen's Circle Sanitarium.

---

[1] Memoir prepared by R. de C. Greene, Esq., New York, N. Y.

In October, 1922, he again returned to the American Cyanamid Company, this time as chief draftsman in charge of design of buildings and chemical plant equipment, drawing up specifications, etc. He was with this company until his death, with the exception of a period in 1923, when he acted as consulting engineer for the Warner Chemical Company, at Carteret, N. J., on building changes.

Mr. Ash was a member of Excelsior Lodge, Free and Accepted Masons 195, in New York, Modern Woodmen of America, Manhattan Camp 11578, and Sigma Alpha Pi Fraternity.

He was married on October 19, 1910, to Anna Sheffield, who, with one son, William, survives him.

Mr. Ash was full of energy and enthusiasm. Even at the outset of his engineering career, he was distinguished by the same qualities that later made him a valued man in directing the important design work of the Air Nitrates Corporation, in the construction of the Muscle Shoals plant during the World War, and his subsequent activities with the parent company.

His good humor was characteristic and his enjoyment of his work contagious. He was studious and developed himself to cover other fields of engineering in addition to the structural work in which he was so well grounded. He had many friends, was always ready to be of service, and his was a life full of endeavor, accomplishment, and courage.

Mr. Ash was elected an Associate Member of the American Society of Civil Engineers on November 25, 1919, and a Member on October 21, 1924.

CALVIN LEWIS BARTON, M. Am. Soc. C. E.[1]

DIED MAY 10, 1939

Calvin Lewis Barton was born in Little Meadows, Pa., on May 16, 1874, the son of Lewis William and Caroline (Beebe) Barton. On both the paternal and maternal sides, his ancestors were among the earliest colonial settlers, one of them being William Bradford, first Governor of the Plymouth (Mass.) Colony.

After his preparatory education at Cazenovia Seminary, Cazenovia, N. Y., Mr. Barton entered Cornell University at Ithaca, N. Y., and was graduated, in 1899, with the degree of Civil Engineer. During his University years, he not only participated in extra-curricular activities, but prepared the tables for "The Transition Curve" by the late Charles Lee Crandall, M. Am. Soc. C. E. Later he was active in alumni affairs, especially those of the Cornell Society of Civil Engineers.

Upon graduation from Cornell, Mr. Barton entered the employ of the Rochester Bridge and Iron Works and, upon the formation of the American Bridge Company, worked at their Pencoyd Plant.

In 1901 he was employed by the late Ira A. Shaler, M. Am. Soc. C. E., on the Park Avenue tunnel section of the original New York subway, and remained on this work until its completion in 1904. He was then employed by S. Pearson and Son on compressed air equipment and caissons for the Pennsylvania Railroad, East River Tunnels, and later acted as General Superintendent for the Phoenix Construction Company on the driving of an experimental tunnel for the Pennsylvania, New York and Long Island Railroad Company.

---

[1] Memoir prepared by Glenn B. Woodruff and Charles E. Fraser, Members, Am. Soc. C. E.

In 1906 Mr. Barton joined Leslie McHarg, Assoc. M. Am. Soc. C. E., and formed the McHarg-Barton Company. Until the dissolution of the firm in 1928, this Company specialized in bridge and harbor construction. Among their larger contracts were Chelsea Pier No. 53 for New York, N. Y., and two of the Stapleton Pier Sheds on Staten Island, New York; Warehouses at Fort Monroe and Charlestown Navy Yard; foundations for the Statue of Liberty; and bridges over the Saugatuck River at Westport, Conn., and the Norwalk River at Norwalk, Conn.

In 1928 Mr. Barton became associated with the Fraser-Brace Engineering Company, Inc., of Montreal, Que., Canada, and New York, in the capacity of Cost Engineer on the New England Power Dam of the 15 Mile Falls hydro-electric power development near St. Johnsbury, Vt. Upon completion of this work in 1930, he established a consulting practice at Norwalk and also devoted part of his time to business interests there, being associated with the Norwalk Savings Society.

During the latter part of his life, Mr. Barton devoted much of his time to civic affairs. He was elected Mayor of Norwalk in 1921 and served two terms. Later he served as a member of the Board of Public Safety and was active in the affairs of the Young Men's Christian Association, serving as its President from 1935 to 1938. In 1938 he was selected to serve on the special grand jury which investigated the Schuyler Merritt Parkway land purchases.

Mr. Barton was a member of the Sigma Xi and Tau Beta Pi fraternities, the Connecticut Society of Civil Engineers (Director from Fairfield County for one term), St. John's Lodge, F. and A. M., the Norwalk Country Club, and the Congregational Church of Norwalk.

In 1908 he was married to Nina Bagley, who, with one son, Bradford, survives him.

Mr. Barton was elected an Associate Member of the American Society of Civil Engineers on April 4, 1906, and a Member on May 6, 1914.

GEORGE ERLE BEGGS, M. Am. Soc. C. E.[1]

DIED NOVEMBER 23, 1939

George Erle Beggs was born in Ashland, Ill., on April 23, 1883. His father and mother, Charles Edwin Beggs and Emma Beggs, were cousins. Edwin

---

[1] Memoir prepared by Carlton S. Proctor, M. Am. Soc. C. E., Elmer K. Timby, Assoc. M. Am. Soc. C. E., Arthur M. Greene, Jr., Dean, School of Eng., and Professor of Mechanical Eng., Princeton Univ., Princeton, N. J., and Kenneth H. Condit, Cons. Editor of *American Machinist* and *Product Engineering*, McGraw-Hill Book Co., Inc., New York, N. Y.

Beggs, as he was known, was a large grain dealer and owned extensive farm lands in Illinois and in the Canadian prairie provinces.

After being graduated from the high school at Ashland, George Beggs matriculated at Northwestern University, at Evanston, Ill., and was graduated with the Bachelor of Arts degree in 1905. He was elected to Phi Beta Kappa at Northwestern. For his engineering education he went to Columbia University, at New York, N. Y., where he was graduated with highest honors in civil engineering in 1910, receiving the Illig Medal. While at Columbia, Mr. Beggs was elected to Tau Beta Pi and Sigma Xi fraternities. During two of his summer vacations he served as instructor in field surveying and geology at the Columbia summer camp near Litchfield, Conn.

For two years after leaving Columbia, Mr. Beggs worked for the American Bridge Company as a draftsman and checker of stress sheets. Here he did his first analysis of stresses in indeterminate structures in which field he was destined to earn distinction. He calculated the erection stresses and deflections of the Bessemer and Lake Erie Railroad bridge, a double-track, indeterminate structure 2,000 ft long, continuous over three spans, and costing about $1,000,000. He also made the design calculations for the Quachee Railway arch near Hanover, N. H.

About this time Union College called on the American Bridge Company for temporary aid in the conduct of courses in kinematics, theoretical mechanics, and surveying, and Mr. Beggs was assigned to the task. Later, he spent four months with Falk and Hager of New York City, consulting engineers, on the design of steel frame buildings, and six months on the Lehigh Valley Railroad valuation survey. He then accepted a position as chief engineer of the Phosphate Mining Company near Lakeland, Fla., and spent nearly two years with that organization on design and construction work.

In 1914 Mr. Beggs was called to Princeton University, at Princeton, N. J., to teach applied mechanics, advanced structures, and mechanics of materials. He was made an assistant professor of civil engineering in 1915, associate professor in 1921, and professor in 1930. In 1921 he became Secretary of the School of Engineering, and was appointed chairman of the Department of Civil Engineering in 1937.

Professor Beggs' special interest was in the field of indeterminate structures. His studies eventually led to his invention of the method of analyzing such structures by means of elastic scale models and the Beggs Deformeter Gauge on which he obtained a patent. He was also granted a patent on a slide-rule bomb sight for airplanes during the World War, and on a pressure control valve in 1939.

After developing the method of analyzing models of structures which could be deformed in two dimensions, Professor Beggs proceeded to apply his methods to three-dimensional analysis, first to a model of a skew concrete arch and then to a model of a concrete arched dam. The investigation of the latter structure, which was a model of the experimental Stevenson Creek Concrete Dam, gave valuable conclusions regarding deflections which were not shown by the experimental dam because of lack of uniformity in the concrete, and were not included in the mathematical analysis because of incorrect assumptions. The stress distribution in general was similar to that from the dam. This investigation demonstrated the applicability of accurate model analysis to such structures with a resulting saving in expense of considerable importance. The experimental dam cost more than $100,000, the model less than $500.

Another pioneering job was the application of model tests in providing the influence lines for the design of the foundations for the Albany (N. Y.) Bell Telephone Building, the first foundation ever designed utilizing the principle of the Vierendeel truss for the distribution of structural loads through a thin slab after excavation of soil materials approximating the weight of the structure to be supported. Although an enormous amount of work on such treatment of foundations has been done since this job, the Albany Telephone Building still stands as a model for foundation treatment of soft and relatively unconsolidated residual clay deposits, and the description of this job together with the important part played by Professor Beggs in his model analyses of a thoroughly indeterminate structure has been translated into every modern language.

Next came the analysis of the stresses in members of suspension bridges by the model method. The first check analysis on the model of an existing bridge on which an intricate mathematical analysis had been made indicated a discrepancy which turned out to be the result of an error in the mathematics. This demonstration of the reliability of the method resulted in the employment of Professor Beggs for a similar analysis of the San Francisco-Oakland Bay suspension bridge for the California Department of Highways.

Somewhat later Professor Beggs was retained to make a model analysis of the towers for the Golden Gate Bridge. Entirely new principles of design had been developed for these towers. The soundness of these principles was demonstrated by the model tests. His last extensive investigation of this kind was for the United States Navy. The model used was a stainless steel type model of a floating structure with a maximum dimension of more than 1,000 ft. In addition to the foregoing investigations, Professor Beggs was frequently consulted by engineers, architects, and manufacturers on structural problems.

In the years 1923, 1926, and 1934, Professor Beggs traveled widely in Europe investigating engineering projects and visiting technical schools. Lectures on his method of analyzing indeterminate structures were given before a conference of the Academy of Sciences in Zurich, Switzerland, in Paris, France, and before engineering conferences in other continental cities.

Professor Beggs was a consultant on the design of the Goodyear-Zeppelin hangar at Akron, Ohio, with special reference to laying out the program for deflections and stresses to allow for expansion and contraction of the structure. He investigated the failure of the Hackensack Bridge and was special consultant for the Raritan River Bridge. He also assisted the Taylor Wharton Iron and Steel Company in developing high tensile steel for railway track work.

Two medals, in addition to the Illig Medal, were awarded to Professor Beggs for his research achievements—the Wason Medal, by the American Concrete Institute, in 1922; and the Howard N. Potts Medal, by the Franklin Institute, in Philadelphia, Pa., in 1927. He was the author of the following technical books and articles: "Live Load Stresses in Railway Bridges," later translated into Spanish; "Mechanical Solution of Indeterminate Structures," American Concrete Institute, later translated into French, German, Spanish, and Russian; "Experimental Design of a Multiple-Arch System and Permissible Simplifications," discussion in *Transactions*, Am. Soc. C. E.[2]; "The Use of Models in the Solution of Indeterminate Structures," *Journal* of the Franklin Institute; "Tests on Structural Models of Proposed San Francisco-Oakland Suspension Bridge"; "Skew Arch Reactions Measured by the Reciprocal Method," *Engineering News-Record;* "Suspension Bridge Stresses Determined by Models," *Engineering News-Record.*

In the course of his professional career Professor Beggs was active in the work of the Society, serving as member and chairman of several important committees. He was also a member of the council of the American Institute of Consulting Engineers, and chairman of the Engineering Foundation.

Professor Beggs entered generously into the social life of the Princeton faculty and of the town community, and he contributed his aid freely to many worthy causes. The minutes of the Princeton faculty regarding his death read: "Not only Princeton University but the Borough of Princeton as well will long remember his kindly life and service extending over a quarter of a century."

Professor Beggs was married to Frances May Ingalls of New York in 1912. He is survived by his widow and their only child, George Erle Beggs, Jr.

Professor Beggs was elected a Junior of the American Society of Civil Engineers on October 3, 1911; an Associate Member on September 2, 1914; and a Member on April 16, 1923.

JAMES C. BELL, M. Am. Soc. C. E.[1]

DIED JUNE 10, 1939

James C. Bell was born in Albany, N. Y., on November 24, 1883, the son of the late James C. Bell, M. Am. Soc. C. E., and Anna (Talcott) Bell. His paternal grandmother was of the Pruyn family of Albany, and his father was of the Bell family who were in business in Newport, R. I., in 1768. His father was also a civil engineer and was graduated from Union College, at Schenectady, N. Y., in the class of 1868.

Mr. Bell attended the old Albany High School and Rensselaer Polytechnic Institute, at Troy, N. Y., where he was graduated in 1906 with the degree of Civil Engineer. He was a member of Sigma Xi and the Rensselaer Society of Civil Engineers.

---

[2] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 1208.

[1] Memoir prepared by Albert R. Reilly, M. Am. Soc. C. E.

During summer vacations while in college and until January, 1908, "Jim" or "Tinkle," as he was known to his many friends, worked his way up from axman to chainman, rodman, and instrumentman in charge of construction and of survey parties with the State Engineer's Department, Division of Highways.

From January, 1908, until September, 1918, he worked up from assistant engineer to resident engineer on many dams, locks, bridges, hydraulic surveys, and designs of the Barge Canal Division of the state engineer's office. Some of the projects were: Dam No. 2 at Crescent, N. Y.; Dam No. 4 at Scotia, N. Y.; Dam No. 3 at Vischer Ferry, N. Y.; Crescent Highway Bridge over the Mohawk River; Vischer's Ferry Viaduct; Freman's Bridge over the Mohawk River; and surveys and studies of lower Hudson River terminals.

Jim Bell had charge of the preliminary survey for Storm King Highway and the first road between McKeever and Old Forge in the Adirondack Mountains, New York. The largest and finest monument standing to James C. Bell's memory, however, is a very unique structure, The Western Gateway Bridge, of twenty-four reinforced-concrete spans built on a curve over the Mohawk River at Schenectady. From September, 1918, to January, 1924, Jim with his assistants and the contractors labored most earnestly to produce the $3,000,000 structure.

In January, 1924, he was appointed commissioner of public works of Schenectady by Mayor William W. Campbell on the recommendation of the late Frank M. Williams,[2] M. Am. Soc. C. E., state engineer. He held this position until January, 1926, when, for a month, he acted as consultant on claim work for the state engineer.

From February, 1926, until June, 1939, he was construction engineer for the Board of Education of Rochester, N. Y., and was in charge of field supervision of the construction of many school buildings, the most outstanding being Charlotte High School and John Marshall High School.

It is most fortunate that occasionally a man is born to this world who can be an inspiration to his fellow men; one who can smile and make others smile, even on those dark days; one whom every one is glad to see and meet, even for only a few words; one who is a welcome friend at any gathering; one whom all men trust and respect and who is so genuinely loyal that he can be rated as a man among men; a man who likes his work well enough to do it thoroughly and who never tires of it enough to spoil his ambition to learn to do it better each time; a man who loves his family genuinely and his neighbor well enough to be every-ready with a helping hand and the kind word; and one who has a true sense of humor, wit, and the gentlemanly instinct to use it properly so that he is envied and loved by all. All of these qualities and more were possessed by Jim Bell, as all affectionately called him. When he passed on, although it was felt that he had gone to a just and well-earned reward, every one felt that the world could ill spare him, that the world needed more like him, and that the profession of engineering had really gained by his membership and record.

James C. Bell was married to Jane Davidson on September 3, 1908. He is survived by his widow; their twin sons, James C. and Gordon R.; and a brother, Roy W. Bell.

---

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 1626.

Mr. Bell was elected an Associate Member of the American Society of Civil Engineers on April 7, 1915, and a Member on October 10, 1927.

JOHN GEORGE BLOOM, M. Am. Soc. C. E.[1]

DIED MARCH 28, 1940

John George Bloom was born on November 25, 1867, at Xenia, Ohio. He was graduated from the Ohio State University, at Columbus, Ohio, with the degree of Civil Engineer in June, 1889.

His first engineering work began in July, 1889, as assistant engineer with the Pennsylvania Railroad Company. From October, 1889, to March, 1890, he served as draftsman with the city engineer of Bucyrus, Ohio, making maps of cities in Ohio and the Indiana gas fields. The next two months he spent with the Hocking Coal and Iron Company at Buchtel, Ohio, as draftsman and instrumentman. Until October, 1890, he was assistant engineer with the Kenova Land Association laying out town sites, grading streets, and making contour maps of Kenova, W. Va.

The following year Mr. Bloom joined the Norfolk and Western Railway Company as assistant supervisor and assistant to the division engineer, Eastern Division, Crewe, Va. Then he returned to his home town as assistant city engineer, and in April, 1892, he entered the service of the Baltimore and Ohio Southwestern Railroad Company at Cincinnati, Ohio, as draftsman. In November, 1893, he was promoted to assistant engineer, in charge of office and field work, and two years later he was promoted to division engineer, Springfield Division, in Flora, Ill., in charge of maintenance of way and structures. In June, 1896, Mr. Bloom was appointed principal assistant engineer of the entire system, in charge of office and field work, design, revision of grades, and new construction; and in February, 1900, he was division engineer of the Ohio Division, at Chillicothe, Ohio, in charge of maintenance of way and structures. He remained in this position until April, 1892, when he became division engineer, Newcastle Division, of the Baltimore and Ohio Railroad Company, in charge of maintenance of way and structures.

Mr. Bloom entered the service of the Chicago, Rock Island and Pacific Railway Company in June, 1903, as principal assistant engineer of the Southwestern District, at Chicago, Ill., and Topeka, Kans., in charge of maintenance and construction. In July, 1905, he was appointed engineer in charge of maintenance and construction of the Southwestern and Choctaw districts and the St. Louis, Colorado and Kansas City Railroad, a subsidiary at Topeka. From January, 1909, to August, 1912, he served as president of the Southern Ballast Company, at Mill Creek, Okla.

---

[1] Memoir prepared by L. C. Fritch, M. Am. Soc. C. E.

A record of his various positions follows:

August, 1912, to December, 1913—superintendent of construction of the John F. Stevens Construction Company on Salmon River Dam and Power House, at Altmar, N. Y.

December, 1913, to October, 1914—with Grace Construction Company on double track work on Illinois Central Railroad at Dyersburg, Tenn.

October, 1914, to June, 1916—valuation work for the Chicago, Rock Island and Pacific Railway Company.

June, 1916, to August, 1918—superintendent of the Chicago, Rock Island and Pacific Railway Company on the Amarillo Division, at Amarillo, Tex.

August, 1918, to May, 1921—superintendent of the Louisiana Division of the same company, at El Dorado, Ark.

May, 1921, to May, 1922—division engineer, Nebraska Division, at Fairbury, Nebr.

May, 1922, to February, 1929—engineer, Maintenance of Way Department, of the entire system of the Chicago, Rock Island and Pacific Railway Company at Chicago.

Resigning voluntarily on February 1, 1929, Mr. Bloom retired from railroad service and moved to Pasadena, Calif.

The writer's acquaintance and friendship extended over almost the entire period of service enumerated previously. The most outstanding characteristic of John George Bloom was his thoroughness, accuracy in his work, and dependability. He was modest and unassuming, but most kind and affable, and endeared himself to all who came in contact with him.

During the forty years of active railroad service he rendered valuable and pioneering service on some of the most prominent railroads of the country. From his large experience in railroad engineering he contributed much to the art of the rehabilitation of the railroads in the revision of line and grades, and did most valuable work in this industry when it was in its infancy.

He spent his retirement years in Pasadena, where he built a fine home and enjoyed his well-earned leisure with his beloved wife. He left no children, and his nearest relative is a niece, Edna Bloom. He lies at rest beside his wife in the beautiful Forest Lawn Memorial Cemetery at Los Angeles, Calif.

Mr. Bloom was a Scottish Rite Mason.

Mr. Bloom was elected a Member of the American Society of Civil Engineers on July 9, 1906.

NORMAN FREED BROWN, M. Am. Soc. C. E.[1]

DIED APRIL 11, 1937

Norman Freed Brown was born at Coudersport, Pa., on March 20, 1878, the son of Thomas Benton and Mary (Housekeeper) Brown.

---

[1] Memoir prepared by a Committee of the Pittsburgh Section composed of Vernon R. Covell, M. Am. Soc. C. E., Chairman, and the late Samuel Eckels and the late Charles M. Reppert, Members, Am. Soc. C. E.

His technical training was obtained at the University of Pennsylvania, at West Philadelphia, Pa., and the Carnegie Institute of Technology, at Pittsburgh, Pa., where he took special work.

He began his engineering career with the Pennsylvania Railroad Company on January 3, 1898, as a rodman and was promoted through the various grades to the position of assistant engineer of construction. As such he was engaged in railroad construction in western Pennsylvania, especially in Pittsburgh and vicinity. Among the projects of which he was in charge were the Brilliant Branch or "Cut-Off"; erection of the Port Perry Bridge over the Monongahela River; construction of the Duquesne (Pa.) Elevated Railroad along the Allegheny River, including warehouses and delivery yard; East Liberty, Pa., and Swissvale, Pa., passenger stations, each requiring change of street grades; and the Pitcairn, Pa., east-bound classification yard.

Mr. Brown was granted a leave of absence on January 31, 1918, and was commissioned in the rank of Major in the Reserve Corps of Engineers, January 24, 1918. He served in France with the American Expeditionary Forces from February, 1918, to April, 1919, attached to the staff of the late Brig. Gen. W. W. Atterbury,[2] M. Am. Soc. C. E., who was Director General of the American Army Transportation Corps. While serving with General Atterbury, Major Brown assisted in the planning and developing of numerous transportation installations and facilities, particularly on the railroads utilized as the first line of communication. He made an extensive study of the French railroad system with regard to traffic density and capacities for the development of the second line of communication and the future development of the third and fourth lines of communication to be used by the United States Army as planned for the peak of combatant troops in action for the planned spring drive of 1919. This study terminated with the signing of the Armistice; however, a complete arrangement had been entered into with the French military services and many plans had been developed for the required facilities for American train operations. Following the signing of the Armistice, Major Brown was assigned to the Inter-Allied Railways Commission, formed under Article 7 of the terms of the Armistice, and by this commission he was delegated as the American representative of the Commission for the reception of German railway equipment with headquarters at Metz, France, and receiving stations at Coblenz and Cologne, in Germany, where the railway equipment was delivered, inspected, accepted, and shipped to France, in accordance with the terms of the Armistice agreement. He was honorably discharged from the U. S. Army on March 1, 1919, and he returned to the service of the Pennsylvania Railroad Company on April 1, 1919, as assistant engineer.

On March 1, 1920, Major Brown was appointed assistant to the chief engineer. During this period he was engaged in adjusting settlements between the federal government and the railroad corporation.

On April 1, 1921, he was again granted a leave of absence and was appointed director of the Department of Public Works of Pittsburgh. During this period the following work was planned, contracts let, and construction completed or started: Beechwood Boulevard Bridge; Center Avenue Bridge over the Pennsylvania Railroad; Island Avenue Bridge over the tracks of the Pittsburgh, Fort Wayne and Chicago Railway; and Boulevard of Allies along the Monongahela River, leading from downtown Pittsburgh to the Oakland district, requiring heavy hillside construction and steel viaduct work.

---

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1518.

On January 1, 1922, he resigned as director and was elected vice-president of the Dravo Corporation, where he had charge of the development and securing of new business in heavy bridge and river work. On May 31, 1923, he organized the company of Brown and Reppert, consulting engineers.

Major Brown was chosen by the county commissioners as director of the newly formed Department of Public Works of Allegheny County on January 1, 1924. He organized this department by combining existing construction units and forming new ones. During the eight years that he held the position, he had charge of the most extensive construction program in the history of Allegheny County, which included 98 highway bridges, twelve of which were large river bridges, and ten of which crossed wide and deep ravines, including the George Westinghouse Memorial Bridge, which has the longest span, reinforced-concrete arch in the Americas; the Allegheny River and Ohio River boulevards, each leading from Pittsburgh to suburban sections; construction or reconstruction of more than 300 miles of county roads; Armstrong highway tunnel; the County Airport; and County Office Building. During this period he had developed and patented a reinforced-concrete railroad tie. On November 15, 1931, he resigned as director in order to devote his time to the further development and promotion of the concrete tie, which has been tested in actual service in a number of localities.

Major Brown was a great organizer and had the faculty of inspiring intense activity and loyalty on the part of those whom he directed.

On April 4, 1906, he was married to Hattie Hoobler who survives him. Also surviving are two daughters, Mary (Mrs. Sydney W. Binns) and Nancy Bell (Mrs. Thomas E. Diehl).

Major Brown was elected an Associate Member of the American Society of Civil Engineers on June 5, 1907, and a Member on November 30, 1909.

EVERETT N. BRYAN, M. Am. Soc. C. E.[1]

DIED DECEMBER 17, 1939

Everett N. Bryan was born in Montezuma, Poweshiek County, Iowa, on April 9, 1884, the son of Neri E. and Mary Elizabeth (Job) Bryan.

He went to California with his parents in 1887, received his education in the public schools of that state, and was graduated in 1907 from the University of California, at Berkeley, Calif., with the degree of Bachelor of Letters.

---

[1] Memoir prepared by a Committee of the Sacramento Section consisting of Edward Hyatt, M. Am. Soc. C. E., as Chairman, and Mark S. Edson and Walter E. Stoddard, Assoc. Members, Am. Soc. C. E.

For two years after his graduation he was employed by the Western Pacific Railroad Company on railroad location in Nevada. He entered the employ of H. D. Hanford, consulting engineer of Seattle, Wash., in 1909, and for a year was engaged in engineering work that involved irrigation and hydroelectric power developments in eastern Washington. In 1910 Mr. Bryan returned to California and until 1921 was actively engaged with irrigation construction and operation. During four years of this period he was chief engineer of the Waterford Irrigation District.

Following a period of nearly a year of private practice in San Francisco, Calif., he accepted a position with the State Water Commission, State of California, on July 1, 1921. The Water Commission Act had been put into effect some years before, but because of the shortage of engineers during the World War period, the work of the Commission had not been standardized. Mr. Bryan immediately took a leading part in perfecting the procedure subsequently put into effect, supervising the initiation and perfection of water rights in accordance with the Water Commission Act. As a result of his wide experience in irrigation and hydroelectric engineering practice, and his activity in the Division of Water Resources, which succeeded to the duties of the State Water Commission, he became one of the most eminent engineering authorities on water rights in California. At the time of his death he was the supervising hydraulic engineer of the Division of Water Resources.

As engineering was his profession, so also was it his avocation. He was keenly interested in everything affecting his work and ever strived to uphold the high standards of the profession. Among many papers which he contributed to leading engineering periodicals on various phases of engineering, perhaps the one which received the highest recognition was that on "Hydro-Electric Power as a By-Product of Agricultural Storage,"[2] published in 1925.

Civic affairs always commanded his attention. In 1928 he was appointed a member of the Sacramento Water Commission for the investigation of a water supply for the City of Sacramento, Calif. He also served on other civic committees, giving freely of his time and counsel.

When the law for the registration of civil engineers was first proposed, Mr. Bryan took a leading part in drafting the proposed act and, as a mark of respect, received Certificate of Registration No. 4 under the act, the members of the first Board of Registration having been assigned the first three numbers.

Mr. Bryan was active in the formation of the Sacramento Section of the Society, was a charter member of this organization, was its fifth president, and served on many of its committees. He labored earnestly and effectively for the best interests of the Section and was one of its acknowledged leaders.

While he was President of the Section, Mr. Bryan, and the committee appointed by him, was instrumental in naming the reservoir on the Merced River, formed by the Merced-Exchequer Dam of the Merced Irrigation District, Lake McClure in honor and in memory of the late Wilbur F. McClure,[3] M. Am. Soc. C. E., state engineer of California from 1912 to 1925.

---

[2] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 910.

[3] For memoir, *loc. cit.*, Vol. 91 (1927), p. 1106.

During Mr. Bryan's term as President of the Section, a movement was also initiated for the erection of a monument on the park fronting the Southern Pacific Railroad station, at Sacramento, to the memory of the late Theodore D. Judah,[4] M. Am. Soc. C. E., first chief engineer of the Central Pacific Railway Company. The monument was later constructed by the employees of the Southern Pacific Railroad Company. If it had not been for the activity of Mr. Bryan and his committee, it is doubtful whether the monument would be standing today.

In later years, Mr. Bryan took an active interest in the topographic mapping program sponsored by the Section and in the activities of the Section's committee on engineering salaries. He served continuously as chairman of the latter committee from 1936 until his death.

Mr. Bryan was a member of Sacramento Lodge No. 40, Free and Accepted Masons of the State of California, and a member of the Sutter Club of Sacramento.

He had a gift of personal charm and affability, was a conscientious worker, loyal to principle, a faithful public servant, and a loyal friend.

He was married in Berkeley, on March 25, 1911, to Gladyes Huber Trumbo of Berkeley. He is survived by his widow and his son, Everett Elgin Bryan. A sister, Myrtle, and five brothers, Ross, Claude, Ellis, Jasper, and Homer Bryan, also survive.

Mr. Bryan was elected an Associate Member of the American Society of Civil Engineers on November 3, 1915, and a Member on July 7, 1925.

WALTER CARL BUETOW, M. Am. Soc. C. E.[1]

Died April 23, 1939

The sudden death of Walter Carl Buetow was a real loss to his many friends and acquaintances. He was a man of outstanding personality and character, true in his friendships, and one who always endeavored to see the good in all persons. For these characteristics he was loved and greatly respected, not only in social life, but also in business life. He possessed a dignified bearing, which always left its impression on new acquaintances at first meeting.

Mr. Buetow was born on August 11, 1884, at Milwaukee, Wis., the son of Charles J. L. and Ida (Bierbauer) Buetow.

His life was entirely devoted to his selected field of civil engineering after graduation from the University of Wisconsin, at Madison, Wis., in 1908. First he served as a design and construction engineer with various firms for approximately three years. Then, when the State of Wisconsin organized a Highway Department, he entered its employ as bridge designer. Service with the state covered nearly sixteen years in various capacities, and included the position of chief engineer for three years. He served with the Stein Construction Company, bridge builders, for four years as chief engineer, during which period he constructed one of the largest and longest "causeway" structures in Florida.

---

[4] For memoir, see *Transactions*, Am. Soc. C. E., Vol. XXXVIII, December, 1897, p. 448.

[1] Memoir prepared by Gordon F. Daggett, M. Am. Soc. C. E.

His hobby was his profession—namely, bridge design—and the highways of Wisconsin have many lasting examples of his work and beautiful layouts which combined art with scientific design.

At the time of his death he was in the employ of the City of Milwaukee, for which city he designed the Cherry Street Bridge, and was in charge of construction work.

Mr. Buetow was married to Marie Endron, of Evansville, Ind., who survives him, together with his mother, and a sister, Wonda Buetow.

One of his intimate friends recently described his death as having taken "a real man and wonderful engineer from our midst."

Mr. Buetow was elected a Member of the American Society of Civil Engineers on August 18, 1930.

RALPH HOWE CADY, M. Am. Soc. C. E.[1]

DIED NOVEMBER 29, 1939

Ralph Howe Cady, the son of George Alonzo Cady and Clara (Cline) Cady, was born in Sault Ste. Marie, Mich., on August 22, 1893. The Cady family ancestors came to Massachusetts from England early in the seventeenth century and a branch of the family later migrated to Michigan. Mr. Cady was educated in the public schools in Sault Ste. Marie and studied at the University of Michigan, at Ann Arbor, Mich., in the class of 1916. In 1918 he received the degree of Bachelor of Science in Civil Engineering from the University of Michigan. He was a member of Sigma Xi and Tau Beta Pi fraternities.

During 1916–1917 Mr. Cady acted as engineering assistant for the Michigan Northern Power Company of Sault Ste. Marie, inspecting pneumatic caisson foundations, investigating compensation works and problems in connection with the control of Lake Superior water levels, canal gaging, and other work. From July, 1918, to January, 1919, he was with the U. S. Army Engineering Section Sanitary Corps, as Sergeant, First Class, on waste disposal experimentation, and later as Second Lieutenant in charge of water supply at Payne Field, West Point, Miss. He was honorably discharged in 1919.

He was associated with Hoad, Decker and Drury, Consulting Engineers of Ann Arbor, from February, 1919, to July, 1920, first as assistant engineer on field work and design of water supply and sewage plants for Highland

---

[1] Memoir prepared by E. S. Skillin, Assoc. M. Am. Soc. C. E.

Park, Owosso, Muskegon, Muskegon Heights, Ann Arbor, and Plymouth, Mich.; and then as designing engineer, which position he held for six months.

Mr. Cady joined the organization of the Frederick Snare Corporation, Engineers and Contractors, New York, in July, 1920. After a few months in the main office he was sent to the office in Havana, Cuba. Until 1924 he was assistant engineer on design and construction, and from then until 1927, he acted as engineer and assistant manager of the Havana office. During that period a number of important projects were developed and constructed, notably the complete steamship terminal for the Ward Line Steamship Company, consisting of a large concrete pier, concrete and steel warehouses, wharves, and bulkheads—costing more than $3,000,000. While in Cuba he had charge of, or assisted with the engineering work on, many other projects in various parts of the island which required investigation, reports, designs, estimates, and the actual construction, of bridges, water supply, power houses, sugar mills, and industrial buildings.

Following his transfer to the main office of this company Mr. Cady made a complete technical, commercial, and financial report with preliminary plans and estimates for a proposed ocean and river terminal for the City of Barranquilla, Colombia, S. A., approximating $5,000,000. He made a similar report on the port of Callao, Peru, for the Peruvian Government, consisting of a steamship terminal with piers, wharves, warehouses, custom house, and railway connection, and the problems involved in the construction of a sheltered harbor by building rock breakwaters, dredging, etc., costing in all more than $8,000,000. After the award of this contract, based on his design, he assisted in supervising the construction work; and later the dock facilities at Callao were enlarged to provide for the increased tonnage that was attracted to the port because of the success of the first terminal. This additional work was also done from designs prepared by Mr. Cady and involved the expenditure of $1,500,000.

From 1928 Mr. Cady traveled extensively throughout South America as foreign representative of the Frederick Snare Corporation making studies, surveys, designs, estimates, and reports for various port works such as Asunción, Paraguay; Arica, Chile; and Turiamo, Venezuela. He designed the ocean and river boat terminal at Cartagena, Colombia, completed at a cost of approximately $3,000,000, and supervised the construction through completion. The new port of Matarani, Peru, for the Peruvian Government, estimated to cost about $4,000,000, was designed by Mr. Cady. This work involved all the problems for developing a port at a location where no harbor of any nature had previously existed and included breakwaters in deep water, bulkheads, warehouses, water supply, electric power plant, and other facilities.

In addition to the foregoing, Mr. Cady was associated with studies, plans, and estimates for port work at Tocopilla, Chile, and water works for Cartagena. While in the New York office he was engaged in the construction problems involved in the building of the large Tygart River concrete dam at Grafton, W. Va., for the U. S. Engineers, a dam costing $8,000,000. The most recent project developed by Mr. Cady was the important terminal for oil shipment near Guanta, Venezuela, completed in 1940 from plans prepared by him. His work was of a highly diversified character and included the studies and designs of many important structures requiring not only the best engineering practice but the consideration of local requirements, consultation, and negotiations with government and municipal authorities, particularly in Latin America. His knowledge of Spanish, familiarity with native customs and social life, his pleasing personality, and wide experience made him unusually well equipped for foreign activities. He was highly respected by all with whom he came in contact, as a thoroughly competent engineer and as a man of the highest integrity.

From time to time Mr. Cady prepared papers on engineering subjects which were published in technical journals, such as the *Proceedings* of the Society, the *Engineering News-Record, Port Works,* and the *Marine News.*

Mr. Cady was interested in outdoor life. He was a keen fisherman and fond of the woods. While in Havana he was a member of the Country Club of Havana, and of the Knickerbocker Country Club of Englewood, N. J. His travels covered practically all of the countries of South America and the West Indies, by ships, planes, and automobiles, so that he was familiar with the principal cities on the coast and in the interior, and in some cases isolated locations far from the main centers.

In 1926, Mr. Cady was married to Carolyn Paxton of Hammond, Ind., and he is survived by his widow, and a son, Paxton; three brothers, Mark S. Cady, G. Fitch Cady, and S. Cline Cady; and a sister, Mrs. Edith Cady Gilhooley.

Mr. Cady was elected an Associate Member of the American Society of Civil Engineers on July 9, 1923, and a Member on July 6, 1936.

THOMAS JENKS CARLILE, M. Am. Soc. C. E.[1]

DIED NOVEMBER 28, 1939

Thomas Jenks Carlile was born on August 20, 1872, at Chattanooga, Tenn., the son of William Rich and Margaret (Wylde) Carlile. He was educated in the public schools of Chattanooga.

He began his engineering career in June, 1888, when he entered railroad service at Poughkeepsie, N. Y., as chainman on the location of the Hudson Connecting Railroad. He served as rodman, transitman, and assistant engineer on the location and construction of the Pennsylvania, Poughkeepsie and Boston Railroad. He went from the Poughkeepsie Bridge Route to the Dutchess County Railroad Company where from March to August, 1890, he was assistant engineer on location.

In August, 1890, he joined the engineer corps of the Norfolk and Western Railway Company as assistant engineer in charge of ten miles of construction in West Virginia on the Ohio Extension, where he remained until February,

---

[1] Memoir prepared by John M. Belknap, M. Am. Soc. C. E.

1891. He was engaged as instrumentman on the location of the Adirondack and St. Lawrence Railroad from April to October, 1891, and went from this work to spend eight months as levelman for the New York and New England Railroad Company. His next assignment was as assistant engineer for T. E. Crimmins on the construction of the Third Avenue Cable Road in New York, N. Y., where he was employed from June to November, 1892.

From November, 1892, to March, 1893, he worked as assistant engineer for the late E. P. Roberts,[2] M. Am. Soc. C. E., on surveys for the location of a highway from West Point, N. Y., around Storm King Mountain to Cornwall, N. Y. In May, 1893, he was appointed to the position of rodman in the Bureau of Surveys in Philadelphia, Pa. Later he became assistant engineer in charge of the field corps on general city survey work and on the construction of the Pennsylvania Avenue Subway and Tunnel for the Philadelphia and Reading Railroad Company and on dredging in the Delaware and Schuylkill rivers.

He resigned from the Bureau of Surveys in June, 1900, to accept the position of principal assistant engineer for the Philadelphia Crude Ore Company on surveys in the Aleutian Islands, Alaska, and remained on this work until October, 1900. He returned to the Bureau of Surveys, in Philadelphia, in December, 1900, and served as assistant engineer on the location of Torresdale Boulevard until May, 1901, when he resigned to become assistant engineer on the location of the Cornwall and Western Railroad. This work was completed in July, 1901, and two months later he returned to the Bureau of Surveys (Filtration Department), in Philadelphia, on the location surveys and as assistant engineer on dredging in the Delaware and Schuylkill rivers.

He remained with the Bureau of Surveys until January, 1903, and from that time until January, 1904, he was engineer in charge of location and construction for the Beaver Creek Railroad Company in West Virginia. He went to San Juan, Puerto Rico, in April, 1904, as locating engineer for the West Indies Construction Company on the surveys for the proposed railroad of the Porto Rico Railway, Light and Power Company across the island from San Juan to Ponce.

Returning to the United States in April, 1905, he accepted a position as locating engineer with the New Jersey Short Line Railroad Company. He completed this work in June, 1905, and in July, 1905, he went to Kansas City, Mo., as assistant chief engineer on the location and construction of the Missouri and Kansas Interurban Railroad from Kansas City to Olathe, Kans. He resigned this position in November, 1905, to become assistant engineer with the Long Island Railroad Company at Jamaica, N. Y.

In April, 1907, he was appointed principal assistant engineer and from that time until his retirement, due to ill health, in April, 1938, he was in charge of all field engineering and in general charge of all construction work, with the exception of the Bay Ridge and Jamaica Improvements.

During his term as principal assistant engineer of the Long Island Railroad Company many important improvements were completed including several major grade crossing eliminations.

---

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXII, June, 1911, p. 593.

Mr. Carlile was a lifelong member of the Protestant Episcopal Church and was one of its most valued and prominent laymen. From 1917 until his death, he was vestryman and warden of St. George's Episcopal Church of Hempstead, N. Y. He was also a member of the executive board of the diocese and held other offices in his church.

The foregoing sketch covers fifty-one years of active work in engineering, largely devoted to railroad location and construction. The writer's friendship and association with Mr. Carlile extended over a period of more than thirty-four years and began when they met in San Juan, Puerto Rico, in April, 1904. This association was continued in July, 1905, on the location and construction of the Missouri and Kansas Interurban Railroad at Kansas City, and was further continued on the Long Island Railroad from January, 1911, to September, 1914. Although their paths in business separated, the friendship continued firm and strong, and the memory of that friendship is one of the finest of a lifetime. He was a true and loyal friend.

He was a very capable construction engineer and a man of splendid executive ability. He possessed a pleasing personality and was a delightful companion. He was married on June 16, 1908, to Alice Phelps Stokes of Philadelphia, who survives him. Also surviving is one son, Thomas Stokes Carlile, born at Flushing, Long Island, N. Y., on July 4, 1909.

Mr. Carlile was a member of the American Railway Engineering Association and the American Association of Engineers.

Mr. Carlile was elected a Junior of the American Society of Civil Engineers on March 3, 1896; an Associate Member on March 6, 1901; and a Member on December 3, 1907.

ALLAN WINTER CUDDEBACK, M. Am. Soc. C. E.[1]

Died December 4, 1938

Allan Winter Cuddeback was born in Cuddebackville, N. Y., in 1868. The family were French in origin, and in this country were farmers in the town which is named for them. One sister survives and the family still owns the farm in Cuddebackville.

Mr. Cuddeback was enrolled with the Freshman Class at Cornell University, at Ithaca, N. Y., in the fall of 1890, and completed the work of one year. Then seeking, as he said, the broadening of view which would come from education in a distant section of the country he entered Stanford University, at Stanford University, Calif., which first opened its doors in the fall of 1891. As a transfer student he completed his course in 1894 and thus became one of Stanford's first engineering graduates under the guidance of Charles D. Marx, Past-President and Hon. M. Am. Soc. C. E., who then directed the engineering course.

---

[1] Memoir prepared by Thomas H. Wiggin, Thaddeus Merriman, and John H. Cook, Members, Am. Soc. C. E. Mr. Merriman died September 26, 1939.

In the summer of 1891 he was leveler for the East Jersey Water Company, one of the group of private companies in New Jersey with which he was later to become closely identified through many years. Between the time of his graduation in 1894 and October, 1898, when he finally became identified with that group of water companies in New Jersey, he had a variety of worthwhile jobs which contributed to the breadth of his engineering experience. These jobs included surveys, plans and specifications for a system of sewers at Palo Alto, Calif. (May to July, 1894); charge of a U. S. Coast and Geodetic Survey party engaged in fixing the boundary line between the states of California and Nevada (July, 1894, to January, 1895); assistant engineer in charge of surveys and construction of irrigation ditch for the Stanislaus and San Joaquin Land and Water Company, California (January to June, 1895); topographer in charge of magnetic observations and reconnaissance on surveys of boundary between the states of California and Nevada (June, 1895, to January, 1896); with East Jersey Water Company on construction of Newark, N. J., conduit No. 2 (January to November, 1896); survey for Passaic Valley, N. J., sewerage system, then topographical surveys of two New York, N. Y., parks (November, 1896, to February, 1897); surveys and construction for Metropolitan Street Railway Company, New York City (March, 1897, to March, 1898).

In March, 1898, Mr. Cuddeback made the connection with the group of water companies in Metropolitan New Jersey which was to endure actively from that time until 1926, and thereafter in a less active way until his death. These companies included the Passaic Water Company, the East Jersey Water Company, Jersey City Water Supply Company, Acquackanonk Water Company, and Montclair Water Company. The territory supplied included Paterson, Passaic, Clifton, Montclair, and Bayonne.

Most of these companies were affiliated with the famous S. U. M. (Society for Establishing Useful Manufactures), founded by Alexander Hamilton, which society is still owner of the water power at the Great Falls on the Passaic River in Paterson, N. J. Of these companies the late Clemens Herschel,[2] Past-President and Hon. M. Am. Soc. C. E., inventor of the venturi meter, was for many years chief engineer and manager, followed by the late J. Waldo Smith,[3] Hon. M. Am. Soc. C. E. The latter engaged Mr. Cuddeback and through the following years gave him the opportunity for training and experience which made him the skilled and rounded water works constructor and operator who was so well known in his field. Mr. Cuddeback often quoted the words with which Mr. Smith introduced him to his work, telling him he was conferring on him no title but for him to go in and make a place for himself. This he did, becoming successively assistant superintendent, superintendent and engineer, and finally, after Mr. Smith was called to take charge of water works construction for New York City, becoming superintendent and vice-president of the combined companies which later were formed into the Passaic Consolidated Water Company. An intimate friendship existed between Mr. Cuddeback and Mr. Smith until ended by the latter's death in 1933.

---

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 1419.

[3] For memoir, *loc cit.*, Vol. 101 (1936), p. 1502.

Among the works which were executed during Mr. Cuddeback's connection with these large New Jersey companies were the Boonton Dam, aqueduct, and the first 72-in. steel pipe line from Boonton to Jersey City, many steel pipe lines which were among the early ones in the country, and the Little Falls rapid sand filter plant of 32 mgd capacity, first of the large masonry plants of this type, for which the late George W. Fuller,[4] M. Am. Soc. C. E., was consulting sanitary engineer and the late William B. Fuller,[5] M. Am. Soc. C. E., was consulting structural and resident engineer.

Mr. Cuddeback made the accuracy and maintenance of water meters a special study, and due in great measure to the skill and energy which he applied to the task of metering all services, gradually and completely, the supply from the Passaic River continued to be adequate for the cities of Paterson, Passaic, Clifton, Bayonne, Montclair, and other communities until the condemnation in 1930 of the works of the Passaic Consolidated Company by certain of the cities supplied. With a rapidly rising population and industrial demand, the consumption actually fell greatly during the several-year period of metering. Mr. Cuddeback's methods of meter testing and repairing are still living and spreading through the younger men whom he trained.

During his many years with the New Jersey companies, Mr. Cuddeback did considerable work as a consulting water works engineer for other plants, and managed some outside companies including that at Massillon, Ohio.

In 1926 Mr. Cuddeback became vice-president and general manager of the Federal Water Service Corporation, a holding company formed at that time, and aided in the appraisal, purchase, and organization into state companies of more than 100 water companies in the states of New York, New Jersey, Pennsylvania, Ohio, Illinois, West Virginia, Alabama, California, Washington, and Oregon. The total value of these companies was more than $100,000,000 and the population supplied was about 2.5 million. Many of the companies were "run down" and the work of rehabilitating them was done under Mr. Cuddeback's general direction. In the four years previous to 1931, the work involved an expenditure of about $7,000,000 each year in extensions, repairs, and new works, and lesser but substantial sums in the following years of depression. A few years before his death he retired from the general managership of Federal Water Service Corporation but remained a director and president of the New York and New Jersey water service companies until a few months before his death, which was much hastened by an attack of pneumonia in the previous year. He died on Sunday, December 4, 1938.

Mr. Cuddeback was a member of the American Water Works Association and New England Water Works Association. He served as president of American Water Works Association from June, 1926, to June, 1927. He was a member of the Hamilton Club of Paterson, N. J., North Jersey Country Club, and Stanford Club.

Mr. Cuddeback had a gift for friendship, and his loyalty and broad human understanding and sympathy endeared him to all his associates. He was fond of watching the by-play of human actions and motives; he understood the strength and was tolerant of the weaknesses of his fellow men and women. At the same time he was firm and direct when action was required. Like many another good engineer, he was very fond of a game of poker with his friends and was a skilful player. It never troubled him if he lost; when he lost it was just a part of the expense of living; when he won he managed to spend his winnings promptly for the benefit of others.

---

[4] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1653.

[5] For memoir, *loc. cit.*, Vol. 87 (1924), p. 1337.

Surviving are his second wife, the former Edith A. Finley; two children of his first marriage, Louise B. and Thomas M. Cuddeback; and a sister, Mrs. F. Ketchum.

Mr. Cuddeback was elected an Associate Member of the American Society of Civil Engineers on April 5, 1899, and a Member on March 1, 1910.

JAMES EUGENE CURTIS, M. Am. Soc. C. E.[1]

DIED AUGUST 4, 1939

James Eugene Curtis was born in Washington, D. C., on October 11, 1883, the son of James M. and Florence A. (Turton) Curtis. His early education was obtained at the Emerson Institute of Washington, and he later attended George Washington University, in Washington.

The first engineering position held by Mr. Curtis was that of draftsman and surveyor with the Pressey and Weller Company of Washington on water works and building construction. In this position he acted as an assistant on design of water distribution systems for various southern cities and was employed on topographical surveying and building location.

In 1905 he entered the employ of the War Department in the Washington Aqueduct section of the U. S. Engineer Office, which has jurisdiction over the collection, purification, pumping, and transmission of water for the City of Washington. This was an important step in Mr. Curtis' life, as he remained with this office continuously until his death, a period of thirty-four years. During this time he was promoted successively from surveyman to senior engineer.

When Mr. Curtis entered the employ of the Washington Aqueduct the city was in the process of completing the construction of the large slow-sand filtration plant at the McMillan Reservation. Mr. Curtis worked directly under E. D. Hardy, the engineer in charge of the work, and was entrusted with the completion of the designs of many of the superstructures. He also participated in the design of a 10-in. suction dredge and the design and construction of an hydraulic sand-washer system for McMillan Reservation, which doubled the efficiency of operation.

In 1910 he was appointed junior engineer and worked on the design and construction for the Washington Aqueduct tunnels, installation of water meters of government services, and the construction of various improvements to McMillan Reservation grounds, including roads, walks, and courts.

---

[1] Memoir prepared by E. A. Schmitt, Esq., and P. O. Macqueen, M. Am. Soc. C. E.

In 1912 Mr. Curtis was appointed assistant superintendent of the Washington Aqueduct under Mr. Hardy, and in this position he had charge of a great many important improvements to the water supply system, including a cast-iron lining for Cabin John Aqueduct Bridge, the design and construction of the remodeling of Georgetown Reservoir, and the investigation of the Patuxent River as a source of water supply for the city.

In 1921 Mr. Curtis was appointed superintendent of operations for the Washington Aqueduct and in 1923 field superintendent supervising the construction of the increase of water supply work. This project included conduits, tunnels, covered reservoirs, pumping stations, transmission water lines, a new rapid sand filtration plant, a hydroelectric generating station, and various minor structures. The total cost of this construction work was approximately $9,000,000. In 1928 Mr. Curtis was appointed civil engineer, and in 1929 senior engineer. He continued in charge of the operations of the Washington Aqueduct and held this position at the time of his death on August 4, 1939.

Mr. Curtis was a man of quiet manners and sound judgment with a high degree of self-confidence. During the entire thirty-four years of his experience in connection with the Washington Aqueduct he had an excellent insight into water-works practices, and his knowledge of all the details of the various structures of the Washington Aqueduct system was of great value to the U. S. Engineer Office.

Mr. Curtis was married to Lillian Gayle on July 21, 1909. There were no children.

He was a member of the Blue Lodge, Consistory and Shrine Masonic Fraternity, and also held memberships in the American Water Works Association, the American Society of Military Engineers, and the Washington Society of Engineers.

Mr. Curtis was elected an Associate Member of the American Society of Civil Engineers on April 30, 1912, and a Member on April 12, 1926.

WALTER WHALEY CURTIS, M. Am. Soc. C. E.[1]

Died June 29, 1940

Walter Whaley Curtis, the son of John Decker and Sarah Catherine (Marston) Curtis, was born in Whaleysville, Md., on September 28, 1863. His early education was acquired in the public schools of Wilmington, Del.

In 1881 Mr. Curtis was employed by the Atchison, Topeka and Santa Fe Railway Company at Las Vegas, N. Mex. Later in the same year he was employed by the late Charles Frederick Loweth,[2] Past-President, Am. Soc. C. E., in the construction of two narrow gage railroads near Des Moines, Iowa.

---

[1] Memoir prepared by Frank O. Ray, Assoc. M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1488.

Upon the termination of this work he returned to the Atchison, Topeka and Santa Fe Railway Company at Topeka, Kans., under the division engineer on work which involved the location and construction of some branch lines and various structures.

In the fall of 1883 he was again employed by Mr. Loweth at St. Paul, Minn., and assisted in the design of bridges. He became greatly interested in this work but found that he did not have sufficient technical education, so he entered the Thayer School of Engineering at Dartmouth College, in Hanover, N. H., and received the degree of Civil Engineer in 1886.

After graduation he joined Mr. Loweth at St. Paul and remained with him until 1887 when he became resident engineer on the construction of the Atchison, Topeka and Santa Fe Railway bridge over the Mississippi River at Ft. Madison, Iowa.

In 1888 he returned to St. Paul where he continued the practice of engineering, designing, and supervising the construction of bridges, water works, and various engineering structures. Among these was one of the first elevated water tanks which has ever been constructed on a steel tower.

In 1891 he moved to Chicago, Ill., to represent the Pittsburg Bridge Company; and among many other structures he erected the Halsted Street Lift Bridge, which was the first high lift bridge ever built.

Severing his connection with the Pittsburg Bridge Company in 1896, he established an engineering and contracting business which he continued until 1913. During this time he investigated the results of railroad tie treatments and assisted in the extension of this industry.

Mr. Curtis made investments in coal lands in Colorado, and in 1913 he found it necessary to devote his entire time to these interests. In 1929 failing health forced his retirement.

He was a student and great reader of engineering literature, and kept up his interest in his profession long after he retired from active practice.

In 1887 he was married to Sarah Marston at Fairlee, Vt. Two sons, Robert C. Curtis and Steven G. Curtis, survive him.

Mr. Curtis was formerly a member of the Western Society of Engineers and of the American Railway Engineering Association.

Mr. Curtis was elected a Member of the American Society of Civil Engineers on September 5, 1888, and served as President of the Colorado Section in 1928–1929.

WILLIAM CHANNING CUSHING, M. Am. Soc. C. E.[1]

DIED JANUARY 11, 1940

William Channing Cushing was born in St. John, New Brunswick, Canada, on March 18, 1863 (of United States citizenship), the second son of

---

[1] Memoir prepared by Allston T. Cushing, M. Am. Soc. C. E.

George Byron and Annie Tilton (Scammell) Cushing. His father was one of the pioneer lumber manufacturers and merchants of St. John, having moved there from Winterport, Me., in 1853. In 1857 he joined his uncle, André Cushing, in the firm André Cushing and Company, in which firm he was a partner until his death in 1888.

William Cushing was educated in the schools of St. John and entered the University of New Brunswick at Fredericton, New Brunswick, in 1881, being graduated with the degree of Bachelor of Arts in 1884 with honors in the Classics, English, and French. He won the English Scholarship of 1883, and half of the French Prize of 1884, there being a tie with his competitor. His examinations entitled him to collegiate "Honors in Classics" in 1881–1882; "Honors in English," 1882–1883; and "Honors in French," 1883–1884. In 1886 he received the degree of Master of Arts from the same institution.

In the fall of 1884, having decided to make Civil Engineering his life work, he entered the Massachusetts Institute of Technology, in Boston, Mass. He was graduated in 1887 with the degree of Bachelor of Science in the Department of Civil Engineering, his graduation thesis being "The Fine Grinding of Cement," in partnership with W. S. Thompson, later division engineer of the Sunbury Division of the Pennsylvania Railroad. It was one of the two theses mentioned in the President's (Gen. Francis A. Walker) Annual Report for 1887 as a contribution to knowledge through research, and the results were used by the late George F. Swain,[2] Past-President and Hon. M. Am. Soc. C. E., then Hayward Professor of the Department of Civil Engineering, in a report to the Society in 1887. The other thesis mentioned by General Walker was that of the late J. Waldo Smith,[3] Hon. M. Am. Soc. C. E., the builder of New York's Catskill water supply system.

At the Commencement exercises at the University of New Brunswick in 1922 Mr. Cushing was the alumni orator and was awarded the honorary degree of Doctor of Laws.

Mr. Cushing left Boston the day after graduation, on May 31, 1887, for Louisville, Ky., where he was engaged for service by George W. Kittredge, Hon. M. Am. Soc. C. E., division engineer, as an assistant in the Maintenance of Way Department of the Louisville Division of the Pennsylvania Railroad System. He held this position until January, 1889, when he became engineer, Maintenance of Way, on the Cincinnati and Muskingum Valley Railway (later the Zanesville Branch of the Pittsburgh Division). From August, 1890, until May, 1894, Mr. Cushing was division engineer on the Indianapolis (later Columbus) Division. From 1894 until December 31, 1900, he held the same position on the Pittsburgh (later Panhandle) Division. During the year 1901 he was superintendent of the Pittsburgh Division. This was followed by a year as superintendent of the Eastern Division. Each of these changes was a promotion, as each division to which Mr. Cushing was assigned as engineer or superintendent was more important than that from which he was brought.

---

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1476.

[3] For memoir, *loc. cit.*, Vol. 101 (1936), p. 1502.

On January 1, 1903, he became chief engineer, Maintenance of Way, on the Southwest System, and on June 1, 1918, was appointed to a similar responsibility on the entire Pennsylvania Lines West of Pittsburgh.

In February, 1920, when the Pennsylvania Lines West of Pittsburgh was merged with the Pennsylvania Railroad Company, Mr. Cushing was transferred to Philadelphia, Pa., as engineer of standards of the Pennsylvania Railroad Company. He held this position until his retirement on June 1, 1932.

Mr. Cushing's life was one of brilliant accomplishment at all stages of his career. During his long service with the Pennsylvania Railroad he was in responsible charge of many important construction projects, such as enlarging and improving facilities by building additional main tracks, separating grades at street and railway crossings, rectifying alinement and grade, designing yards, constructing buildings, stations, and shops, and was engineer-physicist in the study and preparation of specifications for materials and appliances. In his position as division superintendent, he was in charge of all work performed on his division in construction, maintenance, and operation.

He was honored with appointment as a delegate representing the Pennsylvania Railroad Company at the following sessions of the International Railway Congress Association:

1905—Seventh Session, Washington, D. C.
1910—Eighth Session, Berne, Switzerland.
1922—Ninth Session, Rome, Italy, where he was Reporter for America (North and South) on Question III, "Special Steels."
1925—Tenth Session, London, England, where he was Reporter for America (North and South) on Question II, (A) Breaking of Rails and (B) Joints.

Mr. Cushing held membership in the following technical and scientific societies: American Railway Engineering Association, of which he was a charter member in 1889 and president in 1911–1912; Société des Ingénieurs Civils de France; The Institution of Civil Engineers (Great Britain); and The Franklin Institute, in Philadelphia. At the time of his death he was serving on the Committee on Science and Arts of The Franklin Institute.

Following is a list of the technical papers of which he was the author or translator, with the date when each work appeared:

1. "Method of Procedure in Remodeling Yards," *Railroad Gazette*, January 4, 1901.
2. "A New Yard Design," *Railroad Gazette*, March 13, 1903.
3. "Progress in Yard Design," *Railroad Gazette*, May 12, 1905; and *Proceedings*, American Railway Engineering Association, Vol. 7, 1906, and Vol. 9, 1908.
4. English Translation of "Étude sur les Déformations des Voies de Chemins de Fer et les Moyens d'y Rémédier," par M. G. Guënot, published by *Railroad Gazette*, 1907.
5. "The Life of Steel Ties," *Railroad Age Gazette*, June 5, 1908.
6. "The Question of Screw Fastenings to Secure Rails to Ties," *Proceedings*, American Railway Engineering Association, Vol. 10–2, 1909.

7. "Track Superstructure of German Railways," *Proceedings,* American Railway Engineering Association, Vol. 11–2, 1910.
8. "Protecting and Waterproofing Solid Floor Bridges," *Proceedings,* American Railway Engineering Association, Vol. 12–1, 1911.
9. "A Study of Forty Failed Rails," *Proceedings,* American Railway Engineering Association, Vol. 12–2, 1911.
10. "The Question of the Improvement of Rail Design and Specifications from 1893 to the Present Time," *Proceedings,* American Railway Engineering Association, Vol. 13, 1912.
11. "Some of the Causes of Rail Failures," *Proceedings,* American Railway Engineering Association, Vol. 17, 1916; and *Minutes of Proceedings,* The Institution of Civil Engineers (London), Vol. CCI, 1915–1916, Part 1 (awarded "Manby Premium").
12. "Examination of Rails Long in Service for Relation between Internal Stresses and Strains and Transverse Fissures," *Proceedings,* American Railway Engineering Association, Vol. 21, 1920.
13. "Special Steels," *Proceedings,* American Railway Engineering Association, Vol. 17, 1916; and International Railway Association, Ninth Congress, Rome, 1922.
14. "On the Question of Breaking of Rails; Joints" (Subject II for Discussion at the Tenth Session of the International Railway Congress Association); also *Bulletin* of the International Railway Congress Association, Vol. VI, No. 10, October, 1924.
15. "Rail and Wheel"—
    - Part I—"Review of Mathematical and Technical Analysis Applied to Rail Design" (prepared for London Session, 1925, of International Railway Congress Association).
    - Part II—"A Critical Survey of the Physical Requirements for Steel Rails."
    - Part III—"The Genesis of the Transverse Fissure" (American Railway Engineering Association *Bulletin No. 315,* March, 1929).
    - Part IV—"The Heat-Treated Rails in the Tracks" (American Railway Association *Bulletin No. 324,* February, 1930).
    - "Comparison Drawings of Standard Rail Sections," *Proceedings,* American Railway Engineering Association, Vol. 25, 1924, p. 409.

After Mr. Cushing was retired from active service with the Pennsylvania Railroad, he lived a quiet but by no means inactive life in Germantown, a suburb of Philadelphia, where he was doing continuous study and research with The Franklin Institute and other scientific agencies, all with a view to making the world a better place because of his having passed through.

Being very methodical in all that he did, much time in his later years was spent in arranging his personal affairs so that all would be in order when he should be called to leave.

Each year he made a trip to his birthplace, St. John, New Brunswick, to visit with relatives and friends. On many of these trips he visited some of the older family seats in and around Hingham, Mass., and at the time of his death was engaged in collecting data with a view to bringing up to date the genealogy of the Cushing family, the last edition of which was published in 1903.

During his entire life he never missed an opportunity to help a younger engineer or to give credit to an older man who had assisted him. The writer's father was a civil engineer in Canada, and only a few years ago Mr. Cushing produced some papers showing how the writer's father had been helpful to him in his early years. The writer is indebted to Mr. Cushing for obtaining for him his first position in engineering in the United States, when his efforts resulted in the writer's appointment as draftsman and checker on bridges for the Pennsylvania Lines West of Pittsburgh in 1913.

William Cushing was married at Indianapolis, Ind., on February 14, 1894, to Marie Bacon Dye, daughter of John T. and Annie (Glenn) Dye of Indianapolis. He is survived by his only daughter, Annie Theodosia (Mrs. Thomas P. Haviland), and a sister, Emma Helen (Mrs. Alexander Wilson).

A fitting closing for this very inadequate narrative is a sentence from a letter addressed by Mr. Cushing's daughter to the writer on January 29, 1940. Referring to her father, she writes: "He was the most unselfish man that ever lived, and the most wonderful father to me."

Mr. Cushing was elected a Junior of the American Society of Civil Engineers on June 5, 1889; an Associate Member on November 4, 1891; and a Member on May 1, 1901.

CORNELIUS MARK DAILY, M. Am. Soc. C. E.[1]

DIED NOVEMBER 25, 1938

Cornelius Mark Daily, the son of John and Mary Daily, was born on February 23, 1876, in Rolla, Mo. He attended a country school near Rolla and entered the Missouri School of Mines and Metallurgy at Rolla, in 1898, from which he received the degree of Bachelor of Science in General Science in 1902, and the degree of Bachelor of Science in Civil Engineering in 1903.

He served with the U. S. Engineers from 1903 to 1905, first as a surveyman and then as junior engineer on river improvement on both the Mississippi and Missouri rivers. After a brief term as transitman with the St. Louis and North Arkansas Railway Company, he became resident engineer in charge of the installation of machinery in a phosphate mill in Tennessee. After this work was completed, he became associated as assistant engineer with Edward Flad and Company, consulting engineers of St. Louis, Mo., and for the next three years was engaged in the design and construction of such structures as railway yards, shops, retaining walls, water works, and drainage systems.

In 1909 he severed his connection with Edward Flad and Company, and, with Mr. Beckelheimer, formed the firm of Daily and Beckelheimer, consulting engineers, with headquarters in Carmi, Ill. During the next three years he followed a general consulting practice, embracing special investigations and reports, the supervision of a paving project, and the preparation of numerous plans and specifications for roads, drainage districts, and bridges.

---

[1] Memoir prepared by Francis T. Cutts, M. Am. Soc. C. E.

In April, 1911, the firm of Daily and Beckelheimer was dissolved, and Mr. Daily became associated with Pitzman's Company of Surveyors and Engineers as engineer in charge of the construction of a railway incline and protective dike work on the Mississippi River. Upon the completion of this project in January, 1913, he returned to Edward Flad and Company on a special investigation and preliminary plans of slime disposal works for the lead producing companies in the vicinity of Flat River, Mo. In April, 1913, he was appointed assistant engineer, Supply and Purifying Section, St. Louis Water Division, and was engaged in the design of the head-house and other structures that were to be constructed in connection with the Chain of Rocks (Mo.) Filter Plant. On the completion of these designs in November, 1913, Mr. Daily again became associated with Edward Flad and Company in charge of an investigation, survey, and plans for a new water supply for the city of Little Rock, Ark.

In March, 1914, Mr. Daily resumed his position with the Supply and Purifying Section of the St. Louis Water Department as assistant engineer, and was in charge of the operations of the filter plant at Chain of Rocks during the first months of its operation. In 1917 he was made engineer in charge of the Supply and Purifying Section of the St. Louis Water Works.

As engineer in charge of the St. Louis Water Works, Mr. Daily was responsible not only for the operation and maintenance of the supply and purifying division, but also for the design and construction supervision of the new buildings and other structures in connection with the building of the new St. Louis Water Works plant at Howard Bend on the Missouri River. In 1925 he resigned to engage in a general engineering and contracting business, and continued in this business until April, 1932, when he was offered, and accepted, the appointment as water commissioner for the City of St. Louis.

Mr. Daily's earlier professional training and his previous connections with the St. Louis Water Works well qualified him for the position, which he filled with great credit to his profession. In the fall of 1935 he resigned as water commissioner to become identified with the construction industry again, and from that time until his death he was engaged in the construction of many important projects.

Mr. Daily died suddenly in Jacksonville, Ill., on November 25, 1938. He was recognized as an authority in the design and construction of water works. He was a member of the Engineers' Club of St. Louis and the American Water Works Association, and contributed frequently to various technical publications.

On June 19, 1901, Mr. Daily was married to Margaret O. Miller, of St. Charles, Mo., who, with one daughter, Marian Dorothy (Mrs. P. J. Hill), and one son, Austin E. Daily, survives him.

He lived a useful life, and was untiring in his efforts. His loyalty, honesty, and ability were well recognized by all who knew him.

Mr. Daily was elected a Member of the American Society of Civil Engineers on September 11, 1917.

JOHN VIPOND DAVIES, M. Am. Soc. C. E.[1]

DIED OCTOBER 4, 1939

John Vipond Davies was born at Swansea, Wales, on October 13, 1862, the son of Andrew and Emily (Vipond) Davies. He received his education at Wesleyan College, Taunton, England, and at the University of London, London, England.

He worked in Wales as an engineer in the British coal mines and in the manufacture of steel until 1889, when he came to New York, N. Y., on the invitation of the late Charles M. Jacobs,[2] M. Am. Soc. C. E., with whom he later formed the partnership of Jacobs and Davies. In this earlier association Mr. Davies was employed by the Long Island Railroad Company, then under the presidency of Austin Corbin, when Mr. Jacobs was evolving some of the first proposals to connect this railroad with the Pennsylvania Railroad at Jersey City, N. J.

Mr. Davies acted as chief assistant engineer to Mr. Jacobs during the construction of the East River Gas Company tunnel under the East River and Blackwells Island, the first tunnel to pass under any of the harbors of the port of New York.

After the completion of the tunnel in 1893 Mr. Davies and Mr. Jacobs formed the partnership of Jacobs and Davies. Their first assignment was the construction of a pumping plant for the Montauk (N. Y.) Water Company capable of delivering 2,000,000 gal of water per day from artesian wells to the Long Island Railroad.

During the following few years Mr. Davies devoted most of his time to railroad construction. He was chief engineer in entire charge of design and construction of the West Virginia Short Line, the Kanawha and Pocahontas Railroad, and the Atlantic Avenue improvement of the Long Island Railroad, which included 2½ miles of subway and 3 miles of viaduct with the terminal facilities. In 1901 the firm made additional studies when plans were revived to extend the Pennsylvania Railroad into New York City and beyond to Long Island.

In 1902 the firm undertook the project of completing what was later the north uptown tunnel (partly built by De Witt Haskins and also by Pearson and Sons) of the Hudson and Manhattan Railroad and "holed" it through in 1904, completing the first tunnel under the Hudson River at New York. This tunnel was the nucleus of what eventually became the extensive interurban transit system of the Hudson and Manhattan Railroad, with its four tunnels under the Hudson River connecting the main railroads of New Jersey with downtown and uptown New York.

In October, 1909, the firm of Jacobs and Davies was incorporated and continued the extensions of the Hudson and Manhattan Railroad to Newark,

---

[1] Memoir prepared by James Forgie, M. Am. Soc. C. E., and collaborators.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–1920), p. 2236.

N. J., and 33d Street, New York, under the supervision of Mr. Davies as chief engineer. He then became president of his firm, and later chief engineer, vice-president, and a director of the Hudson and Manhattan Railroad Company.

Subsequent to 1909 the firm executed much of the design and construction by its own administration of the following major projects: Hales Bar Dam on the Tennessee River about 20 miles south of Chattanooga, Tenn.; the 20 miles of tunnels, canalage, and intakes of the Laxaxalpam Aqueduct feeding the electric power plant of Necaxa of the Mexican Light and Power Company, about 100 miles northeast of Mexico City, Mexico; the large and hazardous tunnel construction of the Consolidated Gas Company of New York under Hell Gate from its plant in Astoria, Long Island City, N. Y., to 135th Street, Bronx County; also engineering supervision of contract work of the terminal improvement work of the New York Central Railroad Company during the excavation of the site of the Grand Central Station.

Mr. Davies was engaged as consulting engineer for the Brooklyn Rapid Transit Company; the water supply tunnel under the Detroit River for the City of Detroit, Mich.; acted as a member of the board of engineers on the construction of the Moffat Tunnel in the Rocky Mountains near Denver, Colo.; executed construction of the power station intake and discharge tunnels of the New York Edison Company; member of the board of consulting engineers in study and report on a bridge or tunnel crossing the Mississippi River at New Orleans, La.; the New York State Bridge and Tunnel Commission and the New Jersey Interstate Bridge and Tunnel Commission in preliminary studies for a Hudson River highway crossing, which eventually became the Holland Tunnel; he served as a member of the board of consulting engineers during construction—in all, this association covered the period from 1912 to 1922. He made collaborative studies and a report on the proposal for a bridge or tunnel across San Francisco Bay, from San Francisco to Oakland.

John Vipond Davies was a Charter Member of American Institute of Consulting Engineers, and one of the early group of engineers, led by the late Alfred P. Boller,[3] M. Am. Soc. C. E., responsible for the initial organization of the Institute, and the influential and recognized standing of the organization several years in advance of its legal incorporation in 1913.

Mr. Davies was a member of The Institution of Civil Engineers, the American Institute of Mining and Metallurgical Engineers, and president of United Engineering Societies from 1920 to 1923.

He was particularly interested in a Code of Ethics, which should apply in the engineering profession, and a schedule of fees for engineering work. From 1921 until his death he was a member of the committee on Professional Practice and Ethics of American Institute of Consulting Engineers. His advice on this committee was greatly appreciated. In 1929 he served as chairman of a special Committee appointed by the American Society of Civil Engineers on "Charges and Methods of Making Charges for Professional Services." This committee made an elaborate report which was approved by the Board of Direction on July 9, 1939.

---

[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1653.

In 1914 he was awarded the Telford Gold Medal of The Institution of Civil Engineers; the Norman Gold Medal, in 1913, and Thomas Fitch Rowland Prize, in 1917, of the Society; also the Fowler Professional Award in 1930.

Socially he was president of the St. George's Society of New York for the year 1922–1923; president of the Oakland Golf Club, 1914–1919; member of the Pilgrims of the United States, the Engineers' Club, and Railroad Club of New York, of which he was for many years a member of the Board of Governors.

In 1895 he was married to Ruth Ramsey of Pottsville, Pa., who died in 1931. He is survived by two daughters, Margaret (Mrs. C. Olden Davis) and Muriel (Mrs. Malcolm Mackenzie), and a son, J. Vipond Davies, Jr.

Mr. Davies was a man of great energy and courage and risked his life to stop a team of runaway horses from trampling small children. He was dragged for some distance and badly bruised, with the result that he was confined to bed for several weeks.

He was deeply loved and respected by the executive staff and employees of the Hudson and Manhattan Railroad Company, with which his association covered a period of 37 years, and by many others who worked with him or under his direction.

Mr. Davies was elected a Member of the American Society of Civil Engineers on June 6, 1894.

DANIEL YOUNG DIMON, M. Am. Soc. C. E.[1]

Died July 8, 1939

Daniel Young Dimon was born at Riverhead, N. Y., on March 22, 1874. He was the son of John M. and Adelia (Young) Dimon. The Dimon family were early settlers in Riverhead, and Daniel's father was a prosperous farmer during the son's formative years.

Daniel's early life was spent in Riverhead where he was educated in the local public schools. He was graduated from Riverhead High School in 1892. He then spent four years at Cornell University, at Ithaca, N. Y., and was graduated *magna cum laude* with the degree of Civil Engineer in June, 1896.

Immediately following graduation he entered the employ of the Groton (N. Y.) Bridge Company where he had four years of practical formative training in detail drafting, shop and field inspection, and estimating and designing work. Thereafter he followed the career of a structural engineer.

From June, 1900, to September, 1903, he was employed by the American Bridge Company in its Designing Department at Pencoyd, Pa., where he had responsible charge of designing and estimating steel work of highway and railway bridges, buildings, cranes, etc. Subsequently he was employed in the

---

[1] Memoir prepared by Joseph H. O'Brien and Fenwick F. Skinner, Members, Am. Soc. C. E., and Lorenzo C. Dilks, Assoc. M. Am. Soc. C. E.

Designing Department of Milliken Brothers, in New York, N. Y., in charge of design of steel bridge work and steel work of mills and office buildings.

From October, 1904, until 1909 he was employed by Westinghouse, Church, Kerr and Company as office engineer in charge of design under the resident civil engineer of Pennsylvania Terminal improvements in New York City. His work included the design of foundations for the Terminal Station building and surrounding street viaducts, East River tunnel approaches, etc., and the design of all substructures including baggage tunnels, elevator pits, pipe and wire tunnels, underground duct systems, drainage system, etc.

Mr. Dimon then became associated with the Eastern Steel Company as assistant engineer until 1915. He was employed as structural engineer by Milliken Brothers from 1915 to 1917, by the Downey Shipbuilding Corporation from 1917 to July 15, 1918, and by the Foster Wheeler Corporation thereafter until March 26, 1921, when failing eyesight caused him to abandon pursuit of his profession for about two years. He was again employed by Foster Wheeler Corporation in the spring of 1923 and subsequently for two years by the J. G. White Engineering Corporation. This engagement was followed by association with John K. Turton and Company for a period of one year, and association with George A. Fuller Construction Corporation for the period from 1926 through 1929.

From February, 1930, to June, 1938, he was employed by Gibbs and Hill, Inc., as structural engineer in responsible charge of the design of structural steel work for the New York Post Office building on the Pennsylvania Terminal site between Ninth and Tenth Avenues. He was also in charge of structural work connected with important changes made in the 31st Street viaduct and in supports of the original Post Office building at the Terminal site incidental to changes in track layout. He also collaborated in the design of certain important structures in connection with electrification of the Pennsylvania Railroad in the New York zone, and had charge of the design of structural steel for the superstructure of Newark Terminal station improvements for the Pennsylvania Railroad Company, and the structural steel work of certain important power plants for the New York, New Haven and Hartford Railroad Company.

He was employed by the Board of Water Supply for the City of New York from June 1, 1938, until his death.

Mr. Dimon was a brother of the late Henry Dimon, a widely known member of the staff of the American Bridge Company at the time of its formation.

Mr. Dimon, who was highly esteemed by his associates, was a structural engineer of rare ability. He was conscientious, loyal, well poised, and possessed of a real sense of humor; he was ever careful, painstaking, and strong in his determination to perform the work assigned to him in accordance with the best engineering practice. He served his profession well and honorably.

On August 24, 1897, Mr. Dimon was married to Huldah Ann Taft of Riverhead who survives him, as do their three children, Daniel Young Dimon, Jr., John T. Dimon, and Margaret Dimon (Mrs. Benjamin Fisher).

Mr. Dimon was elected an Associate Member of the American Society of Civil Engineers on October 2, 1901, and a Member on March 31, 1908.

WALTER ALEXANDER DOANE, M. Am. Soc. C. E.[1]

Died January 28, 1940

At the time of his death only five living men had been members of the Society, in any grade, longer than Walter A. Doane. He was the son of Isaac S. and Elizabeth (Morse) Doane, and was born in Ellicottville, N. Y., on September 17, 1854.

When he was 10 years old the family settled in East Mead Township, near Wayland. He was educated in the public schools, and then attended for a period a civil engineering college in Connecticut. At the age of 17 young Doane started to work for his father (some time city engineer of Meadville, Pa.) on the Lake Shore Railroad, later known as The Rome, Watertown and Ogdensburg Railroad, and finally a part of the New York Central Railroad system. Mr. Doane was a draftsman and assistant engineer on location from April, 1871, to December, 1873, at which time he became a division engineer in charge of construction. He held this position until September, 1876. It is interesting to recall that, at this time, the late James Ross, M. Am. Soc. C. E., a young Scot, obtained his first job in America on the staff of Walter A. Doane's father and took a liking to young Doane. Mr. Ross proved a remarkably good judge of men, as well as capable in his own right—so much so, that, when he died in 1913, he was reputed to have left an estate of 14 million dollars.

In March, 1878, Mr. Doane accepted the position of chief engineer of the Lehigh and Eastern Railway Company, which position he held until August, 1880, running preliminary lines and location. On August 1, 1880, he started work as principal assistant engineer of the Meadville-Conneaut Lake and Linesville Railroad Company (under his brother Alonzo Doane), which position he held until October, 1880, when he was promoted to the position of chief engineer. The Meadville-Conneaut Lake and Linesville Railroad was completed in 1881 or 1882, permitting Mr. Doane to accept a position as chief of party on the location, in Kansas, of the Santa Fe Railway.

Early in 1883 James Ross, who had just been appointed manager of construction of that section of the Canadian Pacific Railway from Maple Creek, Saskatchewan, some 90 miles east of Medicine Hat, Alberta, to, and through, the Rocky Mountains and over the Selkirk Mountains in British Columbia, sent for Mr. Doane and installed him as his bridge engineer on the Rocky Mountain section. The first office, in May, 1883, was in a tent at Medicine Hat; in August quarters were moved to Calgary, Alberta. On Mr. Doane's staff, from the start, were P. Turner Bone and the writer.

In 1884–1885 Mr. Doane's office was at Laggan, Alberta, the summit of the pass through the Rocky Mountains. In 1884 George Herrick Duggan (later Hon. M. Am. Soc. C. E., and chairman of The Dominion Bridge Company at Montreal, Quebec) joined his staff. The bridges were all of timber, cut and squared at the sites. In those days no steel was available. The only iron used consisted of bolts and rods, with cast-iron angle blocks for the Howe trusses; these materials came from Chicago, Ill.

---

[1] Memoir prepared by T. Kennard Thomson, M. Am. Soc. C. E.

It is doubtful if any engineer had a greater knowledge of the design and construction of wooden bridges than Mr. Doane. Among the structures of this type that he built for the Canadian Pacific Railroad were: The first one over the South Saskatchewan River at Medicine Hat; one over the Bow River at Calgary; many over the Kicking Horse and other rivers in the Rocky Mountain and Selkirk Mountain ranges, including one over Stony Creek (probably the highest wooden trestle, with Howe trusses, in the world); Mountain Creek trestle, a very long and fairly high structure; and Surprise Creek, Cedar Creek, and many other bridges. The notes made by the writer at the time, showing most of these designs, have been presented to the Society. Of course, all of these wooden bridges which had been built in advance of the railroad track were replaced by iron, in time, and these in turn were replaced by steel structures, as the traffic, engines, and cars became heavier and demanded stronger bridges.

On the completion of the Canadian Pacific Railway from Montreal to the Pacific Ocean, in November, 1885, it became the first complete transcontinental road in America; in fact there is still none in the United States under one management. His work finished, Mr. Doane then accepted a position on the Oregon, Washington Railroad and Navigation Company. He acquired the habit here, or possibly earlier, of buying real estate as he moved from place to place. From the start, however, he wisely realized that it does not pay to hold undeveloped land, so his plan was to erect buildings at once which he could rent.

From January, 1890, to the end of 1893 Mr. Doane was principal assistant engineer under the late W. W. Coe,[2] M. Am. Soc. C. E., for the location and construction of the Ohio Extension (to the Ohio River at Kenova, W. Va.) of the Norfolk and Western Railway, later a part of the Pennsylvania Railroad system. The town of Kenova took its spelling from parts of the three surrounding states' names—Kentucky, Ohio, and West Virginia. The Ohio Extension of the Norfolk and Western Railway, built for heavy coal traffic, was on a very difficult location, crossing and recrossing rivers (especially the Big Sandy) many times. It required sharp (12°) curves and steep grades. It is interesting to note that, some 30 or more years later, an up-to-date electric railroad was not able to find a much, if any, better location.

On this line there were 129 iron bridges. Steel was not yet accepted, and for a long time was not considered as safe as iron except for such important members as eyebars. Among these bridges was a large one over the Ohio River at Kenova, with four spans of 300 ft each, one of 520 ft, and a long viaduct approach in Kenova. At this site the low-water depth was 6 ft, and the high water depth, 66 ft. Several severe floods occurred during construction with very little advance warning, so great pains were taken for rapid erection. As a result, the iron was erected so that the falseworks for the 300-ft spans could be removed in three and a half days, and for the 520-ft span in fifteen days—which was quite a record for that time. The bridge was reconstructed[3] for double track in 1912–1913. The Kenova Bridge (Ohio Extension, Norfolk and Western Railway) iron work was designed originally for single track, but the trusses were spaced far enough apart to permit inserting a middle truss later. The foundations and piers also were designed for the future, but when double tracking was decided on, all of the trusses were replaced, using the old piers as originally built.

---

[2] For memoir, see *Transactions*, Vol. LXXXIII (1919–1920), p. 2175.

Although he was a great authority on the design and construction of wooden bridges, Mr. Doane modestly did not claim to be a designer of iron bridges; so he again called on his former associate, the writer, who was then with the Pencoyd Bridge Company, near Philadelphia, Pa., and honored him by an appointment as bridge engineer for the Ohio Extension of the Norfolk and Western Railway. The office, for a short time early in January, 1890, was at the Roanoke, W. Va., headquarters of the Norfolk and Western Railway; then it was moved to a new building in Kenova, the town being merely an undeveloped site extending from Ceredo, W. Va., to the Big Sandy River.

This work demonstrated, as did all of his other work, one of Mr. Doane's sterling characteristics—that he was always courteous and just in his treatment of contractors; and although he was as lenient as he could be, he always insisted on, and obtained, first-class work for his clients. In the early eighties most engineers thought it necessary to swear at contractors, and similarly the contractors did not think that they could manage their men otherwise; but Walter A. Doane always succeeded without the use of profanity. He did not smoke or even drink tea or coffee (another very unusual trait for an engineer in those days); yet he had an active life for 85 years.

From 1893 to 1905 Mr. Doane was city engineer for the City of Meadville, as his father had been before him. He also "acted as city engineer" for several years, in the same city, at a later period. Following this work, in 1905–1907, Mr. Doane was principal assistant engineer in charge of construction of the Carolina, Clinchfield and Ohio Railway.

For the thirty-three remaining years of his life, Mr. Doane maintained his office in Meadville under the name of Walter A. Doane and Son, Consulting and Practicing Engineers. As his son, Arthur W., carries on the business with its valuable records, he perpetuates three generations of engineers by the name of Doane in the city of Meadville.

For more than half a century Mr. Doane honored the writer with several letters a year, the last of which was written on January 16, 1940, when he wrote that he was feeling well and active at 85. He added that he was still using, to prick holes in drawing paper instead of making a pencil point, a piece of white (Rocky Mountain) pine in which had been inserted a needle before it had been shaved down to the size of a pencil. He recalled that the writer had made this for him 57 years before, while on the Canadian work.

---

[3] For description of reconstruction, see *Transactions*, Am. Soc. C. E., Vol. LXXIX (1915), p. 411.

On January 20, 1940, Mr. Doane suffered what was considered to be a slight stroke, from which, in spite of his hopeful optimism, he failed to recover. He died on January 28, 1940.

He was a life member of The Renovation Lodge, Free and Accepted Masons of Albion, N. Y., and belonged to the Meadville Grange, in which he was very active. He was, as in everything else, a faithful member of the Stone Methodist Church, in Meadville, and of the Men's Bible Class. Mr. Doane wrote and published two delightful books of poems, relating to human nature, to his friends, to his family, and to the surrounding country. His writings are evidence of the kindly, home loving, and devoted nature of a sterling man. He wrote about the small intimate things around him, giving them the importance that they deserved. He was proud of the history and the traditions of his home community, which are enriched by these poems.

In physique, he was wiry, strong, and active. As to character, he was absolutely reliable in everything he said or did, true and devoted to his family, his friends, his country, and his church, as well as to the engineering profession and the Society of which he was so proud. Everything Walter A. Doane ever did was well and carefully done.

On May 11, 1882, he was married to Harriet Louise Ellis of West Mead Township, Pa., who was his devoted wife for forty-four years. She died in 1926. Mr. Doane is survived by their four children, Ethel Doane, Morse E. Doane, Arthur W. Doane, and Norman D. Doane. He is also survived by a brother, Dr. L. L. Doane; a sister, Mrs. Mae D. Washburn; a grandchild; and several nieces and nephews.

It is a great achievement to have lived a long and useful life and at the end of 85 years to be esteemed by all as among the foremost of worth-while and respected citizens. Walter A. Doane deserves, as few others have, the epitaph:

> "The memory of a life nobly rendered is immortal."

Mr. Doane was elected a Member of the American Society of Civil Engineers on September 7, 1881. He remained a loyal and enthusiastic member for the remainder of his life.

GUY CARLETON EMERSON, M. Am. Soc. C. E.[1]

Died July 17, 1939

Guy C. Emerson was born on March 12, 1867, at Orland, Me. He was a descendant of Michael Emerson, who was an early settler of Haverhill, Mass.

After graduation from the Eastern Maine Conference Seminary at Bucksport, he prepared at Chauncy Hall School in Boston, Mass., for Massachusetts Institute of Technology (M. I. T.), at Cambridge, Mass. He entered M. I. T.

---

[1] Memoir prepared by Charles W. Sherman and Charles R. Gow, Members, Am. Soc. C. E.

in 1886 and remained for three years, taking the course in civil engineering.

In 1889 the construction of the (Boston) Metropolitan Sewerage System was begun, with the late Howard A. Carson,[2] Hon. M. Am. Soc. C. E., as chief engineer, and Mr. Emerson was given work on the engineering staff. He found this more to his liking than the completion of his course at M. I. T., and remained on the sewerage work as rodman, assistant, transitman, inspector, and assistant engineer, until the original works were substantially completed in 1895. At that time Mr. Carson became chief engineer of the Boston Transit Commission, charged with the construction of the Boston subways—the first structures of their kind in America. Several of the assistant engineers who had been employed on the Metropolitan Sewers accompanied Mr. Carson, among them Mr. Emerson, who became assistant engineer in charge of inspection. Throughout the construction of the original subway he had complete charge of all construction matters in the field. Since the work was the first of its kind ever done in the United States, many of the construction problems were unique and unprecedented, and to Mr. Emerson belonged much of the credit for working out the various methods of procedure adopted in prosecuting the work.

On the completion of the original subway in 1898, the engineering staff was disbanded, and he entered into a partnership with the late E. A. Clark, with the idea of conducting a contracting business. This partnership continued for about two years, but did not obtain any contracts of significance.

In 1900 Mr. Emerson was appointed superintendent of sewers of the City of Boston. At that time all the public works activities of the city were under the control of a superintendent of streets; the late B. T. Wheeler,[3] M. Am. Soc. C. E., held the office at that time, and Mr. Emerson was in charge of the Sewer Division under Mr. Wheeler's general direction. He was designated as deputy superintendent of streets, and for a few months was acting superintendent of streets.

Following a change in administration in 1902, he was replaced, but soon afterward became chief engineer of the Montreal (Quebec, Canada) Subway Company and had charge of surveys, plans, and specifications for tunnels under the St. Lawrence River and subways in city streets. This work lasted about a year, after which he returned to Boston and maintained an office as consulting engineer for about two years.

In August, 1906, he became construction engineer for the Bureau of Reclamation, in charge of one of the original units of the Shoshone Project, located at Powell, Wyo. The work in his charge was the so-called Corbett tunnel, 3½ miles long. In addition to the engineering work, it became necessary for him to act as police chief, postmaster, town clerk, and town treasurer. It has been said that his official connection with the Bureau of Reclamation in its early days was a contributing factor in the necessary pioneer work preliminary to the later extensive program. The "town" was essentially a construction camp, and at that time the necessity of adequate sanitary precautions in such places was not generally recognized. As a result, typhoid fever became prevalent in the camp, and Mr. Emerson was stricken with it shortly before the completion of the job. He resigned his position in December, 1907, and returned to Boston.

---

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1386.

[3] For memoir, *loc. cit.*, Vol. 93 (1929), p. 1919.

After recovering his health, he was appointed in 1908 as superintendent of streets of Boston, but after a year or two a change of administration resulted in his giving up this position. Soon thereafter he was appointed as consulting engineer to the Boston Finance Commission which, under a new city charter, had then been established on a permanent basis, in succession to a temporary body having the same title, which had just completed a 2-yr investigation of the operations of various city departments. This work did not require all his time, and he was able to take other employment. During the next few years he had some other professional engagements, and from time to time he was consulted by other municipalities seeking advice with respect to construction methods and policies. Perhaps the best known of his engagements, other than for the City of Boston, was as engineer of a special commission appointed by the Massachusetts Legislature in 1912 to investigate the water supply needs of certain cities, especially Salem and Beverly. He continued in the service of the Finance Commission until his retirement in 1936.

Mr. Emerson's engineering mind was distinctly of the constructive and creative type rather than of a theoretical character; that is, he was inclined to interest himself in the creative aspects of engineering rather than in analytical and research investigations, and it was in this field that he met his greatest degree of success.

He was a member of the Boston Society of Civil Engineers and the New England Water Works Association. He was interested in the early history of Boston, and was a life member of the Bostonian Society. He was also a member of the Boston City Club.

He is survived by his wife, Perpetia Lillian (Talbot) Emerson.

Mr. Emerson was elected a Member of the American Society of Civil Engineers on May 4, 1904.

JOHN FARRIS, M. Am. Soc. C. E.[1]

DIED APRIL 15, 1939

John Farris was the son of William and Susan (Nixon) Farris, who came to America from Ireland when they were children. He was born in Fennimore, Wis., where he spent his boyhood and received his early education, graduating from the Fennimore High School. In 1901 he was graduated from the Platteville (Wis.) State Normal College.

---

[1] Memoir prepared by Arch W. Nance and Carl J. Jacobsen, Associate Members, Am. Soc. C. E.

His first position was as instructor in science and mathematics in the high schools of Monroe, Wis. In 1905 he entered the University of Wisconsin, at Madison, Wis., and taught in the Wisconsin Academy until his graduation from the University in 1907.

He then joined an older brother as a partner in the Farris Bridge Company, with offices in Pittsburgh, Pa., and Charleston, W. Va. The firm engaged in bridge construction in Pennsylvania, West Virginia, Maryland, and Kentucky.

During 1908 and 1909, Mr. Farris attended the University of Pittsburgh, and received the degree of Civil Engineer. While at the University he designed a set of standards for steel highway bridges varying (in 5-ft lengths) from 40 ft to 200 ft, with both varying loadings and widths of roadways.

In 1915, he became president of the Farris Engineering Company, which company has been active, continuously, in bridge construction in the Tri-State territory (Pennsylvania-Maryland-West Virginia). Also, since 1927, he was president of the Mausoleum Construction Company, builders and owners of Mount Royal Memorial Mausoleum, the first and largest structure of its type and purpose to be offered for above-ground entombment in the Pittsburgh district.

He believed in, and worked diligently for, due recognition of the engineer as a public servant, was a past-president of his local section of the American Society of Civil Engineers and a member of the Pennsylvania Society of Professional Engineers. At the time of his death, he was serving his third year as a member of the State Board for Registration of Professional Engineers of Pennsylvania, and his second year on the Board of Governors of the Constructors Association of Western Pennsylvania.

During his extremely active career, Mr. Farris received consultant appointments on bridges and buildings under three state administrations; and at the time of his death he was engineer for the General State Authority on the Western State Psychiatric Hospital in Pittsburgh, completed in 1939.

He was always loyal to his native state and its university, was twice president of the University of Wisconsin Alumni Club of Pittsburgh and represented the University in local activities. Since the death of his father in 1922, he had taken over the old family homestead near Fennimore and operated a modern dairy farm with a herd of registered Jerseys of which he was justly proud.

Fraternal life was also an active field for his talents. He was a Thirty-Second Degree Scottish Rite Mason, a Knight Templar, and a Shriner; he served seven years on the Divan of Syria Temple, Pittsburgh, and was elected Potentate in 1937.

Possessed of a keen sense of humor and ready wit, he made work and play boon companions during his busy life.

Mr. Farris was elected an Associate Member of the American Society of Civil Engineers on August 31, 1915, and a Member on May 12, 1919.

HOWELL TRACY FISHER, M. Am. Soc. C. E.[1]

DIED NOVEMBER 6, 1938

Howell Tracy Fisher was born on December 10, 1868, in Pottsville, Pa., the son of Howell Fisher and Charlotte Anne (Sheafe) Fisher. He attended private and public schools and was graduated from the Pottsville High School.

From 1886 to 1888 he was a rodman, draftsman, and transitman for the Anthracite Geological Survey of Pennsylvania, and his name is upon a great many of the sheets of the map of the anthracite coal fields. In 1888 and 1889 he served in the engineering department of the Susquehanna Coal Company at Wilkes-Barre, Pa., and from 1889 to 1891 he was city surveyor of Pottsville. From 1891 to 1893 he was principal assistant to A. D. W. Smith, geologist and mining engineer in Philadelphia, Pa., engaged in the preparation of reports for the Anthracite Coal Waste Commission and the compilation of a geological map of Pennsylvania. From 1893 to 1898 he was employed by A. B. Cochran and Son, civil and mining engineers and borough surveyors of Pottsville.

In June, 1898, he took a position with the U. S. Board of Engineers and worked on estimates and surveys for a canal for ships from the Great Lakes to the Atlantic Ocean, on surveys in the Mohawk Valley, New York, and on reports in Detroit, Mich. He became associated with the Isthmian Canal Commission in July, 1900, and was occupied, in Washington, D. C., on drawings, maps, and estimates of cost of the Panama and Nicaragua canal routes. From January, 1902, to August, 1902, he was assistant engineer of the Denver Union Water Company on construction of Cheeseman Dam, in Colorado, then the highest stone arch dam in the world. In August, 1902, he became irrigation engineer of the U. S. Geological Survey in charge of reclamation service of the Grand River Canal Project and cadastral surveys in the Gunnsom Project, Uncompahgre Valley, Colorado.

Mr. Fisher became assistant engineer of the East River Division, Pennsylvania, New York and Long Island Railroad Company, and the Pennsylvania Tunnel and Terminal Railroad Company, in January, 1903, and was engaged on the construction of tunnels under Long Island City, N. Y., and the East River, New York, N. Y. In February, 1909, he became assistant to Henry F. Dose, M. Am. Soc. C. E., in Washington, D. C., on plans and estimates for a railroad in Guatemala, and during 1910 and 1911 he was employed by the Delaware, Lackawanna, and Western Railroad Company on the construction of a cutoff line in New Jersey. He was appointed tunnel engineer with Mackenzie, Mann and Company, Ltd., and the Montreal Tunnel and Terminal Company, Ltd., in April, 1912, and worked on the design and construction of 3⅛ miles of tunnel under Mount Royal for the Canadian Northern Railway at Montreal, Quebec, Canada.

---

[1] Memoir prepared by J. Howell Healy, Esq., Philadelphia, Pa.

From 1915 to 1917 Mr. Fisher was in private engineering practice in New York City, but in the spring of 1917 he took the position of resident engineer with the Federal Ship Building Company and superintended the construction of steel ships on the Hackensack River. After completion of the shipyard work, Mr. Fisher was employed by Fraser, Brace and Company in New York City until the spring of 1919, when he retired.

After his retirement from engineering practice, Mr. Fisher became deeply interested in art. He took up painting and without any previous training or experience did very creditable work in oils and water colors. In recent years he had specialized in miniatures, and for the two years prior to his death he was secretary of the Pennsylvania Miniature Society. Mr. Fisher was also a member of the Pennsylvania Historical Society, and found time, in a very busy life, to write a book on the genealogy of his family. This book was presented to the Pennsylvania Historical Society.

Mr. Fisher was married to Lucy Cobb Williams on June 11, 1913, and they made their home in Germantown, Philadelphia. He died very suddenly on November 6, 1938, and is survived by Mrs. Fisher.

Mr. Fisher will be remembered by those who were privileged to know him for his attractive and gracious personality. He was fortunate in arousing the enthusiasm of all who were brought within the sphere of his friendship.

Mr. Fisher was elected an Associate Member of the American Society of Civil Engineers on November 1, 1905, and a Member on January 7, 1913.

LOUIS PREVOST GASTON, M. Am. Soc. C. E.[1]

Died July 14, 1939

Louis Prevost Gaston died unexpectedly at the summer home of his daughter, Mrs. Milton L. Cornell, at Sea Bright, N. J.; he was seized with a heart attack while engaged in conversation with his wife and daughters.

Mr. Gaston was born in Somerville, N. J., on October 17, 1864. His father was Hugh M. Gaston, a former prosecutor of Somerset County and a civic leader at the time of the Civil War. His mother was Frances Mallet-Prevost. Mr. Gaston inherited French Huguenot ancestry on both sides of his family, as well as some Scotch lineage on his father's side.

His early education was received in the Somerville schools; and he was awarded his Bachelor's Degree from Lehigh University at Bethlehem, Pa., in 1888, and his Civil Engineering Degree from the same institution in 1889.

His first position was with Cofrode and Evans, general contractors, with whom he was engaged from the year of his graduation until 1896. He assisted in the construction of various railroad projects, and designed and supervised the construction of two large coal shipping piers for the New York, Susquehanna and Western Railroad at Edgewater, N. J.

---

[1] Memoir prepared by H. G. Van der Veer, President, Richards & Gaston, Inc., Somerville, N. J.

From 1898 until 1900 Mr. Gaston made examinations and submitted reports on numerous water power developments, including the Great Falls of Catawba River, in South Carolina, Lookout Shoals, in North Carolina, and Trenton Falls, New York.

His mature professional career was devoted essentially to the contracting firm of Richards and Gaston which was organized in 1900, and with which he was associated in an advisory capacity at the time of his death. The first large contract awarded to the firm was the repair of flood damage and the construction of bridge piers and abutments for the Central Railroad of New Jersey at Glen Onoko, Weissport, and Allentown, Pa. From 1905 until 1910 the firm was particularly engaged in the construction of bridges, lakes, and ornamental concrete for James B. Duke, President of the American Tobacco Company, on his private estate in Somerville.

In 1910 the partnership of Richards and Gaston became a corporation with Mr. Gaston as president, a position he held until his retirement from active duties in November, 1925. While he was president the corporation completed railroad contracts amounting to more than $2,000,000, including track elevations for the Central Railroad of New Jersey at Cranford and Somerville, four-tracking of the Reading Railroad from Weston, N. J., to Belle Mead, N. J.; additional tracks for the New York Central Railroad at Staatsburg and Poughkeepsie, N. Y.; also bridge and track changes for the Lehigh Valley Railroad at Buffalo, N. Y., and Roycefield, N. J. The firm also participated in New Jersey State Highway bridge and road construction amounting to more than $1,000,000, and constructed roads, sewers, and dams for counties and municipalities.

Mr. Gaston and his partners formed the Lakewood Sand Company of Lakewood, N. J., in 1905, and in 1922 this company was incorporated with Mr. Gaston as president, a position he held until the business was sold in 1927. At one time this company was the main supplier of sand to dealers along the Central Railroad of New Jersey, and this railroad as well as the State of New Jersey used this sand in its concrete construction.

Mr. Gaston designed and patented a narrow-gage railroad dump car (The Eastern Dump Car) for the use of contractors. He organized a company to build these cars, and many were sold to sand and gravel producers. Others were used for coal stripping and contracting in the eastern part of the United States and in South America.

While Mr. Gaston was known and loved in Somerville for his participation in community affairs, the full extent of his quiet philanthropy was known only to himself. Chief among his interests was the Somerset Hospital, in Somerville, to which he gave generously of his time and means. During the later years of his life he was interested in the development of the Somerville Park lands, and one section of this area was developed and equipped at his expense. He was also President of the Somerville Savings Bank, and a member of the Board of Managers of the New Jersey Sanatorium for Tuberculous Diseases at Glen Gardner, N. J.

On September 5, 1896, Mr. Gaston was married to Maude Safford, of Sharon, Mass., the daughter of the late Edward Stanley Safford, M. Am. Soc. C. E., and a member of the first graduating class of the Massachusetts Institute of Technology. She survives him, as do two sisters, four children, and eight grandchildren.

Mr. Gaston was elected a Member of the American Society of Civil Engineers on February 1, 1905.

GEORGE GIBBS, M. Am. Soc. C. E.[1]

DIED MAY 19, 1940

George Gibbs was born in Chicago, Ill., on April 19, 1861, the son of Francis S. Gibbs and Eliza Gay (Hosmer) Gibbs. He came from a family of English descent, his first American ancestor being James Gibbs who came originally from Bristol, England, and settled in Rhode Island about 1660. His family was identified with the history and growth of Newport, R. I., from the earliest times—dating from a few years after the purchase in 1638 from the Indians by Governor Coddington of the Island of Aquidneck, as Newport was then called. James Gibbs and his wife, Sarah, settled in Newport and later moved to Bristol, R. I. George Gibbs was the fifth descendant in the line of this Rhode Island family to bear the name "George."

George Gibbs' great-grandfather, George Gibbs II, was a shipping merchant in trade with the West Indies, China, St. Petersburg (later Petrograd, U. S. S. R.), and the Mediterranean, and controlled a large fleet of vessels operating out of Newport. He was married to Mary Channing of the Rhode Island family of that name. George Gibbs III, grandfather of George Gibbs, was devoted to scientific studies and acquired a very fine collection of minerals, later in the possession of Yale University at New Haven, Conn. He was married to Laura Wolcott, the granddaughter of Oliver Wolcott, Secretary of the Treasury under George Washington and John Adams, and one of the signers of the Declaration of Independence. George Gibbs' father—the youngest child of George Gibbs III—was Francis Sarason Gibbs, who spent his earlier adult years in Canton and Shanghai, China. He was engaged for the greater part of his life in the grain exporting business. His later years were spent in New York City and Nyack, N. Y.

In his book entitled, "The Gibbs Family of Rhode Island," Mr. Gibbs (the subject of this memoir) says of himself:

> "Throughout a long and active life I have been fortunate in having good friends and owe much to the careful bringing up by a good mother who taught me the importance of faithfulness in the performance of responsibilities. Inherited qualities determined the direction of my career in life."

---

[1] Memoir prepared by E. Rowland Hill, M. Am. Soc. C. E.

Several generations of the Gibbs family had been fond of the natural sciences, and these leanings took the form of a fondness for engineering in Mr. Gibbs' case. He was graduated in 1882 from Stevens Institute of Technology, at Hoboken, N. J., one of the earliest engineering colleges in the country. He was among the first of its graduates. The first work he was able to secure after graduation was as laboratory assistant for Thomas A. Edison, who presently appointed him superintendent of meters in the world's first central power station, Pearl Street, New York. Later he became chief chemist of the Oxford Nickel and Copper Company at Bergen Point, N. J.

George Gibbs' railroad career began in 1885 when, through Roswell Miller, an intimate friend of his father's, he was offered a position with the Chicago, Milwaukee and St. Paul Railway Company to organize and take charge of a new department—that of physical and chemical tests of materials. He soon advanced to the position of mechanical engineer and was placed in charge of all machinery, locomotive, and car designing, in addition to his duties as chief of the Department of Tests. In this capacity he became particularly interested in advancing safety in railroad operation. To this end he devised, after some experimentation, a practical steam-heating system for trains and this, when put into effect, made the St. Paul Road a pioneer in safer and better train heating. For more complete protection of trains from fire, after elimination of the coal stove, it became necessary to abolish the use of the kerosene lamps for lighting cars. To this end Mr. Gibbs designed a system of electric lighting for heavy through trains that proved a great success and came into extensive use on the St. Paul Road. During these years he was active in conducting tests of brakes for freight trains, improving signaling systems, etc., and he patented numerous inventions, notably in the direction of safety appliances. While with the railroad Mr. Gibbs, with his brother, Lucius, organized the Gibbs Electric Company of which he was president. This company devised and manufactured a practical line of electric motors for general factory use.

During these activities in the West, George Gibbs became acquainted with two very prominent men in industry, the late George Westinghouse, M. Am. Soc. C. E., famous inventor of the air-brake and manufacturer of steam and electric machinery and railway signaling equipment, and Samuel M. Vauclain, M. Am. Soc. C. E., of the Baldwin Locomotive Works, Philadelphia, Pa. These men had come to the conclusion that heavy electric traction might soon be a possibility and might become of great importance in their manufacturing fields. At Mr. Vauclain's suggestion they accordingly arranged with Mr. Gibbs to become associated with them in the development of electric locomotives, and to make this arrangement possible Mr. Westinghouse's company purchased the Gibbs Electric Company. Mr. Gibbs established residence in Philadelphia in this new and dual capacity and thus became more intimately associated with George Westinghouse. For the Westinghouse and Baldwin companies Mr. Gibbs developed the low-built type of electric locomotive for restricted headroom service in mines, which has become standard in America.

While thus engaged Mr. Gibbs was asked also to represent the Westinghouse interests abroad in connection with heavy electric traction, and during the next few years (1900–1902) he was chief engineer of the Westinghouse Companies in England and on the Continent. He made many trips to Europe in connection with electrification projects for the Metropolitan Railway (Inner Circle) in London, England; Mersey Railway in Liverpool, England; and certain of the tube railways in London; and also was consultant for the Paris Underground Railway and other projects on the Continent.

The period 1902 to 1906 was one of great development in railroad terminal building and heavy electric traction in the vicinity of New York City, and during this period George Gibbs was first vice-president of Westinghouse, Church, Kerr and Company, then an outstanding firm of engineers. This position enabled him to take on independent consulting work, and he served as a member of the Electric Traction Commission of the New York Central Railroad Company in connection with its new Grand Central Terminal; was consulting engineer for the Rapid Transit Subway Construction Company, which constructed the first New York City subway; designed and installed the electric traction system and equipment for the Long Island Railroad Company; and became consulting engineer for the Pennsylvania Railroad Company. In 1905 George Gibbs was appointed chief engineer of Electric Traction and Station Construction for the Pennsylvania Railroad's New York Tunnel and Terminal Line then being constructed and a member of the Board of Engineers responsible for this entire improvement. This service continued until the completion of the station and terminal lines in 1910–1911.

Mr. Gibbs' interests in promoting the maximum of safety on railroads led him, in connection with his work for the New York subway, to design and patent the first all-steel passenger car ever put into practical use on a railway, a revolutionary idea at the time but one which he regarded as essential for high-speed tunnel and underground operation. The Subway Company adopted his plans and had built initially 300 cars of this new type. The Long Island Railroad Company also adopted it for its new electric lines in Brooklyn, N. Y., as did the New York Central Railroad Company for its Park Avenue tunnel line. The Pennsylvania Railroad Company, in view of the success of the new car, determined to bar all wooden cars from its New York tunnels and station, a far-reaching decision which meant the adoption of the steel car everywhere on its system and ultimately on all American railways.

To continue in the electric traction field after the completion of the Pennsylvania Railroad Terminal, early in 1912 George Gibbs and the writer, who had been closely associated on heavy electric traction development and related projects since 1898, formed a partnership for conducting a general engineering practice. This firm, later incorporated, has been responsible for the design and execution of much of the important electrification work on American railways. Such installations have included the electrification of the Norfolk and Western Railway and the Virginian Railway, mountain grade heavy coal and freight haulage lines—50 and 130 miles long, respectively; extensive additions and changes to electrification equipment and power plants of the New York, New Haven and Hartford Railroad and the New York Connecting Railroad and Long Island Railroad companies' freight line operated by the New Haven

(Hell Gate Bridge Line); the Chicago Suburban electrification of the Illinois Central Railroad; and the suburban lines of the Pennsylvania Railroad in the vicinity of Philadelphia, and more recently the electrification of all through passenger and freight services of the Pennsylvania Railroad between New York and Washington, D. C., and to Harrisburg, Pa. The Pennsylvania lines thus electrified comprise 670 miles of route and a power load of 200,000 kw, and involved a cost of more than $150,000,000; the line operates 260 electric locomotives and 500 multiple unit cars. There also have been consulting or designing engagements for numerous industries, public utility companies, city transit lines, and public service commissions, including the Public Service Commission of New York in connection with the dual subway transit system.

During the World War Mr. Gibbs was a member of the United States Government Advisory Commission of Railway Experts to Russia, and spent seven months on behalf of Russia and the Allies on the reorganization of Russian transportation facilities. This involved the examination in detail of the 30,000 miles of railways in European and Asiatic Russia. Later Mr. Gibbs advised the Ministry of Transport of the British government on electrification projects for lines radiating from the City of London.

In addition to being a Past-Director of the Society, which awarded him the Norman Medal in 1912 and the Wellington Medal in 1931, Mr. Gibbs was a member of the British Institution of Civil Engineers; member and past-president of the American Institute of Consulting Engineers; Fellow of the American Institute of Electrical Engineers; a member of the American Society of Mechanical Engineers, the American Railway Engineering Association, American Railroad Association, and American Mathematical Society. He was the author over a period of many years of numerous technical papers published in the *Proceedings* of the societies and the press.

In the non-technical societies Mr. Gibbs' membership included Descendants of the Signers of the Declaration of Independence (great-great-grandson of Oliver Wolcott), Society of the Cincinnati, and numerous charitable and educational institutions. He was a member of the Board of Trustees of Presbyterian Hospital (Columbia University Medical Center) and vice-president and chairman of the Executive Committee of the Woman's Hospital, New York.

Mr. Gibbs was a Life Trustee of Stevens Institute of Technology, his alma mater, which in 1930 conferred upon him the honorary degree of Doctor of Engineering, with the citation:

> "International authority on the electrification of railways; introducer of the all-steel car; during the Great War a member of the United States Railway Commission to Russia; for many years designer and constructor of equipment mainly electrical, for the intricate surface, underground and underwater arteries of travel that serve and control the concentrations of peoples and industries in the centers of population of this and other countries; human servant of the public through his effective participation in the direction of two great hospitals; a son of this college who has brought to it honor and affectionate service."

Mr. Gibbs' clubs included: The Century Association, New York Union, University, Brook and Downtown, New York, Rittenhouse, Philadelphia, South

Side Sportsmen's (Oakdale), and Long Island and Cascapedia Salmon Club, Canada.

Throughout his career Mr. Gibbs exhibited the desire and determination to excel in the quality of the things he created and in this he possessed masterful skill, balanced and farsighted judgment, and persistence in carrying through to ultimate success the thing he believed to be the best in any situation. There was courtesy, charm, and dignity in his associations with others, both in professional and personal relationships; he had a fondness for art and music, being a regular attendant at the opera and a frequent visitor to the centers of culture abroad; he possessed a keen sense of humor and enjoyed, for recreation and change, nature's open spaces of forest, stream, and sea.

Mr. Gibbs was not married; he was devoted to his brother and sisters and took an unusual interest in their children.

Mr. Gibbs was elected a Member of the American Society of Civil Engineers on March 6, 1895.

RALPH EDWARD GOODWIN, M. Am. Soc. C. E.[1]

DIED DECEMBER 16, 1939

Ralph Edward Goodwin was born in East Hartford, Conn., on September 26, 1886. He was a descendant of Ozias Goodwin who went to Boston, Mass., from England in 1632, and through the wilderness to Hartford with Thomas Hooker's first party in 1635. John Goodwin, in the direct line of descent, went to East Hartford in 1697 and acquired the land on which Ralph Edward Goodwin was born. Ralph and two sisters were the children of Joseph Olcott Goodwin and Harriet Jane (Spencer) Goodwin. The father was town clerk of East Hartford for more than 48 years.

As a boy, Ralph Goodwin attended Miss Loomis' private school and the Hartford Public High School. He received his Bachelor of Arts degree from Yale University at New Haven, Conn., in 1908, and then entered Sheffield Scientific School, from which he was graduated with a Bachelor of Philosophy degree in 1909. He was an excellent student, and at Yale was honored by election to the Phi Beta Kappa and Sigma Xi fraternities.

Mr. Goodwin's first professional work was as a rodman with a United States Government Survey party along the Connecticut River during the summer of 1909 but in the fall he returned to Sheffield Scientific School as assistant in civil engineering.

He taught several civil engineering courses at Sheffield during the next two years, and in turn, as he said, "these years as assistant in civil engineering taught me to work mercilessly without thought of reward and also put me in line for the degree of Civil Engineer, which I much coveted." In June, 1911, at the end of this period of training, he joined the New York Central and Hartford Railroad Company as a rodman, and in one month became a transitman on a variety of surveying and mapping work. He felt that his pay was not sufficient to permit him to buy life insurance and, as he expected to be killed on the tracks before he could get another increase in salary, he left the company in March, 1912. From that date through May, 1912, he was temporary assistant engineer with the New York Board of Water Supply on the New York City pressure tunnel. He gave lines and grades, kept progress records, and had supervision over dynamite. On this job he gathered material to complete his graduate thesis, and he obtained the degree of Civil Engineer from Yale University in 1912.

---

[1] Memoir prepared by William Allan, Assoc. M. Am. Soc. C. E.

While serving as assistant at Sheffield Scientific School Mr. Goodwin formed his decision to become a teacher of civil engineering, and true to this objective, in June, 1912, he accepted the position of assistant in civil engineering at Columbia University, New York, N. Y., and began his work by teaching at Columbia's Summer Surveying Camp. Throughout the school year in his work designing and detailing experimental columns for the testing laboratory, Mr. Goodwin furthered his great interest in column theory and design and subsequently prepared a brochure on that subject. For the summer of 1913 he returned to Yale University as instructor in charge of topographic and mine claim surveys and office work for mining engineering students.

Thereafter Mr. Goodwin turned again to professional work for a period of nine years. For the first three and a half years he was a junior engineer in the Division of Materials Inspection of the New York State Public Service Commission in New York City. He had charge of the inspection of concrete aggregate and of the physical testing laboratory which he equipped for testing and inspecting concrete and aggregate for subway and elevated railroad construction in New York City. For one year following this Mr. Goodwin was an estimator for the Chile Exploration Company of New York City and prepared quantity and cost estimates and wrote specifications for construction materials for the living accommodations and public works of a South American mining plant. From April, 1918, until May, 1922, he served with the United States Shipping Board in the Hull and Machinery Departments as an inspector of construction and repairs on vessels.

In November, 1922, he returned to teaching as instructor in drafting and mathematics in the Veterans' Vocational Training School at the College of the City of New York; and in February, 1923, he joined the staff of the School of Technology of the City College. He rose from the rank of instructor to that of associate professor of civil engineering. In that period Professor Goodwin taught many of the courses in the civil engineering curriculum, and for years directed the Summer Surveying Camp at Van Cortlandt Park and all course work in strength of materials. At the time of his death he was secretary of the School of Technology Faculty and also of The City College Faculty Conference, a member of the Committee on Appointments of the Civil Engineering Department, and program and curriculum adviser to civil engineering students. Of the countless other tasks which he did without flourish or note, many will never be known.

He was a truly competent, scholarly man, whose active and productive work in the School of Technology helped to raise its level of achievement and influence to a high position. He wrote a text book entitled "Advanced Strength of Materials and Structural Design"; manuals for Materials Testing Laboratory, and for First and Second Summer Camp Surveying; and brochures on least squares and on column theory and design. Throughout his professional life he contributed many papers and discussions on technical subjects to civil engineering periodicals.

Within the range of his specialty, Professor Goodwin was a master in a creative way and, along with his strength in science and its applications, he was also a master of that rare art of conveying knowledge in a form and manner suited to the students' powers of assimilation. He further possessed wide general knowledge and interest in all human activities, particularly those of his students.

Special distinction came to Professor Goodwin through the trust reposed in his integrity and through his willingness to be of service. He won the confidence and esteem of the entire school and of its graduates, thereby gaining a reputation closely interwoven with the lives and experiences of those who knew him. He was quiet and unassuming in manner, but independent in thought; and his capacity for calm, unbiased judgment caused his advice to be much sought after. These qualities, together with his sincere and friendly attitude toward all, won him great affection and respect.

In 1912 Mr. Goodwin was married to Gilberta Hopkins Daniels, daughter of Prof. Archibald L. Daniels of the University of Vermont, Burlington, Vt. He is survived by his widow; their son, Robert Daniels; and a sister, Jessie (Mrs. H. A. Preston).

No finer tribute could be expressed in Professor Goodwin's memory than that voiced by several of his friends on learning of his death: "He was the best one among us; surely if all men were as tolerant, as kind and as truly able as he was, the way to Utopia might be found."

Professor Goodwin was elected a Junior of the American Society of Civil Engineers on October 1, 1912; an Associate Member on April 16, 1918; and a Member on June 10, 1935.

ROY COTSWORTH GOWDY, M. Am. Soc. C. E.[1]

DIED JUNE 10, 1939

As evidence of how a young man without the opportunity of a college education can, by energy and ambition, qualify himself as an engineer, Mr. Gowdy's early experiences are given in considerable detail.

Roy Cotsworth Gowdy was born in Washington, Iowa, on September 3, 1878. He was a son of Leander Harper Gowdy, who was born in Monmouth,

---

[1] Memoir prepared by Herbert S. Crocker, Past-President and Hon. M. Am. Soc. C. E.

Ill., and Annie Laurie (Reid) Gowdy, who was born in College Springs, Iowa. He was of Scotch descent on his father's side and of English descent on his mother's side. When he was four years of age his family moved to Colorado Springs, Colo., where he received his early education in public schools, graduating from the high school in 1896. He then attended Colorado College, at Colorado Springs, for one year, but discontinued his courses because of ill health. Thirty-nine years later, in 1935, upon presentation of a thesis, he received the degree of Civil Engineer from the University of Colorado, at Boulder, Colo.

Like most embryo engineers, Mr. Gowdy seems to have been an opportunist. His early experience was as stakeman, axman, and rodman, performed under the direction of H. I. Reid and other engineers and surveyors in the area contiguous to Colorado Springs. In 1899 he was employed by the Colorado Springs and Cripple Creek District Railway Company as calculator with a locating party making surveys for a standard steam railway between those points. Subsequently he became a rodman and instrumentman on the construction of the line. His connection with this company continued intermittently until the fall of 1902. At the same time he was in private practice and doing minor engineering work for various engineers. He was then engaged in similar capacities by the Denver, Northwestern, and Pacific Railway Company (Moffat line) during its survey and construction. This engagement was terminated in 1904.

Mr. Gowdy then returned to the Colorado Springs and Cripple Creek District Railway Company where he was employed as draftsman and assistant engineer of maintenance of way. He remained with that company until the summer of 1905 when it was purchased by the Colorado and Southern Railway Company and its engineering department was discontinued. This work was followed by employment by E. C. van Diest in Colorado Springs as draftsman and clerk.

After his preliminary training in subordinate positions, which had been supplemented by outside reading and study, Mr. Gowdy attained his first important engagement on April 21, 1906, when he was employed by the Ft. Worth and Denver City Railway Company, an affiliate of the Colorado and Southern Railway Company, as engineer in charge of the design and construction of a freight terminal which it was building in Fort Worth, Tex. This work was important in that it involved responsible charge in the field of those many details which must be carefully and progressively worked out in construction of that kind. It also involved considerable designing, under the general direction of the late Herbert W. Cowan,[3] M. Am. Soc. C. E., who was at that time chief engineer of the Colorado and Southern Railway Company. To be associated with Mr. Cowan was an honor as well as a privilege.

In November, 1906, Mr. Gowdy was appointed resident engineer of the Fort Worth and Denver City Railway Company, in which capacity he had charge of all its engineering work, including the completion of its terminal work at Fort Worth. This work also was done under the general direction of Mr. Cowan.

---

[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXX, December, 1916, p. 2124.

In 1908 he was appointed chief engineer of that company and of the Wichita Valley Lines. In this capacity he had charge of all engineering matters for both companies. During this period, and up to 1918, the work which Mr. Gowdy performed was largely the general routine of railway maintenance of track and reconstruction of bridges and other structures, among which were depots and incidental buildings. Following a fire at Childress, Tex., Mr. Gowdy had charge of the construction of a new plant, including locomotive and car repair shops, roundhouse, and other facilities.

In 1918 the United States Government took over the operation of all the railway lines in this country as a war measure. Following this change of administration Mr. Gowdy served for a few months as division engineer under the late Frank Merritt of Galveston, Tex., for a group of lines in northern Texas. However, in August of that year he resigned from that position to become corporate chief engineer of the Colorado and Southern Lines, which included the Colorado and Southern Railway Company, Fort Worth and Denver City Railway Company, Wichita Valley Lines, and their subsidiaries, with headquarters in Denver, Colo. This connection continued until the railway properties were returned to their owners by the government on February 29, 1920, after which Mr. Gowdy was chief engineer of the Colorado and Southern Railway lines, including the Fort Worth and Denver City Railway, with offices in Denver and Fort Worth, until his death.

In 1909, by purchase of a majority of its stock, the Chicago, Burlington and Quincy Railroad Company acquired the Colorado and Southern Railway and from that time forward gradually assumed direction of it and its subsidiaries. However, this did not affect Mr. Gowdy's authority except that he was in frequent consultation with the officers of the Burlington, and conformed to its general policies and budgetary control.

Mr. Gowdy's duties were largely in accordance with the general routine of railroad administration. They included improvements in alinement, renewal of bridges, the construction of the Canadian River bridge in Texas, and the preparation of necessary data, with their presentation as a witness before members and examiners of the Interstate Commerce Commission in cases seeking to procure the approval of the Commission of proposed new lines or the abandonment of existing standard-gage and narrow-gage lines.

Some important exceptions to the routine are certain major constructions which drew upon his powers of judgment in location and designing, and ability in the letting and handling of large contracts. These are probably described better in his own words:

> "During the earlier years of their existence, the Fort Worth & Denver City Railway Co. and the Wichita Valley Lines enjoyed almost a complete monopoly of the territories in which they operated. This condition was due in part to the large areas of the northern part of Texas which had never been developed agriculturally, and in part to the tremendous natural

barrier known as the Cap Rock, extending from the northern part of the State far down towards the Colorado River, and separating the lower Plains country from the drainages of the Red and the Brazos Rivers. Overcoming this barrier presented serious difficulties even for highways, and more serious ones for the location and construction of railroad lines. Opening of the South Plains territory to agriculture brought with it the construction of railway lines from the north and the south, which seriously menaced the tonnage handled heretofore by the Fort Worth & Denver City and Wichita Valley Lines, and this fact, together with a desire to enter a new and growing country, made the extension of those lines desirable and necessary. For several years I worked with the executive and traffic officers of the companies in making traffic and other surveys of the large area lying between the Fort Worth & Denver City Railway Company's main line and the line between the States of Texas and New Mexico. My particular portion of this work had to do with the possibilities of procuring feasible routes from the main line to the foot of the Cap Rock and surmounting the Cap Rock, and their cost, but I also helped in the traffic, financial, and economic studies which were made.

"At about the same time the South Plains country came into prominence agriculturally, the vast territory in the Pan Handle of Texas north of the Canadian River began to open up and was followed by the development of what later became known as the Amarillo oil fields. These developments and the desire of my companies to procure an outlet to the northeast resulted in extensive surveys being made, under my supervision, into Oklahoma, the southern part of Kansas and the southeastern part of Colorado.

"When it was finally decided to build westerly from some point on the main line of the Fort Worth & Denver City into the lower Plains country, I was instructed to ascertain the most feasible and economical route, and to locate and construct a line capable of handling cattle, grain, cotton, and other agricultural products in the most economical and expeditious manner. The Cap Rock, mention of which has been made above, presented the most serious difficulty in the location of a line, and made necessary a detailed investigation of a very large area of country, and the running of many preliminary and exploratory lines before a final route was selected. At one time four locating parties were engaged in this work, and about 1200 miles of preliminary lines were run before the final location of the 65 miles from the main line to the top of the Cap Rock was made.

"The construction of the more than 200 miles of track westerly from the main line of the Fort Worth & Denver City involved some exceedingly heavy work, and unusual construction methods were found necessary. The broken and rugged character of the country in the area immediately below the Cap Rock made it necessary to drive several tunnels from 200 ft. to a little less than 700 ft. in length, erect long and high trestle bridges, and move a very large amount of earth and rock. The construction of this line was unique, in that the engineering forces located, constructed, and turned over to the Operating Department a working machine complete in every detail. All station, roadway, and other buildings were entirely completed, depots contained all furniture, stationery, and other supplies necessary, section gangs had been equipped with tools and were at work, and shop and engine facilities were in operation when the line was released by the engineering forces. This project was also unique in that the line was operated for freight service for several months by the engineering forces under my jurisdiction before it was turned over to the Operating

Department. Operating results obtained since the line was opened for service indicate that its construction was justified and that no serious errors in its location, grades, and general construction were made.

"Surveys for lines running northerly and northwestly from the main line of the Fort Worth & Denver City into Oklahoma and Kansas were carried on at the same time the lines were run into the South Plains country. About the time the new line into the South Plains was placed in operation, the Amarillo oil and gas field became active, and surveys for a line into that field were undertaken and a line was located from Childress, a point on the main line of the Fort Worth & Denver City Company, northerly across the Prairie Dog Town and the Salt Forks of the Red River to the North Fork of the Red River, and thence northerly and westerly up the North Fork to the town of Pampa. Portions of this line were in rough country, but no unusual location or construction difficulties were encountered, except that the river crossing made long bridges necessary. The unusual feature of its construction was the time in which the work was done. With the cooperation of the contractors, the Hamilton & Gleason Company and Roberts Brothers, in a period of less than ten months, more than 2,000,000 cubic yards of earth excavation and embankment and 1,400,000 cubic yards of loose and solid rock were moved. In the same period, and while grading was under way, that firm drove approximately 160,000 lin. ft. of creosoted piling, varying in length from 12 ft. to 100 ft., and placed more than 2100 MBM of creosoted lumber. The track on this line was laid by company forces and was kept so close behind the grading that several times grading would be completed by night shifts, and track forces would start laying track the next morning."

Mr. Gowdy was a man of kindly and generous disposition, loyal to his friends, and of high character. In his business dealings he was conscientious in all things and had the faculty of being able to see clearly both sides of a contention. His decisions were, therefore, wise and fair. He enjoyed a circle of many friends who were attracted to him by his resourceful mind and sympathetic understanding. He was an upright man.

Mr. Gowdy was a Master Mason and a Knight Templar. At the time of his death he was a member of the Plymouth Congregational Church, the Denver Athletic Club, and the Lakewood Country Club.

He had a keen interest in professional matters outside of his business. He was president of the Colorado Engineering Council for two years, as well as its delegate to the American Engineering Council for a corresponding length of time. He was also a member of the American Railway Engineering Association and of the Colorado Society of Engineers. In 1918 he served as President of the Texas Section of the American Society of Civil Engineers and in 1927 as President of the Colorado Section.

He had one brother and three sisters; two of the latter survive him. He was married to Emma Lee Maiden of Colorado Springs on June 27, 1905, who, with a son, Joseph Scott Gowdy, Jun. Am. Soc. C. E., born June 5, 1907, survives him.

Mr. Gowdy was elected an Associate Member of the American Society of Civil Engineers on October 31, 1911, and a Member on January 18, 1916. He served as Director in 1930–1931 and as Vice-President in 1937–1938.

EDWARD FRANCIS HAAS, M. Am. Soc. C. E.[1]

DIED APRIL 13, 1939

Edward Francis Haas was born on August 28, 1870, in Stockton, Calif., the son of Charles Haas and Charlotte (Merk) Haas, who were among the early California pioneer families.

Mr. Haas attended the public schools in Stockton, and was graduated from Stockton High School in 1888. He attended the University of California, at Berkeley, Calif., where in 1892 he was graduated with high honors, receiving the degree of Bachelor of Science in Civil Engineering.

From 1892 to 1893 he was instructor in Civil Engineering at the University of California, after which he attended the Columbia School of Mines of Columbia University, New York, N. Y., receiving his Civil Engineer Degree in 1894. Returning from Columbia University to Stockton, he worked in the City Engineer's Office for one year. Thereafter he was in private practice in the City of Stockton until 1897.

Always deeply interested in the problem of reclaiming the fertile delta lands of the San Joaquin and Sacramento rivers, Mr. Haas soon became engineer in charge of the reclamation of about 20,000 acres of swamp land in these delta regions. He built about twenty miles of levees, three miles of main canal, and large pumping plants for drainage. During this time he was sent to Alaska to make an engineering report for a large mining project which included difficult transportation problems.

In 1898 Mr. Haas took up residence in San Francisco, Calif., where he became affiliated with the City Engineer's Office in preliminary studies for an adequate sewer system.

He was associated with the late Carl E. Grunsky,[2] Past-President, Am. Soc. C. E., from 1899 to 1900 in the design and construction of a sewage system for the City of Belvedere, Calif. Mr. Haas was the engineer in charge of construction.

In 1900 he was engaged, by the Board of Engineers of the City of San Francisco, to design a comprehensive and modern sewage system for the City of San Francisco at an estimated cost of $4,600,000. Mr. Haas was in responsible charge of design for various sections of the system throughout the city. Following this he became assistant engineer in the newly created Board of Public Works, with Mr. Grunsky as city engineer of the City of San Francisco.

In 1901 he entered into partnership with John W. Ferris. The firm, known as Ferris and Haas, engaged in the prosecution of various dredging and reclamation contracts, which often included the necessary engineering. In the winter of 1901–1902 the partnership received from the United States Government the original dredging contract to deepen Pearl Harbor at Honolulu, Hawaii. Mr. Haas, as managing partner, was in Honolulu in charge of the work.

---

[1] Memoir prepared by F. C. Herrmann and Thomas H. Means, Members, Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1591.

During this partnership Mr. Haas maintained independently his private practice of engineering, and was associated with the late Marsden Manson,[8] M. Am. Soc. C. E., in the design and construction of the sewer system of the City of Marysville, Calif.; as associate engineer on design and enlargement of the Placerville (Calif.) Water Works System; as engineer in charge of dredging of the ship channel to Mare Island Navy Yard; and as engineer and manager of the West Coast Canal Company which dredged the navigation channel for the United States Government in San Pablo Bay.

In 1906 Mr. Haas organized the Union Dredging Company, and in 1909 the Caledonia Dredging Company. He acted as president of both of these companies.

The work under the partnership of Ferris and Haas, and as continued by the Union Dredging Company and the Caledonia Dredging Company, consisted largely of river rectification and the work of various reclamation districts and so-called islands of the Sacramento and San Joaquin Delta, as well as marginal marsh lands of San Francisco Bay. Large sums of money were spent on these projects which added many thousands of acres of very fertile land and great wealth to the State of California. Because large portions of these projects consisted of " peat lands," Mr. Haas had to solve many difficult problems in the construction of levees, structures, and canals. By reason of these experiences, Mr. Haas was consulted about the design and construction of the railroad embankment across the San Joaquin River delta lands for the San Joaquin Valley Railroad Company, later the Atchison, Topeka and Santa Fe Railway Company.

Practically all of this work was done with floating dredgers, and Mr. Haas, keenly interested in the improvement of dredging equipment, participated in designing and developing the long-boom clamshell dredger in the Sacramento and San Joaquin regions.

In 1919 Mr. Haas, turning his attention to dry land excavation, formed a partnership with Frank Doughty, Assoc. M. Am. Soc. C. E., known as Haas and Doughty. In 1927 this partnership was enlarged to include Arthur Jones, the firm's name changing to Haas, Doughty and Jones. This partnership continued until Mr. Haas passed away.

Mr. Haas was the dominant figure in these partnerships, which were responsible for important construction work in several of the western states. For the United States Bureau of Reclamation, they constructed the main canals and appurtenant structures on the Minidoka and Boise projects in the State of Idaho; the Yakima Project in the State of Washington; the Vale Project in the State of Oregon; and the Contra Costa Canal of the Central Valley Project in the State of California. In Nebraska they participated in the construction of main canals and appurtenant structures for the Loup River Public Power District, and for the Central Nebraska Public Power and Irrigation District.

---

[8] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 1554.

In addition to the foregoing, in California this firm constructed many miles of large levees of the Sacramento Flood Control Project on rivers and by-passes, as well as levees, irrigation, and drainage canals for many of the reclamation and irrigation districts in that state.

With his continued interest in the efficiency of earth-moving equipment, Mr. Haas, in conjunction with the Stockton Iron Works, developed the excavator with caterpillar treads. This was the forerunner of modern dragline equipment.

The success of these organizations which he dominated was due very largely to the exceptional ability of Mr. Haas to organize and develop a given task or project. This led to the concentration of his energies on contracting work, which continued in the foreground of his activities throughout his life. However he was always deeply interested in all engineering matters, as well as the welfare of the engineer.

Concurrently with engineering and contracting, Mr. Haas became interested in various types of business, outstanding among them being the Snelling Gold Dredging Company of California, of which he was vice-president. The organization and success of this company, like his many interests, was due largely to his ability as an engineer and business executive. This was true also in the organization of the Vermillion Natural Gas Company, later known as the Central Natural Gas Company of South Dakota, of which he was Chairman of the Board.

Aside from his engineering and business activities, Mr. Haas was continuously active as an Alumnus of the University of California and of Sigma Chi fraternity. He was also trustee of the First Presbyterian Church of San Francisco, and for many years was a member of the San Francisco Golf Club, the Commonwealth Club of San Francisco, The San Francisco Chamber of Commerce, Engineers Club of San Francisco, The Pacific Union Club of San Francisco, the Menlo Country Club, and the University Club of San Francisco, serving as president of the latter two clubs for two terms. For many years he was also a member of the Stockton Golf and Country Club, and the Yosemite Club of Stockton.

During the 1920's, Mr. Haas was appointed by the United States Army as engineer in a civilian group to survey the facilities of the Ninth Corps Area, in connection with the National Defense Program. He was charged with the gathering and correlating of various data relative to transportation equipment, water, and fuel supplies, etc., and performed this service in a meritorious manner as gratuitous and patriotic service to the public, and his work was highly commended by the Army officials.

Although he was a stern taskmaster to himself, Mr. Haas was sympathetic and generous to the appeal of others. Highly respected by all who came in contact with him in business, he was deeply loved by his friends and associates.

In June, 1931, Mr. Haas was delegated, together with Daniel W. Mead, Past-President and Hon. M. Am. Soc. C. E., to represent the Society at the Seventy-Fifth Anniversary of the existence of the *Verein Deutscher Ingenieure,* held at Cologne, Germany.

Mr. Haas is survived by his widow, Mabel Thompson Haas, one son, Edward Thompson Haas, one daughter, Alice Haas (Mrs. Frank N. Conner), and two grandchildren, Edward Jewett Conner and Syida Geraldine Haas.

Mr. Haas was elected a Junior of the American Society of Civil Engineers on February 6, 1894; an Associate Member on September 5, 1900; a Member on February 6, 1906, and a Life Member in 1935.

EPHRAIM HARRINGTON, M. Am. Soc. C. E.[1]

DIED JULY 20, 1939

Ephraim Harrington was born in Belmont, Mass., on December 2, 1861. He received his early education in the public schools of Roxbury, Mass. For a short time he attended the School of Mechanic Arts of the Massachusetts Institute of Technology, at Boston, Mass., in the class of 1879, but he did not graduate with his class.

He followed civil engineering work all of his life, starting with the City of Boston in 1877 in the Surveyors Department where he worked two years. Later he transferred to the Engineering Department of the City. From 1883 to 1885 he was transitman and inspector for the Boston, Hoosac and Western Railroad Company. From 1887 to 1888 he was transitman in charge of the work on the reconstruction of roads and bridges for the Toledo, St. Louis and Kansas City Railroad Company.

His most important work was begun in 1888 when he was employed by the Engineering Department of the West End Street Railway Company (later a part of the Boston Elevated Railway). This work continued for five years. In 1893 he and the late Gilbert Hodges formed the engineering firm of Hodges and Harrington, Civil Engineers, with an office in Boston, practicing in steam railroad and electric railway construction and general engineering.

The firm of Hodges and Harrington was probably the most prosperous engineering firm in Boston at that time. It was very active in the rebuilding of old horse car lines to adapt them to electric car operation. As this was a new field there were few precedents.

Hodges and Harrington were the engineers for the majority of the interurban lines in Massachusetts, New Hampshire, Rhode Island, and a part of Vermont. This work included most of the 900 miles of the tracks comprising the Eastern Massachusetts Street Railway, later largely abandoned to make way for buses. The firm was dissolved in 1900 and Mr. Harrington practiced general engineering, especially the construction and layouts of street railways, a comparatively new field. In later years he confined himself to more general work, particularly after the various traction companies organized their own engineering departments.

---

[1] Memoir prepared by Frank B. Walker, M. Am. Soc. C. E. Mr. Walker died on June 3, 1940.

Mr. Harrington was an extremely reserved man and did not "mix" to a great degree with either his assistants or associates, presumably from shyness. His hobby was the Appalachian Mountain Club which he joined in 1892. His loyalty to the club remained to the end of his life. He was a leader of winter parties and organized the Snow Shoe Section for the purpose of encouraging snow shoeing as an aid to mountain climbing. He knew the fundamentals of equipment and gave much attention to its proper selection. In June, 1918, he published a code for signals in the magazine *Appalachia,* and later wrote a "Few Notes on the Uses of the Compass in the Woods" for the same magazine. He initiated the wood chopping enterprise and was one of the outstanding members of this famous mountain club.

Mr. Harrington outlived most of his associates. For many years he lived in West Roxbury, a part of Boston. He was a member of the West Roxbury Citizens Association, the Unitarian Club, the Unitarian Laymen League, and a Trustee of the First Parish for nearly forty years.

He was married to Lina Sophia Weld of Dedham, Mass., on October 10, 1894. He is survived by an older brother, David A. Harrington, a civil engineer of Norwell, Mass.; a sister, Susan T. French; a daughter, Ruth Weld Harrington; and a son, Davis Weld Harrington.

Mr. Harrington was elected a Member of the American Society of Civil Engineers on March 3, 1897.

PHILIP HART, M. Am. Soc. C. E.[1]

DIED NOVEMBER 29, 1938

Philip Hart was born in Portland, Ore., on June 8, 1887, the son of James Dixon Hart and Mary (Goodenough) Hart. The scion of a prominent Portland family, he had his preparatory education in the Portland Academy, graduating with the class of 1905. It was there that he formed friendships with James H. Polhemus, M. Am. Soc. C. E., general manager of the Port of Portland, and Ben S. Morrow, Assoc. M. Am. Soc. C. E., general manager of the water department of the City of Portland. These friendships ripened with the years, and continued with increasing mutual esteem until the day of Mr. Hart's death. Along with many other "Portlanders," Mr. Hart received his university education at the Massachusetts Institute of Technology, at Boston, Mass., graduating in 1910 as a mechanical engineer.

Mr. Hart early gave evidence of the original mind and dauntless spirit which were to be his key to success in later life, and the basis of his life's work. In 1912 he did some minor underwater foundation work at the plant of the St. Helens Lumber Company in St. Helens, Ore. He did his own diving, and all the difficult work with his own hands; perchance while doing work that had the ordinary run of workmen frightened, he conceived the idea of making a business out of work that was too difficult to attract the ordinary contractor, thus enhancing the profit possibilities by elimination of competition.

---

[1] Memoir prepared by F. W. Crocker, M. Am. Soc. C. E.

His first major business venture was in 1914, at the age of 27, in the firm of Sharp and Hart, dealers in transmission and conveying equipment in Portland. This did not provide full scope to Mr. Hart's energy and in 1915 he withdrew. This incident is mentioned to illustrate his daring spirit. In 1914 business in Portland was at the lowest ebb in history—you could hire the entire town for twenty-five cents an hour. Mr. Hart had accepted family responsibilities, and was having plenty of trouble getting on his feet, but he had to have his own business.

His next connection was with the Pacific Bridge Company, then engaged in the construction of the piers for the Interstate Bridge across the Columbia River between Portland, Ore., and Vancouver, Wash.; also upon the paving of a 10-mile section of the Columbia River Highway from Multnomah Falls, Ore., to the Multnomah County line and past the present site of the Bonneville Dam. The company, then under the presidency of the late Charles F. Swigert,[3] M. Am. Soc. C. E., had had an honorable history dating back to its founding in California by R. W. Gorrill in 1869. This is a long life for any construction company engaged in the hazardous undertakings that appealed to the Pacific Bridge Company. However, it had not had an unbroken line of successes; after a brief period of success a crushing setback would usually follow, and these setbacks would be more than any but the dauntless spirit of Mr. Swigert could endure.

The two aforementioned projects proved quite profitable, but the company was to run true to form and dissipate much of its hard won gains in an ill-advised venture in the East in 1916 where the existence of pre-war conditions made successful operations impossible. Mr. Hart would be the last to claim credit, but this reverse set him to thinking, and his function thenceforth was to build an organization for the Pacific Bridge Company that would render such setbacks impossible and that would enhance the company's successes and allow it to attain its so well-deserved place in the construction industry. The realization of this ambition was to take some time, but the leaven was working, and, knowing Mr. Hart, it was only a question of how many years would elapse until he had just the kind of organization he wanted, and the company could make profits with some hope of keeping them.

Then came the war. Mr. Hart enlisted in the Corps of Engineers; served at Ft. Humphreys, Virginia; received a commission as Captain on November 7, 1918; and was mustered out December 31, 1918.

After the war came a vast amount of highway work in Washington and Oregon, at first quite profitable; and of this work the Pacific Bridge Company received its share. Such work offered little opportunity for personal ingenuity, however, and, as usual in state highway work, there was a constant stream of new bidders coming to the front attracted by the frequent public lettings. This had a demoralizing effect on prices and profits, and toward the end of 1921 Mr. Hart was willing to admit that highway building was not the right business for the Pacific Bridge Company. He was as anxious to drop it as he had been to get out of the transmission machinery business six years before. During this period he had built as his own personal venture the Oswego Creek arch, seven miles south of Portland on the Pacific Highway.

---

[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1657.

He was not to languish for long. In the spring of 1922 the lift bridge across the Cowlitz River in the City of Kelso, Wash., was built. Then a bascule bridge across the Lewis and Clark River about seven miles from Astoria, Ore., was constructed. There was an unusual foundation accident on this structure for which Mr. Hart's company had to pay in large part, but for which they were not to blame. However, they were on their way upward then and such setbacks were " all in the day's work."

Enough progress had been made in the position of the Pacific Bridge Company that Mr. Swigert decided to reorganize the company in August, 1919, and give the younger element recognition. At that time Mr. Hart became a member of the company and its treasurer. He had started as assistant engineer and had been advanced to engineer in November, 1917. In 1928 he became vice-president of the company, and in 1935 succeeded Mr. Swigert as president.

In the fall of 1922 the City of Portland issued bonds for the construction of the Burnside and Ross Island bridges across the Willamette River, and in the fall of 1923, under Hedrick and Kremers as consulting engineers, called for bids on the foundation of the Ross Island Bridge. The late Ira G. Hedrick,[3] M. Am. Soc. C. E., had been brought up on Midwest pier construction methods, and insisted on building this bridge on pneumatic caissons. The Pacific Bridge Company had developed quite a technique for building open caisson pile foundations and this had become standard practice in the Northwest. They had all the equipment for building open foundations cheaply and desired to qualify, if possible, as bidders on the pneumatic foundations at Ross Island. Mr. Hart, with the assistance of John Lyle Harrington, Ernest Emanuel Howard, Members, Am. Soc. C. E., and the late Louis Russell Ash, consulting engineers of Kansas City, Mo., prepared a plan for doing all the excavating work in the open, then sinking the pneumatic deck in the hole, sealing it around the outside, weighting it on top, applying the locks, blowing out the air, and allowing the engineers to examine the bottom. Thereafter the work was continued as for a pneumatic foundation. On this the Pacific Bridge Company made the low bid for the work and was promptly rejected as not offering a pneumatic caisson. However, Mr. Hedrick's eyes had been opened as to the possibilities of the open caisson and he proceeded to make new plans for both bridges.

The next letting came in the spring of 1924 and, after considerable controversy due to unscrupulous competition, the contract for the Burnside Bridge was awarded to the Pacific Bridge Company. This work, after three bad setbacks in the spring of 1935, proved quite successful and at this time Mr. Hart may be said to have realized his ambition for the success of the Pacific Bridge Company.

After that came in rapid succession the following contracts:

---

[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 1932.

1925—Foundations for Ross Island Bridge, Portland.
1926—Bridge over Cowlitz River for the City of Longview, Wash.
Harbor wall along the Willamette River in Portland (the Laurgaard plan).
1927—Subaqueous pipe crossings for the Mokelumne pipe line under the three branches of the San Joaquin River (in connection with J. F. Shea Company).
1928—Foundations for Longview Bridge across the Columbia River.
1929—Foundations for St. Johns Bridge across the Willamette River in Portland.
Foundations for Aurora Avenue Bridge across Lake Union, Seattle, Wash.
Submerged pipe line for the Standard Oil Company of California in the Pacific Ocean at Estero Bay, California.
1930—Pipe lines and submerged pipe lines for Everett, Wash.
Headworks and pipe line for Anacortes Water Supply, Anacortes, Wash.
1931—Submerged pipe lines across San Joaquin River on the Hetch Hetchy project.

In 1931 the Pacific Bridge Company became a constituent company of Six Companies, Incorporated, on the main contract for the Hoover Dam, and Mr. Hart was made director. This company, with the cooperation of the J. F. Shea Company, also bid in the contract for the main piers of the Golden Gate Bridge. The award was made, but litigation ensued and no construction could proceed until July, 1932. Previous to that time the Pacific Bridge Company had been released from its bond. It elected to bid again for the work and was low bidder for the main piers in October, 1932.

Difficulties were to arise, however, which were to test the mettle of Mr. Swigert, Mr. Hart, and their associates, as men had rarely been tested before. The construction of the south pier of the Golden Gate Bridge has been characterized by the late D. E. Moran,[4] M. Am. Soc. C. E., as the most difficult foundation job ever undertaken. The specifications required pneumatic caisson construction but allowed only fifteen months for the completion of the work. In the hope of making good on these conflicting requirements, desperate expedients were resorted to in regard to building the caisson away from the site and making all the excavation in the open. The stormy season of 1933 found this company just ready to place the steel fender sections for the south pier in 100 ft of water. The Pacific Ocean, however, was soon to claim its own, and on October 31 the first fender sections were wrecked by a storm while being lowered. It became evident that a drastic revision of plans was necessary for success, but before the plans were completed another storm arose on December 13 and demolished the outer 700 ft of the company's access trestle to the south pier.

With the assistance of their consulting engineers, Moran and Proctor, new plans were made and approval obtained on the principle of making more use of underwater construction. In other words, there was to be no construction project above the water that could be ruined by a storm until the entire pier was completed to 17 ft below water. Then the fender wall was to be built up above the water a little at a time until finally the entire fender ring was completed and after that the shaft could be built inside of it. To Mr. Hart should be given full credit for knowing what his divers could do and for developing the construction methods for getting the fender up to and through the water. In April, 1934, the company succeeded in concreting its first fender section by this method. Quiet water was ahead that summer and Fortune smiled on the company, and the entire pier was completed about January 1, 1935.

---

[4] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1840.

Previous to that time the company had participated in several syndicates organized for the large PWA jobs that were then appearing. They were a constituent company of the Transbay Construction Co. which executed the contract for the piers for the suspension spans of the San Francisco-Oakland Bay Bridge, and for the 85-ft extension of the O'Shaughnessy Dam for the City of San Francisco, Calif., water supply let in January, 1935. They also had an interest in the construction of the Bonneville Dam power-house foundation and power house by the General Construction Company and J. F. Shea Company, and in the construction of the Parker Dam across the Colorado River with the J. F. Shea Company.

The anxieties of the Golden Gate Bridge proved too much for Mr. Swigert and he died on June 23, 1935. Mr. Hart succeeded him as president of the Pacific Bridge Company. Contracts executed by the company under Mr. Hart were:

1935—Subaqueous pipe line for Hetch Hetchy water supply under Dumbarton Straits, San Francisco Bay.

1936—Concrete pier for Naval Ammunition Depot for the Bureau of Yards and Docks, Balboa, Canal Zone.

Concrete deck for the Golden Gate Bridge.

1937—Railroad facilities over the steel spans of the San Francisco-Oakland Bay Bridge.

In 1937 the Pacific Bridge Company participated with the George Pollock Company and other associates in the construction of Drydock No. 3, Mare Island Navy Yard, for the Bureau of Yards and Docks. Mr. Hart's company was the operating company and the work was not completed at the time of his death. In 1937 they became a constituent company of the Consolidated Builders, Inc., later engaged in the completion of the Grand Coulee Dam. They were also a part of the syndicate that constructed the San Francisco railway terminal building of the San Francisco-Oakland Bay Bridge.

In 1938 the Pacific Bridge Company headed the syndicate that held the contract for the Tacoma Narrows Bridge, the syndicate being known as the Pacific-General-Columbia, and was the active operating company for this work.

It was while returning to his headquarters in San Francisco from his home in Portland after signing this contract that he met his untimely death. In the wreck of the Portland-San Francisco plane off Pt. Reyes on the morning of

November 29 the waters he had conquered so often at last claimed him for their own.

Among the keenest ambitions of most constructive minds is to leave an organization that can carry on after the founder of the company is gone. This was so in the case of Mr. Swigert: the company's activities since 1935 prove that he erred not in choosing his successor. It is also true in the case of Mr. Hart, and the successful prosecution of the Tacoma Narrows Bridge contract without his leadership proves that the organization he worked so hard to establish can still carry on.

Mr. Hart's contributions to engineering must be appraised from his actions rather than his words. His professional writings were very meager. He was a pioneer in making a profitable business out of what a diver can do. It was his boast that his divers could do anything under water that the ordinary man could do on the land. He developed new methods for laying submarine pipe and was known as an authority on that subject by every corporation west of the Mississippi River that had submarine pipe to lay. This resulted not only in profits to his company, but in large savings to his company's customers. The company before him, and he himself, had done much to make the open caisson foundation of tremie concrete poured among piles the successful foundation procedure it is today. He had the confidence of many prominent engineers engaged in designing foundation works. How much of the success of such structures as the foundations of the Longview Bridge and the two San Francisco bridges is due to suggestions originating with him he would be far too modest to say. He knew the sawmill business, and two of his most noteworthy successes—the Portland harbor wall and the track construction of the San Francisco-Oakland Bay Bridge—arose from applying sawmill methods to the ordinary contractor's job.

As to Mr. Hart's mental processes, he was somewhat opportunistic; he could scarcely qualify as a high-class technical man familiar with the accomplishments of others through technical literature. Where would he find the time for research? Yet he possessed the ability to follow through a complicated train of thought without benefit of mathematical processes and he usually arrived more or less intuitively at the right answer. This ability to accomplish so much with so little apparent effort was at once the admiration and despair of those who tried to compete with him. The making of a contractor's estimate for a large job, getting all the elements in and properly valued, is a laborious undertaking. Mr. Hart had the faculty of making a few figures which no one else could understand and he usually came out with a low bid which "left nothing on the table." The difficulties he had in nearly every case arose from his relying too strongly on the judgment of others, or in not being able to prevail upon his associates as to what course to follow.

Mr. Hart cannot be judged fairly by ordinary professional standards. He had qualities of courage and dogged pertinacity for which the ordinary engineer can only sigh in vain. To grasp the Golden Gate south pier situation as it was in the winter of 1933, and overcome all the obstacles that stood in the way of success at a time when few people believed the pier could be built at all and in the face of certain ruin for his company in the event of failure, is an accomplishment which won him the admiration of the entire thinking community in San Francisco.

The difficulties others had in following him made him impatient with other men. In later years he became impatient with any but the swiftest means of transportation, and this quality, more than anything else, led to his untimely death. For one who had borne his heavy responsibilities for so many years, he was extremely youthful. May the spirit of eternal youth which he typified never perish!

He was married in 1913 to Milla Wessinger, of Portland, who survives him. He leaves three children, Philip, Jr., Louise, and Dixon. He was a member of the Benevolent and Protective Order of Elks.

Mr. Hart was elected a Member of the American Society of Civil Engineers on April 18, 1927.

RALPH STEVENSON HAWLEY, M. Am. Soc. C. E.[1]

DIED JULY 30, 1939

Ralph Stevenson Hawley was born in Hanford, Calif., on January 11, 1877. He was the son of Luther C. Hawley (born in Ohio on May 4, 1829), a Civil War veteran, and Alice (Muldrew) Stevenson (born in Kentucky on August 15, 1843) who were married in Vandalia, Ill., on November 23, 1865. Ralph was the fifth of ten children. Two brothers and a sister survive him.

Ralph entered the University of California at Berkeley, Calif., in August, 1899, and was graduated from the College of Civil Engineering in May, 1903, at commencement exercises, made memorable by the presence of President Theodore Roosevelt, in the then partly completed Greek Theater of the University.

Immediately after graduation he received an appointment as hydrographic aid (later assistant engineer) in the U. S. Geological Survey, and was assigned to water resources and reclamation studies in Owens Valley, California, from June, 1903, to October, 1904. He developed much of the data upon which the City of Los Angeles (Calif.) later based its estimates for the great Owens River Aqueduct. He was then transferred to Yuma, Ariz., until February, 1905, thence to Los Angeles for three months, and finally to Berkeley, where he served until April, 1907, principally on hydrographic work in northern California under W. B. Clapp.

In April, 1907, he was appointed assistant city engineer in the office of the city engineer of Oakland, Calif. In this employment he prepared specifications and plans for street construction, sanitary and storm sewers, and the salt-water fire protection system. In February, 1908, he was appointed city engineer of the contiguous town (later city) of Emeryville, Calif.

---

[1] Memoir prepared by Harold Farnsworth Gray, M. Am. Soc. C. E.

Mr. Hawley served the City of Emeryville for a little more than thirty-one years as city engineer, and in addition was superintendent of streets and assistant city clerk from 1909. As the city grew, practically all of the storm and sanitary sewers, the street work, and the general municipal improvements of the present city were designed and constructed under his supervision. In the aggregate these works ultimately amounted to an impressive total value, although no individual project of great size was included therein; but if, in the somewhat prosaic development of the steadily growing industrial community, no great works were built, the many smaller projects reflected the character of the man; they were designed on sound principles, were well and honestly built, and have endured effectively in service.

Ralph Hawley, however, was far more to his city than its engineer. Quietly and unobtrusively he accepted other public services which his fellow townsmen placed in his hands because they recognized his integrity and dependability, and turned naturally and confidently to him when there was community work which needed to be done. For others was the limelight and publicity; for him was the modest but effective labor which enabled these community projects to succeed.

For eighteen years he was clerk of the Board of Trustees of the Emery Unified School District. He was one of the organizers of the Emeryville Industrial Association in 1920, and served as its secretary until his death. He was appointed secretary-treasurer of the Emeryville Welfare Commission in 1926 and served as such continuously thereafter. In January, 1936, he was appointed a trustee of the Alameda County Mosquito Abatement District by the City Council. He was also vice-president of the Board of Trustees of the District in 1938, and was elected president of the Board of Trustees in January, 1939.

In June, 1937, he was elected president of the East Bay Engineers Club for the 1937–1938 club year. On January 11, 1939, he was installed as president of the Emeryville Kiwanis Club. In the Kiwanis Club he had the distinction of 100% attendance for ten years.

Mr. Hawley became a member of the First Presbyterian Church in Berkeley in 1899, and was continuously thereafter a member, serving as a Sunday School teacher and as an Elder. He was a sincere and consistent Christian, living the Golden Rule in every action of his daily life. This, with his kind, strong face and gentle manner, brought him affection and admiration, while his fine character inspired respect untinged by envy. At the news of his sudden and utterly unexpected death, the first thought of many was "I have lost a friend."

He was married to Elizabeth Duncan at Bishop, Calif., on November 7, 1903. Mrs. Hawley and their two daughters, Ethel June and Genevieve, survive him.

Mr. Hawley was elected a Member of the American Society of Civil Engineers on December 5, 1911.

WILLIAM CHAUNCEY HAWLEY, M. Am. Soc. C. E.[1]

Died December 10, 1939

William Chauncey Hawley was born in Cambridge, Washington County, N. Y., on August 31, 1865. He was the eldest child of Charles Townsend Hawley and Fannie (Warner) Hawley. The Hawley family came to this country from England in 1630, settling first in Connecticut and later in Vermont and New York. During the Revolutionary War they were Loyalists, and did not fight for the Colonies. However, Mr. Hawley's paternal grandmother and great grandmother brought in strains of Dutch and Scotch-Irish blood, and he had at least five ancestors who fought in the Revolution.

Mr. Hawley's earliest schooling was in Cambridge, but when he was only eight years old he was sent to the Academy at Salem, N. Y., where an uncle was headmaster. It was there that he prepared for college. He entered Rensselaer Polytechnic Institute at Troy, N. Y., and was graduated in 1886, before he was twenty-one, with the degree of Civil Engineer.

His first engineering experience was in the designing and building of the plant for the Cambridge Water Works Company in his home town. The plant is still being operated substantially as Mr. Hawley designed it in 1886–1887, and has proved successful, not only as a safe and dependable source of water supply, but as a sound financial investment for those who were interested in such a supply for a small village in the early days of water works.

From 1887 to 1893 Mr. Hawley was assistant and principal assistant to the late Chester B. Davis, M. Am. Soc. C. E., engaged on surveys, making plans and constructing various water works and sewage systems. The year 1893–1894 he served as assistant engineer on surveys and plans for additional water supply for Troy. The following year he was resident engineer on construction of water works at Grafton, W. Va., for Wilkins and Davison of Pittsburgh, Pa. In 1895, as assistant engineer to Robert Swan of Allegheny City, Pa., he designed the Montrose Pumping Station and 9½ miles of 60-in. riveted steel force main and later was in charge of construction of 6 miles of the latter. From 1896 to 1902, Mr. Hawley was engineer and superintendent for the Water Department of Atlantic City, N. J.; and from 1902 until his death, he acted as chief engineer, superintendent, and vice-president of the Pennsylvania Water Company at Wilkinsburg, Pa.

Mr. Hawley was Past-President of the Engineers' Society of Western Pennsylvania, a member of the American Water Works Association, New England Water Works Association, Past-President and member of the Executive Committee of the Pennsylvania Water Works Association. He was the author of many articles published in the water works journals.

On December 25, 1890, he was married to Nellie N. Newton. He is survived by his widow; one son, William Chauncey Hawley, Jr.; one daughter, Mrs. Margaret Hawley Ely; and four grandchildren.

---

[1] Memoir prepared by Charles L. Fox, M. Am. Soc. C. E.

Mr. Hawley was deeply religious, being a communicant of the Protestant Episcopal Church. At the time of his death he was a Vestryman of the Calvary Protestant Episcopal Church of Pittsburgh. His intimate knowledge of water works management made his advice sought by other leaders. He was a man of upright character and sterling qualities that endeared him to his fellowmen. His untimely sudden death is sincerely regretted by all who have had the privilege of his acquaintance and association.

Mr. Hawley was elected a Junior of the American Society of Civil Engineers on October 1, 1890; an Associate Member on April 6, 1892; and a Member on December 3, 1902.

JAMES CLARK IRWIN, M. Am. Soc. C. E.[1]

Died March 20, 1940

James Clark Irwin was born at Cheyney, Delaware County, Pa., on September 23, 1868, the son of William Henry and Catherine (Browning) Irwin. His preparatory education was received at the Protestant Episcopal Academy at Philadelphia, Pa. Entering the University of Pennsylvania, at Philadelphia, in 1886, he received the degree of Bachelor of Science in 1890 and the degree of Civil Engineer in 1891. While attending the University of Pennsylvania he held various class positions, and upon graduation was instructor in Civil Engineering for the year 1891–1892.

Mr. Irwin entered the service of the New York Central and Hudson River Railroad Company on July 1, 1892, in charge of construction of block signaling between Buffalo, N. Y., and Albany, N. Y. He continued in the service of the New York Central until his retirement from active service on October 1, 1938.

During Mr. Irwin's employment by the New York Central System he held the positions of assistant trainmaster, assistant superintendent of signals, superintendent of the Hudson River Bridge Company, division engineer, signal engineer, assistant to the general superintendent, superintendent of the Lake Erie, Alliance and Wheeling Railroad Company, engineering assistant on the staff of the vice-president, resident engineer and superintendent of construction on the Grand Central terminal improvements, and office engineer in charge of contracts. From 1909 to 1912 he served as chief engineer of the Rutland Railroad Company, and from 1912 to 1938 he was with the Boston and Albany Railroad Company, being valuation engineer from May, 1920.

Mr. Irwin was one of the pioneer engineers in modern railway signaling, one of the charter members of the Railway Signaling Club, predecessor of the Railway Signal Association (later the Signal Section, Association of American Railroads). He continued his active and helpful interest in railway signaling during his entire career.

---

[1] Memoir prepared by E. M. Hastings, M. Am. Soc. C. E.

He was elected a member of the American Railway Engineering Association in 1907 and was very active in its affairs. He served on the Committee on Uniform Contract Forms, being its chairman from 1927 to 1931; on the Committee on Standardization, being its chairman from 1931 to 1935; and was vice-chairman of the Committee on Rules and Organization at the time of his death. He served as a director from 1932 to 1935, vice-president from 1935 to 1937, and president from 1937 to 1938. As president of the A. R. E. A. he was chairman of the Engineering Division of the Association of American Railroads and did much to further the research in engineering matters for that association.

In 1930 Mr. Irwin was appointed to represent the Association of American Railroads Engineering Division on American Standards Association, Standards Council, and was elected chairman of the Standards Council for the three years 1933 to 1936. He was a member of the Board of Directors of the American Standards Association from 1934 to 1936, and ex-officio member of the Board and Council from 1937 to 1939. Mr. Irwin's thoroughness and unusual ability fitted him admirably for the exacting service he performed for the American Standards Association.

In all of the diversified work that he carried on for the railroad by which he was employed, for the associations he served, and for the organizations and committees he assisted, he exercised untiring effort and care; no task was too small nor too great to obtain his undivided interest and attention. Mr. Irwin found much time to devote to those other things of life, outside of his profession, that leave their mark on communities and individuals and live long after a life has passed beyond. For fifteen years he was a vestryman, junior warden or senior warden of St. John's Church, Newtonville, Mass. He was a member of the Episcopalian Club of Massachusetts; he served as a member of its council, as chairman of its membership committee, and as president in 1933. He was Scoutmaster, troop committeeman, district commissioner, and a member of the Greater Boston Council, Boy Scouts of America, from 1913 to 1933.

Mr. Irwin was active in New England affairs during the World War. He was chairman of the New England Committee on the Study of Military Mobilization in 1916 and 1917, and was a member of the Business and Professional Men's Military Battalion. From 1917 to 1920 he was chairman of "Troop C Home Folks" and "Home Association of the 102nd Machine Gun Battalion, 26th Division." In 1918 he was appointed by Governor McCall as a delegate from Massachusetts to the "Win the War" convention in Philadelphia. Three sons of Mr. Irwin served in the World War.

He was active in many other ways; among such activities he was:

Member of the New England Railroad Club, serving on its executive committee and as president;
Member of the New York Railroad Club;
Member of the Railway Guild (railway honorary society);
Life Member of The Episcopal Academy Alumni Society;
Member of The General Alumni Society, University of Pennsylvania, also serving as a member of the board and as vice-president;

Member of the Associated Pennsylvania Clubs, serving as president;
Member of the New England Alumni Society of the University of Pennsylvania, serving as president;
Member of the Committee on Annual Giving of the University of Pennsylvania;
Member of Academy of Political Science;
Member of National Security League; and
Governor of The University Club, Boston, Mass.

He was profoundly interested in young men; his kindly counsel and affectionate interest left its impression on many a young man and helped him over difficult places. Although those who knew him best admired greatly his professional attainments and respected his opinions and judgment, they loved him most for those sterling qualities of Christian character, moral integrity, high and noble purposes, and his love for home and family. It was always a privilege to be with him; truly it may be said that James Clark Irwin made the world a better place in which to live.

On June 5, 1895, he was married to Frances Smith Monaghan of West Chester, Pa. They had four sons, James C., Robert M., William H., and John, and one daughter, Katherine. Three sons, James, Robert, and John, and daughter, Katherine, together with Mrs. Irwin, survive him.

Mr. Irwin was elected a Junior of the American Society of Civil Engineers on April 4, 1893; an Associate Member on December 7, 1898; and a Member on June 6, 1900.

CHARLES WHITE JOHNS, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 16, 1939

Charles White Johns was born in Farmville, Va., on December 28, 1875. His parents were James M. and Pauline (Cassell) Johns, and his early education was received in the public schools of Farmville and Portsmouth, Va. After a term in business college, he concluded that his interest was in engineering and, under the tutorage of his uncles, C. M. Cassell and Virginius O. Cassell, engineers and architects of Norfolk, Va., he obtained his preliminary training.

His engineering experience began in 1891 when he took a position with the Norfolk, Wilmington and Charleston Railway Company, a projected line, as rodman and chainman on surveys. In 1893 he was employed as draftsman in the office of his uncles, and in April, 1898, became instrumentman with Graham and Riddle and later with Haenselman and Smith, both engineering firms of Norfolk. This gave him a varied experience in surveying, subdivision work, and sanitary and other general municipal construction projects.

---

[1] Memoir prepared by A. C. Copland, M. Am. Soc. C. E., and Crosby Miller, Assoc. M. Am. Soc. C. E.

It was in September, 1899, that he accepted employment in the engineering department of the Chesapeake and Ohio Railway Company and continued with that company for forty years. His first assignment was instrumentman on the construction of a grain elevator, a merchandise pier, and a coal pier, at Newport News, Va. In May, 1900, he was transferred to Richmond, Va., as assistant engineer on the joint passenger station of the Chesapeake and Ohio and Seaboard Air Line Railway Companies. This work included grade separation and the construction of a viaduct about 3 miles long which elevated the tracks along the bank of the James River in Richmond. In addition, his work embraced line and grade revisions of the division of the railway extending from Richmond to Clifton Forge, Va., and grade separation at Charlottesville, Va., with construction of yards and sidings.

In December, 1904, Mr. Johns was advanced to assistant division engineer at Hinton, W. Va., in charge of maintenance of branch lines in West Virginia, and in April, 1906, to division engineer of the Huntington and Greenbrier Divisions. Subsequently, he was advanced to the position of assistant engineer of maintenance of way at Huntington, W. Va., in charge of the maintenance of all lines of the railway in West Virginia.

Mr. Johns was promoted to engineer of branch lines, with headquarters at Richmond, in February, 1914. This led to the position of engineer of construction in August, 1920, and in January, 1923, to chief engineer, in which capacity he continued until his death.

Among the many important engineering works which were under his direction as chief engineer were large classification yards and engine terminal facilities at Clifton Forge; replacement of the Ohio River bridge at Cincinnati, Ohio, including separation of grades in Covington, Ky., and Cincinnati; enlarging old tunnels and constructing new ones to provide increased clearances for modern equipment and large shipments; a coal dumper on the dock at Presque Isle, Toledo, Ohio; a coal pier and dumper at Newport News for unloading coal from railroad cars to ships; large locomotive shops at Huntington; and car shops at Russell, Ky. In later years the separation of grades of railroads from highways and city streets occupied his attention in a large measure.

Mr. Johns had an agreeable disposition, always courteous and kindly, which resulted in his having many friends among his acquaintances in the railroad business as well as other interests with which his work brought him in contact. He was considerate of all, especially his immediate associates, and this inspired their greatest respect.

Mr. Johns was an active member of the American Railway Engineering Association, having served on many of its committees and being a member of the committee on rail at the time of his death. He was also a member of the committee on steel of the American Society for Testing Materials. He was actively interested in the Virginia Section of the American Society of Civil Engineers and was vice-president from 1936 to 1938, becoming president in January, 1939. He was a member of St. Pauls Protestant Episcopal Church of Richmond and the Acca Temple Shrine, Ancient Arabic Order, Nobles of the Mystic Shrine.

He was married to Delia H. Brooke of Portsmouth, who, with a daughter, survives him.

Mr. Johns was elected a Member of the American Society of Civil Engineers on June 1, 1925.

ARTHUR NEWHALL JOHNSON, M. Am. Soc. C. E.[1]

DIED JULY 11, 1940

Arthur Newhall Johnson was born in Lynn, Mass., on November 11, 1870, the son of David Newhall and Amanda Malvina (Richardson) Johnson. His technical education was obtained at the Lawrence Scientific School of Harvard University, at Cambridge, Mass., from which he was graduated in 1894 with the degree of Bachelor of Science in Civil Engineering.

During the academic year 1896–1897 he served as instructor in Descriptive Geometry at Harvard. He left the teaching position to become assistant engineer at the Calumet and Hecla Mine at Calumet, Mich. In 1897 he joined the engineering staff of the Massachusetts Highway Commission. At that time the organization was engaged in its pioneer efforts that led to modern highway construction.

In 1898 he was called by the Maryland Geological Survey to make a survey of road conditions and road building materials in that state. In the seven years he spent in Maryland he made a thorough study of the road building resources of the state, built demonstration macadam roads, and so thoroughly promoted the good roads movement that in later years Maryland became one of the " best roaded " states in the Union.

In recognition of his successful work in Maryland, in 1905 Mr. Johnson was invited to become chief engineer of the newly established United States Office of Public Roads in Washington, D. C., later the Public Roads Administration. After serving a year in this position, he was named state highway engineer of Illinois, where he laid the beginnings of the excellent system of highways in that state.

From 1914 to 1916 he was highway engineer of the New York Bureau of Municipal Research. He left that organization to become consulting highway engineer for the Portland Cement Association. After four years in this position, he resigned to become Dean of the College of Engineering at the University of Maryland, at College Park, Md. He reorganized the engineering courses and established the Maryland Engineering Experiment Station. In cooperation with the U. S. Bureau of Public Roads, and the Maryland State Roads Commission, he conducted research studies on highway traffic, and on the strength and elastic properties of concrete, particularly as related to its use in road construction. His ability as a teacher and his kindly manner endeared him to all his students.

---

[1] Memoir prepared by S. S. Steinberg, M. Am. Soc. C. E.

He was author of a number of technical papers and reports. He was a member of the American Society for Testing Materials and of the Highway Research Board of the National Research Council, serving as chairman of the Executive Committee of the latter group from 1923 to 1926. He served as president of the American Association of Engineers in 1922. He was a member of the Cosmos Club in Washington, D. C., and a member of Tau Beta Pi.

On two occasions he represented the United States in conferences held abroad. In 1913 he was delegate to an international highway conference in Paris, France, and in 1925 he was appointed, by President Coolidge, a member of the American delegation to the Pan-American Road Conference in Buenos Aires, Argentine Republic.

In 1924, the University of Maryland, in recognition of his achievements in highway engineering, and particularly of his pioneer work in Maryland, conferred on him the honorary degree of Doctor of Engineering.

In 1932, Dean Johnson was the recipient of the Bartlett Award presented annually "for outstanding contribution to highway progress." The selection for this award is made by a committee consisting of the presidents of the American Road Builders' Association, the Association of State Highway Officials, and the Highway Research Board of the National Research Council.

In the passing of Dean Johnson, the profession has lost one of its pioneers in modern highway development. In the course of his long career in the highway field, it fell to his lot to train many men, some of whom became the outstanding highway engineers in the profession.

In 1936, due to continued ill health, Dean Johnson found it necessary to retire from the University, having served sixteen years as Dean of the College of Engineering.

He was married to Mae Louise Ash on September 12, 1900. He is survived by his widow and three daughters, Louise N. (Mrs. Lewis P. Shanks), Evelyn N., and Mary W. Johnson.

Dean Johnson was elected an Associate Member of the American Society of Civil Engineers on September 2, 1903, and a Member on July 1, 1909.

RICHARD KHUEN, JR., M. Am. Soc. C. E.[1]

DIED SEPTEMBER 24, 1938

Richard Khuen, Jr., was born at Saginaw, Mich., on July 27, 1865, the son of Richard and Caroline (Scheib) Khuen.

After graduating from the University of Michigan, at Ann Arbor, Mich., in 1889, with the degree of Bachelor of Science in Civil Engineering, he entered the employ of the late George Shattuck Morison,[2] Past-President, Am. Soc. C. E., with whom he remained until August, 1891. From July,

---

[1] Memoir prepared by Marshall Williams and C. F. Goodrich, Members, Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LIV, June, 1905, p. 513.

1889, to October, 1890, he was assistant engineer on the construction of the Merchants Bridge, St. Louis, Mo.; from July to October, 1890, assistant engineer on the construction of the Memphis Bridge, Memphis, Tenn.; and from October, 1890, to January, 1891, draftsman in Mr. Morison's Chicago office. From January to June, 1891, he was assistant inspector of steel superstructure of the Memphis and Winona bridges at Union Bridge Company, Athens, Pa., and from June to August, 1891, he was inspector of steel superstructure of Memphis Bridge at Pennsylvania Steel Company, Steelton, Pa.

In August, 1891, he took the position of bridge inspector and assistant engineer for the New York, Lake Erie and Western Coal and Railroad Company. In November, 1892, he became assistant engineer for the Quaker City and North Eastern Elevated Railroads in Philadelphia, Pa., where he remained until July, 1893. During this period the service was under the late Carl Waldemar Buchholz, M. Am. Soc. C. E.

In July, 1893, Mr. Khuen entered the service of the bridge and construction department of the Pencoyd Iron Works (A. and P. Roberts Company) at Pencoyd, Pa., as assistant engineer in charge of estimating and designing, under the late Charles Conrad Schneider,[3] Past-President, Am. Soc. C. E. From February to May, 1898, he was resident engineer for the Pencoyd Iron Works during the erection of the Niagara Falls-Clifton Bridge (840-ft arch span). This bridge was destroyed by the ice in January, 1938.

In February, 1901, following the merging of the Pencoyd Iron Works with the American Bridge Company in May, 1900, Mr. Khuen was transferred to Pittsburgh, Pa., where he held the position of Pittsburgh division engineer for the American Bridge Company. On May 18, 1914, he was appointed general manager of erection, succeeding the late Emil Gerber,[4] M. Am. Soc. C. E. On April 1, 1931, he was appointed chief engineer, succeeding the late James Harvey Edwards,[5] M. Am. Soc. C. E.

He retired from active service with the American Bridge Company on July 1, 1935, shortly before his seventieth birthday. After his retirement, and until his death, he maintained an office as consulting engineer in Pittsburgh.

All branches of bridge and structural engineering benefited from the influence of his efforts and activities. He accommodated himself to the many advancements in design, detail, materials, and methods which took place during his many years of engineering, and, in many instances, took a leading part in the transition. A statement of the outstanding structures that passed through his engineering and erecting administrations is much too long for publication. They cross the streams and dot the landscape in all parts of the United States and in numerous foreign countries. They are representative of the strength of affection and high regard that he built in the hearts of his friends and associates.

His entire professional life was spent in bridge and structural work. He was well founded technically and was also endowed with a keen commercial sense, which made him an ideal engineer—"one who produced the greatest return on the dollar invested." He was freely consulted by his fellow engineers and cheerfully contributed from his great fund of engineering knowledge, gained over a long term of experience. He was a quiet, genial gentleman who enjoyed social contacts with those whom he accepted as worth while. He was rather strong in his likes and dislikes, an attribute which led to a careful selection of those whom he chose to "grapple to thy soul with hoops of steel." He adopted a strict code of social and business integrity, was a capable engineer, a successful administrator, and an honorable, enjoyable gentleman at all times.

---

[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXI, December, 1917, p. 1665.

[4] For memoir, *loc. cit.*, Vol. LXXIX (1915), p. 1418.

[5] For memoir, *loc. cit.*, Vol. 96 (1932), p. 1463.

In 1895 Mr. Khuen was married to Elizabeth Meason, of Greensburg, Pa. She died on February 10, 1929. A son, Richard Khuen, III, two sisters, and a brother survive him.

Mr. Khuen was a member of the American Railway Engineering Association; the Engineering Society of Western Pennsylvania; the Railway Club of Pittsburgh; the Metropolitan Club of Pittsburgh; the Edgeworth Club of Sewickley, Pa.; and the Montour Heights Golf Club, at Coraopolis, Pa. He was a director of the Engineering Society of Western Pennsylvania for the years 1922–1924, and served on a number of important committees during his long term of membership. In 1917 he was the organizer, founder, and first President of the Pittsburgh Section of the American Society of Civil Engineers, and his interest did not relax during his lifetime.

Mr. Khuen was elected Director of the American Society of Civil Engineers for the term 1916–1918, and was the candidate for Vice-President from Zone II on the first ballot at the time of his death. He was a regular attendant at the Society conventions and gave unstintingly of his time and effort to committee and other work that presented itself in the Society's activities.

Mr. Khuen was elected a Junior of the American Society of Civil Engineers on December 3, 1891; an Associate Member on May 1, 1895; and a Member on November 2, 1898.

PHILIP GEORGE LANG, JR., M. Am. Soc. C. E.[1]

Died December 9, 1939

Philip George Lang, Jr., was born in Philadelphia, Pa., on September 24, 1883, the son of Philip George Lang and Marie Louise (Beck) Lang.

He received his early education in the public schools of Philadelphia, and in the North East Manual Training School, graduating from the latter in June, 1901. In the succeeding autumn he entered the University of Pennsylvania, at West Philadelphia, Pa., where he successfully completed the four-year engineering course, and was graduated in June, 1905, with the degree of Bachelor ot Science in Civil Engineering. In June, 1922, he received the postgraduate degree of Civil Engineer from that University.

---

[1] Memoir prepared by a Committee of the Maryland Section consisting of C. E. Sloan, H. A. Lane, and E. J. Dougherty, Members, Am. Soc. C. E.

Shortly after graduation Mr. Lang became a draftsman in the Pencoyd Plant of the American Bridge Company. On March 16, 1906, he entered the service of the South and Western Railroad (later Carolina, Clinchfield, and Ohio Railway Company), at Johnson City, Tenn., as designer. On December 4, 1907, he entered the organization of The Baltimore and Ohio Railroad Company, as assistant engineer, at Baltimore, Md.

During the succeeding decade, while engaged in a variety of duties connected with the design, construction, and maintenance of bridges, he acquired diversified experience and intimate knowledge of every portion of the System. For several years part of his time was devoted to making field measurements and preparing record sketches of bridges. This field experience was alternated with long periods in the Bridge Drafting Room, at Baltimore.

In 1915 construction work was begun on the great Export Coal Pier of The Baltimore and Ohio Railroad Company, at Curtis Bay, near Baltimore. These facilities were unique as to magnitude and character of equipment. Mr. Lang was intimately associated with this work, and also with the design and construction of mechanical trimmers for the stowing of coal and other bulk material in the cargo space of vessels; four such devices were installed at the Curtis Bay Terminal between December, 1919, and April, 1920.

On May 1, 1917, he became chief bridge draftsman, and discharged the duties of that office until August 1, 1918, when he was advanced to the position of assistant engineer of bridges. On May 1, 1921, he was appointed engineer of bridges of The Baltimore and Ohio System.

During the World War railroad bridge work was attended by numerous major difficulties, but, within this trying period, every phase of the tasks committed to Mr. Lang's charge proceeded to a successful conclusion. In 1919 The Baltimore and Ohio Railroad Company commenced the reconstruction of its Main Line bridge across the Allegheny River, at Pittsburgh, Pa. In this work Mr. Lang discharged an active rôle. The structural and operating problems were peculiarly difficult and complex, involving the construction of the new bridge at a level about 15 ft higher than that of its predecessor. Real estate conditions and War Department restrictions limited the working space, and major operations were concluded by the jacking of the main spans a vertical distance of about twelve feet. This project is the subject of an excellent paper by Mr. Lang.[2]

During the summer of 1920 work was commenced on the reconstruction of the railroad bridge across the Great Miami River, Lawrenceburg, Ind. This project was completed in the autumn of 1921. Between 1920 and 1929 The Baltimore and Ohio adopted annual programs of bridge reconstruction which involved a total of more than one thousand separate structures and an aggregate expenditure of approximately $30,000,000. Among the large projects with which he was associated during this period were the reconstruction of the railroad bridge across the Little Kanawha River, Parkersburg, W. Va., and that across the Allegheny River, Foxburg, Pa., and major grade separation projects at Millvale, Pa., Bond Hill, Ohio, and Chicago, Ill. Unique problems were presented by the reconstruction of the bascule bridge across the South Branch of the Chicago River, in Chicago, where the new bridge was built at a low level, and subsequently raised to final elevation, a distance of about 11 ft, by hydraulic jacks.

---

[2] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1152.

Objects of Mr. Lang's care included devices for the transfer of coal, ore, and other bulk materials at ports on the Atlantic Seaboard and Great Lakes. During 1924 he supervised the construction of a large ore bridge at Lorain, Ohio. As a result of his studies new and improved devices, designed to facilitate the transfer and prevent breakage of coal, were installed at several waterside points. He supervised the construction and installation of 115-ft turntables at several points on The Baltimore and Ohio System, where it became necessary to replace shorter tables. For several years he served as chairman of the Turntable Sub-Committee appointed by the Iron and Steel Structure Committee of the American Railway Engineering Association, the result of whose labors is represented in the Turntable Specification of 1924, which is still current.

The depression, which commenced in the autumn of 1929, necessitated the adoption of methods and appliances designed to minimize costs and extend service-life of facilities, and several projects involving the rehabilitation of metallic bridges by welding were completed under Mr. Lang's supervision, including an extensive program of welded repairs to the superstructure of the large bridge crossing the Susquehanna River, at Havre de Grace, Md., completed during the late autumn of 1938. From 1935 to 1939 he directed the designing and erection of more than one hundred and fifty bridges in connection with grade separation projects at various locations on The Baltimore and Ohio System. In central West Virginia the construction of a large dam in the Tygart River necessitated extensive line change, including the construction of numerous bridges. During his final years Mr. Lang devoted his attention to these projects and their associated problems.

Apart from the routine duties of his office, Mr. Lang's activities ranged through an extensive field. For many years he was a member of the American Railway Engineering Association, serving on its Iron and Steel Structure Committee. He was a member of the American Standards Association, directing his efforts especially along the lines of rivet standardization. In March, 1937, that organization published recommended rivet head standards, which seem likely to acquire general acceptance in the United States.

For twelve years he served as Chairman of the Joint Committee on Federal Valuation of Railroad Bridges. Under his skilful guidance this work was conducted through its initial stages, and its creative accomplishment is represented in a series of reports embodying essential data for the use of workers in the field of railroad valuation.

Structural welding early engaged his attention. Entering the American Welding Society, he was appointed chairman (in 1934) of the committee charged with the task of preparing specifications for the design and construction of metallic bridges with welded jointures. Under Mr. Lang's chairmanship this committee selected the best features of American and foreign practice, and embodied these in a specification, published in 1936, and since appearing in new and amended editions. A thorough discussion of the application of welding to metallic bridges, written by Mr. Lang, forms a chapter in the "Welding Handbook," published by the American Welding Society in 1938. In October, 1937, the American Welding Society elected him to serve as its president during the succeeding year.

Mr. Lang was connected with the Society for twenty-eight years and, at the time of his death, held membership in its Committees on Structural Alloy and Heat-Treated Steels and Movable Bridges. During the term 1935–1936 he served as President of the Maryland Section. He was a member of the Engineers Club of Baltimore, serving as its president for the 1937–1938 term. His contributions to professional literature are represented in about seventy papers and articles, appearing in the technical press. He served as editor of the "Railway Engineering and Maintenance Cyclopedia," Editions of 1926, 1929, and 1939, published by Simmons-Boardman Publishing Corporation.

Models of railroad bridge types, representative of practice from the beginning of the railroad era in the United States to the present day, built under Mr. Lang's direction for exhibition at The Baltimore and Ohio Centenary Celebration in 1927, were displayed subsequently at the Century of Progress Exposition in Chicago, and finally at Rockefeller Center, in New York, N. Y.

The social side of Mr. Lang's life was represented by membership in various masonic bodies, and in Boumi Temple, Nobles of the Mystic Shrine, at Baltimore, for which latter organization he served as a Trustee. A lifelong member of the Lutheran Church, he served several terms as an elder.

Because of his mastery of technical details, his integrity, sincerity, and activities in working for the advancement of knowledge and practice with respect to all phases of bridge building, he was recognized by his engineering associates as an outstanding member of the profession.

He was blessed with a genial and companionable personality, and was essentially interested in his home, family, and a wide circle of friends. He was prominent and active in community affairs, and his ability and judgment were always recognized and respected by his associates. He became an enthusiastic and discriminating philatelist, leaving a valuable and well-arranged collection of stamps. He was interested in art, and made a collection of original etchings by leading foreign and American artists. He developed a fine library in the field of general literature, which also included numerous authoritative works on the ancient and modern history of his profession.

In January, 1907, Mr. Lang was married to Eugenie Marie LeFort, of Philadelphia, who, with one daughter, Eugenie (Mrs. Robert Carey Reeder, Jr.), survives him.

Mr. Lang was elected an Associate Member of the American Society of Civil Engineers on October 3, 1911, and a Member on November 21, 1921.

WILLIAM DOMINICK LARRABEE, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 13, 1939

William Dominick Larrabee was born on February 21, 1849, at Horicon, Wis., one of the nine children of William Morse and Mary Margaret (Haight) Larrabee. When he was about one year old his family moved to Chicago, Ill. There William Larrabee's youth was spent in an atmosphere of railroad administration, for his father was secretary of the Galena and Chicago Union Railroad Company, and, later, secretary and treasurer of the Chicago and Alton Railroad Company. Of this latter railroad, William Dominick Larrabee was cashier and paymaster from 1873 to 1885. During this time he studied technical subjects, and advanced what he termed his "self education."

In 1887 Mr. Larrabee went to Los Angeles, Calif., and was employed on survey work by the late Edward Thomas Wright,[2] M. Am. Soc. C. E. In 1888 he was in charge of construction of three miles of cable road for the Temple Street Cable Railway of Los Angeles. In 1889 he was a draftsman in the city engineer's office and was a deputy under the city engineer (Los Angeles). He also, with Wright, Meysenburg and Company, contractors, was in charge of construction of 10 miles of cable railroad for the Los Angeles Cable Railway Company. From 1890 to 1893 he was with the Los Angeles Consolidated Electric Railway Company, first in charge of construction of 20 miles of electric railway, and later chief engineer in charge of maintenance of way and construction departments, including the construction of 30 miles of electric railway. In 1894 and 1895 he was chief engineer of the Pasadena and Los Angeles Electric Railway, having charge of preliminary surveys, location, and construction from Los Angeles through Pasadena, Calif., to Altadena, Calif. (about 40 miles, with local system at Pasadena).

For the next two years he was superintendent and chief engineer of the Pasadena and Los Angeles Electric Railway and the Pasadena and Pacific Railway in charge of the operating and construction departments. From 1897 to 1905 he was superintendent and chief engineer of the Pasadena and Pacific Railway and its successor company, the Los Angeles-Pacific Railroad Company. This was a period of great expansion in the interurban transportation system in Los Angeles and its environs, and Mr. Larrabee supervised all preliminary surveys and locations, had charge of all construction of the system (about 150 miles of track with various branches), made plans and supervised erection of the power station and car shed at Sherman (later West Hollywood), Calif., and supervised erection of various substations. In February, 1903, he was appointed chief engineer of the Beach Land Company, where he designed and constructed a dam across a channel 200 ft wide, with tide and flush gates for holding ocean water in a lagoon, and constructed two piers out into the Pacific Ocean (400 ft and 1,500 ft long). His retirement from this position followed the acquisition of the Los Angeles-Pacific Railroad Company by the Pacific Electric Company in 1905.

---

[1] Memoir prepared by Harry W. Dennis, M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXI, December, 1917, p. 1791.

In January, 1906, Mr. Larrabee joined in the incorporation of the United Railways Company. It was intended to construct a railroad from Portland, Ore., to Peak, Washington County, Ore. In February he became vice-president and general manager. The United Railways projected several electric lines in the vicinity of Portland, and Mr. Larrabee made a location survey for a railroad from Portland to the base of Mount Hood. He resigned from the United Railways Company in August, 1906.

Another electric railway venture in which Mr. Larrabee was interested was known as the Redlands and Yucaipa Electric Railway near Redlands, Calif. Of this project, Mr. Larrabee was general manager and chief engineer, and, although a preliminary location was made, it did not progress beyond the promotional stage.

For several years Mr. Larrabee was in Los Angeles as a consulting engineer in private practice and then, in 1918, he entered the employ of the Southern California Edison Company, Ltd., of Los Angeles. This engagement was for a specific problem of railroad location in rugged country as a part of a power development program in the High Sierras. Although the location work was completed, the line was not constructed. Nevertheless, Mr. Larrabee was retained by that company due, among other things, to his wide acquaintance and his ability and tact in handling many special matters, particularly in public relations, and in the acquisition of rights of way. In 1934, in accordance with the terms of the Edison Company's pension plan, he was relieved of further active duty.

Mr. Larrabee was married, at Parsons, Kans., on July 21, 1873, to Mary Adelia Bemis. To them were born three children, Marion B., Mary W., and John S. Larrabee, no one of whom has survived. Mrs. Larrabee died February 5, 1933.

Mr. Larrabee was a member of John Marshall Lodge No. 636, Free and Accepted Masons (West Hollywood, Calif.), and of the Lions Club (West Hollywood, Calif.). He had served as secretary of the West Hollywood-Sherman Civic Association, and as director of the West Hollywood Chamber of Commerce.

In national politics Mr. Larrabee was a Republican who gloried in the fact that, quoting from one of his own letters, "* * * in 1856 I was old enough to make a big noise for our first presidential candidate, John C. Fremont, and have hollered loudly for all our candidates since."

Mr. Larrabee was elected a Member of the American Society of Civil Engineers on October 5, 1904, and became a Life Member in 1929.

ARTHUR HERMANN MARKWART, M. Am. Soc. C. E.[1]

DIED JANUARY 25, 1940

Arthur Hermann Markwart was born at Duquoin, Ill., on February 13, 1880, the son of Hermann and Sarah (Beck) Markwart. When a boy he was brought to California by his widowed mother, his father having died when the son was eight years of age. His mother died when he was thirteen. There were two younger brothers, the care of whom devolved upon the elder boy.

Mr. Markwart's early education was obtained in the public schools and at the California School of Mechanical Arts, in San Francisco, Calif. In October, 1900, he entered the University of California, at Berkeley, Calif., graduating in December, 1903, with the degree of Bachelor of Science in Civil Engineering. While a student at The California School of Mechanical Arts, young Markwart maintained himself by work at the school. Upon graduation from that institution in 1899 he went on a trip to the Philippine Islands on a transport, where the Spanish War was then being brought to a close. When he returned he was employed as chief draftsman in the Judson Manufacturing Company and retained that position during the three years at the University. With the aid of assistants he worked on bridges, transmission towers, and other structural work.

After graduation, Mr. Markwart was employed by the Pacific Construction Company, contractors and engineers, as engineer on highway bridges (fixed and swing spans), trestles, reservoirs, and general work on both design and estimates of cost.

In September, 1904, Mr. Markwart was employed by the California Gas and Electric Corporation (later the Pacific Gas and Electric Company) as engineer making efficiency tests on water-wheel generator units, and assisted in preparing layouts and installation of three 5,400-hp gas engine generator units. He was also in responsible charge of the plans for the gas, air exhaust, and water piping of this installation. Later he was appointed assistant civil engineer for this corporation and as such had charge of civil engineering work on various structures, pole and pipe lines.

From February, 1906, to January, 1907, Mr. Markwart was chief engineer and manager of the Syndicate Water Company, of Oakland, Calif., and was also vice-president and manager of the Richmond Water Company, an affiliated organization. As such he was engaged in investigation of water resources on San Pablo and Wild Cat Creeks and the design and installation of water works and distribution systems for Richmond and Piedmont, Calif. He organized the water business of these two cities and established them as going concerns.

This engagement terminated when the two water companies were incorporated with an older company and Mr. Markwart became a member of the firm of Howard and Galloway, Architects and Engineers of San Francisco. The firm had charge of the design and construction of buildings in San Francisco and vicinity costing more than $3,500,000. They were architects of the Alaska-Yukon-Pacific Exposition of 1909 in Seattle, Wash., in connection with which three buildings and a power plant were built for the University of Washington at a cost of more than $2,000,000. Mr. Markwart organized and administered the Seattle office of the firm until the completion of the work.

---

[1] Memoir prepared by John D. Galloway and I. C. Steele, Members, Am. Soc. C. E.

The firm of Howard and Galloway was dissolved, and the firm of Galloway and Markwart, Consulting Civil Engineers, was organized in November, 1908, and continued in this form, with intermissions, until November, 1920. During the years 1909 to 1912 the firm made numerous reports of hydroelectric plants, irrigation systems, and similar projects, and designed and constructed bridges and building frames and foundations involving about $4,000,000. The principal hydroelectric work was in connection with the Oro Electric Corporation, later a part of the Pacific Gas and Electric Company.

Mr. Markwart suspended professional activity in the firm in July, 1912, and until April, 1915, had general charge of construction at the Panama-Pacific International Exposition, as chief of construction and assistant director of works. These activities included executive work and engineering management, and involved varied engineering studies, construction programs and estimates, the drawing of construction contracts, the development and control of a construction budget amounting to $15,000,000, and the preparation of the rules and regulations of the Division of Works and their enforcement.

In April, 1915, Mr. Markwart resumed private practice in the firm of Galloway and Markwart, doing engineering work of the same general character as that in the period 1908 to 1912. Among other works the firm designed and constructed a hydroelectric plant for the United States Government in Yosemite Valley.

During the World War, when Mr. Galloway was called to France, Mr. Markwart conducted the business of the firm alone. He completed the construction of the hydroelectric plant in Yosemite Valley and other unfinished work of the firm, did engineering work of a general character, made reports on ship plant financing, as the representative for bondholders, and acted as general manager of the Clyde Company on behalf of the U. S. Shipping Board Emergency Fleet Corporation, spending approximately $1,000,000.

In the years when Mr. Markwart was in practice alone he acted at different times as consulting engineer or did work of engineering and management for the following organizations: The Panama-Pacific International Exposition, United States Department of Interior, the Ordnance Department of the Army and the U. S. Shipping Board Emergency Fleet Corporation, the Northern Electric Railway Company, Tillotson and Wolcott, investment bankers of Cleveland, Ohio, Foundation Company of New York, Syndicate Water Company, Pacific Gas and Electric Company, and the Bethlehem Shipbuilding Corporation.

On the return of his associate from France in February, 1919, the partnership of Galloway and Markwart was resumed. They engaged in the usual practice of civil engineering, involving irrigation system reports, financial representation, design and construction of water works, sewer systems, and similar works. They acted as consulting engineers and rendered engineering and supervisory service of a varied character to the following organizations: Oro Electric Corporation, Great Western Power Company, Turlock Irrigation District, Merced Irrigation District, Northern Electric Railway Company, and the United States Government.

The Pacific Gas and Electric Company engaged Mr. Markwart in November, 1920, and he continued in this service for nearly twenty years—until his death in 1940. The management of the company adopted an extensive program of hydroelectric development. It realized the need of a department, the major activities of which would be directed especially to technical investigations and planning, as distinguished from the construction and operation of the company's properties. As a result the Department of Engineering was organized. Because of his outstanding ability as an executive and an engineer, Mr. Markwart, on November 1, 1920, was employed as director of engineering to organize and conduct this work. Subsequently, in January, 1922, he was promoted to the position of vice-president in charge of engineering, and became an administrative and management officer of the company. He was charged with the responsibility for the design of all hydroelectric plants and water supply, steam plants, and substations; transmission and distribution lines; water pumping and collections systems; street railways; the designs or specifications for all electric switches, transformers, and meters; the preparation of standards governing all phases of engineering work; and research work including testing and inspections. His organization recently contained about 200 engineers and technicians.

In this capacity Mr. Markwart prepared programs for meeting future power demands of the company and developed comprehensive schemes for the production, transmission, and distribution of electric energy. These studies embraced most of the important rivers in northern California and several steam-plant projects.

During the twenty years of his employment, Mr. Markwart directed the engineering on work costing about $300,000,000. Comprehensive studies and location of projects for the Pit, Feather, and Mokelumne River hydroelectric developments were undertaken, including related transmission and distribution facilities. The proposed ultimate capacity of these three developments totals 1,376,000 kva, of which 459,000 kva have been installed.

Major construction work included the following: The completion of the Hat Creek No. 1 and No. 2 and Pit No. 1 hydroelectric plants, construction of which was started prior to Mr. Markwart's employment; the Pit River No. 3 plant of 81,000-kw capacity; the 202-mile 220-kv transmission line from Pit River to a main step-down and synchronous condenser substation near Vacaville, Calif., this being the first 220-kv transmission line ever designed and constructed; the Salt Springs and Tiger Creek projects on the Mokelumne River of 71,000-kw capacity; the Melones tunnel and power house of 27,000-kw capacity; the installation of two 50,000-kva turbo-generators and three boilers to operate at 1,400-lb pressure at steam Station "A," San Francisco, these being the first high pressure installations in the West; the reconstruction of steam Station "C," Oakland, including new high pressure boilers and a 50,000-kva turbo-generator, a group of four substations to supply power for electric train operation over the San Francisco-Oakland Bay Bridge, and power to the Golden Gate International Exposition; the installation of three new 50,000-kva steam turbo-generator plants to be completed in 1940.

Notable among the individual structures built under Mr. Markwart's engineering direction are: The Salt Springs Dam on Mokelumne River, this being the highest rock-fill dam in existence; Pit River Dams No. 3 and No. 4, concrete gravity-type dams on unusual foundations; Tiger Creek Forebay Dam, a reinforced concrete flat slab and buttress type; Tiger Creek Afterbay and Lyons Dams which are concrete arch structures; the Philbrook Dam which is a rolled earth structure; the 4-mile Pit No. 3 concrete-lined tunnel through lava formations; the 11½-mile Stanislaus tunnel; and the seventeen-story general office building of the company in San Francisco.

As an executive, economist, and an engineer, Mr. Markwart's work with the company was outstanding. His integrity, broad vision, brilliance of mind, and fairness commanded the respect and admiration of those who were associated with him during his twenty years of service.

At various times Mr. Markwart contributed valuable papers and discussions to numerous technical and educational organizations. Principal among these are:

"Regional Review of Power Resources, Distribution and Utilization for Pacific Coast States, U. S. A.," prepared jointly with the late H. A. Barre for the World Power Conference of 1924.

"Power in California" for the Franklin Institute of Philadelphia (August, 1927).

"Aspects of Steam Power in Relation to a Hydro Supply" for the American Society of Mechanical Engineers (1926).

"Pumped-Storage Hydro-Electric Plants" for the American Society of Civil Engineers (1929).[2]

"The Philosophy of Private Ownership of Public Utilities" for the Commonwealth Club, San Francisco (1929).

"Recommendations for Legislation and for Application of Law" for the Power Division of the American Society of Civil Engineers (1930).[3]

"The Future of Water Power Development in California" for the National Electric Light Association Convention, San Francisco (1930).

"Power Development on the Pacific Coast, Its Immediate Future" for *Electric Light and Power* (1935).

"Power Surplus in California" for *Electrical West* (1934).

---

[2] Discussion by A. H. Markwart of "Pumped-Storage Hydro-Electric Plants," by William W. K. Freeman, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 911.

[3] Published as part of the Symposium on "Public Supervision of Dams," *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 828.

"Mingled Hydro and Steam Power Production in California—Past, Present and Future" for the American Institute of Electrical Engineers' National Convention, San Francisco (June, 1939).

Mr. Markwart's activities extended into the business world, where he was a member of the Board of Directors of the Russ Building Company, The North American Investment Corporation, and the Commonwealth Investment Company. He was also a director of the San Francisco Remedial Loan Association (a charitable organization). In 1922 he became a member of the Board of Trustees of the California School of Mechanical Arts, from which he had graduated as a young man. In 1933 he became President of the Board, a position that he held at the time of his death.

He was a member of the Astronomical Society of the Pacific, and a member and past-president of the Pacific Coast Electrical Association. He was affiliated with the Phi Delta Theta fraternity and the Sigma Xi honor society; and a member of the Olympic, Bohemian, and Engineers clubs of San Francisco, of the Commonwealth Club of California, and of the Bankers Club of New York.

Mr. Markwart was married to Marie L. Chesebrough, on May 26, 1908, and she, with four sons, Arthur Jr., Henry Chesebrough, John, and Philip Gordon, survives him. A daughter, Elizabeth, is deceased.

This is a brief record of the life work of an intensively active man, whose death occurred all too soon. It is a signal example of how a poor boy, left an orphan at an early age, without relatives or position, worked constantly upward to a high position in the business and social world of the West. Untiring industry and an exceptional mentality, accompanied by a most pleasing personality, explain his success. Wherever he went, and in all of his occupations, his personal charm made friends of all with whom he dealt. One of his most salient characteristics was the sense of absolute honesty, integrity, and justice that pervaded all of his contacts with his associates and with the public. His departure leaves a gap in the engineering and financial world and to his close associates a sense of great loss, especially to those who have prepared this memoir, one of whom was closely associated with Arthur Markwart for more than forty years.

Mr. Markwart was elected an Associate Member of the American Society of Civil Engineers on July 9, 1906, and a Member on January 18, 1916.

FREDERICK MEARS, M. Am. Soc. C. E.[1]

DIED JANUARY 11, 1939

Frederick Mears, Colonel, United States Army (*Retired*), military and civil engineer, was born at Fort Omaha, Nebr., on May 25, 1878. He was the son of Lieut. Col. Frederick Mears, U. S. Army, a veteran of the Civil War who held a distinguished service record, and Elizabeth (McFarland) Mears.

---

[1] Memoir prepared by Ralph Budd, M. Am. Soc. C. E., and R. C. Jamieson, Esq.

He received his education in the public schools of New York, N. Y., and Spokane, Wash., and at Shattuck Military Academy, in Faribault, Minn., from which he was graduated in 1897. He was a distinguished graduate of the U. S. Infantry and Cavalry School, in 1904, and of the U. S. Staff College, in 1905, at Fort Leavenworth, Kans.

Colonel Mears' life and work reflected a unique blending of his military interests and his natural love of engineering construction. His boyhood surroundings of army life implanted in him a loyalty to the service, and his early association with John F. Stevens, Past-President and Hon. M. Am. Soc. C. E., then chief engineer of the Great Northern Railway Company, whose home young Mears often visited, aroused his interest in railroad construction. This twofold interest and double training undoubtedly made possible his engineering assignments and contributed to his success in accomplishing them.

Mr. Stevens gave young Frederick Mears his first railroad job after his graduation from the Shattuck Military Academy in the same class with Mr. Stevens' son, Donald. Mr. Mears went to the Fergus Falls Division of the Great Northern Railway as chainman and in two years worked up through the grades of rodman, levelman, and transitman to resident engineer of the Kootenai Valley, Bedlington and Nelson Railroad (a Great Northern subsidiary) in Northern Idaho and British Columbia.

The Spanish War led to his enlistment in the U. S. Army in 1899, and he was sent to the Philippine Islands during the Insurrection. He was commissioned Second Lieutenant in July, 1901, and continued his service with the 5th Cavalry at San Isidro, Luzon, P. I., for another two years.

After leaving the Army Service Schools at Fort Leavenworth, he held the position of resident engineer on construction of the Little Rock Southern Railroad at Fordyce, Ark., from July to October, 1905.

Army orders then transferred him for a short time to Fort Huachuca, Ariz., and back again to Fort Leavenworth to serve as aide-de-camp to Gen. J. Franklin Bell. He continued to fill this assignment until General Bell's appointment as the first chief of staff of the Army.

In 1906, at the request of Mr. Stevens, then chairman and chief engineer of the Isthmian Canal Commission, Lieutenant Mears was detailed as track foreman in Culebra Cut. When the relocation of the Panama Railroad was undertaken, he was assigned to that by Mr. Stevens and continued the work under the late George W. Goethals,[2] M. Am. Soc. C. E., Colonel, Corps of Engineers, U. S. Army. The work was under direct supervision of Ralph Budd and, in 1909, following Mr. Budd's resignation as chief engineer of the Panama Railroad, Lieutenant Mears was appointed to succeed him. It was during those years of service together in the Canal Zone that a professional and personal friendship that was never broken was formed between Mr. Budd and Lieutenant Mears.

As chief engineer, Lieutenant Mears also had charge of the design and construction of the reinforced concrete terminal docks at Cristobal and of a small boat landing at Gatun, and the construction of the New Washington Hotel on Colon Beach. He also made the survey for the Panama-David Railroad for the Panama Government—350 miles of line in the western part of the republic.

---

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1813.

In 1913 he was appointed general superintendent and chief engineer of the Panama Railroad and Steamship Lines, having entire charge of the operation and maintenance of the railroad and of affairs pertaining to the steamship line on the Isthmus of Panama.

When the Alaskan Engineering Commission for the location and construction of a government railroad in Alaska was created by Congress, Lieutenant Mears was appointed a member on the recommendation of Colonel Goethals. He left the Isthmus of Panama early in 1914, and, after a conference with Secretary of the Interior Lane and the other members of the Commission in Washington, sailed on June 6 for Alaska. The summer of that year was spent in preliminary surveys to determine the best route from an ice-free port to the interior of the Territory. Lieutenant Mears made a daring and hazardous reconnaissance with one man and a pack train from Ship Creek to Fairbanks.

His field notes of those weeks, from June 18 to August 6, 1914, are an exciting day-by-day story of the conquest of 360 miles of wilderness northward from Cook Inlet through Broad Pass to the Tanana River. The regular trains rolling smoothly over the same country today give little indication of the difficulties of that trip. Sucking swamps, swollen rivers crossed on improvised log rafts, struggling horses rolling their packs and riders into the current, miles of cutting trail, shortage of rations, vicious mosquitos, and bruises, sprains, and cuts sewed up with common needle and thread were some of the hardships which Lieutenant Mears and his guide, Jim Wilson, encountered. Their arrival in Fairbanks was made memorable by the news that Great Britain had entered the World War.

The route first covered thus was recommended by the Commission, approved by the Secretary of the Interior, and construction started in the spring of 1915. Commission headquarters were established at the new town of Anchorage and Lieutenant Mears was assigned to that section of the work.

By the time the United States had entered the war, he had passed through the grades of Captain and Major in the regular Army. In January, 1918, after insistent application for active service, he was commissioned Colonel of the 31st Railway Engineers and returned to the United States to recruit the regiment. He made a tour of the training camps and selected twelve hundred experienced railroad men. They sailed for France on June 6 and rendered distinguished service on the French railroads for a year.

Colonel Mears served under the late W. W. Atterbury,[3] M. Am. Soc. C. E., Brigadier-General, U. S. Army, as assistant general manager and then as general manager of transportation at Tours, France, until May, 1919. Subsequently, he received the Distinguished Service Medal with the following citation:

> "For exceptionally meritorious and distinguished services. He served with distinction as Commanding Officer of the 31st Railway Engineers,

---

[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1518.

> and later, as Assistant General Manager and General Manager, Railway Department, Transportation Corps. Due to his remarkable executive ability and skill as an organizer, the railways of the American Expeditionary Forces were operated with rare success and the huge transportation problem involving the carrying of tremendous quantities of supplies from the base ports to the fronts, was satisfactorily solved."

Colonel Mears was also awarded the Grand Cross of the French Legion of Honor, personally conferred upon him by Maréchal Pétain of the French Army.

Secretary Lane recalled Colonel Mears to the Alaska Railroad in May, 1919, and appointed him chairman of the commission in full charge of the work. He remained until the completion of the railroad in 1923.

When President Harding drove the golden spike in the Tanana River Bridge on July 15, 1923, Secretary of the Interior Hubert Work said of Colonel Mears, in part:

> "It has been said that every great achievement is but the shadow of a man. If so, it must in justice be said that the Alaska Railroad is the shadow of an intrepid builder, once a Lieutenant of Cavalry, later a Colonel of Engineers overseas, whose construction, like his character and conception of service, was without flaw. This great builder, clearly, has woven himself into the fabric of this railroad and enshrined in the hearts of the Alaska people, the name of Frederick Mears. Fifty-six millions of dollars were expended by this trusted servant of a generous government, in a wild country, with little supervision, but without avoidable error."

The golden spike was marked with his name and presented to Colonel Mears by the citizens of Anchorage. It remains a cherished possession of his family.

Upon completion of the Alaska work in 1923, Colonel Mears retired from the Army and accepted an appointment as chief engineer of the St. Paul (Minn.) Union Depot Company, succeeding W. C. Armstrong,[4] M. Am. Soc. C. E., who died June 12, 1923. The completion of the modern passenger station and accompanying track layout, which are owned and used by the nine roads entering St. Paul, was accomplished in less than the scheduled time, and in a highly satisfactory manner.

For some years the Great Northern Railway Company had been exploring the possibilities for improving its route across the Cascade Mountains. It had become apparent that some 20 miles of line revision, as well as a long tunnel, would be necessary to provide the same general character of railroad that prevailed elsewhere on the Great Northern main line. It was decided to electrify the entire Cascade Mountain section between Appleyard [Wenatchee] and Skykomish, Wash. (73 miles). The work to be done was in precipitous mountain terrain, and the experience and skill which Colonel Mears possessed to a marked degree fitted him peculiarly for this undertaking. He succeeded O. S. Bowen, M. Am. Soc. C. E., as assistant chief engineer in charge of Lines West, with headquarters at Seattle, Wash., in 1925. (It should be stated here that Mr. Bowen was thoroughly qualified in every way and that his replacement was because of severe arthritis, which necessitated a change in climate, and accordingly he was transferred to St. Paul.) Colonel Mears conducted the investigation for the new route across the Cascade Mountains with great thoroughness, and, late in November, 1926, the work on the 8-mile Cascade Tunnel was begun. The tunnel and the Chumstick Line on the east slope were completed and opened for traffic in January, 1929, establishing a new record for tunnel construction.

---

[4] For memoir, see *Transactions*, Vol. LXXXVII (1924), p. 1305.

In many ways, and possibly in every way, this, more than any other single project, should stand as a monument to Colonel Mears' engineering ability. It has been so thoroughly described by him[5] that it is not necessary to dwell upon details. The 8-mile tunnel tangent was extended over steep mountain sides more than 3,000 ft above the tunnel grade, and work was started at both ends, as well as in both directions from a shaft. Yet when the bore was holed through the alinement was perfect to within a few inches. Colonel Mears personally supervised the engineering work on this great project and deserves the credit for its successful execution. Of course, during his entire service as assistant chief engineer, from 1925 to his death in 1939, considerable improvement and replacement work was being done. The largest railroad extension during that period was the Great Northern Railway from Bend, Ore., to Bieber, Calif., a distance of 235 miles. It involved 160 miles of construction, which was done under Colonel Mears' supervision and completed in 1931.

In acknowledging for the War Department the report of Colonel Mears' death, Gen. Malin Craig, chief of staff of the U. S. Army, and a personal friend, said:

> "Colonel Mears was an officer of fine professional qualifications and attainments, with an exceptional knowledge of railroad and engineering construction, and was at all times found equal to the important and responsible positions in which he was placed during his many years of loyal and efficient service."

As an engineer and executive, Colonel Mears always had a broad view of "the job" and allowed nothing to interfere with it. He often exacted hard service of the men under him, but was so reasonable and kind that he never incurred their resentment.

Throughout his life Frederick Mears stood for the highest and finest in human relationship. He was known first and foremost for fairness and justice in all his dealings, and then for the dignity and graciousness of his quiet personality. The power of his splendid mind, his entire lack of egotism, his absolute honesty of thought and purpose, and his real kindliness and tolerance never failed in their effect upon those with whom he worked or played. A quiet sense of fun shone through his apparent seriousness to endear him to all who really knew him. "He could walk with Kings, nor lose the common touch."

He loved the outdoors with the real simplicity of those who spend most of their lives in the open and who accept and enjoy naturally its treasures and pleasures. He hunted and fished with great enthusiasm and had some fine specimens to show for his skill. Photography was one of his life-long hobbies, and, because of his meticulous precision and his skillful use of light meters and range finders, he has left an excellent picture record of his personal and professional life.

---

[5] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 926.

Perhaps nearest his heart were his books. They were his only extravagance and almost his closest friends. He read history and biography with an atlas at his elbow and found consolation for the trouble of modern society in contemplating those of ages past.

Ever alert to the developments of his profession, during July, 1937, he made a detailed survey and report of the snow-retard work near the Loetchberg Tunnel on the Bernese Alps Railway in Switzerland and of that on the Austrian State Railway near Innsbruck, Austria. He personally scaled the mountain tops at these two sites and took many pictures with which he illustrated his report to the chief engineer of the Great Northern Railway. He received the utmost courtesy and cooperation from the French, Swiss, and Austrian engineers and had much pleasure in the new experience. The report was his last outstanding contribution to engineering.

On April 6, 1907, Frederick Mears, then a Lieutenant, was married to Jennie Wainwright, daughter of the late Maj. R. P. Page Wainwright, U. S. Army. Two children, Josephine Wainwright and Elizabeth McFarland, were born to them in the Canal Zone, and two younger children, Frederick, Jr., and Helen Rogers, were born in Alaska. Helen died in 1929.

Colonel Mears was elected an Associate Member of the American Society of Civil Engineers on February 1, 1910, and a Member on September 2, 1914.

SAMUEL PHILLIPS MITCHELL, M. Am. Soc. C. E.[1]

DIED FEBRUARY 11, 1940

Samuel Phillips Mitchell was born on June 10, 1864, in Richmond, Va., the son of Samuel Phillips and Mary (Fry) Mitchell.

Prior to 1900, the art of designing and manufacturing metal bridges in the United States, as based on the historic treatise[2] of the late Squire Whipple,[3] Hon. M. Am. Soc. C. E., had been marked by a high degree of success. The advance in competent, daring design and economical shop processes had been rapid. However, the methods of building the bridge in the field, although usually carried out with great skill, were generally performed with a very limited amount of engineering direction and in the same manner practised for many years.

First, a temporary timber falsework carrying a movable timber hoisting apparatus, called a traveler, was built. Then the bridge was erected on the falsework, and lastly the traveler and the falsework were removed. When spans were short, the timber falsework was minimized or, if possible, omitted. Large quantities of timber were required and the supply of long-leaf pine, frequently used, was diminishing rapidly and increasing in cost almost as rapidly as it diminished. The timber structures needed many men for construction and operation; and although the field workmen of the time were perhaps unequaled by their successors, still it scarcely can be said that field methods were on a par with those of the shop and office.

---

[1] Memoir prepared by R. Murray, M. Am. Soc. C. E.

[2] "A Work Upon Bridge Building, Consisting of Two Essays, the One Elementary and General, the Other Giving Original Plans and Practical Details for Iron and Wooden Bridges," by Squire Whipple; H. H. Curtiss, Utica, N. Y.

[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. XXXVI, December, 1896, p. 527.

The year 1900 saw the amalgamation of all except a few of the principal bridge companies into one company. This company was officered by a group selected from the staffs of the constituent companies, resulting in a body of men of exceptional ability. The engineer placed in charge of the field work was Samuel Phillips Mitchell.

Mr. Mitchell had entered railroad work, in 1883, for the Danville and New River Railroad Company and the Baltimore and Ohio Railroad Company. His work for the Baltimore and Ohio Railroad Company covered the construction of its extension from Baltimore to Philadelphia, and in the large bridges over the Susquehanna and Schuylkill rivers he found his métier. When his work was completed in 1887, he secured employment with the Edgemoor Bridge Works of Wilmington, Del., where his advancement was rapid. He became manager of the company, in charge of all its operations, in 1896. His service with Edgemoor was marked by the number of large bridges built over many of the principal rivers of the United States. It was found that Edgemoor's competitors might equal or perhaps even surpass in design or manufacture; but few could match the economy and speed of the field organization. Therefore, when Edgemoor joined the amalgamation, Mr. Mitchell was the logical choice to take charge of field operations. It is not too much to say that in this period (1900–1908) as chief engineer of the American Bridge Company of New York he revolutionized field methods of bridge building in the United States.

The use of timber was minimized, and wooden travelers were discarded and replaced by several types of heavy bridge erecting cars designed under his supervision for repeated use for various kinds of bridges. When these were not suitable, steel travelers capable of re-use and locomotive cranes were used. Manila rope and wooden shell blocks were abandoned where possible, and replaced by wire rope and steel blocks. Hand riveting was abandoned and advanced power units were used. Limited engineering preparation and supervision of field work were replaced by general consideration and direction of nearly all work by engineers. In short, the field was brought into equality with the shop. The success of these methods is perhaps best shown by the fact that, although bridgeman's wages rose in this period from 30 cents an hour to 50 cents, the cost of bridge erection did not increase. For some types of work, it actually decreased.

In his relations with labor, Mr. Mitchell won the respect and loyalty of all who knew him. His decisions were firm, but based on sound judgment and justice. The national association of employers which dealt with field labor matters was headed by him as chairman of the Executive Committee from its inception until his death. When questions of wage increases arose, he was usually found on the side of the men, if conditions permitted it.

In 1908 Mr. Mitchell organized and headed the Seaboard Construction Company, formed to exploit the advanced methods of bridge erection instituted by him. During the next twenty-five years, under his direction, this company built several hundred bridges without mishap for many of the principal railroads of the East. Also, Mr. Mitchell organized and headed the American Equipment Company, which company is still engaged in the manufacture of bolts, nuts, and rivets.

His consulting practice began at this time and included many of the large bridges of the United States. Among the number, it is difficult to particularize; but perhaps the Sciotoville Bridge over the Ohio River for the Chesapeake and Ohio Railway Company and the Castleton Bridge over the Hudson River for the New York Central Railroad Company might be mentioned as classic examples of successive-step, heavy cantilever erection. As consulting engineer for the St. Lawrence Bridge Company, he supervised the field construction and brought to successful completion the great Quebec Bridge over the St. Lawrence River.

No account of Mr. Mitchell would be complete which did not include his interests outside his profession. From his French Huguenot forbears he inherited a feeling for art, and he was himself no mean water colorist. In his Edgemoor days it was not uncommon to find him hard at work in some sylvan spot with his easel and brushes. He enjoyed the theater and in his memories he treasured the great names of the past—Bernhardt, Terry, Mansfield, Irving, and Booth. His reputation in gastronomy was secure, and no one refused an invitation to one of Mitchell's dinners. For such an event no trouble was too great to take. The chef was courteously harried until he outdid himself, and the sommelier was persuaded to seek the special reserve. At his place in Germantown, Pa., where he lived for nearly forty years, he spent much time and money in the design and care of the formal garden which was his pride.

Mr. Mitchell was dignified and courteous, but taciturn. That he was modest in a deeply sincere sense is demonstrated by a circumstance affecting his application for a professional engineer's license in Pennsylvania. His statement of professional record was terse—it barely "hit the high spots." When he was asked to elaborate, somewhat, to fill the standard needs, he supplied a record that was several pages long. This file in the archives of the Pennsylvania State Board of Registration is a most valuable biographical document.

The friendships he made were deep and warm, but not many. Those who knew him best will remember him in his characteristic pose, seated at a desk, meticulously groomed, with a rosebud in his lapel, a cigarette in one hand, while he tugged at his little red beard with the other—all the time carefully and thoroughly mentally analyzing the matter before him. His interest in the technical advancement of his profession was great and was shown clearly by his disposition of his residuary estate which he divided between the institution which he attended—the University of Virginia—and that which his son attended—the University of Pennsylvania, at West Philadelphia, Pa.

He was married early in life (February 8, 1888) to Miriam Bond, who, with their only son, Donald, survives him.

His clubs were the Racquet Club, Salmagundi Club, Engineers' Club of Philadelphia, Engineers' Club of New York, and the Wilmington Club; he was also a member of the Engineering Institute of Canada.

Mr. Mitchell was elected a Member of the American Society of Civil Engineers on April 1, 1903.

CHARLES NICHOLAS MONSARRAT, M. Am. Soc. C. E.[1]

Died March 1, 1940

Charles Nicholas Monsarrat was born in Montreal, Quebec, Canada, on July 2, 1871, the son of the late Mark R. and Mary E. G. Monsarrat. He traced his ancestry back through several generations to the north of Ireland where his forbears settled as Spanish Huguenots fleeing from persecution.

As a youth, in 1890, he joined the engineering staff of the then young and vigorous Canadian Pacific Railway Company, and through the usual channels of those days, namely, rodman, chainman, transitman, and draftsman, passed into the office of the chief engineer where he rose by 1902 to the responsible position of engineer of bridges. In this capacity he had charge of the inspection and maintenance of all bridge structures on the system whether timber, concrete, or steel, and in addition he and his staff did the office work incident to the design of new construction and the supervision of erection in the field. He occupied this office until 1911 and devoted his attention to many major projects of the Canadian Pacific Railway during this period of wide expansion. Among these, the most notable is perhaps the Belly River Viaduct, near Lethbridge, Alberta, which is a steel structure more than 300 ft high and a mile long. Other important structures in the west, which were built under Mr. Monsarrat's direction, were the high-level bridge in Edmonton, Alberta, a combined railway and highway structure over the North Saskatchewan River, and the original cantilever bridge over the Fraser River below Lytton in British Columbia. It was during his régime as engineer of bridges that the reconstruction of the famous Lachine Bridge over the St. Lawrence near Montreal was undertaken—the four-span continuous structure designed in 1885 by the late C. Shaler Smith,[2] M. Am. Soc. C. E., was replaced by two independent single-track structures built side by side on common piers to constitute a double-track crossing 3,659 ft long. Before this work had been completed, however, Mr. Monsarrat was appointed by the Department of Railways and Canals to succeed the late H. E. Vautelet as chairman and chief engineer of the Quebec Bridge Board, which Board had been set up by the Dominion Government to undertake the task of designing and supervising the construction of a double-track railway bridge over the St. Lawrence, near Quebec, to carry the National Transcontinental Railway, as it was then called. The original Board of Engineers had been appointed in 1908 following the Report of the Special Commission regarding the 1907 catastrophe, but upon the receipt of tenders preparatory to the award of contracts, the late Mr. Vautelet, who previously had also been engineer of bridges for the Canadian Pacific Railway Company, resigned office.

---

[1] Memoir prepared by P. L. Pratley, Esq., Montreal, Que., Canada.

[2] For memoir, see *Proceedings*, Am. Soc. C. E., Vol. XIII, August, 1887, p. 105.

Several years were required to build the deep-water granite-faced piers and to fabricate and erect this great cantilever span which still retains premier place in its own class of structure; and upon its completion, the services of Mr. Monsarrat were retained by the Department of Railways and Canals as consulting engineer, he and selected members of his staff being moved to Ottawa, Ontario. In 1920, when the Dominion Government decided to take over the Grand Trunk Railway Company, the Department set up a special staff in Montreal to undertake the investigations which were a necessary preliminary to arbitration proceedings, and Mr. Monsarrat was requested to recommend a bridge engineer and to supervise the organization of a bridge inspection office for the examination and valuation of all the bridges on the Grand Trunk System and its subsidiaries. For this appointment he recommended and secured the writer, who was then designing engineer of the Dominion Bridge Company and who had served several years as assistant engineer of design on the Quebec Bridge Board under Mr. Vautelet and later as engineer in charge of design for the St. Lawrence Bridge Company, the successful tenderers for the superstructure. Mr. Monsarrat, although stationed in Ottawa, undertook to visit Montreal weekly during the preparation of the inspection reports, valuation sheets, and other data, in the expectation of giving expert evidence at the sitting of the Arbitration Court. However, the decision of the Court was reached on financial grounds without the presentation of detailed technical evidence, and in due time the Canadian National Railways system came into being. The office of consulting engineer to the department was abolished and Mr. Monsarrat was appointed to the Operating Department of the new organization as consulting engineer on bridges, his staff in Ottawa being absorbed into other divisions of the department. As only part of his time was required by the Canadian National Railways, he and Mr. Pratley formed a professional partnership and entered into private practice as consulting civil engineers in April, 1921.

This partnership continued until the death of Mr. Monsarrat and was productive of much important engineering work, largely in the field of major bridge projects. Among the more outstanding contracts on which they were engaged as consulting, designing, and supervising engineers are the Montreal Harbour Bridge (Jacques Cartier), 1924–1931; the reconstruction of the Second Narrows Bridge in Vancouver, 1933–1934; the suspension bridge at the Isle of Orleans near Quebec, 1933–1935; and the Lions' Gate Bridge over the First Narrows at Vancouver, 1933–1939. They were engaged as consulting engineers on design to the general contractors for the Detroit-Ambassador Bridge, 1928–1929; as Canadian associate engineers on the International structures at Sarnia, Canada, and Port Huron, Mich., 1928–1938, and the Thousand Islands, 1933–1938; and as engineering consultants on a variety of work in the Niagara District for the International Railway Company of Buffalo, N. Y.

In 1929 this partnership became associated as structural engineers in a greater partnership bearing the name of Terminal Architects and Engineers, and included architects and service engineers to develop a professional contract for the design, detail drawing, and supervision during construction of the proposed new passenger terminal for the Canadian National Railways, at the Dorchester Street site in Montreal. This undertaking was part of a very comprehensive railway project (the estimates for which amounted to $50,000,000), and involved extensive relocation of trackage, expansion of facilities, new offices, and a large program of city planning to absorb the numerous new bridges and viaducts necessary for the establishment of a modern uptown railway station. Being subject to political control, this entire scheme was suspended in 1931 by the new administration coming into office after the federal election, and only in 1939 was work resumed at the site. The project was considerably reduced in magnitude, the engineering contract was canceled, and the technical work absorbed by the railway's own staffs.

Mr. Monsarrat was always interested in military affairs and in his early life served with the Victoria Rifles of Canada, one of the oldest militia units in the Montreal District. In 1905 he took out a commission in the Royal Highlanders and rose through all the intervening ranks to the command of the regiment as Lieutenant-Colonel in June, 1915. At the end of the World War in 1918 he retired to the reserve of officers, from which he was released upon reaching the age limit of liability to recall for service.

Colonel Monsarrat was a member of the Engineering Institute of Canada, upon whose Council he served during the years 1910, 1912, 1913, and 1914, and of which he was Vice-President during 1917. He was a charter member of the Corporation of Professional Engineers of the Province of Quebec, and served on its Council during the period 1921–1937, being vice-president during 1926, 1927, and 1933. For many years he was a member of the American Railway Engineering Association.

In recent years he gave freely of his time and effort to the financial and building committees of the Anglican Church of St. James the Apostle in Montreal, where his professional advice was highly appreciated. For many years he was a director of the Canada Foundries and Forgings Limited, whose principal plants are in Welland and Brockville, Ontario. His recreation was golf, but he also enjoyed the pleasures of touring by automobile. He was of a most kindly and companionable disposition, was abundantly endowed with the saving grace of humor, and brought to bear on all problems a wealth of sound judgment almost instinctive in its effectiveness.

He was married on February 9, 1898, to Mary Alice Foster in Montreal. He is survived by his widow, one son and one daughter, and three grandsons.

Colonel Monsarrat was elected a Member of the American Society of Civil Engineers on March 4, 1913.

ASA HALL MORRILL, M. Am. Soc. C. E.[1]

Died December 20, 1938

Asa Hall Morrill was born in the Penacook Section of the City of Concord, N. H., on October 7, 1870, the son of George S. and Clara A. (Moody) Morrill. Following his education in the public schools of Concord, he entered the Massachusetts Institute of Technology, at Boston, Mass., from which he was graduated in 1892 with the degree of Bachelor of Science.

Asa Morrill was reared in an atmosphere of engineering. For thirty-eight years his father, the late George Morrill, M. Am. Soc. C. E., was actively engaged in engineering, first as chief engineer of the Old Colony Railroad Company and later as Division Engineer of the New York, New Haven and Hartford Railroad Company upon the consolidation of the Old Colony with the New Haven Road. Asa Morrill's brother, Harley W. Morrill, also an engineer, attended the Massachusetts Institute of Technology for about three years as a member of the class of 1893, and for ten years thereafter was associated with the New York, New Haven and Hartford Railroad Company as an engineer in maintenance of way service.

As a boy, Asa Morrill, in his spare time, found pleasure, interest, and profit in the local industries and about the railroad rather than in sports or athletics. He took great delight in relating the early experiences and incidents of his boyhood jobs in the print shop, the textile mill, the wood working shop, but more particularly at the railroad station. He considered the practical and diversified experience thus obtained as a valuable part of his education and an important and helpful asset in later life.

Railroading fascinated him, and as a son of a railway chief engineer he was destined to follow it as a career with outstanding success. Upon graduation from college in June, 1892, he entered railroad service as an assistant engineer with the Buffalo, Rochester and Pittsburgh Railroad Company, continuing until August, 1892, when he became assistant engineer of the Old Colony Railroad, later absorbed as a part of the New York, New Haven and Hartford Railroad Company system. From August, 1892, until November, 1912, he remained in the service of the Old Colony and New York, New Haven and Hartford Roads except for a short period, April, 1898, to August, 1898, when he was engaged in railroad contracting service as a superintendent for a construction company. His service with the Old Colony and New Haven railroads included positions of assistant engineer, draftsman, assistant roadmaster, and assistant engineer of construction. He entered the service of the Boston and Maine Railroad in November, 1912, as division engineer of construction on grade-crossing elimination work and he continued in the service of that road through January, 1913.

On February 1, 1913, he was appointed engineer of construction and valuation engineer of the Maine Central Railroad and Portland Terminal companies, and in April, 1928, he was appointed chief engineer of the same roads. On April 15, 1933, he was made assistant chief engineer of the Boston and Maine Railroad, the Maine Central Railroad Company, and the Portland Terminal Company. He served as acting chief engineer of the three roads from April 13, 1936, until July 13, 1936, when he was appointed chief engineer. He served in this capacity until his death.

---

[1] Memoir prepared by W. F. Cummings, Chief Engineer, Boston & Maine Railroad—Maine Central Railroad Co., Boston, Mass.

During his service with the Old Colony and New Haven roads he was in direct charge of four-track improvements and preparation of plans for track elevation through Brockton, Mass. As assistant roadmaster he was in general charge of construction and maintenance forces on one hundred and fifty miles of road including the Boston Freight Terminals. As assistant engineer of construction he had direct charge of preparation of all constructions plans (except steel bridges) for elimination of grade crossings in Boston, New Bedford, Hyde Park, and Worcester, in Massachusetts. In addition, he was in charge of surveys and the preparation of preliminary plans for extensive and varied construction and improvements.

On the Maine Central and Portland Terminal companies he was in charge of engineering, valuation, construction, and maintenance. Outstanding among his accomplishments on these roads was the building of Rigby Yard in Portland, Me., and the construction of extensive wharf and dock facilities also at Portland including modern and efficient equipment for speedy and economical handling of bulk and general cargoes. Passenger station facilities at Ellsworth, Me., were designed and constructed under his direction.

On the Boston and Maine Railroad his initial undertaking as chief engineer was to complete the restoration of flood-damaged lines following the New England floods in March, 1936, which caused widespread destruction to the property of the railroad. By a rather strange coincidence his final accomplishment as chief engineer prior to his death was a similar undertaking following the New England floods and hurricane of September, 1938, which affected approximately 1,000 miles of the Boston and Maine Railroad and caused extensive and severe damage at more than three hundred different locations.

Passenger station facilities at Cambridge, Mass., and White River Junction, Vt., were designed and constructed under his direction.

Asa Morrill, widely known as " Ace," was a natural seeker of facts and a keen analyst. He was intuitive, ingenious, and richly endowed with common sense and good judgment. His natural talents, together with his education and training and an unusually broad engineering and construction experience, gained for him an enviable reputation as a competent and successful engineer. He was a vigorous man possessed of poise and the power of decision. His opinions were valued and respected. He was a man of integrity and devotion to duty, charitable and friendly. He was widely loved and respected and his friends were legion.

He was a member of the American Railway Engineering Association, Terminal Engineers Association, the Maine Association of Engineers, the Portland Club, the Lions Club and Prouts Neck Country Club of Portland,

and the New England Railroad Club, Algonquin Club, and Engineers Club of Boston.

He also served as chairman of the New England Committee on Marine Piling Investigation from 1935 to 1938 and the Valuation Committee of the Maine Central and Portland Terminal companies from 1928 to 1933.

He was a prodigious reader, and his recreations included photography, horticulture, and the building of ship models.

He was married to Elizabeth E. Norris at Dedham, Mass., on June 2, 1902. He is survived by his widow, a daughter, Marjorie N. Morrill, and a brother, Harley W. Morrill.

Mr. Morrill was elected a Member of the American Society of Civil Engineers on May 3, 1910.

RALPH HADLOCK OBER, M. Am. Soc. C. E.[1]

DIED AUGUST 30, 1931

Ralph Hadlock Ober was born in Beverly, Mass., on May 20, 1871, a son of Andrew Kimball and Sarah A. (Hadlock) Ober, and a descendant of an old and prominent Colonial family established in America during the earliest period of Colonial settlement. Some of the original leaders of the Massachusetts Bay Colony are included in the direct family line.

Mr. Ober received his education in the public schools of his birthplace, at Beverly Academy, and at Massachusetts Institute of Technology, at Boston, Mass. He later completed a course in law and was admitted to the bar in the State of Washington, although he never practiced that profession.

In 1892 Mr. Ober served successively as chainman, rodman, instrumentman, and inspector on the location and construction of the Adirondack and St. Lawrence Railroad. In 1893 he was engaged in general surveying work in Beverly, and at the end of that year he transferred his activities to the Pacific Northwest. In 1894 he served as instrumentman on land surveys and as special examiner of surveys for the U. S. Indian Bureau, in the State of Washington. In 1895 he was employed as instrumentman and inspector on the construction of a storage dam at Cedar Lake, Washington. During 1896 and 1897 he made surveys of the islands in the Gulf of Georgia, Washington, for the U. S. Surveyor General. Also, during those two years, as well as during the years 1905 and 1906, as U. S. Deputy Surveyor, he made extensive land surveys in the State of Washington.

For a part of the year 1896, and from 1897 to 1905, he was employed as civilian engineer with the Corps of Engineers, U. S. Army, in the State of Washington. His assignments during that period included, successively: Rodman and instrumentman on general surveys; in charge of river surveys and of preparing maps and reports; on construction of the fortifications at

---

[1] Memoir prepared by Joseph Jacobs and O. A. Piper, Members, Am. Soc. C. E.

Forts Casey and Worden; junior engineer and superintendent of construction of gun and mortar batteries, and of other fortification works at Forts Worden and Flagler; in charge of Drafting Department, making detailed plans and estimates for rapid fire batteries and of central electric power buildings and plants at Forts Worden, Casey, and Flagler; and assistant engineer in charge of fortification work at Forts Worden and Flagler.

Leaving federal employment, Mr. Ober, in 1906, served as consultant for the Wenatchee High Line Canal Company in some of its irrigation development, was connected with the location of the Vancouver, Victoria and Eastern Railway in British Columbia, and was engineer in charge of substructure construction of an electric railway and highway bridge across the Columbia River at Wenatchee, Wash. During 1907 and 1908 he was resident engineer in charge of construction of the main line bridge across the Columbia River at Beverly, Wash., for the Chicago, Milwaukee and St. Paul Railway Company. He also served that company on a special engineering assignment in connection with important matters relating to terminal facilities at Seattle, Wash., Everett, Wash., and other points on the western end of its railway system.

From the latter part of 1908 until 1914 Mr. Ober was in the employ of the City of Seattle, serving as court engineer and as assistant city engineer until 1911. In these capacities he had charge of much important work, including an investigation of the Cedar River Watershed and its reforestation for city water supply purposes, and he represented the city engineer in the condemnation proceedings to acquire control of the watershed. He also made an investigation, and prepared estimates and reports for an enlarged water supply system and for an additional hydroelectric plant for the city. In 1911 he was appointed superintendent of buildings and member of the Board of Public Works, and served as such for a period of three years.

Mr. Ober retired from the city's service in 1914 and thereafter, for a period of nearly five years, he was engaged in private practice as a consulting engineer in the City of Seattle, specializing largely in municipal and in river and harbor work. Following the entry of the United States into the World War, he enlisted in the Army, was commissioned a Captain in the 5th Engineers, and was stationed at Camp Humphreys, Virginia, where he remained until May, 1920. Among his other duties there he served as instructor in civil engineering, giving courses in roads and pavements, timber, steel, and masonry construction, etc. He received high praise as an instructor and upon his discharge from the Army was awarded the Victory Medal.

Following his discharge from war service Captain Ober again returned to the Corps of Engineers, U. S. Army, and for a period of three years had important supervisional assignments with channel rectification, dredging, and the construction of locks and dams along a 250-mile stretch of the Ohio River, with headquarters at Louisville, Ky. Following this service, Captain Ober returned to Seattle to resume his private practice and there, after the lapse of a year, he formed a partnership with Joseph Jacobs, M. Am. Soc. C. E., under the firm name of Jacobs and Ober, in the practice of consulting engineering. This very pleasant partnership continued until the passing of Captain Ober in 1931, and during its continuance the firm (or its individual members) was retained in connection with many important engineering projects. Among these were several federal and state irrigation enterprises—the Skagit River and Cedar River power investigations for the City of Seattle, the street and railway grade separation project for the City of Seattle, and the Lake Union Bridge project for the State of Washington. The Lake Union Bridge was an outstanding highway bridge project of the time. Captain Ober's work on the project was exceptionally skilful under trying requirements and contributed very largely to the success of the project.

He was an able engineer and a gentleman of quiet dignity and fine personality. His outstanding traits of character were his modesty, his uncompromising conscientiousness, his strict devotion to duty, and his capacity for detail thoroughness in all matters with which he concerned himself. He was a student of languages and had made himself quite proficient in Spanish, French, and Russian, particularly the latter. He was a lover of the outdoors, and he spent most of his holidays on fishing and hiking expeditions.

On May 30, 1903, Captain Ober was married, at Port Townsend, Wash., to Mattie Elizabeth Shattuck. She and their adopted daughter, Carol Evelyn, survive him. Besides more distant relatives, he leaves also a sister, Mrs. John P. Fay.

Captain Ober was a member and past vice-president of the Engineers Club of Seattle and of the Pacific Northwest Society of Engineers; an Associate Member of the American Institute of Electrical Engineers; and a member and past-president of the Seattle Local Section of the Society.

Captain Ober was elected a Member of the American Society of Civil Engineers on December 4, 1907.

CHARLES REAL OLBERG, M. Am. Soc. C. E.[1]

DIED APRIL 4, 1938

Charles Real Olberg was born on August 19, 1875, at St. Paul, Minn., his father, John G., having moved there from Norway in 1870; his mother, Ella C. (Lacker) Olberg, was a Virginian. When Mr. Olberg was five years old, his family moved to Washington, D. C., where he was educated in the public schools, Central High School, and Columbia University (later George Washington University). He was graduated in 1900 with the degree of Bachelor of Science in Civil Engineering. Later he took postgraduate work in mathematics and hydraulics at the University of California, at Berkeley, Calif.

Prior to entering college, Mr. Olberg obtained engineering experience as a rodman and instrumentman on surveys and construction of irrigation canals at Crow Indian Reservation in Montana, from June, 1894, to September, 1896. While in college he worked as a structural draftsman in the Supervising Architect's Office, in Washington, D. C., from September, 1896, to April, 1900, at which latter time he became an assistant hydrographer for the U. S. Geological Survey, in connection with irrigation investigations in Arizona. He held this position until June, 1902, when he entered the U. S. Reclamation Service, first as an assistant engineer, and then as engineer, on preliminary work in connection with the Roosevelt Project, in Arizona, and on construction of the Newlands (Nevada) Irrigation Project. In 1905 he became a structural engineer in the office of the Quartermaster General, U. S. Army, in Washington, D. C. While in this position Mr. Olberg was in charge of various structures being built for the army.

---

[1] Memoir prepared by John C. Hoyt, M. Am. Soc. C. E.

In July, 1907, he went to Los Angeles, Calif., where he became supervising engineer, District 4, U. S. Indian Office, in charge of investigations, reports, design, and installation of various irrigation works in Southern California and Arizona.

When the United States entered the World War, Mr. Olberg joined the U. S. Army in August, 1917, and was commissioned a Captain in the Army Engineer Corps. He was overseas from June 15, 1918, to August 12, 1919, in charge of construction and operation of machine shops at Engineer Depot, Base Section No. 2 (he reported for duty at Depot on November 8, 1918). During July and August, 1918, he commanded Company L, 116th Engineers. He attended the 3d Corps School, and Clamecy Bridge School, in Langres, France, during September and October, 1918. On August 29, 1919, he was honorably discharged from the Army, receiving the appointment of Major in the Engineering Section, Officers' Reserve Corps, on January 28, 1920. The commission expired on January 4, 1935.

Returning to the United States, Major Olberg took a position as assistant chief engineer, U. S. Indian Service, on construction and design of miscellaneous irrigation structures and projects, the most notable being the $12,000,000 Coolidge Dam on the Gila River, and the Roosevelt Dam on the Salt River, both in Arizona. The Coolidge Dam was one of his outstanding engineering works. The dam creates a reservoir of 1,250,000 acre-ft of storage capacity. It is of the multiple-dome type and contains many new and heretofore untried features. Major Olberg held this position for ten years (September, 1919, to September, 1929).

His next work was as a consulting engineer for the Russian Government on the design of dams and reclamation projects. These included the Mingichaur Project and various irrigation, hydroelectric, and drainage projects in Georgia (Russia), Armenia, and Azerbaijan. During this period he also served the countries of Peru, Argentina, and Mexico as consulting engineer on irrigation work.

Completing this work in February, 1933, Major Olberg returned to the United States, and from August 17, 1933, to January 15, 1935, he was engineer examiner for the Public Works Administration, after which he was engaged in consulting work and private hydraulic research, a subject in which he had always been especially interested. At the time of his death, he was consultant for the Resettlement Division of the Farm Security Administration, and was working on one of the administration's projects at Amarillo, Tex.

While with the U. S. Reclamation Service and U. S. Engineer Office, Major Olberg designed and had charge of construction of a large variety of irrigation works. He had the rare combination of being both highly theoretical in investigation and design, yet extremely practical in construction and operation. In addition to his regular work, he invented several devices which have extensive application and use in engineering activities.

Major Olberg was especially known for his kindness and helpfulness to all with whom he came in contact. He was never too busy to assist those who presented their problems to him, whether they were personal or technical. He was an Episcopalian and a Mason and a member of the Society of American Military Engineers, the Cosmos Club of Washington, D. C., University Club of Los Angeles, Arizona Club of Phoenix, Ariz., the Kappa Sigma Fraternity, the American Legion, the Concrete Institute of America, and the Military Order of the World War.

He was married to Eloise M. Oliver, of Oswego, N. Y., on February 3, 1904. She died on October 23, 1921. In 1927 he was married to Mary M. Bell, of San Gabriel, Calif., who survives him. He is also survived by two sisters, Mrs. Camille M. Craft and Mrs. Norma Forsyth.

Major Olberg was elected an Associate Member of the American Society of Civil Engineers on May 7, 1902, and a Member on December 4, 1922.

IRWIN SELDEN OSBORN, M. Am. Soc. C. E.[1]

Died November 2, 1935

Irwin Selden Osborn was born on January 24, 1878, in what later was known as Bay Village, Cuyahoga County, Ohio, 12 miles west of Cleveland, the son of Reuben and Eunice (Chadwick) Osborn.

His mother, at the age of 90 yr, is still living in the home on the south shore of Lake Erie, built by his great-grandfather, Reuben Osborn, when he and his wife, Sarah, each took up claims from the Western Reserve Land Company in 1811. The Osborns were the first white settlers to spend a winter west of Cleveland.

Mr. Osborn attended Oberlin Academy, at Oberlin, Ohio, for four years, stopping one year to teach in a public school at Avon Lake, Ohio. He entered Oberlin College at Oberlin, where he spent one year, during which time he played right half-back on the Varsity football team. After his Freshman year at Oberlin, he became interested in sanitary engineering and enrolled at Purdue University, at Lafayette, Ind., as a Sophomore. He was graduated from Purdue in 1904 with a degree of Bachelor of Science in Civil Engineering. In 1909, the University conferred on him the degree of Civil Engineer.

---

[1] Memoir prepared by a Committee of the Cleveland Section, consisting of James H. Herron and George B. Sowers, Members, Am. Soc. C. E.

Mr. Osborn played football throughout his college life and was Captain of the Purdue team in 1903, when sixteen of the squad were killed in a railroad train wreck.

His first engineering work was done in the summer of 1903, for Ritchie and Ruple, Engineers, of Cleveland, on the construction of a bridge at Painesville, Ohio, for the Painesville, Ashtabula and Eastern Electric Railroad Company, and doing some municipal work.

Upon his graduation from Purdue University, Mr. Osborn went to Toledo, Ohio, with the Edson Reduction Machinery Company, of Cleveland, to build and test a new garbage reduction plant for that city. Later, he became Chief Engineer of the Company and prepared plans for other garbage reduction plants.

In 1907, Mr. Osborn became associated with the Roberts and Abbot Company, of Cleveland, and was sent to Baltimore, Md., to take charge of building five miles, into Baltimore, of the Washington, Baltimore and Annapolis Railway. This road was largely elevated tracks with five bridges. Upon the completion of this work he returned to Cleveland as Engineer of Bay Village.

In September, 1908, he was appointed City Engineer of Oberlin, and served a short period, when he was appointed Consulting Engineer for Columbus, Ohio, on the model sanitary plants then being built there. While in the employ of the City of Columbus he designed and built a garbage reduction plant for that city which was a model for odor elimination.

The second plant of this character built by Mr. Osborn was in Philadelphia, Pa., and the third on Staten Island, New York, N. Y., for the Metropolitan By-Products Company of New York City.

Because of the successful operation of the Columbus plant, engineers came from many cities to visit it, including some from England, Germany, France, Wales, Japan, and South America. Among them was the Director of Public Health from Toronto, Ont., Canada, who engaged Mr. Osborn to come to Toronto after a trip abroad made with John Turney Fetherston, M. Am. Soc. C. E., of New York City. Mr. Osborn and Mr. Fetherston visited thirty cities in Germany, Belgium, Holland, France, England, Wales, and Scotland.

In Mr. Osborn's contract with the City of Toronto he retained the right of consulting work with other cities. These included New York City, Chicago, Ill., and Washington, D. C. Mr. Fetherston joined him in a report for Chicago the first year, and Mr. Osborn completed the report for Washington, D. C., the second year. These reports consisted of a survey of the cities with an analysis of quantity, quality, collection, and maintenance of wastes with completed plans for the building of the necessary plants and equipment. In Toronto, plans were completed for both reduction and incineration plants.

During the 1913 flood, in Ohio, Mr. Osborn was placed in charge of the flood emergency and relief work.

In the fall of 1915 he returned to Cleveland and with Mr. Virgil D. Allen organized the Allen-Osborn Company, Consulting Engineers, with offices in both New York City and Cleveland.

In October, 1917, Mr. Osborn was called to Washington, D. C., to take charge of a Department for the Conservation of Food Waste and to make a survey of the entire country under Herbert Hoover, Hon. M. Am. Soc. C. E., then Secretary of Commerce. After four months in the Department of Commerce, the late General George W. Goethals of the Quartermaster Department, United States Army, M. Am. Soc. C. E., asked for Mr. Osborn's release to do the same kind of work for the Quartermaster Department, at which time he received a commission as Major. Major Osborn was ordered overseas on July 1, 1918, but, later, was withheld for special work in the cantonments. He was honorably discharged from the Army and returned to his family and engineering office in Cleveland on January 25, 1919.

In 1922 he became Vice-President of the C. O. Bartlett and Snow Company, of Cleveland, in charge of a new Department for the design and construction of garbage and refuse disposal equipment and plants. Waste surveys were made in many cities; plants were designed, built, and operated in Asheville, N. C., Birmingham, Ala., New Orleans, La., New York City, Syracuse, N. Y., Rochester, N. Y., Montreal, Que., Canada, and Toledo, Philadelphia, and Toronto. He remained with this Company until his death.

Mr. Osborn specialized largely in the solution of municipal refuse problems and was known throughout the United States and Canada as an authority on the subject. His broad knowledge of, and experience in, the disposal of refuse was made available to public officials through his service on local committees and on committees of the engineering societies, in addition to his consulting services. He was never too busy to give a helping hand in the solution of a troublesome refuse disposal problem.

He was a member of the American Road Builder's Association, National Health Council, American Public Works Association, and the American Soy Bean Association. He was also a member of the Cleveland Engineering Association, and Sigma Alpha Epsilon Fraternity.

He was married on April 9, 1906, to Amy L. Reed, of Oberlin, Ohio, who, with two children, survives him.

Mr. Osborn was elected an Associate Member of the American Society of Civil Engineers on December 3, 1912; and a Member on January 18, 1916.

GUY PINNER, M. Am. Soc. C. E.[1]

Died June 25, 1939

Guy Pinner was born on June 7, 1888, in Elizabeth City, N. C., the son of John R. and Melissa (Brickhouse) Pinner. He received his early education in public and private schools, and college preparatory school. He was graduated in 1907 from North Carolina State College of Agriculture and Engineering with the degree of Bachelor of Science in Engineering. In 1912 he received the professional degree of Civil Engineer (C.E.) after five years of experience and a thesis on bridge design and erection.

---

[1] Memoir prepared by R. de C. Greene, Esq., New York, N. Y.

He began his engineering career in June, 1907, with the Phoenix Bridge Company at Phoenixville, Pa., as a draftsman. From 1909 to 1912 he covered detail designing, drawbridge machinery, and emergency dams for the Panama Canal, for the American Bridge Company at Edge Moor, Del., Pencoyd, Pa., Ambridge, Pa., and for Post and McCord in New York, N. Y.

From March, 1912, to June, 1914, he was Assistant Engineer to Herbert C. Keith, M. Am. Soc. C. E., consulting engineer of New York. Mr. Pinner was in charge of office and field work, estimating and checking bridges of steel, reinforced concrete, etc.

From June, 1914, to June, 1915, he organized and had charge of the Safety and Efficiency Engineering Department; and he also conducted contract bond investigations for underwriting, for the American Indemnity Company of Galveston, Tex. He was then asked by the company to take their General Agency in San Antonio, Tex., where he remained until March, 1916; but at that time he preferred to return to the direct engineering field.

From March, 1916, to June, 1918, he was bridge engineer for the Seaboard Airline Railway Company, at Norfolk, Va., in charge of maintenance and construction of all steel bridge structures, etc., for 3,500 miles of track. At the same time, as a member of the firm of Frink and Pinner, Construction Engineers, he designed and supervised construction of a large highway bridge near Camden, S. C., for Kershaw County, South Carolina.

In June, 1918, Mr. Pinner went to James Stewart and Company as engineer in charge of operations and construction at the Naval Operating Base in Norfolk. From June to September, 1921, he was in their New York Office as structural engineer in charge of engineering for piers, docks, buildings, etc. In August, 1921, he was made southern manager of Stewart and Company with headquarters at Norfolk, but before installation in office he was sent by the company to Colombia, South America, as construction engineer for the Andian National Corporation, to survey a harbor for a large oil terminal for shipping products of the International Petroleum Company. Following this inspection he was made chief engineer of the Andian National Company, with headquarters in New York City. Later he was manager in Colombia, where he remained until 1924, purchasing properties, operating the ocean terminal, and reconstructing and planning future developments.

From January, 1925, to October, 1929, he was with The Foundation Company in connection with foreign business; returning to Colombia again, he spent five or six months negotiating with the national government for railroad reconstruction contracts. This assignment was not concluded and in 1926 he returned again to Colombia, Panama, Costa Rica, Venezuela, and Curaçao in connection with new business. In 1927 he was in charge of Latin American work in the New York Office and then transferred to The Foundation Company, Foreign—and in March, 1928, he was elected to the board of directors and made vice-president of the company, in charge of the New York Office for European and Far Eastern affairs.

In 1929 Mr. Pinner resigned from The Foundation Company to become construction engineer for the American Cyanamid Company. In 1931 he was made chief engineer in charge of design and construction of plants for the making of fertilizer, chemicals, explosives, etc. He also held office in three Cyanamid subsidiaries.

He was an indefatigable worker and gave his personal attention to the details of every project, never passing a design until he had acquainted himself with all its features and thoroughly checked its soundness. He was gifted with a remarkable memory, a sound sense of economics, and excellent judgment.

A fine executive, he possessed a keen sense of humanity and justice, and was beloved by all his associates.

Mr. Pinner was a member of the American Railway Engineering Association, the Engineers Club, the Westchester Country Club, Rockefeller Center Luncheon Club, the Valhalla and Sound Beach Skeet Club, and Kappa Sigma Fraternity.

He was married on June 30, 1937, to Barbara Lawson, who survives him.

Mr. Pinner was elected an Associate Member of the American Society of Civil Engineers on November 12, 1913, and a Member on October 12, 1921.

T. FRANK QUILTY, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 16, 1939

T. Frank Quilty, known for forty years for his continuous pioneering in construction engineering, died September 16, 1939. His death occurred, after a brief illness, in the Sacred Heart Hospital, at Milwaukee, Wis.

Mr. Quilty was well known throughout the Middle West. During his entire life he was a believer in the new and progressive, whether in engineering or in technical problems of construction materials. This was the keystone of Mr. Quilty's life—a love of progress.

He was born in Chicago, Ill., on January 31, 1874, the son of Patrick Quilty and Johanna (O'Dea) Quilty. He was educated at St. Ignatius College, in Chicago. His father moved to a ranch at Spaulding, Nebr., and the boy took part not only in the herding of cattle but also in the Sioux War of 1891, at the battle of Wounded Knee, S. Dak. Frank Quilty gained here his life love of battling strenuously with nature. The one object of his vacations as a construction contractor was big game fishing in Florida, moose hunting in Canada, etc.

On his father's death the twenty-year-old Frank Quilty returned to Chicago and began work as a rodman at $1 a day. Then, by passing Civil Service examinations, he entered the Chicago City Hall in the Department of Engineers.

---

[1] Memoir prepared by Joan d'Arc Quilty, Naperville, Ill.

Working under the late John Ernst Ericson,[2] M. Am. Soc. C. E., Frank Quilty advanced rapidly. He was still in his twenties when he became assistant city engineer. Young Quilty was one of the principal supervisors of the gigantic tunnels under the city streets that made Chicago at that time the foremost city in the world in sanitary facilities.

In 1897 Frank Quilty entered private contracting. Specializing in railroad work, he successfully competed in bids for work on the Chicago and Alton Railroad, the Chicago and North Western Railway, Chicago and Eastern Illinois Railroad, and the Chicago, Burlington and Quincy Railroad. At that time he also had completed the Montrose Avenue Crib of Chicago, 5 miles out in Lake Michigan, to insure the city fresh water from depths of the lake. (Twenty years later it was still working "full blast.")

At the same time that he was bidding for the mammoth Kansas City, Mo., railroad terminal, he also bid for the hand of Margaret Lawler Martin, a young schoolteacher of Chicago. After his marriage (and completion of the Kansas City station in 1909), Frank Quilty further narrowed his field of railroad engineering construction by deciding to give his attention to the perfecting of the reinforced-concrete bridge. Some of his most noteworthy bridges at this date were the bridge over the great Rock River and the Okaw Viaduct.

Mr. Quilty forcibly promulgated the reinforced-concrete bridge at a time when iron bridges were just superseding the wooden bridge. He pointed out the strength, utility, and relative "agelessness" of concrete bridges (the beauty of the concrete arch against the green Illinois farm lands was not the least of considerations with him). The use of reinforced concrete in all forms of constructions was a pioneering battle in which Mr. Quilty fought continually. Many were the boards of state, city, and county commissioners he faced and convinced of his ideal—the ultimate in solid, ageless construction.

Mr. Quilty was much concerned at the entrance of America into the World War. When on New Year's Day, 1918, he received a telegram requesting him to accept a commission as Major of Engineers in the United States Army, he accepted; although it meant the breakup of his contracting firm and forfeiting of the railroad work, particularly large bids, from the Pennsylvania and Chicago and North Western railroad companies.

As a Major of Engineers he was commissioned to build a storage depot, comprising many square miles of reinforced concrete for central storage of military and munition materials away from coastal dangers, at Columbus, Ohio. Nine million dollars were allocated to him for this mammoth construction.

After the war Major Quilty was the owner and operator of stone quarries and gravel pits in northern Illinois. In his constant encouragement of the new advances in science, Major Quilty (among other pioneering trials which he fostered while president of the Superior Stone Company) conducted a series of experimental tests. These tests opened a new field for the producer of crushed stone and enriched the agriculture of Illinois. He proved that limestone was most efficacious when used by the farmer as a soil sweetener.

---

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 1686.

Today it is interesting to see each stock truck, after delivering at the city cattle yards, load up with crushed limestone as it passes the quarries on its homeward trip.

In 1936, Major Quilty's interest in new fields of industry came to the fore in a number of chemical tests which he conducted personally in order to prove that sand is as binding as crushed stone or washed gravel in the mixing of concrete. By his championing of this new cause he raised sand from a nuisance material carried at a loss to a scarcity material of high value—this at the height of the 1937 depression!

For some years Major Quilty was a contributor not only to professional magazines but also to historical periodicals. At the time of his death he was completing an illustrated brochure of 50,000 words on the developments and needs of the sanitary districts of Chicago.

In addition to his membership in the Society, Major Quilty was a member of the Western Society of Civil Engineers, the Illinois Society of Civil Engineers, Chicago Engineers' Club, American Legion, Holy Name Society, Calvert Club, St. Ignatius Alumni, and Construction Division Association. He was a fourth-degree Knight of Columbus, and a Major of Engineers in the Reserve Officers of the United States Army. His interests and hobbies ranged from the study of natural history and anthropology to his pistol and gun collection and American history editions.

Major Quilty is survived by his widow, Margaret Quilty; two sons, T. Frank, Jr., and John Pershing; and two daughters, Joan and Margaret. The sons are following in their father's footsteps; both are deeply engrossed in civil engineering study and work.

Major Quilty was elected an Affiliate of the American Society of Civil Engineers on April 5, 1905, and a Member on September 5, 1911.

JOHN MATTHEW RAYBURN, M. Am. Soc. C. E.[1]

DIED JULY 20, 1939

Mr. Rayburn was born September 28, 1868, in Westmoreland County, Pennsylvania, and died July 20, 1939, after a severe illness of several months. His death, no doubt, was hastened by the death of his wife only twelve days before.

Mr. Rayburn's formal education ended with his graduation from the Central High School of Pittsburgh, Pa., a school with a proud record of offering to pupils of native ability, such as Mr. Rayburn, the fundamentals necessary for progression into the learned professions. Shortly thereafter he entered the service of McCully and Taylor, engineers in general practice in Pittsburgh. While with them he studied law and was nearly prepared for the rigid examination for admission to the Allegheny County (Pennsylvania) Bar when he was driven to a milder climate by impending tuberculosis.

---

[1] Memoir prepared by Samuel A. Taylor, M. Am. Soc. C. E., and W. E. Fohl, Esq.

Recovery from this malady was slow, but complete, and at thirty years of age he was able to discharge, successfully, the duties of chief engineer in the construction of the Parral and Durango Railroad in Old Mexico.

Upon his return from this engagement he was employed by the late Selwyn M. Taylor, M. Am. Soc. C. E., whose tragic death in the Harwick Mine disaster is recorded in the *Transactions* of the Society.[2] At the time of Mr. Taylor's death Mr. Rayburn was one of his most valued assistants, and already had achieved an enviable reputation by reason of his important part in the design and construction of the high-grade mining plants of the Eureka Fuel Company and the Pittsburgh Terminal Railroad and Coal Company, among others, in western Pennsylvania.

Immediately after Mr. Taylor's death Mr. Rayburn organized his own successful business which was taken over by his son and three trusted employees who helped him until his death.

A description of his professional activities from this time on, in any detail, would require a volume. Hence, it can only be stated that these activities took him into nearly all the coal producing states of the United States, to Canada, and to South America. In all phases of bituminous coal mining—reconnaissance, development, management and appraisal—he was, in Caesar's words, *facile princeps* (easily the first). This characteristic, with his early legal training and his well-known probity of character, made him a most impressive expert witness, in high esteem in the courts, and sought for in most of the important mining litigation originating in the Pittsburgh region.

Mr. Rayburn was held in high esteem by the members of his profession, by those who employed him, and by the people of the community in which he lived. His sturdy honesty and fine character are well exemplified in the words of Polonius in his advice to Laertes—*Hamlet*—Act I, Scene 3:

"This above all: to thine own self be true,
And it must follow, as the night the day,
Thou canst not then be false to any man."

This is indeed a brief of Mr. Rayburn's character, and accounts for his standing in the community, as well as with all who knew him.

He is survived by one daughter, and one son. A large circle of friends mourn his passing.

Mr. Rayburn was elected a Member of the American Society of Civil Engineers on April 17, 1917.

RALPH JOHN REED, M. Am. Soc. C. E.[1]

DIED JULY 27, 1939

Ralph John Reed, son of Arthur Lucius and Anna Virginia (Kelly) Reed, was born at Port Huron, Mich., on April 30, 1883, and went to Whittier, Calif., at an early age. He attended Pomona College, at Claremont, Calif., where he was graduated in 1905 with the degree of Bachelor of Letters. He also attended the University of Michigan, at Ann Arbor, Mich., where he was graduated in 1908 with the degree of Bachelor of Science in Civil Engineering.

---

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LII, June (1904), p. 552.

[1] Memoir prepared by Harry W. Dennis, M. Am. Soc. C. E.

His subprofessional engineering work included surveying during the summers of 1903–1905 for hydroelectric projects in the mountains of California, instructing in surveying for the summer engineering camps while at the University of Michigan, and, after graduation, topographic surveys on the Huron River for the late Gardner S. Williams, M. Am. Soc. C. E. Also included was his early work with the Union Oil Company of California whose employ he entered in October, 1908.

This latter position, however, soon brought Mr. Reed into work of a professional nature which included the locating of long and important oil transportation lines, with their accessory pumping stations, and the design and construction of large concrete-lined oil storage reservoirs. From 1911 to 1914 he was in charge of the Engineering Division of the Union Oil Company at Los Angeles, Calif. In 1915 he was engineer of the transportation department in charge of rearranging and enlarging five main-line pumping stations, and was acting chief engineer during 1916 and 1917, becoming, in November, 1917, chief engineer. He continued as chief engineer, until he resigned in 1929, in charge of all engineering design and construction, particularly for the departments of sales, manufacturing, and transportation. The work included oil pipe lines, pumping stations, tanks and storage reservoirs, docks and marine terminals, warehouses and refineries, cracking plants, and natural gasoline plants. His work represented expenditures of more than $30,000,000.

In 1929 Mr. Reed became a consulting engineer in private practice. His notable engagements have included: Consultant for the repair and reconstruction of earthquake damage at a large department store following the Long Beach, Calif., earthquake; consultant for various oil companies on appraisals and new work; member of the Board of Economic Survey of the Port of Los Angeles; and consultant for the Los Angeles County Flood Control District on its rock-fill dams in the San Gabriel Canyon.

His scholastic ability was recognized by his election to the honorary societies of Sigma Xi upon his graduation from the University of Michigan, Phi Beta Kappa in 1914 by Pomona College, and Tau Beta Pi in 1928 by the California Institute of Technology, at Pasadena, Calif.

During the twenty-one years of his employment in the petroleum industry, Mr. Reed gained a wide experience in its technical branches. From 1925 he was active in the production division of the American Petroleum Institute (A. P. I.) with special reference to matters of standardization, fire protection, and corrosion study.

He served as national chairman of the A. P. I. Committee on the Standardization of Steel Tanks for Oil Storage from January, 1927, until April, 1930. He was the first national chairman of the committee, and it is largely due to his initiative, vision, and executive ability that the present A. P. I standards on A. P. I. steel tanks were developed to such a high degree of usefulness. He inaugurated the idea of preparing detailed shop drawings which are now (1939) in current use throughout the industry.

After his resignation as national chairman in 1930, he continued as chairman of the California District Subcommittee on Tanks until April, 1932. Upon completion of this period, he continued to serve as a member of the California District Committee until his death.

As national chairman of the A. P. I. Tank Committee, he was also a member of its Central Committee on Standardization, which is the general supervising agency of the Institute on all standardization matters. His high degree of intelligence and business sagacity were a distinct asset to that committee. He also served as a member of the A. P. I. California District Subcommittee on the Standardization of Tubular Goods from June, 1925, to February, 1939.

Mr. Reed was appointed to the State (California) Board of Registration for Civil Engineers on August 13, 1931, became a vice-president thereof on August 28, 1931, and was its president from January 30, 1933, to January 7, 1937. He was again its vice-president from January 7, 1937, until his death. He was also vice-president of the National Council of State Boards of Engineering Examiners from 1933–1934 and president from 1934–1935. He was a member of the Association of Professional Engineers of the Province of British Columbia, and of the American Society of Mechanical Engineers.

Mr. Reed gave freely of his time in connection with civic affairs. He was vitally interested in the development of Los Angeles Harbor and served on the Harbor Subcommittee of the Foreign Trade Committee of the Los Angeles Chamber of Commerce for many years. He was chairman of the subcommittee in 1938. He had a deep interest in matters of fire protection and safety, and was for a long time, until ill health prevented, a valued member of the Los Angeles Traffic Advisory Board. This organization was a volunteer group under the leadership of the Automobile Club of Southern California and was composed of representatives from the various divisions of the city government having jurisdiction over the control or direction of traffic, representatives from organizations in the transportation field, and others interested in, or affected by, the traffic problem. This Board serves as a forum in the interchange of ideas designed to effect improvements in traffic conditions.

Mr. Reed was married on June 9, 1909, to June E. Miller, who died in 1926. Two sons, John Miller Reed, born in 1914, and Robert Pearson Reed, born in 1921, survive. On September 23, 1929, Mr. Reed was married to Alberta P. Mann, who also survives him.

From 1923–1936 Mr. Reed was a director of the Los Angeles Young Men's Christian Association. At the time of his death he was a trustee of Pomona College; a director of the Whittier National Trust and Savings Bank; a director of the Beverly Hills Hotel Company; past master of Arlington Lodge No. 414, Free and Accepted Masons; and a member of Los Angeles bodies Ancient and Accepted Scottish Rite. He also was a member of the University Club (Los Angeles), the Engineer's Club (San Francisco), and the Rotary

Club (Los Angeles). In national politics he was a Republican, and in church affiliation he was a Congregationalist.

During his vacations he looked to mountaineering for recreation. He was a member of the Sierra Club whose purpose is "to explore, enjoy and render accessible the mountain regions of the Pacific Coast; to publish authentic information concerning them; to enlist the support and cooperation of the people and the Government in preserving the forests and other natural features of the Sierra Nevada." Pursuant to that objective, Mr. and Mrs. Reed accompanied the Sierra Club group for many of the excursions into the places where only the hardy and strong may hope to go.

He was President of the Los Angeles Section of the American Society of Civil Engineers in 1922 and was a Director of the Society, elected from District 11, from 1935–1937.

Mr. Reed was elected a Junior of the American Society of Civil Engineers on September 1, 1908; an Associate Member on October 1, 1913; and a Member on December 6, 1920.

EDSON MASON SCOFIELD, M. Am. Soc. C. E.[1]

Died December 28, 1939

Edson Mason Scofield was born in Hermon, St. Lawrence County, N. Y., on September 26, 1867. He was of true American lineage. His great-grandfather was personal aide to General George Washington and served in a continental regiment throughout the Revolutionary War.

His father, George Vincent Scofield, and his mother, Emily Jane (Mason) Scofield, were both descended from pioneer New England families which had lived first in Connecticut and then moved on to northern New York. His father was a hydraulic engineer and owner of a wood-working mill in Hermon where the son began to dream of his future vocation. His mother had been a school teacher, gifted both socially and intellectually, and she instilled into her son her own habits of precision and attention to detail.

Edson Mason Scofield, endowed with exceptional ability, passed with honor through the usual academic courses in the public schools of Hermon, and was graduated from high school in 1884. He then entered Union College, at Schenectady, N. Y., and in 1888 was graduated with the degree of Civil Engineer. In 1934, the degree of Doctor of Science was conferred upon him by Union College.

After teaching mathematics at Rockland College, Nyack, N. Y., for one year, he entered the field of engineering. From June, 1889, until March, 1903, Mr. Scofield was engaged successively as assistant to the late Edwin Thacher,[2] M. Am. Soc. C. E., nationally known bridge engineer of Louisville, Ky.; assistant engineer of the Youngstown Bridge Company, Youngstown, Ohio, then chief engineer and finally manager of the same company; and chief engineer of the National Bridge Company, Pittsburgh, Pa.

---

[1] Memoir prepared by Elisabeth Shaver, San Carlos, Calif.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 918.

In 1903, with his brother, the late Glenn M. Scofield,[3] M. Am. Soc. C. E., he organized the Scofield Engineering Company in Philadelphia, Pa., and served as president. The firm was successful from the start and rapidly won a nation-wide reputation for engineering ability and business integrity.

Mr. Scofield's inventive turn of mind was always ready to meet the opportunity when necessity arose. A notable instance was his invention of a form of slide-rule now in common use among engineers.

The vision, genius, and versatility in the early period of his career may be judged by the following achievements: The designing of a special power plant at Harwood Mines, Pa., capable of using a mixture of coal dust and dirt at the mouth of the mine, making the waste a principal source of revenue for the mine; a gas-operated high-pressure pumping station at Philadelphia, where a 300-lb pressure could be put on the fire mains in less than 40 sec; a power plant erected for the West Jersey and Seashore Railroad Company which furnished power to operate an electrified train running from Philadelphia to Atlantic City, N. J., within six months after work of construction was begun; a concrete dry dock in Mare Island Navy Yard, San Francisco, Calif., requiring the largest timber crib ever used up to that date, where work had to be done in 40 ft of soupy "Tule" mud.

Another important structure was a concrete dry dock at League Island Navy Yard, Philadelphia, the largest in that section at the time of construction, which was built under adverse conditions in a water-bearing gravel formation.

The steel frame for the U. S. Coaling Station at Manila, Philippine Islands, the coal handling equipment of the station at San Francisco Bay and at Guantánamo, Cuba, are among the achievements of the Scofield Engineering Company.

A storage and mixing plant for the Delaware, Lackawanna and Western Railroad at Delanson, N. Y., stored 300,000 tons of coal and enabled the owners to handle 1,000 tons per day out of cars into mixing piles and back into cars, and to ship a fine coal of uniform grade.

In 1918, the Scofield Engineering Company received government recognition and, under orders of the United States Shipping Board, began building reinforced concrete ships at San Diego, Calif., as the Pacific Marine and Construction Company. In 1920, the World War over, the office of the Pacific Marine and Construction Company was closed and the two brothers moved to Los Angeles, Calif.

The organization of the Scofield Engineering-Construction Company marked the passing of the old style contractor in favor of the technically trained expert. This concern was one of the first to feature the trained engineer in business. It was among the first to use the "cost-plus-a-fixed-fee" basis of construction whereby the owner is assured the most favorable cost of work consistent with sound construction and good investment. The company began specializing in large earthquake-proof buildings. Mr. Scofield had made an exhaustive study of earthquakes and earthquake protection. To his satisfaction none of the buildings erected according to methods based upon his research ever sustained more than minor cracks.

---

[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1895.

In the period from 1920 to 1929, his company was responsible, either as engineers or as constructors, for the erection of approximately one hundred large buildings in and about Los Angeles. Some of the best known are the Biltmore Hotel and Theatre, the Pacific Mutual Building, Title Insurance Building, Barker Brothers, the Elks Club, the Good Samaritan Hospital, United Artists Theatre, all buildings of Sears-Roebuck Company, and the Pacific Finance Building.

Early in college days, Mr. Scofield became conscious of his personal responsibility in matters of public interest. That sense of responsibility was ever uppermost in his mind throughout the years crowded with diversified accomplishments. Always a valued public servant awake to the needs of the community in which he lived, Mr. Scofield was called upon by the Mayor of Los Angeles in 1929 to give his expert knowledge to the city of his adoption and he accepted membership on the Board of Water and Power Commission. No man could have been better qualified to appraise the future requirements of his locality and he turned the full force of his varied experience into the business of making the Power Bureau an asset to the public that owned it. In order to do his part, Mr. Scofield cut down his personal activities and gave more than three fourths of his time freely and gladly to this work.

He had watched Los Angeles become the metropolis of the West, the fifth city of the nation, in twenty years. He attributed the growth to the intelligent development of water and power facilities under ownership and control of the people. With characteristic insight, he looked ahead and saw an era of vast expansion in population and manufacturing industries and he was confident that the future growth of the metropolitan district and the complete development of the Pacific Southwest hinged upon the people's decision to take advantage of the Boulder Dam hydroelectric power.

He was among those who had fought with determination to induce the government to build Boulder Dam, and to no small degree must credit be given to his long experience in design, construction, and operation of utilities for the final consummation of this project. When at last the contract was signed for lease of power privileges between the U. S. Department of the Interior and the City of Los Angeles, he followed the immense construction job with satisfaction and gratification.

He responded to constant demand for speeches, for contributions to press and radio addresses. His words never failed to carry weight. His audiences caught his enthusiasm, recognized his rugged adherence to principle, his devotion to the cause he sponsored, and his freedom from selfish motive. He appealed to the people as owners of the Water and Power Department to complete the full program of the Colorado River Project by building a transmission system to bring an abundant supply of cheap power into their homes and factories.

In September, 1930, his connection with the Board of Water and Power was terminated. He had fought valiantly for the public enterprise against the opposition of its competitors and enemies, the private power companies, but was compelled finally to continue his efforts on behalf of the people in an unofficial capacity.

The Municipal Light and Power Defense League was organized by a group of prominent citizens who appreciated the immense benefit of a municipally owned Water and Power Department and who wished to see it protected and promoted to its maximum value. They sponsored a series of radio broadcasts and Mr. Scofield became one of their spokesmen. His weekly radio addresses stressed consideration of water and power as a business belonging to the citizens. To him the radio periods were in the nature of stockholders' meetings to discuss the best interests of business. Cheap light and power were the dividends.

It was highly fitting that he should be designated as a member of the general committee in charge of the Boulder Power Inaugural, on October 9, 1936, to celebrate the placing in service of the 266-mile transmission system—an event which marked the epochal step in the progress of the Southwest of which he dreamed and for which he had labored with diligence and faith.

The key to Mr. Scofield's character and work may perhaps be found in his own words, addressed to a club of young business men: "You are all young men at the acme of inquisitiveness. Don't be a conformist. Keep your faith. Faith should grow with knowledge and courage. There is need for the occasional man with faith and curiosity. There is need for inquiry."

His art of presenting technical facts in popular language is illustrated in his reference to the "romance hidden in the Owens Valley Aqueduct." "The momentum of each drop of water coming from 3800 feet elevation is captured over and over and transformed into electric power."

Mr. Scofield became a life student of unemployment, taxation, and all economic matters after his experiences in the panic of 1893. His research and the importance of his conclusions brought him into contact with the best informed opinion of the country. He believed that the cause of unemployment and the world-wide depression could be found in certain types of special privilege and private monopoly and it was his conviction that a gradual shift of the weight of taxes from the products of labor to the taxation of land values, exclusive of improvements, would discourage speculation in land, increase wages and interest, and so improve the condition of labor and capital. Wise beyond his generation, he knew that the world was not yet ready for this change and that the establishment of justice was not in the immediate future.

Mr. Scofield was married in Louisville, Ky., on February 20, 1894, to Lizziemac Rivers who died in 1930. They had two children, George Vincent and Alberta Bigelow (Mrs. Robert S. Bowen).

On January 21, 1932, Mr. Scofield was married to Ethel O'Shields of Los Angeles.

He was a member of the American Society for the Advancement of Science, the American Concrete Institute, Sigma Xi, California Taxpayers Association, Tax Reform Association, Los Angeles Chamber of Commerce, Orange County Coast Improvement Association, South Coast Improvement Association (was their first president, 1932–1933), X Club of Los Angeles, Los Angeles Country Club, Los Angeles Section of the American Society of Civil Engineers, Jonathan Club, Municipal League of Los Angeles, National Municipal League, Public Ownership League of America, Southern California Council on Earthquake Protection, and the Advisory Committee of California Development Association.

He was appointed by the 1939 Golden Gate International Exposition to be a member of the World's Fair Advisory Commission for the City of Los Angeles.

Mr. Scofield was a sponsor for the Henry George School of Social Science in Los Angeles. He was an able exponent of the Henry George philosophy which he considered a practical and unselfish way of living for the benefit of mankind.

Mr. Scofield managed to find time for recreation even in the midst of his many activities. He was an enthusiastic equestrian, and enjoyed golf, but the relaxation he valued most he found at his lodge in the mountains at Mammoth Lakes and at his seaside cottage on the south coast at Laguna Beach, Calif.

He spent many hours in a fully equipped, soundproof workshop adjoining his home, where he forgot the stress of business while his expert hands obeyed his creative direction.

Mr. Scofield was affectionately known as "E. M." to those who knew him best, and all those who came to him in friendly intercourse found the veil of outward reserve lifted and were at ease in the warmth of his human love and sympathetic understanding.

He was a true and trusted friend never too busy to give his full attention to every question presented; never too hurried to probe to the bottom of every perplexity by his own process of exact analysis. His counsel was sane and sure and he never permitted his tongue to say what his heart denied. In business as well as in his daily dealings with people everywhere he was known to be scrupulously honest.

Structures of stone and steel from Florida to Portland, Ore.—from New York to San Francisco—stand as silent witnesses to his mastery of engineering problems, but Mr. Scofield will be remembered and honored fully as much for the designs he conceived and the service he rendered to benefit the people of his time.

His boyhood ambition, revealed in reminiscent mood, "To be a good engineer and to do some good in the world," has been fulfilled beyond the measure of human standards.

Mr. Scofield was elected an Associate Member of the American Society of Civil Engineers on April 1, 1896, and a Member on May 2, 1905.

HAROLD BEMIS SHATTUCK, M. Am. Soc. C. E.[1]

DIED APRIL 25, 1939

Harold Bemis Shattuck, the son of Gilman C. and Estelle M. (Barnes) Shattuck, was born at Nashua, N. H., on November 17, 1873.

His early education was received in the public schools and high school of Nashua. In 1897 he was graduated from Dartmouth College at Hanover, N. H., with the degree of Bachelor of Science. He had taken the first year of the work of the Thayer School of Civil Engineering during his senior year and intended to complete the required subjects for the civil engineering degree. In accordance with this plan he secured employment as rodman with the Boston Electric Light Company in July, 1897, so as to meet the school's requirement for practical engineering work. However, he became interested in medicine, and in January, 1898, he entered Dartmouth Medical College. A serious illness compelled him to withdraw from college in July of that year and to give up the idea of becoming a physician. For some time he was unable to engage in work of any kind, but in November, 1900, he was employed as clerk by the Pines Hotel, Lakewood, N. J. He resigned from this position in April, 1901, to accept an appointment as assistant in civil engineering at The Pennsylvania State College, at State College, Pa.

Soon after this appointment, in addition to his teaching, he began taking subjects necessary to complete the work required for graduation in civil engineering and, as a result, was graduated from The Pennsylvania State College with the degree of Bachelor of Science in Civil Engineering in 1904. In 1915 he was granted the degree of Civil Engineer by the same institution.

He was promoted to the rank of instructor in July, 1903; assistant professor of civil engineering, in July, 1905; associate professor of railroad engineering, in July, 1908; and professor of railroad engineering, in July, 1924. During much of this time Professor Shattuck was in responsible charge of instruction in railway and highway engineering subjects. He retired from teaching because of physical disability on February 1, 1936, and was appointed professor emeritus of railroad engineering in recognition of his past services.

In addition to his college work, he had been engaged in private engineering practice since 1907—mainly in land surveying, municipal engineering, water supply, and sewerage. He was borough engineer of State College for twenty-seven and a half years, ending January, 1934, and he was borough engineer for several neighboring boroughs for short periods. He was elected county surveyor for Centre County, Pennsylvania, for three successive terms, serving from January, 1921, to January, 1933. He was also bridge engineer for the commissioners of Centre County in 1919, 1922, and 1923.

Professor Shattuck was a member of the American Railway Engineering Association, and he served as a member of Committee XVI, Economic Location. He was also a member of the American Association of University Professors, the Society for the Promotion of Engineering Education, the Pennsylvania Water Works Operators' Association, and the Centre County Engineers' Society.

---

[1] Memoir prepared by Elton D. Walker, M. Am. Soc. C. E.

While he was a student at Dartmouth, he became a member of Alpha Delta Phi and was elected to membership in Phi Beta Kappa in his senior year. At The Pennsylvania State College he became a member of Phi Kappa Phi and of Chi Epsilon. He was a Thirty-Second Degree Mason, a member of the Consistory, the Commandery, and the Shrine. He was active in the organization of the Blue Lodge at State College and served as Worshipful Master. He was a charter member of the Kiwanis Club and served on its board of directors, and was also a charter member of the University Club at State College. He was president of the board of trustees of the Presbyterian Church at State College for six years preceding his death and had been active in the affairs of the church for many years.

Professor Shattuck was married in 1909 to Elizabeth M. Stuart, of State College, who died December 25, 1934. They had no children. He is survived by two brothers, Arthur G. and Roger C., and by two sisters, Helen and Fanny C.

Professor Shattuck was a genial gentleman, a competent engineer, and a faithful teacher, liked and respected by his students and associates.

Professor Shattuck was elected a Member of the American Society of Civil Engineers on May 12, 1930.

BENJAMIN BRUCE SHAW, M. Am. Soc. C. E.[1]

DIED JUNE 2, 1939

Benjamin Bruce Shaw, the son of William Henry Shaw and Emma (Burrell) Shaw, was born at Canton, Ill., on February 21, 1886. His boyhood was spent in his native town where he received his early education, and where his father was the postmaster for twenty years.

Mr. Shaw was graduated in Civil Engineering at the University of Illinois, at Urbana, Ill., in 1911 with the degree of Bachelor of Science and in 1916 he secured the degree of Civil Engineer from the same institution. While still attending school his early technical training began in 1906 when, during summer vacations, he was employed as rodman and draftsman with the Illinois Central (Electric) Railroad Company, the city engineer of Canton, and with the Sanitary District of Chicago, Ill.

In July, 1911, Mr. Shaw began his first regular assignment on railroad construction with the St. Paul and Kansas City Short Line Railroad Company in Iowa and served in the capacity of estimator, rodman, and assistant engineer, respectively. In 1913, upon completion of the construction work, he was transferred to El Reno, Okla., as assistant engineer on the Chicago, Rock Island and Pacific Railway on construction and maintenance work. He was promoted to division engineer in January, 1915, with headquarters at Haileyville, Okla., and in 1916 was transferred to Little Rock, Ark., in the same capacity.

---

[1] Memoir prepared by Fred P. Funda, Assoc. M. Am. Soc. C. E.

During his service with the Chicago, Rock Island and Pacific Railway Company, he constructed extensive trackage, yard, and shop facilities at Little Rock and at El Dorado, Ark. With the discovery of oil at El Dorado in 1920, extensive railroad development in that territory was essential in order to handle the rush of men and materials to the new oil field, and Mr. Shaw planned and executed the construction work in record time. He accepted the position of chief engineer of the Cuba Railroad at Camagüey, Cuba, in December, 1922, and served in that capacity until July, 1925. During this assignment he constructed the Santa Cruz line from Camagüey to Santa Cruz del Sur, Camagüey, Cuba, and completed extensive port improvements at Antilla, Cuba, besides a general rehabilitation program of the Cuba Railroad to enable operation of larger locomotives.

Returning to the United States in July, 1925, Mr. Shaw was employed by several railway supply and contracting companies as sales and contracting engineer. In 1934 he was appointed manager of mills and mines of the Georgia Kaolin Company at Drybranch, Ga., which position he was holding at the time of his sudden death on June 2, 1939.

An engineer of ability, he combined with his technical knowledge and executive qualities a winning personality that would have carried him far in his chosen profession. His loyalty to his employers was outstanding, and his varied contacts among business and professional men gained him many strong friends who respected him for his affability, his conscientious work, and his uprightness of character. His intimate friends will not forget his jovial sense of humor and those rare qualities of mind and heart that so endeared him to all with whom he came in contact. In his professional and social life his courtesy and kindly consideration were never failing.

Mr. Shaw was married on August 4, 1913, to his boyhood friend, Elizabeth Henkle, at Canton. A kind husband, he loved his home and his devoted wife, who survives him. Although he had no children of his own, he believed in youth and giving the boys about him a worthy chance in life, and his kindness found expression in his deep interest in the lives of the individuals and families with whom he worked.

At the time of his death Mr. Shaw was a member of the Sigma Nu Fraternity, which he had served as commandant while at the University of Illinois; he was also a member of the American Railway Engineering Association, the Society of American Military Engineers, Ancient, Free and Accepted Masons, Rotary Club of Macon, Ga., and the Red Cross Board. He was councilman of the Boy Scouts of America.

Mr. Shaw was elected an Associate Member of the American Society of Civil Engineers on August 9, 1920, and a Member on May 8, 1922.

JULIAN CLEVELAND SMITH, M. Am. Soc. C. E.[1]

Died June 24, 1939

Born at Elmira, N. Y., on October 7, 1878, Julian C. Smith was the son of Howard M. and Mary Elizabeth (Joslyn) Smith. He received his education at the central high school, in Buffalo, N. Y., and was graduated from Cornell University at Ithaca, N. Y., with the degree of Mechanical Engineer in Electrical Engineering in 1900.

Gifted with unusual power of concentration and a keen memory, the new graduate was marked for success from the outset. Beginning his professional career, on graduation, as a draftsman with the late Wallace C. Johnson, M. Am. Soc. C. E., consulting engineer at Niagara Falls, N. Y., he remained there two years, when he became assistant to Mr. Johnson at Shawinigan Falls, Que., Canada, at a time when The Shawinigan Water and Power Company was beginning active operations.

This young man thus became associated with one of Canada's most important utilities, the expansion of which gave scope for his unusual talents. At the age of twenty-five he was superintendent of the company. He became vice-president when thirty-six, and three years later was vice-president and general manager. In 1933 he was elected to the presidency and for many years occupied the same position with affiliated and associated companies. He was also in charge, as consulting engineer, of a number of water power projects, including the Manitoba Power Company at Great Falls, Manitoba, and the Cedar Rapids plant on the St. Lawrence River, in addition to the company developments at Shawinigan Falls, La Gabelle, and elsewhere. He saw the company grow from small beginnings to one of the greatest of its kind in the world.

Mr. Smith was the author of several papers on water power development and transmission design and construction, principally in the *Proceedings* of The Engineering Institute of Canada. He was awarded the Sir Casimir Gzowski Medal of the Canadian Society of Civil Engineers (known as The Engineering Institute of Canada after 1918) as the leading contributor in Canada to the literature of the profession of civil engineering during the year 1913, for his paper, " The Design of High Voltage Transmission Lines."

His principal hobby was astronomy and he humored it by the erection of a large telescope on his property at Lake Memphremagog. When the problem was brought to his attention of producing a new and much more efficient type of depth sounding recorder, Mr. Smith undertook to work on the instrument. The type which had been used on naval and mercantile vessels for several years was particularly sensitive and not always reliable. The improvement of this instrument presented a problem, the solution of which required a high degree of technical skill. With the aid of one or two friends he succeeded, just before his death, in developing a machine greatly superior to that in use. It was given a thorough test by the group, who made a chart of Lake Memphremagog waters. The work was finished and the new instrument, marking a great advance on previous practice, was presented to the British Admiralty for use on naval vessels.

---

[1] Memoir prepared by Fraser S. Keith, Esq., Montreal, Que., Canada.

The eminent position held by him in the industrial, financial, scientific, and university life of Canada is evidenced from the following partial list of the positions he held: President of The Shawinigan Water and Power Company, Quebec Power Company, St. Maurice Power Corporation, United Securities Limited, and The Canada Light and Power Company; vice-president of The Royal Bank of Canada, Dominion Bridge Company, and Dominion Engineering Works; Director of Canadian General Electric Company, Consolidated Paper Corporation, Montreal Island Power Company, Montreal Trust Company, and Saguenay Electric Company; governor of the Montreal General Hospital, and McGill University; member of the National Research Council.

The Engineering Institute of Canada honored him with its presidency in 1928. For some time, prior to his death, he had been chairman of the Advisory Committee for Canada of the Institution of Civil Engineers of Great Britain. Two Canadian universities recognized his high standing by conferring on him the degree of Doctor of Laws—Queen's University in 1923 and McGill University in 1928.

He held membership in the following societies: American Institute of Electrical Engineers, Institution of Electrical Engineers of Great Britain, Institution of Civil Engineers of Great Britain, The Engineering Institute of Canada, and professional societies in Quebec and Manitoba. He was a member of important clubs in New York and Montreal.

Big business sought him for his wise counsel, financial organizations for his keen discernment, research and educational institutions for his breadth of vision, and many individuals for his sympathetic advice and generous assistance. Few men have left a greater record of achievement and none has been more sincerely mourned.

Rarely are found in one person such admirable qualities as were possessed by Mr. Smith. His unusual vision and profound technical knowledge were the mainsprings of a great industrial empire. His clarity of judgment enriched many of Canada's leading organizations. Great as were his qualities of mind, his nobility of character and open-hearted generosity of spirit were the attributes that, like a magnet, drew men to him in affectionate regard.

Surviving are Mrs. Smith (Bertha Louise Alexander), two sons, Julian C. and Joslyn A. (both of whom are engineering graduates), two daughters, Mrs. R. Harvey-Jellie and Mrs. J. Desbaillets.

Mr. Smith was elected a Member of the American Society of Civil Engineers on October 2, 1922.

WILLIAM PLINY SNOW, M. Am. Soc. C. E.[1]

Died October 12, 1938

William Pliny Snow was born in Charlton, Mass., on May 19, 1855, the son of Franklin Thurston and Sophia (Kidder) Snow. He was descended from Colonial forbears, some of whom fought in the Revolutionary War.

In 1862 the family moved to the town of Roxbury, Vt. William fitted himself for college at Barre Academy, and was graduated from Dartmouth College at Hanover, N. H., with the degree of Bachelor of Science, in 1881.

He began active engineering work in 1881 as a topographer for the Rocky Mountain Division of the Northern Pacific Railway Company in Montana, which was still largely an uncharted wilderness. When the Northern Pacific Railway was opened, in 1884, Mr. Snow joined the staff of the surveyor general's office at Helena, Mont. There he gathered the material for his monumental map of Montana, the first complete map of the state which had ever been made.

After a short stay in Florida as assistant engineer in charge of construction for the Blue Spring, Orange City and Atlantic Railroad Company, he returned to Montana as assistant engineer in charge of construction for the Great Northern Railway Company. During that period he constructed more than 30 miles of roadbed. In his later years Mr. Snow often referred to his interesting pioneer experiences in Montana as among the most enjoyable of his career.

He left Montana in 1888 and returned to New England and began the type of work which was to engage most of his interest and attention for the rest of his life—that is, mill construction, and hydraulic developments for factories. A list of the important works which Mr. Snow executed with great distinction would be a long one.

His first engagement after returning from Montana was with the Parsons Paper Company at Holyoke, Mass., where he designed and supervised the construction of the raceway, wheel pits, and headgates. In 1889 he built the earthen dam for the water works at Hinsdale, Mass., and repaired the dam for the Sadauga Reservoir at Whitingham, Vt.

In 1890 Mr. Snow attracted the attention of Stephen Greene, then the head of the engineering firm of Lockwood-Greene and Company. Mr. Snow joined the Lockwood-Greene organization at that time and remained with them almost continually until his retirement in 1935 at the age of 80.

Mr. Snow became the most highly trusted construction engineer in this company. He was assigned usually to work that required ingenuity in design, care in construction, and ability to get along well with owners and contractors. Mr. Snow, with his wide experience, careful training, good nature, and unimpeachable integrity, filled the requirements in a very unique way.

---

[1] Memoir prepared by Albert L. Scott, M. Am. Soc. C. E.

The most important projects that Mr. Snow supervised during the 45 years of service with Lockwood-Greene and Company include the following:

1890 to 1891	—The rebuilding of the Dunnell Manufacturing Company plant at Pawtucket, R. I.;
1892	—Design of dams for Pacolet Manufacturing Company, Pacolet, S. C.;
1893 to 1894	—Pelzer Manufacturing Company, Pelzer, S. C., and Tucapau Mills, Tucapau, S. C.;
1896	—Rearrangement of plant of Androscoggin Mills at Lewiston, Me., and building a new plant for the Liondale Bleachery, Rockaway, N. J.;
1898	—Design of a 500-ft canal for Bodwell Water Power Company, Oldtown, Me.;
1900	—Design of stone dam for Palmer Mills, Three Rivers, Mass.; rebuilding and remodeling Lonsdale Bleachery at Lonsdale, R. I.—this operation was executed without curtailing output.

In 1902 Mr. Snow became a general consulting engineer for Lockwood-Greene, assisting in the design and supervision of the dams of the Greenville Carolina Power Company, at Greenville, S. C.; the Rockingham Power Company, at Rockingham, N. C.; and the Central Georgia Power Company, at Flovilla, Ga. He became more and more in demand as an expert in examining, correcting, and modernizing reports made by other engineers. Clients wanted Mr. Snow's opinion on problems considered to be of an unusual or intricate nature, particularly those having to do with water powers, wheel installations, design of canals, forebays, wheel pits, and tailraces. In later years he became the most experienced hydraulic engineer in the Lockwood-Greene organization and found hydraulic work most fascinating. He was the first choice of his superiors and often of the owners for designing and supervising unusual construction and remodeling work.

As previously mentioned in this brief summary, Mr. Snow was particularly felicitous in his dealings with clients and contractors on difficult remodeling, rearranging, and repair work, where output had to be maintained, and changes made, frequently without finished plans or opportunity for bidding. Mr. Snow's character and reputation for fairness generally made both owner and contractor agree to any decision he might make.

Mr. Snow was slight in stature, quick in motion, lean and active to the last. He had a gentle winning smile, rare tact, unfailing urbanity, and a faculty for seeing all sides of a question. He was a sound engineer, an unfailing friend, and unusually able in working with other people, with a long and honorable career in his chosen profession.

Mr. Snow was married on May 14, 1884, to Emma J. Paul. He is survived by his widow, and a daughter, Florence (Mrs. F. I. Miller).

Mr. Snow was elected a Member of the American Society of Civil Engineers on February 5, 1902.

WILLIAM WARBURTON KNOX SPARROW, M. Am. Soc. C. E.[1]

Died November 7, 1939

William Warburton Knox Sparrow was born at Moneymore, County Tyrone, Ireland, on December 30, 1879, the son of the Rev. William and Cecilia Jane (Knox) Sparrow. His father was a clergyman of the Episcopal Church. Mr. Sparrow was educated in England and entered railway service in Ireland in February, 1896, in the engineering department of the Belfast and Northern Counties Railway Company.

In December, 1898, with a desire for adventure, as he once described it, he left Ireland for South Africa, obtaining employment with the Cape Government Railways in February, 1899. From that time until November, 1908, he was engaged continuously on railroad location, and construction and maintenance work. For two years of this period he was with the Charter Company of Rhodesia (Cecil Rhodes' company) on location of railroads and highways.

In 1908, while he was in South Africa, Mr. Sparrow passed the necessary examination and was elected an associate member of The Institution of Civil Engineers, London, England.

Although recognizing the advantages of government service and the feeling of security that it afforded, Mr. Sparrow realized that the opportunities for self-development were limited due, for one thing, to the fact that all positions were under civil service, requiring strict adherence to seniority rules for advancement. After much consideration, he decided to try his fortune in the United States.

He and his wife, the former Mary Batchelor, left Capetown, Union of South Africa, on their wedding day, late in November, 1908, arriving in the United States early in 1909. Neither of them had any acquaintance in the United States, but Mr. Sparrow had, through reading some of his books, heard of the late J. A. L. Waddell, Hon. M. Am. Soc. C. E., consulting engineer of Kansas City, Mo. He decided to go there and present his case, hoping that the fact that he had come from South Africa to see him would make a sufficient impression to gain him work in Mr. Waddell's office. He was given a place in the drafting room of Waddell and Harrington at a salary of $50 a month.

Thus, after having spent almost ten years in South Africa, he started anew in a strange country, later to become the country of his adoption; sustained by a keen desire for self-development and an opportunity to work in the knowledge that the reward would be measured by the worth of one's efforts.

Mr. Sparrow remained with Waddell and Harrington until July, 1912. During his service with that firm he worked as a detailer, checker, and designer on concrete and steel bridges, occupying the position of assistant designing engineer when he left. These three years were indeed trying ones, but his was the determination to solve the problem at hand and overcome obstacles, a characteristic of his entire life. In those years, as in all of the years that followed, he was fortunate in having the encouragement and sympathetic understanding of his ever devoted and loyal wife.

---

[1] Memoir prepared by John J. Roche, Esq., Chicago, Ill.

From July, 1912, to September, 1913, Mr. Sparrow was associated with the late Hans von Unwerth, M. Am. Soc. C. E., consulting engineer of Kansas City, in architectural engineering.

With the creation of the Missouri Public Utilities Commission, Mr. Sparrow's interest turned to the possibilities of service there, and in September, 1913, he took a position as assistant engineer, later becoming assistant chief engineer of that commission. After coming from South Africa to get away from the restrictions of government service, it seemed inconsistent to take a position with another government body, but it gave him what he wanted—an opportunity to get acquainted with people, especially railroad people. His work comprised inspection and valuation of railroad property in the State of Missouri. Through his service with the commission he secured his first railroad position in the United States—that of valuation engineer of the Chicago, Burlington and Quincy Railroad Company, in April, 1916. Here he developed an organization to carry on the valuation work required by the Valuation Act.

In September, 1918, he became corporate chief engineer of the Chicago, Milwaukee and St. Paul Railway Company and continued in that capacity until the termination of federal control of railroads.

The succeeding years of his life were spent in the service of the Milwaukee Road. In June, 1920, he was appointed assistant to the President, in charge of federal valuation and settlement with the United States Railroad Administration for claims arising out of federal control. In November, 1920, he was placed in charge of the Accounting Department which, while indeed a new field to him, was a recognition of his capacity for organization and executive ability. The Real Estate Department, which he was instrumental in establishing in December, 1921, was added to his jurisdiction.

In 1923 Mr. Sparrow was elected vice-president of the company and was assigned to handle financial matters in addition to his accounting, real estate, and valuation work. During the period of receivership of the Chicago, Milwaukee and St. Paul Railway, from 1925 to 1928, he was chief financial and accounting officer for the receivers.

In January, 1928, Mr. Sparrow was elected vice-president and a director of the newly organized Chicago, Milwaukee, St. Paul and Pacific Railroad Company, successor of the Chicago, Milwaukee and St. Paul Railway Company. Upon the appointment of trustees in bankruptcy, on January 1, 1936, Mr. Sparrow was appointed officer in charge of finance, accounting, and real estate, which position he occupied at the time of his death.

Mr. Sparrow was a man of strong convictions and outstanding ability. Trained in the engineering profession, he advanced to positions of responsibility in the broader field of railway management. He embraced responsibility, and to each new problem applied a keen power of analysis, persistence of effort, and intensity of purpose. While exacting in his requirements of others, he was no less exacting in the standards he set for himself, for he strove for perfection in whatever he attempted, to the accomplishment of which he devoted his energies unsparingly. Fair in his judgment and conscientious in his dealings, he was ever thoughtful and considerate of those on whom misfortune had fallen, and was always ready to help alleviate their distress.

For recreation, Mr. Sparrow was fond of golf and shooting and had a keen appreciation of literature, art, and music. He was devoted to home and family.

Mr. Sparrow was a member of the Railway Accounting Officers Association, Glen View Club, Western Railway Club, Electric Club of Chicago, and held a life membership in the Art Institute of Chicago. He was a director of the Mercantile National Bank of Chicago.

In addition to his widow, he is survived by two daughters, Jessie Cecilia Knox Sparrow (Mrs. George L. Green) and Eileen Knox Sparrow (Mrs. Elmer B. Rich, III); one son, Joseph Knox Sparrow; and two brothers.

Mr. Sparrow was elected an Associate Member of the American Society of Civil Engineers on December 6, 1905, and a Member on November 26, 1918. He passed away shortly before he was to have received a certificate of life membership in the Society.

EMBERT HIRAM SPRAGUE, M. Am. Soc. C. E.[1]

Died March 9, 1940

Embert Hiram Sprague, for many years Professor of Civil Engineering, then later Professor of Sanitary Engineering, at the University of Maine, Orono, Me., died very suddenly on March 9, 1940.

Mr. Sprague was born in Haverhill, Mass., on December 20, 1875, the son of Henry S. and Abbie (Nichols) Sprague. He received his early education in the schools of Haverhill, and in 1900 was graduated from Dartmouth College, at Hanover, N. H., with the degree of Bachelor of Science in Civil Engineering. He was a member of Theta Delta Chi, a social fraternity, and Casque and Gauntlet, a senior society.

Upon graduation from Dartmouth, he worked for three years as a draftsman—first for the New England Telephone and Telegraph Company, then the Pencoyd Plant of the American Bridge Company, and finally the Riter Conley Company of Pittsburgh, Pa. In 1903, Mr. Sprague went to South Africa to take the position of bridge engineer for the Central South African Railways. After about a year, he was engaged by the Blaisdell Company of Los Angeles, Calif., to supervise the erection of special sand excavators for mining reduction plants at Johannesburg, Union of South Africa. In March, 1905, he accepted a position with the H. Eckstein Company, mining engineers in Johannesburg, designing layouts for Blaisdell sand-treatment plants. Later he became sampler and surveyor for the Nourse Mine. Following 1908, Mr. Sprague spent much time with the Consolidated Gold Mines of South Africa, where he was in charge of underground workings. It was here that he originated and made a new type of assay plan.

---

[1] Memoir prepared by Weston S. Evans, M. Am. Soc. C. E.

Returning to the United States in 1909, Mr. Sprague accepted the position of chief engineer with the Mulcahy and Gibson Company, later going with Post and McCord of New York, N. Y., on steel building construction. During the latter part of 1914, Mr. Sprague worked as resident engineer for the Maine State Highway Department on bridge erection.

In February, 1915, he was appointed to the position of Associate Professor in the Department of Civil Engineering at the University of Maine. In this capacity, he soon became acting head of the Department of Civil Engineering, and was appointed head of the Department in 1926. He became Professor of Sanitary Engineering in 1934, and served in this capacity until his death.

During Professor Sprague's appointment at the University of Maine, he received many honors, among them election to the honorary societies of Tau Beta Pi and Phi Kappa Phi. He was a member of the Masonic order and several local and national professional societies.

On August 28, 1919, Professor Sprague was married to Addie Stockwell of Bangor, Me., who survives him.

In his chosen field, Professor Sprague had the respect and confidence of his colleagues. He found great pleasure in helping his students, to whom he endeared himself by his friendliness and kindly interest. He was always ready to do anything in his power to help both his students and his associates on the faculty.

Professor Sprague was elected a Member of the American Society of Civil Engineers on March 5, 1928.

WILLIAM SHACKELFORD SULLIVAN, M. Am. Soc. C. E.[1]

DIED DECEMBER 20, 1938

William Shackelford Sullivan, son of William K. Sullivan and Amelia (Shackelford) Sullivan, was born in Chicago, Ill., on May 19, 1885.

The son of a one-time owner and editor of the *Chicago Evening Journal*, and a well-known writer, he was educated in the public schools of Chicago and the Lewis Institute of Chicago. He entered the University of Michigan, at Ann Arbor, Mich., in 1901, joining the Phi Kappa Psi fraternity (Michigan Alpha). His college career was interrupted by ten years of working, but he returned in 1913 and was graduated in 1914 with the degree of Bachelor of Civil Engineering.

Mr. Sullivan served as draftsman with the Indiana Harbor Railroad Company in Chicago from 1905 to 1907, and with the Northern Pacific Railway Company in Montana from 1907 to 1910. From 1910 to 1911 he was resident engineer for the Kettle Valley Railroad Company, and held the same position with the Canadian Northern Pacific Railroad Company from 1912 to 1913. He was associated with Dean Mortimer E. Cooley, Hon. M. Am. Soc. C. E., of Ann Arbor, as chief of party on railroad valuation from 1914 to 1916.

---

[1] Memoir prepared by J. W. Follin, Assoc. M. Am. Soc. C. E.

After a year of private practice in structural engineering at Oakland, Calif., he became structural engineer for the Interstate Commerce Commission, Valuation Department, at San Francisco, Calif. Thereafter he was structural engineer successively for the Nevada State Highway Commission, at Carson City, Nev.; the Liberty Shipyard, at Alameda, Calif.; with Alfred P. Fisher, Assoc. M. Am. Soc. C. E., in San Francisco; and finally with the Pacific Coast Steel Company, in San Francisco.

From May, 1920, to July, 1922, Mr. Sullivan was mechanical inspector for the Pullman Company, at Richmond, Calif. After another year of private practice he became structural designer in the Bridge Engineers Office of the Southern Pacific Railroad Company, in San Francisco, leaving in March, 1924, to become assistant engineer, Department of City Transit, in Philadelphia, Pa., in charge of design and plans in connection with the Broad Street subway. He served this department until 1926. For the next twelve years, Mr. Sullivan resided in Washington, D. C., except for brief periods. He was structural engineer for the Arlington Memorial Bridge Commission from 1926 to 1933, at which time he became associated with the Public Works Department of the Navy, at Pensacola, Fla., as associate engineer (structural), and later with the War Department (Quartermaster Corps), as associate structural engineer. From 1936 to 1937 he was in the U. S. Engineers Office of the War Department on flood control at Bolling Field in Texas, and was later transferred to the flood control project at Cumberland, Md. At the time of his death he was associated with the W. F. Jackson Company, Inc.

An unusually good chess player, he was a member of the Board of Directors of the Capitol City Chess Club. He also won a number of cups in bridge tournaments.

Mr. Sullivan had been a member of the Philadelphia Engineers Club and the University Club of Washington, D. C. He never married and is survived by his sister, Helen (Mrs. James G. Cumming).

He was registered with the State Registration Board for Professional Engineers in Pennsylvania.

Mr. Sullivan was elected a Member of the American Society of Civil Engineers on December 14, 1925.

FREDERICK HORACE TIBBETTS, M. Am. Soc. C. E.[1]

DIED AUGUST 2, 1938

Fred H. Tibbetts will probably be best remembered for his extensive flood-control, reclamation, and irrigation work in the Sacramento Valley and his highly successful water-conservation project in the Santa Clara Valley. However, his field of activity during a period of some thirty years of engineering practice extended well beyond the limits of the State of California, and embraced many of the varied branches of the profession. Few engineers in the history of California have contributed so extensively to the development of its agricultural lands and the control and conservation of its waters.

---

[1] Memoir prepared by Ralph G. Wadsworth, M. Am. Soc. C. E.

Frederick Horace Tibbetts, the elder of the two sons of Horace Albert and Manda (Arnold) Tibbetts, was born at Oshkosh, Wis., on April 28, 1882. The family moved west when he was ten years old and settled in Santa Clara County, California. He attended the College of the Pacific, then at San José, Calif., and received his degree of Bachelor of Science in 1903. He continued his studies during the following three and one half years at the University of California, at Berkeley, Calif., receiving a degree of Bachelor of Science in Civil Engineering in 1904, and a degree of Master of Science in 1907. In the meantime, he had received a degree of Master of Science from the College of the Pacific in 1905.

While doing his graduate work at the University of California, he also served on the faculty as an assistant in civil engineering in the fall of 1904 and again in the academic year 1906–1907. In 1905 and the spring of 1906, he was an instructor in mechanics at the California School of Mechanical Arts in San Francisco. In August, 1907, he was appointed instructor in civil engineering at the University of California, and in July, 1909, became an associate professor, which position he held until 1911. He was elected to membership in three scholarship fraternities: Sigma Xi, Tau Beta Pi, and Sigma Iota Phi.

Mr. Tibbetts' active professional work commenced immediately after he received his first college degree and continued concurrently with his advanced studies and his teaching. Early engagements included surveys in Santa Clara and Alameda counties (1903), irrigation and drainage investigations (1904), and well measurements and surveys (1905). From 1906 to 1909 he was in charge of artesian investigations in Livermore Valley and the Pleasanton Reclamation Project.

In 1909, Mr. Tibbetts entered into a partnership with Perry A. Haviland, county surveyor of Alameda County, which lasted for nine years. The firm operated under the name of Haviland and Tibbetts, except in 1913 and 1914, during which period the name was changed to Haviland, Dozier and Tibbetts. The firm designed and supervised the construction of a wide variety of projects throughout central and northern California. During this period, Mr. Tibbetts was in charge of reports, estimates, designs, and supervision of construction of sewage systems and sewage disposal works for numerous cities and towns. He also handled designs, estimates, and reports on extensive harbor improvements at Richmond and South San Francisco, a highway system in Santa Barbara County, a filtered water supply for the City of Richmond, and all of the larger reclamation projects in Yolo Basin lying west of the Sacramento River above and below the City of Sacramento. The latter projects, particularly Reclamation Districts Nos. 900 and 999, which Mr. Tibbetts designed and in part supervised, included massive levees, drainage canals, and pumping plants. He also supervised extensive improvements of a similar nature in Reclamation District No. 70 in the upper part of Sutter Basin.

In November, 1912, Mr. Tibbetts submitted (on behalf of the firm of Haviland and Tibbetts from a branch office which had been opened in San Franciso) a report on the Knights Landing Ridge Cut, which had a major influence on the reclamation of the upper part of the Sacramento Valley. The cut, which forms an artificial outlet for flood waters in Colusa Basin, was sufficiently completed to be of immense benefit during the great 1915 flood. The project required more than 3,000,000 cu yd of excavation and the construction of highway and railroad bridges and various other structures. This work was followed immediately by the construction of major flood-protection levees and drainage systems in Colusa Basin, including particularly the construction of 50 miles of river levee between Knights Landing and Colusa.

In 1918, the firm of Haviland and Tibbetts was dissolved because of Mr. Haviland's ill health, and Mr. Tibbetts took over the San Francisco office under his own name. He continued as chief engineer of the Colusa Basin projects, previously undertaken for Reclamation District No. 108, the Sacramento River West Side Levee District, and the Knights Landing Ridge Drainage District. These three districts, which overlapped in part, provided complete flood protection for more than 100,000 acres of land which had been subject to frequent flooding by both river overflow and foothill drainage. In addition, Reclamation District No. 108 provided a complete drainage system and five separate irrigation systems for its 58,000 acres.

Mr. Tibbetts also became chief engineer of four additional large reclamation districts; two important water conservation districts; seven irrigation districts, including the two largest ones in the Sacramento Valley; two large land development companies; and a hydroelectric power company at Anchorage, Alaska. For all of these projects, and numerous other smaller ones, he prepared reports, estimates, and designs, and actively supervised construction, performed usually by contract, but in a few instances by force account. Included among the projects were several of the largest gravity intakes and pumping plants on the Sacramento River; the first wood screw pumps in California; irrigation and drainage canals with capacities to 1,500 cu ft per sec, constructed with floating dredges; levees built by the world's largest clamshell dredges, and others built with suction dredges; two steam-electric generating plants; the first modern rock-fill dam in California, 160 ft high; five concrete arch dams; seven earth-fill dams; and innumerable incidental structures such as bridges, headgates, siphons, flumes, roads, etc.

Among the various irrigation projects, two were particularly outstanding for their magnitude, their comprehensive planning, and their widespread public benefit. The Nevada Irrigation District in Nevada County, California, developed a water supply by diversion and storage at high elevation in the mountains and made a long-term contract for sale of the energy content of the falling water on such a basis as to amortize the full cost of the mountain works, thereby giving the agricultural lands at lower elevation what amounted to a free water supply. The mountain works included a 4-mile diversion tunnel, 85,000 acre-ft of storage, and an 11-mile conduit in rough terrain which required numerous flumes and tunnels. The irrigation distribution system included two large concrete diversion dams as well as many miles of canals and numerous structures. Total construction costs amounted to about $7,000,000.

The second outstanding irrigation project was one undertaken for the Santa Clara Valley Water Conservation District for the purpose of replenishing the underground water supply. The district, largely planted to orchards, was irrigated almost entirely by pumping from wells and, to 1934, the ground-water table had been dropping continuously at the rate of about 5 ft per yr until some of the pumping lifts were in excess of 200 ft. This condition was remedied by the construction of six detention reservoirs in the foothills and various regulating and distributing works in stream beds designed to retard runoff and induce percolation into the underground storage basin. A definite rise of the ground-water level has been experienced since completion of this work. The total cost of this project was about $3,000,000.

In addition to his engagements as engineer in complete charge of construction projects, Mr. Tibbetts was also employed as a consultant by many public and private agencies, including two irrigation districts in Nevada, a water conservation district in Arizona, four California cities, several land development companies and contracting firms, and numerous individuals and organizations. His services in connection with these engagements covered a wide variety of activities, including design of dams, investigation of water supplies, both surface and underground, subdivisions, irrigation systems, sewage disposal plans, harbor studies, water-works appraisals, and expert court testimony in water rights and other litigation. He made an extensive appraisal of the water rights and physical works of Miller and Lux, Inc., and the San Joaquin and Kings River Canal Company, and appraised the lands flooded by the Pardee Dam of the East Bay Municipal Utility District. He designed the Hogan flood-control dam on the Calaveras River for the City of Stockton, Calif. He directed final construction work for the contractors on the Wawona tunnel in Yosemite Valley and advised other contractors on tunnel problems. He was employed by the State Department of Public Works as a consultant on the Sacramento Valley unit of the Statewide Water Plan.

In spite of his very active professional practice, Mr. Tibbetts found time for many civic and fraternal duties. He was chairman of the Irrigation Section of the Commonwealth Club of California for three years (1925–1927), and was an occasional speaker for the Regional Planning Association of San Francisco (1925–1926). He was an active Mason, being a Past Commander of Oakland Commandery, Knights Templar, and holding office at the time of his death in Durant Lodge, in Berkeley, and Islam Temple of the Shrine, in San Francisco. He belonged to the University Club, Engineers' Club, and Olympic Club of San Francisco, and at times was a member of the Athens Club of Oakland, the Sutter Club of Sacramento, and the Arizona Club of Phoenix, Ariz. He was also a member of the American Institute of Electrical Engineers and the American Geophysical Union.

Mr. Tibbetts was married, first, to Edith Jean MacKerricher (in 1905), and second to Flora McDonald, who died only a few weeks prior to his own death. He is survived by his son by his first marriage, D. Reginald Tibbetts, and by his brother, Sydney A. Tibbetts.

During his active career, Mr. Tibbetts established a reputation as a keen thinker and a fluent speaker. He was unusually adept in presenting engineering problems to lay clients through both the written and spoken word. He had marvelous capacity for mastering and remembering intricate details of engineering theory and practice. He was a delightful companion and had a host of friends and acquaintances. He read extensively on a wide range of subjects and was particularly well informed on the strategy of the major military engagements of history. His hobbies were farming, photography, and aeronautics.

Mr. Tibbetts took a keen interest in Society affairs and contributed discussions on many subjects. He served on several important committees of the San Francisco Section and was a member of the Executive Committee of the Irrigation Division of the Society in 1927, 1928, and 1929, serving as chairman for the last two years.

Mr. Tibbetts was elected a Junior of the American Society of Civil Engineers on May 1, 1906; an Associate Member on April 6, 1909; and a Member on September 11, 1917.

ROSS KERR TIFFANY, M. Am. Soc. C. E.[1]

DIED JUNE 1, 1939

Ross K. Tiffany was born on June 11, 1879, at Union, Iowa, a son of Walter G. and Harriet (Kerr) Tiffany. After attending the grade and high schools at Union, he entered Cornell College at Mount Vernon, Iowa, from which he was graduated with a degree of Bachelor of Science in Civil Engineering and later was awarded the degree of Civil Engineer. While attending college, in 1899 and 1900, he obtained his first engineering experience in railroad location and construction—first in the Snake River Canyon with the Northwest Railway Company, and later in Missouri and Illinois with the St. Louis and New Orleans Railway Company.

Mr. Tiffany became a resident of the State of Washington in 1901 when he was employed as an engineer by the Washington Irrigation Company on Sunnyside Project, the largest single irrigation undertaking in the state at that time. The company was supplying water to about 43,000 acres of land and was actively engaged in planning storage at the headwaters of the Yakima River to water additional land in the lower Yakima Valley. He became chief engineer of the Washington Irrigation Company in 1903 and resident manager in 1906. From that year until 1910 he supervised all land-development operations undertaken by the Company. In 1905 and 1906 he took an active part in procuring limiting agreements with respect to the use of water among water users in the Yakima Valley, a step that was necessary before the government could be justified in acquiring the Sunnyside Project and undertaking extensive development in other parts of the valley.

---

[1] Memoir prepared by a committee composed of R. C. Knapp and F. H. Rhodes, Jr., Associate Members, Am. Soc. C. E., and G. L. Parker, M. Am. Soc. C. E.

Mr. Tiffany was employed in 1910 by the U. S. Reclamation Service as superintendent of the Sunnyside unit of the Yakima Project. In 1914 he was advanced to manager of the Yakima Project, in charge of operating and maintaining its several separate irrigation units and of its extensive storage developments. While project manager he helped to place collections on a business basis by organizing irrigation districts and was instrumental in discouraging land speculation.

Mr. Tiffany voluntarily left federal service in 1920 to engage in consulting engineering practice at Spokane, Wash. While there he was project engineer and project manager of the Spokane Valley Canal Company. He supervised the refinancing of Post Falls, Hayden Lake, and Dalton irrigation projects and the reconstruction of Lewiston Orchards Canal system. An important engagement during this period was negotiation leading to a final settlement of questions at issue between landowners and the Twin Falls North-Side Canal Company in Idaho.

In April, 1925, Mr. Tiffany was appointed supervisor of hydraulics in the Washington State Department of Conservation and Development. In that capacity he represented the State of Washington on the Columbia River Allocation Board, reorganized the Division of Hydraulics, increased state revenue therefrom fivefold, and handled a large increase of activities without an increase in personnel. He resigned in November, 1929, to undertake the private practice of engineering again with his office at Olympia, Wash.

Mr. Tiffany's second venture into the field of consulting engineering covered a period of more than four years during which he filled a wide range of engagements involving irrigation, drainage, diking, water supply, and water power. He was appointed by President Herbert Hoover, Hon. M. Am. Soc. C. E., to the Commission on Conservation and Administration of the Public Domain. The Commission finished its investigation in the fall of 1930 and rendered a published report in January, 1931. During the worst part of the depression, from March, 1933, to February, 1934, he served as works manager of the Washington Emergency Relief Administration, supervising an unemployment relief program financed by an emergency state bond issue of $10,000,000.

In March, 1934, Mr. Tiffany was appointed executive officer of the then newly created Washington State Planning Council, an office which he held until his death. Perhaps the most constructive accomplishment of his career was with that agency. He was eminently qualified for the office through intimate knowledge of the state and of its varied and abundant natural resources. He visualized extraordinary opportunities for the future use and control of those resources and recognized how vital planning activities are, and will become, in guiding proper development. His enthusiasm, energy, foresight, and tact proved to be invaluable assets to the Planning Council. How well they were appreciated may be judged from the following quotation included in a resolution passed shortly after his death:

> "Mr. Tiffany possessed in a high degree the rugged physique, the capacious mind, the high character, the broad outlook and the exploratory spirit that distinguished those who pioneered in the development of the West. Added to these striking qualities was the imaginative sweep of vision, and the constructive understanding of the needs and opportunities of this Pacific Northwest that made of him a great leader and builder."

Some of Mr. Tiffany's outstanding traits of character, which largely account for his steady professional rise, were his unusual geniality; his capacity to make and hold friends; his loyalty to friends; his effectiveness in promoting team-work and good-will among colleagues, employees, clients, and others; his liberal sense of humor; his fairness and integrity; his technical and administrative competency; and his energetic devotion to the immediate task at hand. All of these were sustained and provided with driving force by exceptional physical vigor and radiant health. His wholesome well-being was responsible to some extent for his untimely death. He was unwilling to remain quiet and inactive long enough to recover properly from a serious automobile accident.

Mr. Tiffany was married to Ella Eicher, of Ainsworth, Nebr., on July 3, 1903. They became the parents of a son, Ross Kerr, Jr., a civil engineer with the Bureau of Reclamation at Grand Coulee Dam, and a daughter, June (Mrs. Robert C. Thurston).

Wherever located, Mr. Tiffany took an active part in civic enterprises. He served as mayor of Sunnyside in 1908, as trustee and president of the Yakima Chamber of Commerce in 1918 and 1919, and as trustee of the Spokane Chamber of Commerce. As a member of the Rotary Clubs of Spokane and Olympia, he helped sponsor many beneficial local undertakings. He was the prime mover in organizing the Washington Irrigation Institute in 1914, served as its president in 1921 and 1922, and thereafter was made an honorary life member. He was a past-president of the Association of Engineers of Spokane. His tactful influence was an important factor in formulating a water code for the State of Washington, in improving irrigation district laws, and in securing wise legislation with respect to reclamation projects and the highest use of water and other natural resources. He was one of the founders of the Washington Natural Resources Association, served on its executive committee for a number of years, and eventually became a life member. He was a member of Sunnyside Lodge No. 138, A. F. and A. M., and of Sunnyside Chapter, R. A. M.

Those who knew Ross Tiffany intimately will cherish the memory of a true friend, the comradeship with a gentleman of fine character, and the association with a capable engineer who was genuinely devoted to the development and upbuilding of his state. He was one of the most highly respected and best loved engineers in the Pacific Northwest.

Mr. Tiffany has been active in the affairs of the Society since his election to membership in 1920. For five years he was a member of the Executive Committee of the Irrigation Division and during the last year of the period (1936) served as its chairman. He was President of the Tacoma Section in

1931 and was elected Director of the Society from District 12 in 1937. Only half of his term had been served when death intervened.

Mr. Tiffany was elected a Member of the American Society of Civil Engineers on October 11, 1920.

WARREN AYRES TYRRELL, M. Am. Soc. C. E.[1]

Died June 29, 1939

Warren Ayres Tyrrell, designing engineer of the Board of Water Supply, City of New York, N. Y., since 1927, had the valuable faculty of inspiring a loyalty and esprit de corps in his associates that made his designing section a coherent, efficient, and enthusiastic group. He was the drainage shaft expert of the Board of Water Supply and was the designer of the unique Shaft 9A of City Tunnel No. 2, a combined drainage and section-valve shaft at the foot of Steinway Street, Astoria, Long Island City, N. Y. He also had under supervision the design of the three drainage shafts for the new Delaware Aqueduct. Such responsibilities involved complete structures both above the ground level and deep down (about 700 ft) in the solid rock. The structures contain the intricate mechanical and electrical equipment necessary for un-watering many miles of sections (45 miles, 25 miles, and 15 miles) of the huge main water tunnel. So wide and varied was his engineering experience that when the position of mechanical engineer was left vacant by the death of Ray C. Ewry, Mr. Tyrrell was designated as acting mechanical engineer in addition to continuing his duties as designing engineer; he held both of these positions at the time of his death.

Mr. Tyrrell was born at Mexico, Mo., on February 16, 1875, the son of George J. Tyrrell, born in London, England, and Frances Shootman, of Mexico, Mo. He attended the elementary and high schools of St. Louis, Mo., and was graduated with the degree of Bachelor of Science in Civil Engineering from Washington University at St. Louis, *summa cum laude* in June of 1897. At Washington University, Mr. Tyrrell had the benefit of personal instruction under such well-known professional engineers as Henry S. Pritchett, C. M. Woodward, Francis E. Nipher, E. A. Engler, and the late J. B. Johnson, M. Am. Soc. C. E.[2] He had the experience of assisting Professor Johnson in the compilation of data for his classical treatise on "Strength of Engineering Materials," and of doing work in the United States Testing Laboratory at Washington University, of which Professor Johnson was in charge.

Because of the diversity and comprehensive character of his practical engineering experience for a period of more than forty years, it is desirable to itemize, briefly, the professional record of Mr. Tyrrell:

---

[1] Memoir prepared by Thomas K. A. Hendrick, Assoc. M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LI, December, 1903, p. 454.

October, 1897, to June, 1899—with the St. Louis, Peoria and Northern Railroad Company (the late Robert Moore,[3] Past-President, Am. Soc. C. E., chief engineer), as a levelman on the preliminary survey, as a topographer and transitman on the location survey, as assistant resident engineer on construction and maintenance, and as a mining surveyor for the Madison Coal Corporation (which was associated with the aforenamed railroad); in this work he was associated with the well-known engineer of St. Louis, Baxter L. Brown, M. Am. Soc. C. E.

June, 1899, to April, 1901—with the St. Louis Water Department as assistant engineer in the Extension Department—computing, designing, and drafting; later inspector and superintendent in the Distribution Department (in this work he was associated with Edward Flad, M. Am. Soc. C. E., of St. Louis).

April, 1901, to March, 1902—structural engineer for Christopher and Simpson, Architectural Iron and Foundry Company, designing, making shop drawings and shop inspections.

March, 1902, to March, 1903—with E. P. Maule, General Contractor, designing for Mauran, Russell and Crowell, Architects; designed the steel work and foundations for the 17-story annex to the Chemical Building in St. Louis, for which he made all the shop drawings, mill and shop inspections, and superintended the erection.

March, 1903, to March, 1914—general practice as a structural engineer in St. Louis. He designed for architects, estimated for contractors, and furnished shop drawings to steel companies. He designed the steelwork and foundations for many buildings, among which are the following: Fine Arts Building, University of Kansas, Lawrence, Kans.; Castner Knott Building, Nashville, Tenn.; Inter Southern Life Insurance Building, Louisville, Ky.; Grand-Leader Building, The Frisco Hospital, The Wall Office Building, Cement Plaster Mill for the Acme Cement Plaster Company, St. Louis, Mo.; and City Hall, Little Rock, Ark. Mr. Tyrrell represented such well-known firms as the Raymond Concrete Pile Company, Noelke-Richards Ironworks of Indianapolis, Ind., The American Bureau of Inspection and Tests, and the Cerosit Water-proofing Company. He was also consulting engineer for the Concessions and Admissions Department of the Louisiana Purchase Exposition Company, and inspector of boilers and elevators for the City of St. Louis for about one year. For his work with the St. Louis Exposition in 1904, Mr. Tyrrell received an engraved bronze medal which he prized very highly throughout the remainder of his life.

March, 1914, to September, 1914—Yellowstone Sulphur Company, Cody, Wyo. (W. F. B. Berger, Denver, Colo.), designing and superintending the erection of a mill for extracting sulfur from geyser deposits.

September, 1914, to June, 1917—with the Commercial Acid Company, East St. Louis, Ill., as construction engineer, and later in charge of office engineering on a large amount of chemical construction—chamber and contact sulfuric acid plants, boiler house, wedge roasting furnaces, sulfur burners, purification systems, etc.

---

[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1618.

June, 1917, to September, 1917—engaged in a series of studies of condensers for zinc vapor for the Metallurgical Laboratories, Incorporated, St. Louis, under C. H. Fulton of the Case School of Applied Science, Cleveland, Ohio.

November, 1917, to June, 1918—office engineer for J. H. Miller, Inc., general contractors, Baltimore, Md., in charge of matters pertaining to subcontractors on work for the United States Government.

June, 1918, to November, 1918—with the Air Nitrates Corporation, New York, N. Y., as assistant to the late Chauncey Grant Williams, M. Am. Soc. C. E., supervisor of purchases.

December, 1918, to October, 1923—with the J. G. White Engineering Corporation of New York City as resident engineer at the Nichols Copper Company at Laurel Hill, Long Island, N. Y., in which work he had varied engineering responsibilities.

October, 1923, to October, 1925—special studies and designs as office engineer for J. G. White Engineering Corporation on complicated structural problems.

November, 1925, to June, 1926—chief engineer and production manager for the Chemical Treatment Company, New York City, engaged in the developing process of electroplating with chromium. The company had plants in New York City, Waterbury, Conn., Chicago, Ill., and many licensees. Mr. Tyrrell was associated with John T. Pratt, L. C. Owens, and F. Hirchland.

July, 1926, to March, 1927—production manager of the Chromium Corporation of America, in New York City, which company was a combination of the Chemical Treatment Company and the Metal and Thermit Corporation.

Mr. Tyrrell's next assignment was with the Board of Water Supply, City of New York, as stated previously.

In addition to his superior ability as a technician, Warren Ayres Tyrrell possessed those qualities of mind and heart that endeared him to his associates (of whom the writer was one). There was always displayed a quiet, unobtrusive, friendly disposition which was extremely effective in promoting harmony among all cooperating engineers and engineering groups. He made a conscious effort to develop an atmosphere in which each individual felt that, while the very best was expected of him, he would be given three chances to make good. This was naturally conducive to a high degree of productivity.

Mr. Tyrrell had some characteristic sayings that he used advantageously and with discretion. He used to quote: "Today is the tomorrow we worried about yesterday, and it never came!" and "It is only by varied iterations that alien conceptions can be fostered in reluctant minds."

He was always ready for the job ahead; and this acted as a spur to stimulate curiosity in and complete the job at hand. A quiet affirmation was oftentimes his sole mark of insistence on orderly procedure. His high efficiency as an engineering executive was due partly to a regard for his associates as a team of engineering specialists, to each of whom certain phases of the analysis of a new problem were to be assigned. He had particular ability in dissecting a complicated design problem into its component parts and of making the most favorable distribution of these components to the engineers available best suited to analyze them. Although he encouraged such specialization, he was also careful to emphasize that the solutions to the various aspects of a problem must be coordinated and cleared into a unified practicable result. He would allow nothing to interfere with the accomplishment of this objective.

A familiar expression of Mr. Tyrrell, when considering a difficult analysis that had been carefully worked up and checked, was: "Can forty engineers be wrong?" Whereupon he would immediately exert a penetrating scrutiny bearing upon the assumptions made, the limitations of the materials used and their fabrication, the practical matters of economical construction, erection, and any other items pertinent to the problem at hand. He weighed those ideas of statute and common law, the exigency of which had directly or indirectly its effect and influence upon design and construction. With his clarity of vision, simplicity of statement, and informal manner of discussion, his ensuing analysis would not only indicate what changes had to be made, but, in general, would lead the way to the finally adopted solution, supported by the "over-the-table" discussions, the checked computations, and, in short, the concurrence of all minds. Certainly this was design leadership of a high order.

What was the basis of such leadership? Unquestionably it was bound up in the man himself. For each decision to be made, Mr. Tyrrell was always careful to inform himself thoroughly on the up-to-the-minute status of all relevant details. Visualization of facts was easy for him; indeed, his associates were often amazed at the clearness and scope of his descriptions. His keen insight always sought "to tie up loose ends" and to anticipate logical sequences envisaging the completed picture of the project as well as the detailed "close-ups" of progressive stages of the work. This close attention to specific detail was so natural to him that he continually elucidated intricate matters with a perspicacity that not only resulted in most excellent expression in clear-cut language, but also gradually established that high measure of inspiration which his associates continually felt in and out of his presence. The natural by-product accruing to his designing group was an increased efficiency in, and an entirely natural enthusiasm for, the work.

Since 1921, Mr. Tyrrell had resided at Merrick, Long Island, spending many winters in New York City. While living in Merrick, he took an interest in its many activities, being for several years a member of the Board of Education, a member of the Advisory Board of the First National Bank of Merrick, and a member of the Board of Trustees of the Merrick Library. At the time of his death, he was President of the Merrick Club, a club devoted to tennis and outdoor sports.

He is survived by his widow, the former Flora B. Hess of St. Louis to whom he was married on June 16, 1909, and a son, Warren Ayres Tyrrell, Jr. Mr. Tyrrell was of the Episcopalian faith.

Mr. Tyrrell was elected a Junior of the American Society of Civil Engineers on October 2, 1900; an Associate Member on October 1, 1902; and a Member on June 16, 1924. He was elected to Life Membership in 1937, and the warmth and depth of his appreciation manifested itself quietly in recurring expressions of the value of the honor bestowed.

AUBREY WEYMOUTH, M. Am. Soc. C. E.[1]

DIED JULY 27, 1939

Aubrey Weymouth was born in Richmond, Va., on November 18, 1872, the son of Edgar Jenkins and Medora (Strode) Weymouth. His parents were of Old Dominion stock.

Mr. Weymouth prepared for college at Kenmore in Amherst, Va. He entered Lehigh University, at Bethlehem, Pa., in September, 1890, and was graduated with the degree of Civil Engineer in June, 1894. He was an honor student in college and also had time for an important part in student activities. He was on the editorial board of the college paper for three years, historian of his class in his sophomore year, and was a member of many student bodies. He displayed a spirit of gentle merriment tempered by dignity and a sturdy character throughout his undergraduate days. These qualities persisted throughout his life and made his undertakings enjoyable to his associates despite the weight of some of them.

Mr. Weymouth spent most of his professional life in building construction. He witnessed the beginning and entire development of the skeleton-frame tall building, and personally took a very important part in the execution of this development. The skeleton-frame building satisfied an urgent economical need in New York, N. Y., and fortunately foundation conditions there are such that they permitted the remarkable development that has occurred.

Mr. Weymouth's first position, as draftsman for Bureau of Yards and Docks, United States Navy, at Port Royal, S. C., lasted for one and one-half years after his graduation from college. In April, 1896, he was employed by Purdy and Henderson, engineers, who specialized and pioneered in the design of steel skeleton frames for buildings.

After one year as draftsman he resigned to enter the employ of Post and McCord. From April, 1897, to April, 1900, the period of his first employment with Post and McCord, he worked on shop drawings and checked the shop drawings of other draftsmen for the steel framework of buildings. In this work he showed great ingenuity and resourcefulness. One of the notable buildings of the time on which Mr. Weymouth worked and took an important part in the preparation of its structural steel drawings was the Washington Life Building of New York City.

---

[1] Memoir prepared by Clarence W. Hudson, M. Am. Soc. C. E.

In April, 1900, Mr. Weymouth was employed by the American Bridge Company, of New York City. He remained with them until January, 1904. For the first part of this period—1900 and 1901—he was in charge of drawings for the Mt. Sinai Hospital buildings and the Corn Exchange Bank, both of New York City; and for the second part—1902 and 1903—he was assistant engineer of the American Bridge Company's Brooklyn plant, while (along with other structural steelwork) the 58th Street Power House of the Interborough Company was being fabricated.

These early experiences with Purdy and Henderson, Post and McCord, and the American Bridge Company gave Mr. Weymouth an insight into the enormous growth of both total tonnage and size of individual members that was taking place in the steel skeleton frame for buildings. He became familiar with the coincident problems of detailing, fabricating, and erecting the frames that had to be solved before the building could be constructed economically and efficiently.

The foregoing experiences and his college training had fitted Mr. Weymouth for the important part he was to take in the future growth of the firm of Post and McCord when he was re-employed by them in January, 1904, as chief draftsman.

He became chief engineer in 1910, and vice-president and chief engineer in 1930. As chief engineer from 1910 to his last illness, which began in September, 1938, he was in charge of engineering in connection with furnishing and erecting the steelwork for hundreds of important buildings in New York City and the surrounding country. A detailed list of these undertakings would be interesting and would show fully the extent of Mr. Weymouth's professional contribution to building construction. However, a short but truly important list will be substituted for it: In New York City—the Metropolitan Tower, erected while Mr. Weymouth was chief draftsman; the Bankers Trust Building, Wall and Nassau Streets; Columbia-Presbyterian Medical Center; Hell Gate Power Station; Aeolian Hall; Barclay-Vesey Telephone Building; Savoy Hotel; Chase National Bank Building at Cedar, Pine, and Nassau streets; Western Union Building; Chrysler Building; Irving Trust Building at No. 1 Wall Street; several buildings for Metropolitan Square Development; Empire State Building; and several Rockefeller Center Buildings, the largest of which is the RCA Building. Also worthy of a place in this list is the New York State National Bank Building, at Albany, N. Y. Many of the buildings listed here were unprecedented in size when undertaken and had to be erected in a metropolitan area, the facilities, safety, and conveniences of which could not be impaired during erection. Mr. Weymouth's courage, serenity of disposition, upright character, and great physical strength were all necessary to enable him to do his part in the undertakings of the Post and McCord Company.

The words of R. C. Post of the Post and McCord Company will show how he was regarded by his associates in the firm:

> "Aubrey was loved and respected by every member of the organization.
>
> "His calm, logical judgment, his keen ability to discern the flaws in any situation under discussion, his honesty and fairness and his fine

> sense of the fitness of things were a source of pride and respect throughout the entire organization.
>
> "Men of his kind are rare, indeed, and his loss as a friend and valued associate will be deeply felt over the years to come."

In order to appreciate better the kind of man Aubrey Weymouth was, an account of some of his activities outside of his regular employment must be considered. Because of professional fitness, good judgment, and his ability to do constructive work when laboring with others he served on many bodies and committees working for the general good of his profession, the industry in which he was employed, and on city and state agencies for promoting sound construction and fair labor relations.

The following partial list of Mr. Weymouth's outside professional, business, and civic services may be considered representative:

For the American Institute of Steel Construction, he was Chairman of its Committee on Technical Research, a member of the Wind Bracing Committee, a member of the Technical Advisory Board, and a member of the Committee on Code of Standard Practice.

For the American Welding Society, he was a member of its Committee on Building Codes and of its Committee on Structural Steel Welding. He served on the Welding Research Committee of The Engineering Foundation.

For the Merchants Association, he served on the Committee on Building Laws and Regulations and on a Sub-Committee on Steel. He was a member of the City-Wide Building Department Committee of New York City.

For the American Standards Association he was a member of a Sub-Committee of the Sectional Committee A-57 on Building Requirements for Iron and Steel.

He was a member of a Special Advisory Committee to the Industrial Board, New York State Department of Labor, on State Standard Building Code.

He was vice-president of the Board of Governors of a New York Building Congress.

He served on a Joint Committee on Structural Safety in Building, which committee was organized by the Architectural and Engineering Societies of the New York Metropolitan District.

He was a member of several committees for both the Iron League of New York City and the Structural Steel Board of Trade.

Of the numerous outside interests in which Mr. Weymouth engaged, none showed the kindliness and generosity of his nature more than his work for his Alma Mater. He took a great interest in everything that pertained to her welfare. He was a strong supporter of the New York Lehigh Club over a period of thirty years. He helped in the incorporation of the Alumni Association and was an important factor in shaping its policies. He realized the part athletics and good living conditions play in a proper development of student education and character, and worked intelligently and fruitfully for clean, honest sport and for the establishment of a real college dormitory and living system on the campus. He was elected an Alumnus Trustee of the University in 1922, and on the expiration of this term, in 1928, he was elected a Corporate

Trustee and in this capacity served until his death. The executive officers of the University leaned heavily on him and had the greatest confidence in his sane judgment. For a number of years he was a member of the Executive Committee of the Board of Trustees. Mr. Weymouth never missed a meeting of the Board of Trustees until his last illness overtook him. Lehigh University conferred the honorary degree of Doctor of Engineering on him in 1935.

Although Mr. Weymouth was able to, and did, greatly enjoy his professional work and his outside activities he loved to get away from it all at times and play. He was an ardent and good golfer and one of his greatest pleasures was to get out on the links with groups of his friends, former college mates, and business and professional associates. Those who were fortunate enough to have had a day's play and recreation with him always came away refreshed and inspired by his spirit of fun and sportsmanship.

Mr. Weymouth was a member of the Pomonok Country Club of Flushing, N. Y., and of the Engineers Club of New York City.

On April 27, 1901, in New York City, he was married to Alice Buer White of Beaufort, S. C., a member of a well-known family of the state.

Mr. Weymouth's first serious illness was in the spring of 1938. However, aided by his great courage he was able to " carry on " until September of that year, after which time he was confined to his home until his death.

He is survived by his widow, and his two daughters, Mrs. A. Hawley Peterson (Medora Weymouth) and Mrs. Russell H. Moock (Martha Weymouth).

Mr. Weymouth was elected a Member of the American Society of Civil Engineers on May 2, 1911.

FRANK WILCOX, M. Am. Soc. C. E.[1]

DIED MAY 25, 1939

Frank Wilcox was born on May 26, 1854, in the old City of Allegheny, Pa., at the site of the present North Side Post Office. He was the son of Lemuel North Wilcox and Eliza Fleming, of English and Irish ancestry.

He received his early education at the old 4th Ward School in Pittsburgh, Pa., later attending Stevens Institute of Technology at Hoboken, N. J. He was graduated with honors in the class of 1880, receiving the degree of Mechanical Engineer.

Upon graduation he associated himself with his brother John in the foundry and machine business in Pittsburgh, the firm being known as Wilcox, Shinkle and Miller. It was in this shop that the first Bessemer Converter was built for the Edgar Thompson Steel Company at Braddock, Pa.

Mr. Wilcox severed his connections with this firm in 1884 to become engineer of the newly organized Peoples Natural Gas Company, and laid the first gas line from the Murraysville Field into the City of Pittsburgh. When the late George Westinghouse, M. Am. Soc. C. E., organized the Philadelphia Company, Mr. Wilcox became engineer, and reported to T. A. Gillespie, who was then general superintendent in charge of construction for Mr. Westinghouse.

---

[1] Memoir prepared by W. C. Hawley, Alfred C. Ackenhell, and Charles L. Fox, Members, Am. Soc. C. E. W. C. Hawley died December 10, 1939.

During this time Mr. Wilcox laid numerous gas pipe lines into various glass houses, iron mills, furnaces, and manufacturing plants in the City of Pittsburgh. When the Philadelphia Company became firmly established, Mr. Wilcox resigned to become superintendent of the Pittsburgh Water Department, and under his supervision were constructed the Brilliant Pumping Station and Highland Park Reservoir.

In 1890 the firm of T. A. and R. G. Gillespie was formed for the construction of large riveted steel pipe lines, and it was at this time that Mr. Wilcox joined the Gillespie Brothers in what proved to be his life work—the construction of pipe lines for gas, water, and air.

Later when the T. A. Gillespie Company was organized, Mr. Wilcox became chief engineer of that large contracting company, and remained with Mr. Gillespie until 1916, when he retired from active duties, but continued his connection with the company as consulting engineer.

His experience with the Gillespie Company was wide and varied, and included the construction in 1895 of one of the first steel water lines in the United States—a 60-in. riveted steel line for water supply to the old City of Allegheny, water lines for the Board of Water Supply for New York and Rochester, N. Y., Jersey City and Atlantic City, N. J., and Detroit, Bay City, and Saginaw, Mich. During his engagement with the Gillespies he built locks and dams, tunnels, and railroads for thirty years.

In the early days of the natural gas business, he was looked upon as an authority in that field; in the introduction of steel pipe lines he stood preeminent as a pioneer; and his advice on design, manufacture, and installation was eagerly sought.

Thomas H. Gillespie, President of the T. A. Gillespie Company, writes of Mr. Wilcox as follows:

> "He was a most lovable character, generous in the extreme, and with a wealth of knowledge and experience, always at the disposal of his associates, and he performed a task as pioneer in the pipe line field that will be long remembered."

All who came in contact with Mr. Wilcox during the many years of his active professional life will agree with this statement.

On November 10, 1886, he was married to Mary Ann Brett of Brookline, Mass. She died in 1929.

Mr. Wilcox died on May 25, 1939, at Mt. Carmel Hospital in Columbus, Ohio, following a two weeks' illness which resulted from a fall on the porch of his home. He is survived by his brother, Charles Wilcox; a son, Winthrop; and a granddaughter, Nancy Wilcox. Mr. Wilcox was a valued member of the Society and his passing will be a distinct loss.

Mr. Wilcox was elected a Member of the American Society of Civil Engineers on October 7, 1903.

SYDNEY BACON WILLIAMSON, M. Am. Soc. C. E.[1]

DIED JANUARY 13, 1939

The Williamson family is of English ancestry and the name is found in various localities of Great Britain. The first of the name to come to Virginia was John Williamson, a native of Kent, who landed on this side of the water in 1707 and settled in Norfolk County, Virginia. His great grandson was Thomas Hoomes Williamson, born in Richmond, Va., on August 13, 1813, and the father of Sydney Williamson.

Thomas Williamson, after early schooling in Norfolk, Va., entered the United States Military Academy at West Point, N. Y., in 1829. He remained nearly four years, but he did not graduate. Adopting civil engineering as a profession, he was engaged in various important railroad and canal works in his native state until 1841. He then accepted a position as instructor of tactics and drawing at the Virginia Military Institute at Lexington, Va., within two years of the founding of this famous school. For forty-seven years he labored here, becoming later not only professor of engineering, but also filling the chairs of drawing and architecture. He also served a 4-yr period as commandant of cadets and another like one as treasurer. During these years he exerted a memorable influence on the hundreds of young men he taught and contributed importantly toward building up an esprit that has made the Institute world famous.

During the Civil War, although 48 years old, he took "time out" to volunteer his services to Virginia, and was appointed a Lieutenant-Colonel of Engineers, C. S. A., serving first as Chief Engineer of the Army of the Potomac at the First Battle of Manassas. He was cited by his commander, General Beauregard, for discharging his duties, which consisted of laying out the lines and field works at Manassas and Centerville, "with untiring energy and devotion, as well as satisfactory skill." He served throughout the war, and it is recorded among other items that he located the batteries along the Potomac and Rappahannock rivers, laid out the defensive lines at Fredericksburg, Va., and was asked for specially by General "Stonewall" Jackson to serve as his Chief Engineer during the McDowell phase of the famous Valley Campaign of 1862. Later he received the honorary rank of General and was so called for the rest of his life.

General Williamson married twice. His second wife was Julia Anna (Lewis) Wharton, whose forbears of Scotch-Irish stock were among the early settlers of Rockbridge County, Virginia. Their son, Sydney Bacon Williamson, was born on April 15, 1865, in Lexington, and was destined to become one of the great engineers of his generation. As his career unfolds, it is easy to trace the influence exerted by his father and by the sterling qualities derived from his ancestry.

---

[1] Memoir prepared by Col. George R. Goethals, U. S. Army (*Retired*), New York, N. Y.

To be reared in such an atmosphere as Lexington must have been inspiring to any boy. Sydney was born in a house at the end of South Randolph Street, which was in sight of the grave of "Stonewall" Jackson in the adjoining Presbyterian Cemetery. Thomas Jackson and Thomas Williamson had been old friends and fellow professors at the Institute, and from the father the boy would hear many tales of the famed Confederate leader. The dignified and beloved figure, Gen. Robert E. Lee, president of Washington College (later Washington and Lee University), at Lexington, in Sydney's earliest years, left an impression that was well remembered as the boy passed on to manhood. Gen. Sam Houston was born only 7 miles away, and many others who became nationally famous in diverse fields were either natives of Lexington or claimed the town as home. Sydney first attended the school conducted by Gen. William N. Pendleton, formerly General Lee's chief of artillery, and his daughter at the Episcopal Rectory, in an atmosphere of inspirational refinement and educational efficiency. Then he entered the public high school of Lexington and was taught by Capt. J. H. B. Jones, famous as leader of one of the few company units to reach Cemetery Ridge in Pickett's immortal charge at Gettysburg. Jones was as good a mathematician as soldier, and to him Sydney owed the fine grounding which he applied so well during his later engineering career.

When only 15 years old, Sydney qualified to enter the Virginia Military Institute, from which he was graduated in June, 1884. A fellow cadet, Capt. Greenlee D. Letcher, of Lexington, reports him as very popular, a fine baseball and football player, a beautiful skater and dancer, with a most delightful personality. In the battalion organization he was cadet adjutant. He belonged to Kappa Alpha fraternity (southern). His standing at graduation was ninth in a class of eighteen. In his own words, "this was mediocre, but I derived some consolation from theory that the hard students stood at the top of the classes, the stupid ones at the foot, and the bright students in the middle."[2]

After graduation, through the influence of his roommate, he accepted an offer as instructor in "Belles Lettres and Commandant of Cadets at the King's Mountain Academy at York, S. C.,"[2] and began his duties in September, 1884. Here he remained for nearly two terms, until February, 1886, when a letter arrived from a classmate urging him to join the former in business at St. Paul, Minn. So he resigned from Kings Mountain Academy, whimsically recording later, "I thought I did well as commandant but not so well as teacher."[2]

On his arrival in St. Paul, the greeting from Camden was disappointing: there was no job, so he had to take whatever he could get, and that quickly. After a few weeks with a real-estate dealer, he was "fired" for refusing to perform menial family tasks for his employer. A few days with an old land surveyor, and, for failing to "prick off" some plots of land tracts to the satisfaction of his boss, he was "fired" again—"fortunately," he records, "because it was then that I really made a start in my chosen profession."[2]

---

[2] From longhand manuscript by Colonel Williamson, in possession of the Williamson family, entitled "Record of Incidents Professional and Otherwise," dated January 16, 1934.

This start was a short assignment of three months from the resident engineer of the Chicago, Burlington and Quincy Railroad Company to compute quantities on a line recently surveyed. From this position he was secured in July, 1886, by C. A. F. Morris, chief engineer of the former St. Paul and Duluth Railway. Here he served as transitman on the relocation of this line from Thompson, Minn., to Duluth, Minn., a survey performed in a bitter winter; then he was in charge of a 10-mile, and soon after a 20-mile, residency; finally he became principal assistant to Mr. Morris on several proposed changes, notably between St. Paul and White Bear Lake, Minn. While on his residency he was much impressed by a visit and inspection by the late James J. Hill,[3] Fellow, Am. Soc. C. E., to whom Mr. Morris introduced him as the engineer in charge.

> "Mr. Hill said, 'Well, young man, I do not want any trestles on that line—only a solid roadway.' After leaving, I asked Mr. Morris what Mr. Hill had to do with it, and he said that Hill only owned about 52%, which was enough for him to dictate as to what he wanted. The road was built as Mr. Hill specified with stone culverts and fills. I think it was the heaviest piece of railroad construction in Minnesota at that time * * *."[2]

When surveys for the proposed changes were completed, a new control came into power, opposed further spending, and, as 1889 began, Mr. Williamson had to seek another job. He found it with the Northern Pacific Railway Company and became assistant to Mr. Kendrick, the chief engineer. He went to Billings, Mont., to look after a 50-mile branch line headed for Cooke City, Mont., from Rocky Fork through Laurel, Mont., and surveyed an alternate route from Cooke City to join the main line via Stillwater Canyon. Later he returned to St. Paul and made an important reconnaissance through northern Minnesota over practically the line later occupied by the Great Northern Railway. After preparing a general map of the country traversed, he received an offer from Montgomery, Ala., too attractive to refuse, and, at the close of 1889, resigned from his railroad work to accept membership in the firm of Williamson and Earl.

His older half brother, the late W. G. Williamson,[4] M. Am. Soc. C. E., had recently resigned as city engineer of Montgomery and had formed the firm of designing engineers bearing his name. After Sydney joined, among their retainers were the design and construction supervision of sewerage systems for several towns, among them Columbus, Ga.; plans and supervision for paving of certain Montgomery streets; and grading for the Capitol grounds. When work slackened in 1891, W. G. Williamson took a position as U. S. assistant engineer on the Tennessee River at Milton's Bluff, Tenn., and soon after (in September) employed Sydney as a transitman to make surveys of all river improvements then contemplated between Milton's Bluff and Decatur, Ala. In due course, the latter was advanced through various ratings and also became assistant engineer. The young Lieutenant of the Corps of Engineers in charge of the district which then comprised all river work below Chattanooga, Tenn., was the late George W. Goethals,[5] M. Am. Soc. C. E. From this mutual assignment arose a friendship never broken and a professional contact extended over many years, which was often interrupted by the turn of fortune, only to be resumed time after time.

---

[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXI (December, 1917), p. 1829.

[4] For memoir, *loc. cit.*, Vol. XLI (June, 1899), p. 645.

Mr. Williamson was soon sent to the district office in Florence, Ala., to take important part on plans and estimates for the lock nearby at Riverton, Ala., designed to by-pass Colbert Shoals, Ala. The design for this lock has historical significance, in that its extreme low-water lift of 26.42 ft was the greatest to be attempted anywhere up to that time. In fact, the district engineer encountered no little opposition in getting the original scheme, for two locks of 13-ft lift each, set aside in his chief's office in Washington, D. C. However, the single lock was finally approved, and with two exceptions[6] remained the highest lift lock in the world until those at Panama were built.

The construction conditions encountered were no less difficult than the design was radical. As Mr. Williamson describes it,

> "The lock site * * * was close to the south bank of the river and the soil consisted of a few feet of loam underlayed by quicksand, and several unsuccessful efforts had been made to sink a shaft through the quicksand to determine the depth to rock foundation. Lieut. Goethals sent me to Riverton to try my hand at getting down to rock. We made heavily braced boxes seven feet square by about six feet long and sunk these as far as they would go or could be driven, which was about ten feet below the surface. From that point down Wakefield sheet piling was used by driving it a few feet at a time by hand, then excavating and bracing over that distance and repeating the operation of driving, excavating and bracing until we reached rock at about 25 feet below the surface. The foundation of the lock was fixed a few feet below this, so we drilled down about 25 feet into the rock to ascertain if it was bedrock or merely a thin stratum, then put in a light shot to get samples. The quicksand exerted great pressure on bracing of our forms and they cracked now and then and additional bracing had to be put in quite frequently. All this frightened the men doing the excavation and I had to stay down in the pit with them to watch the bracing and allay their fears. The labor was darkies except there was one white man named Jim Smith and Jim was a most cheerful liar. When we were loading the hole in the bottom of the pit to shoot out samples the men began to speculate as to what would happen if the dynamite went off prematurely. There wasn't much doubt in my mind. Jim helped the situation by telling of being in a well when the dynamite went off. He said, 'Me and two other niggers were blasting down in a well once when the dynamite exploded and blew them niggers all to pieces.' I asked him, 'what did you do, Jim?' His reply was, 'Oh, I just stepped aside and let her go by.'"[2]

The district engineer inspected one day while Mr. Williamson was with his men at the bottom of the pit, and "from that moment until the day of his death Goethals had Williamson as his lieutenant on every job of importance."[7]

The subsequent construction history saw the contractor fail after incompetent performance and before any masonry was laid. The work then was put on a force account basis with Mr. Williamson in direct charge and thereafter proceeded to successful and uneventful conclusion.

---

[5] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1813.

[6] Lock between Mississippi River and Bayou Plaquemine, lift 37 ft, begun in 1895; and Hales Bar Lock, Tennessee River, lift 37.5 ft, begun in 1905.

[7] "Goethals: Genius of the Panama Canal," by J. B. and F. Bishop, p. 69.

In 1896, when the Riverton Lock was 75% completed, Mr. Williamson was relieved and put in charge of operation and maintenance of the Muscle Shoals Canal. While living on this assignment at Lock 6, he conducted an important series of field tests aimed to check the actual stresses developed in the lock gates against those deduced from the design formulas employed.

When the Spanish War began in 1898, he obtained a leave for the duration and accepted a commission as Captain in the Third Volunteer Engineers. He never served with this organization, for Goethals had applied for him as an assistant, and his service became identified with the staff of the First Army Corps under Maj. Gen. John R. Brooke in Puerto Rico. (After transfer from the Tennessee River in 1894 to the office of chief of engineers, Goethals had become Lieutenant Colonel, U. S. Volunteer Engineers, and Chief Engineer, First Army Corps, during the Spanish War.) Joining the command at Arroyo, Puerto Rico, his first task was the construction of a dock for lighters to land troops and supplies from transports lying out in the practically open roadstead. Soon after, the hope for active campaigning was shattered, for just as General Brooke's command was about to go into action to open the road to San Juan, Puerto Rico, a dispatch announcing the signing of an armistice arrived. Thereafter, the work that fell to Captain Williamson was confined to making the San Juan highway passable by repairing the arch bridges blown up by the Spaniards and removing stone slides which they had blown down across the road in many places. His request to be sent home was then immediately approved, and he left the island without even visiting San Juan, which his efforts had again made accessible. He resumed his former duties on the Tennessee River, with assignment to the Chattanooga office.

In August of 1901, Mr. Williamson was transferred (still as U. S. assistant engineer and again on the application of Major Goethals, then district engineer at Newport, R. I.) to Newport, at that time headquarters of the U. S. Engineer District comprising all federal river and harbor work between Block Island, R. I., and Nantucket, Mass. The most important projects then under way consisted of the new sea-coast defenses of Narragansett Bay and modernization of existing forts. Mr. Williamson was in charge of building the emplacements since embraced in the tactical groups comprising Forts Greble and Getty. Up to that time much trouble had existed, because of the climate of this latitude, in keeping the powder magazines dry. Built of massive concrete, the walls and ceilings used literally to run with moisture caused by condensation induced through the variable humidities and temperatures, not only as between winter and summer, but often as between night and day. A practical solution was found by building an inner room of hollow, porous tile, with narrow air space between its walls and the outer concrete, and a like space between its false tile ceiling and the concrete roof; as fast as condensation against the concrete took place, it drained off through the air spaces, while the tile acted as a blotter, absorbing the moisture in the air of the magazine, draining it off to the air spaces when saturated, and maintaining proper conditions for the storage of powder. Mr. Williamson collaborated in developing this important innovation, and the method came into general use in fortifications located in comparable climates.

With the completion of the new forts, work slowed down to dull routine in the Newport district. Mr. Williamson had been studying with keen interest the rapidly growing use of reinforced concrete, and in the latter part of 1903, after twelve years of unbroken government service, he resigned to become engineer with the Expanded Metal Engineering Company of New York, N. Y. This brought close contact with the Corrugated Bar Company of St. Louis, Mo., for which he shortly went to Baltimore, Md., making his headquarters there right after the big fire. This was the era when few engineers, and fewer architects, were familiar with reinforced-concrete design, and companies handling reinforcing materials were guarding their formulas carefully. Mr. Williamson not only secured work in Baltimore but also in Norfolk, where building opportunities anticipating the Jamestown (Va.) Exposition were very active. In many cases, where structural steel had been specified, Mr. Williamson would persuade the architect to allow an alternate in reinforced concrete. While this compelled the former to prepare his own structural design and estimate, it secured him even more buildings, a number of highway bridges and culverts, a pier at the Jamestown Exposition, and the 14th Street car barn (street railway) in Washington. This activity continued until May, 1907.

Down in the Canal Zone, President Theodore Roosevelt had placed a new Commission in charge of the work, the chairman and chief engineer of which was Mr. Williamson's former chief of Riverton, Puerto Rico, and Newport days. While his private work in Baltimore had been successful and profitable, Mr. Williamson felt he could not refuse the engineering opportunities afforded by the offer Colonel Goethals made him, although, when he reached Cristobal, Canal Zone, on May 17, he had no idea of the important position to which eventually he was to be assigned. That he never regretted the change is shown later by his own words written in 1934 as he looked back on his long career: "All I can say is that it was the most interesting and absorbing piece of engineering I have ever been connected with."[2]

After sufficient opportunity to become thoroughly acquainted with the entire project, Mr. Williamson was first assigned in July as division engineer of La Boca (changed to "Balboa" on April 30, 1909) Lock Construction Division. In December his duties were expanded to include what was then termed the Pacific Locks and Dams Construction Division. He reported at this time to the late Maj. W. L. Sibert,[8] M. Am. Soc. C. E., one of the commissioners, then in charge of all lock and dam work.

On July 1, 1908, the whole canal work began to undergo a complete reorganization which was "brought about gradually through the year with a view * * * to concentrate authority, to expedite the transaction of business, to secure better coordination, to fix more definitely responsibility in any particular case, and to reduce the cost of administration."[9] Out of this reorganization arose the three construction divisions. The Atlantic and the Central divisions were each headed by Army engineers (who, incidentally, were also commissioners). It was Colonel Goethals' plan to have one of the divisions headed by a civilian, not a commissioner, with civilian subordinates. He accomplished this plan on July 15, 1908, by combining the old La Boca Dredging and the Pacific Locks and Dams divisions into the new Pacific Division, extending from mile 39.53 to deep water in the Pacific Ocean, a distance of 11 miles. Mr. Williamson was appointed division engineer, and thereafter reported, as did the other division heads, directly to the chief engineer. Without question the record of the Pacific Division of the Panama Canal was the great achievement of his whole professional career, as it combined all the elements of engineering, construction, and administration over a period of four and a half years, and brought to a peak those qualities which made him a leader in each.

---

[8] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1642.

[9] Annual Report of Isthmian Canal Commission for 1909, p. 1.

How Mr. Williamson organized and completed his assignment through the medium of five subordinate districts has become engineering history that is best presented in his own clear reports and writings.[10] It is pertinent to review here only certain aspects and recollections of the work which emphasize the value of his services.

The original plans of the Canal, set down by the minority members of the Board of Consulting Engineers (the Board appointed by President Theodore Roosevelt on June 24, 1905, whose minority advocated a lock canal—which minority report was adopted by the President), called for a flight of two locks at La Boca and one at Pedro Miguel. Later investigation proved the La Boca site undesirable, not only for military reasons, but because of the unsatisfactory foundation conditions developed for the proposed Sosa-Corozal Dam, a structure required if locks were placed at La Boca. A careful re-examination of the Canal route from Pedro Miguel to the Pacific was undertaken by wash- and diamond-drill borings, and by test pits. Mr. Williamson took direct charge of this investigation soon after it was started, the result of which was to confirm the decision to build a single lift at Pedro Miguel, whereas the La Boca locks were to be moved inland to Miraflores. Aside from the military advantage, the latter location brought about required dams of lower height, shorter length, better foundations, and earlier completion than would have been possible at La Boca. It would have been possible even to locate all three Pacific locks at Miraflores; but if they had been thus located, the foundations of the lower locks there would have had to be carried to a greater depth than was justified economically, and international conditions were bringing increasing clamor for earliest possible completion. Accordingly, arguments favoring lower operating costs for three locks in one place were overruled. Mr. Williamson's foresight and grasp of the essential factors leading to the Miraflores two-lock decision cannot be praised too highly.

The designs common to all the locks and other structures—the lock wall sections and culverts, the gates, the operating and safety machinery, types of buildings, quarters, and the like—were all handled in the office of the chief engineer at Culebra. Questions of design to satisfy conditions peculiar to each division, however, were delegated to the division engineer to draw up and submit for approval. Although Mr. Williamson was primarily the outdoor construction man, he originated many of the features developed in the Pacific Division by his very competent designing staff, improved the details of others, and kept in general touch with all. He would state what he wanted to accomplish, but usually let the designer find the best means. He was always open to suggestion and would insist on his own solution of a problem only when convinced it was better than another. After a long day mostly spent tramping over his division, he might often be seen carrying home an envelop containing some design problem to occupy his evening hours at his quarters in Culebra.

---

[10] See Reports of the Division Engineer, Pacific Division, in Annual Reports of Isthmian Canal Commission for 1909 to 1912, inclusive; see also "Methods of Construction of the Locks, Dams and Regulating Works of the Pacific Division of the Panama Canal," by S. B. Williamson, *Transactions*, International Eng. Cong., 1915, Vol. I, p. 428 *et seq.*

Mr. Williamson contributed ideas for improving the wooden cantilever forms for lock concrete that his division made famous (the original idea is attributed to one of his designing engineers, A. P. Crary, who brought it from a previous job). He originated the idea for the hollow concrete cylinders, sunk open-caisson fashion in superimposed sections, as used for most of the foundations for the Balboa terminal piers and for the north approach wall of the Miraflores locks. Although they were not built until after he had left the Isthmus (excepting the initial dock), Mr. Williamson made the general layout of the piers and docks at Balboa which was later adopted in all essential features. He decided to remove a large amount of blue gum mud in the Canal prism—difficult and expensive to handle in the ordinary dry way—by hydraulic means; he not only reclaimed much waste swamp land, but actually sluiced uphill and deposited the material as core for earth-dam construction by flooding the borrow pit and placing his dredge pumps on floating platforms or barges. The use of hydraulic giants and lock bar pipe for dam construction was rather an innovation for those days. He guided the design for the reinforced concrete barges on which these dredge pumps were mounted. He was the moving spirit in developing the designs for the balanced berm and chamber cranes, used so successfully and economically for mixing and placing the lock concrete, with a foresight that enabled their later dismantling and re-erection for use at the Balboa coaling station. After investigating four possible sites, he selected Ancon Hill for a quarry and made a crusher installation that provided stone in two sizes for concrete—not only for the Pacific locks, but also for the dry dock, the terminals at Balboa and Cristobal, a portion of the east breakwater at Colon, and most of the fortifications, to mention only the more important structures. With all his administrative duties and acknowledged preference for the construction end of the work, his contribution to practical design was marked.

The Pacific Division record for concrete yardage and economy was unsurpassed on the Canal work. The greatest concrete output at any of the locks of the Canal in any one month was that of April, 1912, at Miraflores, when 97,735 cu yd, place measurement, were handled, an average of 3,759 cu yd each 8-hr working day of the month.[11] (The best 8-hr record of the plant, mixer measurement, was as high as 5,700 cu yd.) Comparative concrete unit costs must be read with a clear understanding of all the facts—otherwise they are misleading. Offhand, the difference between those of the Atlantic and Pacific divisions appears to lie just about in the higher cost of stone and sand indicated in the cost reports of the former. This alone makes comparison unfair to the Atlantic Division because the securing of stone and sand from points 20 and 30 miles (respectively) beyond Colon, with a rough sea haul involved, was dictated by interdepartmental considerations lying outside the Atlantic Division (such as the capacity of the Panama Railroad, had rock been hauled from the Pacific side), whereas the Pacific Division obtained its stone with a maximum rail haul of only 7 miles, and its sand after a 20-mile sea haul across water that is invariably calm. The unit-cost performances of the two divisions, therefore, should be compared on the basis of the components entirely within the control of each, such as cost of mixing and placing, reinforcements, forms, power consumption, maintenance of equipment, plant arbitrary, and divisional overhead. In these respects Mr. Williamson's records were unequaled elsewhere.[12]

---

[11] Canal Record, Vol. VI, No. 3, p. 22.

He had no special routine. Those who worked on the locks were sure to have a daily visit from him. Those on the dredging, municipal, and other work would see him less often, but every one welcomed his coming. He was always ready to discuss the work with the youngest foreman, and if the latter had a suggestion which would speed up operations, Mr. Williamson would at once make the suggestion effective, giving proper credit. He was quick to recognize merit in comparative inexperience and to promote accordingly. The superintendent in charge when the Pedro Miguel lock was completed was just out of college when the Pacific Division was organized. The spirit in the division was spontaneous. All pulled together to get the job done. In Mr. Williamson's own words:

> "I had an excellent organization on my Division and there were very few changes in those holding responsible positions throughout the work. All of the force took an intense interest in the work especially since the work in the Atlantic Division was very similar, and due to this there was more or less rivalry between the two divisions, each striving to beat the record of placing concrete in the locks of the other. The yardage placed was published each week in the Canal Record and kept the employees of each division on their toes."[2]

The close of 1912 saw the Pacific Division work in the Canal prism substantially completed, except for minor details. The installation of the lock machinery at all Canal locks was centralized in a special division under the chief engineer. Mr. Williamson had no desire to remain on the permanent operating organization, and an offer had reached him to re-enter private work. He resigned, effective December 12, 1912, and shortly sailed for England to become chief of construction for J. G. White and Company, Ltd., with office in London.

---

[12] See reports of the Chief Cost Accountant in Annual Reports of the Isthmian Canal Commission for the period 1911–1913.

The field of this company at the time was world-wide, and the construction work controlled from London included a power station and street railway at Pernambuco, Brazil; similar work at Buenos Aires, Argentina; and a water supply system for Genoa, Italy. It was essentially a remote-control method of administration, wherein the directing head had little opportunity for the intimate daily contact with his men that the Canal work had afforded; yet Mr. Williamson's associates of that time speak warmly of his ability and accomplishment in the brief period in which he served. His work was suddenly curtailed by the outbreak of the World War in 1914, and as English capital was largely involved in the projects under the London office, most of the work came to a halt. Mr. Williamson returned to the United States.

His ability as a construction administrator had attracted the attention of the Hon. Franklin K. Lane, Secretary of the Interior. Up to that time the U. S. Reclamation Service had been divided into a number of supervisory districts all administered separately by the director and chief engineer from Washington. On December 10, 1914, Mr. Williamson started to make a survey with a view to recommending a reorganization. His views were adopted and on June 15, 1915, Mr. Lane established an executive office in Denver, Colo., with Mr. Williamson as chief of construction. Thenceforth all matters relating to management and execution of the work in the field passed through the Denver office. All other detached offices (except that of supervisor of irrigation, which remained at Billings, Mont.) were incorporated there, as well as the departments of purchasing, disbursing, collections, and examination of accounts, thereby effecting many administrative economies. The chief of construction had direct charge of all employees engaged on construction, operation, and maintenance. All project managers and engineers reported to him, and in the fiscal year, 1915, there were 36 projects under way or in operation, divided among six major divisions. A central designing office was also included to standardize all structures on various projects. One of the most important features was the installation of a uniform system of cost keeping adapted from the one used on the Panama Canal. Mr. Williamson's talent and experience enabled him to accomplish these innovations and economies quickly, and they were all functioning smoothly by the end of the year.

Early in 1916, Guggenheim Brothers were seeking a man of considerable construction experience to handle their interests in Chile. At the request of Harry Guggenheim for a suggestion, General Goethals recommended Mr. Williamson unqualifiedly. The latter resigned from the U. S. Reclamation Service, and began his new duties in February, 1916, as construction engineer, with designation later changed to consulting civil engineer. This assignment occupied him for the next eight years, except for the leave of absence given him during the World War. After he had organized the designing engineering department in the New York office, much of his time was spent in Chile, both at Antofagasta, where the properties of the Chile Exploration Company were located, and at Sewell, the headquarters of the Braden Copper Company. He also spent some time at various other places where important structures were located.

It was work after his own heart. To it he brought men from the old Pacific Division in Panama and from the U. S. Reclamation Service. It involved all features of work from design through construction and the care and feeding of workmen in the field. In his period of service the copper output of the Chile Exploration Company was increased nearly two and a half times, featured by extension of several mine railroads; modernization of equipment; numerous plant extensions; construction of a 9-in. pipe line more than 60 miles long to bring potable water from Toconce to Chuquicamata, Chile; a large power station at Tocopilla, Chile; and modern townsites completely equipped with quarters, churches, hospital, schools, and welfare facilities. Versatility to meet emergency was well illustrated by Mr. Williamson at Chuquicamata. During the World War, no more structural steel could be shipped from the United States, so he immediately altered all plans and programs by changing every major structure to reinforced concrete. For the Braden Copper Company, the output was practically doubled. Featured by new crushers, smelters, and a long aerial tramway, this work included housing and employee welfare facilities as installed for the Chile Exploration Company.

When the United States entered the World War, Mr. Williamson planned his work as far ahead as foresight would permit, and, although then 53 years old, offered his services to the War Department. His employers generously gave him leave for the duration of the war. He was commissioned a Lieutenant-Colonel of Engineers on June 19, 1918, assigned to the 55th Regiment of Engineers, and reached France on July 16. The regiment was sent to Gièvres for work under the section engineer of the Intermediate Section (west), and when the Colonel of the 55th Regiment was relieved to go elsewhere, Colonel Williamson assumed command. Later, on September 22, he relieved Colonel Graves as section engineer, and his duties were soon extended to include the District of Paris. On November 4, he was promoted to the rank of full Colonel. Of the large amount of varied construction work required in the line of communications, the work at Gièvres was the largest single project, and of Colonel Williamson's part therein his immediate superior, Col. Ernest Graves, U. S. Army (*Retired*), notes:

> "The Gièvres project contemplated a depot capable of storing supplies for one million men for thirty days. Its design covered an area 2 miles wide and 6 miles long, and included 213 warehouses, 260 miles of railway track, a large refrigerating and ice plant, two remount depots, a camp hospital, a veterinary hospital, a gasoline distributing station, barracks, a water supply system, shops, roundhouses and motor parks.
>
> "As my assistant at Gièvres Colonel Williamson was the same efficient, capable engineer that he had always been. He was not only skilful and experienced but he was also a first rate executive. He was placed in charge of the Depot construction so that I could give my time to other work of the Intermediate Section (west). Later Colonel Williamson also helped me on this other work. When I was relieved as Section Engineer, there was no doubt that Colonel Williamson should be the one to take charge. This he did and did well. The construction of the Depot was carried on efficiently by Colonel Williamson in spite of continuous diffi-

culties in securing supplies of labor and material. When the Armistice was signed, 174 of the proposed 213 warehouses had been completed and 132 of the 260 miles of track had been laid."[13]

Following the Armistice, his territory was further broadened with activity devoted to road maintenance and buildings at the various centers where troops were concentrated prior to entraining for the ports of embarkation. Early in March, 1919, he was relieved by his old commanding officer, Brig.-Gen. E. L. Daley, M. Am. Soc. C. E., U. S. Army, of the 55th Engineers, who writes of him:

> "The War time service of Colonel Williamson was characterized by great energy and resourcefulness * * *. Hundreds of officers and thousands of soldiers recall the calm and reassuring manner with which he tirelessly met engineering emergency after engineering emergency during the trying days from July, 1918 to March, 1919. His contribution to the winning of the War was valuable beyond measure." [14]

He finally left France on March 9, 1919, and was honorably discharged from the service the day after reaching the United States, resuming at once his duties with Guggenheim Brothers to complete the projects already outlined.

In January, 1924, Colonel Williamson had finished the work he had agreed to undertake for the Guggenheim interests, and his contract with this firm terminated. For the next five years he engaged in private practice, forming a consulting engineering partnership with his son, Lee Hoomes Williamson, M. Am. Soc. C. E., with offices in Birmingham, Ala., and Charlottesville, Va. After reorganizing and finally liquidating a contracting company for a bank in the former city, his activities followed mainly water-supply design, valuations, and appearance as expert trial witness, with occasional retainers in other fields. Important among these assignments were reconstruction of the water supply for Pulaski, Va.; studies for increasing the supply of Front Royal and Lexington, Va.; valuation of the Roanoke, Va., water company plant; similar work for the town of Appalachia, Va.; valuation of Camp Lee, Va., and testimony in connection with the trial of the contractor who built it; design of filtration plant for Fredericksburg, Va.; and design and supervision of a sewage disposal plant for Blacksburg, Va. In addition to this work he also found time to assist General Goethals, in the latter's last undertaking, as consulting engineer on the Lake Worth Inlet project at Palm Beach, Fla. He designed the transit sheds, the rail connections, the water supply, and sewage disposal for this harbor development. When General Goethals' illness prevented his presence in Florida, Colonel Williamson proved his devotion by representing him on the job and closing up the work upon the General's death in 1928. The last piece of private work engaged in by either of these long-time friends found them associated.

In 1929, President Herbert Hoover, Hon. M. Am. Soc. C. E., called Colonel Williamson again to an important government service, this time as one of the three civilian members of the Interoceanic Canal Board, appointed to investigate the cost of building and maintaining a canal across Nicaragua, as well as the practicability and cost of providing additional locks and other facilities at Panama. He was actively engaged on this work from June 12 until the Board submitted its report on November 30, 1931,[15] making one inspection trip to Nicaragua and Panama and spending the remainder of the time in the Washington office of the Board. Of all the valuable work accomplished by this Board, Colonel Williamson's individual contributions comprised a study under the title, "Probable Future Traffic through the Panama and Nicaragua Canals,"[16] and a practical plan for converting the Panama Canal into a sea-level route without interruption of traffic;[17] yet his broad general experience was felt and appreciated throughout all the ramifications of the Board's work, from the office in Washington to the field organization conducting the surveys in Nicaragua.

---

[13] From a letter by Colonel Graves to the writer.

[14] From a letter by General Daley to the writer.

> "Due to his experience on the Panama Canal, Colonel Williamson was a logical choice for appointment to the Interoceanic Canal Board. He brought to the work of that Board a knowledge of canal construction possessed at that time by no other engineer in the United States. He was the only civilian member of the Board who was employed continuously on its work. The meritorious nature of his service is indicated by the fact that, upon completion of the Nicaragua Canal Survey, he was retained as a consultant (Principal Engineer) in the Office of the Chief of Engineers * * *. Too much cannot be said in praise of the thoroughness and accuracy of the engineering work of Colonel Williamson."[18]

For the remainder of his professional career, Colonel Williamson continued in the rating of head engineer, and as such was a member of the Board of Engineers for Rivers and Harbors, devoting a large part of his time to river, harbor, and flood-control work in general, and to the Mississippi River flood-control problem in particular. The remainder of his time was spent in connection with the government's case in litigation regarding railroad, highway, and utility crossings over the Bonnet Carré Floodway. This involved preparing a mass of detail on design and cost analyses of such crossings to enable him to testify at the court sessions held in New Orleans, La. His testimony was of great value in reaching equitable settlement of these cases.

In 1935 Colonel Williamson's health began to fail, and on April 30 he was retired under provision of the Civil Service Retirement Act. Of the more than fifty-one years of his professional career, nearly twenty-seven of them had been devoted to the service of his government. He returned to Lexington, Va., re-established the family home there, and passed the remainder of his days quietly in the picturesque town of his birth. A progressive heart ailment forbade further activity, although he enjoyed and encouraged visits and letters from old friends. Toward the end of 1938 his condition became alarming, and on January 13, 1939, he died in his seventy-fourth year.

---

[15] Rept. of U. S. Army Interoceanic Canal Board, H. R. Doc. No. 139, 72d Cong., 1st Session.

[16] *Loc. cit.*, Appendix C.

[17] *Loc. cit.*, Rept. of Governor of the Panama Canal, Part III.

[18] Col. Ernest Graves, U. S. Army (*Retired*), Chairman of Interoceanic Canal Board, by letter to the writer.

He is survived by his wife, the former Helen C. Davis, a native of St. Paul, Minn., to whom he was married on May 20, 1890; their two children, Lee Hoomes, and Julia Lewis Williamson (Mrs. Julian Smith), and six grandchildren.

Colonel Williamson was an Episcopalian. Besides the American Society of Civil Engineers, he also held membership in the American Society for Testing Materials, the Cosmos Club, and the Army and Navy Club, the latter two in Washington, D. C.

The Virginia Military Institute signalized his passing by a general order[19] summarizing and eulogizing his career. The Board of Visitors also passed a resolution of condolence. In keeping with his characteristic wish for simplicity and avoidance of display of any kind, the funeral was simple and non-military, but, until her distinguished son was buried in Lexington Cemetery on the afternoon of January 14, before a throng gathered in silent tribute, his alma mater kept flags at half-mast.

As an engineer, the outstanding characteristic of Colonel Williamson was breadth of gage. Qualities frequently noted separately in many of his profession were found in him in a combination that occurs but rarely. He possessed marked ability and originality in design, without losing sight of practical considerations and economy in the field wherein so many pure theorists stumble. He was the ideal construction manager, with principles of theory so well grounded that he never relied, as many so-called construction men do, on mere rule-of-thumb methods with which to meet the unexpected problem or field emergency. On all of his peace-time assignments the speed and economy of performance attained were outstanding; yet, in a military situation, where action and speed must rule economic considerations, his versatility could adapt his course instantly to the special demands. He was a born manager of men, getting along with them just as well as they worked for him, and it mattered not whether military discipline was at his elbow. On the civil work he headed, a discipline was there just the same, although the men were never made aware of it. There existed the spirit by which men worked for the job and for him at the same time, whether he was moving from board to board in the drafting room or calming a group of laborers working at the bottom of a test pit and afraid of a cave-in. His attitude toward superiors was just as successful. The younger, ex-line officer generals in France, who would so often send for him to demand, in their innocent ignorance, the "engineering miraculous" to complete something tomorrow, would end the interview with a laugh at Colonel Williamson's exposition of the "engineering possible" and their thanks at having the job done within the time limits of human endeavor, as it always was. In short, every one, civil or military, who ever worked over, with, or under him, and really learned to know him, reached the same appraisal of his well-rounded ability.

---

[19] General Orders No. 14, Virginia Military Inst., January 13, 1939.

He was about 5 ft 10 in. tall, medium complexion, slender but wiry build with erect military carriage, and always neat in dress and appearance. Colonel Williamson possessed a keen wit and sense of humor impossible to describe, and always had command of anecdote to illustrate a point, whether while inspecting his work or sitting as a member of some board. His face was said to resemble greatly that of his father, and, due to the rigorous outdoor life of most of his career, it early became deeply lined and furrowed, giving a very serious, even severe, expression—until he smiled. One of the stories that he loved to tell on himself is of a youngster in Newport, who, seeing him frequently pass by deep in thought about some problem he had not left behind at the office, one day mustered up enough courage to run out, grab him by the hand, look up, and ask, "Mister, what yer cryin' fer?"

Although he was essentially a man devoted to family and friends and shunned society for its own sake, he was the best kind of company in any gathering. He had no foibles and those of others aroused in him amused tolerance. About the only qualities to irritate him were conceit and self-importance, and then he could register a choice sarcasm. A young captain, earmarked by these traits, who had served under him briefly in France, was ordered for service in Paris with the War Damages Board after the Armistice, and told Colonel Williamson on his parting visit that he (the Captain) knew he was badly wanted, "because they were in a fix for good men." "Yes," said the Colonel, looking over the top of his glasses, "and they're going to be in a whole lot worse fix, judging by some of the men they're sending up there."

The Distinguished Service Medal for his work at the Gièvres Depot during the World War was one of the material honors accorded him, but the most signal one was the designation by the governor of the Panama Canal of his likeness to appear on the 30-cent postage stamp of the permanent issue of the Canal Zone (designated for release on April 15, 1940—Colonel Williamson's birthday). It is especially noteworthy in Colonel Williamson's case, for this honor has otherwise been reserved solely for certain of those who held the rank of commissioner during the construction period of the Canal.

No monuments can be greater than the structures he left behind him or the affection in the hearts of those who worked for and with him. Upon his death he was mourned by all these people, wherever found. From Panama, the scene of his greatest achievement, came the word: "There are many men still here who remember his kindly spirit and the tremendously hard work he performed * * *. Of memorials there can be no lack so long as this magnificent work, to the construction of which he contributed in so distinguished a manner, endures."[20]

Colonel Williamson was elected an Associate Member of the American Society of Civil Engineers on January 3, 1894, and a Member on December 2, 1896.

---

[20] Brig.-Gen. C. S. Ridley, Governor of the Panama Canal, by letter to Mrs. Williamson.

FREDERICK NEWTON WILLSON, M. Am. Soc. C. E.[1]

DIED NOVEMBER 15, 1939

Frederick Newton Willson, professor of graphics, emeritus, at Princeton University, at Princeton, N. J., was the son of Thomas Newton Willson and Mary Caroline (Evarts) Willson. He was a direct descendant of Col. John Willson, of the Virginia House of Burgesses, and of Tirzah Humphreys, a niece of Samuel Findley, fifth president of Princeton University.

Professor Willson was born in Brooklyn, N. Y., on December 23, 1855. In 1872 he was graduated from Troy Academy in Troy, N. Y., of which his father was the principal. Immediately after graduation he served for one year as assistant bookkeeper in the Troy City National Bank, entering Rensselaer Polytechnic Institute, at Troy, as a freshman in 1873. After the freshman year he left college to teach mathematics at Troy Academy for the school year 1874–1875, during the illness of his father. He then returned to Rensselaer and was graduated in 1879 with a degree of Civil Engineer.

He took a postgraduate course in psychology and philosophy under his friend, Daniel S. Gregory, president of Lake Forest University, at Lake Forest, Ill. Following his postgraduate year he became assistant foreign correspondent of Drexel, Morgan and Company of New York, N. Y. Early in the winter of 1880 he was called to the College of New Jersey (Princeton) by President James McCosh to organize courses in graphics as part of the John C. Green School of Science.

In 1883 he was made professor of descriptive geometry, stereotomy, and technical drawing, a post created for him in the John C. Green School of Science. In 1896 Princeton University conferred upon him the honorary degree of Master of Arts, and in 1905 his title was changed to professor of graphics.

In 1922 the Department of Graphics and Engineering Drawing was established in the Princeton Engineering School, and Professor Willson was called upon to take the chairmanship and organize the department.

He served as professor of graphics and chairman of the department until he retired from active service in June, 1923, completing nearly forty-three years of university teaching. The courses that he taught included descriptive geometry, stereotomy, technical drawing, kinematics, and machine design.

As a scholar in the field of graphical science, he was recognized as an eminent authority. He developed a collection of models which represented the surfaces with which his teaching was concerned that became a valued possession of the Princeton Engineering School. His well-known treatise on graphics, descriptive geometry, and mechanical drawing, first published in 1897 and entitled "Theoretical and Practical Graphics," was given immediate recognition as an outstanding work.

---

[1] Memoir prepared by Philip Kissam, Assoc. M. Am. Soc. C. E.

In his enthusiasm to utilize his writings he published excerpts from his book while he was still in the process of completing it. These excerpts, together with the book itself, constituted a series of eight texts on various phases of his subject. They are the following:

1. Note-Taking, Dimensioning, and Lettering.
2. The Third Angle Method of Making Working Drawings.
3. Some Mathematical Curves and Their Graphical Construction.
4. Practical Engineering Drawing and Third Angle Projection.
5. Shades, Shadows, and Linear Perspective.
6. Descriptive Geometry—Pure and Applied, with a chapter on "Higher Plane Curves and the Helix."
7. Theoretical and Practical Graphics.
8. The Perspective of Reflections.

He later published "Paraphrased Proverbs," in 1933, and "Graphics and Faith," in 1936.

Professor Willson was married to Miss Mary Hewes Bruere in May, 1884. She died nine years later, and in July, 1895, he was married to Miss A. Russell Albertson. The second Mrs. Willson died in February, 1929.

His survivors include three daughters, M. Louise Willson, Elizabeth Willson (Mrs. Warren Buffum), and Edith Willson (Mrs. Harold F. Pellegrin); one son, Albert N. Willson; and eight grandchildren, Barbara, John, Donald, Paul, and Fred Willson, and Francis, Fred, and Edith Pellegrin.

Professor Willson was an active member of many professional societies and made a particular point of attending every meeting possible. He was a fellow of the American Association for the Advancement of Science, and a member of the American Society of Mechanical Engineers, the American Mathematical Society, Phi Beta Kappa (Princeton), and Sigma Xi (Rensselaer).

His remarkable memory for names and faces endeared him to his professional associates and to his students. He could give on sight the name and class of a former student he had not seen for twenty years or more. He was keenly interested in the development and career of all of them. Earl Anderson, the originator of the "cage" masts which were used for many years to support the fire-control tops of United States warships, was one of Professor Willson's students and his design for the mast had its inspiration in one of the models that Professor Willson had used in class. This occurrence pleased and gratified Professor Willson almost more than some of his own achievements.

Professor Willson was known for his inexhaustible interest in his subject and in his profession as an engineer. He holds the unique record of having crossed the Brooklyn Bridge catwalk in 1878, and the George Washington catwalk in 1929, and finally, on his eightieth birthday in 1935, he stood on the catwalk at the top of one of the towers of the Golden Gate Bridge.

He had a deep and lasting interest in his religion and served as Elder in the First Presbyterian Church of Princeton for fifty-three years, 1886–1939.

He was always characterized by his youthful enthusiasm, by the energy he displayed up to the very end of his life, by the scholarly approach he exhibited toward any problem, and by the comprehensive grasp of his subject; but, despite the contributions he made to the graphical sciences and to the art of teaching, he will be long remembered not for his writings nor for his professional achievements but because of the loyalty and admiration he inspired in the hearts of those who had the good fortune to know him as no others could—his students.

Professor Willson was elected a Junior of the American Society of Civil Engineers on September 5, 1883; an Affiliate on October 4, 1892; and a Member on March 15, 1926.

FRANK EDWARD WINSOR, M. Am. Soc. C. E.[1]

DIED JANUARY 30, 1939

Frank Edward Winsor died at the very height of a distinguished career as an engineer and public servant. He will be remembered not more for his professional achievements than for his important public services, his wide sympathetic interests, his friendly spirit of cooperation, and the high integrity of his character. The success which he achieved in his chosen profession may well be attributed in part to his ancestry. His father was George Henry Winsor, a mechanical engineer. His mother was Lucy Jane Draper, whose father, Alpha Angell Draper, was highway commissioner for fifteen years in the town of Lincoln, R. I. His great-great-grandfather, Israel Angell, was a Colonel in the Revolutionary War and a descendant in the fifth generation of Thomas Angell, who came to Providence, R. I., with Roger Williams.

Mr. Winsor was born on November 16, 1870, in Johnston, R. I., later a part of Providence. He attended Brown University, in that city, from which he was graduated in 1891 with the degree of Bachelor of Philosophy., subsequently receiving the degrees of Civil Engineer in 1892, Master of Arts in 1896, and the honorary degree of Doctor of Science in 1929 from that institution. He was the eldest of four brothers, the surviving three being George A., M. Am. Soc. C. E., William H., and Harry D., Assoc. M. Am. Soc. C. E.

Except for miscellaneous work in surveying parties, prior to graduation, Mr. Winsor began his engineering career in August, 1891, with the Metropolitan Sewerage Commission of Massachusetts, successively as transitman and assistant engineer, mainly on preliminary surveys covering a considerable part of the Northern Metropolitan System, and on the construction of the Shirley Gut Siphon and the Deer Island Pumping Station.

From February to September, 1895, he was employed by the late Samuel M. Gray,[2] M. Am. Soc. C. E., a consulting engineer of Providence, first in designing, and later as resident engineer in constructing a sewerage system for Natick, Mass., which cost, when completed, about $200,000. Association with Mr. Gray was to "bear fruit" later. However, he did not finish the sewer job, but left in October to join the staff of the Metropolitan Water Board of Massachusetts. For about five years he worked as an assistant in the Wachusett Dam and Reservoir and the Wachusett Aqueduct Departments. Then he was promoted to assistant engineer in charge of the Weston Aqueduct Department office in Saxonville (Mass.) for two and one-half years, and division engineer in charge of the construction of Division No. 2 of the Weston Aqueduct in Framingham and Wayland, Mass. This included about 2½ miles of masonry aqueduct and about one mile of 90-in. steel-pipe siphon across Happy Hollow, and cost about $500,000.

---

[1] Memoir prepared by a committee consisting of Karl R. Kennison, M. Am. Soc. C. E., Chairman, and William W. Peabody, M. Am. Soc. C. E., and Harry D. Winsor, Assoc. M. Am. Soc. C. E.

[2] For Memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1679.

In February, 1903, he took a position with the Commission on Additional Water Supply for New York City. He was principal assistant engineer in charge of field investigations in the Aqueduct Department. His work covered more than 100 miles of aqueduct line and about 30,000 acres of reservoir sites. He remained with this Commission less than a year, and in November was called back to Massachusetts by the Charles River Basin Commission, successively as division engineer in charge of the design of dam, locks, flood-control structures, etc., and as principal assistant engineer. Apparently his nine months' work in New York was appreciated because in April, 1906, the New York Board of Water Supply called him back, first as senior division engineer for nearly four years and then for five and one-half years as department engineer in charge of the Southern Aqueduct Department. This included construction of about 31 miles of the Catskill Aqueduct and the Kensico and Hill View reservoirs. The aqueduct includes about 13 miles of tunnel varying in size from 14-ft circular pressure tunnel to 17 ft 6 in. by 13 ft 4 in. horeshoe section grade tunnel, and shafts up to a maximum depth of about 560 ft. The cut and cover sections of the aqueduct were generally of horseshoe section equivalent to 17 ft in diameter. The Kensico Reservoir with a capacity of about 33 billion gallons required a masonry dam about 300 ft high with nearly one million cubic yards of masonry (at the time one of the largest masonry dams in the world), an earth dam, and other appurtenances. The Hill View equalizing reservoir at the New York city line has a capacity of nearly one billion gallons. Expenditures for the department were about $35,000,000 which included about $26,500,000 for construction.

Mr. Winsor never lost touch with the work of the New York Board of Water Supply, and for several years before his death he acted as one of its consulting engineers.

In August, 1915, the newly organized Water Supply Board of the City of Providence had just accepted a report from Mr. Winsor's former employer, Samuel M. Gray, recommending a new water supply, and it called Mr. Winsor to Providence as its chief engineer. The work included the construction of a storage reservoir with a capacity of 37 billion gallons which required an earth dam with a maximum height of 100 ft above the ground and 180 ft above the rock; also 26½ miles of substitute highways, and the establishment of the new cemetery. It also included about 14 miles of aqueduct and pipe lines, a small tunnel about 3½ miles long, a purification plant of 50 mgd capacity of the rapid filter type, and two covered distributing reservoirs. The new supply involved an expenditure of about $21,000,000.

As the work in Providence was nearing completion, the Metropolitan District Water Supply Commission, which had just been named by the Governor of Massachusetts to construct major additions to the water supply sources of the Metropolitan Water District, was looking for a chief engineer. Mr. Winsor's broad experience in this field of work and familiarity with the local conditions made him the logical candidate. He began the work in October, 1926. It consists of an extension westerly to the Ware and Swift rivers from Wachusett Reservoir which he, as a young man, had helped to build. The new Quabbin Reservoir in the Swift River valley will have a capacity of 415 billion gallons, will flood an area of about 39 sq miles, will be 18 miles long, and will have a maximum depth of 150 ft with a shore line of more than 175 miles. The area is so great that it has necessitated relocating the boundaries of six towns and three counties, and wiping four towns out of corporate existence. It also required 36 miles of relocated state highways to replace existing highways in the reservoir area. The variety of the activities in which Mr. Winsor and his organization engaged can be appreciated from the fact that a new cemetery was required in which about 7,500 bodies buried in the area affected by the reservoir required reinterment. About 80,000 acres of land were acquired for sanitary protection and construction purposes in the reservoir area.

The gravity connection to Wachusett Reservoir is by a 24.6-mile tunnel which was practically completed in 1934. This tunnel passes directly beneath the Ware River Intake Works, completed in 1931, by means of which flood flows from the Ware River are dropped into the tunnel and can be diverted in either direction—easterly to Wachusett Reservoir or westerly to Quabbin Reservoir. Two earth dams will impound the waters of Quabbin Reservoir. The smaller dam, called Quabbin Dike, is 2,140 ft long, 135 ft high above the bed of Beaver Brook, and 264 ft above the surface of sound ledge. Mr. Winsor completed it in 1937. The larger main dam is 2,640 ft long, 170 ft high above the bed of Swift River, and 295 ft above sound ledge, and will contain about 4 million cubic yards above the original surface. This dam was nearly completed at the time of Mr. Winsor's death, and the Water Supply Commission very fittingly named it "Winsor Dam" in his memory. Each of these dams was constructed by the hydraulic-fill method, above a deep concrete cutoff wall built by sinking a row of caissons across the valley.

A number of articles in Society publications have covered various aspects of this work. These include a group of four papers, one by Mr. Winsor, in the December, 1929, *Proceedings*; a symposium on Surface and Subsurface Investigations in the 1937 *Transactions*, to which Mr. Winsor contributed a foreword; a general description of the project which he wrote for the June, 1934, *Civil Engineering*; a paper on the Ware River Intake Works by Karl R. Kennison, M. Am. Soc. C. E. (who succeeded Mr. Winsor as chief engineer) in the August, 1934, *Civil Engineering*; a paper on the Quabbin Dike by Stanley M. Dore, M. Am. Soc. C. E., in the 1938 *Transactions*; and a paper on surveys for the Swift River Reservoir by N. LeRoy Hammond, M. Am. Soc. C. E., in the February, 1931, *Proceedings*.

Mr. Winsor's work with the Water Supply Commission involved much more than the Ware-Swift project. In 1927 he developed the Southern Sudbury system of the District as an emergency supply by constructing 7 miles of pipe lines connecting the purer upland sources with the Northern Sudbury system. The Ware-Swift construction had hardly begun, with contracts for shaft sinking, before the State of Connecticut in January, 1928, filed a bill in equity in the United States Supreme Judicial Court against the Commonwealth of Massachusetts objecting to the diversions from the Ware and Swift rivers. Mr. Winsor's energies were thereafter largely devoted to the collection and presentation of data for this important suit. The decree of the court, rendered three years later, dismissed Connecticut's bill of complaint and upheld Massachusetts' right to make the diversions. The successful outcome of this important case was due in no small measure to Mr. Winsor's marshalling and correlation of the facts.

About two and one-half years before his death, Mr. Winsor was asked to investigate methods to improve the distribution of water from the sources of supply of the Metropolitan Water District, and more adequately to prevent the pollution due to the fact that all the water from Quabbin and Wachusett reservoirs must pass through the old Sudbury system and the grade line aqueducts. As a result of his thorough investigations, the legislature gave the Commission the additional task of constructing a pressure aqueduct to by-pass the Sudbury system and deliver the water directly from Wachusett Reservoir to the District, at a sufficiently increased pressure to eliminate much of the present pumping in the District.

Due in no small part to the efficient organization and conduct of the work on the Ware-Swift extension, Mr. Winsor had reported an estimated saving of approximately \$12,000,000 on the original appropriation of \$65,000,000 which was made for the Ware-Swift extension, and in August, 1938, following Mr. Winsor's pressure aqueduct report, the Commission was allowed to spend this sum, supplemented by a federal grant, to take the necessary first steps in the aqueduct construction. Mr. Winsor proceeded to organize the work, and contracts totaling about \$9,000,000 had been awarded, and some of them well underway, at the time of his death.

The District owes a great deal to him personally for the efficient planning and conduct of the work. His contacts with his engineering associates, contractors, and public officials were marked by tactful understanding and fair dealing, which won for him a host of friends. Indicative of the high regard of his associates and the acclaim with which his appointment to the position of chief engineer of the Metropolitan District Water Supply Commission was received, is the complimentary dinner given to him by the entire Affiliated Technical Societies of Boston in January, 1927.

In all the traits that make for a successful engineer and a worth-while man, Frank Winsor was outstanding. His mind could grasp the theoretical as well as the practical details, yet he never lost sight of the larger aspects. He could deal equally well with a lowly subordinate, an important contractor, or an official commissioner. His presence engendered confidence, which grew and grew with acquaintance.

He worked hard, perhaps too hard. Long hours were no novelty to him when he became engrossed in a problem. But he knew equally well how to play and when the occasion suited, he would enter into any hilarity that was afoot with the zest of a boy. The merry twinkle of his eye and the contagious chuckle were characteristic. Thus he was a popular figure at Society gatherings, alumni meetings, or dinners of engineers. He was most punctilious in keeping such engagements, as well as with his more serious duties.

In his home life, be it in the city or at the shore, he was at his best. He and Mrs. Winsor loved company and were marvelous hosts. The social events they sponsored for members and families of his engineering organization were noted.

Not an orator, he was an easy speaker in public. He impressed particularly with his marshalling of facts, his logic, and his great memory for details. Around the conference table this quality was especially effective.

It is not strange, therefore, that his services were in demand on committees dealing with professional matters. He was most generous of his time, as well as his means, in serving engineering, educational, and charitable organizations. Speaking only of his Society activities and mentioning but a few of his services, his appointments included the Executive Committee of the Board of Direction (two years), Committee on Honorary Membership (two years), on National Hydraulic Laboratory, on Federal Reorganization, on Hydraulic Fellowships, on J. Waldo Smith Fund, Society representative on Engineers' Council for Professional Development, Trustee of United Engineering Society.

Following Mr. Winsor's death, the Metropolitan District Water Supply Commission inscribed on its records a memorial which recited his professional achievements and concluded:

> "The forty-seven years of service given to the public by Mr. Winsor and the extensive works constructed under his supervision are a lasting memorial to his vision and industry. To his friends and associates his unlimited kindness and patience, his fidelity to all tasks entrusted to him, and above all his faithfulness to what he conceived to be right constitute his true imperishable memorial."

Mr. Winsor was married to Catharine Holbrook Burton in Providence on October 25, 1893. They had three children: Lucy Burton (Mrs. Hugh B. Killough), associate professor of economics at Wellesley College, Wellesley, Mass.; Edward, a practising lawyer in Providence; and Catharine (Mrs. Hazen H. Ayer) of Winchester, Mass.

Mr. Winsor was a member of the American Water Works Association, president of the Providence Engineering Society in 1922–1923, president of the New England Water Works Association in 1927 and an honorary member at the time of his death, and president of the Boston Society of Civil Engineers in 1929. He also served a term as a director of the Municipal Engineers of the City of New York. He was a Fellow of the American Academy of Arts and Sciences, a Thirty-Second Degree Mason, and a member of Sigma Chi fraternity, Sigma Xi, and the Boston City Club. He was president of the Brown Club of Boston in 1931, a Trustee of Brown University, and an honorary member of the Brown Engineering Association.

Mr. Winsor was elected an Associate Member of the American Society of Civil Engineers on November 4, 1903, and a Member on December 5, 1905. He served as a Director of the Society in 1922–1924, and Vice-President in 1930–1931.

CHARLES NICHOLSON WRIGHT, M. Am. Soc. C. E.[1]

Died September 6, 1939

Charles Nicholson Wright, the second son of the late Edward Thomas Wright, M. Am. Soc. C. E., and Lucy (Nicholson) Wright, was born at Fifth Street and Fort Street (later Broadway), Los Angeles, Calif., on November 29, 1876, his family having moved to Los Angeles from Cobden, Ill., the previous year.

In July, 1875, his father established an engineering office on the site of the present Los Angeles post office and became one of the leaders in the development of Los Angeles and vicinity. He was elected county surveyor in 1880, and afterward was elected twice more to the same office, the last term being 1895–1898, during which period he developed the first complete map of Los Angeles County. During their spare time and vacations, Charles Wright and his brother, G. A. Wright, were tutored by their father in engineering practices, so that it was natural for them to follow in their father's footsteps when they grew up.

After graduating from primary and high schools in Los Angeles, Charles Wright went to the University of California, at Berkeley, and graduated in 1902 with the degree of Bachelor of Science in Mining Engineering. For the next five years he was employed by the Arizona Copper Company and the Lower California Copper Company in the development of their mines in Arizona and Lower California. He then went to Goldfield, Nev., where he was engaged in mining work and the construction of the Las Vegas and Tonopah Railroad.

In the fall of 1909, Mr. Wright was employed as tunnel superintendent on the construction of the Los Angeles aqueduct. From January, 1910, to August, 1911, and from October, 1913, to November, 1917, he was engaged in county highway work in Los Angeles, Riverside, and Ventura counties. This position included design and construction work of the various highway systems. From September, 1911, to September, 1913, he was engaged in the development of lead-silver property in Inyo County, California. From the fall of 1917 until the summer of 1922 he was with the Shell Oil Company, Inc., in Los Angeles, engaged in general engineering work.

---

[1] Memoir prepared by Charles R. Sumner, M. Am. Soc. C. E.

In the summer of 1922 Mr. Wright joined his brother who, since the death of their father in March, 1917, had been continuing the business under the name of The Wright Engineering Company. On the death of his brother, Charles Wright took full charge of The Wright Engineering Company and continued the business until his death. Mr. Wright was also city engineer for the City of San Gabriel, Calif., from May, 1923, until his death, except for the period from May, 1935, to May, 1938.

Mr. Wright was married to Mary Jane Allen of San Francisco on June 2, 1922, and soon after their marriage they moved to their home in San Gabriel. His widow survives him.

Mr. Wright was very accurate and precise in his work, and was an exceptionally good designer and draftsman. He had an even disposition, but was supersensitive of criticism if he thought he was being misjudged. Although somewhat retiring by nature, he was most friendly and helpful to his fellow men. His friends will miss his bubbling laugh and friendly smile, which endeared him to every one with whom he came in contact.

Mr. Wright was elected a Member of the American Society of Civil Engineers on June 9, 1930.

JOHN MORTON BILLING, Assoc. M. Am. Soc. C. E.[1]

DIED JANUARY 9, 1939

John Morton Billing was born on February 1, 1903, in Blackpool, England. He was educated at Bentham Grammar School and at London University, both in England. At the University he received the degree of Bachelor of Science, *cum laude,* in 1922, and later was elected an Associate Member of the Institution of Civil Engineers and also of the Institution of Municipal and County Engineers.

From January to May, 1924, he was lecturer at the Southend Evening Technical School. In June, 1924, he was appointed junior assistant in the County Borough Engineer's Office, Corporation of Southend-on-Sea, England, in Building Inspectors' and Town Planning Sections. In February, 1926, he was appointed assistant in the Borough Engineer's Office in Crewe, England, and worked on surveys, etc.

In September, 1926, he was appointed an assistant engineer in the Malayan Public Works Service, and he remained in the Public Works Department (PWD) until he died at the age of thirty-six.

As assistant engineer, he was stationed in many different parts of Malaya. He was engaged in the sanitation branch in Singapore, on a water supply project in the Province of Wellesley, and on the construction of an aerodrome in the State of Perak. In addition to these posts he acted as assistant engineer in PWD districts in the States of Johore, Perak, and Pahang. In

---

[1] Memoir prepared by A. Wear, Personal Asst. to Director of Public Works, Straits Settlements, Singapore, Straits Settlements.

1935 Mr. Billing was promoted to the rank of executive engineer and, after being in charge of Kuala Lipis district, Pahang, was transferred in 1935 to Segamat in Johore, where he remained in charge until his death.

The work of an engineer in charge of a PWD district does not lend itself easily to brief description of specific works. Road maintenance, the construction of new roads, and the erection of public buildings form part of his normal duties. In addition, he is responsible for water supplies, anti-malarial drainage, and all branches of government engineering work or construction in his district.

Apart from his normal duties, Mr. Billing was an active supporter of the Malayan Association of the Institution of Civil Engineers and of the Engineering Association of Malaya, a society devoted to the more general aspects of engineering. In the *Proceedings* of both of these bodies, many original papers by Mr. Billing have been published, including " Progressive Development of Roads " and " An Approach to the Problem of Improvement of Roads." Mr. Billing did not confine his interests to the practice and theory of engineering, but was keenly interested in science generally. He devoted a considerable portion of his spare time to the study of astronomy.

Mr. Billing was elected an Associate Member of the American Society of Civil Engineers on August 15, 1932.

JOSEPH PATRICK BURNS, Assoc. M. Am. Soc. C. E.[1]

DIED JULY 4, 1939

Joseph Patrick Burns, well-known civil engineer and treasurer of Burns Brothers and Haley, Inc., General Contractors, and only surviving member of the Burns Engineering Company, Consulting Engineers, died at his home at Watertown, N. Y., on July 4, 1939, after a long illness.

Mr. Burns was born at Watertown on March 20, 1875, a son of Patrick and Mary (Kinny) Burns. He was the last surviving member of a family of fourteen children. His father, Patrick Burns, came to Watertown in 1847 from County Sligo in Ireland, and became a prominent contractor.

Mr. Burns received his education in the Public and Parochial Schools of Watertown and was a member of the Class of 1893 but did not graduate on account of illness. He finished his preparatory school work at the Stiles Preparatory School of Ithaca, N. Y., and was graduated in 1894.

For a time he was engaged in construction work and from May, 1894, to June, 1896, he was engaged as architectural draftsman with the late John W. Griffen, Architect of Watertown. From that time until he entered Cornell University at Ithaca he served as a member of the New York City Mounted Police, under the late President Theodore Roosevelt, then New York City Commissioner of Police.

---

[1] Memoir prepared by William T. Field, M. Am. Soc. C. E.

In 1898 he entered Cornell University, taking up Chemical Engineering for one year. Later he transferred to the University of Wisconsin, at Madison, Wis., graduating in 1904 with the degree of Bachelor of Science in Civil Engineering. After graduation he took special work in German and other languages.

During the summer of 1901 he worked as Structural Draftsman in New York City. From June, 1902, to January, 1903, he held the same position with the Louisville and Nashville Railroad Company. In the Summer of 1903 he was engaged in surveying and making property maps for Dane County, Wisconsin.

From May, 1904, to March, 1905, Mr. Burns was bridge draftsman on drawings and computations for canal-lock machinery and bridges in the Department of the New York State Engineer; from March to December, 1905, he was civil engineer and superintendent of construction with the Chief Quartermaster, United States Army, Department of Texas at San Antonio, in responsible charge of designing, estimating, and supervising construction of sewer and water-works systems, buildings, etc. From December, 1905, to July, 1906, he was with the Department of the New York State Engineer, serving as bridge draftsman until February, 1906, then as Assistant Engineer making drawings and computations for locks, dams, retaining walls, etc., in connection with the Barge Canal construction.

Mr. Burns was a member of the firm of Burns Brothers and Haley, Inc., from July, 1906, to November, 1907, constructing New York State roads and investigating and making designs for the improvement of water power along the Black River in New York.

In November he returned to Albany, N. Y., to accept a position as Assistant Engineer on the Barge Canal, and when the Highway Department was created, he was transferred to that department and assigned to the Watertown office.

In April, 1911, he was appointed Deputy Division Engineer under Guy H. Miller, then Division Engineer in the Watertown office, and was in charge of the supervision of construction in St. Lawrence and Franklin counties and the northern part of Jefferson County. He later left the Highway Department to devote his attention to the contracting and engineering business controlled by the Burns Brothers. This firm attained a wide reputation, having constructed numerous power plants, dams, paper mills, and other industrial plants, and churches and schools throughout northern New York, as well as important business buildings.

One of the most ardent advocates of the St. Lawrence Seaway and hydroelectric development, Mr. Burns had delivered many addresses on the subject and had worked in conjunction with the late Delos M. Cosgrove, Vice-Chairman of the Power Authority of the State of New York.

Mr. Burns was a trustee of the Watertown Savings Bank; a member of the Holy Family Church; Watertown Council No. 259, and Watertown Assembly, Fourth Degree, Knights of Columbus. He was an industrious and hard working man, but above all, kindly and lovable, true to his friends, and just to all.

In 1907 he was married to Mary T. Carr, of Madison, Wis., who died on April 26, 1917. A daughter, Mrs. Elizabeth M. (Charles W.) Sloat, survives. On June 30, 1919, Mr. Burns was married to Mary F. Valin of Watertown who, with Robert Celestin Burns, Katherine Anne Burns, and Mary and Joseph Burns, twins, as well as Mrs. Sloat, two grandchildren and several nephews, survives him.

Mr. Burns was elected an Associate Member of the American Society of Civil Engineers on December 1, 1908.

CHARLES PAGE COMEGYS, Assoc. M. Am. Soc. C. E.[1]

DIED JUNE 5, 1939

Charles Page Comegys was born on February 12, 1898, at Easton, Talbot County, Md., the son of Charles Edward and Minna Whitman (Pretzman) Comegys. The Comegys emigrated from Holland to Maryland during the early days of this Republic. Page, as he was known to all his friends, attended the public schools of Easton and Baltimore, Md., and was graduated from the Baltimore Polytechnic Institute, at Baltimore, in June, 1918.

Just prior to his senior year, Mr. Comegys spent one summer—June to October, 1917—as inspector for the Maryland State Roads Commission and, upon completion of his school work, in June, 1918, he reported to Camp Meade (later Fort George G. Meade), Maryland, for service with the 304th Field Service Battalion, and embarked for France with his battalion in July, 1918. His tour of duty with the American Expeditionary Forces lasted one year.

Returning to his native state and the Roads Commission, Mr. Comegys was made inspector on various road and bridge enterprises, until May, 1921, when he was promoted to general inspector, reporting directly to the district engineer. In October, 1924, he resigned from the roads body to accept a position as field and office engineer for Herbert G. Campion, building contractor, of Philadelphia, Pa., on extensive residential and church construction. Here he received training in the building field which was later to prove invaluable to him.

In April, 1928, Mr. Comegys formed a partnership with Arthur Farmer, with offices in Easton, and engaged in general construction on highways, bridges, buildings, and wharves, until 1932, when Mr. Farmer withdrew from the firm. The business was then conducted until Mr. Comegys' death under the name of Charles Page Comegys. During this period he built many beautiful waterfront summer homes in Talbot County for wealthy Pittsburgh and New York families.

On September 18, 1931, Mr. Comegys was married to Mabel Helen Resley, of Hancock, Md., who, with his mother and a sister, survives him. They had no children. His widow has continued his contracting business under the name of Charles Page Comegys Company, Incorporated.

---

[1] Memoir prepared by John K. Flick, M. Am. Soc. C. E.

Mr. Comegys was a member of the Society of American Military Engineers, the American Legion, Masonic Fraternity, and was a Thirty-Second Degree Scottish Rite Mason. His clubs were: Chesapeake Bay Yacht Club, Talbot Country Club, Miles River Yacht Club, and the Tred Avon Yacht Club. He was extremely fond of water sports and most of his week-ends would find him on one or another of the beautiful bodies of water for which Talbot County is noted. He also owned a summer home on an arm of the Tred Avon River.

Mr. Comegys had the faculty of making friends and holding them. His broad smile and warm handshake are remembered by all who met him. His home was his friends' home as well, and his hospitality will never be forgotten. His passing is mourned by all who had any contacts with him in life. He was buried in the graveyard of Old St. Paul's Episcopal Church, near Chestertown, Kent County, Md., one of the oldest houses of worship in the state, antedating the Revolutionary War.

Mr. Comegys was elected a Junior of the American Society of Civil Engineers on January 14, 1924, and an Associate Member on February 10, 1930.

FREDERICK WEBSTER DECK, Assoc. M. Am. Soc. C. E.[1]

DIED SEPTEMBER 26, 1938

Frederick Webster Deck was born in Washington, D. C., on March 28, 1896, the son of William Meade Deck and Annie (Hendricks) Deck. The Deck family, originally from Saxony, Germany, settled in Pennsylvania, and in 1810 migrated to West Virginia (then Virginia). They made their home in the vicinity of Harpers Ferry, a name which was to reverberate throughout the nation two generations later. The Hendricks family originally came from Holland.

Mr. Deck received his education in Washington, D. C., attending the Towers grade school, Eastern High School, and George Washington University. He commenced his studies in grade school in the fourth grade, having received his preliminary training at home from his father. While in high school, he was rewarded with the position most highly desired by his fellow students—the captaincy of the school cadet company. The attainment of this honored post, awarded to him for his outstanding scholastic endeavors, his unmistakable leadership, and his general popularity, indicates clearly to those who knew him later that even in his youth he evinced those attributes which in his maturity brought him a large measure of respect and devotion from all his associates. In 1914 Mr. Deck entered George Washington University, but left in 1917, in his senior year, to enter the U. S. Army.

---

[1] Memoir prepared by J. E. Havelin, Assoc. M. Am. Soc. C. E.

Appointed as a Second Lieutenant of infantry in the regular army in October, 1917, he was under instruction at the army service schools, served with the 48th and 63d Infantry in various camps in the South, and served in France until December, 1919. He was transferred in July, 1920, to the Corps of Engineers, as a First Lieutenant. Shortly afterward he became Captain, and served with the 1st Engineers, a unit in the Army of Occupation at Coblenz, Germany, until March, 1922. It was while in service at Coblenz that Captain Deck was thrown from a horse in a riding hall, seriously injuring his spine and necessitating his retirement from the army. This was a catastrophic blow to him but borne stoically as were the long years of hospitalization and his constant suffering. He was retired from active service with the rank of Captain in the Corps of Engineers in December, 1922.

It was at this point of his career that Captain Deck showed his mettle. His was the example to be pointed to with pride—that somewhat rare achievement of man surmounting excruciating agonies of the flesh through indomitable will power and the determination of mind not to succumb to bodily ills. Perhaps, also, here was another instance of Nature's effort to rectify damage through compensation, such as one sees in the heightened sense of hearing in those bereft of the sense of sight. It acted in his case in rendering more acute an already brilliant mind. In proportion as his bodily activity was restricted by months of lying in hospitals for treatment and convalescence, his mind moved at a feverish pace. He showed his avid thirst for knowledge by the wide range of subjects he studied—philosophy, economics, languages, and all phases of engineering. He picked up his collegiate studies where they had been interrupted in 1917, and in 1924 received the degree of Bachelor of Arts from George Washington University, to be followed in 1925 with the degree of Bachelor of Science in Civil Engineering from the University of Colorado, in Boulder, Colo. He supplemented these studies with miscellaneous courses taken in the University of California Extension School. Later in 1929, after being active in the engineering profession for some time, he received a postgraduate degree of Civil Engineer from the University of Colorado. So keen was his desire for study that in 1936 and 1937 he attended evening classes in the Graduate School at the University of Pennsylvania, in Philadelphia, Pa., with the purpose of obtaining a Doctor's degree in Mathematics. Unfortunately, due to failing health, he was unable to complete these studies.

In August, 1925, having responded to numerous operations and various treatments for his injured spine, but compelled to wear a steel brace, Captain Deck entered active professional work as a designing and supervising engineer in the Steel Tower Section of the Philadelphia Electric Company. His duties in this position called for skill of a high order, not only in ability to plan and direct specialized engineering projects, but also in the handling of delicate negotiations with other corporations for the purpose of coordinating the design of structures for joint use. He never spared himself, although physically handicapped, in his determined effort to carry a project through to a successful conclusion. His associates have had occasion, while accompanying him on many reconnaissance expeditions over exceedingly rough terrain and under a scorching sun, to marvel at the spirit of this unusual man who could continue at an exhausting pace, despite the pain he was obviously suffering.

Captain Deck was responsible in a large measure for the ingenious designs of the supporting structures and their foundations involved in the successful ventures in unusual transmission line construction, known as the Reading Overbuild Lines. These are two 66-kv transmission lines, following the east and west banks of the Schuylkill River, in Pennsylvania, and constructed on steel towers built over the tracks of the Reading Railroad. Approximately forty structures are used on each overbuild section, and because the limitations of the right of way required adaptation of the structures to the ground, rather than the usual practice of standardization of structures, some six individual types of structures were required on each overbuild section. Captain Deck deserves a high place in the ranks of successful engineers when one considers the part he played in solving the complexities of a problem in which the interests of two utilities had to be safeguarded by providing structures of maximum safety erected within the narrow confines of busy railroad rights of way. Attention should also be drawn to his skill in coordinating the manifold phases of the construction involved, not only with normal, heavy railroad traffic but with the concurrent railroad company's construction project. Captain Deck has described the two overbuild transmission lines very ably in several articles.[2, 3, 4]

In 1929, when the Engineering Department of the Philadelphia Electric Company was reorganized, Captain Deck became a senior engineer in the Mechanical Engineering Division, in which position he was in full charge of all civil and structural engineering work. In this position his ever-present originality of mind had full play. It was a cardinal point with him never to accept the standardized methods of attack—the engineering clichés, so common in the profession. He was intrigued by the use of new angles of attack on problems of an empirical nature, but he was fully in his element in solving the unusual, the complex, and the difficult type of problem.

Those familiar with the specialized engineering involved in electric-utility operation will readily appreciate that opportunities for a fecund and aggressive mind are ever present in the transmission-line field, and especially in the structural phases of power-plant design. Captain Deck was given "carte blanche" when, in the course of the reconstruction of one of the company's generating plants, he recommended the use of a new type of suspension coal bunker. That he proceeded with this design on the basis of original formulas, and that its complete success has elicited from the operating engineers the statement that this bunker is vastly superior to any other in the Philadelphia Electric Company's system, is a tribute of a high order to his engineering skill, as well as to his superiors who had unfailing confidence in his genius. Inci-

---

[2] *Engineering News-Record*, May 3, 1928, p. 686.

[3] *Colorado Engineer*, January, 1931.

[4] *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 944.

dentally, Captain Deck left unfinished a paper on the design of this suspension bunker. His intention was to present his theories to those interested so that full discussion of its relative merits might ensue and similar bunkers be erected profitably by others in the industry.

In the transmission-line field, also, his exceptional talents were early recognized, and assignments to national society committees engaged in this field of work followed in quick succession. He was a member of the Power Transmission and Distribution Committee of the American Institute of Electrical Engineers (A. I. E. E.) for many years, and served for several years as chairman of its Subcommittee on Conductors, Towers and Wood Poles. He was chosen as the A. I. E. E. representative on the committee currently working on the revision of the National Electrical Safety Code, serving on two of its subcommittees, one on " Clearances " and the other on " Strength Requirements and Loading." In allied fields, his interest was so keen and his talent so obvious that the company employing him asked him to be its representative in the American Concrete Institute, and also to represent it on Committee A–5 on " Corrosion of Iron and Steel " of the American Society for Testing Materials. All of these responsibilities he handled with his customary thoroughness, and with his unswerving devotion to what he considered to be the best interests of his chosen profession.

Captain Deck was an exceedingly fair man in his dealings with others His was the military manner, the result of army training to receive and execute orders, to obey and be obeyed according to the rank and position eventually reached through one's ability. However, he was not a martinet, and he vehemently scorned blind subserviency and lackey attendance upon superiors. On one occasion his disgust at the actions of an associate was so strong that he penned the following words—words so descriptive of his sense of proportion or perspective in dealing with others that they may well be summed up as typical of his attitude on that subject:

> " A man who constantly trembles for fear of ' what his superior will say ' or ' what people will think ' soon loses self respect, initiative, and individual worth. Respect for the opinion of others, their knowledge, their rank, is conducive to profitable self-discipline. Subservience leads to a contemptible existence. * * * Diplomacy, tact, and understanding patience have their place. Force, firmness, and an unyielding mien have theirs, but brow-beating, taking unfair advantage, and cowardly overbearing have not a place in the repertoire of the successful man."

Captain Deck was married to Mary Evelyn Jolls on February 3, 1921, shortly before he sailed for duty with the Army of Occupation in Germany. Mrs. Deck's forbears were very active politically in Delaware.

Captain Deck is survived by his widow and three sons, Frederick Webster, Jr., William Meade, and John Wheaton; and also by a brother, William H. Deck.

Captain Deck was elected an Associate Member of the American Society of Civil Engineers on July 16, 1928.

CLYDE HENRY FENTON, Assoc. M. Am. Soc. C. E.[1]

DIED AUGUST 7, 1939

Clyde Henry Fenton was born at Neenah, Wis., on March 6, 1897, the son of Merton O. and Anna (McCallum) Fenton. His preliminary education was obtained in the public schools of Kimberly and Appleton, Wis. He then entered Lawrence College, at Appleton, from which he was graduated in 1918 with the degree of Bachelor of Arts. In 1921 Mr. Fenton entered the Colorado School of Mines at Golden, Colo., and was graduated in 1923 with the degree of Bachelor of Science in Mining Engineering.

After receiving his degree at Lawrence College, he spent three years as instructor of chemistry and physics in the public schools of Escanaba, Mich., and Antigo, Wis. In 1923 he took his first position in the engineering field—that of underground foreman in charge of shift in the Creighton Mine, near Sudbury, Ontario, with the International Nickel Company of Canada, Ltd. He continued in this work until 1930, at which time he went to Houston, Tex., as a member of the firm of the Blue Bonnet Tile Company, which was engaged in contracting and general supervision of the installation of tile.

In 1934 Mr. Fenton left the Blue Bonnet Tile Company to become computer-draftsman on a four-county aerial photo-mapping project with the U. S. Geological Survey in the Houston office. When this work was completed, he was employed by the U. S. Forest Service in Lufkin, Tex., as photogrammetrist in connection with the photo-mapping project for land acquisition for the Sam Houston National Forest in the vicinity of that city. While with the same organization he later became senior engineering draftsman. When the Brazos River Conservation and Reclamation District established an engineering and mapping organization in 1935, he left the U. S. Forest Service to join that organization as photogrammetrist on the photo-mapping of thirteen major reservoir areas along the Brazos River and its tributaries. In July, 1937, he was made office engineer in the headquarters office at Temple, Tex. While working in this capacity he did a considerable amount of independent research in connection with the Multiplex Aero-Projector which was being used for the making of topographic maps by his organization. In November, 1938, he took up duties with the Brazos River Conservation and Reclamation District, assisting in the design of earth-fill dams for water conservation purposes in western Texas.

Mr. Fenton passed away on August 7, 1939, at his home in Temple, death being caused by a sudden heart attack. He had been in good health, and his death was most unexpected and a great shock to his many friends.

He was a member of the American Society of Photogrammetry, being Treasurer of the Texas Section. He was also a member of the First Presbyterian Church.

---

[1] Memoir prepared by L. M. Baugh, Assoc. M. Am. Soc. C. E.

He was married to Louise M. Brusse in Holland, Mich., on June 25, 1924. He is survived by his widow; a daughter, Marjorie Jean; and a son, Robert M. His mother and one sister, Dorothy M. Fenton, also survive him.

Mr. Fenton was elected an Associate Member of the American Society of Civil Engineers on May 17, 1937.

GEORGE FARNSWORTH FISK, Assoc. M. Am. Soc. C. E.[1]

Died September 4, 1939

George Farnsworth Fisk was born on September 27, 1878, in Hyde Park, Mass., the son of George Washington Fisk and Eloise Mary (Farnsworth) Fisk. He received his early education in the public schools of Boston, and entered Massachusetts Institute of Technology, at Boston, Mass., in 1897, and finished his sophomore year in June, 1899.

His first employment in his climb to become a distinguished engineer was as a rodman with the New York, New Haven and Hartford Railroad Company. Later, in 1900, he accepted a position as assistant engineer with the U. S. Government under Col. William S. Black, on triangulation survey of fortification in Cuba. Finishing this work in 1903, he was employed as assistant engineer with the United States Appraisal Company of Boston, in charge of surveys of industrial plants and making appraisals.

During this latter work he was transferred to Buffalo, N. Y., to make a survey and appraisal of the large Schoellkopf, Hartford and Hanna Company's chemical works. He was so favorably impressed with Buffalo that he decided to make it his future home and went to work as a transitman with the Pennsylvania Railroad Company. While with this Company he laid out the new railroad shops at Olean, N. Y., and was on the construction of the Goodyear Ship Canal at South Buffalo and the double-span, plate-girder bridge at Hawthorn, Pa.

In May, 1905, Mr. Fisk entered the employ of the City of Buffalo, in the Bureau of Engineering, as a draftsman. During his four years as draftsman, he was assigned to survey and triangulation work. He also appraised all the railroad crossings and bridges in the City of Buffalo. In 1909, he was appointed chief draftsman and took over the investigation of the stability of bridges and viaducts under various loadings.

In 1914 he was appointed assistant engineer in the Bureau of Engineering and was immediately placed in full charge of street paving. During his fourteen years as assistant engineer, Mr. Fisk made many changes in the manner of laying pavements, substituting brick and asphalt for cobblestone and macadam. He established a Paving Laboratory and reorganized the Inspection Division. Under his direction the paving of streets increased yearly. He was chairman of the Brick Committee of the American Society of Municipal Engineers; chairman of the Brick and Block Pavement Committee of the American Society of Testing Materials; director of the Clay Products Institute of America; chairman of the Committee on Sizes and Styles of Paving Brick, Division of Simplified Practice, Department of Commerce, United States of America; and member of the Permanent International Association of Road Congresses.

---

[1] Memoir prepared by Harry M. Glassmire, M. Am. Soc. C. E.

In 1928 Mr. Fisk was appointed commissioner of public works of the City of Buffalo and served three years. During this time the $7,000,000 city hall, the State Normal School of Buffalo, the Hertel Avenue trunk sewer (costing $1,200,000), and the Buffalo city court building were constructed under his supervision. Many changes in engineering procedure and policy were instituted during his administration.

In 1931 Mr. Fisk entered private practice as a consulting engineer, and was often called in consultation in many cities. In 1936 he accepted a position as construction adviser for the U. S. Housing Authority. In this capacity he built two large housing projects, and was completing the third at the time of his death.

With the death of Mr. Fisk the City of Buffalo and the federal government lost a faithful servant. Unfailing courtesy, dignified cordiality, tolerance, and kindliness were George Farnsworth Fisk's characteristics. He was liberal in his views and noted for his fairness and ethical dealings in his engineering contacts with others.

Among his social activities, Mr. Fisk was past master of the Mystic Art Lodge No. 899, Free and Accepted Masons; past high priest of the Triangle Chapter No. 314 of the Royal Arch Masons; a Scottish Rite Mason and a Noble of the Mystic Shrine; and a member of the Buffalo Chamber of Commerce, Rotary, Elks, and the Orioles clubs. He attended the Richmond Avenue Methodist Episcopal Church.

On August 26, 1906, Mr. Fisk was married to Minnie Florence Torge. He is survived by his widow, one son, George William, and one daughter, Dorothy Eloise.

Mr. Fisk was elected an Associate Member of the American Society of Civil Engineers on January 3, 1911.

FRANCIS JAMES FITZPATRICK, Assoc. M. Am. Soc. C. E.[1]

Died July 16, 1939

Francis James Fitzpatrick was born at Chicago, Ill., on June 28, 1884. He was the son of Francis James Fitzpatrick and Frances (Carrol) Fitzpatrick.

---

[1] Memoir prepared by H. E. Anderson, Esq., Senior Engr., U. S. Engr. Office, Pittsburgh, Pa.

His father was captain of sailing vessels for a long period and for many years was master of his own ship. Upon retirement from this occupation he was appointed fire marshal of the City of Chicago. In 1893, during the World's Fair in Chicago, the Cold Storage Building of the Fair was completely destroyed by fire with great loss of life. Fire Marshal Fitzpatrick, while leading his men in the work of rescue, died heroically and was honored accordingly by the city.

The boy's mother died when he was two years old, whereupon he made his home with an aunt, Mrs. Katherine Leonard, to whom he was deeply devoted. Francis James Fitzpatrick was the only child of Frances (Carrol) Fitzpatrick. However, he had an older half brother, Luke Fitzpatrick. After his wife's death, Captain Fitzpatrick spent as much time as practicable with his young son and they were very close to each other in their affection. The tragic death of his father was a great shock to the small boy of nine. His devotion to and admiration for his father made a deep and lasting impression upon him and he too was to become possessed of the courage, strength of character, and integrity of Captain Fitzpatrick, the sailor and hero. He was graduated from Hyde Park High School and then went to Armour Institute of Technology, in Chicago, majoring in civil engineering.

Francis Fitzpatrick, the young man, early became interested in railroad and construction engineering, an interest which he maintained intensively throughout the remainder of his life. In March, 1905, before he had yet attained his twenty-first birthday, he began his engineering career with the Indiana Harbor Railroad Company, first as rodman and later as instrumentman in charge of a residency. In March, 1906, he became topographer for the Cairo and Tennessee River Railway Company. In May of the same year he became instrumentman in charge of a survey party for a fuel company. The next month he returned to railroad work and held the following positions: June to November, 1906, topographer for the Cananea, Yaqui River and Pacific Railway; December, 1906, to January, 1907, leveler and transitman for the Oregon and Washington Railway Company; February to October, 1907, leveler and transitman for the Chicago, Milwaukee and St. Paul Railway Company; December, 1907, with the B. J. Arnold Company on appraisal of railway property; January, 1908, transitman with the Northwestern Elevated Railway Company, in Chicago; and September, 1908, to July, 1909, draftsman and office engineer for the North Coast Railway Company.

Mr. Fitzpatrick then left railroad work to become assistant to the superintendent of the Columbia Canal Company on maintenance and extension of an irrigation system at Attalis, Wash., but again took up railroad work in October, 1909, when he became draftsman on preliminary location surveys for the Great Northern Railway Company. In January, 1910, he became draftsman on maintenance, location, and construction and later served for a short time as transitman for the Oregon and Washington Railway Company. In May, 1910, he went with the Alaska Northern Railway Company as chief draftsman, in charge of the compilation of surveys and estimates for the Alaska Central Railway. In October, 1910, he became draftsman on construction with the Oregon-Washington Railroad and Navigation Company.

Francis Fitzpatrick, the young engineer, left railroad work in August, 1911, to take a position with the Isthmian Canal Commission, and he was successively in charge of construction of two reinforced-concrete retaining walls, the construction of all concrete work at the Pedro Miguel locks of the Pacific Division, and transitman with the First Division in charge of the installation of all lock and gate operation machinery for the Pedro Miguel and Miraflores locks. He later had charge of construction of seacoast fortifications for the Panama Canal and Pier No. 7 for the Panama Railroad. In May, 1916, he left the Canal Zone to take charge of construction of seacoast fortifications at Fort Rosecrans, California, which assignment was completed in May, 1917.

Francis Fitzpatrick, the engineer, qualified for and was commissioned a Captain in the Engineer Reserve Corps of the U. S. Army on February 23, 1917. He was called to active duty on May 8, 1917. Thereupon he reported to assignments as follows: May 8, 1917, presidio of San Francisco; August 29, 1917, Camp Lewis, Washington; November 1, 1917, Camp Pike, Arkansas; and January 23, 1918, American Expeditionary Forces, France. While in France he served as follows: August 15, 1918, Company Commander in 316th Engineer Training Regiment; August 30, 1918, in the office of the chief engineer of the 1st Army, as assistant army roads officer; and November 1, 1918, Company Commander in the 508th Engineer Regiment. He returned to the United States and on June 16, 1919, was stationed at Camp Shelby, Miss., with the 508th Engineer Regiment. The regiment was demobilized and Captain Fitzpatrick was honorably discharged from the service on July 23, 1919.

On July 1, 1920, Captain Fitzpatrick was again commissioned in the U. S. Army, this time as a Captain in the Coast Artillery Corps. Thereupon he reported to assignments as follows: November 27, 1920, on harbor defense works at Puget Sound, with the 14th Coast Artillery, Fort Worden, Washington; September 13, 1923, at Fort Monroe, Virginia, as student, Battery Officers' Course, Coast Artillery School; April 28, 1924, at Fort Stevens, Oregon, with the 14th Coast Artillery Regiment; June 20, 1924, at Fort Eustis, Virginia, with the 52d Coast Artillery Regiment; and on January 4, 1926, in Hawaii, with the 41st Coast Artillery Railway Battery.

Captain Fitzpatrick was transferred from the Coast Artillery Corps to the Corps of Engineers on July 9, 1926. He was assigned to the 3d Engineer Regiment in August, 1926, for service, again in Hawaii. On December 23, 1929, he was assigned to the U. S. Engineer Office, at Vicksburg, Miss., as assistant to the district engineer. While there he also had charge of the Organized Reserves of the 4th Corps Area and later was disbursing officer for the Mississippi River Commission. On July 4, 1932, Captain Fitzpatrick was transferred to Cleveland, Ohio, as instructor for the Ohio National Guard (112th Engineer Regiment), Organized Reserves of the 5th Corps Area. In the spring of 1933, he was construction officer, Civilian Conservation Corps, in Ohio and West Virginia. He was stationed at Chicago on temporary duty with the Civil Works Administration for Cook County, from January to July, 1934. Captain Fitzpatrick was promoted to Major, Corps of Engineers, on August 1, 1935. He was assigned to temporary duty as consulting field engineer with the Works Progress Administration in Philadelphia, Pa., in October, 1935. He was umpire for the 2d Army Maneuver, Fort Knox, Kentucky, in July and August, 1936.

Major Fitzpatrick became assistant to the district engineer, U. S. Engineer Office, in Pittsburgh, Pa., on September 1, 1936. He was on detached service with the Works Progress Administration at Los Angeles, Calif., from October, 1937, to April, 1938, whereupon he resumed his duties at Pittsburgh as chief military assistant to the district engineer.

Major Fitzpatrick was assigned as Professor of Military Science and Tactics at Oklahoma Agricultural and Mechanical College, at Stillwater, Okla., and was to have reported for duty on August 1, 1939. On July 15, 1939, he was en route, with a government chauffeur, to the Tionesta Creek flood control dam, under construction under the supervision of the Corps of Engineers, near Tionesta, Pa. He was particularly interested in construction, highway, and railroad problems in relation to this project. The assignment which required a journey to Tionesta was to have been his last one in the district prior to his transfer. The car in which he was a passenger was struck near Bakerstown, Pa., by another car which came, out of control, from the opposite direction. The occupants of the government car were fatally injured, and Major Fitzpatrick died on July 16, 1939. Before he died he regained consciousness long enough to identify himself and to exonerate the chauffeur of any blame whatever for the accident. Interment was made in the Arlington National Cemetery, Virginia.

The career of Major Fitzpatrick came to a close in the line of duty. He would have had it so. He had been a young railroad engineer from March, 1905, to August, 1911; a government engineer on construction, much of the time in the Panama Canal Zone, from September, 1911, to May, 1917. His service to his country as an army officer began in May, 1917, and ended in July, 1939, a long and honorable service. His profession took him to numerous places, many of which were hazardous. While in France, during the World War, he served in the Defensive Sector (Toul), Seichprey, Xivray, St. Mihiel, and Meuse-Argonne. However, it was a tragic accident on a peaceful countryside that caused his untimely death.

While in the Pittsburgh Engineer District, Major Fitzpatrick served as executive officer. He supervised and correlated, for the district engineer, the numerous civil and military activities of the District. He was an expert on labor conditions and relations as well as regulations relating to Federal projects. It was this particular qualification which led to his being called to temporary duty with the Works Progress Administration at Los Angeles. He was particularly interested in the many engineering problems relating to river and harbor and flood control and protection activities. He was firm in his military duties, but of genial disposition and considerate of all with whom he came in contact.

Major Fitzpatrick was married at Cashmere, Wash., to Jane Ethelyn Clark who, with a daughter, Anne Katherine, and two sons, Francis James, Jr., and Robert Clark, survives him.

Major Fitzpatrick was elected an Associate Member of the American Society of Civil Engineers on October 1, 1913.

HARRY J. HANMER, Assoc. M. Am. Soc. C. E.[1]

DIED MAY 4, 1939

Harry J. Hanmer, the son of Charles and Emma (Crim) Hanmer, was born at Jordanville, N. Y., on December 6, 1881. His preliminary education was received at Jordanville Union School and Richfield Springs High School, after which he entered the College of Applied Science, Syracuse University, Syracuse, N. Y. In June, 1906, he was graduated from the University with the degree of Civil Engineer.

His first assignment was that of inspector on the construction of a sewage system for the Village of Mohawk, N. Y.; and in December, 1906, he entered the employ of the Remington Typewriter Company at Ilion, N. Y., as a draftsman.

Mr. Hanmer was appointed assistant city engineer of Gloversville, N. Y., in March, 1907, and in October, 1913, he became city engineer.

During his long and faithful service with the City of Gloversville, Mr. Hanmer was engaged in the design, construction, and maintenance of city streets, sewers, and the sewage disposal plant. He was in immediate charge of the Gloversville Sewage Experimental Station and of the construction of the Gloversville Sewage Disposal Plant which covered 10 acres and cost $250,000.

In addition to his extensive activities as city engineer, Mr. Hanmer was the author of several articles, one of which was entitled "The Construction and Operation of the Gloversville Sewage Disposal Plant" which has been widely read by sanitary engineers.[2]

On October 23, 1913, he was married to Mabel R. Peck of Gloversville, N. Y. He is survived by his widow, his father, and a sister, Mrs. R. D. Belshaw.

Mr. Hanmer was a member of Phi Gamma Delta Fraternity; the First Methodist Episcopal Church and a member of the Official Board; the Young Men's Christian Association (Y. M. C. A.); the Benevolent and Protective Order of Elks; the Masonic Order, Mohawk Valley Lodge; the New York State Association of City and Village Engineers; the Eccentric Club; the Pine Brook Country Club; and the Republican Club.

Mr. Hanmer was endowed with an attractive personality; his friends were legion in his professional life as well as in the activities of the several social organizations in which he was greatly interested. He was particularly active in church work and in the Y. M. C. A.

He was an indefatigable worker and accomplished each task with diligence and extremely careful endeavor. With these exacting qualities of mind, Mr.

---

[1] Memoir prepared by John A. Wallace, M. Am. Soc. C. E.

[2] *Engineering News*, Part 1, Vol. 74, No. 16, October 14, 1915, p. 744, and Part 2, Vol. 74, No. 17, October 21, 1915, p. 780.

Hanmer retained the esteem and affection of his associates during his entire professional career.

Mr. Hanmer was elected an Associate Member of the American Society of Civil Engineers on October 1, 1912.

HAROLD FREDERICK JANDA, Assoc. M. Am. Soc. C. E.[1]

DIED JULY 29, 1939

Harold F. Janda was born on April 21, 1893, in Portage, Wis. His father, Anthony J. Janda, a printer, had emigrated from Bohemia. His mother, Katherine (Sitterly) Janda, was a native of Madison, Wis. Besides Harold, the family circle included an elder sister, who died in 1920.

The boy attended the public schools of Portage and then entered Carroll College at Waukesha, Wis. A desire for an engineering training caused him to transfer to the University of Wisconsin, at Madison, which he entered as a sophomore with advanced standing. He was graduated in 1916 with the degree of Civil Engineer. He was a good student, but the record shows that one of the poorest grades he received was in highway engineering, the subject which he later taught for many years in the same college.

During the summer following graduation he worked as assistant engineer for the Baraboo Drainage Commission. In the fall he received an appointment as instructor in civil engineering at the University of Cincinnati, at Cincinnati, Ohio, under the late Dean Herman Schneider, Assoc. M. Am. Soc. C. E., who was introducing his famous cooperative system under which the engineering student spends alternate periods in the school and on the job. It was part of Mr. Janda's duty to supervise the work of the students on the job.

The future seemed secure enough to the young instructor to justify matrimony, so he and Margery I. Sawyer, of Menominee, Mich., were married before the end of 1916.

In 1919, Mr. Janda was made an assistant professor at Cincinnati and continued in that position until he went to the University of North Carolina at Chapel Hill, N. C., in 1921 as associate professor of highway engineering. There he found opportunity for research in the field of vertical earth pressures and, as a result of his work, was called to Washington, D. C., in 1924, to serve as associate director of the Highway Research Board of the National Research Council. The university granted him leave of absence for this service. In 1926 he returned to the University of North Carolina as professor of highway engineering and head of the department of civil engineering.

Professor Janda was called to the University of Wisconsin, his alma mater, in 1928, as head of the department of highway engineering and city planning, and served continuously until his death.

---

[1] Memoir prepared by L. F. Van Hagan, M. Am. Soc. C. E.

Through his study of vertical earth pressures, Professor Janda had become interested in the characteristics of soils as they affect engineering work, and the study of soils remained one of his major professional interests. At the time of his return to Wisconsin, in 1928, the Wisconsin Highway Commission was having much difficulty with the phenomenon known as frost boils. Professor Janda was called upon to investigate the problem for the commission and became a recognized authority on soil conditions and characteristics in Wisconsin. From 1929 until his death, he served the commission frequently as a consultant upon a wide range of problems.

Frequently called upon for service on various public committees Professor Janda gave liberally of his time and energy in that work. His familiarity with city planning and zoning and with traffic problems made him an effective worker on such committees. He was open-minded and forward-looking, but always kept his enthusiasms and emotions under control. His ability to visualize the practical possibilities of a situation gave weight to his contributions in council.

He was proud of his profession and was active in promoting its interests. His services were recognized, in 1935, when he was made president of the Engineering Society of Wisconsin. A heart attack at Christmas time in 1935, however, made it impossible for him to preside at the spring meeting of the society in 1936.

Professor Janda liked to teach. He liked the students and enjoyed his contacts with them. He held them to a high standard of performance but was always even-tempered, courteous, and reasonable. His teaching was fresh and stimulating because of his close contact with technical problems in the field. He was deeply interested personally in those problems, seeking to understand them and to find practical solutions. His thesis students (and they were numerous) had the satisfaction of feeling that they were making genuine contributions to professional knowledge, a feeling that was reflected in their earnestness and enthusiasm. He had them out on the job in all parts of the state gathering their data and testing their findings in the "fiery furnace" of practice. His classroom teaching likewise was vibrant with his interest in his subject and the freshness of his material. He prepared carefully for each class period and organized his material with great skill.

An interest in semiprecious stones was Professor Janda's chief hobby. His interest was a scientific one, and he was skilled in the identification of gems. His modest collection was full of interest and furnished material for many an address at student gatherings.

Professor Janda was a member of the honorary societies of Sigma Xi, Chi Epsilon, and Tau Beta Pi. In politics, he was a believer in the American system and could probably have been classed as a liberal conservative. He attended the Episcopal church.

His family, consisting of two daughters, Barbara Sawyer, and Nadia Ann, received much of his attention. His widow and their two children survive him.

In his contacts with people, Professor Janda was frank, easy, and modest. Those who were intimate with him found him to be well-informed in a wide range of subjects. He was an interesting talker, and his cheerfulness and good nature made him a delightful companion.

Professor Janda was elected an Associate Member of the American Society of Civil Engineers on June 7, 1926.

WILLIAM JOSEPH JUDGE, Assoc. M. Am. Soc. C. E.[1]

Died June 30, 1939

William Joseph Judge was born in Hickman, Ky., on April 29, 1878, the son of Henry C. and Julia (Burke) Judge. He received a high school education at Hickman College, at Hickman, Ky. Later he took a course in mechanical engineering with the International Correspondence Schools at Scranton, Pa.

Although little is known about his boyhood days, it was then that his lifelong interest in dredging was inspired. His diversion was to sit on the banks of the Mississippi River which flowed near his home and watch the crews of the giant government dredges deepen the channels and build the levees. He determined then and there that dredging was to be his life work.

Although a year younger than the required age he left home to enlist in the 3d U. S. Volunteer Engineers in the Spanish-American War. He was practically disowned by his father for joining up with the "Damnyankees." However the same father was very proud of his son when he returned as a Sergeant in Company "L."

Immediately after his discharge from the Army, in 1899, he began his work with dredges as a clerk with the Mississippi River Commission on the dredge fleet at Memphis, Tenn. He rose to the grade of foreman in 1902. Then he went with the Atlantic, Gulf and Pacific Company as paymaster and assistant superintendent on work at Vicksburg, Miss., Mobile, Ala., Tampa, Fla., New Orleans, La., and Pascagoula, Miss., and at the Isthmus of Panama where the company erected dredges for the Isthmian Canal Commission until 1907. He then went with the Canal Commission as dredge master until 1910.

From April, 1910, to May, 1917, he was field superintendent with James Stewart and Company and the American Pipe and Construction Company on New York State Barge Canal work, which included dredging, subaqueous drilling, blasting, and rock removal.

In May, 1917, he enlisted for service with the United States Army in the World War. William Judge, previously having received a commission as First Lieutenant of Engineers, was ordered to camp in the spring of 1917. First he was at Madison Barracks, then at Camp American University at Washington, and for a short time at Camp Belvoir, Virginia. At the end of the training period, a very few of the more meritorious were given commissions of higher rank. Among these was Lieutenant Judge, who was commissioned a Captain. He was assigned with thirty others to the 303d Engineers, the Engineer Regiment of the 78th Division, in training at Camp Dix, New Jersey. Captain Judge was put in command of Company " F." Due to the excellent service record of this company, the Commanding Officer of the Regiment, Col. E. M. Markham, M. Am. Soc. C. E., who later became Chief of Engineers, selected Captain Judge for early promotion, made him a Major, and placed him in command of the Second Battalion. He was in charge of troops on the British ship on which they sailed. They landed in England and shortly thereafter went to France and trained behind the lines in the British sector. The Division was brigaded with the British on the northern sector of the front for some time, but moved to the American sector about July, 1918.

---

[1] Memoir prepared by R. H. Keays, M. Am. Soc. C. E.

The Division was in reserve in the St. Mihiel attack, and relieved two regular divisions after the attack, taking over a large sector of the front line. Later the Division was moved to the Argonne where it was held in reserve during the big attack but after the attack it replaced the 77th Division. They occupied the front line, including Grandpré and the region to the east, until shortly before the Armistice. When the last big drive was made the Division led in the attack and was only relieved a day or so before the Armistice.

The Regiment moved into winter quarters after the Armistice and remained in France until the following spring. The men came home about June, 1919. In the meantime Major Judge had been promoted to Lieutenant Colonel and came home in command of the Regiment.

From July, 1920, to May, 1923, Colonel Judge was back with his dredges as field manager for A. H. Cipriani on the Caroni Reclamation Project and Orapouchee Lagoon on the Island of Trinidad.

He returned to the New York State Barge Canal in 1924 and remained until 1926. This time he was field superintendent for The Empire State Dredging Corporation. This project also included much drag-line work.

As field superintendent for the T. T. Sweet Dredging Company of Miami, Fla., in 1927 and 1928, Colonel Judge did dredging for land reclamation. While there the great hurricane of 1927 made much trouble for his dredges.

From 1928 to 1930, Colonel Judge was field superintendent for the National Dredging Company in charge of dredging on the Hudson River and Kings Park, Long Island, New York.

Early in 1930 his old war time comrade, Col. Roy W. Gausmann, M. Am. Soc. C. E., general manager for John Monks and Sons and Ulen and Company in Greece, induced Colonel Judge to come to Macedonia on the Struma project. In this capacity Colonel Judge's career was noteworthy. In Macedonia there was nothing with which to begin—no organization, no plant, no trained men. The project was flood control, land reclamation, and irrigation in the Struma Plain, the drainage of the Pravi Marsh in the Plains of Philippi, and miscellaneous other work. The estimated cost was about

$20,000,000, which would have been about twice as much in American money. The company established their headquarters in Serres, Macedonia, a city of about 30,000 people.

More than $2,000,000 worth of equipment was bought in England, Germany, Holland, and the United States, including five large drag-lines, three large hydraulic dredges, a dipper dredge, many small drag-lines, and an excavating machine. The orders for plant were distributed by the Greek Government in this way, for diplomatic reasons. Men were brought from America, England, and Germany to operate the plant. It was necessary to redesign the dredges in Macedonia. Camps, roads, and bridges were built as well as railroads, machine shops, docks, and launching pits.

After a year or so the dirt began to move in ever increasing quantities so that in the end about 80,000,000 cu m had been handled. Long before the work was completed all the foreign operators had been replaced by native Greek operators trained on the job.

The most exciting times of Colonel Judge's stay in Macedonia, however, were his experiences in the attempted revolution of 1935. The project was started under the Liberal Government of Venizelos and most of the inhabitants of Macedonia were Venizelists. In 1933, however, the Royalists obtained control of the government under Tsaldaris and General Kondylis.

A revolutionary movement was started in 1935 in Macedonia with headquarters in Serres. This was very embarrassing for Colonel Judge and the companies he represented, for most of the employees were Venizelists.

All communications were soon cut off from the outside world and from the companies' main office in Athens, Greece. The companies' transportation equipment and telephone system were seized by the revolutionists. Bombs were dropped in the city by Royalist aviators and the city was full of troops on their way to the west to their defense lines along the Struma River. On the advance of the Royalist troops, however, the revolution quickly collapsed and the Royalists took over the surrounding territory.

Colonel Judge has been commended highly for the diplomatic skill with which he maintained absolute neutrality in handling the delicate situation and the affairs of his employers during the trying period.

While still in Macedonia, Colonel Judge became seriously ill in the summer of 1936. He was taken to Vienna, Austria, for treatment. His health did not improve, however, and in January, 1937, he was brought to the United States to his home in Cooperstown, N. Y., where he died on June 30, 1939. Funeral services were held from Christ Church in Cooperstown, and he was buried in Lakewood Cemetery.

William Judge was a Colonel in the Engineer Reserve and a Charter Member of the Society of American Military Engineers.

On April 28, 1917, Colonel Judge was married at Cooperstown to Margery Bunn, who survives him.

Colonel Judge was elected an Associate Member of the American Society of Civil Engineers on August 13, 1934.

ARTHUR SYLVANUS KEMMAN, Assoc. M. Am. Soc. C. E.[1]

Died October 31, 1939

Arthur Sylvanus Kemman was born at New Hampton, Iowa, on August 30, 1886, the son of Alvino Kemman and Louise (Schert) Kemman. His father was county engineer of New Hampton for many years.

Mr. Kemman was graduated from the New Hampton High School, the valedictorian of his class. He attended the U. S. Naval Academy, Annapolis, Md., from June, 1904, to May, 1908, and spent two years at sea. For two years subsequent to this period he assisted his father on various engineering projects at New Hampton. In September, 1912, he went to California where he worked for slightly more than a year on plant and structural design for the Colton Cement Company. The next four years he was employed as draftsman by the California State Highway Commission.

He went to Washington, D. C., in May, 1916, and compiled specifications for ships in the Bureau of Construction and Repair, Navy Department. Between April and December, 1917, he was employed in the capacity of general assistant to the superintendent constructor, U. S. Navy, on submarines L-6 and L-7 at the Long Beach (Calif.) Shipbuilding Company. Mr. Kemman assisted on trial runs and inclining experiments during this assignment.

In December, 1917, he received an appointment as Lieutenant in the U. S. Naval Reserve Force, and was sent to the U. S. Navy Yard at Bremerton, Wash. His assignment there included duties as office superintendent and material officer. He supervised requisition and inspection of materials and supplies for industrial plants, and inspected and valued the vessels taken over by the Navy. This appointment terminated in October, 1919, and Mr. Kemman immediately secured the position of marine inspector for the Union Oil Company. Here he supervised construction of tank steamships at the Southwestern Shipbuilding Company in San Pedro, Calif., until May, 1921. Following this came a period of private practice in surveying and land subdivision work.

He was employed from 1922 to 1925 as draftsman by the Southern California Edison Company at Los Angeles, Calif. Subsequently he spent a short time as Junior Civil Engineer with the Storm Drain Design Division, City Engineer's Office, at Los Angeles.

In July, 1925, he took a position as draftsman with the Los Angeles County Flood Control District and remained with this organization continuously until his death. His work was quite varied. After assisting in designing some of the first structures constructed by the District, he transferred to the Hydrographic Department shortly after its inception in 1927 and was active in the expansion of this work. He was particularly interested in the conception and extension of the District's mountain telephone lines and radio system and served for a period as head of the Communications Department. Mr. Kemman was a very capable engineer in any work assigned, and was an excellent draftsman. He was very active in all employees' affairs and popular with his fellow engineers.

---

[1] Memoir prepared by Walter J. Wood, Assoc. M. Am. Soc. C. E.

Mr. Kemman served as secretary and president of the Naval Academy Alumni Association at Los Angeles. He was a member of the William H. Upton Naval and Military Lodge, No. 206, Free and Accepted Masons; Post 457, American Legion, and served as Commander during the year 1938–1939; the American Association of Engineers and Architects; the Southern California Historical Society; E. Clampus Vitus; and the Los Angeles Yacht Club. His principal diversions were golf, yachting, and tennis.

The promotion of the Ross-Loos Clinic in Los Angeles for the benefit of public employees was aided by Mr. Kemman. He gave freely of his time in assisting the organization of the Los Angeles Civic Center Employees Federal Credit Union and served for many months as the first Secretary-Treasurer, without compensation, to get the Union going.

Personally, Arthur Kemman was a kindly, generous, considerate gentleman. He made friends easily and was ever ready to do them a favor.

He was married to Ruth Holly of Westfield, N. Y., on October 21, 1913. He is survived by his widow, two sisters, Martha Kemman and Elva Kemman (Mrs. James E. Bezpalec), and a brother, Alvin Kemman.

Mr. Kemman was elected an Associate Member of the American Society of Civil Engineers on April 15, 1929.

JOHN KYLSTRA, Assoc. M. Am. Soc. C. E.[1]

DIED MARCH 18, 1939

A long way from the pastoral scene of his boyhood, dependent entirely on his own energy and resources for his advancement, John Kylstra entered "on the ground floor" an industry that he grew to love, and grew with it to a position at the top of his profession, as chief engineer of the Boeing Aircraft Company.

The son of the Rev. S. C. Kylstra and Mrs. A. V. (Willebrands) Kylstra, Jack was born on February 3, 1895, in Engleum, The Netherlands. He attended grade school in four or five different villages in the land of dikes and windmills, clumping in wooden shoes along the country roadways to school. Then he graduated to leather shoes to receive his secondary education in the city school of Leeuwarden, Province of Friesland.

One day in 1913 the taste of an imported Yakima (Wash.) apple enticed the elder Kylstra to consider moving to the United States, and four weeks later the family arrived in the Yakima Valley. For five years Jack helped with the irrigating and marketing of crops, while his younger brother finished high school in Yakima and prepared for college.

---

[1] Memoir prepared by Robert J. Minshall, Esq., Seattle, Wash.

Jack's eye, too, turned toward higher education, and he forthwith took his Holland school records and a few choice compositions to the registrar at the University of Washington, Seattle, Wash., where he was at once enrolled.

He was graduated from the University in 1922 with a Bachelor of Science degree in civil engineering. His industry and aptitude had won him a *cum laude* rating with his degree, and membership in Sigma Xi, science honorary fraternity, and Tau Beta Pi, engineering honorary fraternity.

Promptly came Mr. Kylstra's opportunity to put his engineering training to work. He joined Lovsted and Company, a railway material firm of Seattle, in August, 1922, working afternoons for four months and then full time until September, 1923. As production manager of the firm, his job included the drafting and design of gears, bearings, etc., for logging locomotives and sugar plantation equipment.

From September, 1923, to October, 1924, he was a fieldman for the Pacific Telephone and Telegraph Company in Seattle. He served as instrumentman, was in charge of small maintenance projects, and prepared estimates on larger projects.

After these two years of varied experience, in October Mr. Kylstra joined the Boeing Aircraft Company of Seattle and was assigned to the stress section of the engineering department. He began checking the structural strength of the big PB-1 patrol boat that was being developed for the U. S. Navy. He became known for alert resourcefulness, meticulous attention to details, and dependability, and before long he was advanced to chief of the stress section. In this work he became thoroughly acquainted with airplane construction of all types common to the period.

In 1929 Mr. Kylstra was made project engineer for a new type airplane that proved to be far more significant in the advancement of aviation than even its makers anticipated. It was the Boeing Monomail, first successful all-metal, smooth-skin, monoplane transport, incorporating retractable landing gear and the streamlining that has since become an inherent feature of modern airplane design.

Upon the successful conclusion of this assignment he became project engineer of the XP-936 experimental pursuit plane for the U. S. Army, a type that proved as significant to the pursuit airplane field as the Monomail was to the air transport field. The XP-936 was the forerunner of a long series of high-speed Boeing low-wing monoplane pursuits known as the P-26 type pursuit planes widely used by the U. S. Army Air Corps. Subsequently, Mr. Kylstra served as project engineer for the YP-29, an advanced type experimental pursuit plane, and the XF7B-1, an advanced fighter airplane built experimentally for the U. S. Navy.

In 1934 Mr. Kylstra was called upon to "shoulder a job" that was to measure the fullest capacities of the man. The Boeing Company had contracted to build experimentally for the Army Air Corps a four-engine bombardment airplane far larger than any previously conceived—the now-famous Boeing

XB-15 Super Flying Fortress. It was a project actually started before the development of the smaller B-17 type Flying Fortress, but because of its scope, it required substantially more time for completion. At the helm of this secret project was Jack Kylstra in his familiar rôle as project engineer. Directly or indirectly, he had his finger on some 315,000 man-hours of engineering time that went into the development of the 35-ton super ship, now the largest landplane in service in the world and holder of two official international aviation records, one for maximum load-carrying and the other for speed with a pay load of 2,000 kg for a distance of 5,000 km, or 3,100 miles.

With his wide experience in the military airplane field, Mr. Kylstra, in 1937, advanced to become engineer in charge of all military projects for his company. As such he was overseer of work on the initial fleet of B-17B Flying Fortresses produced for the Army Air Corps, and was a frequent commuter between the Seattle factory and Wright Field in Dayton, Ohio, where the Air Corps Matériel Division is located.

It was in Dayton that he met Peg Luetta Platte, who became Mrs. Kylstra on October 9, 1938.

Mr. Kylstra became chief engineer of the Boeing Aircraft Company on January 1, 1939, while the company was in the midst of producing and testing its various new types of large four-engine airplanes for both commercial and military service. This was his position at the time of his death.

John Kylstra was elected a Junior of the American Society of Civil Engineers on July 9, 1923, and an Associate Member on June 6, 1927.

JOHN RAYMOND LAPHAM, Assoc. M. Am. Soc. C. E.[1]

DIED OCTOBER 2, 1939

John Raymond Lapham was born at West Medway, Mass., on April 1, 1886, the son of Frank D. and Alfarata (Scott) Lapham. His family was of old New England stock on both sides. He attended public school and determined to have a college education. By dint of hard work and "stick-to-itiveness" he managed to put himself through Brown University, at Providence, R. I., where he was graduated with the degree of Bachelor of Science in Civil Engineering in 1909. He had only moderate help from home, and this fact influenced him throughout his life in his attitude toward those who were always asking others for help. "Help," he would say, "help? I helped myself. Why can't you?" And then, after laying down the law in no uncertain terms, and denouncing the human race in general and particular, he would help, aid, and assist all sorts and conditions of men.

At college he was an average student; the acclaim and honors that went to the brilliant man meant nothing to him and he simply did not try to achieve what seemed to him unnecessary merit. He did his work passably well and was entirely satisfied. Also he was always more than glad to help his fellows with their studies and took no end of satisfaction when they received more credit than he did himself. In the classroom, when he was in the proper mood, his innate ability would suddenly shine and as quickly disappear, as once when he had a special assignment in mechanics and acquitted himself so remarkably that his fellow students were aghast and the professor could only give the infrequent recognition, "Very good!"

---

[1] Memoir prepared by J. S. Crandell and Sydney Wilmot, Members, Am. Soc. C. E.

Some things in his college life were not learned from books. Perhaps the outstanding relationship of his last year, at least to judge by the repeated references that he and other "old grads" were wont to make, was a certain hilarious group meeting familiarly known as the "42 Hope" club. This was named after the room in which the club met irregularly, but frequently and protractedly, as the demands of the college schedule permitted. The group confined itself diligently and boisterously to research into the ancient, if not honorable, game of "penny ante." Most if not all of the members were senior civil engineers and all have made subsequent favorable records, including half a dozen now members of the Society.

Upon graduation John Lapham "got a job" with the New York, New Haven, and Hartford Railroad Company. He liked the work fairly well, but thought he could do better at teaching, although there was no one in his family who ever had devoted his or her life to the profession. After two years of railroad life, during which he proudly announced that he learned much more about straight flushes and full houses than he did about engineering, he "got a position" as instructor in civil engineering at The Pennsylvania State College, at State College, Pa.

It is no secret that many of his acquaintances were a bit dubious about his prospects of success in this new work. To his intimate friends, however, the issue was never in doubt, for they recognized only too well the real ability that lay behind his simple and rather offhand manner. The fact was that after a period of intellectual drifting he was about to "set sail" seriously for a definite "port" in the educational world. Within the short space of fifteen years he was to become Dean of an important engineering school.

In those days of the early nineteen hundreds the life of a faculty member at the State College was particularly rough and tough on those whom the students did not like. But they took to "Lap" immediately; and he was equally popular with the faculty. He was appointed to numerous committees and attended many a conference that many older men never even knew about. Yet no one ever became jealous of him. His methods were always "open and above board," so that not a shred of envy clouded his colleagues' minds—a rather remarkable experience in a college community.

Seeking more rapid advancement, in 1916 he went to George Washington University, at Washington, D. C., to serve as assistant professor of civil engineering. He soon became professor, and eventually, in 1926, he was made, first, Acting Dean of the College of Engineering, and Dean, a year later. In these positions he became acquainted with many of the leading men of the country, and, because of the favorable circumstances that only the Capital can afford, he met various prominent engineers from all over the world. This was of benefit alike to himself and to his classes.

He did considerable consulting work both for the manufacturer as well as the user of portland cement. His duties as Dean, however, prevented him from giving as much of his time to such work as he otherwise might well have done. In addition, he was made a member of the Board of Zoning Adjustment of the District of Columbia. In this he was particularly well fitted to take an active part, for one of his peculiarities was his inborn ability to " see quite through the deeds of men." Consequently, when obstreperous Washington real-estate manipulators tried to force some piece of chicanery past the Board, Mr. Lapham instinctively put his finger on the iniquity and checked it.

From his father, Dean Lapham inherited the gift of telling stories. His classroom work sometimes was largely devoted to the relating of yarns that were apropos of the subject under discussion. The students absorbed these tales, and increased their own knowledge out of all proportion by remembering what had been told them in so entertaining a manner. He was very careful never to exaggerate, yet the stories lost nothing in his telling. He never knew that he gave instruction in such a manner; it simply was just natural for him to do so.

His fund of anecdotes for social gatherings was almost limitless, and his ability to play jokes on others was amazing. Often his friends woke up, long afterward, to the fact that they had been the objects of the tricks; but there was no resentment. " Oh, that was just Lap again," would be the inevitable remark.

His railroad experience, plus a New England conscience, gave him as astonishing a command of well-nigh irrepressible expletives as any American other than Mark Twain ever loosed upon his friends. And he would " cuss out " the preacher to his face as eloquently as if he were the garbage man. It was all the same to Dean Lapham so long as he thought the person addressed needed it. These were no common, ordinary, everyday, over-used " cuss words," however; they were vibrant, cogent, exhilarating phrases, the like of which have seldom been heard before. One prominent structural engineer, a member of the Society, spent an entire afternoon trying to repeat, with suitable tones, a particularly novel example that the Dean had used with marked effect, only to give up finally and say, " Well, only Lap could say that and get away with it." Needless to say, the students knew nothing of this hidden gold mine of humor, for humor it was. But the outside world frequently was exposed to its fury.

Probably few of his colleagues knew that he had no use for structural engineering. To him it was a branch of the civil engineer's work that he supposed had to be carried on by somebody, but he was certain that that somebody would not be he. He was greatly interested in sewage disposal, water supply, and concrete testing. He made numerous trips about the country examining works of this character—usually the whole family drove together—and he was planning on writing a book concerning them. He was temperamentally unsuited to be an author, however, because he had too much caution to write. He would talk very freely, if no reporters were present, but almost never would he put his remarks on paper. Hence, he never wrote a book, in which respect he differed from most of his fellows.

He took his work as Dean very seriously as regards both students and faculty. He thought that he owed it to the students to supply them with incentives for further study, for perfecting themselves in their chosen field, and for high scholarship. His faculty members always had "a friend at court," because he would battle for his men with much greater aggressiveness than he would for his own personal advancement. The students always found him to be just. There was never a suggestion of favoritism in any of his decisions. The faculty knew he had a sympathetic ear, and that their troubles were his. In spite of a forthrightness that was inborn, Dean Lapham developed the utmost of tact in dealing with those college situations that frequently become "ticklish." He was careful not to stir up animosities. Beyond that, his fairness, his generosity, and his hospitality were bywords. As a result the administrative side of his deanship was remarkably free from misunderstandings and heartrendings. It was a genuine success.

He hated all pretence and sham. The engineer in practice who failed to give a good account of himself felt Dean Lapham's vituperative denunciations most severely. He was painfully honest himself, and he expected that same quality in others if they were engineers. "If they are not honest, let them try something other than engineering," was his slogan.

In 1916 he was married to Evelyn M. Ayres, secretary of the civil engineering department at The Pennsylvania State College. In so doing he showed the same keen judgment that marked his dealings with men. By virtue of conscientious team work their home life typified everything that intelligent people are supposed to attain. Almost all of it was in Washington, at first in an apartment, then in a small house of their own, and eventually into a more spacious home, where Dean Lapham lived for the last ten years. There could be found students, neighbors, relatives, out-of-town friends—it was continuous "open house." No home could have been more profitably used.

In the home was real hospitality and in the hearts of the Dean and his family there was much genuine friendship, the kind that showed itself in many concrete acts of kindness or beneficence, as many a needy student found out to his satisfaction. It was only natural that many friends should congregate there, where one was sure to find good fellowship, good food, good books, and good fun. On the frequent occasions when the Dean would set out on a foraging expedition for the family, he was almost sure to return with a bouquet of flowers, if nothing else. He loved them, although not himself particularly fond of gardening.

Dean Lapham was a member of Acacia fraternity, in which he took great interest. He was an early member of Sigma Tau, an honorary engineering fraternity; a member of the Washington Society of Engineers; the Society for the Promotion of Engineering Education; the American Concrete Institute; and the Charles River Lodge of Masons in West Medway, Mass. He was much interested in the Student Chapter of the Society at the University and served for a number of years as its faculty adviser.

Dean Lapham's sense of strict integrity was such that for many years he saw no need of affiliating with any church. As his own children were growing up, he faced the matter again with equal candor. He joined the Methodist Episcopal Church. For years thereafter he was a loyal member and at various times an officer of the Hamline Methodist Church in Washington. In eulogizing him, his pastor gave this penetrating picture of his character:

> "He was not saintly, nor would anyone call him pious. But he was genuine, he was sincere, he was fair, he was generous, he was wholesome. In a word, he was just plain good—this jovial Dean."

The funeral service of Dean Lapham was a revelation, and a reflection of how great an investment of friendliness can be put into one abbreviated lifetime.

Thanks to his good constitution and abstemious habits, Dean Lapham enjoyed continuous good health. He died on October 2, 1939, after a week's illness, following an operation. He is survived by his mother, Mrs. Alfarata Lapham, and a brother, Mark Albert Lapham, both of West Medway, Mass.; also by his widow, Mrs. Evelyn Ayres Lapham; a daughter, Evelyn; a son, John Raymond, Jr.,

Dean Lapham was elected a Junior of the American Society of Civil Engineers on November 8, 1909, and an Associate Member on April 16, 1918.

HENRY AMBROSE LARSEN, Assoc. M. Am. Soc. C. E.[1]

DIED APRIL 1, 1939

Henry Ambrose Larsen was born at Manistee, Mich., in 1884. His early education was received in the grade and high schools at Manistee. In 1906 he was graduated from the University of Michigan at Ann Arbor, Mich., with the degree of Bachelor of Science in Civil Engineering, and immediately accepted a position with the Great Lakes Engineering Company at Detroit, Mich.

In 1908 he headed for the Pacific Northwest. In Portland, Ore., he accepted a position with the firm of Doyle and Beach, handling their structural engineering work. Many of the older steel frame structures in this city are the result of his endeavors.

Mr. Larsen was married to Alice Gregory, in Bay City, Mich., on January 18, 1911, and returned with his bride to Portland.

In 1915 he accepted a position with the L. A. Norris Company at San Francisco, Calif., and in 1918 was transferred to the Los Angeles (Calif.) office as manager. During these years he became identified with the sale and distribution of "Clinton" electrically welded fabric. When the Wickwire

---

[1] Memoir prepared by Cris Cron, Esq., Los Angeles, Calif.

Spencer Steel Company decided to merchandise this commodity in 1919, Mr. Larsen transferred to the Wickwire Spencer organization.

In the early 1920's he accepted a position with the National Steel Fabric Company, a subsidiary of Pittsburgh Steel Company, as its Pacific Coast Manager with headquarters in San Francisco. His duty was to merchandise "Steeltex" and Pittsburgh Fabric. His friends and competitors all agreed he did a most commendable job.

In 1933 he again became associated with the Wickwire Spencer Steel Company and in 1937 became the Pacific Coast Manager, a position which he occupied at the time of his death.

As an old business rival writes—

> "Heinie was a man's man, good to know, a worthy competitor with a sense of fairness and honesty and integrity in his relationships with others, which made him outstanding and respected among his business associates, friend or foe. The fabric industry of the Pacific Coast has been fortunate for two decades in having such a man in its midst and his passing will be sorely felt."

Mr. Larsen's sudden passing on April 1, 1939, in San Francisco, came as a shock to his associates of Wickwire Spencer Steel Company and his multitude of friends on the Pacific Coast, as well as in eastern circles.

He is survived by his widow and one daughter, Ethel (Mrs. John M. Evans).

Mr. Larsen was elected an Associate Member of the American Society of Civil Engineers on April 2, 1913.

DANIEL COLLINS LIPSCOMB, Assoc. M. Am. Soc. C. E.[1]

Died December 23, 1939

Daniel Collins Lipscomb was born in Fort Worth, Tex., on November 30, 1887, the son of William Lipscomb and Virginia Collins Lipscomb, who were both well-known educators in the South. The William Lipscomb School, a grade school in Dallas, Tex., is named for his father. His mother was principal of this school for many years. Mr. Lipscomb received his early education in the public schools of Dallas. Later, he entered the University of Texas at Austin, Tex., from which he was graduated in 1909 with the degree of Bachelor of Science in Civil Engineering.

From June, 1909, to September, 1909, he worked as a draftsman for the Chicago, Milwaukee and St. Paul Railway Company of Chicago, Ill. In September, 1909, he became an instructor at the University of Texas, teaching in the engineering department for one year. In May, 1910, he accepted the position of superintendent for the Standard Engineering and Contracting Company of Dallas, working on concrete bridge construction. In October, 1910, at the age of 23, he became assistant engineer for Hedrick and Cochrane, engineers of Dallas, in the construction of the Oak Cliff Viaduct, an outstanding feat of engineering at the time of its erection.

---

[1] Memoir prepared by a Committee of the Fort Worth Branch of the Texas Section consisting of W. O. Jones, Chairman, Joe J. Rady, Members, Am. Soc. C. E., and C. J. Harkrider, Jun. Am. Soc. C. E.

Mr. Lipscomb worked as assistant engineer on the Board of Engineers in charge of the Fort Worth water supply during the year of 1911. In 1912, he became resident engineer for the Southern Traction Company at Waco, Tex., in charge of bridge construction, but in 1913 went back to his job with the Board of Engineers as assistant engineer under Maj. John B. Hawley, M. Am. Soc. C. E., in the construction of the Lake Worth Dam. In 1914, he became engineer for J. C. Field, consulting engineer of Denison, Tex., and held this position until 1918 when he was employed as masonry supervisor for the Missouri, Kansas and Texas Railway Company of Denison.

Mr. Lipscomb resigned from this position to become city engineer of Durant, Okla., a position which he held for two years. He was in charge of supervising street paving, and sewer and water lines. Then, in 1921, he transferred his services as city engineer and water superintendent to the City of Denison, where he worked for two years. In 1923, he became an inspector for the United States Fidelity and Guaranty Company of Dallas.

In 1924, he worked for five months as assistant engineer for the Bureau of Public Roads and then worked the next five months as maintenance foreman of the State Highway Department of Texas. In April, 1925, he was employed by the American Road Company of Dallas as superintendent but gave up this position in December to become superintendent of streets for the City of Fort Worth, which position he held until his death. His work included the supervision, construction, improvement, and maintenance of all streets, and the supervision of garbage collection and disposal. He was a licensed engineer in the State of Texas.

Mr. Lipscomb was an excellent engineer and a man of fine personality and high ideals. He was highly esteemed as an efficient public servant and his loss is deeply regretted by his many friends.

He is survived by his wife, Frances (Wolfe) Lipscomb, to whom he was married in 1917. Mrs. Lipscomb was the daughter of Frank and Sarah Wolfe.

Mr. Lipscomb was elected an Associate Member of the American Society of Civil Engineers on August 31, 1915.

HARRY SEYMOUR McKIBBEN, Assoc. M. Am. Soc. C. E.[1]

DIED APRIL 12, 1939

Harry Seymour McKibben was born January 6, 1873, in Braceville Township, Trumbull County, Ohio. The eldest son of Seymour M. and Lydia (Smith) McKibben, he received his early education at the local schools of the township and the Newton Falls, Ohio, grade schools. He was graduated from Ohio Northern University, at Ada, Ohio, in 1896, and took postgraduate study at Wooster College, at Wooster, Ohio.

---

[1] Memoir prepared by Wendell P. Brown, M. Am. Soc. C. E.

Before graduation, and continuing until 1901, he was engaged in survey work during the summer, and teaching in the local country schools during the winter seasons. During 1901 and 1902, he taught school in Oklahoma, in addition to engineering and survey work for the Missouri, Kansas and Texas Railway Company and the Rock Island Railroad Company, and for irrigation projects in Oklahoma. From 1903 to 1905, he was engaged in survey work for the Baltimore and Ohio Railroad Company in Ohio, and as assistant county surveyor of Trumbull County.

From 1905 to 1909, Mr. McKibben was in private engineering practice in Trumbull County in charge of reservoir and contour surveys and all preliminary and field work in connection with the construction of three hydroelectric plants and about 18 miles of high tension lines of the Hydro-Electric and Gas Company of Warren, Ohio. He then served from 1909 to 1913 as county engineer in Trumbull County, and during 1914 and 1915 was city engineer at Warren. He was again in private engineering practice from 1916 to 1929, but acted as engineer for the Village of Newton Falls from 1929 to 1935. He returned to private engineering practice at Warren in 1935, and continued in this work until his death. While engaged in a field survey in the county, he suddenly passed away on April 12, 1939.

In addition to his professional practice, Mr. McKibben was interested in civic affairs, and served from 1922 to 1933 on the Board of Education of the City of Warren. He was also an active member of the Second Christian Church, and, among other activities, was a valued member of the church choir.

He was married on November 15, 1902, to Bertha Musson of Ligonier, Ind., who, with four sons, four daughters, and nine grandchildren, survives him.

He will be remembered as a conscientious and dependable engineer, a devoted husband, father, and grandfather, a worthy citizen, and one that recognized his obligations to the community and to his profession, and fulfilled them faithfully.

Mr. McKibben always had a deep interest in and appreciated his membership in the American Society of Civil Engineers, although he did not take an active part in its affairs. He was elected an Associate Member of the Society on December 2, 1914.

WILLIAM FRANKLIN MILLER, Assoc. M. Am. Soc. C. E.[1]

DIED DECEMBER 18, 1939

William Franklin Miller was born on June 21, 1880. He spent the earlier years of his life in Philadelphia, Pa., and came from a family that schooled him thoroughly along religious and civic responsibilities, and throughout his entire life he responded conscientiously to this earlier training.

---

[1] Memoir prepared by R. R. Nace, H. H. Garrigues, and R. G. Develin, Members, Am. Soc. C. E.

He obtained his engineering education from courses of the International Correspondence School, and through attendance at night classes of the Wharton School of the University of Pennsylvania, at West Philadelphia, Pa. In addition to these courses he was tutored by engineers working for the Pennsylvania Railroad Company, who were attracted to him by his energies and application as a young man. He was a hard and conscientious worker during his entire career, and was highly regarded by his Pennsylvania Railroad associates for his character and ability.

Mr. Miller spent his entire professional career, commencing in 1903, with the Pennsylvania Railroad Company. He passed through the grades of rodman, transitman, assistant supervisor (track), division engineer, and engineer of maintenance of way. He was assistant to the chief engineer, maintenance of way, Eastern Region, at Philadelphia at the time of his death. While occupying these positions Mr. Miller was engaged in a great variety of engineering work, which ranged all the way from simple surveys and design of small structures to the prosecution of reconnaissance surveys in the mountain territory of the Pennsylvania System; and design of such major structures as reservoirs, branch lines, plans for new freight yards, and changes in existing freight yards. Following this experience, Mr. Miller devoted his attention to maintenance of way work in which he demonstrated outstanding ability, and which resulted in his being advanced to the position of engineer of maintenance of way.

With the passage of the Valuation Law in 1913, he was called upon by the Pennsylvania Railroad Company to perform special duty in conjunction with the Federal Valuation Officers, which first involved field inventory of the property, and later the compilation of considerable cost data. His wide experience and knowledge was of considerable value, both to the railroad companies and the Bureau of Valuation. During the disastrous fire at the Broad Street Station, in Philadelphia, on June 11, 1923, he rendered valuable service of an emergency nature in restoring traffic at that terminal.

Mr. Miller, wherever located, always engaged in matters of civic interest, and made close contacts with engineering officers of the different communities. He was president of the Engineers' Society of Pennsylvania in 1932, and in November of that year he had the pleasure and honor of inducting President Herbert Hoover into the Engineers' Society of Pennsylvania at the White House in Washington, D. C., in the presence of more than one hundred engineers.

Mr. Miller died on December 18, 1939, at the age of fifty-nine at Ardmore, Pa.

Mr. Miller had two sons to whom he was devoted, and the interest he displayed in them reflected his own home training.

Mr. Miller was elected an Associate Member of the American Society of Civil Engineers on May 31, 1916.

ERI OLMSTEAD, Assoc. M. Am. Soc. C. E.[1]

Died April 13, 1939

Eri Olmstead was born in Portland, Mich., on October 12, 1885, the son of Allen and Elvira Olmstead. He received his early school training in that city and was graduated from the Portland High School in 1905. He entered the University of Michigan, at Ann Arbor, Mich., in 1907, and completed the civil engineering course in 1911.

Mr. Olmstead began his engineering career in July, 1911, as a surveyor and map maker for the Bermudez Company, a subsidiary of the General Asphalt Company, in Venezuela, South America. This temporary assignment was completed in December, 1911. He returned to the United States and accepted a position the following month with Stone and Webster Engineering Corporation of Boston, Mass. For the following two years he acted as draftsman, surveyor, and designer for many important undertakings which this company had in progress at that time. From 1914 to the middle of 1916 he acted in the capacity of chief checker for Stone and Webster on the new buildings which the Massachusetts Institute of Technology at Cambridge, Mass., was sponsoring at that time. Immediately thereafter he represented this firm as resident engineer on the construction of factory buildings for the Simplex Wire and Cable Company, and the Boston Woven Hose and Rubber Company. Leaving Stone and Webster in the fall of 1916, he accepted a position with the Harry M. Hope Engineering Corporation, also of Boston, as resident engineer in charge of construction of a complicated boiler house and pumping station for the Fisk Rubber Company, a power station for the Union Street Railway, at New Bedford, Mass., and a factory for the Schuhle Grape Juice Company at Highland, N. Y. This work occupied his complete time until February, 1920.

It was at this time that the C. B. Roberts Engineering Company of Boston had a contract calling for the construction of a refinery for the Louisiana Oil and Refining Company, at Shreveport, La., and Mr. Olmstead was appointed superintendent and engineer in charge of construction. This work was Mr. Olmstead's last assignment with eastern engineering firms.

Leaving Boston in the fall of 1921, he went to Detroit, Mich., and accepted the position of general superintendent of construction for the J. A. Utley Company of that city. He was with this company until the spring of 1925 and completed many fine factory buildings and residences of the better class.

In March, 1925, the general contracting firm of Beyster and Olmstead, Inc., was founded in Detroit with Mr. Olmstead as president. This contracting organization, although no longer existent, is well remembered among the construction fraternity in and around Detroit where most of their work was done.

In September, 1934, Mr. Olmstead accepted a position as resident engineer inspector with the Federal Emergency Administration of Public Works

---

[1] Memoir prepared by Milo F. Ohr, M. Am. Soc. C. E.

(PWA). In this capacity he represented the federal government on the complete waterworks projects at Port Hope, and Standish, and on the construction of the new elementary school building at Mount Pleasant, all in Michigan. Following this, his work covered the new hospital addition built for the City of Hamtramck, Mich., and the sewage disposal plant at Utica, Mich.

For more than three years preceding his death, Mr. Olmstead served as a traveling engineer inspector for the PWA within the State of Michigan. He visited construction jobs throughout the state, passing upon the work being done on the many projects, instructing and acquainting new engineer inspectors with their duties and responsibilities, and in general acting as a liaison man between the field and its construction problems and the office and its personnel. It was through this work in the latter part of his life that the helpfulness and cheerfulness ever present within this man was brought to so many people who enjoyed the opportunity to be associated with him. His passing will be a distinct loss to his many friends and associates, both within, and aside from, the business world.

He was married to Pearl Osborn on January 4, 1910, and is survived by his widow; his daughter, Barbara; and a sister, Mrs. George Erdman.

Mr. Olmstead was elected an Associate Member of the American Society of Civil Engineers on July 12, 1937.

VICTOR SMITH PERSONS, Assoc. M. Am. Soc. C. E.[1]

DIED MARCH 5, 1940

Victor Smith Persons, the son of the Rev. Greenleaf Elvero Persons, D.D. and Gertrude (Reed) Persons, was born on December 5, 1878, at Ashland, Ohio. His ancestors, on both sides, came from England and settled in New England early in the seventeenth century.

After completing his early school training, Mr. Persons entered Ohio Wesleyan University at Delaware, Ohio, in 1895, and was graduated with the degree of Bachelor of Arts in 1899. Later he attended Armour Institute of Technology at Chicago, Ill., and was graduated with the degree of Bachelor of Science in 1902. In this same year he was given the degree of Master of Arts at Ohio Wesleyan University. A few years later he served as instructor in Civil Engineering at Armour Institute and during this period he prepared for and received his Master's degree of Civil Engineering in 1907.

Immediately after graduation from Armour Institute he entered the employ of Noble and Modjeski, and for two years was connected with the construction of the Thebes bridge over the Mississippi River at Thebes, Ill., first as inspector, then as assistant engineer.

---

[1] Memoir prepared by C. B. Hopkins, Assoc. M. Am. Soc. C. E., and Otto G. Hintermann, Esq., Certified Architect, San Francisco, Calif.

In 1904 he joined the staff of the University of Chicago, Archæological and Research Department, and was detailed as engineer of excavation for the expedition in Mesopotamia and Egypt, remaining there for two years.

Upon his return to the United States he was employed by the City of Chicago as engineer on tunnel construction. He became interested in the then rapidly developing reinforced concrete industry and joined the staff of the Corrugated Bar Company of St. Louis, Mo., first as estimator, then as designer of all classes of reinforced concrete structures. Later he was sent to Portland, Ore., by the Concrete Steel Products Company of Chicago, there becoming their Pacific Coast representative in charge of design of reinforced concrete structures and the sale of steel reinforcing products.

In 1912 Mr. Persons settled in San Francisco, Calif., and continued his work in reinforced concrete design and development of steel reinforcing materials, his principal connection being with the L. A. Norris Company in the promotion and sale of Clinton Wire Cloth Company products. The development of many new uses for these products, and allied ones, was the result of his imagination and ingenuity.

From 1920 until his death Mr. Persons operated his own business, continuing his engineering design and doing his sales engineering work under the firm names of Persons, Dwan and Company and finally as The Persons Company. In his sales engineering work he represented, on the Pacific Coast, some of the outstanding national building specialty manufacturers.

Mr. Persons' career as a student, teacher, and engineer was marked by his devotion to the problem at hand, and by his conscientiousness and accuracy. In his business relationships he assumed a broad spirit of fairness and was above reproach in all his dealings. He was a bachelor, and is survived by a sister, Alice Lora Persons, and two brothers, C. E. and V. J. Persons. His recreations were simple and wholesome, and included bridge, golf, fishing, riding, and camping.

Mr. Persons was a member of Delta Tau Delta, social fraternity, and Phi Beta Kappa, scholastic honor society. He belonged to the Olympic Club, the Fraternity Club, the Producers Council, and the Mechanics Institute, all of San Francisco.

Mr. Persons was elected an Associate Member of the American Society of Civil Engineers on June 11, 1917.

CHARLES ARNER RUGGLES, Assoc. M. Am. Soc. C. E.[1]

DIED SEPTEMBER 6, 1938

Charles Arner Ruggles was born in Canfield, Ohio, on May 30, 1868, the son of Horace Greeley Ruggles, of Canfield, and Amanda C. (Hoyle) Ruggles, of Berlin, Ohio.

---

[1] Memoir prepared by E. W. Robinson, Assoc. M. Am. Soc. C. E.

The family is descended from John Ruggles of Nansing, Essex, England (the son of Thomas of Sudbury, Suffolk, England), who came to the United States with his wife, Barbara, and son, John, in the ship, *Hopewell,* and settled in Roxbury, Mass., in 1635.

Mr. Ruggles' great grandfather, Azor Ruggles, was born and brought up in Brookfield, Litchfield County, Conn. He was a millwright and went to Ohio on horseback in 1810 for the purpose of doing some work for Judge Canfield. After remaining a year he returned to Connecticut and in 1813 took his wife and six children to Ohio in two wagons and settled permanently in Canfield.

His grandfather, Charles, was a land surveyor and lawyer and his father was also a member of the Ohio bar. The latter served as a corporal in Company H, 105th, Ohio Volunteer Infantry, in the Civil War, and after practising law in Canfield for a number of years, moved to Wichita, Kans., with his family in 1874, where he was admitted to the Kansas bar and became a practising attorney.

Charles Arner Ruggles received his early education in the schools of Wichita, was graduated from Lewis Academy, and took an engineering course at Garfield University, at Wichita. On completion of his schooling, about 1890, he secured a position as architect and assistant engineer with a firm in Middlesboro, Ky., and was transferred from there to its office in Knoxville, Tenn. When a better opportunity was offered him by the Chicago Ornamental Iron Company in 1892, he joined their forces, remaining with them about three years, and becoming foreman draftsman.

From Chicago Mr. Ruggles came east, in 1895, and held positions as draftsman and designer with various ornamental iron works until 1897, when he became manager of the New York office of the Brown Ketcham Iron Works of Indianapolis, Ind., in charge of all work east of Pittsburgh, Pa.

He remained with the Brown Ketcham Iron Works for five years and in 1902 became manager of the New York office of the Columbian Fireproofing Company of Pittsburgh. In 1907 he was made manager of the New York office of Cramp and Company, of Philadelphia, Pa.

In 1910 Mr. Ruggles had an opportunity to go into business on his own account, in New York, N. Y., and he organized the firm of Ruggles-Robinson Company to do general contracting and building business. He became president of this firm, retaining the office until his death. During his last years he was not active in business, but devoted most of his time to the management of his personal and family affairs.

Mr. Ruggles was a man of modest and self-effacing character; he had few close friends and was always reticent about his personal activities and contributions to the well being of his fellowmen. On rare occasions he would unbosom himself and one might hear of his love for horses, as a boy in Kansas; of his escapades during vacation in Indian Territory; of his Militia service in the Tennessee National Guard; and of his membership in choral societies in Chicago. His one hobby in later life was his garden and orchard, to which he devoted much time, and where he raised specimen apples, mostly of the old-fashioned varieties. In his relations with clients and fellow workers, his ability and fairness in technical and business matters was outstanding.

Mr. Ruggles was married to Florence G. Clark on March 3, 1908, in New York City. He is survived by his widow, a daughter, Jean Campbell, and a brother, Rollin P. Ruggles.

At the time of his death he was a director of Charles and Company, a member of the Apawamis and Manursing Island clubs of Rye, N. Y., and, since 1903, a member of Albion Lodge No. 26, Free and Accepted Masons.

Mr. Ruggles was elected an Associate Member of the American Society of Civil Engineers on June 4, 1902.

HERMAN SCHNEIDER, Assoc. M. Am. Soc. C. E.[1]

Died March 28, 1939

Herman Schneider was born at Summit Hill, Pa., on September 12, 1872, the son of Anton and Sarah (Wintersteen) Schneider. At an early age he started to work as a breaker boy in the coal mines and, before entering college, he had worked as a helper on a construction gang and on survey parties.

He attended Lehigh University, at Bethlehem, Pa., from which he was graduated in 1894, with the Degree of Bachelor of Science in Architecture. In order to defray his expenses at college he worked in architectural offices during vacations and after school hours. His work was first as a blueprint boy and later, by time of graduation, as a designer. This practical work was extremely beneficial in helping him to utilize the theories taught in the classroom, and gave him a marked advantage at graduation over those students who had pursued academic work alone. This fact so impressed itself upon him that he never forgot it, and it was the idea born from this joint work and school program which influenced all the remainder of his life and led to the origin of cooperative education.

After graduation, Herman Schneider entered private practice as an architect in Cumberland, Md. During the three years that followed, he designed about three hundred buildings and superintended the construction of about seventy-five of them. In October of 1897, he accepted employment with the Oregon Short Line Railway, and from that time until June, 1898, he designed several hotels and depots for the railway. In June, 1898, he returned to South Bethlehem, Pa., where he worked as superintendent of construction for A. W. Leh, Architect.

During all of this experience, there was one thing that impressed itself upon young Schneider. That was the lack of practical knowledge shown by the young college graduates with whom he was associated. With the keen instinct of the scientist, he was already seeking a solution for this problem, and the answer seemed to him to be found in some combination of practice and theory, such as had been his own personal experience when he worked after school and on Saturdays in the architect's office at South Bethlehem, while attending Lehigh University. He decided to place this idea before qualified educational authorities and have it tried.

---

[1] Memoir prepared by a committee of the Cincinnati Local Section consisting of Raymond W. Renn, Assoc. M. Am. Soc. C. E., assisted by Prof. Clyde W. Park, Head of English Dept., Univ. of Cincinnati, Cincinnati, Ohio.

Hence he welcomed the opportunity of becoming connected with the faculty of his alma mater, and accepted an appointment as instructor in the Department of Civil Engineering in September, 1899. For four years, he continued as instructor at Lehigh, and during most of this time he actively sought the adoption of his idea of practical training to supplement the academic work at the university. The task of changing the educational system of an established engineering college was too much for a young instructor, however, and so in 1903, Schneider decided that the best procedure would be to carry his idea to a newer institution. The obvious fact that such an idea stood a better chance of success if applied in an industrial center, where the school and the shops existed side by side, became more evident.

So, in casting about for the best location for such a school, he found a vacancy existing in the University of Cincinnati, at Cincinnati, Ohio, and in 1903 he accepted an appointment as Professor of Civil Engineering at that institution. For three years he worked diligently, losing no chance to forward his idea of cooperative education. Finally, in 1906, the Board of Directors of the University of Cincinnati approved the adoption of the cooperative system as an experiment, the continuation of the approval to depend upon its success. At the same time, Professor Schneider was appointed Dean of the College of Engineering and was given full responsibility for the execution of his plan.

In September, 1906, the first class of cooperative students was formed under Dean Schneider's guidance. It consisted of 27 students in chemical, mechanical, and electrical engineering. As proof of the vigor and strength with which he attacked his problem, the biography of Herman Schneider, from this point, is largely the history of the cooperative system of education, and the history of the College of Engineering of the University of Cincinnati.

In 1911, the University of Pittsburgh, at Pittsburgh, Pa., conferred upon Dean Schneider the degree of Doctor of Science. From 1911 to 1913, he was a member of the Committee on School Inquiry of the City of New York. An important part of this work was the introduction of the cooperative plan in the New York, N. Y., high schools. Because of ill health, it was necessary for Dr. Schneider to terminate this work in 1915.

In September, 1917, Dr. Schneider accepted the appointment as Chief of the Industrial Service Section of the Ordnance Department, a new bureau which handled for the Ordnance Department all labor and employment matters relating to production in arsenals and private manufacturing plants. In this department he had to deal with labor problems which arose from handling contracts approximating $2,000,000,000 in connection with the procurement of material for the U. S. Army.

In February, 1918, he was chosen to represent engineering schools as a member of the advisory committee for the Committee on Education and Special Training. This Committee, appointed by the Secretary of War, was charged with the responsibility of training 90,000 men of the National Army for technical and skilled work of various kinds.

After the war, there followed a rapid succession of events in the educational and scientific development at the University of Cincinnati, for which Dr. Schneider was responsible. In 1919, cooperative courses in commerce were inaugurated, and the College of Engineering became the College of Engineering and Commerce. In 1920, organized research work was begun by the opening of the Leather Research Laboratory of the Tanners' Council of America, and the Institute of Scientific Research was established. In 1922 the School of Applied Arts was added to the cooperative group of the University by the introduction of a Department of Architecture. In succession were added the Departments of Landscape Architecture, Interior Decoration, Art Applied to Industry, and Ceramics. In 1924, a Resource Survey was begun which covered the territory within a radius of 300 miles of Cincinnati. In March, 1925, the Lithographic Research Laboratory was started, and in 1926 the Basic Science Research Laboratory which dealt with the combining and correlating of the natural sciences was established. During this era of development, all under the close personal supervision of Dr. Schneider, he became, in addition to the Dean of the College of Engineering and Commerce, the Director of the School of Applied Arts, the Director of the Institute of Scientific Research, and the Director of the Evening College.

In 1928, after much persuasion, Dr. Schneider transferred the seat of his activities from the Dean's Office in the Engineering College to the President's Office. Although he agreed to this change only upon the condition that he was to be relieved of the duties as soon as possible and returned to the activities in which he was most interested—scientific development and cooperative education—nevertheless he served as President of the University of Cincinnati from 1928 until 1932.

In 1931, Dr. Schneider and Prof. George Sperti announced the discovery of the method of irradiation of vitamin D by the application of selective ultraviolet rays, a discovery which has important bearing on increasing the nutritive and health building properties of foods.

The University of Cincinnati conferred the Degree of Doctor of Laws on Dr. Schneider in 1933 and made him President-Emeritus of the University. At the time of his death, Dr. Schneider headed the College of Engineering and Commerce, the Institute of Scientific Research, the School of Applied Arts, and the Evening College of the University of Cincinnati. In addition to these duties, he found time to serve with Charles F. Kettering, M. Am. Soc. C. E., President and Director of General Motors Research Corporation, as a consultant on the Walter P. Murphy Foundation, in the establishment of a new Institute of Technology at Northwestern University, at Evanston, Ill.

Although Dr. Schneider's life was essentially a life of action he still found time to do a great deal of writing. He was the author of "Arthur McQuaid, American," consisting of six short stories published in the *American Magazine*. Two of them were placed on the Honor Roll in Edward J. O'Brien's list of the best American short stories in 1917. Other works consisted of "Education and Industrial Peace," published in 1912; "Education for Industrial Workers," published in 1915; "The Problem of Vocational Guidance," May, 1938; essays and contributed editorials for the *Engineering News-Record* and the *Cooperative Engineer;* and numerous articles on educational subjects.

In 1931, the Cincinnati Institute of Fine Arts awarded Dr. Schneider the Sachs Prize for outstanding achievement during the year 1931 in the field of literature, art, and science. In 1936, he was awarded the Lamme medal by the Society for the Promotion of Engineering Education. He also served on the Advisory Council on Applied Physics of the American Institute of Physics in 1936.

Dr. Schneider was a member of the Society for the Promotion of Engineering Education; the Engineers' Club of Cincinnati; and the Commercial Club and Carbon Lodge No. 242, F. and A. M., of Mauch Chunk, Pa.

On July 27, 1904, Herman Schneider was married to Louise Bosworth at Cassopolis, Mich. They had no children. His widow survives him.

Dr. Schneider was a Presbyterian. A brief glimpse of his philosophy of life may be had from the following public statement:[3]

> "You can't go very far in modern science, groping toward an understanding of its absolutely perfect laws, without being forced—to your knees, I almost said—to the conviction that back of the whole scheme of things there is a most amazing Divine Intelligence.
>
> "Try to grasp some of the facts of astronomy and of astrophysics: of little patches of our sky containing hundreds of thousands of suns much greater than our own, of our galaxy of stars one hundred million light years across, and some billions of light years beyond, other galaxies and super-galaxies; then consider the solar systems which we call atoms—and let your mind run through the whole pulsating mystery of what we call 'matter' from the whirling electron to the super-galaxies, all held in a bewildering balance and movement by absolutely perfect law * * * well, if you grasp only a little of that, you just slough off your egocentric trends to personal acquisition of this and that, and try to get into the swing of the great movement of which you are a little part; and you strive to make your work conform. You get the feeling of infinite progress in accordance with a wonderful pattern which you can barely sense but which you can't understand."

Throughout an arduous and busy life, "Dean" Schneider, as he was always referred to by the students and by his associates, remained accessible to the least of them. Nothing gave the "Dean" more pleasure than to have a student come to his office and there, in quiet and privacy, pour out his troubles, and in turn listen to the timely and wise advice which Dean Schneider was able to give from the wealth of his research and experience.

The opening paragraph of the resolution adopted by the Cincinnati Section of the Society, upon his death, fitly pays tribute to his qualities:

> "Herman Schneider,—man, philosopher, dreamer, practical molder of lives and character, friend and counsellor of youth, confidant and arbiter in man's estate, has responded to the beckoning hand of The Infinite, whose might and wisdom were to his eyes so evident in the scheme of the universe."

Dr. Schneider was elected an Associate Member of the American Society of Civil Engineers on April 2, 1902.

---

[3] *Engineering News-Record,* June 10, 1926.

REGINALD CROWLEY SCOTT, Assoc. M. Am. Soc. C. E.[1]

DIED SEPTEMBER 30, 1939

Reginald Crowley Scott was born at Philadelphia, Pa., on September 23, 1892, the son of Valentine and Sara A. Scott. He was educated in the Philadelphia public schools and was graduated from the Central Manual Training High School. Later he attended evening classes in engineering subjects at Drexel Institute of Technology, at Philadelphia, from which he received a Certificate in the Collegiate Course in Engineering.

In June, 1910, he was employed as a rodman by J. H. Young, a civil engineer of Philadelphia. In October, 1911, he became a rodman and later a chainman with the Pennsylvania State Highway Department in the Philadelphia District, being subsequently transferred to the Bloomsburg District as a chainman.

Two years later Mr. Scott was employed by the City of Philadelphia in the Department of City Transit as a rodman. He advanced to the ranks of assistant engineer and junior assistant engineer. His work consisted of establishing lines, grades, and layouts necessary for the construction of the Frankford Elevated high-speed electric passenger line and for future transit routes.

In January, 1924, he was engaged by the Real Estate and Construction Company of Philadelphia as managing engineer on general construction work.

He returned to service in the Department of City Transit, in June, 1924, advancing in June, 1936, to the rank of senior assistant engineer, which rank he held until his death. As assistant to the division engineer he was responsible for the supervision of lines, grades, estimates, records, and construction of portions of the Broad Street, Ridge Avenue-Eighth Street, Locust Street and South Broad Street subways and appurtenant work.

Mr. Scott's career in the engineering profession was notable because of his strict attention to details of his work, careful study of construction problems, loyalty to his employer and associates, and a broad spirit of fairness to all who through business came in contact with him. He possessed a fine character and a splendid personality.

He took an active interest in the work of the Philadelphia Section, having been elected to the office of Secretary-Treasurer, which office he held at the time of his death. He was a Founder Member of the Pennsylvania Chapter of the American Society of Professional Engineers.

On August 25, 1926, Mr. Scott was married to Alice A. Newis, who survives him. Also surviving are his mother, his sister Emma, and a brother Howard.

Mr. Scott was elected an Associate Member of the American Society of Civil Engineers on October 11, 1920.

---

[1] Memoir prepared by Charles H. Stevens, M. Am. Soc. C. E., aided by Charles S. Shaughnessy and Howard S. Hipwell, Members, Am. Soc. C. E.

ALEXANDER JENIFER TAYLOR, Assoc. M. Am. Soc. C. E.[1]

DIED FEBRUARY 27, 1940

Alexander Jenifer Taylor was born in the town of Dover, Del., on February 9, 1875, the son of Alexander J. and Wilhelmina (Baynard) Taylor. His early education was received in the private and public schools of his home town. In 1889 he entered Delaware College (later the University of Delaware), at Newark, Del., and was one of the thirteen who graduated four years later, being valedictorian of his class and the youngest in his class to receive a degree.

After his graduation Mr. Taylor began work as a rodman with the Park Department of the City of Wilmington, and a year later became the assistant engineer of the Street and Sewer Department in charge of the drafting room, surveying, and field work.

At the outbreak of the Spanish-American War, Mr. Taylor joined the Delaware volunteers and became Captain of Company "M," Sixth Regiment. At the close of the war, he resumed his engineering studies at Massachusetts Institute of Technology, at Boston, Mass., graduating in 1901 with a Bachelor of Science degree.

He returned to Wilmington, but soon became interested in the possibilities of sulfur deposits in Alaska. He joined a group of engineers to make a survey there and acted as assistant to the engineer in charge of that survey. Due to financial difficulties the group returned to Wilmington after five months in Alaska. Mr. Taylor immediately resumed his engineering work with the Street and Sewer Department. From September, 1902, to December, 1903, he was with the late T. Chalkley Hatton, M. Am. Soc. C. E., consultant engineer, as assistant engineer on design of water and sewerage systems for several towns, having had charge of construction of water works in Hammonton, N. J., and Matawan, N. J.; street paving in Frederick, Md.; and Price's Run Sewer, in Wilmington (a $100,000 project). From December, 1903, to September, 1906, Mr. Taylor was engineer and superintendent for park commissioners, in Wilmington, in charge of maintenance work and of designing and constructing driveways, paths, swimming pools, and bath houses.

In 1910 he was appointed city engineer for Wilmington, which position he retained until 1915 when he entered the employ of E. I. du Pont de Nemours and Company. His engineering ability and executive ability were soon recognized by the du Pont Company, and he was placed in charge of construction of the Kennett Highway, one of the early reinforced concrete roads in the state, improved by P. S. du Pont. He acted as assistant engineer and as resident engineer at Williamsburg, Va. (Penniman plant), and as resident engineer at Gibbstown, N. J. (Repauno plant), and Arlington, N. J., in charge of construction work on industrial plants. At this time, Mr. du Pont was looking for a capable man to help administer the state-wide important school building program planned for Delaware. That post was filled by Mr. Taylor, who entered into the work with much zeal and a desire to establish a project that would forever be a monument in the State of Delaware.

---

[1] Memoir prepared by J. L. Warner, M. Am. Soc. C. E.

Although Mr. Taylor's life work was varied, his principal contribution to his state was his keen interest in the construction of public schools throughout Delaware during the period when the school building program was being improved by the Delaware School Foundation and Delaware School Auxiliary. Mr. Taylor's thorough knowledge of civil engineering and his experience in this branch of the work established him as an outstanding authority on school building. He became vice-president and chief engineer of the Delaware School Auxiliary which was linked with the Service Citizens Association of Delaware. In 1927, when the Service Citizens went into history, the Delaware School Auxiliary continued under the name of the Delaware School Foundation; and upon the death of its president, Joseph H. Odell, in 1929, Mr. Taylor became the president. He continued as chief engineer, although the major work of the Foundation had been completed.

As an alumnus of the University of Delaware, with degrees of Bachelor of Science in Civil Engineering and Civil Engineer, he was one of its most active members. His keen interest and open enthusiasm in all activities and his readiness to promote all worthy projects will always be remembered by his associates and those interested in the educational work of Delaware. The well-equipped gymnasium at the university was named in his honor, and he was serving as a trustee of the university at the time of his death.

It was the custom of Alexander J. Taylor to devote himself energetically to each task which fell to his lot. His keen interest continued until each job was done, and done satisfactorily. He was a capable engineer who served well both public and private interests during his entire career.

As president of the Delaware School Foundation, Mr. Taylor's name will go down in Delaware History as the able director of the program of that organization for modernizing the state's public schools. This undertaking, financed by Pierre S. du Pont, resulted in the construction of fine new schools in all parts of the state. As a citizen, Mr. Taylor was respected and appreciated by his fellow citizens. He was a willing, cheerful, and helpful participant in those public undertakings that seemed to him to be in the interest of the general good.

It was without hesitancy that, with this record of reliability and competency, he was appointed executive secretary to the Delaware Liquor Commissioner after the repeal of prohibition. Although Mr. Taylor neither drank nor smoked, he entered into his new duties with close attention and sincerity in guiding the state out of prohibition. Where he had once moved in architectural and engineering circles, Mr. Taylor, as the liquor commissioner's secretary, found himself in almost a new world, but he quickly became respected and admired by all with whom he came in contact in this new and strange work for his fairness and intelligent grasp of the problems. In 1938 his health became impaired, but he continued in his position as executive secretary to the liquor commissioner. His sudden death, despite the fact that his health had not been perfect, came as a shock to the citizens of Wilmington. His dealings with the citizens of his state leave with them a knowledge of Alexander Taylor as a man who was never unwilling to try to solve any problem; and in his engineering career his sound and unerring judgment in engineering matters, especially in the State of Delaware's public schools, stands out as the excellent work in engineering of Alexander Taylor.

Mr. Taylor was married to Isabelle Macallister, on January 7, 1902. His wife, a son, Alexander J., and a daughter, Eleanor (Mrs. L. Samuel Vadner), survive him.

Mr. Taylor, a Thirty-Second Degree Mason, was a member of the Masonic Club of Delaware, a member of the Shrine, past-president of the Knights of the Round Table, and a member of the Engineers Club of Philadelphia.

Alexander J. Taylor was elected a Junior of the American Society of Civil Engineers on October 3, 1899, and an Associate Member on April 3, 1907.

GEORGE COPELAND URQUHART, Assoc. M. Am. Soc. C. E.[1]

Died January 10, 1939

George Copeland Urquhart, the son of Capt. Moses Johnston and Susan (Copeland) Urquhart, was born at Steubenville, Jefferson County, Ohio, on Saturday, March 24, 1866. His family moved to Mansfield, Ohio, in 1873, and to Mount Vernon, Ohio, in 1875, returning to Steubenville in 1877. Mr. Urquhart received his early education in Miss Brown's Private School at Steubenville and the public schools at Mansfield and Mount Vernon. He completed his education at Steubenville, graduating from Steubenville High School in June, 1883.

Mr. Urquhart's first experience on engineering work was during his school vacation in 1882, when he was employed on the engineering force of the Maintenance of Way Department of the Pittsburgh, Cincinnati, and St. Louis Railway Company. Upon graduation the next year he was appointed to the engineering force of the Cincinnati and Muskingum Valley Railway Company, with headquarters at Zanesville, Ohio. In 1884 he was transferred to the Pittsburgh, Cincinnati, and St. Louis Railway Company with office at Pittsburgh, Pa.; and on July 5, 1886, he was promoted to Assistant Engineer. In addition to railroad work during this time he conducted the surveys in connection with the erection of a water-works system for the villages of Dennison and Uhrichsville, Ohio.

On April 20, 1888, he was appointed Assistant Engineer in Charge of new construction, with headquarters at Steubenville. His work included double tracking 12 miles of the Panhandle Division of the Pittsburgh, Cincinnati and St. Louis Railway between Bulger and Colliers, W. Va., the completion of Dinsmore Tunnel for double track, and the widening of the bridge over the Ohio River at Steubenville for double track.

---

[1] Memoir prepared by a Committee consisting of L. J. Riegler, M. Am. Soc. C. E., Paul J. Urquhart, Aluminum Co. of America, and James L. Mason and A. R. Meredith, Pennsylvania Railroad Co., Pittsburgh, Pa.

In 1890 he moved his headquarters to Bowerstown, Ohio, and supervised the widening of tunnel No. 10 and the construction of double track from Philadelphia Road to Bowerston (6 miles). This work included street paving, sewering, and municipal improvements in the village of Dennison. In 1893 he was appointed Assistant Engineer in the office of the Chief Engineer of the Pittsburgh, Cincinnati, and St. Louis Railway, in charge of tax and real estate matters.

On April 1, 1899, the Real Estate Department of the Pennsylvania Lines West of Pittsburgh was organized and Mr. Urquhart was appointed Assistant Real Estate Agent for the Southwest System, Mr. Charles D. Law being the head of the entire department. On January 1, 1903, Mr. Urquhart was appointed Real Estate Agent of the Lines West succeeding Mr. Law who died in November, 1902. On March 1, 1920, the organization of the Pennsylvania Railroad Company was changed into a Regional System and Mr. Urquhart was appointed Real Estate Agent for the Central Region. By reason of further reorganization of the System he was appointed Assistant to General Real Estate Agent on February 1, 1923, and on July 1, 1924, General Agent, Real Estate Department, both positions with headquarters at Pittsburgh.

While head of the Real Estate Department of the Pennsylvania Lines West of Pittsburgh and its subsidiary corporations, Mr. Urquhart was in full charge of the departmental activities covering consultation upon land survey work, and other engineering related thereto, in connection with acquisition and administration of land. Upon assuming office in 1903, he was confronted with the largest program of new construction work that the railroad had ever undertaken, including many new lines in Pennsylvania, Ohio, Indiana, and Illinois, double tracking of a large part of the entire system, the construction of many yards and terminals, and track elevation in the larger cities of the Lines West of Pittsburgh.

During the earlier period, while in charge of construction projects, particularly the double tracking of the Ohio River bridge at Steubenville, and the widening of the tunnels (all of which had to be done along with the maintenance of an exacting main line railroad traffic) he earned a reputation of great dependability and reliability and won the confidence of the railroad's operating officers. To his later duties he brought a wealth of knowledge and skill in analyzing and solving intricate title and boundary problems involved in the purchase and administration of real estate on a large scale, and under many diversified conditions. He was considered an expert on land values and on taxation. Mr. Urquhart retired from active service of the Pennsylvania Railroad Company under its pension regulations on January 31, 1932.

He was married on September 3, 1896, to Mrs. Alberta V. Thompson (née Alberta V. Mills) who died May 22, 1913. On April 11, 1916, he was married to Mary Anna McCombs, who survives him.

Mr. Urquhart was a member of Dallas Lodge No. 508, F. and A. M.; Pennsylvania Consistory, A. A. S. R.; and Syria Temple, A. A. O. N. M. S. He was a member of Christ Methodist Episcopal Church and served on its Board of Trustees; for some years he was its Secretary, and was a member of the Board at the time of his death. He was a member of the Engineers Society of Western Pennsylvania from 1893.

To those who knew him best he was a kindly man of great personal charm and of the most lovable characteristics. He was regarded by all who knew him as an able and conscientious worker and a delightful friend. He was a man of clear vision and of the staunchest integrity.

Mr. Urquhart was elected an Associate Member of the American Society of Civil Engineers on May 6, 1891.

HAROLD EMIL VON BERGEN, Assoc. M. Am. Soc. C. E.[1]

DIED FEBRUARY 28, 1940

Harold Emil von Bergen was born in Petersburg, Nebr., on July 7, 1904, the son of J. F. and Anna (Schultz) von Bergen.

He received his elementary and high school education in the public schools of Nebraska. After graduation from high school, in 1924, he went to California and worked for a year in pipe-line maintenance with the Pacific Gasoline Company in Taft. In 1925, Mr. von Bergen entered Oregon State College, at Corvallis, Ore., and was graduated in the class of 1929, with the degree of Bachelor of Science in Civil Engineering. He was affiliated with Theta Chi social fraternity in college.

While attending college, he worked each summer with the City of Klamath Falls and the California-Oregon Power Company in Oregon and the Iowa-Nebraska Power Company in Nebraska in order to gain experience in surveying and construction. After graduation from Oregon State College, he was employed for nine months by the Reclamation Commission of the State of Oregon on drafting and dam analysis.

Mr. von Bergen returned to California in March, 1930, and accepted a position as junior engineer of hydraulic structure design with the Division of Water Resources, Department of Public Works, State of California, in connection with the administration of the "Act Governing the Supervision of Dams," enacted in 1929.

He left the Division of Water Resources in July, 1933, and took a position with the U. S. Forest Service at Quincy, Calif., as chief of a surveying party. He returned to the Division of Water Resources in March, 1935, as water master on the Pit River stream system.

Mr. von Bergen was particularly well qualified to make involved structural and hydraulic analyses, as he possessed an outstanding and rare power of independent and logical thinking. He was the author or co-author of several papers[2, 3] and discussions[4, 5] which have appeared in Society publications.

---

[1] Memoir prepared by a Committee of the Sacramento Section consisting of Gordon Zander, M. Am. Soc. C. E., Chairman, and T. R. Simpson and I. M. Nelidov, Assoc. Members, Am. Soc. C. E.

His geniality, personal charm, and outstanding ability as a pianist made him an indispensable participator in the programs of the Rotary Club at Alturas, Calif.; the Sacramento (Calif.) Section of the Society; and other musical and theatrical events in Sacramento. He was a man of strong physique and gloried in the outdoor functions of his work. His untimely and accidental death by drowning occurred in the line of duty when a boat, from which he was making a measurement of emergency flood flow in the Sacramento by-pass, capsized.

Mr. von Bergen was married in Reno, Nev., on April 1, 1934, to Frances Evelyn Rooney of Sacramento. He is survived by his widow and his daughter, Della Ann von Bergen; his mother, Anna S. von Bergen; five sisters, Alice, Katherine (Mrs. Albert Ohlman), Grace (Mrs. Otto Jensen), Elizabeth (Mrs. True Phillips), and Clara (Mrs. Ed Meyers); and two brothers, Arthur and Max.

Mr. von Bergen was elected a Junior of the American Society of Civil Engineers on July 14, 1930, and an Associate Member on December 14, 1936.

RALPH WATTS WARDWELL, Assoc. M. Am. Soc. C. E.[1]

Died June 28, 1939

Ralph Watts Wardwell was born in San Francisco, Calif., on February 26, 1879, the son of Atwater Murdock and Louise (Watts) Wardwell. He was educated in the public schools of Berkeley, Calif., and the University of California, in the same city, where he pursued a course in Civil Engineering. During his school years he took an energetic part in all activities, and was a member of Delta Upsilon fraternity.

His first professional work was as assistant engineer, U. S. Reclamation Service, on the Truckee-Carson Project in Nevada, and the Klamath Project in Oregon. This work included the location and construction of irrigation and drainage canals with accompanying tunnels, diversion dams, and headworks, and covered the period from July, 1904, to January, 1908. Through June of the latter year he was instrumentman and dredge inspector for the Southern Pacific Railroad Company on construction work in the vicinity of San Francisco Bay. Then followed a year as assistant engineer with Viele, Blackwell and Buck, building a timber rock-filled dam on the North Fork of the Feather River for the Great Western Power Company, and preparing maps, plans, and

---

[2] "Graphical Analysis of Stress in a Reinforced Concrete Section Under an Eccentric Load," by Harold E. von Bergen, *Civil Engineering*, February, 1933, p. 93.

[3] "Earthquake Stresses in an Arch Dam," by Ivan M. Nelidov and Harold E. von Bergen, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 133.

[4] *Loc. cit.*, Vol. 98 (1933), p. 1017.

[5] *Loc. cit.*, Vol. 102 (1937), p. 661.

[1] Memoir prepared by N. R. McKay, Birmingham, Ala.

estimates for a proposed power development on the North Slope of Pikes Peak in Colorado.

Mr. Wardwell's work for the Power Company attracted the attention of the Colorado Springs Heat, Light and Power Company, a public utility serving the cities of Colorado Springs, Colorado City, and Manitou, Colo., with electricity, gas, and steam heat, and in April, 1909, he accepted the position of assistant superintendent. Soon he was advanced to chief engineer, which position he held until March, 1914, when he resigned to become chief engineer in charge of the Geological Department and Water Supply System for the Fuel Oil Department of the Atchison, Topeka and Santa Fe Railway Company.

In May, 1917, he left the railway company to become chief engineer of the Western Water Company of Taft, Calif., and in October of the same year he was made general manager. This company served industrial and domestic consumers in the Midway-Sunset Oil Fields, Kern County, California.

Meanwhile, the United States entered the World War, and the call for capable engineer officers became more and more insistent; so in October, 1918, Mr. Wardwell was commissioned a Captain in the U. S. Engineers. He trained at Camp Humphreys, Virginia (later known as Ft. Belvoir), and for a time was detailed on special duty with the Coast Artillery. Receiving his honorable discharge in October, 1919, he was employed by the Bureau of Internal Revenue in the Oil and Gas Valuation Section. He was valuation engineer until August, 1921, and was then advanced to the office of Chief of California and Foreign Sub-Section. This work consisted of the estimation of oil and gas reserves, and the appraisal of oil and gas producing properties for income tax purposes. In February, 1923, he left the Bureau and became consulting engineer for a group of clients in the presentation and handling of engineering matters before the various government departments.

In February, 1926, Mr. Wardwell moved to Florida where he spent twelve years, the time divided between various engineering enterprises and fruit growing. As superintendent of the Coral Gables Construction Company he was engaged in the design and construction of the water and electric systems of that community and later as local manager for the Florida Power and Light Company. The years from 1931 to April, 1936, were spent in the employ of the U. S. Engineers, War Department, as civilian engineer on the Caloosahatchee River, Lake Okeechobee Drainage Project. In March, 1938, he was called to assist in the survey of the Passaic River Watershed. He took charge of the section on flood damage and reservoir valuations and organized the work of compiling data and collecting hydrologic data. This was his last work. Returning to his home in Providence, R. I., on the evening of June 27, 1939, he was taken ill and died very suddenly soon after midnight. His body was cremated and the ashes buried in Arlington National Cemetery, Virginia.

He is survived by his widow, the former Ruth Letitia Halpenny of Washington, D. C., and three children by a former marriage, Ralph Watts Wardwell, Jr., Robert S. Wardwell, and Louise Wardwell Spence.

Outside his professional activities, Mr. Wardwell experimented with candid camera photography; golf was his diversion; and during his years in Florida he was particularly interested in, and devoted time and attention to, Sea Scouting for boys.

Ralph Watts Wardwell was one of the vast army of engineers who never experience wide recognition—those who work with no idea of display, yet leave their mark on each structure they build. Drawing from a wide experience with fundamentals, Mr. Wardwell solved the problems he encountered by careful, thorough analysis, and the application of sound engineering principles—no ostentation, but persevering, diligent efficiency. One of a team, sometimes the leader, sometimes a wheel horse, he always pulled his share of the load—appreciated and respected by his fellows.

Mr. Wardwell was elected an Associate Member of the American Society of Civil Engineers on May 6, 1914.

WILLIAM TIBBITTS WEBB, Assoc. M. Am. Soc. C. E.[1]

DIED JUNE 13, 1939

William Tibbitts Webb was born in Cambridge, Mass., on August 13, 1879. He was the son of William W. and Emeline D. Webb, and obtained his early education in Providence, R. I. From 1898 to 1899, he was a special student at Brown University, in Providence, and later took special extension courses at Columbia University in New York, N. Y., in 1909 and 1910.

From August, 1896, to 1902 he was a student engineer, transitman, and inspector in the office of the city engineer at Providence. In December, 1902, he accepted a position as transitman on location on the Norfolk and Western Railway in Virginia and West Virginia, and remained at this work until October, 1903. From September, 1904, to December, 1905, he was chief inspector in charge of masonry work on the new sewerage system and treatment plant for the town of Dansville, N. Y., leaving this work to go with Westinghouse, Church, Kerr Company, on the construction of the Pennsylvania Terminal in New York. He worked here for some months as inspector, and, until December, 1907, as chief of field party. In this latter capacity he was in charge of the excavation, foundations, and steel erection for the Eighth Avenue Viaduct, and of the location and erection of the power plant, reinforced concrete tunnels, and a drainage system.

On completion of the work on the terminal, he entered the Civilian Engineering Department of the U. S. Army. From January, 1908, until May, 1911, he was engaged as a civil engineer and draftsman at the U. S. Military Academy at West Point, N. Y., where he made the necessary topographical surveys and maps for the location of new buildings, highways, and retaining walls, and also designed a new drainage and sewerage system for the north end of the academy. The remainder of his time on this assignment was occupied as structural engineer on foundations, design, and checking of steel and reinforced concrete construction of the chapel, gymnasium, guard house, and riding hall.

---

[1] Memoir prepared by James M. Bell, M. Am. Soc. C. E.

In May, 1911, he was appointed engineer to the quartermaster at West Point, and in this capacity had charge of all surveys and engineering in connection with the enlargement of the Academy. This included the new East Academic Building constructed at that time.

On completion of this work, in May, 1913, and until the end of that year, he was again with the Corps of Engineers of the U. S. Army, principally engaged in specifications and similar work. Toward the latter part of 1913 he was sent to San Juan, Puerto Rico, for the Army Engineers. There he made a survey for the establishment of pierhead lines and extensive dredging work. He also completed an inspection of other Puerto Rican harbors and had the supervision of the removal of certain wrecks along the coast.

In March, 1916, he left the Army Engineering Corps to go to J. G. White and Company as engineer on the installation of a water supply and distribution system for the cities of Remedios and Caibarién, Cuba. This work completed in September, 1916, he took a position with the West Indian Sugar Finance Corporation in charge of the erection of a large sugar mill at Alto Cedro, Cuba. This work was completed in March, 1917, and until January, 1918, Mr. Webb remained with the same company as resident engineer at San Pedro de Macorís, Dominican Republic. Here he built considerable railway extensions and was in charge of the maintenance of all railroads and structures of this group; also, he acted as consulting engineer for the City of San Pedro de Macorís in the extension of their drainage and sewerage system.

From January, 1918, to May, 1918, Mr. Webb was with the Public Works Department of the Dominican Republic as designing engineer, and in this capacity he designed a reinforced concrete dock for San Pedro de Macorís and four large reinforced concrete bridges for the 130-mile main highway across the island.

In July, 1918, he returned to the United States and entered the Army with the rank of Captain—Quartermaster's Corps. He worked in the Construction Department as cost engineer on the construction of a picric acid plant at Brunswick, Ga. While in this post he had considerable incidental work in connection with the design and layout of railroad, telephone, drainage, and water supply systems, the latter part of his time being principally occupied in the disposal of salvage material after the Armistice.

On giving up his commission, he was transferred to the Reserve Corps, and in July, 1919, went to Brazil, where he spent the remainder of his life. Here he was first engaged as representative for Livermore, Rojas Company, exporters, New York, in developing an organization for the sale of railroad construction machinery and supplies.

From August, 1922, to February, 1927, he was manufacturing agent for many manufacturers of machinery and construction equipment—principally, Koehring Mixers, C. L. Best Tractors, Russel Road Machinery, International Trucks, Magor Car Corporation, and Dupont Explosives. The latter representation occupied most of his time, as up to that date all explosives were imported and the representative of these firms was of necessity in constant contact with all of the major construction work done in the country.

In February, 1927, Mr. Webb gave up his representations of these manufacturers, and took a position as purchasing agent for the Emprezas Electricas Brasileiras, the Brazilian Subsidiary of the American and Foreign Power Company who, in the latter part of 1926, began to purchase many electrical operating properties in Brazil and shortly expanded until they owned public service operating properties in almost every state of the country. In March, 1929, Mr. Webb was put in charge of the Land Title Investigation of the same organization, a position which he occupied until his resignation in 1931.

From that date until his death, Mr. Webb was principally occupied in enterprises of interest to the American Colony of Rio de Janeiro, Brazil. He was a charter member of the Gavea Golf and Country Club, and was to no small degree responsible for the successful development and expansion of that enterprise. He was an ardent golfer and to the day of his death was one of the most regular patrons of the club, his broad engineering knowledge and experience having been freely utilized by the club on many occasions in working out their problems in connection with roads, water supply, etc., not to mention the preparation and maintenance of the course itself.

He was also always interested in educational matters and was very helpful in the consultations and work which have been done for the establishment of an American school in the community. He was a member of the Rio de Janeiro American Chamber of Commerce, and for several years was a director of it.

In 1922 he became a member of the committee headed by D. C. Collier, U. S. Commissioner to the Brazilian Centenary Exposition in Rio de Janeiro in 1922.

Mr. Webb was married on April 6, 1910, to Mary Washbun Murtha of New York, who survives him. There were no children, and his only other surviving relatives are two nieces, Mrs. Allan H. Chalmers, wife of the Rector of the Broadway Tabernacle Church, New York, and Mrs. Dudley Shannon Mackenzie, of Washington, D. C.

Mr. Webb was highly esteemed by all those who had personal contact with him, and his integrity and fairness in business dealings made him particularly admired. His death from acute indigestion in June of this year came as a great shock to all of his friends.

Mr. Webb was elected an Associate Member of the American Society of Civil Engineers on May 7, 1913.

NATHAN DAVIS WHITMAN, Assoc. M. Am. Soc. C. E.[1]

DIED SEPTEMBER 29, 1939

Nathan Davis Whitman was born in Boston, Mass., on March 23, 1878, the son of Nathan Davis Whitman and Helen (Mitchell) Orr. His paternal ancestry is coincident with the staunch history of New England and dates back to John Whitman, one of the earliest settlers in Weymouth, Mass. (1638). The Orr family, too, was prominent in New England, having established the first triphammer shop in the area, which shop supplied many cannons and muskets to the American forces in the Revolutionary War.

---

[1] Memoir prepared by Ernest F. Bent, Esq., Am. Concrete and Steel Pipe Co., Los Angeles, Calif.

Mr. Whitman was educated in the public schools of Boston. He then attended the Massachusetts Institute of Technology, at Cambridge, Mass., for one year, transferring at that time to the Massachusetts State College at Amherst, Mass. He received a degree of Bachelor of Science from the latter college in 1901. During this period, he spent his summers as draftsman in various offices in Boston and was an instructor in drawing at college.

After graduation, Mr. Whitman was employed by George S. Pierson, M. Am. Soc. C. E., consulting engineer of Kalamazoo, Mich., becoming his assistant engineer. His experience in this office was varied and formed an excellent background for all his subsequent work. Here he worked on the layout and design of sewer systems, water supply developments, substructure design and construction for bridges, and sewage disposal plants. After finishing the sewage disposal plant for Jackson, Mich., he left Mr. Pierson in 1905 to become engineer for the Reinforced Concrete Pipe Company of Michigan, at Jackson.

The reinforced-concrete pipe industry in the United States was then in the formative period and furnished Mr. Whitman an opportunity to grow with, and contribute to, a new field. To this end, he was more than successful, attaining an unquestioned national position as an engineering authority on concrete pipe. He developed several types of joints, improvements in form design, methods of reinforcing, and numerous machines and devices essential to the manufacture of pipe. His study and analyses of the problems of pipe production contributed much to the achievement of high quality pipe which is standard today.

In 1907 Mr. Whitman was transferred to Los Angeles, Calif., where the Reinforced Concrete Pipe Company of Los Angeles was formed, and for this company, as engineer, he was in charge of all engineering and design of reinforced-concrete pipe manufactured for water-pressure lines, sewers, and drains.

In 1913, Mr. Whitman returned to the East to continue his work with the Reinforced Concrete Pipe Company of Chicago, Ill., which company he served as secretary, director, and engineer until 1919.

For the next several years, Mr. Whitman left professional engineering to engage in sales work in San Francisco, Calif., and northern California where he conducted his own business.

In 1923 he returned to his chosen work in the concrete pipe industry, becoming chief engineer for the Bent Concrete Pipe Company in Los Angeles. He retained this position when this company merged with the Western Concrete Pipe Company to form the American Concrete and Steel Pipe Company, and was active until a short time before his death. During this period he supervised the design and construction of many large pipe lines in the Western States for water supply, irrigation, and sanitation.

In 1927, his company was the first in the United States to use, on a large commercial basis, the centrifugal process for the manufacture of reinforced-concrete pipe. The advantages of this superior pipe for water-pressure lines became apparent at once, and in the many developments necessary, Mr. Whitman's skill and knowledge as a pipe engineer played an important part in the design, construction, and successful performance of many such lines. Among these were a 30-mile domestic water supply line for Phoenix, Ariz., in 1931, and important lines for the California Water Service Company in the San Francisco Bay area in 1929, and for the La Mesa, Lemon Grove and Spring Valley Irrigation District near San Diego, Calif., in 1934.

Later his company undertook the manufacture of cylindrical reinforced-concrete pipe for pressure lines of high head and large diameter. Here again Mr. Whitman's abilities as the company's chief engineer were indispensable in the intricate problems of the design of such structures, which involve pressure, surge, contraction and expansion, load bearing, thrust, etc. Notable among these lines were 5 miles of 62-in. pipe (operating head 400 ft) for the Hetch Hetchy Water Supply for San Francisco in 1934, and several sections aggregating 15 miles for the City of Tacoma, Wash., from 1934 to 1939. Mr. Whitman's ability, experience, and contribution culminated in his cooperation with the designing engineers of the Metropolitan Water District of Southern California in the manufacture and installation in 1937 of 17 miles of 12-ft 8-in. and 11-ft 8-in. diameter reinforced-concrete pipe (heads to 230 ft) for the Colorado River Aqueduct, this being the largest pipe line of its kind constructed to that date.

So outstanding had Mr. Whitman's integrity, experience, and engineering ability become through these many years in this industry that often he was called in by consulting and public body engineers as an expert for their designs and to make recommendations.

Mr. Whitman was regarded with esteem and affection by his office colleagues, plant superintendents, foremen, and shop workers. His quiet humor, his wealth of general knowledge, and his friendly personality endeared him to all with whom he came in contact. His work was his hobby and he kept abreast, if not ahead, of the progress made in the pipe industry and engineering; nor did the fact that he had chosen a special field of engineering limit his viewpoint for he maintained an active interest in and had a profound knowledge of all engineering.

Mr. Whitman was married to Rose Marie Dickert of Milwaukee, Wis., on May 7, 1908, in Los Angeles. He is survived by his widow; a son, Nathan Davis Whitman, Jr., Assoc. M. Am. Soc. C. E.; and a grandson, Nathan Davis Whitman, IV.

Mr. Whitman was elected an Associate Member of the American Society of Civil Engineers on December 5, 1906.

JOHN THAD WHITNEY, Assoc. M. Am. Soc. C. E.[1]

DIED SEPTEMBER 30, 1938

John Thad Whitney was born on September 13, 1891, at Granville, Vt. He was the only child of Horace Harvey Whitney, who was descended from the early English settlers of New England, and Louise (Kerr) Whitney, whose Irish ancestors came to the United States in 1852.

His father, who was a merchant, operated a general country store at Northfield Falls, Vt. Both of John's parents had the outstanding qualities of sterling character, lovable personality and excellent judgment, and were virtually bulwarks of stability in their community.

Young John was reared in Northfield, Vt., where he received his elementary and high school education, graduating in 1909. During his early years, the excellent environment of clean living, honesty, and religious affiliation afforded him by his parents and New England tradition were to have a great effect on shaping his life.

He was graduated from Norwich University, at Northfield, with the degree of Bachelor of Science in Civil Engineering in June, 1913. Norwich University is a State-endowed military school and at the time that John Whitney attended, the students, in so far as military training was concerned, were members of the First Cavalry, Vermont National Guard. Throughout his life he maintained a sincere interest in his University. In later years he completed three graduate courses in advanced mechanics and mathematics at Carnegie Institute of Technology at Pittsburgh, Pa. At the time of his death he had nearly completed two theses for graduate degrees at Norwich University as follows: "The Transmission of Vertical Pressures Through Earth" and "Correlation Between Less-Than-One-Day Rainfalls and One-Day Rainfalls in the Upper Ohio River Basin."

In August, 1913, Mr. Whitney became connected with the U. S. War Department and served for two years as Inspector on the construction of Ohio River locks and dams in the Wheeling, W. Va., Engineer District. This was followed by two years as Field Engineer with the H. Koppers Company, of Pittsburgh, on the construction of a by-product coke plant at Follansbee, W. Va.

In September, 1917, Mr. Whitney entered the Officers' Training Camp, Fort Benjamin Harrison, Indiana. He was commissioned Second Lieutenant, Field Artillery, O. R. C., on November 27, 1917, and assigned to the 89th Division. He served as a member of the American Expeditionary Forces from July 10, 1918, to July 6, 1919. During this period he was engaged principally in the transportation of artillery supplies in connection with the 2d Corps Artillery Park and participated in five major offensives as follows: Aisne Marne, St. Mihiel, Oise-Aisne, Meuse-Argonne, and Vesle-Sector. He was discharged on July 11, 1919, at Camp Dix, New Jersey, as a Second Lieutenant, Field Artillery.

---

[1] Memoir prepared by a Committee of the Pittsburgh Section consisting of Harry A. Hickman, M. Am. Soc. C. E., and Ely G. Fenton, Assoc. M. Am. Soc. C. E.

Upon return to civil life Mr. Whitney was connected with the Vermont State Engineer Department (later combined with the State Highway Department) as Assistant Engineer. In this connection, he served successively as Inspector and Resident Engineer on highway construction, in charge of bridge design, and as Assistant to the State Engineer. During the last nine months of this engagement he served as Office Engineer in charge of the main office.

In May, 1923, Mr. Whitney again entered the service of the Koppers (H. Koppers) Company, of Pittsburgh, with whom he remained for the next four years. One of these years he served as Field Engineer on the construction of a water-gas plant at Kokomo, Ind.; a by-product coal-gas plant at Fort Wayne, Ind.; and a liquid purification plant for gas at Milwaukee, Wis. During the remaining three years Mr. Whitney was in charge of all foundation and concrete design. Much of his time was devoted to the supervision of design and preparation of plans, specifications, and estimates for structures for the handling and storage of coal, coke, and ashes for by-product coke plants, and other plants constructed by the Koppers Company. Mr. Whitney's knowledge of the latest in engineering procedures—their thorough practical application—and his engineering foresight are indicated in that the specifications for such work, as prepared by Mr. Whitney, are today used by the Koppers Company, as standards.

His next engagement, from November, 1927, to December, 1931, was with the City of Pittsburgh, where he served as Designing Engineer in the Bureau of Bridges and Structures and Assistant Engineer with the City Transit Commission. In this period he was closely connected with the major municipal improvements and in responsible charge of much of the heavy structural design for such projects as the East Street and Aiken Avenue bridges, extension of Mount Washington Roadway and Bigelow Boulevard, and the repairs to several bridges. He also made the preliminary design and estimates for a 500-ft span, reinforced concrete, arch bridge for a proposed extension of the Boulevard of the Allies over Nine-Mile Run. In June, 1932, Mr. Whitney submitted designs and estimates for single-track and double-track sections for proposed municipal subways to the City Transit Commission of Pittsburgh.

From May, 1933, until his death Mr. Whitney was again employed by the U. S. War Department, being attached to the Pittsburgh Engineer District, where he was engaged on general construction, design of river structures, and hydraulic studies pertaining to flood control reservoirs.

John Thad Whitney was an excellent engineer, measured not only by the volume and variety of his technical knowledge or its practical application, but still more by his ability to work with others both as a subordinate and as a superior. He was a man of strong convictions; nevertheless, he conscientiously observed the rights and feelings of others. He was always so tremendously in earnest about everything that he undertook. He had another even more outstanding characteristic in that he lived "with malice toward none." There was nothing petty or mean about him. Few people have been less critical of their fellow beings. His disposition was that of the typical engineer, and his hopes, aspirations, failures, and successes strike a responsive chord with most of us.

John Whitney was a lover of animals, especially horses. He was deeply interested in outdoor life, good literature, music, and poetry. His more harsh commitments were often tempered at the end with some witticism or verse of poetry. He was an ardent student of current events. His personal library of many hundred volumes was very diversified and indicative of a sincere and widely read scholar. In accordance with his wish the technical part of Mr. Whitney's library was given to Norwich University.

He always took a profound interest in the younger engineer. He was willing to assist others in acquiring the engineering information which they desired. He was a virtual encyclopedia for many of his friends and working associates. He would have made an excellent teacher. He had a keen sense of humor, and a rare faculty of enjoying a good joke on himself as well as on the other fellow.

He was a registered engineer in the State of Pennsylvania, a member of the American Legion, and Theta Chi Fraternity. He had long been affiliated with the Methodist Episcopal Church and later became a member of the Mount Lebanon Presbyterian Church, Mount Lebanon, Pa.

Mr. Whitney died in Pittsburgh, Pa., on September 30, 1938. Interment was made in Elmwood Cemetery, in Northfield. He returned from whence he came, to the land he loved so well.

Mr. Whitney was married at Huntington, W. Va., on February 23, 1921, to Mary Wigal who survives. He is also survived by one daughter, Linda Louise Whitney.

Despite his conscientious absorption in his profession he was most devoted to his home and family. His sterling personal character is indicated in his true devotion as a son, husband, father, and friend. In passing he left voids in many lives, never to be filled. However, his memory will have a lasting effect on those who knew him well and now mourn one of earth's gentlemen.

John Thad Whitney was elected a Junior of the American Society of Civil Engineers on April 1, 1914, and an Associate Member on January 19, 1920.

HARRY McKEAN CONNER, Jun. Am. Soc. C. E.[1]

DIED SEPTEMBER 1, 1939

Harry McKean Conner was born at Oakfield, N. Y., on March 1, 1907. His father, H. McKean Conner, a widely known consulting mining engineer, was born at Mauch Chunk, Pa., on March 31, 1875, and died on December 1, 1919. He was a member of the American Institute of Mining and Metallurgical Engineers, a member of the Engineers Club of Northeastern Pennsylvania, and a prominent Mason. He collaborated as an associate editor with Robert Peele in preparing the "Mining Engineers' Handbook" and was responsible for the section dealing with the "Preparation and Coking of Bituminous Coal."

---

[1] Memoir prepared by John C. Buhrendorf and R. B. Strang, Board of Water Supply, Scarsdale, N. Y.

His mother, Sarah G. (Wagner) Conner, born at Newville, Pa., on January 10, 1874, was a daughter of State Senator Samuel C. Wagner and Laura Ege (Woodburn) Wagner. Samuel C. Wagner served as a Captain, during the Civil War, in the Third Pennsylvania Cavalry. Mrs. Conner died at Newville on April 18, 1940.

His uncle was Eli Taylor Conner of Scranton, Pa. Like Harry's father, he was a prominent mining engineer, serving as consulting engineer for the United States Bureau of Mines and as Chairman of the Coal Division of the American Institute of Mining and Metallurgical Engineers.

Harry's youth was spent mainly at Scranton, where he obtained his early education. He was graduated from Central High School, in 1925, having taken a scientific course. Immediately thereafter he secured his first regular employment, and for the thirteen years that followed he worked during the day on engineering projects and pursued engineering studies at night. Due to a change in employment in the summer of 1929 which brought him to New York, N. Y., he was afforded the opportunity of attending the School of Engineering of Columbia University. From this time until his enrollment for the Civil Engineering Degree course at New York University, in New York City, he took extension courses at Columbia pertaining to some of the phases of engineering that were pertinent to his work. Prior to leaving Scranton, he had studied various phases of coal mining as presented by the International Correspondence School. In September, 1931, he enrolled for a degree course in the College of Engineering of New York University. During his attendance at New York University he excelled in all his studies and his brilliant work was recognized by the University when it conferred upon him, in 1937, the degree of Bachelor of Science in Civil Engineering, *cum laude*.

Still in search of further technical knowledge, he enrolled in the Graduate School of the College of Engineering of New York University in September, 1938, for courses leading to a Master's degree in Sanitary Engineering.

On February 4, 1938, he obtained a license to practice Professional Engineering from the University of the State of New York which, previously, on May 11, 1933, had granted him a license to practice land surveying.

With the engineering background furnished by his father and uncle, there was never any question about his vocation in life, and his first practical engineering work started during a summer vacation, in 1924, when he was with a survey party on field work for the Hudson Coal Company, of Scranton, Pa., in connection with anthracite coal mining by the stripping method. After graduation from high school in 1925, he was again engaged by the same company in a similar capacity, and in addition made surveys in their underground mine workings. In September, 1927, he resigned from this firm to join the United Gas Improvement Contracting Company in charge of the surveying on the construction of a plant for the General Electric Company in Philadelphia, Pa. Upon the completion of this work in June, 1928, he became associated with W. W. Young, a consulting engineer of New York City, as an assistant engineer in charge of repairing a sewer system for the Town of Metuchen, N. J. When this project was finished he continued with Mr. Young as an assistant engineer, the next work being for the Westchester County Sanitary Sewer Commission on the preliminary surveys and the construction of sewer trunk lines in Westchester County and on the design of, and specifications for, two sewage disposal plants.

In 1929, he was appointed to the Board of Water Supply of the City of New York on a part of the construction of City Tunnel No. 2. This included the following procedure: A shaft was sunk through earth and rock to a depth of 500 ft, a reinforced concrete caisson being used for the first 100 ft through water-bearing material; then the rock shaft was sunk another 400 ft and lined with concrete; a tunnel, about two miles long, was driven and lined with concrete; the concrete lining of the shaft and tunnel was grouted; and the shaft operating equipment was installed for controlling the flow of water from the tunnel, through the shaft, and into the distribution system. Mr. Conner was in charge of a field survey party on this work until April, 1933, when the project was completed.

From November, 1933, to December, 1936, he was respectively an assistant engineer with the Civil Works Administration (CWA); a section engineer with the Department of Public Welfare; a senior engineer and a supervising engineer with the Works Progress Administration (WPA); and Chief of the Planning Section for the WPA in Bronx County, covering all phases of municipal engineering.

In December, 1936, he was recalled to the Board of Water Supply of New York City to assist in the construction of a portion of the new Delaware Aqueduct System. His first assignment on this project put him in charge of sinking a shaft, 450 ft deep, in rock, lining this shaft with concrete, and driving about 400 ft of tunnel in rock from the bottom of the shaft. In May, 1938, he was appointed assistant section engineer on another contract of the Board of Water Supply which consisted of driving 29,138 ft of deep rock tunnel. He was engaged on this work at the time of his death.

Along with his work during the day and his studies during the evening, Mr. Conner found time to be actively engaged in a wide variety of interests allied to the engineering profession. In 1929 he became a charter member of the New York State Sewage Works Association. While at New York University he was elected to membership in the honorary engineering society of Iota Alpha. On April 21, 1938, he was elected a member of the American Institute of Mining and Metallurgical Engineers. He also served as Secretary and Director of the Bronx County Chapter of Professional Engineers. Upon his graduation from New York University he was appointed the Alumni Member of the University Employment Committee, College of Engineering, and was very active on this committee.

With his varied activities, Harry Conner met a great number of people, and such was his personality that those who were associated with him, both in his work and in his outside interests, became his firm friends, all recognizing the rare combination of ability with the highest personal attributes. His untimely passing deprived the profession of one of its most promising engineers.

On June 27, 1928, Mr. Conner was married to Louise Sanderson Fuller of Scranton, daughter of Ray W. Fuller and Grace (Sanderson) Fuller, and their married life was extremely happy.

Like many men who devote most of their time to solving difficult problems, Mr. Conner derived great pleasure from his camp at Lake Mombasha, New York, where during week ends and vacation periods he partook of the social life of the small community, and attended to a vegetable and flower garden with the aid of his wife. Here he also did odd jobs of carpentry work incidental to making a comfortable lodge.

Harry Conner is survived by his widow and two brothers, William Young Conner and George Wagner Conner.

Mr. Conner was elected a Junior of the American Society of Civil Engineers on November 23, 1931.

FRANKLIN MECHTLEY FELKER, Jun. Am. Soc. C. E.[1]

Died May 29, 1939

Franklin Mechtley Felker was born at Lewistown, Pa., on October 17, 1908, the son of Frank F. and Mary (Mechtley) Felker. He attended the Lewistown grade school and high school. In 1927 he entered the University of Pittsburgh, at Pittsburgh, Pa., to study Civil Engineering, and was graduated in 1931 with the degree of Bachelor of Science. He was a member of Sigma Pi Fraternity. During his college days he worked as transitman for the Standard Steel Company of Burnham, Pa., and worked again for the same firm after graduation. His work as transitman included the laying out of a power plant and annealing furnace, and the mapping of the works and its railroad.

In 1933 he was appointed a junior engineer in the U. S. Forest Service and was assigned to Civilian Conservation Corps (CCC) Camp S58. Later he was transferred to Camp S57. As the man responsible for the engineering work required at these camps, his duties ranged from boundary surveys for the state forest map to construction of camp water supplies, roads, and bridges. During this time he not only endeared himself to the men under him, but impressed the district forester with his efficiency and fitness for his job. In 1936 he was made truck trail locator for twenty of the CCC camps in northern Pennsylvania, a position of considerable responsibility. The following year, when the state was divided into five districts, Mr. Felker was made district engineer of the northern district, with headquarters at Galeton, Pa.

---

[1] Memoir prepared by Charles F. Ruff, M. Am. Soc. C. E.

In April, 1933, he was married to Sarah E. Alexander in Warrenton, Va. Like many another young engineer, Mr. Felker was never able to establish a permanent home but followed the job from Paradise Furnace, Pa., near Camp S57, to Galeton.

In December, 1937, he left the Forest Service to join the newly established Flood Control Bureau of the State of Pennsylvania. Here he was assigned to the hydrologic section, the particular rôle of which was to estimate how large and how often floods might be expected. Besides making valuable contributions to a general study of floods in Pennsylvania, he was also actively engaged on the flood control project for York, Pa., and several other cities in the Susquehanna Basin. Mr. Felker was well liked at the Bureau, both for his unassuming good nature and the quiet efficiency with which he tackled the work in hand.

When the Flood Control Bureau was closed early in 1939, he secured a position as junior engineer with the U. S. Engineer Corps, at Pittsburgh, doing similar work on the Pittsburgh Flood Control Project. He had had this position scarcely a month when a severe cold developed into pneumonia. A week later, on May 29, 1939, he died at Elizabeth Magee Hospital, in Pittsburgh.

The sudden death of this promising engineer, who, despite starting his career during the depression, had managed to keep himself busy and useful, was a great shock to all who knew him. He is survived by his wife and by his mother and father.

Mr. Felker was elected a Junior of the American Society of Civil Engineers on October 26, 1931.

GLENN WILLIAM HOLMES, Jun. Am. Soc. C. E.[1]

Died January 19, 1940

A descendant of a long line of pioneer teachers, engineers, and statesmen, Glenn William Holmes was born in Syracuse, N. Y., on February 1, 1910. His father, Glenn D. Holmes, M. Am. Soc. C. E., was sanitary engineer for the City of Syracuse, and for many years has been engaged in consulting practice in that city. His great-grandfather, Joseph Holmes, was one of the pioneer settlers in western New York; and his grandfather, Joseph W. Holmes, a student of mathematics and astronomy, was the inventor of the Solar Theodolite, the basic principles of which are to be found in the present-day instruments. His mother, Adah (Brett) Holmes, traces her ancestry from the House de la Brette, which produced kings of Navarre, and fought for William the Conqueror in 1066 A.D. Madam Brett, daughter of Francis Rombouts, mayor of the City of New York, N. Y., in 1679, was the first white woman to establish her home (in 1709) in the highlands of the Hudson River. This manor, known as " The Crary House," was later owned and occupied by Robert Fulton Crary, eldest grandson of Robert Fulton and his wife, a descendant of Madam Brett.

---

[1] Memoir prepared by C. E. Ramser, C. S. Jarvis, Members, Am. Soc. C. E., and W. D. Potter, Assoc. M. Am. Soc C. E.

The early boyhood days of Glenn Holmes were spent in Syracuse where he was graduated as an honor student of Central High School. He entered the University of Michigan, at Ann Arbor, Mich., where he pursued an engineering course and received the degree of Bachelor of Science in Civil Engineering in 1932.

With the exception of a summer spent as assistant sanitary inspector for the Lake George Association and a short engagement as assistant engineer for the Onondaga County Sanitary Sewer and Public Works Commission, the first three years of Mr. Holmes' professional career were spent as assistant engineer in his father's office. He assisted in the design and operation of the Newark, New York, Sewage Treatment Plant and conducted various scientific investigations and research, probably the most important of which was his adaptation of the principles of light absorption for the determination of suspended solids in sewage effluent.[2]

In 1934, Mr. Holmes was employed by the firm of Metcalf and Eddy as assistant engineer. He assisted in the preparation of preliminary reports on sewage treatment for various municipalities and more especially in an investigation of the operation of the North Toronto Sewage Treatment Works for the purpose of improving treatment efficiency.

Mr. Holmes resigned from Metcalf and Eddy to take a position in the government service as hydrologist for the newly created Soil Conservation Service, United States Department of Agriculture. While stationed at the operations project of the Service at Bath, N. Y., he conducted the studies reported in an unpublished manuscript entitled " Hydrologic Studies of the July 1935 Flood in South Central New York State." This manuscript later was released to the United States Geological Survey and became the basis for their *Water Supply Paper No. 773–E.*

In November, 1935, he transferred from the Bath project to take a position as assistant, and later as associate hydraulic engineer, on the Washington staff of the Hydrologic Division of the Service. Besides preparing critical reviews of such reports as the State Soil Conservation Programs and the National Resources Committee's Drainage Basin Reports, he was engaged at the time of his death in making a correlation and analysis of hydrologic data collected by the United States Geological Survey on eight of the Demonstration Projects of the Soil Conservation Service.

To all of Mr. Holmes' many friends his philosophy of life was an outstanding characteristic. His fairness in all dealings with his fellow men was an exemplification of the Golden Rule, and his relations as husband and father were always those of the cheerful, loving companion, patient teacher, and guide. In spite of physical handicaps suffered in his early youth, he lived life to its fullest extent up to the very moment of his sudden and untimely death.

---

[2] "Determination of Suspended Solids in Sewage by Light Absorption," by Glenn W. Holmes, *Sewage Works Journal,* July, 1935, pp. 642–657.

On December 22, 1934, he was married to Elma Lyons, who, with two children, Martha and Barbara Jean, survives him. He is also survived by his father and mother, and by his brother, Brett D. Holmes.

Mr. Holmes was a member of the Syracuse South Presbyterian Church. He was also a member of the American Geophysical Union.

Mr. Holmes was elected a Junior of the American Society of Civil Engineers on August 16, 1937.

JOHN MONKS, Affiliate Am. Soc. C. E.[1]

Died October 17, 1939

John Monks was born in New York, N. Y., on October 16, 1864, the son of John Monks and Mary Ann (O'Leary) Monks. He received his education at De LaSalle Institute and Seton Hall College, at South Orange, N. J. Upon leaving college, he entered his father's contracting business which had been established by his grandfather in 1863.

The firm had been largely engaged in building docks about New York harbor, but when his father retired from active participation in the business to accept an appointment from Mayor Strong as dock commissioner, Mr. Monks was elected President of John Monks and Sons and proceeded to extend the firm's activities in other directions. Under the title of General Contractors, the firm specialized in harbor development, and heavy and pneumatic foundations. In record time, the firm built the piers for the landing of the troops of the Spanish-American War, at Montauk Point, Long Island, N. Y., and, upon the invitation of General Alger, then Secretary of War, Mr. Monks visited and advised him regarding the development of the Harbor at San Juan, Puerto Rico.

Under Mr. Monks' direction, the firm also built numerous piers and bulkheads along the Hudson and East rivers, as well as extensive work of the same character in the West Indies. Among the many outstanding structures built by the firm under his régime were the deep pneumatic foundations for the Hecker-Jones-Jewel Mill, pneumatic foundation for the Cincinnati Water Works, foundations for the West Street Building, pneumatic foundations for the Baltimore and Ohio Railroad Bridge over the Susquehanna River at Havre de Grace, Md., and others.

Mr. Monks was also a director in the Old Maine Steamship Line and president of the New York and Baltimore Steamship Company. As a member of the Chamber of Commerce of the State of New York, he took an active interest in civic affairs.

---

[1] Memoir prepared by Emott D. Buel, M. Am. Soc. C. E.

Because of ill health, he was forced, in 1905, to relinquish active participation in the contracting business. He entrusted the business to his brothers, but retained his position as president of the firm and acted in an advisory capacity in all matters of importance. In the meantime, he traveled extensively in Europe and South America and, in 1916, believing his health sufficiently restored, resumed his former duties.

Unfortunately, in late years, his health failed him completely, and in 1932 he was compelled to retire from the firm, his clubs, and all social activities. He spent the last few years of his life on his country estate at East Hampton, Long Island, N. Y., where he had collected a very fine library. He found one of his greatest pleasures among his books.

He was a member of the Metropolitan Club of Washington, D. C., the American Geographic Society, and the National Audubon Society. Until his retirement, he had been a member of the Racquet and Tennis Club, the Manhattan Club, the Grolier Club, the Downtown Association, and the Long Island Country Club. Mr. Monks was greatly beloved by his friends and known as a man of high ideals and unswerving integrity.

He was married to Alice Parsons in 1903, and is survived by his son, John Monks, Jr., and three brothers, Richard A., Robert, and Jerome Monks.

On the evening of October 17, 1939, Mr. Monks suffered a heart attack and passed away peacefully.

Mr. Monks was elected an Affiliate of the American Society of Civil Engineers on May 31, 1898.

LOUIS ALOYSIUS SLATTERY, Affiliate Am. Soc. C. E.[1]

Died February 24, 1939

Louis Aloysius Slattery was born in Chatham, N. Y., on May 12, 1887, the son of John Slattery and Mary (Gilmore) Slattery. He came to New York, N. Y., when still a boy and received his early education in the public schools of Manhattan.

In 1904 he entered the employment of the J. C. Rodgers Contracting Company as a timekeeper and material checker. Later he was connected with the contracting organization of Rogers and Hagerty, Incorporated, in the same capacities. During his period of employment with the latter company, he advanced to the position of assistant superintendent.

In September, 1913, he accepted the position of assistant superintendent of construction with the U. S. Realty and Improvement Company on the construction of a section of the Brooklyn-Manhattan Transit subway along Broadway, from West 28th Street to West 38th Street, New York.

During the World War Mr. Slattery was superintendent of construction with the Federal Shipbuilding Company at Kearney, N. J. In this position he did outstanding work in the erection of shipways, docks, machine foundations, and industrial buildings incidental to a large shipbuilding yard.

---

[1] Memoir prepared by Walter Caccia, Assoc. M. Am. Soc. C. E.

From January, 1920, to January, 1922, he was engaged by the Frederick Snare Corporation as superintendent of construction in the building of the Hunters Brook, Turkey Mountain, and Indian Brook siphons for the New York Board of Water Supply. Following this assignment he returned to New York and took charge of foundations, fill, and grading of the Interborough Rapid Transit subway storage yard on Westchester Avenue, Borough of the Bronx, for the P. T. Cox Company, Inc. He had essentially completed this work when the D. C. Serber Company, Inc., of New York was awarded the contract for the erection of the car barns, storage yard facilities, and steel elevated railroad structure in the same yard. Mr. Slattery took charge of this work for the Serber Company and completed it successfully.

In July, 1925, Mr. Slattery and Martin J. Lyons organized their own contracting organization under the name of Lyons-Slattery Company, Inc. As construction manager, Mr. Slattery took an active part in all field operations, in addition to his duties as secretary and treasurer of this organization.

Some of the outstanding contracts prosecuted by the Lyons-Slattery Company, Inc., included the bridges and decked areas of Fordham Plaza over the Harlem Division tracks of the New York Central Railroad in the Borough of the Bronx; the elimination of the New York Central grade crossing at Dyckman Street, Borough of Manhattan; and the elimination of a grade crossing on the Putnam Division of the New York Central Railroad at Nepperhan Heights, N. Y. Following these contracts, the Lyons-Slattery Company was awarded the contract for Section 5, Route 106, of the Grand Boulevard and Concourse Rapid Transit subway, extending from 183d Street to Kingsbridge Road in the Borough of the Bronx. This contract was completed successfully and considerable credit must be given to Mr. Slattery for the expeditious manner in which the work was done.

In May, 1931, Mr. Slattery participated in the formation of the Hudson River Stone Corporation with Mr. Lyons and the executives of the O'Brien Brothers, Inc., wholesale producers and distributers of sand and gravel in New York. As vice-president and general manager of this new organization, he was in responsible charge of the construction and operation of the company's quarry and crushing plant at Coldspring, N. Y., from the time of formation of the Company until the late fall of 1938, when a cardiac condition became too critical to permit him to continue his work.

Mr. Slattery possessed an unusual faculty in solving the various problems of construction work; he had a natural ability for leading men and selecting his staff executives. His personality drew his associates into a close relationship and encouraged them to give to their work the best they had to offer.

He died on February 24, 1939, in New York, and is survived by his wife, the former Maud Kamper.

Mr. Slattery was elected an Affiliate of the American Society of Civil Engineers on July 6, 1936.

GEORGE OLIVER TENNEY, Affiliate Am. Soc. C. E.[1]

Died November 6, 1938

George Oliver Tenney, the son of Charles B. and Abby (Lerned) Tenney, was born in Littleton, Mass., on September 11, 1861.

His long and brilliant career in engineering and construction began at the early age of sixteen when he was employed by the City Engineer of the City of Somerville, Mass. His early ability and hard study are attested by the fact that after three years in the City Engineer Department of Somerville, as Rodman and Inspector on Sewer Work, he was given a position, in 1880, as Assistant Engineer, by the New York, West Shore and Buffalo Railway Company. Mr. Tenney had remarked he was once the youngest member of a field party of which he was in charge. In 1883 he returned to the City of Somerville as Assistant Engineer, where he remained until 1886.

About this time the City of Decatur, Ala., became the center of what Mr. Tenney described as the "Alabama Boom." He accepted a position as Assistant Engineer on Sewer Construction for the City of Decatur in 1886 and remained there until 1892. It was here that he gave up his engineering practice to enter the contracting field on sewer construction in which he continued successfully in Decatur until the yellow fever epidemic, which caused the overnight abandonment of all construction activities and a general exodus from the city.

Continuing in the contracting field, he formed a partnership (Crass and Tenney) in 1892, and was engaged in the construction of the Chickamauga National Park, Chattanooga, Tenn., for the United States National Park Commission. On completion of this work, about 1895, he moved to Spartanburg, S. C., and engaged in the construction of dams. Among those constructed were six at the following points: Clifton, Greenville, Portman Shoals, and Neil Shoals, in South Carolina, and McAdenville and Harden in North Carolina.

He continued in this work until 1906, when the Atlantic Bitulithic Company was organized by Warren Brothers Company of Boston, Mass., for street and road construction in the South. Mr. Tenney was made President at its organization and so continued, for a third of a century, until his death. Under his direction many of the important streets in cities throughout the South Atlantic States have been constructed.

His honesty and fairness were recognized throughout the country, by both his fellow contractors and the officials for whom the work was performed. His contribution to the industry, in this respect, is priceless. His ability to organize was remarkable, as was the loyalty he received from his employees.

He is survived by his widow, Mrs. Minna L. Tenney, to whom he was married on February 2, 1929.

Mr. Tenney was elected a Junior of the American Society of Civil Engineers on September 7, 1887, and an Affiliate on February 5, 1901.

---

[1] Memoir prepared by M. J. Johnson, Pres., Atlantic Bitulithic Co., Washington, D. C.

TRANSACTIONS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

INDEX

VOLUME 105

1940

Subject Index, Page 2030

Author Index, Page 2041

Titles of papers are in quotation marks when given with the author's name.

VOLUME 105

SUBJECT INDEX

ACCIDENTS.

Stopping the increase in night accidents. 326.

ADDRESSES.

Presidential Address at the Annual Convention at Denver, Colorado, July 24, 1940. John P. Hogan. 1781.

ALUMINUM.

"Tests on Built-Up Columns of Structural Aluminum Alloys." Marshall Holt. (With Discussion.) 196.

AQUEDUCTS.

See **PIPE LINES.**

BEACHES.

See **SHORE PROTECTION.**

BORINGS.

Interpretation of borings. 455.

"Large Core Drills Aid Construction at Chickamauga Dam." James S. Lewis, Jr. (With Discussion.) 850.

BOUNDARIES.

Boundary location. 733.

BRACING.

"Wind Bracing in Steel Buildings." Final Report of Sub-Committee No. 31, Committee on Steel, of the Structural Division. 1713.

BRIDGES.

"Reconstruction of the Walpole-Bellows Falls Arch Bridge." H. E. Langley and Edward J. Ducey. 1675. Discussion: J. B. French, Lewis D. Rights, Conde B. McCullough, and H. M. Nelson. 1701.

BUILDINGS.

"Settlement Studies of Structures in Egypt." Gregory P. Tschebotareff. (With Discussion.) 919.

"Simplified Wind-Stress Analysis of Tall Buildings." Otto Gottschalk. (With Discussion.) 1019.

"Wind Bracing in Steel Buildings." Final Report of Sub-Committee No. 31, Committee on Steel, of the Structural Division. 1713.

"Wind Forces on a Tall Building." J. Charles Rathbun. (With Discussion.) 1.

BULKHEADS.

Design for a simple bulkhead. 885.

CHANNELS.

"Design of an Open-Channel Control Section." Karl R. Kennison. (With Discussion.) 744.

"Lateral Spillway Channels." Thomas R. Camp. (With Discussion.) 606.

COLUMNS.

Relative resistance values of building columns to unit lateral displacement. 1031.

Special treatment of nose pier. 863.

"Tests on Built-Up Columns of Structural Aluminum Alloys." Marshall Holt. 196. Discussion: Alfred S. Niles. 218.

CONSTRUCTION PLANT.

See **CONTRACTOR'S PLANT.**

CONTRACTOR'S PLANT.

"Reconstruction of the Walpole-Bellows Falls Arch Bridge." (With Discussion.) 1675.

CONTRACTS.

"The Risk of the Unexpected in Sub-Surface Construction Contracts." Oren Clive Herwitz. 454. Discussion: Alonzo J. Hammond, Frederick W. Newton, David A. Molitor, F. B. Marsh, Evan S. Martin, C. Maxwell Stanley, Francis J. Morgan, Frederick C. Zeigler, Lazarus White, T. Kennard Thomson, and Carroll A. Farwell. 476.

COSTS OF WORK.

Cost of main sewage pumping station and sewage treatment plant. 1622.

Cost of treatment works for Boston sewerage system. 1007.

DAMS.

"Engineering Geology Problems at Conchas Dam, New Mexico." Irving B. Crosby. 581. Discussion: H. L. Johnson. 600.

"Graphical Arch Analysis Applicable to Arch Dams." Carl H. Heilbron, Jr., and William H. Saylor. 765. Discussion: I. M. Nelidov. 790.

"Large Core Drills Aid Construction at Chickamauga Dam." James S. Lewis, Jr. 850. Discussion: J. B. Newsom, J. G. Tripp, James B. Hays, and O. N. Floyd. 865.

Trends in dam construction (earth, masonry, rock-fill) 1930–1939. 1744.

DOWEL BARS.

"Design of Dowels in Transverse Joints of Concrete Pavements." Bengt F. Friberg. (With Discussion.) 1076.

DRILLING.

"Large Core Drills Aid Construction at Chickamauga Dam." James S. Lewis, Jr. (With Discussion.) 850.

EARTH PRESSURES.

"Field Test of a Shale Foundation." August E. Niederhoff. (With Discussion.) 1519.

EARTHQUAKES.

"Earthquakes and Structures." Leander M. Hoskins and John D. Galloway. 269. Discussion: Homer M. Hadley, R. McC. Beanfield, R. S. Chew, Jacob J. Creskoff, Arthur C. Ruge, Walter L. Huber, A. A. Eremin, Henry D. Dewell, and Lydik S. Jacobsen. 298.

ENGINEERS AND ENGINEERING.

"The Relation of the Society to National Defense." Address at the Annual Convention, Denver, Colorado, July 24, 1940. John P. Hogan.

EROSION.

"Beach Erosion Studies." Earl I. Brown. (With Discussion.) 869.

Control of soil erosion along Yellow River. 407.

EVAPORATION.

"Hydrology of the Great Lakes": A Symposium. C. R. Pettis and Harold C. Hickman. 794. Discussion: A. A. Young, Alfred J. Cooper, Jr., Adolph F. Meyer, S. T. Harding, O. E. Meinzer, R. W. Davenport, and G. H. Hickox. 818.

FILTRATION.

Progress in water treatment and filtration in the decade, 1930–1939. 1775.

FLANGES.

Flange stresses measured. 1470.

FLASH BOARDS.

"Flash-Board Pins." Chilton A. Wright and Clifford A. Betts. 1412. Discussion: William P. Creager, Lincoln W. Ryder, E. T. Schuleen, John E. Field, Julian H. White, and Harry H. Hatch. 1445.

FLOODS.

Outline of floods in U. S. during decade from 1930 to 1939. 1741.

"The Yellow River Problem." O. J. Todd and S. Eliassen. (With Discussion.) 346.

Use of the unit hydrograph in estimating flood flows. 1175.

FOUNDATIONS.

"Engineering Geology Problems at Conchas Dam, New Mexico." Irving B. Crosby. (With Discussion.) 581.

"Field Tests of a Shale Foundation." August E. Niederhoff. 1519. Discussion: Harry H. Hatch, Jacob Feld, and Jack R. Rountree. 1535.

"Large Core Drills Aid Construction at Chickamauga Dam." James S. Lewis, Jr. (With Discussion.) 850.

"Settlement Studies of Structures in Egypt." Gregory P. Tschebotareff. 919. Discussion: L. C. Wilcoxen, Trent R. Dames, Edwin J. Beugler, W. S. Hanna, and D. P. Krynine. 946.

"The Risk of the Unexpected in Sub-Surface Construction Contracts." Oren Clive Herwitz. (With Discussion.) 454.

FRICTION.

Correction for friction in open-channel control section. 747.

Experimental evaluation of friction factor in lateral spillway channels. 614.

GAGES.

Measuring wind pressure on a tall building. 14.

See also **METERS.**

GEODESY.

"Combining Geodetic Survey Methods with Cadastral Surveys." Carl M. Berry. 1300. Discussion: William L. Sawyer, Philip Kissam, and Gordon Macleish. 1331.

Establishing a system of rectangular coordinates. 534.

GEOLOGY.

"Engineering Geology Problems at Conchas Dam, New Mexico." Irving B. Crosby. (With Discussion.) 581.

GIRDERS.

"Effective Moment of Inertia of a Riveted Plate Girder." Scott B. Lilly and Samuel T. Carpenter. 1462. Discussion: William R. Osgood, Clyde T. Morris, B. R. Leffler, E. Neil W. Lane, Lewis E. Moore, W. E. Black, L. E. Grinter, Charles M. Spofford, E. Mirabelli, Edward Godfrey, Walter H. Weiskopf, C. D. Williams, Alvin B. Auerbach, Jonathan Jones, R. L. Moore, J. M. Garrelts, I. E. Madsen, Francis P. Witmer, A. W. Fischer, J. R. Shank, Harold D. Hussey, Charles U. Kring, and Thomas C. Shedd. 1477.

GRAPHICAL CHARTS.

"Graphical Arch Analysis Applicable to Arch Dams." Carl H. Heilbron, Jr., and William H. Saylor. (With Discussion.) 765.

GROUND-WATER.

Progress in knowledge of ground-water, 1930–1939. 1754.

HARBORS.

Great Lakes harbors. 172.

"Pollution of Boston Harbor." Arthur D. Weston and Gail P. Edwards. (With Discussion.) 973.

HIGHWAYS.

Highway developments in the United States. 222.

HYDRO-ELECTRIC POWER PLANTS.

See **POWER PLANTS.**

INSTRUMENTS.

Experiment visualizing the direction of column reactions. 1026.

IRRIGATION.

"Development of the Colorado River in the Upper Basin." Thomas C. Adams. (With Discussion.) 1345.

LAW.
  "Proposed Improvements for Land Surveys and Title Transfers." Philip Kissam. (With Discussion.) 731.
  The land court and its engineering procedure. 554.
  "The Risk of the Unexpected in Subsurface Construction Contracts." Oren Clive Herwitz. (With Discussion.) 454.

LEVEES.
  Earth dikes on the Yellow River. 391.

LEVELING.
  "An Improved Method for Adjusting Level and Traverse Surveys." Clarence Norris and Julius L. Speert. 1376. Discussion: George H. Dell, Howard S. Rappleye, R. M. Wilson, Cleveland B. Coe, George D. Whitmore, C. C. Miner, W. O. Byrd, and Robert C. Sheldon. 1395.

MATERIALS OF CONSTRUCTION.
  See **ALUMINUM.**

MEMOIRS OF DECEASED MEMBERS.
  See name of member in Author Index.

METERS.
  Types of water meters in use, 1930–1939. 1769.

MODELS.
  Model of Empire State Building used in wind pressure studies. 32.

NATIONAL DEFENSE.
  "The Relation of the Society to National Defense." Address at the Annual Convention, Denver, Colorado, July 24, 1940. John P. Hogan. 1781.

NAVIGATION.
  Improvement of navigation on the Yellow River. 412.

PAVEMENT.
  "Design of Dowels in Transverse Joints of Concrete Pavements." Bengt F. Friberg. 1076. Discussion: L. E. Grinter, and W. O. Fremont. 1096.

PHOTOELASTICITY.
  Photoelastic study of stress distribution around a tunnel. 1149.
  "Specification and Design of Steel Gusset-Plates." T. H. Rust. (With Discussion.) 142.

PIPE LINES.
  Trends in construction of pipe lines, 1930–1939. 1756.

POWER PLANTS.
  The Yellow River power developments at Hu-kou Falls. 409.

PUBLIC UTILITIES.
  Trends in valuation of utilities. 1778.

PUMPS AND PUMPING.

Ten years' progress in centrifugal pumps. 1771.

RAILROADS.

"Transportation Developments in the United States." Fred Lavis. (With Discussion.) 220.

RAINFALL.

"Analysis of Run-Off Characteristics." Otto H. Meyer. 83. Discussion: Victor H. Cochrane, Bertram S. Barnes, LeRoy K. Sherman, Richmond T. Zoch, Merrill Bernard, Franklin F. Snyder, W. G. Hoyt, C. O. Clark, C. S. Jarvis, and Howard M. Turner. 101.

Effect of storms on quantity of suspended solids in sewage. 1610.

"The Unit Hydrograph Principle Applied to Small Water-Sheds." E. F. Brater. 1154. Discussion: Franklin F. Snyder, LeRoy K. Sherman, Raphael G. Kazmann, and Waldo E. Smith. 1179.

REINFORCED CONCRETE.

"Design of Circular Concrete Tanks." George S. Salter. (With Discussion.) 504.

"Design of Dowels in Transverse Joints of Concrete Pavements." Bengt F. Friberg. (With Discussion.) 1076.

REPORTS OF COMMITTEES.

"Important Events, Developments, and Trends in Water Supply Engineering During the Decade Ending With the Year 1939." Report of the Committee on Water Supply of the Sanitary Engineering Division. F. A. Barbour, J. R. Baylis, W. W. Brush, L. R. Howson, J. A. Wade, and Thomas H. Wiggin. 1740.

"Wind Bracing in Steel Buildings." Final Report of Sub-Committee No. 31, Committee on Steel, of the Structural Division. Clyde T. Morris, N. A. Richards, Francis P. Witmer, and C. R. Young. 1713.

RESERVOIRS.

"Functional Design of Flood Control Reservoirs." C. J. Posey and Fu-Te I. 1638. Discussion: Edward Soucek, J. C. Stevens, Edward J. Bednarski, Roland A. Kampmeier, Edgar E. Foster, Sherman M. Woodward, M. Kindinger, P. Wilhelm Werner, Waldo E. Smith, J. O. Jones, and W. C. Hammatt. 1648.

RIVERS.

"Development of the Colorado River in the Upper Basin." Thomas C. Adams. 1345. Discussion: C. C. Elder. 1366.

"The Yellow River Problem." O. J. Todd and S. Eliassen. 346. Discussion: J. W. Beardsley, Elliott J. Dent, Herbert Chatley, H. van der Veen, C. S. Jarvis, E. W. Lane, and George Higgins. 417.

See also **WATER, FLOW OF, IN OPEN CHANNELS.**

RIVETED JOINTS.

Design of riveted connections. 657.

"Tension Tests of Large Riveted Joints." Raymond E. Davis, Glenn B. Woodruff, and Harmer E. Davis. 1193. Discussion: Charles F. Goodrich, Frederick P. Shearwood, Jonathan Jones, C. C. Winter, W. M. Wilson, J. M. Garrelts, A. E. Richard De Jonge, E. C. Hartmann, and Marshall Holt. 1246.

RUN-OFF.

See **RAINFALL.**

SAFETY DEVICES AND MEASURES.

"Traffic Problems in Metropolitan Areas." Earl J. Reeder. (With Discussion.) 323.

SEDIMENTATION.

"The Yellow River Problem." O. J. Todd and S. Eliassen. (With Discussion.) 346.

Transportation of suspended material by turbulence. 1562.

SEWAGE DISPOSAL.

"Pollution of Boston Harbor." Arthur D. Weston and Gail P. Edwards. (With Discussion.) 973.

"Sewage Disposal Project of Buffalo, New York." Samuel A. Greeley. 1601. Discussion: C. A. Holmquist, Arthur J. Bulger, C. R. Velzy, and Thomas M. Niles. 1626.

SEWAGE WORKS.

See **SEWAGE DISPOSAL.**

SEWERAGE.

"Sewage Disposal Project of Buffalo, New York." Samuel A. Greeley. (With Discussion.) 1601.

SHALES.

"Field Tests of a Shale Foundation." August E. Niederhoff. (With Discussion.) 1519.

SHIPPING.

Shipping on the Great Lakes. 174.

SHIPS.

Growth in tonnage of Great Lakes vessels. 180.

SHORE PROTECTION.

"Beach Erosion Studies." Earl I. Brown. 869. Discussion: Morris N. Lipp, George A. Soper, James J. O'Rourke, Elliott J. Dent, Ralph F. Rhodes, and Charles T. Leeds. 891.

SILT.

See **SEDIMENTATION.**

SIPHONS.

"Design of a High-Head Siphon Spillway." Elmer Rock. (With Discussion.) 1050.

SOILS.

"Settlement Studies of Structures in Egypt." Gregory P. Tschebotareff. (With Discussion.) 919.

SPILLWAYS.

"Design of a High-Head Siphon Spillway." Elmer Rock. 1050. Discussion: J. C. Stevens, B. E. Torpen, G. E. Hyde, R. B. Cochrane, and I. M. Nelidov. 1062.

"Flash-Board Pins." Chilton A. Wright and Clifford A. Betts. (With Discussion.) 1412.

"Lateral Spillway Channels." Thomas R. Camp. 606. Discussion: W. E. Howland, Lewis V. Carpenter, John K. Vennard, Fenner H. Whitley, F. Knapp, Carl Rohwer, and Harold Allen Thomas, Jr. 618.

STATISTICS.

"Relation of the Statistical Theory of Turbulence to Hydraulics." A. A. Kalinske. (With Discussion.) 1547.

STEEL PLATES.

"Specification and Design of Steel Gusset-Plates." T. H. Rust. 142. Discussion: R. H. Sherlock, L. E. Grinter, Russell C. Brinker, and C. W. Wixom. 159.

STRESS AND STRAIN.

"Effective Moment of Inertia of a Riveted Plate Girder." Scott B. Lilly and Samuel T. Carpenter. (With Discussion.) 1462.

"Specification and Design of Steel Gusset-Plates." T. H. Rust. (With Discussion.) 142.

"Stress Distribution Around a Tunnel." Raymond D. Mindlin. (With Discussion.) 1117.

"Tension Tests of Large Riveted Joints." Raymond E. Davis, Glenn B. Woodruff, and Harmer E. Davis. (With Discussion.) 1193.

STRUCTURES, THEORY OF.

"Design of Circular Concrete Tanks." George S. Salter. (With Discussion.) 504.

"Graphical Arch Analysis Applicable to Arch Dams." Carl H. Heilbron, Jr., and William H. Saylor. (With Discussion.) 765.

"Simplified Wind-Stress Analysis of Tall Buildings." Otto Gottschalk. (With Discussion.) 1019.

"Theory of Limit Design." J. A. Van den Broek. 638. Discussion: John H. Meursinge, I. K. Silverman, Edward Godfrey, Basil Sourochnikoff, E. Mirabelli, C. M. Goodrich, George Winter, Francis E. Simpson, Joseph A. Wise, Alfred Freudenthal, Hans H. Bleich, Alfred S. Niles, A. Floris, L. H. Nishkian, F. G. Eric Peterson, E. S. Fabian, A. A. Eremin, and L. H. Donnell. 662.

SURVEYING.

"An Improved Method for Adjusting Level and Traverse Surveys." Clarence Norris and Julius L. Speert. (With Discussion.) 1376.

"Combining Geodetic Survey Methods With Cadastral Surveys." Carl M. Berry. (With Discussion.) 1300.

"Proposed Improvements for Land Surveys and Title Transfers." Philip Kissam. 731. Discussion: George D. Whitmore. 741.

"State-Wide Surveying Practice in Massachusetts": A Symposium. Elmer C. Houdlette and Clarence B. Humphrey. 533. Discussion: H. J. Shea, Philip Kissam, Lloyd G. Frost, William Bowie, R. M. Wilson, and Elliott B. Roberts. 565.

TANKS.

"Design of Circular Concrete Tanks." George S. Salter. 504. Discussion: Carl A. Rosengarten, I. K. Silverman, Frank J. McCormick, H. B. Muckleston, Robert B. B. Moorman, Dana Young, L. J. Mensch, Jens Egede Nielsen, Lloyd S. Dysland, Basil Sourochnikoff, and Regis F. Fey. 514.

See also **WATER TANKS.**

TOWERS.

Design of a steel tower. 651.

TRAFFIC.

"Traffic Problems in Metropolitan Areas." Earl J. Reeder. 323. Discussion: George H. Herrold, Thomas Buckley, D. Grant Mickle, Robert A. Mitchell, and Cleveland B. Coe. 331.

TRANSPORTATION.

"Great Lakes Transportation." C. M. Tyler. (With Discussion.) 167.

Mass transportation in cities. 329.

"Transportation Developments in the United States." Fred Lavis. 220. Discussion: Milton Harris, Ralph Budd, L. Alfred Jenny, William J. Cunningham, S. R. Truesdell, David A. Molitor, and C. A. Hogland. 237.

TRIANGULATION.

"An Improved Method for Adjusting Level and Traverse Surveys." Clarence Norris and Julius L. Speert. (With Discussion.) 1376.

"Combining Geodetic Survey Methods with Cadastral Surveys." Carl M. Berry. (With Discussion.) 1300.

TURBULENCE.

"Relation of the Statistical Theory of Turbulence to Hydraulics." A. A. Kalinske. 1547. Discussion: Hunter Rouse, Martin A. Mason, J. C. Stevens, Boris A. Bakhmeteff, Clyde W. Hubbard, John S. McNown, Samuel Shulits, Paul Nemenyi, and Bennie N. Netzer. 1567.

TUNNELS.

"Stress Distribution Around a Tunnel." Raymond D. Mindlin. 1117. Discussion: W. O. Richmond, Jacob Feld, and J. H. A. Brahtz. 1141.

VALUATION.

"Transportation Developments in the United States." Fred Lavis. (With Discussion.) 220.

Trends in valuation of public utilities. 1778.

VIBRATION.

Theory of forced and free oscillations. 276.

WATER.

Developments in water treatment, 1930–1939. 1775.

WATER, FLOW OF, IN OPEN CHANNELS.

"Design of an Open-Channel Control Section." Karl R. Kennison. 744. Discussion: V. L. Streeter, Emery H. Willes, Robert O. Thomas, Frank S. Bailey, and George E. Barnes. 752.

"Development of the Colorado River in the Upper Basin." Thomas C. Adams. (With Discussion.) 1345.

Handling heavy flows of water. 860.

"Lateral Spillway Channels." Thomas R. Camp. (With Discussion.) 606.

"Relation of the Statistical Theory of Turbulence to Hydraulics." A. A. Kalinske. (With Discussion.) 1547.

"The Yellow River Problem." O. J. Todd and S. Eliassen. (With Discussion.) 346.

WATER, FLOW OF, OVER DAMS AND WEIRS.

"Flash-Board Pins." Chilton A. Wright and Clifford A. Betts. (With Discussion.) 1412.

WATER, FLOW OF, THROUGH ORIFICES.

"Design of a High-Head Siphon Spillway." Elmer Rock. (With Discussion.) 1050.

WATER HAMMER.

Study of. 1772.

WATER METERS.

See **METERS.**

WATER POLLUTION.

"Pollution of Boston Harbor." Arthur D. Weston and Gail P. Edwards. 973. Discussion: E. Sherman Chase, F. E. DeMartini, A. M. Rawn, Harrison P. Eddy, Jr., Samuel A. Greeley, and George A. Soper. 1009.

WATER PURIFICATION.

See **WATER.**

WATER SUPPLY.

See **WATER-WORKS.**

WATER TANKS.

Growth in use of elevated tanks, 1930–1939. 1765.

WATER TERMINALS.

Terminals on the Great Lakes. 178.

WATERWAYS.

As a means of transportation in the United States. 225.

"Great Lakes Transportation." M. C. Tyler. 167. Discussion: T. Kennard Thomson, Harland C. Woods, W. L. R. Haines, and L. C. Sabin. 186.

"Hydrology of the Great Lakes": A Symposium. C. R. Pettis and Harold C. Hickman. (With Discussion.) 794.

Waterways affected by sewage disposal project of Buffalo, New York. 1605.

WATER-WORKS.

"Important Events, Developments, and Trends in Water Supply Engineering During the Decade Ending With the Year 1939." Report of the Committee on Water Supply of the Sanitary Engineering Division. 1740.

WEIRS.

See **WATER, FLOW OF, OVER DAMS AND WEIRS.**

WIND.

Wind records for beach erosion studies. 875.

WIND PRESSURE.

"Simplified Wind-Stress Analysis of Tall Buildings." Otto Gottschalk. 1019. Discussion: Samuel T. Carpenter, and Charles B. Winick. 1029.

"Wind Bracing in Steel Buildings." Final Report of Sub-Committee No. 31, Committee on Steel, of the Structural Division. 1713.

"Wind Forces on a Tall Building." J. Charles Rathbun. 1. Discussion: David Cushman Coyle, Clyde T. Morris, Robins Fleming, F. P. Shearwood, Lydik S. Jacobsen, Francis L. Castleman, Jr., J. B. Wilbur, R. D. Spellman, David A. Molitor, Walter J. Gray, K. L. DeBlois, Albert Smith, Victor R. Bergman, Otto Gottschalk, and Colin Skinner.

AUTHOR INDEX

ADAMS, THOMAS C.
" Development of the Colorado River in the Upper Basin." 1345.

ALVORD, HENRY BISSELL.
Memoir of. 1794.

ANGLE, JAMES MACFARLANE.
Memoir of. 1796.

ASH, WILLIAM JOHN.
Memoir of. 1798.

AUERBACH, ALVIN B.
Moment of inertia of girder. 1496.

BAILEY, FRANK S.
Open-channel control section. 757.

BAKHMETEFF, BORIS A.
Theory of Turbulence. 1572.

BARBOUR, F. A.
" Report of the Committee on Water Supply Engineering of the Sanitary Engineering Division: Important Events, Developments, and Trends in Water Supply Engineering During the Decade Ending with the Year 1939." 1740.

BARNES, BERTRAM S.
Analysis of run-off characteristics. 104.

BARNES, GEORGE E.
Open-channel control section. 758.

BARTON, CALVIN LEWIS.
Memoir of. 1799.

BAYLIS, J. R.
" Report of the Committee on Water Supply Engineering of the Sanitary Engineering Division: Important Events, Developments, and Trends in Water Supply Engineering During the Decade Ending with the Year 1939." 1740.

BEANFIELD, R. McC.
Earthquakes and structures. 300.

BEARDSLEY, J. W.
Yellow River problem. 417.

BEDNARSKI, EDWARD J.
Flood control reservoirs. 1652.

BEGGS, GEORGE ERLE.
Memoir of. 1800.

BELL, JAMES C.
Memoir of. 1803.

BERGMAN, VICTOR R.
Wind forces on a tall building. 64.

BERNARD, MERRILL.
Analysis of run-off characteristics. 117.

BERRY, CARL M.
" Combining Geodetic Survey Methods with Cadastral Surveys." 1300.

BETTS, CLIFFORD A.
" Flash-Board Pins." 1412.

BEUGLER, EDWIN J.
Settlement studies of structures in Egypt. 950.

BILLING, JOHN MORTON.
Memoir of. 1963.

BLACK, W. E.
Moment of inertia of girder. 1482.

BLEICH, HANS H.
Theory of limit design. 690.

BLOOM, JOHN GEORGE.
Memoir of. 1805.

BOWIE, WILLIAM.
Surveying practice in Massachusetts. 574.

BRAHTZ, J. H. A.
Stress distribution. 1143.

BRATER, E. F.
" The Unit Hydrograph Principle Applied to Small Water-Sheds." 1154.

BRINKER, RUSSELL C.
Steel gusset-plates. 161.

BROWN, EARL I.
" Beach Erosion Studies." 869.

BROWN, NORMAN FREED.
Memoir of. 1806.

BRUSH, W. W.
" Report of the Committee on Water Supply Engineering of the Sanitary Engineering Division: Important Events, Developments, and Trends in Water Supply Engineering During the Decade Ending with the Year 1939." 1740.

BRYAN, EVERETT N.
Memoir of. 1808.

BUCKLEY, THOMAS.
Traffic problems. 333.

BUDD, RALPH.
Transportation developments. 239.

BUETOW, WALTER CARL.
Memoir of. 1810.

BULGER, ARTHUR J.
Sewage disposal. 1628.

BURNS, JOSEPH PATRICK.
Memoir of. 1964.

BYRD, W. O.
Method for adjusting surveys. 1405.

CADY, RALPH HOWE.
Memoir of. 1811.

CAMP, THOMAS R.
" Lateral Spillway Channels." 606.

CARLILE, THOMAS JENKS.
Memoir of. 1813.

CARPENTER, LEWIS V.
Spillway channels. 621.

CARPENTER, SAMUEL T.
" Effective Moment of Inertia of a Riveted Plate Girder." 1462.
Wind-stress analysis. 1029.

CASTLEMAN, FRANCIS L., JR.
Wind forces on a tall building. 52.

CHASE, E. SHERMAN.
Pollution of Boston Harbor. 1009.

CHATLEY, HERBERT.
Yellow River problem. 425.

CHEW, R. S.
Earthquakes and structures. 303.

CLARK, C. O.
Analysis of run-off characteristics. 128.

COCHRANE, R. B.
High-head siphon spillways. 1063.

COCHRANE, VICTOR H.
Analysis of run-off characteristics. 101.

COE, CLEVELAND B.
Method for adjusting surveys. 1401.
Traffic problems. 342.

COMEGYS, CHARLES PAGE.
Memoir of. 1966.

CONNER, HARRY McKEAN.
Memoir of. 2018.

COOPER, ALFRED J., JR.
Hydrology of the Great Lakes. 821.

COYLE, DAVID CUSHMAN.
Wind forces on a tall building. 42.

CREAGER, WILLIAM P.
Flash-board pins. 1445.

CRESKOFF, JACOB J.
Earthquakes and structures. 305.

CROSBY, IRVING B.
"Engineering Geology Problems at Conchas Dam, New Mexico." 581.

CUDDEBACK, ALLAN WINTER.
Memoir of. 1815.

CUNNINGHAM, WILLIAM J.
Transportation developments. 256.

CURTIS, JAMES EUGENE.
Memoir of. 1818.

CURTIS, WALTER WHALEY.
Memoir of. 1819.

CUSHING, WILLIAM CHANNING.
Memoir of. 1820.

DAILY, CORNELIUS MARK.
Memoir of. 1824.

DAMES, TRENT R.
Settlement studies of structures in Egypt. 946.

DAVENPORT, R. W.
Hydrology of the Great Lakes. 835.

DAVIES, JOHN VIPOND.
Memoir of. 1826.

DAVIS, HARMER E.
"Tension Tests of Large Riveted Joints." 1193.

DAVIS, RAYMOND E.
"Tension Tests of Large Riveted Joints." 1193.

DeBLOIS, K. L.
Wind forces on a tall building. 60.

DECK, FREDERICK WEBSTER.
Memoir of. 1967.

DELL, GEORGE H.
Method for adjusting surveys. 1395.

DeMARTINI, F. E.
Pollution of Boston Harbor. 1010.

DENT, ELLIOTT J.
Yellow River problem. 418.
Beach erosion studies. 898.

DEWELL, HENRY D.
Earthquakes and structures. 315.

DIMON, DANIEL YOUNG.
Memoir of. 1828.

DOANE, WALTER ALEXANDER.
Memoir of. 1830.

DONNELL, L. H.
Theory of limit design. 713.

DUCEY, EDWARD J.
"Reconstruction of the Walpole-Bellows Falls Arch Bridge." 1675.

DYSLAND, LLOYD S.
Circular concrete tanks. 527.

EDDY, HARRISON P., JR.
Pollution of Boston Harbor. 1012.

EDWARDS, GAIL P.
"Pollution of Boston Harbor." 973.

ELDER, C. C.
Colorado River. 1366.

ELIASSEN, S.
"The Yellow River Problem." 346.

EMERSON, GUY CARLETON.
Memoir of. 1833.

EREMIN, A. A.
Earthquakes and structures. 314.
Theory of limit design. 713.

FABIAN, E. S.
Theory of limit design. 710.

FARRIS, JOHN.
Memoir of. 1835.

FARWELL, CARROLL A.
Sub-surface contracts. 499.

FELD, JACOB.
Stress distribution. 1143.
Tests of a shale foundation. 1537.

FELKER, FRANKLIN MECHTLEY.
Memoir of. 2021.

FENTON, CLYDE HENRY.
Memoir of. 1971.

FEY, REGIS F.
Circular concrete tanks. 529.

FIELD, JOHN E.
Flash-board pins. 1451.

FISCHER, A. W.
Moment of inertia of girder. 1505.

FISHER, HOWELL TRACY.
Memoir of. 1837.

FISK, GEORGE FARNSWORTH.
Memoir of. 1972.

FITZPATRICK, FRANCIS JAMES.
Memoir of. 1973.

FLEMING, ROBINS.
Wind forces on a tall building. 44.

FLORIS, A.
Theory of limit design. 700.

FLOYD, O. N.
Core drills at Chickamauga Dam. 867.

FOSTER, EDGAR E.
Flood control reservoirs. 1656.

FREMONT, W. O.
Dowels in concrete pavements. 1102.

FRENCH, J. B.
Walpole-Bellows Falls Bridge. 1701.

FREUDENTHAL, ALFRED.
Theory of limit design. 686.

FRIBERG, BENGT F.
"Design of Dowels in Transverse Joints of Concrete Pavements." 1076.

FROST, LLOYD G.
Surveying practice in Massachusetts. 569.

GALLOWAY, JOHN D.
"Earthquakes and Structures." 269.

GARRELTS, J. M.
Moment of inertia of girder. 1501.
Tests of riveted joints. 1275.

GASTON, LOUIS PREVOST.
Memoir of. 1838.

GIBBS, GEORGE.
Memoir of. 1840.

GODFREY, EDWARD.
Moment of inertia of girder. 1491.
Theory of limit design. 667.

GOODRICH, CHARLES F.
Tests of riveted joints. 1246.

GOODRICH, C. M.
Theory of limit design. 672.

GOODWIN, RALPH EDWARD.
Memoir of. 1844.

GOTTSCHALK, OTTO.
"Simplified Wind-Stress Analysis of Tall Buildings." 1019.
Wind forces on a tall building. 65.

GOWDY, ROY COTSWORTH.
Memoir of. 1846.

GRAY, WALTER J.
Wind forces on a tall building. 60.

GREELEY, SAMUEL A.
Pollution of Boston Harbor. 1012.
"Sewage Disposal Project of Buffalo, New York." 1601.

GRINTER, L. E.
Dowels in concrete pavements. 1096.
Moment of inertia of girder. 1484.
Steel gusset-plates. 159.

HAAS, EDWARD FRANCIS.
Memoir of. 1851.

HADLEY, HOMER M.
Earthquakes and structures. 298.

HAINES, W. L. R.
Great Lakes transportation. 188.

HAMMATT, W. C.
Flood control reservoirs. 1670.

HAMMOND, ALONZO J.
Sub-surface contracts. 476.

HANMER, HARRY J.
Memoir of. 1977.

HANNA, W. S.
Settlement studies of structures in Egypt. 951.

HARDING, S. T.
Hydrology of the Great Lakes. 829.

HARRINGTON, EPHRAIM.
Memoir of. 1854.

HARRIS, MILTON.
Transportation developments. 237.

HART, PHILIP.
Memoir of. 1855.

HARTMANN, E. C.
Tests of riveted joints. 1291.

HATCH, HARRY H.
Flash-board pins. 1458.
Tests of a shale foundation. 1535.

HAWLEY, RALPH STEVENSON.
Memoir of. 1861.

HAWLEY, WILLIAM CHAUNCEY.
Memoir of. 1863.

HAYS, JAMES B.
Core drills at Chickamauga Dam. 866.

HEILBRON, CARL H., JR.
"Graphical Arch Analysis Applicable to Arch Dams." 765.

HERROLD, GEORGE H.
Traffic problems. 331.

HERWITZ, OREN CLIVE.
"The Risk of the Unexpected in Subsurface Construction Contracts." 454.

HICKMAN, HAROLD C.
"Hydrology of the Great Lakes: A Symposium: Evaporation Experiments." 807.

HICKOX, G. H.
Hydrology of the Great Lakes. 838.

HIGGINS, GEORGE.
Yellow River problem. 441.

HOGAN, JOHN P.
"The Relation of the Society to National Defense." Address at the Annual Convention, Denver, Colorado, July 24, 1940. 1781.

HOGLUND, C. A.
Transportation developments. 263.

HOLMES, GLENN WILLIAM.
Memoir of. 2022.

HOLMQUIST, C. A.
Sewage disposal. 1626.

HOLT, MARSHALL.
"Tests on Built-Up Columns of Structural Aluminum Alloys." 196.
Tests of riveted joints. 1291.

HOSKINS, LEANDER M.
"Earthquakes and Structures." 269.

HOUDLETTE, ELMER C.
"State-Wide Surveying Practice in Massachusetts: A Symposium: Establishing a System of Rectangular Co-Ordinates." 534.

HOWLAND, W. E.
Spillway channels. 618.

HOWSON, L. R.
"Report of the Committee on Water Supply Engineering of the Sanitary Engineering Division: Important Events, Developments, and Trends in Water Supply Engineering During the Decade Ending with the Year 1939." 1740.

HOYT, W. G.
Analysis of run-off characteristics. 127.

HUBBARD, CLYDE W.
Theory of turbulence. 1574.

HUBER, WALTER L.
Earthquakes and structures. 309.

HUMPHREY, CLARENCE B.
" State-Wide Surveying Practice in Massachusetts: A Symposium: The Land Court and Its Engineering Procedure." 554.

HUSSEY, HAROLD D.
Moment of inertia of girder. 1508.

HYDE, G. E.
High-head siphon spillways. 1063.

I, FU-TE.
" Functional Design of Flood Control Reservoirs." 1638.

IRWIN, JAMES CLARK.
Memoir of. 1864.

JACOBSEN, LYDIK S.
Earthquakes and structures. 319.
Wind forces on a tall building. 50.

JANDA, HAROLD FREDERICK.
Memoir of. 1978.

JARVIS, C. S.
Analysis of run-off characteristics. 131.
Yellow River problem. 433.

JENNY, L. ALFRED.
Transportation developments. 243.

JOHNS, CHARLES WHITE.
Memoir of. 1866.

JOHNSON, ARTHUR NEWHALL.
Memoir of. 1868.

JOHNSON, H. L.
Engineering geology. 600.

JONES, JONATHAN.
Moment of inertia of girder. 1498.
Tests of riveted joints. 1253.

JONES, J. O.
Flood control reservoirs. 1669.

JONGE, A. E. RICHARD de.
Tests of riveted joints. 1275.

JUDGE, WILLIAM JOSEPH.
Memoir of. 1980.

KALINSKE, A. A.
" Relation of the Statistical Theory of Turbulence to Hydraulics." 1547.

KAMPMEIER, ROLAND A.
Flood control reservoirs. 1655.

KAZMANN, RAPHAEL G.
Unit hydrograph. 1183.

KEMMAN, ARTHUR SYLVANUS.
Memoir of. 1983.

KENNISON, KARL R.
" Design of an Open-Channel Control Section." 744.

KHUEN, RICHARD, JR.
Memoir of. 1869.

KINDINGER, M.
Flood control reservoirs. 1658.

KISSAM, PHILIP.
Geodetic and cadastral surveys. 1331.
" Proposed Improvements for Land Surveys and Title Transfers." 731.
Surveying practice in Massachusetts. 568.

KNAPP, F.
Spillway channels. 625.

KRING, CHARLES U.
Moment of inertia of girder. 1511.

KRYNINE, D. P.
Settlement studies of structures in Egypt. 954.

KYLSTRA, JOHN.
Memoir of. 1984.

LANE, E. NEIL W.
Moment of inertia of girder. 1480.

LANE, E. W.
Yellow River problem. 434.

LANG, PHILIP GEORGE, JR.
Memoir of. 1871.

LANGLEY, H. E.
" Reconstruction of the Walpole-Bellows Falls Arch Bridge." 1675.

LAPHAM, JOHN RAYMOND.
Memoir of. 1986.

LARRABEE, WILLIAM DOMINICK.
Memoir of. 1875.

LARSEN, HENRY AMBROSE.
Memoir of. 1990.

LAVIS, FRED.
"Transportation Developments in the United States." 220.

LEEDS, CHARLES T.
Beach erosion studies. 905.

LEFFLER, B. R.
Moment of inertia of girder. 1478.

LEWIS, JAMES S., JR.
"Large Core Drills Aid Construction at Chickamauga Dam." 850.

LILLY, SCOTT B.
"Effective Moment of Inertia of a Riveted Plate Girder." 1462.

LINDENTHAL, GUSTAV.
Memoir of. 1790.

LIPP, MORRIS N.
Beach erosion studies. 891.

LIPSCOMB, DANIEL COLLINS.
Memoir of. 1991.

McCORMICK, FRANK J.
Circular concrete tanks. 517.

McCULLOUGH, CONDE B.
Walpole-Bellows Falls Bridge. 1702.

McKIBBEN, HARRY SEYMOUR.
Memoir of. 1992.

McNOWN, JOHN S.
Theory of turbulence. 1576.

MACLEISH, GORDON.
Geodetic and cadastral surveys. 1333.

MADSEN, I. E.
Moment of inertia of girder. 1501.

MARKWART, ARTHUR HERMANN.
Memoir of. 1877.

MARSH, F. B.
Sub-surface contracts. 489.

MARTIN, EVAN S.
Sub-surface contracts. 490.

MARX, CHARLES DAVID.
Memoir of. 1785.

MASON, MARTIN A.
Theory of turbulence. 1568.

MEARS, FREDERICK.
Memoir of. 1881.

MEINZER, O. E.
Hydrology of the Great Lakes. 834.

MENSCH, L. J.
Circular concrete tanks. 524.

MEURSINGE, JOHN H.
Theory of limit design. 662.

MEYER, ADOLPH F.
Hydrology of the Great Lakes. 825.

MEYER, OTTO H.
"Analysis of Run-Off Characteristics." 83.

MICKLE, D. GRANT.
Traffic problems. 336.

MILLER, WILLIAM FRANKLIN.
Memoir of. 1993.

MINDLIN, RAYMOND D.
"Stress Distribution Around a Tunnel." 1117.

MINER, C. C.
Method for adjusting surveys. 1405.

MIRABELLI, E.
Moment of inertia of girder. 1489.
Theory of limit design. 670.

MITCHELL, ROBERT A.
Traffic problems. 339.

MITCHELL, SAMUEL PHILLIPS.
Memoir of. 1886.

MOLITOR, DAVID A.
Sub-surface contracts. 488.
Transportation developments. 259.
Wind forces on a tall building. 58.

MONKS, JOHN.
Memoir of. 2024.

MONSARRAT, CHARLES NICHOLAS.
Memoir of. 1889.

MOORE, LEWIS E.
Moment of inertia of girder. 1481.

MOORE, R. L.
Moment of inertia of girder. 1499.

MOORMAN, ROBERT B. B.
Circular concrete tanks. 519.

MORGAN, FRANCIS J.
Sub-surface contracts. 492.

MORRILL, ASA HALL.
Memoir of. 1892.

MORRIS, CLYDE T.
" Final Report of Sub-Committee No. 31, Committee on Steel of the Structural Division, on Wind Bracing in Steel Buildings." 1713.
Moment of inertia of girder. 1477.
Wind forces on a tall building. 44.

MUCKLESTON, H. B.
Circular concrete tanks. 518.

NELIDOV, I. M.
Graphical arch analysis. 790.
High-head siphon spillways. 1066.

NELSON, H. M.
Walpole-Bellows Falls Bridge. 1707.

NEMENYI, PAUL.
Theory of turbulence. 1581.

NETZER, BENNIE N.
Theory of turbulence. 1591.

NEWSOM, J. B.
Core drills at Chickamauga Dam. 865.

NEWTON, FREDERICK W.
Sub-surface contracts. 477.

NIEDERHOFF, AUGUST E.
" Field Tests of a Shale Foundation." 1519.

NIELSEN, JENS EGEDE.
Circular concrete tanks. 526.

NILES, ALFRED S.
Tests on aluminum columns. 218.
Theory of limit design. 693.

NILES, THOMAS M.
Sewage disposal. 1634.

NISHKIAN, L. H.
Theory of limit design. 702.

NORRIS, CLARENCE.
"An Improved Method for Adjusting Level and Traverse Surveys." 1376.

OBER, RALPH HADLOCK.
Memoir of. 1894.

OLBERG, CHARLES REAL.
Memoir of. 1896.

OLMSTEAD, ERI.
Memoir of. 1995.

O'ROURKE, JAMES J.
Beach erosion studies. 895.

OSBORN, IRWIN SELDEN.
Memoir of. 1898.

OSGOOD, WILLIAM R.
Moment of inertia of girder. 1477.

PERSONS, VICTOR SMITH.
Memoir of. 1996.

PETERSON, F. G. ERIC.
Theory of limit design. 706.

PETTIS, C. R.
"Hydrology of the Great Lakes: A Symposium: Typical Quantitative Analysis as Applied to Lake Superior." 795.

PINNER, GUY.
Memoir of. 1900.

POSEY, C. J.
"Functional Design of Flood Control Reservoirs." 1638.

QUILTY, T. FRANK.
Memoir of. 1902.

RAPPLEYE, HOWARD S.
Method for adjusting surveys. 1398.

RATHBUN, J. CHARLES.
"Wind Forces on a Tall Building." 1.

RAWN, A. M.
Pollution of Boston Harbor. 1012.

RAYBURN, JOHN MATTHEW.
Memoir of. 1904.

REED, RALPH JOHN.
Memoir of. 1905.

REEDER, EARL J.
"Traffic Problems in Metropolitan Areas." 323.

RHODES, RALPH F.
Beach erosion studies. 901.

RICHARDS, N. A.
"Final Report of Sub-Committee No. 31, Committee on Steel of the Structural Division, on Wind Bracing in Steel Buildings." 1713.

RICHMOND, W. O.
Stress distribution. 1141.

RIGHTS, LEWIS D.
Walpole-Bellows Falls Bridge. 1701.

ROBERTS, ELLIOTT B.
Surveying practice in Massachusetts. 579.

ROCK, ELMER.
"Design of a High-Head Siphon Spillway." 1050.

ROHWER, CARL.
Spillway channels. 626.

ROSENGARTEN, CARL A.
Circular concrete tanks. 514.

ROUNTREE, JACK R.
Tests of a shale foundation. 1538.

ROUSE, HUNTER.
Theory of turbulence. 1567.

RUGE, ARTHUR C.
Earthquakes and structures. 307.

RUGGLES, CHARLES ARNER.
Memoir of. 1997.

RUST, T. H.
"Specification and Design of Steel Gusset-Plates." 142.

RYDER, LINCOLN W.
Flash-board pins. 1445.

SABIN, L. C.
Great Lakes transportation. 193.

SALTER, GEORGE S.
" Design of Circular Concrete Tanks." 504.

SAWYER, WILLIAM L.
Geodetic and cadastral surveys. 1331.

SAYLOR, WILLIAM H.
" Graphical Arch Analysis Applicable to Arch Dams." 765.

SCHNEIDER, HERMAN.
Memoir of. 1999.

SCHULEEN, E. T.
Flash-board pins. 1447.

SCOFIELD, EDSON MASON.
Memoir of. 1908.

SCOTT, REGINALD CROWLEY.
Memoir of. 2003.

SHANK, J. R.
Moment of inertia of girder. 1506.

SHATTUCK, HAROLD BEMIS.
Memoir of. 1913.

SHAW, BENJAMIN BRUCE.
Memoir of. 1914.

SHEA, H. J.
Surveying practice in Massachusetts. 565.

SHEARWOOD, FREDERICK P.
Tests on riveted joints. 1249.
Wind forces on a tall building. 49.

SHEDD, THOMAS C.
Moment of inertia of girder. 1512.

SHELDON, ROBERT C.
Method for adjusting surveys. 1407.

SHERLOCK, R. H.
Steel gusset-plates. 159.

SHERMAN, LeROY K.
Analysis of run-off characteristics. 106.
Unit hydrograph. 1181.

SHULITS, SAMUEL.
Theory of turbulence. 1578.

SILVERMAN, I. K.
Circular concrete tanks. 515.
Theory of limit design. 665.

SIMPSON, FRANCIS E.
Theory of limit design. 679.

SKINNER, COLIN.
Wind forces on a tall building. 65.

SLATTERY, LOUIS ALOYSIUS.
Memoir of. 2025.

SMITH, ALBERT.
Wind forces on a tall building. 61.

SMITH, JULIAN CLEVELAND.
Memoir of. 1916.

SMITH, WALDO E.
Flood Control reservoirs. 1665.
Unit hydrograph. 1184.

SNOW, WILLIAM PLINY.
Memoir of. 1918.

SNYDER, FRANKLIN F.
Analysis of run-off characteristics. 122.
Unit hydrograph. 1179.

SOPER, GEORGE A.
Beach erosion studies. 893.
Pollution of Boston Harbor. 1016.

SOUCEK, EDWARD.
Flood control reservoirs. 1648.

SOUROCHNIKOFF, BASIL.
Circular concrete tanks. 528.
Theory of limit design. 668.

SPARROW, WILLIAM WARBURTON KNOX.
Memoir of. 1920.

SPEERT, JULIUS L.
"An Improved Method for Adjusting Level and Traverse Surveys." 1376.

SPELLMAN, R. D.
Wind forces on a tall building. 57.

SPOFFORD, CHARLES M.
Moment of inertia of girder. 1488.

SPRAGUE, EMBERT HIRAM.
Memoir of. 1922.

STANLEY, C. MAXWELL.
Sub-surface contracts. 490.

STEVENS, J. C.
Flood control reservoirs. 1649.
High-head siphon spillways. 1062.
Theory of turbulence. 1571.

STREETER, V. L.
Open-channel control section. 752.

SULLIVAN, WILLIAM SHACKELFORD.
Memoir of. 1923.

TAYLOR, ALEXANDER JENIFER.
Memoir of. 2004.

TENNEY, GEORGE OLIVER.
Memoir of. 2027.

THOMAS, HAROLD ALLEN, JR.
Spillway channels. 627.

THOMAS, ROBERT O.
Open-channel control section. 755.

THOMSON, T. KENNARD.
Great Lakes transportation. 186.
Sub-surface contracts. 497.

TIBBETTS, FREDERICK HORACE.
Memoir of. 1924.

TIFFANY, ROSS KERR.
Memoir of. 1928.

TODD, O. J.
"The Yellow River Problem." 346.

TORPEN, B. E.
High-head siphon spillways. 1063.

TRIPP, J. G.
Core drills at Chickamauga Dam. 865.

TRUESDELL, S. R.
Transportation developments. 257.

TSCHEBOTAREFF, GREGORY P.
"Settlement Studies of Structures in Egypt." 919.

TURNER, HOWARD M.
Analysis of run-off characteristics. 134.

TYLER, M. C.
" Great Lakes Transportation." 167.

TYRRELL, WARREN AYRES.
Memoir of. 1931.

URQUHART, GEORGE COPELAND.
Memoir of. 2006.

VAN DEN BROEK, J. A.
" Theory of Limit Design." 638.

VAN DER VEEN, H.
Yellow River problem. 427.

VELZY, C. R.
Sewage disposal. 1629.

VENNARD, JOHN K.
Spillway channels. 621.

VON BERGEN, HAROLD EMIL.
Memoir of. 2008.

WADE, J. A.
" Report of the Committee on Water Supply Engineering of the Sanitary Engineering Division: Important Events, Developments, and Trends in Water Supply Engineering During the Decade Ending with the Year 1939." 1740.

WARDWELL, RALPH WATTS.
Memoir of. 2009.

WEBB, WILLIAM TIBBITTS.
Memoir of. 2011.

WEISKOPF, WALTER H.
Moment of inertia of girder. 1493.

WERNER, P. WILHELM.
Flood control reservoirs. 1662.

WESTON, ARTHUR D.
" Pollution of Boston Harbor." 973.

WEYMOUTH, AUBREY.
Memoir of. 1935.

WHITE, JULIAN H.
Flash-board pins. 1452.

WHITE, LAZARUS.
Sub-surface contracts. 495.

WHITLEY, FENNER H.
Spillway channels. 621.

WHITMAN, NATHAN DAVIS.
Memoir of. 2013.

WHITMORE, GEORGE D.
Method for adjusting surveys. 1405.
Title transfers. 741.

WHITNEY, JOHN THAD.
Memoir of. 2016.

WIGGIN, THOMAS H.
"Report of the Committee on Water Supply Engineering of the Sanitary Engineering Division: Important Events, Developments, and Trends in Water Supply Engineering During the Decade Ending with the Year 1939." 1740.

WILBUR, J. B.
Wind forces on a tall building. 55.

WILCOX, FRANK.
Memoir of. 1938.

WILCOXEN, L. C.
Settlement studies of structures in Egypt. 946.

WILLES, EMERY H.
Open-channel control section. 753.

WILLIAMS, C. D.
Moment of inertia of girder. 1494.

WILLIAMSON, SYDNEY BACON.
Memoir of. 1940.

WILLSON, FREDERICK NEWTON.
Memoir of. 1955.

WILSON, R. M.
Method for adjusting surveys. 1399.
Surveying practice in Massachusetts. 578.

WILSON, W. M.
Tests of riveted joints. 1264.

WINICK, CHARLES B.
Wind-stress analysis. 1030.

WINSOR, FRANK EDWARD.
Memoir of. 1957.

WINTER, C. C.
Tests of riveted joints. 1261.

WINTER, GEORGE.
Theory of limit design. 673.

WISE, JOSEPH A.
Theory of limit design. 680.

WITMER, FRANCIS P.
" Final Report of Sub-Committee No. 31, Committee on Steel of the Structural Division, on Wind Bracing in Steel Buildings." 1713.
Moment of inertia of girder. 1504.

WIXOM, C. W.
Steel gusset-plates. 162.

WOODRUFF, GLENN B.
" Tension Tests of Large Riveted Joints." 1193.

WOODS, HARLAND C.
Great Lakes transportation. 188.

WOODWARD, SHERMAN M.
Flood control reservoirs. 1658.

WRIGHT, CHARLES NICHOLSON.
Memoir of. 1962.

WRIGHT, CHILTON A.
" Flash-Board Pins." 1412.

YOUNG, A. A.
Hydrology of the Great Lakes. 818.

YOUNG, C. R.
" Final Report of Sub-Committee No. 31, Committee on Steel of the Structural Division, on Wind Bracing in Steel Buildings." 1713.

YOUNG, DANA.
Circular concrete tanks. 521.

ZEIGLER, FREDERICK C.
Sub-surface contracts. 492.

ZOCH, RICHMOND T.
Analysis of run-off characteristics. 109.